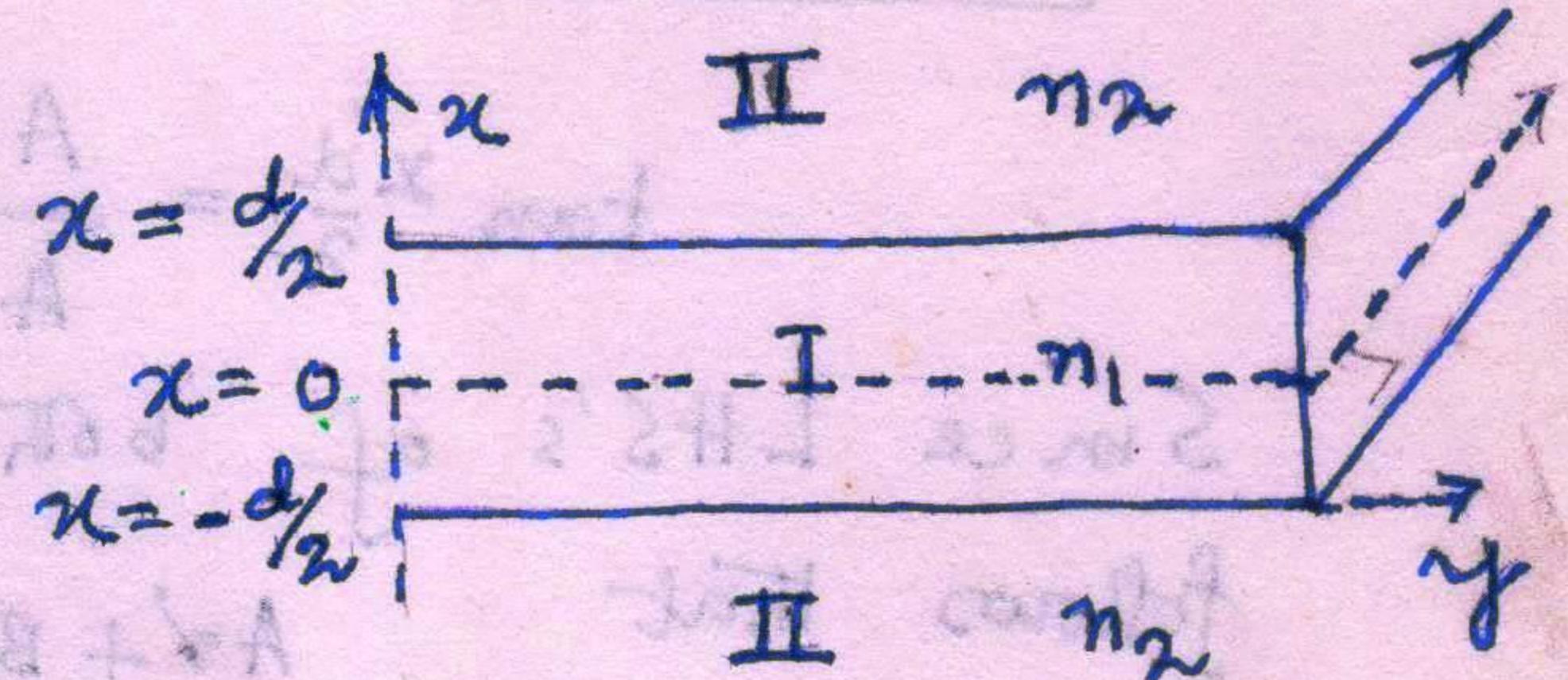


Planar waveguide (dielectric-dielectric interface)

Symmetric structure : TE-modes

The wave eqn will be satisfied by each homogeneous layer of the waveguide :

$$\frac{d^2 E_y}{dx^2} + (k_0 n^2(x) - \beta^2) E_y = 0;$$



$$\text{where } n^2(x) = \begin{cases} n_1^2 & : \pm \frac{d}{2} > |x| \\ n_2^2 & : \text{elsewhere} \end{cases}$$

∴ for region -I :

$$\frac{d^2 E_y}{dx^2} + (k_0 n_1^2 - \beta^2) E_y = 0 : \text{Core}$$

and for region -II :

$$\frac{d^2 E_y}{dx^2} - (\beta^2 - k_0^2 n_2^2) E_y = 0 : \text{cladding}$$

Define mode parameters :

$$\left. \begin{array}{l} x^2 = k_0^2 n_1^2 - \beta^2 \\ \text{and} \quad \gamma^2 = \beta^2 - k_0^2 n_2^2 \end{array} \right\}$$

These are actually x-compt of the propagation

With these,

$$\frac{d^2 E_y}{dx^2} + x^2 E_y = 0 \quad (1)$$

$$\frac{d^2 E_y}{dx^2} - \gamma^2 E_y = 0 \quad (2)$$

Solutions of (1) & (2) are

$$E_y(x) = A'e^{ix} + B'e^{-ix} \quad \text{for (1)}$$

$$E_y(x) = C'e^{-\gamma x} + D'e^{\gamma x} \quad \text{for (2)}$$

$$E_y(x) = A \cos \lambda x + B \sin \lambda x \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$E_y(x) = C e^{-\gamma x} + D e^{\gamma x} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Sols.}$$

Now, at $x = \pm \frac{d}{2}$: interface

$$E_y(x = \pm \frac{d}{2}) \Big|_{\text{region-I}} = E_y(x = \pm \frac{d}{2}) \Big|_{\text{region-II}} \Rightarrow A \cos \lambda \frac{d}{2} + B \sin \lambda \frac{d}{2} = C e^{-\frac{\gamma d}{2}}$$

$$\frac{\partial E_y}{\partial x} \Big|_{(x = \pm \frac{d}{2})} = \frac{\partial E_y}{\partial x} \Big|_{(x = \pm \frac{d}{2})} \Rightarrow -A \lambda \sin \lambda \frac{d}{2} + B \lambda \cos \lambda \frac{d}{2} = -\gamma C e^{-\frac{\gamma d}{2}}$$

$$\text{Multiplying (1) by } x: \tan \lambda \frac{d}{2} = \frac{Ax + Bx}{Ax - Bx}$$

At $x = -\frac{d}{2}$ interface

$$A \cos k \frac{d}{2} - B \sin k \frac{d}{2} = D e^{-k \frac{d}{2} r}$$

$$A k \sin k \frac{d}{2} + B k \cos k \frac{d}{2} = r D e^{-k \frac{d}{2} r}$$

Also use $D = C$ as
the waveguide is symmetric

$$\tan k \frac{d}{2} = \frac{Ar - Bx}{Ax + Br}$$

Since LHS's of both the conditions are the same, it follows that

$$\frac{Ar + Bx}{Ax - Br} = \frac{Ar - Bx}{Ax + Br}$$

This on simplification gives $2AB(x^2 + r^2) = 0$

i.e., either $x^2 = -r^2$ [which is true for
or $AB = 0$: saying that $A = 0$
or, $B = 0$ homogeneous layers]

So, if we start from any of the condition —

Say $B = 0$, then

$$\tan k \frac{d}{2} = \frac{r}{x}$$

And for $A = 0$

$$\tan k \frac{d}{2} = -\frac{x}{r}$$

For $B = 0$:

$$E_y(x) = A \cos kx \quad |x| < \frac{d}{2} \\ = C e^{-r|x|} \quad |x| > \frac{d}{2}$$

symmetric field distribution

Eigen value eqn

$$\tan k \frac{d}{2} = \frac{r}{x} \Rightarrow$$

$$\frac{x \frac{d}{2}}{2} \tan k \frac{d}{2} = \frac{r \frac{d}{2}}{2}$$

Define v - number:

$$v = R_0 \frac{d}{2} \cdot \sqrt{n_1^2 - n_2^2}$$

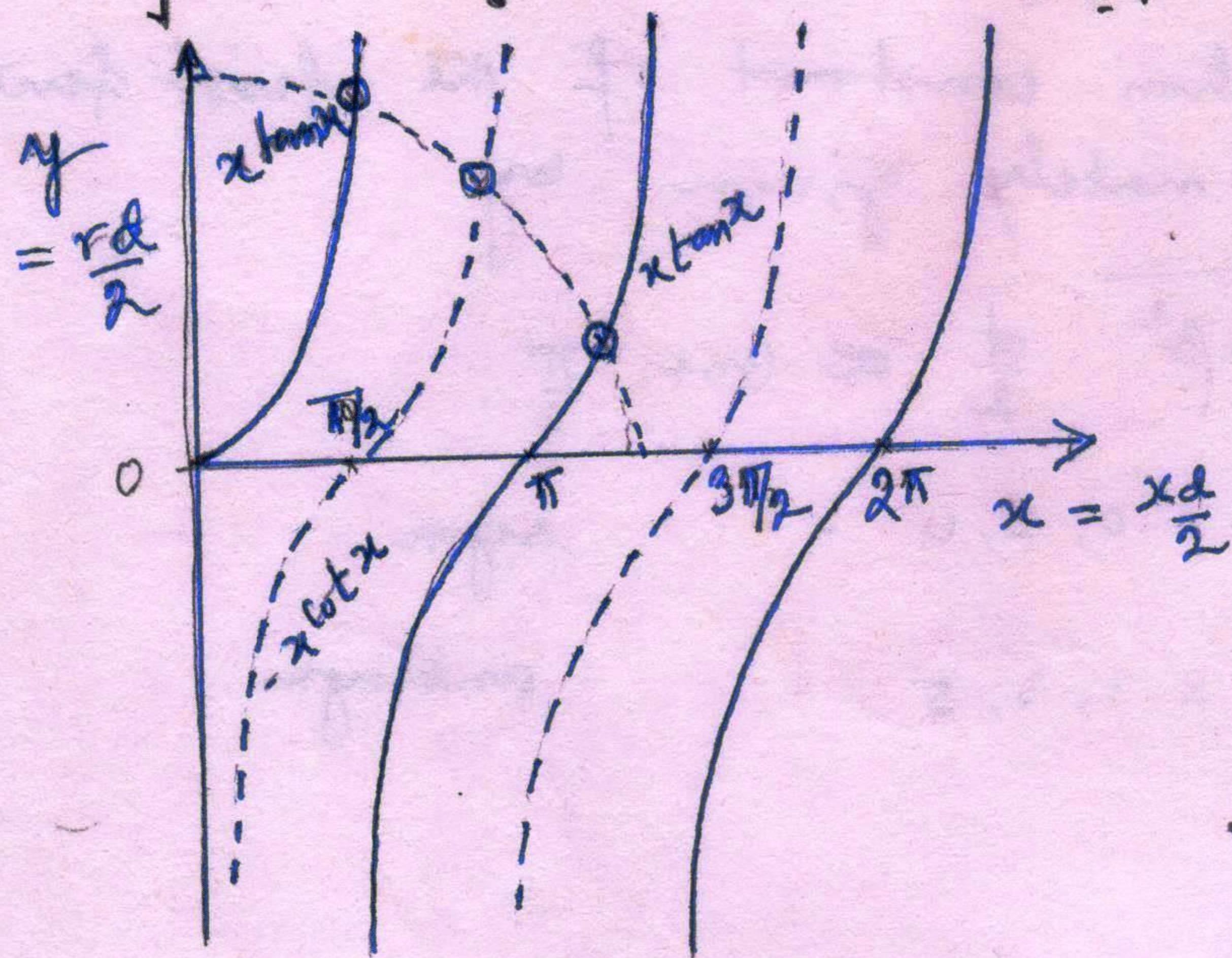
$$\text{Now, } v^2 = \frac{d^2}{4} (R_0 n_1^2 - R_0 n_2^2) = \frac{d^2}{4} x^2 + \frac{d^2}{4} r^2$$

$$v^2 = \left(\frac{x d}{2}\right)^2 + \left(\frac{r d}{2}\right)^2$$

$$y = x \tan k$$

$$v^2 = x^2 + y^2 \quad \text{--- (A)}$$

Eqn (A) is that of a circle of radius V . The intersection of the circles with curves given by (B) are the solutions of the eigen value eqn.



In the same way for antisymmetric modes, $A=0$

$$Ey(x) = B \sin kx \\ = \frac{x}{|x|} C e^{-r|x|}$$

and the eigen value eqn will be

$$-x \cot x = y$$

Now from this figure,

$$0 < V < \frac{\pi}{2} \rightarrow \text{one sym}$$

$$\frac{\pi}{2} < V < 2\frac{\pi}{2} \rightarrow \text{one sym + one antisym}$$

In general, $(2m+1)\frac{\pi}{2} < V < (2m+2)\frac{\pi}{2} \Rightarrow (m+1)$ sym + $(m+1)$ antisym

Total of $(2m+2)$ modes

$$V < (2m+2)\frac{\pi}{2} \Rightarrow (2m+2) > \frac{2V}{\pi}$$

$$N > \frac{2V}{\pi}$$

And if

$$2m\frac{\pi}{2} < V < (2m+1)\frac{\pi}{2} \Rightarrow (m+1)$$
 sym + m antisym

$$(2m+1) > \frac{2V}{\pi}$$

$$N > \frac{2V}{\pi}$$

Total $\sim (2m+1)$ modes

\therefore Total no. of guided modes will be integer closest to and greater than

$$N = \frac{2V}{\pi}$$

When the wguide supports many modes, (ii, $\epsilon \gg 1$) then the points of intersection for solutions will be very close to $\beta = \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \dots$ etc.

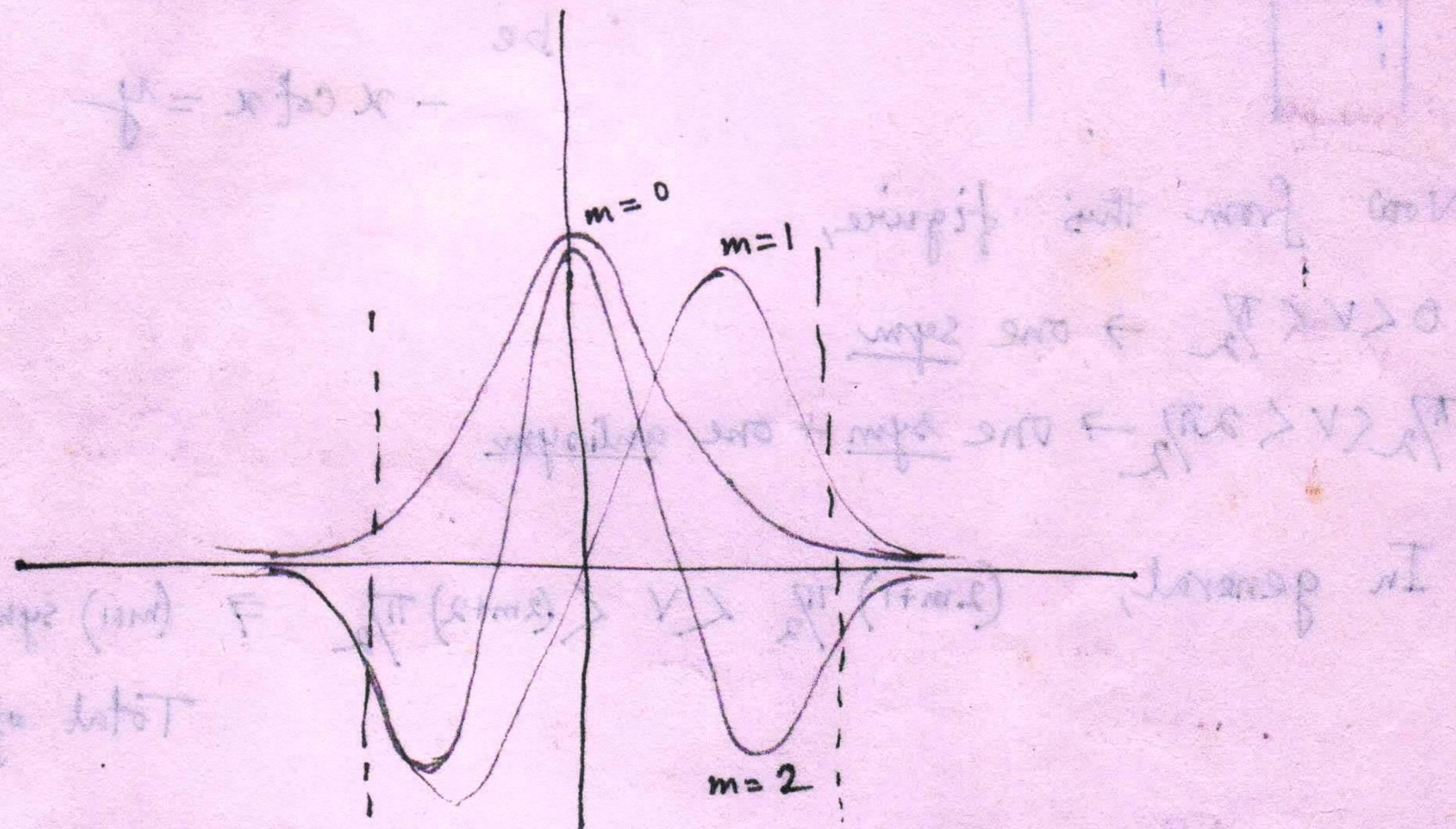
Thus, the propagation constant of the first few modes will be approximately given by

$$\frac{x_d}{2} = \beta = \beta_m = \sqrt{k_0 n_1^2 - \beta^2} \cdot \frac{d}{2} \approx (m+1) \frac{\pi}{2}$$

where $m = 0, 2, 4, \dots$ sym

$= 1, 3, 5, \dots$ antisym.

the wgs overlaps at the



For guided modes, $n_2^2 < \beta^2 / k_0^2 < n_1^2$. That means for a guided mode β cannot be less than $k_0 n_2$. At this value of β , the mode ceases to propagate i.e., the cut-off of the mode. Hence, $\beta = k_0 n_2$ $\gamma = 0$.

Then $\beta + \tan \beta = 0$ ii, $\frac{\pi}{2} + \tan \frac{\pi}{2} = 0$ for sym

and

$\frac{\pi}{2} \cot \frac{\pi}{2} = 0$ for anti-sym.

This means that the cut-off values of various modes are given by $N_c = m\pi$, $m = 0, 1, 2, 3, 4, 5, \dots$

sym

$$0 \leftrightarrow \underline{\underline{\eta_2}} \Rightarrow 1 \text{ sym.}$$

$$\underline{\underline{\eta_2}} \leftrightarrow \underline{\underline{2\pi\eta_2}} \Rightarrow \frac{1}{=} \text{sym} + \frac{1}{=} \text{anti sym.}$$

$$2 \cdot \frac{\pi}{2} \leftrightarrow \underline{\underline{3\eta_2}} \Rightarrow \underline{\underline{2\text{sym}}} + \underline{\underline{1\text{anti sym}}}$$

$$3\eta_2 \leftrightarrow \underline{\underline{4\eta_2}} \Rightarrow \underline{\underline{2\text{sym}}} + \underline{\underline{2\text{anti sym}}}$$

$$(2m+1)\eta_2 \leftrightarrow \underline{\underline{(2m+2)\eta_2}} \quad \underbrace{\underline{\underline{(m+1)\text{sym}}} + \underline{\underline{(m+1)\text{anti}}}}$$

$$(2m+1)\eta_2 < v < (2m+2)\eta_2 \quad 2m+2 = N = \text{no. of modes}$$

$$v = (2m+2)\eta_2 \rightarrow N \text{ no. of modes.}$$

$$v = N\eta_2$$

$$N = \frac{2v}{\pi}$$

⇒ no. of modes.

If $v \leq \eta_2 \rightarrow$ only one mode

single-mode w/g.

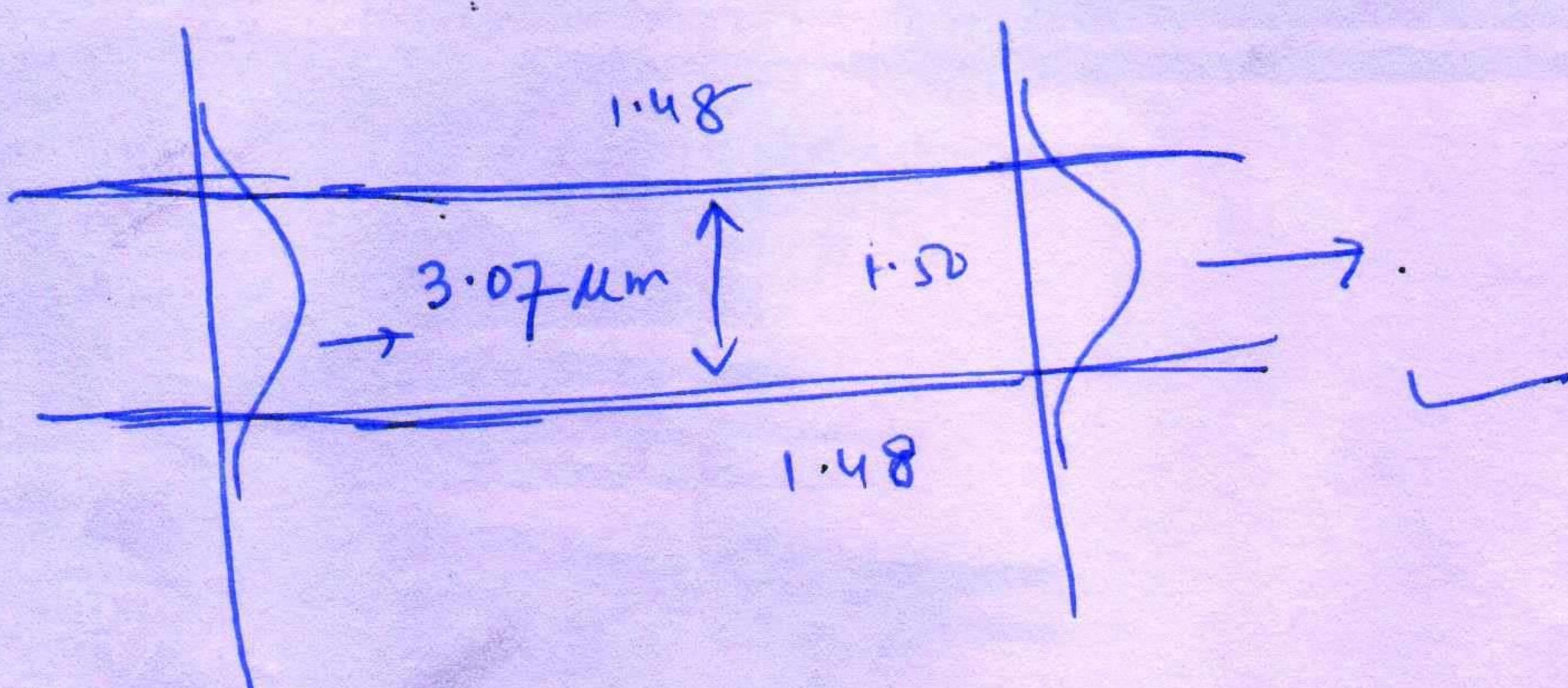
$$\lambda_0 = 1.5 \mu m = 1.5 \times 10^{-6} m.$$

$$n_1 = 1.50$$

$$n_2 = 1.48$$

$$V = \frac{2\pi}{1.5} \times d \sqrt{(1.5)^2 - (1.48)^2} = \frac{\pi}{2} \Rightarrow d = \frac{1.5}{2\sqrt{1 - (1.48/1.5)^2}}$$

$$= \frac{3.07}{2} \mu m$$



10a

Example: $\lambda_0 = 1.5 \mu\text{m} = 1.5 \times 10^{-6} \text{ m}$

$$n_1 = 1.50$$

$$n_2 = 1.48$$

$$v = \frac{2\pi}{1.5} \times d \sqrt{\left(\frac{(1.50)^2 - (1.48)^2}{(1.50)^2}\right)}$$

we must have

For this waveguide to support only one mode i.e., with $0 < v < \pi$ condition for fundamental symmetric mode

$$v < \pi \text{ i.e., } d < 3.07 \mu\text{m}$$

So, the waveguide with λ_0, n_1, n_2 and $d = 3.07 \mu\text{m}$, will be a single-mode guide.

Now, if the operating w/l is changed the same guide can support more no. of modes. for example,

if $\lambda_0 = 0.6 \mu\text{m}$, then $v = 2.5\pi$.

And then we will have 3 modes, i.e., two symm + one antisym.

$$\begin{aligned} &= \frac{\bar{n} \times 1.5}{0.6} \\ &= 2.5\pi \end{aligned}$$