

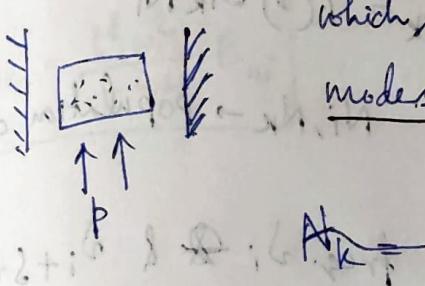
LASER

Light amplification by stimulated emission of radiation
 Continuous wave gas laser. (e.g. - He-Ne Laser)

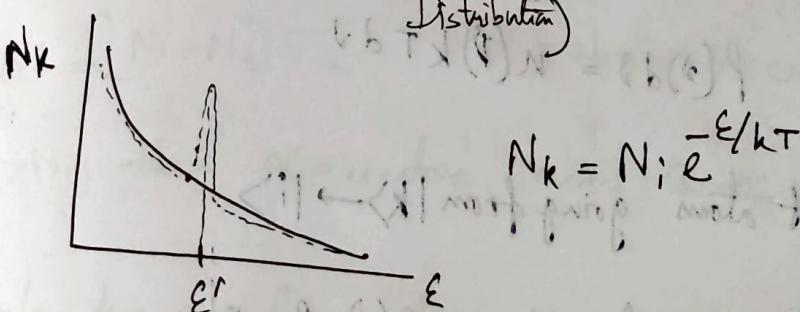
13/01/2025

- 1) Active medium: amplifies the intensity of E.M. radn.
- 2) Pump: populates selected energy levels of the active medium. (Population inversion)

3) Optical Resonator: two parallel high reflective mirror which stores energy of few resonator modes by providing feedback.



② Population Inversion → (Classically we use Boltzmann Distribution)



So the Laser setup works as (kind of Black Body we can say in one direction) closed resonator.

$$E = E_0 e^{i(\omega t - kx)} + E_0 e^{i(\omega t + kx)}$$

$$k = \frac{\pi}{L} \sqrt{n_1^2 + n_2^2 + n_3^2}^{1/2}$$

$$x = \frac{L}{\pi} \frac{1}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$$

for laser we need open cavity, cause we'll keep only few of the resonator modes.

1st time possibility was told by Einstein.

Einstein's A & B coefficient →

|k⟩ (a) No. of atoms going from |i⟩ to |k⟩

$$R_{i \rightarrow k} = P(\nu) B_{ik} N_i$$

$P(\nu)$ = spectral density

N_i, N_k → population density

No. of modes within the freq. ν_i & $\nu_i + \Delta\nu$.

$$N(\nu) d\nu = \frac{8\pi \nu^2}{c^3} d\nu$$

$$P(\nu) d\nu = N(\nu) kT d\nu$$

(b) No. of atoms going from |k⟩ → |i⟩

$$R_{k \rightarrow i} = A_{ki} N_k + P(\nu) B_{ki} N_k$$

(c) At equilibrium →

$$R_{i \rightarrow k} = R_{k \rightarrow i}$$

$$P(\nu) B_{ik} N_i = A_{ki} N_k + P(\nu) B_{ki} N_k$$

$$P(\nu) B_{ik} N_i = N_i e^{-E/kT} [A_{ki} + B_{ki} P(\nu)]$$

$$P(i) \left[B_{ik} - B_{ki} e^{\epsilon/kT} \right] = A_{ki} e^{\epsilon/kT}$$

$$P(i) = \frac{A_{ki} e^{-\epsilon/kT}}{B_{ik} - B_{ki} e^{\epsilon/kT}} = \frac{A_{ki}}{B_{ik} e^{\epsilon/kT} - B_{ki}}$$

$$= \frac{\hbar^2 \omega^3}{\pi^2 c^3} \frac{1}{e^{\epsilon/kT} - 1} (\epsilon - h\nu)$$

$\checkmark B_{ik} = B_{ki}$

$\checkmark A_{ik} = \frac{\hbar^2 \omega^3}{\pi^2 c^3} B_{ki}$

Intensity $I = I_0 \bar{e}^{\alpha L}$ α = absorption coefficient.

$$\alpha = (N_i - N_k) \sigma_{ik} \quad \sigma_{ik} = \text{absorption cross-section.}$$

$N_k > N_i$ then $\alpha = \text{negative}$ means $I = I_0 \bar{e}^{\alpha L}$.

To keep only few resonator modes we need to have a loss →

loss coefficient in a round trip ($2L$): γ

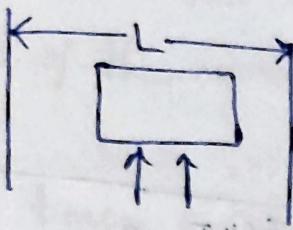
$$I_{loss} = I_0 \bar{e}^{-\gamma}$$

(we consider round trip)

$$I = I_0 \bar{e}^{\alpha \cdot 2L} \bar{e}^{-\gamma}$$

so for increase in intensity $\alpha \cdot 2L \gg \gamma$

$$\boxed{\Delta N > \frac{\gamma}{2L \sigma_{ik}}}$$



Suppose \rightarrow

$$L = 10 \text{ cm}$$

$$\gamma = 10\%$$

$$\sigma = 10^{-12} \text{ cm}^2$$

$$\Delta N = \frac{\gamma}{2L\sigma} = \frac{10}{2 \times 10 \times 10^{-12}} = \frac{10^{12}}{2 \times 100} = 0.5 \times 10^9$$

$$\Delta N \sim 10^{10} \text{ cm}^{-3}$$

we have to get this to get Laser.

② Rate Eqn.

∇P \rightarrow P: pump rate : no. of atoms of on molecule at 12 per cm^3 per sec by pump

∇R_i \rightarrow R_i : no. of atoms / cm^3 in state i.

n : photon density \rightarrow (12) get laser in two phase

$N_i R_i$ = No. of atoms which are removed from i by relaxation

(collision)

$$i) \frac{dN_1}{dt} = -\beta_{12} nhv N_1 + \beta_{12} nhv N_2 + A_{12} N_2 - N_1 R_1$$

$$ii) \frac{dN_2}{dt} = \beta_{12} nhv N_1 - \beta_{12} nhv N_2 - A_{21} N_2 + P - N_2 R_2$$

$$iii) \frac{dn}{dt} = -\beta n + \beta_{12} nhv (N_2 - N_1)$$

n = photon density
 ↓
 (rate eqn)
 (for photon)

β = loss coefficient

$$\gamma = \beta \cdot T = \beta \cdot \frac{2d}{c}$$

General loss coefficient due to round trip.

We cannot combine any gas to get the laser. 14/01/2025

~~(iii)~~ Stationary state that does not change with time.

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = \frac{dn}{dt} = 0$$

$$\text{Add (i) and (ii)} \quad -N_1 R_1 + P - N_2 R_2 = 0$$

$$\Rightarrow P = N_1 R_1 + N_2 R_2 \quad \dots \dots \text{--- (iv)}$$

$$\text{Add (ii) and (iii)} \quad \beta n - A_{21} N_2 + P - N_2 R_2 = 0$$

$$\Rightarrow P = \beta n + N_2 (R_2 + A_{21}) \quad \dots \dots \text{--- (v)}$$

R_2 = relaxation

A_{21} = spontaneous emission coefficient $| (R_2 + A_{21}) \rightarrow \text{incoherent loss rate.}$

From (iv) and (v)

$$N_1 R_1 + N_2 R_2 = \beta n + N_2 R_2 + N_2 A_{21}$$

$$\Rightarrow N_1 R_1 = \beta n + N_2 A_{21} \quad \text{--- (vi)}$$

$R_2 \times \text{eqn (i)}$

$$-B_{12} nh \beta N_1 R_2 + B_{12} nh \beta N_2 R_2 + A_{21} N_2 R_2 - N_1 R_1 R_2 = 0$$

$R_1 \times \text{eqn (ii)}$

$$B_{12} nh \beta N_1 R_1 - B_{12} nh \beta N_2 R_1 - A_{21} N_2 R_1 - N_2 R_2 R_1 + P R_1 = 0$$

$$\Rightarrow -P R_1 + R_1 R_2 \Delta N + A_{21} N_2 R_1 + A_{21} (P - N_1 R_1) + B_{12} nh \beta N_1 (R_1 - B_{12} nh \beta N_2 (R_1 + R_2)) = 0 \quad (\Delta N = N_2 - N_1)$$

$$\Rightarrow -P R_1 + R_1 R_2 \Delta N + A_{21} P + A_{21} R_1 \Delta N + B_{12} nh \beta (R_1 + R_2) \Delta N = 0$$

$$\Rightarrow \Delta N = \frac{P(R_1 - A_{21})}{B_{12} nh \beta (R_1 + R_2) + R_1 R_2} \quad \text{as } \Delta N > 0$$

$$R_1 > A_{21}$$

$$\frac{1}{\tau_1} > A_{21} \Rightarrow \left[\frac{\tau_1}{\tau_1 + A_{21}} < \frac{A_{21}}{A_{21} + R_1} \right] \quad \text{--- (vii)}$$

(iv), (v), (vi), (vii) has to satisfy to get the laser.

Let us consider if state $|1\rangle$ is your ground state then there will be no N_{1R} .

W_k : Energy stored for a particular mode k .

Loss rate of the mode $\frac{dW_k}{dt} = -\beta_k W_k$

$$W_k = W_k(0) e^{-\beta_k t}$$

Quality factor $\mathcal{Q} = \frac{\text{Energy stored}}{2\pi}$

$$\begin{aligned} & \text{as per cycle} \\ & \frac{2\pi}{T} \\ & \frac{W_k}{\frac{dW_k}{dT} T} \\ & = 2\pi \frac{W_k}{\beta_k W_k T} = \frac{2\pi}{\beta_k} \end{aligned}$$

higher the β value, lesser will be the effect of particular mode.

When we talk about the loss, we have two types of loss \rightarrow ① Reflection loss: ② Diffraction loss.

① Reflection Loss \rightarrow , Reflectivity of mirrors are very high, 99%, 99.99% reflectivity and it has to have a finite reflectivity. (practically $1\text{cm} \times 1\text{cm}$).

$$\text{For round trip} \rightarrow I = I_0 R_1 R_2 = I_0 e^{-\gamma_L d}$$

$$\gamma_L = -\ln(R_1 R_2)$$

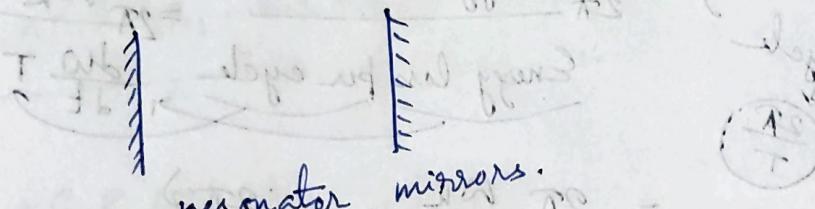
$$\gamma_L = \beta_k \frac{2d}{c} = |\ln(R_1 R_2)| \Rightarrow \beta_k = \frac{c}{2d} |\ln(R_1 R_2)|$$

$$\Rightarrow \frac{1}{I} = \frac{2d}{c[\ln R_1 R_2]}$$

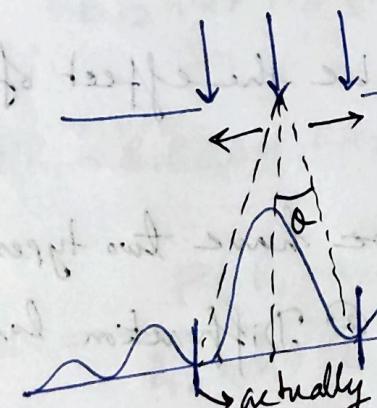
for any wavelength reflectivity is high, denominator high
fraction low, $\frac{1}{I}$ low, I high.

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② Diffraction Loss :- It is because of resonator mirrors.



resonator mirrors.



$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2}$$

$$\alpha = \frac{\pi b \sin \alpha}{\lambda}$$

actually the central maxima is inside two mirrors.

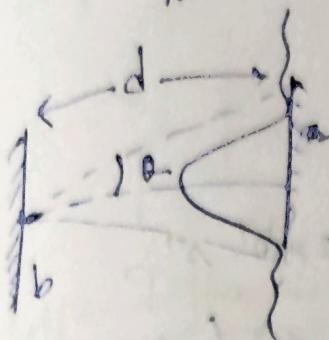
Babinet principle :- Diffraction by an open body

$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2}$$

$$\alpha = \frac{\pi b \sin \alpha}{\lambda} \approx \frac{\pi b \alpha}{\lambda} = \pi$$

$$\Rightarrow \alpha = \frac{\lambda}{b} \quad (\text{for central maxima})$$

Diffraction loss is defined by something called Fresnel Number : $N = \frac{b^2}{\lambda d}$



$$a = \frac{b}{2d}$$

for n transits

$$a = \frac{b}{2nd}$$

$$\Rightarrow n = \frac{b}{2ad} = \frac{b^2}{2d^2} = \frac{N}{2}$$

$\boxed{Y_{\text{diff}} \sim \frac{1}{N}}$ (Diffraction Loss)

② Fabry Perot Interferometer \rightarrow two mirrors kept at very small distance and

$$b = 3 \text{ cm} \quad d = 1 \text{ cm} \quad \lambda = 500 \text{ nm} = 500 \times 10^{-7} \text{ cm}$$

$$N = \frac{b^2}{\lambda d} = \frac{9 \text{ cm}^2}{5 \times 10^{-5} \times 1 \text{ cm}} = 9 \times 10^5$$

③ Laser Resonator :-

$$b = 0.1 \text{ cm} \quad d = 50 \text{ cm} \quad \lambda = 500 \text{ nm} = 5 \times 10^{-5} \text{ cm}$$

$$N = \frac{b^2}{\lambda d} = \frac{0.1 \times 0.1}{50 \times 5 \times 10^{-5}} = \frac{0.1 \times 0.1 \times 10^4}{25}$$

$$= \frac{10^4}{25} = 4$$

$Y_{\text{diff}} \sim \frac{1}{N}$, Fabry Perot interferometer has less diff. loss though the construction is same. It's the matter of construction size.

Laser mirror size should be very small as compared to the length.

①

$$N_2 = 10^{10} \text{ cm}^{-3}$$

$$\text{Incoherent Loss rate: } (A_{21} + R_2) = 2 \times 10^7 \text{ s}^{-1}$$

$$\text{Total incoherent loss: } N_2 \times (A_{21} + R_2) / \text{cm}^3 \propto$$

$$= 2 \times 10^{17} \times 10^{10} \text{ cm}^{-5}$$

$$= 2 \times 10^{17} \text{ cm}^{-3} \text{ s}^{-1}$$

$$\text{Suppose } L = 10 \text{ cm} \quad b = 1 \text{ mm}$$

total incoherent loss for cylinder per second \rightarrow

$$\approx 10^{16} \text{ s}^{-1}$$

Total no. of photons in the excited state = $N_2 \times \text{volume}$.
at the same time we need the decays around 10^{18} s^{-1}
~~then~~ then only we can get laser.

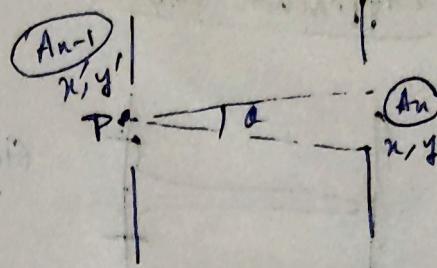
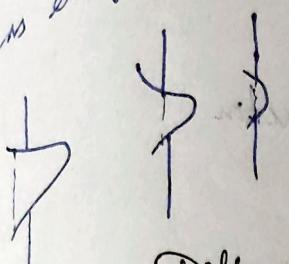
$$\frac{\text{no. of photons}}{\text{time}} = \frac{\text{no. of photons}}{10 \times 10^{-3}} = \frac{10 \times 10^3}{10 \times 2 \times 10^{-3}} = \frac{10^4}{2 \times 10^{-3}} = 5 \times 10^6$$

$$\rho = \frac{10^4}{2 \times 10^{-3}}$$

What will be the intensity of laser?

Intensity of interference with direct and

Some more about diffraction loss →
 In every back and forth reflection, there must be a loss every time.



Kirchhoff's Diffraction Theory →

$$A(u, y) = \frac{i}{\lambda} \iint A_{n-1}(x', y') \frac{e^{ikP}}{P} \cos \phi \, dx' dy'$$

Stationary Condition \rightarrow phase factor.

$$A_n(u, y) = C A_{n-1}(u, y) \quad C = \sqrt{1 - r_0} e^{i\phi}$$

$$A_n(u, y) = -\frac{i}{\lambda} \iint \frac{A_n(u, y)}{C} \frac{e^{ikP}}{P} \cos \phi \, dx' dy'$$

$$= -\frac{i}{\lambda} (1 - r_0)^{1/2} e^{i\phi} \iint A_n(u', y') \frac{e^{ikP}}{P} \cos \phi \, dx' dy'$$

phase factor

and $r_0 = \frac{1}{2} \pi$

$$P_s = \frac{d^2}{4} \pi^2$$

relative permittivity

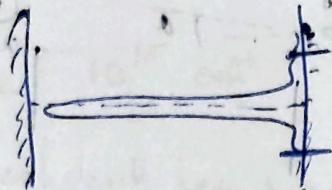
$$\text{and } r_0 = \sqrt{\epsilon_r}$$

$$P_s = \frac{d^2}{4} \pi^2 \epsilon_r$$

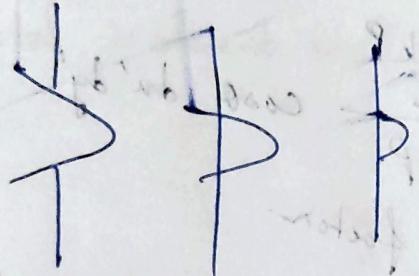
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Monday

Diffraction Loss →



single slit diffraction



because of the diffraction law

$$A(x, y) = \int_{-\infty}^{\infty} A_{n-1}(x', y') \frac{e^{ikr}}{r} e^{i\phi} d\theta' d'y'$$

$$A(x, y) = \int_{-\infty}^{\infty} A_{n-1}(x', y') \frac{e^{ikr}}{r} e^{i\phi} d\theta' d'y'$$

Kirchhoff's Law

$$A_n(x, y) = -\frac{i}{\lambda} \iint A_{n-1}(x', y') \frac{e^{ikr}}{r} \cos \theta d\theta' d'y'$$

In stationary condition

$$A_n(x, y) = C A_{n-1}(x, y) \quad C = \sqrt{1 - \gamma_0} e^{i\phi}$$

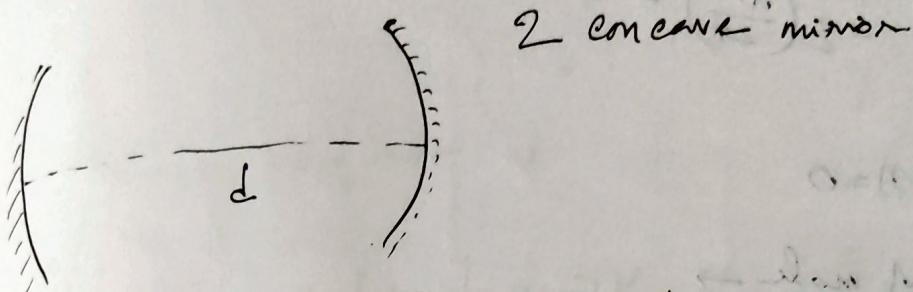
γ_D = diffraction loss for
round trip

$$A_n(x, y) = -\frac{i}{\lambda} (1 - \gamma_b)^{1/2} e^{i\phi} \int A_n(x', y') \frac{e^{ikr}}{r} \cos \theta d\theta dy'$$

field distribution.

for particular case \rightarrow for this geometry if one finds field distribution \rightarrow

A_n for confocal minor resonator



radii of curvature $\rightarrow R_1 = R_2 = R = d$

Field distribution becomes

$$A_{mn} \stackrel{(r, R, z)}{=} C^* H_m(x^*) H_n(y^*) e^{-r^2/\omega^2} \exp(-i\phi(r, R, z))$$

$$x^* = \frac{\sqrt{2}x}{\omega} \quad y^* = \frac{\sqrt{2}y}{\omega} \quad \omega = \frac{\pi d}{2\pi} \left(1 + \left(\frac{2z}{d}\right)^2\right)$$

(M, n) gives you different modes \rightarrow

$$m = n = 0 \quad H_m = H_n = 1$$

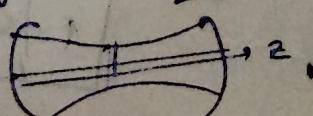
$$A_{00} = C^* e^{-r^2/\omega^2} e^{-i\phi}$$

$$\downarrow I_{00} = I_0 e^{-2\pi^2/\omega^2} \quad (\text{shape of the wave} \rightarrow \text{Gaussian})$$

$$\text{at } r = \omega \rightarrow I_{00} = I_0 \omega^{-2} \doteq \text{beam radius}$$

$$\text{when } z = 0, \text{ min value of } \omega^2 = \frac{\pi d}{2\pi}$$

b = beam waist



beam waist is min, at $z=0$

beam radius keep on changing.

$$w = R_w = \left(\frac{\pi R}{2\pi} \right)^{1/2} \text{ at } z=0$$

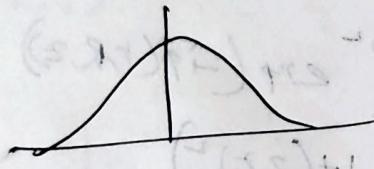
$$R_w = R_{1/2} = \left(\frac{\pi R}{2\pi} \right)^{1/2} \left(1 + \left(\frac{2R}{\lambda} \right)^2 \right)$$

$$= 2 \left(\frac{\pi R}{2\pi} \right)^{1/2} = 2 R_w$$

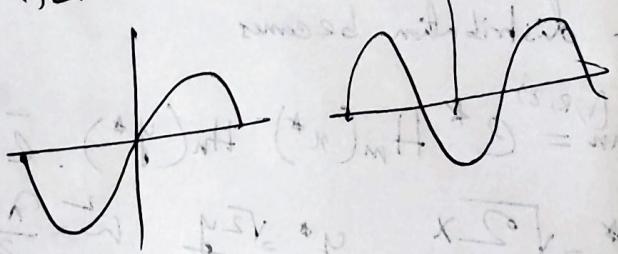
$$M=0, n=0$$

TEM mode \rightarrow

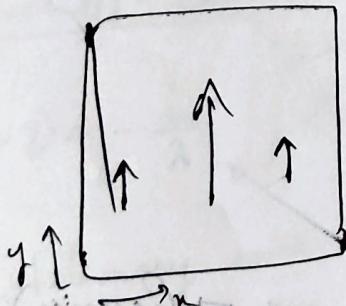
1D TEM₀₀



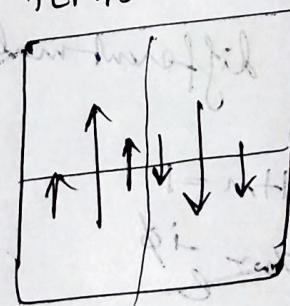
TEM₁₁



2D TEM₀₀

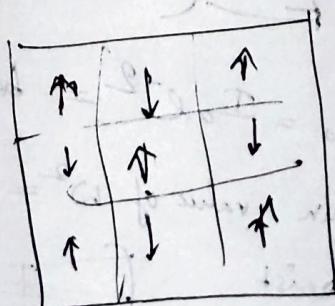
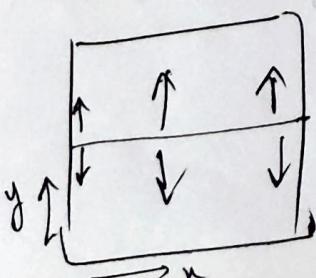


TEM₁₀

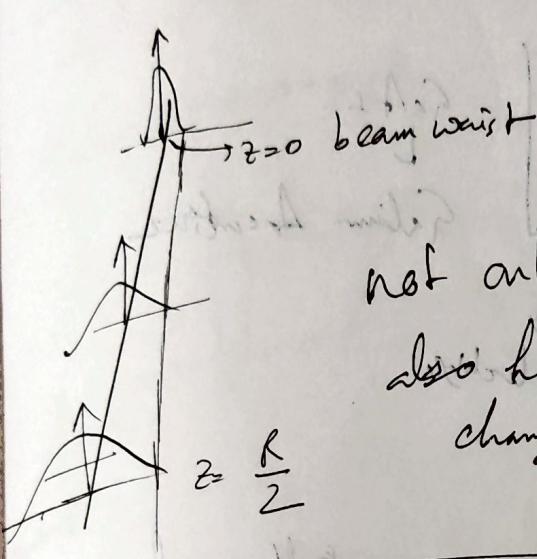
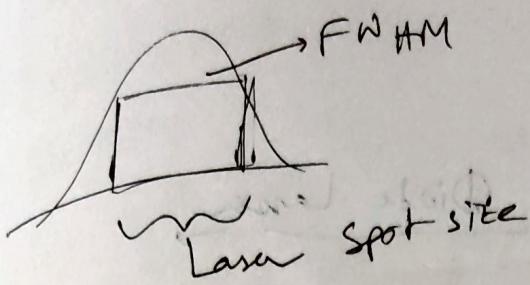


TEM₂₀

TEM₀₁

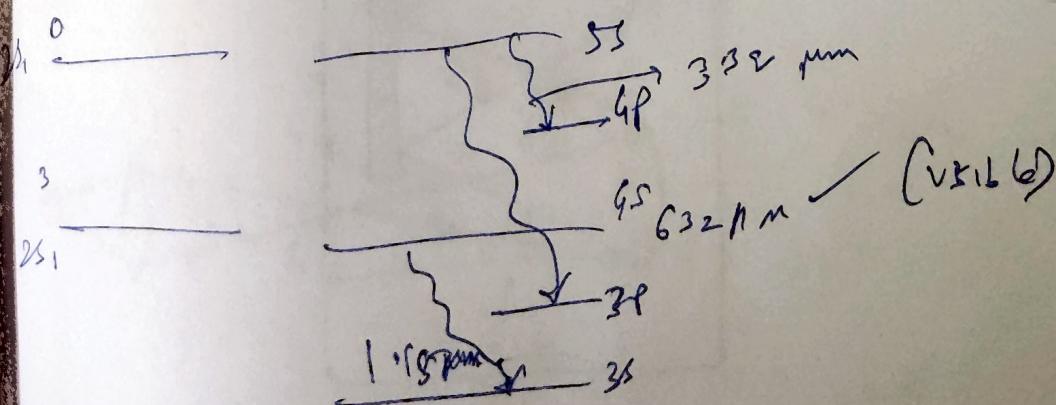
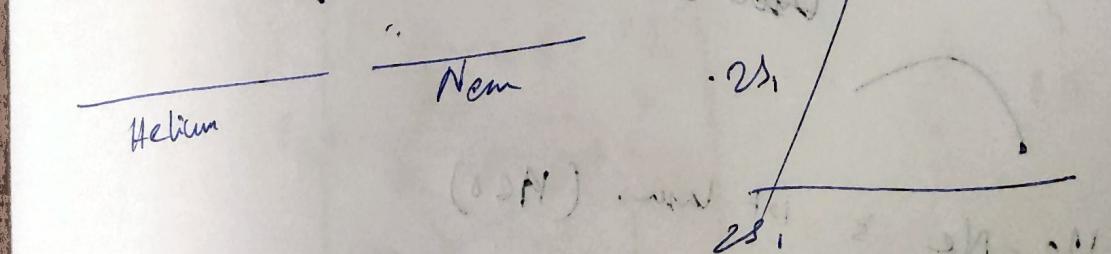


On the screen we get circular region and intensity decreased Gaussian.



not only width changed, phase also has changed. Phase varies with distance.

He-Ne laser \rightarrow use this for stability.



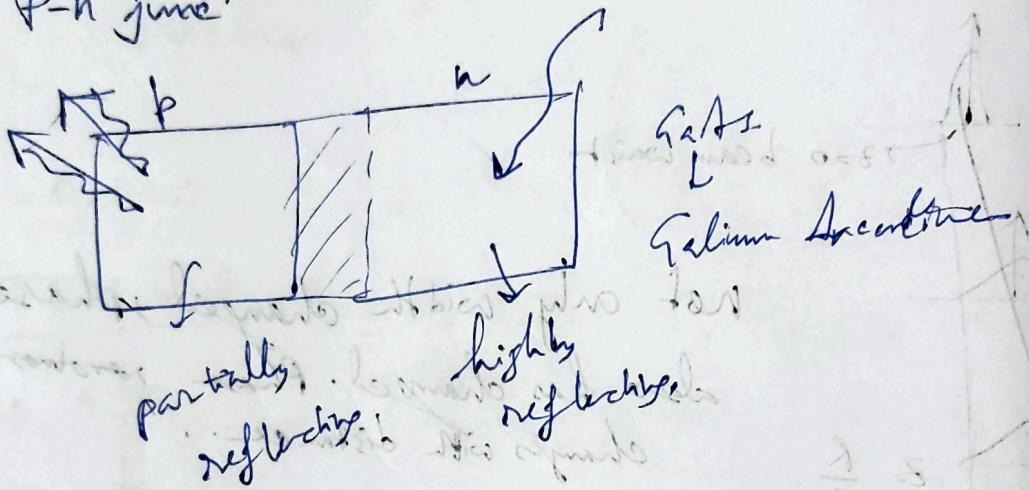
Ar laser. laser operation by excitation

Continuous wave gas laser - (CW Laser)

Semi-Conductor

Semi Conductor laser / Diode Laser

P-n junction



Laser pointer is actually a diode laser

laser diode laser

He-Ne is 1st laser. (1960)

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Tuesday

Basic units of Spectroscopy

Source

① Dispersion unit

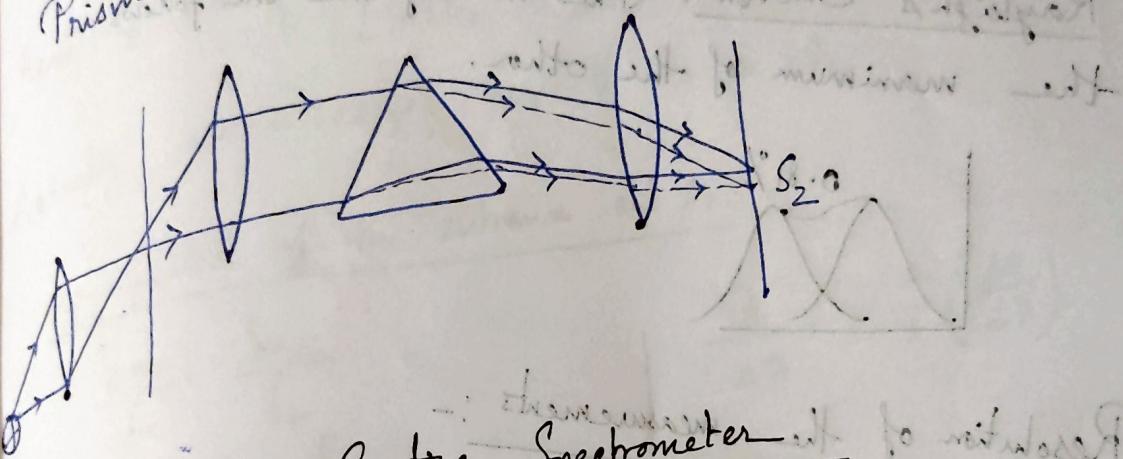
② Detector

Dispersion unit :-

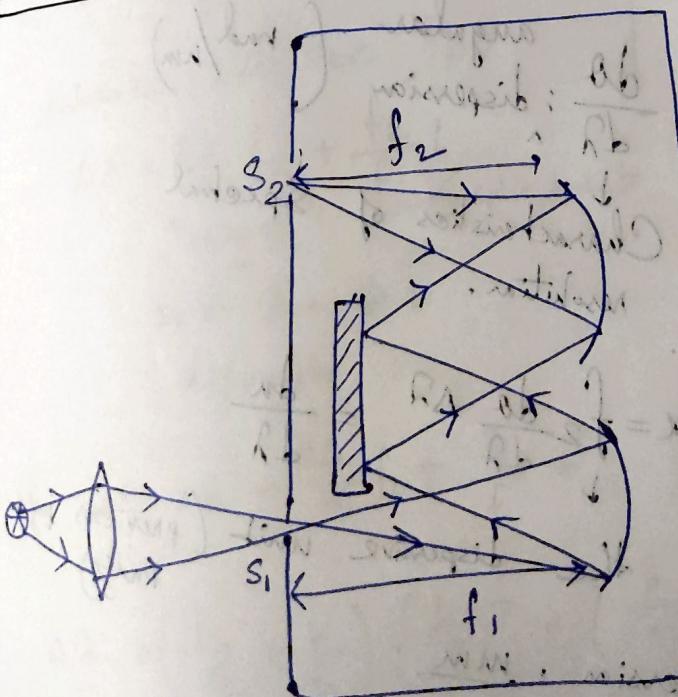
Spectrometer / Spectrograph

which gives image of a slit S_1 and S_2

Prism Spectrograph



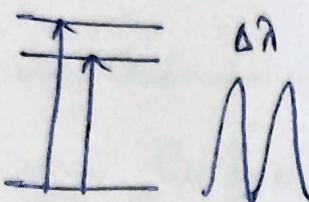
We mostly use Grating Spectrometer



$$R_D = \frac{\theta_b}{\theta_a} = 0.1$$

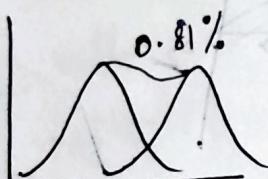
Spectroscopy is a quantum phenomena.

Resolution means the close lines/ close energy levels that we want to measure.



Two spectral lines can be resolved if they satisfy Rayleigh's Criteria.

Rayleigh's Criterion → The min. of one follows on the maximum of the other.



Resolution of the measurements :-

angular speed

$$\Delta\Omega = \frac{d\Omega}{d\lambda} \Delta\lambda$$

angular dispersion (rad/nm)

$$\frac{d\theta}{d\lambda}$$

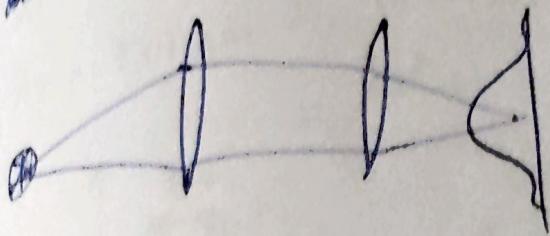
Characteristics of Spectral resolution.

$$\text{linear dispersion } \Delta x = f_2 \frac{d\theta}{d\lambda} \Delta\lambda = \frac{dn}{d\lambda}$$

f_2 → focal length of the dispersive unit (previous page image).

$$\frac{dn}{d\lambda} : \text{linear dispersion, } \frac{\text{mm}}{\text{nm}}$$

Linear dispersion: $\Delta\lambda = f_2 \frac{da}{d\lambda}$



a = size of the grating/prism element

$$\text{diffraction} \quad \Delta\lambda_2 = \frac{2\lambda}{a} f_2 \Rightarrow f_2 \frac{da}{d\lambda} \Delta\lambda \geq \frac{2\lambda}{a} f_2$$

Resolving power: $\left| \frac{\lambda}{\Delta\lambda} \right| \leq 2a \left(\frac{da}{d\lambda} \right)$

Actually we're seeing image of S_1 or S_2

magnification of the entrance slit S_1

$$\frac{\delta x_1}{f_1} = \frac{\delta x_2}{f_2}$$

$$\delta x_2 = \frac{f_2}{f_1} \delta x_1$$

entrance slit size : b

$$\Delta x = \underbrace{\frac{2\lambda}{a} f_2}_{\text{split due to magnification}} + \underbrace{\frac{f_2}{f_1} b}_{\text{split due to diffraction}}$$

split due to
diffraction

$$\frac{\lambda}{\Delta\lambda} = \frac{a}{3} \left(\frac{da}{d\lambda} \right)$$

$$f_2 \frac{da}{d\lambda} \Delta\lambda = \frac{2\lambda}{a} f_2 + \frac{f_2}{f_1} b$$

$$\Delta\lambda = \left(\frac{\lambda}{a} + \frac{b}{f_1} \right) \left(\frac{da}{d\lambda} \right)^{-1} = \left(\frac{\lambda}{a} + \frac{2\lambda f_1}{a f_1} \right) \left(\frac{da}{d\lambda} \right)^{-1}$$

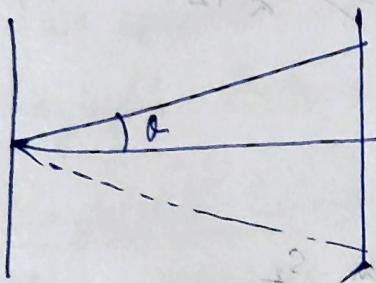
$$\Delta\lambda = \frac{3\lambda}{a} \left(\frac{da}{d\lambda} \right)^{-1}$$

b = entrance slit.

there is a min limit of b :

$$\frac{2\lambda}{b} f_1 = a$$

$$b_{\min} = \frac{2\lambda f_1}{a}$$



$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2}$$

$$\left(\frac{dI}{d\alpha}\right)_{\alpha=0} = \frac{2\lambda b \alpha}{\lambda^2} = 0$$

$$\Rightarrow \alpha = \frac{2\lambda}{b}$$

$$\left(\frac{dI}{d\alpha}\right)_{\alpha=0} = -\frac{2}{\alpha^2}$$

$$\frac{dI}{d\alpha} = \frac{dI}{d\theta} \frac{d\theta}{d\alpha}$$

$$\frac{dI}{d\theta} = \frac{dI}{d\alpha} \frac{d\alpha}{d\theta}$$

if $dI/d\theta < 0$ then

$\frac{dI}{d\theta} + d\frac{dI}{d\theta} = d\frac{dI}{d\theta}$
the figure of which is

if $dI/d\theta > 0$
then

$$\frac{dI}{d\theta} + d\frac{dI}{d\theta} = d\frac{dI}{d\theta}$$

$$\left(\frac{dI}{d\theta}\right) \left(d\frac{dI}{d\theta} + d\frac{dI}{d\theta}\right)$$

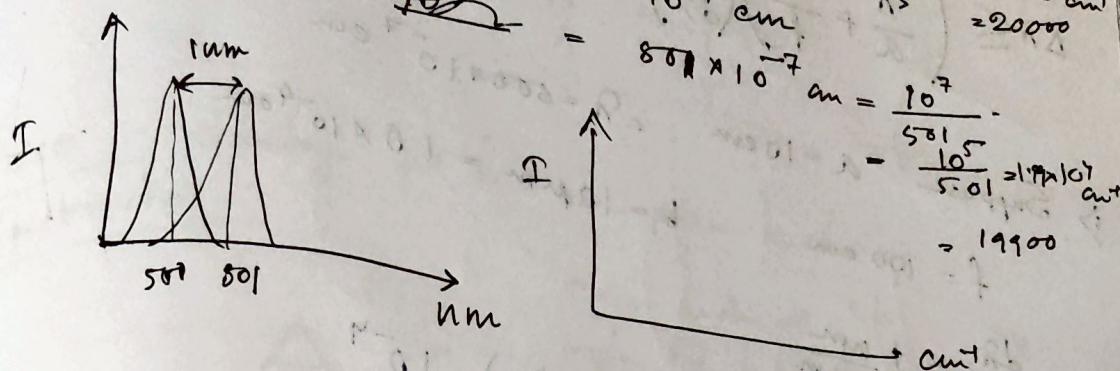
$$\left(\frac{dI}{d\theta}\right) \left(d\frac{dI}{d\theta} + d\frac{dI}{d\theta}\right)$$

$$\left(\frac{dI}{d\theta}\right) \frac{dI}{d\theta} = 0$$

22/1/25

$$\lambda_1 = 500 \text{ nm} = 500 \times 10^{-7} \text{ cm}$$

$$\lambda_2 = 501 \text{ nm} \quad \Delta\lambda = 1 \text{ nm} = \frac{10^7}{500} = \frac{10^7}{501} = 2 \times 10^8 \text{ cm}^{-1}$$



$$\Delta\lambda = 20000 - 19900$$

$$= 100 \text{ cm}^{-1}$$

$$\frac{19900}{20000} = 0.995$$

~~Q~~ $E_V = \frac{1240}{500} \text{ eV} = 2.48 \text{ eV}$

$E_V = \frac{1240}{501} = 2.475 \text{ eV}$

~~Q~~ $\lambda_1 = 500 \text{ nm}$

$\lambda_2 = 501 \text{ nm}$

$\Delta\lambda \text{ in cm}^{-1}$

$= 100 \text{ cm}^{-1}$

I



we plot when we do Raman

Spectroscopy as it is spread.

$$\Delta \lambda \leq \left(\frac{\lambda}{a} + \frac{b}{f} \right) \left(\frac{da}{d\lambda} \right)^{-1}$$

$\frac{da}{d\lambda} = f \cdot \frac{da}{d\lambda}$
a = grating element

Suppose $a = 10 \text{ cm}$ $\lambda = 500 \times 10^{-7} \text{ cm}$

$$f = 100 \text{ cm} \quad b = 10 \mu\text{m} = 10 \times 10^{-4} \text{ cm}$$

$$\frac{da}{d\lambda} = f \frac{\text{nm}}{\text{mm}}$$

$$\Delta \lambda = \left(\frac{500 \times 10^{-7}}{10} + \frac{10 \times 10^{-4}}{100} \right) 10^{-4} = 2 \times 10^{-4} \text{ nm}$$

$$\frac{dx}{d\lambda} = f \frac{db}{d\lambda}$$

$$\left(\frac{da}{d\lambda} \right)^{-1} = f \left(\frac{dx}{d\lambda} \right) = 1 \times \frac{\text{nm}}{\text{mm}} \times 100 \times 10 \frac{\text{nm}}{\text{mm}} = 10^4 \text{ nm}^{-1}$$

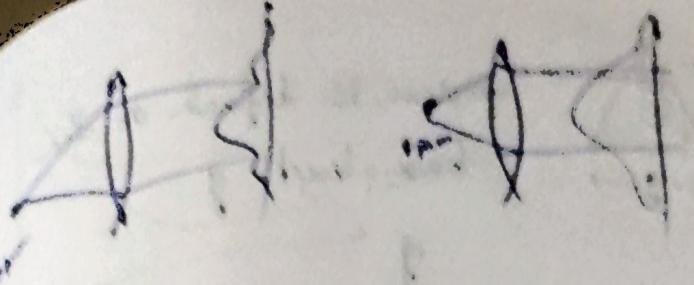
$$= 100 \times 10^{-7} \frac{\text{cm}}{10^{-2}} = 10^{-4} \text{ cm}$$

$$\Delta \lambda = \left(50 \times 10^{-7} + 10^{-5} \right) \times 10^{-4} = 1.5 \times 10^{-6} \times 10^{-4} \text{ cm}$$

$$= \frac{6 \times 10^{-7} \times 10^{-4}}{6 \times 10^{-2}} = 1.5 \times 10^{-6} \times 10^{-4} \text{ cm}$$

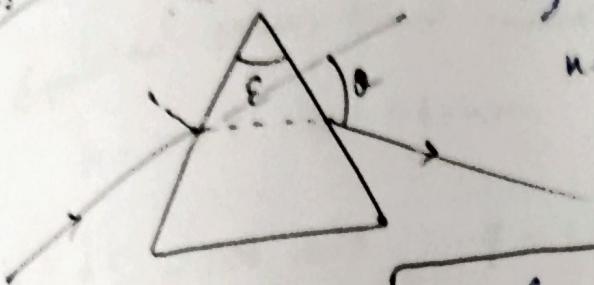
$$= 1.5 \times 10^{-7} \times 10^{-10} \text{ cm} = 1.5 \times 10^{-3} \text{ nm} = 0.015 \text{ nm}$$

* If $b = 5 \mu\text{m}$ $\Delta \lambda = 0.01 \text{ nm}$

No. 
Normal angle of deviation from prism.

Prism spectrometer

prism angle α
angle of deviation = δ



for min deviation

$$n \sin \frac{\delta}{2} = \sin \frac{\alpha + \epsilon}{2}$$

$\frac{d\delta}{d\lambda}$ is characteristics of diffraction element.

we have use $\frac{dn}{d\lambda}$

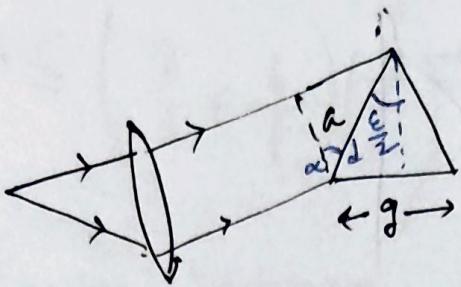
$$n \sin \frac{\delta}{2} = \sin \frac{\alpha + \epsilon}{2}$$

$$\frac{1}{2} \cdot \cos \frac{\alpha + \epsilon}{2} d\delta = \sin \frac{\epsilon}{2} dn$$

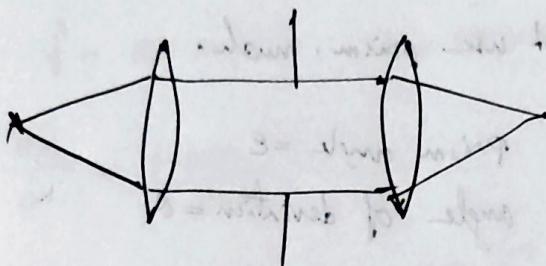
$$\frac{d\delta}{dn} = \frac{2 \sin \frac{\epsilon}{2}}{\sqrt{1 - n^2 \sin^2 \frac{\epsilon}{2}}}$$

$$\frac{d\delta}{d\lambda} = \frac{d\delta}{dn} \cdot \frac{dn}{d\lambda} = \frac{2 \sin \frac{\epsilon}{2}}{\sqrt{1 - n^2 \sin^2 \frac{\epsilon}{2}}} \cdot \frac{dn}{d\lambda}$$

$$\Delta \lambda \leq \frac{a}{3} \left(\frac{d\delta}{d\lambda} \right)$$



Prism is defined on the base length g .



$$\frac{g}{d} = \sin \frac{\epsilon}{2}$$

$$d = \frac{g}{2 \sin \frac{\epsilon}{2}}$$



$$a = d \cos \alpha$$

$$a = \frac{g \cos \alpha}{2 \sin \frac{\epsilon}{2}}$$

$$= \frac{g}{2 \sin \frac{\epsilon}{2}} \cos \alpha$$

for angle of min. deviation

$$n \sin \frac{\epsilon}{2} = \sin \frac{(\alpha + \epsilon)}{2} = \sin \frac{\alpha}{\frac{n-1}{n+1}}$$

$$\cos \alpha = \sqrt{1 - \sin^2 \alpha}$$

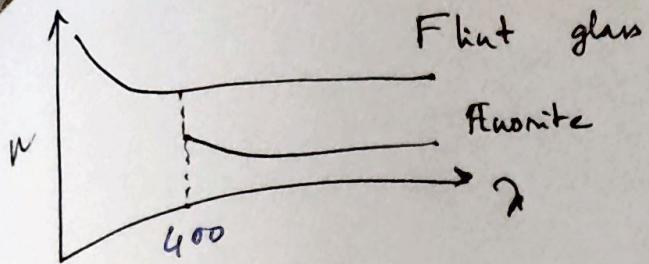
$$\cos \alpha = \sqrt{1 - n^2 \sin^2 \frac{\epsilon}{2}}$$

$$a = \frac{g \sqrt{1 - n^2 \sin^2 \frac{\epsilon}{2}}}{2 \sin \frac{\epsilon}{2}}$$

$$\frac{\Delta a}{a} \leq \frac{a}{3} \left(\frac{da}{d\lambda} \right)$$

$$\leq \frac{1}{3} \frac{g \sqrt{1 - n^2 \sin^2 \frac{\epsilon}{2}}}{2 \sin \frac{\epsilon}{2}} \cdot \frac{2 \sin \frac{\epsilon}{2}}{\sqrt{1 - n^2 \sin^2 \frac{\epsilon}{2}}} \cdot \frac{dn}{d\lambda}$$

$$\frac{2 \sin \frac{\epsilon}{2}}{\sqrt{1 - n^2 \sin^2 \frac{\epsilon}{2}}} \cdot \frac{dn}{d\lambda}$$



We take Flint glass below 400 nm
below 400 nm $\frac{dn}{d\lambda} < 0$ we get
but above 400 $\frac{dn}{d\lambda} > 0$.

$$\Delta\lambda \leq \frac{g}{3} \frac{dn}{d\lambda}$$

Equilateral prism, Fused quartz

$$n = 1.47 \quad \lambda = 400 \text{ nm} \quad \varepsilon = 60^\circ$$

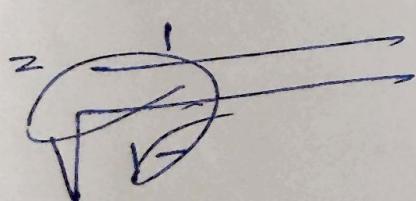
$$\frac{dn}{d\lambda} = 1100 \text{ cm}^{-1} \quad q = 1 \text{ cm}$$

$$\Delta\lambda = ? \quad \frac{\lambda}{\Delta\lambda} = \frac{q}{3} \times 1100 = 0.33 \times 1200 \\ = 33 \times 11 \\ = 330 + 33 \\ = 363 \cdot$$

$$\Delta\lambda = \frac{\lambda}{3c_3} = \frac{400}{363} \text{ nm} \\ = 1.10 \text{ nm}$$

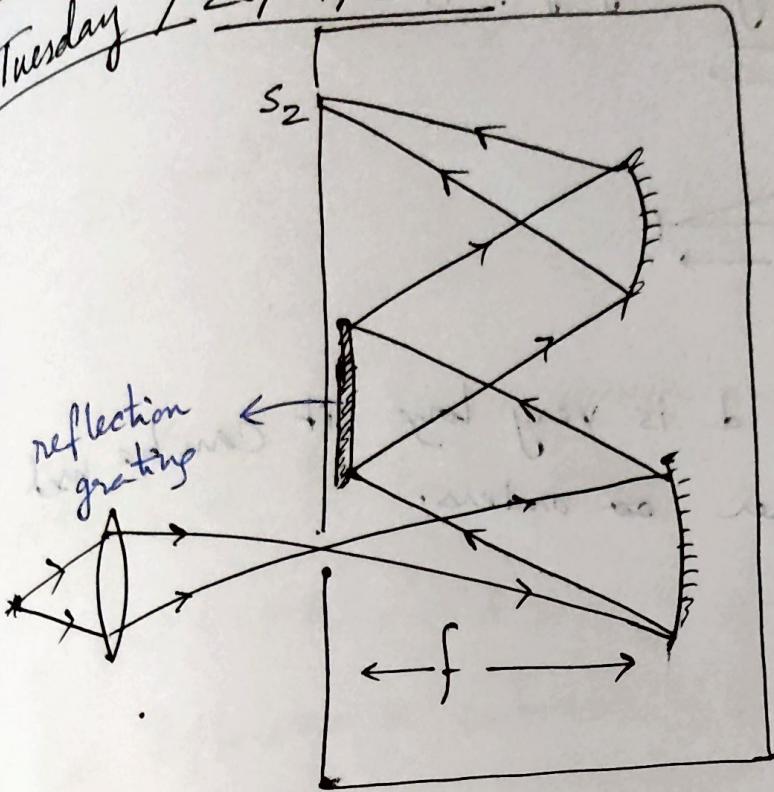
$$\frac{d\alpha}{d\lambda} = -\frac{2 \sin \frac{\varepsilon}{2}}{q} \quad \frac{d\alpha}{dn} \quad \frac{dn}{d\lambda}$$

$$\therefore \frac{d\alpha}{dn} = \frac{2 \sin \frac{\varepsilon}{2}}{\sqrt{1 - n^2 \sin^2 \frac{\varepsilon}{2}}} = \frac{2 \times \frac{1}{2}}{\sqrt{1 - (1.47)^2 \frac{1}{4}}} \\ = \frac{1}{\sqrt{0.45977}}$$



$$= \frac{1}{0.6780}$$

Tuesday / 28/01/25



$$d(\sin\alpha \pm \sin\beta) = m\lambda$$

grating normal
groove normal

Littrow grating

angle can be
anything

incident angle = reflection angle

$$\alpha = 90^\circ, \beta = 90^\circ$$

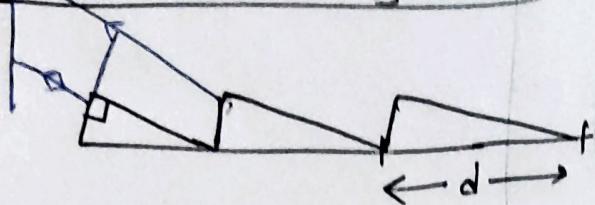
$$d(2\sin\alpha) = m\lambda$$

$$2d\sin\alpha = m\lambda$$

it basically chooses different wavelength for different order. So it is called wavelength selective grating.

There are another type of grating.

Echelle Grating :-



$$2d \sin \alpha = m\lambda$$

here as comparatively d is very large it can be used to determine higher ∞ orders.

$$\text{as } d \gg \lambda$$

$$\theta = 90^\circ - \alpha$$

$$2d \cos \alpha = m\lambda$$

grating size: $10\text{cm} \times 10\text{cm}$ $10^3 \text{ mm} \times 10^3 \text{ mm} = 10^6 \text{ mm}^2$

$$10^3 \text{ grooves/mm} \Rightarrow m=2$$

$$\text{Resolving power of the grating. } R = \frac{\lambda}{\Delta \lambda} = 10^6$$

N = total no. of groove which will be illuminated.

$$R = \frac{\lambda}{\Delta \lambda} = mN = 2 \times 10^5$$

$$\frac{1 \text{ mm}}{10^{-1} \text{ cm}} = 10^3 \quad \frac{10^3}{10^{-1}} = 10^4 \quad \frac{10^3 \times 10^3}{10^{-1}} = 10^3 \times 10^2 = 10^5 = N$$

$$MN = 2 \times 10^5 = \frac{\lambda}{\Delta \lambda}$$

$$\lambda = 500 \text{ nm}$$

$$\Delta \lambda = \frac{500}{2 \times 10^5} = 2.5 \times 10^{-3} \text{ nm} = 0.0025 \text{ nm}$$

$$\Delta \lambda = \left(\frac{\lambda}{a} + \frac{b}{f} \right) \left(\frac{d\alpha}{d\lambda} \right)^{-1}$$

$$d(\sin \alpha + \sin \beta) = m \lambda$$

here diffraction is controlled by β .

$$a = \text{size}, \quad \alpha = \beta \quad \text{so} \quad \frac{dn}{d\lambda} = f \frac{d\alpha}{d\lambda}$$

$$\Delta \lambda = \left(\frac{\lambda}{a} + \frac{b}{f} \right) \left(\frac{d\beta}{d\lambda} \right)^{-1}$$

$$b = 50 \mu\text{m} \quad f = 1 \text{ m} \quad a = 10 \text{ cm}, \quad \lambda = \beta = 30^\circ$$

$$\Delta \lambda = ?$$

$$\frac{d\beta}{d\lambda} = \frac{m}{d \cos \beta} = \frac{2}{10^{-3} \times 18^6 \times \frac{\sqrt{3}}{2}}$$

$$\Delta \lambda = ?$$

$$\frac{d\beta}{d\lambda} =$$

$$\Delta \lambda = \left(\frac{50 \times 10^{-9}}{10 \times 10^{-2}} + \frac{50 \times 10^{-6}}{1} \right) \left(\frac{d\beta}{d\lambda} \right)^{-1} = \frac{2}{\sqrt{3}} \times 10^{-3} = 0.0023$$

$$= \left(5 \times 10^{-6} + 50 \times 10^{-6} \right) \left(\frac{d\beta}{d\lambda} \right)^{-1} = 434 \text{ nm}$$

$$= 55 \times 10^{-6} \left(\frac{d\beta}{d\lambda} \right) = 55 \times 10^{-6} \times \frac{42 \times \sqrt{3}}{9 \times 10^6}$$

$$= 23.8 \times 10^{-12} \text{ nm}$$

$$= 23.8 \times 10^{-12} \times 10^9 = 0.023 \text{ nm}$$

$$d(\sin \alpha + \sin \beta) = m\lambda$$

10^3 grooves/mm

$$d\left(\frac{1}{2} + \frac{1}{2}\right) = m\lambda$$

1 mm 10^3 g

$$d = m\lambda$$

1 g

$$10^{-3} \times 10^{-3} = m \times 500 \times 10^{-9}$$

10^{-3} mm

$$\frac{10^{-6}}{500} \times 10^9 = m$$

$$m = \frac{10^3}{500} = m = 2$$

Least

to get more resolution we need to use higher focal length.

small slit - b

higher size of gratings. α

higher focal length $\sim f$

characteristics of the gratings will be governed by

$$\left(\frac{d\beta}{d\lambda} \right)$$

$$\Delta\lambda = \left(\frac{\lambda}{a_0} + \frac{b}{f} \right) \left(\frac{d\beta}{d\lambda} \right)^{-1}$$

$$d \cos \beta d\beta = m d\lambda$$

$$\frac{d\beta}{d\lambda} = \frac{m}{d \cos \beta}$$

$$\frac{d\beta}{d\lambda} = 0.023 \text{ nm}^{-1}$$

$$f = \lambda^2 / m = 10^9 \text{ nm}$$

$$\frac{dn}{d\lambda} = f \cdot \frac{d\beta}{d\lambda} = 10^9 \text{ nm} \cdot 0.023 / \text{nm} \\ = 23 \times 10^6$$

Detector :-

- ① Thermal Detector
- ② Photo Detector

① Thermal Detector

- ▷ Does not depend on wavelength.
- ▷ Depends on the power.
- ▷ Insensitive
- ▷ Works on change in material properties with temperature.

② Photo Detection

- ▷ 2. Insensitive
- ▷ Works on response of material for a given photon energy.

Characteristics of the Detectors :-

Wednesday
Date - 29/1/25

1) Spectral response ($R(\lambda)$) \rightarrow Spectral range.

2) Two: relative intensity

$$\frac{I(\lambda_1)}{I(\lambda_2)} = \frac{R(\lambda_1)}{R(\lambda_2)}$$

3) Sensitivity : $\frac{\text{Volts}}{\text{Watt}}$ or $\frac{\text{Amp}}{\text{Watt}}$ $\frac{\text{Volt}}{\text{A.P.} \downarrow \text{area}}$, impedance

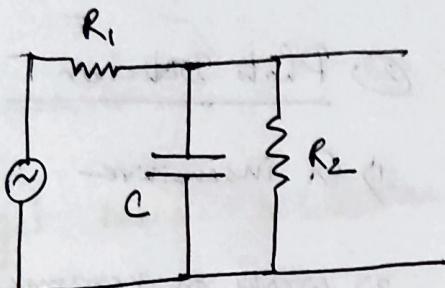
3) Signal to noise ratio ($\frac{S}{N}$)

Source of noise \rightarrow electronic noise,
thermal noise

② Noise equivalent of input power

\rightarrow power level for which $\frac{S}{N} = 1$

④ of time response



$$R_2 C \ll \tau : \tau = \text{pulse time}$$

\downarrow
must be faster, so that

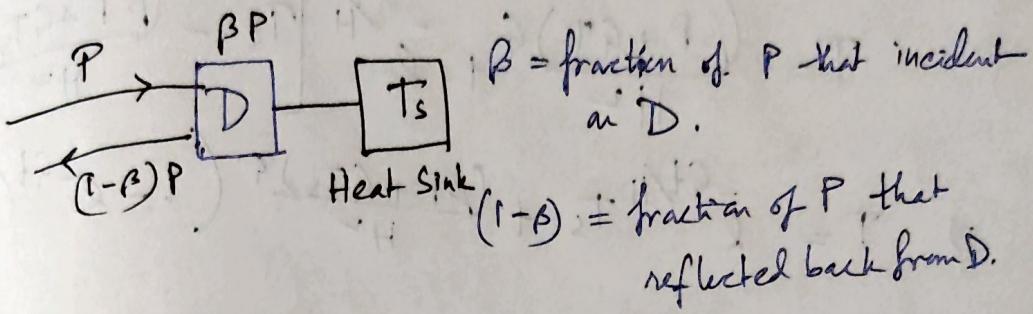
$$R_2 C \ll \frac{1}{f}$$

~~$$f \ll (R_2 C)^{-1}$$~~

5) Cost of the detector

Thermal Detector → does not depend on wavelength.
 works with a material property that changes with temp..
~~Thermistor~~

Usage → \triangleright Power measurement of continuous wave laser.
 2) Output energy of the pulsed laser.



H: heat capacity of the detector

G: thermal conductance of the heat sink.

$$\beta P = H \frac{dT}{dt} + G(T - T_s)$$

$$\beta P + GT_s = H \frac{dT}{dt} + GT$$

$$\frac{\beta P + GT_s}{H} = \frac{dT}{dt} + \frac{G}{H}T$$

$$a = \frac{\beta P + GT_s}{H}$$

$$\frac{dT}{dt} + bT = a$$

$$b = \frac{G}{H}$$

$$IF = e^{\int b dt} = e^{\frac{Gt}{H}}$$

$$\int_a^b \left(e^{\frac{Gt}{H}} \right) = \int \frac{\beta P + GT_s}{H} e^{\frac{Gt}{H}} dt$$

$$T e^{\frac{Gt}{H}} = \frac{\beta P + GT_s}{H} \frac{G}{H} e^{\frac{Gt}{H}} + C$$

$$\Rightarrow T = \frac{(\beta P + GT_s) G}{H^2} + C e^{-\frac{Gt}{H}}$$

$$\# \quad \beta P = H \frac{dT}{dt} + G(T - T_s)$$

$$\Rightarrow \beta P = H \frac{dT}{dt} + GT - GT_s$$

$$\Rightarrow \frac{(\beta P + GT_s)}{H} = \frac{dT}{dt} + \frac{G}{H} T$$

$$\Rightarrow \frac{dT}{dt} + \frac{G}{H} T = \frac{(\beta P + GT_s)}{H}$$

$$\Rightarrow \mu = \left(e^{\int \frac{G}{H} dt} \right) \frac{e^{-\int \frac{G}{H} dt}}{H} = e^{Gt/H}$$

$$\Rightarrow \int d(e^{Gt/H}) = \int e^{Gt/H} \left(\frac{\beta P + GT_s}{H} \right) dt$$

$$\Rightarrow e^{Gt/H} T = \left(\frac{\beta P + GT_s}{H} \right) \frac{e^{Gt/H}}{G} + C$$

$$\Rightarrow T = \left(\frac{\beta P + GT_s}{G} \right) + C e^{-Gt/H}$$

$$\Rightarrow T = \frac{\beta P}{G} + T_s + C e^{-Gt/H}$$

at $t=0 \quad T=T_s$

$$\Rightarrow T_s = T_s + \frac{\beta P}{G} + C e^{-G \cdot 0 / H}$$

$$\Rightarrow C = -\frac{\beta P}{G}$$

$$\Rightarrow T = T_s + \frac{\beta P}{G} - \frac{\beta P}{G} e^{-Gt/H}$$

$$T = T_s + \frac{\beta P}{G} \left(1 - e^{-Gt/H} \right)$$

$\frac{dy}{dx} + P y = Q$
 $\mu = I \cdot F = e^{\int P dx}$
 $\int d(\mu \cdot y) = \int Q \mu dx$

$$T = T_s + \frac{\beta P}{G} \left(1 - e^{-\frac{G}{H} t} \right)$$

$$= T_s + \Delta T \left(1 - e^{-\frac{G}{H} t} \right)$$

$\Delta T = \frac{\beta P H}{G}$, it is independent of H .

time constant / response time $= \frac{H}{G}$, $= ?$

Small values of G make a thermal detector sensitive, small values of both quantities.

No. instead of constant power

$$P = P_0 (1 + \alpha \text{ const})$$

$$T = T_0 + \Delta T (1 + \alpha \cos(\omega t + \phi))$$

$$\Delta T = \frac{\alpha P}{\sqrt{1 + \frac{\partial^2 H}{G^2}}} \quad \tan \phi = \frac{\partial H}{G}$$

Pulsed Laser

$$\int_0^t \beta P dt = H \int_0^t dT \quad (G=0)$$

~~$$\Rightarrow \beta P t = H \Rightarrow \Delta T = \frac{1}{H} \int_0^t \beta P dt$$~~

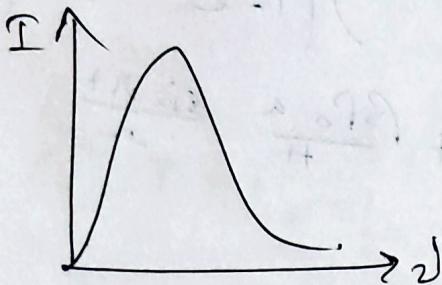
There are different types of thermal detector.

① Thermistor ② Bolometer ③ Golay Cell.

④ Thermocouple detector

* ~~graph~~

Spectral Line Shape



Natural Line broadening. \rightarrow particular state has life time. and depending on that it has a $\Delta E \Delta \tau \sim \hbar$ natural width.

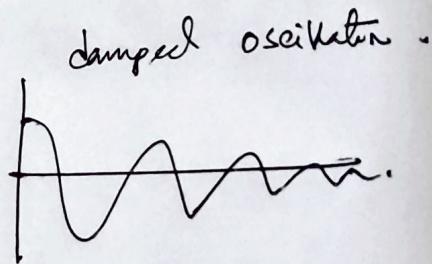
$$\Delta E \sim \frac{\hbar}{\Delta \tau} \quad \Delta \tau = \text{life time.}$$

Oscillator Model

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = 0$$

↓
dampng.

$$I = \frac{1}{(\omega - \omega_0)^2 + (\frac{\gamma}{2})^2} \quad (\text{Lorentzian fn})$$



$$\text{Energy Response} = E = \frac{1}{2} \tilde{x}^* \tilde{x}$$

for visible light $\Delta \tau_{\text{vib}} \approx 0.1 \text{ sec}$

$$\Delta E \Delta \tau = \hbar$$

$$\Delta \nu = \frac{\hbar}{\Delta \tau} \Rightarrow \Delta \nu = \frac{\hbar}{\hbar \Delta \tau} = \frac{1}{2\pi \Delta \tau}$$

$$= \frac{1}{2\pi \times 0.1} \text{ sec}^{-1}$$

Line shape of Absorption Spectra :-

Now, we have external force, forced oscillation.

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \frac{q E_0 e^{i\omega t}}{m}$$

$$x = x_0 e^{i\omega t}$$

$$(\omega)^2 x_0 e^{i\omega t} + \gamma i\omega x_0 e^{i\omega t} + \omega_0^2 x_0 e^{i\omega t} = \frac{q E_0}{m} e^{i\omega t}$$

$$-\omega^2 x_0 + i\gamma\omega x_0 + \omega_0^2 x_0 = \frac{q E_0}{m}$$

$$x_0 = \frac{q E_0}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

$$\text{Dipole moment } \vec{P} = qx_0 = q \cdot \frac{q \vec{E}}{m(\omega_0^2 - \omega^2 + i\gamma\omega)}$$

$$\text{polarizability : } \vec{P} = \frac{\text{dipole moment}}{\text{volume}}$$

N = density of oscillation \rightarrow electron.

$$\vec{P} = \frac{N q^2 \vec{E}}{m(\omega_0^2 - \omega^2 + i\gamma\omega)} = X E \vec{\epsilon}_0 \quad \text{--- (1)}$$

↓
susceptibility.

$$\text{Direct Dielectric constant } \epsilon = (1 + X) \epsilon_0$$

$$X = \epsilon_r - 1 = \frac{\epsilon}{\epsilon_0} - 1 = n^2 - 1$$

$$\text{for } n=1 \quad X = 2(n-1) \quad \text{--- (2)}$$

↓
refractive
index.

From ① we can write

$$X = \frac{Nq^2}{m\epsilon_0(\omega_0^2 - \omega^2 + i\gamma\omega)} = 2(n-1).$$

$$n = 1 + \frac{Nq^2}{2m\epsilon_0(\omega_0^2 - \omega^2 + i\gamma\omega)}.$$

$$\text{Now } n = n' - ik = 1 + \frac{Nq^2}{2m\epsilon_0} \frac{(\omega_0^2 - \omega^2 - i\gamma\omega)}{[(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2]}$$

$$n' = 1 + \frac{Nq^2}{2m\epsilon_0} \frac{(\omega_0^2 - \omega^2)}{[(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2]} \rightarrow @$$

Electric field of EM wave in vac.

$$\vec{E} = E_0 e^{i(\omega t - k_0 z)},$$

Electric field in medium:

$$\vec{E} = E_0 e^{i(\omega t - nk_0 z)}.$$

$$= E_0 e^{i(\omega t - (n-ik)k_0 z)}$$

$$= E_0 e^{-k k_0 z} e^{i(\omega t - n' k_0 z)},$$

Lambert Beer's Law

$$I = I_0 e^{-\alpha z} \quad \alpha = \text{abs. coefficient}$$

$$(\text{Amp})^2 = \text{Intensity}.$$

$$I = E_0^2 e^{2Kk_0 z} e^{2i(\omega t - k_0 z^2)}.$$

$$I = I_0 e^{-\alpha^2}$$

$$\alpha = 2Kk_0$$

$$\boxed{\alpha = 2k_0 \cdot \frac{Nq^2 \gamma \omega}{2m\epsilon [(\omega_0^2 - \omega^2) + (\gamma \omega)^2]}} \rightarrow \textcircled{1}$$

$$\omega = ck \rightarrow k_0 = \frac{\omega_0}{c}$$

Kramers-Kronig ~~Eqn.~~ @, \textcircled{b}
Relation.

gives real part of RI and absorption coefficient of light following this oscillator model.

$$n' = 1 + \frac{Nq^2(\omega_0^2 - \omega^2)}{2m\epsilon [(\omega_0^2 - \omega^2) + (\gamma \omega)^2]}$$

$$\alpha = 2k_0 \frac{Nq^2 \gamma \omega}{2m\epsilon [(\omega_0^2 - \omega^2) + (\gamma \omega)^2]}$$

Close to resonance $\omega_0 \approx \omega$

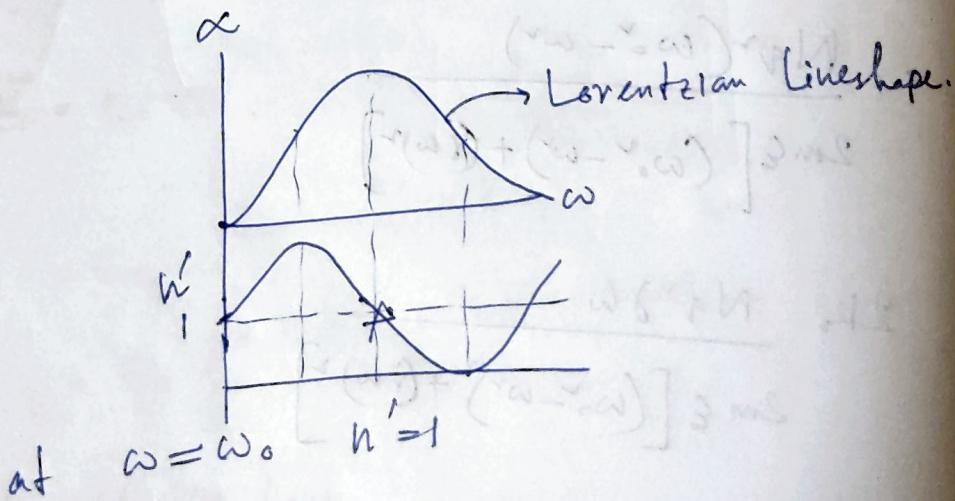
$$n' = 1 + \frac{Nq^2 (\omega_0 + \omega) (\omega_0 - \omega)}{2m\epsilon [(\omega_0 + \omega)^2 (\omega_0 - \omega)^2 + (\gamma \omega)^2]} + \frac{Nq^2 2\omega_0 (\omega_0 - \omega)}{2m\epsilon [4\omega_0^2 (\omega_0 - \omega)^2 + (\gamma \omega)^2]}$$

$$n' = 1 + \frac{N g^2 (\omega_0 - \omega)}{4 m \epsilon_0 \omega_0 \left[(\omega_0 - \omega)^2 + \left(\frac{\gamma}{2}\right)^2 \right]}$$

$$\alpha = 2 k_0 \frac{N g^2 \omega \gamma}{2 m \epsilon_0 \left[4 \omega_0^2 (\omega_0 - \omega)^2 + (\gamma \omega)^2 \right]}$$

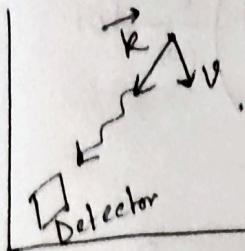
$$= \frac{2 k_0}{2 m \epsilon_0} \frac{N g^2 \omega_0 \gamma}{4 \omega_0^2 \left[(\omega_0 - \omega)^2 + \left(\frac{\gamma}{2}\right)^2 \right]} \\ = \frac{N k_0 g^2 \gamma}{4 m \epsilon_0 c \omega_0 \left[(\omega_0 - \omega)^2 + \left(\frac{\gamma}{2}\right)^2 \right]}$$

$$= \frac{N g^2 \gamma}{4 m \epsilon_0 c \left[(\omega_0 - \omega)^2 + \left(\frac{\gamma}{2}\right)^2 \right]}$$



Doppler Broadening :-

Natural line broadening is for static atom.



$$\omega = \omega_0 \pm k \cdot v$$

+ : source is moving towards detector

- : source is moving away from the detector.

Assume $k \rightarrow k_z$ ($0, 0, k_z$) $k = \frac{\omega}{c}$,

$$v \rightarrow v_z$$

$$(0, 0, v_z) \quad \omega = \omega_0 \left(1 + \frac{v_z}{c}\right) \quad \text{--- (1)}$$

~~$\omega_0 \pm k_z$~~

E_i : energy of atom per unit vol/m in the i th state

No. of molecules having energy E_i moving with velocity v_z and $v_z + dv_z$

$$n_i(v_z) dv_z = \frac{N_i}{\sqrt{\pi} v_p} \exp\left(-\left(\frac{v_z}{v_p}\right)^2\right) dv_z \quad \text{--- (2)}$$

$$v_p = \text{most probable speed} = \sqrt{\frac{2kT}{m}}$$

$$\text{Total no. of atom in the } i\text{th state} \quad N_i = \int n_i(v_z) dv_z$$

From (1) $\rightarrow d\omega = \omega_0 \frac{dv_z}{c}$

$$\rightarrow v_z = \left(\frac{\omega}{\omega_0} - 1\right) c = \frac{(\omega - \omega_0)c}{\omega_0}$$

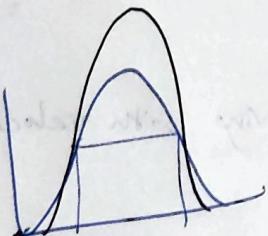
From ②

$$N(\omega) d\omega = \frac{Ni}{\sqrt{\pi} V_p} \exp\left(-\left(\frac{(\omega - \omega_0)^2 c^2}{\omega_0^2 V_p^2}\right)\right) \frac{c}{\omega_0} d\omega$$

$$= \frac{Ni c}{\sqrt{\pi} V_p \omega_0} \exp\left[-\frac{(\omega - \omega_0)^2 c^2}{\omega_0^2 V_p^2}\right] d\omega$$

Intensity $I = I_0 \exp\left[-\frac{(\omega - \omega_0)^2 c^2}{\omega_0^2 V_p^2}\right]$ (Gaussian fit)

intensity of the atoms in the frequency range ω and $\omega + d\omega$ or in energy range E and $E + dE$ following Maxwell-Boltzmann distribution.



FWHM

$$\exp\left[-\frac{(\omega - \omega_0)^2}{\sigma^2}\right] \quad \exp\left[-\frac{(\omega - \omega_0)^2}{\omega_0^2 V_p^2}\right]$$

$$\text{FWHM} = 2\sqrt{\ln 2} \sigma$$

$$\sigma = \frac{\omega_0}{c}$$

$$\Delta\omega_0 = 2\sqrt{\ln 2} \frac{\omega_0 V_p}{c}$$

$$= 2\sqrt{\ln 2} \frac{\omega_0}{c} \sqrt{\frac{2kT}{m}}$$

$$= \frac{\omega_0}{c} \sqrt{\frac{8kT \ln 2}{m}} = \frac{\omega_0}{c} \sqrt{\frac{8T N_A k \ln 2}{N_m}}$$

$$= \frac{\omega_0}{c} \sqrt{\frac{8RT \ln 2}{M}} \boxed{D = 7.16 \times 10^{-7} \frac{\omega_0}{c} \sqrt{\frac{T}{M}}}$$

M = molar mass.

for Na D line $\tau = 16 \text{ ns}$

~~So~~ $\delta\omega_N = 10 \text{ MHz}$

~~so~~ $\Delta E \Delta t = h$

$$\Delta E = \frac{h}{\Delta t}$$

$$h\Delta\omega = \frac{h}{\Delta t} \Rightarrow \Delta\omega = \frac{h}{h\Delta E} = \frac{1}{\Delta E \times 2\pi}$$

$\delta\omega_D$ at 500k M: 23

$$\lambda = 589 \text{ nm} \quad v = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{589 \times 10^{-9} \text{ m}}$$

$$\omega = 2\pi v$$

$$\delta\omega = 2\pi \delta\nu$$

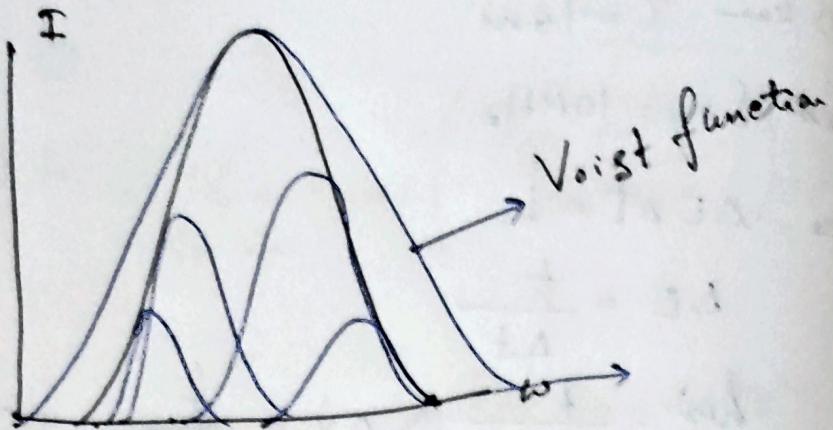
$$\delta\nu = \frac{1}{2\pi} \delta\omega = \frac{1}{2\pi} [7.16 \times 10^{-7}] = 5 \times 10^{19} \text{ s}^{-1}$$

$$\delta\omega_D = 7.16 \times 10^{-7} \nu_0 \sqrt{\frac{T}{M}}$$

$$= 7.16 \times 10^{-7} \times 0.509 \times 10^{15} \sqrt{\frac{500}{23}}$$

$$= 7.16 \times 0.509 \times 10^8 \times 4.66$$

$$= 16.98 \times 10^8 = 1.6 \times 10^9 \text{ Hz}$$



for each ω you'll have a Lorentzian broadening,
you get gaussian fn for doppler broadening.

for natural broadening you get Lorentzian fn.

B. Combining both we get Voigt fn.

$$I = C \int \exp \left[-\frac{(\omega_0 - \omega)^2 c^2}{\omega_0^2 v_p^2} \right] \frac{1}{(\omega - \omega')^2 + (\gamma/2)^2} d\omega'$$

If in instrument we get