Linearly graded R.I. profile slab

Symmetric Triangular profile waveguide / scalar modes

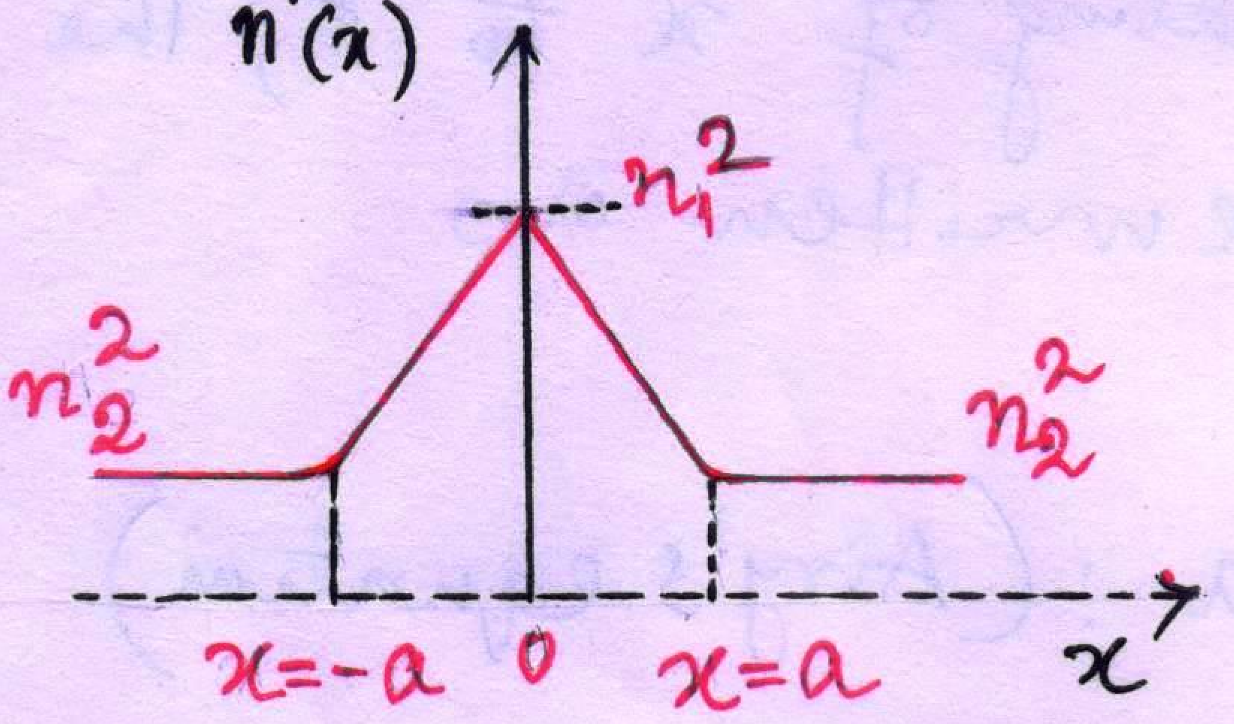
The scalar field distribution $\psi(x)$ of an optical waveguide satisfies the Helmoltz's equation which for a planar structure can be expressed as

$$\frac{d^2\psi}{dx^2}+\left[k_0^2 n^2(x)-\beta^2\right]\psi(x)=0$$

For a linearly graded RI profile (triangular slab) of the planar waveguide, we write

$$n^2(x) = n_1^2\left[1-2\Delta\frac{|x|}{a}\right] \quad : |x|\le a$$

$$= n_2^2 \quad : |x|>a \qquad \text{and} \quad \Delta=\frac{n_1^2-n_2^2}{2n_1^2}$$



Here 'a' is the core radius n_1, n_2 are the core & cladding R.I's.

For this RI profile, the Helmoltz's equation takes the form:

$$\frac{d^2\psi}{dx^2}+\left[k_0^2n_1^2-\beta^2-\frac{k_0^2n_1^2 2\Delta|x|}{a}\right]\psi=0 \quad \text{for the core.}$$

Let us put $C^2=k_0^2n_1^2-\beta^2$ and $D=\left[\frac{k_0^2n_1^2 2\Delta}{a}\right]^{2/3}$ in the above equation to get

$$\frac{d^2\psi}{dx^2}+\left[C^2-D^{3/2}|x|\right]\psi(x)=0$$

$$\text{or} \quad \frac{d^2\psi}{dx^2}-\left[D^{3/2}|x|-C^2\right]\psi(x)=0.$$

Again we make a substitution:

$$P = \frac{D^{3/2}|x| - e^2}{D} \qquad \frac{dP}{dx} = \sqrt{D}$$

Then $$\frac{d\psi}{dx} = \frac{d\psi}{dP}\cdot\frac{dP}{dx} = \sqrt{D}\,\frac{d\psi}{dP}.$$

and $$\frac{d^2\psi}{dx^2} = \frac{d}{dx}\left(\sqrt{D}\,\frac{d\psi}{dP}\right) = \sqrt{D}\cdot\frac{d}{dP}\left(\frac{d\psi}{dP}\right)\cdot\frac{dP}{dx}$$

$$= \sqrt{D}\,\frac{d^2\psi}{dP^2}\cdot\sqrt{D}$$

For the triangular core, $\frac{d^2\psi}{dx^2} = D\,\frac{d^2\psi}{dP^2}$.

With this shift in the coordinate of the core position by scaling of x to P, the Helmoltz's equation can be rewritten as

$$\frac{d^2\psi}{dP^2} - P\psi = 0 \; : |x| \leq a \; : (\text{Airy's equation})$$

The Helmoltz's equation for the cladding, using the R.I. profile $n^2(x) = n_2^2$

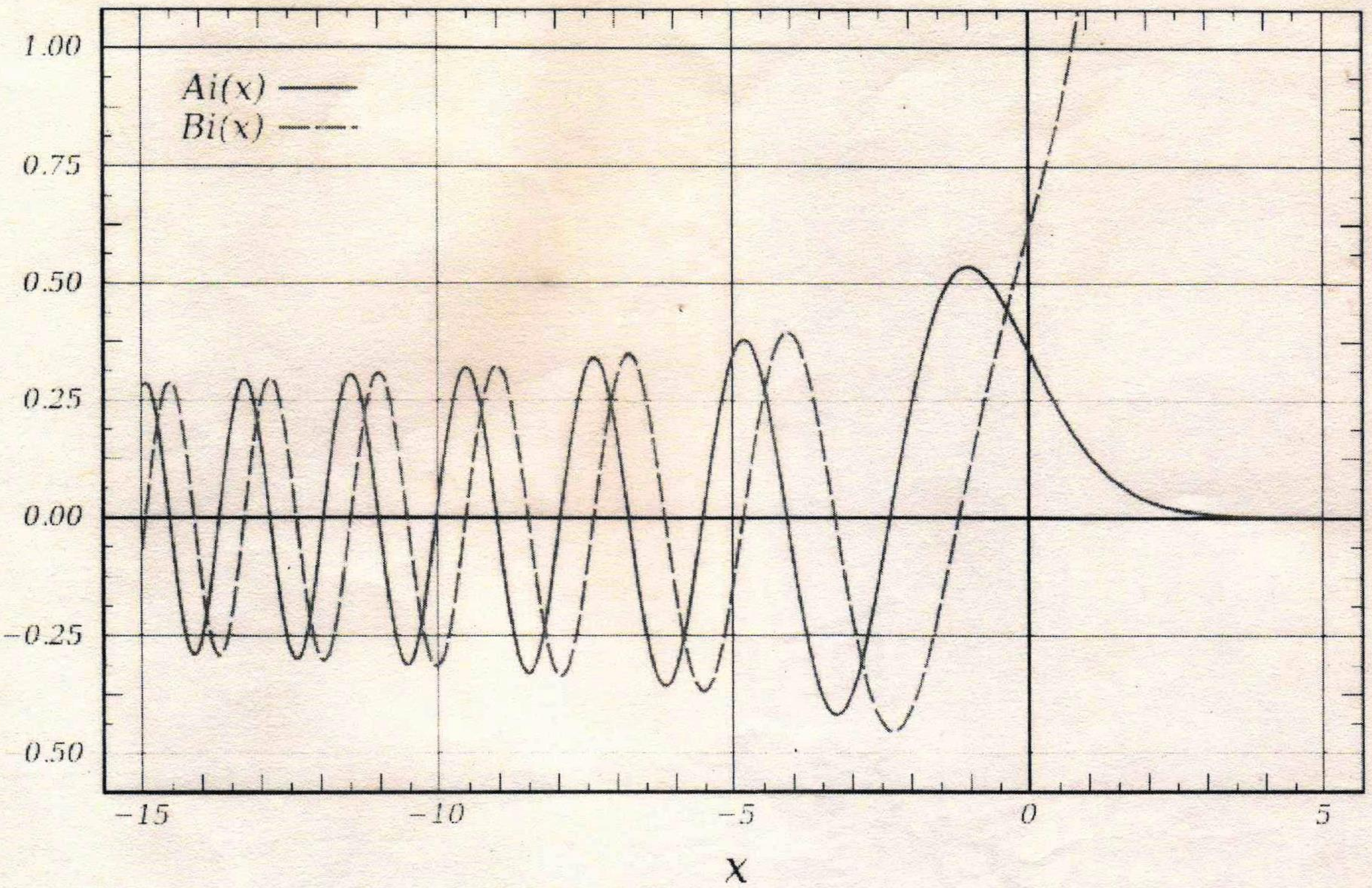
$$\frac{d^2\psi}{dx^2} - \gamma^2\psi = 0 \; : \; |x| > a$$

with $\gamma^2 = \beta^2 - k_0^2 n_2^2$

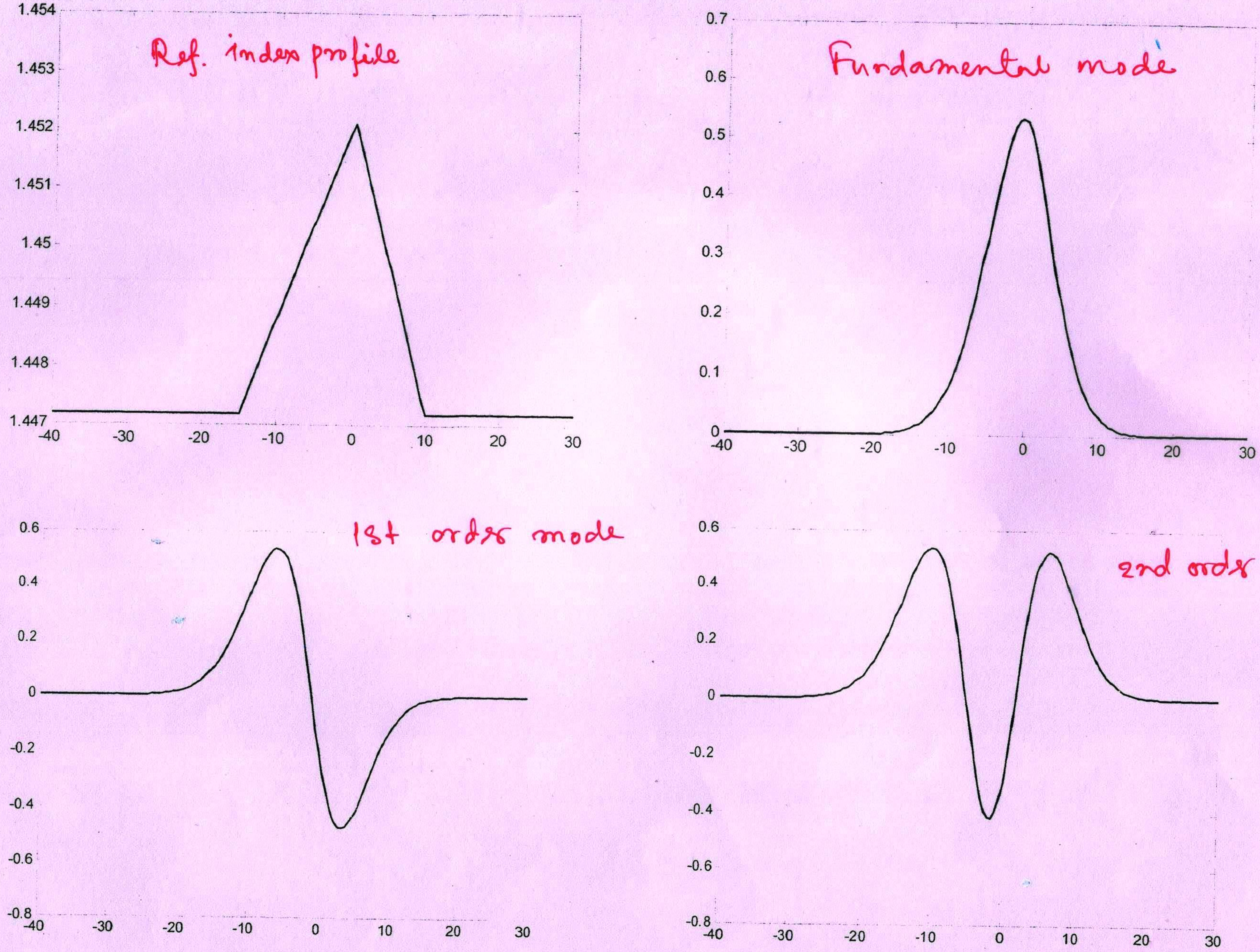
Now the solution in the core is given by the Airy's functions. The solution in the cladding is described by conventional exponentially decaying function.

$$\psi(x) = A_{\pm}\,A_i(P) + B_{\pm}\,B_i(P) : |x| \leq a$$

$$= C_{\pm}\,e^{-\gamma|x|} \qquad : |x| > a$$



1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
Ai(x)
Bi(x)
−15
−10
−5
0
5
X



Planar symmetric triangular profile w/guide.