1. Show that the wave equation describing magnetic field H in an inhomogeneous dielectric medium is

$$\nabla^2 H + \frac{\nabla n^2}{n^2} \times [\nabla \times H] - \frac{n^2}{c^2} \cdot \frac{d^2 H}{dt^2} = 0$$

2. Write down the electric and magnetic field components for TE and TM modes. Use the TE-equations to obtain the wave equation satisfied by the E_y field as

$$\frac{d^2 E_y}{dx^2} + [k_0^2 n^2(x) - \beta^2] E_y = 0$$

Also, use TM- equations to show that H_y component in the TM-mode satisfies a different relationship:

$$\frac{d^2 H_y}{dx^2} - \left[\frac{1}{n^2(x)} \cdot \frac{d\, n^2(x)}{dx}\right] \cdot \frac{d\, H_y}{dx} + [k_0^2 n^2(x) - \beta^2] H_y = 0$$

3. Consider a symmetric step-index planar waveguide of core-width d. derive the transcendental equation for the symmetric and anti-symmetric modes, and the field expressions in the core and cladding regions.

4. A symmetric step-index planar waveguide has the parameters: $n_1 = 1.503$, $n_2 = 1.500$, $d = 4$ μm.

(*a*) Obtain V-parameter at $\lambda_0 = 1.0$ μm. How any modes will be supported by this waveguide at this wavelength?

[Ans: V = 2.385 : $n_{eff} = 1.50159$]

(*b*) If the same waveguide is operated at $\lambda_0 = 0.5$ μm, what is the new V-number? How many modes will then be supported by the waveguide? Also calculate the effective indices of the modes in this case.

[Ans: V = 4.771 : $n_{eff\,0} = 1.5023$: $n_{eff\,0} = 1.5007$]

5. (*a*) For a step-index planar waveguide with $n_1 = 1.5$, $n_2 = 1.0$, $d = 0.555$ μm, operated at $\lambda_0 = 1.3$ μm, calculate the effective indices for TE- and TM-modes.

[Ans: TE: $n_{eff} = 1.3361$, TM: $n_{eff} = 1.2495$]

(*b*) If a linearly polarized light is launched at 45^0 with x-axis into this waveguide, then calculate the beat-length of the waveguide at which the polarization state will be repeated. Also calculate the length of the waveguide when the output light is circularly polarized.

[Ans : $L_B = 15$ μm, $L = L_B/4$]

6. Write down the Helmholtz's equation in different layers of an asymmetric slab waveguide for TM modes. Solve the equations for the eigenvalue equation and sketch the field profiles in the transverse plane of the waveguide.

7. For an asymmetric step-index waveguide, show that the power associated with TM-modes is

$$P = \frac{A^2 \beta}{2\omega\varepsilon_0 n^2}\left[\frac{d}{2} + \frac{(n_1 n_2)^2}{\gamma} \frac{k_0^2 (n_1^2 - n_2^2)}{(n_2^4 \kappa^2 + n_1^4 \kappa^4)}\right]$$

for both symmetric and anti-symmetric modes.