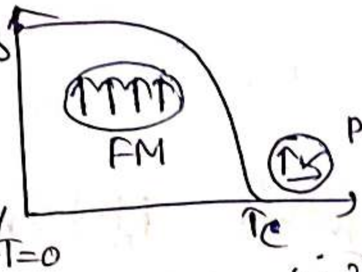
SPIN WAVES (MAGNON)

FM Ground State → $|0\rangle = \uparrow\uparrow\uparrow\uparrow\uparrow$ completely ordered
$|\alpha\rangle_1 |\alpha\rangle_2 |\alpha\rangle_3 |\alpha\rangle_4 |\alpha\rangle_5$

$E_0 = -NS^2 zJ$

Individual elementary magnetic moments in a magnetic crystal are essentially completely ordered at $t = 0\,K$.

But an increasing the temperature the M decreases towards zero at the critical (T_C, T_N, T_f) since the atomic moments acquire enough thermal energy to turn against effective molecular field



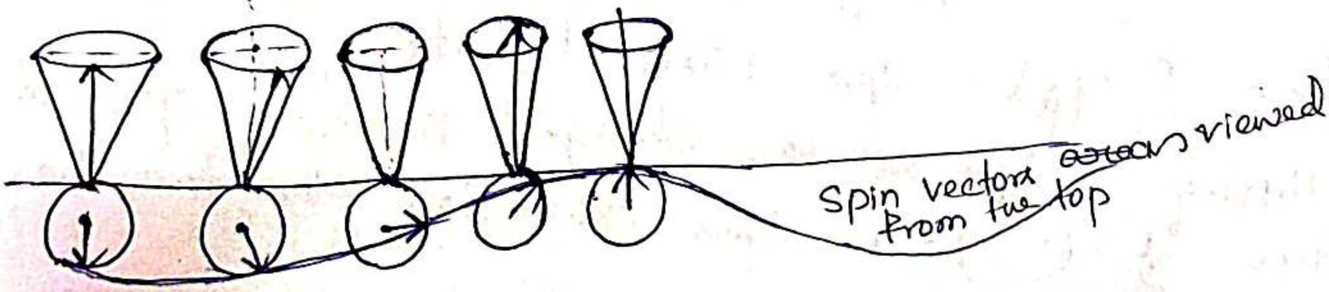
Now suppose temperature is raised slightly so that ~~all~~ one spin is reversed

$M = M_S(1 - \alpha T^{3/2})$
spin wave
$= M_S - \beta T^{3/2}$
$= M(T)$

$$|j\rangle = \uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow$$

We are dealing with collective excitation of all spins. The reversed spin will not remain localised at one atom. Spins are coupled through exchange interaction. A state in which one or more spins are reversed relative to their orientation at $t=0$ represents an excited state of the system.

In a simplest spin system with all its atomic magnetic moment in parallel alignment at absolute zero $T=0$ will $S_i^z = S$, where S_i^z denotes the component of spins site i that may change the value of S_i^z from S, $(S-1)$ & so on. The excitation does not remain localised at the site. It is infact transferred to the neighbouring spin setting its onward propagation in the crystal. ~~Varial~~ Deviation of the spin direction at different site follow a certain pattern such that the ~~peak~~ tips of the spin vectors convolute into wave like envelope.



The curve joining the tips of spin vectors has a wave like shape. A spin wave is shown here as an oscillation in the relative orientation of spin on a linear space. The wave thus generated is spin wave.
The quantized unit of ~~g~~ spin wave energy is called Magnon. To derive the excitation state we begin with spin Hamiltonian $\mathcal{H} = -2J_{ex} \sum_{i,j} \hat{S}_i \cdot \hat{S}_j$

$$\mathcal{H} = -2J_{ex} \sum_{i,j} S_i^z \cdot S_j^z + \frac{1}{2}\left(S_i^+ S_j^- + S_i^- S_j^+ \right)$$

ground state $|\phi\rangle = |\alpha\rangle_1 |\alpha\rangle_2 |\alpha\rangle_3 \ldots$

$$|\phi\rangle = \prod_n |\alpha\rangle_n = |\alpha\rangle_1 |\alpha\rangle_2 |\alpha\rangle_3 \ldots$$

$|\alpha\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|\beta\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$S_j^- |\phi\rangle = |\alpha\rangle_1 |\beta\rangle_2 |\alpha\rangle_3 \ldots$$

$$S_j^- |\phi\rangle = S_j^- \prod_n |\alpha\rangle_n$$

$$|j\rangle = \uparrow\uparrow\uparrow\underset{j}{\downarrow}\uparrow\uparrow\uparrow$$

$$= |\alpha\rangle_1 |\alpha\rangle_2 \ldots |\beta\rangle_j \ldots |\alpha\rangle_n$$

$\mathcal{H}|\phi\rangle = E_0 |\phi\rangle$

$E_0 = -NZJS^2$

This $|j\rangle$ is not an eigen state of the $\mathcal{H}$ because applying $S_j^+ S_{j+\delta}^-$ inside $\mathcal{H}$ would shift the reversed spin to the atom $(j+\delta)$ and create a different state.

G.S = $|\phi\rangle$

$$\hat{\mathcal{H}} = -2J \sum_i \left[S_i^z S_{i+1}^z + \frac{1}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ \right) \right]$$ only nearest neighbour interaction

$$\hat{\mathcal{H}}|\phi\rangle = -NS^2 J \cancel{2}\cancel{\tfrac{1}{2}} |\phi\rangle$$

$\uparrow\uparrow\uparrow$
Nearest neighbour interaction.

$$= -NS^2 J |\phi\rangle$$

$|j\rangle = \hat{S}_j^- |\phi\rangle$ = spin flipped at j site.

By flipping a spin we have changed the total spin of the system $\frac{1}{2} - \left(-\frac{1}{2}\right) = 1$. This excitation therefore has integer spin and is a boson.

$$\hat{H}|j\rangle = 2\left[\left(-NS^2J + 2SJ\right)|j\rangle - SJ|j+1\rangle - SJ|j-1\rangle\right]$$

$$\hat{H}|\phi\rangle = E_0|\phi\rangle = -NS^2J|\phi\rangle$$

1) $|\phi\rangle = \uparrow\uparrow\uparrow\uparrow \ldots$

$|j\rangle = \uparrow\uparrow\uparrow\downarrow\uparrow\uparrow \ldots$

$$\mathcal{H} = -2J_{ex}\sum_i S_i^z S_i^z + \frac{1}{2}\left(S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+\right)$$

1st term

$$\sum_i S_i^z S_{i+1}^z|j\rangle = S_1^z S_2^z|j\rangle + S_2^z S_3^z|j\rangle + \cdots + \left(S_{j-1}^z S_j^z + S_j^z S_{j+1}^z\right)|j\rangle$$

$$= \left(S^2 + S^2 + \cdots + S(S-1) + (S-1)S + \cdots S^2\right)|j\rangle$$

$$= \left\{(N-2)S^2 + S^2 - S^2 + S^2 - S\right\}|j\rangle$$

$$= \left(NS^2 - 2S\right)|j\rangle$$

2nd term

$$2S|j+1\rangle \leftarrow \left(\sum S_i^+ S_{i+1}^-\right)|j\rangle$$

$$S^+|m\rangle = \sqrt{(S-m)(S+m+1)}\,|m+1\rangle$$

$$S^-|m\rangle = \sqrt{(S+m)(S-m+1)}\,|m-1\rangle$$

3rd term

$$\left(\sum_i S_i^- S_{i+1}^+\right)|j\rangle = 2S|j-1\rangle$$

Result. $\hat{H}|j\rangle = 2\left[\left(-NS^2J + 2SJ\right)|j\rangle - SJ|j+1\rangle - SJ|j-1\rangle\right]$ not a constant multiplied $|j\rangle$

Which is not an eigen state of spin Hamiltonian.

So, this state is not an eigen state is

$$|q\rangle = \frac{1}{\sqrt{N}}\sum_j e^{iqR_j}|j\rangle$$

$$H|q\rangle = E(q)|q\rangle$$

This state $|q\rangle$ is essentially a flipped spin delocalized across all the sites.

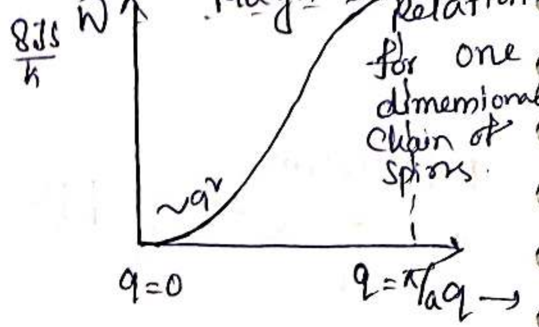
$$E(q) = -2NS^2J + 4JS(1-\cos qa)$$

The energy of the excitation is then

$$\hbar\omega = 4JS(1-\cos qa)$$

$$\omega = \frac{4JS}{\hbar}(1-\cos qa) = \frac{4JS}{\hbar}\left(2\sin^2\frac{qa}{2}\right)$$

$$= \frac{8JS}{\hbar}\sin^2 qa/2$$



Magnon dispersion Relation for one dimensional Chain of spins.

The Bloch $T^{3/2}$ law

at small q: $\hbar\omega = 8JS\frac{q^2a^2}{4}$

$$\hbar\omega \approx 2JS\,q^2a^2$$

$q \propto \omega^{1/2}$
$dq \propto \frac{1}{2}\omega^{-1/2}d\omega$
$\omega \cdot \omega^{-1/2}d\omega$
$- \omega^{1/2}d\omega$

So, $\omega \propto q^2$

<u>In 3-d</u>: the density of states $g(q)dq \propto q^2dq$

which leads to $g(\omega)d\omega \propto \omega^{1/2}d\omega$

At low temp. where only small q & small ω are important

The no. magnon modes excited at temperature T

$$n_{magnon} = \int_0^\infty \frac{g(\omega)d\omega}{\exp\left(\frac{\hbar\omega}{k_BT}\right)-1}$$

write $x = \frac{\hbar\omega}{k_BT}$

At low temp. $g(\omega)\,\cancel{d\omega} \propto \omega^{1/2}$

$$n_{magnon} = \int_0^\infty \frac{x^{1/2}dx}{e^x-1}\cdot\left(\frac{k_BT}{\hbar}\right)^{3/2}$$

$$= \left(\frac{k_BT}{\hbar}\right)^{3/2}\int_0^\infty \frac{x^{1/2}dx}{e^x-1}$$

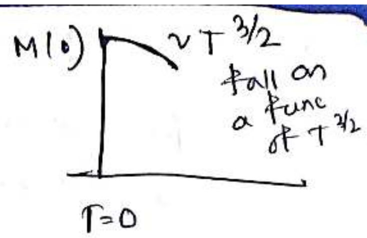
Since, each magnon mode which thermally excited reduces the total ~~spin~~ magnetization by $S=1$ $\quad \frac{1}{2}-(-\frac{1}{2})$

Then at low temp reduction in sponteneous scale Magnetization from $T=0K$

$$\frac{M(0)-M(T)}{M(0)} = (\quad)\,T^{3/2}$$

$$\propto T^{3/2}$$

$$M(T) = M(0)\left[1-\alpha T^{3/2}\right]$$ (Spin wave Magnon)



Bloch $T^{3/2}$ law

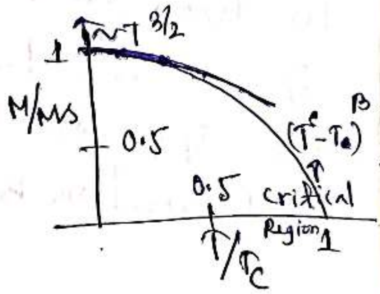
The energy magnon modes is given by

$$E_{magnon} = \int_0^\infty \frac{\hbar\omega\, g(\omega)\, d\omega}{\exp\left(\frac{\hbar\omega}{k_BT}\right)-1} \quad \propto T^{5/2}$$

So, heat capacity

$$c = \frac{\partial E_{magnon}}{\partial T} \propto T^{3/2}$$

$$\boxed{c \propto T^{3/2}}$$



$M \propto (T_c - T)^\beta \quad T<T_c$ where $\beta = 0.5$

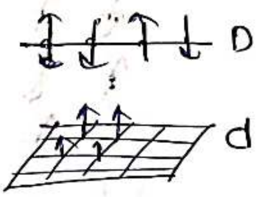
$\chi \propto (T-T_c)^{-\gamma} \quad T>T_c$

$M \propto H^{1/\delta} \quad T=T_c$

$c \propto (T-T_c)^\alpha \quad T>T_c$

~~Model Mean field~~

Mermin theorem



Model	Mean field	Ising	Ising	Heisenberg
D (Dim. of the order parameter)	any value	1	1	3
d (Dim of lattice)	$d \geq 4$	2 (2d Ising)	3 (3d Ising)	3
β	$1/2$	$1/8$	0.326	0.367
γ	1	$7/8$	1.24	1.388
δ	3	15	4.78	4.78

Critical exponent value

Scaling law:

$$2 = \alpha + 2\beta + \gamma$$
$$\delta = 1 + \gamma/\beta = (1 + 1/1_2 = 3 = \cdot 8)$$
$$\alpha = 2 - \delta\gamma$$