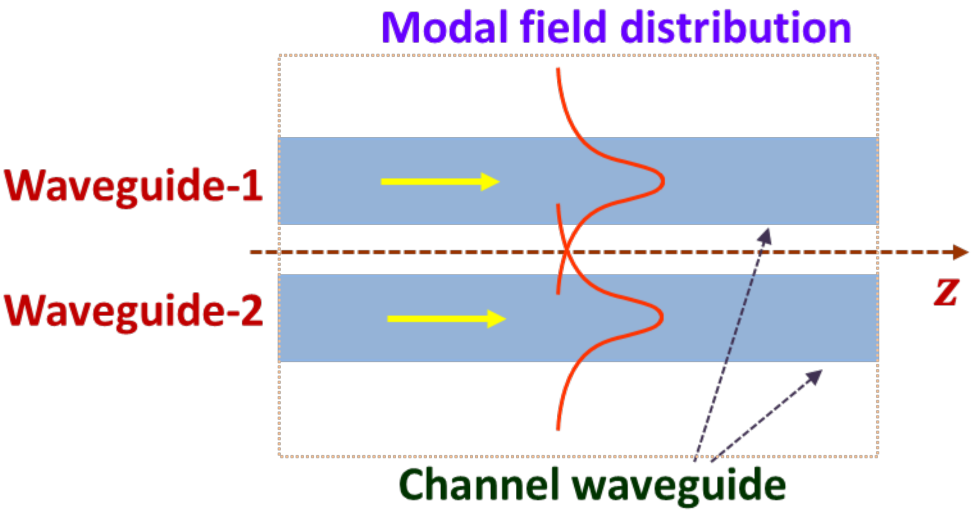
Mode coupling in directional coupler

Directional Coupler

one very important device used in Integrated optics, Fiber optics for various applications

optical amplitude modulator, power tapping, power divider, wavelength filter, optical switch, optical multiplexer, optical cross-connect and many more..

consists of two channel optical waveguides placed close to each other so that their fields can interact



with each other

Qualitative understanding

If we excite only the waveguide-1

- ✓ The modal field of the waveguide-1 will be intercepted by waveguide-2
- ✓ Due to continuity of fields at the boundary of waveguide-2, some fields get excited inside this waveguide
- ✓ Being a bound structure, field induced in waveguide-2 must be modal field
- ✓ The modal field follows distribution similar to that of waveguide-1
- ✓ The induced field of waveguide-2 interacts back with waveguide-1

On the whole

- ✓ the fields of the two waveguides start interacting
- ✓ Waveguide-2 taps power from waveguide-1
- ✓ As interaction goes on power tap occurs from waveguide-2 to Waveguide-1

- ✓ There is exchange of power between two waveguides due to overlapping of the field outside the waveguide
- ✓ This is evanescent mode coupling

<u>this qualitative understanding</u>

- ✓ **helps investigate analytically the power exchange between two channel waveguides**
- ✓ **now we can use coupled mode equations**

Analysis of directional coupler

When non-interacting

For individual waveguide field varies with z as: $\boldsymbol{a = a_0 e^{-i\beta_1 z}}$

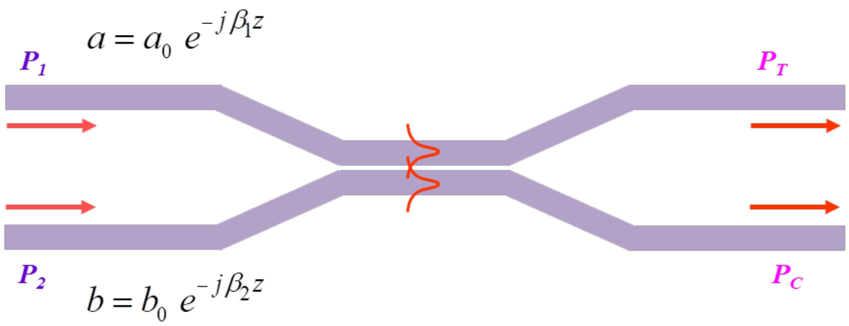
$\boldsymbol{b = b_0 e^{-i\beta_2 z}}$

the amplitude of a mode at z is $\dfrac{\partial a}{\partial z} = -i\beta_1 a$

the amplitude of a mode at z is $\dfrac{\partial b}{\partial z} = -i\beta_2 b$

This is true as long as the waveguides are not interacting
The fields of waveguides do not overlap, optically isolated

Parallel waveguides



Coupled mode equations

When the waveguides are in the vicinity

- ✓ The modes overlap through evanescent tails
- ✓ So waveguides are then interacting optically

the amplitude of a mode at z is $\frac{\partial a}{\partial z} = -i\beta_1 a - i\kappa_{12} b(z)$

the amplitude of a mode at z is $\frac{\partial b}{\partial z} = -i\beta_2 b - i\kappa_{21} a(z)$

→ (1)

κ_{12} and κ_{21} are the strength of interaction- called coupling constants

Coupling constant

- ✓ In absence of any interaction: $\kappa_{12} = \kappa_{21} = 0$
- ✓ In presence of interaction, the amplitude of mode in a waveguide depends on that of the other
- ✓ coupling coefficient is a complicated function of the parameters:

  the width of waveguide, RI of the waveguide, separation between the waveguides, the mode and importantly the wavelength of operation

- ✓ For a pair of identical waveguides: $\kappa_{12} = \kappa_{21}$

Postulate

There exists a wave in the composite structure travelling with phase constant β

The wave of the composite system is the superposition of modes of the individual waveguides

The waves in waveguide-1 and waveguide-2 are then

$$\left.\begin{aligned} a(z) &= a_0 e^{-i\beta z} \\ b(z) &= b_0 e^{-i\beta z} \end{aligned}\right\} \longrightarrow (2)$$

Substituting equations (2) in the coupled mode equations (1)

$$-i\beta a_0 = -i\beta_1 a_0 - i\kappa_{12} b_0$$

$$-i\beta b_0 = -i\beta_2 b_0 - i\kappa_{21} a_0$$

The equations simplify to the following matrix equation

$$(\beta - \beta_1) a_0 - \kappa_{12} b_0 = 0$$

$$(\beta - \beta_2) b_0 - \kappa_{21} a_0 = 0$$

$$\begin{pmatrix} \beta - \beta_1 & -\kappa_{12} \\ -\kappa_{21} & \beta - \beta_2 \end{pmatrix} \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = 0$$

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For non-trivial solutions: $\begin{vmatrix} \beta-\beta_1 & -\kappa_{12} \\ -\kappa_{21} & \beta-\beta_2 \end{vmatrix} = 0$

This yield the equations: $\beta^2-\beta(\beta_1+\beta_2)+\beta_1\beta_2-\kappa^2=0$

where $\sqrt{\kappa_{12}\kappa_{21}}=\kappa^2$

The two solutions of β : $\beta_{s,a}=\frac{1}{2}(\beta_1+\beta_2)\pm\sqrt{\frac{1}{4}\left((\beta_1-\beta_2)\right)^2+\kappa^2}$

The composite waveguide has a set of two independent modes

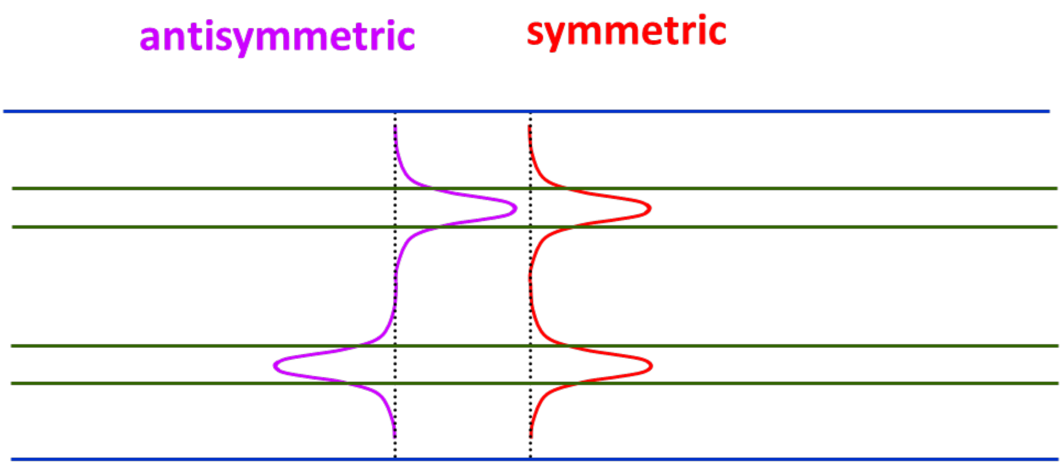
One propagating with β_s and the other with β_a

Therefore the general solutions to equations (1)

$$a(z)=a_s e^{-i\beta_s z}+a_a e^{-i\beta_a z}$$

$$b(z)=\frac{\beta_s-\beta_1}{\kappa_{12}}a_s e^{-i\beta_s z}+\frac{\beta_a-\beta_1}{\kappa_{12}}a_a e^{-i\beta_a z}$$

s and a denote symmetric and antisymmetric modes respectively



Existence of symmetric and antisymmetric modes

Assume that

At $z = 0$ unit power is launched into waveguide-1
and no power launched in waveguide-2

Therefore the initial conditions are at $z = 0$:

$$a_s + a_a = 1$$

$$\frac{\beta_s - \beta_1}{\kappa_{12}} a_s + \frac{\beta_a - \beta_1}{\kappa_{12}} a_a = 0$$

This yields $a_s = \dfrac{\beta_1 - \beta_a}{\beta_s - \beta_a}$ and $a_s = -\dfrac{\beta_1 - \beta_s}{\beta_s - \beta_a}$

The power in waveguide-1 and waveguide-2 is then respectively

$$|a(z)|^2 = 1 - \frac{\kappa^2}{\frac{1}{4}\Delta\beta^2 + \kappa^2} \sin^2\left\{\left(\sqrt{\frac{1}{4}\Delta\beta^2 + \kappa^2}\right) z\right\}$$

$$|b(z)|^2 = \frac{\kappa^2}{\frac{1}{4}\Delta\beta^2 + \kappa^2} \sin^2\left\{\left(\sqrt{\frac{1}{4}\Delta\beta^2 + \kappa^2}\right) z\right\}$$

$$\Delta\beta = \beta_1 - \beta_2$$

Power transfer equations

$$|a(z)|^2 = 1 - \frac{\kappa^2}{\frac{1}{4}\Delta\beta^2 + \kappa^2} \sin^2\left\{\left(\sqrt{\frac{1}{4}\Delta\beta^2 + \kappa^2}\right) z\right\}$$

$$|b(z)|^2 = \frac{\kappa^2}{\frac{1}{4}\Delta\beta^2 + \kappa^2} \sin^2\left\{\left(\sqrt{\frac{1}{4}\Delta\beta^2 + \kappa^2}\right) z\right\}$$

A periodic exchange of power takes place
between waveguide-1 and waveguide-2

Identical waveguides

If the two waveguides are identical

Then $\beta_1 = \beta_2 = \beta_0$

therefore $\beta_s = \beta_0 + \kappa$

$$\beta_a = \beta_0 - \kappa$$

Then for the symmetric mode with β_s : $b_0 = a_0$

And for antisymmetric mode with β_a : $b_0 = -a_0$

Power transfer equations

So if two waveguides are identical

Field amplitudes: $a(z) = e^{-i\beta_0 z} \, sin(\kappa z)$

$$b(z) = e^{-i\left(\beta_0 z + \frac{\pi}{2}\right)} \, sin(\kappa z)$$

The power in waveguide-1 and waveguide-2 is then respectively

$$|a(z)|^2 = cos^2\{kz\}$$

$$|b(z)|^2 = sin^2\{kz\}$$

Coupling length

In power transfer equation, if we put

$$z = \frac{\pi}{2\sqrt{\left(\frac{1}{4}\Delta\beta^2 + \kappa^2\right)}}$$

This length corresponds to maximum power transfer
and is called the coupling length L_c

Maximum coupled energy

Over the coupling length L_c the maximum transfer of energy

$$|b(z)|^2 = \frac{\kappa^2}{\frac{1}{4}\Delta\beta^2 + \kappa^2}$$

Thus maximum transfer of energy depends on $\Delta\beta$, the mismatch of propagation constants

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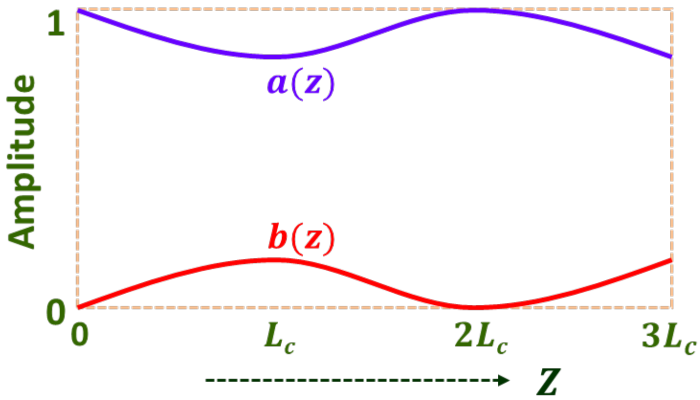
Dissimilar waveguides

If two waveguides are dissimilar ($\beta_1 \neq \beta_1$) and if $\frac{\Delta\beta}{2} \gg \kappa$ field amplitude of waveguide-1 remains almost constant

field amplitude of waveguide-2 is much less than 1

most of power remains with waveguide-1 and a small power is transferred to waveguide-2

Coupling in dissimilar waveguides



Identical waveguides

When $\Delta\beta = 0$, the coupling constant becomes

$$\kappa = \frac{\beta_s - \beta_a}{2}$$

Coupling length L_c for maximum power transfer

$$L_c = \frac{\pi}{2\kappa} = \frac{\pi}{\beta_s - \beta_a}$$

Identical waveguides

Then power transfer equations take the forms

$$|a(z)|^2 = cos^2\{\kappa z\}$$

$$|b(z)|^2 = sin^2\{\kappa z\}$$

Identical waveguides

When $L_c = \frac{\pi}{2\kappa}$ the power coupled into waveguide-2 is

$$|b(z)|^2 = 1$$

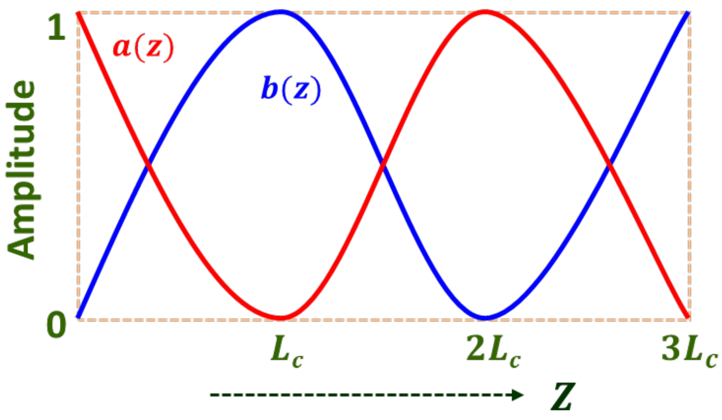
Also the power in waveguide-1 under this condition is

$$|a(z)|^2 = 0$$

That means entire power gets transferred from waveguide-1 to waveguide-2

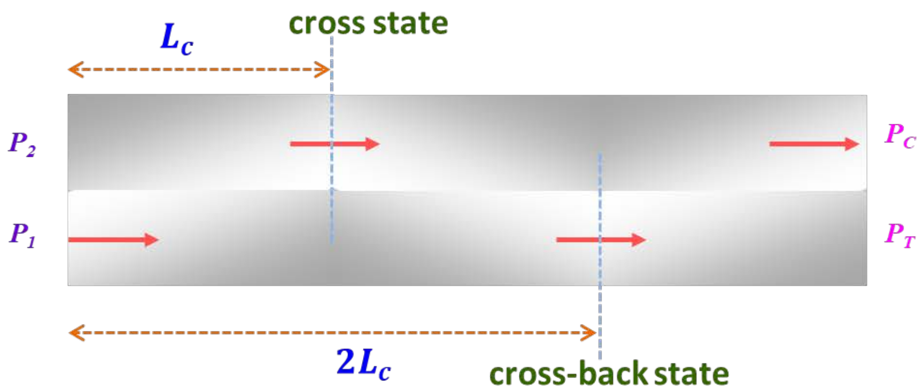
Beyond this distance power couples back from waveguide-2 to waveguide-1

Coupling in identical waveguides



Periodic exchange of Power

$$P_T = P_I \cos^2(\kappa l)$$
$$P_C = P_I \sin^2(\kappa l)$$



Mode coupling

Supermodes' Beating