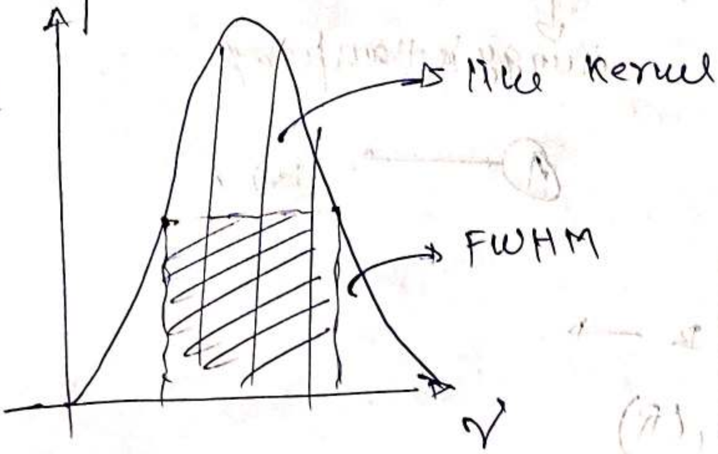
Spectral line shape [Natural broadening]. 17/3/25.



Natural line broadening.

$\Delta E \, \Delta t \approx \hbar$.

$\Delta \nu = \frac{1}{2\pi (\Delta t)}$ → lifetime.

Oscillator model.

$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = 0.$

$x = x_0 e^{-\gamma t/2} \cos(\omega_0 t).$

$[\omega_0 \gg \gamma]$.

$x(t) = \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{\infty} A(\omega) e^{i\omega t} d\omega.$

$A(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t) e^{-i\omega t} d\omega.$

$= \frac{1}{\sqrt{2\pi}} \int x_0 e^{-\gamma t/2} \cos(\omega_0 t) d\omega.$

Kramers-Kronig relations

$I = A(\omega) A^*(\omega)$

$= \frac{1}{2\pi} \frac{\gamma}{(\omega - \omega_0)^2 + (\gamma/2)^2}$ → Lorentzian

Avg power → $\frac{d\overline{W}}{dt}$.

Doppler broadening.

$n_i(v_z) \, dv_z = \frac{N_i}{\sqrt{\pi}\, v_p} \exp\left(-\left(\frac{v_z}{v_p}\right)^2\right) dv_z$

Collision broadening [inelastic collision]

$\downarrow$ energy is transferred

Atom A $\rightarrow E_k \rightarrow E_i$

$A \longrightarrow B$

$\downarrow$

B

[without B]. $\Delta E = E_k - E_i$

in presence of B $\longrightarrow$

$\Delta E_R = E_k(R) - E_i(R)$

R : separation b/w A & B.

Lets take intensity : $I(\omega) : \int A_{ik}(R)\, P_{col}(R) \left[E_k(R) - E_i(R)\right] dR$

$P_{col}(R)$: probability / unit time for atom to be at R & R+dR from B.

$[E_k(R) - E_i(R)]$: Lennard Jones potential.

$$I_{col} : \frac{C}{(\omega - \omega_0)^2 + \left(\frac{\gamma_n + \gamma_{col}}{2}\right)^2}$$

$\boxed{\gamma_{col}}$.

~~Transit time~~

Transit time broadening

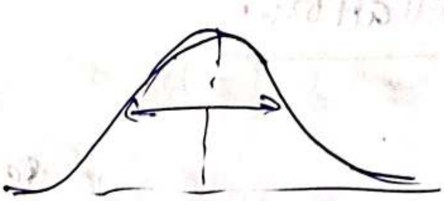
Speed of atom = 500 m/sec

Beam size = ·1 mm.

$$T = d/v = \frac{.1 \times 10^{-3}}{500} = \boxed{.2 \times 10^{-?}\ \mu s}$$

Vibrational lifetime $\approx$ millisec $\rightarrow$ This lifetime is too large for laser time. Hence integration time is NOT $\int_{-\infty}^{\infty}$ but $\int_0^T$.

$$A(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^T x_0\, e^{-\gamma t/2} \cos\omega_0 t\, e^{-i\omega t}\, dt.$$



$$I = A^*(\omega)\, A(\omega).$$

$$I = C\, \frac{\sin^2\left[\frac{(\omega-\omega_0)T}{2}\right]}{(\omega-\omega_0)^2}.$$

$$\frac{(\omega-\omega_0)T}{2} = \pi$$

$$2\Delta\omega = \frac{4\pi}{T} = \frac{12.6}{T}.$$

COLD Atom expts.
Doppler broadening >> Transit, normal

$$\boxed{\Delta\omega_{tt} = \frac{5.6}{T}.}$$

Case: For gaussian beam: $\boxed{I_G = I_0 \cdot \exp\left(-\frac{r^2}{W^2}\right)}$

$$I(\omega) = C \exp\left[-\left(\frac{(\omega-\omega_0)W}{2v}\right)^2\right].$$

$$\delta\omega_{tt} = 2\sqrt{\ln 2}\, \frac{v}{W}$$

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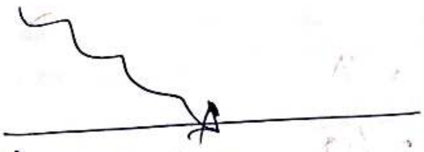
Rabi Oscillation.



(b)

(a)

semiclassical theory.

atomic states $\longrightarrow$ quantum state.

$$E = E_0 \cos(\omega t + kr).$$

$$\simeq E_0 \cos(\omega t).$$

$$\simeq A_0\left(e^{i\omega t} + e^{-i\omega t}\right).$$

$$A_0 = \frac{E_0}{2}.$$

$$\phi_a = U_a(r)\, e^{-iE_a t/\hbar}$$

$$\phi_b = U_b(r)\, e^{-iE_b t/\hbar}.$$

without perturbation.

$$H_0 \phi_a = i\hbar \frac{d\phi_a}{dt}.$$

$$= i\hbar\, U_a(r)\left(\frac{-iE_a}{\hbar}\right) e^{-iE_a t/\hbar}.$$

$$H_0 \phi_b = i\hbar \frac{d}{dt}\phi_b.$$

$$= i\hbar\, U_b(r)\left(\frac{-iE_b}{\hbar}\right) e^{-iE_b t/\hbar}$$

with perturbation,

$$H = H_0 + V.$$

$$V = pE = -e\,\vec{r}\cdot\vec{E}.$$

$$\psi = a(t)\, U_a(r)\, e^{\frac{-iE_a t}{\hbar}} + b(t)\, U_b(r)\, e^{\frac{-iE_b t}{\hbar}}$$

$$H\psi = (H_0 + V)\psi = i\hbar\, \partial_t \psi.$$

$$H_0\psi = H_0\left[a(t)\, U_a(r)\, e^{\frac{-iE_a t}{\hbar}} \ldots\right.$$

$$= \cancel{a(t) E_a U_a(r)\, e^{\frac{-iE_a t}{\hbar}}}$$

$$+ \cancel{b(t) E_b U_b(r)\, e^{\frac{-iE_b t}{\hbar}}}.$$

$$V\psi = -V\left[a(t)\, U_a(r)\, e^{\frac{iE_a t}{\hbar}} + b(t)\, U_b(r)\, e^{\frac{-iE_b t}{\hbar}}\right]$$

$$i\hbar\, \partial_t \psi = i\hbar\left[\dot{a}(t)\, U_a(r)\, e^{\frac{-iE_a t}{\hbar}}\right.$$

$$+ \cancel{a(t)\, U_a(r)\left(\frac{-iE_a}{\hbar}\right) e^{\frac{-iE_a t}{\hbar}}} + \dot{b}(t)\, U_b(r)\, e^{\frac{-iE_b t}{\hbar}}.$$

$$\left. + \cancel{b(t)\, U_b(r)\left(\frac{-iE_b}{\hbar}\right) e^{\frac{-iE_b t}{\hbar}}}\right)$$

from S.E.

$$\dot{a}(t)\, U_a(r)\, e^{-iE_at/\hbar} + \dot{b}(t)\, U_b(r)\, e^{-iE_bt/\hbar} = a(t)\, V\, U_a(r)\, e^{-iE_at/\hbar} + b(t)\, V\, U_b(r)\, e^{-iE_bt/\hbar}$$

from orthogonality $\int U_a^* U_a\, d\tau = \delta_{ab}$

multiply with U_a^* & integrate over space.

$$\dot{a}(t)\, e^{-iE_at/\hbar} = V_{aa}\, a(t)\, e^{-iE_at/\hbar} + V_{ab}\, e^{-iE_bt/\hbar}\, b(t).$$

$$V_{aa} = -eE \int U_a^*\, r\, U_a\, d\tau \;[\text{odd parity}] = 0.$$

$$\boxed{\begin{array}{l} V_{aa} = \int U_a^* V U_a\, d\tau \\ V_{ab} = \int U_a^* V U_b\, d\tau \end{array}}$$

$$\dot{a}(t) = V_{ab}\, b(t)\, e^{i(E_a-E_b)t/\hbar}\, (-i/\hbar)$$

$$\dot{b}(t) = V_{ab}\, a(t)\, e^{i(E_b-E_a)t/\hbar}\, (-i/\hbar).$$

$$\boxed{-e\int U_a^*\, r\, U_b\, d\tau = D_{ab}}$$

matrix element of dipole moment D_{ab} / exp value

$$D = -e\int \left[a^*(t)\, U_a^*(r)\, e^{iE_at/\hbar} + b^*(t)\, U_b^*(r)\, e^{iE_bt/\hbar}\right] r \left[a(t)\, U_a(r)\, e^{-iE_at/\hbar} + b(t)\, U_b(r)\, e^{-iE_bt/\hbar}\right] d\tau.$$

$\boxed{2\times 2}$

$$\boxed{\begin{array}{l} (1,1) \longrightarrow 0\;[\text{odd parity}]. \\ (2,2) \longrightarrow 0\;[\text{odd parity}]. \end{array}}$$

$\begin{pmatrix} 1\times 2 \\ 2\times 1 \end{pmatrix}$

$$= -e\int \left[a^* b\, (U_a^*\, r\, U_b)\, e^{i(E_a-E_b)t/\hbar} + b^* a\, (U_b^*\, r\, U_a)\, e^{-i(E_a-E_b)t/\hbar}\right] d\tau.$$

$$= D_0 \cos(\omega_{ab} t + \phi).$$

$Z + Z^*$

to $\cos(x)$.

$$\boxed{\omega_{ab} = \frac{E_a - E_b}{\hbar} = -\omega_{ba}.}$$

Anti symmetric.

Rabi oscillation freq: $\Omega_{ab} = D_{ab} E_0/\hbar$.

$\dot{a}$ & $\dot{b}$ in terms of $\boxed{\text{dipole}}$ - Ω.

$$\dot{a}(t) = -e\int t\, U_a^* \, r\, U_b \, e^{i\omega_{ab}t} \, dt \cdot b(t)$$

$$= D_{ab}\left[A_0\left(e^{i\omega t}+e^{-i\omega t}\right)e^{i\omega_{ab}t}\right] b(t)$$

$$= D_{ab}\left[A_0\left(e^{i(\omega+\omega_{ab})t}+e^{-(\omega-\omega_{ab})t}\right)\right] b(t).$$

$$\dot{a}(t) = -i\frac{\Omega_{ab}\hbar}{E_0} A_0\left[e^{i(\omega+\omega_{ab})t/\hbar}+e^{-i(\omega-\omega_{ab})t/\hbar}\right]$$

$$= -i\frac{\Omega_{ab}}{E_0} A_0\left[\qquad\right] b(t).$$

$$\therefore \dot{a}(t) = -i\frac{\Omega_{ab}}{2}\left[e^{i(\omega+\omega_{ab})t}+e^{-i(\omega-\omega_{ab})t}\right] b(t).$$

$$\dot{b}(t) = -i\frac{\Omega_{ba}}{2}\left[e^{i(\omega+\omega_{ba})t}+e^{-i(\omega-\omega_{ba})t}\right] a(t).$$

$$= -\frac{\Omega_{ab}}{2}\left[e^{-i(\omega-\omega_{ab})t}+e^{i(\omega+\omega_{ab})t}\right] a(t).$$

How $a(t)$ & $b(t)$ related?