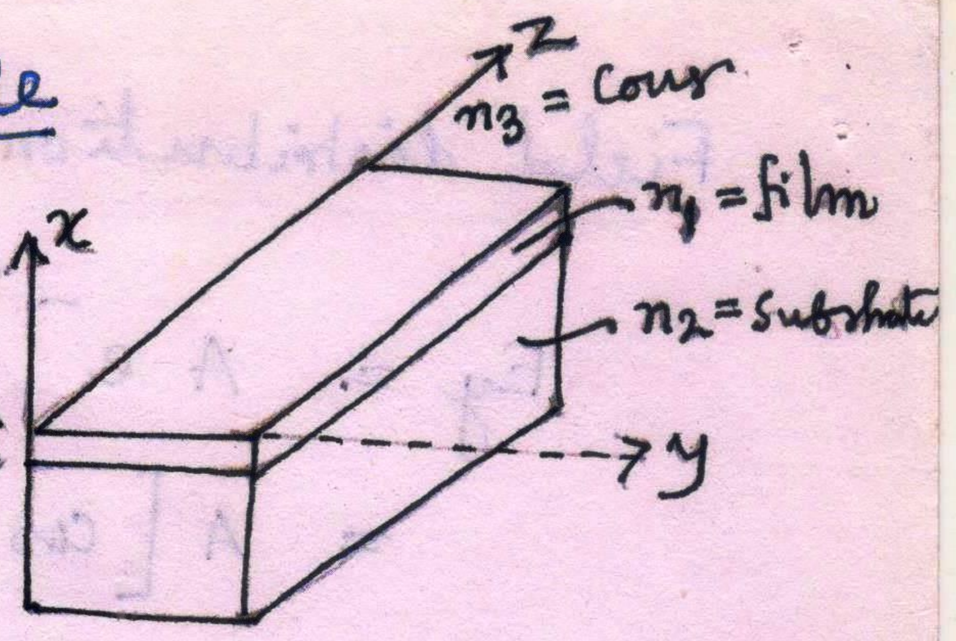
Asymmetric Planar Waveguide

TE - modes



Index profile: $n^2(x) = n_3^2 : x > 0$: cover

$= n_1^2 : -d < x < 0$: film

$= n_2^2 : x < -d$: substrate

$n_1 > n_2 > n_3$

guided mode Condition

$k_0^2 n_2^2 < \beta^2 < k_0^2 n_1^2$

Helmholtz's eqns: TE-modes

In the homogeneous layers —

$$\frac{d^2E_y}{dx^2} - \gamma_3^2 E_y = 0 \quad : \text{cover}$$

$$\frac{d^2E_y}{dx^2} + \chi^2 E_y = 0 \quad : \text{film}$$

$$\frac{d^2E_y}{dx^2} - \gamma_2^2 E_y = 0 \quad : \text{substrate}$$

Soln: $E_y(x) = A e^{-\gamma_3 x} \quad : x > 0$

$= B e^{i\chi x} + C e^{-i\chi x} \quad : 0 > x > -d$

$= D e^{\gamma_2 x} \quad : x < -d$

Continuity:

E_y : $x = 0 : E_y|_I = E_y|_{II} \Rightarrow A = B + C \quad -(1)$

$x = -d : \Rightarrow B e^{-i\chi d} + C e^{i\chi d} = D e^{-\gamma_2 d} \quad -(2)$

$\frac{\partial E_y}{\partial x}$: $x = 0 : -\gamma_3 A = i\chi B - i\chi C \quad -(3)$

$x = -d : i\chi B e^{-i\chi d} - i\chi C e^{i\chi d} = \gamma_2 D e^{-\gamma_2 d} \quad -(4)$

Eliminate A from (1) and (3) $\Rightarrow (\gamma_3 + i\chi) B + (\gamma_3 - i\chi) C = 0$

" D " (2) and (4) $\Rightarrow (\gamma_2 - i\chi) B e^{-i\chi d} + (\gamma_2 - i\chi) e^{i\chi d} = 0$

The last two eqns give

$$\tan(\chi d) = \frac{\gamma_2/\chi + \gamma_3/\chi}{1 - \gamma_2\gamma_3/\chi^2} \quad : \text{TE: modes}$$

Field distribution :

$$E_y = A e^{-\gamma_3 x} \quad : \text{cover}$$

$$= A\left[\cos \kappa x - \left(\frac{\gamma_3}{\kappa}\right) \sin \kappa x\right] \quad : \text{film}$$

$$= A\left[\cos \kappa d + \left(\frac{\gamma_3}{\kappa}\right) \sin \kappa d\right] e^{\gamma_2 (x+d)} \quad : \text{substrate}$$

TM: modes: Asymmetric w/g (above)

Eigen value equ:

$$\tan(\kappa d) = \frac{\frac{n_1^2}{n_2^2}\frac{\gamma_2}{\kappa} + \frac{n_1^2}{n_3^2}\frac{\gamma_3}{\kappa}}{1 - \frac{n_1^4}{n_2^2 n_3^2}\frac{\gamma_2 \gamma_3}{\kappa^2}}$$

Field distribution :

$$H_y(x) = A e^{-\gamma_3 x} \quad : \text{cover}$$

$$= A\left[\cos \kappa x - \left(\frac{n_1^2}{n_3^2}\frac{\gamma_3}{\kappa}\right) \sin \kappa x\right] \quad : \text{film}$$

$$= A\left[\cos \kappa d + \left(\frac{n_1^2}{n_3^2}\frac{\gamma_3}{\kappa}\right) \sin \kappa d\right] e^{\gamma_2 (x+d)} \quad : \text{substrate}$$