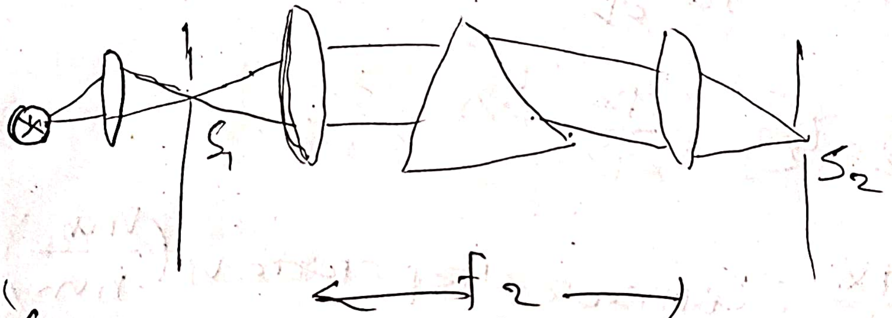
Basic unit of spectroscopy.

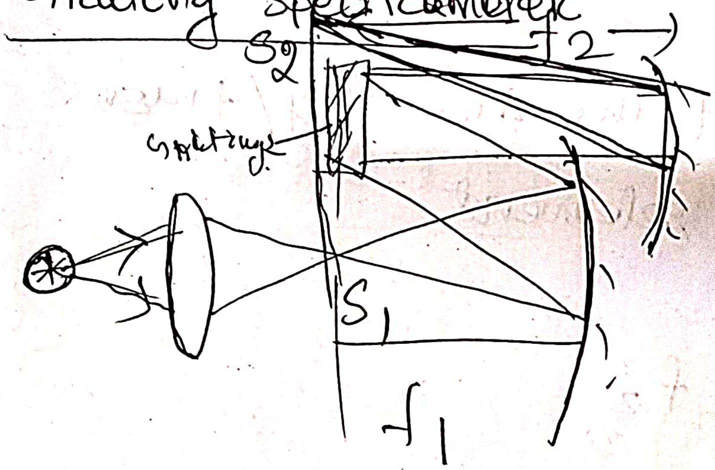
1. Source.
2. Dispersion unit.
3. Detector.

Dispersion unit

Spectrometer → which gives image of a slit S_1 on $S_2(\lambda)$

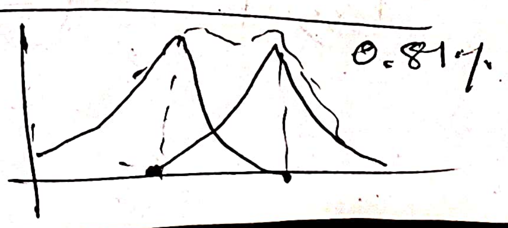


Grating Spectrometer



Resolution of Measurements.

Rayleigh's criterion :-



Angular speed.

$$\Delta\theta = \frac{d\theta}{d\lambda}\Delta\lambda$$

$\frac{d\theta}{d\lambda}$:- Angular dispersion; $\frac{rad}{nm}$

$2d\sin\theta = n\lambda$

Linear dispersion

$$\Delta x = f_2 \frac{d\theta}{d\lambda}\Delta\lambda$$

$$= \frac{dx}{d\lambda}\Delta\lambda$$

$\frac{dx}{d\lambda}$: Linear dispersion $\left(\frac{mm}{nm}\right)$

a : size of the grating/prism element.

$$\Delta x_2 = \frac{\lambda}{a} f_2$$

$$\frac{d\theta}{d\lambda}\Delta\lambda \geq \frac{\lambda}{a} . f_2$$

Resolving power : $\left|\frac{\lambda}{\Delta\lambda}\right| \leq a\left(\frac{d\theta}{d\lambda}\right)$

Magnification of entrance slit S_1:

$$\frac{\delta x_1}{f_1} = \frac{\delta x_2}{f_2}$$

$$\Rightarrow \delta x_2 = \frac{f_2}{f_1}\delta x_1$$

entrance slit size: b

$$\Delta x = \frac{\lambda}{a} f_2 + \frac{f_2}{f_1} b$$

$$f_2 \frac{d\theta}{d\lambda} \delta\lambda = \frac{\lambda}{a} f_2 + \frac{f_2}{f_1} b$$

$$\delta\lambda = \left(\frac{\lambda}{a} + \frac{b}{f_1}\right)\left(\frac{d\theta}{d\lambda}\right)^{-1}$$

Limit of b:

$$\frac{2\lambda}{b} f_1 = a$$

$$\boxed{b_{min} = \frac{2\lambda f_1}{a}}$$

$$\delta\lambda = \left(\frac{\lambda}{a} + \frac{2\lambda f_1}{a f_1}\right)\left(\frac{d\theta}{d\lambda}\right)^{-1}$$

$$= \frac{3\lambda}{a}\left(\frac{d\theta}{d\lambda}\right)^{-1}$$

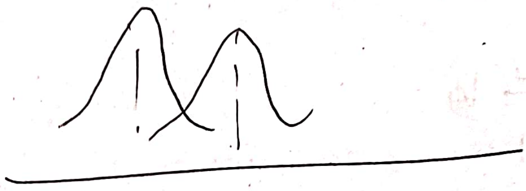
$$\Rightarrow \frac{\lambda}{\delta\lambda} = \frac{a}{3\lambda}\left(\frac{d\theta}{d\lambda}\right)$$

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2}$$

$$\alpha = \frac{\pi b \theta}{\lambda}$$

$$\boxed{\theta = \frac{2\lambda}{b}}$$

Date – 22.01.025



500 nm   501 nm e ...

$$\theta = \frac{2\lambda}{b}$$

$$= \frac{2 \times 500\, nm}{10^{-6}\, m}$$

$$= 1000 \times 10^{-3}\ Rad$$

$$\Delta\lambda \le \left(\frac{\lambda}{a} + \frac{b}{f}\right)\left(\frac{d\theta}{d\lambda}\right)^{-1}$$

$a = 10\, cm$

$\lambda = 500 \times 10^{-7}\, cm$

$f = 100\, cm$

$$\frac{d\lambda}{dx} = \frac{1\ nm}{mm} = 10^{-9} \times 10^{3} = 10^{-6}$$

$b = 10\, \mu m = 10 \times 10^{-4}\, cm$

$\Delta x\ \theta =$

$$\frac{dx}{dx} = f_o \frac{d\theta}{dx} \quad \left| \frac{dx}{d\theta} = f_e \frac{dx}{d\alpha} \right.$$

$$\Delta x = \left(\frac{500 \times 10^{-7} \times 10^{-2}\, m}{10 \times 10^{-2}\, m} + \frac{10^{-6}\, m}{100 \times 10^{-2}\, m} \right)$$

$$\times 100 \times 10^{-2}\, m$$

$$\times 10^{-6}$$

$$= \left(\frac{500 \times 10^{-9}\, m}{10^{-1}\, m} + 10^{-6} \right) 10^{-6}\, m$$

$$= \left(5 \times 10^{-6} + 10^{-6} \right) \times 10^{-6}\, m$$

$$= 6 \times 10^{-6} \times 10^{-6}\, m$$

$$= 6 \times 10^{-12}\, m$$

$$= 6 \times 10^{-3} \times 10^{-9}$$

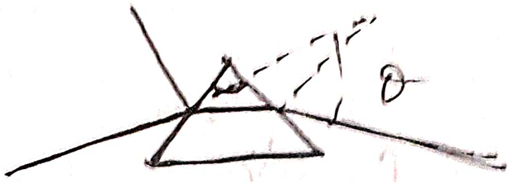
$$= \frac{6}{1000} \times 10^{-9}\, m$$

$= \cancel{0.006} \cancel{\times 10^{-}}$

$= 0.006\, nm$. $(0.015\, nm)$

$= 0.01\, nm$

Prism spectrometer



For minimum deviation:

$$n\sin\frac{\varepsilon}{2} = \sin\frac{\theta+\varepsilon}{2}$$

$$\Rightarrow dn\,\sin\frac{\varepsilon}{2} = \frac{1}{2}\cos\left(\frac{\theta+\varepsilon}{2}\right)d\theta$$

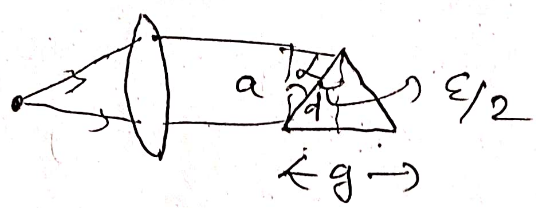
$$\Rightarrow \frac{d\theta}{dn} = \frac{2\sin\varepsilon/2}{\cos\left(\frac{\theta+\varepsilon}{2}\right)}$$

$$= \frac{2\sin\frac{\varepsilon}{2}}{\sqrt{\left(1-n^2\sin^2\frac{\varepsilon}{2}\right)}}$$

$$\frac{d\theta}{d\lambda} = \frac{d\theta}{dn}\cdot\frac{dn}{d\lambda}$$

$$= \frac{2\sin\varepsilon/2}{\sqrt{1-n^2\sin^2\varepsilon/2}}\,\frac{dn}{d\lambda}$$

$$\Delta\lambda \le \frac{a}{3}\left(\frac{d\theta}{d\lambda}\right)$$



↳ consider

$$\frac{g}{2d} = \sin \varepsilon/2$$

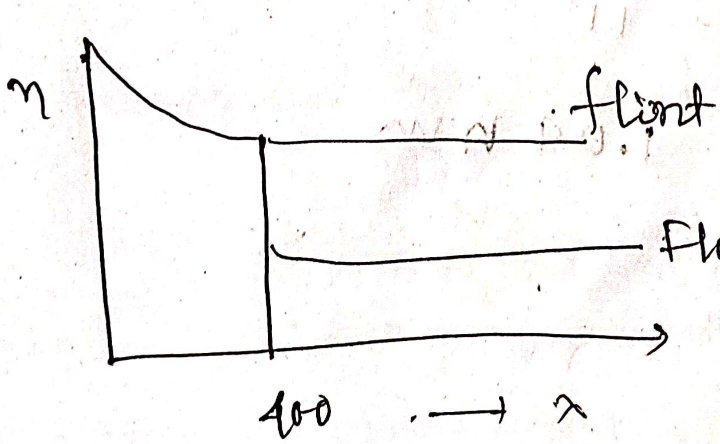
$$\Rightarrow \boxed{d = \frac{g}{2\sin\varepsilon/2}} \qquad a = \frac{g\cos\alpha}{2\sin\varepsilon/2}$$

angle of min. deviation $\quad n\sin\varepsilon/2 = \sin\frac{\delta+\varepsilon}{2}$
$= \sin\alpha$

$$a = \frac{g\sqrt{1-n^2\sin^2\varepsilon/2}}{2\sin\varepsilon/2}$$

$$\frac{\lambda}{\Delta\lambda} \le \frac{g\sqrt{1-n^2\sin^2\varepsilon/2}}{\cancel{2}\sin\varepsilon/2} \cdot \frac{2\sin\varepsilon/2}{\sqrt{1-n^2\sin^2\varepsilon/2}} \cdot \frac{dn}{d\lambda}$$

$$\boxed{\frac{\lambda}{\Delta\lambda} \le \frac{g}{3}\frac{dn}{d\lambda}}$$



Example

Fused quartz

$\eta = 1.47$ at $\lambda = 400\,nm$

$\frac{dn}{d\lambda} = 1100\ cm^{-1}$

$\frac{d\lambda}{dn} = \frac{1}{1100}\ cm$

$g = 1\,cm$

$\frac{\lambda}{\Delta\lambda} \leq \frac{g}{3}\frac{dn}{d\lambda} = \frac{10^{-2}\cancel{m}}{3} \times \frac{1100}{10^{-2}\cancel{m}}$

$= \frac{\cancel{1100}^{\cancel{3}6}}{\cancel{3}}$

$\frac{\lambda}{\Delta\lambda} = \frac{g}{3}\frac{dn}{d\lambda}$

$\Rightarrow \Delta\lambda = \frac{3\lambda}{g}\frac{d\lambda}{dn}$

$= \frac{3 \times 400 \times 10^{-9}\,m}{\cancel{10^{-2}}\,\cancel{m}} \times \frac{1}{1100} \times \cancel{10^{-2}}\,\cancel{m}$

$\Rightarrow \frac{12}{11} \times 10^{-9}\,m$

$= 1.09\,nm$

$$\frac{d\theta}{d\lambda} =$$

Grating spectrometer.

Path diff.

$$d(\sin\alpha \pm \sin\beta) = m\lambda$$

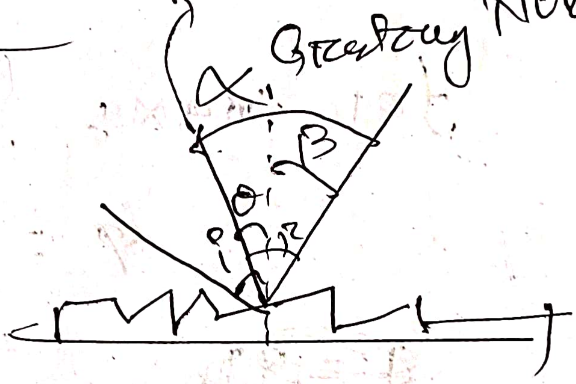
Maximum reflection

$i = r$

$$\alpha - \theta = \beta + \theta$$

$$\theta = \frac{\alpha - \beta}{2}$$

Blaze angle.



Intensity

$$I = I_0 \frac{\sin^2 \frac{N\phi}{2}}{\sin^2 \phi/2}$$

$$\phi = \frac{2\pi}{\lambda} d(\sin\alpha \pm \sin\beta)$$

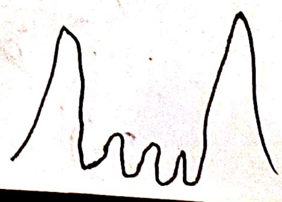
N → no. of groove

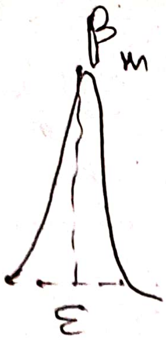
Maximum $\phi = 2m\pi$

minima $\frac{N\phi}{2} = l\pi$ $l = 1, 2 \ldots (N-1)$

If $N = 5$

then minima will $N-1$





$$\sin(\beta_m+\varepsilon) = \sin\beta_m\cos\varepsilon + \cos\beta_m\sin\varepsilon$$

for small ε

$$= \sin\beta_m + \cos\beta_m \cdot \varepsilon$$

$$\phi = \frac{2\pi}{\lambda} d\left[\sin\alpha + \sin\beta_m + \varepsilon\cos\beta_m\right]$$

for max:-

$$\phi = 2m\pi + \frac{2\pi}{\lambda} d\,\varepsilon\cos\beta_m$$
$$= 2m\pi + \delta$$

$$I = RI_0 \frac{\sin^2\frac{N\delta}{2}}{\sin\delta/2}$$

$$= RI_0 \frac{N^2\sin^2\frac{N\delta}{2}}{\left(\frac{N\delta}{2}\right)^2}$$

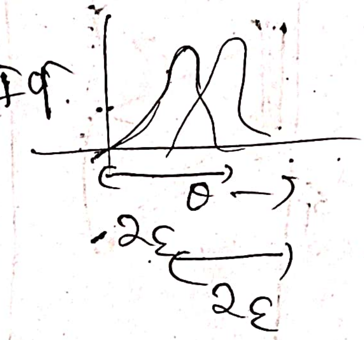
$$\Rightarrow \frac{N\delta}{2} = \pi$$

$$\Rightarrow \frac{N}{\cancel{2}}\cdot\frac{\cancel{2}\pi}{\lambda} d\,\varepsilon\cos\beta_m = \cancel{\pi}$$

$$\Rightarrow \varepsilon = \frac{\lambda}{Nd\cos\beta_m}$$

Resolution of Grating

$$d(\sin\alpha + \sin\beta) = m\lambda$$



Ang. dispersion.

$$\frac{d\beta}{d\lambda} = \frac{m}{d\cos\beta}$$

$$\frac{d\beta}{d\lambda}\,\Delta\lambda = \frac{\lambda}{Nd\cos\beta\, m}$$

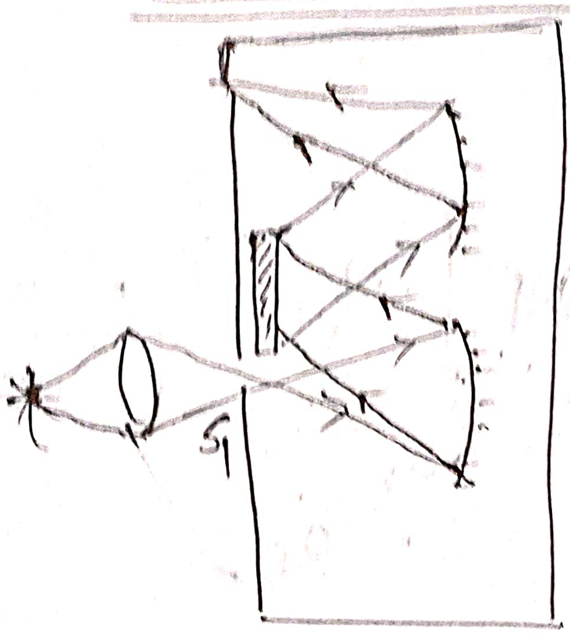
$$\cancel{\Rightarrow \Delta\lambda = \frac{\lambda}{Nd\cdot\cos\beta\, m}\left(\frac{d\beta}{d\lambda}\right)^{-1}}$$

$$\Rightarrow \frac{\lambda}{\Delta\lambda} = Nd\cos\beta\, m\left(\frac{d\beta}{d\lambda}\right)$$

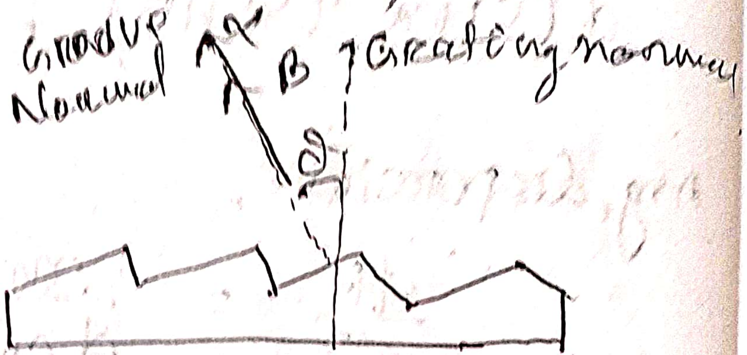
$$= N\cancel{d\cos\beta}\, m\,\frac{m}{\cancel{d\cos\beta}}$$

$$= mN$$

Date - 28.01.2023



$(\sin\alpha \pm \sin\beta) = m\lambda$

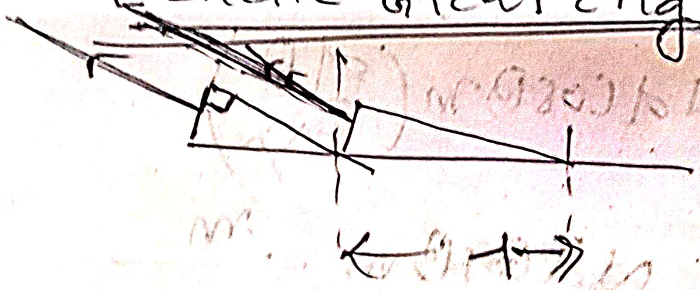


Littrow Grating

$2d\sin\alpha = m\lambda$

wavelength selective grating.

Echelle Grating



$2d\sin\alpha = m\lambda$

as $d \gg \lambda$

$\theta = 90 - \alpha$.

$2d\cos\theta = m\lambda$

grating size = $100 cm^2$

$N = 10^3$ groove/mm $\boxed{m = 2}$

Resolving Power of the grating.

$R = \frac{\lambda}{\Delta\lambda} = 2 \times 10^3$ groove/mm $\times 10^2 cm^2$

$= 2 \times 10^5$ groove $\times \frac{(10^{-2} m)^2}{10^{-3} m}$

$= 2 \times 10^5$ groov $= \cancel{2 \times 10^5}$

$\lambda = 500 nm$

$\Delta\lambda = 2.5 \times 10^{-3} nm$

$= 0.0025 nm$ ✗ $= 0.025 nm$

$\Delta\lambda = \left(\frac{\lambda}{2} + \frac{b}{f}\right)\left(\frac{d\theta}{d\lambda}\right)^{-1}$

$b = 50 \mu m$

$\alpha = \beta = 30^\circ$

$f = 1 m$

$\cancel{\Delta\lambda}$

$d(\sin\alpha + \sin\beta) = m\lambda$

$2d \sin\beta = m\lambda$

$\Rightarrow \sin\beta = \frac{m\lambda}{2d}$

$\Rightarrow \cos\beta \frac{d\beta}{d\lambda} = \frac{m}{2d}$

$$\frac{d\beta}{d\lambda} = \frac{m}{d\cos\beta}$$

$$\Delta\lambda = \frac{2d\sin\theta}{\lambda d\cos\beta}$$

$\frac{1}{5}/\frac{\sqrt{3}}{2}$

$$\Rightarrow \frac{2\tan\beta}{\lambda}$$

$$= \frac{2\times}{500\times\sqrt{3}\ nm} = \frac{2}{5\times100\sqrt{3}}$$

$$= \frac{0.4}{100\sqrt{3}}\ nm^{-1}$$

$5\overline{)2.0}$ 0.4

$$= \frac{0.4}{100\times1.732}\ nm^{-1}$$

$$= 0.006928\ nm^{-1}$$

$$= \frac{6928}{1000000} = 0.006928$$

$$= 6928\times10^{-6}$$

$$= \cancel{6.928\times10^{-3}}$$

$$= 0.028\ nm^{-1}$$

$$= 0.006928\ nm^{-1}$$

$\Delta\lambda =$

$$\frac{b}{f} = \frac{50\,\mu m}{1\,m} = 50 \times 10^{-6}$$

$$\frac{\lambda}{a} = 5 \times 10^{-6}$$

$\Delta\lambda =$

$$\Delta\lambda = \left(5\times10^{-6} + 50\times10^{-6}\right) 0.0069$$

$$= 55\times10^{-6} \times 0.006928 \text{ nm}$$

$$= 0.38104 \times 10^{-6}$$

$=$

$$\frac{d\beta}{dx} \times 100\,mm = 0.023\,nm^{-1} \times 100\,mm$$

$=$

$$\frac{dx}{d\lambda} = \frac{0.023 \times 1000\,mm}{nm}$$

$$= 0.023 \times 1000 \times 10^{-3} \times 10^{9}$$

$$= \frac{23}{1000} \times 1000 \times 10^{-3} \times 10^{9}$$

$$= 23 \times 10^{6}$$

$$\frac{dy}{dx} =$$

$$100 \frac{10^{-7} \times 10^{-2}}{}$$

$= 100$ nm

Detector

→ Thermal detector

→ Photodetector.

Thermal detector

→ λ insensitive.

→ works on change in material properties with the temperature.

Photo detector

→ λ sensitive

→ work on response of material for given photon energy.

characteristics of the detector

Date- 29.01.025

Characteristics of detector.

1. spectral response. $[R(\lambda)]$

(i) (spectral range)

(ii) Two relative intensity.

$$\frac{I(\lambda_1)}{I(\lambda_2)} = \frac{R(\lambda_1)}{R(\lambda_2)}$$

2. sensitivity of the detector.

$\left(\frac{\text{Volt}}{\text{watt}}\right)$ or $\left(\frac{\text{Ampere}}{\text{watt}}\right)$

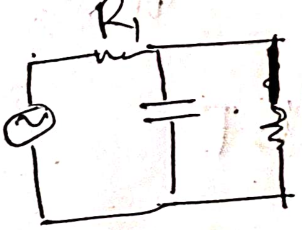
or $\frac{\text{Volt}}{A\,I}$ → irradiance
↳ Area

3. Signal to Noise Ratio $\left(\frac{S}{N}\right)$

Noise equivalent of input power.

→ Power level for which $\frac{S}{N} = 1$

4. Time Response.



$R_2C \ll \tau$

τ: Pulse time

$R_2C \ll \frac{1}{f}$

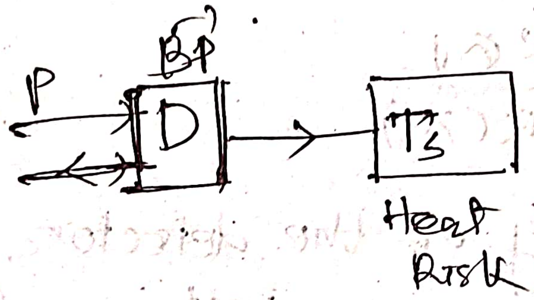
$\Rightarrow \boxed{f \ll \frac{1}{R_2C}}$

⑤ Cost of the detector.

Thermal detection

→ Power Measurement of C.W laser.

→ Output energy of the Pulse laser.



H: heat capacity of the detector

G: Thermal conduct to the heat Rsk

$$\beta P = H\frac{dT}{dt} + G(T - T_s)$$

$$\Rightarrow \beta P = H\frac{dT}{dt} + GT - GT_s$$

$$\Rightarrow \frac{\beta P + GT_s}{H} = \frac{dT}{dt} + \frac{GT}{H}$$

$$\Rightarrow a = \frac{dT}{dt} + bT$$

$$\Rightarrow \frac{dT}{dt} = a - bT$$

$$\Rightarrow \frac{dT}{a - bT} = dt$$

$\Rightarrow$

$\Rightarrow$ ..

$a - bT = x$

$-b\,dT = dx$

$dT = \frac{dx}{-b}$

$\Rightarrow \frac{dx}{-b\,x} = dt$

$\Rightarrow -\frac{1}{b} \ln x = t + c$

$\Rightarrow -\frac{1}{b} \ln (a - bT) = t + c$

$\Rightarrow \ln (a - bT)^{-1/b} = t + c$

$\Rightarrow \ln \frac{1}{(a - bT)^{1/b}} = t + c$

$\Rightarrow \frac{1}{(a - bT)^{1/b}} = e^{t + c}$

$\Rightarrow (a - bT)^{1/b} = e^{-(t + c)}$

$\Rightarrow (a - bT) = e^{-(t + c)b}$

$\Rightarrow a - e^{-(t + c)b} = bT$

$\Rightarrow T = \frac{1}{b}\left[a - e^{-(t + c)b}\right]$

$T_S = \frac{H}{G}\left[a - e^{-\frac{cG}{H}}\right]$

let $t = 0, T = T_s$

$$T_S = \frac{1}{b}[$$

$$\frac{T - T_s}{1 - e^{Ht/G}} = \Delta T$$

$$T_s = \frac{H}{G}\left[\frac{BP + GT_s}{H} - e^{-\frac{Hc}{G}}\right]$$

$$= \left[\frac{BP}{G} + T_s - \frac{H}{G} e^{-He/G}\right]$$

$$= \frac{BP}{G}\left(1 + \frac{G}{BP} - \frac{H}{BP} e^{-HC/G}\right.$$

$$T = T_s + \frac{\beta P}{G}\left(1 - e^{-H/G\,t}\right) \qquad \Delta T = \frac{\beta P}{G}$$

Response time of detection $\frac{G}{H}$

$$P = P_0\left(1 + \cos\omega t\right)$$

$$T = T_s + \Delta T\left[1 + a\cos(\Omega t + \phi)\right]$$

$$\Delta T = \frac{a\beta P}{\sqrt{1 + \frac{\Omega^2 H^2}{G^2}}} \qquad \tan\phi = \frac{-\Omega H}{G}$$

Pulse width

$$\int_0^t \beta P\,dt = H\int_0^t dT \qquad G = 0$$

$$\Delta T = \frac{1}{H}\int_0^t \beta P\,dt$$

Thermal Detector

Thermistor.

Bolometer

Golay cell

Thermocouple detector.

Date - 4.02.025

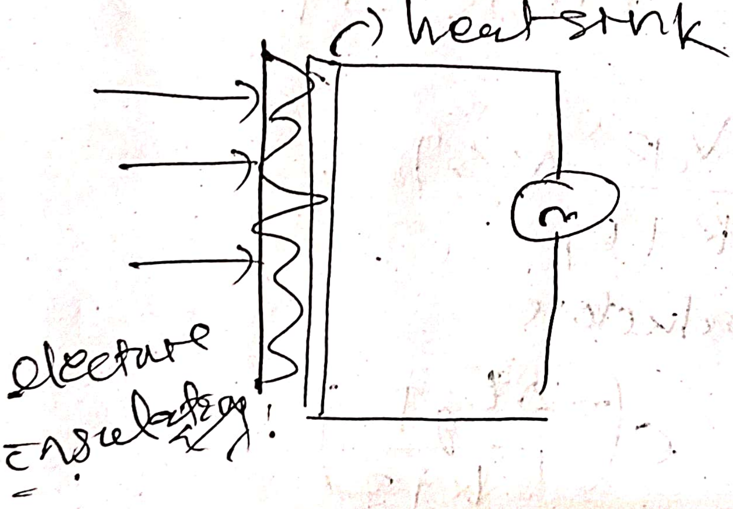
Thermal detector

$$\xrightarrow{P} \boxed{D} \longrightarrow \boxed{T_s}$$

Bolometer

Thermal imageing.

Infra red ~~astronogy~~ astronomy



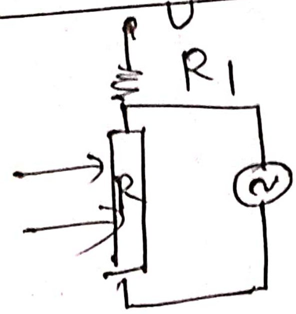
change in voltage

$$\Delta V = N \frac{dV}{dT} \Delta T$$

N: no. of thermocouple.

$\frac{dV}{dT}$: Sensitivity of each thermocouple

Thermistor



temp. co-eff. of resistance.

$$\alpha = \frac{\frac{dR}{dT}}{R}$$

$$\delta U = i\delta R = i\alpha \cdot R \,\Delta T$$

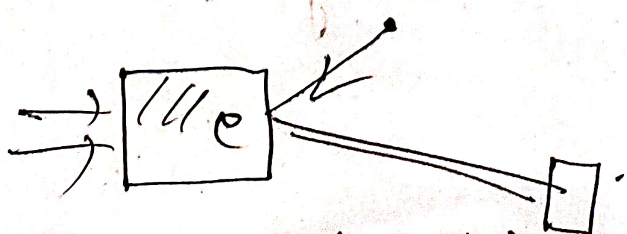
$$= \frac{V_0 R}{R+R_1} \alpha \Delta T$$

for semiconductor

$$\frac{n_e(T)}{n_e(T+\Delta T)} = e^{\left(-\frac{\Delta E_g T}{k T^2}\right)}$$

Golay cell

→ use for photoacoustic spectroscopy



e: cell with membrane

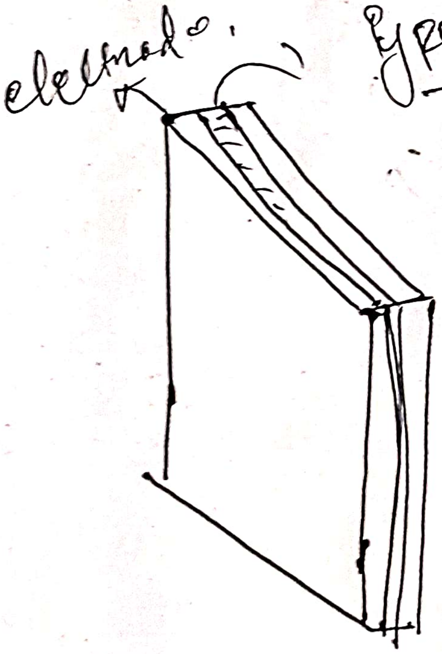
filled with gase.

$$PV = NRT$$

$$\delta V = \frac{nR\delta T}{P}$$

→ change in reflected position of light → change in capacitance

Pyroelectric detector.

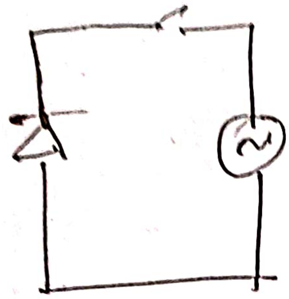


it has intrinsic dipole moment. which arrange with temp.

↳

to measure → change in surface charge.

Photodetector

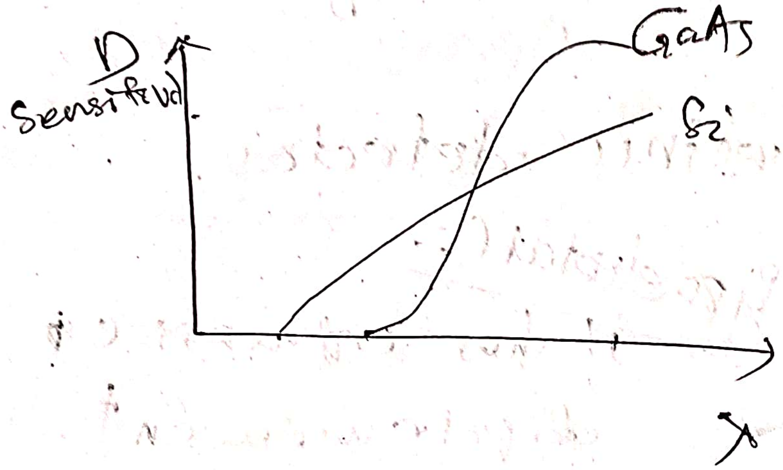


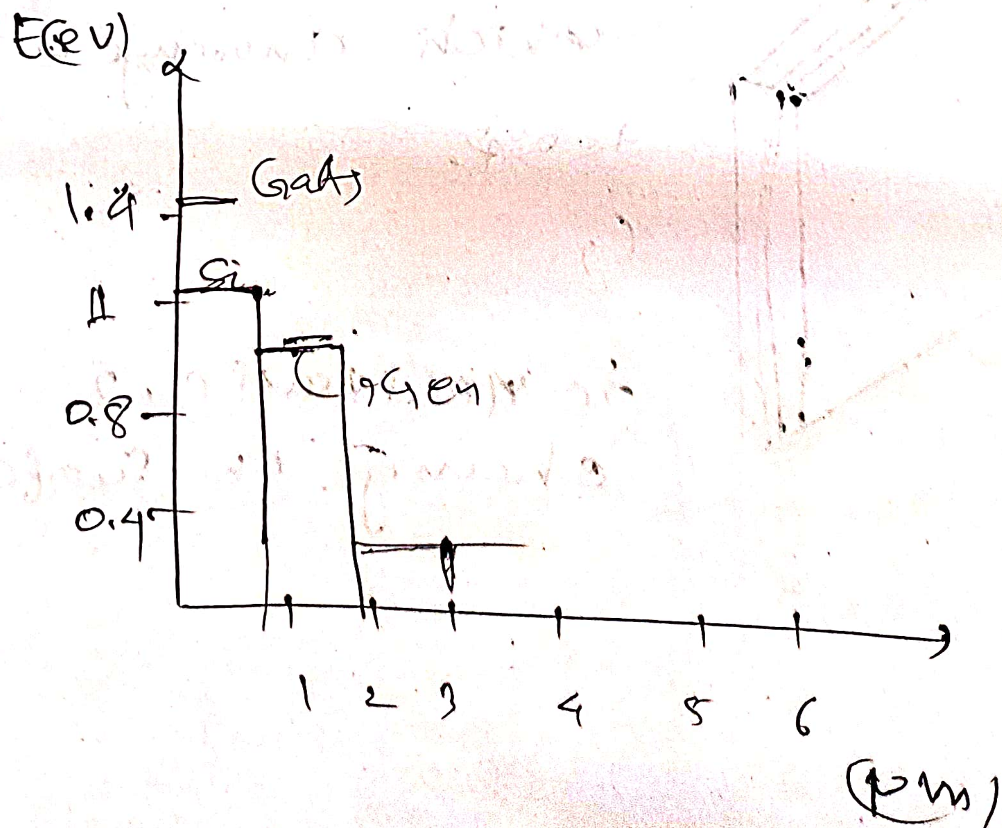
absorption coefficient

$\alpha \propto (h\nu - E_g)^{1/2}$

$\hookrightarrow h\nu > E_g$

$\alpha \approx 0 \qquad h\nu < E_g$





Ge : 0.6 eV $\approx$ 1.9 μm

GaAs : 1.42 eV $\approx$ 1.88 μm

PbS : 0.37 eV $\approx$ 3.356 μm.

Photodiode array.

Length of each diode : 15 μm (L)

spacing between diode : 10 μm (d)

1024 ~~x~~ diode array.

Length of detector (15+10) 1024

= 25 mm.

Linear dispersion for each diode.

$= \frac{d\lambda}{dx} = 5 \frac{nm}{mm}$

Spectral Resolution : $\frac{d\lambda}{dx} \times L$

$= 5 \times 15 \times 10^{-3}$

$= 0.125$ nm.

spectral range

$$\frac{d\gamma}{dx}(L+9) = \frac{5 \times 10^{-9} m \times 25 \times 10^{-6} m}{10^{-3} m}$$

$=$

Also called Multi-Chan

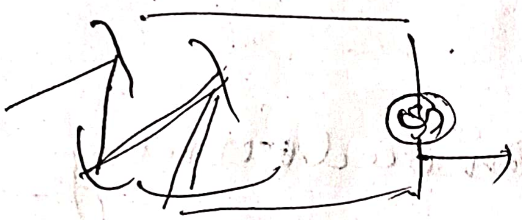
CCD

Array MOS diode / Pixels

Date - 05.02.025

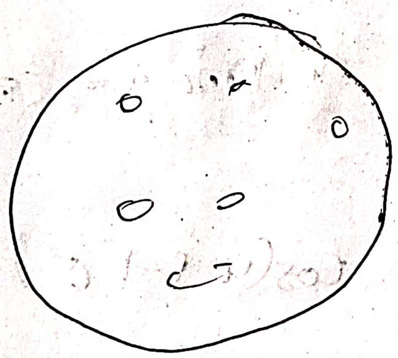
PMT

series of dynodes.



$$e = \frac{Q}{V} = \frac{q^N e}{V}$$
$$= \frac{G e}{V}$$

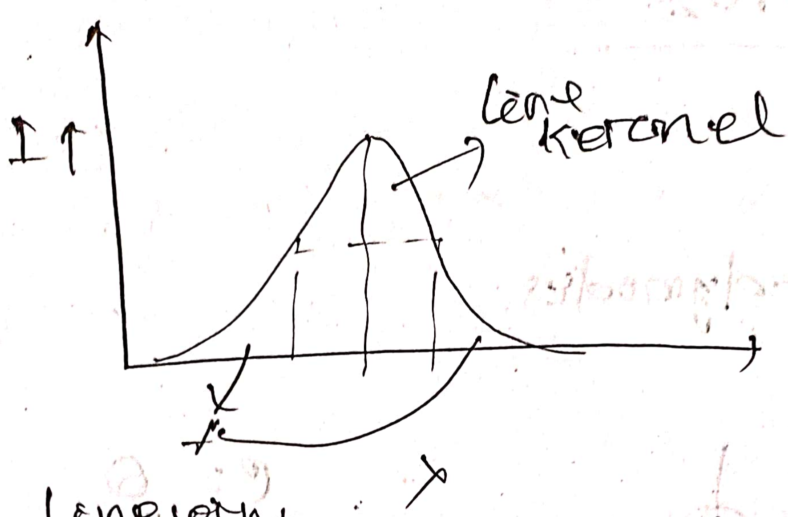
Microchannel plate



60% holes

Source of Noise

1. Thermal Noise → Richardson eqn — also called dark noise.
   $I = q T^2 e^{-(2\phi/T)}$
2. shot noise → discrete nature of ele charge.
3. Johnson noise → Thermal aggitation of charge.
4. stray light →



Lene wind
Natural line broadening.

oscillator model

damp oscillat. $\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$

$$x(t) = x_0 e^{-\gamma t/2} \cos(\omega t + \phi)$$

$$\omega = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2}$$

$$\gamma \ll \omega_0$$

$$\simeq x_0 e^{-\gamma t/2} \cos\omega_0 t$$

Superposition of oscillation of difference frequency.

$$x(t) = \frac{1}{2\sqrt{2\pi}} \int A(\omega)\, e^{i\omega t}\, d\omega$$

$$A(\omega) = \frac{1}{\sqrt{2\pi}} \int x(t)\, e^{-i\omega t}\, d\omega$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} x_0 \cos(\omega_0 t)\, e^{-i\omega t}\, d\omega$$

$$A(\omega) = \frac{x_0}{\sqrt{8\pi}} \left[\frac{1}{i(\omega-\omega_0) + \frac{\gamma}{2}} + \frac{1}{(\omega+\omega_0) + \frac{\gamma}{2}} \right.$$

$$I(\omega) = A(\omega)\, A^*(\omega)$$

$$I(\omega) = \frac{C\,(\omega-\omega_0)^2}{(\omega-\omega_0)^2 + \left(\frac{\gamma}{2}\right)^2} \quad \text{for } \omega = \omega_0$$

$$\rightarrow \frac{1}{2\pi} \frac{\gamma}{(\omega-\omega_0)^2 + \left(\frac{\gamma}{2}\right)^2}$$

$$\int I(\omega-\omega_0)\, d\omega = 1$$

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = 0$$

multiply by $m\dot{x}$

$$m\dot{x}\ddot{x} + \gamma m \dot{x}^2 + m\omega_0^2 \dot{x} x = 0$$

$m\dot{x}\ddot{x} + m\omega_0^2 x\dot{x} = -\gamma m\dot{x}^2$

$\Rightarrow \frac{d}{dt}\left[\frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega_0^2 x^2\right] = -\gamma m\dot{x}^2$

$\Rightarrow \frac{dw}{dt} = -\gamma m\dot{x}^2$

$x = x_0 e^{-\gamma t}\cos\omega_0 t$

$\cancel{\dot{x} = x_0\left(-\frac{\gamma}{2}\right)e^{-\gamma t/2}\cos\omega t}$

$\dot{x} = x_0\left[\cos\omega_0 t\,(-\gamma)e^{-\gamma t} + e^{-\gamma t}\omega_0\sin\omega_0 t\right.$

$\approx x_0 e^{-\gamma t/2}\omega_0\sin\omega_0 t$

$\gamma = \frac{1}{\tau}$

$\left\langle\frac{dw}{dt}\right\rangle = \langle\gamma m\dot{x}^2\rangle$

$= -\langle\gamma m x_0^2\omega_0^2 e^{-\gamma t}\rangle$