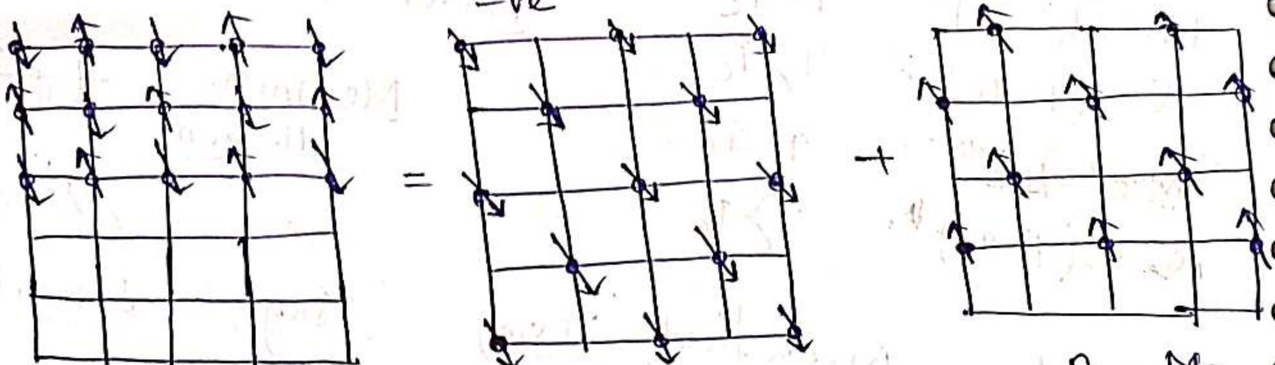
Antiferromagnetism

13/03/25

In 1936 Neel showed theoretically that if exchange integral $J_{ij} = -ve$, there a state of lowest energy is obtained where the spin of neighbouring atoms have opposite orientation. Such materials are known as antiferromagnetic materials.

$$\hat{H}_{spin} = -2 J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

J_{ij} : $-ve$



A M_A B M_B

Two interpenetrating sublattice A & B

nearest neighbour interaction is AFM

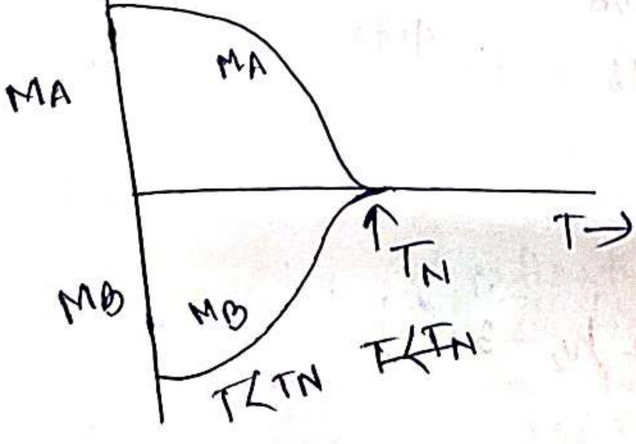
next " " " is FM

$M = M_A + M_B \approx 0$

$|M_A| = |M_B|$

M_A = A-sublattice magnetization

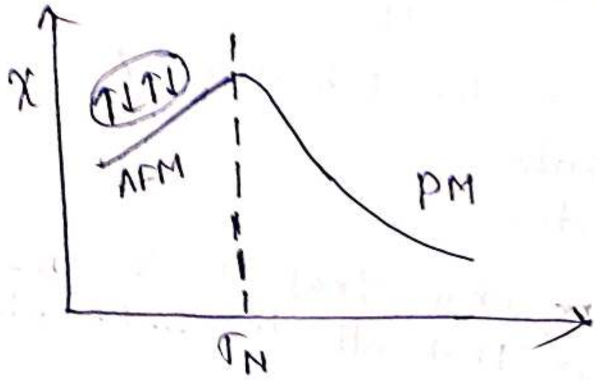
M_B = B-sublattice "

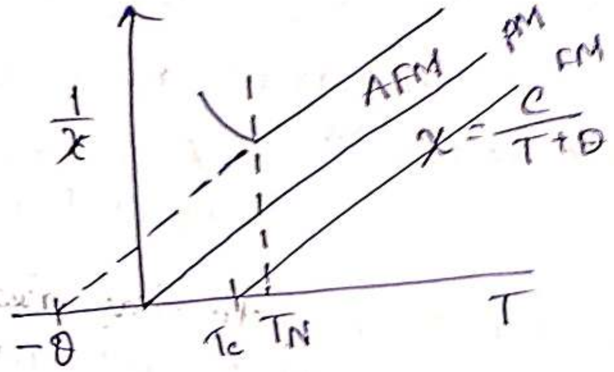


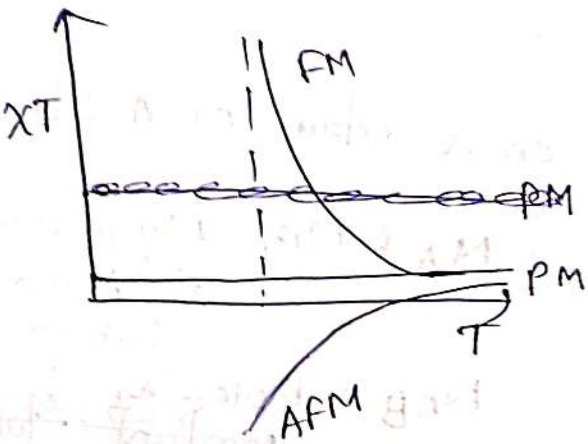
T_N = Neel temperature

$M = M_A + M_B = 0$

$|M_A - M_B|$ = Staggered Magnetization is used as order parameter.







$\chi = \frac{C}{T}$

$\chi = \frac{C}{T-\theta}$

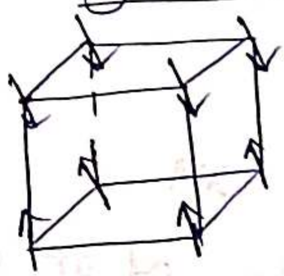
$\chi T - \chi\theta = C$

$\theta = 0$ PM
$\theta > 0$ FM
$\theta < 0$ AFM

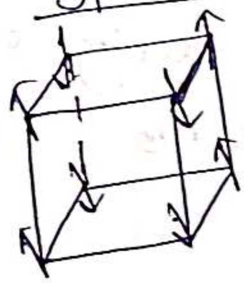
Types of AFM

Cubic Crystal

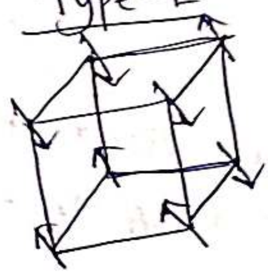
Type A



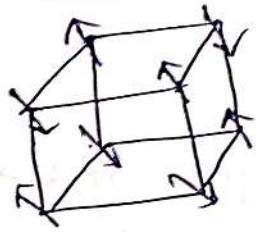
Type-C



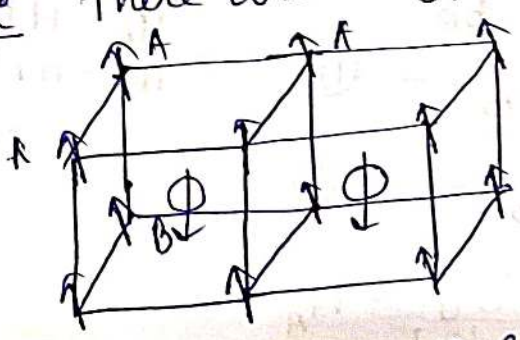
Type-E



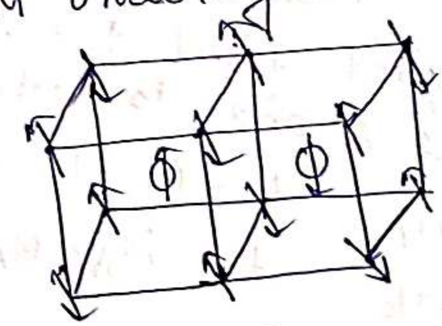
Type-G

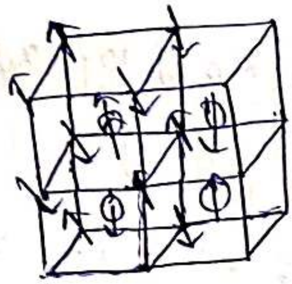


Bcc there are 3-types of AFM ordering



B sublattice.





Wiess molecular field theory for AFM

Consider magnetic materials with two sublattices A & B

Bcc lattice — A → Corner points

B → Bcc position

An atom at A-site has nearest neighbours that all lie on B-sites & next nearest neighbour that all lie on A-sites.

$$H_{M_+} = -\lambda M_-$$

$$H_{M_-} = -\lambda M_+$$

The molecular field H_{mA} acting on a atom at A-site

$$H_{mA} = -N_{AA} M_A - N_{AB} M_B$$

M_A & M_B = Magnetizations of A and B Sublattice

N_{AB} = Molecular field constant for the Nearest Neighbour Interaction.

N_{AA} = " For Next Nearest Neighbour Int.

The molecular field H_{mB} acting on the atom B-site

$$H_{mB} = -N_{BA} M_A - N_{BB} M_B$$

$$N_{AA} = N_{BB} = N_{ii}$$

$$N_{BA} = N_{AB}$$

$H + \lambda M$

If H applied field the, the field H_A and H_B at an atom on the A and B lattice would be

$H_A = H + H_{mA}$

$H_B = H + H_{mB}$

$$H_A = H - N_{ii} M_A - N_{AB} M_B \quad (1)$$

$$H_B = H - N_{AB} M_A - N_{ii} M_B \quad (2)$$

$$M_A = N g \mu_B J B_J(x_A) \qquad x_A = \frac{J g \mu_B}{k_B T} H_A$$

$$M_B = \frac{N}{2} g \mu_B J B_J(x_B) \qquad x_B = \frac{J g \mu_B}{k_B T} H_B$$

$$B_J(x_A) = \frac{2J+1}{2J} \coth \frac{2J+1}{2J} x_A - \frac{1}{2J} \coth\left(\frac{x_A}{2J}\right)$$

Case - 1 : Behaviour above T_N . $T > T_N$

$$B_J(x) \to \frac{3J+1}{3J}x$$

$x \to$ small
$$M_A = \frac{N}{2} g \mu_B J \cdot \frac{J+1}{3J} \cdot \frac{J g \mu_B}{k_B T} H_A$$
$$= \frac{N g^2 \mu_B^2 J(J+1)}{6 k_B T} H_A = \frac{N g^2 \mu_B^2 J(J+1)}{6 k_B T} \cdot \left\{ H - N_{ii} M_A - N_{AB} M_B \right\}$$

$$M_B = \frac{N g^2 \mu_B^2 J(J+1)}{6 k_B T} H_B = \frac{N g^2 \mu_B^2 J(J+1)}{6 k_B T} \cdot \left\{ H - N_{AB} M_A - N_{ii} M_B \right\}$$

$$M = M_A + M_B$$
$$M = \frac{N g^2 \mu_B^2}{6 k_B T} J(J+1) \left[2H - N_{AB}(M_A + M_B) - N_{ii} M \right]$$

$$M \left[1 + (N_{AB} + N_{ii}) \frac{N g^2 \mu_B^2}{6 k_B T} J(J+1) \right] = \frac{N g^2 \mu_B^2}{6 k_B T} J(J+1) \cdot 2H$$

$$\chi = \frac{M}{H} = \frac{\dfrac{N g^2 \mu_B^2 \cdot J(J+1)}{3 k_B T}}{1 + (N_{AB} + N_{ii}) \dfrac{N g^2 \mu_B^2 J(J+1)}{6 k_B T}}$$

$$\chi = \frac{\dfrac{C}{T}}{1 + \dfrac{(N_{AB} + N_{ii})}{2} \cdot C/T}$$

$$\chi = \frac{C/T}{1 + \frac{C}{2T}(N_{AB} + N_{ii})}$$

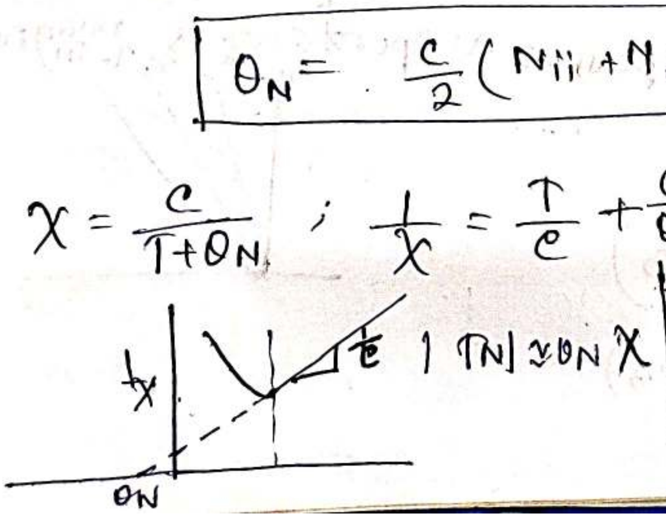
$$\chi = \frac{C}{T + \frac{C}{2}(N_{AB} + N_{ii})}$$

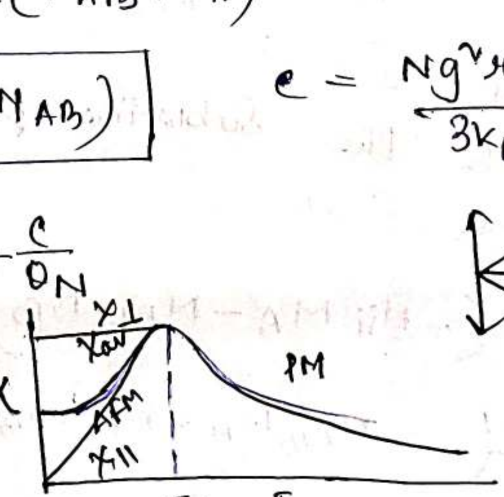
$$\chi = \frac{C}{T + \theta} \to \text{Curie Weiss law for the AFM in the PM.}$$

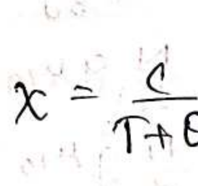
$$\boxed{\theta_N = \frac{C}{2}(N_{ii} + N_{AB})}$$

$$C = \frac{N g^2 \mu_B^2 J(J+1)}{3 k_B}$$

$$\chi = \frac{C}{T + \theta_N} ; \quad \frac{1}{\chi} = \frac{T}{C} + \frac{\theta_N}{C}$$







Scaling law

$2 = \alpha + 2\beta + \gamma$

$\delta = 1 + \left(\frac{\gamma}{\beta}\right)$

$\alpha = 2 + \delta\gamma$

**

(#) Fe $T_c = 1043K$

$\beta = 0.36 \pm 0.02$

$\gamma = 1.33 \pm 0.02$

$\delta = 4.35 \pm 0.02$

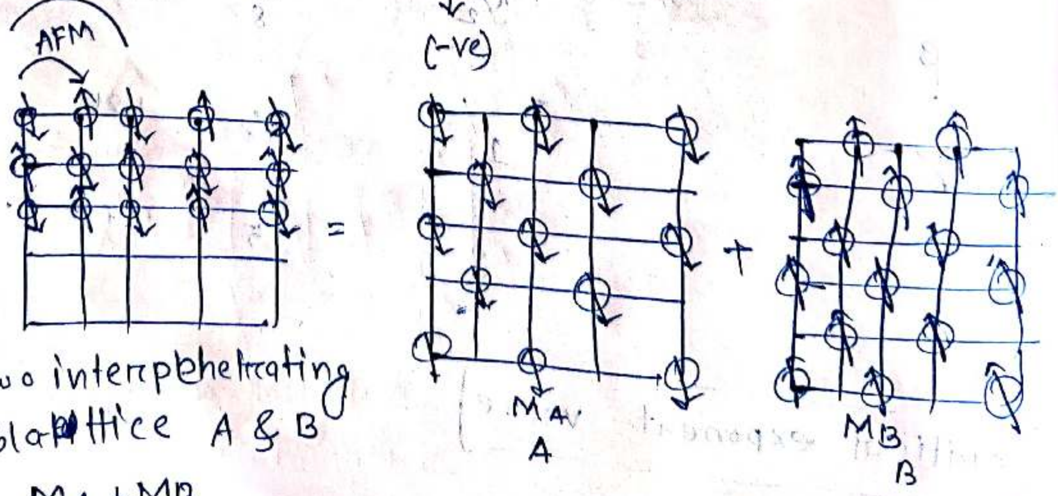
19/03/2025

Antiferromagnetism

In 1936 Neel shown theoritically that if exchange integral $J_{ex} = -ve$, there a state of lowest energy is obtained where the spins of neighbouring atoms have opposite orientation. Such material are known as AFM.

$$H_{spin} = -2J_{in}\, \vec{S_i} \cdot \vec{S_j}$$

J_{in} (-ve)

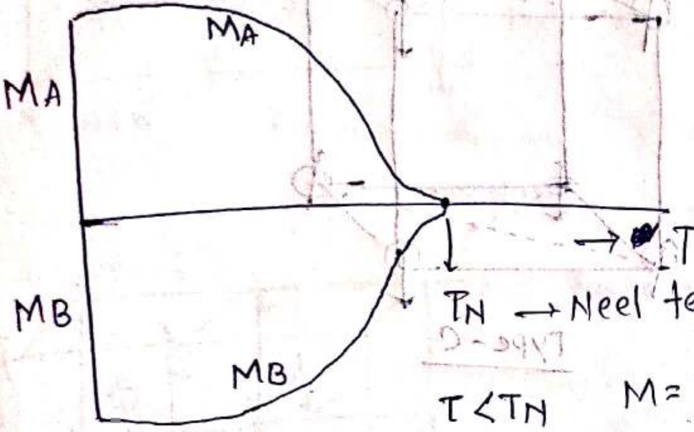


Two interpenetrating sublattice A & B

$M_A + M_B$

nearest neighbour interation is AFM
next " " " FM

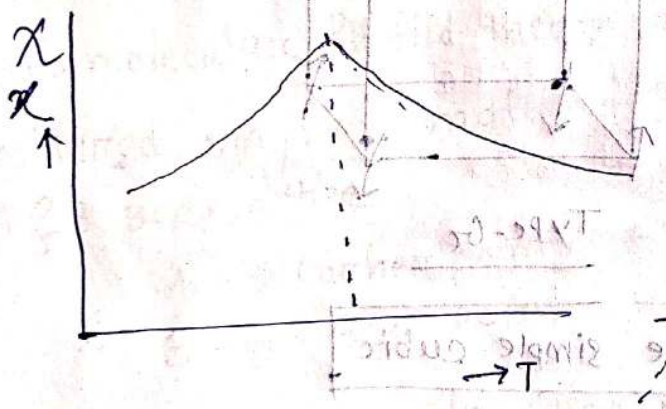
M_A = A-sublattice magnetisation
M_B = B sublattice magnetisation

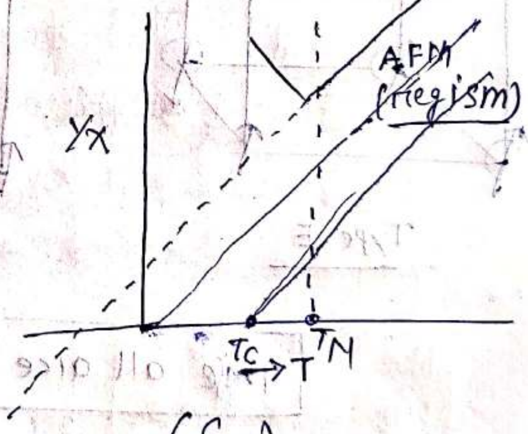


T_N → Neel temperature

$T < T_N$ $M = m_A + m_B = 0$

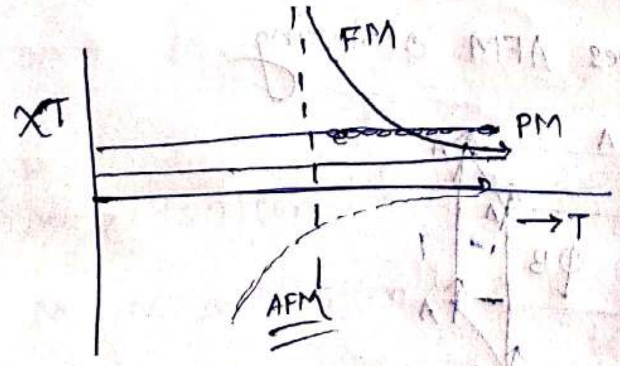
$|M_A - M_B|$ = staggered magnetisation is used as order parameter.





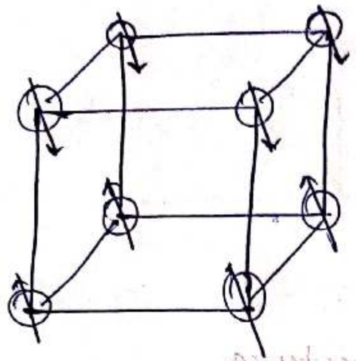
$$\chi = \left(\frac{C}{T+\theta}\right)$$

curie weiss-law

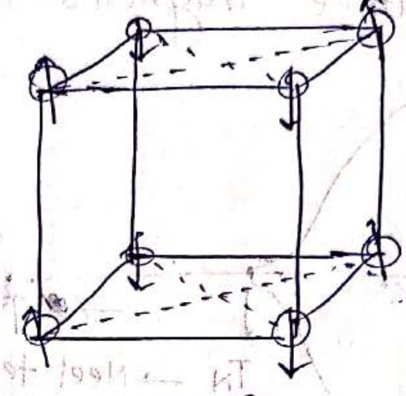


$\theta = 0$ PM
$\theta > 0$ FM
$\theta < 0$ AFM

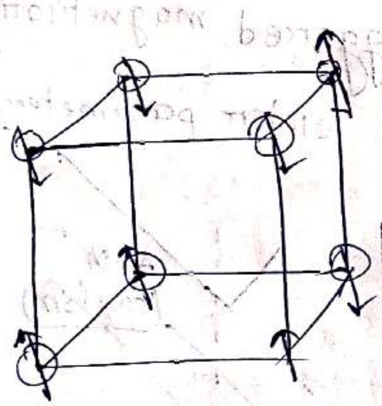
Types of AFM
cubic crystal



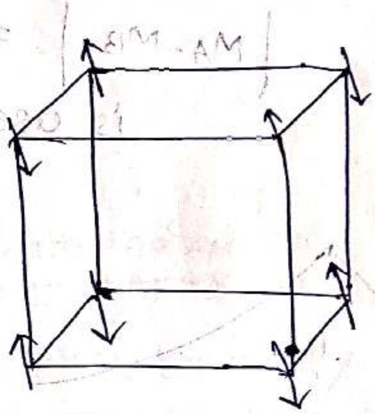
Type-A



Type-C



Type-E

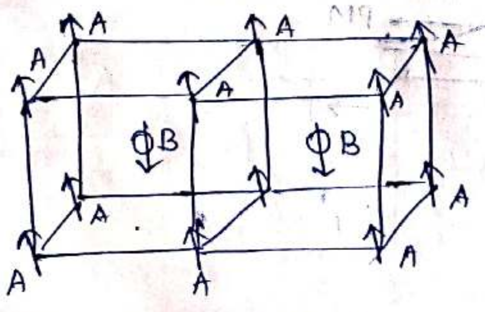


Type-G

$\boxed{\text{These all are simple cubic}}$

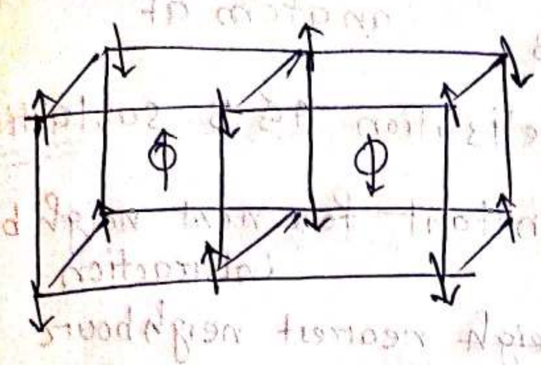
(#) In Bcc Structure

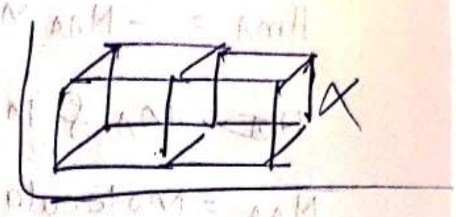
there are 3 types AFM ordering

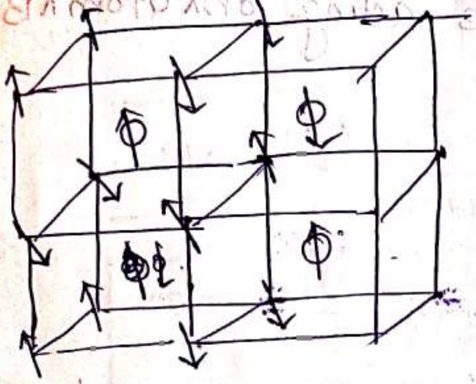


$\downarrow$ B-sublattice

$\uparrow$ A → sublattice







weiss moleculare ~~fil~~ fild theory for AFM

considered magnetic materials ~~with~~ with two sublattice A & B Bcc lattice

A $\longrightarrow$ Corner point

B $\longrightarrow$ Bcc point

An atom ~~has~~ at A-site has nearest neighbour that all lie on B-sites & next neighbour that all lie on A site.

M_+ Magnetisation of A sublattice

M_- Magnetisation of B sublattice.

$H_{M+} = -\lambda M_-$

$M_{M-} = -\lambda M_+$

The molecular field H_{mA} acting on an atom at A site /

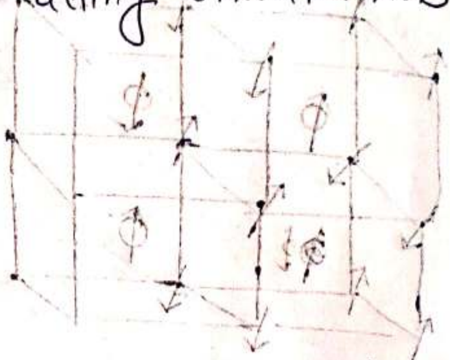
$$H_{mA} = -N_{AA} M_A - N_{AB} M_B$$

~~MAD~~ M_A & M_B = Magnetisation A & B sublattice

N_{AA} = Molecular field constant for next neighbour interaction

N_{AB} = Molecular field constant for ~~neigh~~ nearest neighbour interaction.

Simillarly the molecular field H_{mB} acting on an atom at B-site



$$H_{mB} = -N_{BA} M_A - N_{BB} M_B$$

$$N_{AA} = N_{BB} = N_{ii}$$

$$N_{BA} = N_{AB}$$

If H field is applied the held H_A & H_B at an atom on two A & B lattice would be

$$H_A = H - N_{ii} M_A - N_{AB} M_B \quad \text{(I)}$$

$$H_B = H - N_{AB} M_A - N_{ii} M_B \quad \text{(II)}$$

$$\boxed{\begin{aligned} H_A &= H + H_{mA} \\ H_B &= H + H_{mB} \end{aligned}}$$

\# Magnetisation $(M_A) = \frac{N}{2} g \mu_B J B_J (x_A)$

$$(M_B) = \frac{N}{2} g \mu_B J B_J (x_B)$$

$$x_A = \frac{J g \mu_B}{K_B T} (H_A)$$

$$x_B = \frac{J g \mu_B}{K_B T} H_B$$

$$B_j(x_A) = \cancel{\frac{1}{2j}} \frac{(2j+1)}{2j} \coth \frac{(2j+1)}{2j} x_A - \frac{1}{2j} \coth \frac{1}{2j} x_A$$

<u>case-I</u> Behaviour above T_N , $T > T_N$

$$\boxed{B_j(x) \longrightarrow \frac{(J+1)}{3j}(x)}$$

$x \to$ Become small

$$M_A = \left(\frac{N}{2}\right) g \mu_B (J) \frac{(J+1)}{3J} \left(\frac{g\mu_B}{K_B T}\right) \cdot (H_A)$$

$$\boxed{M_A = \left(\frac{N}{2}\right) \frac{g^2 \mu_B^2 j(J+1)}{6K_B T} (H_A)}$$

$$\boxed{M_B = \frac{N g^2 \mu_B^2 j(J+1)}{6K_B T} H_B}$$

$$M_A = \frac{N g^2 \mu_B^2 j(J+1)}{6K_B T} \left(H - N_{ii} M_A - N_{AB} M_B\right)$$

$$M_B = \frac{N g^2 \mu_B^2 j(J+1)}{6K_B T} \left(H - N_{AB} M_A - N_{ii} M_B\right)$$

Total magnetisation

$$M = (M_A + M_B)$$

$$M = \frac{N g^2 \mu_B^2 j(J+1)}{6K_B T} \left[2H - (N_{ii} + N_{AB}) M\right]$$

$$M \left[1 + \frac{N g^2 \mu_B^2 j(J+1)}{6K_B T} (N_{ii} + N_{AB})\right] = \frac{N g^2 \mu_B^2 j(J+1) \cdot H}{3K_B T} = \left(\frac{C}{T}\right) H$$

$$\chi = \frac{M}{H} = \frac{(C/T)}{1 + \frac{C}{2T}(N_{ii} + N_{AB})}$$

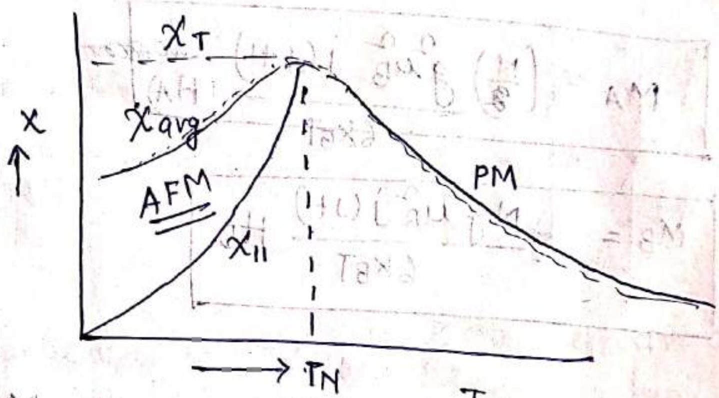
$$\boxed{\chi = \frac{C}{T + \frac{C}{2}(N_{ii} + N_{AB})}}$$

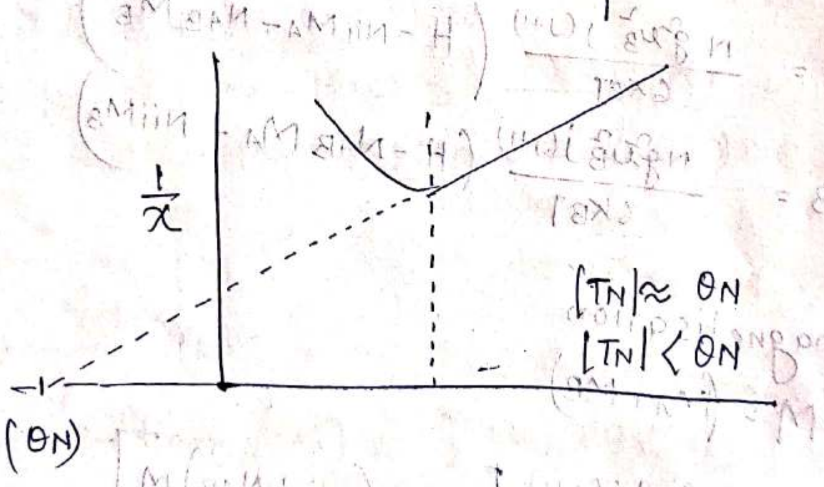
$$\boxed{\chi = \frac{C}{T + \theta}}$$ (curie-weiss for AFM in the paramagnetic regism)

where $\theta = \frac{C}{2}\left(N_{ii} + N_{AB}\right)$

$$= \frac{N g^2 \mu_B^2 J(J+1)}{(3K_B T)\,2}\left[N_{ii} + N_{AB}\right]$$

$$\chi = \left(\frac{C}{T + \theta_N}\right) \Rightarrow \frac{1}{\chi} = \frac{T}{C} + \left(\frac{\theta_N}{C}\right)$$





A sublattice & B-sublattice ~~.~~ $\uparrow\downarrow\uparrow\downarrow$

case-II

$T < T_N$ (doesn't apply ~~an~~ magnetic field)

below T_N both sublattice posses spontinous magnetisation.

$$H = 0$$

$$M_A = \frac{c}{2T}\left(-N_{ii} M_A - N_{AB} M_B\right)$$

$$M_B = \frac{c}{2T}\left(-N_{AB} M_A - N_{ii} M_B\right)$$

$$M_A\left[1 + \frac{c}{2T} N_{ii}\right] + M_B\left(\frac{c}{2T} N_{AB}\right) = 0$$

$$M_A\left(\frac{c}{2T}\right) N_{AB} + M_B\left[1 + \frac{c}{2T} N_{ii}\right] = 0.$$

$$\begin{vmatrix} x & y \\ y & x \end{vmatrix} = 0$$

$$x = 1 + \frac{c}{2T} N_{ii}$$

$$y = \frac{c}{2T} N_{AB}$$

$$x = 1 + \frac{c}{2T} N_{ii}$$

$$y = \frac{c}{2T} N_{AB}$$

$$x^2 - y^2 = 0$$

$$(x+y)(x-y) = 0$$

$$\boxed{x \neq -y} \quad \boxed{x = y}$$

not possible

$$1 + \frac{c}{2T} N_{ii} = \frac{c}{2T} N_{AB}$$

$$1 + \frac{c}{2T_N}(N_{ii} - N_{AB}) = 0$$

$$\frac{c}{2T_N}(N_{AB} - N_{ii}) = 1$$

$T_N = \frac{c}{2}\left(N_{AB} - N_{ii}\right)$

$\theta = \frac{c}{2}\left(N_{ii} + N_{AB}\right)$

$$\frac{T_N}{\theta} = \frac{N_{AB} - N_{ii}}{N_{ii} + N_{AB}}$$

if $N_{ii} = 0$ $\quad \frac{T_N}{\theta} = \frac{N_{AB}}{N_{AB}}$

$T_N = \theta$

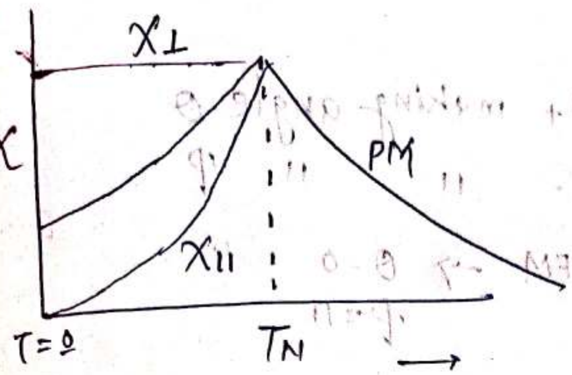
if $N_{ii} \neq 0$ $\quad T_N < \theta$

T_N increase if AFM AB interaction N_{AB} become stronger but decreasing with increasing AA & BB.

Material	$T_N(K)$	$\theta(K)$	J
MnF_2	67	-80	5/2
MnO	116	-510	5/2
CoO	292	-330	3/2
FeO	116	-610	2
Cr_2O_3	307	-485	3/2
Fe_2O_3	950	-2000	5/2

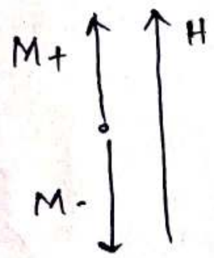
It is seen that $T_N < \theta$

AFM

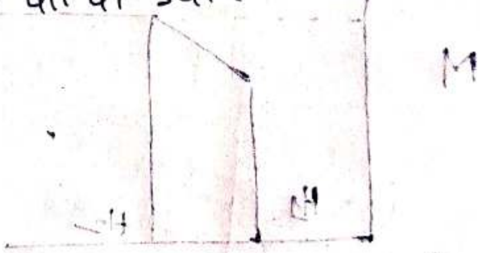


$$\chi_{avg} = \frac{1}{3}\chi_{\parallel} + \frac{2}{3}\chi_\perp$$

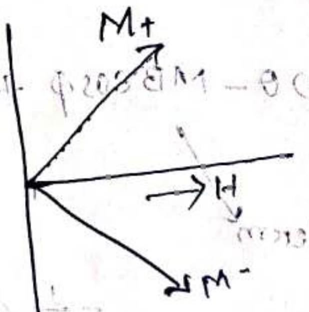
small magnetic field



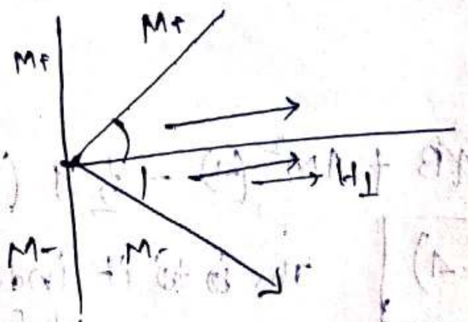
magnetisation for M+ & M- are all at saturated, state.

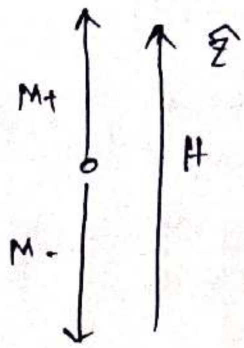


(small magnetic field)



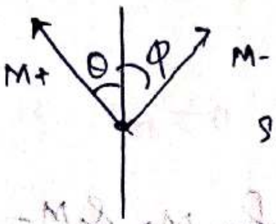
Now for strong magnetic field.



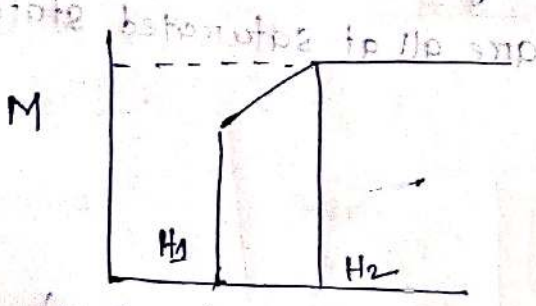


let M+ making angle θ
M- " " φ

AFM $\rightarrow$ $\theta = 0$
$\varphi = \pi$



spin-flop transition
$\theta = \varphi$



Total energy $E = -MB\cos\theta - MB\cos\varphi + \underbrace{AM^2\cos(\theta+\varphi)}_{\text{due to interacti}}$

exchange coupling

Zeman term

$-\frac{1}{2}A(\cos^2\theta + \cos^2\varphi)$

(Anistropy term)

AFM case
$\theta = 0, \varphi = \pi$

$E = -\cancel{M}B + \cancel{M}B + AM^2(-1) - \frac{1}{2}A(1+1)$

$\boxed{E = (-AM^2 - A)}$ The ~~is~~ ~~to~~ it independent of field

$\theta = \varphi$ In spin flop state

$$E = -2MB\cos\theta + AM^2\cos(2\theta) - \frac{1}{2}4\cos^2\theta$$

Minimum energy configuration

$$\frac{dE}{d\theta} = 0$$

$$+2MB\sin\theta + AM^2(-1)\sin 2\theta\,(2) + \frac{1}{2}\,2\cos\theta\sin\theta\,(1) = 0$$

$$\Rightarrow 2MB\sin\theta - 2AM^2\sin 2\theta + 24\sin\theta\cos\theta = 0$$

$$\Rightarrow (2MB - 4AM^2\cos\theta + 24\cos\theta)\sin\theta = 0$$

$$\therefore \sin\theta = 0 \quad \boxed{\theta = 2n\pi}$$

$$2MB + (24 - 4AM^2)\cos\theta = 0$$

$$\boxed{\cos\theta = \frac{2MB}{(4AM^2 - 24)}}$$

$4 << 1$

$$\cos\theta \approx \frac{2MB}{4AM^2}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{B}{2AM}\right)}$$

$$E = -2MB\left(\frac{B}{2AM}\right) + 2AM^2\,\frac{B}{2AM} - AM^2$$

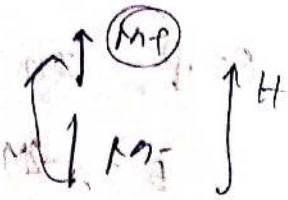
$$E = -2MB\cos\theta + AM^2\,2\sin\theta\cos\theta - 4\cos^2\theta$$

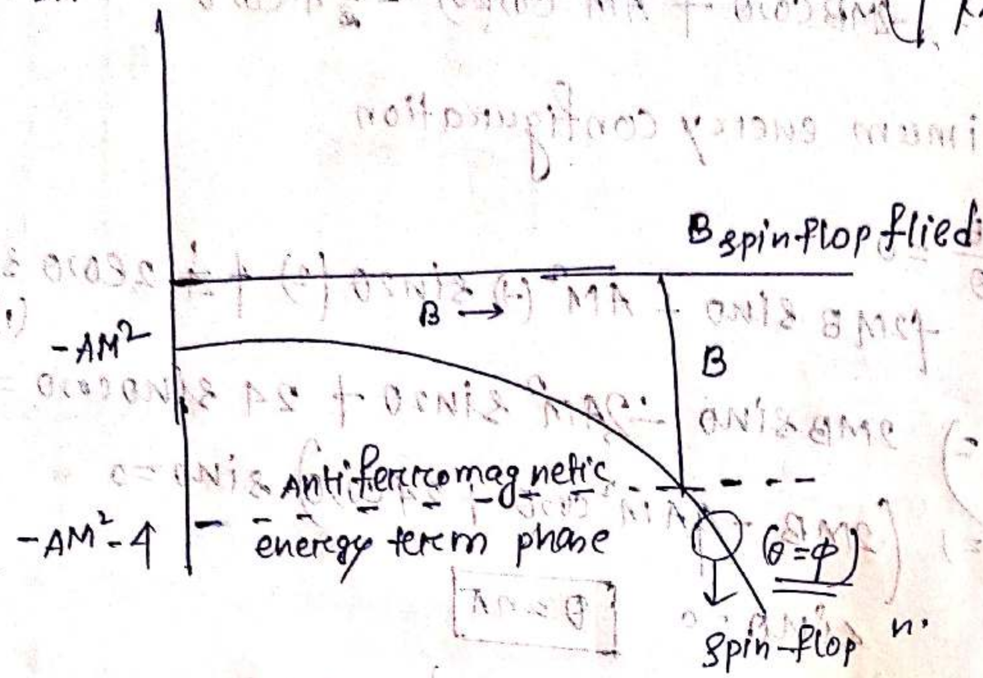
$$= -\cancel{2M}B\left(\frac{B}{\cancel{2AM}}\right) + \cancel{AM^2}\,2\,\frac{B^2}{\cancel{4}A^{\cancel{2}}\cancel{M^2}} - \left(4\cdot\frac{B^2}{4A^2M^2}\right)$$

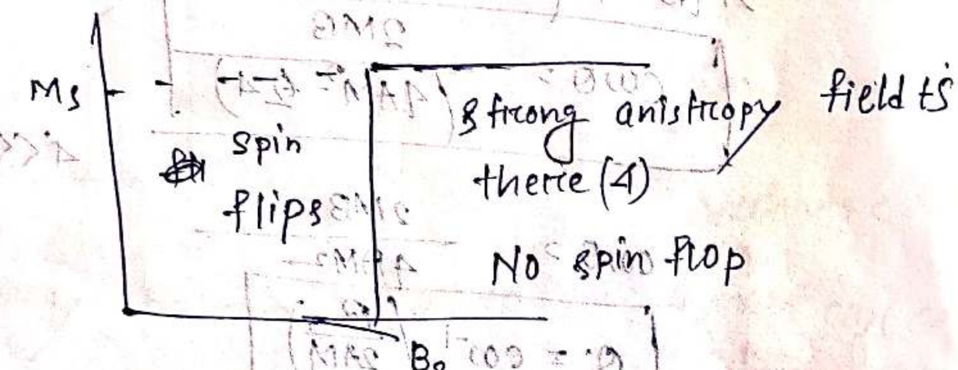
$$\Rightarrow -\frac{B^2}{2A} - \frac{B^2}{2A}$$

$$\Rightarrow -\left(\frac{B^2}{A}\right)$$

$$E = -\frac{B^2}{2A} - AM^2$$







Direct exchange:

$$H_{m} = -2J\, \vec{S_i} \cdot \vec{S_2}$$

FM

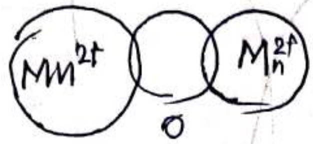
AFM

PM

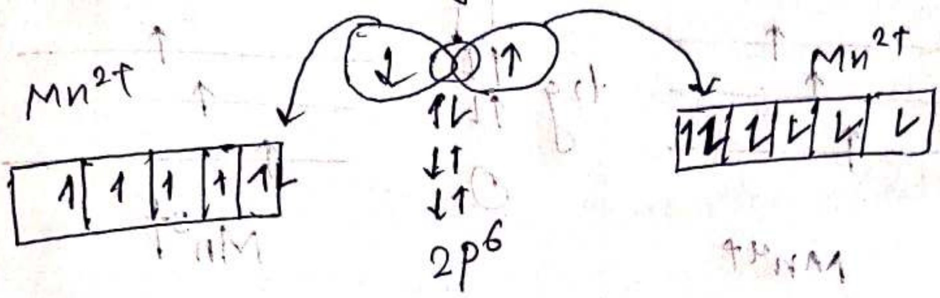
Indirect exchange

- superexchang
- Doeble exchange
- DM interaction
- RKKY interaction

MnO $Mn^{2+} - O^{2-} - Mn^{2+}$

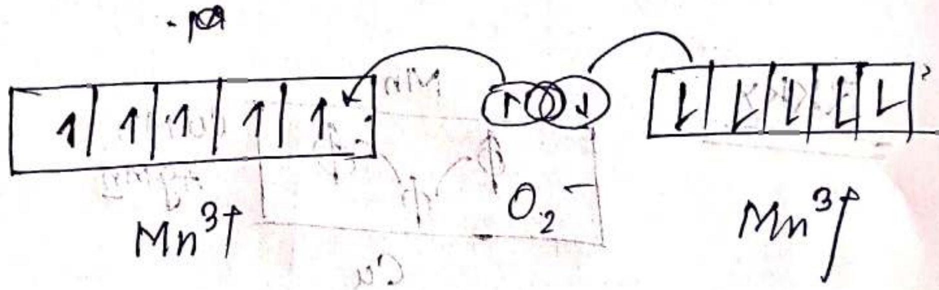


ligand



AFM superexchange

(#) M^{3+}



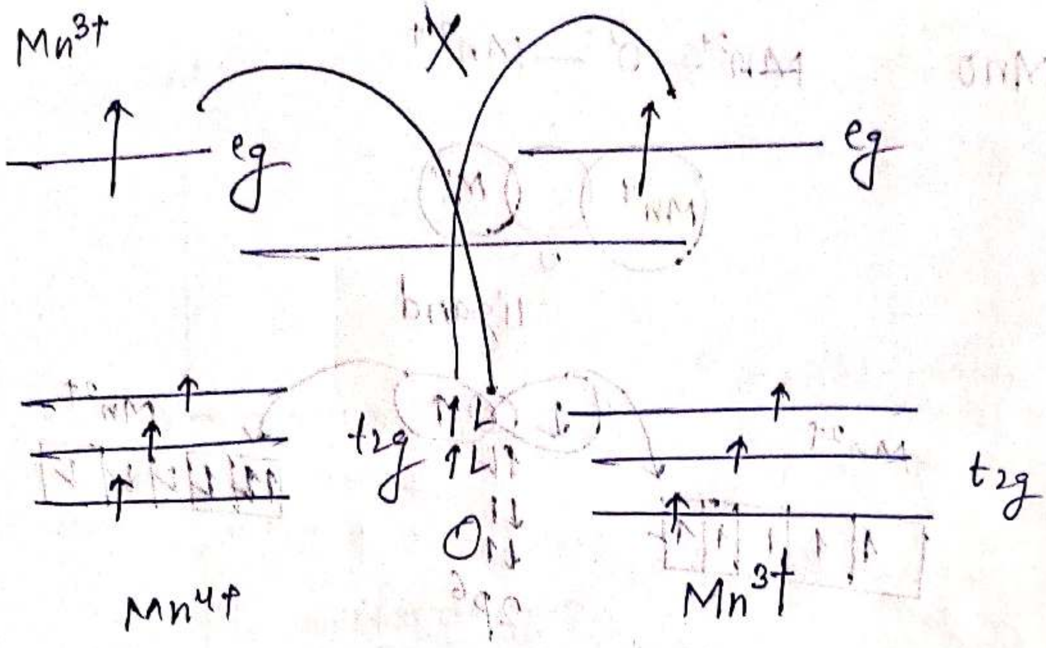
Superchange APM interaction

(#) double exchange interaction

$\overset{3+}{La}\,\overset{3+}{Mn}\,\overset{2-}{O_3}$

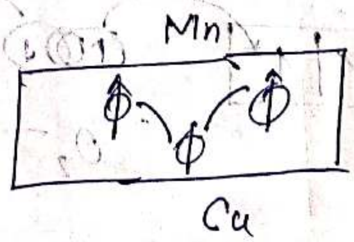
$La^{3+}_{0.7}Sr^{2+}_{0.3}$ Mn^{3+}, Mn^{4+} O_3^{2-}

$La_{0.7}Sr_{0.3}MnO_3$ $Mn^{3+} - O - Mn^{4+}$

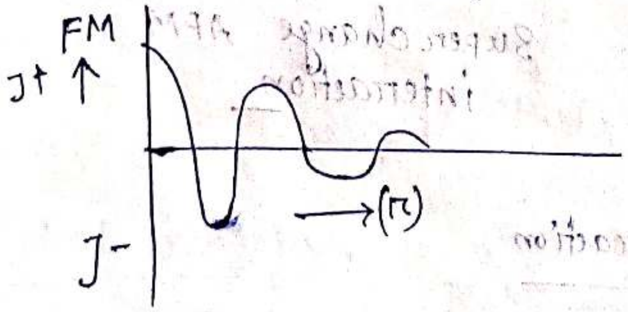


$\boxed{\text{CMR materials DF interaction}}$

RKKY



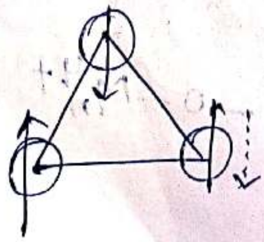
CuMn
AgMn



$$J(r) \propto \frac{\cos(K_F r)}{r^3}$$

$$\boxed{H_{DM} = D\, \vec{S_1} \times \vec{S_2}}$$

DM interaction



spin ~~frasted~~

frusta state

spin glass

~~Superconductivity~~

HRTEM (32)
STEM
FEM (22)

The critical temperature can be found by approaching from the high temp. state $T > T_N$

$$M_A = \frac{c}{2T}\left(H - N_{ii} M_A - N_{AB} M_B\right)$$

$$M_B = \frac{c}{2T}\left(H - N_{AB} M_A - N_{ii} M_B\right)$$

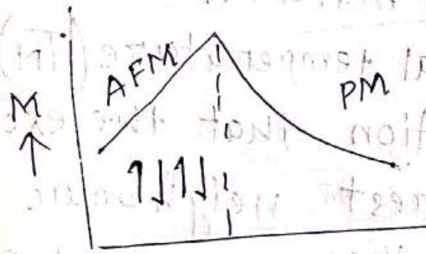
In the visinity of T_N (saturation effects are unimportant.)

$H = 0$

$$M_A = \frac{c}{2T}\left(-N_{ii} M_A - N_{AB} M_B\right)$$

$$M_B = \frac{c}{2T}\left(-N_{AB} M_A - N_{ii} M_B\right)$$

for Non-zero value of M_A & M_B the determinant the position of M_A & M_B must be zero.



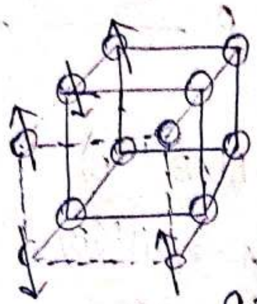
$M_A = 0$
$M_B = 0$

$$T_N = \frac{c}{2}\left(N_{AB} - N_{ii}\right)$$

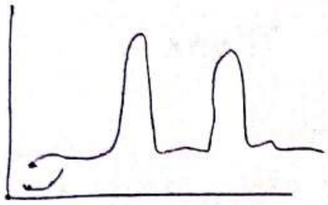
$$\frac{T_N}{\theta} = \frac{N_{AB} - N_{ii}}{N_{AB} + N_{ii}} \qquad \boxed{T_N < \theta}$$

AFM ↑↓↑↓

$T<T_N$



$a = 8.85 Å$

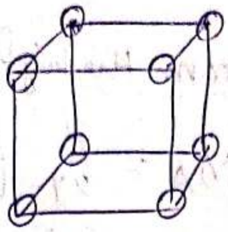


Neutron diffraction

it give lattice constant and also spin structure

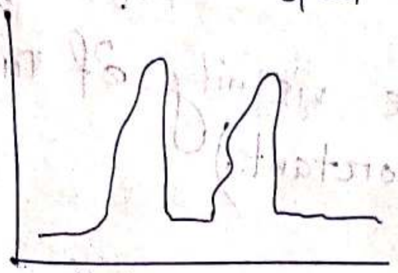
PM ↗↑↑↖

$T>T_N$



unmagnetised Mn

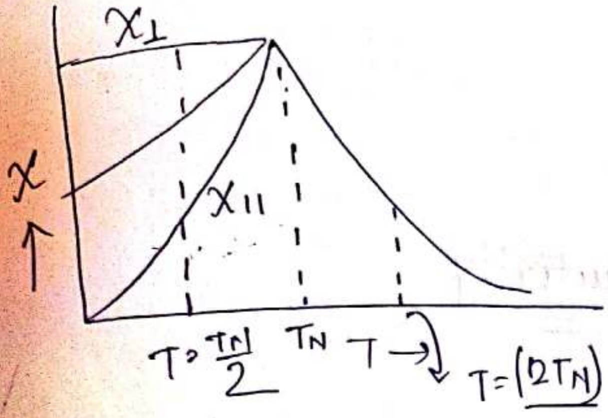
$a = 4.45 Å$



(X-ray diffraction)

problem:- (1) Consider an AFM material which has susceptibility χ_0 at its neel temperature (T_N). Assuming X-ray diffraction that the exchange interaction between nearest neighbour A & B ions much larger than those A-A and B-B pairs calculate the values of susceptibilities which would be measured under the application of field's perpendicular to the magnetisation direction at $T=0$, $T=\left(\frac{T_N}{2}\right)$ & $T=2T_N$

AFM. AFM PM



$$\frac{T_N}{\theta} = \left(\frac{N_{AB}-N_{ii}}{N_{AB}+N_{ii}}\right) \quad (N_{AB} > N_{ii})$$

$$\boxed{T_N = \theta}$$

$$X = \left(\frac{C}{T+\theta}\right)$$

$$= \left(\frac{C}{T+T_N}\right)$$

$$X = \frac{C}{2T_N+T_N} = \left(\frac{C}{3T_N}\right)$$

$$\boxed{X = \frac{C}{3T_N}}$$

$$\cancel{X = \frac{C}{T_N+\frac{T_N}{2}}}$$

$$\cancel{\boxed{X = \frac{2C}{3T_N}}}$$

$$X(T_N) = X_0$$

$$X_0 = \frac{C}{T_N+T_N}$$

$$\boxed{X_0 = \frac{C}{2T_N}}$$

$$C = 2T_N X_0$$

$$X = \frac{2\cancel{T_N}X_0}{3\cancel{T_N}}$$

$$\boxed{X = \frac{2}{3}X_0} \quad (1)$$

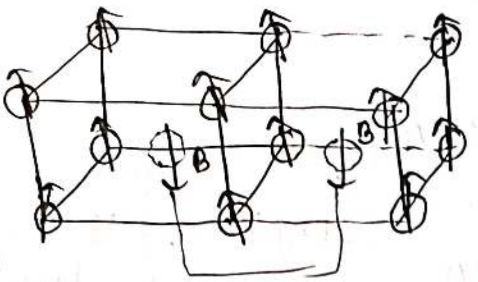
for (~~AFM~~) PM

$$\cancel{X = \frac{2\times 2T_N X_0}{3T_N}}$$

$$\cancel{\boxed{X = \frac{4}{3}X_0}}$$

~~for ~~PM~~ APM~~

20/03/25

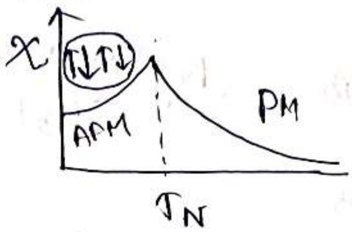


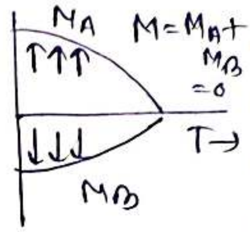
A Sublattice and B Sublattice.

$$M = M_A + M_B \approx 0$$

Stagged Magnetization $= |M_A - M_B|$

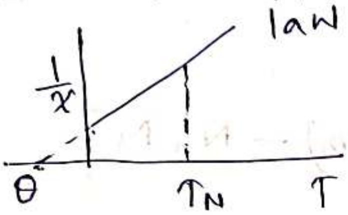
Order parameter





$T > T_N$

$$\chi = \frac{C}{T+\theta} \quad \text{Curie Weiss law}$$



$$H_A = H - N_{ii} M_A - N_{AB} M_B$$
$$H_B = H - N_{AB} M_A - N_{ii} M_B$$

N_{AB} = Internal molecular field co-efficient for n.n. interaction

N_{AA} or N_{BB} or N_{ii} = Internal molecular field co-efficient for n.n.n interaction.

$$M_A = \frac{Ng^2\mu_B^2 J(J+1)}{6K_BT}\{H - N_{ii}M_A - N_{AB}M_B\} \qquad M = M_A + M_B$$

$$M_B = \frac{Ng^2\mu_B^2 J(J+1)}{6K_BT}(H - N_{AB}M_A - N_{ii}M_B) \qquad \chi = \frac{C}{T+\theta_N}$$

$$B_J(x) = \frac{J+1}{3J}x$$

$$M_A = \frac{N}{2} g\mu_B J\, B_J(x_A) \qquad C = \frac{Ng^2\mu_B^2 J(J+1)}{3K_B}$$

$$M_B = \frac{N}{2} g\mu_B J\, B_J(x_B) \qquad \theta_N = \frac{C}{2}(N_{ii} + N_{AB})$$

Case-II : $T < T_N$

Below T_N, both the Sublattice posses a spontaneous magnetization.

$H = 0$

$$M_A = \frac{C}{2T}(-N_{ii}M_A - N_{AB}M_B)$$

$$M_B = \frac{C}{2T}(-N_{AB}M_A - N_{ii}M_B)$$

$$M_A\left[1+\frac{c}{2T_N}N_{ii}\right]+M_B\left(\frac{c}{2T_N}N_{AB}\right)=0$$

$$M_A\left(\frac{c}{2T_N}N_{AB}\right)+M_B\left[1+\frac{c}{2T_N}N_{ii}\right]=0$$

$$\begin{vmatrix} x & y \\ y & x \end{vmatrix}=0 \qquad x=1+\frac{c}{2T_N}N_{ii}, \quad y=\frac{c}{2T_N}N_{AB}$$

$x^2-y^2=0$

$(x-y)(x+y)=0$

$x=y$ valid soln

$x=-y$ ✗ opposite magnetization.

So take negative also as opposite.

$x=y$

$$\Rightarrow 1+\frac{c}{2T_N}N_{ii}=\frac{c}{2T_N}N_{AB}$$

$$\Rightarrow \frac{c}{2T_N}(N_{AB}-N_{ii})=1$$

$$\boxed{T_N=\frac{c}{2}(N_{AB}-N_{ii})}$$

$$\theta=\frac{c}{2}(N_{ii}+N_{AB})$$

$$\frac{T_N}{\theta}=\frac{N_{AB}-N_{ii}}{N_{AB}+N_{ii}}$$

if $N_{ii}=0$, $\boxed{T_N=\theta}$

If consider ~~Nor~~ Next Nearest Neighbour int.

$T_N<\theta$

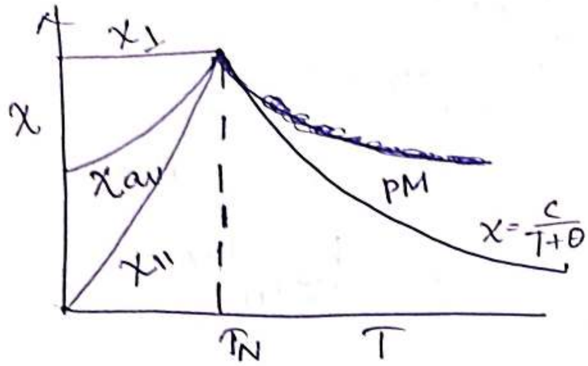
if $N_{ii}>0$

T_N increases if AFM, AB int (N_{AB}) because stronger but decreases with increasing AA or BB.

Material	T_N (K)	θ (K)	J
MnF_2	67	−80	5/2
MnO	116	−510	5/2
CoO	292	−330	3/2
FeO	116	−610	2
Cr_2O_3	307	−485	3/2
α-Fe_2O_3	950	−2000	5/2

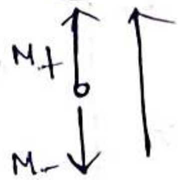
It is seen that $T_N<\theta$

AFM



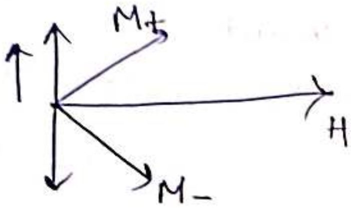
powder sample $\to \chi_{av}$

$$\chi_{avg} = \frac{1}{3}\chi_{\parallel} + \frac{2}{3}\chi_\perp$$



Small Magnetic field

$H_\perp \to$



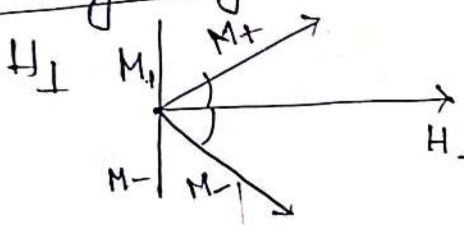
$H_{\parallel} \to$ Magnetisation for M_+ & M_- sublattice all are at saturated state.

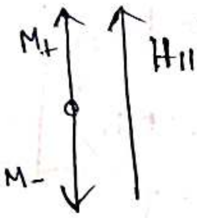
~~$\chi H_\perp$ ~~ ~~ven~~

$\chi_\perp$ remain const till $T = T_N$

Strong Magnetic field

$H_\perp$

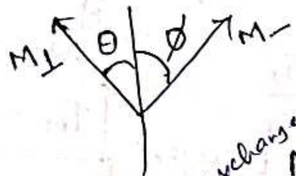


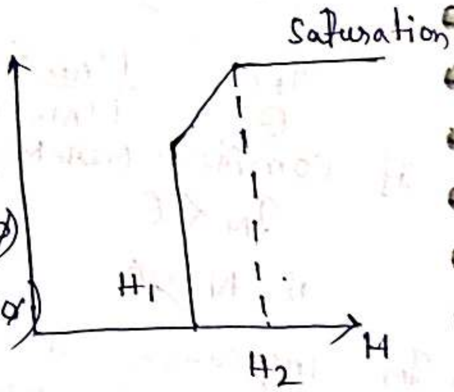


at M_+ making angle θ M_- making angle ϕ

AFM $\to$ $\theta = 0$
$\phi = \pi$

Spin - flop transition $\theta = \phi$

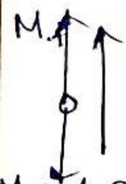




Total Energy:

$$E = -MB\cos\theta - MB\cos\phi + AM^2\cos(\theta+\phi) - \frac{1}{2}\Delta(\cos^2\theta + \cos^2\phi)$$

zeeman, zeeman, exchange, anisotrophy



AFM, $\theta = 0$, $\phi = \pi$

$$E_{AFM} = -MB + MB - AM^2 - \Delta$$
$$= -AM^2 - \Delta$$

This is independent of field.

In the spin flop state $\theta = \phi$

$$E_{AFM} = -2MB\cos\theta + AM^2\cos 2\theta - \Delta\cos^2\theta$$

Minima Energy Configuration $\frac{\partial E}{\partial \theta} = 0$

$$+2MB\sin\theta \mp 2AM^2\sin 2\theta + 2\Delta\cos\theta\sin\theta = 0$$

$\Delta\cos^2\theta$ is small

$$\cancel{\sin\theta(2MB \mp 2AM^2\cos\theta + 2\Delta(\cos\theta)) = 0}$$

$$2MB\sin\theta - 2AM^2\sin 2\theta = 0$$

$$\Rightarrow 2\sin\theta\,(MB - 2AM^2\cos\theta) = 0$$

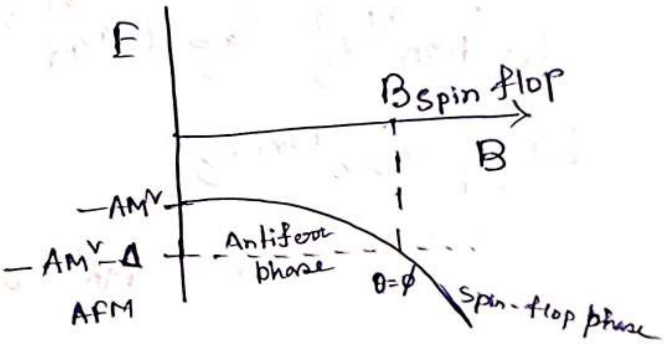
$\sin\theta = 0$, $\theta = 0$ | $B - 2AM\cos\theta = 0$, $\cos\theta = \dfrac{B}{2AM}$, $\theta = \cos^{-1}\left(\dfrac{B}{2AM}\right)$

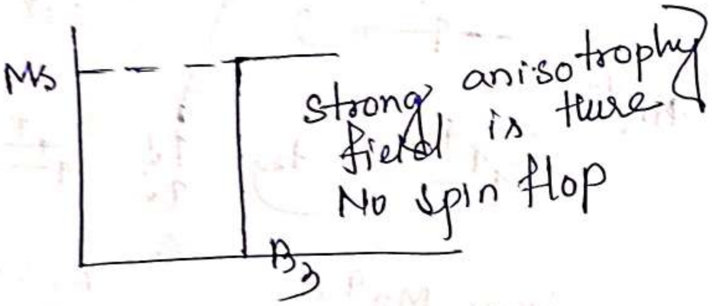
$\sin\theta =$

$$E = -2MB\,\cancel{\cos\theta}\cdot\frac{B}{2AM} + 2AM^2\cdot 2\cancel{\sqrt{1-\frac{B}{2AM}}}\;\frac{2\times B^2}{4A^2M^2} - AM^2$$

$$E = -\frac{B^2}{A} + \frac{B^2}{2A} - AM^2$$

$$E = -\frac{B^2}{2A} - AM^2$$





Direct Exchange:

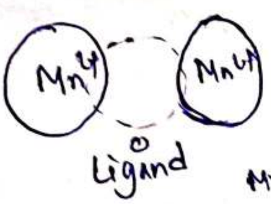
$$H_{ex} = -2J\,\vec{S_i}\cdot\vec{S_j}$$

FM
AFM
Ferri

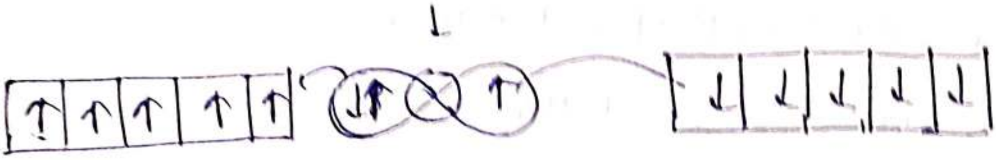
Indirect Exchange

- Superexchange
- Double exchange
- DM inter
- RKKY int

MnO → $Mn^{2+} - O^{2-} - Mn^{2+}$ far apart, No direct exch.

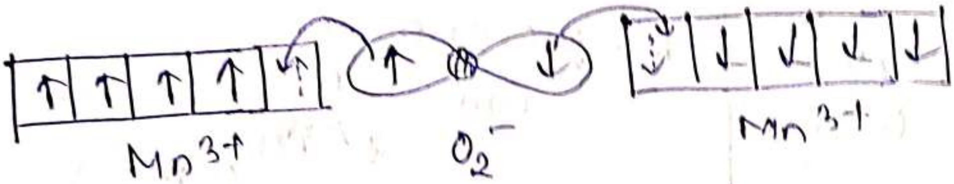


Mn^{2+}: ↑ ↑ ↑ ↑ ↑ $3d^5$ ↓↑ ↓↑ ↓↑ $2p^6$ Mn^{2+}: ↑ ↑ ↑ ↑ ↑ $3d^5$



AFM superexchange.

Mn 3+



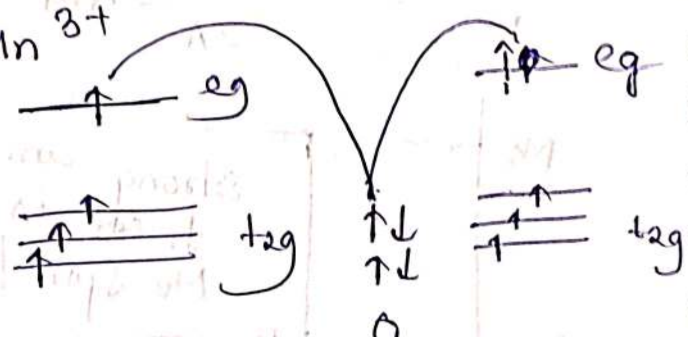
Superexchange
AFM interaction.

Double Exchange :

$La^{3+}Mn^{3+}O_3^{2-}$, $La^{3+}_{0.7}Sr^{2+}_{0.3}Mn^{3+}Mn^{4+}O_3^{2-}$

$La_{0.7}Sr_{0.3}MnO_3$

$Mn^{3+}-O-Mn^{4+}$

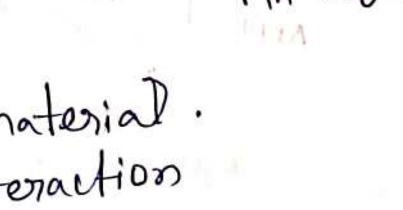


CMR material.
DE interaction

become Mn^{4+}

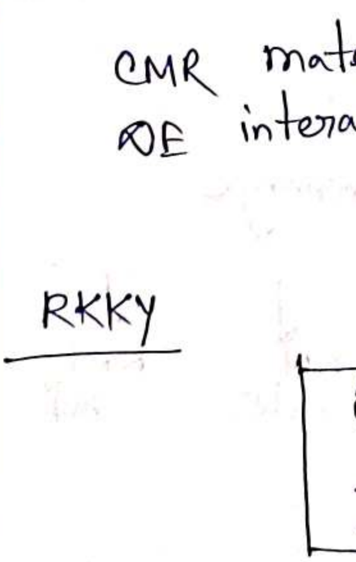
e^- - hoping can take place

RKKY



communicate via conduction electron
cu have lots of conduction e^-.

CuMn, AgMn



$J(r) \propto \frac{\cos k_F r}{r^3}$

21/03/25

The critical temperature can be found by approaching from the high temp side $T > T_N$

$$M_A = \frac{C}{2T}(H - N_{ii} M_A - N_{AB} M_B)$$

$$M_B = \frac{C}{2T}(H - N_{AB} M_A - N_{ii} M_B)$$

In the vicinity of T_N (saturation effects are unimportant)

$H = 0$

$$M_A = \frac{C}{2T}[-N_{ii} M_A - N_{AB} M_B]$$

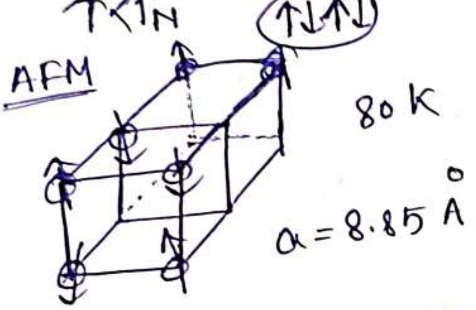
$$M_B = \frac{C}{2T}[-N_{AB} M_A - N_{ii} M_B]$$

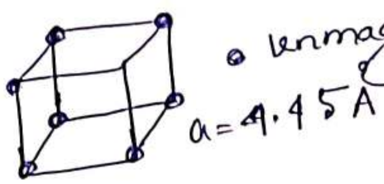
For non zero values of M_A and M_B the determinant of M_A & M_B must be zero.

$T > T_N$

$T < T_N$

AFM





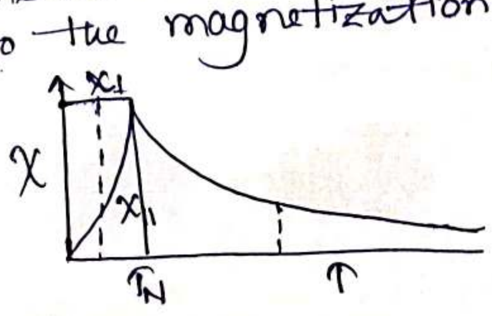
x-ray ~~Neutron~~ diffraction XRD, NPD

Lattice structure.

Neutron diffraction, NPD

Spin structure

Problem → Consider an AFM material which has susceptibility χ_0 at its Neel temp. T_N. Assuming that the exchange interaction between nearest neighbour A & B ions are much larger than those A-A and B-B pairs. Calculate the values of susceptibilities which would be measured under the application of fields perpendicular to the magnetization direction at $T=0$, $T = T_N/2$ and $T = 2T_N$



$N_{AB} \gg N_{ii}$ $\quad \chi = \frac{C}{T+\theta} = \frac{C}{T+T_N}$

$T_N = \theta = C N_{AB}$

at $T = 2T_N$ $\quad \chi = \frac{C}{3T_N}$