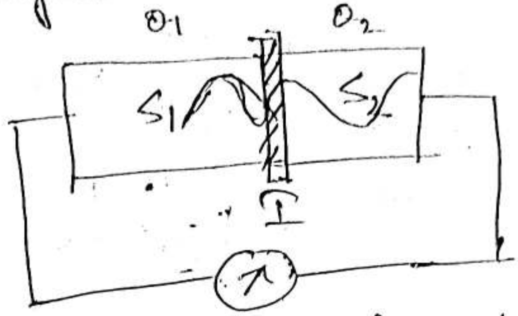
10/4/25

Josephson Junction Tunnelling :-

$J = J_0 \sin(\theta_1 - \theta_2)$



SQUID

$|\Psi\rangle = \alpha|\Psi_1\rangle + \beta|\Psi_2\rangle$

highly coherent state $\Rightarrow$ single quantum state. (macroscopic)

AC J : $J = J_0 \sin\left(\delta_0 - \frac{2eV}{\hbar}t\right)$

$\delta = \delta_0 - \frac{2eVt}{\hbar}$ $\rightarrow$ can change phase by just changing biasing.

+ super current flowing b/w 2 points of the wave function are different as these 2 points are not same.

The Josephson effect are due to the fact that phases can be changed by $\vec{B}$ or V. Josephson effects are the manifestation of quantum interference phenomenon on a macroscopic scale. entire SC is on a single quantum state.

Quantum $\Rightarrow$ Interference $\Rightarrow$ the measured properties depend on the phase of the state which can be tuned by a $\vec{B}$ or V. and states w/ diff. phases can be made to interfere. The interference results in current oscillation which can be detected electrically.

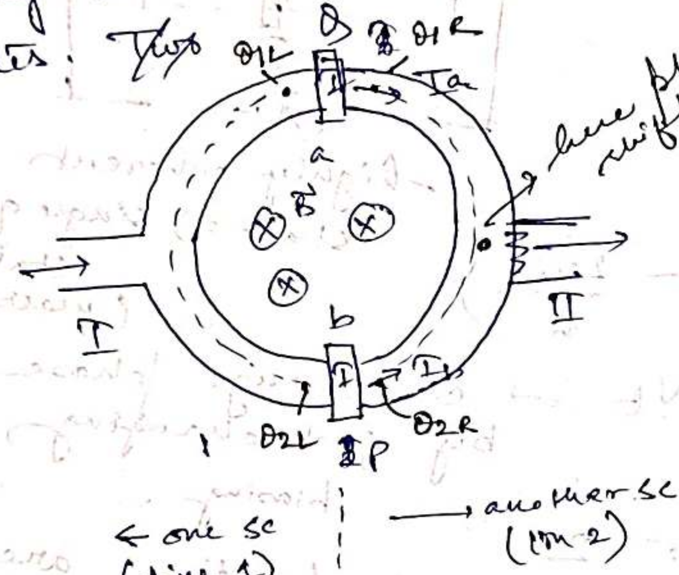
SQUID $\rightarrow$ Superconducting QUantum Interferometry Device

$\Phi_0 = \frac{h}{2e} = 2 \times 10^{-15}$ Wb $(T\text{-}m^2)$

interference pattern $\equiv$ oscillating electric field
classical quantum

Supercurrent Interference

The JJ tunneling in presence of $\vec{B}$ provides strong evidence for highly coherent nature of SC states. Two



$I = I_a + I_b$.

$\delta_a = \delta_b = \delta_0$ for same type of SC and same JJ. (same I, same thickness)

Two Josephson junction are arranged in parallel combn and are placed in a region in which magnetic field $\vec{B}$ is imposed. A supercurrent starting in region I is divided into two parts and get to flow parallel paths, each of which contains tunnel junction.

The current I_a and I_b ~~carrying~~ crossing the tunnel barrier a and b respectively unit in R-II. The combined current shows oscillation characteristics of an interference pattern produced by 2 coherent sources.

By analogy, I_a and I_b are regarded as two sources of current whose disturbances when superposed by the wave of recombination producing interference pattern. In view of Josephson tunneling, tunneling of Cooper pair causes a phase shift to tot wave fn of the superconducting state in R-II relative to R-I.

If phase shift at the two barriers in absence of magnetic field be δ_a and δ_b

$$\delta_a = \theta_{1L} - \theta_{2R}$$

$$\delta_b = \theta_{2L} - \theta_{2R}.$$

Then supercurrent through the 2 junctions ~~Ia~~

$I_a = I_0 \sin \delta_a$ and $I_b = I_0 \sin \delta_b$ — this is without magnetic field.

The phase diff. b/w two regions I. and II in ~~phase~~ presence of magnetic field of vector potential A, $J_s = 0$ and deep inside material → and $\vec{\nabla} \times \vec{A} = \vec{B}$.

$$\vec{\nabla}\theta = \frac{2e}{\hbar}\vec{A}$$

Line integral →

$$\int_P^Q \vec{\nabla}\theta \, dl = \frac{2e}{\hbar}\int_P^Q \vec{A}\cdot\vec{dl}$$

total phase shift →

$$\nabla\theta_I^{II} = \delta_a + \frac{2e}{\hbar}\int_a \vec{A}\cdot\vec{dl}.$$

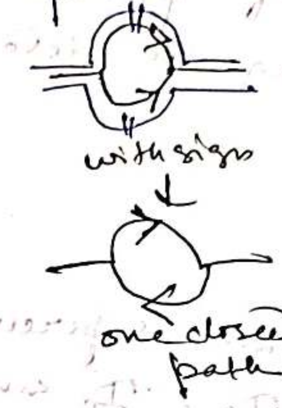
$$\nabla\theta_I^{II} = \delta_b - \frac{2e}{\hbar}\int_b \vec{A}\cdot\vec{dl}.$$

The two phase shifts coming back to same point ~~the~~ must be same as ψ is uniquely valued at each point.

$$\delta_a + \frac{2e}{\hbar}\int_a \vec{A}\cdot\vec{dl} = \delta_b - \frac{2e}{\hbar}\int_b \vec{A}\cdot\vec{dl}$$

$$\Rightarrow \delta_a - \delta_a = \frac{2e}{\hbar}\left[\int_a \vec{A}\cdot\vec{dl} + \int_b \vec{A}\cdot\vec{dl}\right]$$

Taken together with sign ≡ one closed path



$$\delta_b - \delta_a = \frac{2e}{\hbar} \oint \vec{A} \cdot \vec{dl}$$

$$= \frac{2e}{\hbar} \cdot \int_S \underbrace{(\vec{\nabla} \times \vec{A})}_{\vec{B}} \cdot \vec{dS}$$

$$\delta_b - \delta_a = \frac{2e}{\hbar} \Phi \quad \text{(flux enclosed by ring)}$$

The above relation states the total phase difference around loop can be controlled by varying magnetic field $\vec{B}$.

General: $\delta_a = \delta_0 - \frac{e}{\hbar} \int \vec{B} \cdot \vec{ds}$

$\delta_b = \delta_0 + \frac{e}{\hbar} \int \vec{B} \cdot \vec{ds}$

$\delta_a = \delta_b = \delta_0$ when $B = 0$ (initial phase diff)

$\delta_b - \delta_a = 0$ when $B = 0$ (this is when same type of SC used)

$\therefore \delta_b - \delta_a = \frac{2e}{h} \cdot 2\pi \Phi$

$= (2\pi \Phi / \Phi_0)$ = in units of flux quantum

∴ Total current after recombining ⇒ total recombined super-current

$I = I_a + I_b$

$= I_0 \sin \delta_a + I_0 \sin \delta_b$

$= I_0 \sin\left(\delta_0 - \frac{e}{\hbar} \cdot \Phi\right) + I_0 \sin\left(\delta_0 + \frac{e}{\hbar} \Phi\right)$

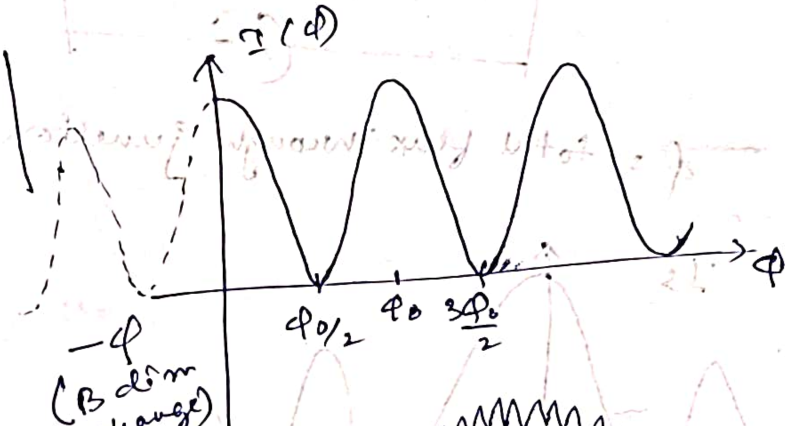
$$\therefore I = I_0 \left(2 \sin \frac{\delta a + \delta b}{2} \cdot \cos \frac{\delta b - \delta a}{2} \right)$$

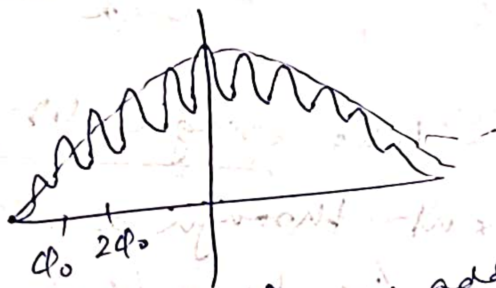
$$= 2I_0 \left[\sin \delta_0 \cdot \cos \frac{2\pi \cdot \phi}{2\phi_0} \right]$$

$$= 2I_0 \sin \delta_0 \cdot \cos(\pi \phi / \phi_0)$$

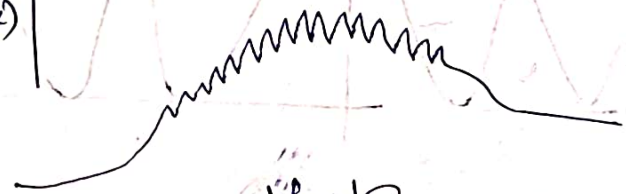
$$\therefore I(\phi) \propto \left| \cos\left(\pi \phi / \phi_0\right) \right|$$

with sin term





$\equiv$



output

as $\phi \to \phi_0, 2\phi_0, \dots$ additional peaks appear

[optics
sin = diffraction term
cos = interference

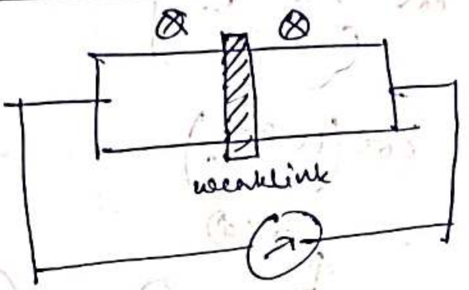
This modulation of observed SQUID ring critical current is shown. The current is essentially interference pattern observed in an ideal fraunhoffer double slit expt. optics with Young's

Here, the two Josephson Junction play the role of two slits and interference is b/w the supercurrent passing through 2 halves of the ring. The supercurrent acquire different phases due to $\vec{B}$.

The SQUID provides a simple but highly accurate system for measuring $\vec{B}$. magnetic flux. Since $\phi \sim 10^{-15}$ T-m^2 one can make device ~ 1 cm^2, we can measure $B \sim 10^{-10}$ T (very small field or moment can be measured = bio fields)

SQUID = higest accuracy magnetometer

Single Josephson Junction



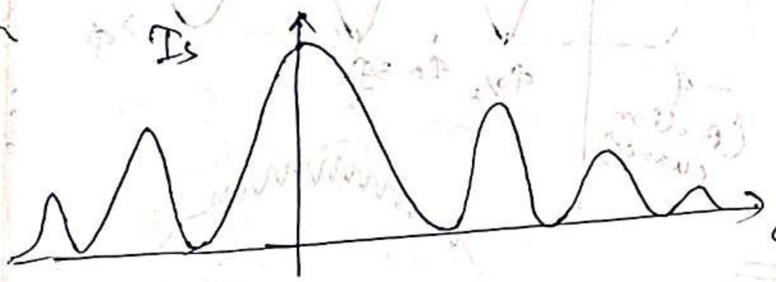
Tunnel current through one junction →

$$I_s = I_0 \frac{\sin(\pi\phi/\phi_0)}{\pi\phi/\phi_0}$$

$= I_0 \,\mathrm{sinc}\,(\pi\phi/\phi_0)$

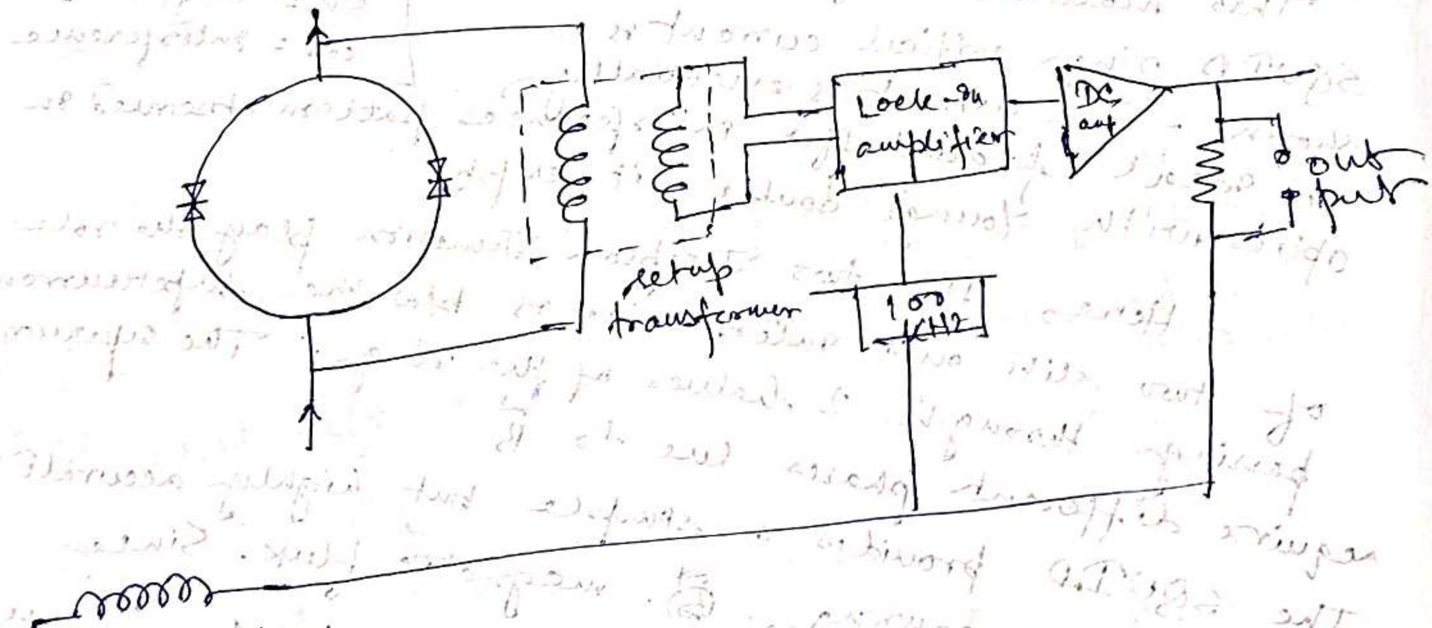
$\mathrm{sinc} \rightarrow \frac{\sin x}{x}$

— ϕ = total flux through junction.



like single slit pattern.

(magnetic field dependant current through a single J-J) (Sn/SnO/Sn)



Feedback.

Simple DC Squid

11/04/25

BCS Theory [Bardeen, Cooper & Schrieffer Theory] (1957)

BCS theory — well accepted for low T_c SC.

① Isotope effect $T_c \propto M^{-1/2}$ ⇒ phonons are involved in superconductivity

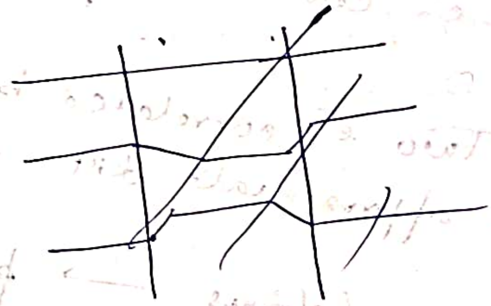
② cooper remodeled frolich's idea on e^- - phonon interaction into the philosophy of an $e^- - e^-$ interaction mediated by phonon — cooper pair.

Cooper pair formation :- Cooper demonstrated that the creation condn favourable for net attractive interactions b/w 2 electrons in a conductor, conductor transforms from normal state to SC state.

Electron-phonon interaction → coulombic repulsion and also

→ $e^- - e^-$ interaction ⇒ $e^- - e^-$ int. (retarded int) mediated by instanteneous

— e^- - lattice - e^- interaction → phonon can give rise to attraction



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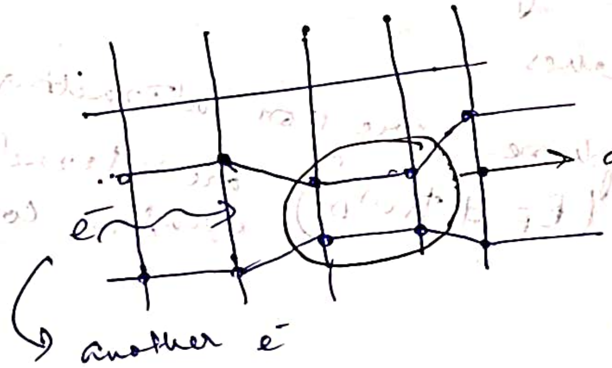


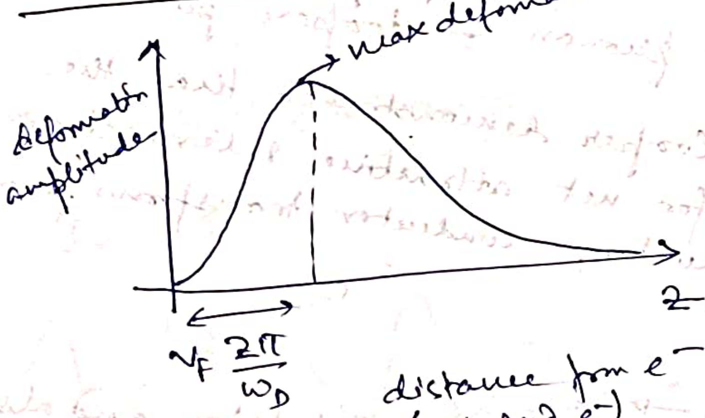
- electron travelling in a lattice, leaves behind a disorted trail which can be regarded as an accumulation of +vely charged ion cores.

- area of enhanced +ve charges compared to neutral crystal is created behind e^- and exerts an attractive force on 2nd e^-

- e^- moves very fast but ions cannot move as fast, so next e^- behind that one feels attractive force.

- at low T, $e^- - e^-$ coulombic repulsion is weaker (due to screening)
e^- - phonon - e^- $\longrightarrow$ stronger ~~effects~~

Instability of fermi sea at cooper pair formation $\rightarrow$



phonon - vibration period $= \frac{2\pi}{\omega_D} \sim 10^{-13}$ s

$v_F = 10^8$ cm/s

$v_F \cdot \frac{2\pi}{\omega_D} \approx 1000$ Å

This is a long distance process. At 1000 Å distance interaction can take place. At this far away distance $e^- - e^-$ coulomb repulsion is weak.

$\Rightarrow$ Two e^- correlated by lattice distortion will have an approximate dist. of 1000 Å.

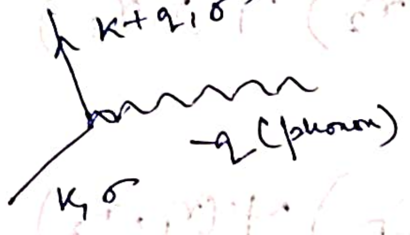
$e^- \longrightarrow$ deforms $\longrightarrow$ phonon created $\longrightarrow$ absorbed by next e^-.

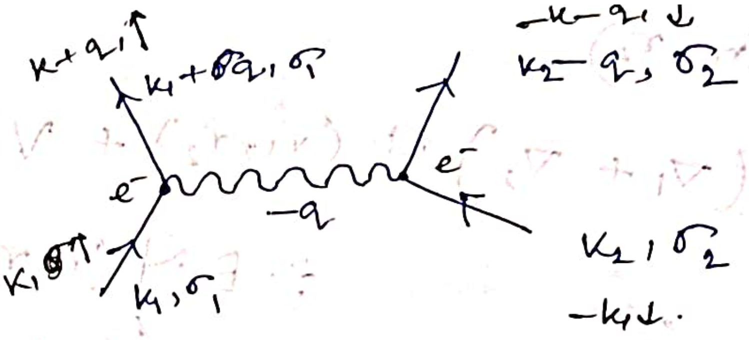
- An e^- travelling through crystal lattice leaves behind the deformation trail which can be regarded as an accumulation of +vely charged cores.

$\Rightarrow$ compression of lattice planes

- e^- well within fermi sphere are non interacting
e^- close to fermi level $(E_F \pm \hbar\omega_D)$ are interacting and can form cooper pair.

e-p vertex





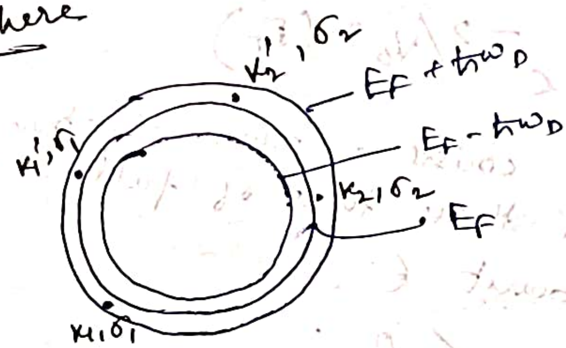
$k \to \uparrow$ $\quad\sigma =$ spin (up / down)

$-k \to \downarrow$

most favourable ~~spin~~ pair = opposite momentum and oppo. spin.

ideal $\begin{cases} k_1 + k_2 = k_1' + k_2' = k = 0 \\ \Rightarrow k_1 = -k. \end{cases}$

Fermi Sphere



$k_1' = k_1 + q$

$k_2' = k_2 - q.$

The e^- act at (k_1, σ_1) and (k_2, σ_2) are scattered to $(k_1 + q, \sigma_1)$ and $(k_2 - q, \sigma_2)$. The int is attractive provided all wave vectors lie in the range b/w $E_k \pm \hbar\omega_D$

$$V_{eff} = |g_q|^2 \cdot \frac{1}{\omega^2 - \omega_q^2}$$

$$g_q = \sqrt{\frac{m}{M}} \sim 0.01$$

and $\omega < \omega_D$.

$$V_{eff}(q, \omega) = |g_{eff}|^2 \cdot \frac{1}{\omega^2 - \omega_D^2}.$$

$$= -V_0.$$

$\psi(r_1, r_2)$

$$-\frac{\hbar^2}{2m}(\nabla_1 + \nabla_2)\psi(r_1, r_2) + V(r_1 r_2)\psi(r_1 r_2)$$

$$= E\,\psi(r_1 r_2)$$

$$= (\epsilon + 2E_{F_0})\psi(r_1, r_2)$$

$$\psi(r_1, r_2) = \frac{1}{\sqrt{L^3}} e^{i\vec{k}\cdot\vec{r}_1 + i\tilde{k}\cdot\vec{r}_2}$$

$$= \frac{1}{\sqrt{L^3}} \cdot e^{i\vec{k}\cdot(\vec{r}_1 + \vec{r}_2)}.$$

$$= \frac{1}{L^3} \cdot e$$

and $\epsilon = -2\hbar\,\omega_D\, e^{-2/V_0 Z(E_F)}$

Then we means there exists a two electron bound state whose energy is lower than that of fully occupied fermi sea by an amount ϵ.

$$\epsilon = E - E_{F_0} < 0$$

The ground state of non. interacting free e^- → becomes unstable when any attractive interaction b/w. electrons is switched on. (well within fermi sphere) close to E_F

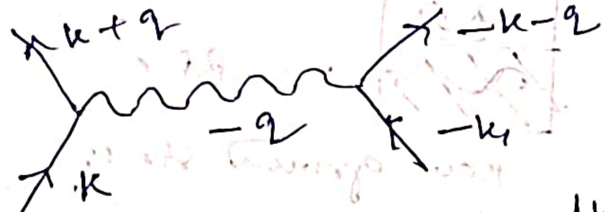
The instability leads to formation of such electron pairs having momentum $(k\uparrow, -k\downarrow)$ — cooper pairs having lower energy

And system tries to reach new lower energy ground state. $(k\uparrow, -k\downarrow)$, $(k'\uparrow, -k'\downarrow)$ and so on.

cooper pairs w/ opposite k vectors and spin

Cooper pair

feynmann diagram →



Coulomb: int is reduced due to the screening (presence of other e⁻ in fermi sphere)

Something new occurs → the two e⁻ may attract e/o

The two e⁻ would then form bound state very close to fermi surface.

The binding energy is strongest when e⁻ forming the pair have opposite momentum and oppo. spins

All e⁻ in neighbourhood of fermi surface condense into a system of many cooper pairs.

The binding b/w e⁻ (1 and 2) — an energy gap appears in the spectrum of e⁻

e⁻ polarizes lattice (k↑) ⟶ creates phonon — another (−k↓)

e⁻ absorbs phonon → result → (k−q ↑, −k+q ↓)

Bose Einstein condensate →

At low temp cooper pairs are formed with the favourable cond. The pair wavefn all have form and the superposn of pair wavelength describes ⟶ BEC.

BCS energy gap relation →

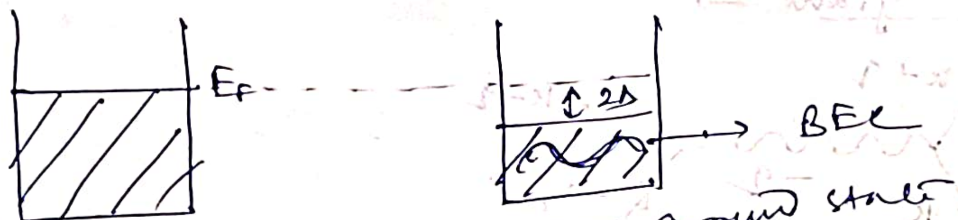
In simples form →

$$K_B T_c = 1.14\,\hbar\omega_D \exp\left[\frac{1}{-V N(E_F)}\right] \quad \text{—(1)}$$

V = effective e⁻ photon int potn

$N(E_F)$ = density of states at E_F

ω_D = debye freq.



normal metal — band gap

new ground state

$$2\Delta(T=0) = 4\hbar\omega_D \exp\left[-\frac{1}{VN(E_F)}\right] \quad (2)$$

$\hbar\omega_D \sim 10^{-2}\,eV - 10^{-4}\,eV$

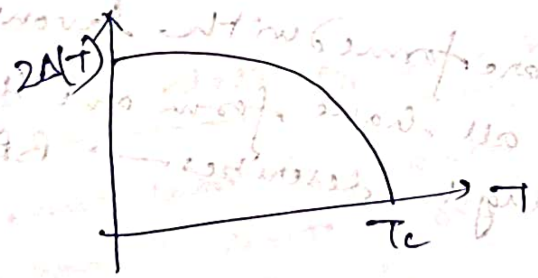
$$\frac{2\Delta(0)}{k_B T_c} = 3.53$$

energy to give to break cooper pair

element	$\frac{2\Delta(0)}{k_B T_c}$
In	4.1
Sn	3.6
Hg	4.6
Pb	4.1

measured values close to typical value.

at $T = T_c$, band gap vanishes.



$$2\Delta(T) = 2\Delta(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

BCS ground state →

$\psi(r, s, r', s')$ — r = electronic position, s = spin.

In a system of N e^- the e^- are grouped into $N/2$ pairs

$$\phi(r_1 s_1, r_2 s_2, \ldots) = \psi(r_1 s_1, r_2 s_2)\,\psi(r_3 s_3\, r_4 s_4)\,\psi(r_5 s_5\, r_6 s_6) \ldots$$

(ground ~~eg~~ state)

$\phi_{BCS} = a\phi$ — a = antisymmetrizer; ϕ_{BCS} have to antisym

$$|\phi_{BCS}\rangle = \prod_k \left(u_k |0\rangle + v_k |1\rangle\right)$$

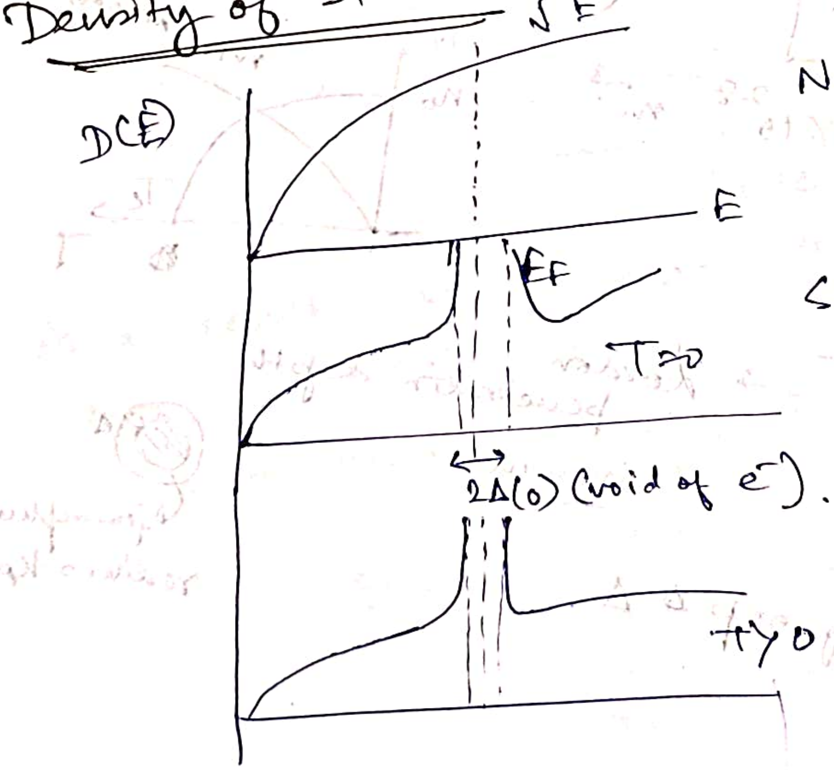
created somewhere

annihilated somewhere

normalisation: $u_k^2 + v_k^2 = 1$.

Book
- Ibach & Luth
- Kittel (Theory of solids)

Density of states



at $T > 0$
$2\Delta(T) < 2\Delta(0)$