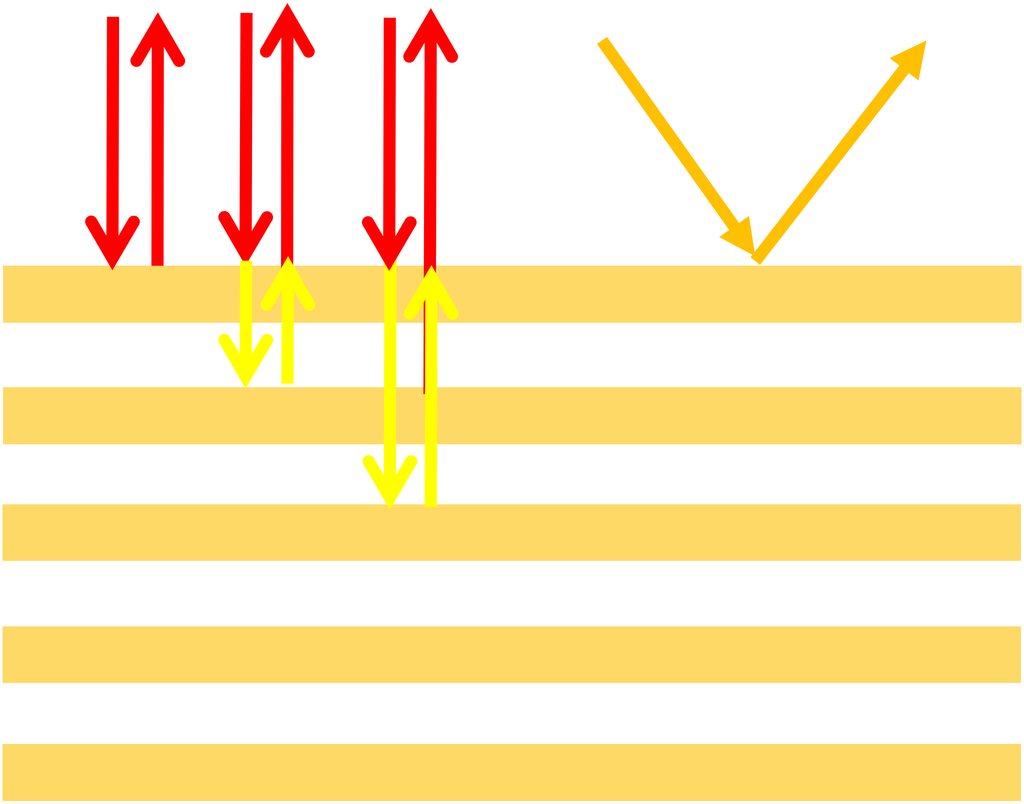
All-fiber Components

- The necessary function of signal processing/ manipulation is performed whilst the signal is still guided by fiber
- Components realized from fiber in the form of fiber itself
- Components can be readily spliced to signal carrying circuit with a common fiber handling tool
- No significant insertion loss due to geometry mismatch or mismatch in overlap of modal fields

Major Devices

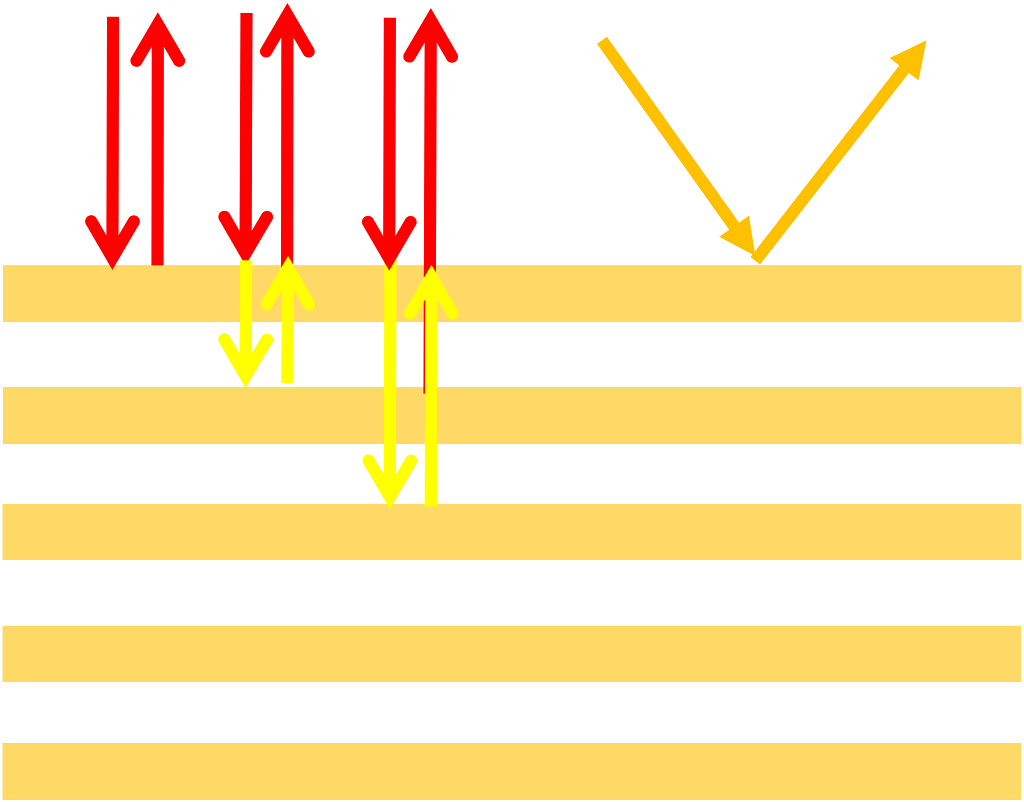
- **Fiber Couplers: FFC**
- **Fiber Amplifier: EDFA**
- **Fiber Bragg Grating: FBG**

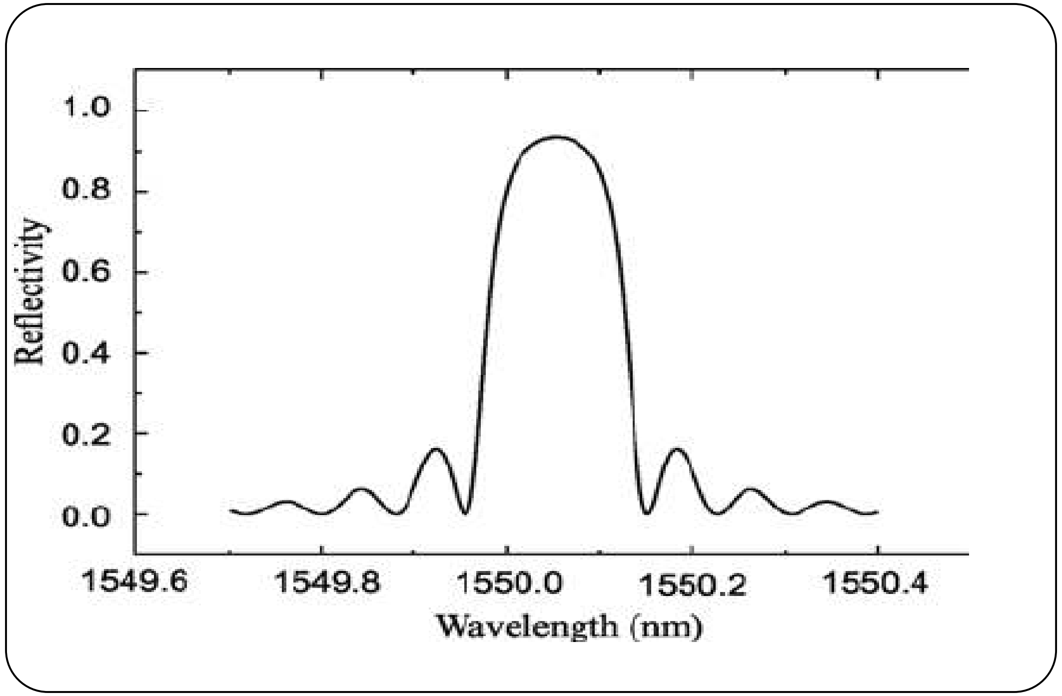
Bragg Reflection



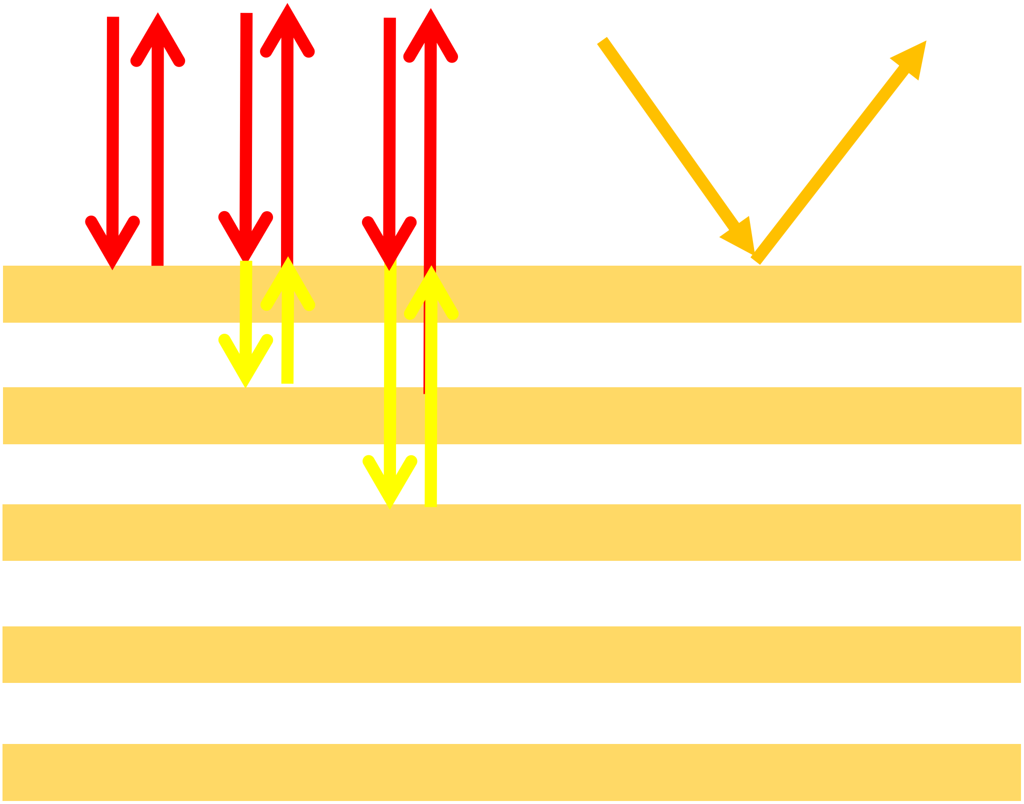
$$\Delta\phi = k_0 2\Lambda n_{eff} = m2\pi$$
$$\rightarrow \frac{2\pi}{\lambda_B} 2\Lambda n_{eff}$$

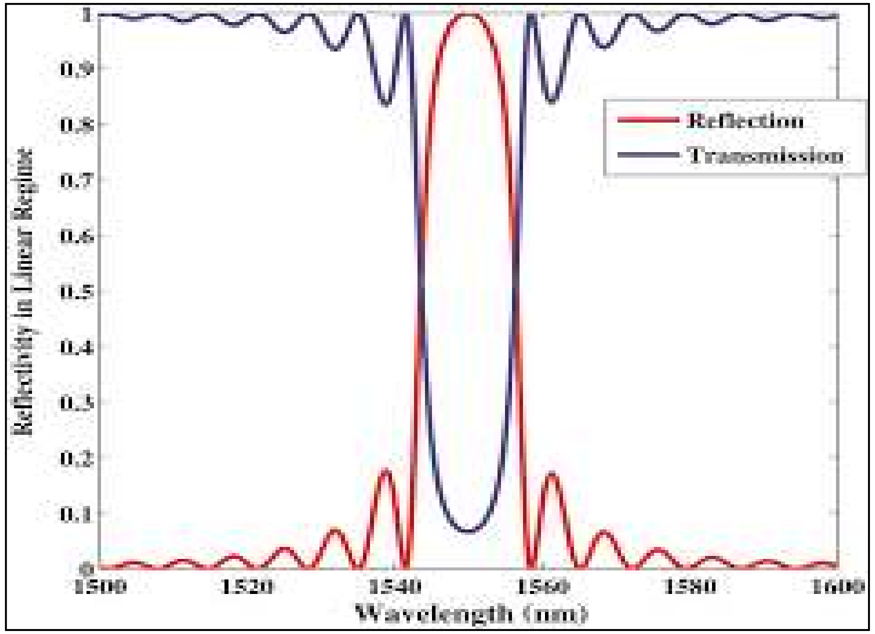
Bragg Reflection



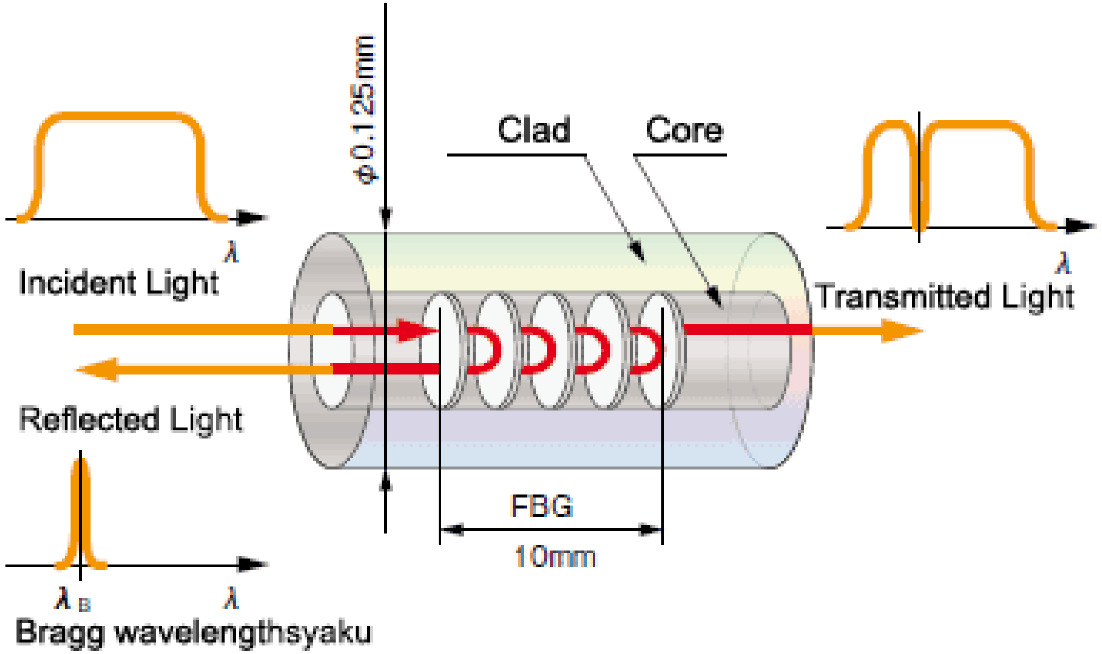


Bragg Reflection





Fiber Bragg Grating

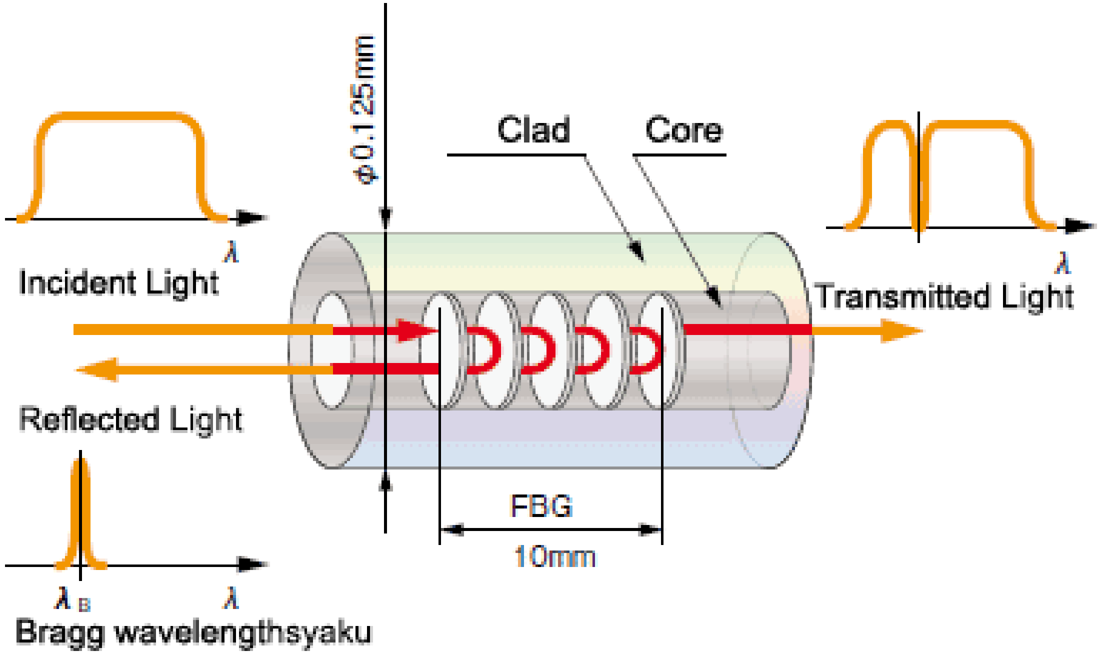


A **Fiber Bragg Grating** is

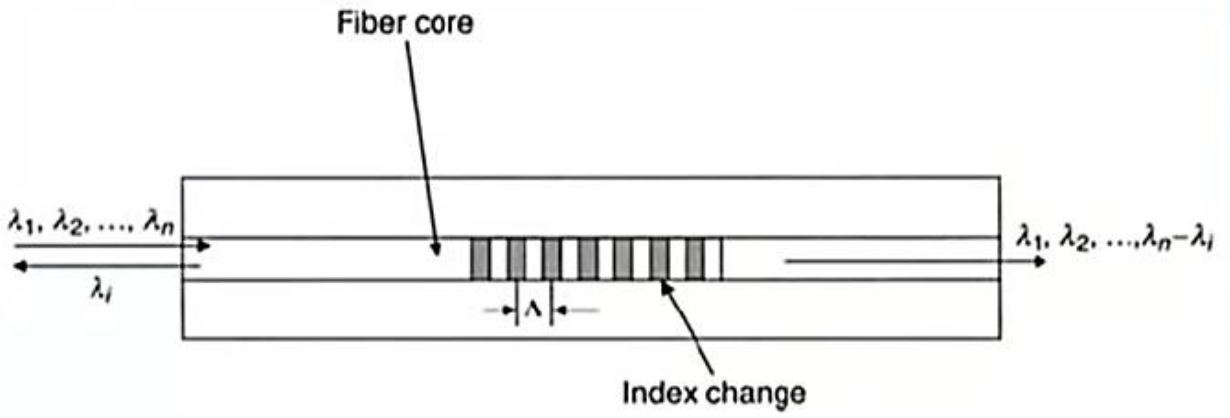
a distributed Bragg reflector that reflects specific wavelengths and transmits all other.

constructed by a **periodic variation** of refractive index at the core of fiber.

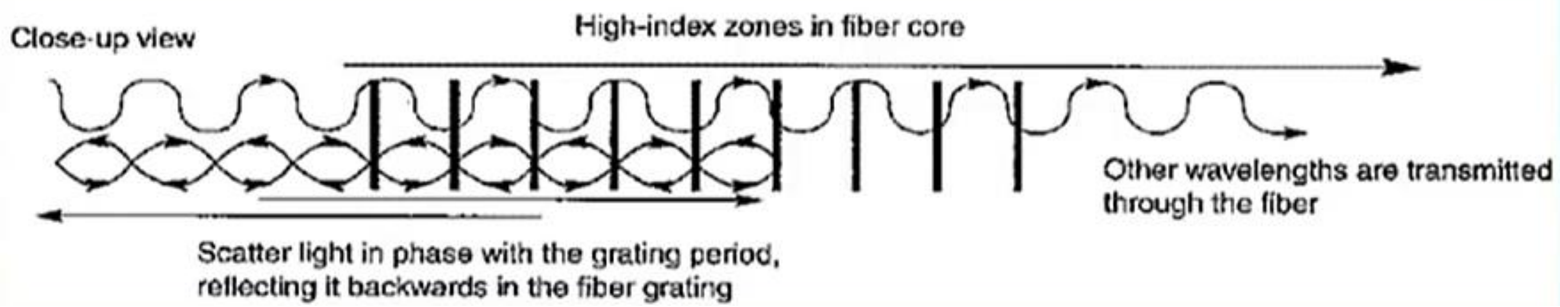
Fiber Bragg Grating



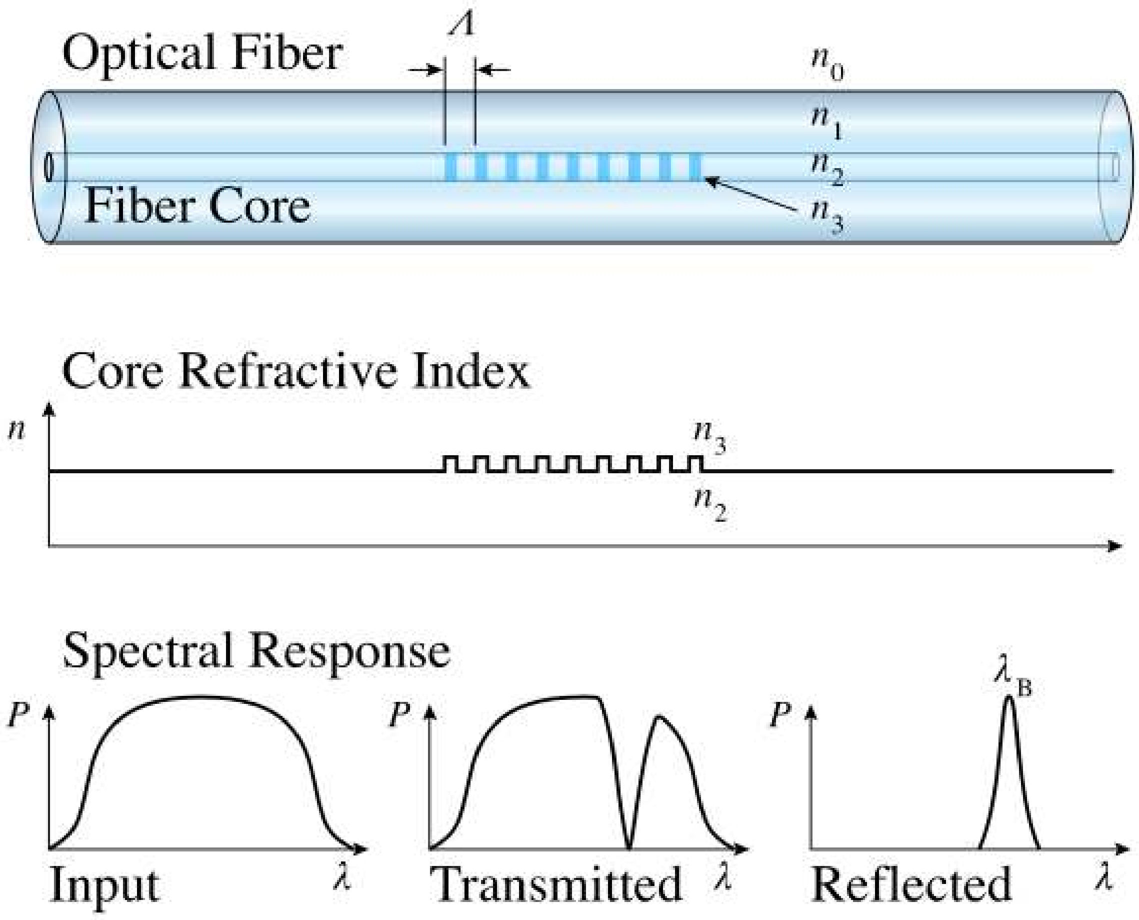
FBG’s wavelength filter characteristics



Schematic diagram of a fiber Bragg grating.



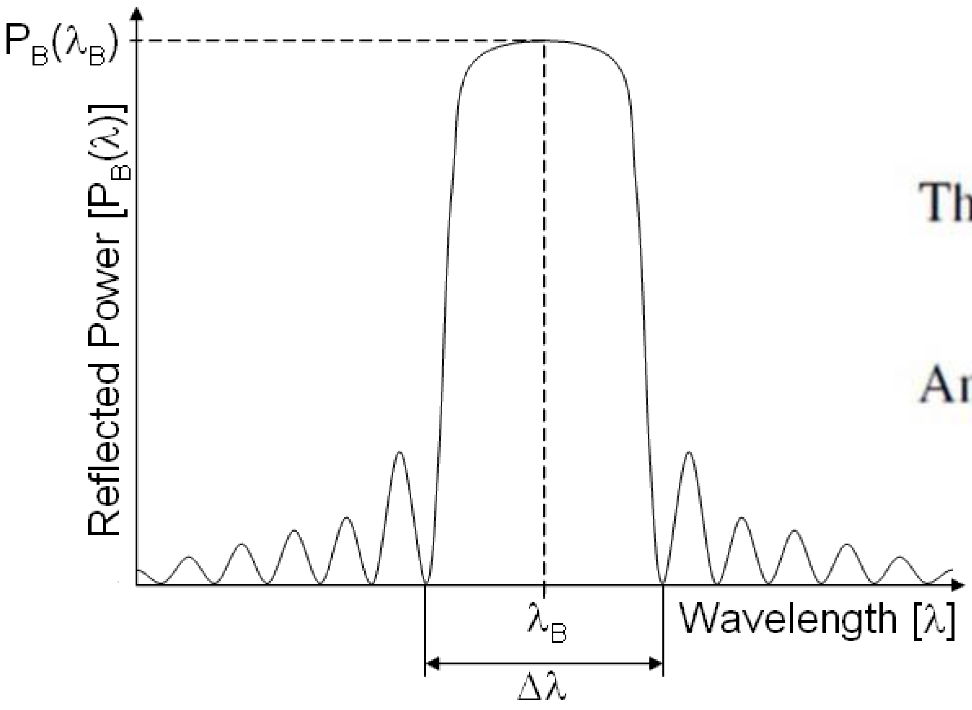
Fiber Bragg Grating used as inline wavelength filter



$$\lambda_B = 2n_e\Lambda$$

$$\Delta\lambda = \left[\frac{2\delta n_0 \eta}{\pi}\right]\lambda_B$$

FBG's Reflection Spectrum



$$R_{max} = \tanh^2(\kappa L).$$

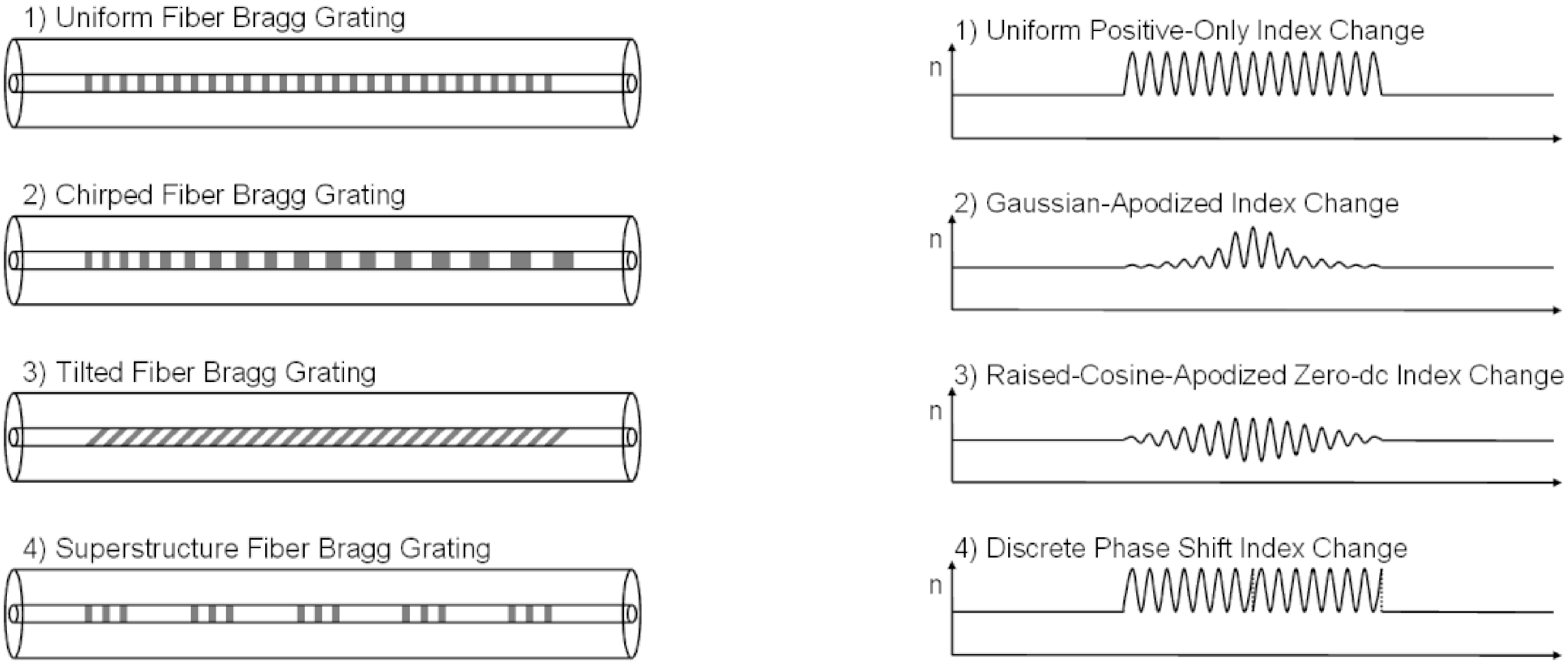
The full bandwidth $\Delta\lambda$ over which R_{max} holds is

$$\Delta\lambda = (\lambda^2_{Bragg}/\pi n_{eff} L)[(\kappa L)^2 + \pi^2]^{1/2}$$

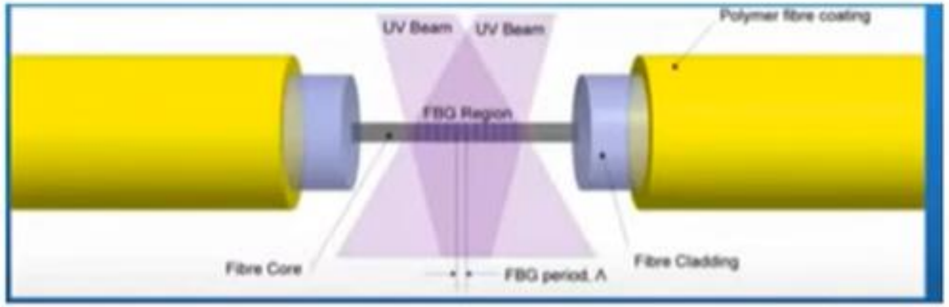
An approximation for the FWHM bandwidth is

$$\Delta\lambda_{FWHM} = \lambda_{Bragg}\, s[(\delta n/2n_{core})^2 + (\Lambda/L)^2]^{1/2}$$

Fiber Bragg Grating used as inline wavelength filter



How to fabricate a Fiber Bragg Grating



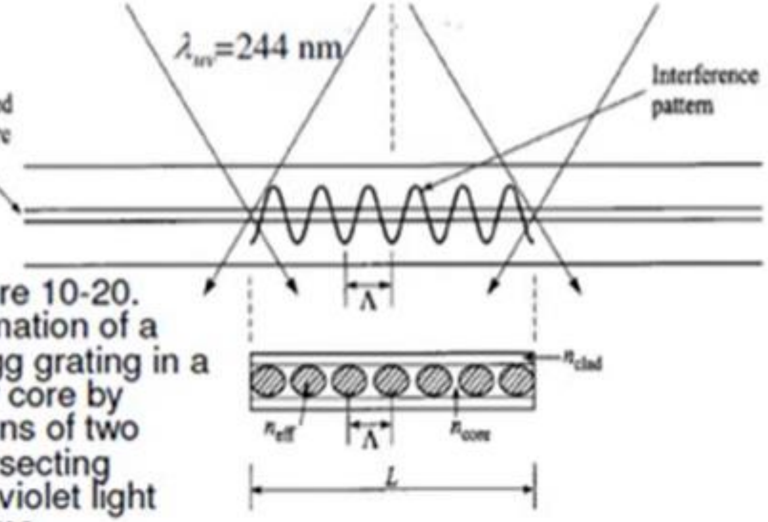
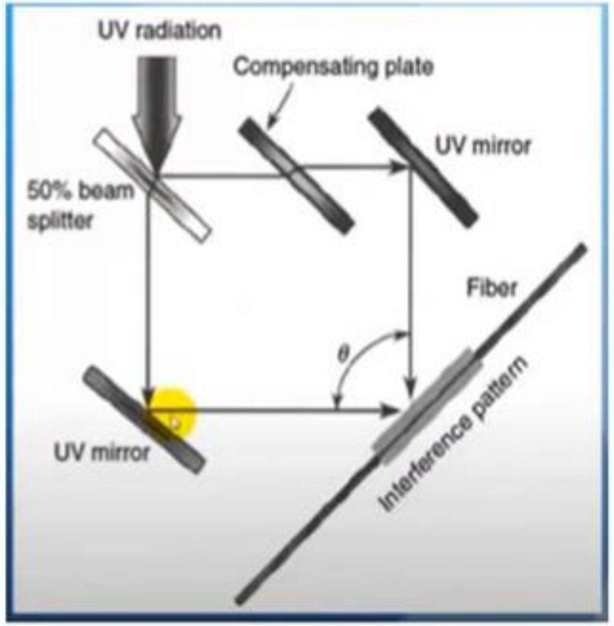


Figure 10-20. Formation of a Bragg grating in a fiber core by means of two intersecting ultraviolet light beams.



Fiber Bragg Grating

Coupled-mode Equations
for axially periodic perturbation

Consider an optical fiber with RI profile as $n^2(x,y)$ in which there is a periodic z-dependent perturbation in the RI profile as $\Delta\mathrm{n}^2(x,y,z)$

It could be a periodic stress

OR a periodic undulation of this fiber axis.

For a sinusoidal z-perturbation: $\Delta n^2(x,y,z) = \Delta n^2(x,y)\sin kz$

$$K = \frac{2\pi}{\Lambda}; \Lambda = \text{spatial period}$$

If $\psi_1(x,y)$ and $\psi_2(x,y)$ are the two modes of the fiber, then the total field under perturbation may be written as

$$\psi(x,y,z) = A(z)\psi_1 e^{-i\beta_1 z} + B(z)\psi_2 e^{-i\beta_2 z} \qquad (1)$$

$\beta_1, \beta_2 \rightarrow$ are mode propagation constants without perturbation.

$A(z), B(z) \rightarrow$ are the amplitudes of the modes.

- Without perturbation *A*, *B* are constant.
- In absence of any perturbation:

$$\nabla_{xy}^2\Psi_1 + [k_0^2 n^2(x,y) - \beta_1^2]\Psi_1 = 0 \qquad (2)$$

$$\nabla_{xy}^2\psi_2 + [k_0^2 n(x,y) - \beta_2^2]\psi_2 = 0$$

- Perturbation couples power among modes, hence A, B are z-dependent.

Since modes are orthogonal,

$$\iint_{-\infty}^{+\infty} \psi_1^* \psi_2 dxdy = 0 \qquad (3)$$

Under perturbation, the wave equation to be satisfied by $\psi(x, y, z)$ is then

$$\nabla_{xy}^2 \psi + \frac{\partial^2 \psi}{\partial z^2} + [k_0^2 n^2(x, y) + \Delta n^2(x, y, z)]\psi = 0 \qquad (4)$$

Substituting (1) in (4):

$$\Rightarrow \left[\frac{\partial^2 \psi_1}{\partial x^2} + \frac{\partial^2 \psi_1}{\partial y^2}\right] A(z)e^{-i\beta_1 z} + \left[\frac{\partial^2 \psi_z}{\partial x^2} + \frac{\partial^2 \psi_2}{\partial y^2}\right] B(z)e^{-i\beta_2 z} = \nabla_{xy}^2 \psi_1 \cdot Ae^{-i\beta_1 z} + \nabla_{xy}^2 \psi_2 Be^{-i\beta_1 z}$$

1st term:

$$\nabla_{xy}^2 \psi = \left[\frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2}\right]\left\{A(z)\psi_1 e^{-i\beta_1 z} + B(z)\psi_2 e^{-i\beta_2 z}\right\}$$

$$= A(z)e^{-i\beta_1 z} \cdot \frac{\partial^2 \psi_1}{\partial x^2} + B(z)e^{-i\beta_2 z} \cdot \frac{\partial^2 \psi_2}{\partial x^2} + A(z)e^{-i\beta_1 z} \cdot \frac{\partial^2 \psi_1}{\partial y^2} + B(\mathrm{z})e^{-i\beta_2 z} \frac{\partial^2 \psi_2}{\partial y^2}$$

$$\frac{\partial \psi}{\partial z} = \frac{\partial A(z)}{\partial z} \cdot \psi_1 e^{-i\beta_1 z} - i\beta_1 A\psi_1 e^{-i\beta_1 z} + \frac{\partial B}{\partial z}\psi_2 e^{-i\beta_2 z} - i\beta_2 B\psi_2 e^{-i\beta_1 z}$$

$$\textbf{2}^{\textbf{nd}}\textbf{ term:}\quad \frac{\partial^2\psi}{\partial z^2} = \frac{\partial^2 A}{\partial z^2}\psi_1 e^{-i\beta_1 z} - i\beta_1 \frac{\partial A}{\partial z}\psi_1 e^{-i\beta_1 z} - i\beta_1 \frac{\partial A}{\partial z}\psi_1 e^{-i\beta_1 z} - \beta_1^2 A\psi_1 e^{-i\beta_1 z}$$
$$+\frac{\partial^2 B}{\partial z^2}\psi_2 e^{-i\beta_1 z} - i\beta_2 \frac{\partial B}{\partial z}\cdot\psi_2 e^{-i\beta_2 z} - i\beta_2 \frac{\partial B}{\partial z}\psi_2 e^{-i\beta_2 z} - \beta_2^2 B\psi_2 e^{-i\beta_2 z}$$
$$= \frac{\partial^2 A}{\partial z^2}\psi_1 e^{-i\beta_1 z} - 2i\beta_1 \frac{\partial A}{\partial z}\psi_1 e^{-i\beta_1 z} - \beta_1^2 A\psi_1 e^{-i\beta_1 z}$$
$$+\frac{\partial^2 B}{\partial z^2}\psi_2 e^{-i\beta_2 z} - 2i\beta_2 \frac{\partial \beta}{\partial z}\psi_2 e^{-i\beta_2 z} - \beta_2^2 B\psi_2 e^{-i\beta_2 z}$$

(=0; slowly varying approx.)

$$\textbf{3}^{\textbf{rd}}\textbf{ term:}\quad k_0^2 n^2(x,y)A\psi_1 e^{-i\beta_1 z} + k_0^2 n^2(xy)B\psi_2 e^{-i\beta_2 z} + k_0^2 \Delta n^2(x,yz)A\psi_1 e^{-i\beta_1 z}$$
$$+k_0^2 \Delta n^2(x,yz)B\psi_2 e^{-i\beta_2 z}$$
$$\text{I}:\ \underline{\nabla_{xy}^2\psi} = \underbrace{\nabla_{xy}^2\Psi_1}\cdot Ae^{-i\beta_1 z} + \nabla_{xy}^2\psi_2\cdot Be^{-i\beta_2 z}$$
$$\text{II}:\ \frac{\partial^2\psi}{\partial z^2} = -2i\beta_1\frac{\partial A}{\partial z}\psi_1 e^{-i\beta_1 z}\underbrace{-\beta_1^2\psi_1}\cdot Ae^{-i\beta_1 z} - 2i\beta_2\frac{\partial\beta}{\partial z}\psi_2 e^{-i\beta_2 z} - \beta_2^2\psi_2 Be^{-i\beta_2 z}$$
$$\underbrace{k_0^2 n^2(xy)\psi_1}\cdot Ae^{-i\beta_1 z} + \underbrace{k_0^2 n^2(xy)\psi_2}\cdot Be^{-i\beta_2 z} \qquad \text{by (2)}$$
$$+k_0^2\Delta^2(xu_2)\psi_1\cdot Ae^{-i\beta_1 z} + k_0^2\Delta^2(xyz)\psi_2\cdot Be^{-i\beta_2 z}$$

So,

$$-2i\beta_1\frac{\partial A}{\partial z}\psi_1 e^{-i\beta_1 z}2i\beta_2\frac{\partial B}{\partial z}\psi_2 e^{-i\beta_2 z} + k_0^2\Delta^2(xyz)\left[A\psi_1 e^{-i\beta_1 z} + B\psi_2 e^{-i\beta_2 z}\right] = 0$$

Dividing by $e^{-i\beta_1 z}$ all-throughout: $\Delta\beta = \beta_1 - \beta_2$

$$-2i\beta_1\frac{\partial A}{\partial z}\psi_1 - 2i\beta_2\frac{\partial B}{\partial z}\psi_2 e^{i\Delta\beta z} + k_0^\gamma \Delta \mathrm{n}^\nu(xyz)\left[A\psi_1 + \beta\psi_2 e^{i\Delta\beta z}\right] = 0 \qquad (5)$$

Multiply eqn (5) by ψ^* from left and integrating over the whole space across the fiber cross-section:

$$-2i\beta_1 \frac{\partial A}{\partial z} \int \psi_1^* \psi_1 dxdy - 2i\beta_2 \frac{\partial B}{\partial z} \cdot \int \psi_1^* \psi_2 dxdy \qquad + k_0^2 A \int \psi_1^* \Delta n^2 \psi_1 dxdy$$

$$+ k_0^2 B \int \psi_1^* \Delta \mathrm{n}^2 \psi_2 dxdy \cdot e^{i\Delta\beta z} = 0.$$

using (3)

Hence, $\frac{dA}{dz} = -ic_{11}A - ic_{12}Be^{i\Delta\beta z} - (6.1)$

where we have used

$$\frac{k_0^2}{2\beta_1} \frac{\int \psi_1^* \Delta n^2 \psi_1 dxdy}{\int \psi_1^* \psi_1 dxdy} = c_{11}$$

and $\frac{k_0^2}{2\beta_1} \frac{\int \psi_1^* \Delta n^2 \psi_2 dxdy}{\int dxdy} = c_{12}$

Similarly, multiply equation (5) from the right and integrating, we shall obtain

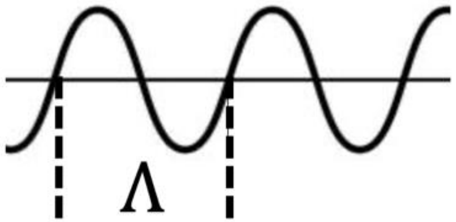
$$\frac{d\beta}{dz} = -ic_{22}B - ic_2 Ae^{-i\Delta\beta z}(6.2)$$

Eqn (6) are the coupled mode eqns. describing z-dependents of amplitudes A, B.

$$C_{21} = \frac{k_0^2}{2\beta_2^2} \cdot \frac{\int 2\Delta \mathrm{n}^2 1}{\int 22}$$

$$C_{22} = \frac{k_0^2}{2\beta_2^2} \cdot \frac{\int 2\Delta \mathrm{n}^2 1}{\int 22}$$

So far, we have considered perturbation $\Delta\mathrm{n}^2(x, y, z)$ which is general and weak.
For a periodic & z-dependent perturbation such as in fiber Bragg grating,



we may write

$$\Delta\mathrm{n}^2(x, y, z) = \Delta\mathrm{n}^2(x, y) \cdot \sin kz \quad \text{where } k = \frac{2\pi}{\Lambda}$$

And then we have,

$$c_{11} = \frac{k_0^2}{2\beta_1} \cdot \frac{\int |\Delta\mathrm{n}^2|}{\int \ 11} \cdot \sin kz = 2\chi_{11} \sin k_z.$$

$$i_1 k_{11} = \frac{k_0^2}{4\beta_1} \cdot \frac{\int |\Delta\mathrm{n}^2|}{\int \ 11}$$

and similarly,

$$c_{12} = 2\chi_{12} \sin kz \qquad c_{22} = 2\chi_{22} \sin kz$$

$$c_{21} = 2\chi_{21} \sin kz$$

So, coupled-mode equations take the form:

$$\frac{dA}{dz} = -2ik_{11}A\sin kz - k_{12}Be^{i(\Delta\beta+k)z} + k_{12}Be^{i(\Delta\beta-k)z}$$

Integrating above eq. over a length L, which is small compared with the length over which A and B change appreciably,

$$A\left(z+\frac{L}{2}\right) - A\left(z-\frac{L}{2}\right)$$

$$= +\,4ix_{11}A\cos K_z\frac{\sin KL/2}{K}$$

$$-2ix_{12}Be^{i(\Delta\beta+K)z}\left\{\frac{\sin(\Delta\beta+K)L/2}{\Delta\beta+K}\right\}$$

$$+2ix_{12}Be^{i(\Delta\beta-K)z}\left\{\frac{\sin(\Delta\beta-K)L/2}{\Delta\beta-K}\right\}$$

Since $\Delta\beta = \frac{2\pi}{\lambda_0}.\,\Delta h_{\text{eff}}$

Δh_{eff} $\approx$ index difference between core-cladding.

$$\approx 0.005 \text{ for } \lambda_0 = 1.0 \text{ mm.}$$

$$\Delta\beta \approx 3 \times 10^4 \text{ m}^{-1}$$

If we choose $K \approx \Delta\beta$ and $L \approx 2 \times 10^{-3}$ m (typical values)

$$\left|\frac{\sin(\Delta\beta - K)L/2}{(\Delta\beta - K)}\right| \approx \frac{L}{2} = 10^{-3} \text{ m}$$

Then,

$$\left|\frac{\sin(\Delta\beta + K)L/2}{\Delta\beta + K}\right| \leqslant \frac{1}{\Delta\beta + K} \approx \frac{1}{2\Delta\beta} \approx 1.7 \times 10^{-5} \text{ m}$$

$$\left|\frac{\sin KL/2}{K}\right| \leqslant \frac{1}{K} \approx \frac{1}{\Delta\beta} \approx 3 \times 10^{-5} \text{ m.}$$

Thus, for $k \approx \Delta\beta$, the 1^{st} & 2^{nd} terms are negligible.

And for $\Delta\beta = -K$, 2^{nd} term would have made significant contribution.
1^{st} & 3^{rd} terms are negligible.
So, coupling takes place if $\Delta\beta \approx K \; or - K$.
Thus, if we choose $K = \frac{2\pi}{\Lambda} \simeq \Delta\beta = \beta 1 - \beta 2$: but $\underline{\Delta\beta - K = \Gamma}$

$$\xrightarrow{\beta_1}$$
$$\xrightarrow[\beta_2]{}$$

$$\left.\begin{aligned} \frac{dA}{dz} &= \kappa_{12} B e^{i\Gamma z} \\ and\ \frac{dB}{dz} &= -\kappa_{21} A e^{-i\Gamma z} \end{aligned}\right\} ----------- -(7.1)\&(7.2)$$

Under weakly guiding approximation, the modes ψ_1, ψ_2 can be normalized as

$$\left.\begin{aligned} &\frac{\beta_1}{2\omega\mu_0} \iint \psi_1^* \psi_1 dxdy = 1 \\ \text{and} \quad &\frac{\beta_2}{2\omega\mu_0} \iint \psi_2^* \psi_2 dxdy = 1 \end{aligned}\right\} ----------- -(8)$$

Using this

$$\kappa_{12} = \frac{\omega\epsilon_0}{8} \iint \psi_1^* \Delta \mathrm{n}^2 \psi_2 dxdy \;\&\; \kappa_{21} = \frac{\omega\epsilon_0}{8} \iint \psi_2^* \Delta n^2 \psi_1 dxdy$$

yielding that $\kappa_{12} = \kappa_{21} = \kappa$ (say).
Hence

$$\left.\begin{aligned} \frac{dA}{dz} &= \kappa B e^{i\Gamma z} \\ \text{and} \quad \frac{dB}{dz} &= -\kappa A e^{-i\Gamma z} \end{aligned}\right\} ----------- -(9)$$

Equations (9) describe the coupling between two modes propagating along the same direction is $(\beta_1$ and β_2 are along +z direction) <u>CODIRECTIONAL COUPLING</u>.

For <u>CONTRADIRECTIONAL COUPLING</u>, coupling occurs between the modes traveling in the opposite direction. There we start form $\psi(x, y, z) = A(z)\psi_1(x, y)e^{-i\beta_1 z} + B(z)\psi_2(x, y)e^{i\beta_2 z}$.

Thus, following a same procedure we can obtain the CME as

$$\left.\begin{aligned} \frac{dA}{dz} &= \kappa B e^{i\Gamma z} \\ \text{and} \quad \frac{dB}{dz} &= +\kappa A e^{-i\Gamma z} \end{aligned}\right\} ----------- -(10)$$

$$\underset{\longleftarrow}{\beta_1} \quad \underset{\longrightarrow}{\beta_2}$$

where $\Gamma = \beta_1 + \beta_2 - K$
Since $\Delta\beta = \beta_1 - (-\beta_2)$

CONTRADIRECTIONAL COUPLING: between same modes

We have the coupled-mode eques for this case as

$$\frac{dA}{dz} = \kappa B e^{i\Gamma z}$$

$$\frac{dB}{dz} = \kappa A e^{-i\Gamma z} \text{ and } \Gamma = \beta_1 + \beta_2 - K, K = \frac{2\pi}{\Lambda}.$$

If the coupling between the two identical modes traveling in opposite direction, then $\beta_1 = \beta_2 = \frac{2\pi}{\lambda_0} n_{eff}$,

n_{eff} = mode-index

So, $\Lambda = \frac{\lambda_0}{2n_{\text{eff}}}$ Compare this with the case of codirectional case, see the periodicity required here is much smaller.

When the modes are phase-matched, i.e., $\Gamma = 0$, we obtain the equations as

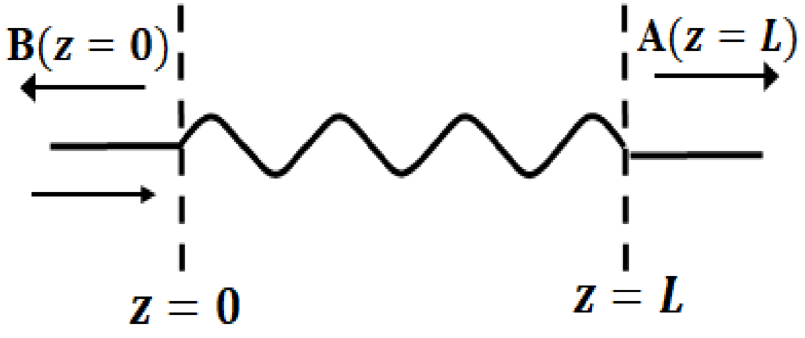
$$\frac{d^2B}{dz^2} = \kappa^2 B$$

Whose solution is $\boxed{B(z) = b_1 e^{\kappa z} + b_2 e^{-\kappa z}}$

(the solutions are not oscillatory)

And then $\boxed{A(z) = b_1 e^{\kappa z} - b_2 e^{-\kappa z}}$

<u>Boundary condition</u>: A unit power is incident in mode A propagating through a periodic wavelength of length L.
i.e., $A(z=0)=1$
Since there is no back-coupled wave beyond $z=L$, $B(z=L)=0$
Thus, $b_1 e^{\kappa L}+b_2 e^{-\kappa L}=0$; $b_1-b_2=1$

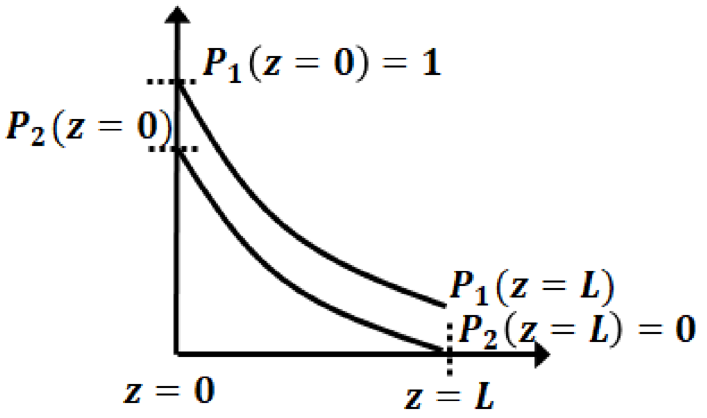


This gives

$$b_1=\frac{e^{-\kappa L}}{2\cosh\ \kappa L}\ \parallel\ b_2=\frac{-e^{\kappa L}}{2\cosh\ \kappa L}$$

$$\therefore B(z)=\frac{sinh\,\kappa(z-L)}{\cosh\ \kappa L},A(z)=\frac{\cosh\ \kappa(z-L)}{\cosh\ \kappa L}$$

Note that $|A(z)|^2 - |B(z)|^2 = (\cosh^2 \kappa L)^{-1} =$ const.
⇒ energy Conservation.



The reflection coefficient

$$r = \frac{B(z=0)}{A(z=0)} = -\tanh \kappa L$$

So, the energy reflection coefficient is

$$\boxed{R = \tanh^2 \kappa L}$$

for a medium of index variation as $n(z) = n_0 + \Delta n \sin \kappa z$, the coupling coefficient can be shown to be

$$\kappa = \frac{\pi \Delta n}{\lambda_0}$$

For fiber then assuming a similar expression,

$$R = \tanh^2 \left(\frac{\pi \Delta n L}{\lambda_0} \right)$$

⇒ Thus, if we wish a reflection centered around 1550 mm, then the required period is

$$\Lambda = \frac{\lambda_0}{2n_{\text{eff}}} = \frac{1550}{2 \times 1.46} = 513 \text{ nm}$$

$$\approx \frac{1}{2} \mu\text{m}$$

⇒ Typical UV written gratings have $\Delta n = 0.4 \times 10^{-3}$.
For a grating length of $L = 2$ mm $= 2 \times 10^{-3}$ m, the reflectivity is

$$R = tanh^2\left(\frac{\pi \times 0.4 \times 10^{-3} \times 2 \times 10^{-3}}{1.55 \times 10^{-3}}\right) = 0.85$$

⇒ The corresponding BW of reflection is

$$\Delta\lambda_0 = \frac{\lambda_B^2}{\pi \eta_{eff}} \sqrt{\kappa^2 L^2 + \pi^2} = 0.8\, nm$$

CODIRECTIONAL COUPLING: <u>Phase-Matched</u>

We have coupled mode equation for this case as

$$\left.\begin{aligned} \frac{dA}{dz} &= \kappa B e^{i\Gamma z} \\ \frac{dB}{dz} &= -\kappa A e^{-i\Gamma z} \end{aligned}\right\}$$

where $\Gamma = \beta_1 - \beta_2 - K$ is the phase mismatch parameter and β_1, β_2 are the propagation constants of the modes between which the coupling is to take place.

We first consider the coupling under a phase matching condition i.e., $\Gamma = 0$ i.e., the periodic perturbation has a spatial period

$$\Lambda = \frac{2\pi}{\beta_1 - \beta_2} = \frac{\lambda_0}{n_{\text{eff }1} - n_{\text{eff }2}}.$$

Under this condition,

$$\frac{dA}{dz} = \kappa B \text{ and } \frac{dB}{dz} = -\kappa A$$

which yields on differentiation

$$\frac{d^2B}{dz^2} = -\kappa^2 B.$$

The Solution of this differential equation is

$$B(z) = b_1 \cos \kappa z + b_2 \sin \kappa z;$$

And

$$A(z) = -\frac{1}{\kappa}\frac{dB}{dz} \Rightarrow A(z) = b_1 \sin \kappa z - b_2 \cos \kappa z$$

<u>Boundary conditions:</u>
At $z = 0$, mode 1, $\{E_1, \beta,\}$ is excited with unit power, $A(z = 0) = 1$ and $B(z = 0) = 0$.

$$\therefore b_1 = 0 \text{ and } b_2 = -1$$

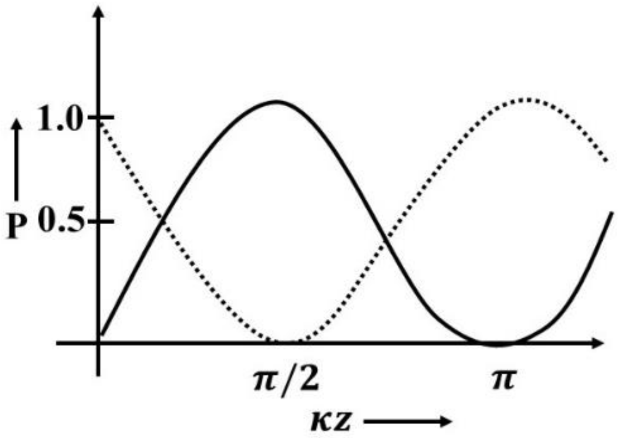
So, $$A(z) = \cos \kappa z$$
$$B(z) = -\sin \kappa z$$

So,

$$A(z) = \cos \kappa z$$
$$B(z) = -\sin \kappa z$$

Hence, the power carried by modes $\{E_1, \beta_1\}$ and $\{E_2, \beta_2\}$ vary with z as

$$P_1 = |A(z)|^2 = \cos^2 \kappa z$$
$$P_2 = |B(z)|^2 = \sin^2 \kappa z$$



Thus, we see that a periodic exchange of power between the modes takes place. Under phase-matching condition, complete transfer of power is possible.
The length of interaction required for complete transfer is $z = L_c = \frac{\pi}{2\kappa}$.

Problem: Consider a planar waveguide: $n_f = 1.51; n_s = 1.50; n_c = 1.0$ and $d = 4\ \mu$m. Solve it for eigen modes at $\lambda_0 = 0.6\mu$m.

⇒ Two TE -mode will be supposed: $n_{\text{eff }1} = 1.50862$ $n_{\text{eff }2} = 1.50460$

For a phase-matching condition to achieve complete transfer of power, we need the pitch :

$$\Lambda = \frac{2\pi}{\Delta\beta} = \frac{\lambda_0}{\Delta n_{\text{eff}}} = \frac{\lambda_0}{n_{\text{eff}} - n_{\text{eff }2}} = 149.3\mu\text{m}$$

For a planar waveguide: sinusoidal perturbation:

Coupling coefficient:

$$\kappa \simeq \frac{\pi}{\lambda_0} \cdot \frac{h}{\sqrt{d_1 d_2}} \cdot \sqrt{\frac{\left(n_f^2 - n_{ef_1}^2\right)\left(n_f^2 - n_{eff_2}^2\right)}{n_{ef_1} \cdot n_{eff2}}}$$

$$d_1 = d + \frac{1}{k_0\sqrt{n_1^2 f_1 - n_s^2}} + \frac{1}{r_0\sqrt{n_e^2 f_1^2 - n_c^2}}.$$

$$d_2 = d + \frac{1}{k_0\sqrt{n_e^2 f_2 - n_s^2}} + \frac{1}{k_0\sqrt{n_e f_2^2 - n_c^2}}.$$

Here h = amplitude of periodic thickness variation.

$d_1, d_2 \rightarrow$ effective waveguide thickness for the two modes

$n_f, n_c, n_s \rightarrow$ are the indices.

Here $d_1 = 4.678$ µm, $d_2 = 4.897$ µmand assume $h = 0.01$ um ⇒ $x = 0.598\ \text{cm}^{-1}$.

So, the coupling length $L_C = \dfrac{\pi}{2\kappa} = 2.63$ cm

CODIRECTIONAL COOPLING: **<u>Phase Mismatched</u>**

Here $\Gamma = \beta_1 - \beta_2 - K \neq 0$

So, from $\dfrac{dA}{dz} = \kappa B e^{i\Gamma z} \qquad \dfrac{dB}{dz} = -kAe^{-i\Gamma z}$

give together $\dfrac{d^2B}{dz^2} = -k\dfrac{dA}{dz}e^{-i\Gamma z} + ik\Gamma e^{-i\Gamma z}$

i.e., $\dfrac{d^2B}{dz^2} + \kappa^2 B + i\Gamma\dfrac{dB}{dz} = 0$

General sols: $B(z) = e^{-i\frac{\Gamma}{2}\cdot z}\left(b_1 e^{i\gamma z} + b_2 e^{-i\gamma z}\right) \qquad r^2 = x^2 + \dfrac{\Gamma^2}{4}$

Thus, $A(z) = \dfrac{i}{\kappa}e^{i\Gamma/2\cdot z}\left[\left(\dfrac{\Gamma}{2} - \gamma\right)b_1 e^{i\gamma z} + \left(\dfrac{\Gamma}{2} + \gamma\right)b_2 e^{-i\gamma z}\right]$

<u>Boundary Conditions:</u> $A(z = 0) = 1$ and $B(z = 0) = 0$

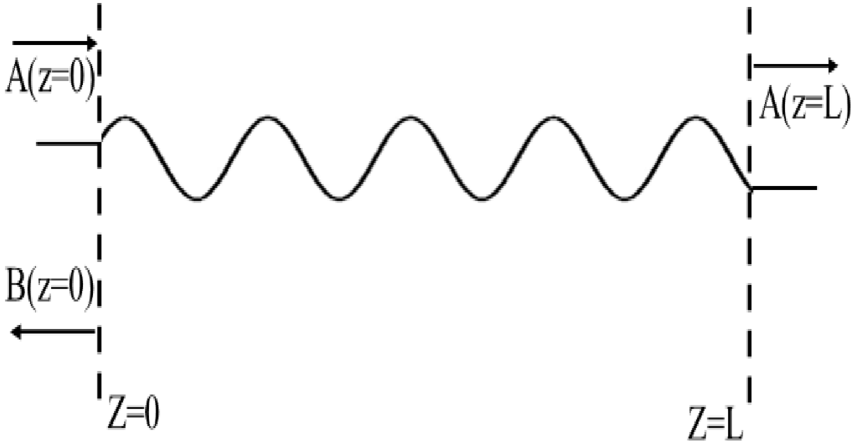
So, $b_1 + b_2 = 0 \;\Rightarrow\; b_1 = -b_2$

and, $\dfrac{i}{\kappa}\left[\left(\dfrac{\Gamma}{2} - \gamma\right)b_1 + \left(\dfrac{\Gamma}{2} + \gamma\right)b_2\right] = 1$

Solving $$b_1 = \frac{i\kappa}{2\gamma} = -b_2$$

So, $$B(z) = -\frac{\kappa}{\gamma} e^{-\frac{i}{2}\cdot z} \sin\gamma z$$

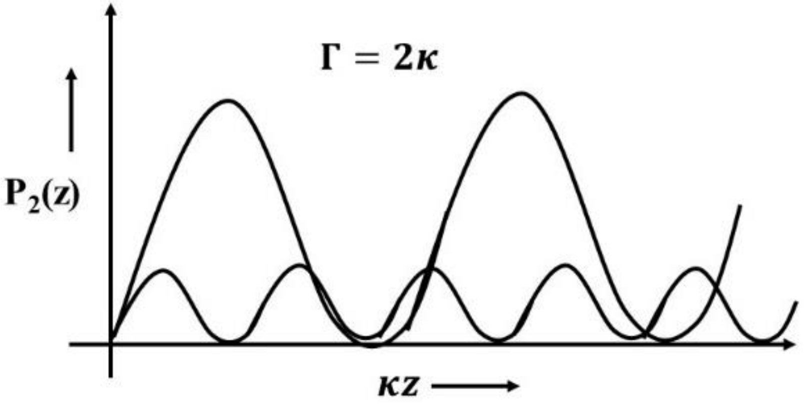
$$A(z) = e^{i\frac{\Gamma}{2}\cdot z}\left[\cos\gamma z - i\frac{\Gamma}{2\gamma}\sin\gamma z\right]$$



Thus, power in modes 1 and 2 at any value of z will be,

$$P_1 = |A(z)|^2 = \cos^2\kappa z + \frac{\Gamma^2}{4\gamma^2}\sin^2\kappa z$$

$$P_2 = |B(z)|^2 = \frac{\kappa^2}{\gamma^2}\sin^2\kappa z$$



<u>Contra-directional coupling: phase-mismatched</u> ($\Gamma \neq 0$)

We have the coupled mode equation for this case as

$$\left.\begin{aligned}\frac{dA}{dz} &= \kappa B e^{i\Gamma z}\\ \frac{dB}{dz} &= -\kappa A e^{-i\Gamma z}\end{aligned}\right\} \ldots\ldots(1)$$

where $\Gamma = \beta_1 + \beta_2 - K$ and $K = \frac{2\pi}{\lambda}$

Differentiating equations (1), we get $$\frac{d^2A}{dz^2} - i\Gamma\frac{dA}{dz} - \kappa^2 A = 0 \quad \ldots\ldots(2)$$

General solution of equation (2) is $A(z) = e^{\frac{i\Gamma}{2}z}\,[Pe^{gz} + Qe^{-gz}]$ where, $g^2 = \kappa^2 - \frac{\Gamma^2}{4}$

From (1), $B(z) = e^{-\frac{i\Gamma}{2}z}\left[\frac{(g+i\frac{\Gamma}{2})}{\kappa} Pe^{gz} - \frac{(g-i\frac{\Gamma}{2})}{\kappa} Qe^{-gz}\right]$

Now, use the boundary conditions, $A(z=0) = 1$ (Unit power launched at input) & B (z = L) = 0 (no coupling beyond z = L)

We obtain

$$\left.\begin{aligned} P &= \frac{(g - i\frac{\Gamma}{2})\, e^{-gL}}{2\{g\cosh(gL) \ + \ i\frac{\Gamma}{2}\sinh(gL)\}} \\ Q &= \frac{(g + i\frac{\Gamma}{2})\, e^{-gL}}{2\{g\cosh(gL) \ + \ i\frac{\Gamma}{2}\sinh(gL)\}} \end{aligned}\right\} \ldots\ldots(3)$$

So, the reflectivity of the periodic structure is

$$\left.\begin{aligned} R &= \frac{|B(0)|^2}{|A(0)|^2} = \frac{\kappa^2\ sinh^2(gL)}{g^2\ cosh^2(gL)\ +\ \frac{\Gamma^2}{4}\ sinh^2(gL)} \\ \&\ \ T &= \frac{|A(L)|^2}{|B(L)|^2} = \frac{g^2}{g^2\ cosh^2(gL)\ +\ \frac{\Gamma^2}{4}\ sinh^2(gL)} \end{aligned}\right\} \ldots\ldots(4)$$

$\Delta\lambda$: wavelength spacing between the minima

$g^2 = +ve$ if $\kappa^2 > \frac{\Gamma^2}{4}$ (center wavelength λ_B for which $\Gamma = 0$, $g = \kappa$)

As we deviate from λ_B, Γ increases.

And when $\Gamma^2 > 4\kappa^2$, g^2 becomes negative. i.e., $g^2 = -ve$ when $\Gamma^2 > 4\kappa^2$

when $g^2 = -ve$, hyperbolic functions in R & T become ordinary sin & cosine functions.

Thus, for $\Gamma^2 > 4\kappa^2$,

$$R = \frac{\kappa^2 \sin^2(\tilde{g}L)}{\tilde{g}^2 \cos^2(\tilde{g}L) + \frac{\Gamma^2}{4}\sin^2(\tilde{g}L)} \quad \ldots\ldots..(1)$$

(where $\tilde{g}^2 = -g^2$)

The reflectivity R becomes zero when- $\sin(\tilde{g}L) = 0 \Rightarrow \tilde{g}L = m\pi$; $m = 1,2,3\ldots.$ (Zeros in reflected spectrum)

Substituting for $\tilde{g}$,

$$\frac{\Gamma^2}{4} - \kappa^2 = \frac{m^2\pi^2}{L^2}$$

Contra directional coupling:

$$\psi(x,y,z) = A(z)\psi_1(xy)e^{-i\beta_, z} + B(z)\psi_2(x,y)e^{i\beta_2 z}$$

$$\frac{dA}{dz} = \kappa B e^{i\Gamma z} \qquad \frac{dB}{dz} = \kappa A e^{-i\Gamma z}$$ ---- co-directional

$$\psi(x,y,z) = A(z)\psi_1(xy)e^{-i\beta_, z} + B(z)\psi_2(x,y)e^{i\beta_2 z}$$

<u>Co-directional: Solution under phase-matching:</u>

$$\Gamma = 0\ , \quad \Lambda = \frac{2\pi}{\beta_1 - \beta_2} = \frac{\lambda_0}{n_{eff1} - n_{eff2}}$$

<u>CME:</u>

$$\frac{dA}{dz} = \kappa B \qquad \frac{dB}{dz} = -\kappa A$$

$$\frac{d^2 B}{dz^2} = \kappa^2 B$$

soln.

$$B(z) = b_1 \cos \kappa z + b_2 \sin \kappa z$$

$$\frac{dB}{dz} = -x b_1 \sin \kappa z + x b_2 \cos \kappa z = -\kappa A$$

$$\Rightarrow A(z) = b_1 \sin \kappa z - b_2 \cos \kappa z$$

Assume at $z = 0$,

E_1 is launcled wits unit power:

$$A|_{z=0} = 1, \quad B|_{z=0} = 0 \qquad \therefore b_1 = 0 \quad b_2 = -1$$

$$A(z) = cos\,\kappa z, \qquad B(z) = -sin\,\kappa z$$

Thus, $P_1 = |A(z)|^2 = cos^2\,\kappa z, \qquad P_2 = |B(z)|^2 = sin^2\,\kappa z$

Periodic exchange of power: $z = L_c = \frac{\pi}{2\kappa}$ (coupling length)

General case:

$$\frac{d^2B}{dz^2} = -\kappa\frac{dA}{dz}e^{-i\Gamma z} + i\kappa\Gamma A e^{-i\Gamma z} = -\kappa^2 B - i\Gamma\frac{dB}{dz}$$

or

$$\frac{d^2B}{dz^2} + i\Gamma\frac{dB}{dz} + \kappa^2 B = 0$$

Solution: $B(z) = e^{-\frac{i\Gamma z}{2}}\left[b_1 e^{i\gamma_z} + b_2 e^{-i\gamma z}\right]$

$$\gamma^2 = \kappa^2 + \frac{\Gamma^2}{4}$$

$$A(z) = \frac{i}{\kappa} e^{i\frac{\Gamma}{2}\cdot z}\left[\left(\frac{\Gamma}{2} - \gamma\right) b_1 e^{i\gamma z} + \left(\frac{\Gamma}{2} + \gamma\right) b_2 e^{-i\gamma_z}\right]$$

$$A|_{z=0} = 1 \qquad\qquad B|_{z=0} = 0$$

$$b_1 + b_2 = 0$$

$$\frac{i}{k}\left[\left(\frac{\Gamma}{2} - \gamma\right) b_1 + \left(\frac{\Gamma}{2} + \gamma\right) b_2\right] = 1 \Rightarrow\ b_1 = i\frac{\kappa}{2\gamma} = -b_2$$

$$\therefore B(z) = -\frac{\kappa}{\gamma} e^{-i\frac{\Gamma}{2}\cdot z} sin\,\gamma z$$

$$A(z) = e^{i\frac{\Gamma}{2}z}\left[cos\,\gamma z - i\frac{\Gamma}{2\gamma} sin\,\gamma z\right]$$

$$P_1(z) = |A(z)|^2 = cos^2\gamma z + \frac{\Gamma}{4\gamma^2}\cdot sin^2\gamma z$$

$$P_2(z) = \frac{\kappa^2}{\gamma^2} sin^2\gamma z$$

Contra directional coupling: phase matched:

$$\frac{dA}{dz} = \kappa B e^{i\Gamma z} \qquad \Gamma = 0 \qquad \beta_1 + \beta_2 = \kappa$$

$$\frac{dB}{dz} = \kappa A e^{i\Gamma z} \qquad \Lambda = \frac{\lambda_0}{2n_{eff}}$$

$$\frac{d^2B}{dz^2} = \kappa^2 B$$

$$B(z) = b_1 e^{kz} + e^{-kz}$$

$$A(z) = b_1 e^{kz} - b_2 e^{-kz}$$

$$A|_{z=0} = 1 \qquad B|_{z=0} = 0$$

Unit power in mode A of periodic wavelength of length L

$$b_1 e^{\kappa L} + b_2 e^{-\kappa L} = 0;\ b_1 - b_2 = 1$$

$$b_1 = \frac{e^{-\kappa L}}{2\, cos\, h\kappa L}, \qquad b_2 \quad = \frac{-e^{\kappa_L}}{2cos\, h\kappa L}$$

$$B(z) = \frac{\sin \kappa(z-L)}{cos\, h\kappa L}, \qquad A(z) = \frac{cos\, \kappa(z-L)}{cos\, h\kappa L}$$

$$|A(z)|^2 - |B(z)|^2 = (cos^2 h\kappa L)^{-1} = const.$$

$$\gamma = \frac{B(z=0)}{A(z=0)} = -\, tan\, h\kappa L$$

$$R = |\gamma|^2 = tanh^2\, kL$$

$$R = tanh^2 \left(\frac{\pi \Delta n L}{\lambda_0}\right)$$

Fabricate a reflector centered around 1550 mm.

$$\Lambda = \frac{\lambda_0}{2\, n_{eff}} = \frac{1550}{2 \times 1.46} \simeq 531 \text{ nm.}$$

NN written grating $m = 0.4 \times 10^{-3}$, grating length 2 mm.

$$R = tanh^2\left(\frac{\pi \Delta L}{\lambda} L\right)$$

$$= tanh^2\left(\frac{3.14 \times 0.4 \times 10^{-3}}{1550 \times 10^{-6}} \times 2 \times 10^{-3}\right)$$

$$= 0.85$$

Corresponding Bandwidth

$$\Delta\lambda_0 = \frac{\lambda_B^2}{\pi n_{\text{eff}} L} (\kappa^2 L^2 + \pi^2)^{\frac{1}{2}}$$

$$\simeq 0.8 \text{ nm}.$$

Centre wavelength: 1092 nm, BW(FWHM) = 0.8 nm

Length $= 1$ mm, $R = 0.98$. Calculate the Λ = ? Assume $n_{\text{eff}} = 1.46$.

$$R = tan^2\, h\kappa L = 0.98$$

$$\Rightarrow \kappa = \frac{1}{2L} \ln\left(\frac{1+\sqrt{R}}{1-\sqrt{R}}\right) = 2.64 \text{ mm}^{-1}.$$

$$\Lambda = \frac{\lambda_0}{2n_{\text{eff}}} = \frac{1.092}{2 \times 1.46} \simeq 0.37 \text{ μm}.$$

$$A(z) = \frac{i}{\kappa} e^{i\frac{\Gamma}{2}\cdot z}\left[\left(\frac{\Gamma}{2} - \gamma\right) b_1 e^{i\gamma z} + \left(\frac{\Gamma}{2} + \gamma\right) b_2 e^{-i\gamma_z}\right]$$

$$A|_{z=0} = 1 \qquad\qquad B|_{z=0} = 0$$

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$$\frac{i}{k}\left[\left(\frac{\Gamma}{2} - \gamma\right) b_1 + \left(\frac{\Gamma}{2} + \gamma\right) b_2\right] = 1 \Rightarrow b_1 = i\frac{\kappa}{2\gamma} = -b_2$$

$$\therefore B(z) = -\frac{\kappa}{\gamma} e^{-i\frac{\Gamma}{2}\cdot z} si\,n\,\gamma z$$

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$$\frac{d^2B}{dz^2} = \kappa^2 B$$
$$B(z) = b_1 e^{kz} + e^{-kz}$$
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