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Date 3/4/25.

Assignment SUPERCONDUCTIVITY.

(1.) $\lambda_L = \sqrt{\frac{m_e}{n_s e^2 \mu_0}}$ Assumption: (1) All valence e^- are SC type $\therefore n_s = n$.

(2) All e^- in $E_F \pm \Delta$.

(a) $\lambda_L = \sqrt{\frac{m_e}{e^2} \cdot \frac{1}{\mu_0} \cdot \frac{1}{n_{Al}}}$

For, 0K (absolute zero) there is no boltzmann thermal distribution

$n = \left(Z_v \times \left(\frac{\rho}{M_{Al}} \right) \times N_{A} \right)$ Z : Valency. ρ : Density

$\rho = \frac{\text{Mass}}{\text{Volume}}$, $\rho / \frac{}{\text{Mass mole}} = \#$

$= 3 \times \frac{2700 \text{ kg m}^{-3}}{27 \text{ g mol}^{-1}} \times 6.022 \times 10^{23}$

$= 3 \times \frac{27 \times 10^2 \times 10^3 g}{27 g} \text{ m}^{-3} \times 6.022 \times 10^{23}$

$= 18.066 \times 10^{28} = \underline{1.806 \times 10^{29} \text{ m}^{-3}}$

$14-29$
$-15.$

$\lambda_L = \sqrt{\frac{9.11 \times 10^{-31}}{(4\pi \times 10^{-7}) \times (1.8 \times 10^{29}) \times (1.6 \times 10^{-19})^2}}$

$= 0.39664 \times 10^{\frac{-31+7-29+38-1}{2}} \times \sqrt{10}$

$\lambda_L = 1.254 \times 10^{-8} \text{ m} = \boxed{12.54 \text{ nm}}$ (ans)

(b) Fraction of the valence e^- concentration that contributes to this phenomenon is :

This case says that a fraction of the total concentration is available in the supercurrent in the energy interval $E = [E_F - \Delta, E_F + \Delta]$.

This would give a different value for λ_L.

n = number of electrons = INTEGRAL of density of states from $E=0$ to $E=E_F$.

$\therefore n = \int_0^{E_F} N(E)\,dE = \int^{E_F} \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \sqrt{E}\,dE$

ROUGH.

$N(E) = \xi(m)\, \frac{2}{3} E^{3/2}$

$\therefore n(E_F) = \xi(m)\, \frac{2}{3}\left[E^{3/2}\right]_0^{E_F} = \xi(m)\, \frac{2}{3}\, E_F^{3/2}$

$\Delta n = \xi(m)\, \cancel{\frac{2}{3}} \cdot \cancel{\frac{3}{2}}\, E^{1/2}\, \Delta E$.

10^6

$\therefore$ In the neighbourhood Δ of E_F.

$n(E_F) + n(\Delta E_F) - (n(E_F) - n(\Delta E_F)) = 2n(\Delta E_F)$

$\therefore$ Number $= \cancel{2} \cdot \left(\frac{1}{\cancel{2}\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \cdot E_F^{1/2}\, \Delta\right) = \boxed{\frac{1}{\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E_F^{1/2}\, \Delta}$

$\therefore$ Fraction $= \dfrac{2 \times \cancel{\xi(m)}\, E_F^{1/2}\, \Delta}{\cancel{\xi(m)}\, \frac{2}{3}\, E_F^{3/2}} = \left(\dfrac{3\Delta}{E_F}\right)$

$\Rightarrow$ Fraction $= \dfrac{3 \times .18 \times 10^{-3}}{11.7}$

For Al, $\Delta \sim 0.18$ meV

$E_F \sim 11.7$ meV.

$= .046 \times 10^{-3}$

$= \boxed{4.6 \times 10^{-5}}$

~~Number in $E_F \pm \Delta$ = 8.29 × 10~~

~~$n_{E_F \pm \Delta}$ = 0.21 × 10^{19} = 31 ppm~~

1(b) Number of electrons in $(E_F - \Delta$ & $E_F + \Delta)$ $= n \cdot f$.

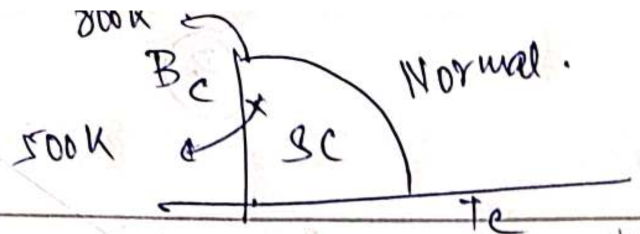
$= 1.806 \times 10^{29} \times 4.6 \times 10^{-5}$ m^{-3}.

$\therefore \lambda_{\Delta} = \dfrac{\lambda_L}{\sqrt{f}} = \dfrac{1.254 \times 10^{-8}}{\sqrt{4.6 \times 10^{-5}}} = 0.184 \times 10^{-8+3}$

$= 0.184 \times 10^{-5}$

$= \boxed{1.848\ \mu m.}$

The penetration depth increases for fewer number of electrons as $\boxed{\lambda_L \propto \frac{1}{\sqrt{n}}}$



(2) Pb is a superconductor with $\boxed{T_c = 7.2K}$ &

~~current~~ $\boxed{B_{c,0} = 800\,G}$

$B_c = B_{c_0}\left[1 - \left(\frac{T}{T_c}\right)^2\right]$ $\therefore$ At $T = 0.1\,K$.

$B_c \approx 800\,G$.

Magnetic energy density

$\rho = -\frac{1}{2}\chi \frac{B^2}{\mu_0}$

Now @ 7.5 K, both Cu & Pb act as Normal metals.

But @ $T < 7.2K$, Cu acts as metal while Pb acts as SC.

$\therefore$ @ 7.5 K.

$$Cu = +\frac{1}{2} \times \frac{+9.63 \times 10^{-6} \times 500 \times 500 \times 10^{-8}\ T.}{4\pi \times 10^{-7}\ H/m}\ Jm^{-3} = 9.57 \times 10^{4-7} = \underline{9.57 \times 10^{-3}\ Jm^{-3}}$$

$$Pb = +\frac{1}{2} \times \frac{+1.58 \times 10^{-5} \times 500 \times 500 \times 10^{-8}}{4\pi \times 10^{-7}} = 1.571 \times 10^{-2}\ Jm^{-3}$$

Magnetic Energy (ρ) : $\boxed{\begin{aligned} \rho_{Cu} &= 9.57 \times 10^{-3}\ Jm^{-3} \\ \rho_{Pb} &= 1.57\cancel{8} \times 10^{-2}\ Jm^{-3} \end{aligned}}$

For, Pb ($T < T_c$ @ 0.1K) $\longrightarrow$ $\boxed{\chi = -1}$

$$Pb = -\frac{1}{2}(\chi = -1) \times \frac{B^2}{\mu_0} = 9.947 \times 10^{+3+6-7}$$

$$= \boxed{9.947 \times 10^{2}\ J/m^3}$$

$\therefore$ @ 0.1K $\longrightarrow$ $\rho_{Cu} = \boxed{9.57 \times 10^{-3}\ J/m^3}$, $\rho_{Pb} = \boxed{9.94 \times 10^{2}\ J/m^3}$

(3.) BCS gap ~~voltage~~ energy of Pb @ 4K = 2meV.

$\therefore$ Gap voltage = ϕV = 2 meV
= 2 mV = 2×10^{-3} V.

$\therefore$ frequency $= \dfrac{2 \times e \times V}{h} = \dfrac{2e \times 2\,mV}{h}$

$= \dfrac{2 \times 1.6 \times 10^{-19} \times 2 \times 10^{-3}}{6.68 \times 10^{-34}} = 0.96 \times 10^{-19+34-3}$

$= 0.96 \times 10^{12}$.

$\therefore f = 958 \times 10^{9}$ Hz
= 958 GHz.

Gap @ 4.2 K $\Rightarrow$ $\Delta(T) = \Delta_0 \left[1 - \left(T/T_c\right)^4\right]^{1/2}$

$= \boxed{1.88 \text{ meV}}$

$\left[\begin{array}{l} T = 4.2K \\ T_c = 7.2K \\ \Delta_0 = 2\,meV \end{array}\right]$

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(4) $B = B_0 \hat{z}$.

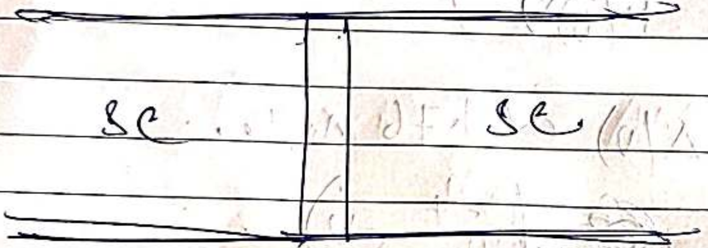
In the absence of magnetic field, $\Delta\phi = 0$. [DC josephson]

In the presence of a magnetic field, $\Delta\phi \neq 0$.

Temporal phase gradient:

$$\cancel{\phi} = \cancel{\phi_0} + \frac{2eVt}{\hbar} \Rightarrow \Delta\cancel{\phi} = \frac{2eVt}{\hbar}.$$

$$\Phi_B = B \times \text{Area} = B_0 b d.$$



$$\Delta\cancel{\phi} = \frac{2\pi\Phi}{\Phi_0} \qquad \left[\Phi_0 = h/2e\right]$$

$$\Delta\cancel{\phi} = \frac{2\pi}{\Phi_0} B_0 d b$$

Spatial phase gradient:

$$\cancel{\phi(y)} = \phi_0 + \frac{2\pi}{\Phi_0} d B_0 (y)$$

At any point (y).

$$\Delta\phi = \frac{2\pi}{h/2e} B_0 d b = \left(\frac{2eB_0 d b}{\hbar}\right)$$

$$\therefore \phi = \cancel{\phi_0} + \left(\frac{2eB_0 d b}{\hbar}\right).$$

$$I = \int_{-b/2}^{b/2} J(y)\, dy = \int_{-b/2}^{b/2} j_c \sin[\phi(y)] = j_c \int_{-b/2}^{b/2} \sin\left(\phi_0 + \frac{2\pi d B_0 y}{\Phi_0}\right) dy.$$

$$\therefore I = j_c \int_{-b/2}^{b/2} \sin(\phi_0 + Ky)\, dy = j_c \left[\frac{\cos(\phi_0 - Ky) - \cos(\phi_0 + Ky)}{K}\right]$$

Let $\frac{2\pi d B_0 \cdot 2e}{h} = \left(\frac{2eB_0 d}{\hbar}\right) = K$

$$= j_c \frac{2\sin(\phi_0)\sin(Ky)}{K} \qquad [\phi_0 = \pi/2]$$

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$$= \left[2 j_c \frac{\sin(Ky)}{K} \right]^{b/2} \qquad K = \frac{2eB_0 d}{\hbar}$$

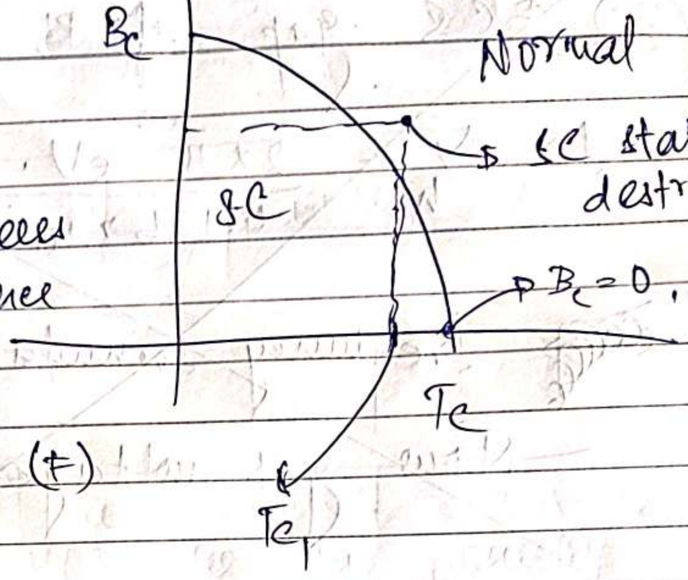
$$= \frac{\cancel{2} j_c \sin\left(\frac{\cancel{2}eB_0 d}{\hbar} \cdot \frac{b}{\cancel{2}}\right)}{\frac{\cancel{2} eB_0 d}{\hbar}\left(b \cdot \frac{1}{b}\right)} = \frac{(j_c \cdot b) \sin\left(\frac{eB_0 db}{\hbar}\right)}{\frac{eB_0 db}{\hbar}}$$

I_0

$$\boxed{I = I_0 \frac{\sin\left(\frac{eB_0 db}{\hbar}\right)}{\left(\frac{eB_0 db}{\hbar}\right)}} \text{ (ans)}$$

(5) Initial $T_{c_1} = \alpha T_c$.

We need to calculate the temperature drop. The process is ADIABATIC, and hence entropy



change in free energy (F) is given by:

$$F_n(T) - F_s(T) = \frac{B_c^2(T)}{2\mu_0}$$

$$\therefore S_n(T) - S_s(T) = -\frac{d}{dT}\left[F_n(T) - F_s(T)\right] = -\frac{d}{dT}\left[\frac{B_c^2(T)}{2\mu_0}\right].$$

$$\longrightarrow B_c^2(T) = B_c^2(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]^2. \text{ (we know).}$$

Also, $T = \alpha T_c$. ($\alpha < 1$).

$$\therefore \frac{d}{dT}\left[B_c^2(T)\right] = \frac{d}{dT}\left[B_c^2(0)\left(1-\frac{T^2}{T_c^2}\right)^2\right] = B_c^2(0)\cdot 2\left[1-\left(\frac{T}{T_c}\right)^2\right]\left(-\frac{2T}{T_c^2}\right).$$

$$= 2B_c^2(0)\left(\frac{-2\alpha T_c}{T_c \cdot T_c}\right)(1-\alpha^2).$$

$$= -4\alpha B_c^2(0)(1-\alpha^2).$$

$$\therefore \Delta S = -\frac{1}{2\mu_0}\left[\frac{d}{dT}B_c^2(T)\right] = \frac{2\alpha B_c^2(0)}{\mu_0 T_c}(1-\alpha^2).$$

$\boxed{\text{Phase transi}^{n}\text{ latent heat}}$

$\boxed{\Delta T = 0, \text{ since Heat change happening at } T = T_c}$

Let. Specific Heat $\longrightarrow C_v$ & Heat released $= Q$.

In Normal state ($T > T_c$).

$\therefore C_v\, dT = T\, dS$. $\boxed{C_v = \gamma T.} \longrightarrow$ Specific Heat coeff.

$$\therefore \Delta S = \int_0^T \frac{C_v(T')}{T'}\, dT'.$$

$\boxed{\text{latent Heat.}}$

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$$\therefore \Delta S = \int_0^T \frac{\gamma T' \, dT'}{T'} = \gamma T \;\rightarrow\; S_n$$

$\therefore S_n = \gamma T + S_n^0$ [Normal state is more disordered]

$$\therefore S_S = \gamma T - \frac{2\alpha B_c^2(0)}{\mu_0 T_c}(1-\alpha^2).$$

We know the process is adiabatic.
Hence, $S_S(T_f) = S_N(T_f)$.

$$\therefore S_S(T_f) = S_n(T_f) = \gamma T_f.$$

$$\therefore \gamma T_f = \gamma T_i - \frac{2\alpha B_c^2(0)}{\mu_0 T_c}(1-\alpha^2).$$

$$\Rightarrow \gamma \Delta T = \frac{2\alpha B_c^2(0)}{\mu_0 T_c}(1-\alpha^2).$$

$$\therefore \boxed{\Delta T = \frac{2\alpha B_c^2(0)}{\mu_0 \gamma T_c}(1-\alpha^2). \quad \text{(ans).}} \rightarrow \text{temperature drop}$$

(c) By BCS theory

$$\Delta(0) \sim 2\hbar\omega_D \exp\left(-\frac{1}{N(0)V}\right),$$

Using approximation $\Delta(0) \sim 1.76\, k_B T_c$.

Usually this value $\sim (10^{-4}\ eV)$.

$\therefore\ hf_{low} = 10^{-4}$

$$f_{low} = .151 \times 10^{34-4} \times 1.6 \times 10^{-19}$$

$$\approx 2.41 \times 10^{10}\ Hz$$

$$\approx \boxed{24.1\ GHz}$$

$$\cancel{hf_{high} = 10^{-3}}$$

$$\cancel{f_{high} = .151 \times 10^{34-3} \times 1.6 \times 10^{-19}}$$

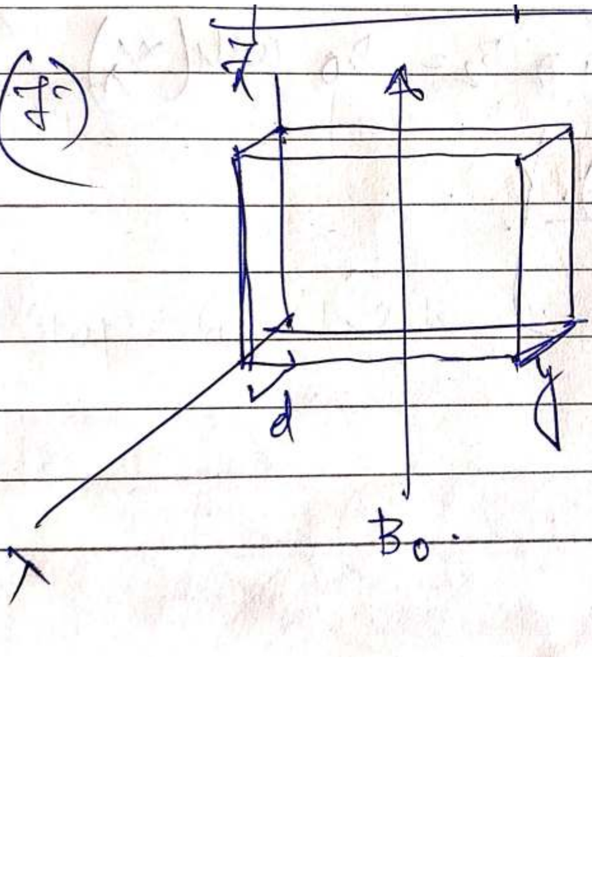
$$\cancel{\approx 2.41 \times 10^{11}\ Hz}$$

$$\cancel{\approx 241.5\ GHz}$$

Microwave frequency $\rightarrow$ 300 MHz - 300 GHz.

Hence, radiation to break cooper pair lies in microwave frequency range.

(f)



$$\nabla^2 B(x) = \frac{1}{\lambda^2} B(x).$$

solving we gets

$$B(x) = B_1 e^{-x/\lambda} + B_2 e^{+x/\lambda}.$$

$$B(d/2) = B(-d/2) = B \quad \text{[boundary)}.$$

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$B(-d/2) = B_1 e^{-d/2\lambda} + B_2 e^{d/2\lambda} = B$

$B(d/2) = B_1 e^{d/2\lambda} + B_2 e^{-d/2\lambda} = B$.

Solving this set of equations, we get

$$B_1 = B_2 = \frac{B}{2\cosh\left(\frac{d}{2l}\right)}.$$

Similarly for any x the value of B_1 & B_2 will be symmetric, since the slab geometry is symmetric, isotropic & uniform.

$\therefore B(x) = B(-x)$.

$\therefore B_1 e^{-x/\lambda} + B_2 e^{x/\lambda} = B_1 e^{x/\lambda} + B_2 e^{-x/\lambda}$.

$$= B_1\left(\frac{e^{-x/\lambda} - e^{x/\lambda}}{2}\right) = B_2\left(\frac{e^{-x/\lambda} - e^{x/\lambda}}{2}\right)$$

$$\Rightarrow -B_1 \cosh\left(x/\lambda\right) = -B_2 \cosh\left(x/\lambda\right).$$

If $B_1 = B_2$, then $B_1 = B_2 = B_0 \cosh(x/\lambda)$.

$$\therefore \boxed{B(x) = \frac{B_0 \cosh(x/\lambda)}{2\cosh\left(d/2\lambda\right)}}.$$

Expansion of $\cosh(z) = 1 + z^2/2$.

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$$B(x) = \frac{B_0\left(1+x^2/2\lambda^2\right)}{\left(1+\frac{d^2}{8\lambda^2}\right)} \to B_0\left(1+\frac{x^2}{2\lambda^2}\right)\left(1-\frac{d^2}{8\lambda^2}\right). \quad [d \ll \lambda]$$

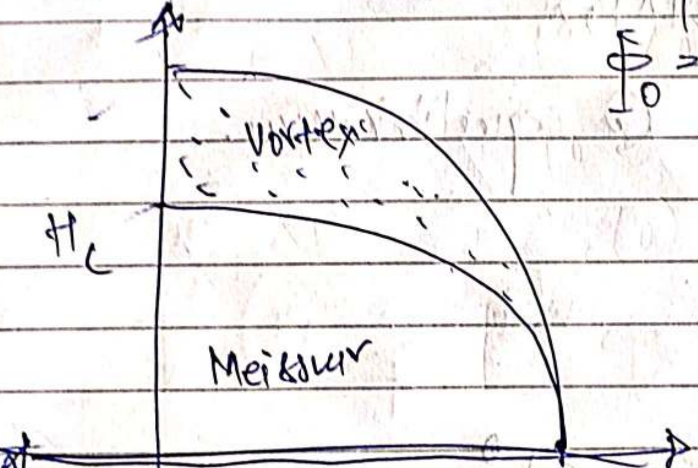
$$= B_0\left(1+\frac{x^2}{2\lambda^2}-\frac{d^2}{8\lambda^2}\right).$$

$$\mu_0 M = B(x) - B_0.$$

$$\mu_0 M = B_0\left(\frac{x^2}{2\lambda^2}-\frac{d^2}{8\lambda^2}\right) = \frac{B_0}{8\lambda^2}\left(4x^2-d^2\right).$$

$\therefore$ effective magnetization : $\boxed{M(x) = \frac{B_0}{2\mu_0\lambda^2}\left(4x^2-d^2\right).}$

(b) critical field required for onset of Type 2 vortex



$$\Phi_0 = h/2e, \quad \Phi = n\Phi_0.$$

$$\nabla^2 B = B/\lambda^2.$$

for cylindrical symmetry

$$B = B_z(r).$$

$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}B_z\right) = \frac{B_z}{\lambda^2}.$$

$$\Rightarrow B_z(r) = B_0 K_0\left(r/\lambda\right)$$ modified Bessel func^n.

Total flux:

$$\Phi_0 = \int_0^\infty B_z(r)\, 2\pi r\, dr$$

$$= \int_0^\infty \frac{1}{\cancel{r}}\frac{d}{\cancel{dr}}\left(r\frac{d}{dr}B_z\right)2\pi \cancel{r}\,\cancel{dr} = \lambda^2\pi\int_0^\infty d\left(r\frac{d}{dr}B_z\right) \simeq 2\pi\lambda^2 B_0.$$

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$$B_0 \approx \frac{\Phi_0}{2\pi\lambda^2} \quad \text{--- (i)}$$

$$\therefore \cancel{\frac{B_c^2}{2\mu_0} = \frac{B_0^2}{2\mu_0} = \frac{\Phi_0^2}{8\pi^2\lambda^2}}$$

Now, energy lost by the vortices (/vortex) =

$P(B) \times$ [Area of (vortex)].

$$= \frac{B_c^2}{2\mu_0} \times \pi \xi_g^2$$

magnetic energy stored in the vortex (/vortex)

$$= \frac{\Phi_0^2}{4\pi\mu_0\lambda^2}.$$

equating loss & gain @ equilibrium.

$$\frac{B_c^2}{\cancel{2\mu_0}} \pi \xi_g^2 = \frac{\Phi_0^2}{\cancel{2}\,4\pi\cancel{\mu_0}\lambda^2}.$$

$$B_c^2 = \frac{\Phi_0^2}{2\pi^2\lambda^2\xi_g^2} \Rightarrow B_c = \frac{\Phi_0}{\sqrt{2}\pi\lambda\xi_g}.$$

$$\therefore \frac{B_c^2}{2\mu_0} = \frac{\Phi_0^2}{4\pi^2\lambda^2\xi_g^2\mu_0} \quad \text{--- (ii)}$$