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Home assignment on Superconductivity

(Due on 15th May, 2020)

1. Calculate the London penetration depth in Al at 0 K under the assumption (a) that all the valence electrons are responsible for the superconducting property, and (b) that only electrons in an energy interval Δ in the neighborhood of the Fermi energy contribute to the supercurrent. To what fraction of the valence electron concentration does the latter case correspond?

The metals Cu and Pb have diamagnetic susceptibilities χ = -9.63×10^{-6} and -1.58×10^{-5} respectively. Furthermore, Pb is a superconductor with T_C = 7.2 K and B_{c0} = 800 G. Compare the magnetic energies of Cu and Pb in an external field of 500 G at 7.5 K and 0.1 K; assume the long-wire geometry with the field parallel to the wire. What is the principle difference in the free energy of the Pb sample at the two temperatures and in the presence of the field?

12.7 nm
2.2 μm.
3.2×10^{-5}

The BCS gap of Pb at 4 K is about 2 meV. Calculate the frequency of the tunnel current in Pb Josephson junction when it is biased with voltage equal to one half of the gap potential. Calculate the gap energy at 4.2 K.

A Josephson junction of rectangular cross-section and of thickness d is placed in the region of a magnetic field B_0, applied normal to an edge of width b. If the phase difference between the two superconductors be $\pi/2$ when $B_0 = 0$, prove that the dc current in the presence of the field is given by $I = I_0 \frac{\sin(bdB_0e/\hbar)}{bdB_0e/\hbar}$.

5. Estimate the drop in temperature of a superconducting specimen, initially at T = αT_C (α<1), in which the superconducting state is destroyed by the application of magnetic field B > B_C(T).
6. Show that the wavelength of a photon required to destroy a Cooper pair is in the microwave region.
7. Consider a superconducting plate perpendicular to the x-axis and of thickness d placed in a region in an external magnetic field $\vec{B}_0 = B_0\vec{e}_z$. If the penetration of field inside the plate is described by $\nabla^2 B(x) = \frac{1}{\lambda^2} B(x)$, show that the magnetic field at any point x inside the plate can be expressed as $B(x) = B_0 \frac{\cosh(x/\lambda)}{\cosh(d/2\lambda)}$, where the center of the plate is at x = 0 and λ denotes the penetration depth. Show that the effective magnetization M(x) in the superconducting slab of thickness d << λ, is given by $\mu_0 M(x) = B(x) - B_0 = \frac{B_0}{8\lambda^2}(4x^2 - d^2)$.
8. Find the critical field required for the onset of a vortex region in the Type-II superconducting material. You may assume that (a) the magnitude of the order parameter drops rapidly to zero in the core of the vortex, which contains a thin filament of magnetic flux and (b) the induction field decays exponentially with distance away from the core, as described by the second London equation.

Assignment – Superconductivity

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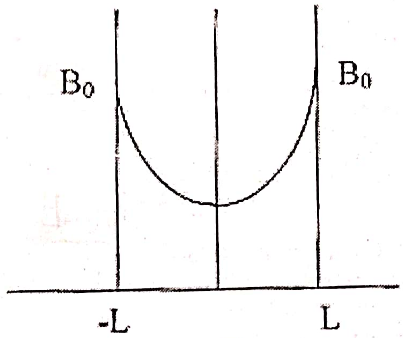
Due date 15th April. 2025

(1) Estimate the drop in temperature of a superconducting specimen per unit volume, initially at $T = \alpha\, T_c$, $(\alpha < 1)$, in which the superconducting state is destroyed by the application of field $[B > B_c(T)]$ showing first the change in entropy per unit volume between normal state and superconducting state at $T = \alpha T_c$ is

$$\left[S_S - S_N\right]_{T=\alpha T_C} = \frac{1}{\mu_0}\left[B_C \frac{dB_C}{dT}\right]_{T=\alpha T_C} = 2\alpha(1-\alpha^2)\frac{B_C^{\,2}(0)}{\mu_0 T_C}$$

The drop in temperature from $\alpha T_C \Rightarrow T_f$ must be calculated considering only the electronic specific heat $(C_V = \gamma\, T)$ at low temperature.

(2) Consider a thin superconducting slab, of thickness $2L \ll \lambda_L$, as shown below:



If an external parallel magnetic field, B_0 is applied parallel to the slab surface and λ is the London's penetration depth, show that inside the slab the magnetic field becomes

$$B_Z(x) = B_0 \frac{\cosh(x/\lambda)}{\cosh(L/\lambda)}$$

(3) Show also the magnetization M(x) inside the superconducting slab in an external magnetic field $\vec{B} = B_0\, \vec{e}_z$ is $M(x) = (B_0 / 8\,\mu_0\, \lambda_L^2)\,[4x^2 - (2L)^2]$, considering $x/\lambda_L \ll 1$.

4. A vortex in a superconductor can be modeled as having a cylindrical core of normal metal of radius ξ_0. Use $\nabla\times(\nabla\times \mathbf{B}) = -\,\mathbf{B}/\lambda^2$ and assuming $dB_Z/dr = a/r$, show that the super current corresponding to the field $B_Z(r)$ is equal to

$j_s(r) = -\dfrac{a}{\mu_0\, r} e_\varphi$, similar to the superfluid current in a ^{4}He vortex . a is a constant.

Hence find the vector potential A and find the constant a as a function of the magnetic flux enclosed by the vortex core, ϕ.

(5) Show that the critical field required for the onset of a vortex region in the superconducting material is

$$B_c = \frac{\phi_0}{2\pi\lambda_L^2} = \frac{\hbar}{2e\lambda_L^2}, \qquad \text{where } \phi_o \text{ is the flux quantum..}$$

Assume that the magnitude of the order parameter drops rapidly to zero in the core of the vortex, which contains a thin filament of magnetic flux and the induction field decays exponentially as $B(r) = B_0 e^{-r/\lambda_L}$. (Cylindrical symmetry of the vortex). The required critical field can be calculated by considering minimum flux enclosed in the vortex region is the one unit quantum of flux ϕ_o.

(6)

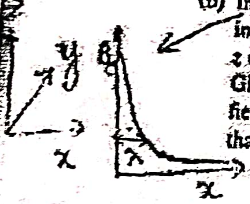
(6) (a) Using the London equation show that

$$\nabla \times (\nabla \times \mathbf{B}) = -\frac{1}{\lambda^2}\mathbf{B}$$

in a superconductor.

(b) In Fig. A, the surface of the superconductor lies in the y–z plane. A magnetic field is applied in the z direction parallel to the surface, $\mathbf{B} = (0, 0, B_0)$. Given that inside the superconductor the magnetic field is a function of x only, $\mathbf{B} = (0, 0, B_z(x))$ show that

$$\frac{d^2B_z(x)}{dx^2} = \frac{1}{\lambda^2}B_z(x).$$



FigA

(c) Solving the ordinary differential equation in (b) show that the magnetic field near a surface of a superconductor has the form

$$B = B_0 \exp(-x/\lambda)$$

as shown in Fig. A.

(7) (7) Consider a thin superconducting slab, of thickness $2L$, as shown in Fig. B. If an external parallel magnetic field, B_0, is applied parallel to the slab surfaces, show that inside the slab the magnetic field becomes

8(a)

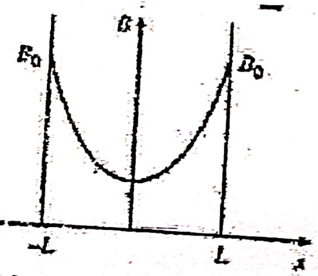
$$B_z(x) = B_0 \frac{\cosh(x/\lambda)}{\cosh(L/\lambda)}.$$

(8) (8) (a) A vortex in a superconductor can be modeled as having a cylindrical core of normal metal of radius ξ. Use $\nabla \times (\nabla \times \mathbf{B}) = -\mathbf{B}/\lambda^2$ and the expression

8(a)

8(a)

Fig.B



B

Fig. B Exercise 3.2: the magnetic field inside a superconducting slab of thickness $2L$.

for curl in cylindrical polar coordinates (Eq. 2.43) to show that the magnetic field $B_z(r)$ outside of the core obeys the Bessel equation:

8(b)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dB_z}{dr}\right) = \frac{B_z}{\lambda^2}.$$

(b) Show that the current corresponding to the field $B_z(r)$ found in (b) is equal to

$$\mathbf{J} = -\frac{a}{\mu_0 r}\mathbf{e}_\phi$$

similar to the superfluid current in a ^{4}He vortex. Hence find the vector potential $\mathbf{A}$ and find a as a function of the magnetic flux enclosed by the vortex core, Φ.