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LASER SPECTROSCOPY

Rabi Oscillation.

$|b\rangle$ ———

$|a\rangle$ ———

$E = E_0 \cos(\omega t)$,

$= A_0 \left[e^{i\omega t} + e^{-i\omega t}\right]$.

$\left(A_0 = \frac{E_0}{2}\right)$.

$\dot{a}(t)$

$\dot{b}(t)$

→ probability Amp

$\dot{a}(t) = -\frac{i}{2}\Omega_{ab}\left[e^{i(\omega_{ab}+\omega)t} + e^{i(\omega_{ab}-\omega)t}\right] b(t)$

$\dot{b}(t) = -\frac{i}{2}\Omega_{ab}\left[e^{-i(\omega_{ab}-\omega)t} + e^{-i(\omega_{ab}+\omega)t}\right] a(t)$.

weak field Approximation: (thermal source).

$|b(t)|^2 \ll 1$. (along with rotating wave approximation)

also $|b(t)|^2 = \left(\frac{\Omega_{ab}}{2}\right)^2 \left[\frac{\sin\left[\left(\frac{\omega_{ab}-\omega}{2}\right)t\right]}{\left(\frac{\omega_{ab}-\omega}{2}\right)}\right]^2$.

close to resonance condition →

$|b(t)|^2 = \left(\frac{\Omega_{ab}}{2}\right)^2 t^2$.

$|b(t)|^2 \leq 1$

$$\boxed{t < \left(\frac{2}{\Omega_{ab}}\right)}$$

Probability of transition.

$P(\omega)$ spectral density: $\int P(\omega)\,d\omega = \frac{\epsilon_0}{2}E_0^2 = 2\epsilon_0 A_0^2$.

Probability transition from $|a\rangle \rightarrow |b\rangle$.

$P_{ab} = \int |b(t)|^2\,d\omega = \int \left(\frac{\Omega_{ab}}{2}\right)^2 \frac{\sin^2\left(\frac{\omega_{ab}-\omega}{2}\right)t}{\left(\frac{\omega_{ab}-\omega}{2}\right)^2}\,d\omega$.

$$\Omega_{ab} = \left(\frac{D_{ab} \cdot E_0}{\hbar}\right).$$

$$\therefore \left(\frac{\Omega_{ab}}{2}\right)^2 = \left(\frac{D_{ab} E_0}{\hbar\, 2}\right)^2$$

$$\longrightarrow \int \frac{D_{ab}^2 E_0^2}{\hbar^2 4} \left[\frac{\sin^2\left(\left(\frac{\omega_{ab}-\omega}{2}\right)t\right)}{\left(\frac{\omega_{ab}-\omega}{2}\right)^2}\right] d\omega$$

$\boxed{\text{At} \quad \omega = \omega_{ab}.}$

$$P_{ab} = \frac{D_{ab}^2}{2\epsilon_0 \hbar^2} \rho(\omega_{ab})\, 2\pi t. \longrightarrow \int_{-\infty}^{\infty} \frac{\sin^2 xt}{x^2} = 2\pi t$$

$$\boxed{P_{ab} = \frac{D_{ab}^2 \pi}{3\epsilon_0 \hbar^2} \rho(\omega_{ab}) t.}$$

Dipole moment of atom under isotropic radiation.
$\langle p_x\rangle^2 = \langle p_y\rangle^2 = \langle p_z\rangle^2 = \frac{\langle p\rangle^2}{3}$.
D = dipole moment.

Rate of transition probability,

$$\frac{d}{dt} P_{ab} = \text{const} = \left[\frac{D_{ab}^2 \pi}{3\epsilon_0\hbar^2}\right] \rho(\omega_{ab}).$$

$|a\rangle = u_a \exp(-iE_a t/\hbar).$
$|b\rangle = u_b \exp(-iE_b t/\hbar).$

$$\frac{d}{dt} P_{ab} = B_{ab}\, \rho(\omega_{ab}).$$

$$B_{ab} = \frac{D_{ab}^2 \pi}{3\epsilon_0 \hbar^2} = \frac{\pi}{3\epsilon_0\hbar^2}\left|\int u_a^* r u_b\, d\tau\right|$$

These are for Non degenerate states.

Now, considering degeneracy: ⟶ Same energy eigenval diff state.

$$B_{ki} = \frac{\pi}{3\epsilon_0\hbar^2} \frac{1}{g_i} \sum_{m=1}^{g_i} \sum_{n=1}^{g_k} \left| D_{mn\,ki} \right|^2.$$

Transition from $m \to g_i$ (i degen)

Degeneracy

(*) Rabi Oscillation [strong field]

$$\dot{a}(t) = -\frac{i\Omega_{ab}}{2}\left[e^{i(\omega_{ab}-\omega)t}\right] b(t) \quad \ldots (i)$$

$$\dot{b}(t) = -\frac{i}{2}\Omega_{ab}\left[e^{-i(\omega_{ab}-\omega)t}\right] a(t) \quad \ldots (ii).$$

Trial solution: $a(t) = e^{i\mu t}$

$$\dot{a}(t) = i\mu e^{i\mu t}.$$

$$\cancel{i}\mu e^{i\mu t} = -\frac{\cancel{i}\,\Omega_{ab}}{2} e^{i(\omega_{ab}-\omega)t} b(t).$$

$$b(t) = \frac{-2\mu e^{i\mu t}}{\Omega_{ab} e^{i(\omega_{ab}-\omega)t}} = \left(\frac{2\mu}{-\Omega_{ab}}\right) e^{i[\mu - (\omega_{ab}-\omega)]t}.$$

$$\dot{b}(t) = -\frac{2\mu}{\Omega_{ab}} i\left(\mu - (\omega_{ab}-\omega)\right) e^{i[\mu-(\omega_{ab}-\omega)]t}.$$

$$= \frac{i2\mu}{\Omega_{ab}}\left(\omega_{ab}-\omega-\mu\right) e^{-i[(\omega_{ab}-\omega)-\mu]t}$$

From (2) $\rightarrow$ $-\frac{\cancel{i}}{2}\Omega_{ab} e^{-i(\omega_{ab}-\omega)t} a = \frac{i2\mu}{\Omega_{ab}}(\omega_{ab}-\omega-\mu)$ $\exp\left[-i[(\omega_{ab}-\omega)-\mu]t\right]$

$$\cancel{\frac{-i}{2}\Omega_{ab} = \frac{i}{2}}$$

$$\Rightarrow -\left(\frac{\Omega_{ab}}{2}\right)^2 e^{-i\mu t} = \mu(\omega_{ab}-\omega-\mu)\cancel{e^{i\mu t}}$$

$$\Rightarrow \mu^2 - \mu(\omega_{ab}-\omega) - \left(\frac{\Omega_{ab}}{2}\right)^2 e^{-i\mu t} = 0.$$

$$\therefore \mu = \frac{(\omega_{ab}-\omega) \pm \sqrt{(\omega_{ab}-\omega)^2 + \cancel{4}\frac{(\Omega_{ab})^2}{\cancel{4}}\cancel{e^{-i\mu t}}\cancel{e^{+i\mu t}}}}{2}$$

$$\boxed{\mu = (\omega_{ab}-\omega) \pm \sqrt{(\omega_{ab}-\omega)^2 + (\Omega_{ab})^2}} \rightarrow C_+,\ C_-$$

$$a(t) = c_1 e^{i\mu_1 t} + c_2 e^{i\mu_2 t}.$$

$$\dot a(t) = c_1 i\mu_1 e^{i\mu_1 t} + c_2 i\mu_2 e^{i\mu_2 t}.$$

$$= \frac{i}{2}\Omega_{ab}\left[e^{i(\omega_{ab}-\omega)t}\right] b(t); \text{ --- from (i).}$$

$$\therefore b(t) = \frac{-i\,2\,e^{-i(\omega_{ab}-\omega)t}}{\Omega_{ab}}\left(c_1 i\mu_1 e^{i\mu_1 t} + c_2 i\mu_2 e^{i\mu_2 t}\right)$$

@ $t=0$ $a=1$ $b=0$.

$$\therefore c_1 + c_2 = 1, \qquad c_1 - c_1\left(\frac{\mu_1}{\mu_2}\right) = 1.$$

$$c_1\mu_1 + c_2\mu_2 = 0, \qquad c_1\left(\frac{\mu_2-\mu_1}{\mu_1}\right) = 1.$$

$$\cancel{c_2 = c_1\left(\frac{\mu_1}{\mu_2}\right)}$$

$$c_2 = \left(\frac{\mu_1}{-\mu_2+\mu_1}\right)$$

$$c_1\mu_1 + c_2\mu_1 = \mu_1$$

$$c_1\mu_1 + c_2\mu_2 = 0.$$

$$\cancel{c_2 = \frac{\mu_1}{\mu_2-\mu_1}}$$

$$c_2(\mu_1-\mu_2) = \mu_1$$

$$\boxed{c_2 = \frac{\mu_1}{\mu_1-\mu_2}}.$$

$$\cancel{= \frac{\mu_2-\mu_1-\mu_1}{\mu_2-\mu_1}}$$

$$\cancel{= \frac{\mu_2-2\mu_1}{\mu_2-\mu_1}}$$

$$c_1 = 1 - \frac{\mu_1}{\mu_1-\mu_2}$$

$$= \frac{\cancel{\mu_1}-\mu_2-\cancel{\mu_1}}{\mu_1-\mu_2} = \frac{-\mu_2}{\mu_1-\mu_2}$$

$$\boxed{c_1 = \frac{-\mu_2}{\mu_1-\mu_2}}$$

$$\mu_1 - \mu_2 = \pm\sqrt{(\omega_{ab}-\omega)^2 + \Omega_{ab}^2}\ \cancel{\times 2}$$

$$\mu_1\mu_2 = -\left(\frac{\Omega_{ab}}{2}\right)^2 = \frac{1}{4}\left[\cancel{(\omega_{ab}-\omega)^2} - \cancel{(\omega_{ab}-\omega)^2} + \Omega_{ab}^2\right].$$