(7) The power slitting ratios (SR) of a directional coupler at 1300 nm and 1550 nm are 9:16 and 16:9 respectively for a given length of interaction. When the interaction length is increased to achieve an SR of 16:9 at 1300 nm, what would be the SR then at 1550 nm? {Ans. SR = 9:2}

Solution:

Let 1300 nm $=\lambda_1$ and 1550 nm = λ_2.

Also assume at λ_1 and λ_2, the coupling coefficients are respectively κ_1 and κ_2.

Let the original interaction length be L_1 and that after increasing it be L_2.

So, for original interaction length:

At 1300 nm: $\frac{P_T'}{P_C'} = \frac{cos^2 \kappa_1 L_1}{sin^2 \kappa_1 L_1} = \frac{9}{16} \Rightarrow cot^2 \kappa_1 L_1 = \frac{9}{16} \Rightarrow tan\, \kappa_1 L_1 = \frac{4}{3} \Rightarrow \kappa_1 L_1 = 0.92729$

At 1550 nm: $\frac{P_T''}{P_C''} = \frac{cos^2 \kappa_2 L_1}{sin^2 \kappa_2 L_1} = \frac{16}{9} \Rightarrow cot^2 \kappa_2 L_1 = \frac{16}{9} \Rightarrow tan\, \kappa_2 L_1 = \frac{3}{4} \Rightarrow \kappa_2 L_1 = 0.64350$

Later for increased interaction length:

At 1300 nm: $\frac{P_T'''}{P_C'''} = \frac{cos^2 \kappa_1 L_2}{sin^2 \kappa_1 L_2} = \frac{16}{9} \Rightarrow cot^2 \kappa_1 L_2 = \frac{16}{9} \Rightarrow tan\, \kappa_1 L_2 = \frac{3}{4} = tan\, \kappa_2 L_1$

(from previous 1550 nm). So $\kappa_1 L_2 = \kappa_2 L_1$

At 1550 nm: $\frac{P_T''''}{P_C''''} = \frac{cos^2 \kappa_2 L_2}{sin^2 \kappa_2 L_2} = cot^2 \kappa_2 L_2 =?$ Now, $cot\, \kappa_2 L_2 = cot\left(\frac{\kappa_1 L_2 \times \kappa_2 L_1}{\kappa_1 L_1}\right)$

Required Splitting Ratio:

$$cot^2 \kappa_2 L_2 = cot^2\left(\frac{\kappa_1 L_2 \times \kappa_2 L_1}{\kappa_1 L_1}\right) = cot^2\left\{\frac{(\kappa_2 L_1)^2}{\kappa_1 L_1}\right\} = cot^2\left\{\frac{(0.6435)^2}{0.92729}\right\} = (2.1)^2$$
$$= 4.41 = \frac{45}{10} = 9:2$$

(20) The measured transmission spectrum of an FBG having grating length $L = 4.8\, mm$ shows the peak reflectivity of $R = 0.93$ at the corresponding Bragg wavelength $\lambda_B = 1532.1\, nm$. Calculate the required effective index modulation Δn_{eff}. If the core-overlap integral $I = 0.7$, then estimate the actual index modulation Δn. If the effective mode index of this LP_{01} mode is $n_{eff} = 1.4517$, then calculate the bandwidth of the reflected spectrum. {Ans. $\Delta n_{eff} \approx 2 \times 10^{-4}, \Delta n \approx 2.6 \times 10^{-4}, \Delta\lambda \approx 0.4$ mm}

Solution:

An FBG of length L=4.8 mm shows the measured transmission spectra as

Peak reflectivity $R = 0.93$ at the wavelength $\lambda_B = 1532.1\, nm$

We need to calculate the required effective index modulation Δn_{eff}

Now, $R = tanh^2 \kappa L$ or $\tanh \kappa L = \sqrt{R}$

$$tanh\, \kappa L = \frac{\sinh \kappa L}{\cosh \kappa L} = \frac{e^{\kappa L} - e^{-\kappa L}}{e^{\kappa L} + e^{-\kappa L}}.$$

by componendo – dividendo one obtains $\dfrac{1 + \tanh \kappa L}{1 - \tanh \kappa L} = e^{2\kappa L}$

Therefore $\kappa = \dfrac{1}{2L}\, ln\, \dfrac{1 + tanh\, \kappa L}{1 - tanh\, \kappa L}$

So $\kappa = \dfrac{1}{2 \times 4.8} \ln \dfrac{1 + \sqrt{0.93}}{1 + \sqrt{0.93}} = \dfrac{1}{9.6} \ln(55.1247) = \dfrac{1}{9.6} \times 4.0 = 0.4167\, mm^{-1}$

Therefore, $\quad kL \approx 0.4167 \times 4.8 \approx 2$

One can calculate Δn_{eff} by using the approximate formula $\kappa \approx \dfrac{\pi \Delta n_{eff}}{\lambda_B}$

$$\Delta n_{eff} = \frac{\kappa \lambda_B}{\pi} = \frac{0.4167 \times 1532.1}{3.14159} \approx 2.0 \times 10^{-4}$$

Now $\Delta n_{eff} = \Delta n I$ and I is the overlap integral factor

therefore $\Delta n = \dfrac{\Delta n_{eff}}{I} = \dfrac{2.0 \times 10^{-4}}{0.7} \approx 2.6 \times 10^{-4}$

To calculate the bandwidth of reflected spectrum, we'll use

$$\Delta\lambda = \frac{\lambda_B^2}{\pi n_{eff} L}\sqrt{\kappa^2 L^2 + \pi^2} = \frac{(1532.1)^2}{3.14159 \times 1.4571 \times 4.8} \times \sqrt{2^2 + 9.8696} \approx 0.4 \text{ nm}$$

(24) Assuming a sinusoidal RI modulation that is uniform with the core of the fiber, I corresponds to the fractional power in the core of the fiber and is approximately given by $I \approx 1 - exp\left[-2\left(\frac{a}{\omega_0}\right)^2\right]$. Consider a fiber with $a = 5\ \mu m$ and $NA = 0.09$ operated at $\lambda = 1.3\ \mu m$. Using the empirical relation $\frac{\omega}{a} \approx \left(0.65 + \frac{1.619}{V^{\frac{3}{2}}} + \frac{2.879}{V^6}\right)$, estimate the overlap integral I. {Ans. $\frac{\omega}{a} \approx 1.182,\ I \approx 0.76$}

Solution:

Given that overlap integral: $I \approx 1 - exp\left[-2\left(\frac{a}{\omega_0}\right)^2\right]$ and the fiber parameters as $a = 5\ \mu m$ and $NA = 0.09$ operated at $\lambda = 1.3\ \mu m$.

$$\text{Now} \quad V = \frac{2\pi}{\lambda}. a. \sqrt{n_1^2 - n_2^2} = \frac{2\pi}{\lambda}. a. NA = \frac{6.2832}{1.3} \times 5 \times 0.09 = 2.1749$$

Using approximate relation $\dfrac{\omega}{a} \approx 0.65 + \dfrac{1.619}{V^{\frac{3}{2}}} + \dfrac{2.879}{V^6}$,

we obtain $\dfrac{\omega}{a} \approx 1.182$

And therefore $I = 1 - e^{-2(1.182)^2} \approx 0.76$

(34) Calculate the broadening of a narrow pulse (in ns) at the output of a 1 km long step-index multimode fiber having $n_1 = 1.5$ and $\Delta = 0.01$. What will be broadening of that pulse when it travels through 1 km in a multimode parabolic index profile fiber having $n_1 = 1.45$ and $\Delta = 0.01$?

Solution:

For a 1 km long step-index multimode fiber having $n_1 = 1.5$ and $\Delta = 0.01$, the pulse broadening will be: $\Delta\tau = \frac{n_1 L}{c}\Delta = \frac{1.5\times1000\times0.01}{3\times10^8} = 50$ ns.

For a 1 km long, multimode parabolic index profile fiber having $n_1 = 1.45$ and $\Delta = 0.01$, the pulse broadening will be: $\Delta\tau = \frac{n_2 L}{2c}\Delta^2 = \frac{1.45\times1000\times(0.01)^2}{2\times3\times10^8} = 0.25$ ns.