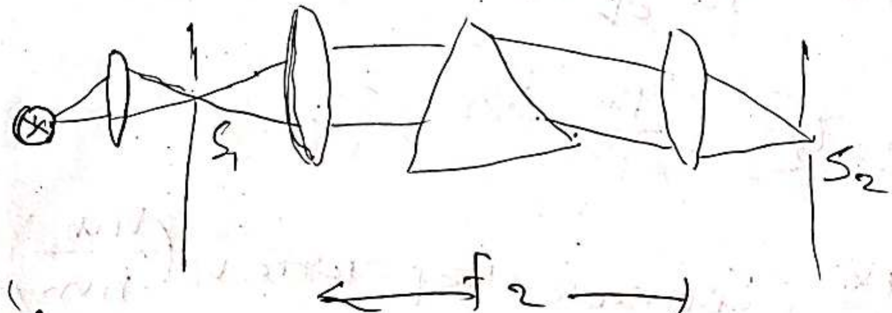
Basic unit of spectroscopy.

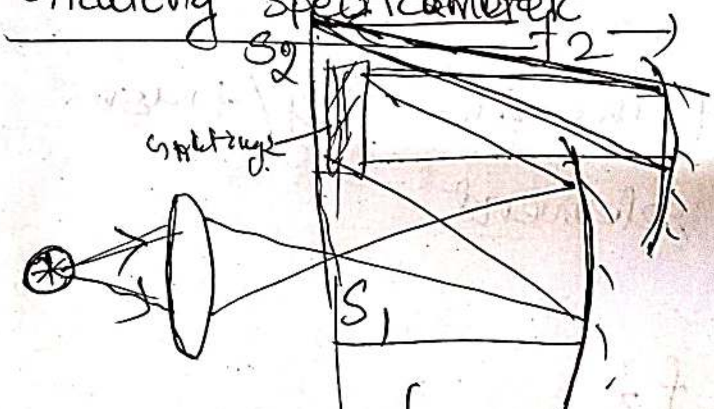
1. Source.
2. Dispersion unit.
3. Detector.

Dispersion unit

Spectrometer → which gives image of a slit S_1 on $S_2(\lambda)$

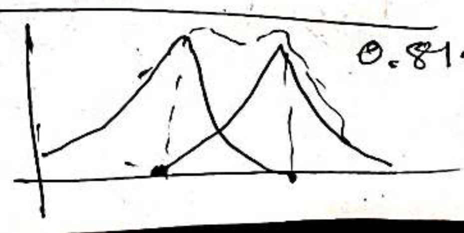


Grating Spectrometer



Resolution of Measurements.

Rayleigh's criterion :-



Angular speed.

$$\Delta\theta = \frac{d\theta}{d\lambda}\Delta\lambda$$

$\frac{d\theta}{d\lambda}$:- Angular dispersion; $\frac{rad}{nm}$

$2d\sin\theta = n\lambda$

Linear dispersion

$$\Delta x = f_2 \frac{d\theta}{d\lambda}\Delta\lambda$$

$$= \frac{dx}{d\lambda}\Delta\lambda$$

$\frac{dx}{d\lambda}$: Linear dispersion $\left(\frac{mm}{nm}\right)$

a : size of the grating/prism element.

$$\Delta x_2 = \frac{\lambda}{a} f_2$$

$$\frac{d\theta}{d\lambda}\Delta\lambda \geq \frac{\lambda}{a}.f_2$$

Resolving Power : $\left|\frac{\lambda}{\Delta\lambda}\right| \leq a\left(\frac{d\theta}{d\lambda}\right)$

Magnification of entrance slit S_1:

$$\frac{\delta x_1}{f_1} = \frac{\delta x_2}{f_2}$$

$$\Rightarrow \delta x_2 = \frac{f_2}{f_1}\delta x_1$$

entrance slit size: b

$$\Delta x = \frac{\lambda}{a} f_2 + \frac{f_2}{f_1} b$$

$$f_2 \frac{d\theta}{d\lambda}\delta\lambda = \frac{\lambda}{a} f_2 + \frac{f_2}{f_1} b$$

$$\delta\lambda = \left(\frac{\lambda}{a} + \frac{b}{f_1}\right)\left(\frac{d\theta}{d\lambda}\right)^{-1}$$

limit of b:

$$\frac{2\lambda}{b} f_1 = a$$

$$\boxed{b_{min} = \frac{2\lambda f_1}{a}}$$

$$\delta\lambda = \left(\frac{\lambda}{a} + \frac{2\lambda f_1}{a f_1}\right)\left(\frac{d\theta}{d\lambda}\right)^{-1}$$

$$= \frac{3\lambda}{a}\left(\frac{d\theta}{d\lambda}\right)^{-1}$$

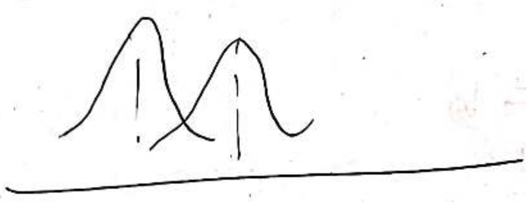
$$\Rightarrow \frac{\lambda}{\delta\lambda} = \frac{a}{3\lambda}\left(\frac{d\theta}{d\lambda}\right)$$

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2}$$

$$\alpha = \frac{\pi b \theta}{\lambda}$$

$$\boxed{\theta = \frac{2\lambda}{b}}$$

Date — 22.01.025



500 nm 501 nm

$$\theta = \frac{2\lambda}{b}$$

$$= \frac{2 \times 500\,nm}{10^{-6}\,m}$$

$$= 1000 \times 10^{-3}\ Rad$$

$$\Delta\lambda \le \left(\frac{\lambda}{a} + \frac{b}{f}\right)\left(\frac{d\theta}{d\lambda}\right)^{-1}$$

$a = 10\,cm$

$\lambda = 500 \times 10^{-7}\,cm$

$f = 100\,cm$

$$\frac{d\lambda}{dx} = \frac{1\ nm}{mm} = 10^{-9} \times 10^{3} = 10^{-6}$$

$b = 10\,\mu m = 10 \times 10^{-4}\,cm$

$\Delta\lambda\ \theta =$

$$\frac{dx}{d\lambda} = f_e \frac{d\theta}{d\lambda} \quad \left(\frac{dx}{d\theta} = f_e \frac{dx}{d\alpha} \right.$$

$$\Delta x = \left(\frac{500\times10^{-7}\times10^{-2}\,m}{10\times10^{-2}\,m} + \frac{10^{-6}\,m}{100\times10^{-2}\,m} \right)$$

$$\times 100\times10^{-2}\,m$$

$$\times 10^{-6}$$

$$= \left(\frac{500\times10^{-9}\,m}{10^{-1}\,m} + 10^{-6} \right) 10^{-6}\ m$$

$$= \left(5\times10^{-6} + 10^{-6} \right) \times 10^{-6}\ m$$

$$= 6\times10^{-6} \times 10^{-6}\ m$$

$$= 6\times10^{-12}\ m$$

$$= 6\times10^{-3}\times10^{-9}$$

$$= \frac{6}{1000}\times10^{-9}\ m$$

$$= \cancel{0.006} \cancel{\times10^{-}}$$

$= 0.006\,nm.$ $(0.045\,nm)$

$= 0.01\,nm$

Prism spectrometer

for minimum deviation:

$$n \sin\frac{\varepsilon}{2} = \sin\frac{\theta+\varepsilon}{2}$$

$$\Rightarrow dn \sin\frac{\varepsilon}{2} = \frac{1}{2}\cos\left(\frac{\theta+\varepsilon}{2}\right) d\theta$$

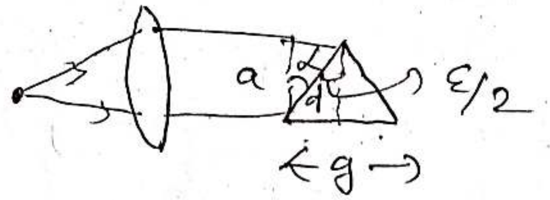
$$\Rightarrow \frac{d\theta}{dn} = \frac{2\sin\varepsilon/2}{\cos\left(\frac{\theta+\varepsilon}{2}\right)}$$

$$= \frac{2\sin\frac{\varepsilon}{2}}{\sqrt{\left(1-n^2\sin^2\frac{\varepsilon}{2}\right)}}$$

$$\frac{d\theta}{d\lambda} = \frac{d\theta}{dn} \cdot \frac{dn}{d\lambda}$$

$$= \frac{2\sin\varepsilon/2}{\sqrt{1-n^2\sin^2\varepsilon/2}} \frac{dn}{d\lambda}$$

$$\Delta\lambda \leq \frac{a}{3}\left(\frac{d\theta}{d\lambda}\right)$$



$\hookrightarrow$ consider

$$\frac{g}{2d} = \sin \varepsilon/2$$

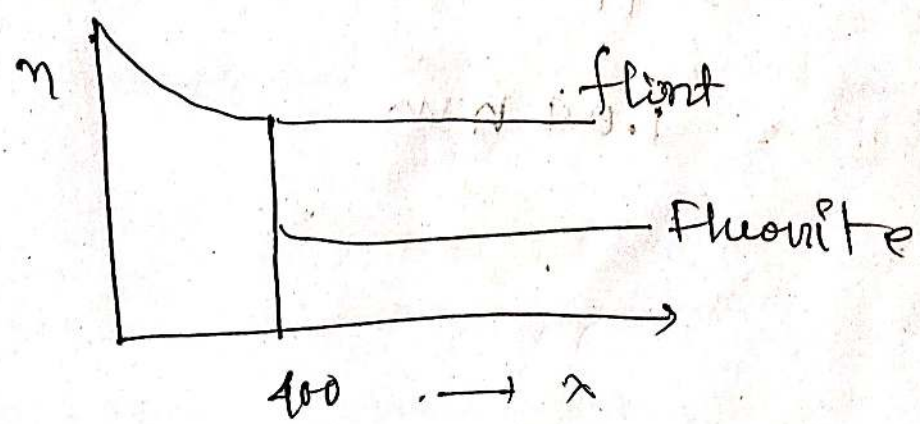
$$\Rightarrow \boxed{d = \frac{g}{2\sin \varepsilon/2}} \qquad a = \frac{g \cos\alpha}{2\sin \varepsilon/2}$$

angle of min. deviation $\quad n \sin \varepsilon/2 = \sin \frac{\delta + \varepsilon}{2}$
$= \sin \alpha$

$$a = \frac{g\sqrt{1 - n^2 \sin^2 \varepsilon/2}}{2\sin \varepsilon/2}$$

$$\frac{\lambda}{\Delta\lambda} \leq \frac{g\sqrt{1-n^2\sin^2\varepsilon/2}}{2\sin\varepsilon/2} \cdot \frac{2\sin\varepsilon/2}{\sqrt{1-n^2\sin^2\varepsilon/2}} \cdot \frac{dn}{d\lambda}$$

$$\boxed{\frac{\lambda}{\Delta\lambda} \leq \frac{g}{3}\frac{dn}{d\lambda}}$$



Example

Fused quartz

$n = 1.47$ at $\lambda = 400\,nm$

$$\frac{dn}{d\lambda} = 1100\ cm^{-1} \qquad \frac{d\lambda}{dn} = \frac{1}{1100}\ cm$$

$g = 1\,cm$

$$\frac{\lambda}{\Delta\lambda} \le \frac{g}{3}\frac{dn}{d\lambda} = \frac{\cancel{10^{-2}m}}{3} \times \frac{1100}{\cancel{10^{-2}m}}$$

$$= \frac{\cancel{1100}^{36}}{\cancel{3}}$$

$$\frac{\lambda}{\Delta\lambda} = \frac{g}{3}\frac{dn}{d\lambda}$$

$$\Rightarrow \Delta\lambda = \frac{3\lambda}{g}\frac{d\lambda}{dn}$$

$$= \frac{3 \times 4\cancel{00} \times 10^{-9} m}{\cancel{10^{-2}}\ \cancel{m}} \times \frac{1}{11\cancel{00}} \times \cancel{10^{-2}}\ \cancel{m}$$

$$\Rightarrow \frac{12}{11} \times 10^{-9}\ m$$

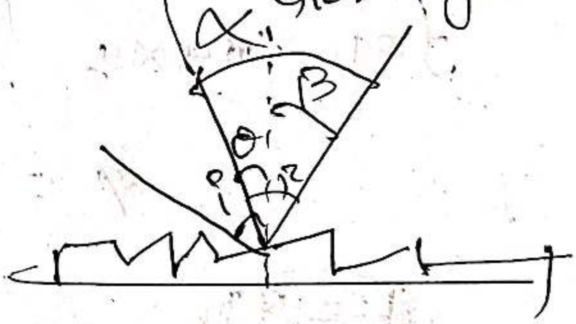
$$= 1.09\ nm$$

$\frac{d\theta}{d\lambda} =$

Gating spectrometer.

Path diff.

$d(\sin\alpha \pm \sin\beta) = m\lambda$



Maximum reflection

$i = r$

$\alpha - \theta = \beta + \theta$

$\theta = \frac{\alpha - \beta}{2}$

Blaze angle.

Intensity

$I = I_0 \frac{\sin^2 \frac{N\phi}{2}}{\sin^2 \phi/2}$

$\phi = \frac{2\pi}{\lambda} d (\sin\alpha \pm \sin\beta)$

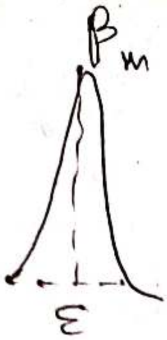
N → no. of groove.

Maximum $\phi = 2m\pi$

minima $\frac{N\phi}{2} = l\pi$ $\quad l = 1, 2 . (N-1)$

If $N = 5$

then minima will $N-1$



$$\sin(\beta_m+\varepsilon) = \sin\beta_m\cos\varepsilon + \cos\beta_m\sin\varepsilon$$

For small ε

$$= \sin\beta_m + \cos\beta \cdot \varepsilon$$

$$\phi = \frac{2\pi}{\lambda} d\left[\sin\alpha + \sin\beta_m + \varepsilon\cos\beta_m\right]$$

For max:

$$\phi = 2m\pi + \frac{2\pi}{\lambda} d\,\varepsilon\cos\beta_m$$
$$= 2m\pi + \delta$$

$$I = RI_0 \frac{\sin^2\frac{N\delta}{2}}{\sin\delta/2}$$

$$= RI_0 \frac{N^2\sin^2\frac{N\delta}{2}}{\left(\frac{N\delta}{2}\right)^2}$$

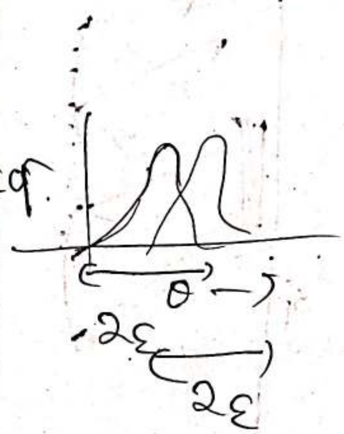
$$\Rightarrow \frac{N\delta}{2} = \pi$$

$$\Rightarrow \frac{N}{2}\cdot\frac{2\pi}{\lambda} d\,\varepsilon\cos\beta_m = \pi$$

$$\Rightarrow \varepsilon = \frac{\lambda}{Nd\cos\beta_m}$$

Resolution of Grating

$$d(\sin\alpha + \sin\beta) = m\lambda$$



Ang. dispersion.

$$\frac{d\beta}{d\lambda} = \frac{m}{d\cos\beta}$$

$$\frac{d\beta}{d\lambda}\Delta\lambda = \frac{\lambda}{Nd\cos\beta\, m}$$

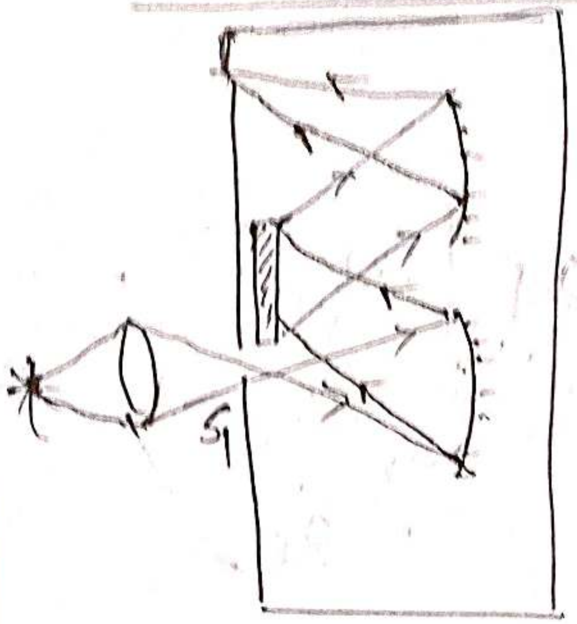
$$\Rightarrow \cancel{\Delta\lambda = \frac{\lambda}{Nd\cos\beta\, m}\left(\frac{d\beta}{d\lambda}\right)^{-1}}$$

$$\Rightarrow \frac{\lambda}{\Delta\lambda} = Nd\cos\beta\, m\left(\frac{d\beta}{d\lambda}\right)$$

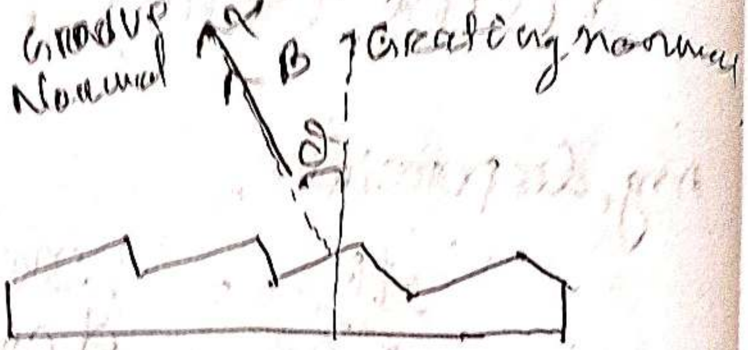
$$= N\cancel{d\cos\beta}\, m\,\frac{m}{\cancel{d\cos\beta}}$$

$$= mN$$

Date - 28.01.2025



$(d\sin\alpha \pm \sin\beta) = m\lambda$

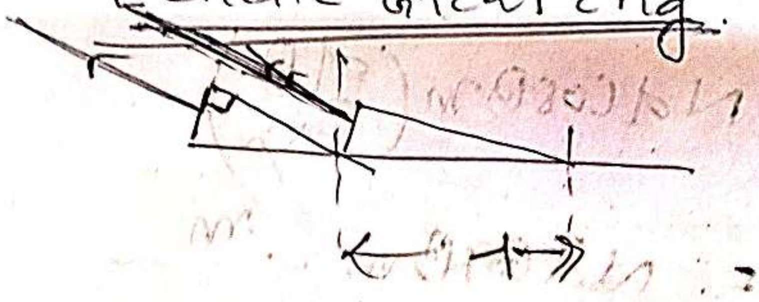


Littrow Grating

$2d\sin\alpha = m\lambda$

wavelength selective grating.

Echelle Grating



$2d\sin\alpha = m\lambda$

as $d \gg \lambda$

$\theta = 90 - \alpha$.

$2d\cos\theta = m\lambda$

grating size $= 100\,cm^2$

$N = 10^3$ groov/mm $\boxed{m = 2}$

Resolving Power of the grating.

$$R = \frac{\lambda}{\Delta\lambda} = 2 \times 10^3 \text{ groove/mm} \times 10^2\,cm^2$$

$$= \frac{2 \times 10^5 \text{ groove} \times (10^{-2}\,m)^2}{10^{-3}\,m}$$

$$= 2 \times 10^5 \text{ groov} \quad = \cancel{2 \times 10^5}$$

$\lambda = 500\,nm$

$$\Delta\lambda = 2.5 \times 10^{-3}\,nm$$

$$= 0.0025\,nm \times \quad = 0.025\,nm$$

$$\Delta\lambda = \left(\frac{\lambda}{2} + \frac{b}{f}\right)\left(\frac{d\alpha}{d\lambda}\right)^{-1}$$

$b = 50\,\mu m$

$\alpha = \beta = 30^\circ$

$f = 1\,m$

$\cancel{\Delta\lambda}$

$$d(\sin\alpha + \sin\beta) = m\lambda$$

$$2d\sin\beta = m\lambda$$

$$\Rightarrow \sin\beta = \frac{m\lambda}{2d}$$

$$\Rightarrow \cos\beta \frac{d\beta}{d\lambda} = \frac{m}{2d}$$

$$\frac{d\beta}{d\lambda} = \frac{m}{d\cos\beta}$$

$1/\sqrt{3}/2$

$$\Delta\lambda = \frac{2d\sin\beta}{\lambda d\cos\beta}$$

$$\Rightarrow \frac{2\tan\beta}{\lambda}$$

$$= \frac{2\times}{500\times\sqrt{3}\ nm} = \frac{2}{5\times 100\sqrt{3}}$$

$$= \frac{0.4}{100\cdot\sqrt{3}}\ nm^{-1}$$

$5\overline{)20}\ 0.4$

$$= \frac{0.4}{100\times 1.732}\ nm^{-1}$$

$$= 0.006928\ nm^{-1}$$

$$= \frac{6928}{1000000} = 0.006928$$

$$= 6928\times 10^{-6}$$

$= \cancel{6.928\times 10^{-8}}$

$$= 0.023\ nm^{-1}$$

$$= 0.006928\ nm^{-1}$$

$\Delta\lambda =$

$\frac{b}{f} = \frac{50\,\mu m}{1\,m} = 50\times10^{-6}$

$\frac{\lambda}{a} = 5\times10^{-6}$

$\Delta\lambda =$

$\Delta\lambda = (5\times10^{-6} + 50\times10^{-6})\, 0.0069$

$= 55\times10^{-6} \times 0.006928$ nm

$= 0.38104\times10^{-6}$

$=$

$\frac{d\lambda}{dx}$ $\frac{d\beta}{dx} \times 100\,mm = 0.023\ nm^{-1} \times 100\,mm$

$=$

$\frac{dx}{d\lambda} = \frac{0.023 \times 1000\,mm}{nm}$

$= 0.023\times1000\times 10^{-3}\times10^{9}$

$= \frac{23}{1000}\times 1000\times10^{-3}\times10^{9}$

$= 23\times10^{6}$

$$\frac{dy}{dx} = $$

$$100 \frac{10^{-7} \times 10^{-2}}{ }$$

$= 100\ nm$

Detector

→ Thermal detector

→ Photodetector.

Thermal detector

→ λ insensitive.

→ works on change in material properties

with the temperature.

Photo detector.

→ λ sensitive

→ work on response of

material for given photon

energy.

characteristics of the detector

Date- 29.01.025

Characteristics of detector.

1. spectral response. [R(λ)]

(i) (spectral range)

(ii) Two relative intensity.

$$\frac{I(\lambda_1)}{I(\lambda_2)} = \frac{R(\lambda_1)}{R(\lambda_2)}$$

2. sensitivity of the detector.

$\left(\frac{\text{volt}}{\text{watt}}\right)$ or $\left(\frac{\text{Ampere}}{\text{watt}}\right)$

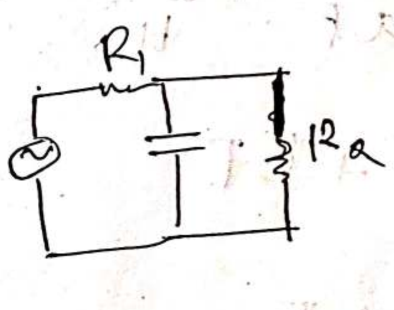
or $\frac{\text{Volt}}{A \cdot I}$ → irradiance, A → Area

3. Signal to Noise Ratio $\left(\frac{S}{N}\right)$

Noise equivalent of input power.

→ Power level for which $\frac{S}{N} = 1$

4. Time Response.



$R_2C \ll \tau$

τ: Pulsetime

$R_2C \ll \frac{1}{f}$

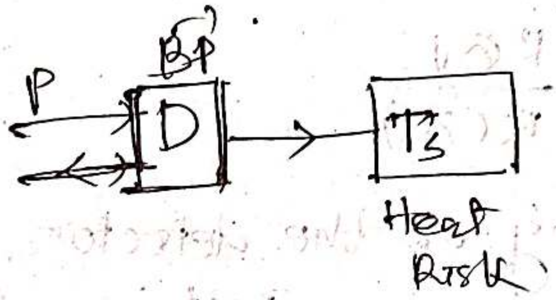
$\Rightarrow \boxed{f \ll \frac{1}{R_2C}}$

⑤ Cost of the detector.

Thermal detection

→ Power Measurement of C.W laser.

→ Output energy of the Pulse laser.



H: heat capacity of the detector

G: Thermal conductance to the heat Rsk

$$\beta P = H\frac{dT}{dt} + G(T - T_s)$$

$$\Rightarrow \beta P = H\frac{dT}{dt} + GT - GT_s$$

$$\Rightarrow \frac{\beta P + GT_s}{H} = \frac{dT}{dt} + \frac{GT}{H}$$

$$\Rightarrow a = \frac{dT}{dt} + bT$$

$$\Rightarrow \frac{dT}{dt} = a - bT$$

$$\Rightarrow \frac{dT}{a - bT} = dt$$

$\Rightarrow$ $\Rightarrow$

$a - bT = x$

$-b\,dT = dx$

$dT = \frac{dx}{-b}$

$\Rightarrow \frac{dx}{-b\,x} = dt$

$\Rightarrow -\frac{1}{b} \ln x = t + c$

$\Rightarrow -\frac{1}{b} \ln (a - bT) = t + c$

$\Rightarrow \ln (a - bT)^{-1/b} = t + c$

$\Rightarrow \ln \frac{1}{(a - bT)^{1/b}} = t + c$

$\Rightarrow \frac{1}{(a - bT)^{1/b}} = e^{t + c}$

$\Rightarrow (a - bT)^{1/b} = e^{-(t + c)}$

$\Rightarrow (a - bT) = e^{-(t + c)b}$

$\Rightarrow a - e^{-(t + c)b} = bT$

$\Rightarrow T = \frac{1}{b}\left[a - e^{-(t + c)b}\right]$

$T_S = \frac{H}{G}\left[a - e^{-\frac{cG}{H}}\right]$

let $t = 0, T = T_s$

$$T_s = \frac{1}{b}[$$

$$\frac{T - T_s}{1 - e^{Ht/G}} = \Delta T$$

$$T_s = \frac{H}{G}\left[\frac{BP + GT_s}{H} - e^{-\frac{Hc}{G}}\right]$$

$$= \left[\frac{BP}{G} + T_s - \frac{H}{G} e^{-Hc/G}\right]$$

$$= \frac{BP}{G}\left(1 + \frac{G}{BP} - \frac{H}{BP} e^{-Hc/G}\right.$$

$$T = T_s + \frac{\beta P}{G}\left(1 - e^{-t/(H/G)}\right) \qquad \Delta T = \frac{\beta P}{G}$$

Response time of detection $\frac{G}{H}$

$$P = P_0\left(1 + \cos\omega t\right)$$

$$T = T_s + \Delta T\left[1 + a\cos(\Omega t + \phi)\right]$$

$$\Delta T = \frac{a\beta P}{\sqrt{1 + \frac{\Omega^2 H^2}{G^2}}} \qquad \tan\phi = \frac{-\Omega H}{G}$$

Pulse width

$$\int_0^t \beta P\,dt = H\int_0^t dT \qquad G = 0$$

$$\Delta T = \frac{1}{H}\int_0^t \beta P\,dt$$

Thermal Detector

Thermistor

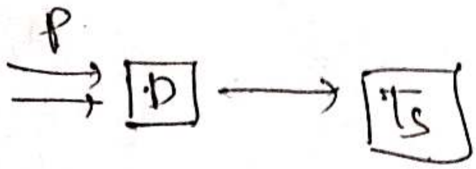
Bolometer

Golay cell

Thermocouple detector.

Date - 4.02.025

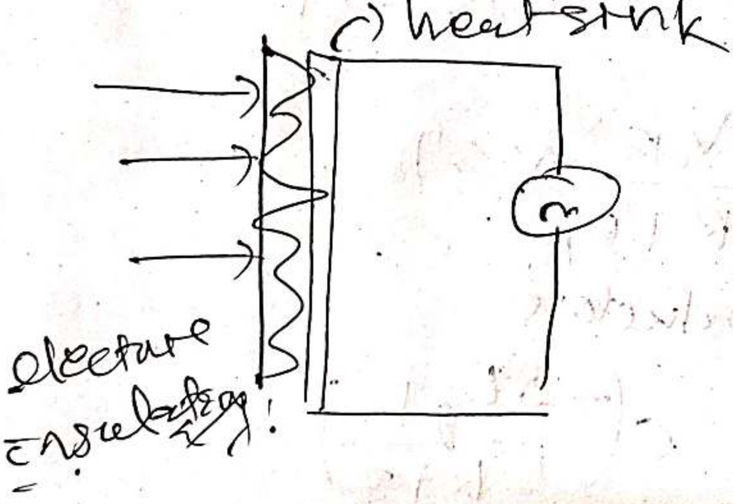
Thermal detector



Bolometer

Thermal imageing

Infrared ~~astronology~~ astronomy



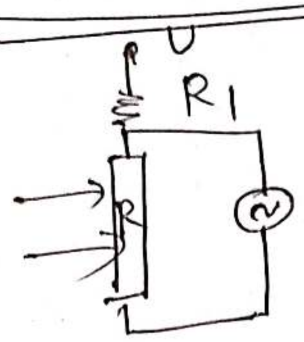
change in voltage

$$\Delta V = N \frac{dV}{dT} \Delta T$$

N: no. of thermocouple

$\frac{dV}{dT}$: sensitivity of each thermo couple

Thermistor



temp. co-eff. of resistance.

$$\alpha = \frac{\frac{dR}{dT}}{R}$$

$$\delta U = i\,\delta R = i\,\alpha \cdot R\,\Delta T$$

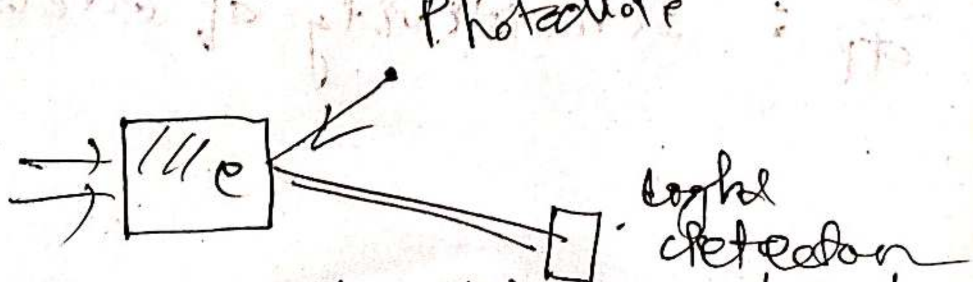
$$= \frac{V_0 R}{R+R_1}\,\alpha \Delta T$$

for semiconductor

$$\frac{n_e(T)}{n_e(T+\Delta T)} = e^{\left(-\frac{\Delta E_g T}{k T^2}\right)}$$

Golay cell

→ use for : photoacoustic spectroscopy



c : cell with membrane

filled with gase.

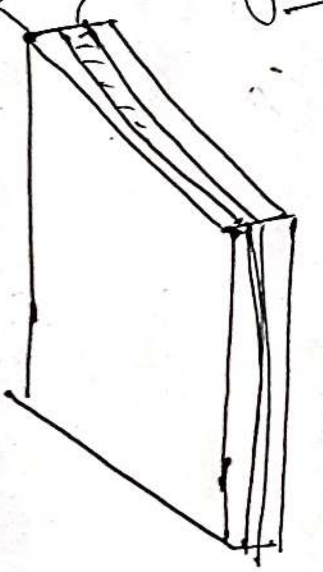
$$PV = NRT$$

$$\delta V = \frac{nR\delta T}{P}$$

→ change in reflected position of light ~~→~~ change in capacitance

Pyroelectric detector.

electrode. Pyroelectric..

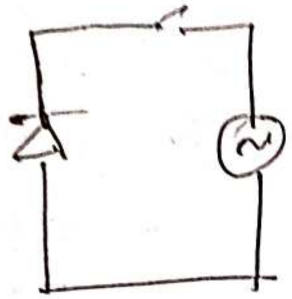


it has intrinsic dipolemoment. which arrange with temp.

C,

to measure → change in surface charge.

Photodetector

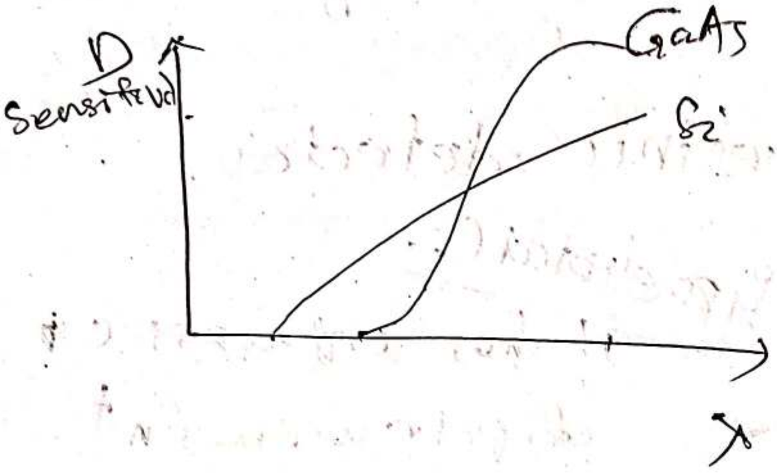


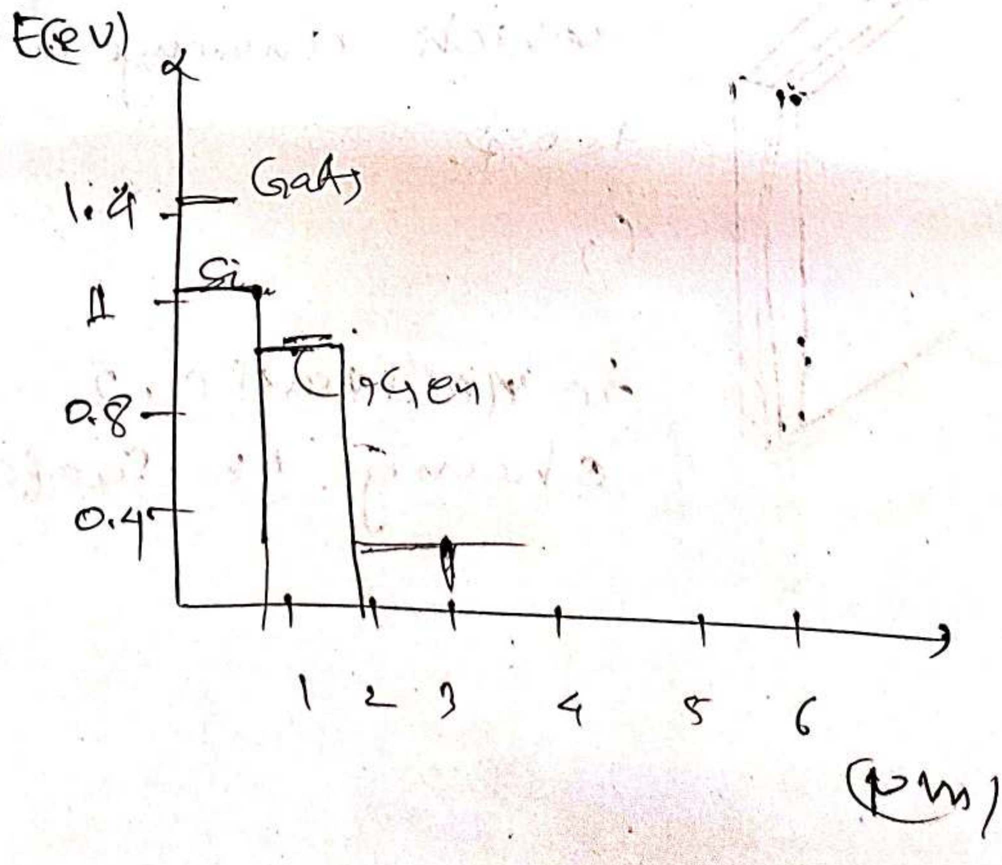
absorption Coefficient

$\alpha \sim (h\nu - E_g)^{1/2}$

$\hookrightarrow h\nu > E_g$

$\sim 0 \quad h\nu < E_g$





Ge: 0.6 eV $\approx$ 1.9 μm

GaAs: 1.42eV $\approx$ 1.88 μm

PbS: 0.37eV $\approx$ 3.356 μm.

Photodiode array.

Length of each diode: 15 μm (L)

spacing between diode: 10 μm (a)

1024 diode array.

Length of detector (15+10) 1024

= 25 mm.

Linear dispersion for each diode.

$= \frac{d\lambda}{dx} = 5 \frac{nm}{mm}$

Spectral Resolution: $\frac{d\lambda}{dx} \times L$

$= 5 \times 15 \times 10^{-3}$

= 0.125 nm.

spectral range

$$\frac{d\gamma}{dx}(L+q) = \frac{5 \times 10^{-9} m \times 25 \times 10^{-6} m}{10^{-3} m}$$

=

Also called Multi-channel

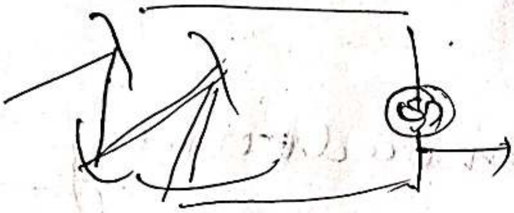
CCD

Array MOS diode / Pixels

Date - 05.02.025

PMT

series of dynodes.



$$e = \frac{Q}{V} = \frac{q^N e}{V}$$

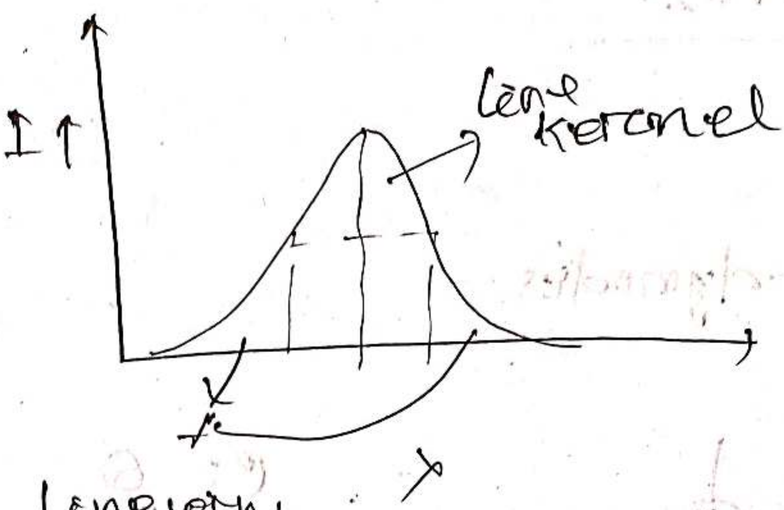
$$= \frac{G e}{V}$$

Microchannel plate

60% holes

Source of Noise

1. Thermal Noise → Richardson eqn (also called dark noise.)
   $I = a T^2 e^{-(2\phi/T)}$
2. shot noise → discrete nature of ele charge.
3. Johnson noise → Thermal aggitation of charge.
4. Stray light →



Kernel dominating.

oscillator model

damp oscillat. $\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$

$$x(t) = x_0 e^{-\frac{\gamma t}{2}} \cos(\omega t + \sigma)$$

$$\omega = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2}$$

$\gamma << \omega_0$

$$\simeq x_0 e^{-\gamma t/2} \cos\omega_0 t$$

Superposition of oscillation of difference frequencies.

$$x(t) = \frac{1}{2\sqrt{2\pi}} \int A(\omega)\, e^{i\omega t}\, d\omega$$

$$A(\omega) = \frac{1}{\sqrt{2\pi}} \int x(t)\, e^{-i\omega t}\, d\omega$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^\infty x_0 \cos(\omega_0 t)\, e^{-i\omega t}\, d\omega$$

$$A(\omega) = \frac{x_0}{\sqrt{8\pi}} \left[\frac{1}{i(\omega-\omega_0)+\frac{\gamma}{2}} + \frac{1}{(\omega+\omega_0)+\frac{\gamma}{2}} \right.$$

$$I(\omega) = A(\omega)\, A^*(\omega)$$

$$I(\omega) = \frac{C\, \cancel{(\omega-\omega_0)^2}}{(\omega-\omega_0)^2 + (\frac{\gamma}{2})^2} \quad \text{for } \omega = \omega_0$$

$$\rightarrow \frac{1}{2\pi} \frac{\gamma}{(\omega-\omega_0)^2 + (\frac{\gamma}{2})^2}$$

$$\int I(\omega-\omega_0)\, d\omega = 1$$

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$$

multiplying by $m\dot{x}$

$$m\dot{x}\ddot{x} + \gamma m\dot{x}^2 + m\omega_0^2 \dot{x} x = 0$$

$m\ddot{x}\dot{x} + m\omega^2 x\dot{x} = -r \cdot m\dot{x}^2$

$\Rightarrow \frac{d}{dt}\left[\frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega_0^2 x^2\right] = -rm\dot{x}^2$

$\Rightarrow \frac{dW}{dt} = -rm\dot{x}^2$

$x = x_0 e^{-rt} \cdot \cos\omega_0 t$

~~$\dot{x} = x_0\left(-\frac{r}{2}\right) e^{-rt/2} \cos\omega t$~~

$\dot{x} = x_0\left[\cos\omega_0 t\,(-r)\,e^{-rt} + e^{-rt}\,\omega_0 \sin\omega_0 t\right]$

$= x_0 e^{-rt/2}\,\omega_0 \sin\omega_0 t$

$r = \frac{1}{\tau}$

$\left\langle \frac{dW}{dt} \right\rangle = \langle rm\dot{x}^2 \rangle$

$= -\langle rm x_0^2 \omega_0^2 e^{-rt} \ldots \rangle$