3/4/25 Magle

Critical current in a SC.

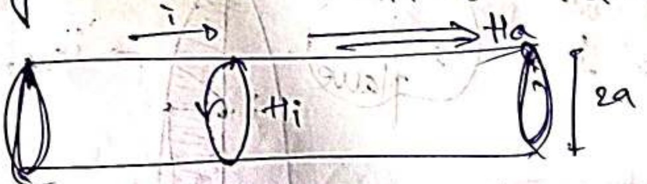
A superconducting wire along which a current is passed from an external source is transport current (J_i). If the wire is in magnetic field, screening current (J_H) so as to cancel the flux inside the S.C.

Total current : $J = J_i + J_H$

If J @ any point exceeds the critical current density J_c, $(J > J_c)$ superconductivity is lost.

In other words, a SC loses its zero resistance state, when at any point on the surface, the total magnetic field strength due to transport current 4 applied magnetic field exceeds critical field strength H_c.

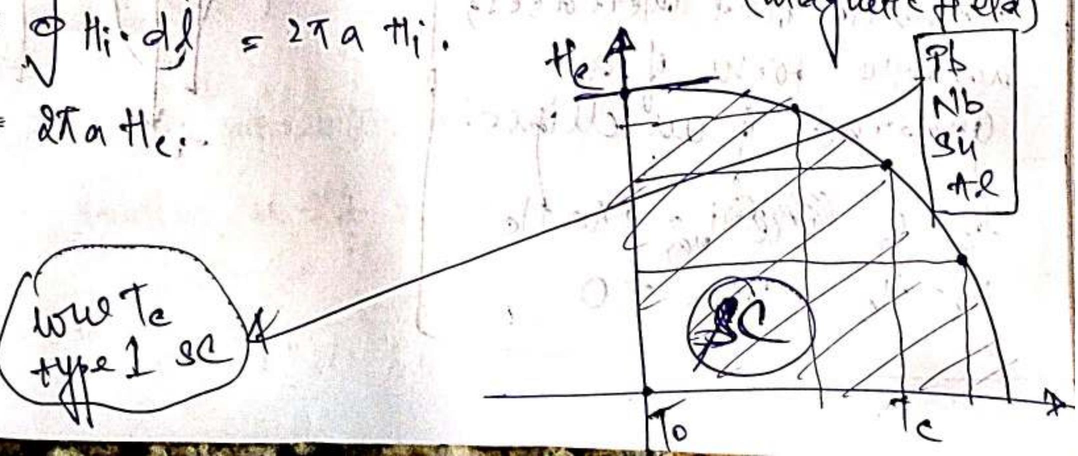
The maximum amount of transport current which can be passed along a piece of SC without any resistance appearing is called critical current.



A cylindrical SC wire of radius 'a', kept in SC.
i current is passed through the wire → H_i (magnetic field)

$i = \oint H_i \cdot dl = 2\pi a H_i$.

$\therefore i_c = 2\pi a H_c$.



Let us now consider to what extent the critical current is reduced by the presence of an external applied field (B_a).

B_a is applied parallel to the dirn of current carrying wire.

$B_a = \mu_0 H_a$.

For i current passed it generates:

$$H_i = \frac{i}{2\pi a} \quad \text{(as per Ampere law).}$$

The field H_i & H_a both act vectorially and they are at right $\perp$ to the strength H of the resultant field:

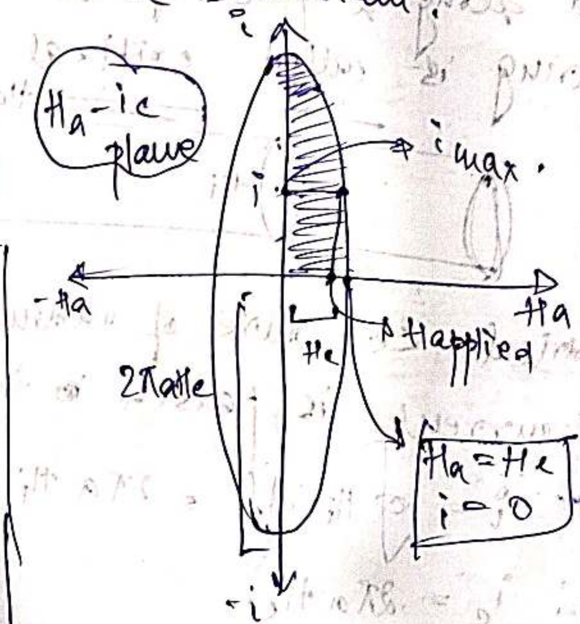
$$H^2 = H_a^2 + H_i^2$$

$$= H_a^2 + \left(\frac{i}{2\pi a}\right)^2.$$

The critical value H_c occurs when $H = H_c$.

$$H_c^2 = H_a^2 + \left(\frac{i}{2\pi a}\right)^2. \quad H_c \text{ is constant.}$$

$$\left(\frac{H_a}{H_c}\right)^2 + \left(\frac{i}{2\pi a H_c}\right)^2 = 1$$



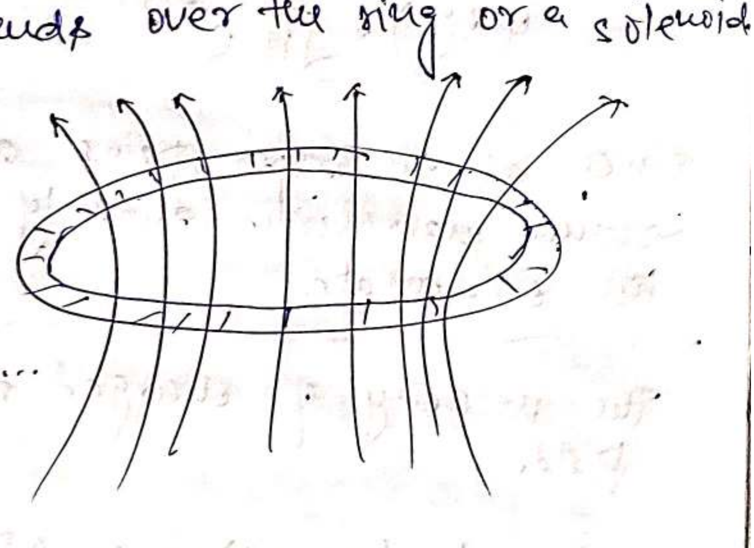
The decrease in critical current as the strength of the longitudinal applied magnetic field increases, has the form of a quadrant of an ellipse.

$H_a = 0 \quad i_{max} = 2\pi a H_c$

$H_a = H_c \quad i_{max} = 0$

FLUX QUANTIZATION

* Long range quantum effect in which the coherence of the superconducting state extends over the ring or a solenoid

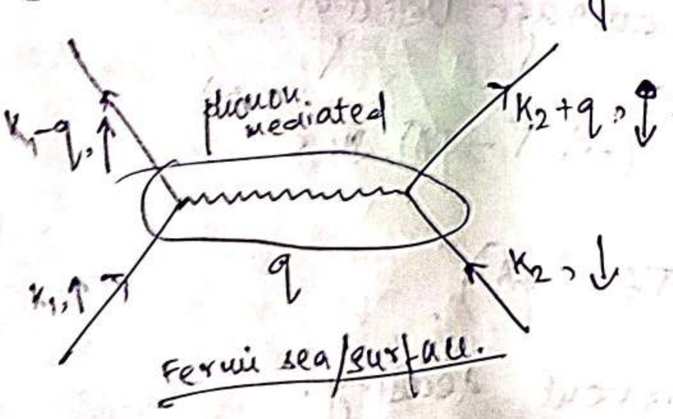


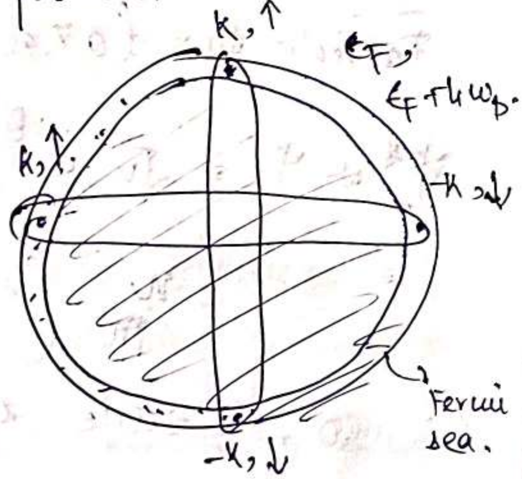
$\Phi = n \boxed{\Phi_0}$

$\Phi_0 = \frac{h}{2e}$; n = integer $= 1, 2, 3, 4 \ldots$

$\boxed{\text{fluxon or fluxoid.}}$

A charged boson gas (cooper pair) obeys london equation
$e^- - e^-$ interaction mediated by a phonon.





All the cooper pairs have to go to the single quantum state
$\Longrightarrow$ BEC.

e-e interaction mediated by phonons forms a cooper pair: All cooper pairs have proper phase relation and all of them go to single quantum state. The material then goes to SC state.

1. same phase (θ)
2. same Amplitude.
3. same frequency.

let $\psi(r)$ = particle probability of pair wf

The pair concentration $n = \frac{1}{2}$ of the concentration of e^- in CB.

= cooper pair concentration.

∴ Pair wave function/particle amplitude:

$$\psi(r) = \sqrt{n}\, e^{i\theta(r)}$$
$$\psi^*(r) = \sqrt{n}\, e^{-i\theta(r)}$$
$\theta(r)$ = phase.

Since all the cooper pairs are phase locked, θ becomes macroscopic quantity characterizing the condensate.

The velocity of charged particle in magnetic field.

$$v = \frac{1}{m}(p - qA) = \frac{1}{m}(-i\hbar\nabla - qA).$$

Particle flux (aka average velocity):

$$\psi^* v \psi = \sqrt{n}\, e^{-i\theta(r)} (-i\hbar\nabla - qA) \sqrt{n}\, e^{i\theta(r)}$$

$$= \frac{n}{m}(\hbar\nabla\theta - qA).$$

∴ so, electric current density.

$$J = q(\psi^* v \psi) = \frac{nq}{m}(\hbar\nabla\theta - qA).$$

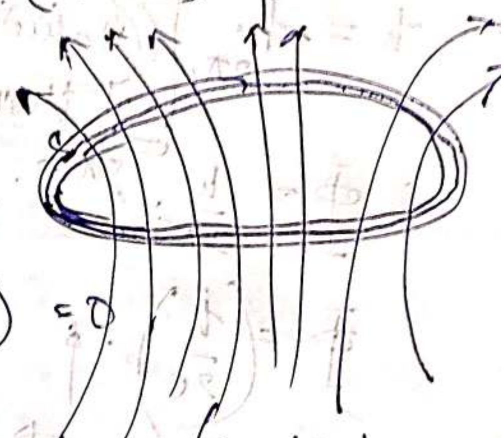
$$\nabla\times J_s / \nabla\times J = \frac{nq}{m}\left[\cancel{\hbar(\nabla\times\nabla\theta)} - q(\nabla\times A)\right]$$

$$\nabla\times J = -\frac{nq^2}{m}(\nabla\times A)$$ $\rightarrow \vec{B}$.

$$\boxed{\nabla\times J = -\frac{nq^2}{m}B.}$$ London's equation.

Let us take a closed path through the interior of a superconducting material ring well away from the surface. $\boxed{J_s = J_0 \exp(-x/\lambda_L)}$

The Meissner effect tells us that B & J are effectively 0 (zero) in this region (interior) of the SC ring material.



(C : closed path)

$\therefore$ If $J = 0$ in interior then,

$$J = q\psi^* v\psi = \frac{n q}{m}(\hbar \nabla\theta - qA) = 0$$

$$\therefore \hbar \nabla\theta = qA.$$

Take line integral on both sides over a closed path C.

$$\oint_C \nabla\theta . dl = \frac{q}{\hbar} \oint_C A . dl$$

The LHS: $\oint_C \nabla\theta . dl = (\theta_2 - \theta_1) \longrightarrow$ change of phase on going once around the ring.

But the phase @ our common starting and finishing point MUST be uniquely defined. [closed line integral].

So, phase integral must be equal to zero or integral multiple of 2π.

$\psi(r)$ must be single valued.

$\therefore \theta_2 - \theta_1 = 2\pi p$, where p = integer $= 0, 1, 2, 3, 4, \ldots$

The RHS:

$$\oint A . dl = \int \nabla \times A . ds = \int_S \underline{B . ds}$$

$$\frac{q}{\hbar} \oint A . dl = \frac{q}{\hbar} \int_S B . ds = \frac{q}{\hbar} \Phi$$

$$\therefore \frac{q}{\hbar} \Phi = 2\pi p.$$

$$\therefore \Phi = \frac{\hbar}{q} 2\pi p$$

$\Phi = \frac{\hbar}{q} 2\pi p.$

$q = -2e \longrightarrow$ charge of bosonic cooper pair.

$\hbar = \frac{h}{2\pi}$. $\boxed{\text{Value of fluxon: } \frac{6.626\times10^{-34}}{2\times1.6\times10^{-19}} = 2.07\times10^{-15}\ T\text{-}m^2\ (Wb) = 2.07\times10^{-7}\ G\text{-}cm^2\ (maxwell)}$

$\therefore \Phi = \frac{h}{\cancel{2\pi}}\cdot\cancel{2\pi}p\cdot\frac{1}{2e}$

$\Phi = \left(\frac{h}{2e}\right)\cdot p$

$\Rightarrow \Phi = p\cdot\Phi_0 \rightarrow$ quantum of flux

$\rightarrow \boxed{\text{quantiza}^n}$

$\Phi_0 = \frac{h}{2e}$

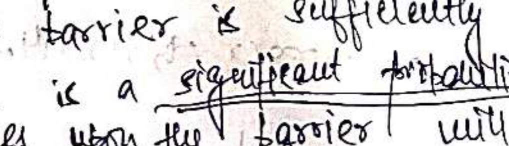
also

$\Phi_0 = \frac{h\pi}{e}$

fluxoid / fluxon

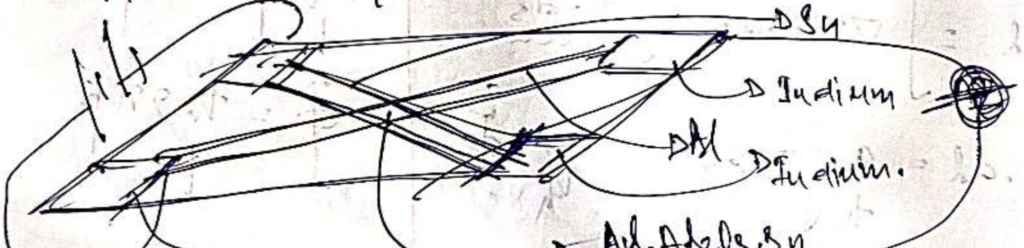
Thus, the flux through the SC ring or the flux trapped within the SC ring is quantized in integral multiple of units $\left(\Phi_0 = \frac{h\pi}{e} / \frac{h}{2e}\right)$.

QUANTUM TUNNELING

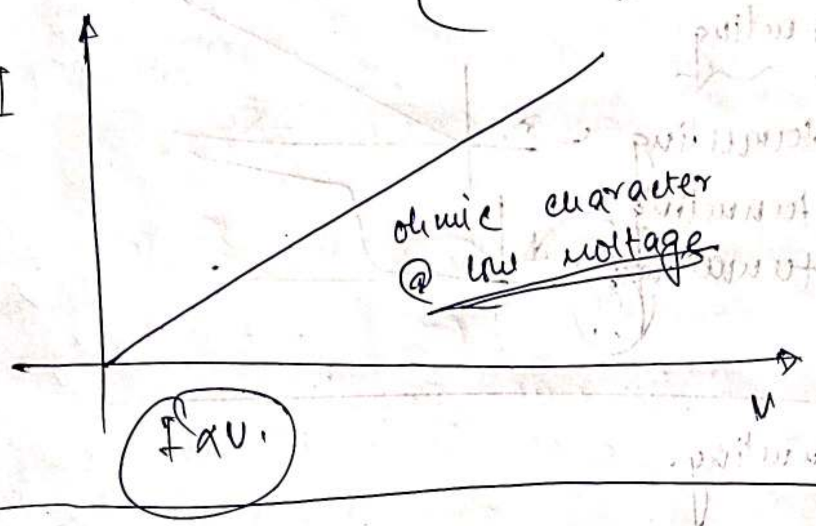


Single particle tunneling

Consider 2 metals separated by an insulator. The insulator normally acts like a barrier to the flow of conduction electrons from one metal to another. If the barrier is sufficiently thin (i.e less than 10 Å), there is a significant probability that an electron which impinges upon the barrier will pass from one metal to another. This phenomena is called tunneling.

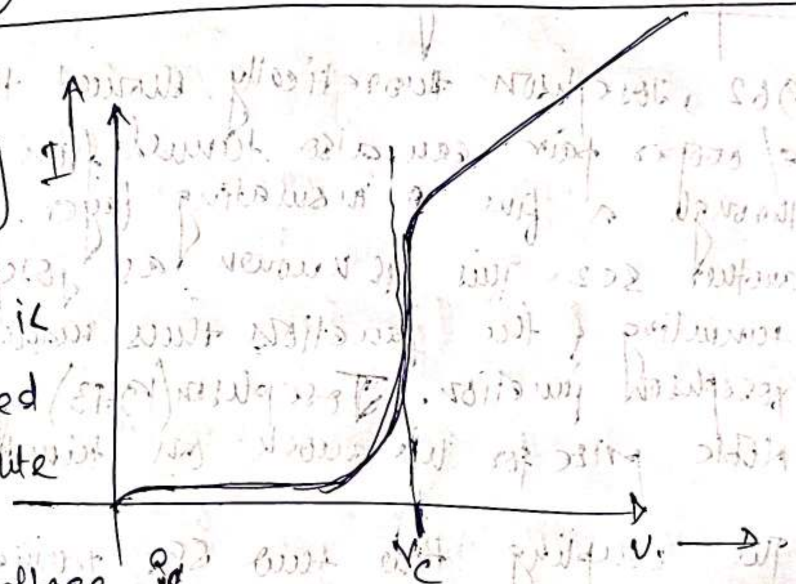


IV characteristic of metal-insulator-metal junction. (MOxide…) (MIM junction)



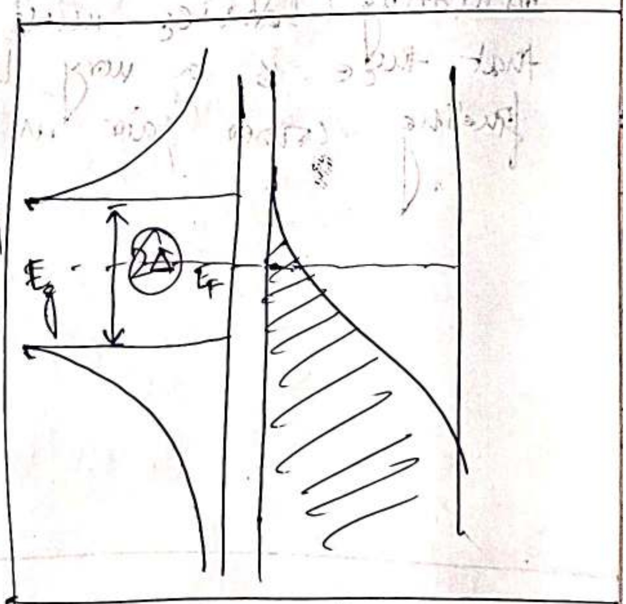
Giaever (1960)

metal	Insul	S.C.



In the SC there is an energy gap centered at the E_F. At absolute zero no current can flow until applied voltage is

$$V = \frac{E_g}{2e} = \frac{2\Delta}{2e}$$



E_g (energy gap) is energy required to break off cooper pair e's in the SC state with the formation of 2 electron, or e & h in the normal state.

The current starts when:

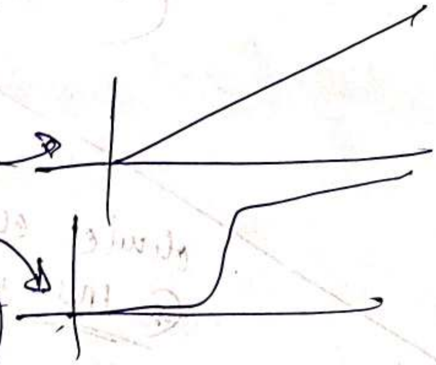
$eV = \Delta \quad (T=0)$

$V = \Delta/e$

At finite temp ($T \neq 0$) there is a small current flow even at low voltages, because electrons in the SC that are thermally excited across the energy gap.

4|4|25 Mag SC.

Quantum Tunneling

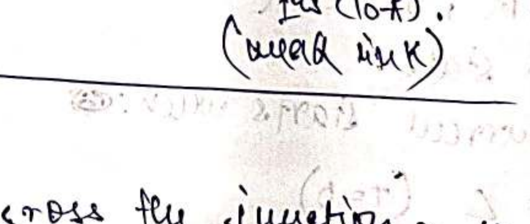


(i) MIM tunneling
(ii) MIS tunneling
(iii) SIS tunneling ??

Josephson Tunneling.

1962, Josephson theoretically showed than an e/ cooper pair can also tunnel from one SC1 through a fine insulating layer into another SC2. This is known as Josephson tunneling & the junction thus made is the Josephson junction. Josephson (1973) got the Noble prize for his work on tunneling in SC.

The coupling b/w two SCs provided by thin insulating barrier must be very weak so that, there is a very low probability of finding cooper pair in the insulating region.



DC Josephson effect:

A dc current flows across the junction in the absence of any electric & magnetic field.

Let ψ_1 : prob amplitude/wavefunc^n of cooper pair.
ψ_2 : other SC.

Let H_1 & H_2 be Hamiltonian of the respective isola[ted] ... let SC1 = SC2 and ... potentia[l]

Time dependent SE :-

In SC_1: $i\hbar \frac{\partial}{\partial t}\psi_1 = \hbar T \psi_2$ .... ①

& in SC_2:

$i\hbar \frac{\partial}{\partial t}\psi_2 = \hbar T \psi_1$ .... ②

"$\hbar T$" → effects of e⁻ pair coupling (coupling force) or transfer interaction across the insulator.

T has the dimension of rate or frequency. It is a measure of the leakage of ψ_1 into region 2 & ψ_2 into region 1. If insulator is very thick & there is no cooper pair tunnelling $T \to 0$.

Let $\psi_1 = \sqrt{n_1}\, e^{i\theta_1}$

$\psi_2 = \sqrt{n_2}\, e^{i\theta_2}$

cooper pair concentration: $\psi_1^* \psi_1 = (\sqrt{n_1})^2 = n_1$.

$n = \frac{1}{2}$ concentration of e⁻ in C.B. (conduction band)

$\theta(r)$ = phase. All cooper pairs are in same-phase single macroscopic quantum state in a SC.

$\frac{\partial \psi_1}{\partial t} = \frac{e^{i\theta_1}}{2\sqrt{n_1}} \frac{\partial}{\partial t} n_1 + i\psi_1 \frac{\partial}{\partial t}\theta_1 = -iT\psi_2$ .... (i)

$\frac{\partial \psi_2}{\partial t} = \frac{e^{i\theta_2}}{2\sqrt{n_2}} \frac{\partial}{\partial t} n_2 + i\psi_2 \frac{\partial}{\partial t}\theta_2 = -iT\psi_1$ .... (ii)

multiply (i) by $\sqrt{n_1}\, e^{-i\theta_1}$ & (ii) by $\sqrt{n_2}\, e^{-i\theta_2}$

let $\theta_2 - \theta_1 = \delta$.

$\frac{\partial n_1}{\partial t} + i\,\underbrace{\sqrt{n_1}\sqrt{n_1}}_{n_1}\, \cancel{e^{-i\theta_1}}\cancel{e^{i\theta_1}} \frac{\partial}{\partial t}\theta_1 = -i\sqrt{n_1}\, e^{-i\theta_1}\, T \sqrt{n_2}\, e^{i\theta_2}$ .... (i)

$= -iT\sqrt{n_1 n_2}\, e^{i(\theta_2-\theta_1)}$ .... (i)

$\frac{\partial n_2}{\partial t} + i n_2 \frac{\partial}{\partial t}\theta_2 = -iT\sqrt{n_1 n_2}\, e^{i(\theta_1-\theta_2)}$

$\dot{n}_1 + i n_1 \dot{\theta}_1 = -iT\sqrt{n_1 n_2}\, e^{i\delta}$ | $\dot{n}_2 + i n_2 \dot{\theta}_2 = -iT\sqrt{n_1 n_2}\, e^{-i\delta}$.

$$\frac{\dot n_1}{n_1} + i\dot\theta_1 = -iT\frac{\sqrt{n_2}}{\sqrt{n_1}}\, e^{i\delta} \rightarrow (\cos\delta + i\sin\delta) \rightarrow -i\cos\delta + \sin\delta .$$

$$\therefore \frac{\dot n_1}{n_1} = T\frac{\sqrt{n_2}}{\sqrt{n_1}}\,.\sin\delta .$$

$$\dot n_1 = \left(T\sqrt{n_2}\sin\delta\right)\sqrt{n_1} .$$

$$\dot n_2 = -\left(T\sqrt{n_1}\sin\delta\right)\sqrt{n_2} .$$

if $n_2 = n_1$.

$$\dot\theta_1 = -T\left(\frac{n_2}{n_1}\right)^{1/2}\cos\delta .$$

$$\dot\theta_2 = -T\left(\frac{n_1}{n_2}\right)^{1/2}\cos\delta .$$

$$\frac{d}{dt}(\theta_2-\theta_1) = -T\cos\delta + T\cos\delta = \boxed{0} .$$

$\therefore$ δ remains constant.

This implies that phase difference b/w 2 SCs $\delta = \theta_2 - \theta_1$ remains const.

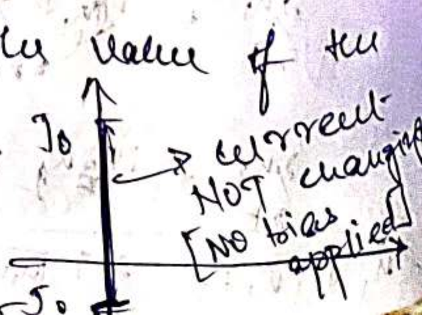
$-\dot n_1 = \dot n_2 = 2Tn\sin\delta$ & the current flow is ~~equal~~ proportional to $n = \sqrt{n_1 n_2}$ & to $\frac{\partial n_1}{\partial t}$ & $\frac{\partial n_2}{\partial t}$

$\rightarrow \underline{(n_1 = n_2) = n}$.

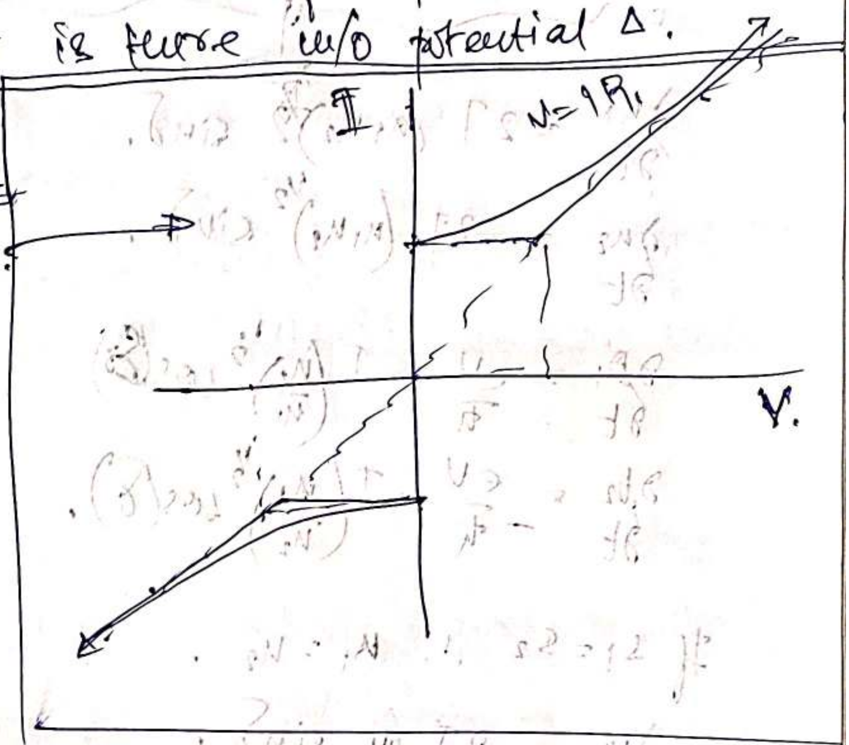
$$\therefore J = J_0\sin(\delta) ,$$

$\boxed{J_0 = 2Tn}$ $\quad = J_0\sin(\theta_2-\theta_1)$. $\quad \boxed{J \propto T}$ (transfer interaction)

With no applied voltage, a dc current will flow across the junction with a value between $+J_0$ to $-J_0$, according to the value of the phase difference $[\delta = \theta_2 - \theta_1]$.



Because of the weak link enabling electrons to tunnel from 1 to 2, with a small probability, there is however, a tendency for a phase 1 equalize. So a current depending on $(\theta_2 - \theta_1)$ will flow. current will be zero if $\theta_1 = \theta_2$
flow of current is there w/o potential Δ.



A C Josephson junc^n:

Let a DC voltage V be applied across the junction. we can do this because the junction is an insulator.

An e^- pair experiences a potential energy $\Delta = qV$ on passing the junction. One can say that a pair on one side is at PE = $-eV$ ~~Δ~~, on other side @ PE = $+eV$.
The equation of motion can then be written as

$$i\hbar \frac{\partial \psi_1}{\partial t} = \hbar T \psi_2 - eV\psi_1$$

$$i\hbar \frac{\partial \psi_2}{\partial t} = \hbar T \psi_1 + eV\psi_1$$

$$\boxed{\begin{aligned} \psi_1 &= \sqrt{n_1}\, e^{i\theta_1} \\ \psi_2 &= \sqrt{n_2}\, e^{i\theta_2} \end{aligned}}$$

$$\sqrt{n_1}e^{-i\theta_1} \times \frac{e^{i\theta_1}}{2\sqrt{n_1}} \dot{n}_1 + i\psi_1 \dot{\theta}_1 = -iT\psi_2 + \frac{ieV}{\hbar}\psi_1$$

$$\sqrt{n_2}e^{-i\theta_2} \times \frac{e^{i\theta_2}}{2\sqrt{n_2}} \dot{n}_2 + i\psi_2 \dot{\theta}_2 = -iT\psi_1 - \frac{ieV}{\hbar}\psi_2$$

$$\frac{1}{2}\frac{\partial u_1}{\partial t} + i u_1 \frac{\partial \theta_1}{\partial t} = -iT(u_1 u_2)^{1/2} e^{i\delta} + \frac{ieV}{\hbar} u_1.$$

$$\frac{1}{2}\frac{\partial u_2}{\partial t} + i u_2 \frac{\partial \theta_2}{\partial t} = -iT(u_1 u_2)^{1/2} e^{-i\delta} - \frac{ieV}{\hbar} u_2.$$

$$\frac{\partial u_1}{\partial t} = 2T(u_1 u_2)^{1/2} \sin\delta.$$

$$\frac{\partial u_2}{\partial t} = -2T(u_1 u_2)^{1/2} \sin\delta.$$

$$\frac{\partial \theta_1}{\partial t} = \frac{eV}{\hbar} - T\left(\frac{u_2}{u_1}\right)^{1/2} \cos(\delta)$$

$$\frac{\partial \theta_2}{\partial t} = -\frac{eV}{\hbar} - T\left(\frac{u_1}{u_2}\right)^{1/2} \cos(\delta).$$

If $s_1 = s_2$ $\therefore$ $u_1 = u_2$.

$$\frac{\partial u}{\partial t} = 2Tu \sin\delta.$$

$$\frac{\partial \theta_1}{\partial t} = \frac{eV}{\hbar} - T\cos\delta.$$

$$\frac{\partial \theta_2}{\partial t} = -\frac{eV}{\hbar} - T\cos\delta.$$

$$\frac{\partial (\delta)}{\partial t} = -\frac{2eV}{\hbar}$$

$$\frac{\partial (\delta)}{\partial t} = -\frac{2eV}{\hbar}.$$

$2uT$

$$\therefore \delta = \left(-\frac{2eV}{\hbar}\right)t + c.$$

At $t=0$, $\delta = \delta_0$ (initial phase Δ).

$$\delta(t) = \left(-\frac{2eV}{\hbar}\right)t + \delta(0).$$

$$\therefore J = J_0 \sin[\delta(t)] = J_0 \sin\left(\delta(0) - \frac{2eV}{\hbar}t\right).$$

$$= J_0 \sin(\delta(0) - \omega t).$$

$$\left[\omega = \frac{2eV}{\hbar}\right].$$

The current oscillates with L frequency $\left[\omega = \frac{2eV}{\hbar}\right]$,
This is AC Josephson effect.
A DC voltage applied externally ($1\mu V$) produces a frequency 483.6 MHz.

photon energy $\hbar\omega = 2eV$ is emitted when the e^- pair crosses the barrier.
By measuring the frequency & voltage it is possible to obtain the very precise value of $\left(e/\hbar\right)$.

when the voltage [DC] is applied across the weak link, an AC current of frequency $\omega = \frac{2eV}{\hbar}$
This AC current can be detected by the EM radiation of photon (microwave or infra) of same frequency emitted by the ckt.

$$J = J_0 \sin\left(\frac{2eV}{\hbar}t - \Delta\right).$$

J vs t is oscillatory.
But J(t) vs V(t) is const.