Numerical optimization algorithms: Broyden's method

Choose $x^{(0)}$
$B^{(0)} = B(x^{(0)})$
$d^{(0)} = -B_0^{-1} \nabla f(x^{(0)})$
$x^{(1)} = x^{(0)} + d^{(0)}$
$k = 0$
while not converged **do**
  $u^{(k)} = B_k^{-1} \nabla f(x^{(k+1)})$
  $c_k = d^{(k)} \cdot (d^{(k)} + u^{(k)})$
  $B_{k+1}^{-1} = B_k^{-1} - \frac{1}{c_k}[u^{(k)} \otimes d^{(k)}] B_k^{-1}$
  $k = k + 1$
  $d^{(k)} = -B_k^{-1} \nabla f(x^{(k)})$
  $x^{(k+1)} = x^{(k)} + d^{(k)}$
end while

Advantages and disadvantages

Advantages:

1. Computation of derivates is not needed unlike the Newton's method, thus avoids expensive computation of full Hessian matrix
2. Not sensitive to the choice of the initial guess
3. Memory-efficient, especially in situations where the Jacobian matrix is large and sparse.

Disadvantages:

1. Sensitive to the choice of the initial approximation to the Jacobian.
2. Convergence issues where the objective function has multiple minima or when the solution is in the vicinity of a saddle point.