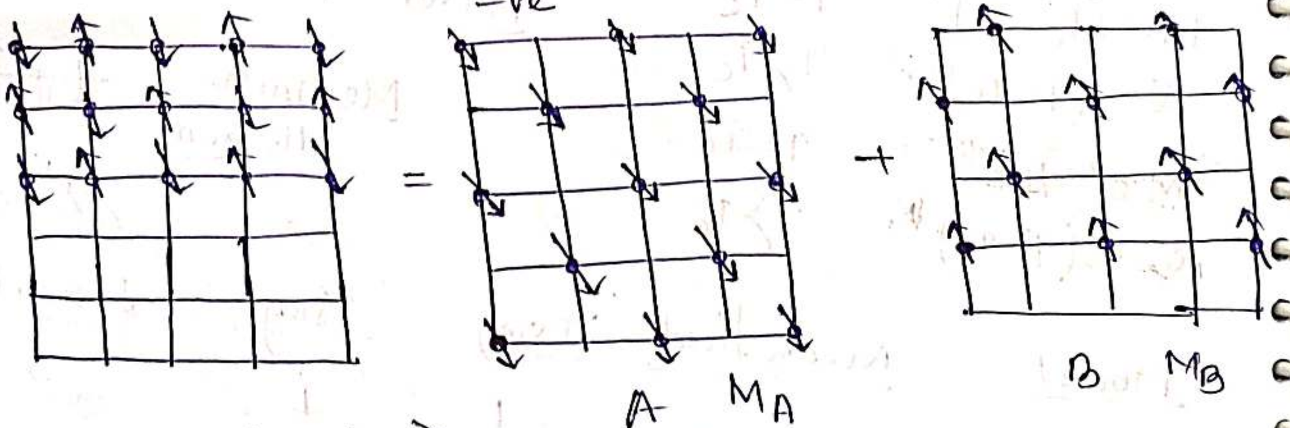
Antiferromagnetism

13/03/25

In 1936 Neel showed theoretically that if exchange integral $J_{ij} = -ve$, there a state of lowest energy is obtained where the spin of neighbouring atoms have opposite orientation. Such materials are known as anti-ferromagnetic materials.

$$\hat{H}_{spin} = -2 J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$-ve$



Two interpenetrating sublattice A & B

$M = M_A + M_B$

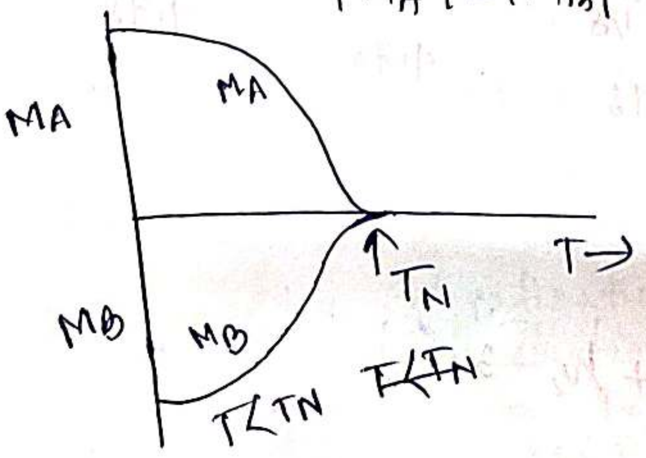
≈ 0 $\quad |M_A| = |M_B|$

nearest neighbour interaction is AFM

next " " " is FM

M_A = A-sublattice magnetization

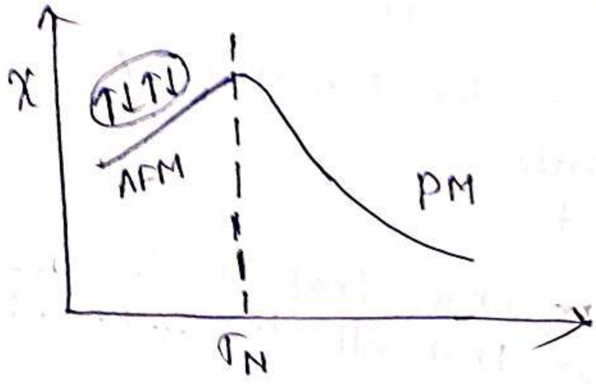
M_B = B-sublattice "

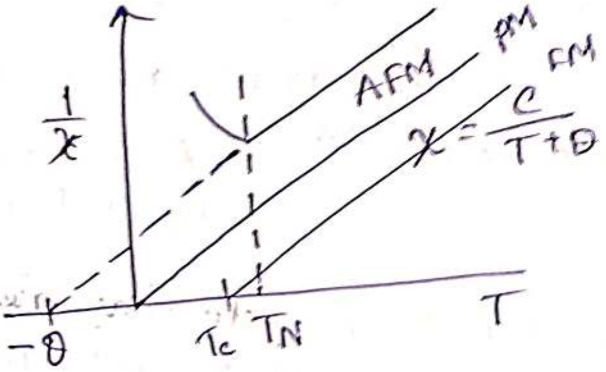


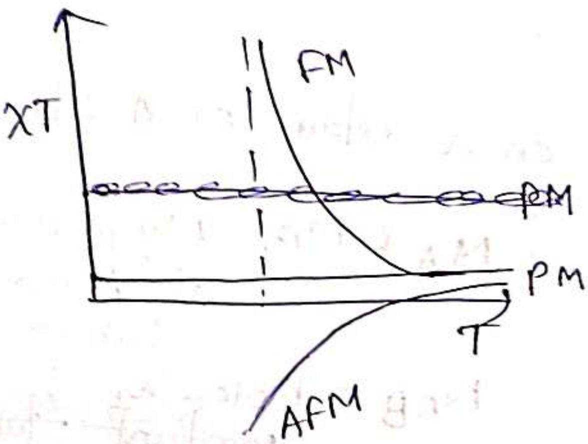
T_N = Neel temperature

$M = M_A + M_B = 0$

$|M_A - M_B|$ = Staggered Magnetisation is used as order parameter.







$\chi = \frac{C}{T}$

$\chi = \frac{C}{T-\theta}$

$\chi T - \chi\theta = C$

$\theta = 0$ PM

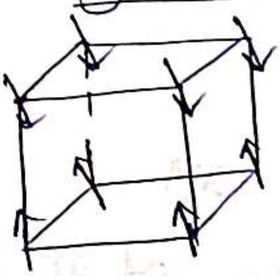
$\theta > 0$ FM

$\theta < 0$ AFM

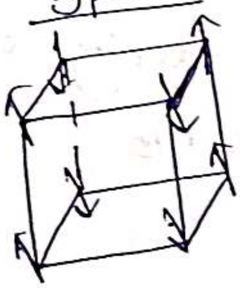
Types of AFM

Cubic Crystal

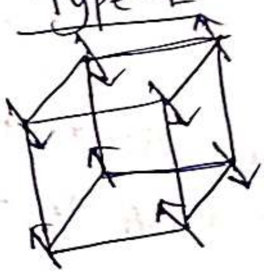
Type A



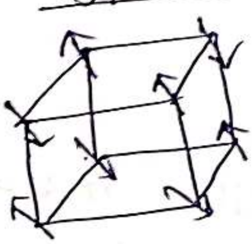
Type-C



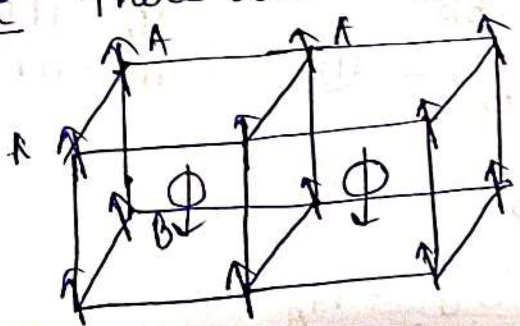
Type-E

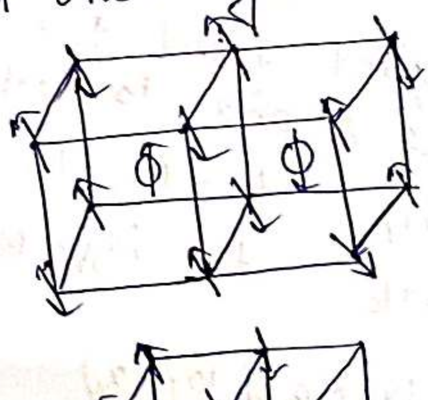


Type-G

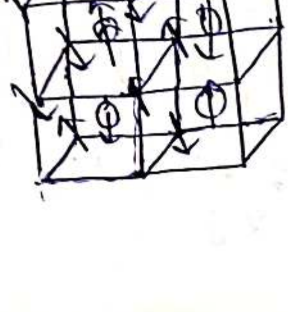


Bcc there are 3-types of AFM ordering





B sublattice.



Wiess molecular field theory for AFM

Consider magnetic materials with two sublattices A & B

Bcc lattice — A → Corner points
B → Bcc position

An atom at A-site has nearest neighbours that all lie on B-sites & next nearest neighbours that all lie on A-sites.

$$H_{M+} = -\lambda M_-$$

$$H_{M-} = -\lambda M_+$$

The molecular field H_{mA} acting on a atom at A-site

$$H_{mA} = -N_{AA}M_A - N_{AB}M_B$$

M_A & M_B = Magnetizations of A and B Sublattice

N_{AB} = Molecular field constant for the Nearest Neighbour Interaction.

N_{AA} = " For Next Nearest Neighbour Int.

The molecular field H_{mB} acting on the atom B-site

$$H_{mB} = -N_{BA}M_A - N_{BB}M_B$$

$$N_{AA} = N_{BB} = N_{ii}$$

$$N_{BA} = N_{AB}$$

$H + \lambda M$

If H applied field the, the field H_A and H_B at an atom on the A and B lattice would be

$H_A = H + H_{mA}$
$H_B = H + H_{mB}$

$$H_A = H - N_{ii}M_A - N_{AB}M_B \quad \text{——— (1)}$$

$$H_B = H - N_{AB}M_A - N_{ii}M_B \quad \text{——— (2)}$$

$$M_A = Ng\mu_B J\, B_J(x_A) \qquad x_A = \frac{Jg\mu_B}{k_BT}H_A$$

$$M_B = \frac{N}{2}g\mu_B J\, B_J(x_B) \qquad x_B = \frac{Jg\mu_B}{k_BT}H_B$$

$$B_J(x_A) = \frac{2J+1}{2J}\coth\frac{2J+1}{2J}x_A - \frac{1}{2J}\coth\left(\frac{x_A}{2J}\right)$$

Case-1 : Behaviour above T_N · $T > T_N$

$$B_J(x) \to \frac{3J+1}{3J} x$$

$x \to$ small
$$M_A = \frac{N}{2} g \mu_B J \cdot \frac{J+1}{3J} \cdot \frac{J g \mu_B}{k_B T} H_A$$

$$= \frac{N g^2 \mu_B^2 J(J+1)}{6 k_B T} H_A = \frac{N g^2 \mu_B^2 J(J+1)}{6 k_B T} \cdot \left\{ H - N_{ii} M_A - N_{AB} M_B \right\}$$

$$M_B = \frac{N g^2 \mu_B^2 J(J+1)}{6 k_B T} H_B = \frac{N g^2 \mu_B^2 J(J+1)}{6 k_B T} \cdot \left\{ H - N_{AB} M_A - N_{ii} M_B \right\}$$

$$M = M_A + M_B$$

$$M = \frac{N g^2 \mu_B^2}{6 k_B T} J(J+1) \left[2H - N_{AB}(M_A + M_B) - N_{ii} M \right]$$

$$M \left[1 + (N_{AB} + N_{ii}) \frac{N g^2 \mu_B^2}{6 k_B T} J(J+1) \right] = \frac{N g^2 \mu_B^2}{6 k_B T} J(J+1) \cdot 2H$$

$$\chi = \frac{M}{H} = \frac{\dfrac{N g^2 \mu_B^2 \cdot J(J+1)}{3 k_B T}}{1 + (N_{AB} + N_{ii}) \dfrac{N g^2 \mu_B^2 J(J+1)}{6 k_B T}}$$

$$\chi = \frac{\frac{C}{T}}{1 + \frac{(N_{AB} + N_{ii})}{2} \cdot \frac{C}{T}}$$

$$\chi = \frac{C/T}{1 + \frac{C}{2T}(N_{AB} + N_{ii})}$$

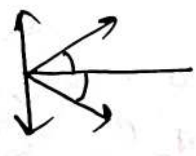
$$\chi = \frac{C}{T + \frac{C}{2}(N_{AB} + N_{ii})}$$

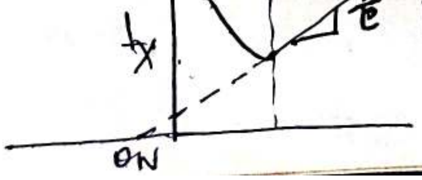
$\chi = \dfrac{C}{T + \theta}$ → Curie Weiss law for the AFM in the PM.

$$\boxed{\theta_N = \frac{C}{2}(N_{ii} + N_{AB})}$$

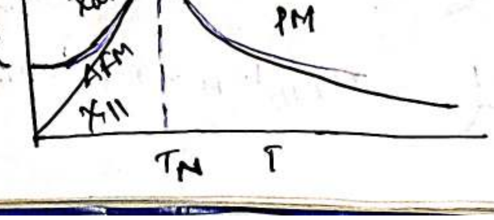
$$C = \frac{N g^2 \mu_B^2 J(J+1)}{3 k_B}$$

$$\chi = \frac{C}{T + \theta_N} \; ; \quad \frac{1}{\chi} = \frac{T}{C} + \frac{\theta_N}{C}$$





$|T_N| \approx \theta_N$



Scaling law **

$$2 = \alpha + 2\beta + \gamma$$

$$\delta = 1 + \left(\frac{\gamma}{\beta}\right)$$

$$\alpha = 2 + \delta\gamma$$

(#) Fe $T_c = 1043K$

$\beta = 0.36 \pm 0.02$

$\gamma = 1.33 \pm 0.02$

$\delta = 4.35 \pm 0.02$

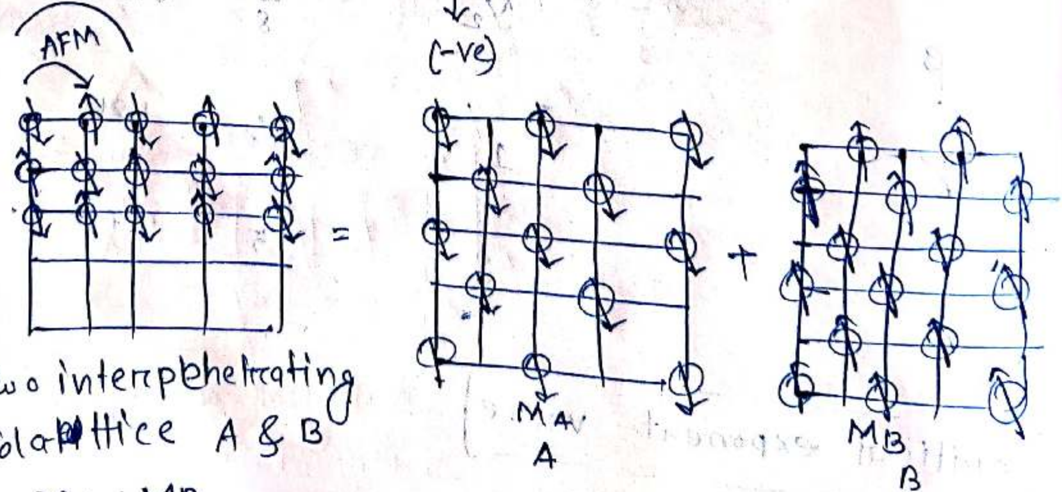
19/03/2025

Antiferromagnatism

In 1936 Neel shown theoritically that if exchange integral $J_{ex} = -ve$, there a state of lowest energy is obtained where the spins of neighbouring atoms have opposite orientation. Such material are known as AFM.

$$H_{spin} = -2J_{in}\, \vec{S_i} \cdot \vec{S_j}$$

(-ve)

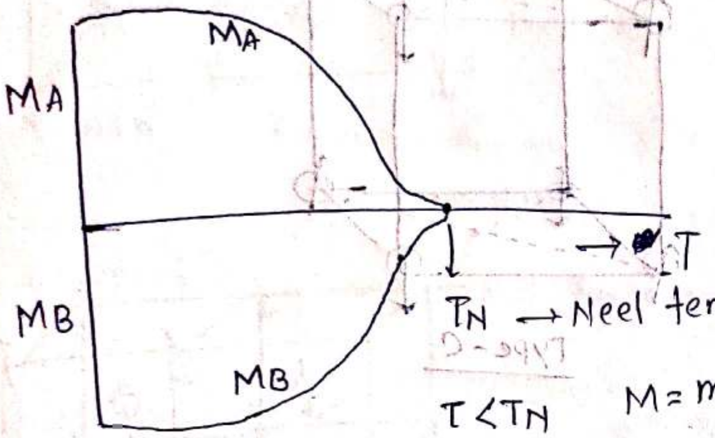


Two interpenetrating sublattice A & B

$M_A + M_B$

nearest neighbourre interaction is AFM
next " " " FM

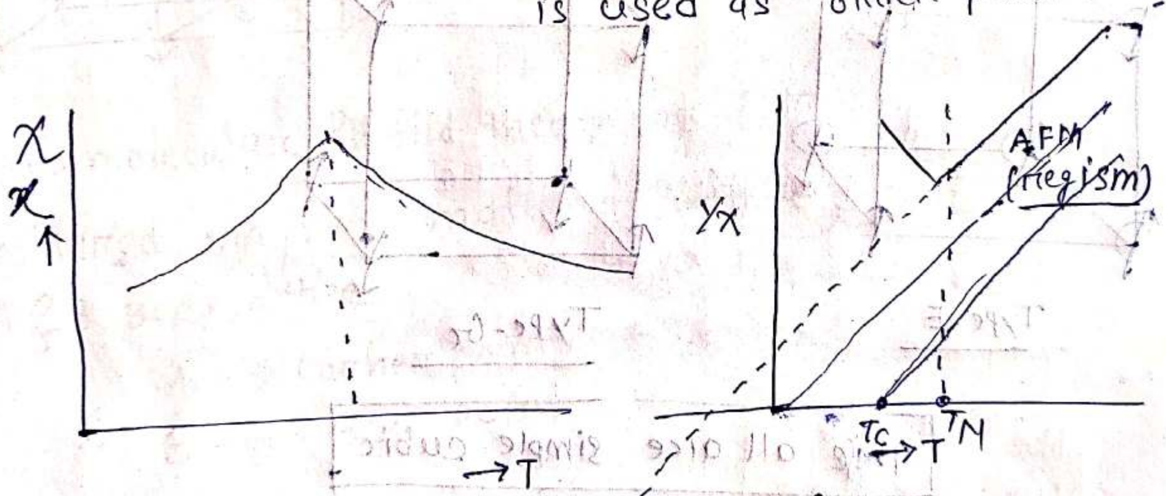
M_A = A-sublattice magnetisation
M_B = B sublattice magnetisation



$T_N \to$ Neel temperature

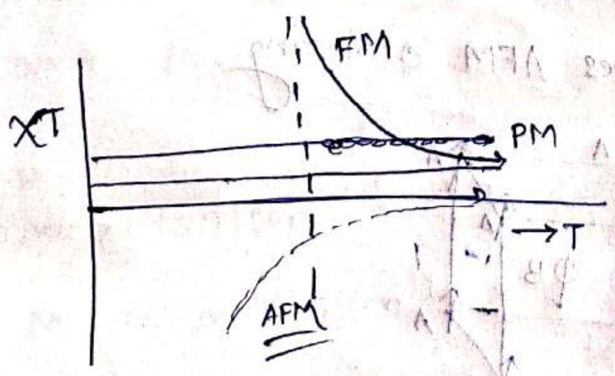
$T < T_N$ $\quad M = m_A + m_B = 0$

$|M_A - M_B|$ = staggered magnetisation is used as order parameter.



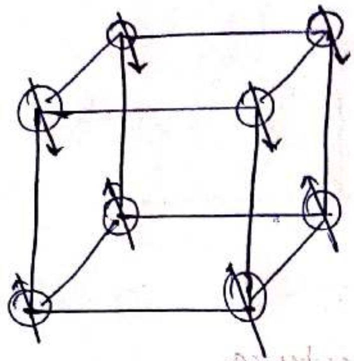
$$\chi = \left(\frac{C}{T+\theta}\right)$$

Curie weiss-law

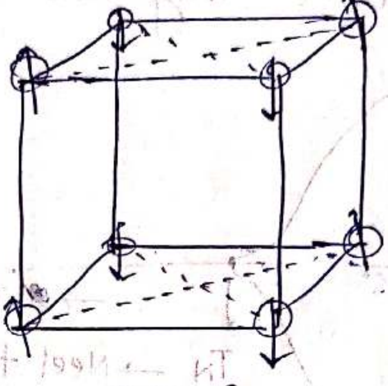


$\theta = 0$ PM
$\theta > 0$ FM
$\theta < 0$ AFM

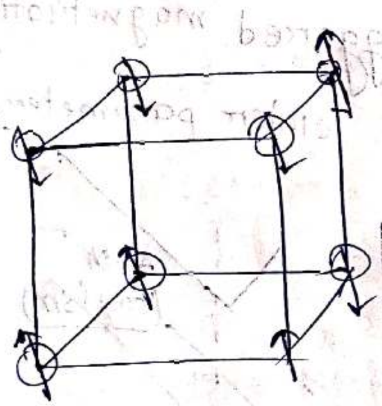
Types of AFM
cubic crystal



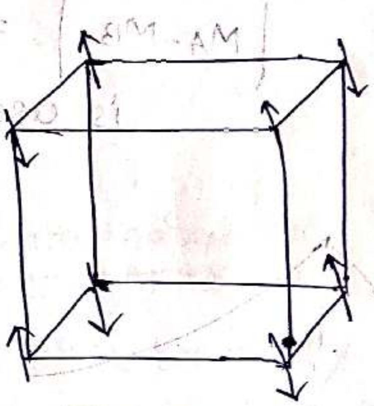
Type-A



Type-C



Type-E

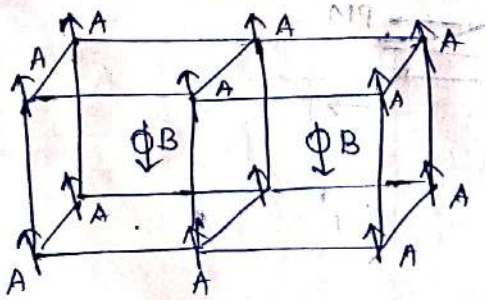


Type-G

$\boxed{\text{These all are simple cubic}}$

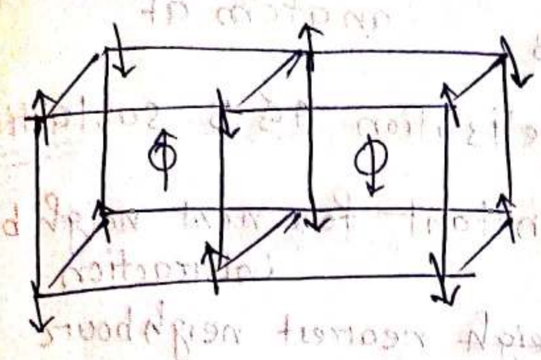
(#) In BCC Structure

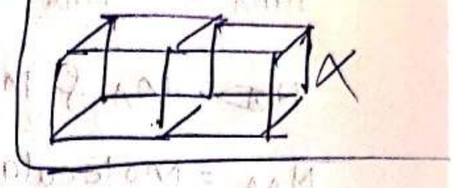
there are 3 types AFM ordering

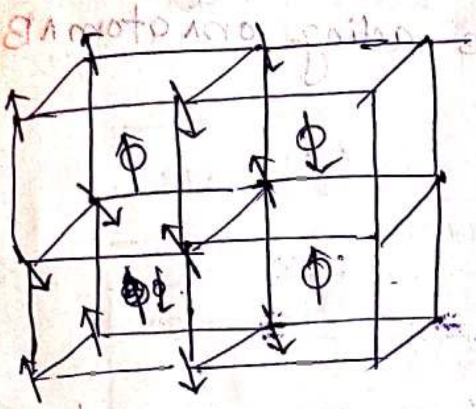


$\updownarrow$ B-sublattice

$\updownarrow$ A $\rightarrow$ sublattice







weiss moleculare ~~fd~~ fild theory for AFM

considered magnetic materials ~~with~~ with two sublattice A & B Bcc lattice

$A \longrightarrow$ Corner point

$B \longrightarrow$ Bcc point

An atom ~~has~~ at A-site has nearest neighbour that all lie on B-sites & next neighbour that all lie on A site.

M_+
Magnetisation of A sublattice

M_- Magnetisation of B sublattice.

$H_{M+} = -\lambda M_-$

$M_{M-} = -\lambda M_+$

The moleculare field H_{mA} acting on an atom at A site /

$$H_{mA} = -N_{AA} M_A - N_{AB} M_B$$

M_A & M_B = Magnetisation A & B sublattice

N_{AA} = Moleculare field constant for next neighbour interaction

N_{AB} = Moleculare " for ~~neigh~~ nearest neighbour interaction.

simillarly the moleculare field H_{mB} acting on an atom at B-site

$$H_{mB} = -N_{BA} M_A - N_{BB} M_B$$

$$N_{AA} = N_{BB} = N_{ii}$$

$$N_{BA} = N_{AB}$$

If H field is applied the held H_A & H_B at an atom on two A & B lattice would be

$$H_A = H - N_{ii} M_A - N_{AB} M_B \quad \text{—— (I)}$$

$$H_B = H - N_{AB} M_A - N_{ii} M_B \quad \text{—— (II)}$$

$$\boxed{\begin{aligned} H_A &= H + H_{mA} \\ H_B &= H + H_{mB} \end{aligned}}$$

Magnetisation $(M_A) = \frac{N}{2} g \mu_B J B_J(x_A)$

$(M_B) = \frac{N}{2} g \mu_B J B_J(x_B)$

$$x_A = \frac{J g \mu_B}{K_B T}(H_A)$$

$$x_B = \frac{J g \mu_B}{K_B T} H_B$$

$$B_j(x_A) = \frac{(2j+1)}{2j} \coth \frac{(2j+1)}{2j} x_A - \frac{1}{2j} \coth \frac{1}{2j} x_A$$

Case-I Behaviour above T_N, $T > T_N$

$$\boxed{B_j(x) \longrightarrow \frac{(J+1)}{3j}(x)}$$

$x \rightarrow$ Become small

$$M_A = \left(\frac{N}{2}\right) g \mu_B (J) \frac{(J+1)}{3\cancel{J}} \left(\frac{\cancel{J} g \mu_B}{K_B T}\right) \cdot (H_A)$$

$$\boxed{M_A = \left(\frac{N}{2}\right) \frac{g^2 \mu_B^2 \, j(J+1)}{6K_BT} (H_A)}$$

$$\boxed{M_B = \frac{N g^2 \mu_B^2 \, j(J+1)}{6K_BT} H_B}$$

$$M_A = \frac{N g^2 \mu_B^2 \, j(J+1)}{6K_BT} \left(H - N_{ii} M_A - N_{AB} M_B\right)$$

$$M_B = \frac{N g^2 \mu_B^2 \, j(J+1)}{6K_BT} \left(H - N_{AB} M_A - N_{ii} M_B\right)$$

Total magnetisation

$$M = (M_A + M_B)$$

$$M = \frac{N g^2 \mu_B^2 \, j(J+1)}{6K_BT} \left[2H - (N_{ii} + N_{AB}) M\right]$$

$$M \left[1 + \frac{N g^2 \mu_B^2 \, j(J+1)}{6K_BT} (N_{ii} + N_{AB})\right] = \frac{N g^2 \mu_B^2 \, j(J+1) \cdot H}{3K_BT} = \left(\frac{C}{T}\right) H$$

$$\chi = \frac{M}{H} = \frac{(C/T)}{1 + \frac{C}{2T}(N_{ii} + N_{AB})}$$

$$\boxed{\chi = \frac{C}{T + \frac{C}{2}(N_{ii} + N_{AB})}}$$

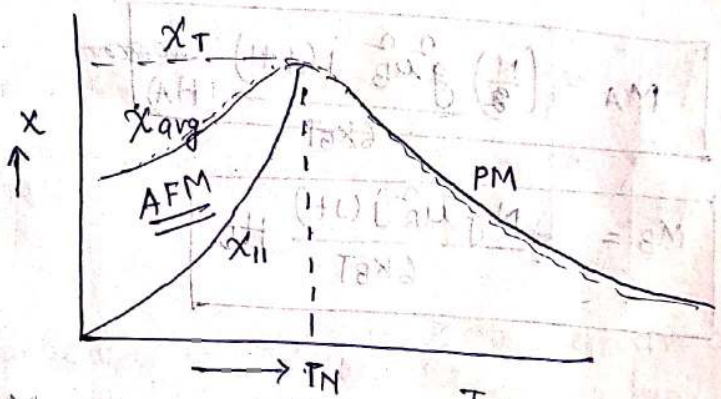
Scanned with OKEN Scanner

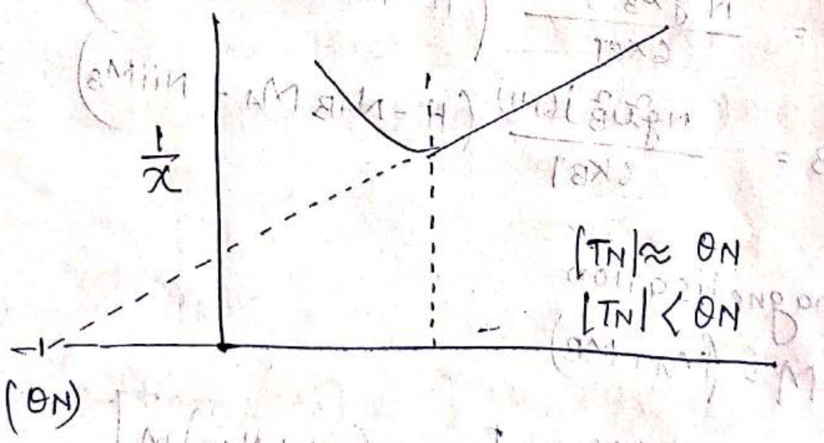
$$\boxed{\chi = \frac{C}{T + \theta}}$$ (curie-weiss for AFM in the paramagnetic regism)

where $\theta = \frac{C}{2}\left(N_{ii} + N_{AB}\right)$

$$= \frac{N g^2 \mu_B^2 J(J+1)}{(3K_B T)\,2}\left[N_{ii} + N_{AB}\right]$$

$$\chi = \left(\frac{C}{T + \theta_N}\right) \Rightarrow \frac{1}{\chi} = \frac{T}{C} + \left(\frac{\theta_N}{C}\right)$$





A sublattice & B-sublattice ↑↓↑↓

case-II

$T < T_N$ (doesn't apply magnetic field)

below T_N both sublattice posses spontinous magnetisation.

$$H = 0$$

$$M_A = \frac{c}{2T}\left(-N_{ii} M_A - N_{AB} M_B\right)$$

$$M_B = \frac{c}{2T}\left(-N_{AB} M_A - N_{ii} M_B\right)$$

$$M_A\left[1 + \frac{c}{2T} N_{ii}\right] + M_B\left(\frac{c}{2T} N_{AB}\right) = 0$$

$$M_A\left(\frac{c}{2T}\right) N_{AB} + M_B\left[1 + \frac{c}{2T} N_{ii}\right] = 0.$$

$$\begin{vmatrix} x & y \\ y & x \end{vmatrix} = 0$$

$$x = 1 + \frac{c}{2T} N_{ii}$$

$$y = \frac{c}{2T} N_{AB}$$

$$x = 1 + \frac{c}{2T} N_{ii}$$

$$y = \frac{c}{2T} N_{AB}$$

$$x^2 - y^2 = 0$$

$$(x+y)(x-y) = 0$$

$$\boxed{x \neq -y} \quad \boxed{x = y}$$

not possible

$$1 + \frac{c}{2T} N_{ii} = \frac{c}{2T} N_{AB}$$

$$1 + \frac{c}{2T_N}\left(N_{ii} - N_{AB}\right) = 0$$

$$\frac{c}{2T_N}\left(N_{AB} - N_{ii}\right) = 1$$

$$T_N = \frac{c}{2}\left(N_{AB} - N_{ii}\right)$$

$$\theta = \frac{c}{2}\left(N_{ii} + N_{AB}\right)$$

$$\frac{T_N}{\theta} = \frac{N_{AB} - N_{ii}}{N_{ii} + N_{AB}}$$

if $N_{ii} = 0$ $\quad \frac{T_N}{\theta} = \frac{N_{AB}}{N_{AB}}$

$T_N = \theta$

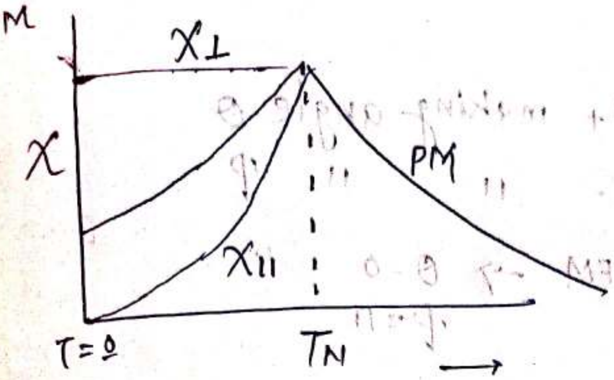
if $N_{ii} \neq 0$ $\quad T_N < \theta$

T_N increase if AFM AB interaction N_{AB} become stronger but decreasing with increasing AA & BB.

Material	T_N(K)	θ(K)	J
MnF_2	67	-80	5/2
MnO	116	-510	5/2
CoO	292	-330	3/2
FeO	116	-610	2
Cr_2O_3	307	-485	3/2
Fe_2O_3	950	-2000	5/2

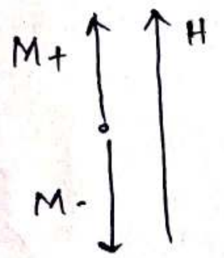
It is seen that $T_N < \theta$

AFM

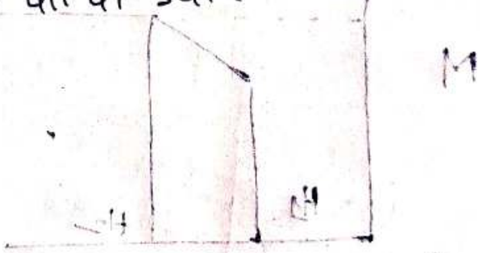


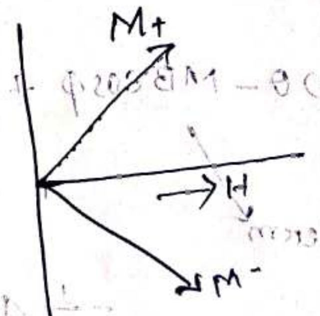
$$\chi_{avg} = \frac{1}{3}\chi_{\parallel} + \frac{2}{3}\chi_\perp$$

small magnetic field



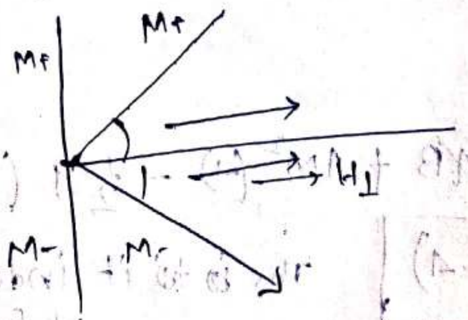
magnetisation for M+ & M-
are all at saturated, state.

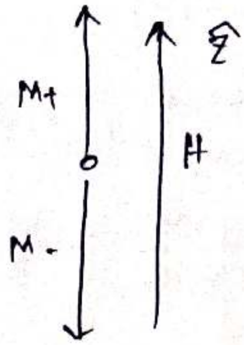




(small magnetic field)

Now for strong magnetic field.



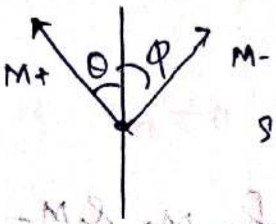


let M_+ making angle θ

M_- 〃 〃 ϕ

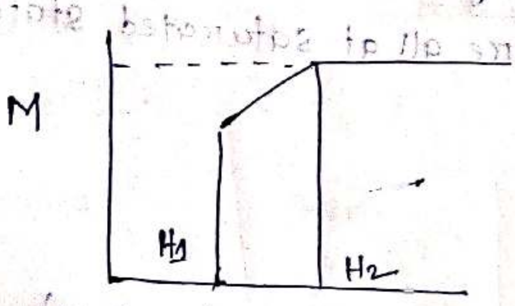
AFM $\rightarrow$ $\theta = 0$

$\phi = \pi$



spin-flop transition

$\theta = \phi$



Total energy $E = -MB\cos\theta - MB\cos\phi + \underbrace{AM^2\cos(\theta+\phi)}_{\text{due to interacti}}$ $\overset{\text{exchange coupling}}{}$

$-MB\cos\theta - MB\cos\phi$ → ~~Zeem~~ Zeman term

$-\frac{1}{2}\Delta(\cos^2\theta + \cos^2\phi)$ → (Anistropy term)

AFM case

$\theta = 0, \phi = \pi$

$E = -\cancel{MB} + \cancel{MB} + AM^2(-1) - \frac{1}{2}\Delta(1+1)$

$\boxed{E = (-AM^2 - \Delta)}$ The ~~is~~ ~~to~~ it independent of field

$\theta = \varphi$ In spin flop state

$E = -2MB\cos\theta + AM^2\cos(2\theta) - \frac{1}{2}4\cos^2\theta$

Minimum energy configuration

$$\frac{dE}{d\theta} = 0$$

$$+2MB\sin\theta + AM^2(-1)\sin2\theta\,(2) + \frac{1}{2}4\,2\cos\theta\sin\theta\,(1) = 0$$

$$\Rightarrow 2MB\sin\theta - 2AM^2\sin2\theta + 24\sin\theta\cos\theta = 0$$

$$\Rightarrow (2MB - 4AM^2\cos\theta + 24\cos\theta)\sin\theta = 0$$

$\therefore \sin\theta = 0$ $\boxed{\theta = 2n\pi}$

$$2MB + (24 - 4AM^2)\cos\theta = 0$$

$$\boxed{\cos\theta = \frac{2MB}{(4AM^2 - 24)}}$$

$4 << 1$

$$\cos\theta \approx \frac{2MB}{4AM^2}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{B}{2AM}\right)}$$

$$E = -2MB\left(\frac{B}{2AM}\right) + 2AM^2\cdot\frac{B}{2AM} - AM^2$$

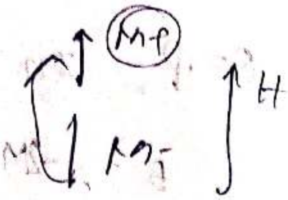
$$E = -2MB\cos\theta + AM^2\,2\sin\theta\cos\theta - 4\cos^2\theta$$

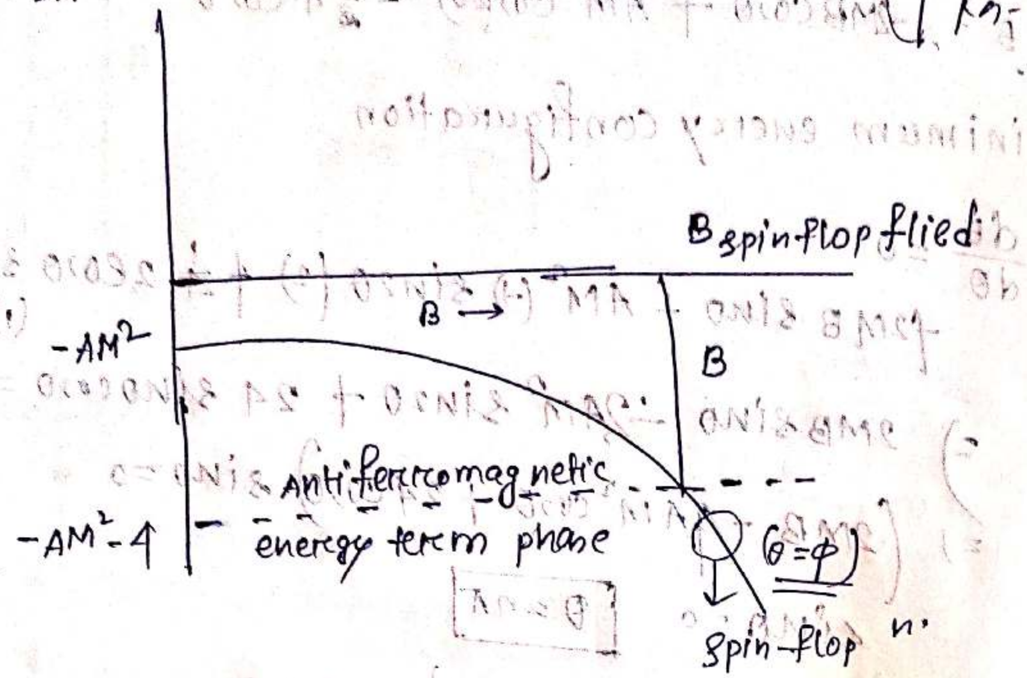
$$= -\cancel{2M}B\left(\frac{B}{\cancel{2AM}}\right) + \cancel{AM^2}\,2\,\frac{B^2}{\cancel{4A^2M^2}} - \left(4\cdot\frac{B^2}{4A^2M^2}\right)$$

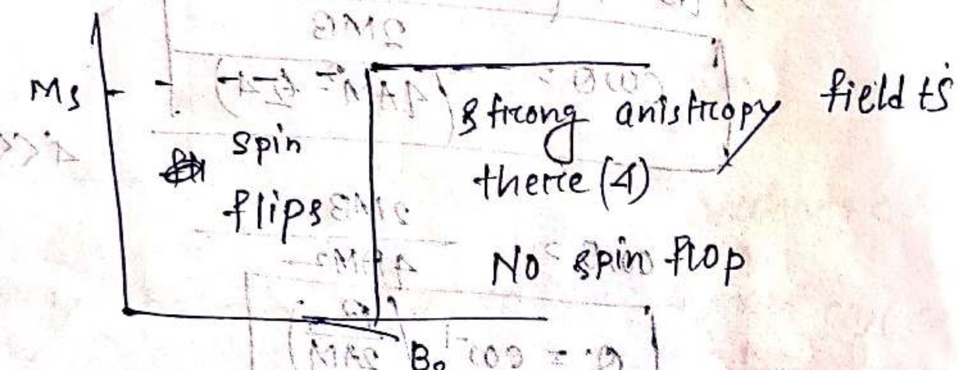
$$= -\frac{B^2}{2A} - \frac{B^2}{2A}$$

$$= -\left(\frac{B^2}{A}\right)$$

$$E = -\frac{B^2}{2A} - AM^2$$







Direct exchange :-

$H_{m_i} = -2J\,\vec{S_i}\cdot\vec{S_2}$

FM

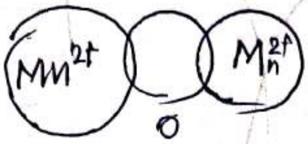
AFM

PM

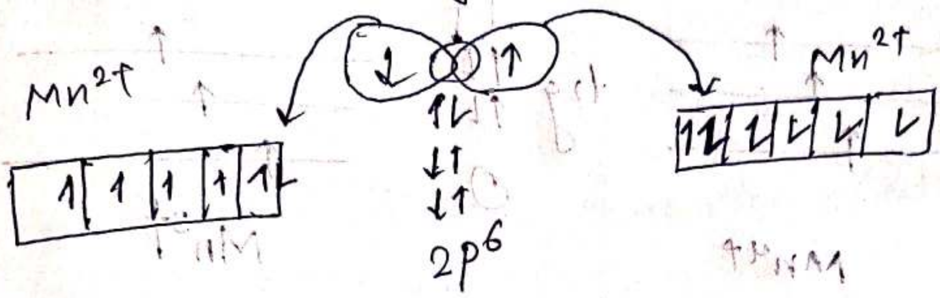
Indirect exchange

- Superexchang
- Double exchange
- DM interaction
- RKKY interaction

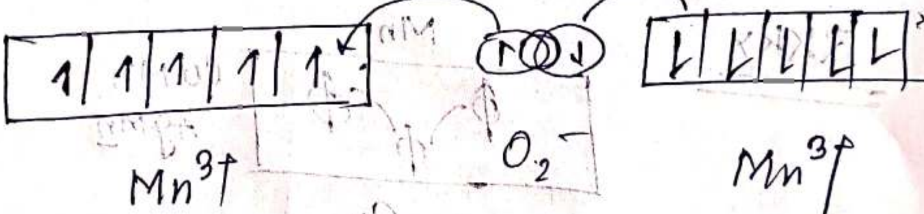
MnO $Mn^{2+} - O^{2-} - Mn^{2+}$



ligand



AFM superexchange



(#) M^{3+}

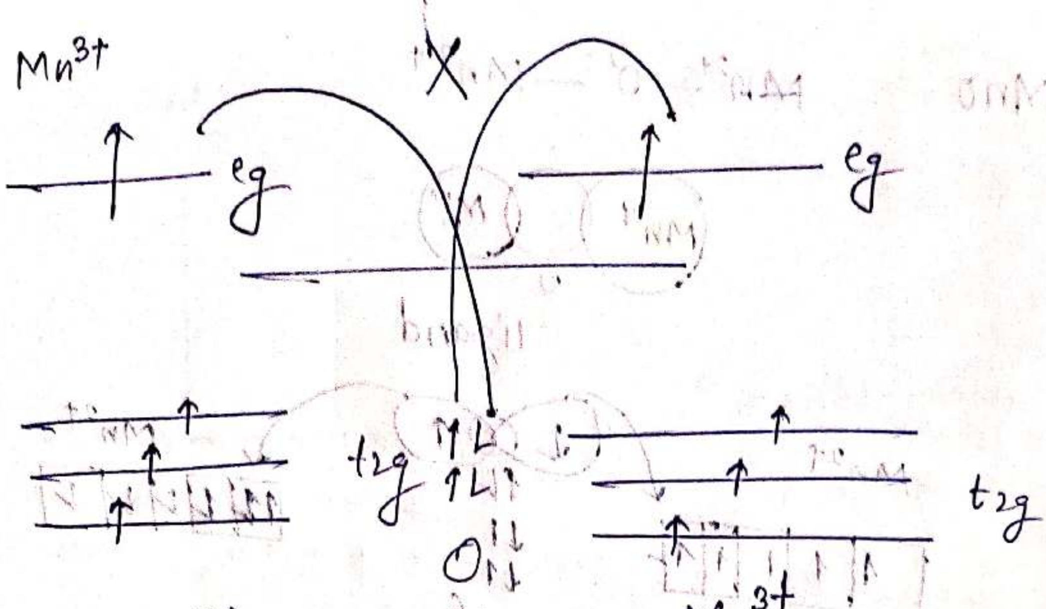
Superchange APM interaction

(#) double exchange interaction

$\overset{2+}{La}\ \overset{3+}{Mn}\ \overset{2-}{O_3}$

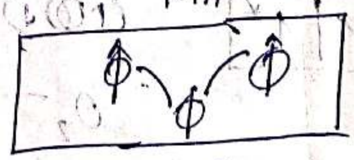
$La^{3+}_{0.7} Sr^{2+}_{0.3}\ Mn^{3+}, Mn^{4+}\ O_3^{2-}$

$La_{0.7} Sr_{0.3} MnO_3$ $\quad Mn^{3+} - O - Mn^{4+}$

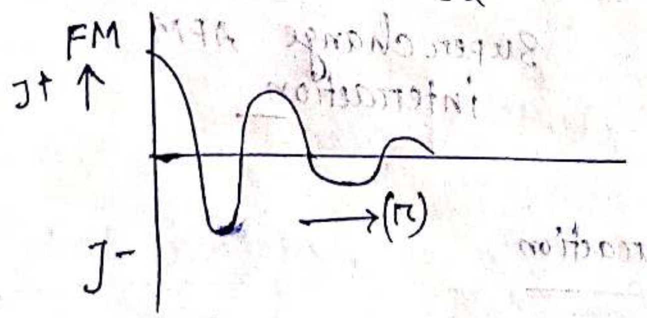


CMR materials DF interaction

RKKY



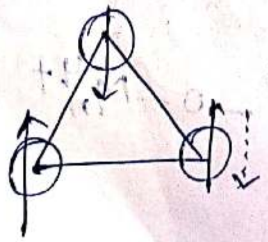
CuMn
AgMn



$$J(r) \propto \frac{\cos(K_f r)}{r^3}$$

$$\boxed{H_{DM} = D\,\vec{S_1} \times \vec{S_2}}$$

DM interaction



spin ~~frasted~~
frusta state
spin glass

~~superconductivity~~

HRTEM (32)

STEM

TEM (22)

The critical temperature can be found by approaching from the high temp. state $T > T_N$

$$M_A = \frac{c}{2T}\left(H - N_{ii}M_A - N_{AB}M_B\right)$$

$$M_B = \frac{c}{2T}\left(H - N_{AB}M_A - N_{ii}M_B\right)$$

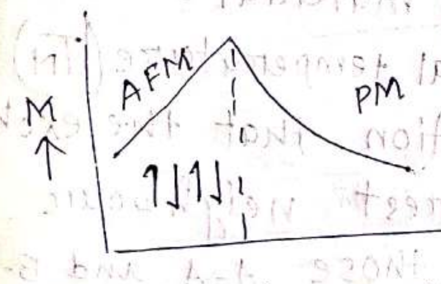
In the visinity of T_N (saturation effects are unimportant.)

$H = 0$

$$M_A = \frac{c}{2T}\left(-N_{ii}M_A - N_{AB}M_B\right)$$

$$M_B = \frac{c}{2T}\left(-N_{AB}M_A - N_{ii}M_B\right)$$

for Non-zero value of M_A & M_B the determinant the position of M_A & M_B must be zero.



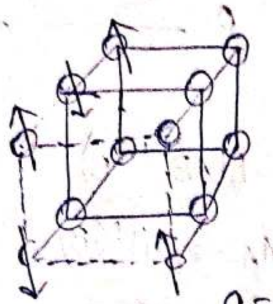
$M_A = 0$
$M_B = 0$

$$T_N = \frac{c}{2}\left(N_{AB} - N_{ii}\right)$$

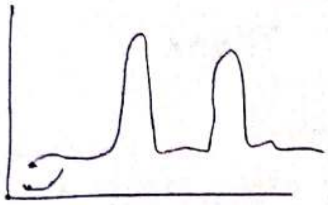
$$\frac{T_N}{\theta} = \left(\frac{N_{AB} - N_{ii}}{N_{AB} + N_{ii}}\right) \quad \boxed{T_N < \theta}$$

AFM $\uparrow\downarrow\uparrow\downarrow$

$T<T_N$



$a = 8.85 Å$

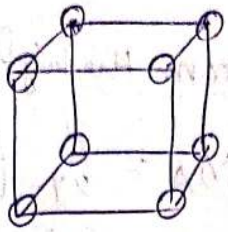


Neutron diffraction

it give lattice constant and also spin structure

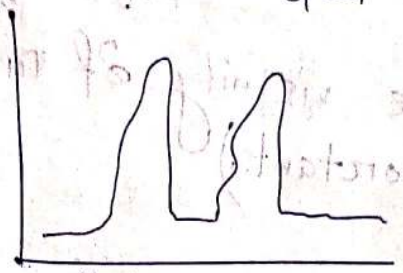
PM $\nearrow \uparrow \uparrow \nwarrow$

$T>T_N$



unmagnetised Mn

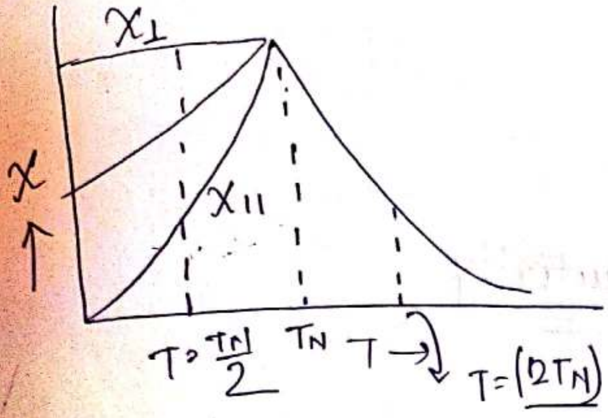
$a = 4.45 Å$



(X- ray diffraction)

problem:- (1) Consider an AFM material which has a susceptibility χ_0 at its neal temperature (T_N). Assuming X-ray diffraction that the exchange interaction between nearest neighbour A & B ions much larger than those A-A and B-B pairs calculate the values of suceptibilities which would be measured under the application of field's perpendicular to the magnetisation direction at $T=0$, $T=\left(\frac{T_N}{2}\right)$ & $T=2T_N$

AFM. AFM PM



$$\frac{T_N}{\theta} = \left(\frac{N_{AB} - N_{ii}}{N_{AB} + N_{ii}}\right) \qquad (N_{AB} > N_{ii})$$

$$\boxed{T_N = \theta}$$

$$\chi = \left(\frac{C}{T+\theta}\right)$$

$$= \left(\frac{C}{T+T_N}\right)$$

$$\chi = \frac{C}{2T_N + T_N} = \left(\frac{C}{3T_N}\right)$$

$$\boxed{\chi = \frac{C}{3T_N}}$$

$$\cancel{\chi_P = \frac{C}{T_N + \frac{T_N}{2}}}$$

$$\cancel{\boxed{\chi = \frac{2C}{3T_N}}}$$

$$\chi(T_N) = \chi_0$$

$$\chi_0 = \frac{C}{T_N + T_N}$$

$$\boxed{\chi_0 = \frac{C}{2T_N}}$$

$$C = 2T_N\chi_0$$

$$\chi = \frac{2\cancel{T_N}\chi_0}{3\cancel{T_N}}$$

$$\boxed{\chi = \frac{2}{3}\chi_0} \quad (1)$$

for (~~AFM~~) PM

$$\cancel{\chi_P = \frac{2 \times 2T_N\chi_0}{3T_N}}$$

$$\cancel{\boxed{\chi = \frac{4}{3}\chi_0}}$$

~~for PM APM~~