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Optimization of coil design for Wireless Power Transfer System

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***Abstract*—This report is regarding Wireless Power Transfer using the Inductive Power Transfer technique. In Inductive Power Transfer the coils used will play a crucial role. We need to transfer power wirelessly with the help of coils thus losses should be minimal in this process. This involves achieving a target mutual inductance between the coils by varying different parameters of the coils and finding out which set of parameters allows us to hit the target mutual inductance with minimal losses. In this paper we will be looking at both the designing and optimization of the coils for a given target mutual inductance in cases of coils placed at a distance from each other and also when placed on top of each other.**

I. Introduction

Wireless Power Transfer: The main objective in this report is to light a bulb with known ratings using wireless power transfer system. It can be done by using Inductive Power Transfer. Inductive power transfer works in the following manner. The circuit consists of transmitting and receiving coils on the primary and secondary sides respectively. An input AC supply is given to the transmitting coil and the power gets transmitted to the receiving coil. High-frequency resonant capacitors are used on both sides for compensating the impedances of the coils. For the transmitting coil, instead of giving AC supply directly it is passed through a rectifier and then inverted and the obtained output is fed into the transmitting coil. This is done for frequency matching and safety measures. On the secondary side, there is a receiving coil followed by a rectifier for converting AC obtained to DC and it is given to the load, which is a bulb in this case.

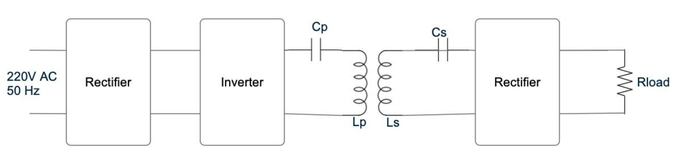

Fig. 1. Circuit Diagram of Wireless Power Transfer System

Initially the parameters considered are of a 40W, 220V bulb. Thus the output parameters being $P_o = 40W$ and $V_o = 220V$. The input, which is supply voltage is taken as 220V, 50 Hz AC. Since high frequency capacitors are needed, capacitors with frequency 85 KHz are considered and coupling coefficient(K) between coils to be 0.2 for analysis purposes. Further analysing the above circuit it can be simplified into the circuit shown below

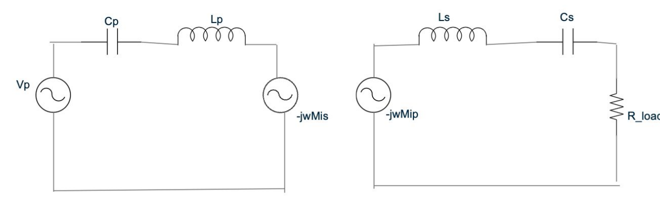

Fig. 2. Analysis of the circuit

Let, K be coupling co-efficient between coils assumed as 0.2
$V_{in} = V_p = 220V$
V_o be output voltage = 220V
P_o be output power = 40W
i_p and i_s be the current in primary and secondary side
L_p and L_s be the inductance of primary and secondary coils
M be the mutual inductance between two coils
f be the resonant frequency = 85kHz
As show in fig 2, a back emf of $-jwMi_s$ and $-jwMi_p$ is induced in the primary side and secondary side.
Calculating R_L:

$$R_L = \frac{V_o^2}{P_o} = \frac{220^2}{40} = 1210\Omega \tag{1}$$

The secondary side current can be calculated as:

$$i_s = \frac{P_o}{V_o} = \frac{40}{220} = 0.1818A \tag{2}$$

Applying KVL in the primary loop, we get:
Impedances of C_p and L_p balance out, so

$$V_p = -jwMi_s \tag{3}$$

$$|M| = \frac{V_p}{wi_s} = \frac{220}{2\pi \cdot 85 \cdot 10^3 \cdot 0.1818} = 2.265 \cdot 10^{-3} H \tag{4}$$

Applying KVL in the secondary loop, we get:
Impedances of C_s and L_s balance out, so

$$i_s \cdot R_L = -jwMi_p \tag{5}$$

$$i_p = \frac{i_s \cdot R_L}{Mw} = \frac{0.1818 \cdot 1210}{2.265 \cdot 10^{-3} \cdot 2\pi \cdot 85 \cdot 10^3} = 0.186A \tag{6}$$

The relation between M, L_p, L_s is given as

$$M = K \cdot \sqrt{L_p \cdot L_s} \tag{7}$$

For simplicity let us assume $L_p = L_s$. Substituting this in equation 7, we get

$$L_p = L_s = \frac{M}{K} = \frac{2.265 \cdot 10^{-3}}{0.2} = 11.325 \cdot 10^{-3} H \tag{8}$$

Since the impedance of resonant capacitor and inductor are same $jwL_p = 1/jwC_p$

$$C_p = \frac{1}{L_p \cdot w^2} = \frac{1}{11.325 \cdot 10^{-3} \cdot (2\pi \cdot 85 \cdot 10^3)^2} \tag{9}$$

$$C_p = C_s = 0.309 \cdot 10^{-9} F \tag{10}$$

The circuit in MATLAB simulink is:

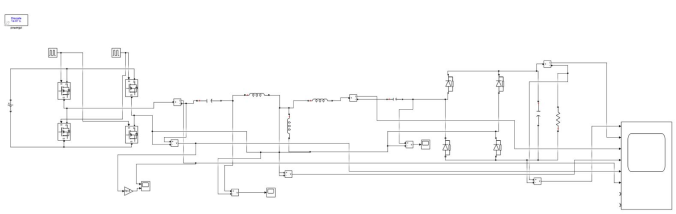

Fig. 3. Circuit in Matlab Simulink

The circuit is simulated in MATLAB simulink and the following are the obtained waveforms:

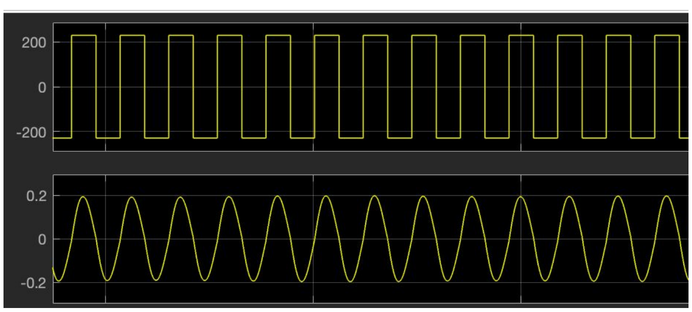

Fig. 4. Upper waveform = V_p and Lower waveform = i_p

Both V_p and i_p are in phase in the figure 3

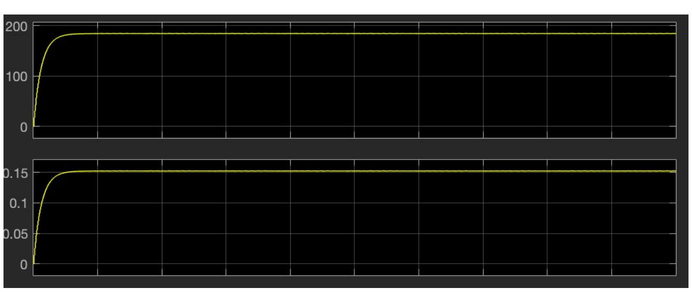

Fig. 5. Upper waveform = V_s and Lower waveform = i_s

II. Coil Parameters

A. Coils are placed at a distance from each other

Design of Coils: For initial design of coils that is for determining number of turns, inner and outer diameters. The coils are considered to be concentric and spiral. Observing the pattern we can make the following observation regarding the relation between outer radius, inner radius, width of the coil, spacing between the coils.

$$D_{out} = D_{in} + 2w + (T + w) \cdot (2N - 1) \tag{11}$$

$$R_{out} = R_{in} + w + (T + w) \cdot (N - \frac{1}{2}) \tag{12}$$

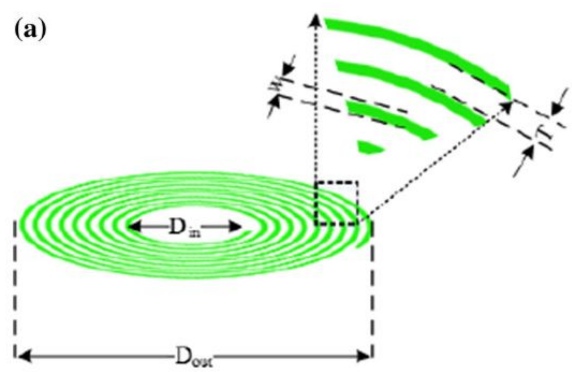

Fig. 6. Physical representation of a coil [1]

Here R_{in} is inner radius, R_{out} is outer radius, w is width of the coil, T is spacing between the turns, N is no. of turns.
Now let us consider that the coils are placed at a distance from each other. It is assumed that flux density is constant throughout the coils. For calculating mutual inductance between those two coils, secondary side is considered to be a point and primary side to be concentric spiral rings then using Biot Savarts, law flux is calculated for one ring and then it is summated for all other rings and we get mutual inductance for one secondary ring to primary rings and then by superposition of individual M between primary coils and each secondary coil we get our final expression for mutual inductance in terms of no. of turns.
Using Biot–Savart's law, the magnetic flux density B_0 (RMS value) at the center of the secondary coil due to each primary current-carrying coil can be given by

$$B_0 = \frac{\mu_0}{4\pi} \cdot i_{prms} \cdot \frac{2\pi \cdot R_p^2}{(R_p^2 + d^2)^{\frac{3}{2}}} \tag{13}$$

Total flux density at the center of the secondary coil due to all primary coils is given by summation of each flux density by each primary current coil

$$B_0 = \frac{\mu_0}{4\pi} \cdot i_{prms} \left(\sum_{p=1}^{Np} \frac{2\pi \cdot R_p^2}{(R_p^2 + d^2)^{\frac{3}{2}}} \right) \tag{14}$$

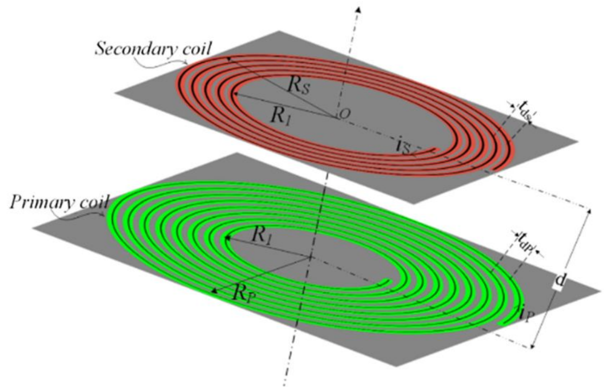

Fig. 7. Coils placed at a distance d [1]

Mutual inductance between primary and secondary coils can be calculated as

$$M = \frac{B_0 \cdot A_s}{I_{prms}} \tag{15}$$

$$A_s = \pi \cdot R_s^2 \tag{16}$$

By substituting equation (15) and (16) in (14) we get mutual inductance for one secondary coil due to all primary coils

$$M = \frac{\mu_0 \pi}{2} \cdot R_s^2 \left(\sum_{p=1}^{Np} \frac{R_p^2}{(R_p^2 + d^2)^{\frac{3}{2}}} \right) \tag{17}$$

The total mutual inductance between primary and secondary coils can be given by the superposition of individual M between the primary coil and each secondary coil is

$$M = \frac{\mu_0 \pi}{2} \sum_{s=1}^{Ns} \left(\sum_{p=1}^{Np} \frac{R_s^2}{(R_p^2 + d^2)^{\frac{3}{2}}} \right) \tag{18}$$

For design purposes we considered:
width of the coil w to be 0.01m.
spacing between turn T as 0.005m.
distance between the coils as 0.01m.
inner radius R_{in} = 0.075m.
Symmetric Case: For simplicity let us first consider the case where both primary and secondary coils are identical . That is having same number of turns and inner, outer radius. From the previous calculations M is obtained as $2.265 \cdot 10^{-3} H$. This is taken as target mutual inductance M for the above formula. Then the desired N value is calculated by checking the summation value for N starting from 1 until a point where target- $M[n-1] > 0$ and target- $M[n] < 0$. This is depicted in the graph in Fig. 8.
We observe that for N=23 this is getting satisfied. Outer radius of the coils is obtained as 0.4225m
Now we calculate the power losses (conduction losses) for these coils. Losses are give by the equation:

$$Losses = i_p^2 \cdot R_p + i_s^2 \cdot R_s \tag{19}$$

$$R = \frac{\rho \cdot L}{A} \tag{20}$$

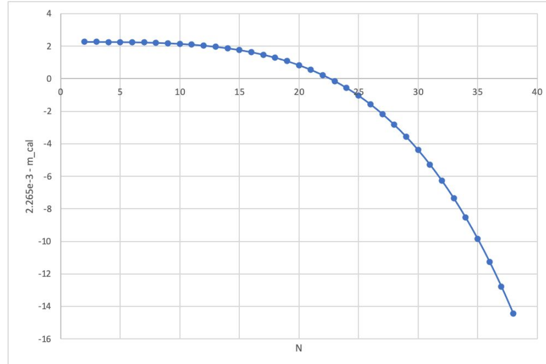

Fig. 8. Symmetric case: N=23

$$L = 2\pi \cdot n \cdot (R_{in} + \frac{n-1}{2} \cdot T + \frac{n+1}{2} \cdot w) \tag{21}$$

We obtain the losses to be 0.523 $\cdot 10^{-3}$W. This might not be the most efficient way for doing this. Thus now we look at the case of asymmetric coils that is primary and secondary coils not being identical and check for the cases where the losses are minimum.
Asymmetric coils: Now we consider the case where both coils are not identical. For finding the values we use the same M equation. In that equation we vary N1 starting from 1 to 50 and the corresponding N2 value is recorded. This is obtained by using the same technique as above target- $M[n1, n2-1] > 0$ and target- $M[n1, n2] < 0$.The computation is done in the following manner. A function for computing N2 value of input N1 value is given is written which is a similar function to that of symmetric case. Then for each value of N1 from 1 to 50, the corresponding N2 value is computed using the function and recorded in a matrix. The graph below depicts the N2 corresponding to each N1.

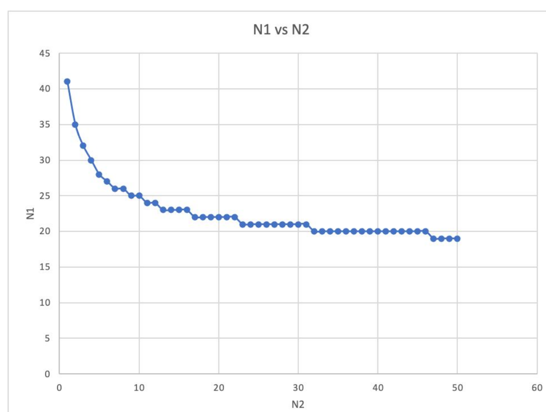

Fig. 9. Asymmetric case: $N_1 vs N_2$

Now by calculating losses corresponding to each (N1, N2) pair using the formulae (19),(20) and (21). Primary coil outer radius = 0.2125m and secondary coil outer radius = 0.4525m. These values are plotted and represented by the graph below. From the graph we can observe that for (N1=9,N2=25) pair

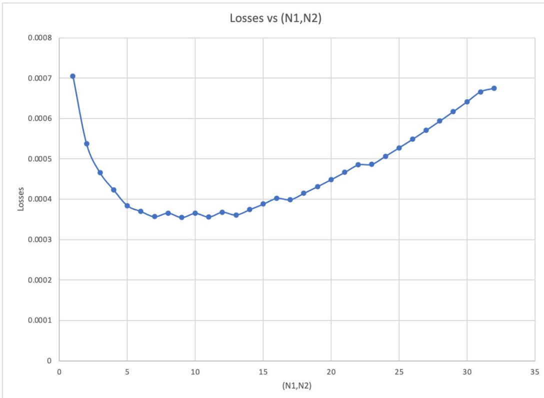

Fig. 10. Losses vs $N_1 N_2$

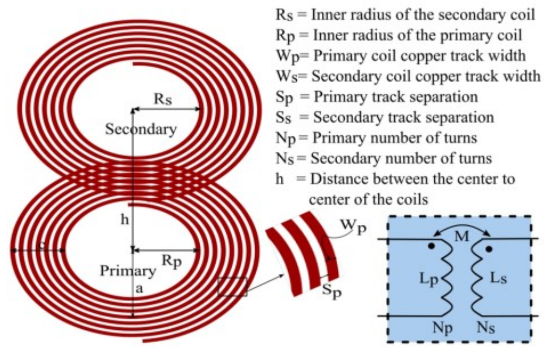

Fig. 11. Coils with a distance h [2]

the losses are minimum which is 0.355 $\cdot 10^{-3}$W
The mutual inductance formula used above is for the case where coils are placed at a distance from each other. The key assumption here is flux density is uniform throughout the coils and secondary side of the coils are considered as a point. But this assumption does not hold true when coils are placed very close to each other.

B. Coils are placed very close to each other

We are now considering that coils which are placed very close to each other or being placed on top of each other. Then the distance between coils is almost equal to zero. In this case while calculating the flux linkage between the coils, each coil turn on the secondary side is consider as infinitely small elements and the flux linkage between each of these small elements and the primary coils is calculated and integrated over for getting the total flux linkage and thus mutual inductance formula is obtained by the following equation:

$$M = \sum_{p=1}^{N_p} \sum_{s=1}^{N_s} \frac{2\mu_0 \cdot \sqrt{R_p R_s}}{k_{ps}} [(1 - \frac{{k_{p_s}}^2}{2}) \cdot K_{ps}(k) - E_{ps}(k)] \tag{22}$$

$$\alpha = \frac{R_s}{R_p} \tag{23}$$

$$\beta = \frac{h}{R_p} \tag{24}$$

$${k_{p_s}}^2 = \frac{4\alpha}{(1+\alpha)^2 + \beta} \tag{25}$$

where $K_{ps}(k)$ = complete elliptical integral of first kind and $E_{ps}(k)$ = complete elliptical integral of second kind
Intially we considered a 40W, 220V bulb but we could not find bulbs with the given high output 220 DC voltage and the model is not feasible because of high load resistance.
Thus we considered a bulb of voltage 12V and 9W power from now on.
We obtain the new value of mutual inductance value as $48.57 \cdot 10^{-6}$H
Here we considered 25/40 SWG litz wire:

Diameter of the wire(w) = $\sqrt{n \cdot (p.f)} \cdot d$ = 0.689mm
where, n = no. of strands = 25
p.f = packing factor =1.28
d = conductor diameter =0.1219mm
spacing between turn T = 0m.
distance between the coils(h) = w.
outer radius R_{out} = 0.075m.
a) Symmetric Case:
Similarly here we first consider the symmetric case when primary and secondary coils are identical. Elliptical integrals are imported from a library
Then the desired N value is calculated by checking the summation value for N starting from 1 until a point where target- $M[n-1] > 0$ and target- $M[n] < 0$. This is depicted in the graph below.

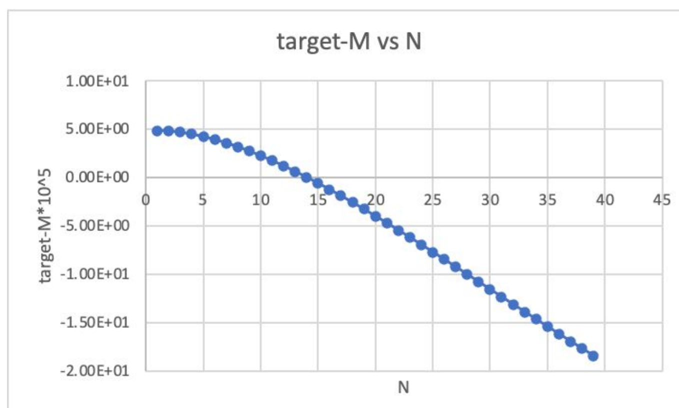

Fig. 12. Symmetric case: N=13

We observe that for N=13 this is getting satisfied. Inner radius of the coils is obtained as 0.065m
From equations (12) and (21) we get L in terms of R_{out}as

$$L = 2\pi n \cdot (R_{out} - \frac{n}{2} \cdot (T + w) + w) \tag{26}$$

From equation (19) losses are calculated to be 0.225 W.
b) Asymmetric Case:
Now we consider the case where both coils are not identical.

For finding the values we use the same M equation. In that equation we vary N_2 starting from 1 to 50 and the corresponding N2 value is recorded. This is obtained by using the same technique as above target- $M[n1-1, n2] > 0$ and target- $M[n1, n2] < 0$. The computation is done in the following manner, a function for computing N_1 value of input N_2 value is written which is a similar function to that of symmetric case.
First we are fixing R_{out} and varying R_{in} as $R_{out} - (T + w) \cdot (n - 0.5)$ and the graph in Fig. 12 depicts the N2 corresponding to each N1.

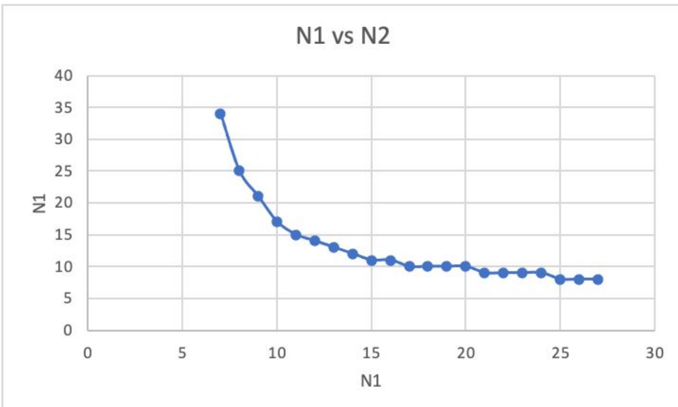

Fig. 13. Asymmetric case: N_1 vs N_2

Now by calculating losses corresponding to each (N1, N2) pair using the formulae (19),(20) and (26).These values are plotted and represented by the graph in Fig. 13.

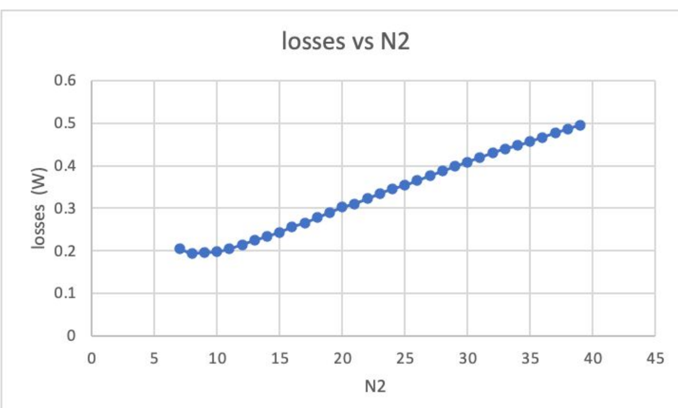

Fig. 14. Losses vs N_1,N_2

From the graph in Fig. 13 we can observe that for (N1=25,N2=8) pair the losses are minimum which is 0.196 $\cdot 10^{-3}$W

C. LTspice simulations for the new model

Earlier we made assumptions to many things like coupling co-efficient(K), L_p, L_s, ratings of Mosfet, because we were focused on the approach rather than the accurate result. But

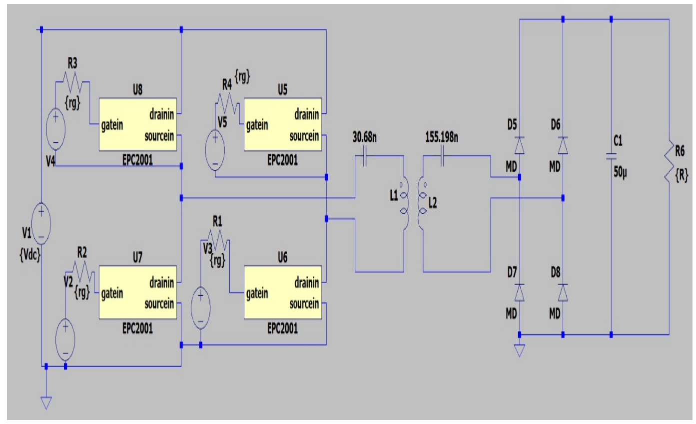

Fig. 15. Complete Circuit in LTspice

now we need to make sure each and every parameter is accurate enough to support and summarise the whole physical application.

Simulation results:

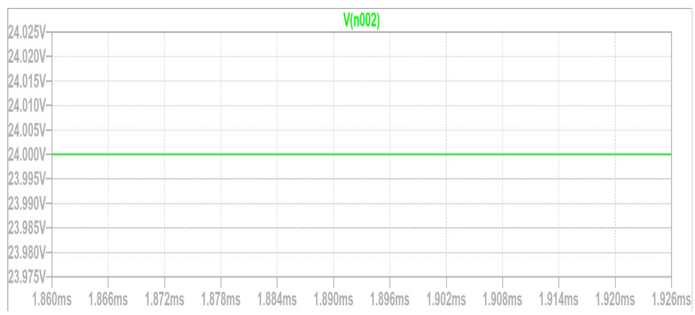

Fig. 16. Input to inverter

Maximum voltage that can be sent into the current inverter available to us is 32V. For safety measures and calculation purposes we have taken 24V as the input voltage to the inverter.

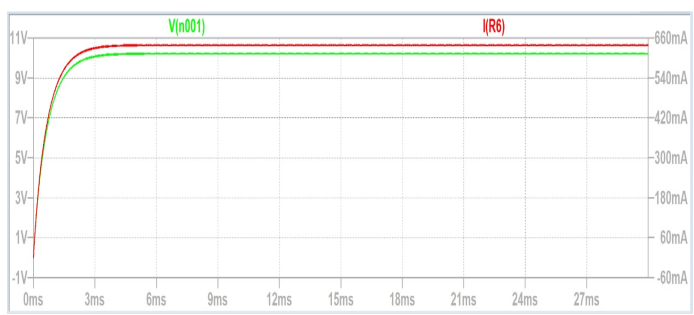

Fig. 17. Output voltage across R and output current through R

Output voltage across the load resistance is coming to be 10.7V and output current through the load resistance is coming to be 0.61A.

III. Physical Model of Coils

These coils are made using SWG 25/40 wire wounded in a concentric manner on a cardboard. And firmed to the cardboard with the help of thread and needle.

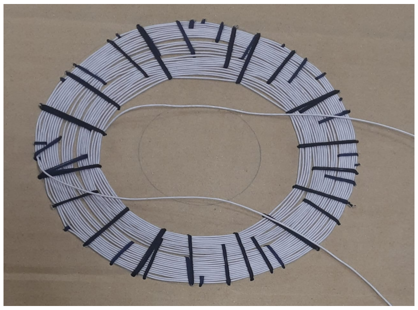

Fig. 18. Primary coil,N_p=25

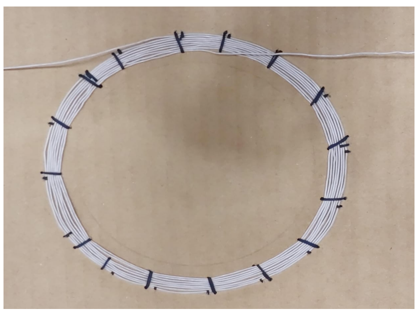

Fig. 19. Secondary coil,N_s=8

Expected parameters of the coil:
spacing between turn T = 0m.
distance between the coils(h) = w.
outer radius R_{out} = 0.075m.
But the spacing between the turns practically cannot be 0m. It is slightly deviated because of the human error. And distance between the coils is not exactly w. Because cardboard width is not considered.
$L_p = 114.26\mu H$ and $L_s = 22.59\mu H$ are measured experimentally using LCR meter

A. Observations

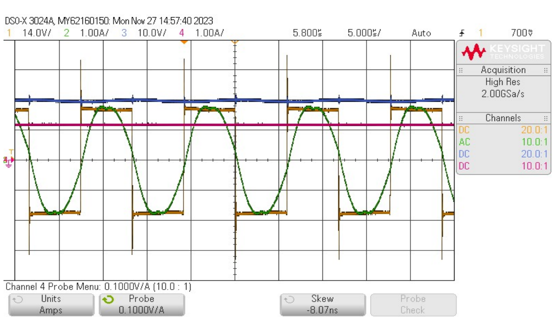

Fig. 20. $V_{in} = 24V$, Input Voltage, Input current, Output current

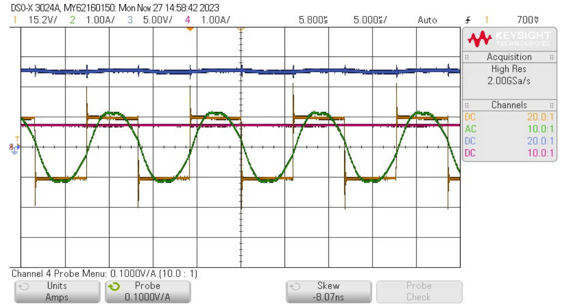

Fig. 21. For $V_{out} = 12V$, Input voltage, Input current and output current

We observed that input voltage and input current are in phase.

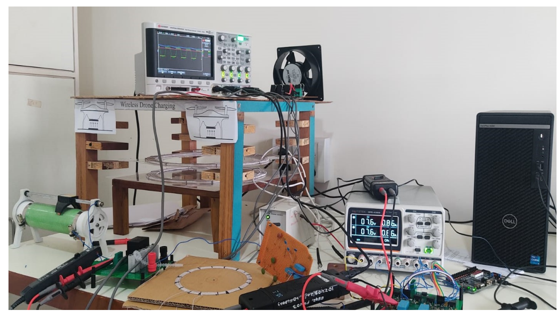

Fig. 22. Experimental Setup

IV. Calculating Partial turns

Up until now we have calculated no. of turns in integer values or assumed coils to completely circular. So we are getting some deviation from the actual target Mutual inductance. Now we are trying to minimize the deviation further by calculating the additional partial turn if our target M is greater than experimental M or remove one turn partially if our target M is lower than experimental M.

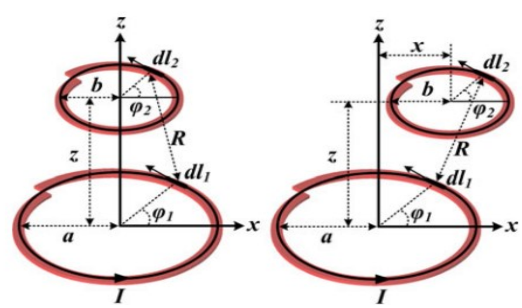

Fig. 23. Calculating M for a single turn primary and secondary coil[3]

$M = \frac{\mu_0 ab}{4\pi\sqrt{a^2+b^2+z^2+x^2}} \cdot \int_{\Phi_2=0}^{2\pi} \int_{\Phi_1=0}^{2\pi} \cos(\Phi_1 - \Phi_2)$
$\cdot[1 - \gamma\cos(\Phi_1 - \Phi_2) + \alpha\cos\Phi_1 - \beta\cos\Phi_2]^{\frac{-1}{2}}\, d\Phi_1\, d\Phi_2$. This equation is obtained from taking one element in both partial and secondary turn and integrating it from $\Phi = 0$ to $\Phi = 2\pi$ and using Biot Savart's law.

If we need to calculate for partial turn, then consider fixing secondary turn and varie primary turn for simple calculations. So, we need to varie Φ_1 from Φ to 0 and Φ_2 from 2π to 0. where, x = 0, α = 0, β = 0 and $\gamma = \frac{2ab}{a^2+b^2+z^2}$. Now after applying tailor series expansion on $[1-\gamma\cos(\Phi_1-\Phi_2)]^{\frac{-1}{2}}$ multiplying it with $cos(\Phi_1-\Phi_2)$ and finally integrating it we get the following equation for 1 turn secondary coil and partial turn primary coil

$$M = \boxed{\frac{\mu_0 ab}{4\pi\sqrt{a^2+b^2+z^2}}} \cdot \pi\Phi[\frac{\gamma}{2} + \frac{15\gamma^3}{64} + \frac{63\gamma^5}{512} + --] \quad (27)$$

Now for calculating M for n secondary turns and 1 partial primary turn we get the following equation

$$M = \sum_{s=1}^{n} \boxed{\frac{\mu_0 R_p R_s}{4\sqrt{R_p^2+R_s^2+d^2}}} \cdot \Phi[\frac{\gamma}{2} + \frac{15\gamma^3}{64} + \frac{63\gamma^5}{512} + \frac{429\gamma^7}{8192} --] \quad (28)$$

V. Conclusion

. 1. We could not transfer the desired power completely because of human errors and experimental limitations. But the simulations yielded the desired output for our given coil specifications.

2. Those human errors are fabricating the coil with space between two turns as 0, misalignment of coils when placed on top of each other and availability of thin SWG wire.

VI. References

1. Analytical design of Archimedean spiral coils used in inductive power transfer for electric vehicles application Kunwar Aditya

2. Optimization of Circular Coil Design for Wireless Power Transfer System in Electric Vehicle Battery Charging Applications Ravi Kumar Yakala1 · Sumit Pramanick1 · Debi Prasad Nayak1 · Manish Kumar1

3. Modeling of Mutual Coupling Between Planar Inductors in Wireless Power Applications Salahuddin Raju, Student Member, IEEE, Rongxiang Wu, Member, IEEE, Mansun Chan, Fellow, IEEE, and C. Patrick Yue, Senior Member, IEEE