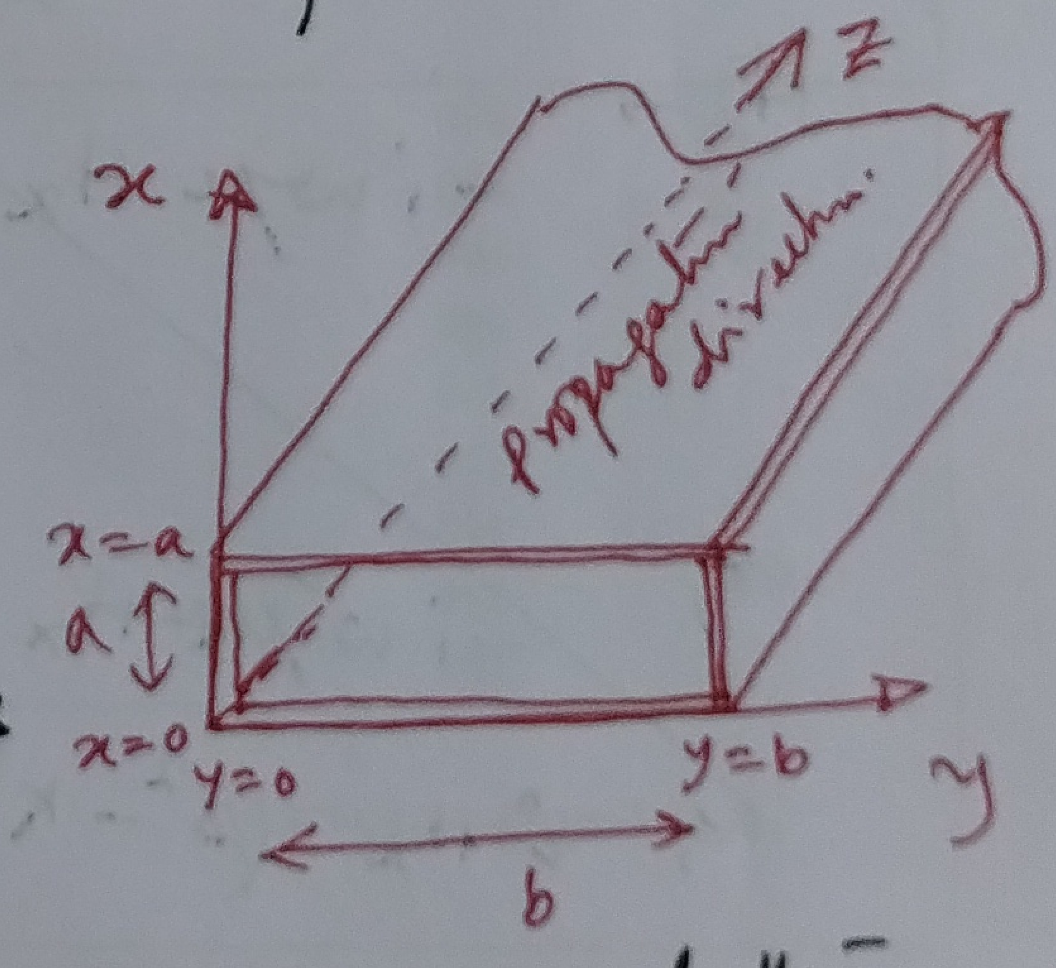
Rectangular metallic waveguide

We shall now investigate the propagation characteristics of EM waves through a perfectly conducting rectangular cross-section pipe, known as rectangular conducting waveguide.

Boundary conditions: The EM field in the space between the conducting walls must obey the physical laws: The electric field is always perpendicular to the magnetic field. Now at the surface of the conducting walls any component of electric field parallel to the surface must vanish. Magnetic field, being perpendicular to electric field, any component of magnetic field normal to the surface of conducting walls must also vanish. Thus,

$$E_{tangential} = 0 \quad \text{and} \quad H_{normal} = 0 .$$

Subject to these boundary conditions, Maxwell's eqns can determine the propagation of the EM waves in the waveguide.



The waveguide has two pairs of conducting plates — one pair at $x=0$ and $x=a$ and the other pair at $y=0$ and $y=b$. The wave propagates along z-direction and the guide is finite in both x and y-directions, ie, ie, $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ are no longer zero.

Considering the z-dependence of the wave as $\sim e^{-i\beta z}$, we have $\frac{\partial}{\partial z} \equiv -i\beta$ and the Maxwell's curl eqns:

$$\nabla \times H = i\omega\epsilon E$$

and

$$\nabla \times E = -i\omega\mu H$$

yields the following six relations: —

$$\left.\begin{array}{l|l}
\frac{\partial H_z}{\partial y} + i\beta H_y = i\omega\epsilon E_x & \frac{\partial E_z}{\partial y} + i\beta E_y = -i\omega\mu H_x \quad (4) \\
(2) \rightarrow \quad \frac{\partial H_z}{\partial x} + i\beta H_x = -i\omega\epsilon E_y & \frac{\partial E_z}{\partial x} + i\beta E_x = i\omega\mu H_y \\
\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i\omega\epsilon E_z & \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -i\omega\mu H_z .
\end{array}\right\} \quad (1)$$

From (2)

$$E_y = \frac{i}{\omega\epsilon}\frac{\partial H_z}{\partial x} - \frac{\beta}{\omega\epsilon} H_x$$

Substituting in (4)

$$-i\omega\mu H_x = i\beta\left[\frac{i}{\omega\epsilon}\frac{\partial H_z}{\partial x} - \frac{\beta}{\omega\epsilon}H_x\right] + \frac{\partial E_z}{\partial y}$$

$$= -\frac{\beta}{\omega\epsilon}\frac{\partial H_z}{\partial x} - \frac{i\beta^2}{\omega\epsilon}H_x + \frac{\partial E_z}{\partial y}$$

$$-i\omega^2\mu\epsilon H_x + i\beta^2 H_x = -\beta\frac{\partial H_z}{\partial x} + \omega\epsilon\frac{\partial E_z}{\partial y}$$

$$i(\beta^2 - K^2)H_x = -\beta\frac{\partial H_z}{\partial x} + \omega\epsilon\frac{\partial E_z}{\partial y}$$

$$H_x = \frac{-\beta}{i(\beta^2-K^2)}\cdot\frac{\partial H_z}{\partial x} + \frac{\omega\epsilon}{i(\beta^2-K^2)}\frac{\partial E_z}{\partial y}$$

$$= \frac{i\beta}{(\beta^2-K^2)}\frac{\partial H_z}{\partial x} - \frac{i\omega\epsilon}{(\beta^2-K^2)}\frac{\partial E_z}{\partial y}$$

$$H_x = -\frac{i\beta}{\chi^2}\frac{\partial H_z}{\partial x} + \frac{i\omega\epsilon}{\chi^2}\frac{\partial E_z}{\partial y}$$

In the same way, we get the other eqns as.

$$\left.\begin{aligned} H_x &= \frac{-i\beta}{\chi^2}\frac{\partial H_z}{\partial x} + \frac{i\omega\epsilon}{\chi^2}\frac{\partial E_z}{\partial y} \\ H_y &= -\frac{i\beta}{\chi^2}\frac{\partial H_z}{\partial y} - \frac{i\omega\epsilon}{\chi^2}\frac{\partial E_z}{\partial x} \\ E_x &= \frac{-i\beta}{\chi^2}\frac{\partial E_z}{\partial x} + \frac{i\omega\mu}{\chi^2}\frac{\partial H_z}{\partial y} \\ E_y &= \frac{-i\beta}{\chi^2}\frac{\partial E_z}{\partial y} - \frac{i\omega\mu}{\chi^2}\frac{\partial H_z}{\partial x} \end{aligned}\right\} \quad (2)$$

where $\chi^2 = k^2 - \beta^2$

$k^2 = k_0^2\epsilon_r$

$= \omega^2\mu\epsilon$

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The various electric and magnetic field components in the guide are related through the above eqns. Interestingly all the transverse components of fields namely, E_x, E_y, H_x, H_y can be determined if the longitudinal components E_z & H_z are known.

Since the guide is finite in $x-$ and $y-$ directions, we have the wave eqns for E_z and H_z as

$$\left.\begin{aligned} \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (k^2-\beta^2)E_z = 0. \\ \text{and} \quad \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (k^2-\beta^2)H_z = 0. \end{aligned}\right\} \quad (3)$$

Now we shall analyze the propagation in the guide for two possible field configurations

1. TE - wave for which $E_z = 0$
2. TM - wave for which $H_z = 0$.

For this rectangular wave guide the boundary conditions are

$E_{tangential} = 0 \Rightarrow E_x = 0$ and $E_z = 0$ at $x = 0$ and $x = a$

$E_{tangential} = 0 \Rightarrow E_y = 0$ at $y = 0$ and $y = b$ and $E_z = 0$.

TE-wave of rectangular waveguide.

For TE-mode, $E_z = 0$ while H_z will be a function of x and y. The z-dependence is given by

$$H_z = H_z(x,y)\, e^{-i\beta z}$$

where $H_z(x,y) = X(x)\,Y(y)$. Using the method of separation of variables

[For analysis of TE-mode propagation, we shall solve the eqn. for H_z:

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (k^2-\beta^2) H_z = 0 \,] \quad \text{— (4)}$$

we write eqn (4) as.

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -(k^2-\beta^2) = -\chi^2$$

where $\chi^2 = k^2 - \beta^2$

or, $$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \chi^2 = -\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}$$

In any equation a function, purely dependent on x, can not be equated to some other function which, on the other hand, purely a function of another independent coordinate y unless each of them is equal to a constant. Therefore,

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \chi^2 = -\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = k_y^2 \text{ (say)}$$

Thus, we now solve two equations:

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -\chi^2 + k_y^2 = -k_x^2 \text{ (say)}$$

and $$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k_y^2$$

The solutions of these two equations are :

$$X(x) = A\cos k_x x + B\sin k_x x$$

and $Y(y) = C\cos k_y y + D\sin k_y y$

Thus the solution of H_z field is given by

$$H_z = XY = AC\cos k_x x\cos k_y y + AD\cos k_x x\sin k_y y + BC\sin k_x x\cos k_y y + BD\sin k_x x\sin k_y y \quad \text{— (5)}$$

Now we need to apply boundary conditions :

$E_y = 0$ at $x=0$ and $E_x = 0$ at $y=0$.

From the relation of E_y and H_z and E_z, given in (2)

$$E_y = \frac{-i\beta}{\chi^2}\frac{\partial E_z}{\partial y} - \frac{i\omega\mu}{\chi^2}\frac{\partial H_z}{\partial x}$$, using $E_z = 0$ (TE-mode),

$$E_y = -\frac{i\omega\mu}{\chi^2}\frac{\partial H_z}{\partial x}.$$

So, to apply the condition $E_y = 0$ at $x=0$, we find from (5)

$$\frac{\partial H_z}{\partial x} = -ACk_x\sin k_x x\cos k_y y - ADk_x\sin k_x x\sin k_y y + BCk_x\cos k_x x\cos k_y y + BDk_x\cos k_x x\sin k_y y$$

using $x=0$, $E_y=0$ and $\frac{\partial H_z}{\partial x} = 0$; we obtain

$$0 = BCk_x\cos k_y y + BDk_x\sin k_y y.$$

This is valid for all values of y if $B=0$.

Therefore eqn (5) reduces to

$$H_z = +AC\cos k_x x\cos k_y y + AD\cos k_x x\sin k_y y \quad \text{— (6)}$$

To apply the condition $E_x = 0$ at $y = 0$,

we use the relation given in ② for connecting E_x and H_z :

$$E_x = \frac{-i\beta}{K^2}\frac{\partial E_z}{\partial x} + \frac{i\omega\mu}{K^2}\frac{\partial H_z}{\partial y}$$ with $E_z = 0$ (TE-mode)

$$E_x = \frac{i\omega\mu}{K^2}\frac{\partial H_z}{\partial y}$$

Therefore, from ⑥

$$\frac{\partial H_z}{\partial y} = -AC\,k_y \cos k_x x \sin k_y y + AD\,k_y \cos k_x x \cos k_y y$$

for $E_x = 0$ at $y = 0$, $\frac{\partial H_z}{\partial y} = 0$ gives

$$0 = AD\,k_y \cos k_x x .$$

This shows that for all values of x, E_x will be zero if $D = 0$. Hence eqn ⑥ becomes

$$\therefore \boxed{H_z = AC \cos k_x x \cos k_y y} \quad — ⑦$$

Now we apply the boundary condition: $x = a$ $E_y = 0$.

i.e, $\frac{\partial H_z}{\partial x} = 0$.

This will be satisfied for all values of y if $\longrightarrow -AC\,k_x \sin k_x a \cdot \cos k_y y = 0$

$\longrightarrow k_x a = m\pi$ or, $k_x = \frac{m\pi}{a}$

And at the boundary $y = b$, $E_x = 0$ and

$\frac{\partial H_z}{\partial y} = 0$ will be satisfied.

ie,
$$\frac{\partial H_z}{\partial y} = -ACk_y \cos k_x x \cdot \sin k_y y \Big|_{y=b} = 0.$$

implying that $k_y b = n\pi$

This condition will be satisfied for all values of x. or, $k_y = \frac{n\pi}{b}$.

Hence we write the field soln. of H_z as

$$\boxed{H_z(x,y) = H_{0z} \cos \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y} \quad \text{--- (8)}$$

The other fields can be obtained as:

$$H_x = \frac{-i\beta}{\chi^2} \frac{\partial H_z}{\partial x} + \frac{i\omega\epsilon}{\chi^2} \underbrace{\frac{\partial E_z}{\partial y}}_{=0}$$

$$= +\frac{i\beta}{\chi^2} \cdot \frac{m\pi}{a} \cdot H_{0z} \cdot \sin \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y$$

$$\boxed{H_x = H_{0x} \sin \frac{m\pi}{a} x \cos \frac{n\pi}{b} y} \quad \text{--- (9)}$$

$$H_y = \frac{-i\beta}{\chi^2} \frac{\partial H_z}{\partial y} - \frac{i\omega\epsilon}{\chi^2} \frac{\partial E_z}{\partial x} = 0.$$

$$\boxed{H_y = H_{0y} \cos \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y} \quad \text{--- (10)}$$

$$E_x = \frac{-i\beta}{\chi^2} \cdot \overbrace{\frac{\partial E_z}{\partial x}}^{=0} + \frac{i\omega\mu}{\chi^2} \frac{\partial H_z}{\partial y}$$

$$\boxed{E_x = E_{0x} \cos \frac{m\pi}{a} x \sin \frac{n\pi}{b} y} \quad \text{--- (11)}$$

$$E_y = \frac{-i\beta}{\chi^2} \underbrace{\frac{\partial E_z}{\partial y}}_{=0} - \frac{i\omega\mu}{\chi^2} \frac{\partial H_z}{\partial x}$$

$$\boxed{E_y = E_{0y} \sin \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y} \quad \text{--- (12)}$$

Equations 8 – 12 represent the field components of the TE-modes in a rectangular conducting waveguide.

mode designation, m, n
To write here.

Cut-off wavelengths:

The propagation of the wave is characterized by the continuous phase change along the propagation direction z, given as the

$$Re \cdot e^{-i\beta z}.$$

Thus, for propagation β must be real.

$$\mu\epsilon\omega^2 = K^2 = k_x^2 + k_y^2 + \beta^2$$

But $\beta^2 = \mu\epsilon\omega^2 - k_x^2 - k_y^2$

$$= \mu\epsilon\omega^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

$$\beta = \sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}.$$

Hence, when $\beta = 0$, there cannot be any phase shift in the propagation direction in the tube, ie, no wave motion along the guide for the frequencies corresponding to $\beta \leq 0$. Thus, the cut-off frequency is given by $\omega_c^2\mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{a}\right)^2$

or,

$$\boxed{\omega_c = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{a}\right)^2}}$$

Therefore wave having frequency above ω_c the propagation will be possible. So,

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{a}\right)^2} = \frac{1}{2} \times c\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$c = \frac{1}{\sqrt{\mu\epsilon}}$ = Speed of light in the hollow tube

So, the cut-off wavelength is

$$\boxed{\lambda_c = \frac{c}{f_c} = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}}}$$

One can note that the cut-off wavelength λ_c depends on the size (a, b) of the waveguide and the order of modes (m, n). Propagation is possible only if $\lambda < \lambda_c$.

wavelength inside the waveguide

The speed of wave propagation in the guide will be given by

$$v_g = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}} = \frac{\omega}{\sqrt{\omega^2\mu\epsilon - \omega_c^2\mu\epsilon}}$$

$$= \frac{\omega}{\omega\sqrt{\mu\epsilon}} \cdot \frac{1}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} = \frac{c}{\sqrt{1-\left(\omega_c/\omega\right)^2}}$$

This shows that the velocity of wave propagation in the guide is greater than the phase velocity in free space. At or near cut-off the guide speed is infinitely large. As the frequency is increased above the cut-off v_g decreases and approaches the value of phase velocity in free space

at large values of frequency the guide velocity approachs!!

The guide wavelength is given by

$\lambda_g = \frac{v_g}{f}$. As v_g is greater than corresponding free-space velocity, λ_g is longer than the free-space wavelength. Thus,

$$\lambda_g = \frac{v_g}{f} = \frac{\omega}{f\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$$

$$= \frac{2\pi}{\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$$

Also, $\lambda_g = \frac{v_g}{f} = \frac{c}{f\sqrt{1-(\omega_c/\omega)^2}} = \frac{\lambda_0}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$

or, $\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$

$$1 - \frac{\lambda_0^2}{\lambda_c^2} = \frac{\lambda_0^2}{\lambda_g^2}$$

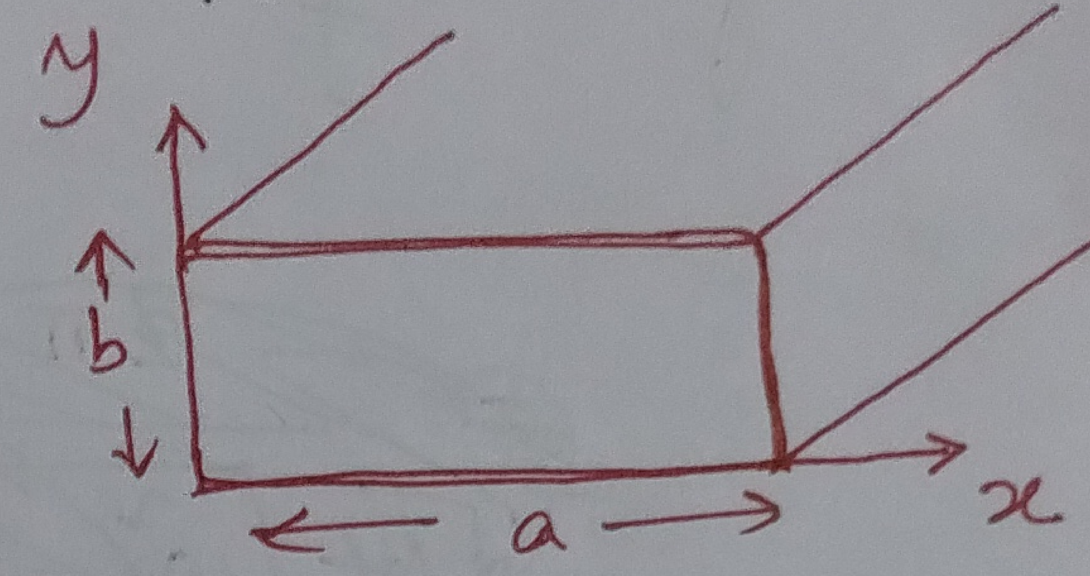
$$\frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2} = \frac{1}{\lambda_g^2}$$ (dividing by λ_0^2 throughout)

So, $\boxed{\frac{1}{\lambda_0^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}}$. guid wavelength equation.

The two subscripts m and n are used to designate a particular mode. The first one m indicates the number of half-period variations of E-field (also in TM-modes) along the x-direction in the guide which is taken along the dimension of the waveguide. The second one n counts the number of half period variations of E-field along y-axis along the width of the guide.

We see from the field solutions [eqn (8)-(12)] of TE-mode that if $m=0$ or $n=0$ or both, then all the field components donot become zero (unlike the TM-case).

Thus, for this case TE_{01} or TE_{10} modes will exist. Since m corresponds to the wide dimension of the waveguide, TE_{10} wave has a lowest cut-off frequency than TE_{01} wave.



The cutoff frequency

$$\omega_c^2 \mu\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \quad \text{or} \quad f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

For TE_{10} - mode $m = 1$, $n = 0$ so, $f_{c,10} = \frac{c}{2a} \Rightarrow \lambda = 2a$

And for TE_{01} - mode $m = 0$ $n = 1$ so $f_{c,01} = \frac{c}{2b} \Rightarrow \lambda = 2b$

since $a > b$ $f_{c,10} < f_{c,01}$.

The wave which has the lowest cut-off frequency is known as the dominant mode. In this case, TE_{10} is the dominant mode.

We see, for TE_{10} mode, the cut-off frequency is that for which the corresponding free space half-wavelength $\left(\frac{\lambda}{2}\right)$ is equal to the width of the guide. ie, $\boxed{\lambda_c = 2a}$.

The waves with wavelength longer than λ_c can not be transmitted through the guide. Thus, the waveguide is a form of high pass filter for transmitting a wave of frequency greater than $f_c = \frac{c}{\lambda_c}$. For TE_{10} - mode the guide wavelength is

$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{2a}\right)^2}}.$$

TM-mode : Rectangular waveguide

For TM-mode, $H_z = 0$ and $E_z = E_z(x,y)$. with z-dependence of propagation we can write

$$E_z(x,y,z) = E_z(x,y)\, e^{-i\beta z}$$

where $E_z(x,y) = X(x)\,Y(y)$

Substituting in the wave equation —

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} = -(k^2-\beta^2) = -\chi^2$$

or $$\frac{1}{X}\frac{d^2X}{dx^2} + \chi^2 = -\frac{1}{Y}\frac{d^2Y}{dy^2} = k_y^2 \text{ (say)}$$

where $\chi^2 = k^2 - \beta^2$ transverse compt of prop. vector.

Thus, we now solve two equations

$$\frac{1}{X}\frac{d^2X}{dx^2} = -\chi^2 + k_y^2 = -k_x^2 \text{ (say)}$$

and $$\frac{1}{Y}\frac{d^2Y}{dy^2} = -k_y^2 .$$

The solutions of these differential equations are

$$\left.\begin{aligned} X(x) &= A\cos k_x x + B\sin k_x x \\ Y(y) &= C\cos k_y y + D\sin k_y y \end{aligned}\right\}$$

Hence, the E_z field solution will be

$$E_z = AC\cos k_x x\cos k_y y + AD\cos k_x x\sin k_y y + BC\sin k_x x\cos k_y y + BD\sin k_x x\sin k_y y$$

Since $E_z = 0$ at $x = 0$ and $x = a$, we obtain

$$E_z = BC\sin k_x x\cos k_y y + BD\sin k_x x\sin k_y y$$

For $E_z = 0$ at $y = 0$ $C = 0$ and we get

$$E_z = BD\sin k_x x\sin k_y y = E_{0z}\sin k_x x\sin k_y y$$

Applying the conditions, $E_z = 0$ at $x = a$ and at $y = b$,

$$E_z = E_{0z} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

Using the Maxwell's eqns, we have the other field components

$$E_x = E_{0x} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad E_y = E_{0y} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b}$$

$$H_x = H_{0x} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \qquad H_y = H_{0y} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

<u>Cut-off wavelength</u>:

we have $\beta^2 = k^2 - \varkappa^2 = \omega^2\mu_0\epsilon - k_x^2 - k_y^2$

$$= \omega^2\mu_0\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2.$$

for propagation within the guide $\beta > 0$, giving the condition for cut-off frequency:

$$\omega_c^2\mu_0\epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

or,

$$\omega_c = \frac{1}{\sqrt{\mu_0\epsilon}} \cdot \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}.$$

or, $$f_c = \frac{1}{2\cancel{\pi}\sqrt{\mu_0\epsilon}} \cdot \cancel{\pi}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\therefore f_c = \frac{c}{2} \cdot \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

c = free space velocity of em waves.

So, $$\lambda_c = \frac{c}{f_c} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

The above is the condition of cut-off wavelength. If the wavelength is larger than this, the propagation will take place. Also, the cut-off wavelength depends on the size of the waveguide (a, b) and also on the mode order (m, n).

wavelength inside the guide: v_{guide}.

$$v_{guide} = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{\mu\epsilon}\sqrt{\left(1 - \frac{\omega_c^2}{\omega^2}\right)}}$$

$$\boxed{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega_c^2}$$

$$\boxed{v_{guide} = \frac{c}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}}$$

This shows that the wave propagation in the guide takes place at a larger speed than the free space speed of wave. At cut-off frequency (near cut-off) the phase velocity of the wave in the guide approaches infinity. Then the speed decrease as the frequency increase. At a very large frequency, the speed approaches that of light in free space.

The guide wavelength: λ_{guide}

$$\lambda_{guide} = \frac{v_{guide}}{f} = \frac{c}{f\sqrt{1 - (\omega_c/\omega)^2}} = \frac{\lambda_0}{\sqrt{1 - \frac{\lambda_0^2}{\lambda_c^2}}}.$$

$$\left[\ c/f = \lambda_0 = \text{Free Space}\ \right]$$

Hence

$$\frac{1}{\lambda_0^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}.$$

This is called guide equation.

The two subscripts m, n are used to designate a particular mode. The m indicates the no. of half period variations of E-field (even in TM) along x-axis which is taken along the narrow dimension of wave guide. The n indicates the no. of half period variations along y-axis.

We see for $m = 0$ or $n = 0$ all fields vanish. Thus the lowest order mode is only TM_{11}.