Precision Spectroscopy of Trapped Hydrogen: A Simplified Walkthrough of Two-Photon Spectroscopy in Magnetic Traps

1. Introduction and Research Goal

Objective:

- To **perform high-precision two-photon spectroscopy** of the **1S–2S transition** in **trapped atomic hydrogen**.
- This transition is of profound significance due to its extremely **narrow natural linewidth (1.3 Hz)**, making it a candidate for:
  - Testing **Quantum Electrodynamics (QED)**
  - Determining the **Rydberg constant**
  - Measuring the **proton charge radius**
  - Investigating potential **temporal variations** of fundamental constants

2. Background Theory

2.1 Two-Photon Spectroscopy

- Involves the **simultaneous absorption** of two photons (in this case, 243 nm) to excite hydrogen from the **1S (ground state)** to the **2S (metastable state)**.
- **Second-order electric dipole process**: forbidden in one-photon transitions but allowed in two-photon processes.
- The transition is **Doppler-free** due to counter-propagating photons with the same frequency.

$$\Delta E = 2h\nu = E_{2S} - E_{1S}$$

2.2 Hydrogen's Importance

- As the **simplest atom**, hydrogen is ideal for **fundamental physical tests**.
- The **1S–2S transition** has one of the **longest lifetimes**, allowing for **extremely narrow linewidths**, which increases measurement precision.

3. Experimental Setup

3.1 Atomic Hydrogen Source

- **Hydrogen atoms** are generated by **dissociating molecular hydrogen** in an RF discharge.
- A **cryogenic nozzle** cools the atoms to 80 K.
- The beam passes through a **quartz nozzle** into a **superconducting magnetic trap**.

3.2 Magnetic Trapping

- Magnetic traps are used to **confine low-field seeking states** (i.e., atoms whose magnetic moments align opposite to the local field).
- A **Ioffe-Pritchard trap** is used:
  - Provides a **quadrupole radial field**
  - Has an **axial bias field** to avoid spin-flip losses at the center (Majorana transitions)

3.3 Cooling

- Hydrogen atoms are cooled by **evaporative cooling**, allowing temperatures to reach below **100 μK**.
- This reduces **Doppler broadening** and improves **spectral resolution**.

3.4 Laser System

- A **frequency-doubled dye laser** produces 243 nm light.
- Output is **stabilized to an optical cavity** and **referenced to a cesium clock**, allowing precise frequency measurements.

3.5 Spectroscopy Method

- Counter-propagating photons allow for **Doppler-free** two-photon excitation.
- **2S state** is metastable and decays only via **two-photon emission or collisions**.
- Detection is done by **quenching** 2S to 2P with a microwave field, which rapidly decays to 1S, emitting **Lyman- (121.6 nm)** photons, detected by **photomultiplier tubes (PMTs)**.

4. Energy Levels and Zeeman Effect

4.1 Zeeman Splitting

- The presence of a magnetic field splits energy levels due to the **Zeeman effect**:

$$\Delta E = \mu_B g_F m_F B$$

- Transition is done between the **1S(F=1, mF=1)** and **2S(F=1, mF=1)** hyperfine states.

5. Key Experimental Considerations

5.1 AC Stark Shift (Light Shift)

- The intense spectroscopy laser field **shifts energy levels** via the **AC Stark effect**.
- The shift must be corrected to obtain the **true resonance frequency**.

$$\Delta\nu_{\text{Stark}} \propto I$$

5.2 Magnetic Field Shifts

- Even small inhomogeneities in the magnetic field can cause **frequency shifts**.
- These are carefully **mapped and accounted** for using spectroscopy across the trap.

5.3 Collisional Effects

- 2S atoms can undergo **quenching collisions** with 1S atoms, leading to non-radiative decay.
- **Density of atoms** in the trap must be monitored to account for this.

6. Measurement Procedure

6.1 Excitation and Detection

- Atoms are exposed to **pulsed or continuous-wave 243 nm light**.
- **Quenched fluorescence** is recorded after excitation to detect 2S state populations.

6.2 Frequency Scan

- The laser frequency is scanned across the **expected resonance region**.
- Observed signal as a function of frequency yields the **spectral line shape**.

7. Line Shape Analysis

7.1 Doppler Effects

- Residual Doppler broadening is minimal due to the **two-photon nature** and low temperatures.
- Linewidth is mainly governed by **transit-time broadening**, **power broadening**, and **natural linewidth**.

7.2 Theoretical Line Shape

- A **Voigt profile** (convolution of Lorentzian and Gaussian) is used to model the spectrum.
  - **Lorentzian**: from natural linewidth and collisions
  - **Gaussian**: from residual Doppler broadening

8. Corrections Applied to Measured Frequency

Effect	Magnitude (Hz)	Nature
AC Stark shift	10–100	Linear in power
Zeeman shift	30–40	Depends on field
Second-order Doppler	1	Relativistic
Gravitational redshift	0.1	Negligible here
Line pulling	1–5	Line shape artifacts

All shifts are **measured and subtracted** to yield the **unperturbed transition frequency**.

9. Results

9.1 Measured Transition Frequency

$$\nu_{1S-2S} = 2\,466\,061\,413\,187\,103\ (46)\ \text{Hz}$$

- Uncertainty: 46 Hz
- Relative uncertainty: 1.9×10^{-14}

9.2 Comparison to Other Measurements

- In **excellent agreement** with previous beam-based measurements.
- Slight improvements in **systematic uncertainty** due to magnetic trap stability and reduced atomic motion.

10. Significance of the Results

10.1 Fundamental Constants

- Transition frequency used to **derive the Rydberg constant**:

$$R_\infty = \frac{E_{1S-2S}}{hc}\,(1 + \text{QED corrections})$$

10.2 Testing QED

- Precise agreement confirms **bound-state QED calculations**.
- No significant deviation found → **Standard Model holds** at this precision.

10.3 Constraints on New Physics

- Results limit models that propose **temporal variation** of α (fine-structure constant).
- Can be combined with **atomic clocks** to test for **variation in fundamental constants**.

11. Experimental Challenges and Innovations

11.1 Trap Lifetime and Background Gas

- Long trap lifetimes (several minutes) achieved by maintaining **ultra-high vacuum**.
- Background gas collisions minimized to prevent **loss or quenching** of 2S atoms.

11.2 Efficient Detection

- **Lyman- photon detection** is challenging due to **UV optics** and low efficiency.
- Innovative use of **PMTs** and **quench radiation** improved detection.

11.3 Laser Stability

- Laser linewidth and stability are **crucial for precision**.
- Locked to **ultra-stable cavity** and referenced to **cesium clock** via **frequency combs**.

12. Future Prospects

12.1 Improved Measurement

- Reducing **AC Stark uncertainties** with **intensity extrapolation** methods.
- Further cooling (e.g., **Bose-Einstein condensation**) could eliminate Doppler effects entirely.

12.2 Fundamental Constant Studies

- Use the 1S–2S transition in combination with other **hydrogen-like systems** (e.g., muonic hydrogen) to refine values of:
  - Proton charge radius
  - Fine-structure constant

12.3 Cross-disciplinary Applications

- Fundamental symmetry tests
- Metrology (redefining SI units)
- Space-time variation studies

13. Conclusion

- This landmark experiment marks one of the **most precise frequency measurements ever performed**.
- The innovative use of **magnetic traps**, **ultra-cold atoms**, and **two-photon spectroscopy** allows for extraordinary **control over systematic effects**.
- Results match theoretical expectations, supporting **QED and Standard Model predictions**.
- Paves the way for **future ultra-precise tests** of atomic structure, fundamental physics, and metrology.

Trapped Ion Quantum Computation with Transverse Phonon Modes

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Term Project
Laser Spectroscopy

Contents

1 Introduction and Research Goal

- **Primary Aim**: To propose a method for implementing high-fidelity quantum logic gates in trapped ion systems using **transverse phonon modes** rather than conventional **axial phonon modes**.
- **Motivation**: Axial modes become problematic with system scaling due to increasing mode density, spatial inhomogeneity, and cross-talk. The paper demonstrates that **transverse modes** offer a better route to scalable and robust quantum gate implementations.

2 Background Theory

2.1 Qubits in Trapped Ions

- Qubits are encoded in two hyperfine ground states of a trapped ion, typically denoted as $|\uparrow\rangle$ and $|\downarrow\rangle$.
- The ions are confined in a **linear Paul trap** with strong confinement in the transverse directions and weak confinement in the axial direction.

2.2 Ion Trap Geometry

- Confining potential:

$$V(x, y, z) = \frac{1}{2}M(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \tag{1}$$

where $\omega_x, \omega_y \gg \omega_z$.

2.3 Phonon Modes

- Ions oscillate in normal modes:
  1. N axial modes (along z) with lower frequencies
  2. $2N$ transverse modes (along x and y) with higher frequencies

2.4 Lamb-Dicke Regime

- The system operates in the Lamb-Dicke limit:

$$\eta_k = \Delta k \sqrt{\frac{\hbar}{2M\omega_k}} \ll 1 \tag{2}$$

where η_k is the Lamb-Dicke parameter.

3 Laser-Ion Interaction: Hamiltonian Framework

3.1 Basic Interaction Hamiltonian

$$H = \hbar\Omega \sum_j \sigma_j^x \cos(\Delta k \cdot x_j - \mu t) \tag{3}$$

3.2 Position Operator Expansion

$$x_j = \sum_k b_j^k \sqrt{\frac{\hbar}{2M\omega_k}} (a_k^\dagger + a_k) \tag{4}$$

3.3 Simplified Hamiltonian

$$H \approx \hbar\Omega \sum_{j,k} \eta_k b_j^k \sigma_j^x (a_k e^{-i\omega_k t} + a_k^\dagger e^{i\omega_k t}) \cos(\mu t) \tag{5}$$

4 Quantum Gates via Spin-Dependent Forces

4.1 Conditional Dynamics

$$H_{\text{int}} = \hbar\Omega\eta_k b_j^k \sigma_j^x (a_k e^{i(\mu-\omega_k)t} + a_k^\dagger e^{-i(\mu-\omega_k)t}) \tag{6}$$

4.2 Time Evolution Operator

$$U(\tau) = \exp\left[i \sum_j \phi_j(\tau)\sigma_j^x + i \sum_{i<j} \phi_{ij}(\tau)\sigma_i^x \sigma_j^x \right] \tag{7}$$

4.3 Gate Condition

- $\phi_j(\tau) = 0$ (no residual entanglement)
- $\phi_{ij}(\tau) = \pi/4$ (entangling phase)

5 Transverse vs. Axial Mode Characteristics

5.1 Transverse Mode Advantages

- Higher frequency: $\omega_{x(k)} \sim \omega_x \gg \omega_z$
- More uniform eigenvectors b_j^k
- Less sensitive to heating and spectral crowding

5.2 Axial Mode Disadvantages

- Densely spaced frequencies
- Sensitive to ion spacing and imperfections

5.3 Mode Participation

- Axial: higher modes are localized
- Transverse: more uniformly delocalized

6 Experimental Implementation

6.1 Trap Parameters

- $\omega_z \approx 1$ MHz, $\omega_x, \omega_y \approx 5$ MHz

6.2 Raman Beam Configuration

- Bichromatic beams tuned near $\omega_k \pm \delta$
- Pulse shaping to ensure closed phonon loops:

$$\int_0^\tau \alpha_k(t) dt = 0 \tag{8}$$

7 Results and Analysis

7.1 Gate Fidelity

- Achieved fidelity $> 99\%$
- Robust to ion spacing, thermal motion, and trap errors

7.2 Robustness to Scaling

- High fidelity retained with increasing ion number

7.3 Heating and Decoherence

- Strong confinement reduces heating
- Easier cooling requirements

8 Comparison Table

Feature	**Axial Modes**	**Transverse Modes**
Mode frequency spacing	Narrow	Wide
Sensitivity to ion spacing	High	Low
Thermal robustness	Low	High
Gate fidelity under scaling	Degrades fast	Remains high
Experimental complexity	Higher	Lower

9 Key Insights and Deviations

- **Insight**: Transverse modes, despite higher frequency, yield better entangling gates due to uniform coupling.
- **Observation**: Less sensitive to localization effects than axial modes.
- **Implication**: Calls for design paradigms using transverse control for better scalability.

10 Conclusion

- The paper demonstrates a novel approach using transverse modes for scalable, high-fidelity trapped-ion gates.
- This method shows resilience against scaling issues and environmental noise.
- Opens a promising path toward practical, large-scale quantum computation using trapped ions.