Scaling law **

$2 = \alpha + 2\beta + \gamma$

$\delta = 1 + \left(\frac{\gamma}{\beta}\right)$

$\alpha = 2 + \delta\gamma$

(#) Fe $T_c = 1043K$

$\beta = 0.36 \pm 0.02$

$\gamma = 1.33 \pm 0.02$

$\delta = 4.35 \pm 0.02$

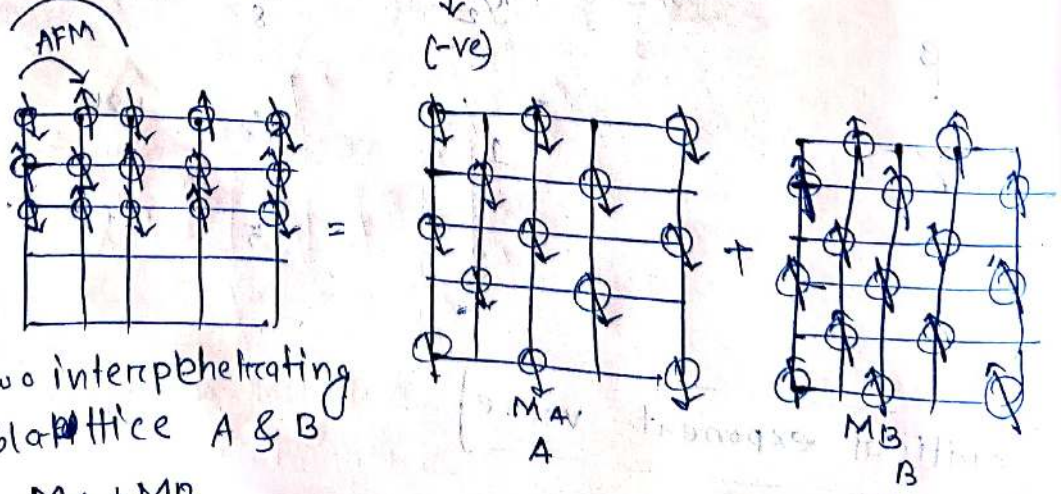
19/03/2025

Antiferromagnatism

In 1936 Neel shown theoritically that if exchange integral J_{ex} = -ve, there a state of lowest energy is obtained where the spins of neighbouring atoms have opposite orientation. Such material are known as AFM.

$$H_{spin} = -2J_{in}\ \vec{S_i} \cdot \vec{S_j}$$

(-ve)



Two interpenetrating sublattice A & B

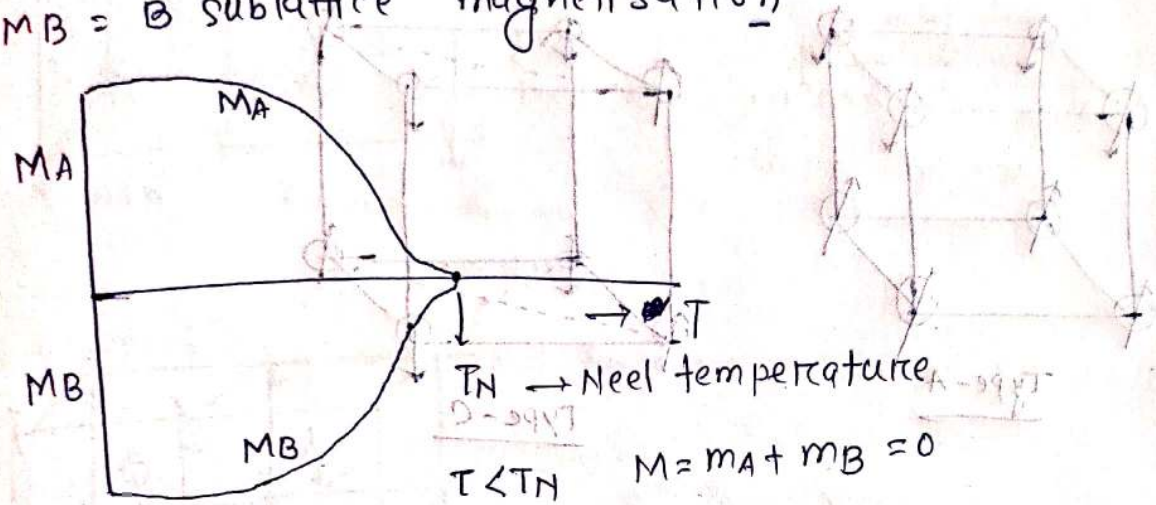
$M_A + M_B$

nearest neighbour interaction is AFM

next " " " FM

M_A = A-sublattice magnetisation

M_B = B sublattice magnetisation

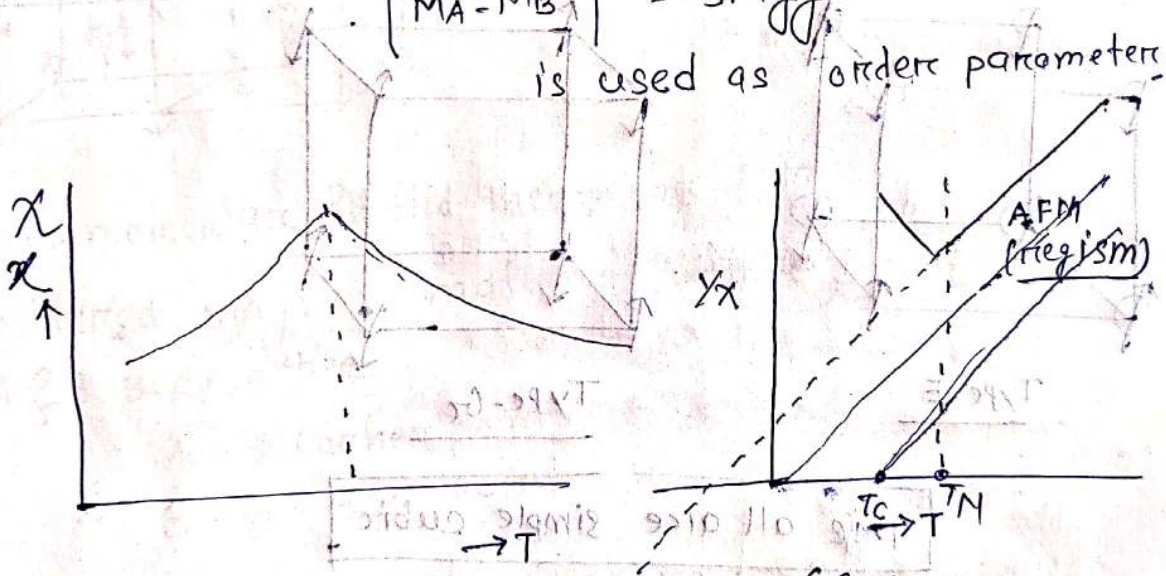


T_N → Neel temperature

$T < T_N$ $M = m_A + m_B = 0$

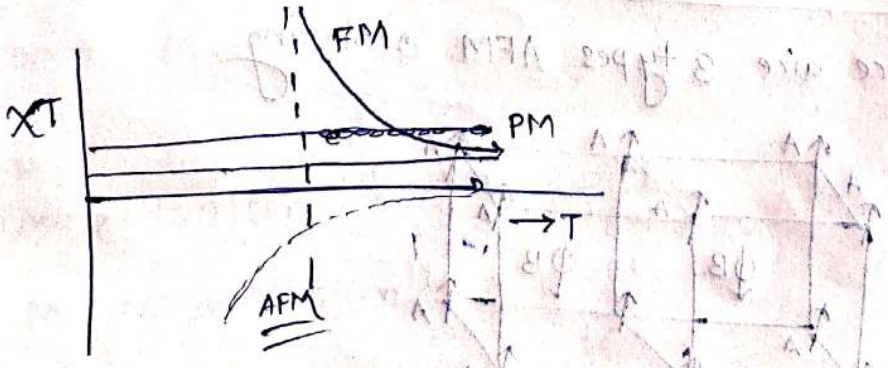
$|M_A - M_B|$ = staggered magnetisation

is used as order parameter.



$\chi = \left(\frac{C}{T+\theta}\right)$

curie weiss-law

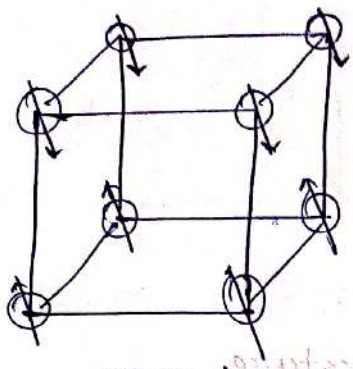


$\theta = 0$ PM

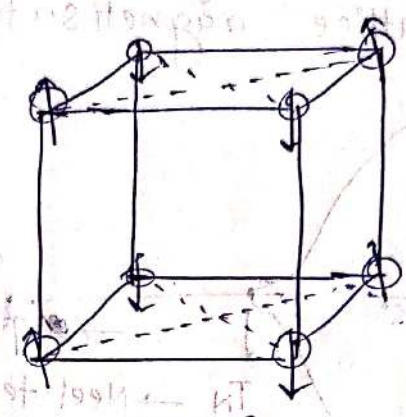
$\theta > 0$ FM

$\theta < 0$ AFM

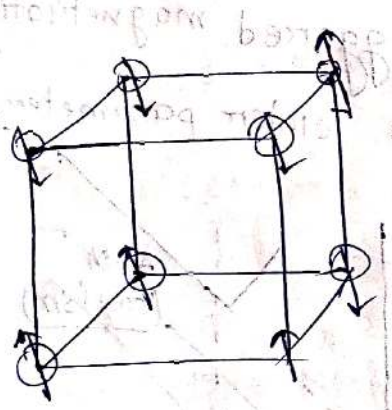
Types of AFM
cubic crystal



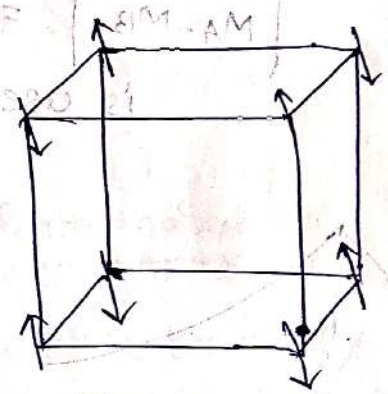
Type-A



Type-C



Type-E

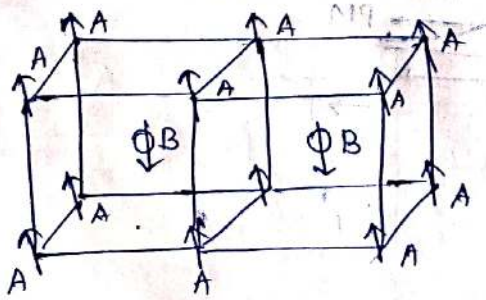


Type-G

These all are simple cubic

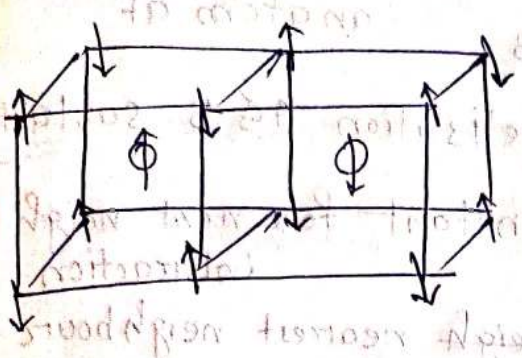
(#) In Bcc structure

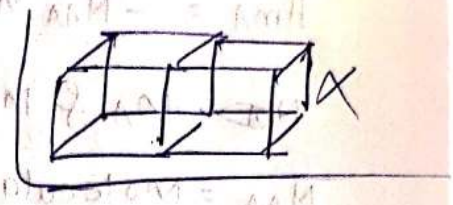
There are 3 types AFM ordering

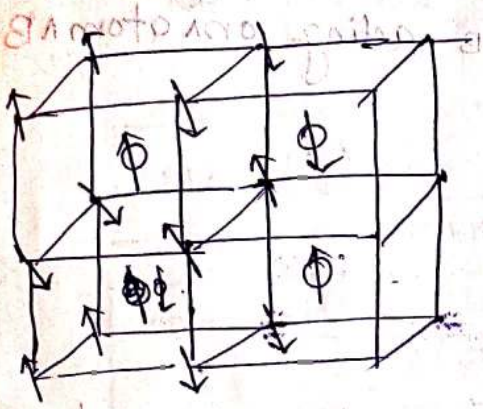


ϕ B-sublattice

ϕ A → sublattice







weiss molecular field theory for AFM considered magnetic materials with two sublattice A & B BCC lattice

A → Corner point

B → BCC point

An atom at A-site has nearest neighbour that all lie on B-sites & next neighbour that all lie on A site.

M+ Magnetisation of A sublattice

M- Magnetisation of B sublattice.

$H_{M+} = -\lambda M-$

$M_{M-} = -\lambda M+$

The molecular field H_{mA} acting on A site / an atom at

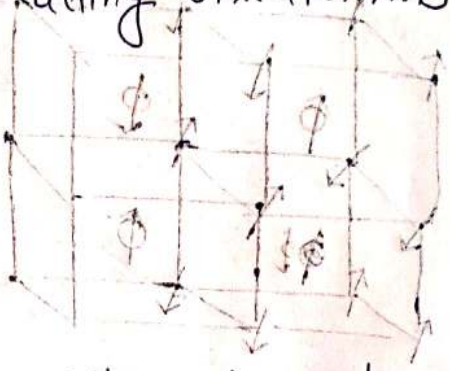
$$H_{mA} = -N_{AA} M_A - N_{AB} M_B$$

M_A & M_B = Magnetisation A & B sublattice

N_{AA} = Molecular field constant for next neighbour interaction

N_{AB} = Molecular 2 for nearest neighbour interaction.

similarly the molecular field H_{mB} acting on an atom at B-site



$$H_{mB} = -N_{BA} M_A - N_{BB} M_B$$

$$N_{AA} = N_{BB} = N_{ii}$$

$$N_{BA} = N_{AB}$$

If H field is applied the field H_A & H_B at an atom on two A & B lattice would be

$$H_A = H - N_{ii} M_A - N_{AB} M_B \quad \text{---(I)}$$

$$H_B = H - N_{AB} M_B - N_{ii} M_B \quad \text{---(II)}$$

$$\boxed{\begin{aligned} H_A &= H + H_{mA} \\ H_B &= H + H_{mB} \end{aligned}}$$

(#) Magnetisation $(M_A) = \frac{N}{2} g \mu_B J B_j(x_A)$

$$(M_B) = \frac{N}{2} g \mu_B J B_j(x_B)$$

$$x_A = \frac{J g \mu_B}{K_B T} (H_A)$$

$$x_B = \frac{J g \mu_B}{K_B T} H_B$$

$$B_j(x_A) = \frac{(2j+1)}{2j}\coth\frac{(2j+1)}{2j}x_A - \frac{1}{2j}\coth\frac{1}{2j}x_A$$

case-I Behaviour above T_N, $T > T_N$

$$B_j(x) \longrightarrow \frac{(J+1)}{3j}(x)$$

$x \rightarrow$ Become small

$$M_A = \left(\frac{N}{2}\right) g\mu_B(J)\frac{(J+1)}{3J}\left(\frac{Jg\mu_B}{K_BT}\right)\cdot(H_A)$$

$$\boxed{M_A = \left(\frac{N}{2}\right)\frac{g^2\mu_B^2 j(J+1)}{6K_BT}(H_A)}$$

$$\boxed{M_B = \frac{Ng^2\mu_B^2 j(J+1)}{6K_BT}H_B}$$

$$M_A = \frac{Ng^2\mu_B^2 j(J+1)}{6K_BT}\left(H - N_{ii}M_A - N_{AB}M_B\right)$$

$$M_B = \frac{Ng^2\mu_B^2 j(J+1)}{6K_BT}\left(H - N_{AB}M_A - N_{ii}M_B\right)$$

Total magnetisation

$$M = (M_A + M_B)$$

$$M = \frac{Ng^2\mu_B^2 j(J+1)}{6K_BT}\left[2H - (N_{ii}+N_{AB})M\right]$$

$$M\left[1 + \frac{Ng^2\mu_B^2 j(J+1)}{6K_BT}(N_{ii}+N_{AB})\right] = \frac{Ng^2\mu_B^2 j(J+1)\cdot H}{3K_BT} = \left(\frac{C}{T}\right)H$$

$$\chi = \frac{M}{H} = \frac{(C/T)}{1 + \frac{C}{2T}(N_{ii}+N_{AB})}$$

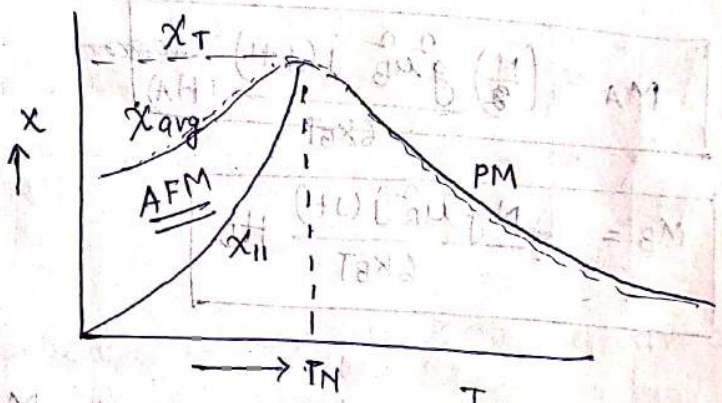
$$\boxed{\chi = \frac{C}{T + \frac{C}{2}(N_{ii}+N_{AB})}}$$

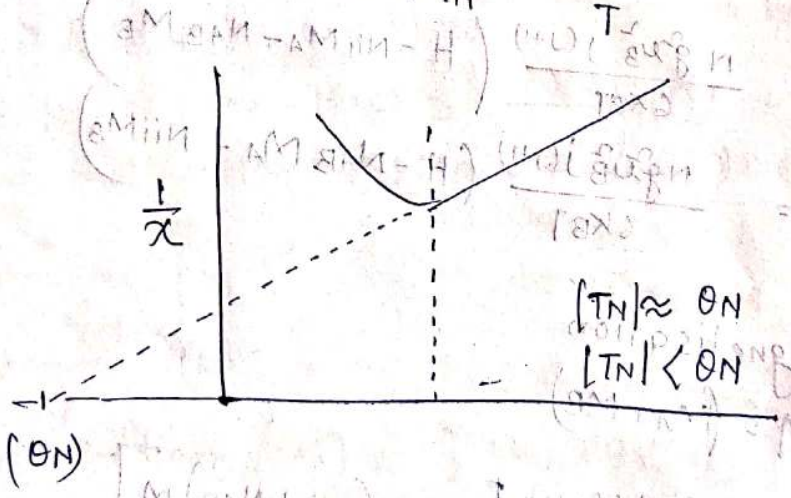
$$\chi = \frac{C}{T + \theta}$$ (curie-weiss for AFM in the paramagnetic regism)

where $\theta = \frac{C}{2}(N_{ii} + N_{AB})$

$$= \frac{N g^2 \mu_B^2 j(j+1)}{3(3K_BT)2} [N_{ii} + N_{AB}]$$

$$\chi = \frac{C}{T + \theta_N} \Rightarrow \frac{1}{\chi} = \frac{T}{C} + \left(\frac{\theta_N}{C}\right)$$





A sublattice & B-sublattice ↑↓↑↓

case-II $T < T_N$ (doesn't apply magnetic field)

below T_N both sublattice posses spontinous magnetisation.

$H = 0$

$$M_A = \frac{C}{2T}\left(-N_{ii} M_A - N_{AB} M_B\right)$$

$$M_B = \frac{C}{2T}\left(-N_{AB} M_A - N_{ii} M_B\right)$$

$$M_A\left[1 + \frac{C}{2T} N_{ii}\right] + M_B\left(\frac{C}{2T} N_{AB}\right) = 0$$

$$M_A\left(\frac{C}{2T}\right) N_{AB} + M_B\left[1 + \frac{C}{2T} N_{ii}\right] = 0$$

$$\begin{vmatrix} x & y \\ y & x \end{vmatrix} = 0$$

$$x = 1 + \frac{C}{2T} N_{ii}$$

$$y = \frac{C}{2T} N_{AB}$$

$$x^2 - y^2 = 0$$

$$(x+y)(x-y) = 0$$

$x \neq -y$ (not possible) | $x = y$

$$1 + \frac{C}{2T} N_{ii} = \frac{C}{2T} N_{AB}$$

$$1 + \frac{C}{2T_N}\left(N_{ii} - N_{AB}\right) = 0$$

$$\frac{C}{2T_N}\left(N_{AB} - N_{ii}\right) = 1$$

$$T_N = \frac{C}{2}\left(N_{AB} - N_{ii}\right)$$

$$\theta = \frac{C}{2}\left(N_{ii} + N_{AB}\right)$$

$$\frac{T_N}{\theta} = \frac{N_{AB} - N_{ii}}{N_{ii} + N_{AB}}$$

if $N_{ii} = 0$ $\quad \frac{T_N}{\theta} = \frac{N_{AB}}{N_{AB}}$

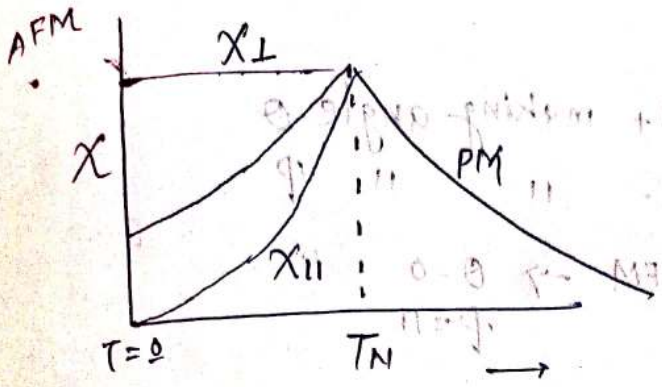
$T_N = \theta$

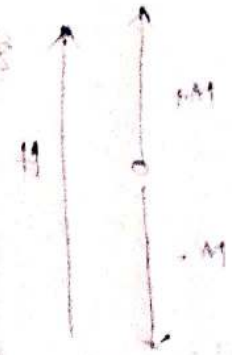
if $N_{ii} \neq 0$ $\quad T_N < \theta$

T_N increase if AFM AB interaction N_{AB} become stronger but decreasing with increasing AA & BB.

Material	T_N(K)	θ(K)	J
MnF_2	67	-80	5/2
MnO	116	-510	5/2
CoO	292	-330	3/2
FeO	116	-610	2
Cr_2O_3	307	-485	3/2
Fe_2O_3	950	-2000	5/2

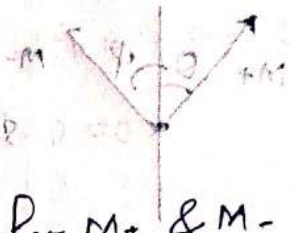
It is seen that $T_N < \theta$



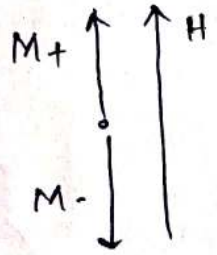


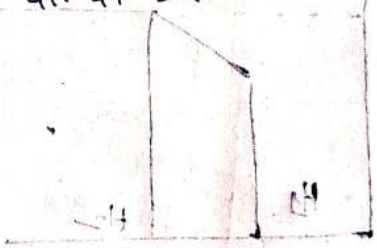
$$\chi_{avg} = \frac{1}{3}\chi_{||} + \frac{2}{3}\chi_\perp$$

small magnetic field

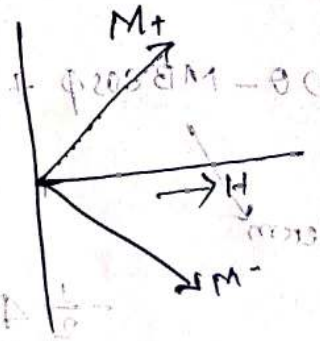


magnetisation for M+ & M- are all at saturated state.

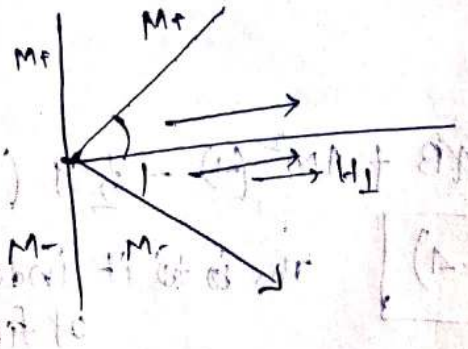


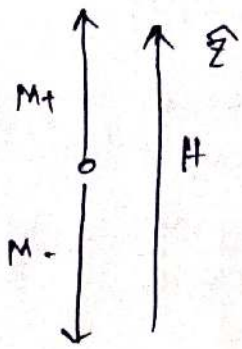


(small magnetic field)



Now for strong magnetic field.



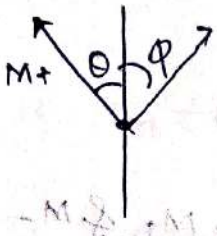


let M+ making angle θ

M- " " ϕ

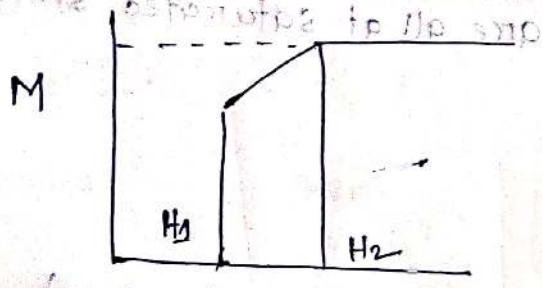
AFM → $\theta = 0$

$\phi = \pi$



spin-flop transition

$\theta = \phi$



exchange coupling

Total energy $E = -MB\cos\theta - MB\cos\phi + AM^2\cos(\theta+\phi)$

↓ Zeman term

↓ due to interaction

$-\frac{1}{2}A(\cos^2\theta + \cos^2\phi)$

↓ (Anistropy term)

AFM case

$\theta = 0, \phi = \pi$

$E = -MB + MB + AM^2(-1) - \frac{1}{2}A(1+1)$

$\boxed{E = (-AM^2 - A)}$

This is the energy independent of field

$\theta = \phi$ In spin flop state

$$E = -2MB\cos\theta + AM^2\cos(2\theta) - \frac{1}{2}A\cos^2\theta$$

Minimum energy configuration

$$\frac{dE}{d\theta} = 0$$

$$+2MB\sin\theta + AM^2(-1)\sin 2\theta\,(2) + \frac{1}{2}\,2\cos\theta\sin\theta \quad (1) = 0$$

$$\Rightarrow 2MB\sin\theta - 2AM^2\sin 2\theta + 2A\sin\theta\cos\theta = 0$$

$$\Rightarrow (2MB - 4AM^2\cos\theta + 2A\cos\theta)\sin\theta = 0$$

$$\therefore \sin\theta = 0 \qquad \boxed{\theta = n\pi}$$

$$2MB + (2A - 4AM^2)\cos\theta = 0$$

$$\boxed{\cos\theta = \frac{2MB}{(4AM^2 - 2A)}}$$

$A \ll 1$

$$\cos\theta = \frac{2MB}{4AM^2}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{B}{2AM}\right)}$$

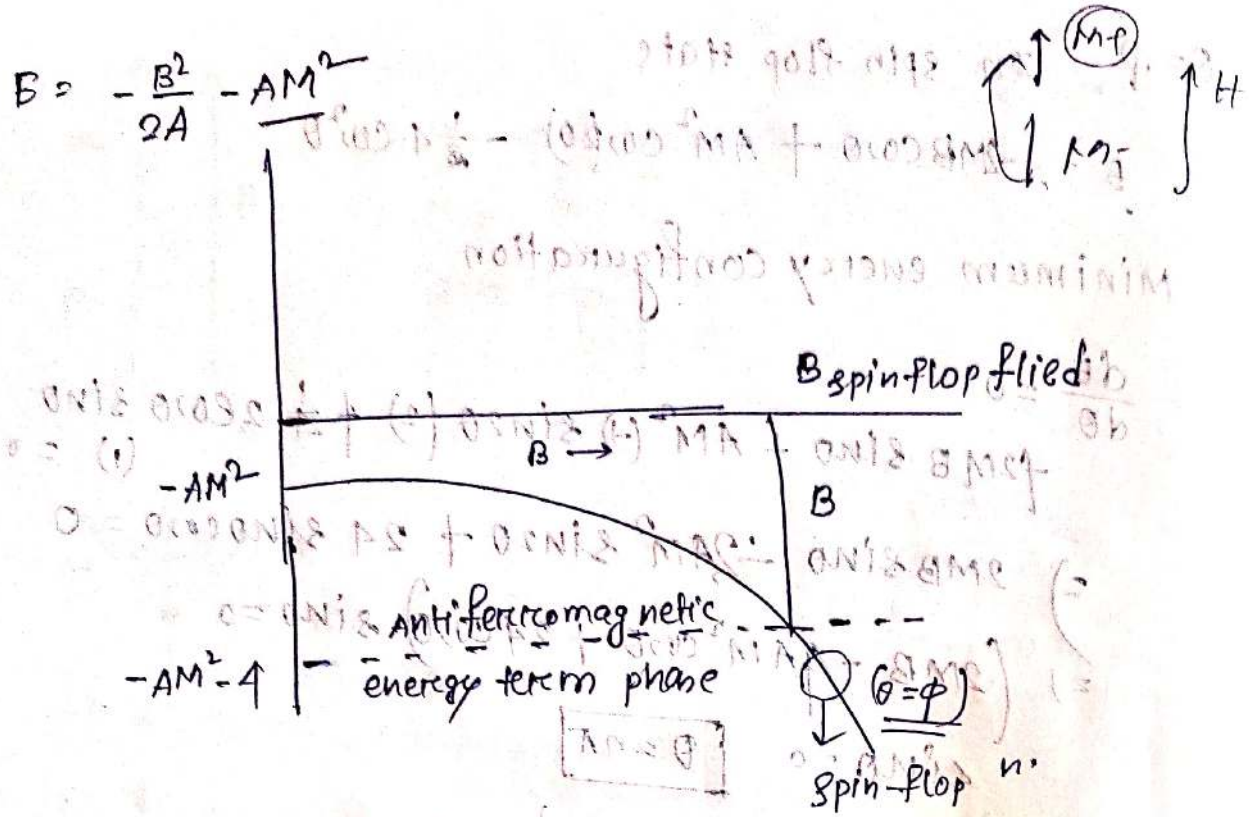
$$E = -2MB\left(\frac{B}{2AM}\right) + 2AM^2\left(\frac{B}{2AM}\right)^2 - AM^2$$

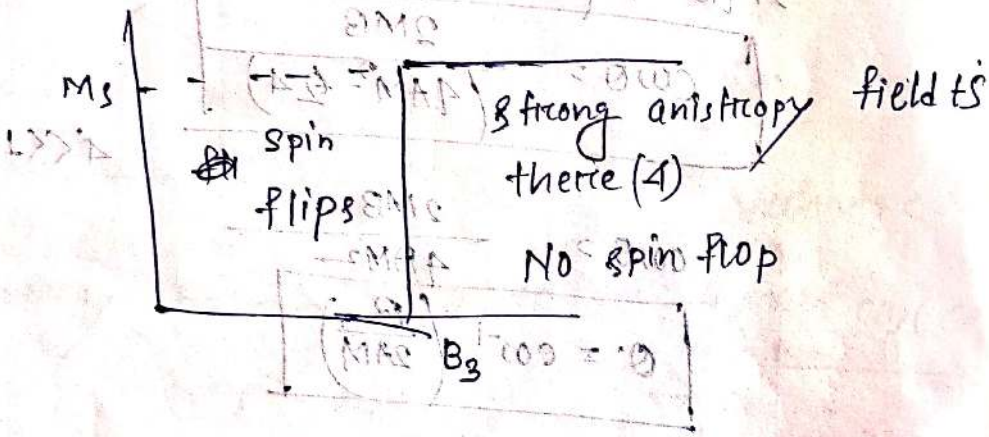
$$= -\frac{B^2}{A} + \frac{B^2}{2A} - A\left(\frac{B^2}{4A^2M^2}\right)$$

$$= -\frac{B^2}{2A} - \frac{B^2}{2A}$$

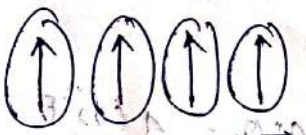
$$= -\left(\frac{B^2}{A}\right)$$

$E = -\frac{B^2}{2A} - AM^2$





Direct exchange:



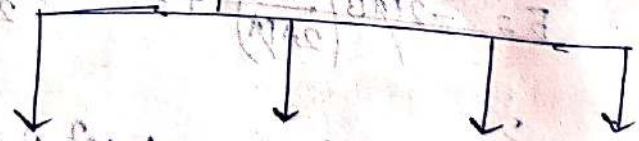
$H_m = -2J \vec{S_1} \cdot \vec{S_2}$

FM

AFM

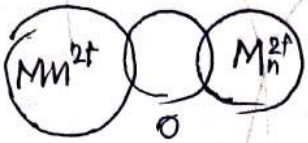
PM

Indirect exchange

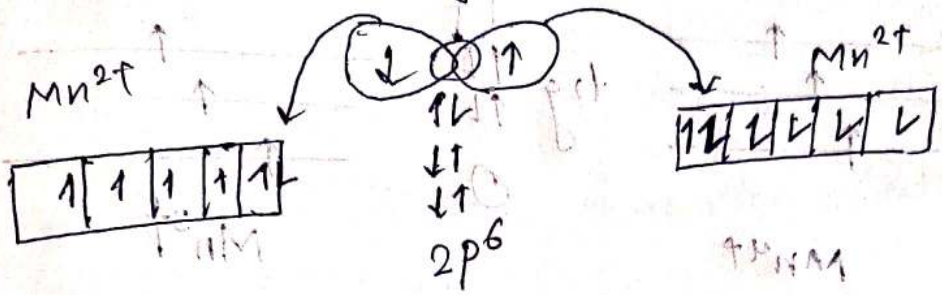


Superexchang | Doceble exchange | DM interaction | RKKY interac

MnO $Mn^{2+} - O^{2-} - Mn^{2+}$

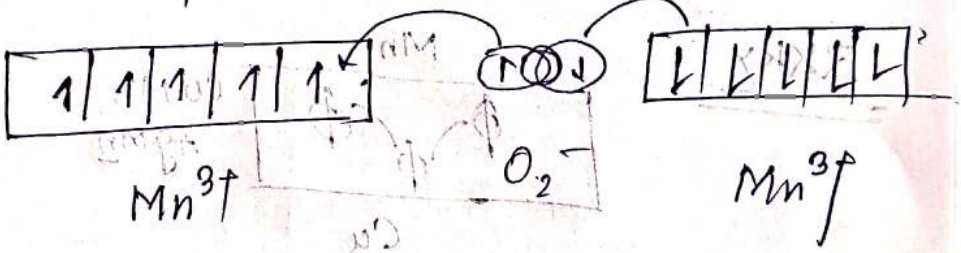


ligand



AFM superexchange

(#) M^{3+}

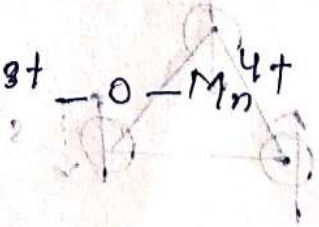


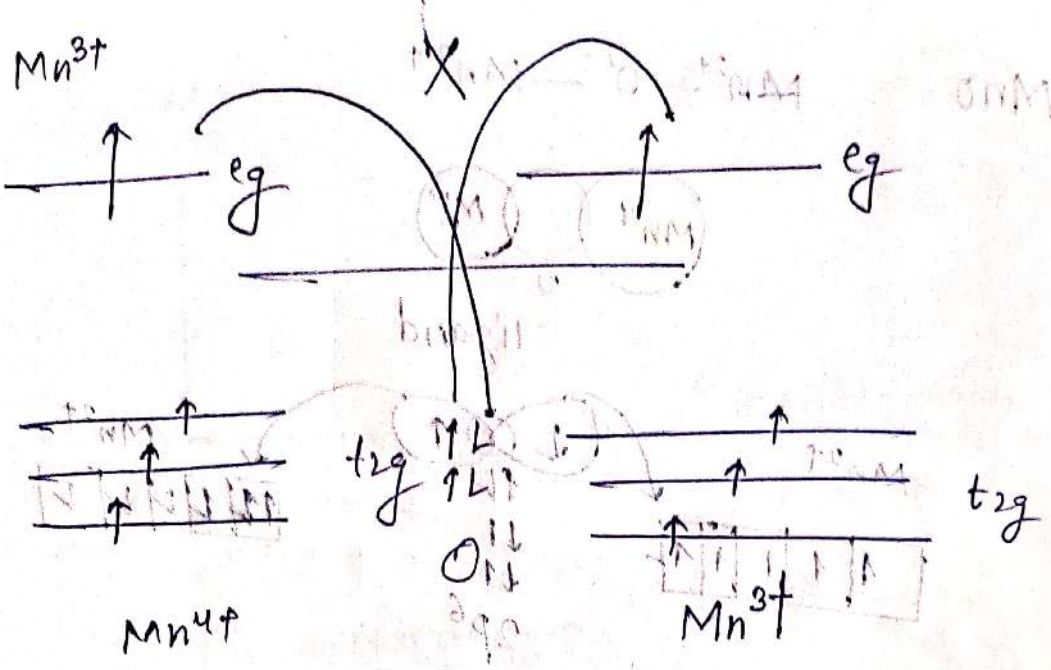
Superchange AFM interaction

(#) double exchange interaction

$La^{3+}_{0.7} Sr^{2+}_{0.3} Mn^{3+}, Mn^{4+} O_3^{2-}$

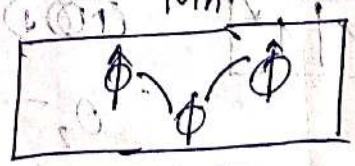
$La_{0.7} Sr_{0.3} Mn O_3$



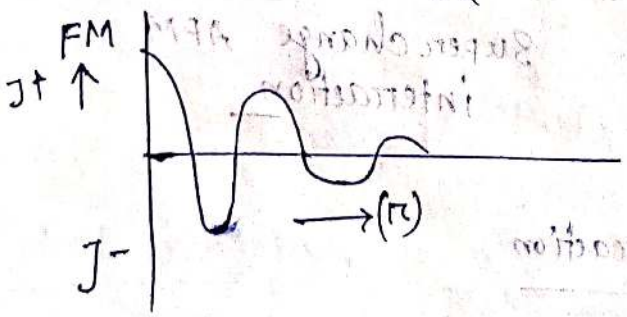


CMR materials DF interaction

RKKY



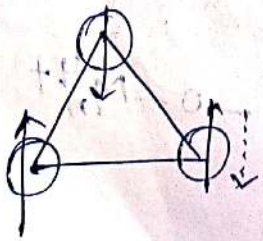
CuMn
AgMn



$$J(r) \propto \frac{\cos(k_f r)}{r^3}$$

$$H_{DM} = D\,\vec{S_1} \times \vec{S_2}$$

DM interaction :-



spin frustated state

spin glass is

~~Superconductivity~~

The critical temperature can be found by approaching from the high temp.

state $T > T_N$

$$M_A = \frac{C}{2T}(H - N_{ii}M_A - N_{AB}M_B)$$

$$M_B = \frac{C}{2T}(H - N_{AB}M_A - N_{ii}M_B)$$

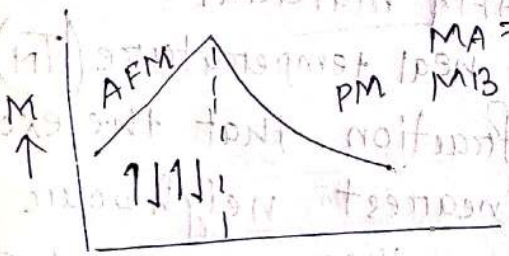
In the vicinity of T_N (saturation effects are unimportant.)

$H = 0$

$$M_A = \frac{C}{2T}(-N_{ii}M_A - N_{AB}M_B)$$

$$M_B = \frac{C}{2T}(-N_{AB}M_A - N_{ii}M_B)$$

for Non-zero value of M_A & M_B, the determinant the position of M_A & M_B must be zero.

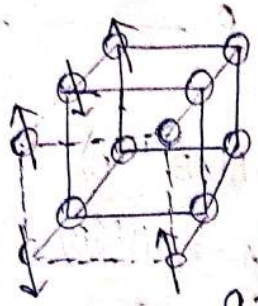


$$T_N = \frac{C}{2}(N_{AB} - N_{ii})$$

$$\frac{T_N}{\theta} = \left(\frac{N_{AB} - N_{ii}}{N_{AB} + N_{ii}}\right) \qquad \boxed{T_N < \theta}$$

AFM ↑↓↑↓

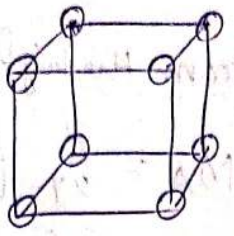
T<TN



a = 8.85Å

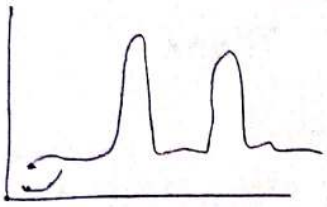
PM → ↑ ↑ ←

T>TN



unmagnetised Mn

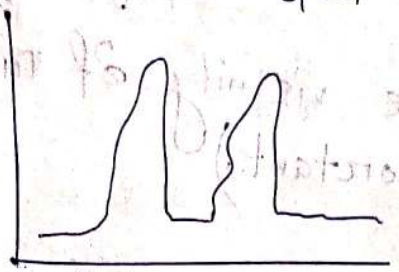
a = 4.45Å



Neutron diffraction

it give lattice constant and also spin structure

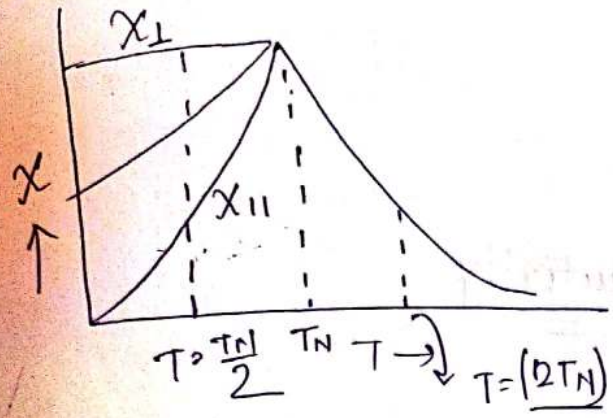
↓



(X-ray diffraction)

problem:- (1) Consider an AFM material which has susceptibility χ_0 at its neal temperature (T_N). Assuming X-ray diffraction that the exchange interaction between nearest neighbour A & B ions much larger than those A-A and B-B pairs calculate the values of susceptibilities which would be measured under the application of field's perpendicular to the magnetisation direction at $T = 0$, $T = \left(\frac{T_N}{2}\right)$ & $T = 2T_N$

AFM. AFM PM



$$\frac{T_N}{\theta} = \left(\frac{N_{AB} - N_{ii}}{N_{AB} + N_{ii}}\right) \quad (N_{AB} > N_{ii})$$

$$\boxed{T_N = \theta}$$

$$\chi = \left(\frac{C}{T+\theta}\right)$$

$$= \left(\frac{C}{T+T_N}\right)$$

$$\chi = \frac{C}{2T_N + T_N} = \left(\frac{C}{3T_N}\right)$$

$$\boxed{\chi = \frac{C}{3T_N}}$$

$$\chi(T_N) = \chi_0$$

$$\chi_0 = \frac{C}{T_N + T_N}$$

$$\boxed{\chi_0 = \frac{C}{2T_N}}$$

$$C = 2T_N\chi_0$$

$$\chi = \frac{2T_N\chi_0}{3T_N}$$

$$\boxed{\chi = \frac{2}{3}\chi_0} \quad (1)$$

for (~~AFM~~) PM

$$\chi = \frac{2 \times 2T_N\chi_0}{3T_N}$$

$$\boxed{\chi = \frac{4}{3}\chi_0}$$

for ~~PM~~ APM