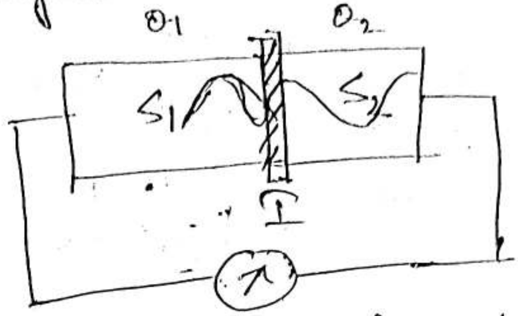
10/4/25

Josephson Junction Tunnelling :—

$J = J_0 \sin(\theta_1 - \theta_2)$



SQUID

$|\Psi\rangle = \alpha|\Psi_1\rangle + \beta|\Psi_2\rangle$

highly coherent state ⇒ single quantum state. (macroscopic)

AC J : $J = J_0 \sin\left(\delta_0 - \frac{2eV}{\hbar}t\right)$

$\delta = \delta_0 - \frac{2eV t}{\hbar}$ → can change phase by just changing biasing.

+ If super current flowing b/w 2 points of the wave function are ~~pts~~ different as these 2 points are not same phases of the wave function are not same.

The Josephson effect are due to the fact that phases can be changed by $\vec{B}$ or V. Josephson effects are the manifestation of quantum interference phenomenon on a macroscopic scale. or on a single quantum states.

Quantum ⇒ entire SC

Interference ⇒ the measured properties depend on the phase of the state which can be tuned by a $\vec{B}$ or V. and states w/ diff. phases can be made to interfere. The interference results in current oscillation which can be detected electrically.

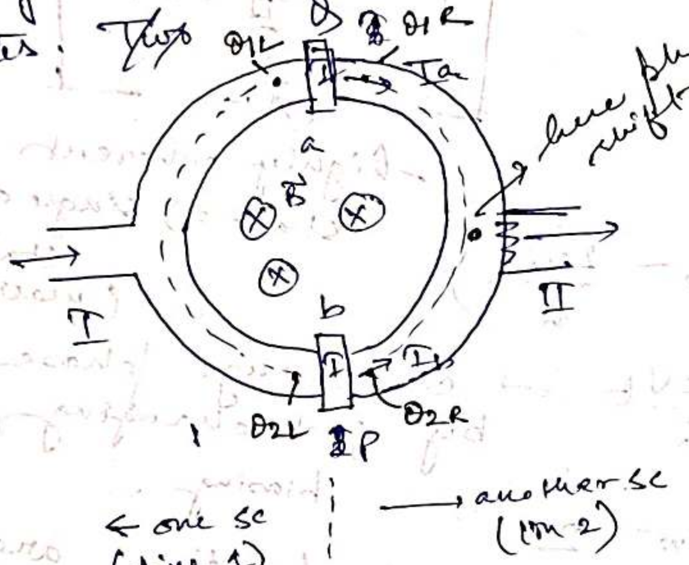
SQUID → Superconducting QUantum Interferometry Device

$\Phi_0 = \frac{h}{2e} = 2 \times 10^{-15}$ Wb $(T\text{-}m^2)$

interference pattern ≡ oscillating electric field
classical quantum

Supercurrent Interference

The JJ tunneling in presence of $\vec{B}$ provides strong evidence for highly coherent nature of SC states. Two



$$I = I_a + I_b$$

$\delta_a = \delta_b = \delta_0$ for same type of SC and same JJ (same I, same thickness)

Two Josephson junction are arranged in parallel combn and are placed in a region in which magnetic field $\vec{B}$ is imposed. ~~..~~ A supercurrent starting in region I is divided into two parts and get to flow parallel paths, each of which contains tunnel junction.

The current I_a and I_b ~~carrying~~ crossing the tunnel barrier a and b respectively unit in R-II. The combined current shows oscillation characteristics of an interference pattern produced by 2 coherent sources.

By analogy, I_a and I_b are regarded as two sources of current whose disturbances when superposed by the wave of recombination producing interference pattern. In view of Josephson tunneling, tunneling wave fn of of Cooper pair causes a phase shift to that wave fn of the superconducting state in R-II relative to R-I.

If phase shift at the two barriers in absence of magnetic field be δ_a and δ_b

$$\delta_a = \theta_{1L} - \theta_{2R}$$

$$\delta_b = \theta_{2L} - \theta_{2R}.$$

Then supercurrent through the 2 junctions ~~Ia~~

$I_a = I_0 \sin \delta_a$ and $I_b = I_0 \sin \delta_b$ — this is without magnetic field.

The phase diff. b/w two regions I. and II in ~~phase~~ presence of magnetic field of vector potential A, $J_s = 0$ and deep inside material → and $\vec{\nabla} \times \vec{A} = \vec{B}$.

$$\vec{\nabla} \theta = \frac{2e}{\hbar} \vec{A}$$

Line integral →

$$\int_P^Q \vec{\nabla} \theta \, dl = \frac{2e}{\hbar} \int_P^Q \vec{A} \cdot \vec{dl}$$

total phase shift →

$$\nabla \theta_I^{II} = \delta_a + \frac{2e}{\hbar} \int_a \vec{A} \cdot \vec{dl}.$$

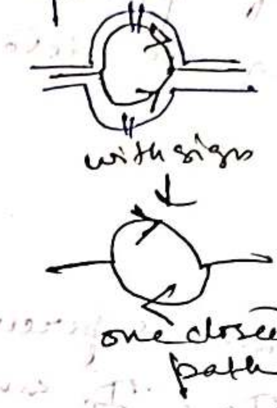
$$\nabla \theta_I^{II} = \delta_b - \frac{2e}{\hbar} \int_b \vec{A} \cdot \vec{dl}.$$

The two phase shifts coming back to same point ~~the~~ must be same as ψ is uniquely valued at each point.

$$\delta_a + \frac{2e}{\hbar} \int_a \vec{A} \cdot \vec{dl} = \delta_b - \frac{2e}{\hbar} \int_b \vec{A} \cdot \vec{dl}$$

$$\Rightarrow \delta_a - \delta_a = \frac{2e}{\hbar} \left[\int_a \vec{A} \cdot \vec{dl} + \int_b \vec{A} \cdot \vec{dl} \right]$$

Taken together with sign $\Rightarrow$ one closed path



$$\delta_b - \delta_a = \frac{2e}{\hbar} \oint \vec{A} \cdot \vec{dl}$$

$$= \frac{2e}{\hbar} \int_S \underbrace{(\vec{\nabla} \times \vec{A})}_{\vec{B}} \cdot \vec{dS}$$

$$\delta_b - \delta_a = \frac{2e}{\hbar} \Phi \quad \text{(flux enclosed by ring)}$$

The above relation states the total phase difference around loop can be controlled by varying magnetic field $\vec{B}$.

General : $\delta_a = \delta_0 - \frac{e}{\hbar} \int \vec{B} \cdot \vec{ds}$

$\delta_b = \delta_0 + \frac{e}{\hbar} \int \vec{B} \cdot \vec{ds}$

$\delta_a = \delta_b = \delta_0$ when $B = 0$ (initial phase diff)

$\delta_b - \delta_a = 0$ when $B = 0$ (this is when same type of SC used)

$$\therefore \delta_b - \delta_a = \frac{2e}{h} \cdot 2\pi \Phi$$

$$= (2\pi \Phi / \Phi_0) \Rightarrow \text{in units of flux quantum}$$

. Total current after recombining $\Rightarrow$ total recombined supercurrent

$$I = I_a + I_b$$

$$= I_0 \sin \delta_a + I_0 \sin \delta_b$$

$$= I_0 \sin\left(\delta_0 - \frac{e}{\hbar} \cdot \Phi\right) + I_0 \sin\left(\delta_0 + \frac{e}{\hbar} \Phi\right)$$

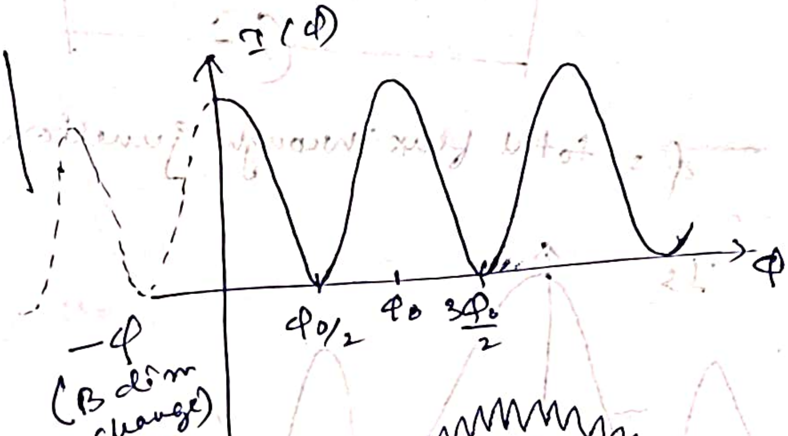
$$\therefore I = I_0 \left(2 \sin \frac{\delta a + \delta b}{2} \cdot \cos \frac{\delta b - \delta a}{2} \right)$$

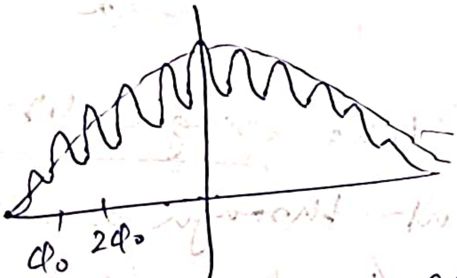
$$= 2I_0 \left[\sin \delta_0 \cdot \cos \frac{2\pi \cdot \Phi}{2\Phi_0} \right]$$

$$= 2I_0 \sin \delta_0 \cos (\pi \Phi / \Phi_0)$$

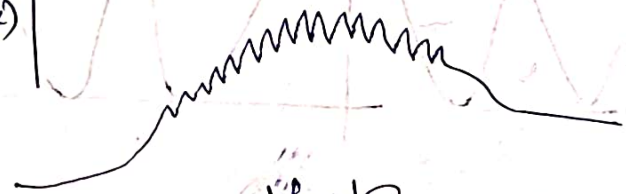
$$\therefore I(\Phi) \propto \left| \cos \left(\pi \Phi / \Phi_0 \right) \right|$$

with sin term





$\equiv$



output

as $\Phi \to \Phi_0, 2\Phi_0, \ldots$ additional peaks appear

[optics
sin = diffraction term
cos = interference

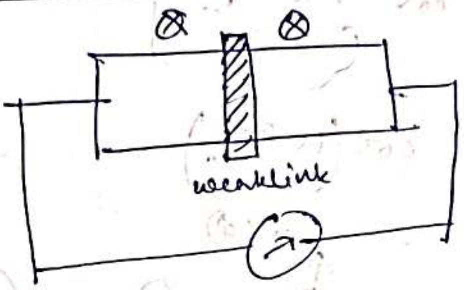
This modulation of observed SQUID ring critical current is shown. The current is essentially interference pattern observed in an ideal fraunhoffer optics with Young's double slit expt.

Here, the two Josepson Junction play the role of two slits and interference is b/w the supercurrent passing through 2 halves of the ring. The supercurrent acquire different phases due to $\vec{B}$.

The SQUID provides a simple but highly accurate system for measuring $\vec{B}$. magnetic flux. Since $\Phi \sim 10^{-15}$ T-m^2 one can make device ~ 1 cm^2, we can measure $B \sim 10^{-10}$ T (very small field or moment can be measured = bio fields)

SQUID = higest accuracy magnetometer

Single Josephson Junction



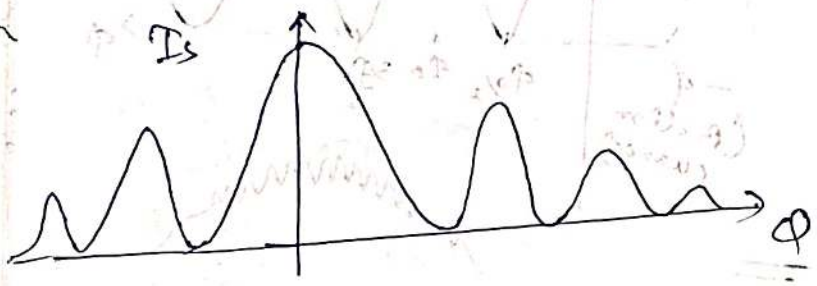
Tunnel current through one junction →

$$I_s = I_0 \frac{\sin(\pi\phi/\phi_0)}{\pi\phi/\phi_0}$$

$= I_0 \, \text{sinc}\,(\pi\phi/\phi_0)$

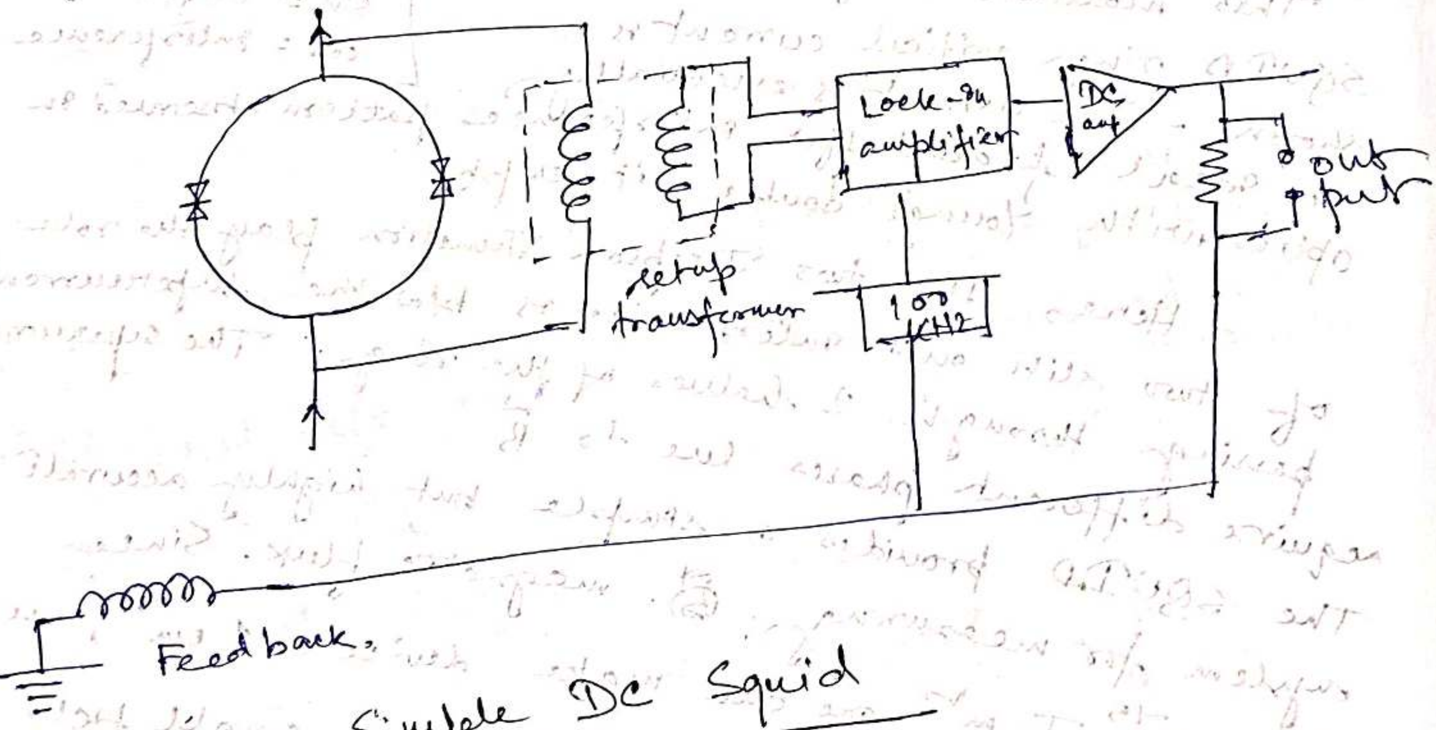
$\text{sinc} \rightarrow \frac{\sin x}{x}$

— ϕ = total flux through junction.



like single slit pattern

(magnetic field dependant current through a single J-J) (Sn/SnO/Sn)



Simple DC Squid

11/04/25

BCS Theory [Bardeen, Cooper & Schrieffer Theory] (1957)

BCS theory — well accepted for low T_c SC.

① Isotope effect $\quad T_c \propto M^{-1/2} \Rightarrow$ phonons are involved in superconductivity

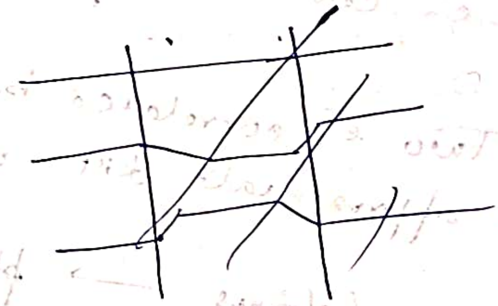
② cooper remodeled frolich's idea on the philosophy of an $e^- - e^-$ interaction into the e^- - phonon interaction mediated by phonon — cooper pair.

Cooper pair formation :- Cooper demonstrated that the creation condn. favourable for net attractive interactions b/w 2 electrons in a conductor, conductor transforms from normal state to SC state.

Electron-phonon interaction → coulombic repulsion and also
→ $e^- - e^-$ interaction ⇒ instanteneous
⇒ $e^- - e^-$ int. (retarded int) mediated by phonon
— e^- - lattice - e^- interaction can give rise to attraction

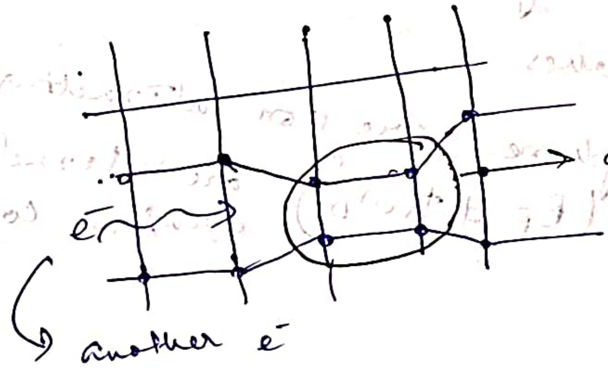
attraction to e^- from ion at lattice site

distorts lattice



↓

- electron travelling in a lattice leaves behind a disorted trail which can be regarded as an accumulation of +vely charged ion cores.



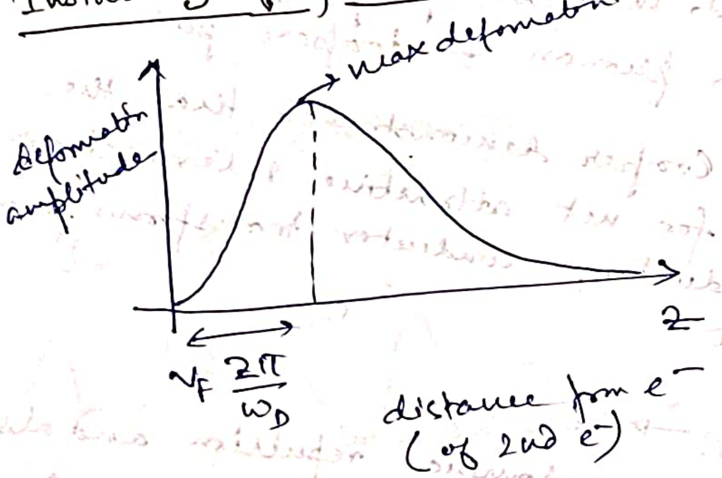
- area of enhanced +ve charges compared to neutral crystal is created behind e^- and exerts an attractive force on 2nd e^-

- e^- moves very fast but ions cannot move as fast, so next e^- behind that one feels attractive force.

- at low T, $e^- - e^-$ coulombic repulsion is weaker (due to screening)

e^- - phonon - e^- $\longrightarrow$ stronger ~~effects~~

<u>Instability of fermi sea at cooper pair formation</u> $\rightarrow$



phonon - vibration period $= \frac{2\pi}{\omega_D} \sim 10^{-13}$ s

$v_F = 10^8$ cm/s

$v_F \cdot \frac{2\pi}{\omega_D} \approx 1000\,\text{Å}$

This is a long distance process. At 1000 Å distance interaction can take place. At this far away distance $e^- - e^-$ coulomb repulsion is <u>weak</u>.

$\Rightarrow$ Two e^- correlated by lattice distortion will have an approximate dist. of 1000 Å.

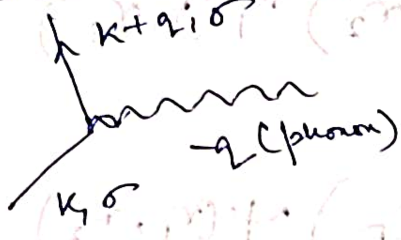
$e^- \longrightarrow$ deforms $\longrightarrow$ phonon created $\longrightarrow$ absorbed by next e^-.

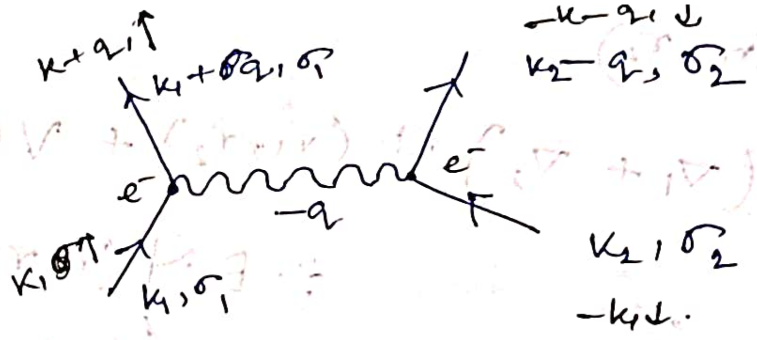
- An e^- travelling through crystal lattice leaves behind the deformation trail which can be regarded as an accumulation of +vely charged cores.

$\Rightarrow$ compression of lattice planes

- e^- well within fermi sphere are non interacting
e^- close to fermi level $(E_F \pm \hbar\omega_D)$ are interacting and can form cooper pair.

e-p vertex





$k \to \uparrow$ σ = spin (up / down)

$-k \to \downarrow$

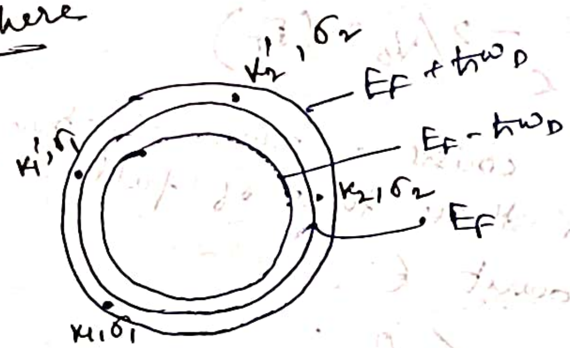
most favourable ~~spin~~ pair = opposite momentum and oppo. spin.

ideal $\left\{ \begin{array}{l} k_1 + k_2 = k_1' + k_2' = k = 0 \\ \Rightarrow k_1 = -k. \end{array} \right.$

$k_1' = k_1 + q$

$k_2' = k_2 - q$.

Fermi Sphere



The e⁻ act at (k_1, σ_1) and (k_2, σ_2) are scattered to (k_1+q, σ_1) and (k_2-q, σ_2). The int is attractive provided all wave vectors lie in the range b/w $E_k \pm \hbar\omega_D$

$$V_{eff} = |g_q|^2 \cdot \frac{1}{\omega^2 - \omega_q^2}$$

$$g_q = \sqrt{\frac{m}{M}} \sim 0.01$$

and $\omega < \omega_D$.

$$V_{eff}(q, \omega) = |g_{eff}|^2 \cdot \frac{1}{\omega^2 - \omega_D^2}$$

$$= -V_0.$$

$\psi(r_1, r_2)$

$$-\frac{\hbar^2}{2m}(\nabla_1 + \nabla_2)\psi(r_1, r_2) + V(r_1 r_2)\psi(r_1 r_2)$$

$$= E\,\psi(r_1 r_2)$$

$$= (\epsilon + 2E_{F_0})\psi(r_1, r_2)$$

$$\psi(r_1, r_2) = \frac{1}{\sqrt{L^3}}\, e^{i\vec{k}\cdot\vec{r}_1}$$

$$= \frac{1}{\sqrt{L^3}}\, e^{+i\vec{k}\cdot\vec{r}_2}$$

$$i\vec{k}\cdot(\vec{r}_1 + \vec{r}_2).$$

$$= \frac{1}{L^3}\cdot e$$

and $\epsilon = -2\hbar\omega_D\, e^{-2/V_0 Z(E_F)}$

Then we means there exists a two electron bound state whose energy is lower than that of fully occupied fermi sea by an amount ϵ.

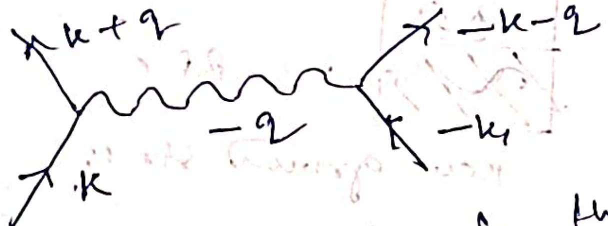
$$\epsilon = E - E_{F_0} < 0$$

The ground state of non. interacting free $e^- \rightarrow$ — becomes unstable when any attractive interaction b/w. electrons is switched on. (well within fermi sphere) close to E_F

The instability leads to formation of such electron pairs — cooper pairs having momentum $(k\uparrow, -k\downarrow)$

And system tries to reach new lower energy ground state. $(k\uparrow, -k\downarrow)$, $(k'\uparrow, -k'\downarrow)$ and so on.

cooper pairs w/ opposite k vectors and spin feynmann diagram →



Coulomb: int is reduced due to the screening (presence of other e^- in fermi sphere)

Something new occurs → the two e^- may attract e/o

The two e^- would then form bound state very close to fermi surface.

The binding energy is strongest when e^- forming the pair have opposite momentum and oppo. spins

All e^- in neighbourhood of fermi surface condense into a system of many cooper pairs.

The binding b/w e^- (1 and 2) — an energy gap appears in the spectrum of e^-

e^- polarizes lattice $(k\uparrow)$ → creates phonon — another $(-k\downarrow)$

e^- absorbs phonon → result → $(k-q\uparrow, -k+q\downarrow)$

Bose Einstein condensate →

At low temp cooper pairs are formed with favourable condn. The pair wavefn all have form and the superposn of pair wavelength describes → BEC.

BCS energy gap relation →

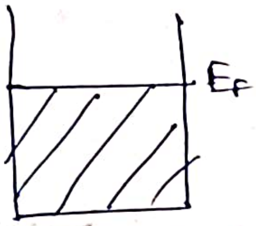
In simples form →

$$K_B T_c = 1.14\,\hbar\omega_D \exp\left[\frac{1}{-V N(E_F)}\right] \quad \text{—(1)}$$

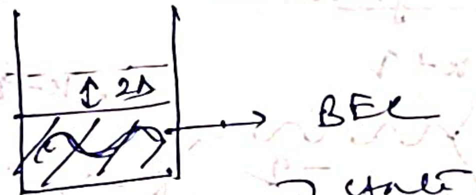
V = effective e^- photon int. potn.

$N(E_F)$ = density of states at E_F

ω_D = debye freq.



normal metal



new ground state

band gap

$$2\Delta(T=0) = 4\hbar\omega_D \exp\left[-\frac{1}{VN(E_F)}\right] \quad \text{—(2)}$$

$$\hbar\omega_D \sim 10^{-2}\,eV - 10^{-4}\,eV$$

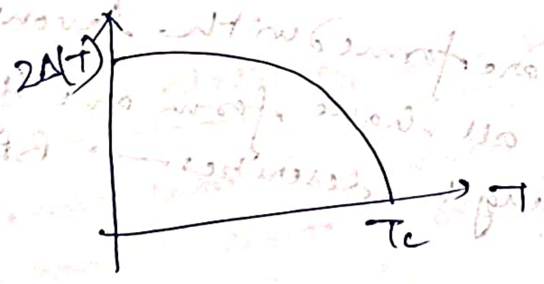
$$\frac{2\Delta(0)}{k_B T_c} = 3.53$$

energy to give to break cooper pair

element	$\frac{2\Delta(0)}{k_B T_c}$
In	4.1
Sn	3.6
Hg	4.6
Pb	4.1

measured values close to typical value.

at $T = T_c$, band gap vanishes.



$$2\Delta(T) = 2\Delta(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

BCS ground state →

$\psi(r_1 s_1 r' , s')$

r = electronic position
s = spin.

In a system of $N e^-$ the e^- are grouped into $N/2$ pairs

$$\phi(r_1 s_1, r_2 s_2, \ldots) = \psi(r_1 s_1, r_2 s_2)\psi(r_3 s_3 r_4 s_4) \psi(r_5 s_5 r_6 s_6) \ldots$$

(ground state)

$\phi_{BCS} = a\phi$

a = antisymmetrizer
ϕ_{BCS} have to antisym

created somewhere

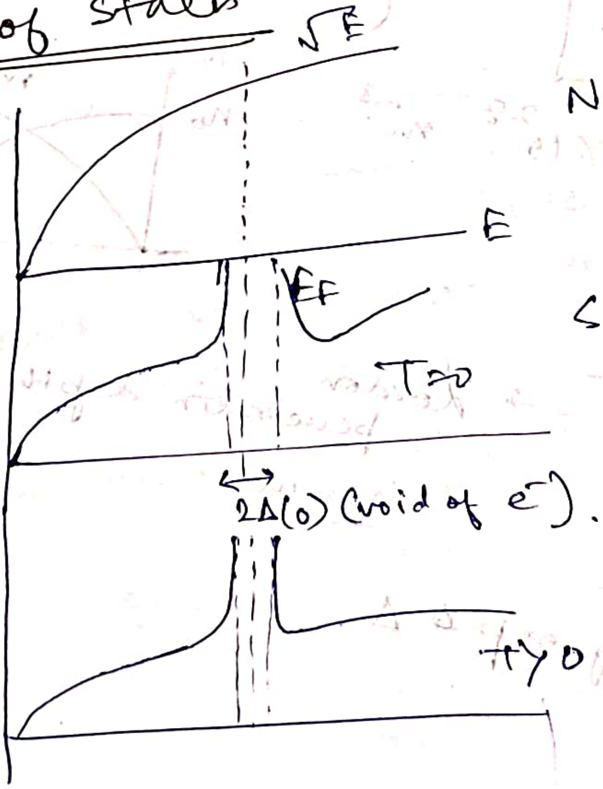
$$|\phi_{BCS}\rangle = \prod_k (u_k |0\rangle + v_k |1\rangle)$$

annihilated somewhere

normalisation : $u_k^2 + v_k^2 = 1$.

Book
Ibach & Luth
Kittel (Theory of solids)

Density of states

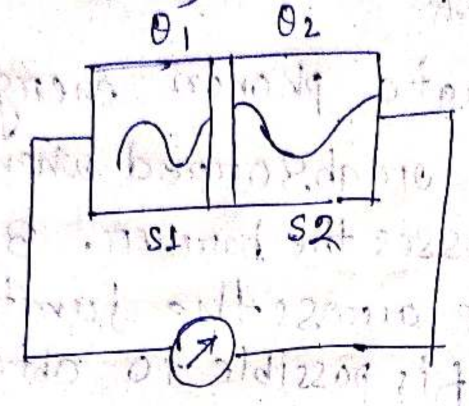


at $T>0$
$2\Delta(T) < 2\Delta(0)$

~~10/04/2025~~ 10/04/2025

Josephson junction tunneling

$J = J_0 \sin(\theta_1 - \theta_2)$



Highly coherent = single quantum state (macroscopic scale)

ACC $J = J_0 \sin\left(\delta(0) - \frac{2eVt}{\hbar}\right)$

$\delta(t) = \delta(0) - \frac{2eVt}{\hbar}$

A super ~~con~~ current flows between two points in the phase of the wave function are different as the points are not same.

The Josephson effect due to the fact that the phases can be changed by $\vec{B}$ & V.

The Josephson effect are manifestation of quantum interference phenomena on a macroscopic scale

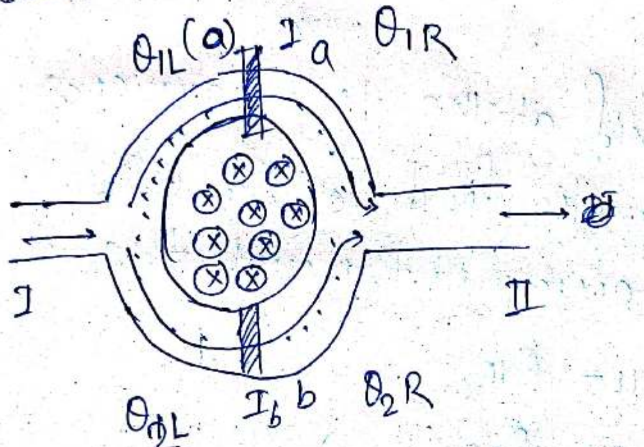
Quantum mean:- ~~E~~ Entire superconductor is in ~~the~~ single quantum state.

Interference - The measured properties depend on the phase of the state which can be tuned by a $\vec{B}$ or other agencies and states with different phases can be made by interference the interferences results in current oscillation. which can be detected electrically.

SQUID → ~~superconding~~ super conducting quantum Interferomemetric Device

Supercurrent Interference :-

The Josephson tunneling in presence of magnetic field provide strong evidence for the highly coherent Nature & superconducting state.



$I = I_a + I_b$

. two Josephson Junctions are arranged in parrallel combination and are placed in a region in which magnetic field ($\bar{B}$) is imposed. A supercurrent starting in region (I) is devided into two parts and made to flow along parrallel path each of contain josephson tunnel junction.

The current I_a and I_b crossing the tunnel barrier a and b respectively reunit in region II.. The combine current ~~source~~ shows oscillations characterics of an interferrence pattern produced by two coherent ~~sour~~ sources. By analosy of of interference of light I_a & I_b are regarded as two coherent sources of current whose distributions when superposed by the way of recombination produce an interference pattern.

In view of relation JJ tunneling the tunneling of cooper pair causes a phase shift & total wavefunction of the superconducting state in region II related to the region I.

If the phase shift at the two barriers in absence of magnetic field be δa & δb then the super~~con~~ current through the junction $\delta a = \theta_{1L} - \theta_{1R}$

$$\delta_b = \theta_{2L} - \theta_{2R}$$

$$\left. I_a = I_0 \sin \delta a \quad I_b = I_0 \sin \delta_b \right\}$$ without applying magnetic field.

The phase difference between two region I & II In the presence of magnetic field of vector product A

$J_s = 0$ deep inside the superconducting material

$$\nabla\theta = \frac{2e}{\hbar} A \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

taking the line integral of above equation.

$$\int_P^Q \nabla\theta \cdot dl = \frac{2e}{\hbar}\int_P^Q A\cdot dl$$

$$\boxed{\int \vec{B}\cdot ds = \int (\vec{\nabla}\times A) ds = \int A\cdot dl}$$

Scanned with OKEN Scanner

Therefore the total phase shift wavefunction along two paths from region I to II can be expressed

$$\nabla\theta_{I}^{II} = \delta a + \frac{2e}{\hbar}\int_a \vec{A}\cdot d\vec{l}$$

$$\nabla\theta_{II}^{I} = \delta_b - \frac{2e}{\hbar}\int_b \vec{A}\cdot dl$$

Two phase shift must be identical because wavefunction has a unique value at every point

so $$\delta a + \frac{2e}{\hbar}\int_a \vec{A}\cdot d\vec{l} = \delta_b - \frac{2e}{\hbar}\int_b \vec{A}\cdot d\vec{l}$$

$$\cancel{\frac{4e}{\hbar}\int \vec{A}\cdot d\vec{l} = (\delta_b - \delta a)}$$

$$\frac{2e}{\hbar}\int_a \vec{A}\cdot d\vec{l} + \frac{2e}{\hbar}\int_b \vec{A}\cdot d\vec{l} = (\delta_b - \delta a)$$

$$\delta_b - \delta a = \frac{2e}{\hbar}\oint A\cdot dl$$

Two line integrals are in opposite dirrection Therefore when taken together they give integral over a closed path

$$\delta_b - \delta a = \frac{2e}{\hbar}\int_S \vec{B}\cdot d\vec{s}$$

$$= \frac{2e}{\hbar}\phi$$ (using the stoke's theoream).

The above relation states that the total ~~field~~ phase difference aroand the loop can be controlled by varying the magnetic field $\vec{B}$.

The general expression for δa & δ_b

$$\delta a = \delta_0 \mp \frac{e}{\hbar}\int \vec{B}\cdot d\vec{s}$$

$$\delta_b = \delta_0 + \frac{e}{\hbar}\int \vec{B}\cdot d\vec{s}$$

$\delta = \delta_b - \delta_a = 0$ [when $B = 0$]

$\delta_b - \delta_a$

when $B \neq 0$

$$\delta_b - \delta_a = \frac{2e}{\hbar} \int \vec{B} \cdot d\vec{s} = \frac{2e}{\hbar}\phi \qquad \phi_0 = \left(\frac{h}{2e}\right)$$

$$= \frac{2e}{\hbar}\phi$$

$$= 2\pi \left(\frac{2e}{h}\right)\phi$$

$$= (2\pi)\left(\frac{\phi}{\phi_0}\right)$$

$= 2\pi \times$ times the flux in unit of flux quantum.

now the total current after re... $\Rightarrow$ total recombined supercurrent

$$I = I_a + I_b$$

$$= I_0 \sin\delta_a + I_0 \sin\delta_b$$

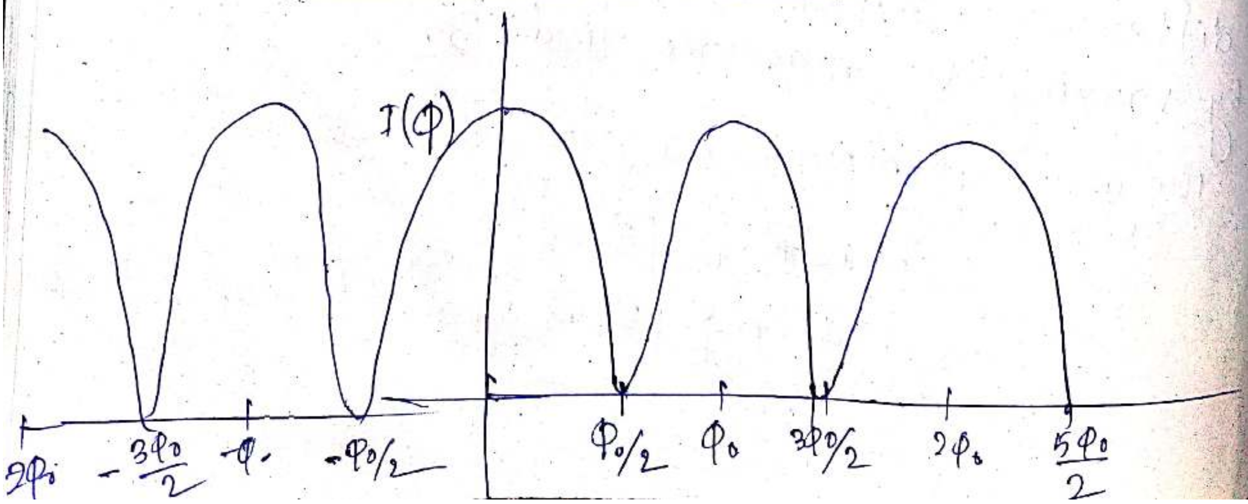
$$= I_0 \sin\left[\delta_0 - \frac{e}{\hbar}\int \vec{B}\cdot d\vec{s}\right] + I_0 \sin\left[\delta_0 + \frac{e}{\hbar}\int \vec{B}\cdot d\vec{s}\right]$$

$$= I_0 \sin\left[\delta_0 - \frac{e}{\hbar}\phi\right] + I_0 \sin\left[\delta_0 + \frac{e}{\hbar}\phi\right]$$

$$I = I_0 2 \sin\delta_0 \cos\left(\frac{e\phi}{\hbar}\right)$$

$$I = 2I_0 \sin\delta_0 \cos\left(\frac{e\phi}{\hbar}\right)$$

$$\boxed{I = 2I_0 \sin\delta_0 \cdot \cos\left(\pi \cdot \frac{\phi}{\phi_0}\right)}$$



$\Delta\theta_1 = \Delta\theta + \frac{\pi\Phi}{\Phi_0}$

~~$\Delta\theta_2 =$~~

$\Delta\theta_2 = \Delta\theta - \frac{\pi\Phi}{\Phi_0}$

$$I = I_0 \sin(\Delta\theta_1) + I_0 \sin(\Delta\theta_2)$$

$$= I_0 \{\sin\left(\Delta\theta + \frac{\pi\Phi}{\Phi_0}\right) + I_0 \sin\left(\Delta\theta - \frac{\pi\Phi}{\Phi_0}\right]$$

$$= 2I_0 \sin(\Delta\theta) \cos\left(\frac{\pi\Phi}{\Phi_0}\right)$$

This modulation observed SQUID ring critical current is shows the current ~~ems~~ essentially an ideal ~~form~~ Fraunhoffer interference pattern exactly analogoes to the interference pattern on observed in optics with young's two slit experiment.

Here two Josephson junctions are playing the role of slits & the interferrence is between the supercurrent passing through the two halves of the ring. The super ~~co~~ current acquire different phases due to the magnetic field. The SQUID Device provides a simple but highly accurate system for measuring magnetic flux. since the flux quantum Φ_0 only about 2×10^{-15} Wb or Tesla $-m^2$ in SI unit & are can make SQUID devices $1\,cm^2$ area $B \approx 10^{-10}$ T.

highly possible magnetometer → [SQUID magnetometer]

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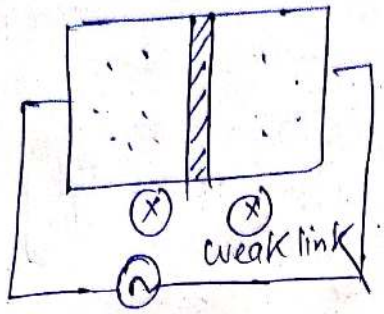
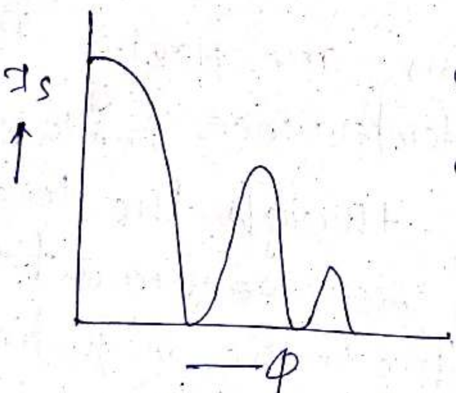


Tunnel current through are junction

$$I_s = I_0 \frac{\sin\left(\frac{\pi\phi}{\phi_0}\right)}{\cancel{\sin}\left(\frac{\pi\phi}{\phi_0}\right)}$$

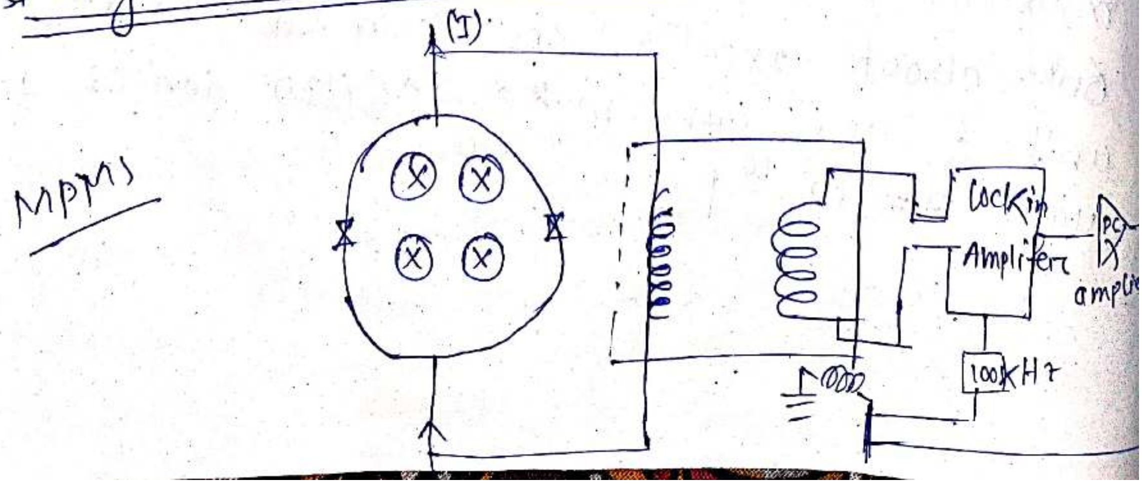
ϕ = Total flux throug the junction



single state light diffraction.

magnetic field dependent current through a single JJ B|| tunneling.

diagram of ~~Squi~~ SQUID



11/04/2025

BCS Theory (1957) Bardeen, Cooper & Schrieffer

BCS theory is well accepted for low T_c superconductor

(1) Isotope effect $T_c \propto M^{-1/2}$

$\Rightarrow$ phonons are involved in superconductivity

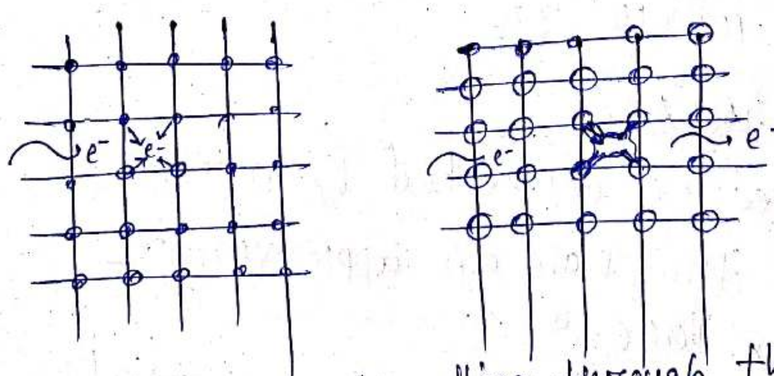
(2) Cooper remoduled idea an electron - phonon interaction into the philosophy of a (electron-phonon -electron)

cooper ~~demostra~~ demonstrad that with the creation of condition favourable for a net attractive interaction between two electrons in a conductor, conductor transfer from Normal state to — SC state.

Electron-phonon interaction —

— e-e interaction coloumbic repulsion (~~instaneous~~ instantaneous)

— e-i-e i.e e-e interaction mediated by phonon (retarded) attraction —



An electron travelling through the crystal lattice leaves behind the ~~deformodation~~ deformation trail which can be regarded as an accumulated of

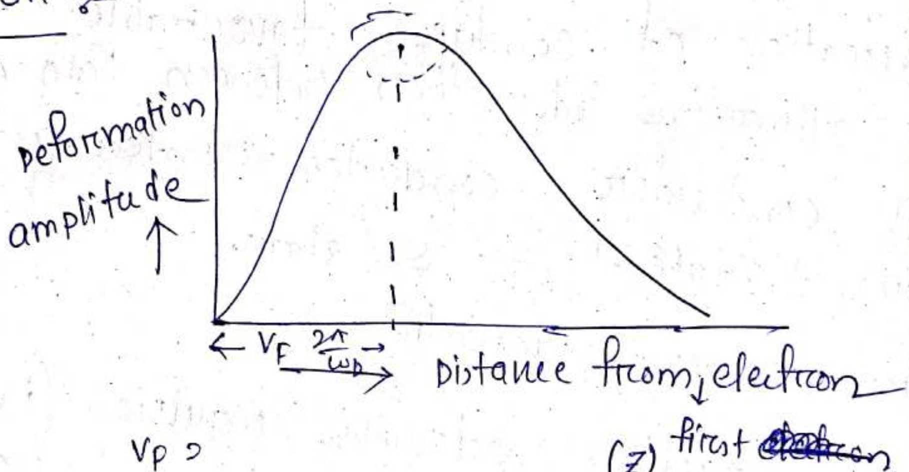
~~very~~ '+' value charge ion core.

→ Area of enhanced +ve charge compared to neutral crystal is increased behind the electron and exerts an attractive force on a second electron behind the 1st electron.

e-e ~~co~~ coloumb repulsion (weak)

e-p-e cooper pair (strong)

Instability of Fermi sea and cooper pair formation :-



$v_p =$

phonon vibration period $\frac{2\pi}{\omega_D} = 10^{-13}$

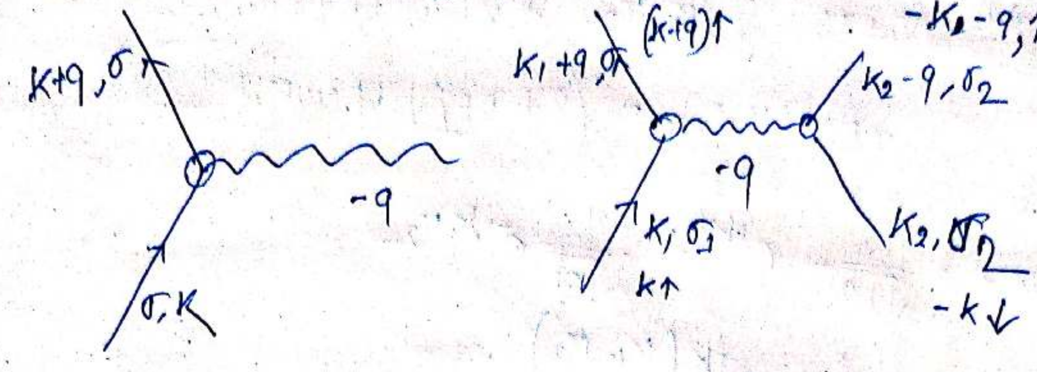
$v_F = 10^{8}$ cm/s

$10^8 \times \frac{2\pi}{\omega_D} = 10^8 \times 10^{-13}$ cm

$= 1000 A^0$

The two electron correated by lattice deformation thus have an approximate seperated by $1000 A^0$

An electron travelling through crystal lattice leaves behind information tell which can be regarded as accomulation of positive charge ion cores



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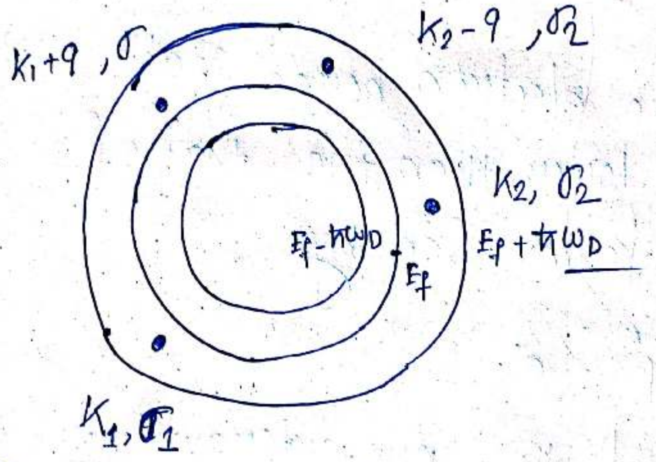


The effective e-e interaction near the Fermi-surface the electron at $k_1\sigma_1$ & $k_2\sigma_2$ are scatted to k_1+q, σ_1 & k_2-q, σ_2. The interaction is attracted provided all the wave vectors lie in the range $\varepsilon_K \pm \hbar\omega_D$ of the Fermi sphere.

$$V_{eff}(q,\omega) = |g_q|^2 \frac{1}{(\omega^2 - \omega_q^2)} \qquad g_q = \sqrt{\frac{m}{M}} \sim 0.01$$

$$V_{eff}(q,\omega) = |g_{eff}|^2 \frac{1}{\omega^2 - \omega_D^2}$$

$$= -|g_{eff}|^2 \qquad \omega < \omega_D$$

$$= -V_0 \qquad (\omega < \omega_D)$$

Two particle wave function

~~$\psi(r_1, r_2)$~~,

$$-\frac{\hbar^2}{2m}\left(\nabla_1 + \nabla_2\right)\psi(r_1, r_2) + V(r_1, r_2)\,\psi(r_1, r_2)$$

$$= E\,\psi(r_1, r_2) = \left(\varepsilon + 2E_f^0\right)\psi(r_1, r_2).$$

$$\psi(r_1, r_2) = \frac{1}{\sqrt{L^3}}\, e^{ikr_1} \cdot \frac{1}{\sqrt{3!}}\, e^{ikr_2}$$

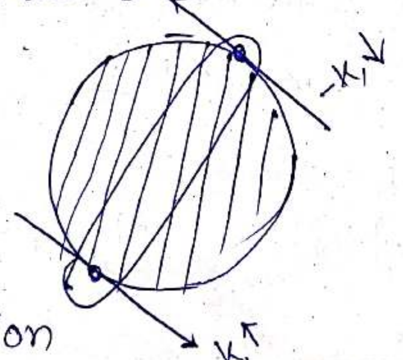
$$= \frac{1}{3!}\, e^{ik(r_1 - r_2)}$$

$$\varepsilon = -2\hbar\,\omega_D\, e^{-2/V_0 Z(E_f^0)}$$

There exist a two electron bound state whose energy is lower than the that of fully occupied fermi sea by

$$\varepsilon = E - 2E_f^0 < 0$$

The ground state of the non-interacting free electron gas becomes unstable when any attractive interaction between electron is switched on. The instability leads to the



formation of such electron pairs such as cooper pair $(k\uparrow, -k\downarrow)$ and the system tries to reach the new lower energy ground state

$(k\uparrow, -k\downarrow)$ $(k'\uparrow, -k'\downarrow)$ with

& so on the cooper pair and opposite. K-vector

& spin.

coloumb interaction is reduced due to Screeinig (presence of the electron in E_F)

Some thing new occurs the two electron may attracted each other, the two electron wall then from bound state very close to the E_f surface

$E_f \pm \hbar\omega_D$ the binding energy is shortest when electrons forming the pair have oppositive momentum & opposite Spin $K\uparrow$, & $-K\downarrow$. All the electrons in the neighbourhood of the fermi surface. A system of many cooper pairs.

The binding energy of electrons (1 & 2) an energy gap appears in the spectrum of the electron

An electron $(K\uparrow)$ polarised the lattice creating phonon (q). Another electron with wave vector $(-K\downarrow)$ absorbs the phonon. The end result is two electrons $(k-q,\uparrow)$ $(-K+q,\uparrow)$

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condensate :-

At low temp. cooper pairs are formed a favourable condition. The pair wavefunction all have form & the superposition of pair wavefunction describes ⟶ condensate (BEC)

BCS energy gap relation :-

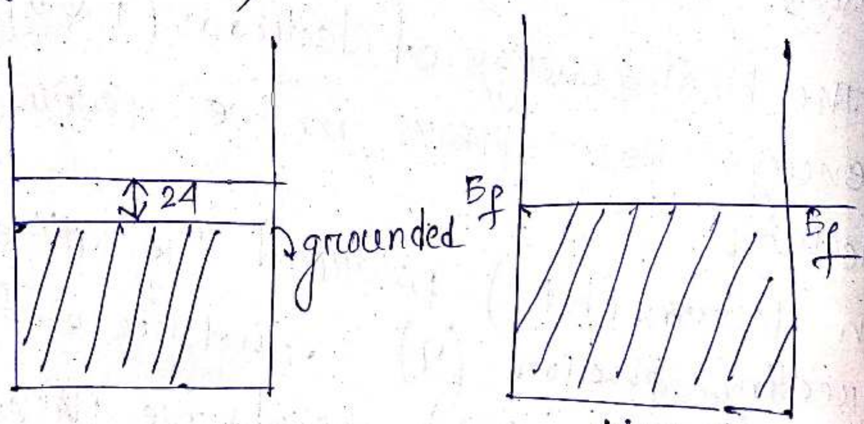
In the simplest form BCS theory

$$K_B T_c = 1.14\, \hbar\omega_D \exp\left[-\frac{1}{V\cdot N(E_f)}\right] \quad \text{—} \ (1)$$

V = effective electron - phonon interaction

$N(E_f)$ = DOS at fermi level

ω_D = Debye frequency.



BEC condensate.

Normal metal

from BCS calculation

$$2\Delta(0) = 4\hbar\omega_D \exp\left(-\frac{1}{V\,N(E_f)}\right] \quad \text{—} \ (2)$$

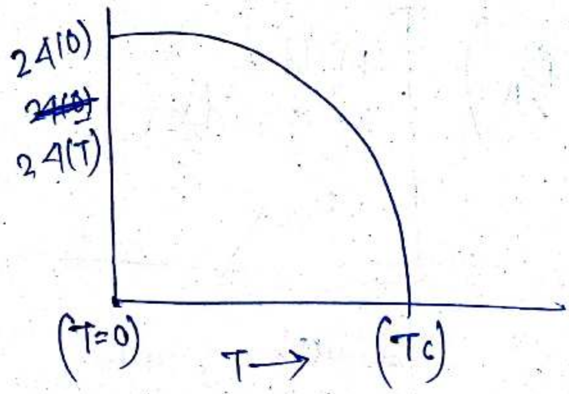
$\hbar\omega_D \sim 10^{-2} eV \sim 10^{-4} eV$

$$\frac{2\Delta(0)}{K_B T_c} = \frac{4\cancel{\hbar\omega_D}\,\cancel{\exp\left(-\frac{1}{VN(E_F)}\right)}}{1.14\,\cancel{\hbar\omega_D}\,\cancel{\exp\left(-\frac{1}{VN(E_F)}\right)}}$$

$= 3.53$

	$\frac{2\Delta(0)}{K_B T_c}$
In	4.1
Sn	3.6
Hg	4.6
Pb	4.1

Experimental measured values clearly show types of values of of $\frac{2\Delta(0)}{K_B T_c} = 3.53$ as predicted by BCS.



$$2\Delta(T) = 2\Delta(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

BCS ground state.

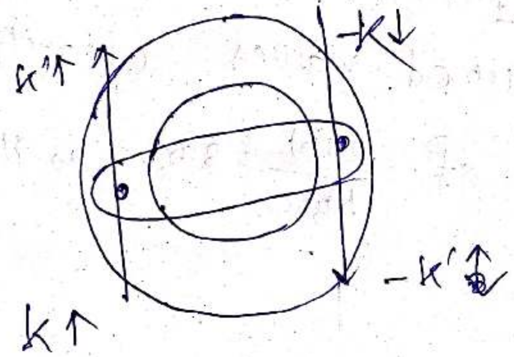
$\psi(r, s, r', s')$ r = electron position

s = spin

In a system of N electrons, the electrons are grouped into $N/2$ pairs

$$\phi = (r_1, s_1, r_2, s_2, r_3, s_3) = \psi(r_1, s_1, r_2, s_2) \cdot \psi(r_3, s_3, r_4, s_4) \cdot \psi(r_5, s_5, r_6, s_6)$$

$\phi_{BCS} = a\phi$ a = antisymmetrization



$$|\phi_{BCS}\rangle = \prod (u_k|0\rangle_k + v_k|1\rangle_k)$$

Field of of solids Book

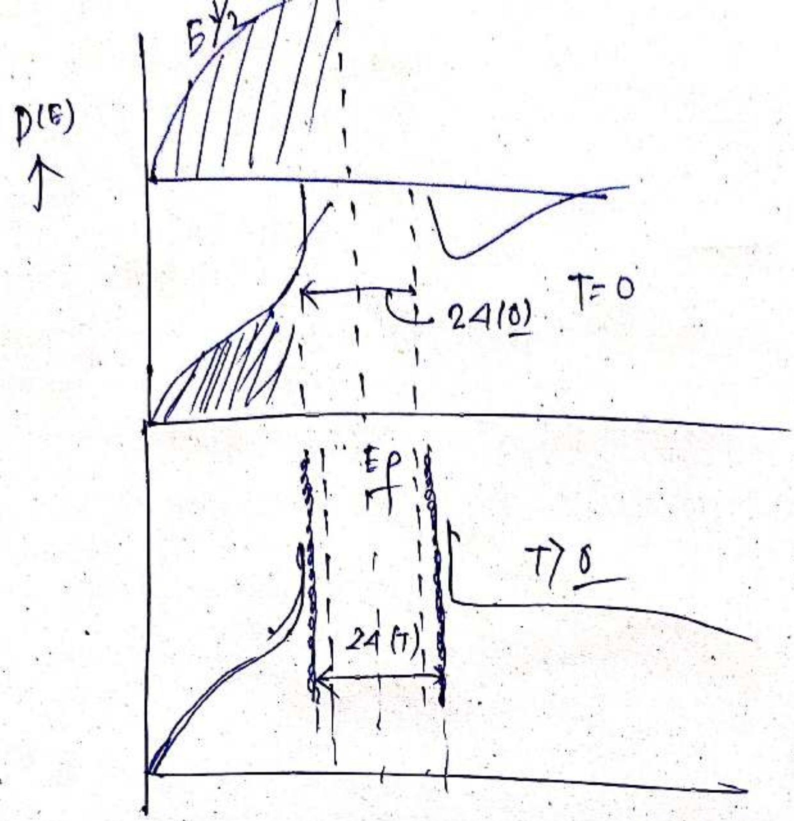
$$\langle \phi_{BCS} | \phi_{BCS} \rangle = 1$$

$u_k^2 + v_k^2 = 1$

[Ibach & luth solid state physics]

u_k, v_k are normalised constants.

cooper pair breaks if $2\Delta(0)$ is a supplied



$D(E)$
$E^{1/2}$
$2\Delta(0)$
$T=0$
E_f
$2\Delta(T)$
$T>0$