Pulse broadening and dispersion in optical fibers

We consider the time dependence of the pulse corresponding to the qth mode. The pulse propagates with the velocity

$$v_{\mathrm{q}} = \frac{dw}{d\beta}$$

Please remember, $v_q = \frac{d\omega}{d\lambda}$ is the group velocity of the mode.

Thus, the time taken by the mode to travel a distance L of the fiber is

$$t_{\mathrm{q}} = \frac{L}{v_{\mathrm{q}}} = L \cdot \frac{d\beta_{\mathrm{q}}}{d\omega}.$$

$$= L\frac{d\beta_{\mathrm{q}}}{d\lambda} \cdot \frac{d\lambda}{d\omega}$$

But $\qquad \frac{d\lambda}{d\omega} = -\frac{\lambda^2}{2\pi c} \Rightarrow \qquad = -\frac{L\lambda^2}{2\pi c} \cdot \frac{d\beta_{\mathrm{q}}}{d\lambda}$

$$\left.\begin{aligned} \omega &= \frac{2\pi}{\lambda} \cdot c \\ \frac{d\omega}{d\lambda} &= -\frac{2\pi c}{\lambda^2} \end{aligned}\right\} \qquad t_q = -\frac{1}{2\pi c} \cdot \lambda^2 \frac{d\lambda}{d\lambda}$$

Therefore, we see that the group velocity of different modes are different. Thus, if all guide modes are exited at the input $z = 0$ of the fiber, then the time taken by the modes will be different. Hence there will be dispersion. This dispssein is known as intermodal dispersion.

Now if the source of light at the fiber input characterized by a spectral width $\Delta\lambda_0$, each wavelength component takes, in general, different times to traverse the length of the fiber, even when only me mode is excited. Thus, the pulse describing the mode will be broadened. The spread is given by

$$\Delta t = \left(\frac{dt}{d\lambda_0}\right) \Delta\lambda_0$$

Now $\boldsymbol{t = \frac{L\lambda_0^2}{2\pi c}\left(\frac{d\beta}{d\lambda_0}\right)}$ (we have dropped the qth mode suffix)

$$\frac{dt}{d\lambda_0} = \frac{L}{2\pi c}\left[2\lambda_0 \frac{d\beta}{d\lambda_0} + {\lambda_0}^2 \frac{d^2\beta}{d{\lambda_0}^2}\right]$$

Dispersion

Intermodal Intramodal

$$\Delta t = \frac{L\Delta\lambda_0}{2\pi c}\left[2\lambda_0 \frac{d\beta}{d\lambda_0} + {\lambda_0}^2 \frac{d^2\beta}{d{\lambda_0}^2}\right]$$

The brooding of the pulse of a particular mode due to finite spectral width of the source is called intramodal dispersion.

(1) Highly multimode step index fiber – intermodal: very high & intramodal: negligible

(2) Multimode graded index fiber – intermodal: very low & intramodal: significantly important

(3) Single mode step index fiber – intramodal: not present & intramodal: significant important

Intramodal dispersion: - two components

Material dispersion Waveguide dispersion

Material dispersion is due to the dependence of RI on λ.

Waveguide dispersion is due to structure of the waveguide.

Material Dispersion

$$To\ evaluate\ \frac{d\beta}{d\lambda_0}:$$

Consider a well guided mode: $\beta = k_0 n_{eff} \approx \frac{2\pi}{\lambda_0}.n_1$ [For $n_1 \sim n_2$]

$$\Delta t = \left(\frac{dt}{d\lambda}\right)\Delta\lambda_0 = -\frac{L\Delta\lambda_0}{2\pi c}\left[2\lambda_0\frac{d\beta}{d\lambda} + \lambda_0^2\frac{d^2\beta}{d\lambda_0^2}\right]\Delta\lambda_0$$

Therefore, $\frac{d\beta}{d\lambda_0} = 2\pi\left[-\frac{n_1}{\lambda_0^2} + \frac{1}{\lambda_0}.\frac{dn_1}{d\lambda_0}\right] \Rightarrow 2\lambda_0\frac{d\beta}{d\lambda_0} = \left[-\frac{4\pi n_1}{\lambda_0} + 4\pi.\frac{dn_1}{d\lambda_0}\right]$

In other form (Group Index) $\frac{d\beta}{d\lambda_0} = -\frac{2\pi}{\lambda_0^2}\left[n_1 - \lambda_0\frac{dn_1}{d\lambda_0}\right] = -\frac{2\pi}{\lambda_0^2}\cdot N_1$

where $N_1 = n_1 - \lambda_0\frac{dn}{d\lambda_0}$ = Group Index

$$\left.\begin{aligned} \omega &= \frac{2\pi}{\lambda}\cdot c \\ \frac{d\omega}{d\lambda_0} &= -\frac{2\pi c}{\lambda^2} \end{aligned}\right\} \Rightarrow v_g = \frac{d\omega}{d\beta} = \frac{d\left(\frac{2\pi}{\lambda_0}\cdot c\right)}{d\left(\frac{2\pi}{\lambda_0}n_1\right)} = c/n_1$$

$$\frac{d\omega}{d\beta} = \frac{d\omega}{d\lambda_0}\frac{d\lambda_0}{d\beta} = \frac{2\pi c}{\lambda_0^2}\cdot\frac{\lambda_0^2}{2\pi\left[n_1 - \lambda_0\frac{dn_1}{d\lambda_0}\right]}$$

$$\frac{d\omega}{d\beta} = \frac{d\omega}{d\lambda_0}\frac{d\lambda_0}{d\beta} = \frac{2\pi c}{\lambda_0^2}\cdot\frac{\lambda_0^2}{2\pi\left[n_1 - \lambda_0\frac{dn_1}{d\lambda_0}\right]} = \frac{c}{\left[n_1 - \lambda_0\frac{dn_1}{d\lambda_0}\right]} = \frac{c}{N_1}$$

$To\ evaluate\ \frac{d^2\beta}{d\lambda^2}$:

$$\frac{d}{d\lambda_0}\left(\frac{d\beta}{d\lambda_0}\right) = \frac{d}{d\lambda_0}\left[-\frac{2\pi n_1}{\lambda_0^2} + \frac{2\pi}{\lambda_0}\cdot\frac{dn_1}{d\lambda_0}\right]$$

$$= -\frac{2\pi n_1}{\lambda_0^3}\times(-2) - \frac{2\pi}{\lambda_0^2}\cdot\frac{dn_1}{d\lambda_0} - \frac{2\pi}{\lambda_0^2}\cdot\frac{dn_1}{d\lambda_0} + \frac{2\pi}{\lambda_0}\cdot\frac{d^2n_1}{d\lambda_0^2}$$

$$\frac{d^2\beta}{d\lambda_0^2} = \frac{4\pi n_1}{\lambda_0^3} - \frac{4\pi}{\lambda_0^2}.\frac{dn_1}{d\lambda_0} + \frac{2\pi}{\lambda_0}\cdot\frac{d^2n_1}{d\lambda_0^2}$$

$$\lambda_0^2\frac{d^2\beta}{d\lambda_0^2} = \frac{4\pi n_1}{\lambda_0} - 4\pi.\frac{dn_1}{d\lambda_0} + 2\pi\lambda_0\cdot\frac{d^2n_1}{d\lambda_0^2}$$

$$Also\ \frac{d\beta}{d\lambda_0} = 2\pi\left[-\frac{n_1}{\lambda_0^2} + \frac{1}{\lambda_0}.\frac{dn_1}{d\lambda_0}\right] \Rightarrow 2\lambda_0\frac{d\beta}{d\lambda_0} = \left[-\frac{4\pi n_1}{\lambda_0} + 4\pi.\frac{dn_1}{d\lambda_0}\right]$$

$$\Delta\tau = -\frac{\mathrm{L}\Delta\lambda_0}{2\pi c}\underbrace{\left[2\lambda_0\frac{d\beta}{d\lambda_0} + \lambda_0^2\frac{d^2\beta}{d\lambda^2}\right]}_{2\pi\lambda_0\frac{d^2n_1}{d\lambda_0^2}} = -\frac{1}{c}\cdot\left(\frac{\Delta\lambda_0}{\lambda_0}\right)\lambda_0^2\cdot\frac{d^2n_1}{d\lambda_0^2} = -\frac{\lambda_0}{c}\frac{d^2n_1}{d\lambda_0^2}$$

$\Delta\tau_m$ is defined in the following way:

$$\Delta\tau_m = \frac{\Delta t_m}{\Delta\lambda_0}\cdot\frac{1}{L} = -\frac{\lambda_0}{c}\frac{d^2n_1}{d\lambda_0^2}\quad ps/km - nm$$

Sellmeire equation:

$$n(\lambda) = c_0 + c_1\lambda_0^2 + c_2\lambda_0^4 + \frac{c_3}{(\lambda_0^2 - l)} + \frac{c_4}{(\lambda_0^2 - l)^2} + \frac{c_5}{(\lambda_0^2 - l)^3}$$

$c_0 = 1.4508554$ $c_1 = -0.0031268$ $c_2 = -0.0000381$ $c_3 = 0.0038270$
$c_4 = 0.0000779$ $c_5 = 0.0000018$

$$\tau_m = \begin{cases} \sim 85\mathrm{ps/Km.nm} - 0.85\ \mu\mathrm{m} \\ \sim 0.1\mathrm{ps/km.nm} - 1.279\mathrm{pm} \\ \sim 20\mathrm{ps/km.nm} - 1.50\ \mathrm{m} \end{cases}$$

Waveguide Dispersion

Equation to time of transit:

$$t = \frac{L}{v} = L\frac{d\beta}{d\omega} = -L\frac{\lambda_0^2}{2\pi c}\frac{d\beta}{d\lambda_0}$$

For waveguide dispersion we need to evaluate $\frac{d\beta}{d\lambda_0}$ in terms of waveguide dependent parameters, V.

$$\frac{d\beta}{d\lambda_0} = \frac{d\beta}{dV}\cdot\frac{dV}{d\lambda_0} = -\frac{V}{\lambda_0}\cdot\frac{d\beta}{dV}$$

$$\Leftarrow \quad \mathrm{V} = \frac{2\pi}{\lambda_0}a\cdot n_1\sqrt{2\Delta}, \qquad \frac{dV}{d\lambda_0} = -\frac{2\pi a n_1\sqrt{2\Delta}}{\lambda_0^2} = -\frac{V}{\lambda_0}$$

We define the normalised frequency : $b = \dfrac{\left(\frac{\beta}{k_0}\right)^2 - n_2^2}{n_1^2 - n_2^2} \approx \dfrac{\frac{\beta}{k_0} - n_2}{n_1 - n_2} = \dfrac{\frac{\beta}{k_0} - n_2}{n_2\Delta}$

So we may write: $\dfrac{\beta}{k_0} = n_2\Delta b + n_2$

Hence, $\beta = k_0 n_2\Delta\cdot b + k_0 n_2 = k_0 n_2(1+\Delta b) = \dfrac{2\pi}{\lambda_0}n_2(1+b\Delta)$

So, $\dfrac{d\beta}{d\lambda_0} = -\dfrac{2\pi n_2}{\lambda_0^2}(b\Delta+1) + \dfrac{2\pi}{\lambda_0}n_2\Delta\dfrac{db}{d\lambda}$

For the 2nd term:

$$\frac{db}{d\lambda} = \frac{db}{dV}\cdot\frac{dV}{d\lambda} = -\frac{db}{dV}\cdot\frac{V}{\lambda_0}$$

Therefore,
$$\frac{d\beta}{d\lambda_0} = -\frac{2\pi n_2}{\lambda_0^2}(b\Delta+1) - \frac{2\pi n_2}{\lambda_0^2}\cdot V\Delta\frac{db}{dV}$$
$$= -\frac{2\pi n_2}{\lambda_0^2}\left[1 + b\Delta + V\Delta\cdot\frac{db}{dV}\right]$$

Hence, $t_\omega = -L\dfrac{\lambda_0^2}{2\pi c}\dfrac{d\beta}{d\lambda_0} = \dfrac{L\lambda_0^2}{2\pi c}\cdot\dfrac{2\pi n_2}{\lambda_0^2}\cdot\left(1 + b\Delta + \Delta V\dfrac{db}{dV}\right)$

$$= \frac{Ln_2}{c}\cdot\left(1 + b\Delta + V\Delta\frac{db}{dV}\right)$$

The broadening of a pulse due to waveguide dispersion is

$$\Delta t_\omega = \left(\frac{dt}{d\lambda_0}\right)\Delta\lambda_0 = \frac{Ln_2}{c}\left[\Delta\frac{db}{d\lambda_0} + \Delta\frac{d}{d\lambda_0}\left(V\frac{db}{dV}\right)\right]\cdot\Delta\lambda_0$$

$$= \frac{Ln_2}{c}\left[\Delta\frac{db}{d\lambda_0} + \Delta\frac{d}{d\lambda_0}\left(V\frac{db}{dV}\right)\right]\cdot\Delta\lambda_0 = \frac{Ln_2}{c}\left[\Delta\frac{db}{dV}\cdot\frac{dV}{d\lambda_0} + \Delta\frac{d}{dV}\left(V\frac{db}{dV}\right)\frac{dV}{d\lambda_0}\right]\Delta\lambda_0$$

$$= \frac{Ln_2}{c}\left[-\frac{\Delta V}{\lambda_0}\frac{db}{dV} - \frac{\Delta V}{\lambda_0}\left(\frac{db}{dV} + V\frac{d^2b}{dV^2}\right)\frac{dV}{d\lambda_0}\right]\Delta\lambda_0 = \mathrm{c}\left[\frac{2V\mathit{\Delta}}{\lambda_0}.\frac{db}{dV} + V^2\frac{\mathit{\Delta}}{\lambda_0}.\frac{d^2b}{dV^2}\right].\Delta\lambda_0$$

$$= -\frac{Ln_2}{c}\cdot\left(\frac{\Delta\lambda_0}{\lambda_0}\right).\left(2V\frac{db}{dV} + V^2\frac{d^2b}{dV^2}\right)$$

Therefore, $\Delta t_\omega = -\dfrac{Ln_2}{c}\left(\dfrac{\Delta\lambda_0}{\lambda_0}\right)\cdot V\dfrac{d^2(bV)}{dV^2}$

Broadening is defined as ps/km-nm

So,

$$\frac{\Delta t_w}{L\cdot\Delta\lambda_0} = -\frac{n_2}{c\lambda_0}\cdot V\frac{d^2(bV)}{dV^2}$$

Now we use an empirical formula for *b* as

$b = \left(A - \dfrac{B}{V}\right)^2$ where $A = 1.1428$ and B= 0.996

using this

$$\frac{\Delta t_\omega}{\Delta\lambda_0 L} = -\frac{2n_2\Delta}{C\lambda_0}\cdot\frac{B^2}{V^2} = -\frac{2n_2\Delta}{c\lambda_0}\cdot\frac{B^2}{\left(\frac{4\pi^2a^2\cdot n_2^2 2\Delta}{\lambda_0^2}\right)} \quad (V = k_0an_1\sqrt{2\Delta})$$

$$= -\frac{\lambda_0}{ca^2}\cdot\frac{B^2}{4\pi^2n_2C} \qquad B = 0.996$$

Tolal Dispersion $\boldsymbol{\Delta\tau = \Delta\tau_m + \tau_w}$

Case 1: $\lambda_0 = 0.8\ \mu$m, $\frac{d^2 n_1}{d\lambda_0^2} \approx -4 \times 10^{-2} \mu m^{-2}$

Material Dispersion: Depends on λ

$$\frac{\Delta\tau_m}{L\Delta\lambda_0} = -\frac{\lambda_0}{c}\frac{d^2 n_1}{d\lambda_0^2} = -\frac{\lambda_0}{c}\frac{d^2 n_1}{d\lambda_0^2}$$

$$\frac{\Delta\tau_m}{L\Delta\lambda_0} \approx 100\mathrm{ps/km-nm}$$

Case-2:

For an SIF $\Delta = 0.00154,$ $a = 3\mu m,\ n_2 = 1.45$ $V \rightarrow 1.9$

Waveguide Dispersion:

$$\frac{\Delta\tau_\omega}{L\Delta\lambda_0} \approx -5\mathrm{ps/km-nm}$$

$\lambda_0 = 1.3\ \mu m$; $\frac{d^2 n}{d\lambda_0^2} = -5.5 \times 10^{-4} \mu m^{-2}$

Material Dispersion:

$$\frac{\Delta t_m}{L\Delta\lambda_0} \approx 2.4\ \mathrm{ps/km.mm.}$$

For an SIF $a = 5.6\mu\mathrm{m}\ \Delta = 0.00117.\ n_2 = 1.45$ V $\longrightarrow 1.9.$

$$\frac{\Delta t_\omega}{L\Delta\lambda_0} = -2.4\mathrm{ps/km.nm.}$$

Case 3:

For SIF Material Dispersion:

$\lambda_0 = 1.55\ \mu\text{m}$ $\qquad$ $\dfrac{d^2 n_1}{d\lambda_0{}^2} = -4.2 \times 10^{-3} \mu m^{-2}$

$$\frac{\Delta t_m}{L\Delta\lambda_0} = 22\text{ps/km. nm}$$

For SIF waveguide dispersion:

$a = 2.02\mu\text{m}$ $\qquad$ $\Delta = 0.013$ $\qquad$ $n_2 = 1.45$ $\qquad$ $V \longrightarrow 1.9$

$$\frac{\Delta t_\omega}{L\Delta\lambda_0} = -22\text{ps/km. nm}$$