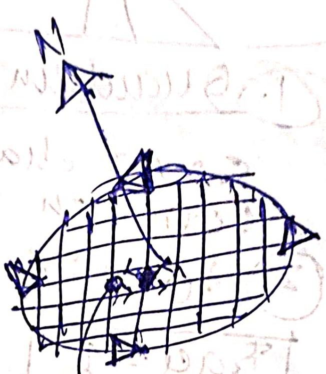
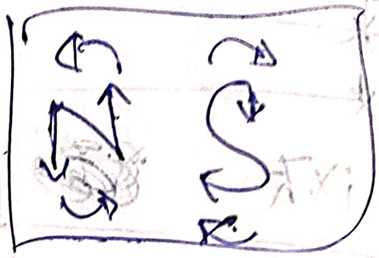
MAGNETISM & SC Materials

TM Nath.

Magnetic moment.

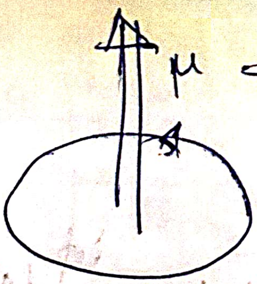
$$\mu = \int d\mu = I \int ds$$
$$= I A \quad (A\text{-}m^2)$$





Magnetic moment & gyromagnetic ratio & momentum.

A current carrying loop occurs as a consequence of motion of one or more electrical charges.



$$\mu = \gamma L$$

$\gamma \rightarrow (e/2m)$

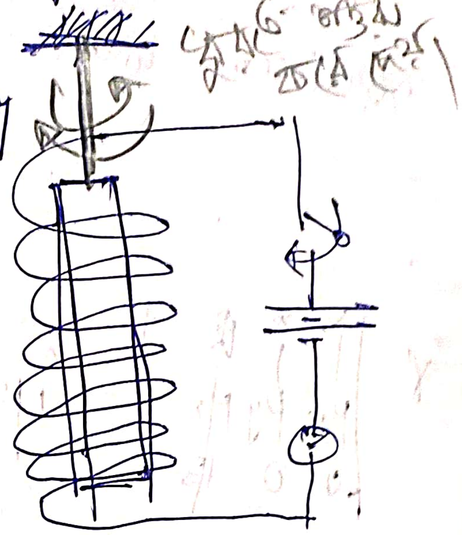
$\mu \rightarrow$ moment

$L \rightarrow$ momentum

Einstein - de Haas effect.

① A (fm) [ferromagnetic] rod is suspended from a thin fibre

② A coil provides a magnetic field which magnetizes the fm vertically along its length.



③ To conserve net angular momentum a rotation is produced due to net magnetization.

④ Frequency of rotation direction flipping = frequency of change in current flow direction.

Demo of rela^n b/w μ & L.

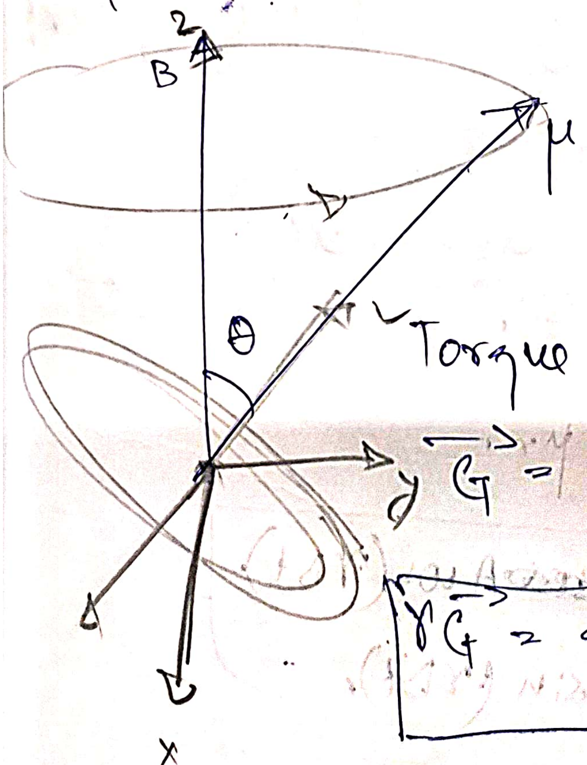
Precession: Potential Energy of $\vec{\mu}$ in $\vec{B}$

$$E = \vec{\mu \cdot B}.$$

$\mu_z = \mu\cos\theta$

$\mu_y = \mu\sin\theta$.

$\mu_x = 0$.



$\vec{U}$ is min when $\mu \parallel B$.

Torque in moment due to $\vec{B}$.

$$\vec{G} = \vec{\mu} \times \vec{B} = \frac{d\vec{L}}{dt}.$$

$$\cancel{\gamma(\vec{\mu} \times \vec{B}) = \frac{d\mu}{dt}}.$$

$$\boxed{\gamma \vec{G} = \frac{d\mu}{dt}}$$

$$\boxed{\gamma(\mu \times B) = \frac{d\mu}{dt}.}$$

Rate of change of μ is $\perp$ to μ.

Therefore, circular motion.

The magnetic moment in a magnetic field precesses around the field @ the larmor frequency (ω_L) and a cone of semi angle θ.

consider when

$$\gamma \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \mu_x & \mu_y & \mu_z \\ 0 & 0 & B \end{vmatrix} = \hat{i}\mu_x + \hat{j}\mu_y + \hat{k}\mu_z.$$

$$\gamma B \cdot (\mu_y \hat{i} - \mu_x \hat{j}) = \frac{d}{dt}\mu_x + \frac{d}{dt}\mu_y + \frac{d}{dt}\mu_z,$$

$$\left.\begin{aligned} \dot{\mu}_x &= \gamma B \mu_y, \\ \dot{\mu}_y &= -\gamma B \mu_x. \end{aligned}\right\} \text{ solve.}$$

$\boxed{\dot{\mu}_z = 0}$

$\mu_x = Ae^{i\omega t} + Be^{-i\omega t},$

$\mu_z = \text{const}$ $\quad \boxed{\omega = \gamma B}$ @ $t=0$ $\quad A+B=0$

$\mu_y = Ce^{i\omega t} + De^{-i\omega t}.$ $\quad \mu_y = \mu \sin\theta$. @ $t=0$.

$\ddot{\mu}_x = \gamma B \dot{\mu}_y$ $\quad \boxed{\mu_y = \mu \sin\theta \cos(\gamma B t).}$

$\ddot{\mu}_x = -(\gamma^2 B^2)\mu_x \Rightarrow \ddot{\mu}_x + (\gamma B)^2 \mu_x.$

$\frac{d}{dt}\left(\frac{d}{dt}\mu_x\right) + (\gamma B)^2 \frac{d}{dt}(\mu_x) \Rightarrow$ $\quad \boxed{\begin{aligned} \mu_x &= A\cos(\gamma B t) \\ \mu_y &= B \sin(\gamma B t). \end{aligned}}$

$\boxed{\mu_x = \mu \sin\theta \sin(\gamma B t)}$

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$\mu_z = |\mu| \cos\theta$.

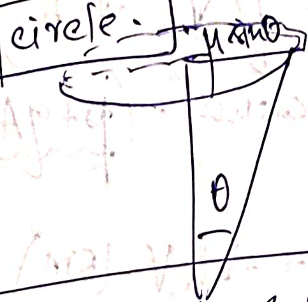
$\mu_x = |\mu| \sin\theta \cos(\gamma B t)$.

$\mu_y = |\mu| \sin\theta \sin(\gamma B t)$.

Dynamics of electron in B mag field & non ‖ magnetic moment.

$\mu_x^2 + \mu_y^2 = \mu^2 \sin^2\theta$.

circle.



$\omega_L = \gamma B$.

larmor frequency.

Size of ATOMIC magnetic moment.

let us consider a hydrogen atom, (e, m_e) electron circulating circularly around the nucleus with orbit radius (r) with speed (v). How much Magnetic moment?

$|\vec{\mu}| = I \cdot A$.

$T = \frac{2\pi r}{v}$. $f = \frac{v}{2\pi r}$.

$I = qf = \frac{-ev}{2\pi r}$

$A = \pi r^2$.

$|\vec{\mu}| = -\frac{ev}{2\pi r} \cdot \pi r^2 = \frac{-evr}{2}$ Magnetic moment.

$L = m_e v r = m_e v r$.

$v = \frac{L}{m_e r}$

$I = -\frac{e}{2\pi r} \cdot \frac{L}{m_e r} = \left(\frac{-eL}{2\pi r^2 m_e}\right)$.

$|\mu| = -\frac{e r}{2} \cdot \frac{L}{m_e r} = \left(\frac{-e}{2m}\right) L$

γ

$\boxed{\mu = \gamma L}$

$L = n\hbar$.

$\mu = \gamma \cdot n\hbar$
$= n(\gamma\hbar)$.
$= n \cdot \left(\frac{e\hbar}{2m}\right)$
$\mu = -n \mu_B$

$\mu_B = \frac{e\hbar}{2m}$.

"Bohr Magneton"

$1\,BM = 9.274 \times 10^{-24}\ A\,m^2$.

Momentum in a magnetic field [canonical moment].

$F = q[\vec{E} + \vec{v} \times \vec{B}]$ lorentz force.

$\vec{v}$ → velocity

$\vec{F} = m\frac{d\vec{v}}{dt}$

Electric field $(\vec{E}) = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}$.

Mag field $(\vec{B}) = \vec{\nabla} \times A$.

$m\frac{d\vec{v}}{dt} = q\left[-\vec{\nabla}V - \frac{\partial A}{\partial t} + \vec{v} \times \vec{\nabla} \times \vec{A}\right]$.

$$m\frac{d\vec{v}}{dt} = q\left[-\nabla V + \nabla(\vec{v}\cdot A)\right] - q\left[(\vec{v}\cdot\nabla)A + \frac{\partial}{\partial t}A\right].$$

electric potential

velocity

$$\left(\frac{d}{dt}\vec{A}\right)$$

$$(v_x \partial_x A_x) + (v_y \partial_y A_y) + (v_z \partial_z A_z) + \partial_t A_t.$$

$$\Rightarrow \partial_t A_x + \partial_t A_y + \partial_t A_z + \partial_t A_t.$$

$$= \partial_t [A_x + A_y + A_z + A_t].$$

$$= \frac{d}{dt}[A(x,y,z,t)]. \rightarrow$$ total derivative [convective derivative].

New momentum "μ"

$$\mu = p + qA.$$

Effective P.E.

$$PE = q(V - \vec{v}\cdot\vec{A}).$$

$$\text{Momentum} = (p + qA).$$

$$\mu = -i\hbar\nabla \cdot (QM).$$

$$\frac{d}{dt}(mv) + \frac{d}{dt}(qA) = -q\left[\nabla V - \nabla(v\cdot A)\right].$$

$$\frac{d}{dt}\underbrace{(mv + qA)}_{\text{canonical momentum}}. = -\nabla\left[qV - qv\cdot A\right].$$

$$KE = \frac{mv^2}{2} = \frac{p^2}{2m}.$$

$$= \frac{1}{2m}(\mu - eA)^2.$$

mechanical KE.

BOHR- Van Leewen theorem.

In a classical system, @ thermal equilibrium, there is NO magnetization possible.

Thus the magnetization must be zero in a classical system.

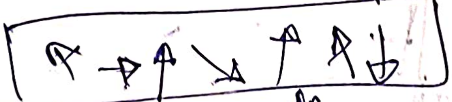
Magnetism is an entirely QUANTUM phenomena & cannot be ~~explained~~ explained to classically.

several types of magnetic order.

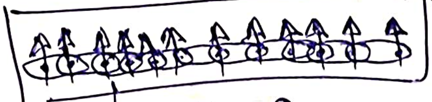
(a) Diamagnetism.
(b) Paramagnetism.
(c) Ferromagnetism.
(d) Antiferromagnetism.
(d') canted AF.
(e) Ferrimagnetism.
(f) Helimagnetic.

SC is a perfect Diamagnet.

Dia ⟹ All materials (many) diamagnet.

Para :  $\chi = \frac{M}{H}$ (dimensionless No unit). Am^{-1}

(random alignment).

Ferro :  "collective phenomena" $\chi \gg 1$.

$H = 0$. $\mu_r = 1 + \chi$.

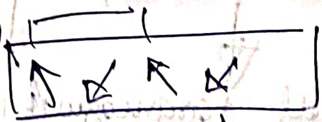
interacting (in absence of field) $\langle M_s \rangle \neq 0$.

aligned correctly. "spontaneous magnetiz^n"

AF →  $H = 0$. $M = 0$ $M_A - M_B \approx 0$

"collective" pheno

"interacting"

canted 

sub lattice

Ferri →  ~~$M_A - M_B \gg 0$.~~

$M_A - M_B \neq 0$.

sub lattice.

For any phase transition, there should be some order parameter.

Ferri → Ferro
second order
PT (phase transition)
Even if ext mag field = 0.

Heli : spins vectors align themselves according to a helix. "Helical pattern".

FAMILY TREE OF MAGNETISM.

Diamagnetism (properties of all matter). Curie's law.

uncompensated Orbital & spin angular momentum (in all solids).

Permanent atomic moments

Independent atomic moments

Ideal paramagnets

Electron energy bands in Metals

Pauli spin paramagnetism

Band ferromagnetism

Band AFerromagnetism.

"collective phenomena" Cooperative atomic moments

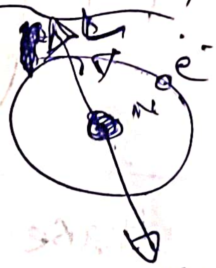
Ferromagnetism

Ferrimagnetism

A Ferromagnet

ORIGIN of permanent dipole moments in materials.

An atom is an elementary magnet.



(*) In atom we find the following 3 contribution:

(i) Orbital motion of an electron contributes OAM. μ

$L = \vec{r} \times \vec{p} = -i\hbar(\vec{r} \times \vec{\nabla})$.

$\boxed{\mu = \gamma L}$

$L^2|\psi\rangle = \hbar^2 l(l+1)|\psi\rangle$.

$L_z|\psi\rangle = m_l\hbar|\psi\rangle$.

Dipole moment due to the orbital motion of the electron.

Dipole moment $\Rightarrow \mu = \gamma L$. $\rightarrow$ Bohr magneton.

$\Rightarrow \mu_L = \left(\frac{-e}{2m}\right)\hbar \sqrt{l(l+1)}$. (orbital Mag moment)

② Dipole moment due to spin of electron.

$\Rightarrow \mu_s \longrightarrow$ similar treatment

$$\mu_s = -g\left(\frac{e\hbar}{2m}\right)\sqrt{s(s+1)}.$$

Bohr magneton

Lande 'g' factor.

$S^2 |\chi_s\rangle = \hbar^2 s(s+1) |\chi_s\rangle$

$S_z |\chi_s\rangle = m_s \hbar |\chi_s\rangle$.

$m_s = \pm \frac{1}{2}$ (spin half).

$S_z |\chi_s\rangle = \pm \frac{\hbar}{2} |\chi_s\rangle$.

$\frac{M_N}{M_e} = 1836$.

③ Dipole moment due to spin of nucleus.

$$\mu_N = +g_N \left(\frac{e\hbar}{2M_N}\right) \cdot \sqrt{s_N(s_N+1)}.$$

Big mass of nucleus.

neglected

~~Magnetic~~ Proof that (Lie algebra). [H.W].

(a) $[L^2, L_i] = 0$ (b) $[L_i, L_j] = i\hbar L_k$.

(c) $L_z = -i\hbar \frac{\partial}{\partial\phi} = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$.

(d) $S^2 = S_x^2 + S_y^2 + S_z^2$.

$= \frac{1}{2}(S_+S_- + S_-S_+) + S_z^2$.

(e) $[S^2, S_z] = 0$.

(f) $[S_z, S_\pm] = \pm S_\pm$.