RONIT DUTTA . (1) Date 3/4/25.

Magsc — Superconductivity Assignment

(1)(a) $\lambda_L = \sqrt{\dfrac{m_e}{n_s e^2 \mu_0}}$

Assumption: (1.) All valence e^- are SC type. $\therefore n_s = n$.

$n = n_{Al} = Z_{Al} \times N_A \times \left(\dfrac{\rho}{M_{Al}}\right)$

$= 3 \times \dfrac{\cancel{2700}\, 10^2 \times 10^3\ gm^{-3}}{\cancel{27}\ g\,mol^{-1}} \times 6.022 \times 10^{23}\ \cancel{mol^{-1}}$

$= 1.8066 \times 10^{24+2+3} = \boxed{1.806 \times 10^{29}\ m^{-3}}$

$\therefore \lambda_L = \left[\dfrac{9.11 \times 10^{-31}}{(4\pi \times 10^{-7}) \times (1.8 \times 10^{29}) \times (1.6 \times 10^{-19})^2}\right]^{1/2}$

$= 1.254 \times 10^{-8}\ m. = \boxed{12.54\ nm.}$

(b) Fraction of valence electrons @ the energy level Δ of the neighbourhood of E_F (Fermi energy)

Number of electrons: Integral of density of states from $E = 0$ to $E = E_F$.

$N(E) = \dfrac{1}{2\pi^2}\left(\dfrac{2m}{\hbar^2}\right)^{3/2} E_F^{3/2}$.

$n(E) = \int N(E)\, dE$.

$\delta n(E) = N(E)\, \delta E$.

$\therefore \Delta n(E) = N(E) (\Delta E)$.

$\therefore \delta n(E) = N(E) \cdot \Delta$. $\longrightarrow$ $\boxed{\text{For us, } \Delta E = \Delta}$.

In the neighbourhood : $[E_F - \Delta,\ E_F + \Delta]$.

Our, total number of electrons $= \boxed{2 \cdot \Delta \cdot N(E)}$.

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$$\therefore n(E_F) = \int_0^{E_F} \frac{1}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{3/2}\, dE.$$

$$= \frac{1}{3\pi^2}\cdot\left(\frac{2m}{\hbar^2}\right)^{3/2}\left[\frac{E^{5/2}}{5/2}\right]_0^{E_F} = \frac{2}{15\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E_F^{5/2}.$$

$$\therefore \text{Fraction i.e.} \left(\frac{\delta n}{n}\right) \Rightarrow \frac{\cancel{2}\times\Delta\times\frac{1}{\cancel{3\pi^2}}\cdot\cancel{\left(\frac{2m}{\hbar^2}\right)^{3/2}} E^{3/2 - (5/2)}}{\frac{\cancel{2}}{5\,\cancel{15\pi^2}}\times\cancel{\left(\frac{2m}{\hbar^2}\right)^{3/2}} E^{5/2}}.$$

$$\therefore \text{Fraction} \Rightarrow \left(\frac{5\Delta}{E_F}\right)$$

[For Al, $\Delta \sim 0.18$ meV, $\epsilon_F \sim 11.7$ eV.]

$$\therefore f\ (\text{fraction}) = \frac{5\times .18\times 10^{-3}}{11.7}$$

$$= 0.0776\times 10^{-3} = \boxed{7.76\times 10^{-5}}.$$

$$\therefore n = (n_{Al}\times f)$$

$$n(E_F\pm\delta) = 1.806\times 10^{29}\times 7.76\times 10^{-5} = 14.012\times 10^{24}$$

$$= \boxed{1.401\times 10^{25}}.$$

$$\therefore \lambda(E_F\pm\Delta) = \sqrt{\frac{m_e}{n(E_F\pm\Delta)\, e^2\mu_0}} = \sqrt{\frac{m_e}{n.e^2\mu_0.(f)}} = \left(\frac{\lambda_L}{\sqrt{f}}\right).$$

$$= \frac{12.54\times 10^{-9}}{\sqrt{.776\times 10^{-4}}} = 14.235\times 10^{-9+2=-7}$$

$$= \boxed{1.423\ \mu m.}$$

This is 3 orders of magnitude greater. This is because $\lambda \propto \frac{1}{\sqrt{n}}$

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2) Pb is a SC with $T_c = 7.2K$ & $B_{c,0} = 800 G$.

$$B_c = B_{c,0}\left[1 - \left(\frac{T}{T_c}\right)^2\right].$$ At $T = 0.1 K$
$B_c \approx 800 G$.

Magnetic Energy density formula $(\rho) = \boxed{-\frac{1}{2}\chi \frac{B^2}{\mu_0}}$

(*) At, 7.5 both Cu & Pb are metal.
(*) At, $T < 7.2K$ (or at 0.1K), Cu is metal, Pb is SC.
(*) Hence, Cu & magnetic energy density will remain UNCHAINED @ 0.1 K.
(*) But, for Pb @ 0.1 K, $\chi = -1$ [Meissner state
because @ meissner state susceptibility $= -1$
as material becomes perfect diamagnet.

At. 7.5 K,

$$\rho(Cu) \longrightarrow \frac{+\frac{1}{2} \times 9.63 \times 10^{-6} \times (500)^2 \times (10^{-8})}{4\pi \times 10^{-7}} = \boxed{+9.57 \times 10^{-3}\ Jm^{-3}}$$

$$\rho(Pb) \longrightarrow \frac{+\frac{1}{2} \times 1.58 \times 10^{-5} \times (500)^2 \times 10^{-8}}{4\pi \times 10^{-7}} = \boxed{+1.571 \times 10^{-2}\ Jm^{-3}}.$$

At 0.1K.

$\rho(Cu) = +9.57 \times 10^{-3}\ Jm^{-3}$ [Unchanged].

$$\rho(Pb) = \frac{-\frac{1}{2} \times -1 \times (500)^2 \times 10^{-8}}{4\pi \times 10^{-7}} = \boxed{9.947 \times 10^{2}\ Jm^{-3}}.$$

AC Josephson effect.

$$\boxed{J = J_0 \sin(\delta(t)) = J_0 \sin\left(\delta(0) - \left(\frac{2eV}{\hbar}\right)t\right)}.$$

frequency

(3) BCS gap (Δ) of Pb @ 4K = 2meV.

$\therefore$ Gap voltage = $\boxed{2\text{ mV}}$. $\boxed{\text{Half} = 1\text{mV}}$.

$$\left(\omega/f = \frac{2eV}{\hbar}\right).$$

(ii) $\cancel{\quad} = \cancel{\quad}$ $2eV$

$$\Rightarrow = \frac{2 \times 1.6 \times 10^{-19} \times 1 \times 10^{-3}}{1.05 \times 10^{-34}}$$

$$= 3.047 \times 10^{34-22} = \boxed{3.047 \times 10^{12}}.$$

$\therefore$ ω/f = 3047 GHz.

Gap at 4.2K $\Rightarrow$ $\Delta(T) = \Delta(0)\left[1 - \left(\frac{T}{T_c}\right)^4\right]^{1/2}$.

$(T = 4.2K,\ T_c = 7.2K,\ \Delta(0) = 2\text{meV})$

$\therefore$ $\boxed{\Delta(T = 4.2K) = 1.88\text{meV}.}$

(4.) $B = B_0\,\hat{z}$.

Temporal variation $\Rightarrow$ $\delta(t) = \delta_0 + \frac{2eV}{\hbar}t$.

Spatial variation $\Rightarrow$ $\delta(y) = \delta(0) + \frac{\pi\,\Phi}{\Phi_0}$, let

$\Phi = bd\,B_0$.

$\Phi_0 = h/2e$

$\therefore$ $\delta(y)\big|_b = \delta(0) + (b)\left[\frac{d B_0 \pi 2e}{h}\right]$

$\therefore$ $\delta(y)\big|_b = \delta(0) + (b)\cdot[\beta]$.

Let, $\beta = \frac{\pi d B_0 2e}{h}$.

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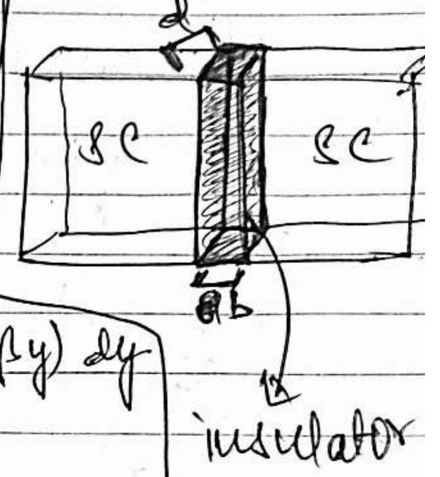
$$\delta(y) = \delta(0) + \frac{\pi d B_0}{h/2e}\, y = \delta(0) + \beta y. \qquad \delta(0) = \pi/2.$$



$$\therefore\ I = \int_{-b/2}^{b/2} J(y)\, dy = \int_{-b/2}^{b/2} j_c \sin[\phi(y)]\, dy$$

$$= j_c \int_{-b/2}^{b/2} \sin\left(\frac{\pi}{2} + \beta y\right) dy = j_c \int_{+b/2}^{-b/2} \cos(\beta y)\, dy$$

$$= \frac{j_c}{\beta}\left[-\sin\left(\beta \frac{b}{2}\right) - \sin\left(-\frac{\beta b}{2}\right)\right]$$

$$= 2\frac{j_c}{\beta}\sin\left(\frac{\beta b}{2}\right) = 2(j_c b)\,\frac{\sin\left(\frac{\beta b}{2}\right)}{\left(\frac{\beta b}{2}\right)}$$

$$I = I_0 \frac{\sin\left(\frac{\cancel{2}\pi d B_0 e b}{\cancel{2} h}\right)}{\frac{\cancel{2}\pi d B_0 e b}{\cancel{2} h}} \Rightarrow \boxed{I = I_0 \frac{\sin\left(\frac{\pi d B_0 e b}{h}\right)}{\left(\frac{\pi d B_0 e b}{h}\right)}\ \text{(ans)}}$$

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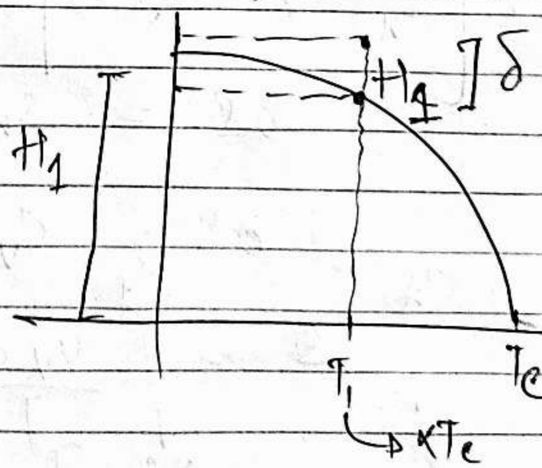
(5) Initial $T_1 = \alpha T_c$.



Phase transition process is Adiabatic & happens in a isolated isotropic reservoir. Hence, Δ in entropy = 0.

$$F_n(T) - F_s(T) = \frac{B_1^2(T)}{2\mu_0}.$$

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$\therefore S_n(T) - S_s(T) = -\frac{d}{dT}\left[F_n(T) - F_s(T)\right]$

Formula for $B_c(T) = B_{c,0}\left[1 - \left(\frac{T}{T_c}\right)^2\right]$.

$\therefore B_c^2(T) = B_{c,0}^2\left[1 - \left(\frac{T}{T_c}\right)^2\right]^2$

$\boxed{\begin{array}{l} T = \alpha T_c. \\ \therefore \frac{T}{T_c} = \alpha. \end{array}}$

$\frac{d}{dT}\left(B_c^2\right) = B_{c,0}^2 \, 2\left[1 - \left(\frac{T}{T_c}\right)^2\right] - 2\frac{T}{T_c^2}$.

$-\frac{d}{dT}B_c^2 = \frac{4B_0^2\,\alpha\left(1-\alpha^2\right)}{T_c}$.

$\therefore -\frac{d}{dT}\left(\frac{B_c^2(T)}{2\mu_0}\right) = \Delta S = \boxed{S_n(T) - S_s(T) = \frac{2B_0^2\,\alpha\left(1-\alpha^2\right)}{\mu_0 T_c}}$

Now some temperature change is observed. This must be equal to the change in entropy since TOTAL entropy change $= 0$.

$\therefore \left[S_n(T) - S_s(T)\right] + \delta S = 0.$

$\therefore \delta S = S_n(T) - S_s(T).$

$\therefore T\delta S = C_v\, dT.$

$\int_{T_i}^{T_f} \delta S = \int_{T_i}^{T_f} \frac{C_v\, dT}{T}$ $\quad \boxed{C_v = \gamma T}.$

$= \int_{T_i}^{T_f} \frac{\gamma \cancel{T}}{\cancel{T}}\, dT = \boxed{\gamma\, \Delta T} \rightarrow \Delta T = (T_f - T_i)$

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$\therefore \gamma \Delta T = \delta S = S_S(T) - S_N(T).$

$$\gamma \Delta T = \frac{-2B_0^2 \alpha(1-\alpha^2)}{\mu_0 T_c}.$$

$\therefore$ Temperature drop $= T_f - T_i$ (which is negative since $T_f < T_i$).

$$\therefore T_i - T_f = \boxed{\frac{2B_0^2 \alpha(1-\alpha^2)}{\mu_0 \gamma T_c}} \text{ (ans)}$$

(b) Usually, gap energy value is around $\sim \boxed{10^{-4} eV}$

$\therefore$ Photon energy $= h\nu$

$\therefore h\nu = 10^{-4}$

$$\nu = \frac{10^{-4} \times 1.6 \times 10^{-19}}{6.626 \times 10^{-34}} = 0.2414 \times 10^{11}.$$

$$\approx \boxed{2.4 \times 10^{10} \text{ Hz.}}$$

Microwave frequency $\longrightarrow$ $\boxed{24\,0 \text{ GHz.}}$

(Range = 300 MHz – 300 GHz).

Hence radiation/photon energy required to break cooper pair lies in microwave range.

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(i) $\nabla^2 B(x) = \frac{B(x)}{\lambda^2}$.

solving $\longrightarrow B(x) = B_1 e^{-x/\lambda} + B_2 e^{x/\lambda}$.

$B(-d/2) = B_1 e^{d/2\lambda} + B_2 e^{-d/2\lambda} = B$.

$B(d/2) = B_1 e^{-d/2\lambda} + B_2 e^{d/2\lambda} = B$

$\cancel{B_1} + B_2 e^{-d/\lambda} = B e^{-d/2\lambda}$.

$-\quad \cancel{B_1} + B_2 e^{d/\lambda} = B e^{+d/2\lambda}$.

$B_2 (e^{-d/\lambda} - e^{d/\lambda}) = B (e^{-d/2\lambda} - e^{d/2\lambda})$.

$B_2 = B \left(\frac{e^{d/2\lambda} - e^{-d/2\lambda}}{e^{d/\lambda} - e^{-d/\lambda}} \right)$

$B_1 = B \left[\frac{e^{d/2\lambda} - e^{-d/2\lambda}}{e^{d/\lambda} - e^{-d/2\lambda}} \right]$

$B_1 = B_2 = \tilde{B}$

$\left(\frac{a - b}{a^2 - b^2} \right) = \frac{1}{a+b}$,

$\therefore B = \tilde{B} \left[e^{x/\lambda} + e^{-x/\lambda} \right]$.

$= B \frac{(e^{x/\lambda} + e^{-x/\lambda})}{e^{d/2\lambda} + e^{-d/2\lambda}}$

$$\boxed{B(x) = B \frac{\cosh(x/\lambda)}{\cosh(d/2\lambda)}}.$$

Approx:

$\cosh(z) = 1 + z^2/2$.

$B = \mu_0 (H + M)$

$B = B_0 + \mu_0 M$

$\mu_0 M(x) = B(x) - B_0$.

$\therefore M(x) = \frac{1}{\mu_0} [B(x) - B_0]$.

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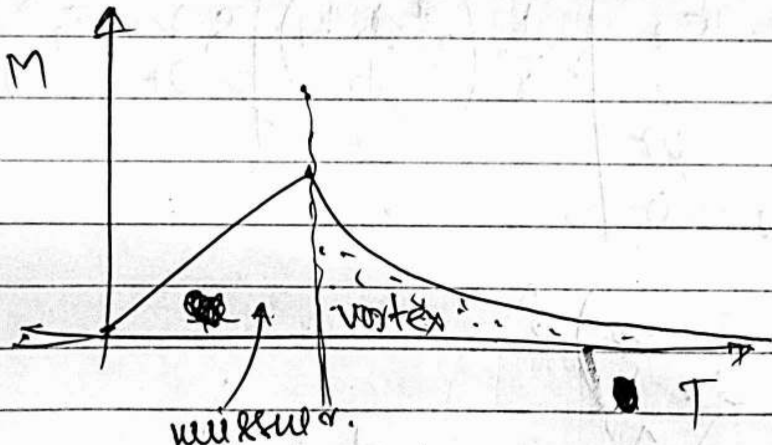
$$\therefore B(x) = B\frac{\left(1+\frac{x^2}{2\lambda^2}\right)}{\left(1+\frac{d^2}{8\lambda^2}\right)} = B_0\left(1+x^2/2\lambda^2\right)\left(1 - d^2/8\lambda^2\right)$$

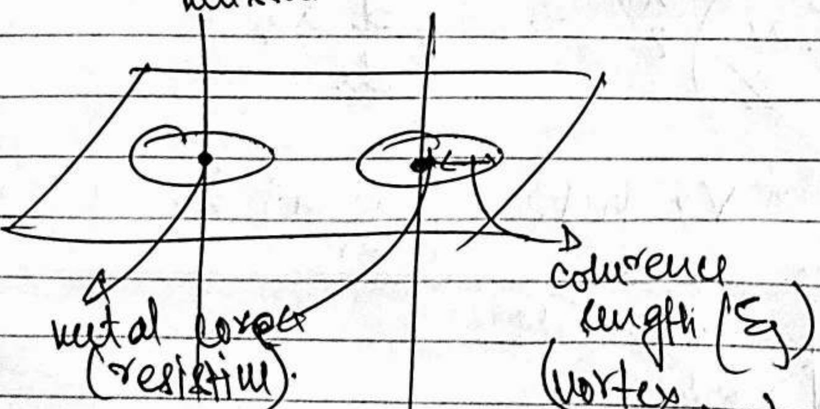
$$= B_0\left(1+x^2/2\lambda^2 - d^2/8\lambda^2\right).$$

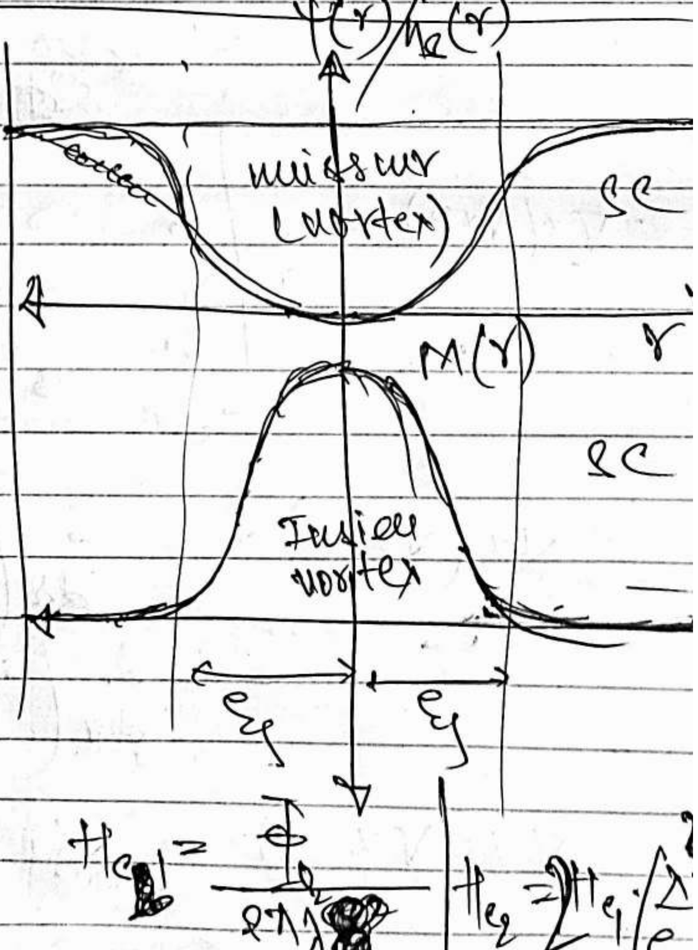
$$M(x) = \frac{1}{\mu_0}\left(B(x)-B_0\right) = \frac{B_0}{\mu_0}\left(x^2/2\lambda^2 - d^2/8\lambda^2\right).$$

$$\boxed{M(x) = \frac{B_0}{8\mu_0\lambda^2}\left(4x^2-d^2\right).}$$

(8) Type 2 SC







$$H_{c_1} = \frac{\Phi_0}{2\pi\lambda^2}$$

$$H_{c_2} = \frac{\Phi_0}{\pi\xi^2}$$

$$H_{c_2} = 2H_{c_1}\left(\frac{\lambda}{\xi}\right)^2$$

$$\frac{H_{c_2}}{H_{c_1}} = 2\left(\frac{\lambda}{\xi}\right)^2.$$

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Assignment 2. [Questions 1–3 already done].

(4.) A vortex has core radius ξ_0 [cylindrical geometry]

$\frac{d}{dr} B_z = a/r.$ Field $\longrightarrow B_z(r)$.

$B_z(r)$ depends only on r

$$\nabla \times [\nabla \times B] \quad \nabla \times B = \frac{1}{r}\begin{vmatrix} e_r & r e_\phi & e_z \\ \partial/\partial r & \partial/\partial\phi & \partial/\partial z \\ B_r = 0 & r B_\theta = 0 & B_z \end{vmatrix} = \cancel{\frac{1}{r}\left(B_z(r)\right)\left(e_r \frac{\partial}{\partial\phi} - r e_\phi \frac{\partial}{\partial r}\right)}$$

$$= \left(\frac{d}{dr} B_z\right)\hat{\phi}.$$

$B_\phi(r)$

$$\nabla \times (\nabla \times B) = \frac{1}{r}\begin{vmatrix} e_r & r e_\phi & e_z \\ \partial/\partial r & \partial/\partial\phi & \partial/\partial z \\ 0 & r\frac{d}{dr}B_z & 0 \end{vmatrix} = \cancel{\frac{1}{r}\left(r\frac{d}{dr}B_z\right)\left[e_r\frac{\partial}{\partial z} - e_z\frac{\partial}{\partial r}\right]}$$

$$\nabla \times (\nabla \times B) = -\frac{d}{dr}\left(r\frac{d}{dr}B_z\right)\hat{z}.$$

$$= -\frac{d}{dr}\left(\cancel{r}\cdot\frac{a}{\cancel{r}}\right)\hat{z} = \left(\frac{d}{dr}a\,\hat{z}\right)$$

$$\nabla \times (\nabla \times B) = -B/\lambda^2 = \nabla \times \mu_0 J. = \left(\frac{d}{dr}a\right)\hat{z}. = 0 \quad \lambda$$

~~$\iint (\nabla \times \mu_0 J)\, ds = \iint \frac{d}{dr}a\, ds$~~

~~$\oint \mu_0 J \cdot dl =$~~

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$$(\nabla\times B) = \frac{d}{dr} B_z\,\hat{\phi}.$$

$$\mu_0 J = \frac{a}{r}\,\hat{\phi}.$$

$$\therefore \boxed{J_s(\text{supercurrent}) = \frac{-a}{\mu_0 r}\,\hat{\phi}}.$$

In London gauge, $\nabla\cdot A = 0$, the supercurrent is given by:

$$J_s = +\frac{1}{\mu_0\lambda^2} A = +\frac{a}{\mu_0 r}\,\hat{\phi}.$$

$$\therefore \vec{A} = \frac{\lambda^2 a}{r}\,\hat{\phi}.$$

Flux $(\Phi) \Rightarrow \oint_S B\cdot ds$

$$= \int(\nabla\times A)\cdot ds.$$

$$\cancel{\text{Now } B_z(r) = \frac{1}{r}\frac{d}{dr}(rA_\phi) = \frac{1}{r}\frac{d}{dr}(\lambda^2 a) = 0.}$$

$$\Phi = \oint A\cdot dl = \oint \frac{\lambda^2 a}{\cancel{r}}\cancel{r}\,d\theta = \lambda^2 a\left[\oint d\theta\right] \rightarrow 2\pi$$

$$= \lambda^2 a \cdot 2\pi.$$

$$\therefore \Phi = \lambda^2 a\, 2\pi.$$

$$\therefore \boxed{a(\text{constant}) = \frac{\Phi}{2\pi\lambda^2}}$$

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(5) Flux $(\Phi) = \int B \cdot ds$ $\quad B = B_0 e^{-r/\lambda_L}$ (given).

flux per vortex.

$$= B_0 \int e^{-r/\lambda_L} r \, d\theta \, dr.$$

$$= B_0 \left(\int_0^{2\pi} d\theta\right)\left(\int_0^{r} e^{-r/\lambda_L} \frac{r}{\lambda_L}\frac{dr}{\lambda_L}\right) \lambda_L^2.$$

$$\Phi = B_0 (2\pi) \frac{\Gamma(2)}{\cancel{\ }} \lambda_L^2 = B_0 \frac{2\pi}{\cancel{\ }} \lambda_L^2.$$

$$B_c = \frac{\Phi_0}{2\pi \lambda_L^2} = \frac{h}{2e} \cdot \frac{1}{2\pi \lambda_L^2} = \frac{h}{2e\lambda_L^2} \text{ (ans)}$$

[proved]

Questions 6 – 8 are repeated.