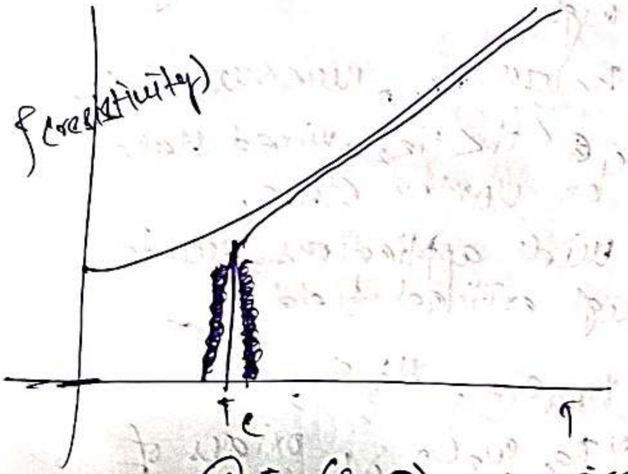
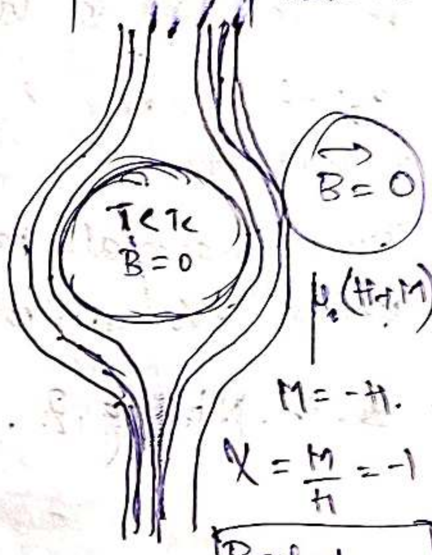
SUPER CONDUCTIVITY. 27/3/24

Superconductivity is a macroscopic quantum phenomenon.



@ T_c ($\rho = 0$) → meissner effect.



$\mu_0(H+M)$

$M = -H$.

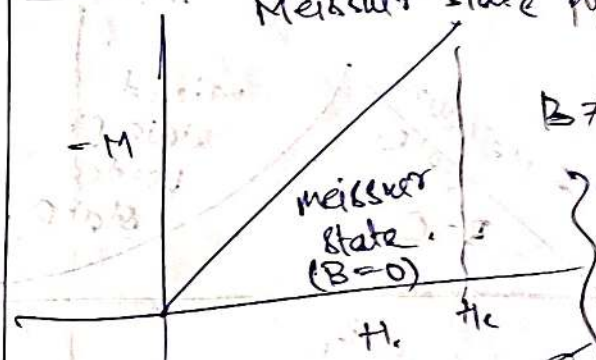
$\chi = \frac{M}{H} = -1$.

$\boxed{\text{Perfect diamagnet}}$

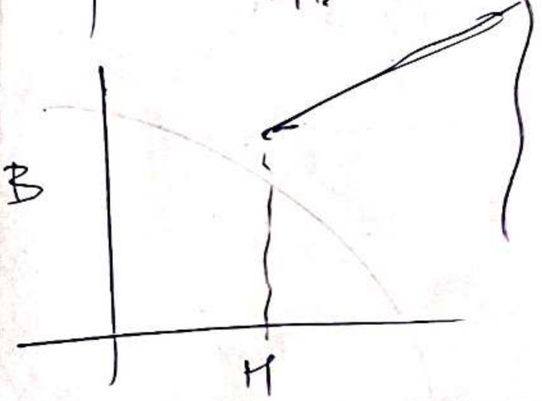
$Y_1 Ba_2 Cu_3 O_{7-d}$

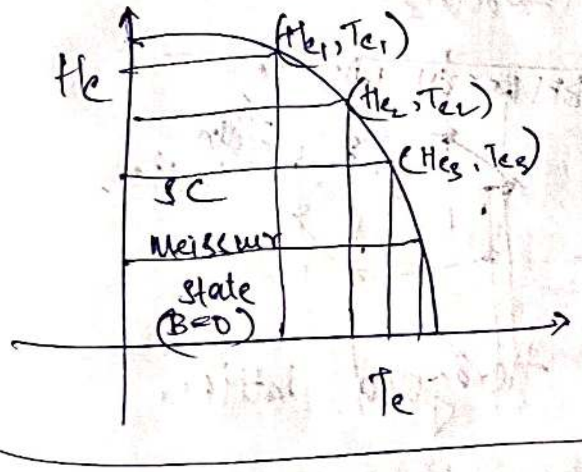
maglev trains
500 km/h.

Meissner state plot.



B ≠ 0.



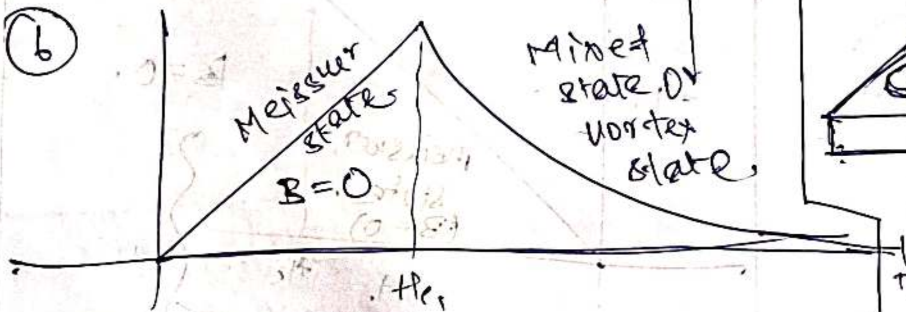


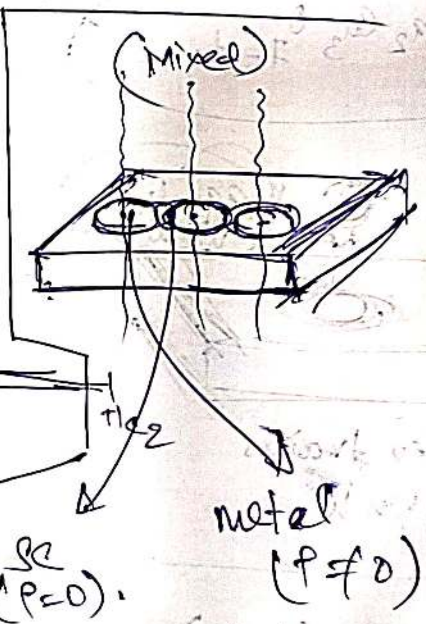
$$H_c = H_0\left[1 - \left(\frac{T_a}{T_c}\right)^2\right].$$

Type 1	Type 2
(1) Only 1 critical field exists (H_c)	(1) Two critical fields H_{c_1} & H_{c_2}
(2) critical field is low.	(2) critical field H_{c_2} is very high.
(3) Exhibit complete & perfect meissner state.	(3) Below H_{c_1}, meissner state. $H_{c_1} < H < H_{c_2}$, mixed state or vortex state.
(4) These materials have limited tech application.	(4) wide applications due to high critical field.
(5) Sn, Pb, Nb, Hg, Zn.	(5) Nb_3Ge, Nb_3Si, $YBa_2Cu_3O_{7-\delta}$, oxides of $[Bi, Sr, Ca, Cu]\,O_{n+\delta}$.

Type 2

(6)





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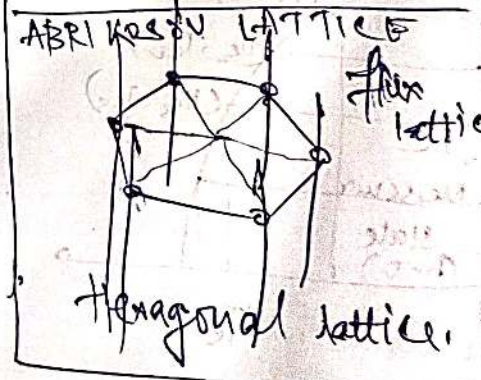

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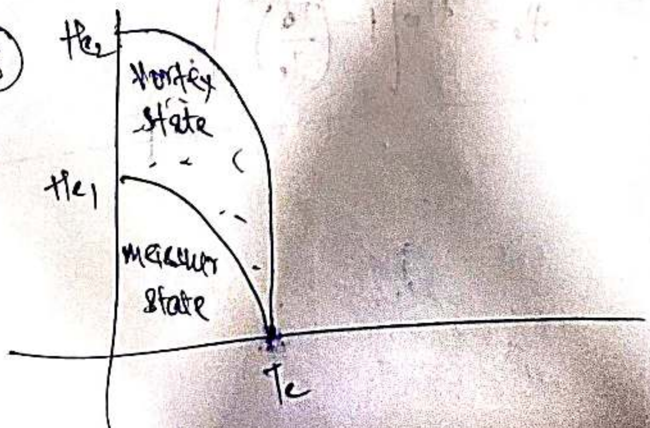
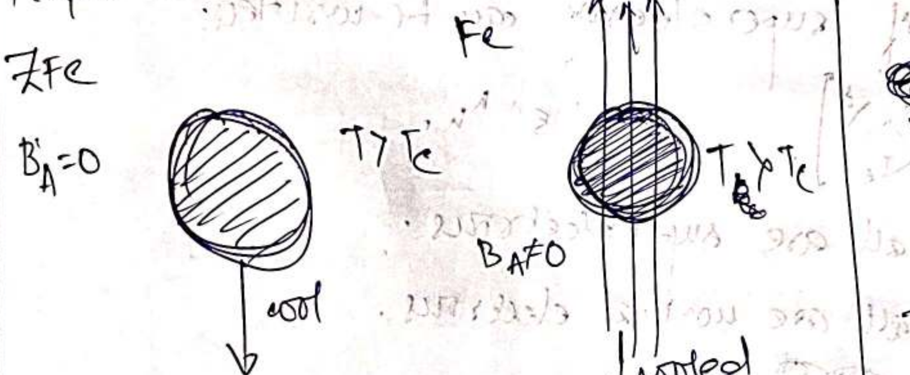
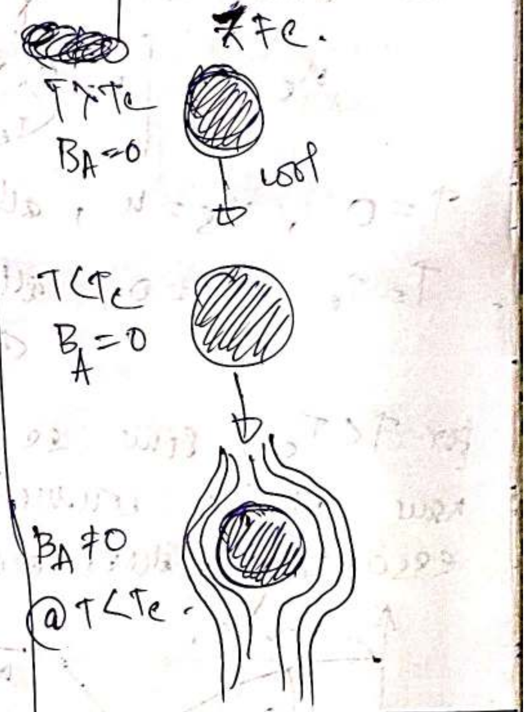


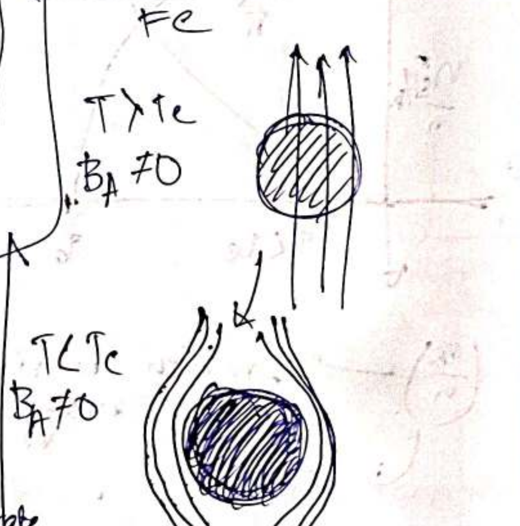
Perfect conductor.	Superconductor.	Type 1	Type 2.
$\rho = 0$ $E = \rho J$ $\nabla \times E = 0$ $-\frac{\partial B}{\partial t} = 0.$ $\Rightarrow B = \text{const.}$	$\rho = 0$, $B = 0$ 0! Not only const but zero.	$\xi_g > \lambda$ ξ_g: coherence length λ: penetration depth $\kappa = \frac{\xi_g}{\lambda} > 1$	$\xi_g < \lambda$ $\kappa < 1$.

Perfect conductor



super conductor.





The magnetic state of perfect conductor is dependent on the order in which the field is applied and the state of perfect conductivity is achieved.

If the field is first switched on PC, field won't penetrate.

Sequence of applied magnetic field is immaterial for a superconductor.

LONDON - Pippard theory:

(1) 1934, two different types of e^- (conduction e^-):
(1) superelectrons (2) Normal electrons.

(2) $n = n_s + n_n$ = total e^- density.

(3) The super electron do not experience any scattering, have zero entropy (perfect order) & long coherence length (ξ).

(4) Normal e^- behave in usual fashion & experience resistive scattering due to collision.

(5) The concentration of superelectron can be written:

$$n_s = n\left[1-\left(\frac{T}{T_c}\right)^4\right], \quad n = n_s + n_n.$$

$T=0$, $n_s = n$, all are superelectrons.

$T = T_c$, $n_s = 0$ all are normal electrons.

(6) for $T<T_c$, some sc electrons are present since they have infinite conductivity (no scattering), they essentially short circuit the normal electrons.

$$\boxed{J_n = -n_n e v_n = \sigma_n E}$$

(normal electron)

For the superelectron are considered to be not affected by scattering interaction.

$$\boxed{m_e \frac{dv_s}{dt} = -eE}$$

[if damping term is dropped from classical eqn].

$\left(\frac{n_s}{n}\right) \rightarrow$ fraction carries electrical energy.

$\rightarrow$ below T_c

$J_s = -n_s e v_s$.

$$\frac{dJ_s}{dt} = -n_s e \frac{dv_s}{dt} = -\frac{n_s e^2 E}{m_e} \rightarrow \text{first London eqn for perfect conductor.}$$

Take curl on both sides,

$$\nabla \times \frac{dJ_s}{dt} = \nabla \times \left(\frac{n_s e^2 E}{m_e}\right) \qquad (\nabla \times E = -\frac{dB}{dt})$$

$$\frac{d}{dt}(\nabla \times J_s) = -\frac{n_s e^2}{m_e}\frac{dB}{dt}$$

$$\therefore \frac{d}{dt}\left[(\nabla \times J_s) + \left(\frac{n_s e^2 \vec{B}}{m_e}\right)\right] = 0.$$

contribution

London chose the const to be zero.

$$\therefore \boxed{\nabla \times J_s + \frac{n_s e^2}{m_e}\vec{B}} = C = \boxed{0}.$$

$$\boxed{\begin{aligned} B &= \nabla \times A, \\ E &= \nabla \phi. \end{aligned}} \qquad \frac{d}{dt}\left[(\nabla \times J_s) + \nabla \times \left(\frac{n_s e^2}{m_e} A\right)\right] = 0$$

$$\boxed{\frac{dJ_s}{dt} = \frac{n_s e^2}{m_e} E.}$$

First London equ.

$$\Rightarrow \frac{d}{dt}\left[\nabla \times \left\{ J_s + \frac{n_s e^2}{m_e} A \right\}\right] = 0.$$

$$\therefore \boxed{J_s = -\frac{n_s e^2}{m_e} A}$$

second London equ

$$\nabla \times (\nabla \times J_s) = \frac{n_s e^2}{m_e}(\nabla \times B)..$$

$$\because \boxed{\nabla \times B = \mu_0 J_s.}$$

$$\therefore \boxed{\nabla \times (\nabla \times J_s)} = \boxed{-\frac{n_s e^2}{m_e} \mu_0 J_s}. = \boxed{-\frac{1}{\lambda_L^2} J_s.}$$

Penetration depth: $(\lambda_L) = \sqrt{\frac{m_e}{n_s e^2 \mu_0}}$

$$\Rightarrow \nabla \cdot (\nabla \cdot J_s) - \nabla^2 J_s =$$

OR

$$\boxed{\nabla (\nabla \cdot B) - \nabla^2 B = -\frac{1}{\lambda_L^2} B.}$$

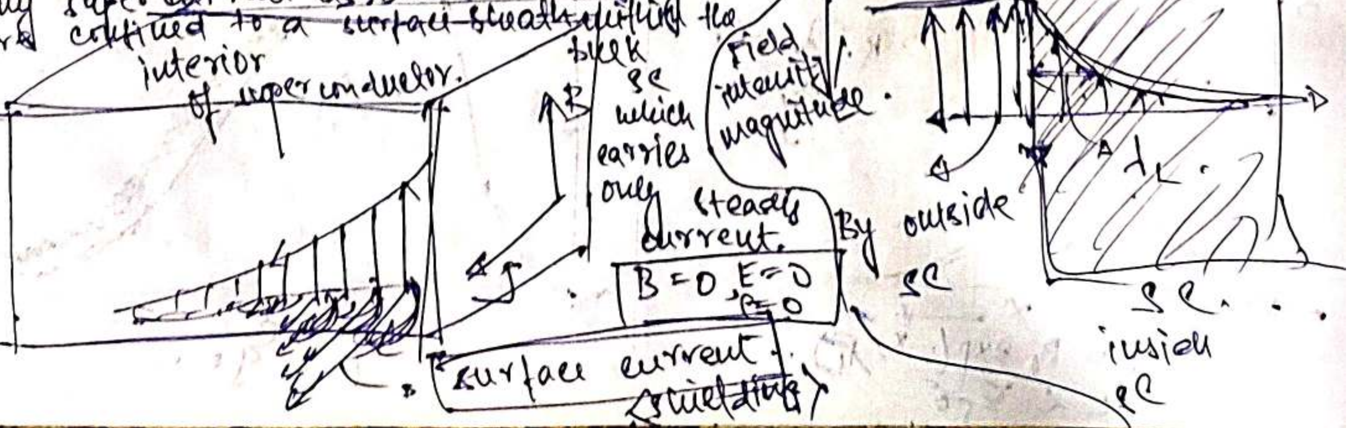
$$\boxed{\begin{aligned} \nabla \times (\nabla \times J) &= -\frac{1}{\lambda_L^2} J_s \\ \nabla \times (\nabla \times B) &= -\frac{1}{\lambda_L^2} B. \end{aligned}}$$

B inside SC (meissner state = 0).

$$\Rightarrow \boxed{\nabla^2 B = \frac{1}{\lambda_L^2} B.} \xrightarrow{\text{solution}} \boxed{\begin{aligned} B_y(x) &= B_0(x) \exp\left(-x/\lambda_L\right)\hat{e}_y \\ J_s(x) &= -J(x) \exp\left(-x/\lambda_L\right). \end{aligned}}$$

meissner state.

Any super current associated with B_y are confined to a ~~surface sheath within the~~ interior of superconductor.

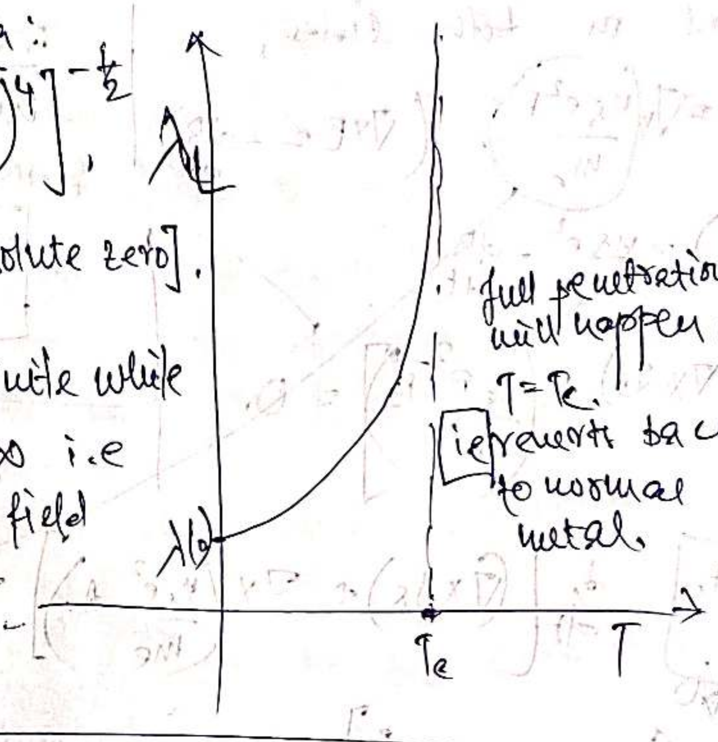


Empirical formula :

$$\lambda_L = \lambda_L(0)\left[1-\left(\frac{T}{T_c}\right)^4\right]^{-\frac{1}{2}},$$

$\lambda_L \sim 500\,\text{Å}$ [@ absolute zero].

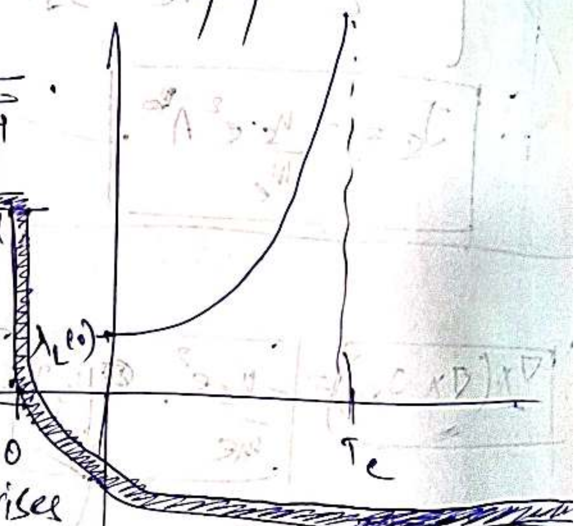
λ_L @ $T=0$ is finite while @ $T \gg T_c$, $\lambda_L \to \infty$ i.e full penetration & field penetrates wholly.



COHERENCE length.

2/3/25

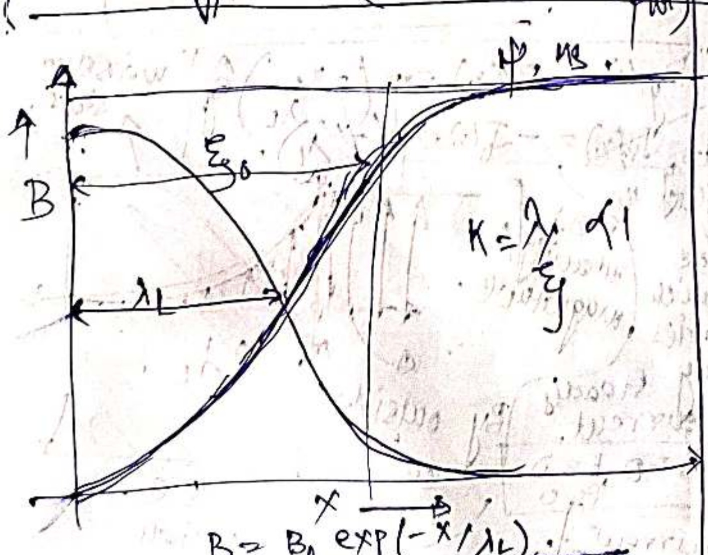
Penetration depth $\lambda_L = \dfrac{\lambda_L(0)}{\sqrt{1-\left(\frac{T}{T_c}\right)^4}}$.



coherence length

→ It is the measure of the natural length scale over which the spatial variation of the order parameter costs superconductivity energy.
The order parameter which is zero at the axis (highest field there) rises to its equilibrium value in radial distance ξ_0.
n_s cannot change rapidly but change appreciably within a distance $\sim 10^{-4}$ cm (for SC).

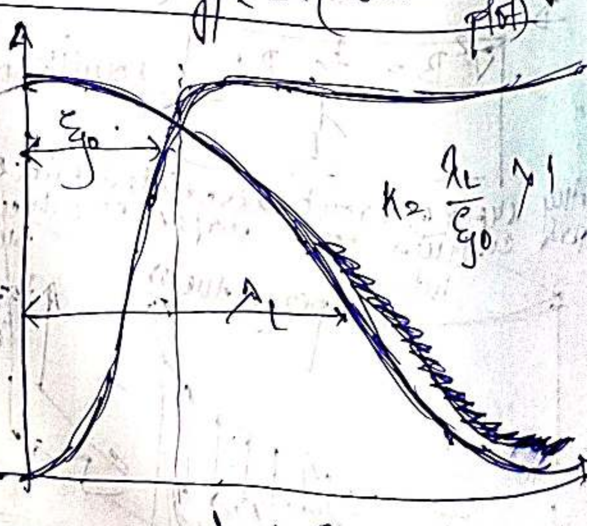
Type 1. (coherence length plot)



$B = B_0 \exp(-x/\lambda_L)$.

$\lambda_L < \xi_0$

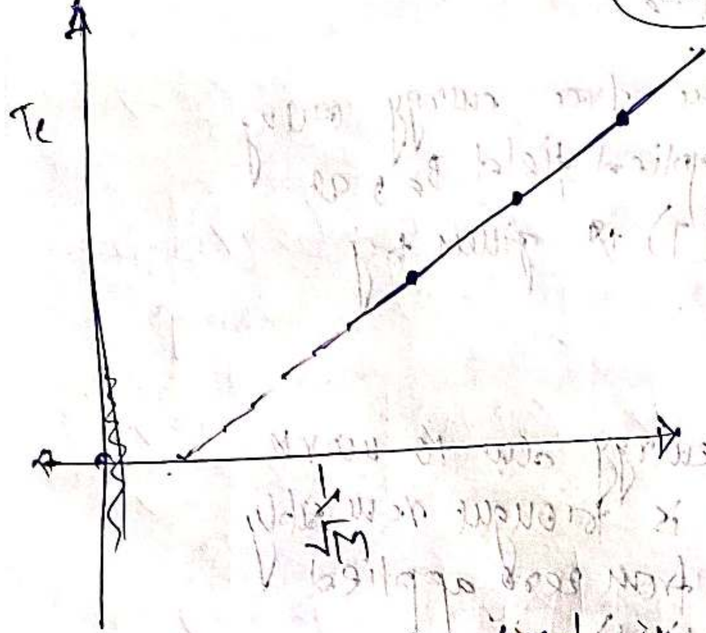
Type 2. (coherence length plot)



$\lambda_L > \xi_0$.

Isotope theory:-

$T_c \propto \frac{1}{\sqrt{M}}$ expt observation.



$T_c M^{\beta} = const$

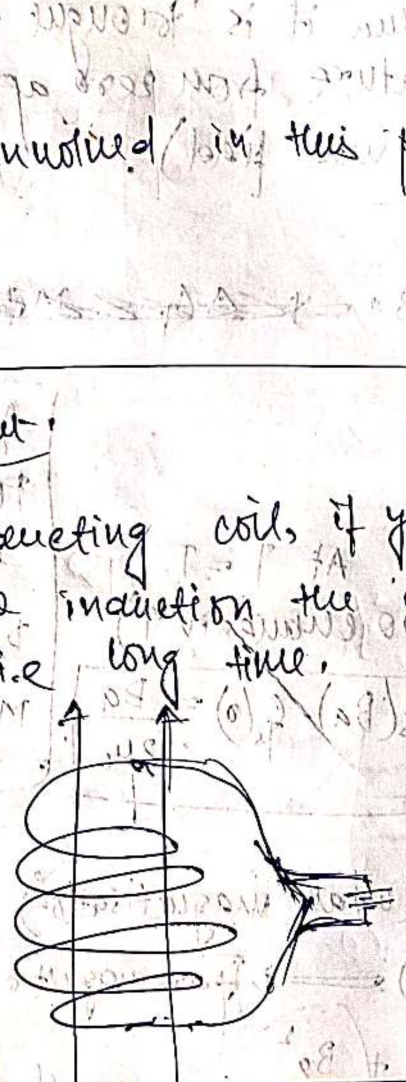
the value of critical temp or transition temperature T_c of a superconductor is found to vary with its isotopic mass.

Lattice is involved in this particular observation.

Persistent current:

In a superconducting coil, if you introduce some current due to induction the current stays for a persistent while i.e long time.

$\frac{d\phi}{dt} = \varepsilon$



$i = i_0 e^{-tR/L}$

$R \to 0$,

thus, $i = i_0 \approx 10^5$ years.

Thermodynamics of a superconductivity transition.

The transition between normal & SC state is thermodynamically reversible just like a transition b/w liquid & vapour of a substance is reversible.

- Normal state entropy
- Superconductive state entropy.

$S_N - S_S$: Gibbs free energy

$$G = U - TS - M \cdot B_a$$

Any small change in the free energy resulting to a small change in applied field B_a, at a constant temperature (T) is given by

$$\boxed{dG = -M\, dB_a}$$

The change in Gibbs free energy due to work done on a SC when it is brought reversibly at constant temperature from zero applied field to $B_a = B_a$ (finite field).

$$\int_{T,0}^{T,B_a} dG = \int_0^{B_a} -M\, dB \Rightarrow \cancel{\Delta G = -M B_a}$$

$$\Delta G = + \int_0^{B_a} \frac{B}{\mu_0}\, dB$$

$\chi = -1$.
For a SC material
$M = -H$.
$B = 0 = \mu_0 (M + H)$
$M = -H = -\frac{B}{\mu_0}$.

$$G_S(B_a, T) - G_S(0,T) = \frac{B_a^2}{2\mu_0}$$

[critical/maximum field can be applied to SC @ T_1]

$\therefore$ At $T = T$ (No change in T)

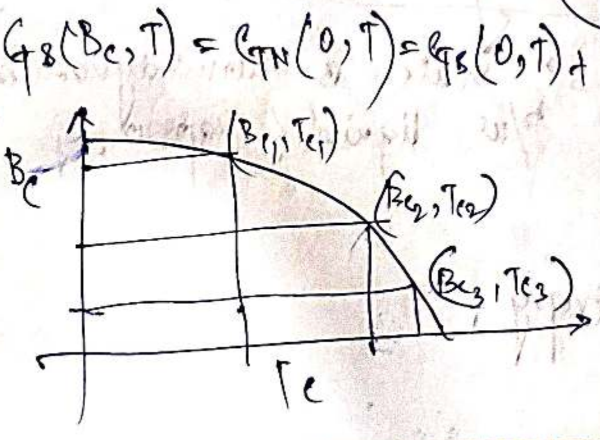
$$\boxed{G_S(B_a) - G_S(0) = \frac{B_a}{2\mu_0}}$$

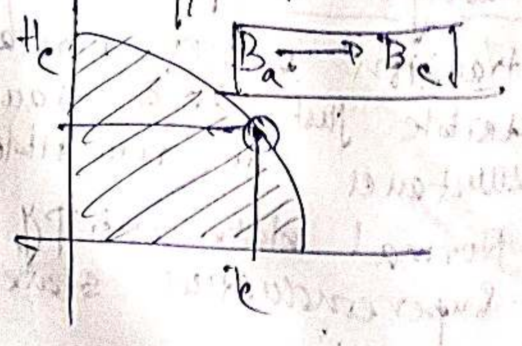
$\therefore$ For Normal state (weak magnetisation)

Normal state $G_N(B,T) = G_N(0,T) \Longrightarrow$ Paramagnetic character

$$G_S(B,T) = G_S(0,T) + \frac{B_a^2}{2\mu_0} \longrightarrow \text{small change in applied field.}$$

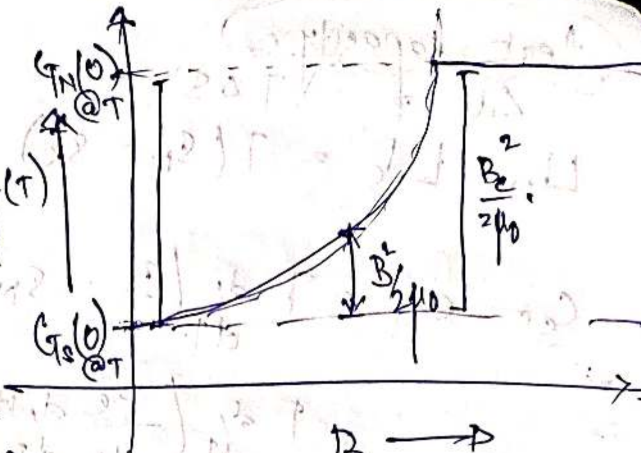
$$G_S(B_c, T) = G_N(0,T) = G_S(0,T) + \frac{B_c^2}{2\mu_0}$$





$B_a \rightarrow B_c$

$G_N(0,T) - G_s(0,T) = \frac{B_c^2}{2\mu_0}$.



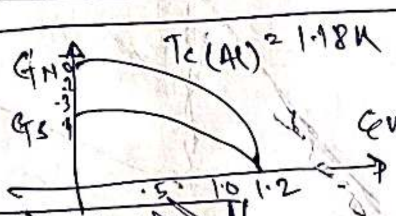
- In the $B=0$, SC state is lower in free energy by $\frac{B_c^2}{2\mu_0}$ from the normal state per unit volume. In the presence of small field below T_c the specimen has to choose B/μ_0 gaining in energy by forcing all the magnetic field out (retaining SC character) and gaining in energy by letting the flux in (going to normal). The superconductivity is found to be energetically favourable for the small field but NOT for large field.

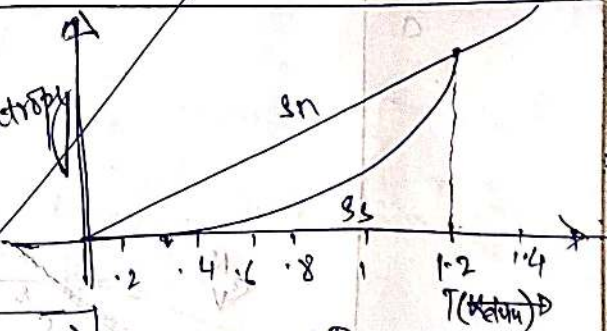
Criteria

$$\boxed{G_N(T_1) - G_S(0,T_1)} = \frac{B_c^2(T_1)}{2\mu_0}$$

$$\boxed{B_c(T_1) = \sqrt{2\mu_0\left[G_{N_0}(T_1) - G_{S0}(T_1)\right]}}.$$

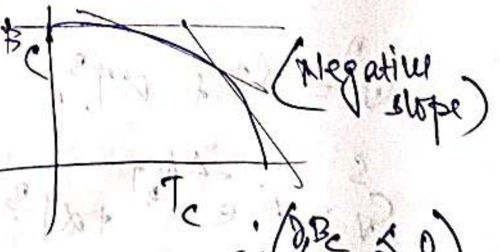
Entropy





$$S = -\frac{\partial}{\partial T} G = \boxed{\left(-\frac{\partial}{\partial T}G_N\right) + \left(\frac{\partial}{\partial T}G_S\right)} = -\frac{B_c}{\mu_0}\frac{\partial B_c}{\partial T}$$

$\Rightarrow S_N - S_s = -\frac{B_c}{\mu_0}\left(\frac{\partial B_c}{\partial T}\right)$ → Boundary (limiting)



(Negative slope)

$\therefore S_N - S_S \geq 0 = \frac{B_c}{\mu_0}(-)(\text{slope})$.

$\therefore \left(\frac{\partial B_c}{\partial T} \leq 0\right)$

$\therefore S_N \geq S_s$ → superconducting state is MORE ordered.

Heat capacity :

$\Delta U = T\Delta S$

$U_N - U_s = T(S_N - S_s)$

$C_s - C_N = T \frac{d}{dT}(S_s - S_N)$

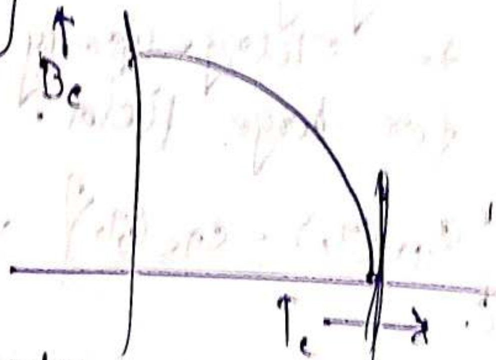
$= T\frac{d}{dT}\left(\frac{B_c}{\mu_0}\frac{d}{dT}B_c\right).$

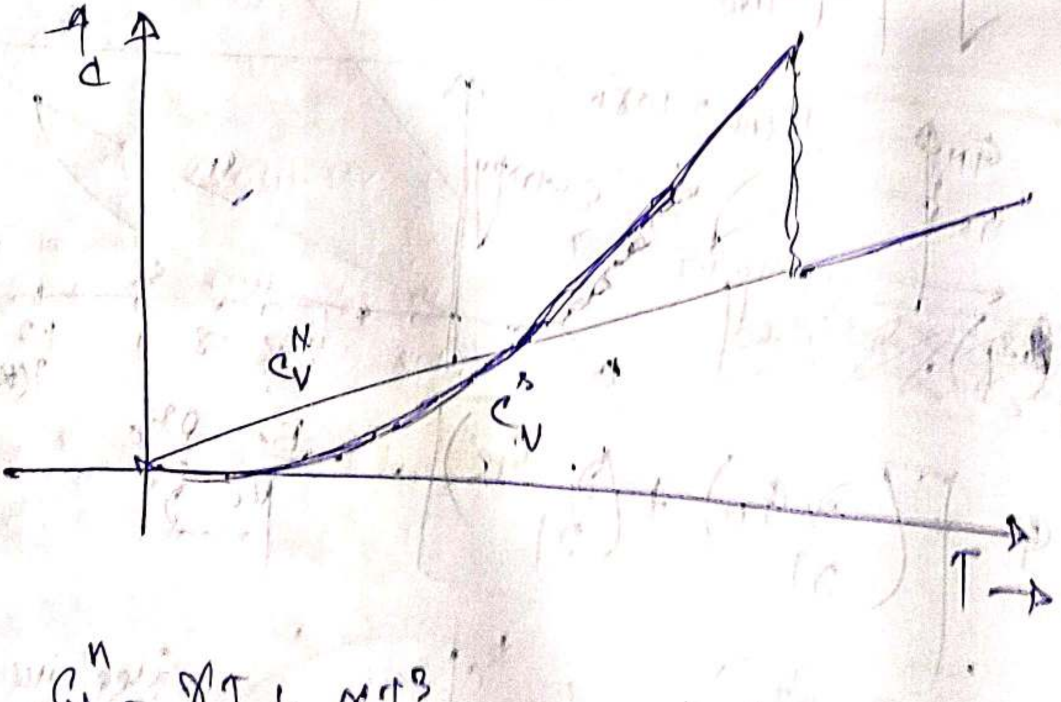
$= \frac{T}{\mu_0}\left(\frac{d}{dt}B_c\right)^2 + \frac{TB_c}{\mu_0}\frac{d^2}{dT^2}B_c$

$\boxed{\text{Near } B_c = 0 \text{ neighbourhood}}$

slope $\left(\frac{\partial B_c}{\partial T_c}\right)$ is huge.

$\therefore \boxed{C_s - C_N = \frac{T}{\mu_0}\left(\frac{d}{dt}B_c\right)^2}$ dominates.





$C_V^n = \gamma T + \alpha T^3.$

$C_V^S = A.e^{-\Delta/k_BT} + \alpha T^3$