

## 02. Assignment

- (1) A parallel-plate waveguide is known to have a cutoff wavelength for the  $m = 1$  TE and TM modes of  $\lambda_{c1} = 0.4$  cm. The guide is operated at wavelength  $\lambda = 1$  mm. How many modes propagate? The cutoff wavelength for mode  $m$  is  $\lambda_{cm} = 2nd/m$ , where  $n$  is the refractive index of the guide interior.
- (2) A parallel-plate guide is to be constructed for operation in the TEM mode only over the frequency range  $0 < f < 3$  GHz. The dielectric between plates is to be teflon ( $\epsilon'_R = 2.1$ ). Determine the maximum allowable plate separation,  $d$ .
- (3) A lossless parallel-plate waveguide is known to propagate the  $m = 2$  TE and TM modes at frequencies as low as 10GHz. If the plate separation is 1 cm, determine the dielectric constant of the medium between plates. Use
- (4) A  $d = 1$  cm parallel-plate guide is made with glass ( $n = 1.45$ ) between plates. If the operating frequency is 32 GHz, which modes will propagate? For a propagating mode, we require  $f > f_{cm}$
- (5) For the guide of Problem 14.4, and at the 32 GHz frequency, determine the difference between the group delays of the highest order mode (TE or TM) and the TEM mode. Assume a propagation distance of 10 cm. From Problem 14.4, we found  $m_{max} = 3$ . The group velocity of a TE or TM mode for  $m = 3$  is
- (6) The cutoff frequency of the  $m = 1$  TE and TM modes in a parallel-plate guide is known to be  $f_{c1} = 7.5$  GHz. The guide is used at wavelength  $\lambda = 1.5$  cm. Find the group velocity of the  $m = 2$  TE and TM modes. First we know that  $f_{c2} = 2f_{c1} = 15$  GHz. Then  $f = c/\lambda = 3 \times 10^8 / .015 = 20$  GHz. Now,
- (7) A parallel-plate guide is partially filled with two lossless dielectrics (Fig. 14.23) where  $\epsilon'_{R1} = 4.0$ ,  $\epsilon'_{R2} = 2.1$ , and  $d = 1$  cm. At a certain frequency, it is found that the  $TM_1$  mode propagates through the guide without suffering any reflective loss at the dielectric interface.
  - a) Find this frequency. The ray angle is such that the wave is incident on the interface at Brewster's angle.
  - b) Is the guide operating at a single TM mode at the frequency found in part a?
- (9) A rectangular waveguide has dimensions  $a = 6$  cm and  $b = 4$  cm.
  - a) Over what range of frequencies will the guide operate single mode?
  - b) Over what frequency range will the guide support both  $TE_{10}$  and  $TE_{01}$  modes and no others?
- (10) Two rectangular waveguides are joined end-to-end. The guides have identical dimensions, where  $a = 2b$ . One guide is air-filled; the other is filled with a lossless dielectric characterized by  $\epsilon'_R$ .
  - a) Determine the maximum allowable value of  $\epsilon'_R$  such that single mode operation can be simultaneously ensured in both guides at some frequency.
  - b) Write an expression for the frequency range over which single mode operation will occur in both guides; your answer should be in terms of  $\epsilon'_R$ , guide dimensions as needed, and other known constants.
- (11) An air-filled rectangular waveguide is to be constructed for single-mode operation at 15 GHz. Specify the guide dimensions,  $a$  and  $b$ , such that the design frequency is 10% while being 10% lower than the cutoff frequency for the next higher-order mode.
- (12) Using the relation  $P_{av} = (1/2)\text{Re}\{\mathbf{E}_s \times \mathbf{H}_s^*\}$ , and Eqs. (44) through (46), show that the average power density in the  $TE_{10}$  mode in a rectangular waveguide is given by

$$P_{av} = \frac{\beta_{10}}{2\omega\mu} E_0^2 \sin^2(\kappa_{10}x) \mathbf{a}_z \quad \text{W/m}^2$$