Assignment : Optical waveguides.

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(1) show : $\nabla^2 H + \frac{\nabla^2 n^2}{n^2} \times (\nabla \times H) - \frac{n^2}{c^2} \frac{d^2 H}{dt^2} = 0.$

$\Rightarrow \nabla^2 H - \frac{1}{v^2} \frac{d^2 H}{dt^2} + \frac{\nabla^2 n^2}{n^2} \times (\nabla \times H) = 0.$

$\Rightarrow \cancel{\Box^2 H + \frac{\nabla^2 n^2}{n^2} \times (\nabla \times H) = 0}$ . $\nabla^2 H - \frac{1}{v^2} \frac{\partial^2 H}{\partial t^2} + \frac{1}{n^2} \nabla n^2 \times (\nabla \times H)$

Ans. $\nabla \times (\nabla \times H) = \nabla \times (\epsilon \frac{\partial}{\partial t} E)$.

$\cancel{\nabla(\nabla \cdot H)}^{0} - \nabla^2 H = \nabla \times (\epsilon \frac{\partial}{\partial t} E) = \epsilon(\nabla \times \frac{\partial}{\partial t} E) + (\nabla \epsilon \times \frac{\partial}{\partial t} E)$.

$= \epsilon \frac{\partial}{\partial t}(\nabla \times E) + (\nabla \epsilon \times \frac{\partial}{\partial t} E)$.

$= \epsilon \frac{\partial}{\partial t}(-\mu_0 \frac{\partial}{\partial t} H) + \nabla \epsilon \times \frac{\partial}{\partial t} E$.

Now $\boxed{\epsilon \frac{\partial}{\partial t} E = \nabla \times H.}$

$\therefore -\nabla^2 H = -\mu_0 \epsilon \frac{\partial^2}{\partial t^2} H + \frac{\nabla \epsilon}{\epsilon} \times (\nabla \times H)$

$\Rightarrow \left[\nabla^2 H - \frac{n^2}{c^2} \frac{\partial^2}{\partial t^2} H\right] + \frac{\nabla \epsilon}{\epsilon} \times (\nabla \times H)$

$\epsilon = \epsilon_0 n^2.$
$\nabla \epsilon = \epsilon_0 \nabla n^2.$

$\Rightarrow \boxed{\left[\nabla^2 H - \frac{n^2}{c^2} \frac{\partial^2}{\partial t^2} H\right] + \frac{\nabla n^2}{n^2} \times (\nabla \times H) = 0}$

(proved)

(2) TE/TM modes.

$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t}$ ①

$\nabla \times H = \epsilon_0 n^2 \frac{\partial E}{\partial t}$ ②

$E = E_0(x) \exp[i(\beta z - \omega t)]$.
$H = H(x) \exp[i(\beta z - \omega t)]$.

$\nabla \times E = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ E_x & E_y & E_z \end{vmatrix} = -\mu_0 \{H_0(-i\omega) \exp[i(\beta z - \omega t)]\}$

$= i \mu_0 \omega H.$

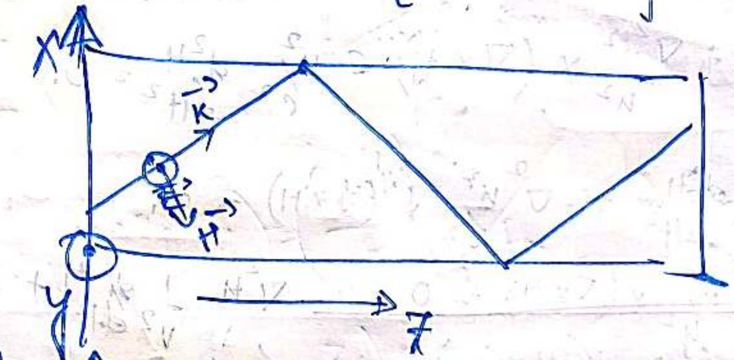
$\nabla \times H = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ H_x & H_y & H_z \end{vmatrix} = +\epsilon_0 n^2 \{E_0(-i\omega) \exp[i(\beta z - \omega t)]\}$

$= -i \epsilon_0 n^2 \omega E.$

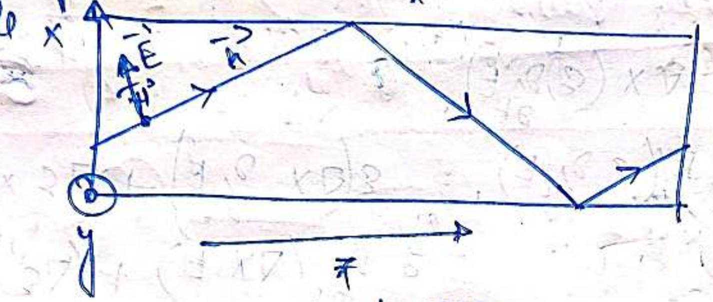
Optical waveguides

$$\cancel{\nabla \times E = i\beta \varepsilon_y \hat{x} + \hat{y}\left[i\beta \varepsilon_x - \partial_x \varepsilon_z\right] + \hat{z}\,\partial_x \varepsilon_y}$$

TE mode ($E_z = 0$)




TM mode ($H_z = 0$)




$\therefore\ \nabla \times E = i\mu_0\omega H$ (i)

$\nabla \times H = -i\varepsilon_0 n^2 E.$

from (i)

$$\left.\begin{aligned}
\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &= i\mu_0\omega H_x \\
\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} &= i\mu_0\omega H_y \\
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= i\mu_0\omega H_z
\end{aligned}\right\}$$

$$\Downarrow$$

$\frac{\partial E_z}{\partial y} - i\beta E_y = i\mu_0\omega H_x.$ $(i\varepsilon_0 n^2 E_y)$

$i\beta E_x - \frac{\partial E_z}{\partial x} = i\mu_0\omega H_y.$

$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\mu_0\omega H_z.$

from (ii)

$$\left.\begin{aligned}
\frac{\partial H_z}{\partial x} - \frac{\partial H_y}{\partial z} &= -i\varepsilon_0 n^2 E_x \\
\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= -i\varepsilon_0 n^2 E_y \\
\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= i\varepsilon_0 n^2 E_z
\end{aligned}\right\}$$

$$\Downarrow$$

$\frac{\partial}{\partial x} H_z - i\beta H_y = -i\varepsilon_0 n^2 E_x$

$i\beta H_x - \frac{\partial}{\partial x} H_z = -i\varepsilon_0 n^2 E_y$ $(i\beta)$

$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = -i\varepsilon_0 n^2 E_z.$

$(i\varepsilon_0 n^2)\frac{\partial}{\partial y} E_z - i\mu_0\omega H_x = i\beta E_y (i\varepsilon_0 n^2)$

$(i\beta)\, i\beta H_x - \frac{\partial}{\partial x} H_z = -i\varepsilon_0 n^2 E_y (i\beta)$

$i\varepsilon_0 n^2 \frac{\partial}{\partial y} E_z + (\varepsilon_0 n^2 \mu_0)\omega H_x$

$- \beta^2 H_x - i\beta \frac{\partial}{\partial x} H_z = 0.$

$n^2 = \cancel{\beta^2 / (\mu_0 \varepsilon_0)}$

$$H_x = \left(i\beta \frac{\partial}{\partial x} H_z - i\varepsilon_0 n^2 \frac{\partial}{\partial y} E_z\right)$$

$-i\beta \mathcal{E}_y = i\omega\mu_0 H_x$ ---- (i)

$(i\beta \mathcal{E}_x - \partial_x \mathcal{E}_z) = i\omega\mu_0 H_y$ --- (ii).

$\partial_x \mathcal{E}_y = i\omega\mu_0 H_z$ ----- (iii)

TM modes.

$\beta H_y = \omega \epsilon_0 n^2 \mathcal{E}_x$ ---- (iv)

$\partial_x H_z - i\beta H_x = i\omega\epsilon_0 n^2 \mathcal{E}_y$ --- (v)

$\partial_x H_y = -i\omega \epsilon_0 n^2 \mathcal{E}_z$ ---- (vi)

For TE modes: $\mathcal{E}_y = -\frac{\omega\mu_0}{\beta} H_x$ .... (i)

$i\omega\epsilon_0 n^2 \mathcal{E}_y = \partial_x H_z - i\beta H_x$ ... (v)

$\partial_x \mathcal{E}_y = i\omega\mu_0 H_z$ (iii)

$\therefore$ For TM modes: $H_y = \frac{\epsilon_0 \omega n^2}{\beta} \mathcal{E}_x$ (iv)

$\partial_x H_y = -i\omega\epsilon_0 n^2 \mathcal{E}_z$ (vi)

$i\omega\mu_0 H_y = i\beta \mathcal{E}_x - \partial_x \mathcal{E}_z$ (ii).

$$\boxed{\begin{aligned}\mathcal{E}_j &= E_j\, e^{i(\beta z - \omega t)}\\ H_j &= H_j\, e^{i(\beta z - \omega t)}\end{aligned}}$$

$\therefore$ For TE modes $\rightarrow$

$-i\beta \mathcal{E}_y = -\frac{\omega\mu_0}{\beta}\cancel{(i\beta)} H_y$

$i\omega\epsilon_0 n^2 \mathcal{E}_y \left(\frac{\omega\mu_0}{\beta}\right) = \left(\frac{\omega\mu_0}{\beta}\right)\left[\partial_x H_z - i\beta H_x\right]$

$\Rightarrow -i\beta \mathcal{E}_y + i\omega^2 \left(\frac{\epsilon_0\mu_0 n^2}{\beta}\right)\mathcal{E}_y = \left(\frac{\omega\mu_0}{\beta}\right)\partial_x H_z.$

$\Rightarrow -i\beta \mathcal{E}_y + i\omega^2 \left(\frac{\epsilon_0\mu_0 n^2}{\beta}\right)\mathcal{E}_y = \left(\frac{\omega\mu_0}{\beta}\right)\partial_x\left[\frac{\partial_x \mathcal{E}_y}{i\omega\mu_0}\right].$

$\Rightarrow +\beta^2 \mathcal{E}_y - \omega^2(\epsilon_0\mu_0 n^2)\mathcal{E}_y = \partial_x^2 \mathcal{E}_y.$

$\therefore \frac{\partial^2}{\partial x^2}\mathcal{E}_y - (\beta^2 - k^2 n^2)\mathcal{E}_y = 0.$

$$\therefore \boxed{\frac{\partial^2}{\partial x^2}\mathcal{E}_y + (k^2 n^2(x) - \beta^2)\mathcal{E}_y = 0}$$

Similarly:

$$(-i\beta)\, H_y = (i\beta)\, \cancel{\frac{\epsilon_0 \omega n^2}{\beta}}\, E_x$$

$$\left[\frac{\epsilon_0 \omega n^2}{\beta}\right](i\omega\mu_0 H_y) = \left(\frac{\epsilon_0 \omega n^2}{\beta}\right)\left[i\beta \cancel{E_x} - \partial_x E_z\right]$$

$$\Rightarrow -i\beta H_y + \left(\frac{\epsilon_0 \omega n^2}{\beta}\right)(i\omega\mu_0 H_y) = \frac{\epsilon_0 \omega n^2}{\beta}\left(-\partial_x E_z\right)$$

$$= \frac{\epsilon_0 \omega n^2}{\beta}\left(\partial_x \left[\frac{\partial_x H_y}{-i\omega\epsilon_0 n^2}\right]\right).$$

$$= \left(\frac{\epsilon_0 \omega n^2}{\beta}\right)\left[\frac{1}{-i\omega\epsilon_0}\right]\partial_x\left[\frac{\partial_x H_y}{n^2}\right].$$

$$\Rightarrow -\beta^2 H_y + \omega^2 n^2 (\mu_0\epsilon_0) H_y = n^2\left(\frac{\partial^2 H_y}{n^2} - \frac{1}{n^2}\frac{\partial(n^2)}{\partial x}\frac{\partial H_y}{\partial x}\right).$$

$$\Rightarrow \boxed{\frac{\partial^2 H_y}{\partial x^2} + (k^2 n^2 - \beta^2) H_y - \left[\frac{1}{n^2}\frac{\partial n^2}{\partial x}\right]\frac{\partial H_y}{\partial x} = 0}.$$

(3) The WG supports TE modes where $E_y$ is transverse to $\vec{k}$.

From above: $\frac{\partial^2 E_y}{\partial x^2} + (k^2 n^2 - \beta^2) E_y = 0.$ $\boxed{k_0 = \frac{2\pi}{\lambda_0}}$.

Now $n(x)$ → $n_1$ for $|x| < d/2$ [core].
→ $n_2$ for $|x| > d/2$ [cladding].

→ Solutions: $E_y = A\cos(k_x x)$ $\quad k_x = \sqrt{k^2 n_1^2 - \beta^2}$.
[core]. ⟨symmetric modes⟩ &

$E_y = A\sin(k_x x)$
⟨Antisymmetric modes⟩ $\boxed{k^2 n_1^2 - \beta^2 > 0}$

Solutions: $E_y = B e^{-\{\gamma\}|x|}$ [Cladding]. ⟨symmetric⟩ $\gamma = \sqrt{\beta^2 - n_2^2 k^2}$.

$= \pm B e^{-\gamma |x|}$.

Boundary conditions for $|x| = d/2$: continuity of $E_y$ & $\frac{d}{dx}E_y$.

$$A\cos\left(k_x \frac{d}{2}\right) = B e^{-\gamma d/2}.$$

$$-A k_x \sin\left(k_x \frac{d}{2}\right) = B(-\gamma) e^{-\gamma d/2}.$$

$$+k_x \tan\left(k_x \frac{d}{2}\right) = +\gamma \Rightarrow \boxed{k_x \tan\left(k_x \frac{d}{2}\right) = \gamma}$$

symmetric mode transcendental equation

for AS modes:

$$A\sin\left(k_x \frac{d}{2}\right) = B e^{-\gamma d/2}.$$

$$A k_x \cos\left(k_x \frac{d}{2}\right) = B(-\gamma) e^{-\gamma d/2}.$$

$$k_x \cot\left(k_x \frac{d}{2}\right) = -\gamma. \Rightarrow \boxed{k_x \cot\left(k_x \frac{d}{2}\right) = -\gamma.}$$

Antisymmetric mode transcendental equa$^n$.

(4) (a) $V = \frac{2\pi d}{\lambda}\sqrt{n_1^2 - n_2^2}$.

$n_1 = 1.503$, $\lambda_0 = 1\,\mu m$. $\therefore V = 2\pi \frac{4}{1}\sqrt{(1.503+1.5)(.003)}$

$n_2 = 1.5$

$d = 4\,\mu m$.

$= \boxed{2.3854}$. $\frac{\pi}{2} \times (0.949)$

Modes $= \left[\frac{V}{\pi}\right] + 1 = 0 + 1 = 1$ [symmetric].

(b) $\lambda_0 = .5\,\mu m$. Everything is same $\lambda_2 = \frac{\lambda_1}{2}$.

$\therefore V' = 2V_1 = 4.7708$

$\approx 4.78$.

Symmetric modes $\Rightarrow \left[\frac{V}{\pi}\right] + 1 = 2$.

5(a) For SIP waveguide, with $n_1 = 1.5$, $n_2 = 1$, $d = 0.555\ \mu m$, $\lambda_0 = 1.3\ \mu m$, calculate $n_{eff}$.

$$V = \frac{2\pi d}{\lambda_0}\sqrt{n_1^2 - n_2^2}$$

$$= \frac{2\times\pi\times .555}{1.3}\times\sqrt{2.5\times .5}$$

$1.25$

$125\times10^{-2}$

$\frac{5\sqrt{5}}{10}$

$$= \frac{\cancel{2}\times .5\times .555\times\pi}{1.3}\times\sqrt{5}$$

$\approx 3$

$n_{eff}$ $\Rightarrow$ $K_0 = \frac{2\pi}{\lambda_0} = \frac{2\pi}{1.3}\times10^{6}$

$n_{eff}$ $(K_x)$ : $\sqrt{K_0^2 n_1^2 - \beta^2}$

$K_x \tan\left(K_x \frac{d}{2}\right) = \gamma$

& $\beta = K_0 n_{eff}$

$\cancel{n_{eff_1} = K^2 n_1^2 - \beta^2 = K_x}$

$\cancel{n_{eff_2} = K^2 n_2^2 - \beta^2 = \gamma}$

$\cancel{-K_x^2 + \gamma^2 = K^2 n_2^2 - \beta^2 - K^2 n_1^2 + \beta^2}$

$\cancel{\gamma^2 - K_x^2 = K_0^2 (n_2^2 - n_1^2)}$

$\cancel{= K_0^2 V^2 \times \frac{\lambda_0^2}{(2\pi d)^2}}$

$\therefore K_x = \sqrt{K_0^2 n_1^2 - K_0^2 n_{eff}^2}$

$\gamma = \sqrt{K_0^2 n_{eff}^2 - K_0^2 n_2^2}$

$\therefore K_x = K_0\sqrt{n_1^2 - n_{eff}^2}$

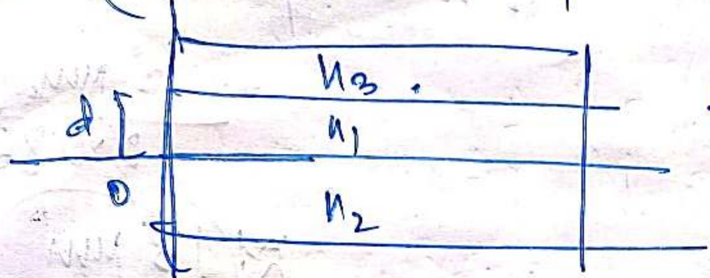
$\gamma = K_0\sqrt{n_{eff}^2 - n_2^2}$

From transcendental eqn: $\tan\left(\frac{K_0 d}{2}\sqrt{n_1^2 - n_{eff}^2}\right) = \frac{\sqrt{n_{eff}^2 - n_2^2}}{\sqrt{-n_{eff}^2 + n_1^2}}$

TE mode (neff) $\longrightarrow$ $\boxed{1.3361}$

TM mode (neff) $\longrightarrow$ $\boxed{1.2495}$

(b) Helmholtz equations.




As slab step index waveguide.

TM modes ($H_y$ transverse to $\hat{n}$).

$$\frac{d^2 H_y}{dx^2} + (k^2 n_i^2 - \beta^2) H_y = 0 \quad \left\{ \frac{1}{n_i^2} \frac{d}{dx} n_i^2 \to 0 \right\}$$

{ $n$ is not constantly varying. $\frac{d}{dx}[\text{const}] = 0$ }

core:

$$H_y(x) = A\cos(k_x x) + B\sin(k_x x).$$

{ $n_i$ = constant for each layer }.

Cladding:

$$H_y(x) \to \begin{cases} Ce^{+\gamma_2 x} & (x < 0) \\ De^{-\gamma_3 (x-d)} & (x > d). \end{cases}$$

$$k_x = \sqrt{k^2 n_1^2 - \beta^2}$$

$$\gamma_2 = \sqrt{\beta^2 - k^2 n_2^2}$$

$$\gamma_3 = \sqrt{\beta^2 - k^2 n_3^2}.$$

BOUNDARY conditions [$H_y$ & $\frac{1}{n^2}\frac{d}{dx} H_y$ both continuous].

@ $x = 0$. $\longrightarrow$ $A = C$.

$$\frac{1}{n_1^2}\left[A(k_x)\sin(k_x x) + Bk_x\cos(k_x x)\right]\Big|_0 = \frac{C}{n_2^2}\left[\gamma_2 e^{\gamma_2 x}\right]\Big|_0.$$

$$\Rightarrow \frac{Bk_x}{n_1^2} = \frac{C\gamma_2}{n_2^2} = A\frac{\gamma_2}{n_2^2}.$$

@ $x = d$: $\to$ $A\cos(k_x d) + B\sin(k_x d) = D.$

$$\frac{-k_x A\sin(k_x d) + Bk_x\cos(k_x d)}{n_1^2} = -\frac{D\gamma_3}{n_3^2}.$$

solving the above set of equations we get:

$$\tan(k_x d) = \left[\frac{\frac{k_x}{n_1^2}\left(\frac{\gamma_2}{n_2^2} + \frac{\gamma_3}{n_3^2}\right)}{\frac{k_x^2}{n_1^4} - \frac{\gamma_2\gamma_3}{n_2^2 n_3^2}}\right].$$

5(b) Beat length: $L_B = \frac{\lambda_0}{|n_{TE} - n_{TM}|} = \frac{1.3\ \mu m}{|1.3361 - 1.2495|}$.

$= \frac{1.3\ \mu m}{0.0866}$.

$\approx 15\ \mu m$.

circular polarization: $\frac{L_B}{4} = \frac{15}{4}\ \mu m = 3.75\ \mu m$

$\cos^2 = \frac{1+\cos 2x}{2}$

(f) Power: $\frac{1}{2} Re\left[\int_{-\infty}^{\infty} (E_x H_y^*) dx\right]$.

$E_x = \frac{\beta}{\omega\epsilon} H_y \longrightarrow P = \frac{1}{2}\int_{-\infty}^{\infty} \frac{\beta |H_y|^2 dx}{\omega \epsilon_0 n^2(x)} = \frac{\beta}{2\omega\epsilon_0}\int \frac{|H_y|^2 dx}{n^2(x)}$.

Symmetric modes $H_y \to A\cos(K_x x)$.

$n \longrightarrow n_1$

$P_{core} = \frac{\beta A^2}{2\omega\epsilon_0 n_1^2}\left[\int_0^d \frac{dx}{2} + \int_0^d \frac{\cos(2K_x x)}{2} dx\right]$

$= \frac{\beta A^2}{2\omega\epsilon_0 n_1^2}\left[\frac{d}{2} + \frac{\sin(2K_x d)}{2(2K_x)}\right]$

$= \frac{\beta A^2}{4\omega\epsilon_0 n_1^2}\left[d + \cancel{\frac{\sin(2K_x d)}{(2K_x)}}\right] \to 0$.

For guided modes, second term $= 0$.

In cladding:

$P_{clad} = \frac{\beta C^2}{2\omega\epsilon_0}\int_{-\infty}^{0} e^{2\gamma x} dx$.

$= \frac{\beta C^2}{4\omega\epsilon_0 n_2^2 \gamma}$.

boundary cond$^n$: $C = A\cos(K_x d)$.

$\therefore \underbrace{\frac{\beta A^2 \cos^2(K_x d)}{4\omega\epsilon_0 n_2^2 \gamma}}_{\text{cladding 1}} + \underbrace{\frac{\beta A^2}{4\omega\epsilon_0}\frac{d}{n_1^2}}_{\text{core}} + \underbrace{\frac{\beta A^2 \cos^2 K_x d}{4\omega\epsilon_0 n_2^2\gamma}}_{\text{cladding 2}} = P_{total}$.

$\cos^2(K_x d) = \frac{1}{1+\tan^2(K_x d)} = \frac{n_2^4 K_x^2}{n_1^4\gamma^2 + n_2^4 K_x^2}$.

$\therefore P_{tot} = \frac{A^2\beta}{2\omega\epsilon_0 n_1^2}\left[\frac{d}{2} + \frac{(n_1^2 n_2^2)}{\gamma}\left[\frac{K_0^2(n_1^2 - n_2^2)}{n_1^4\gamma^2 + n_2^4 K_x^2}\right]\right]$ (proved)

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# || plate waveguides.

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(1) $\lambda_{cm} = \frac{2nd}{m}$ $\lambda < \lambda_{cm}$

$\lambda_{c1} = 2 \times d = 1.4$ cm.
$\Rightarrow d = 0.2$ cm.

$\lambda_{op} = \boxed{0.1 \text{ cm.}}$

$\frac{2 \times 1 \times 0.2}{m} = \lambda_{cm}$

$\Rightarrow \frac{0.4}{m} = \lambda_{cm}$.

$\Rightarrow \lambda_{op} < \lambda_{cm}$

$\Rightarrow \lambda_{op} = 0.1$ cm $< \frac{0.4}{m}$ cm

$m < 4. \longrightarrow m = 1, 2, 3.$

$\therefore$ TE/TM modes $= 3 \times 2 = 6$ total modes.

(2) $f > 3$ GHz $\Rightarrow \frac{mc}{2d\sqrt{\epsilon_r}} > 3 \times 10^9$.

$d < \frac{(m=1) \times 3 \times 10^8}{2 \times 3 \times 10^{10} \times \sqrt{2.1}}$ [m = 1 (lowest cutoff)].

$d < 0.0345 \Rightarrow \boxed{d_{max} = 3.45 \text{ cm.}}$

TEM has no cutoff. But, TE / TM modes DO have a cutoff frequency & all operating frequencies MUST be GREATER than the TE/TM cutoff. So, if we keep our upper bound as the TE/TM cutoff, the wave will never propagate in TE/TM mode.

(3) $f_c = \frac{mc}{2d\sqrt{\epsilon_r}}$ $\left[ \begin{array}{l} m = 2 \quad d = 10^{-2} \\ f_c = 10 \text{ GHz. find } \epsilon_r \end{array} \right]$.

$\cancel{10} \times \cancel{10^9}^{10} = \frac{\cancel{2} \times 3 \times \cancel{10^8}}{\cancel{2} \times \cancel{10^{-2}} \times \sqrt{\epsilon_r}} \Rightarrow \sqrt{\epsilon_r} = 3.$

$\therefore \boxed{\epsilon_r = 9}$.

(4) $d = 10^{-2}$ cm. $f_{op} = 32 \times 10^{9}\ s^{-1}$.

$n = 1.45$

$f > f_{cm} \Rightarrow f > \frac{mc}{2d\,n}$

$\therefore 32 \times 10^{9} > \frac{m \times 3 \times 10^{8}}{2 \times 10^{-2} \times 1.45}$

$m < \frac{32 \times 10 \times 2 \times 10^{-2} \times 1.45}{3}$

$m < 3.09$ $\therefore m = 1, 2, 3$

$\therefore$ modes $\rightarrow$ $TE_{c1}, TE_{c2}, TM_{c1}, TM_{c2}, TE_{c3}, TM_{c3}$

(5) $V_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$ $f_{c2} = \frac{2 \times 3 \times 10^{8+2}}{2 \times 10^{-2} \times 1.45} = 31.03\ GHz$

$\therefore \left(\frac{f_c}{f}\right)^2 = \left(\frac{31.03}{32}\right)^2$

$v_p = 4.1 \times c.$ $\therefore v_g \cdot v_p = c^2.$

$v_g = \frac{c^2}{v_p} = \frac{c^2}{4.1\,c} = \frac{c}{4.1}$

$\therefore$ group delay $\Rightarrow \frac{L}{v_g} = \frac{10 \times 10^{-2}}{3 \times 10^{8}} \times 4.1$

$= 13.6 \times 10^{-10}\ s = 1.36 \times 10^{-9}\ s.$

Highest mode group delay $= 1.36\ ns.$

TEM delay $= \frac{nL}{c} = \frac{1.45 \times 10^{-1}}{3 \times 10^{8}}$ $\left[ n = \frac{c}{v} \therefore v = \frac{c}{n} \right]$

$\therefore$ TEM delay $= 0.483 \times 10^{-6} = .483\,\mu s$

$\therefore \delta = (1.36 - .48)\,\mu s$

$= \boxed{0.876\,\mu s}$

(c) $f_{c1} = 7.5\,GHz = \frac{c}{2nd}$. $\lambda_{op} = 1.5$ cm.

$f_{op} = c/\lambda$

$= \frac{3 \times 10^8}{3/2 \times 10^{-2}} = 20\,GHz$.

$v_{g1} = \left(\sqrt{1 - \left(\frac{f_{c1}}{f}\right)^2}\right) \cdot c$.

$\therefore v_{g2} = \sqrt{1 - \left(\frac{f_{c2}}{f}\right)^2} \cdot c$. $\boxed{f_{c2} = 2 \times f_{c1}}$.

$\therefore v_{g2} = \sqrt{1 - \left(\frac{f_c}{f_{op}}\right)^2}\, c$.

$= 0.661 \times 3 \times 10^8$

$= \boxed{1.98 \times 10^8\ m/s}$.

(D) Rectangular WG: Dominant mode is $TE_{10}$, & then it is $TE_{20}$ / $TE_{01}$. (lowest cutoff).

$f_{c_{mn}} = \frac{c}{2\sqrt{\epsilon_r}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$ $a = 2b \longrightarrow \frac{K}{b}\sqrt{\frac{m^2}{4} + n^2}$.

$\therefore TE_{20} \longrightarrow \frac{K}{b}$, $TE_{01} \longrightarrow \frac{K}{b}$

$\therefore \frac{c}{2a} < f < \frac{c}{a}$ [Air filled], $\therefore \boxed{TE_{20} = TE_{01}}$.

$\frac{c}{2a\sqrt{\epsilon}} < f < c/a\sqrt{\epsilon}$.

Optimal solutions $\boxed{\frac{c}{2a\sqrt{\epsilon}} < f < \frac{c}{a}}$

OR $c/a\sqrt{\epsilon} = c/2a$.

8) $a = 6$ cm
$b = 4$ cm.

$f > f_c$

$$f > \frac{c}{2a\sqrt{\epsilon_r}} = \frac{3\times10^8}{2\times 6}$$

$TE_{10}$

$$= .25\times10^8$$
$$= 0.025\ GHz$$

$TE_{01}$ mode.

$$f < \frac{c}{2b} = \frac{3\times10^8}{2\times4} = .0375\times10^9$$
$$= 0.0375\ GHz.$$

(a) ∴ frequency for operating in single mode
i.e. $TE_{10} \longrightarrow TE_{01}$ $\boxed{25\ MHz < f_0 < 37.5\ MHz}$

(b) for frequencies greater than 37.5 GHz. the guide will support both $TE_{01}$ & $TE_{10}$.

7) $\tan\theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \approx 0.7246$ $\theta_B \approx 36^\circ$.

$$f = \frac{mc}{d\sqrt{\epsilon_1+\epsilon_2}} = \frac{1\times3\times10^8\times10^2}{1\times\sqrt{4+21}} \approx 12.15\ GHz.$$

(b) The cutoff frequency for higher order modes $(m>1) \longrightarrow (mf_1)$. will exceed the value for single mode frequency. Hence, only $m=1$ mode is allowed/ permitted.

(10) continued

$$\frac{d}{\lambda\sqrt{\epsilon}} = \frac{c}{2a} \Rightarrow \boxed{\epsilon = 4.}$$

$\therefore$ frequency range: $\boxed{\frac{c}{2a\sqrt{\epsilon_r}} < f < \frac{c}{2a}}$.

(11). Operating freq: 15 GHz $\rightarrow$ $TE_{10}$.

$TE_{10} < TE_{20} < TE_{01}$.

$TE_{20} = \frac{TE_{10}}{0.9} = 16.67$.

$f_{20} = \frac{c}{a} \therefore a = \frac{c}{f_{20}}$

$\therefore a = \frac{3 \times 10^8}{16.67 \times 10^9}$

$= 0.018m$

To suppress $TE_{01}$, we ensure $f_{c_{01}} > 16.67$ Hz.

$b < \frac{c}{2f_{c_{01}}} = \frac{3 \times 10^8}{2 \times 16.67 \times 10^9} = \boxed{0.009 cm}$

$\boxed{b_{max} = 0.009 \text{ cm.}}$

(12) To prove

$$P_{av} = \frac{\beta_{10}}{2\omega\mu} E_0^2 \sin^2(k_{10}x) a_z \quad W/m^2$$

using $\boxed{P_{av} = \frac{1}{2} Re[E_s \times H_s^*]}$.

For $TE_{10}$.

$$E_y = -j\frac{\omega\mu}{(\cancel{\pi/a})^2}\left(\frac{\pi}{a}\right) H_0 \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_{10} z}.$$

$$E_y = -j\frac{\omega\mu}{(\pi/a)} H_0 \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_{10} z}.$$

$$H_x = \frac{j\beta_{10}}{(\pi/a)^2}\left(\frac{\pi}{a}\right) H_0 \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_{10} z}$$

$$\beta_{10} = \sqrt{\omega^2\mu_0\epsilon_0\mu - (\pi/a)^2}.$$

$$\beta_{10} = \sqrt{k_0^2 n^2 - (\pi/a)^2}.$$

$$P_{av} = \frac{1}{2}\mathrm{Re}\left[\int \frac{\omega\mu\beta_{10}}{(\pi/a)^2} |H_0|^2 \sin^2\left(\frac{\pi x}{a}\right) dx\right].$$

From $E_y \Rightarrow$ $|E_y| = \frac{\omega\mu a}{\pi}|H_0| \sin\left(\frac{\pi x}{a}\right) \Rightarrow \left(\frac{E_y \pi}{\omega\mu a} = H_0\right)$

$$P_{av} = \frac{1}{2}\left[\frac{\beta_{10}\cancel{\pi^2}}{\omega\mu \cancel{\pi^2/a^2}\, a^2} E_0^2 \sin^2\frac{\pi x}{a}\right] = \left[\frac{\beta_{10}}{2\omega\mu} E_0^2 \sin^2\frac{\pi x}{a}\right].$$

$$k_{10} = \pi/a$$

$$\therefore \boxed{P_{av} = \frac{\beta_{10}}{2\omega\mu} E_0^2 \sin^2(k_{10} x).}$$ (proved).

১৩ আষাঢ় বৃহস্পতি[illegible]
একাদশী রাঃ ১০/৪৮
Saka - 8 Ashar 1945
অহম - ১৩ আহার ১৪৩[illegible]
Sunrise - 4.56 A.M.

JUNE 29 THURSDAY
Id-Uz-Zuha

১১ आषाढ़ शुक्ल गुरूवार २०८०
एकादशी रा० १०/४८
Hizri - 10 Zilhazza 1444
२९ जुन २०२३
Sunset - 6.22 P.M.

Metallic WG

[Pg-1]

Rohit Dutta.
20PH20035.

Metallic waveguides.

Assignment

(1) $TE_{11}$ mode : $m=1$, $n=1$.

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}\ \ cm^{-1} \qquad \left[n=\sqrt{\epsilon_r}\right].$$

$$= \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} \qquad \left[\frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}}\right] \left[\begin{matrix} m=1,\ n=1 \\ \epsilon_r = 4\end{matrix}\right].$$

$$= \frac{3\times 10^8}{2\times\sqrt{4}} \times \sqrt{\left(\frac{1}{20/3}\right)^2+\left(\frac{1}{20/4}\right)^2} = \frac{3\times 10^8}{2\times 2} \times \frac{c}{\frac{20}{4}}$$

$$\lambda_c = \frac{c}{f_c} = \frac{c}{\frac{c}{2\times 2}\times\frac{1}{4}} = 16.$$

$$\therefore \lambda < \lambda_c \quad [\because f > f_c].$$

$$\therefore \lambda_{max} = \lambda_c = \boxed{16\ cm.}\ (ans)$$

(2) Minimum frequency for metallic waveguides happens for dominant mode i.e $TE_{10}$ mode. i.e $m=1$, $n=0$.

| S | S | M | T | W | T | F | S | S | M | T | W | T | F | S | S | M | T | W | T | F | S | S | M | T | W | T | F | S | S | M | JUL |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 2023 |

[Pg - 2]

$TE_{10}$ mode: $f_c = \dfrac{c}{2a\sqrt{\epsilon_r}} = \dfrac{3\times10^8\times10^2}{2\times10\times\sqrt{1}}$ cycles/s $^{-1}$

($\epsilon_r$ : 1 for air)

$= 1.5\times10^9\ s^{-1}$.

$\boxed{(a)} \longleftarrow = 1.5\ GHz$ (ans)

3.) TM modes CANNOT have $m, n, p = 0$.

(c) $m, n, p > 0$. Hence $TM_{110}$ is impossible

4.) $v_p \cdot v_g = c^2$. $v_p = \dfrac{c}{\sqrt{1-\left(f_c/f\right)^2}}$.

(a)

Now, $f > f_c$ [ALWAYS].

$\therefore v_p = c/\alpha$ [where $0<\alpha<1$].

$\therefore \boxed{v_p > c}$ & $v_p \cdot v_g = c^2 \Longrightarrow \boxed{v_g < c}$

$\therefore v_p$ is greater than velocity of light in free space.

Pg-3

5-
(a) For $\boxed{\text{ideal}}$ rectangular wave guide, dominant $TE_{10}$ mode has zero attenuation

(d) It also has the highest cutoff wavelength & consequently the lowest cutoff frequency.

(6) $\boxed{u_p > c}$ for air filled rectangular waveguide. $\boxed{c}$

(7) In $TE_{10}$ mode also (dominant), $u_p > c$. $\boxed{d}$

(8) $$c_\epsilon = \frac{1}{\sqrt{\mu\epsilon}} \Rightarrow \frac{c_0}{\sqrt{\epsilon_r}} = \frac{3\times10^8}{3/2} = \boxed{2\times10^8\ m/s}$$ $\boxed{n = \sqrt{\epsilon_r}}$

Derivation for $\boxed{n = \sqrt{\epsilon_r}}$

$$n = \frac{c_0}{c_\epsilon} = \frac{\sqrt{\epsilon\mu}}{\sqrt{\epsilon_0\mu_0}} = \frac{\sqrt{\epsilon_r\mu_r}\cancel{\sqrt{\epsilon_0\mu_0}}}{\cancel{\sqrt{\epsilon_0\mu_0}}} = \sqrt{\epsilon_r}$$

$\therefore \boxed{n = \sqrt{\epsilon_r}}$ $[\because \mu_r = 1]$

$$\lambda = c/f = \frac{2\times10^8}{10^{14}} = 2\times10^{-6} = 2\,\mu m \quad \boxed{c}$$

9) (b) $TM_{10}$ mode does NOT exist
∵ [m, n > 0] for $TM_{mn}$ modes in rectangular metallic waveguide.

10) $f_c = \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times10^{8}\times10^{2}}{2\times1}\sqrt{\left(\frac{2}{5}\right)^2+\left(\frac{1}{3}\right)^2}$

$= 0.7810249\times10^{10}$

∴ In MHz = 7810.249 MHz
≈ $\boxed{7810.25\ \text{MHz}}$ (ans)

11) cutoff frequency = $\frac{c}{2\sqrt{\epsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$

$\boxed{(a)(b)}$ Dielectric & mode number (m,n) & waveguide dimensions (a×b).

12) $v_p > c$, $v_p > v_g$ $\boxed{(a),(b)}$

13) $H = H_0\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left[\omega t - \sqrt{\left(\frac{\omega}{c}\right)^2-\left(\frac{m\pi}{a}\right)^2-\left(\frac{n\pi}{b}\right)^2}\,z\right]$

→ Field (H) equation for TE mode. → (b)

JULY
01
SATURDAY

১৫-১৬ আষাঢ় ১৪৩০
Ashar 1945
Sunrise - 4.57-4.58 A.M.

१३-१४ आषाढ़ शुक्ल शनि-रवि २०८०
त्रयोदशी रा० ८/५४/चतुर्दशी रा० ७/२०
Hizri - 12-13 Zilhazza 1444
१-२ जुलाई २०२३
Sunset - 6.22-6.22 P.M.

$\boxed{Pg-5}$

$\boxed{v_p = c/\beta}$ $\boxed{\beta = \sqrt{(\omega/c)^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$ AKA $\sqrt{k^2 - k_c^2}$.

$\frac{m\pi}{a} = 2.094 \times 10^2 \ m^{-1}$ $\omega = 6.283 \times 10^{10} \ s^{-1}$.

$\frac{n\pi}{b} = 2.618 \times 10^2 \ m^{-1}$ $a = 3 \ cm, b = 1.2 \ cm$.

$m = 1.999 \approx 2$. $\therefore$ Mode $= TE_{21}$. $\boxed{\beta k \beta}$

$n = 1$.

$\beta = \sqrt{(209.43)^2 - (209.4)^2 - (261.8)^2}$ ~~= 395.302 m~~

$\therefore$ ~~$v_p$~~ $\boxed{k < k_c}$ $\left[k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}\right]$ [For propagating wave $\boxed{k_0 > k_c}$ $Re(\beta) > 0$]

Sunday 02

The wave is $\boxed{\text{evanescent}}$ since $\boxed{\beta \text{ is imaginary}}$ & hence would NOT propagate.

(14) $Z = \frac{E_t}{H_t}$ For $Z_{TE} = \frac{\omega\mu}{\beta}$, $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$,

$\therefore \eta_0 = \sqrt{\frac{\mu_0 \epsilon_0}{\epsilon_0^2}} = \frac{1}{c} \cdot \frac{1}{\epsilon_0} = \frac{1}{\omega c}$ $Z_{TM} = \frac{\beta}{\omega \epsilon}$ $\approx \boxed{377 \Omega}$

T W T F S S M T W T F S S M T W T F S S M T W T F S S M T W T AUG
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 2023

$\eta_{air} = \frac{1}{c}$

Pg-6

$$Z = \frac{\eta_0}{\left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2}} = \frac{1}{c\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Also, $\eta = \frac{\eta_0}{u_2}$

$(u > 1)$

$\therefore (\eta > \eta_0)$ Also

$$\therefore \eta_0 < \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \quad \therefore \eta_0 < \eta$$

(b) $\therefore$ wave impedance > Free space impedance.

(c) Phase velocity $(v_p)$ > Free space velocity $(c)$.

(*) TEM mode is NEVER possible in metallic waveguide. Hence option (D) is WRONG.

(a) Guided wavelength is ~~ALWAYS~~ NEVER less than free space wavelength. $(\lambda > \lambda_c)$. Hence option (A) is correct.

$$K_c = \frac{2\pi}{\lambda_c} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \Rightarrow \lambda_c = \frac{2\pi}{\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}} = \frac{2\pi}{K_c}$$

$\lambda = \frac{2\pi}{K}$ $\quad K > K_c$

$$\frac{2\pi}{K} = \lambda < \frac{2\pi}{K_c} = \lambda_c \quad \text{i.e. } \lambda < \lambda_c$$

guided wavelength

(a), (b), (c)

JULY

08

SATURDAY

Pg-7

...৩০০

.../১৮/সপ্তমী রাঃ ১/১২

Ashar 1945

২২-২৩ আহার ১৪৩০

Sunrise - 5.00-5.00 A.M.

৫/৬-৭ श्रावण कृष्ण शनि-रवि २०८०

पंचमी प्रा० २/१५/षष्ठी रा० ३/१८/सप्तमी रा० १/१२

Hizri - 19-20 Zilhazza 1444

८-९ जुलाई २०२३

Sunset - 6.22-6.22 P.M.

15) $H_0$ $H = (0.1)\cos(25\pi x)\cos(30.3\pi y)\cos(12\pi \times 10^9 t - \beta z)$.

$\frac{m\pi}{a}$ $\frac{n\pi}{b}$ $\omega$

C.

1) $\frac{m\pi}{a} = 25\pi$

2) $\frac{n\pi}{b} = 30.3\pi$.

3) $a = .08$

4) $b = .033$

5) $\omega = 12\pi \times 10^9$.

$m = 1$.

$n = 2$

$TE_{21}$ (c)

16) (c)

$a = 1$ cm

$b = .5$ cm.

$\therefore f_c = \frac{c}{2a\sqrt{\epsilon_r}} = \frac{3\times10^8 \times 10^2}{2\times1\times\sqrt{1}} = 1.5\times10^{10}$

$= 15$ GHz (ans)

17) (a) $f_c = \frac{c}{2a\sqrt{\epsilon_r}} = \frac{3\times10^{10}}{2\times3\times2} = .25\times10^{10}$

$= 2.5$ GHz (ans)

18) $TE_{10}$ ($f_c = 18$ GHz). $f_c \propto \frac{1}{a}$.

(c) $TE_{30} = ?$

$f_{c1}$ $a_2$ 3

$f_{c2}$ $a_1$ 1

$f_{c2}$ 6 GHz.

Sunday 09

$f = \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$,

$6 = \frac{c}{2a}$

$a = \frac{3\times10^{10}}{2\times6\times10^9}$

$a = .25\times10^{10}$

$= 2.5$ cm

$f_{TE_{30}}(m=3, n=0) = \frac{cm}{2a\sqrt{\epsilon_r}}$ $(\epsilon_r = 1)$. $= \frac{cm}{2a}$

$= f_D\, m$

$\therefore f_{TE_{10}} = f_{TE_{30}}/m = 6$ GHz.

ANS

T W T F S S M T W T F S S M T W T F S S M T W T F S S M T W T AUG
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 2023

Pg-8

constant

(19) $E = S H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin(\omega t - \beta z)$.

m=0, therefore no variation along x direction.
Therefore, answer is (a).

(21) (a) Since $\cos\left(\frac{n\pi y}{b}\right)$ component is not available in the electric field equation for TE mode, thus n=0.
TM mode cannot have m, n=0, therefore, we have $TE_{20}$

(22) (a) $f_c = \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = \frac{3\times10^{8+2}}{2\times1}\times\sqrt{\frac{5^2}{3^2\cdot4^2}} = \frac{3\times10^{10}\times5}{2\times3\times4} = \boxed{6.25\text{ GHz}}$ (ans)

(23) $TE_{10} = \frac{c}{2a\sqrt{\epsilon_r}}$ $TE_{20} = \frac{c\cdot2}{2a\sqrt{\epsilon_r}} = \frac{c}{a\sqrt{\epsilon_r}}$ $\boxed{TE_{11} = \frac{\lambda+\lambda/2}{2} = \frac{3\lambda}{4}}$

$\therefore \frac{3}{4}\frac{c}{2a\sqrt{\epsilon_r}} = \frac{c}{2\sqrt{\epsilon_r}}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} \Rightarrow \frac{9}{16a^2} = \frac{1}{a^2}+\frac{1}{b^2}$

$\frac{3}{4}\frac{1}{a} = \sqrt{\frac{1}{a^2}+\frac{1}{b^2}} \Rightarrow \left(\frac{4}{3}\right)^2 a^2 = a^2+b^2 \Rightarrow \left[\left(\frac{4}{3}\right)^2-1\right]a^2 = b^2$

$\therefore b = a\sqrt{\frac{16-9}{9}} = \sqrt{5/9}\times7 = \boxed{1.972}$ (ans)

[Pg-9]

(25) $\frac{c}{2a} = 6\text{ GHz}$ $\qquad \frac{c}{2b} = ?$

$\frac{c}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = 15\text{ GHz}$ $\quad\Big|\quad a = \frac{3\times 10^{8}}{2\times 6\times 10^{9}} = \frac{1}{4}\times 10^{-1}$

$\frac{\frac{1}{a}}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} = \frac{\cancel{6}\,2}{\cancel{15}\,5} = \frac{\frac{1}{a}}{\frac{\sqrt{a^2+b^2}}{ab}} = \frac{b}{\sqrt{a^2+b^2}} = \frac{2}{5}$

$\Rightarrow 25b^2 = 4a^2 + 4b^2$

$\Rightarrow 21b^2 = 4\times a^2 \quad (a = 2.5\times 10^{-1+2}\text{ cm})$

$\therefore \boxed{b} = \sqrt{\frac{4\times(2.5)^2}{21}} = \boxed{1.091\text{ cm}}$

$\frac{3\times 10^{10}}{2\times 1.09108945} = 1.3747\times 10^{10} = \boxed{13.74\text{ GHz}}$

$\therefore f_c\,[\text{for } TE_{01}] = \boxed{13.74\text{ GHz}}$ (ans)

(26) (a, b) are automatically wrong since $f_c$ is minimum for $TE_{10}$.

we know $TE_{10} < TE_{20} < TE_{20}$. Hence (c) is wrong ($TE_{11} \rightarrow \sqrt{1/a^2+1/b^2}$, $TE_{20} = \sqrt{4/a^2}$ & $b < (a/2)$)

(d) $\longrightarrow$ correct : ~~$TE_{10} < TE_{11} < TE_{20} < TE_{01}$~~

$\boxed{TE_{10} < TE_{20} < TE_{01} < TE_{11}}$

Pg-10

(25) Propagation const = $\beta$.

$\omega = 10\ GHz$
$c = 3\times10^8\ m/s$.

$\beta = \sqrt{K^2 - K_c^2}$

$a = 2.286\ cm$
$b = 1.016\ cm$.

Since, no mode is specified, we can assume dominant $TE_{10}$ mode.

$\therefore K_c = \frac{m\pi}{a}\ (m=1) = \frac{\pi}{a} = 1.3743\ cm^{-1}$.

$K = \omega/c = \frac{10\times10^8}{3\times10^8} = 33.33\ m^{-1} = 0.33\ cm^{-1}$.

Again: $K < K_c$, Hence evanescent wave.

For propagating wave $K > K_c$ & $\boxed{Re(\beta) > 0}$

(20) $f_c @ TE_{20} = \frac{c}{2}\sqrt{\left(\frac{2}{a}\right)^2} = \frac{c}{2}\cdot\frac{2}{a} = c/a$, $2f_D$.

$\therefore f_c/f = \frac{1}{2}$

$\eta_z = \eta_0 \frac{K}{\beta} = \frac{\eta_0}{\sqrt{1-(f_c/f)^2}}$

$\eta = \frac{\eta_0}{\sqrt{1-1/2^2}} = \frac{2}{\sqrt{3}}\ \eta_0 = 435\,\Omega$