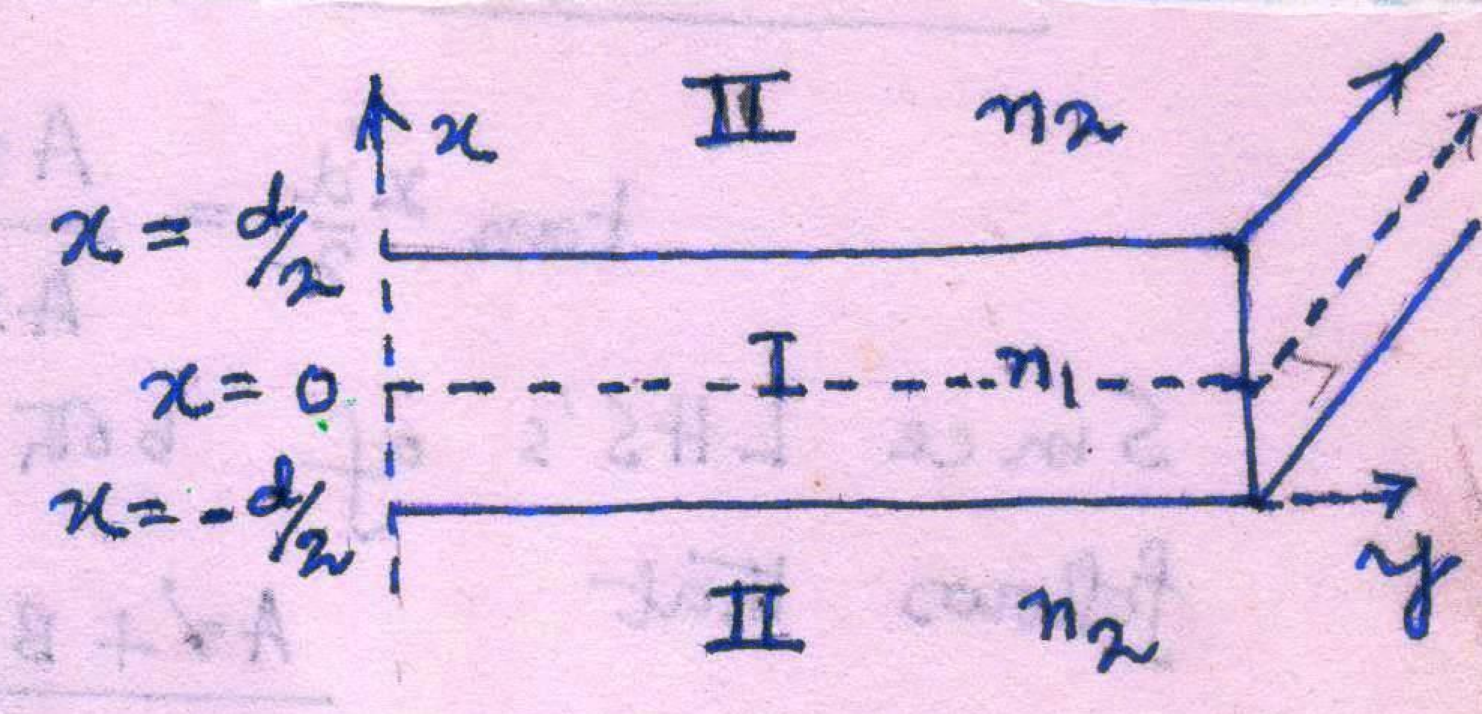
Planar waveguide (dielectric - dielectric interface)

Symmetric structure : TE - modes

The wave eqn will be satisfied by each homogeneous layer of the waveguide :



$$\frac{d^2E_y}{dx^2} + \left(k_0^2 n^2(x) - \beta^2\right) E_y = 0 \; ;$$

where $n^2(x) = n_1^2 \; : \; \pm d/2 > |x|$
$\qquad\qquad = n_2^2 \; : \;$ elsewhere

$\therefore$ for region - I : $\quad \dfrac{d^2E_y}{dx^2} + \left(k_0^2 n_1^2 - \beta^2\right) E_y = 0 \;$: Core

and for region - II : $\quad \dfrac{d^2E_y}{dx^2} - \left(\beta^2 - k_0^2 n_2^2\right) E_y = 0 \;$: cladding

Define mode parameters :

$$\left.\begin{aligned} \chi^2 &= k_0^2 n_1^2 - \beta^2 \\ \text{and} \quad \gamma^2 &= \beta^2 - k_0^2 n_2^2 \end{aligned}\right\} \text{These are actually } x\text{-compt of the propagation}$$

With these,

$$\frac{d^2E_y}{dx^2} + \chi^2 E_y = 0 \quad \text{—— (1)}$$

$$\frac{d^2E_y}{dx^2} - \gamma^2 E_y = 0 \quad \text{—— (2)}$$

Solutions of (1) & (2) are

$$E_y(x) = A' e^{i\chi x} + B' e^{-i\chi x} \qquad \text{for (1)}$$

$$E_y(x) = C e^{-\gamma x} + D e^{\gamma x} \qquad \text{for (2)}$$

$$\left.\begin{aligned} E_y(x) &= A\cos \chi x + B \sin \chi x \\ E_y(x) &= C e^{-\gamma x} + D e^{\gamma x} \end{aligned}\right\} \text{Solns.}$$

Now, at $x = +\frac{d}{2}$: interface

$$E_y\left(x = +\tfrac{d}{2}\right)\Big|_{\text{region-I}} = E_y\left(x = +\tfrac{d}{2}\right)\Big|_{\text{region-II}} \Rightarrow A\cos \frac{\chi d}{2} + B \sin \frac{\chi d}{2} = C e^{-\frac{d}{2}\gamma}$$

$$\frac{\partial E_y}{\partial x}\Big|^{(x = +\frac{d}{2})}_{\text{region-I}} = \frac{\partial E_y(x = +\frac{d}{2})}{\partial x}\Big|_{\text{region-II}} \Rightarrow -A\chi \sin \frac{\chi d}{2} + B\chi \cos \frac{\chi d}{2} = -\gamma C e^{-\frac{d}{2}\gamma}$$

Multiplying (1) by γ : $\quad \tan \dfrac{\chi d}{2} = \dfrac{A\gamma + B\chi}{A\chi - B\gamma}$

At $x=-\frac{d}{2}$ interface

$$A\cos\chi\tfrac{d}{2} - B\sin\chi\tfrac{d}{2} = D\, e^{-\frac{d}{2}\gamma}$$
$$A\chi\sin\chi\tfrac{d}{2} + B\chi\cos\chi\tfrac{d}{2} = \gamma D\, e^{-\frac{d}{2}\gamma}$$

Also use $\underline{D=C}$ as the waveguide is symmetric

$$\tan\chi\frac{d}{2} = \frac{A\gamma - B\chi}{A\chi + B\gamma}$$

Since LHS's of both the conditions are the same, it follows that

$$\frac{A\gamma + B\chi}{A\chi - B\gamma} = \frac{A\gamma - B\chi}{A\chi + B\gamma}$$

This on simplification gives $2AB(\chi^2+\gamma^2) = 0$

ie, either $\gamma^2 = -\chi^2$ [which is true for homogeneous layers

or $AB = 0$: saying that $A=0$ or, $B=0$

So, if we start from any of the condition — say $B=0$, then $\tan\chi\frac{d}{2} = \frac{\gamma}{\chi}$

And for $\boxed{A=0}$ $\tan\chi\frac{d}{2} = -\frac{\chi}{\gamma}$

For $B=0$:

$$\left.\begin{aligned} E_y(x) &= A\cos\chi x & |x| < \tfrac{d}{2} \\ &= C\, e^{-\gamma|x|} & |x| > \tfrac{d}{2} \end{aligned}\right\} \text{Symmetric field distribution}$$

Eigen value eqn

$$\tan\frac{\chi d}{2} = \frac{\gamma}{\chi} \Rightarrow \boxed{\frac{\chi d}{2}\tan\frac{\chi d}{2} = \frac{\gamma d}{2}}$$

Define V-number:

$$V = k_0\frac{d}{2}\sqrt{n_1^2 - n_2^2}$$

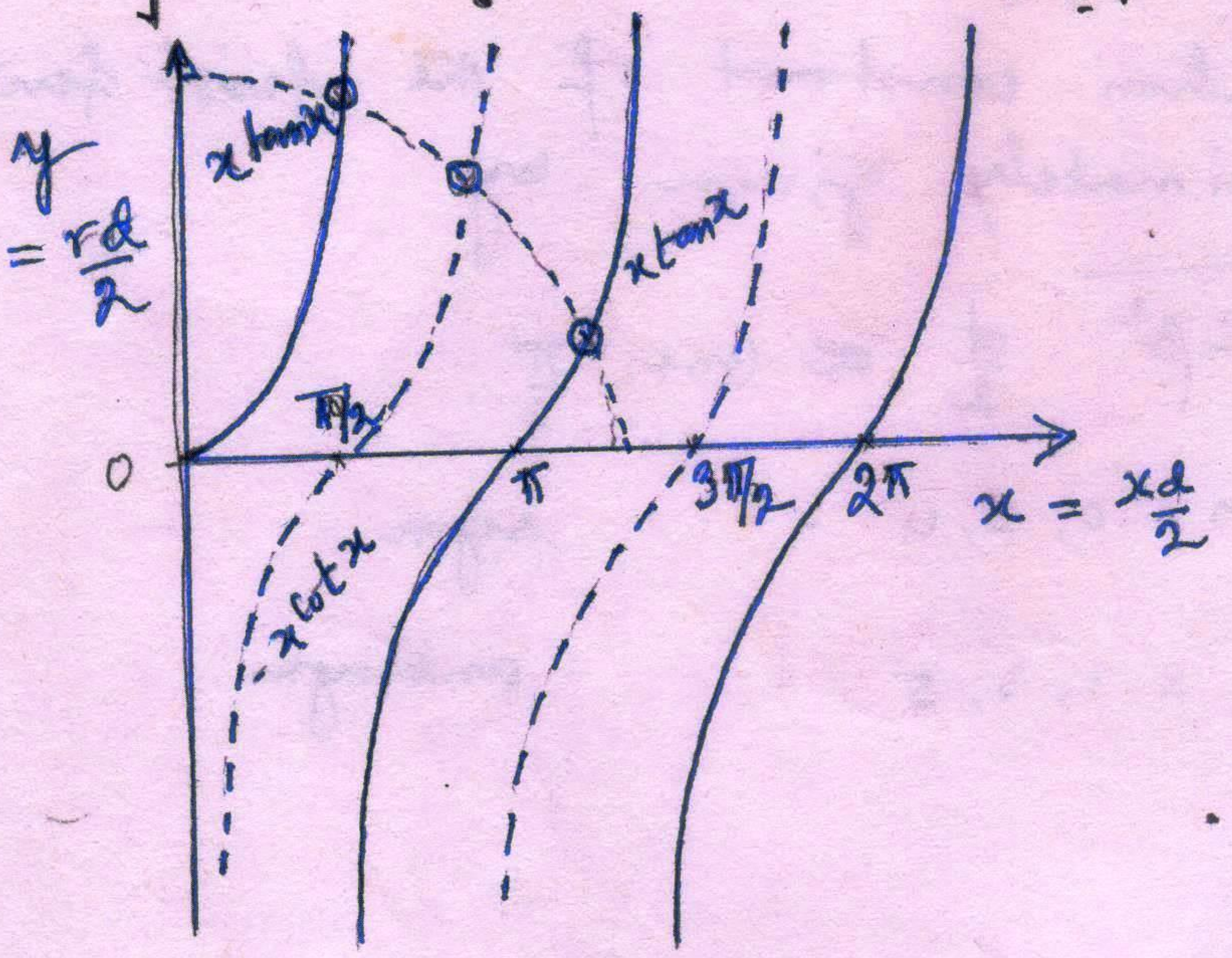
Now, $V^2 = \frac{d^2}{4}\left(k_0^2 n_1^2 - k_0^2 n_2^2\right) = \frac{d^2}{4}\chi^2 + \frac{d^2\gamma^2}{4}$

$$V^2 = \left(\frac{\chi d}{2}\right)^2 + \left(\frac{\gamma d}{2}\right)^2$$

$$y = x\tan x$$

$$V^2 = x^2 + y^2 \quad \text{— (A)}$$

Eqn (A) is that of a circle of radius V. The intersection of the circles with curves given by (B) are the solutions of the eigen value eqn.



In the same way for antisymmetric modes, $A = 0$

$$E_y(x) = B \sin \chi x$$
$$= \frac{x}{|x|} C e^{-\gamma|x|}$$

and the eigen value eqn. will be

$$-x \cot x = y$$

Now from this figure,

$0 < V < \pi/2 \rightarrow$ one sym

$\pi/2 < V < 2\pi/2 \rightarrow$ one sym + one antisym

In general, $(2m+1)\pi/2 < V < (2m+2)\pi/2 \;\; \exists \;\; (m+1)$ sym $+ (m+1)$ antisym

Total of $(2m+2)$ modes

$V < (2m+2)\pi/2 \qquad \exists \qquad (2m+2) > \frac{2V}{\pi}$

$$\boxed{N > \frac{2V}{\pi}}$$

And if

$2m\,\pi/2 < V < (2m+1)\pi/2 \;\; \exists \;\; (m+1)$ sym $+$ m antisym

$(2m+1) > \frac{2V}{\pi}$ Total (2m+1) modes.

$\boxed{N > \frac{2V}{\pi}}$ $\therefore$ Total no. of guided modes will be integer closest to and greater than $N = \frac{2V}{\pi}$.

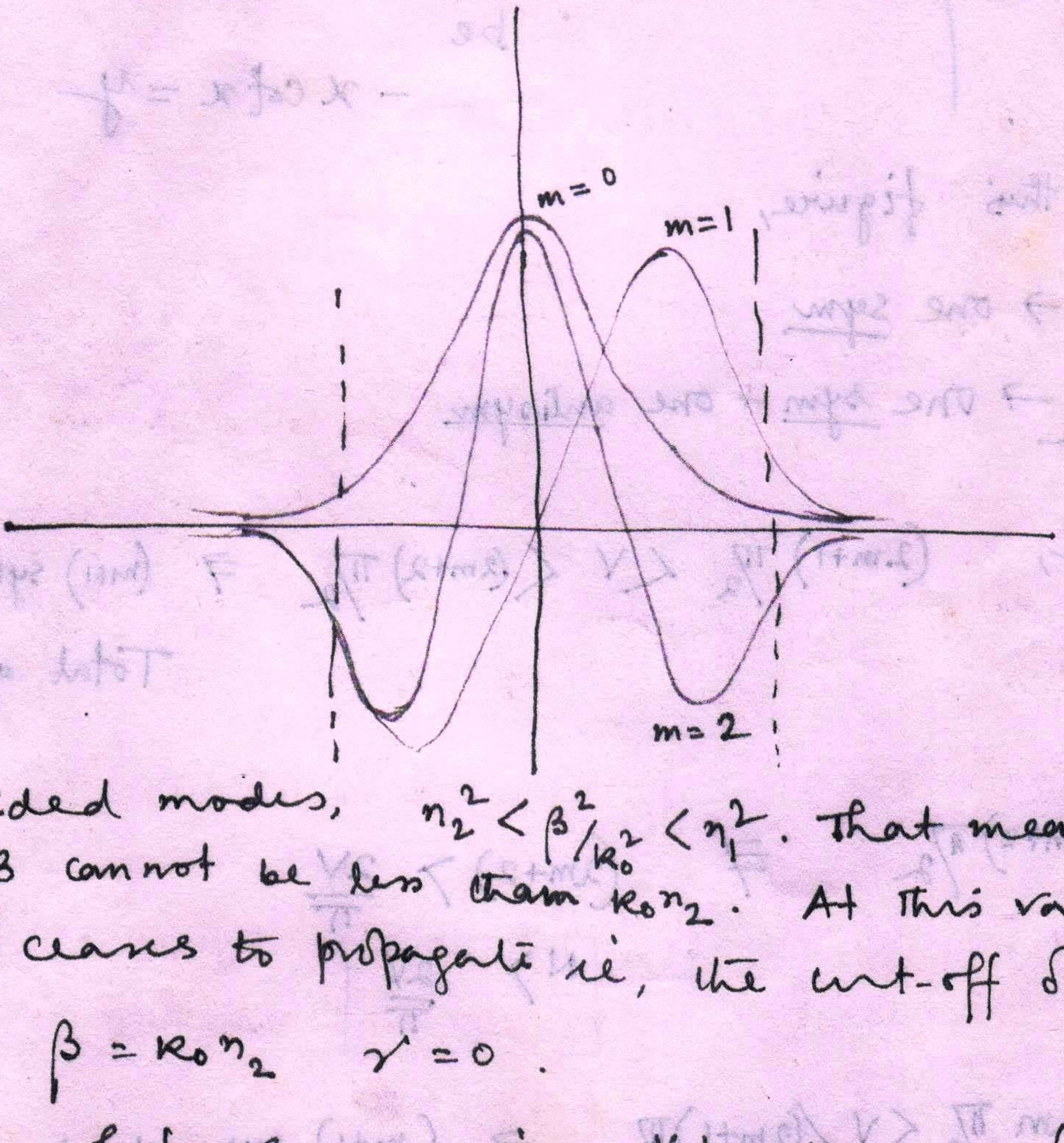
when the w/guide supports many modes, (ie, $V >> 1$) then the points of intersection for solutions will be very close to $\xi = \pi/2, \pi, 3\pi/2, \cdots$ etc.

Thus, the propagation constant of the first few modes will be approximately given by

$$\frac{\alpha d}{2} = \xi = \xi_m = \sqrt{k_0^2 n_1^2 - \beta^2} \cdot \frac{d}{2} \approx (m+1)\frac{\pi}{2}.$$

where $m = 0, 2, 4 \cdots$ sym

$= 1, 3, 5 \cdots$ antisym.



For guided modes, $n_2^2 < \beta^2/k_0^2 < n_1^2$. That means for a guided mode β cannot be less than $k_0 n_2$. At this value of β, the mode ceases to propagate ie, the cut-off of the mode. Hence, $\beta = k_0 n_2$ $\gamma = 0$.

Then $\xi \tan \xi = 0$ ie, $\frac{V}{2} \tan \frac{V}{2} = 0$ for sym

and $\frac{V}{2} \cot \frac{V}{2} = 0$ for anti sym.

This means that the cut-off values of various modes are given by $V_c = m\pi$; $m = \underbrace{0, 2, 4}_{\text{sym}}$; $\overbrace{1, 3, 5}^{\text{antisym.}} \cdots$

Actually written: $m = 0, 1, 2, 3, 4, 5 \cdots$

$0 \xleftrightarrow{V} \pi/2 \quad \Rightarrow \quad 1$ sym.

$\pi/2 \xleftrightarrow{V} 2 \cdot \pi/2 \quad \Rightarrow \quad \underline{1 \text{ sym} + 1}$ anti sym.

$\frac{2 \cdot \pi}{2} \xleftrightarrow{V} 3\pi/2 \quad \Rightarrow \quad \underline{2 \text{ sym} + 1}$ antisym

$3\pi/2 \xleftrightarrow{V} 4\pi/2 \quad \Rightarrow \quad \underline{2 \text{ sym} + 2}$ antisym.

$(2m+1)\pi/2 \xleftrightarrow{V} (2m+2)\pi/2 \quad \underbrace{(m+1) \text{ sym.} + (m+1) \text{ anti.}}$

$(2m+1)\pi/2 < V < (2m+2)\pi/2 \quad 2m+2 = N =$ no. of modes

$V = (2m+2)\pi/2 \longrightarrow N$ no. of modes.

$V = N\pi/2 \quad N = \frac{2V}{\pi} \quad \Rightarrow$ no. of modes.

If $V \leq \pi/2 \rightarrow$ only one mode

single-mode w/g.

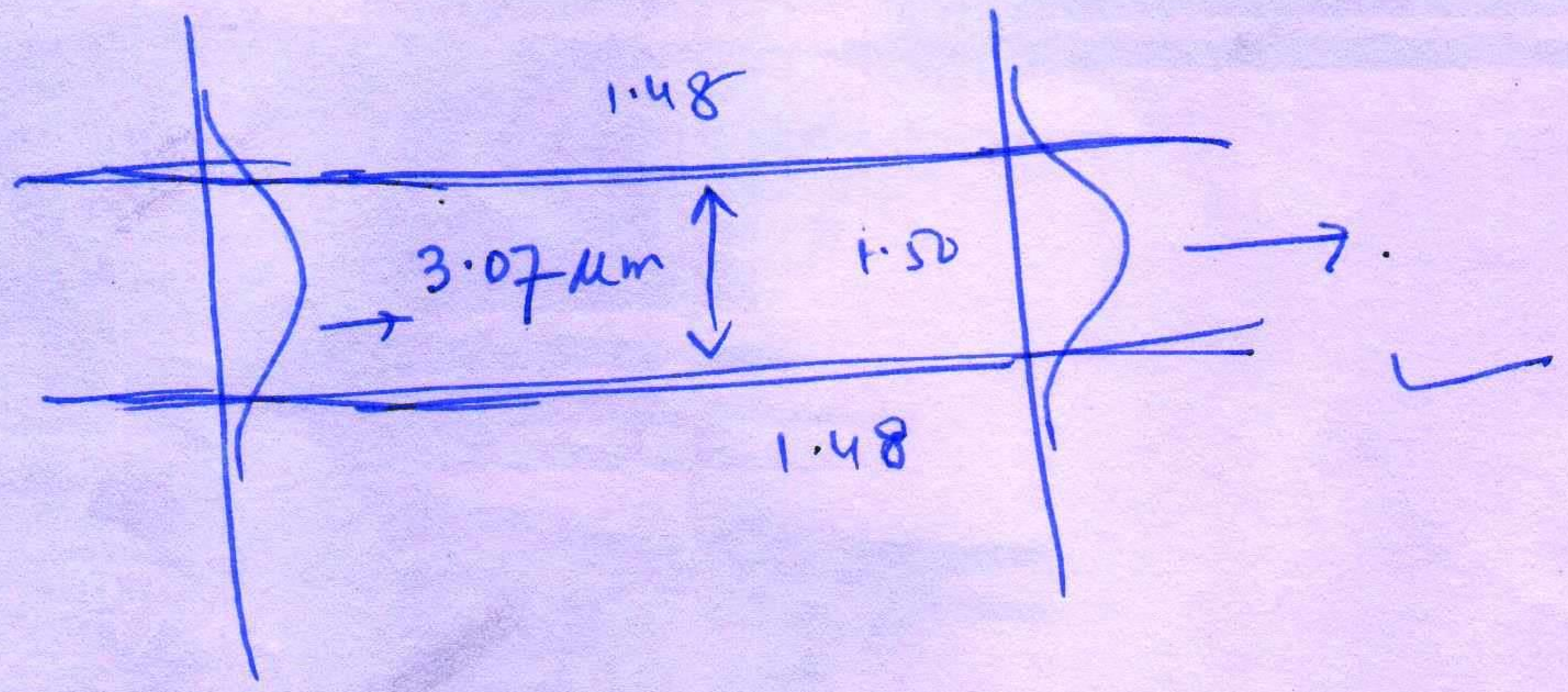
$\lambda_0 = 1.5\,\mu m = 1.5 \times 10^{-6}$ m.

$n_1 = 1.50$

$n_2 = 1.48$

$V = \frac{2\pi}{1.5} \times \frac{d}{2} \sqrt{(1.5)^2 - (1.48)^2} = \frac{\pi}{2} \quad \Rightarrow d = \frac{1.5}{2\sqrt{(\quad)}}$

$= 3.07\,\mu m$



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Example: $\lambda_0 = 1.5\ \mu m = 1.5 \times 10^{-6}\ m$

$n_1 = 1.50$

$n_2 = 1.48$

$$V = \frac{2\pi}{1.5} \times d\sqrt{\left[(1.50)^2 - (1.48)^2\right]}$$

For this waveguide to support only one mode i.e., with $0 < V < \pi$ condition for fundamental symmetric mod

we must have $V < \pi$ i.e., $\boxed{d < 3.07\ \mu m}$

So, the waveguide with λ_0, n_1, n_2 and $d = 3.07\ \mu m$, will be a single-mode guide.

Now, if the operating w/L is changed the same w/guide can support more no. of modes. for example, if $\lambda_0 = 0.6\ \mu m$, then $V = 2.5\pi$. $\left[= \frac{\pi \times 1.5}{0.6} = 2.5\pi\right.$

And then we will have 3 modes, i.e., two symm + one antisym.