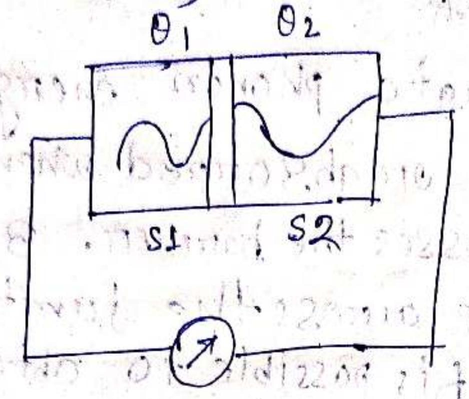
~~#~~ ~~[illegible]~~ 10/04/2025

Josephson junction tunneling

$J = J_0 \sin(\theta_1 - \theta_2)$



Highly coherent = single quantum state (macroscopic scale)

ACC $J = J_0 \sin\left(\delta(0) - \frac{2eVt}{\hbar}\right)$

$\delta(t) = \delta(0) - \frac{2eVt}{\hbar}$

A super ~~con~~current flows between two points in the phase of the wave function are different as the points are not same.

The josephson effect due to the fact that the phases can be changed by $\vec{B}$ & V.

The josephson effect are menifestation of quantum interference phenomena. on a macroscopic Scale

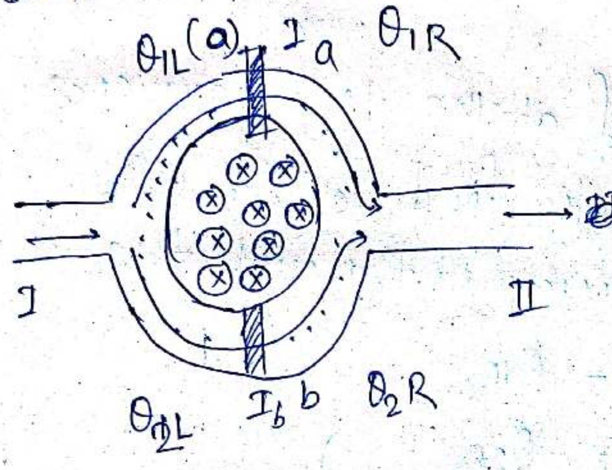
Quantum mean:- ~~e~~ Entire superconductor. Is in ~~the~~ single quantum state.

Interference - The measured properties depend on the phase of the state which can be tuned by a $\vec{B}$ or other agencies and states with different phases can be made by interference the interferences results in current oscillation. which can be detected electrically.

SQUID → ~~superconding~~ super conducting quantum Interferometric Device

Supercurrent Interference :-

The Josephson tunneling in presence of magnetic field provide strong evidence for the highly coherent Nature & superconducting state.



$I = I_a + I_b$

. two Josephson Junctions are arranged in parrallel combination and are placed in a region in which magnetic field ($\vec{B}$) is imposed. A supercurrent starting in region (I) is devided into two parts and made to flow along parrallel path each of contain josephson tunnel junction.

The current I_a and I_b crossing the tunnel barrier a and b respectively reunit in region II. The combine current ~~source~~ shows oscillations charactorics of an interferrence pattern produced by two coherent ~~sour~~ sources. By analosy of of interference of light I_a & I_b are regarded as two coherent sources of current whose distributions when superposed by the way of recombination produce an interference pattern.

In view of relation JJ tunneling the tunneling of cooper pair causes a phase shift & total wavefunction of the superconducting state in region II related to the region I.

If the phase shif at the two barriers in absence of magnetic field be δ_a & δ_b then the supercurrent through the junction

$$\delta_a = \theta_{1L} - \theta_{1R}$$

$$\delta_b = \theta_{2L} - \theta_{2R}$$

$$\left.\begin{array}{l} \\ I_a = I_0 \sin\delta_a \quad I_b = I_0 \sin\delta_b \\ \\ \end{array}\right\} \text{without applying magnetic field.}$$

The phase differenw between two region I & II In the presence of magnetic field of vector product A

$J_s = 0$ deep inside the superconducting material

$$\nabla\theta = \frac{2e}{\hbar} A \qquad \vec{B} = \vec{\nabla} \times \bar{A}$$

taking the line integral of above equation.

$$\int_P^Q \nabla\theta \cdot dl = \frac{2e}{\hbar}\int_P^Q A \cdot dl$$

$$\boxed{\int \vec{B} \cdot ds = \int (\vec{\nabla} \times A) ds = \int A \cdot dl}$$

Therefore the total phase shift wavefunction along two paths from region I to II can be expressed

$$\nabla\theta_{I}^{II} = \delta a + \frac{2e}{\hbar}\int_a \vec{A}\cdot d\vec{l}$$

$$\nabla\theta_{II}^{I} = \delta_b - \frac{2e}{\hbar}\int_b \vec{A}\cdot dl$$

Two phase shift must be identicall because wavefunction has a unique value at every point

so $$\delta a + \frac{2e}{\hbar}\int_a \vec{A}\cdot d\vec{l} = \delta_b - \frac{2e}{\hbar}\int_b \vec{A}\cdot d\vec{l}$$

$$\cancel{\frac{4e}{\hbar}\int \vec{A}\cdot d\vec{l} = (\delta_b - \delta a)}$$

$$\frac{2e}{\hbar}\int_a \vec{A}\cdot d\vec{l} + \frac{2e}{\hbar}\int_b \vec{A}\cdot d\vec{l} = (\delta_b - \delta a)$$

$$\delta_b - \delta a = \frac{2e}{\hbar}\oint A\cdot dl$$

Two line integrals are in opposite dirrection Therefore when taken together they give integral over a closed path

$$\delta_b - \delta a = \frac{2e}{\hbar}\int_s \vec{B}\cdot d\vec{s}$$

$$= \frac{2e}{\hbar}\phi$$ (using the stoke's theorcam).

The above relation states that the total ~~fiel~~ phase difference around the loop can be controlled by varying the magnetic field $\vec{B}$.

The general expression for δa & δ_b

$$\delta a = \delta_0 \mp \frac{e}{\hbar}\int \vec{B}\cdot d\vec{s}$$

$$\delta_b = \delta_0 + \frac{e}{\hbar}\int \vec{B}\cdot d\vec{s}$$

$\delta = \delta_b - \delta_a = 0$ [when $B = 0$]

$\delta_b - \delta_a$

when $B \neq 0$

$$\delta_b - \delta_a = \frac{2e}{\hbar}\int \vec{B}\cdot d\vec{s} = \frac{2e}{\hbar}\phi \qquad \phi_0 = \left(\frac{h}{2e}\right)$$

$$= \frac{2e}{\hbar}\phi$$

$$= 2\pi\left(\frac{2e}{h}\right)\phi$$

$$= (2\pi)\left(\frac{\phi}{\phi_0}\right)$$

$= 2\pi \times$ times the flux in unit of flux quantum.

now the total current after re... $\Rightarrow$ total recombined supercurrent

$$I = I_a + I_b$$

$$= I_0 \sin\delta_a + I_0 \sin\delta_b$$

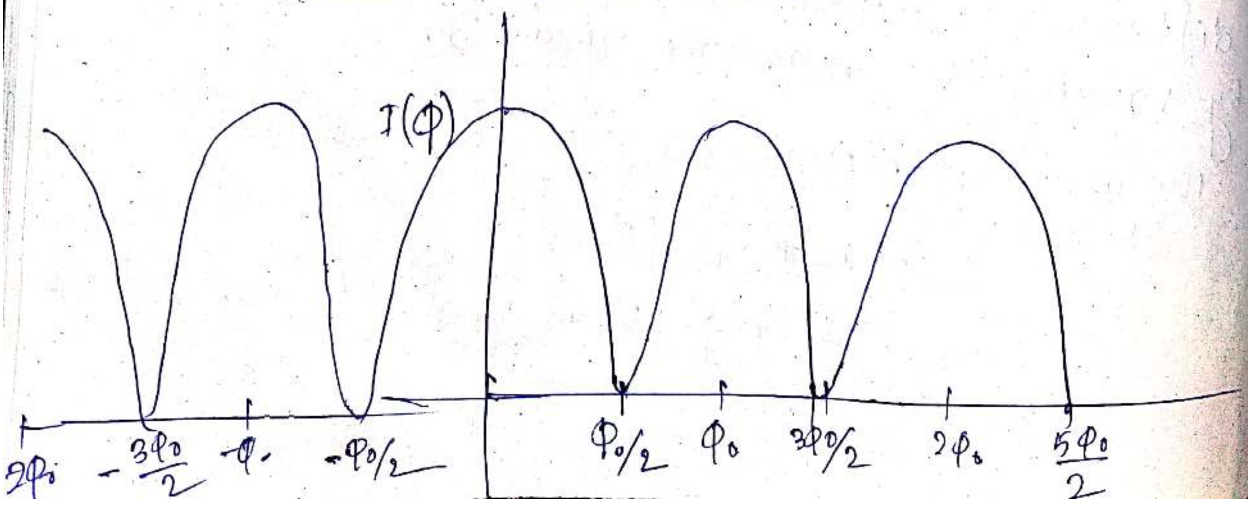
$$= I_0 \sin\left[\delta_0 - \frac{e}{\hbar}\int \vec{B}\cdot d\vec{s}\right] + I_0 \sin\left[\delta_0 + \frac{e}{\hbar}\int \vec{B}\cdot d\vec{s}\right]$$

$$= I_0 \sin\left[\delta_0 - \frac{e}{\hbar}\phi\right] + I_0 \sin\left[\delta_0 + \frac{e}{\hbar}\phi\right]$$

$$I = I_0\, 2\sin\delta_0 \cos\left(\frac{e\phi}{\hbar}\right)$$

$$I = 2I_0 \sin\delta_0 \cos\left(\frac{e\phi}{\hbar}\right)$$

$$\boxed{I = 2I_0 \sin\delta_0 \cdot \cos\left(\pi \cdot \frac{\phi}{\phi_0}\right)}$$



$$\Delta\theta_1 = \Delta\theta + \frac{\pi\Phi}{\Phi_0}$$

$$\Delta\theta_2 = \Delta\theta - \frac{\pi\Phi}{\Phi_0}$$

$$I = I_0 \sin(\Delta\theta_1) + I_0 \sin(\Delta\theta_2)$$

$$= I_0 \left\{\sin\left(\Delta\theta + \frac{\pi\Phi}{\Phi_0}\right) + I_0 \sin\left(\Delta\theta - \frac{\pi\Phi}{\Phi_0}\right]\right.$$

$$= 2I_0 \sin(\Delta\theta) \cos\left(\frac{\pi\Phi}{\Phi_0}\right)$$

This modulation observed SQUID ring critical current is shows the current ~~cns~~ essentially an ideal ~~form~~ Fraunhoffer interference pattern exactly analogoes to the interference pattern on observed in optics with young's two slit experiment.

Here two Josephson junctions are playing the role of slits & the interferrence is between the supercurrent passing through the two halves of the ring. The super ~~co~~ current acquire different phases due to the magnetic field. The SQUID Device provides a simple but highly accurate system for measuring magnetic flux. since the flux quantum Φ_0 only about 2×10^{-15} Wb or Tesla-m^2 in SI unit & are can make SQUID devices $1\,cm^2$ area $B \approx 10^{-10}$ T.

highly possible magnetometer → [SQUID magnetometer]

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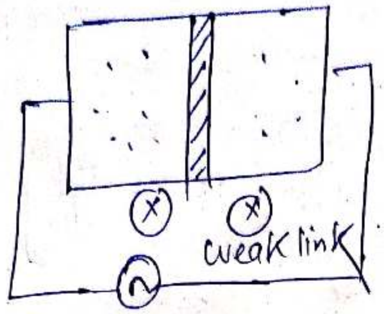
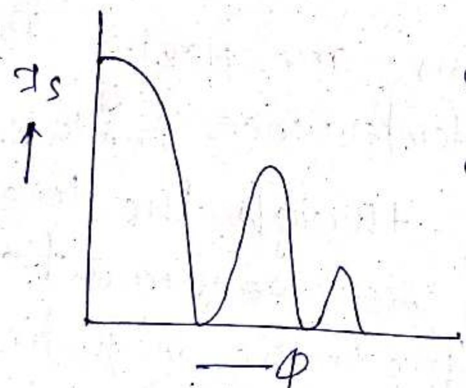


Tunnel current through are junction

$$I_s = I_0 \frac{\sin\left(\frac{\pi\phi}{\phi_0}\right)}{\cancel{\sin}\left(\frac{\pi\phi}{\phi_0}\right)}$$

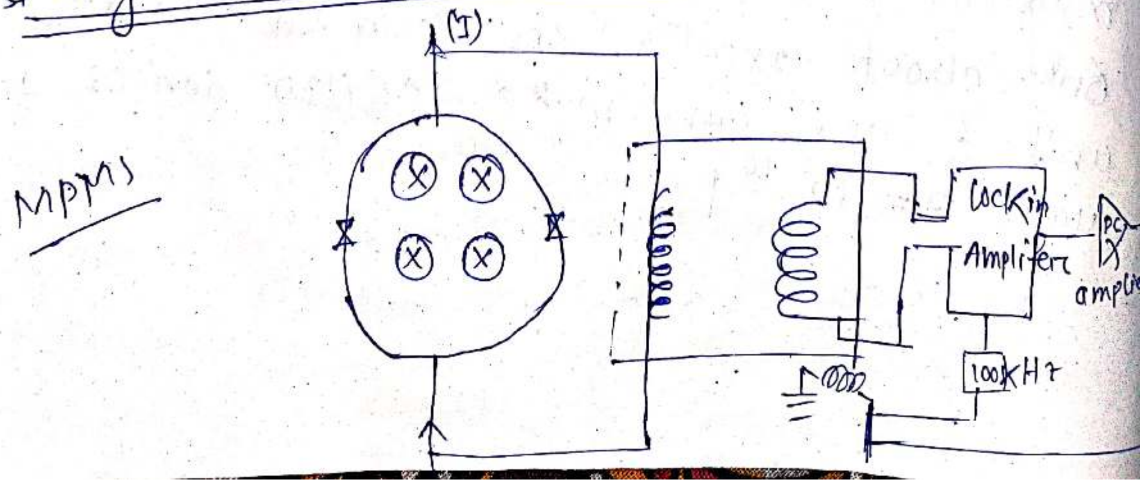
ϕ = Total flux throug the junction



single state light diffraction.

magnetic field dependent current through a single JJ B|| tunneling.

diagram of ~~squi~~ SQUID



11/04/2025

BCS Theory (1957) Bardeen, Cooper & Schrieffer

BCS theory is well accepted for low T_c superconductor

(1) Isotope effect $T_c \propto M^{-1/2}$

$\Rightarrow$ phonons are involved in superconductivity

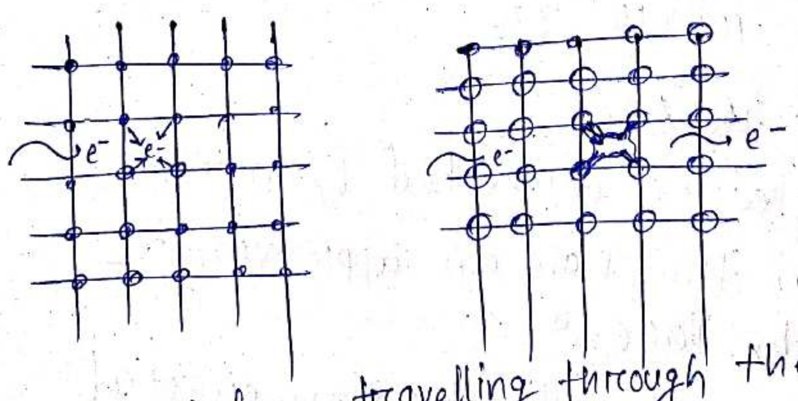
(2) Cooper remoduled idea an electron-phonon interaction into the philosophy of a (electron-phonon-electron)

Cooper ~~demostra~~ demonstrated that with the creation of condition favourable for a net attractive interaction between two electrons in a conductor, conductor transfer from Normal state to SC state.

Electron-phonon interaction

- e-e interaction coloumbic repulsion (~~instaneous~~ instantaneous)

- e-i-e i.e e-e interaction mediated by phonon (retarded) attraction.



An electron travelling through the crystal lattice leaves behind the ~~deformadation~~ deformation trail which can be regarded as an accumulated of

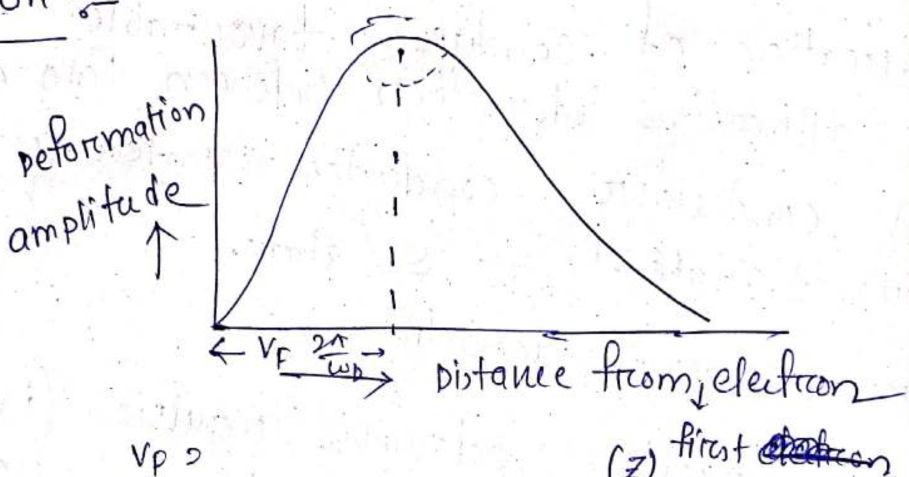
~~v~~ '+' value charge ion core.

→ Area of enhanced +ve charge compared to neutral crystal is increased behind the electron and exerts an attractive force on a second electron behind the 1st electron.

e-e ~~co~~ coloumb repulsion (weak)

e-p-e cooper pair (~~s~~ strong)

Instability of Fermi sea and cooper pair formation :-



$V_P =$

phonon vibration period $\frac{2\pi}{\omega_D} = 10^{-13}$

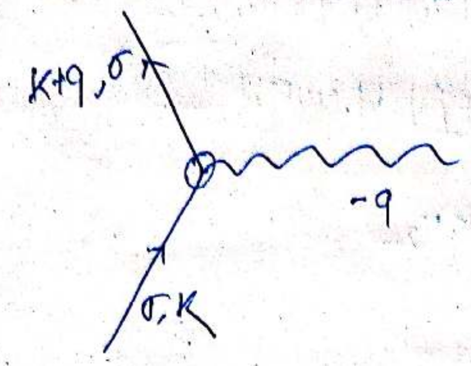
$V_F = 10^{8}$ cm/s.

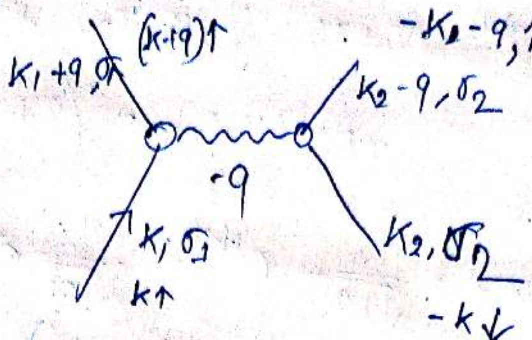
$10^8 \times \frac{2\pi}{\omega_D} = 10^8 \times 10^{-13}$ cm

$= 1000 A^0$

The two electron correated by lattice deformation thus have an approximate seperated by $1000A^0$

An electron travelling through crystal lattice leaves behind information tell which can be regarded as accomulation of positive charge ion cores





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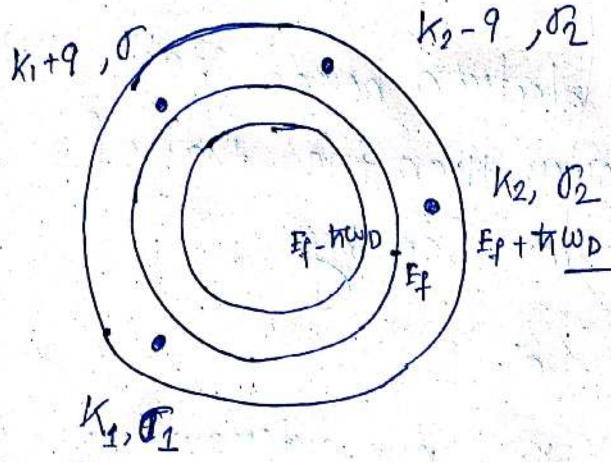


The effective e-e interaction near the fermi-surface the electron at $k_1\sigma_1$ & $k_2\sigma_2$ are scatted to k_1+q, σ_1 & k_2-q, σ_2. The interaction is attracted provided all the wave vectors lie in the range $\varepsilon_K \pm \hbar\omega_D$ of the fermi sphere.

$$V_{eff}(q,\omega) = |g_q|^2 \frac{1}{(\omega^2 - \omega_q^2)} \qquad g_q = \sqrt{\frac{m}{M}} \sim 0.01$$

$$V_{eff}(q,\omega) = |g_{eff}|^2 \frac{1}{\omega^2 - \omega_D^2}$$

$$\omega < \omega_D$$

$$= -|g_{eff}|^2$$

$$= -V_0 \quad (\omega < \omega_D)$$

(#) Two particle wave function

~~$\psi(r_1, r_2)$~~,

$$-\frac{\hbar^2}{2m}(\nabla_1 + \nabla_2)\,\psi(r_1, r_2) + V(r_1, r_2)\,\psi(r_1, r_2)$$

$$= E\,\psi(r_1, r_2) = (\varepsilon + 2E_f^0)\,\psi(r_1, r_2).$$

$$\psi(r_1, r_2) = \frac{1}{\sqrt{L^3}} e^{ikr_1} \cdot \frac{1}{\sqrt{3}} e^{-ikr_2}$$

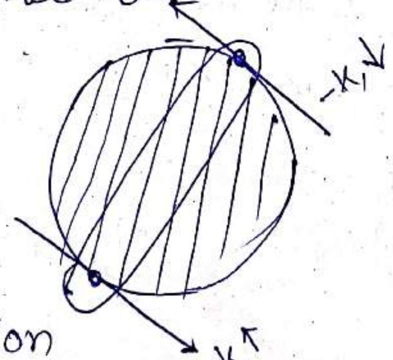
$$= \frac{1}{3!} e^{ik(r_1 - r_2)}$$

$$\varepsilon = -2\hbar\omega_D\, e^{-2/V_0 Z(E_f^0)}$$

There exist a two electron bound state whose energy is lower than the that of fully occupied fermi Sea by

$$\varepsilon = E - 2E_f^0 < 0$$

The ground state of the non-interacting free electron gas becomes unstable when any attractive interaction between electron is switched on. The instability leads to the



formation of such electron pairs such as cooper pair $(k\uparrow, -k\downarrow)$ and the system tries to reache the new lower energy ground state

$(k\uparrow, -k\downarrow)$ $(k'\uparrow, -k'\downarrow)$ with

& so on the cooper pair and opposite. k-vector & spin.

coloumb interaction is reduced due to screeinig (presence of the electron in E_F)

Some thing new occurs the two electron may attracted each other, the two electron wall then from bound state very close to the E_f surface

the binding energy is shortest when $E_f \pm \hbar\omega_D$ electrons forming the pair have oppositive momentum & opposite spin $k\uparrow$, & $-k\downarrow$. All the electrons in the neighbourhood of the fermi surface. A system of many cooper pairs.

The binding energy of electrons (1 & 2) an energy gap appears in the spectrum of the electron

An electron $(k\uparrow)$ polarised the lattice creating phonon (q). Another electron with wave vector $(-k\downarrow)$ absorbs the phonon. The end result is two electrons $(k-q, \uparrow)$ $(-k+q, \uparrow)$

condensate:-

At low temp. Cooper pairs are formed a favourable condition. The pair wavefunction all have form & the superposition of pair wavefunction describes $\longrightarrow$ condensate (BEC)

BCS energy gap relation:-

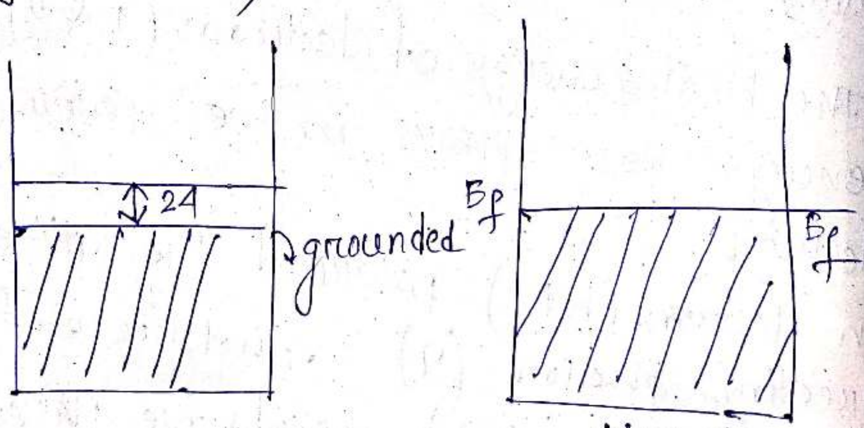
In the simplest form BCS theory

$$K_B T_c = 1.14\, \hbar\omega_D \exp\left[-\frac{1}{V \cdot N(E_f)}\right] \quad \text{--- (1)}$$

V = effective electron-phonon interaction

$N(E_f)$ = DOS at fermi level

ω_D = Debye frequency.



BEC condensate.

Normal metal

from BCS calculation

$$2\Delta(0) = 4\hbar\omega_D \exp\left(-\frac{1}{V N(E_f)}\right] \quad \text{--- (2)}$$

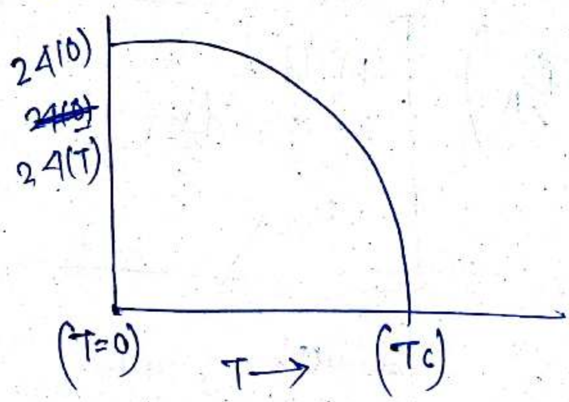
$\hbar\omega_D \sim 10^{-2}\,eV \sim 10^{-4}\,eV$

$$\frac{2\Delta(0)}{K_BT_c} = \frac{4\cancel{\hbar\omega_D}\,\cancel{\exp\left(-\frac{1}{VN(E_F)}\right)}}{1.14\,\cancel{\hbar\omega_D}\,\cancel{\exp\left(-\frac{1}{VN(E_F)}\right)}}$$

$$= 3.53$$

	$\frac{2\Delta(0)}{K_BT_c}$
In	4.1
Sn	3.6
Hg	4.6
pb	4.1

Experimental measured values clearly show types of values of of $\frac{2\Delta(0)}{K_BT_c} = 3.53$ as predicted by BCS.



$$2\Delta(T) = 2\Delta(0)\left[1-\left(\frac{T}{T_c}\right)\right]^{2}$$

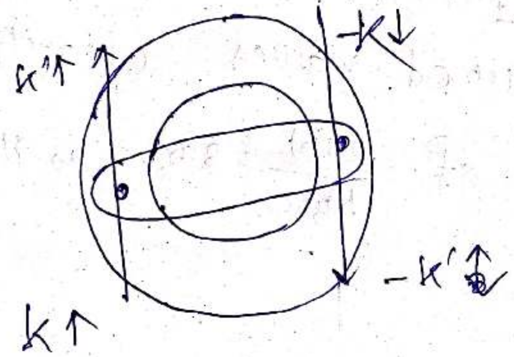
BCS ground state.

$\psi(r, s, r', s')$ r = electron position
s = spin

In a system of N electrons, the electrons are grouped into N/2 pairs

$$\phi = (r_1, s_1, r_2, s_2, r_3, s_3) = \psi(r_1, s_1, r_2, s_2) \cdot \psi(r_3, s_3, r_4, s_4) \cdot \psi(r_5, s_5, r_6, s_6)$$

$\phi_{BCS} = a\phi$ a = antisymmetrician



$$|\phi_{BCS}\rangle = \prod (u_k|0\rangle_k + v_k|1\rangle_k)$$

Field of solids Book

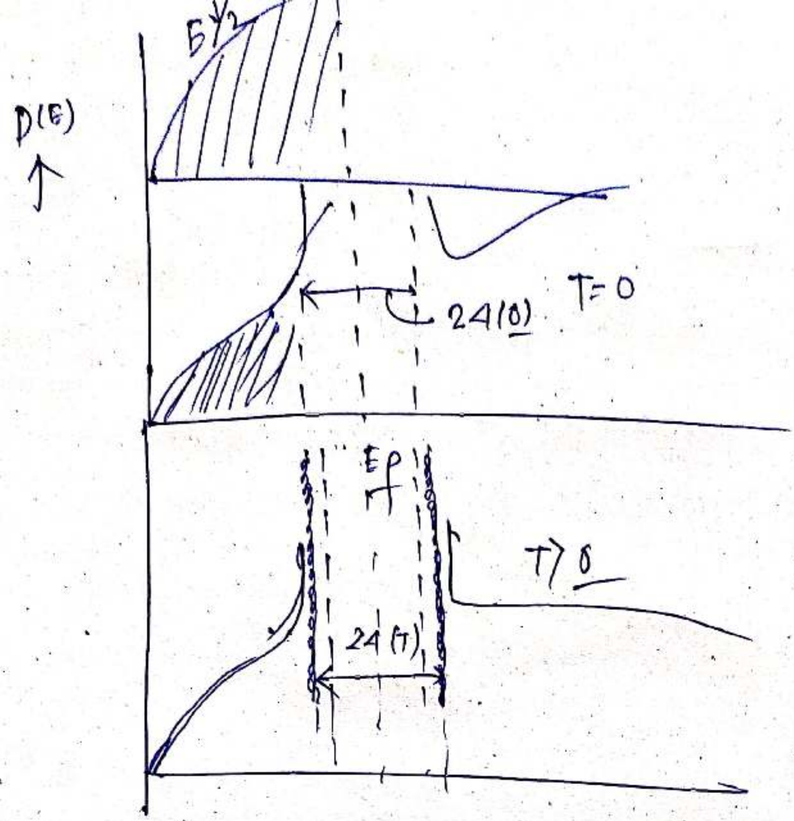
$$\langle \phi_{BCS} | \phi_{BCS} \rangle = 1$$

[Ibach & luth solid state physics]

$$u_k^2 + v_k^2 = 1$$

u_k, v_k are normalised constants.

cooper pair breaks if $2\Delta(0)$ is supplied


$E^{1/2}$
D(E)
2Δ(0)
T=0
E_f
2Δ(T)
T>0