

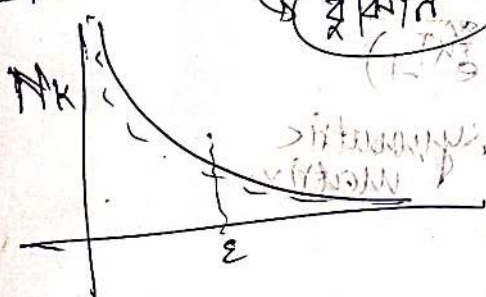
# Laser Spectroscopy

LASER = Light Amplification by Stimulated Emission of Radiation.

- (1) Active medium amplifies the intensity of EM radiation.
- (2) Pump: populates selected energy levels of the Active medium
- (2) Optical resonator: Two // high reflection mirror which stores energy of few resonator waves by providing feedback.

Population inversion

[Classically we use Boltzmann]



$$N_k = N_i e^{-\frac{E_k - E_i}{kT}}$$

Boltzmann rule

So the laser setup works as kind of black body w/o can (closed resonator).

$$E = \vec{E}_0 e^{i(\omega t - Kx)} + \vec{E}_0 e^{-i(\omega t + Kx)}$$

$$K = \frac{\pi}{L} (n_1^2 + n_2^2 + n_3^2)^{1/2}$$

$$\chi = \frac{1}{4\pi} \frac{1}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$$

$\frac{N_k}{N_i}$

Laser:

Einstein's A & B coefficients

No. of atoms going from  $i \rightarrow k$

$$P_{i \rightarrow k} = \rho(\nu) B_{ik} N_i$$

population @  $i$  state

absorption coefficient

What is spectral density?

(Ans) No of modes within freq:  $\nu$  to  $\nu + d\nu$  is  $\frac{8\pi\nu^2}{c^3} d\nu$

$$\rho(\nu) d\nu = u(\nu) kT d\nu$$



④ NO of Atoms going from  $|k\rangle \rightarrow |i\rangle$  (reverse)

$$R_{k \rightarrow i} = A_{ki} N_k + P(\nu) B_{ki} N_k$$

⑤ At equilibrium.

CLOSED  
CAVITY  
LASER

$$R_{i \rightarrow k} = R_{k \rightarrow i}$$

$$P(\nu) B_{ik} N_i = A_{ki} N_k + P(\nu) B_{ki} N_k$$

$$P(\nu) B_{ik} N_i = N_i e^{-\epsilon/kT} (A_{ki} + P(\nu) B_{ki})$$

$$P(\nu) N_i [B_{ik} - B_{ki} e^{-\epsilon/kT}] = N_i A_{ki} e^{-\epsilon/kT}$$

$$P(\nu) = \frac{A_{ki}}{B_{ik} e^{\epsilon/kT} - B_{ki}} = \frac{A_{ki}}{B_{ik} (e^{\epsilon/kT} - 1)}$$

symmetric matrix.

$$\frac{A_{ki}}{B_{ik}} = \frac{\hbar^2 \omega^2}{\pi^2 c^2} \frac{1}{e^{\epsilon/kT} - 1}$$

LASER

Intensity  $I = I_0 e^{-\alpha L}$  (Lambert law)

$\alpha \rightarrow$  absorption coefficient.

$$\alpha = (N_i - N_k) \sigma_{ik}$$

absorption cross section.

For intensity to increase  $N_k > N_i$ .

Loss coefficient in a round trip (2L)

$$\therefore \text{Net intensity} \equiv I = I_0 e^{-\alpha(2L)} \cdot e^{-\gamma}$$



for intensity to increase.

$$\Delta N_{in} \gg \gamma$$

sign about matter

$$\Delta N_{in} \gg \frac{\gamma}{2L\sigma_{ik}}$$

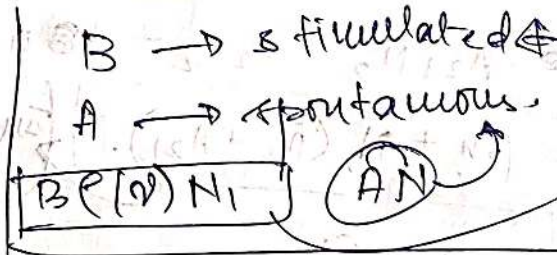
$$\Delta N_{in} \gg \frac{\gamma}{2L\sigma_{ik}} \rightarrow \text{threshold condition}$$

# OPEN CAVITY LASER THEORY

$$L = 10 \text{ cm.}$$

$$\gamma = 10\% \rightarrow 0.1$$

$$\sigma = 10^{-12} \text{ cm}^2$$



$$\Delta N_{\text{thres}} = \frac{10^{-1+12-1}}{2 \times 0.1 \times 10^{-12}} = \frac{10^{10}}{2} \text{ cm}^{-3}$$

$$= 0.5 \times 10^{10} \text{ cm}^{-3}$$

$(N_A - N_i)$

He-Ne laser

Ar-Kr laser

Why?

Rate equation  $\Rightarrow$

$$\frac{dN_1}{dt} = -B_{12} n h \nu N_1 + B_{21} n h \nu N_2 + A_{21} N_2 - N_1 R_1$$

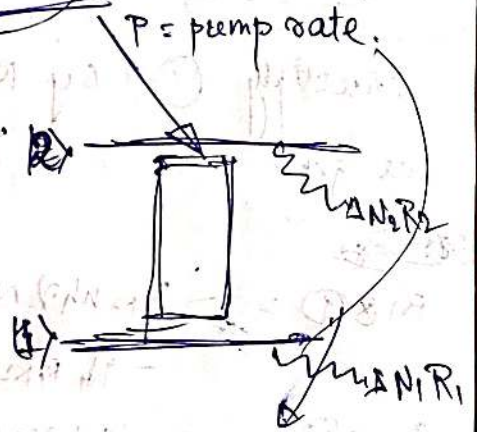
emission

$$\frac{dN_2}{dt} = B_{12} n h \nu N_1 - B_{21} n h \nu N_2 - A_{21} N_2 + P - N_2 R_2$$

pump

freq of resonator mode.

$\Delta$  relax.



no of atoms @  $12\%$  per  $\text{cm}^3$  per sec

$N_i = \#$  of atoms  $\text{cm}^{-3}$  in state.

$n \equiv$  photon density.

$N_i R_i \equiv$  no. of atoms which are removed from  $i$  by relaxation (collision)

$$\frac{dn}{dt} = -\beta n + B_{12} n h \nu (N_2 - N_1)$$

loss coeff

stimulated

$\gamma = \beta + \frac{1}{2L} \left( \frac{c}{\nu} \right)$

of open system  $[0.8\%]$ .

Laser spectroscopy



14/1

@ Stationary state.

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = \frac{dP}{dt} = 0.$$

Add (i) & (ii)

$$P = N_1 R_1 + N_2 R_2$$

Add (iii) & (iv)

$$P = A_2 N_2 + N_2 R_2 - \beta N = 0.$$

$$P = \beta N + N_2 (R_2 + A_2)$$

INCOHERENT LOSS RATE

Add (v) & (vi)

$$\beta N + N_2 (R_2 + A_2) = N_1 R_1 + N_2 R_2$$

$$N_1 R_1 = \beta N + N_2 A_2$$

$$N_2 R_2 = P - \beta N - N_2 A_2$$

Multiply (i) by  $R_2$  & (ii) by  $R_1$  & subtract.  
we get

$$R_1 \otimes (i) = -B_{12} N_1 N_2 R_2 + B_{12} N_1 N_2 R_1 + A_2 N_2 R_2 - N_1 R_1 R_2 = 0$$

$$R_2 \otimes (ii) = +B_{12} N_1 N_2 R_1 - B_{12} N_1 N_2 R_2 - A_2 N_2 R_1 - N_2 R_1 R_2 + P R_1 = 0$$

subtract

$$= -P R_1 + R_1 R_2 \Delta N + A_2 N_2 R_1 + A_2 (P - N_1 R_1) - B_{12} N_1 N_2 (R_1 + R_2) + B_{12} N_1 N_2 (R_1 + R_2) = 0$$

$$= -P R_1 + A_2 P + R_1 R_2 \Delta N + A_2 N_1 \Delta N + B_{12} N_1 N_2 (R_1 + R_2) \Delta N = 0$$

$$\Rightarrow \Delta N = \frac{P(R_1 - A_2)}{R_2 R_1 + A_2 R_1 + B_{12} N_1 N_2 (R_1 + R_2)}$$



$$\Delta N = N_2 - N_1$$

We need  $\Delta N > 0$ .

$$\therefore R_1 > A_{21}$$

$$\frac{1}{L} > A_{21}$$

$$L < A_{21}$$

For laser in open system

Loss & laser modes.

$W_k$ : energy stored for a particular mode  $k$ .

loss rate =

$$\frac{dW_k}{dt} = -\beta_k W_k$$

$$W_k = W_0 \exp(-\beta_k t)$$

$$Q \text{ (quality factor)} / Q = \frac{\text{Energy stored}}{\text{Energy loss per cycle}} = \frac{W_k}{T \frac{dW_k}{dt}}$$

$$= \frac{1}{\beta_k T}$$

$$nh\nu = P(\nu)$$

HELP.

Loss Types.

- (1) Reflection loss
- (2) Diffraction loss

"Special type of mirror"

Reflection loss for round trip.

$$R_1 R_2 I_0 = I_0 \exp(-\gamma)$$

$$\gamma = -\ln(R_1 R_2)$$

$$\gamma_R = \beta_R \frac{2d}{c} = |\ln(R_1 R_2)|$$

$$\beta_R = \frac{c}{2d} |\ln(R_1 R_2)|$$

$$\frac{1}{T} \geq \frac{2d}{c} |\ln(R_1 R_2)|$$

When will mode NOT survive?

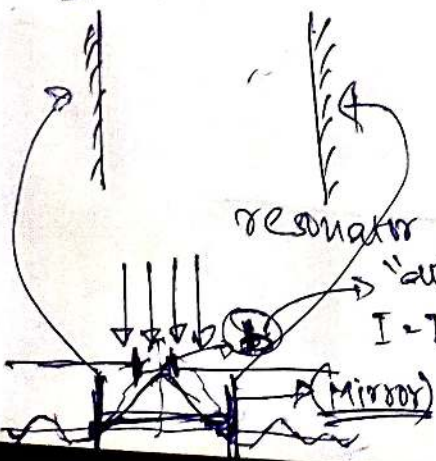
Ans: Low R will have low T. Low R will NOT survive.

$R \rightarrow$  reflectivity of mirror

Diffraction loss.

Babinet principle.

Diffraction by mirror ~ diffraction by slit.



resonator mirrors

"diffraction slit width"

$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2}$$

"Fraunhofer diffraction"

$$\alpha = \frac{\pi b \sin \theta}{\lambda}$$

Approx (small) condition.

$$\frac{\pi b}{\lambda} \approx \pi$$

dist b/w source & mirror  $\infty$ .

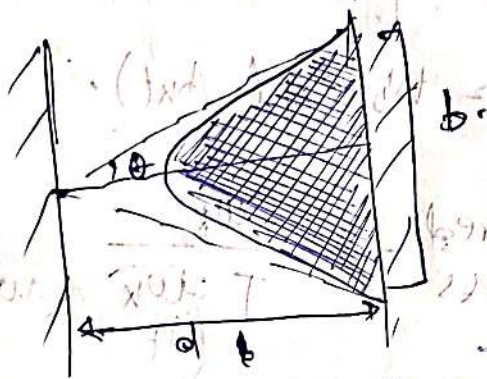
$$\theta = \lambda/b$$



Diffraction loss is described by FRESNEL NO.

Fresnel No.  $N = \frac{b^2}{\lambda d}$

Fraunhofer diffraction gives Fresnel radius & Fresnel zones.



resonator mirrors

$\gamma(\text{loss}) \propto \frac{1}{N}$   
Diffraction

Diffraction loss is defined as:

Fresnel no.  $N = \frac{b^2}{\lambda d}$

$\theta = \frac{b}{2d}$

for in transit:

$\theta = \frac{b}{2d}$

$N = \frac{b^2}{\lambda d}$

$N = \frac{b^2}{\lambda d} = \left(\frac{N}{2}\right)^2$

$\cos \approx \frac{1}{N}$

### FABRY - PEROT INTERFERO

$b = 3 \text{ cm}$

$d = 1 \text{ cm}$

$\lambda = 500 \text{ nm}$

$N = ?$

1.8 (N) LASER

$8.8 \times 10^5 \times 10^5$

$10^2 \times 10^2$

$10^2 \times 10^2$

$8.8 \times 10^5 \times 8.8 \times 10^5$

$10^2 \times 10^2$

$10^2 \times 10^2$

$10^2 \times 10^2$

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$10^2 \times 10^2$

$10^2 \times 10^2$

$1.8 \text{ cm}$

$1 \text{ cm}$

$8.8 \times 10^5 \times 10^5$

$10^2 \times 10^2$

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$10^2 \times 10^2$

Many modes

$1.8 \times 10^5$

laser modes

$\gamma \propto \frac{1}{N}$

$N = 25 \text{ / Loss}$



$\rightarrow 10^{10} \text{ cm}^{-3}$

$\rightarrow 12 \times 10^2$

Incoherent loss Rate:  $A_2 + R_2$

(I & R)

$\rightarrow 12 \times 10^2$

$$I \& R = 2 \times 10^{17} \text{ s}^{-1}$$

Let  $L = 10 \text{ cm}$   
 $b = 1 \text{ mm}$   
 $= 0.1 \text{ cm}$

Total "incoherent" loss  $= 2 \times 10^{17} \text{ cm}^{-3} \text{ s}^{-1}$

Volume loss rate  $= 2 \times 10^{17} \text{ cm}^{-3} \text{ s}^{-1} \times \frac{2\pi}{100} \times 10^8 \text{ cm}^3$

$$= 4\pi \times 10^{16} \text{ s}^{-1}$$

Kirchhoff Diffraction Theory

$$\text{Amplitude: } A(x, y) = -\frac{i}{\lambda} \iint A_{n-1}(x', y') \frac{e^{i k r}}{r} \cos \theta \, dx' dy'$$

"spherical waves"

Stationary condition.

$$A_n(x, y) = c A_{n-1}(x, y)$$

$$A_n(x, y) = -\frac{i}{\lambda} \sqrt{1 - \gamma_D} e^{i\phi} \iint A_{n-1}(x', y') \frac{e^{-i k r}}{r} \cos \theta \, dx' dy'$$