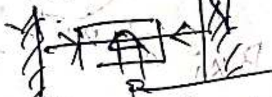
~~Quantum ML / Qchem~~

Laser Spectroscopy — 13/1/25

LASER = Light Amplification by Stimulated Emission of Radiation.

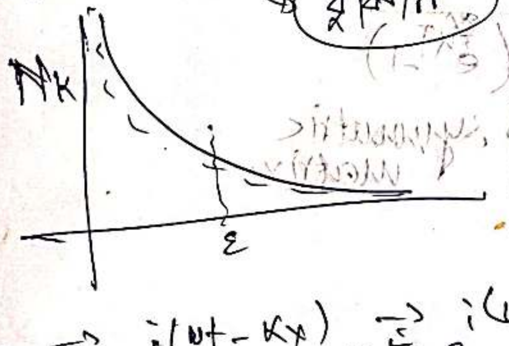
(1) Active medium amplifies the intensity of EM radiation.
(2) Pump: populates selected energy levels of the Active medium
(3) Optical resonator: Two ∥ high reflective mirror which stores energy of few resonator modes by providing feedback.



→ pump

Population inversion [Classically we use Boltzmann].

→ ব্রহ্মাণ্ড



Boltzmann rule

~~$N_K = N_i\, e^{ikT}$~~

$$\boxed{N_K = N_i\, e^{-\Delta E/KT}} \to \text{temp.}$$

∴ So the laser setup works as kind of black body we can say in 1D. [closed resonator].

$$E = \vec{E}_0\, e^{i(wt-Kx)} + \vec{E}_0\, e^{i(wt+Kx)}$$

$$K = \pi/L\,(n_1^2 + n_2^2 + n_3^2)^{1/2}$$

$$\lambda = 4_\pi \frac{1}{\sqrt{n_1^2+n_2^2+n_3^2}}.$$

→ ব্রহ্মাণ্ড !!

(laser) Einstein's A & B coefficients

No. of atoms going from $|i\rangle \to |k\rangle$.

$|k\rangle$ ——

$|i\rangle$ ——

$$\boxed{P_{i\to k} = \rho(\nu)\, B_{ik}\, N_i}$$

N_i → population @ $|i\rangle$ state.

B_{ik} → absorption coefficient

What is spectral density?

(Ans) No of modes within freq: ν & $\nu + d\nu$ | $n(\nu)\,d\nu = \frac{8\pi\nu^2}{c^3}\, d\nu$.

$$\rho(\nu)\,d\nu = n(\nu)\,KT\,d\nu$$

(2). No of Atoms going from $|k\rangle \to |i\rangle$ (reverse).

$$R_{k\to i} = A_{ki} N_k + \rho(\nu) B_{ki} N_k$$

CLOSED CAVITY LASER

At equilibrium.

$$R_{i\to k} = R_{k\to i}.$$

$$\rho(\nu) B_{ik} N_i = A_{ki} N_k + \rho(\nu) B_{ki} N_k.$$

$$\rho(\nu) B_{ik} \cancel{N_i} = \cancel{N_i} e^{-E/KT}\left(A_{ki} + \rho(\nu) B_{ki}\right)$$

$$\rho(\nu) \cancel{N_i}\left[B_{ik} - B_{ki} e^{-E/KT}\right] = \cancel{N_i} A_{ki} e^{-E/KT}.$$

$$\rho(\nu) = \frac{A_{ki}}{B_{ik} e^{E/KT} - B_{ki}} = \frac{A_{ki}}{B_{ik}\left(e^{E/KT}-1\right)}$$

B_{ik} → symmetric matrix.

$$\frac{A_{ki}}{B_{ik}} = \frac{h^2\omega^2}{\pi^2 c^2}\cdot\frac{1}{e^{E/KT}-1}$$

LASER

Intensity $I = I_0 e^{-\alpha L}$ (lambert law).

$\alpha \to$ absorption coefficient.

$$\alpha = (N_i - N_k)\,\sigma_{ik}$$

σ_{ik} → absorption cross section.

For intensity to increase $N_k > N_i$.

Loss coefficient in a round trip $(2L)$: γ

$\therefore$ Net intensity $= I = I_0 e^{-\alpha(2L)} \cdot e^{-\gamma}$.

for intensity to increase.

$$(-\alpha) 2L \gg \gamma.$$

sign dont matter

$$\Delta N \, \sigma_{ik} \, 2L \gg \gamma.$$

$$\boxed{\Delta N_{ik} \gg \frac{\gamma}{2L\sigma_{ik}}} \longrightarrow \text{threshold condition}$$

OPEN CAVITY LASER THEORY

B → stimulated

A → spontaneous.

$$\boxed{B\rho(\nu) N_1} \quad AN$$

$L = 10$ cm.

$\gamma = 10\% \to 0.1$.

$\sigma = 10^{-12}$ cm^2.

$$\Delta N_{thres} = \frac{10^{-1+12-1}}{2 \times 10} = \frac{10^{10}}{2} \text{ cm}^{-3}.$$

$$(N_k - N_i) = \boxed{0.5 \times 10^{10} \text{ cm}^{-3}} \longrightarrow (N_k - N_i).$$

He-Ne laser // Ar - Kr Layer. (WHY?)

Rate equation ⇒

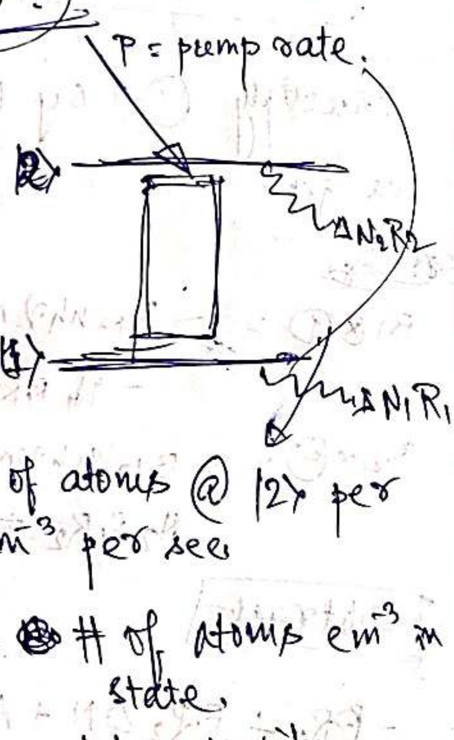
freq of resonator mode.

$$\frac{dN_1}{dt} = -B_{12} n h\nu N_1 + B_{21} n h\nu N_2 + A_{21} N_2 - N_1 R_1$$

emission — relax.

$$\frac{dN_2}{dt} = B_{12} n h\nu N_1 - B_{21} n h\nu N_2 - A_{21} N_2 + P - N_2 R_2$$

pump — relax.



no of atoms @ |2⟩ per cm^{-3} per sec

$$\frac{dn}{dt} = -\beta n + B_{12} h\nu (N_2 - N_1).$$

$\rho(\nu)$ [spectral ρ].

loss coeff — stimulated

$\gamma = \beta T$

$= \beta \left(\frac{2L}{c}\right)$

∴ of open system [loss].

N_i = # of atoms cm^{-3} in state.

n ≡ photon density.

$N_i R_i$ ≡ No. of atoms which are removed from i by relaxation (collision)

Laser spectro; Wpemtroder.

14/1

@ Stationary state.

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = \frac{dn}{dt} = 0.$$

Add (i) & (ii)

$$\boxed{P = N_1 R_1 + N_2 R_2}$$

Add (ii) & (iii)

$$P - A_{21} N_2 - N_2 R_2 - \beta n = 0.$$

$$\boxed{P = \beta n + N_2 (R_2 + A_{21})}$$

INCOHEREN LOSS RATE.

Add (iv) & (v).

$$\beta n + N_2(\cancel{R_2} + A_{21}) = N_1 R_1 + \cancel{N_2 R_2}$$

$$\boxed{N_1 R_1 = \beta n + N_2 A_{21}}$$

$$\boxed{N_2 R_2 = P - \beta n - N_2 A_{21}}$$

Multiply (1) by R_1 & (2) by R_2 & subtract.

we get.

$$R_1 \times (1) = -B_{12} nh\nu N_1 R_2 + B_{12} nh\nu N_2 R_2 + A_{21} N_2 R_2 - N_1 R_1 R_2 = 0$$

$$R_2 \times (2) = + B_{12} nh\nu N_1 R_1 - B_{12} nh\nu N_2 R_1 - A_{21} N_2 R_1 - N_2 R_1 R_2 + P R_1 = 0.$$

subtract

$$= -PR_1 + R_1 R_2 \Delta N + A_{21} N_2 R_1 + A_{21}(P - N_1 R_1) - B_{12} nh\nu N_1 (R_1 + R_2) + B_{12} nh\nu N_2 (R_1 + R_2) = 0,$$

$$= -PR_1 + A_{21} P + R_1 R_2 \Delta N + A_{21} R_1 \Delta N + B_{12} n h\nu (R_1 + R_2) \Delta N = 0$$

$$\Rightarrow \Delta N = \frac{P(R_1 - A_{21})}{R_2 R_1 + A_{21} R_1 + B_{12} nh\nu (R_1 + R_2)}$$

$\Delta N = N_2 - N_1$

we need $\Delta N > 0$.

$\therefore R_1 > A_{21}$.

$\frac{1}{\tau} > A_{21}$

$$\boxed{\tau < A_{21}}$$

Loss & laser modes.

W_K : energy stored for a particular mode K.

loss rate =

$$\frac{d}{dt} W_K = -\beta_K W_K$$

$$W_K = W_0 \exp(-\beta_K t).$$

For laser in open system.

$$F \text{ (quality factor)} / Q = \frac{\text{Energy stored}}{\text{energy loss per cycle}} = \frac{W_K}{T \frac{dW_K}{dt}} \quad \left(\frac{dW_K}{dt} = \beta_K W_K\right)$$

$$= \boxed{\frac{1}{\beta_K T}}$$

$$\boxed{nh\nu = f(\nu)}$$

HELP.

Loss Types.

"Special type of mirror".

(1) Reflection loss
(2) Diffraction loss

Reflection loss for round trip.

$\rightarrow R_1 R_2 I_0 = I_0 \exp(-\gamma)$.

$\rightarrow \gamma = -\ln(R_1 R_2)$.

$$\gamma_R = \beta_R \frac{2d}{c} = |\ln(R_1R_2)|$$

$$\beta_R = \frac{c}{2d} |\ln(R_1 R_2)|$$

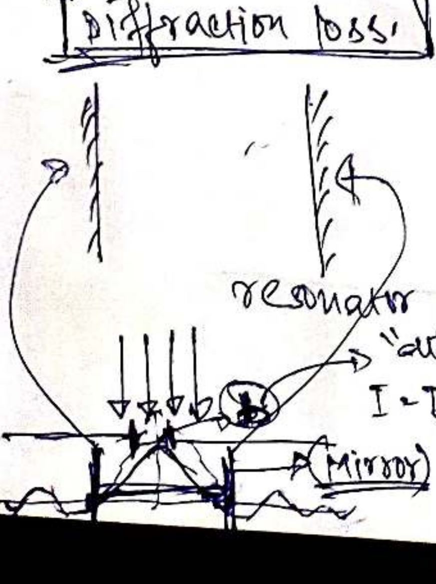
$$\boxed{\frac{1}{\tau} = \frac{2d}{c\,|\ln(R_1R_2)|}}$$

When will mode NOT survive?

ANS: Low R will have low τ. Low R will NOT survive.

$R \rightarrow$ reflectivity of mirror

Diffraction loss.



Babinet principle.
Diffraction by mirror ~ diffrac^n by slit.

$\rightarrow$ "diffraction slit width"

$I = I_0 \frac{\sin^2\alpha}{\alpha^2}$ "Fraunhofer Diffrac^n" $\rightarrow$ dist b/w source & mirror ∞.

$$\boxed{\alpha = \frac{\pi b \sin\theta}{\lambda}}$$

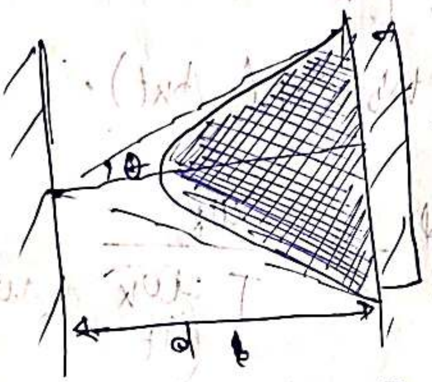
Approx (small) condition.

$\frac{\pi b\theta}{\lambda} \approx \pi$ $\quad \theta = \lambda / b$

Diffraction law is described by FRESNEL NO.

Fresnel No. $\boxed{N = \frac{b^2}{\lambda d}}$.

Fraunhofer Diffrac^n gives Fresnel radius & fresnel zones.



resonator mirrors

$$\boxed{\gamma(\text{loss})_{\text{Diffrac}^n} \sim 1/N}$$

Diffrac^n loss is defined as:

Fresnel no.: $\frac{b^2}{\lambda d}$.

$\theta = \frac{\lambda b}{2d}$

for 'n' transits:

$\theta = \frac{b}{2nd}$

$n = \frac{b}{2\theta d}$ $\boxed{\cancel{\frac{b^2}{2d\lambda}}}$

$= \frac{b^2}{2d\lambda} = \frac{N}{2}$

$\boxed{\gamma_{loss} \propto \frac{1}{N}}$

→ Many modes

FABRY-PEROT INTERFERO 1.8 (N3) LASER

b = 3 cm.
d = 1 cm.
λ = 500 nm.

N = ?

(I) $\frac{9}{\cancel{500}\times10^{-9}\times10^{-2}}$ 1.8 $\frac{9\,cm^2}{1\,cm\times\cancel{500}\times10^{-9}}$ $\boxed{>1.8\times10^5}$ → Many modes

(II) $\frac{10^{-2}}{\cancel{500}\times10^{-9}\times\cancel{50}\times10^{-2}} = \frac{100}{25} = \boxed{4}$ → $\boxed{\text{laser modes.}}$

$\gamma_{diff} \sim \frac{1}{4}$

N $\boxed{25\%\ \text{LOSS.}}$

$\rightarrow 10^{10}\ cm^{-3}$.

$|2\rangle$ "N_2" Incoherent loss Rate: $\boxed{A_{21} + R_2.}$
(ICR)

$|1\rangle$ "N_1" $ICR = 2 \times 10^{+7}\ s^{-1}$.

Let $L = 10\,cm$
$b = 1\,mm$
$= 0.1\,cm$

Total "incoherent" loss $= 2 \times 10^{17}\ cm^{-3} s^{-1}$.

volume loss rate $= 2 \times 10^{17-1} \cancel{cm^{-3}} s^{-1} \times \frac{2\pi}{\cancel{100}} \times 1\cancel{0}\ \cancel{cm^3}$

$= \boxed{4\pi \times 10^{16}\ s^{-1}}$

$\boxed{\text{Kirchhoff Diffraction Theory:}}$

Amplitude: $A(x,y) = -\frac{i}{\lambda} \iint A_{n-1}(x',y') \frac{e^{ik\rho}}{\rho} \cos\theta\, dx'dy'$.

"spherical wave"

Stationary condition.

$A_n(x,y) = c\, A_{n-1}(x,y)$ $c = \sqrt{1-\gamma_D}\ e^{i\phi}$

$A_n(x,y) = -\frac{i}{\lambda} \sqrt{1-\gamma_D}\ e^{i\phi} \iint A_{n-1}(x',y') \frac{e^{-ik\rho}}{\rho} \cos\theta\, dx'dy'$.
