Coupling of waves and optical couplers

Mode coupling

Until now, we have been concerned with

the characteristics of perfect, uniform waveguides- axially invariant structures

We have seen

these structures to support a set of characteristic modes having specific field patterns that propagates in the waveguide

the field pattern of modes does not change with propagation in the waveguide but only a change in phase occurs

We'll encounter some exceptions

at waveguide tapers, bends, gratings and beam couplers, where mode amplitudes also change due to power redistribution amongst the guided modes

OR mode amplitudes change due to conversion from guided modes to radiation modes

Now we'll be concerned with any quantitative calculations of the amplitude changes

Introduction of a perturbation to an otherwise perfect guide can induce an interchange of energy amongst the modes

This interchange can be highly significant, if a number of conditions are satisfied

In fact, one individual mode can be converted into another with close to 100% efficiency

This principle is known as mode coupling

Mode coupling

Occurs in many branches of physics other than guided wave optics (most notably, in quantum mechanics)

The principle helps understand the performance of waveguides suffering from imperfection or distortion

The principle is also well exploited for use in many guided wave components and devices

We will begin with one of the most useful and versatile devices in both integrated and fibre optics- the directional coupler

which works by coupling together two modes travelling in the same direction- co-directional coupling?

In its simplest form, this acts as a beam splitter

But more complicated devices are achieved as two-way switches or modulators; further variants can be used as filters or polarizers

Directional coupler

Broadly, directional coupler works as follows

An evanescent field extending outside any dielectric waveguide

If two parallel guides are placed sufficiently close together, these parts

of the field must overlap spatially

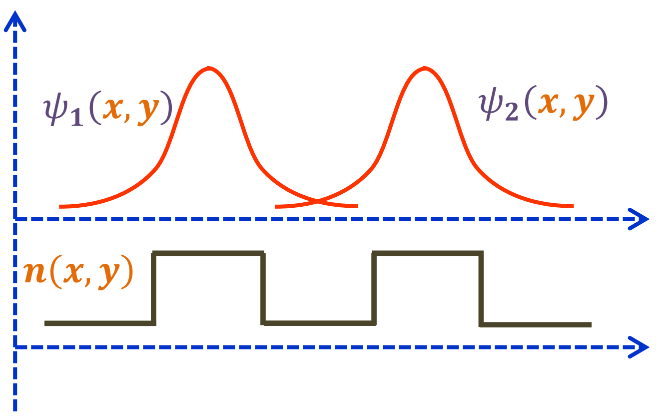
Usually, the inter-waveguide gap required for this overlap to be significant is of the order of the waveguide width

The overlap of modes through evanescent field induce redistribution of energy between the waveguides

If the two waveguides run parallel for sufficient distance, the exchange of power can be highly significant, reaching almost 100%

The field then starts coupling back

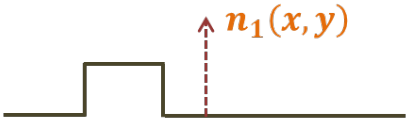
The power transfer is periodic with the distance or the interaction length



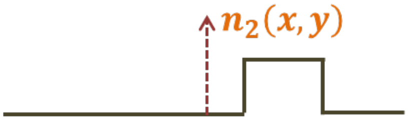
Coupled mode analysis: coupled mode equations

Consider two waveguides in close vicinity lying parallel to each other

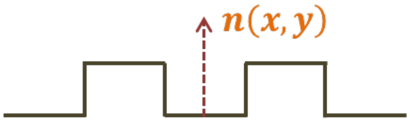
$n_1(x,y) \rightarrow R.I.\,profile\ of\ waveguide\ 1$ (a)



$n_2(x,y) \rightarrow R.I.\,profile\ of\ waveguide\ 2$ (b)



$n(x,y) \rightarrow that\ of\ composite\ waveguide$ (c)



If β_1 and β_2 respectively denote the propagating constants of the modes of the individual waveguides, then

$$\nabla_t^2\psi_1 + [\,k_o^2\, n_1^2(x,y) - \beta_1^2]\psi_1 = 0 \longrightarrow (1)$$

$$\nabla_t^2\psi_2 + [\,k_o^2\, n_2^2(x,y) - \beta_2^2]\psi_2 = 0 \longrightarrow (2)$$

here $\psi_1(x,y)$ and $\psi_2(x,y)$ represent the transverse mode field patterns of the individual waveguides, and

$$\nabla_t^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Now, if $\psi(x,y,z)$ denote the total field of the composite waveguide , then we have,

$$\nabla_t^2\,\psi + k_o^2 n^2(x,y)\,\psi + \frac{\partial^2\psi}{\partial z^2} = 0 \longrightarrow (3)$$

We shall now express $\psi(x,y,z)$ as a linear combination of the individual waveguide modes ψ_1 and ψ_2 *i.e.*,

$$\psi(x,y,z) = a(z)\psi_1(x,y) + b(z)\psi_2(x,y)$$

Where $\boldsymbol{a}(z)$ and $\boldsymbol{b}(z)$ can be expressed as

$$a(z) = A(z)e^{-i\beta_1 z}$$

$$b(z) = B(z)e^{-i\beta_2 z}$$

Then the composite waveguide mode

$$\psi(x, y, z) = a(z)\psi_1(x, y) + b(z)\psi_2(x, y)$$

takes the form: $\psi(x, y, z) = A(z)\psi_1(x, y)e^{-i\beta_1 z} + B(z)\psi_2(x, y)e^{-i\beta_2 z}$

This is valid when the two waveguides are not very strongly interacting

So we have $\quad \nabla_t^2\psi + k_o^2 n^2(x, y)\psi + \dfrac{\partial^2\psi}{\partial z^2} = 0 \longrightarrow$ (3)

$$\psi(x, y, z) = A(z)\psi_1(x, y)e^{-i\beta_1 z} + B(z)\psi_2(x, y)e^{-i\beta_2 z}$$

In order that this $\psi(x, y, z)$ will satisfy equation (3), we now consider the individual terms of equation (3) as the following:

First term: $\quad \nabla_t^2\psi = A(z)\, e^{-i\beta_1 z}\, \nabla_t^2\psi_1 + B(z)e^{-i\beta_2 z}\, \nabla_t^2\psi_2$

Third term:

$$\frac{\partial^2\psi}{\partial z^2} = \psi_1\frac{\partial}{\partial z}\left[\left(\frac{\partial A}{\partial z}e^{-i\beta_1 z}\right) + \left(-i\beta_1 e^{-i\beta_1 z}A\right)\right] + \psi_2\frac{\partial}{\partial z}\left[\left(\frac{\partial B}{\partial z}e^{-i\beta_2 z}\right) + \left(-i\beta_2 e^{-i\beta_2 z}B\right)\right]$$

$$= \psi_1\frac{\partial^2 A}{\partial z^2}e^{-i\beta_1 z} - i\beta_1\psi_1\frac{\partial A}{\partial z}e^{-i\beta_1 z} - i\beta_1\psi_1 e^{-i\beta_1 z}\frac{\partial A}{\partial z} + (-i\beta_1)^2 A\psi_1 e^{-i\beta_1 z}$$

$$+ \psi_2\frac{\partial^2 B}{\partial z^2}e^{-i\beta_2 z} - i\beta_2\psi_2\frac{\partial B}{\partial z}e^{-i\beta_2 z} - i\beta_2\psi_2 e^{-i\beta_2 z}\frac{\partial B}{\partial z} + (-i\beta_2)^2 B\psi_2 e^{-i\beta_2 z}$$

$$= \psi_1 e^{-i\beta_1 z}\left(\frac{\partial^2 A}{\partial z^2} - 2i\beta_1\frac{\partial A}{\partial z} - {\beta_1}^2 A\right) + \psi_2 e^{-i\beta_2 z}\left(\frac{\partial^2 B}{\partial z^2} - 2i\beta_2\frac{\partial B}{\partial z} - {\beta_2}^2 B\right)$$

Neglecting $\frac{\partial^2 B}{\partial z^2}$ *and* $\frac{\partial^2 B}{\partial z^2}$ *terms as* $A(z)$ *and* $B(z)$ *are slowly varying with* z

$$\frac{\partial^2 \psi}{\partial z^2} = \psi_1 e^{-i\beta_1 z}\left(-2i\beta_1 \frac{\partial A}{\partial z} - \beta_1^2 A\right) + \psi_2 e^{-i\beta_2 z}\left(-2i\beta_2 \frac{\partial B}{\partial z} - \beta_2^2 B\right)$$

$$\frac{\partial^2 \psi}{\partial z^2} = -2i\beta_1 \psi_1 e^{-i\beta_1 z}\frac{\partial A}{\partial z} - \beta_1^2 A\psi_1 e^{-i\beta_1 z} - 2i\beta_2 \psi_2\, e^{-i\beta_2 z}\frac{\partial B}{\partial z} - \beta_2^2 B\psi_2 e^{-i\beta_2 z}$$

Second term: $k_o^2 n^2(x,y)\psi = k_o^2 n^2 A\psi_1 e^{-i\beta_1 z} + k_o^2 n^2 B\psi_2 e^{-i\beta_2 z}$

So, from equation (3) we obtain

$$Ae^{-i\beta_1 z}\left(\nabla_\perp^2 \psi_1 - \beta_1^2 \psi_1 + k_o^2 n^2 \psi_1\right) + Be^{-i\beta_2 z}\left(\nabla_\perp^2 \psi_2 - \beta_2^2 \psi_2 + k_o^2 n^2 \psi_2\right)$$

$$-2i\beta_1 \psi_1 e^{-i\beta_1 z}\frac{\partial A}{\partial z} - 2i\beta_2 \psi_2 e^{-i\beta_2 z}\frac{\partial B}{\partial z} = 0 \longrightarrow \text{(3.1)}$$

Now, we consider the following correction terms:

$$\Delta n_1^2 = n^2(x,y) - n_1^2(x,y)$$

$$\Delta n_2^2 = n^2(x,y) - n_2^2(x,y) \qquad \textit{and} \quad \Delta\beta = \beta_1 - \beta_2$$

Using these terms, we rewrite the equation (3.1) as:

$$Ae^{-i\beta_1 z}\overbrace{\left(\nabla_\perp^2 \psi_1 - \beta_1^2 \psi_1 + k_o^2\, n_1^2\, \psi_1\right)}^{=0} + Be^{-i\beta_2 z}\overbrace{\left(\nabla_\perp^2 \psi_2 - \beta_2^2 \psi_2 + k_o^2\, n_2^2\, \psi_2\right)}^{=0}$$

$$+\, k_o^2 \Delta n_1^2 A\psi_1 e^{-i\beta_1 z} + k_o^2 \Delta n_2^2 B\psi_2 e^{-i\beta_2 z} - 2i\beta_1 \frac{\partial A}{\partial z}\psi_1 e^{-i\beta_1 z} - 2i\beta_2 \frac{\partial B}{\partial z}\psi_2 e^{-i\beta_2 z} = 0$$

So equation (3.1) takes the reduced form as:

$$Ak_o^2 \Delta n_1^2 \psi_1 + Bk_o^2 \Delta n_2^2 \psi_2 e^{i\Delta\beta z} - 2i\beta_1 \frac{\partial A}{\partial z} \psi_1 - 2i\beta_2 \frac{\partial B}{\partial z} \psi_2 e^{i\Delta\beta z} = 0 \longrightarrow (3.2)$$

Multiplying equation (3.2) by ψ_1^* from the left and integrating over the whole cross-section of the composite system, we write

$$Ak_o^2 \iint_{-\infty}^{+\infty} \psi_1^* \Delta n_1^2 \psi_1 dxdy + Bk_o^2 e^{i\Delta\beta z} \iint_{-\infty}^{+\infty} \psi_1^* \Delta n_2^2 \psi_2 dxdy - 2i\beta_1 \frac{\partial A}{\partial z} \iint_{-\infty}^{+\infty} \psi_1^* \psi_1 dxdy - 2i\beta_2 e^{i\Delta\beta z} \frac{\partial B}{\partial z} \iint_{-\infty}^{+\infty} \psi_1^* \psi_2 dxdy = 0$$

$$Ak_o^2 \iint_{-\infty}^{+\infty} \psi_1^* \Delta n_1^2 \psi_1 dxdy + Bk_o^2 e^{i\Delta\beta z} \iint_{-\infty}^{+\infty} \psi_1^* \Delta n_2^2 \psi_2 dxdy - 2i\beta_1 \frac{\partial A}{\partial z} \iint_{-\infty}^{+\infty} \psi_1^* \psi_1 dxdy - \underbrace{2i\beta_2 e^{i\Delta\beta z} \frac{\partial B}{\partial z} \iint_{-\infty}^{+\infty} \psi_1^* \psi_2 dxdy}_{} = 0$$

We'll neglect this term as the overlap of the modes ψ_1 and ψ_2 are very small compared to $\iint_{-\infty}^{+\infty} \psi_1^* \psi_1 dxdy$

Therefore, $$\frac{\partial A}{\partial z} = -\frac{Aik_o^2}{2\beta_1} \frac{\iint_{-\infty}^{+\infty} \psi_1^* \Delta n_1^2 \psi_1 dxdy}{\iint_{-\infty}^{+\infty} \psi_1^* \psi_1 dxdy} - \frac{Bik_o^2}{2\beta_1} \frac{\iint_{-\infty}^{+\infty} \psi_1^* \Delta n_2^2 \psi_2 dxdy}{\iint_{-\infty}^{+\infty} \psi_1^* \psi_1 dxdy} e^{i\Delta\beta z}$$

We write $$\frac{\partial A}{\partial z} = -i\kappa_{11} A - i\kappa_{12} B e^{i\Delta\beta z}$$

Similarly, $$\frac{\partial B}{\partial z} = -i\kappa_{22} B - i\kappa_{21} A e^{-i\Delta\beta z}$$

$\longrightarrow$ (4)

Here we have defined the coupling coefficients as

$$\kappa_{11} = \frac{k_o^2}{2\beta_1} \frac{\iint_{-\infty}^{+\infty} \psi_1^* \Delta n_1^2 \psi_1 dxdy}{\iint_{-\infty}^{+\infty} \psi_1^* \psi_1 dxdy} \qquad \kappa_{22} = \frac{k_o^2}{2\beta_2} \frac{\iint_{-\infty}^{+\infty} \psi_2^* \Delta n_2^2 \psi_2 dxdy}{\iint_{-\infty}^{+\infty} \psi_2^* \psi_2 dxdy}$$

$$\kappa_{12} = \frac{k_o^2}{2\beta_1} \frac{\iint_{-\infty}^{+\infty} \psi_1^* \Delta n_2^2 \psi_2 dxdy}{\iint_{-\infty}^{+\infty} \psi_1^* \psi_1 dxdy} \qquad \kappa_{21} = \frac{k_o^2}{2\beta_2} \frac{\iint_{-\infty}^{+\infty} \psi_2^* \Delta n_1^2 \psi_1 dxdy}{\iint_{-\infty}^{+\infty} \psi_2^* \psi_2 dxdy}$$

Now writing back $a(z)$ and $b(z)$ again

$$a(z) = A(z)e^{-i\beta_1 z}$$

$$b(z) = B(z)e^{-i\beta_2 z}$$

we can write equation (4) as following

$$\left.\begin{aligned} \frac{\partial a}{\partial z} &= -i(\beta_1 + \kappa_{11})a - i\kappa_{12} b \\ \frac{\partial b}{\partial z} &= -i(\beta_2 + \kappa_{22})b - i\kappa_{21} a \end{aligned}\right\}$$

These are Coupled mode equations

It reveals that κ_{11} and κ_{22} are corrections to the propagation constants of the individual waveguide modes due to the presence of the other waveguide

These correction factors are usually neglected in the analysis as it is too small

$$\boxed{\begin{aligned} \frac{\partial a}{\partial z} &= -i\beta_1 a - i\kappa_{12} b \\ \frac{\partial b}{\partial z} &= -i\beta_2 b - i\kappa_{21} a \end{aligned}}$$

These are Coupled mode equations used in coupler analysis