LASER :

13/01/2025

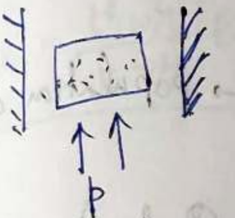
Light amplification by stimulated emission of radiation

Continuous wave gas laser. (e.g. - He-Ne Laser)

1) **Active medium:** amplifies the intensity of E.M radⁿ.

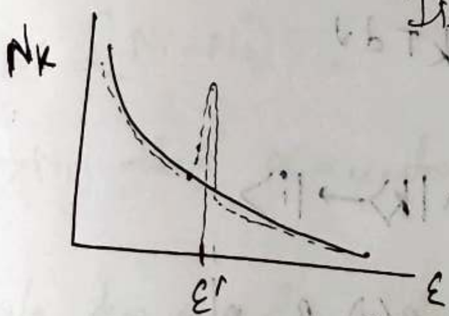
2) **Pump:** populates selected energy levels of the active medium. (Population inversion)

3) **Optical Resonator:** two parallel high reflective mirror which stores energy of few resonator modes by providing feedback.



$A_k =$

① **Population Inversion** → (Classically we use Boltzmann Distribution)



$$N_k = N_i \, e^{-\mathcal{E}/kT}$$

↳ So the Laser setup works as (kind of Black Body we can say in one direction) closed resonator.

$$\vec{E} = \vec{E_0}\, e^{i(\omega t - kx)} + \vec{E_0}\, e^{i(\omega t + kx)}$$

$$k = \frac{\pi}{L}\left(n_1^2 + n_2^2 + n_3^2\right)^{1/2}$$

$$\lambda = \frac{L}{\pi}\frac{1}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$$

for Laser we need open cavity, cause we'll keep only few of the resonator modes

1st time possibility was told by Einstein.

⊙ Einstein's A & B coefficient →

$|k\rangle$ (a) No. of atoms going from $|i\rangle$ to $|k\rangle$

$|i\rangle$ ✓ $R_{i\to k} = \rho(\nu)\, B_{ik} N_i$

$N_i, N_k \to$ population density

$\rho(\nu)$ = spectral density

no. of modes within the freq. ν_i & $\nu_i + \delta\nu$.

$$n(\nu)\, d\nu = \frac{8\pi\nu^2}{c^3}\, d\nu$$

$$\rho(\nu)\, d\nu = n(\nu)\, kT\, d\nu$$

(b) No. of atoms going from $|k\rangle \to |i\rangle$

✓ $R_{k\to i} = A_{ki} N_k + \rho(\nu)\, B_{ki} N_k$

(c) At equilibrium →

$$R_{i\to k} = R_{k\to i}$$

$$\rho(\nu)\, B_{ik} N_i = A_{ki} N_k + \rho(\nu)\, B_{ki} N_k$$

$$\rho(\nu)\, B_{ik} N_i = N_i\, e^{-\varepsilon/kT}\left[A_{ki} + B_{ki}\, \rho(\nu)\right]$$

$$\rho(\nu)\left[B_{ik} - B_{ki}\, e^{-\epsilon/kT}\right] = A_{ki}\, e^{-\epsilon/kT}$$

$$\rho(\nu) = \frac{A_{ki}\, e^{-\epsilon/kT}}{B_{ik} - B_{ki}\, e^{-\epsilon/kT}} = \frac{A_{ki}}{B_{ik}\, e^{\epsilon/kT} - B_{ki}}$$

$$= \frac{\hbar^{\ell}\omega^{3}}{\pi^{2}c^{3}}\,\frac{1}{e^{\epsilon/kT}-1} \quad (\epsilon = \hbar\nu)$$

$$\boxed{\begin{aligned} B_{ik} &= B_{ki} \\ A_{ik} &= \frac{\hbar^{\ell}\omega^{3}}{\pi^{2}c^{3}}\, B_{ki} \end{aligned}}$$

Intensity $I = I_0\, e^{-\alpha L}$ α = absorption coefficient.

$\alpha = (N_i - N_k)\,\sigma_{ik}$ σ_{ik} = absorption cross-section.

$N_k > N_i$ then α = negative means $I = I_0\, e^{+ve}$.

To keep only few resonator modes we need to have a loss →

Loss coefficient in a round trip (2L) : γ

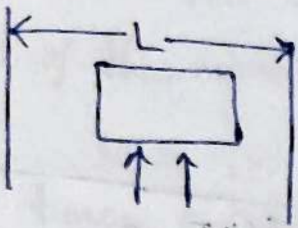
$I_{Loss} = I_0\, e^{-\gamma}$

(we consider round trip)

$I = I_0\, e^{-\alpha \cdot 2L}\, e^{-\gamma}$

So for increase in intensity $\alpha \cdot 2L \gg \gamma$

$$\boxed{\Delta N_{threshold} \gg \frac{\gamma}{2L\,\sigma_{ik}}}$$



Suppose → $L = 10\,\text{cm}$

$\gamma = 10\%$

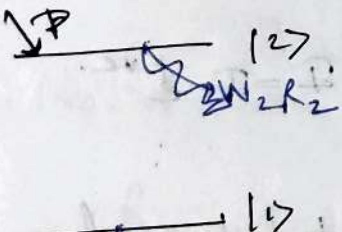
$\sigma = 10^{-12}\ \text{cm}^2$

$$\Delta N = \frac{\gamma}{2L\sigma} = \frac{\frac{10}{100}}{2\times 10\times 10^{-12}} = \frac{10^{12}}{2\times 100} = 0.5\times 10^{10}$$

$\Delta N \sim 10^{10}\ \text{cm}^{-3}$

we have to get this to get Laser.

⊙ Rate Eqn



P: pump rate : no. of atoms ~~of~~ or molecules at |2> per cm³ per sec by pump

N_i : no. of atoms /cm³ in state i.

n = photon density

N_iR_i = No. of atoms which are removed from i by relaxation (collision)

i) $$\frac{dN_1}{dt} = -B_{12}\, n h\nu N_1 + B_{12}\, n h\nu N_2 + A_{21} N_2 - N_1 R_1$$

ii) $$\frac{dN_2}{dt} = B_{12}\, n h\nu N_1 - B_{12}\, n h\nu N_2 - A_{21} N_2 + P - N_2 R_2$$

n = photon density

iii) $$\frac{dn}{dt} = \frac{-\beta n}{\tau} + B_{12}\, n h\nu (N_2 - N_1)$$

↓ (rate eq^n for photon)

It is loss due to open cavity.

β = loss coefficient

$$\gamma = \beta \cdot \tau = \beta \frac{2d}{c}$$

↳ general loss coefficient due to round trip.

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We cannot combine any gas to get the Laser.

Stationary state that doesnot change with time.

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = \frac{dn}{dt} = 0$$

Add (i) and (ii) $\quad -N_1R_1 + P - N_2R_2 = 0$

$$\Rightarrow \boxed{P = N_1R_1 + N_2R_2} \quad \text{------ (iv)}$$

Add (ii) and (iii) $\quad -\beta n - A_{21} N_2 + P - N_2 R_2 = 0$

$$\Rightarrow \boxed{P = \beta n + N_2(R_2 + A_{21})} \quad \text{------ (v)}$$

R_2 = relaxation

A_{21} = spontaneous emission coefficient | $(R_2 + A_{21}) \rightarrow$ incoherent loss rate.

From (iv) and (v)

$$N_1R_1 + \cancel{N_2R_2} = \beta n + \cancel{N_2R_2} + N_2A_{21}$$

$$\Rightarrow \boxed{N_1R_1 = \beta n + N_2A_{21}} \text{ - - - - - (vi)}$$

$R_2 \times$ eqn (i)

$$-B_{12}nh\nu N_1R_2 + B_{12}nh\nu N_2R_2 + A_{21}N_2R_2 - N_1R_1R_2 = 0$$

$R_1 \times$ eqn (ii)

$$B_{12}nh\nu N_1R_1 - B_{12}nh\nu N_2R_1 - A_{21}N_2R_1 - N_2R_2R_1 + PR_1 = 0$$

$$- \quad + \quad + \quad + \quad -$$

$$\Rightarrow -PR_1 + R_1R_2\Delta N + A_{21}N_2R_1 + A_{21}(P - N_1R_1) + B_{12}nh\nu N_1(R_1+R_2)$$

$$+ B_{12}nh\nu N_2(R_1+R_2) = 0 \quad (\Delta N = N_2 - N_1)$$

$$\Rightarrow -PR_1 + R_1R_2\Delta N + A_{21}P + A_{21}R_1\Delta N + B_{12}nh\nu(R_1+R_2)\Delta N = 0$$

$$\rightarrow \Delta N = \frac{P(R_1 - A_{21})}{B_{12}nh\nu(R_1+R_2) + R_1R_2} \quad \text{as } \Delta N > 0$$

$$\boxed{R_1 > A_{21}}$$

$$\frac{1}{\tau_1} > A_{21} \Rightarrow \boxed{\tau_1 < \frac{1}{A_{21}}} \text{ - - - - (vii)}$$

(iv), (v), (vi), (vii) has to satisfy to get the Laser.

Let us consider if state $|1\rangle$ is your ground state then there will be no N.R..

W_k : Energy stored for a particular mode k.

Loss rate of the mode $\frac{dW_k}{dt} = -\beta_k W_k$

$$W_k = W_k(0)\, e^{-\beta_k t}$$

Quality factor $Q = 2\pi \frac{\text{Energy stored}}{\text{Energy loss per cycle}} = 2\pi \frac{W_k}{\frac{dW}{dt} T}$

as per cycle $\frac{2\pi}{T}$

$$= 2\pi \frac{W_k}{\beta_k W_k T} = \frac{2\pi\nu}{\beta_k}$$

$(\nu = \frac{1}{T}) \longrightarrow$

higher the β value, lesser will be the effect of particular mode.

Q When we talk about the loss, we have two types of loss $\longrightarrow$ ① Reflection loss: ② Diffraction loss.

① Reflection Loss $\rightarrow$ Reflectivity of mirrors are very high, 99%, 99.99% reflectivity and it has to have a finite reflectivity. (practically 1cm × 1cm).

For round trip $\rightarrow$ $I = I_0 R_1 R_2 = I_0 e^{-\gamma_R}$

$$\gamma_R = -\ln(R_1R_2)$$

$\gamma_R = \beta_R \frac{2d}{c} = |\ln(R_1R_2)| \Rightarrow \beta_R = \frac{c}{2d}[\ln(R_1R_2)]$

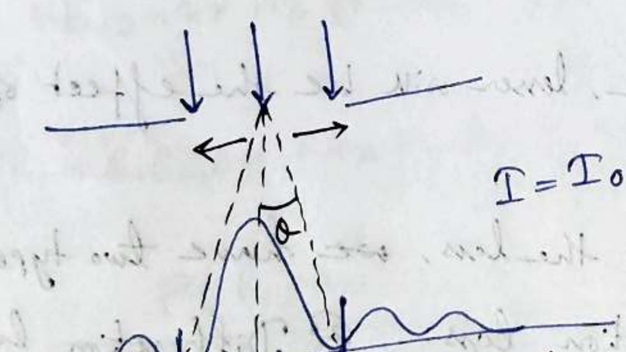
$$\Rightarrow \frac{1}{\tau} = \frac{2d}{c[\ln R_1R_2]}$$

for any wavelength Reflectivity is high, denominator high fraction low, $\frac{1}{\tau}$ low, τ high.

15/1/25

② **Diffraction Loss :-** It is because of resonator mirrors.

resonator mirrors.



$$I = I_0 \frac{\sin^2\alpha}{\alpha^2}$$

$$\alpha = \frac{\pi b \sin\theta}{\lambda}$$

actually the central maxima is inside two mirrors.

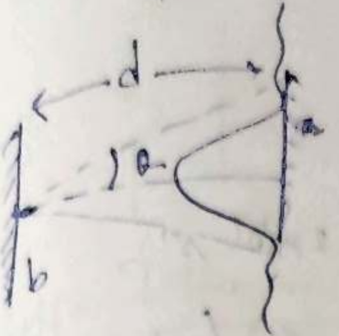
Babinet principle :- Diffraction by an open body

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2}$$

$$\alpha = \frac{\pi b \sin\theta}{\lambda} \simeq \frac{\pi b\theta}{\lambda} = \pi$$

$$\Rightarrow \theta = \frac{\lambda}{b} \text{ (for central maxima)}$$

Diffraction loss is defined by something called Fresnel Number : $N = \frac{b^2}{\lambda d}$



$\theta = \frac{b}{2d}$

for n transits

$\theta = \frac{b}{2nd}$

$\Rightarrow n = \frac{b}{2\theta d} =$  $\frac{b^2}{2d\lambda} = \frac{N}{2}$

$\boxed{\gamma_{diff} \sim \frac{1}{N}}$ (Diffraction Loss.)

① Fabry Perot ~~Fabry Perot~~ Interferometer → two mirrors kept at very small distance and

$b = 3\ cm \quad d = 1\ cm \quad \lambda = 500\ nm = 500 \times 10^{-7}\ cm$

$N = \frac{b^2}{\lambda d} = \frac{9\ cm^2}{5 \times 10^{-5} \times 1\ cm^2} = \frac{9}{5} \times 10^5$

② Laser Resonator :-

$b = 0.1\ cm \quad d = 50\ cm \quad \lambda = 500\ nm = 5 \times 10^{-5}\ cm$

$N = \frac{b^2}{\lambda d} = \frac{0.1 \times 0.1}{50 \times 5 \times 10^{-5}} = \frac{0.1 \times 0.1 \times 10^4}{25}$

$= \frac{10^2}{25} = 4$

$\gamma_{diff} \sim \frac{1}{N}$, Fabry Perot interferometer has less diff. loss though the construction is same. It's the matter of construction size.

Laser mirror size should be very small as compared to the length.

(2)

$N_2 : 10^{10}\ cm^{-3}$

Incoherent Loss rate : $(A_{21} + R_2) : 2\times10^{7}\ s^{-1}$

Total incoherent loss : $N_2 \times (A_{21} + R_2) / cm^3\ s$

$= 2\times10^{+7} \times 10^{10}\ /cm^3\ s$

$= 2\times10^{17}\ cm^{-3}\ s^{-1}$

Suppose $L = 10$ cm $b = 1$ mm

total incoherent Loss for cylinder per second →

$\approx 10^{16}\ s^{-1}$

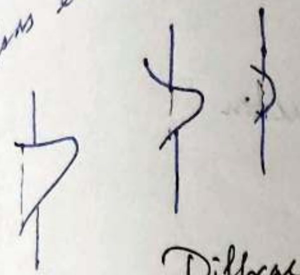
Total no. of photons in the excited state = $N_2 \times$ volume.

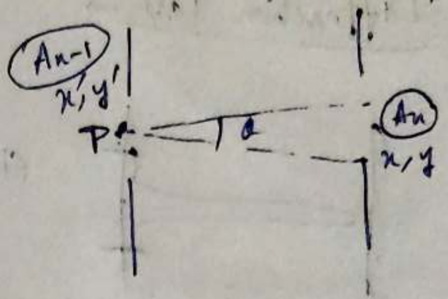
at the same time we need the decays around $10^{16}\ s^{-1}$

~~them~~ then only we can get Laser.

Some more about diffraction Loss →

In every back and forth reflection, there must be a loss every time.





Kirchoff's Diffraction Theory →

$$A_n(x,y) = -\frac{i}{\lambda} \iint A_{n-1}(x',y') \frac{e^{-ik\rho}}{\rho} \cos\theta \, dx' dy'.$$

Stationary Condition

$$A_n(x,y) = C A_{n-1}(x,y) \qquad C = \sqrt{1-\gamma_D}\, e^{i\phi}$$

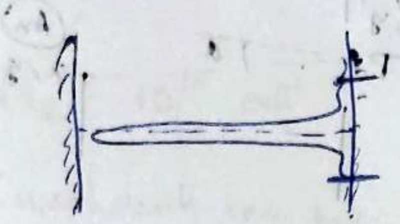
phase factor

$$A_n(x,y) = -\frac{i}{\lambda} \iint \frac{A_n(x,y)}{C} \frac{e^{-ik\rho}}{\rho} \cos\theta \, dx' dy'.$$

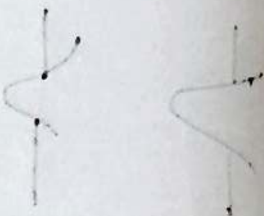
$$= -\frac{i}{\lambda} (1-\gamma_D)^{1/2} e^{-i\phi} \iint A_n(x',y') \frac{e^{-ik\rho}}{\rho} \cos\theta \, dx' dy'$$

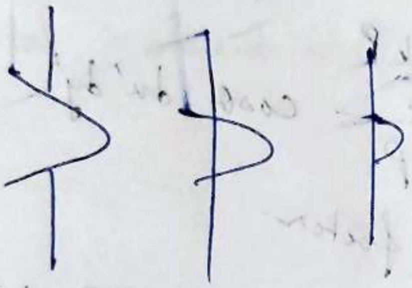
20/1/25 Monday

Diffraction Loss →

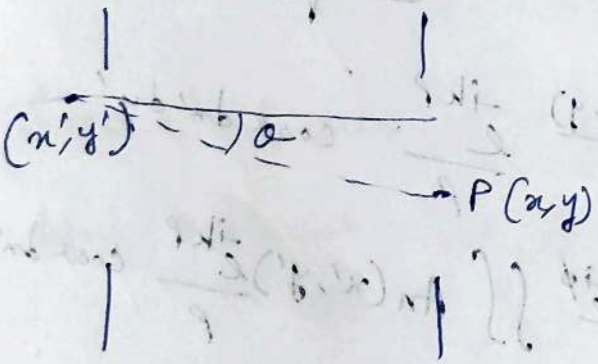


Single slit diffraction





because of the diffraction Law



Kirchoff's Law

$$A(x,y) = -\frac{i}{\lambda} \iint A_{n-1}(x', y') \frac{e^{-ik\rho}}{\rho} \cos\theta \, dx' dy'$$

In stationary condition

$$A_n(x,y) = C A_{n-1}(x,y) \qquad C = \sqrt{1-\gamma_D}\, e^{i\phi}$$

γ_D = diffraction loss for round trip

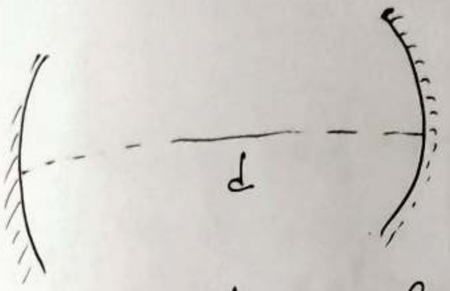
$$A_n(x,y) = -\frac{i}{\lambda}(1-\gamma_D)^{-1/2} e^{-i\phi}\int A_n(x',y')\frac{e^{-ik\rho}}{\rho}\cos\theta \, dx'dy'$$

field distribution.

for particular case → for this geometry if one find field distribution →

A_n for confocal mirror resonator

2 concave mirror



radii of curvature → $R_1 = R_2 = R = d$

Field distribution becomes

$$A_{mn}(r,R,z) = C^* H_m(x^*)\, H_n(y^*) \cdot e^{-r^2/\omega^2} \exp(-i\phi(r,R,z))$$

$$x^* = \frac{\sqrt{2}\,x}{\omega} \qquad y^* = \frac{\sqrt{2}\,y}{\omega} \qquad \omega^2 = \frac{\lambda d}{2\pi}\left(1+\left(\frac{2z}{d}\right)^2\right)$$

(m,n) gives you different modes →

$m=n=0$ $H_n = H_m = 1$

$$A_{00} = C^* e^{-r^2/\omega^2} e^{-i\phi}$$

$I_{00} = I_0 e^{-2r^2/\omega^2}$ ← (shape of the wave → Gaussian)

at $r=\omega$ → $I_{00} = I_0 e^{-2}$ ⇒ beam radius

when $z=0$, min value of $\omega^2 = \frac{\lambda d}{2\pi}$

r = beam waist

beam waist is min, at $z=0$

beam radius keep on changing.

$$w = r_w = \left(\frac{\lambda R}{2\pi}\right)^{1/2} \text{ at } z=0$$

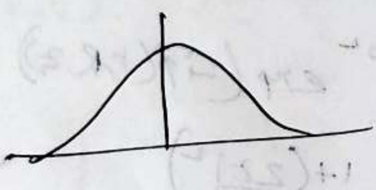
$$\text{for } r_{R/2} = \left(\frac{\lambda R}{2\pi}\right)^{1/2}\left(1+\left(\frac{2R}{2R}\right)^2\right)$$

$$= 2\left(\frac{\lambda R}{2\pi}\right)^{1/2} = 2r_w$$

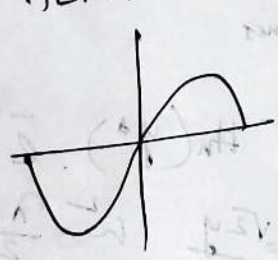
$m=0, n=0$

TEM mode →

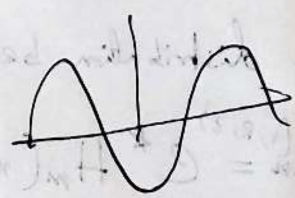
1D TEM0



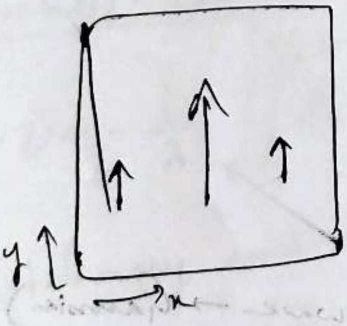
TEM1



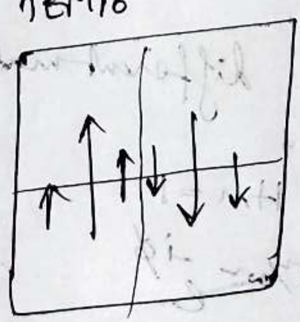
TEM2



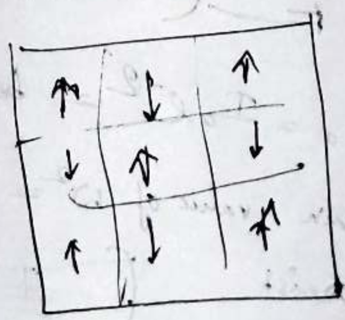
2D TEM00



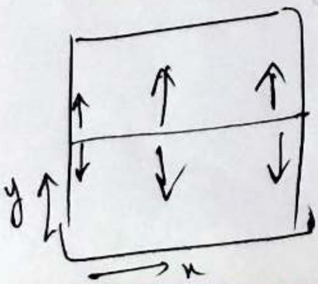
TEM10



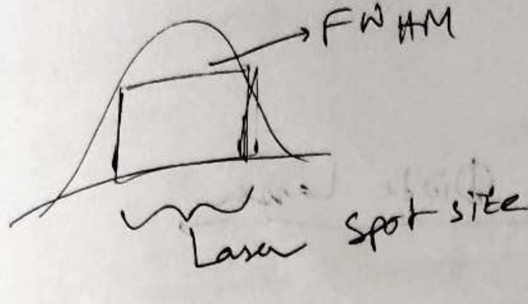
TEM22

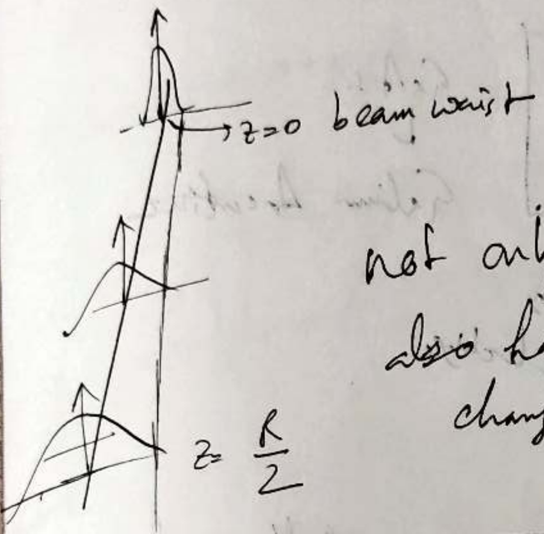


TEM01



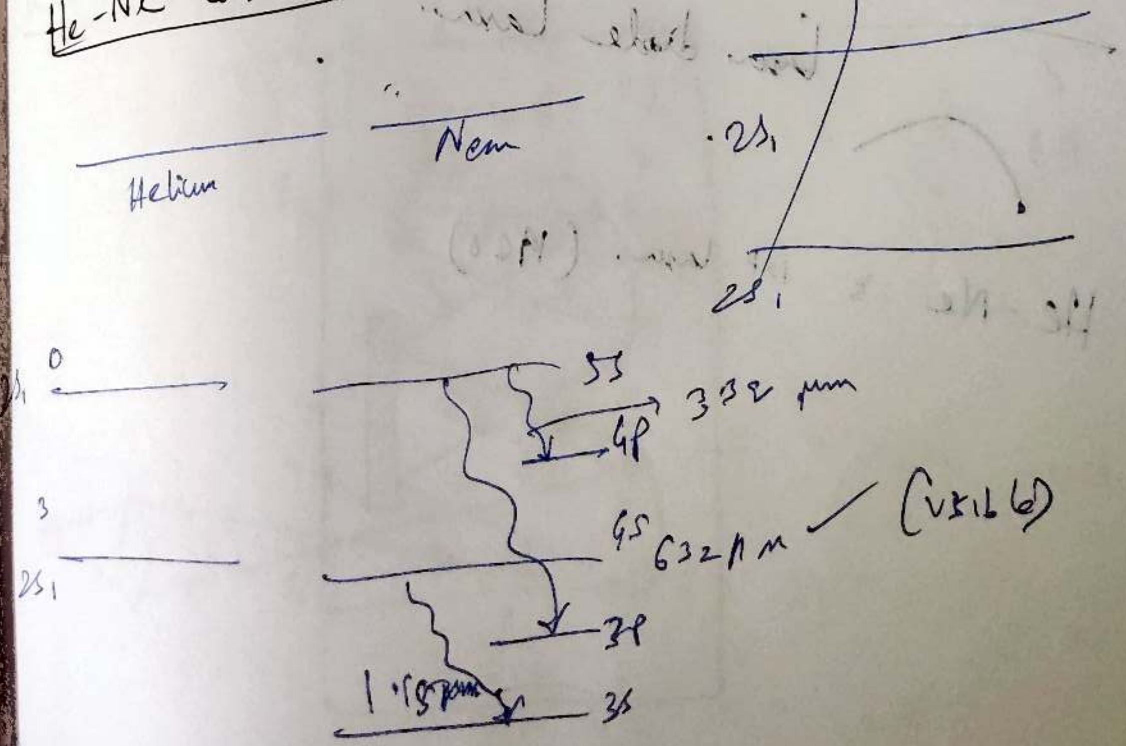
On the screen we get circular region and intensity decreases gaussian.





not only width changed, phase also has changed. Phase ~~revolves~~ changes with distance.

He-Ne laser → use this for stability.



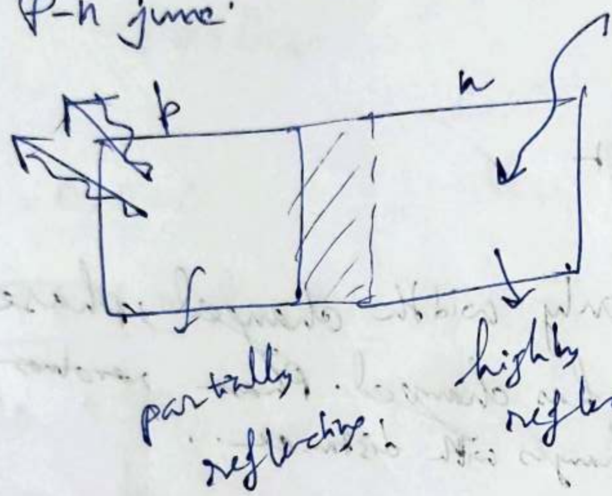
Ar laser:

Continuous wave gas laser. (CW laser)

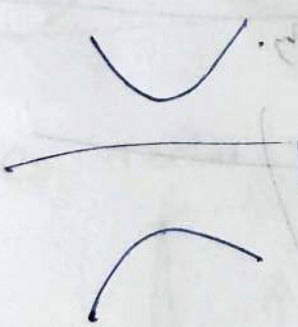
~~Semi Conductor laser~~

Semi Conductor laser / Diode laser

P-n junc.



GaAs
Galium Arcentine



Laser pointer is actually a ~~diode~~
laser diode laser.

He-Ne is 1st laser. (1960)

21/1/25 Tuesday

Basic units of Spectroscopy

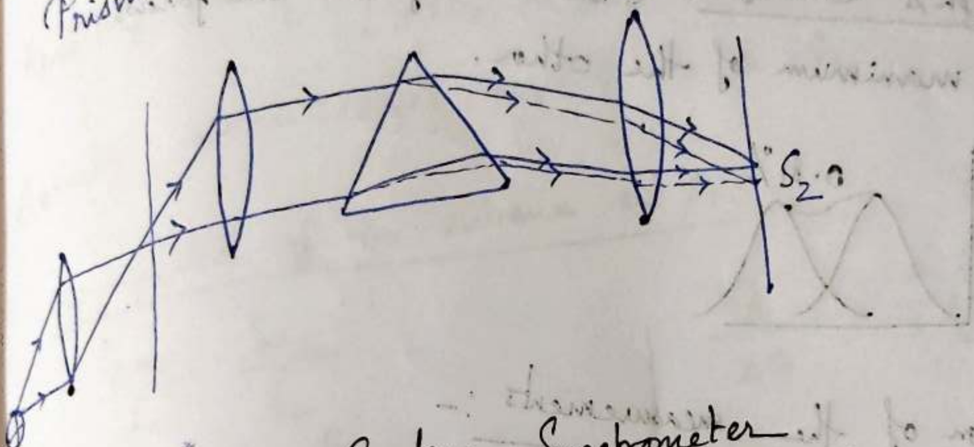
1. Source
2. Dispersion unit
3. Detector

Dispersion unit :-

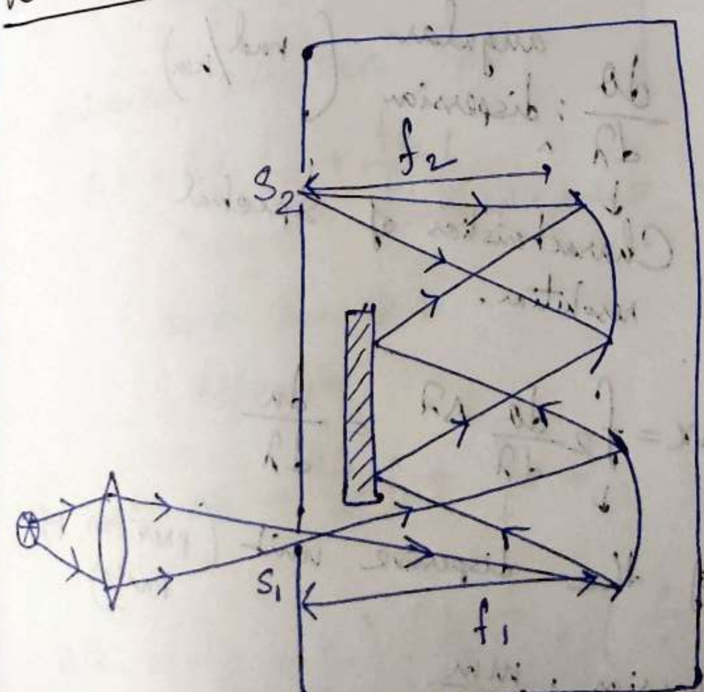
Spectrometer / Spectrograph

which gives image of a slit S_1 and $S_2(\lambda)$

Prism Spectrograph

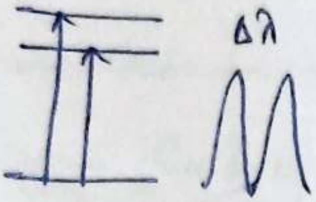


We mostly use Grating Spectrometer.



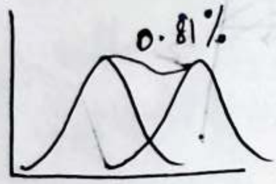
Spectroscopy is a q.m phenomena.

Resolution means the close lines/ close energy levels that we want to measure.



Two spectral lines can be resolved if they satisfy Rayleigh's Criteria.

Rayleigh's Criterion → The min. of the one follows on the maximum of the other.



Resolution of the measurements :-

angular speed

$$\Delta\theta = \frac{d\theta}{d\lambda}\,\Delta\lambda$$

$\frac{d\theta}{d\lambda}$: angular dispersion (rad/nm)

↓

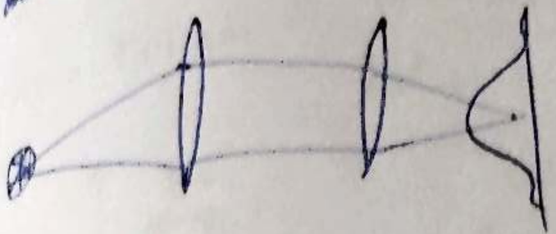
Characteristics of Spectral resolution.

linear dispersion $\Delta x = f_2 \frac{d\theta}{d\lambda}\,\Delta\lambda = \frac{dx}{d\lambda}$

f_2 → focal length of the dispersive unit (previous page image).

$\frac{dx}{d\lambda}$: linear dispersion, $\frac{mm}{nm}$

~~Linear dispersion: $\Delta x = f_2\, d\theta$~~



a = size of the grating / prism element

diffraction

$\Delta x_2 = \frac{2\lambda}{a} f_2 \Rightarrow f_2 \frac{d\theta}{d\lambda} \Delta\lambda \geq \frac{2\lambda}{a} f_2$

Resolving power : $\left|\frac{\lambda}{d\lambda}\right| \leq 2a\left(\frac{d\theta}{d\lambda}\right)$

Actually we're seeing image of S_1 an S_2

magnification of the entrance slit S_1

$\frac{\delta x_1}{f_1} = \frac{\delta x_2}{f_2}$

$\delta x_2 = \frac{f_2}{f_1} \delta x_1$

$\frac{\lambda}{\Delta\lambda} = \frac{a}{3}\left(\frac{d\theta}{d\lambda}\right)$

entrance slit size : b

$\Delta X = \underbrace{\frac{2\lambda}{a} f_2}_{\text{split due to diffraction}} + \frac{f_2}{f_1} b$ → split due to magnification.

$f_2 \frac{d\theta}{d\lambda} \Delta\lambda = \frac{2\lambda}{a} f_2 + \frac{f_2}{f_1} b$

$\Delta\lambda = \left(\frac{\lambda}{a} + \frac{b}{f_1}\right)\left(\frac{d\theta}{d\lambda}\right)^{-1} = \left(\frac{\lambda}{a} + \frac{2\lambda \cancel{f_1}}{a \cancel{f_1}}\right)\left(\frac{d\theta}{d\lambda}\right)^{-1}$

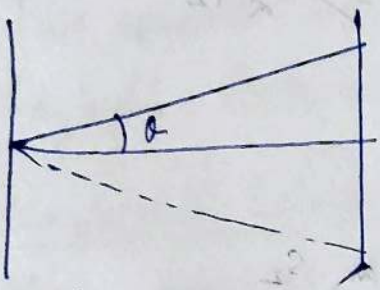
$\Delta\lambda = \frac{3\lambda}{a}\left(\frac{d\theta}{d\lambda}\right)^{-1}$

b = entrance slit.

there is a min limit of b.

$$\frac{2\lambda}{b} f_1 = a$$

$$b_{min} = \frac{2\lambda f_1}{a}$$



$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2}$$

$$\alpha = \frac{\pi b \theta}{\lambda} = \pi$$

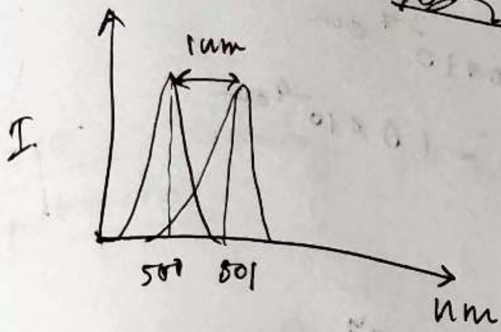
$$\Rightarrow \theta = \frac{2\lambda}{b}$$

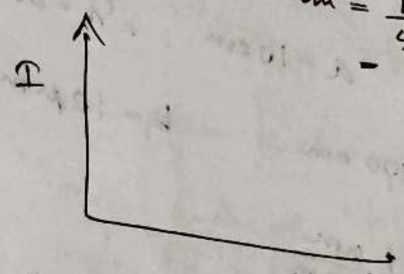
22/1/25

$\lambda_1 = 500\,nm = 500 \times 10^{-7}\,cm$ $= \frac{10^7}{500} = \frac{10^5}{5} = 2\times10^4\,cm^{-1} = 20000$

$\lambda_2 = 501\,nm$ $\Delta\lambda = 1\,nm = 10^{-7}\,cm$

$= 501 \times 10^{-7}\,cm = \frac{10^7}{501} = \frac{10^5}{5.01} = 1.99\times10^4\,cm^{-1} = 19900$





$\Delta\lambda = 20000 - 19900$

$= 100\,cm^{-1}$

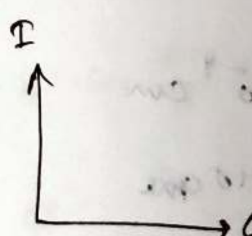
19900
$\underline{100}$
20000

$E_v = \frac{1240}{500}\,eV = 2.48\,eV$

$E_v = \frac{1240}{501} = 2.475\,eV$ $\Delta E_v = 0.005\,eV$

$\lambda_1 = 500\,nm$

$\lambda_2 = 501\,nm$ $\Delta\lambda$ in cm^{-1} $= 100\,cm^{-1}$



we plot when we do Raman Spectroscopy as it is spreaded.

$$\Delta\lambda \leq \left(\frac{\lambda}{a} + \frac{b}{f}\right)\left(\frac{d\theta}{d\lambda}\right)^{-1}$$

$\frac{dx}{d\lambda} = f \cdot \frac{d\theta}{d\lambda}$

a = grating element

i) Suppose $a = 10$ cm $\quad \lambda = 500 \times 10^{-7}$ cm

$f = 100$ cm $\quad b = 10\,\mu m = 10 \times 10^{-4}$ cm

$\frac{d\lambda}{dx} = 1 \frac{nm}{mm}$

$$\Delta\lambda = \left(\frac{50\cancel{0} \times 10^{-7}}{1\cancel{0}} + \frac{1\cancel{0} \times 10^{-4}}{10\cancel{0}}\right) 10^{-4}$$

$$\frac{dx}{d\lambda} = f\frac{d\theta}{d\lambda}$$

$$\left(\frac{d\theta}{d\lambda}\right)^{-1} = f\left(\frac{d\lambda}{dx}\right) = 1 \times \frac{nm}{mm} \times 100 \times 10 \text{ mm}$$

$= 1000 \times 10^{-7}$ cm

$= 10^{-4}$ cm

$$\Delta\lambda = \left(50 \times 10^{-7} + 10^{-5}\right) 1 \times 10^{-4}$$

$(5 \times 10^{-6} + 1.0 \times 10^{-5}) \times 10^{-4}$

$= \cancel{6 \times 10^{-7} \times 10^{-4}}$

$= \cancel{6 \times 10^{-11}}$

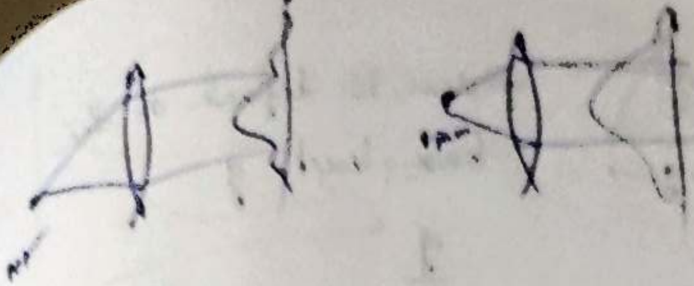
$= 15 \times 10^{-6} \times 10^{-4}$ cm

$= 15 \times 10^{7} \times 10^{-10}$ nm

$= 15 \times 10^{-3}$ nm

$= 0.015$ nm

* if $b = 5\,\mu m$ $\quad \Delta\lambda = 0.01$ nm



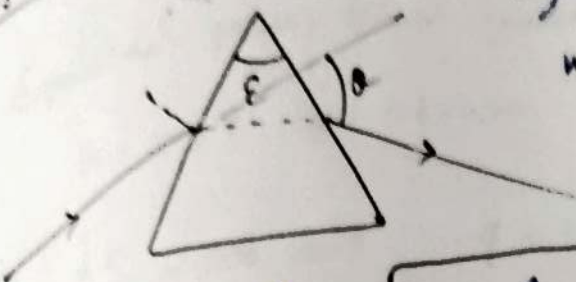
Prism :- Normally we dont use prism much.

Spectrometer

prism angle = ε

angle of deviation = θ

$n =$



for min deviation $\boxed{n \sin\frac{\varepsilon}{2} = \sin\frac{\theta+\varepsilon}{2}}$

$\frac{d\theta}{d\lambda}$ is characteristics of diffraction element.

we here use $\frac{dn}{d\lambda}$

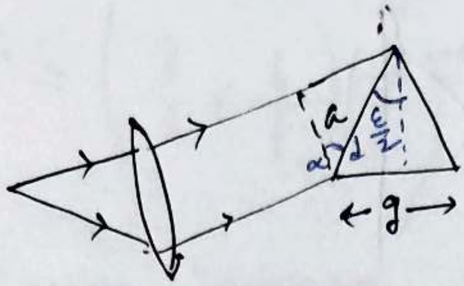
$$n \sin\frac{\varepsilon}{2} = \sin\frac{\theta+\varepsilon}{2}$$

$$\frac{1}{2}\cdot\cos\frac{\theta+\varepsilon}{2}\, d\theta = \sin\frac{\varepsilon}{2}\, dn$$

$$\frac{d\theta}{dn} = \frac{2\sin\frac{\varepsilon}{2}}{\sqrt{1-n^2\sin^2\frac{\varepsilon}{2}}}$$

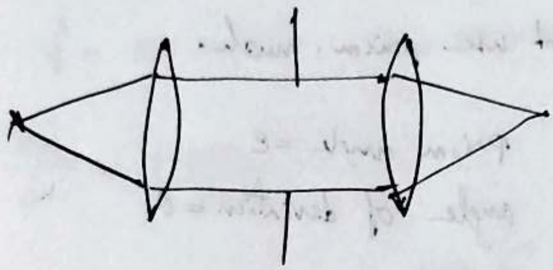
$$\frac{d\theta}{d\lambda} = \frac{d\theta}{dn}\cdot\frac{dn}{d\lambda} = \frac{2\sin\frac{\varepsilon}{2}}{\sqrt{1-n^2\sin^2\frac{\varepsilon}{2}}}\cdot\frac{dn}{d\lambda}$$

$$\Delta\lambda \leq \frac{a}{3}\left(\frac{d\theta}{d\lambda}\right)$$

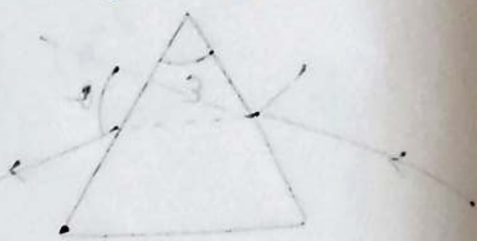


prism is defined on the base length g.

$$\frac{g/2}{d} = \sin\frac{\varepsilon}{2}$$



$$d = \frac{g}{2\sin\frac{\varepsilon}{2}}$$



$$a = d\cos\alpha$$

$$a = \frac{g\cos\alpha}{2\sin\frac{\varepsilon}{2}}$$

for angle of min. deviation

$$n\sin\frac{\varepsilon}{2} = \sin\frac{(\theta+\varepsilon)}{2} = \sin\alpha$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha}$$

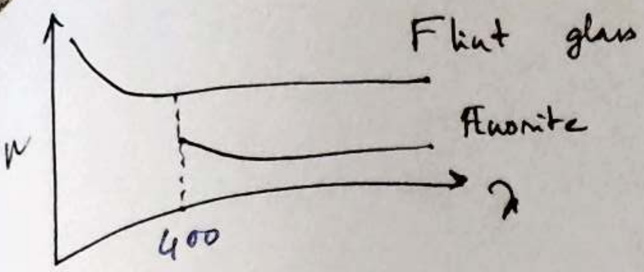
$$\cos\alpha = \sqrt{1-n^2\sin^2\frac{\varepsilon}{2}}$$

$$a = \frac{g\sqrt{1-n^2\sin^2\frac{\varepsilon}{2}}}{2\sin\frac{\varepsilon}{2}}$$

$$\frac{\lambda}{\Delta\lambda} \le \frac{a}{3}\left(\frac{d\alpha}{d\lambda}\right)$$

$$\le \frac{1}{3}\,\frac{g\sqrt{1-n^2\sin^2\frac{\varepsilon}{2}}}{2\sin\frac{\varepsilon}{2}}\cdot\frac{2\sin\frac{\varepsilon}{2}}{\sqrt{1-n^2\sin^2\frac{\varepsilon}{2}}}\cdot\frac{dn}{d\lambda}$$

We use Flint glass below 400 nm
below 400 nm $\frac{dn}{d\lambda}$ we get
but above 400 $\frac{dn}{d\lambda} \approx 0$.



$$\Delta\lambda \leq \frac{g}{3}\frac{dn}{d\lambda}$$

Equilateral prism, Fused quartz

$n = 1.47 \quad \lambda = 400\,nm \qquad \varepsilon = 60°$

$\frac{dn}{d\lambda} = 1100\ cm^{-1} \qquad g = 1\ cm$

$\Delta\lambda = ?$

$$\frac{\lambda}{\Delta\lambda} = \frac{g}{3} \times 1100 = 0.33 \times 1100$$
$$= 33 \times 11$$
$$= 330 + 33$$
$$= 363.$$

$$\Delta\lambda = \frac{\lambda}{363} = \frac{400}{363}\,nm$$
$$= 1.10\,nm$$

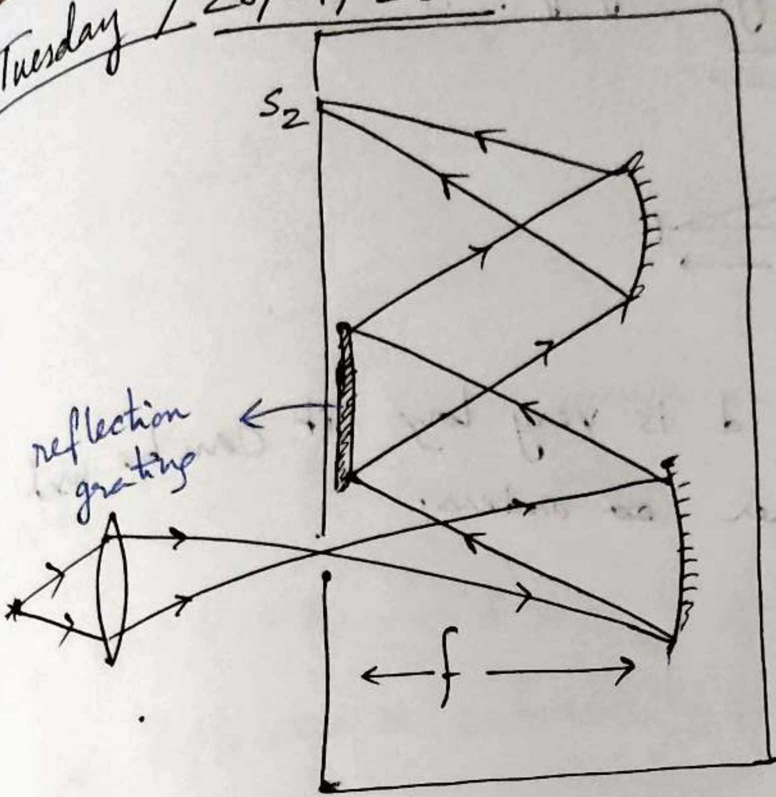
$$\frac{d\alpha}{d\lambda} = \cancel{\frac{2\sin\frac{\varepsilon}{2}}{1}} \quad \frac{d\alpha}{dn}\frac{dn}{d\lambda}$$

$$\frac{d\alpha}{dn} = \frac{2\sin\frac{\varepsilon}{2}}{\sqrt{1 - n^2\sin^2\frac{\varepsilon}{2}}} = \frac{2 \times \frac{1}{2}}{\sqrt{1 - (1.47)^2\frac{1}{4}}}$$

$$= \frac{1}{\sqrt{0.45977}}$$

$$= \frac{1}{0.6780}$$

Tuesday / 28/01/25

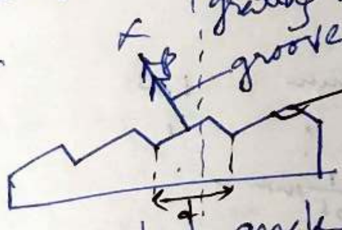


$$d(\sin\alpha \pm \sin\beta) = m\lambda$$

grating normal
groove normal

here angle can be anything.

Littrow grating



incident angle = reflection angle

$\alpha = 90°$ $\beta = 90°$

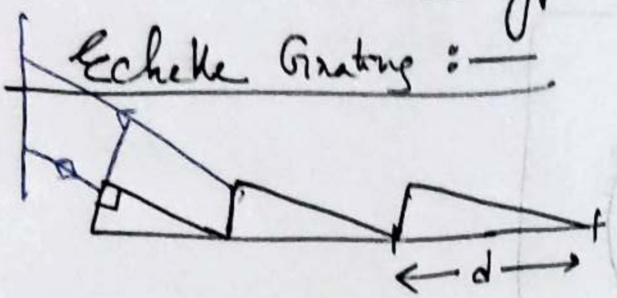
$$d(2\sin\alpha) = m\lambda$$

$$2d\sin\alpha = m\lambda$$

it basically chooses different wavelength for different order. So it is called wavelength selective grating.

There are another type of grating :

Echelle Grating :—



$2d \sin\alpha = m\lambda$

here as comparatively d is very long it can be used to determine higher orders.

as $d \ggg \lambda$

$\theta = 90° - \alpha$

$2d \cos\theta = m\lambda$

Q grating size : $10\,cm \times 10\,cm$

10^3 grooves/mm $m = 2$ 1 mm 10^3 $\frac{1}{10^3}$ mm

Resolving power of the grating. $R = \frac{\lambda}{\Delta\lambda} = Nm$

N = total no. of groove which will be illuminated

$R = \frac{\lambda}{\Delta\lambda} = mN = 2 \times 10^5$

1 mm 10^3

10^{-1} cm 10^3 $\frac{10 \times 10^3}{10^{-1}} = 10^3 \times 10^2 = 10^5 = N$

10 cm ?

$mN = 2\times10^{5} = \frac{\lambda}{\Delta\lambda}$

$\lambda = 500$ nm

$\Delta\lambda = \frac{500}{2\times10^{5}} = 2.5\times10^{-3}$ nm $= 0.0025$ nm.

$$\Delta\lambda = \left(\frac{\lambda}{a} + \frac{b}{f}\right)\left(\frac{d\theta}{d\lambda}\right)^{-1}$$

$d(\sin\alpha + \sin\beta) = m\lambda$

here diffraction is controlled by β.

a = size $\theta \cong \beta$ $\frac{dx}{d\lambda} = f\,\frac{d\theta}{d\lambda}$

$$\Delta\lambda = \left(\frac{\lambda}{a} + \frac{b}{f}\right)\left(\frac{d\beta}{d\lambda}\right)^{-1}$$

$b = 50\ \mu m$ $f = 1m$ $a = 10$ cm, $\alpha = \beta = 30^\circ$

$\Delta\lambda = ?$

$\frac{d\beta}{d\lambda} = \frac{m}{d\cos\beta} = \frac{2}{10^{-3}\times 18^{6}\,\frac{\sqrt{3}}{2}}$

$= \frac{4}{\sqrt{3}\times10^{-6}}$

$= \frac{4\times10^{6}}{\sqrt{3}}$

$\cancel{\frac{d\theta}{d\lambda}=}$

$\cancel{\Delta\lambda = f}$

$\Delta\lambda = \left(\frac{500\times10^{-9}}{10\times10^{-2}} + \frac{50\times10^{-6}}{1}\right)\left(\frac{d\beta}{d\lambda}\right)^{-1} = \frac{4}{\sqrt{3}}\times10^{-3}$

$= 0.0023$

$= \left(5\times10^{-6} + 50\times10^{-6}\right)\left(\frac{d\beta}{d\lambda}\right)^{-1}$ $\left(\frac{d\beta}{d\lambda}\right)^{-1} = 434$ nm

$= 55\times10^{-6}\left(\frac{d\beta}{d\lambda}\right) = 55\times10^{-6}\times\frac{4\times\sqrt{3}}{4\times10^{6}}$

$= 23.8\times10^{-12}$

$= 23.8\times10^{-12}\times10^{9} = 0.023$ nm

$$d(\sin\alpha + \sin\beta) = m\lambda$$

10^3 grooves/mm

$$d\left(\frac{1}{2} + \frac{1}{2}\right) = m\lambda$$

1 mm 10^3 g

1 g

$$d = m\lambda$$

10^{-3} mm

$$10^{-3} \times 10^{-3} = m \times 500 \times 10^{-9}$$

$$\frac{10^{-6} \times 10^{9}}{500} = m$$

$$m = \frac{10^3}{500} = m = 2$$

~~$d\cos\beta$~~

to get more resolution we need to use higher focal length.

small slit - b

higher size of grating. - a

higher focal length - f

Characteristics of the grating will be governed by

$$\frac{d\beta}{d\lambda}$$

$$\Delta\lambda = \left(\frac{\lambda}{a} + \frac{b}{f}\right)\left(\frac{d\beta}{d\lambda}\right)^{-1}$$

$$d\cos\beta\, d\beta = m\, d\lambda$$

$$\frac{d\beta}{d\lambda} = \frac{m}{d\cos\beta}$$

$$\frac{d\beta}{d\lambda} = 0.023 nm^{-1}$$

$$f = \cancel{10}^{2}\ 1m = 10^{9} nm$$

$$\frac{dx}{d\lambda} = f\frac{d\beta}{d\lambda} = 10^{9} nm \times 0.023\ {}^{1}/_{nm}$$

$$= 23 \times 10^{6}$$

Detector :- ① Thermal Detector
② Photo Detector

① Thermal Detector

1) Doesnot depend on wavelength. Depends on the power.
λ insensitive

2) works on change in material properties with ~~p~~ temperature.

② Photo Detector

1) λ insensitive

2) works on response of material for a given photon energy.

Characteristics of the Detectors :- Date - Wednesday 29/1/25

1) Spectral response $(R(\lambda))$ → 1) Spectral range.
2) Two relative intensity

$$\frac{I(\lambda_1)}{I(\lambda_2)} = \frac{R(\lambda_1)}{R(\lambda_2)}$$

2) Sensitivity ; $\frac{\text{Volts}}{\text{Watt}}$ or $\frac{\text{Amp}}{\text{Watt}}$ $\frac{\text{Volt}}{A\ I}$ I → irradiance, A → area

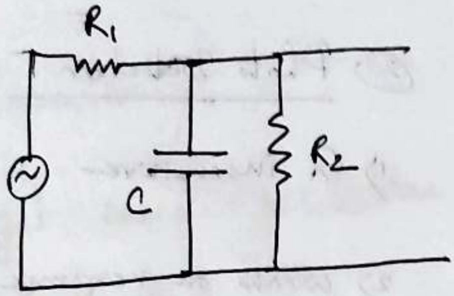
3) Signal to noise ratio $\left(\frac{S}{N}\right)$

Source of noise → electronic noise,
thermal noise

• Noise equivalent of input power

→ power level for which $\frac{S}{N} = 1$

4) Time response



$R_2 C \ll \tau$; τ = pulse time

↓

must be faster, so that

$R_2 C \ll \frac{1}{f}$

$f \ll (R_2 C)^{-1}$

5) Cost of the detector

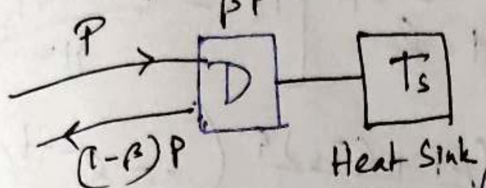
② **Thermal Detector** → doesnot depend on wavelength.

works with a material property that changes with temp..

Thermistor

Usage → 1) Power measurement of continuous wave laser.

2) Output energy of the pulsed laser.



β = fraction of P that incident on D.

(1-β) = fraction of P that reflected back from D.

H: heat capacity of the detector

G: Thermal conductance of the heat sink.

$$\boxed{\beta P = H\frac{dT}{dt} + G(T - T_s)}$$

$$\beta P + GT_s = H\frac{dT}{dt} + GT$$

$$\Rightarrow \frac{\beta P + GT_s}{H} = \frac{dT}{dt} + \frac{G}{H}T \qquad a = \frac{\beta P + GT_s}{H}$$

$$b = \frac{G}{H}$$

$$\Rightarrow \frac{dT}{dt} + bT = a$$

$$IF = e^{\int b\,dt} = e^{\frac{Gt}{H}}$$

$$\int d\left(e^{Gt/H}T\right) = \int \frac{\beta P + GT_s}{H} e^{Gt/H}\,dt$$

$$\Rightarrow Te^{\frac{Gt}{H}} = \frac{\beta P + GT_s}{H}\frac{G}{H}e^{Gt/H} + C$$

$$\Rightarrow T = \frac{(\beta P + GT_s)G}{H^2} + Ce^{-Gt/H}.$$

$\beta P = H \frac{dT}{dt} + G(T - T_s)$

$\Rightarrow \beta P = H \frac{dT}{dt} + GT - GT_s$

$\Rightarrow \frac{(\beta P + GT_s)}{H} = \frac{dT}{dt} + \frac{G}{H} T$

$\Rightarrow \frac{dT}{dt} + \frac{G}{H} T = \frac{(\beta P + GT_s)}{H}$

$\frac{dy}{dx} + Py = Q$

$\mu = I \cdot F = e^{\int P dx}$

$\int d(\mu \cdot y) = \int Q \mu \, dx$

$\Rightarrow \mu = \left(e^{\int \frac{G}{H} dt}\right) = e^{Gt/H}$

$\Rightarrow \int d\left(e^{Gt/H} T\right) = \int e^{Gt/H} \left(\frac{\beta P + GT_s}{H}\right) dt$

$\Rightarrow e^{Gt/H} T = \left(\frac{\beta P + GT_s}{H}\right) \frac{e^{Gt/H}}{\frac{G}{H}} + C$

$\Rightarrow T = \left(\frac{\beta P + GT_s}{G}\right) + C e^{-Gt/H}$

$\Rightarrow T = \frac{\beta P}{G} + T_s + C e^{-Gt/H}$

at $t = 0$ $T = T_s$

$\Rightarrow T_s = T_s + \frac{\beta P}{G} + C e^{-G \cdot 0/H}$

$\Rightarrow C = -\frac{\beta P}{G}$

$T = T_s + \frac{\beta P}{G} - \frac{\beta P}{G} e^{-Gt/H}$

$T = T_s + \frac{\beta P}{G}\left(1 - e^{-Gt/H}\right)$

$$T = T_s + \frac{\beta P}{G}\left(1 - e^{-\frac{G}{H}t}\right)$$

$$= T_s + \Delta T\left(1 - e^{-\frac{G}{H}t}\right).$$

$$\Delta T = \frac{\beta P}{G}.$$ it is independent of H.

time constant / response time $= \frac{H}{G} = \tau$

Small values of G make a thermal detector sensitive, small values of both quantities.

Now, instead of constant power

$$P = P_0 (1 + a \cos \Omega t)$$

$$T = T_s + \Delta T \left(1 + a \cos(\Omega t + \phi)\right)$$

$$\Delta T = \frac{a \beta P}{\sqrt{1 + \frac{\Omega^2 H^2}{G^2}}} \qquad \tan\phi = \frac{\Omega H}{G}$$

Pulsed Laser

$$\int_0^t \beta P \, dt = H \int_0^t dT \quad (G = 0)$$

$$\rightarrow \cancel{\beta P t = H} \Rightarrow \Delta T = \frac{1}{H} \int_0^t \beta P \, dt$$

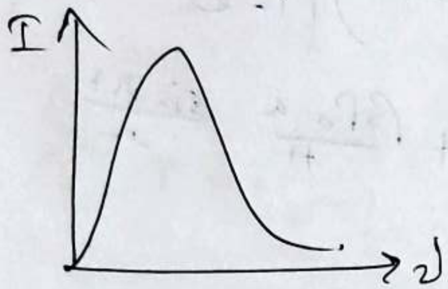
There are differenta types of thermal detector.

① Thermistator ② Bolometer ③ Golay cell

④ Thermocouple detector

$\ast$ $\cancel{\beta P = H}$

Spectral Line Shape



Natural Line broadening $\rightarrow$ particular state has life time. and depending on that it has a natural width.

$\Delta E \, \Delta\tau \sim \hbar$

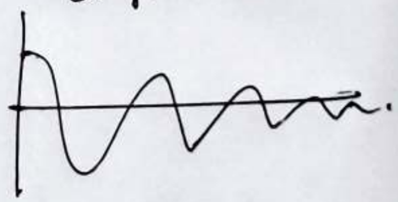
$\Delta E \sim \dfrac{\hbar}{\Delta\tau}$ $\quad \Delta\tau =$ life time.

Oscillator Model

$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = 0$

damping.

damped oscillator.



$$I = \frac{1}{(\omega-\omega_0)^2 + \left(\frac{\gamma}{2}\right)^2} \quad \text{(Lorentzian } f^n\text{)}$$

Energy response $= E = \frac{1}{2} \tilde{x}^* \tilde{x}$

for visible light ~~*~~ $\Delta\tau_{vib} \approx 0.1$ sec

$\Delta E \, \Delta\tau = \hbar$

$$h\nu = \frac{\hbar}{\Delta\tau} \Rightarrow \Delta\nu = \frac{\hbar}{h\,\Delta\nu} = \frac{1}{2\pi\,\Delta\tau}$$

$$= \frac{1}{2\pi \times 0.1} \ \text{sec}^{-1}$$

Line shape of Absorption Spectra :-

Now, we have external force, forced oscillation.

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \frac{qE_0 e^{i\omega t}}{m}$$

$$x = x_0 e^{i\omega t}$$

$$(i\omega)^2 x_0 \cancel{e^{i\omega t}} + \gamma i\omega x_0 \cancel{e^{i\omega t}} + \omega_0^2 x_0 \cancel{e^{i\omega t}} = \frac{qE_0}{m} \cancel{e^{i\omega t}}$$

$$-\omega^2 x_0 + i\gamma\omega x_0 + \omega_0^2 x_0 = \frac{qE_0}{m}$$

$$x_0 = \frac{qE_0}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}.$$

Dipole moment $\vec{P} = qx = q \cdot \frac{q\vec{E}}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}.$

polarizibility : $\vec{P} = \frac{\text{dipole moment}}{\text{volume}}.$

N = density of oscillator → electron.

$$\vec{P} = \frac{Nq^2\vec{E}}{m(\omega_0^2 - \omega^2 + i\gamma\omega)} = \underset{\substack{\downarrow \\ \text{susceptibility}}}{\chi} E \varepsilon_0 \quad \text{——— (1)}$$

~~Direct~~ Dielectric constant $\varepsilon = (1+\chi)\varepsilon_0.$

$$\chi = \varepsilon_r - 1 = \frac{\varepsilon}{\varepsilon_0} - 1 = \underset{\substack{\downarrow \\ \text{refractive index}}}{n}^2 - 1$$

for $n \approx 1$ $\quad \chi = 2(n-1)$. ——— (2)

from ① we can write

$$\chi = \frac{Nq^2}{m\varepsilon_0(\omega_0^2 - \omega^2 + i\gamma\omega)} = 2(n-1)$$

$$n = 1 + \frac{Nq^2}{2m\varepsilon(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

$$n = n' - ik = 1 + \frac{Nq^2}{2m\varepsilon_0} \frac{(\omega_0^2 - \omega^2 - i\gamma\omega)}{[(\omega_0^2 - \omega^2)^2 + (\gamma^2\omega^2)]}$$

$$\boxed{n' = 1 + \frac{Nq^2}{2m\varepsilon_0} \frac{(\omega_0^2 - \omega^2)}{[(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2]}} \longrightarrow ⓐ$$

Electric field of EM radn in vac.

$$\vec{E} = E_0 e^{i(\omega t - k_0 z)}$$

Electric field in medium:

$$\vec{E} = E_0 e^{i(\omega t - nk_0 z)}$$

$$= E_0 e^{i(\omega t - (n' - ik)k_0 z)}$$

$$= E_0 e^{-kk_0 z} e^{i(\omega t - n'k_0 z)}$$

Lambert Beer's Law

$I = I_0 e^{-\alpha z}$ α = abs. coefficient

$(Amp)^2$ — Intensity.

$$I = E_0^2 \, e^{-2K k_0 z} \, e^{2i(\omega t - n' k_0 z)}.$$

$$I = I_0 e^{-\alpha z}$$

$$\alpha = 2 K k_0.$$

$$\boxed{\alpha = 2 k_0 \cdot \frac{N q^2 \gamma \omega}{2 m \varepsilon \left[(\omega_0^2 - \omega^2)^2 + (\gamma \omega)^2 \right]}} \longrightarrow (b)$$

$$\omega = c k \longrightarrow k_0 = \frac{\omega_0}{c}$$

Kramer's - Kronig eqn. (a), (b).
Relation.

gives real part of RI and absorption coefficient of light following this oscillator model.

$$n' = 1 + \frac{N q^2 (\omega_0^2 - \omega^2)}{2 m \varepsilon_0 \left[(\omega_0^2 - \omega^2) + (\gamma \omega)^2 \right]}$$

$$\alpha = 2 k_0 \frac{N q^2 \gamma \omega}{2 m \varepsilon \left[(\omega_0^2 - \omega^2)^2 + (\gamma \omega)^2 \right]}$$

Close to Resonance $\omega_0 \approx \omega$

$$n' = 1 + \frac{N q^2 (\omega_0 + \omega)(\omega_0 - \omega)}{2 m \varepsilon \left[(\omega_0 + \omega)^2 (\omega_0 - \omega)^2 + (\gamma \omega)^2 \right]} \approx 1 + \frac{N q^2 \, 2\omega_0 (\omega_0 - \omega)}{2 m \varepsilon \left[4 \omega_0^2 (\omega_0 - \omega)^2 + (\gamma \omega)^2 \right]}$$

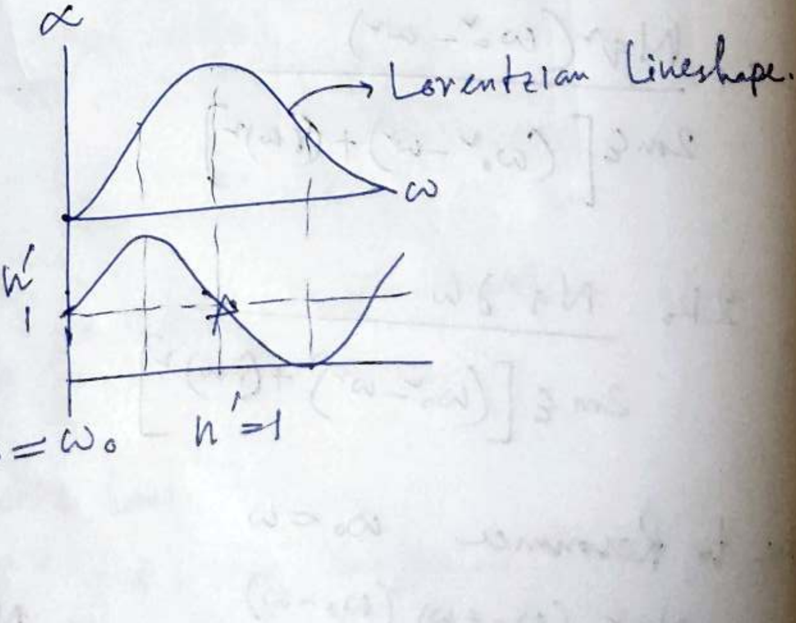
$$n' = 1 + \frac{Nq^2(\omega_0-\omega)}{4m\varepsilon\,\omega_0\left[(\omega_0-\omega)^2+\left(\frac{\gamma}{2}\right)^2\right]}$$

$$\alpha = 2k_0 \frac{Nq^2\,\omega\gamma}{2m\varepsilon\left[4\omega_0^2(\omega_0-\omega)^2+(\gamma\omega)^2\right]}$$

$$= \frac{\cancel{2}k_0}{\cancel{2}m\varepsilon_0}\,\frac{Nq^2\cancel{\omega_0}\gamma}{4\omega_0^{\cancel{2}}\left[(\omega_0-\omega)^2+\left(\frac{\gamma}{2}\right)^2\right]}$$

$$= \frac{N\cancel{k_0}q^2\gamma}{4m\varepsilon_0 c\cancel{\omega_0}\left[(\omega_0-\omega)^2+\left(\frac{\gamma}{2}\right)^2\right]}$$

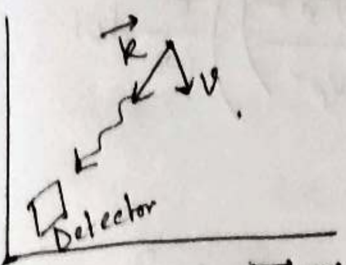
$$= \frac{Nq^2\gamma}{4m\varepsilon_0 c\left[(\omega_0-\omega)^2+\left(\frac{\gamma}{2}\right)^2\right]}$$



at $\omega = \omega_0$ $n' = 1$

Doppler Broadening :-

Natural line broadening is for static atom.



$\omega = \omega_0 \pm \vec{k}\cdot\vec{v}$

+ : source is moving towards detector

− : source is moving away from the detector.

Assume $k \to k_z$ $(0, 0, k_z)$ $\quad k = \frac{\omega}{c}$

~~$\omega = \omega_0 \pm k_z$~~

$v \to v_z$

$(0, 0, v_z)$ $\quad \omega = \omega_0\left(1 + \frac{v_z}{c}\right)$ — ①

E_i: energy of atom per unit volm in the i th state

No. of molecules having energy E_i moving with velocity v_z and $v_z + dv_z$

$$n_i(v_z)\, dv_z = \frac{N_i}{\sqrt{\pi}\, v_p} \exp\left(-\left(\frac{v_z}{v_p}\right)^2\right) dv_z \quad \text{— ②}$$

v_p = most probable speed. $= \sqrt{\frac{2kT}{m}}$

Total no. of atom in the ith state $N_i = \int n(v_z)\, dv_z$

From ① $\longrightarrow d\omega = \omega_0 \frac{dv_z}{c}$

$\longrightarrow v_z = \left(\frac{\omega}{\omega_0} - 1\right)c = \frac{(\omega - \omega_0)c}{\omega_0}$

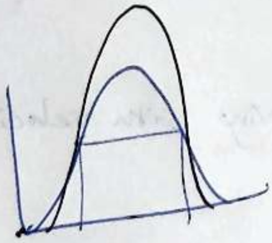
From (2)

$$n(\omega)\,d\omega = \frac{N_i}{\sqrt{\pi}\,V_p} \exp\left(-\left(\frac{(\omega-\omega_0)^2 c^2}{\omega_0^2 V_p^2}\right)\right)\frac{c}{\omega_0}\,d\omega$$

$$= \frac{N_i c}{\sqrt{\pi}\,V_p\,\omega_0} \exp\left[-\frac{(\omega-\omega_0)^2 c^2}{\omega_0^2 V_p^2}\right] d\omega$$

Intensity $I = I_0 \exp\left[-\frac{(\omega-\omega_0)^2 c^2}{\omega_0^2 V_p^2}\right]$ (gaussian fn)

intensity of the atoms in the frequency range ω and $\omega + d\omega$ or in energy range E and $E + dE$ following Maxwell - Boltzmann distribution.



FWHM

$\exp\left[-\frac{(x-x_0)^2}{\sigma^2}\right]$ | $\exp\left[-\frac{(\omega-\omega_0)^2}{\omega_0^2 V_p^2}\right]$

$\sigma = \frac{\omega_0 V_p}{c}$

$$FWHM = 2\sqrt{\ln 2}\,\sigma$$

$$\delta\omega_D = 2\sqrt{\ln 2}\,\frac{\omega_0 V_p}{c}$$

$$= 2\sqrt{\ln 2}\,\frac{\omega_0}{c}\sqrt{\frac{2kT}{m}}$$

$$= \frac{\omega_0}{c}\sqrt{\frac{8kT\ln 2}{m}} = \frac{\omega_0}{c}\sqrt{\frac{8T N_A k \ln 2}{N_A m}}$$

$$= \frac{\omega_0}{c}\sqrt{\frac{8RT\ln 2}{M}}$$

$$\boxed{\delta\nu_D = 7.16\times 10^{-7}\,\nu_0\sqrt{\frac{T}{M}}}$$

M = molar mass.

for Na D line $\tau = 16\,ns$

$\delta\nu_N = 10\,MHz$

$\Delta E \Delta t = \hbar$

$$\Delta E = \frac{\hbar}{\Delta t}$$

$$h\Delta\nu = \frac{\hbar}{\Delta t} \Rightarrow \Delta\nu = \frac{\hbar}{h\,\Delta t} = \frac{1}{\Delta\tau \times 2\pi}$$

$\delta\nu_D$ at $500K$ $M = 23$

$$\lambda_0 = 589\,nm \qquad \nu_0 = \frac{c}{\lambda} = \frac{3\times10^{8}\,m/s}{589\times10^{-9}\,m}$$

$$= 0.00509\times10^{17}\,s^{-1}$$

$\omega = 2\pi\nu$

$$= 0.509\times10^{15}\,s^{-1}$$

$\delta\omega = 2\pi\,\delta\nu$

$$= 5\times10^{14}\,s^{-1}$$

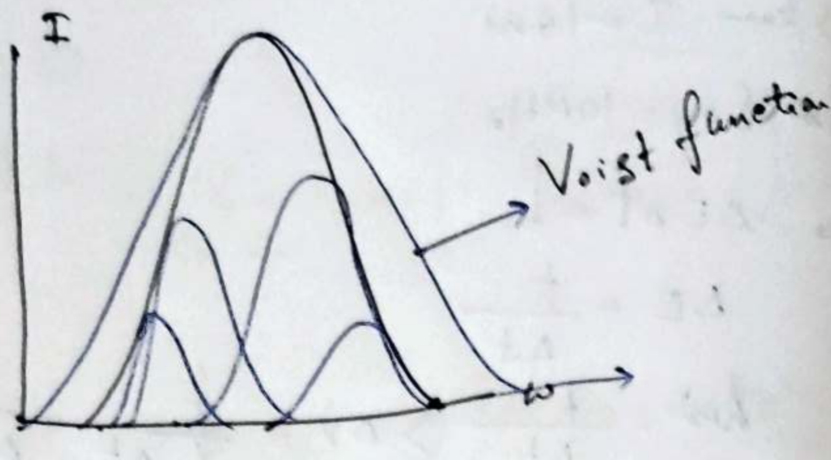
$$\cancel{\delta\nu = \frac{1}{2\pi}\,\delta\omega = \frac{1}{2\pi}\left[7.16\times10^{-7}\right]}$$

$$\delta\nu_D = 7.16\times10^{-7}\,\nu_0\sqrt{\frac{T}{M}}$$

$$= 7.16\times10^{-7} \times 0.509\times10^{15}\sqrt{\frac{500}{23}}$$

$$= 7.16\times0.509\times10^{8}\times4.66$$

$$= 16.98\times10^{8} = 1.6\times10^{9}\,Hz$$



for ~~each~~ each ω you'll have a Lorentzian broadening, you get gaussian fn for doppler broadening.

for natural broadening you get Lorentzian fn

B. Combining both we get Voigt fn.

$$I = C\int \exp\left[-\frac{(\omega_0-\omega')^2 c^2}{\omega_0^2 v_p^2}\right]\frac{1}{(\omega-\omega')^2+(\gamma/2)^2}\,d\omega'$$

~~I~~ In instrument we get