Basic Electrical and Electronics Engineering

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LECTURE 2

Module 4
Digital Systems
Lecture 2
Topics to be covered

- **Logical Operations**
- **Boolean variables**
- **Truth Table**
- **Rules of Boolean Algebra**

BOOLEAN ALGEBRA

A set of rules to perform logical addition and multiplication

The basis of Boolean algebra lies in the operations of logical addition, or the OR operation; and logical multiplication, or the AND operation. Both of these find a correspondence in simple logic gates.

The variables in a Boolean, or logic, expression can take only one of two values, usually represented by the numbers 0 and 1.

These variables are sometimes referred to as true (1) and false (0). This convention is normally referred to as positive logic.

There is also a negative logic convention in which the roles of logic 1 and logic 0 are reversed.

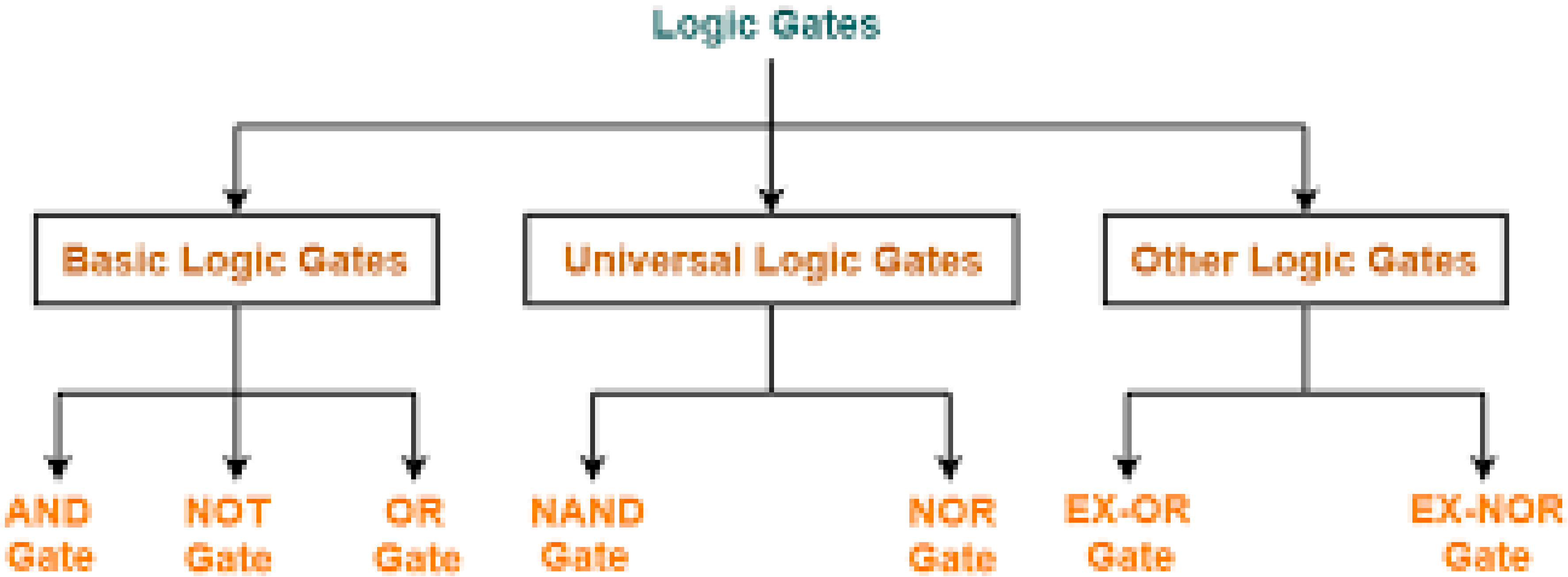
Truth Table

Analysis of logic functions, that is, functions of logical (Boolean) variables, can be carried out in terms of truth tables. A truth table is a listing of all the possible values each of the Boolean variables can take, and of the corresponding value of the function.

The rules that define a logic function are often represented in tabular form by means of a truth table

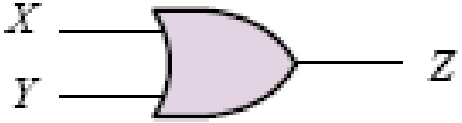
Logic Gates

Logic gates are physical devices that can be used to implement logic functions.



Types of Logic Gates

OR GATE



OR gate

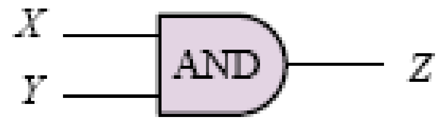
X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

Truth table

AND GATE

Rules
for logical
multiplication (AND)

$0 \cdot 0 = 0$
$0 \cdot 1 = 0$
$1 \cdot 0 = 0$
$1 \cdot 1 = 1$

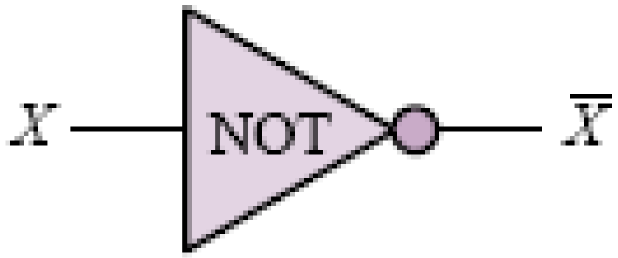


AND gate

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

Truth table

NOT GATE

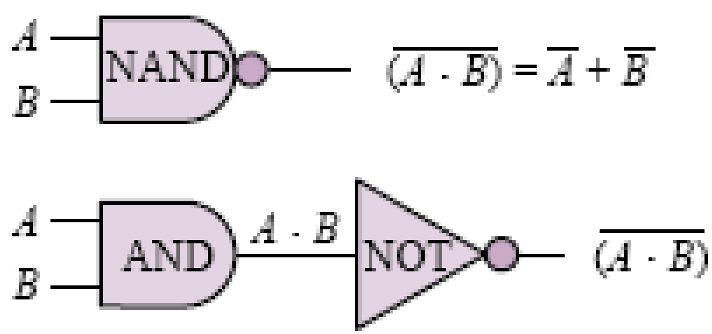


NOT gate

X	$\overline{X}$
1	0
0	1

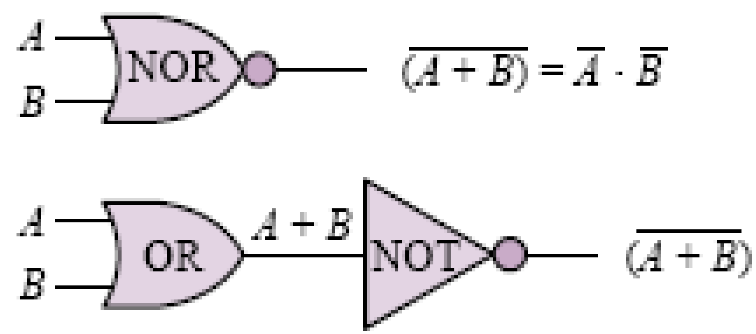
Truth table for NOT gate

NAND and NOR Gates



A	B	$\overline{A}$	$\overline{B}$	$\overline{(A \cdot B)}$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

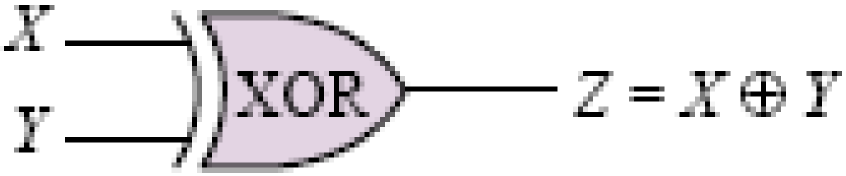
NAND gate

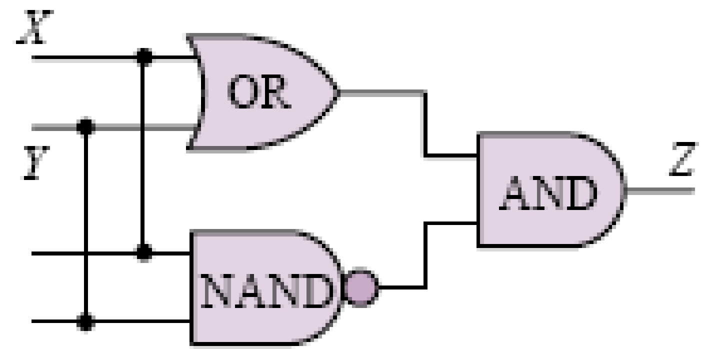


A	B	$\overline{A}$	$\overline{B}$	$\overline{(A + B)}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

NOR gate

XOR (Exclusive OR) Gate





X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

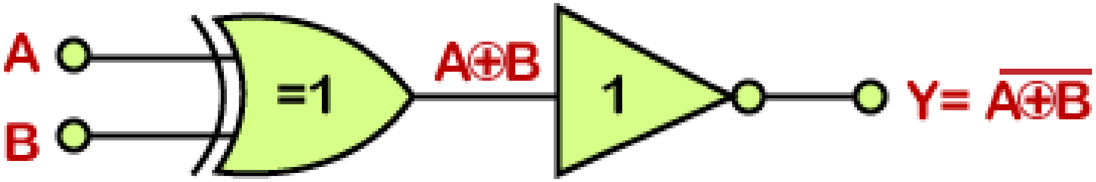
Truth table

$$Z = X \oplus Y = (X + Y) \cdot (\overline{X \cdot Y})$$

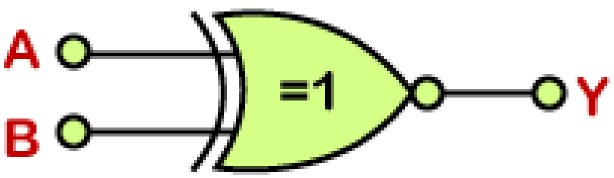
$$\boldsymbol{Z = \overline{X}Y + X\overline{Y}}$$

XOR gate is used in half and full adder circuits

XNOR (Exclusive NOR) Gate



2-input "Ex-OR" Gate plus a " NOT" Gate



2-input "Ex-NOR Gate

Truth Table		
A	B	Q
0	0	1
0	1	0
1	0	0
1	1	1

$$z = \overline{XY} + XY$$

Rules of Boolean algebra

1. $0 + X = X$
2. $1 + X = 1$
3. $X + X = X$
4. $X + \overline{X} = 1$
5. $0 \cdot X = 0$
6. $1 \cdot X = X$
7. $X \cdot X = X$
8. $X \cdot \overline{X} = 0$
9. $\overline{\overline{X}} = X$
10. $X + Y = Y + X$ } Commutative law
11. $X \cdot Y = Y \cdot X$ } Commutative law
12. $X + (Y + Z) = (X + Y) + Z$ } Associative law
13. $X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z$ } Associative law
14. $X \cdot (Y + Z) = X \cdot Y + X \cdot Z$ Distributive law
15. $X + X \cdot Z = X$ Absorption law
16. $X \cdot (X + Y) = X$
17. $(X + Y) \cdot (X + Z) = X + Y \cdot Z$
18. $X + \overline{X} \cdot Y = X + Y$
19. $X \cdot Y + Y \cdot Z + \overline{X} \cdot Z = X \cdot Y + \overline{X} \cdot Z$

De Morgan's theorems

These are stated here in the form of logic functions:

$$(\overline{X+Y}) = \overline{X}\cdot\overline{Y} \quad \textbf{or} \quad (X+Y) = \overline{\overline{X}\cdot\overline{Y}}$$

$$(\overline{X\cdot Y}) = \overline{X}+\overline{Y} \quad \textbf{or} \quad (X\cdot Y) = \overline{\overline{X}+\overline{Y}}$$

These two laws state a very important property of logic functions:

DeMorgan's first theorem is stated as follows:
The complement of two or more ORed variables is equivalent to the AND of the complements of the individual variables,

DeMorgan's Second theorem is stated as follows:
The complement of two or more ANDed variables is equivalent to the OR of the complements of the individual variables.

De Morgan's Shortcut

- **BREAK THE LINE, CHANGE THE SIGN**
- Break the *LINE* over the two variables,
- and change the *SIGN* directly under the line.

$$\overline{A+B} = \overline{A}\cdot\overline{B}$$

For Theorem 1, break the line, and change the OR function to an AND function.
Be sure to keep the lines over the variables.

$$\overline{A\cdot B} = \overline{A}+\overline{B}$$

For Theorem 2, break the line, and change the AND function to an OR function.
Be sure to keep the lines over the variables.

De Morgan's theorems

These are stated here in the form of logic functions:

$$(\overline{X + Y}) = \overline{X} \cdot \overline{Y}$$

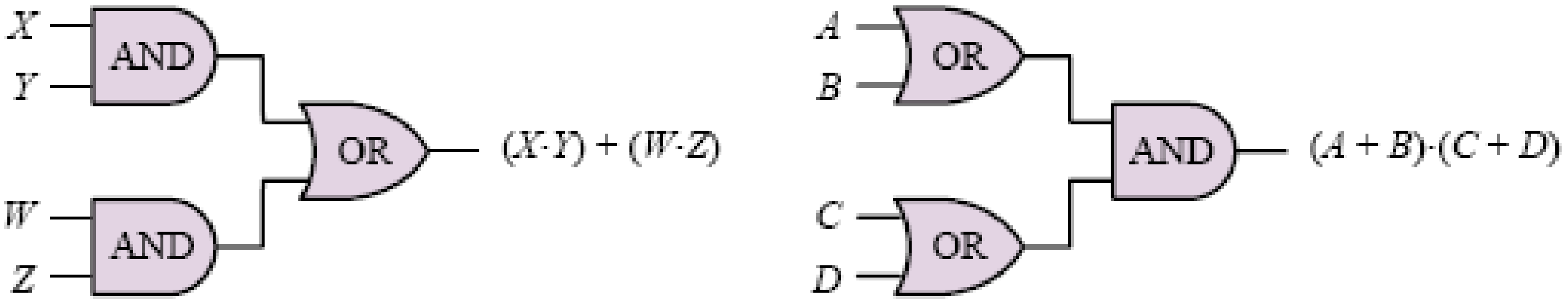
$$(\overline{X \cdot Y}) = \overline{X} + \overline{Y}$$

These two laws state a very important property of logic functions:

Any logic function can be implemented using only OR and NOT gates, or using only AND and NOT gates.

DUALITY

The importance of De Morgan's laws is in the statement of the duality that exists between AND and OR operations: any function can be realized by just one of the two basic operations, plus the complement operation. This gives rise to two families of logic functions: sums of products and product of sums.



Sum of products expression $(X \cdot Y) + (W \cdot Z)$

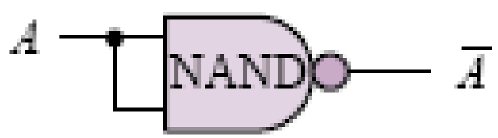
Product of sums expression $(A + B) \cdot (C + D)$

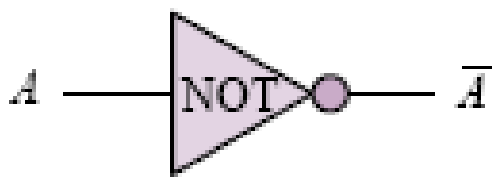
Any logical expression can be reduced to either one of these two forms. Although the two forms are equivalent, it may well be true that one of the two has a simpler implementation (fewer gates).

UNIVERSAL GATES

Inverter using NAND

A	$B(=A)$	$A \cdot B$	$\overline{(A \cdot B)}$
0	0	0	1
1	1	1	0

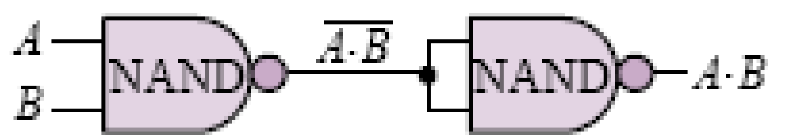


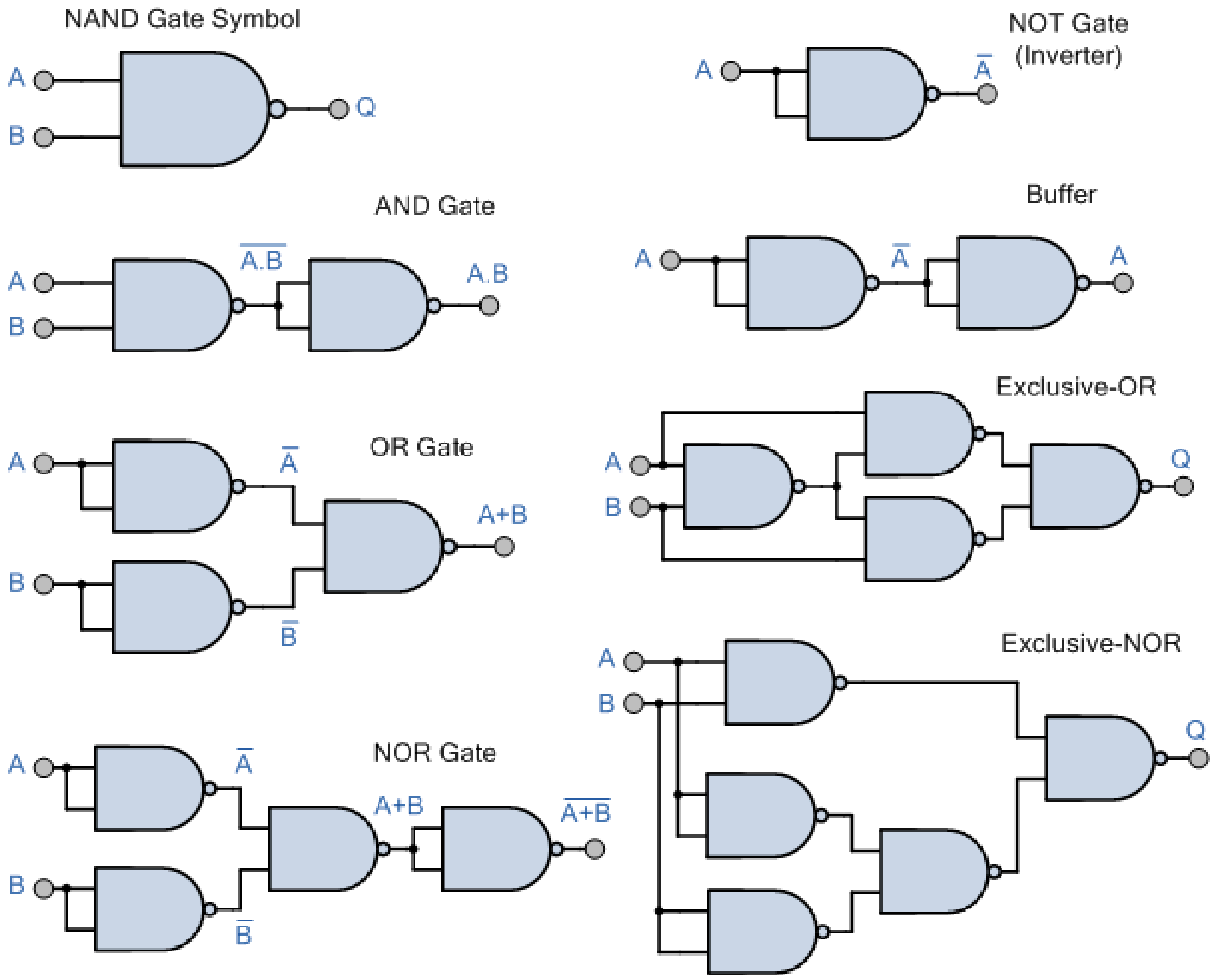


NAND gate
as an inverters

AND using NAND

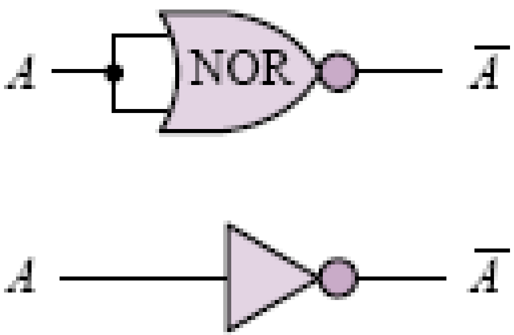
A	B	NAND $\overline{A \cdot B}$	AND $A \cdot B$
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1




NAND Gate Symbol
A
B
Q
NOT Gate
(Inverter)
A
$\overline{A}$
AND Gate
A
B
$\overline{A.B}$
A.B
Buffer
A
$\overline{A}$
A
OR Gate
A
B
$\overline{A}$
$\overline{B}$
A+B
Exclusive-OR
A
B
Q
NOR Gate
A
B
$\overline{A}$
$\overline{B}$
A+B
$\overline{A+B}$
Exclusive-NOR
A
B
Q

UNIVERSAL GATES

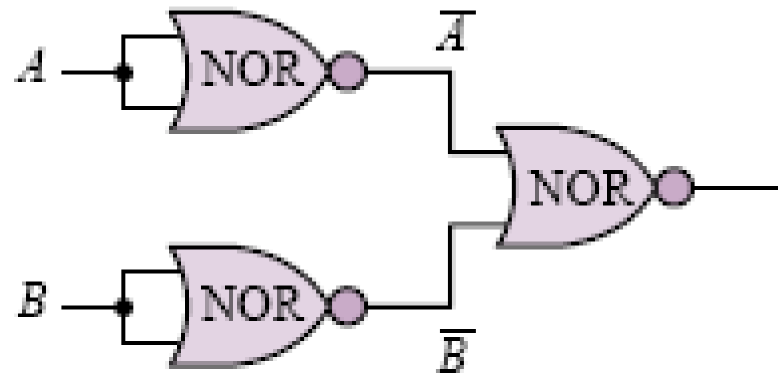
Inverter using NOR



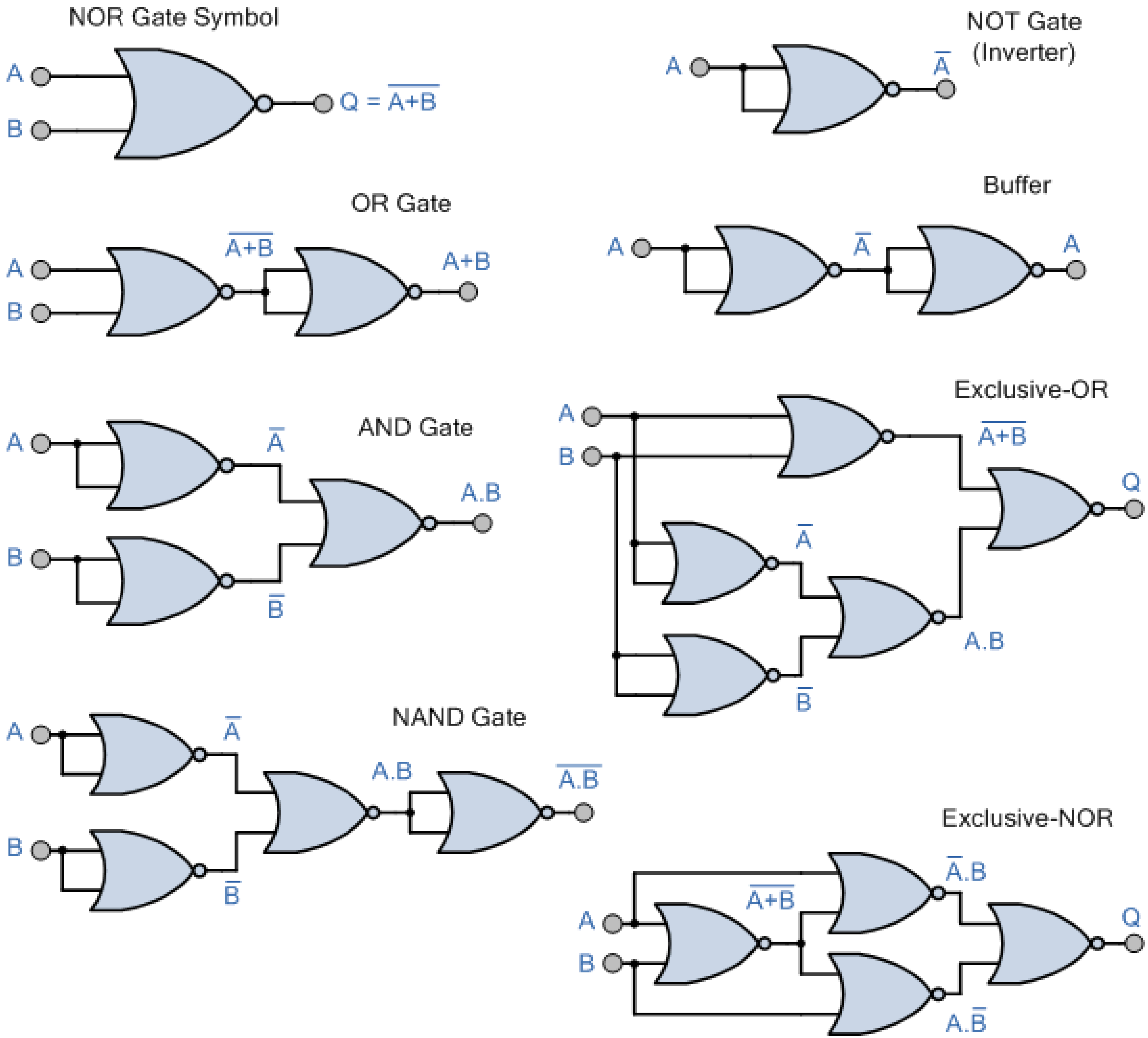
A	$B(=A)$	$(A+B)$	$\overline{(A+B)}$
0	0	0	1
1	1	1	0

NOR gate as
an inverter

AND using NOR



$$\overline{(\overline{A}+\overline{B})} = A \cdot B$$


NOR Gate Symbol
A
B
Q = A+B
NOT Gate (Inverter)
A
A
OR Gate
A
B
A+B
A+B
Buffer
A
A
A
AND Gate
A
B
A
B
A.B
Exclusive-OR
A
B
A+B
A
B
A.B
Q
NAND Gate
A
B
A
B
A.B
A.B
Exclusive-NOR
A
B
A+B
A.B
A.B
Q

Exercise:

Realize the following function using only NAND and NOR gates:

$$y = \overline{\overline{(A \cdot B)} + C}$$

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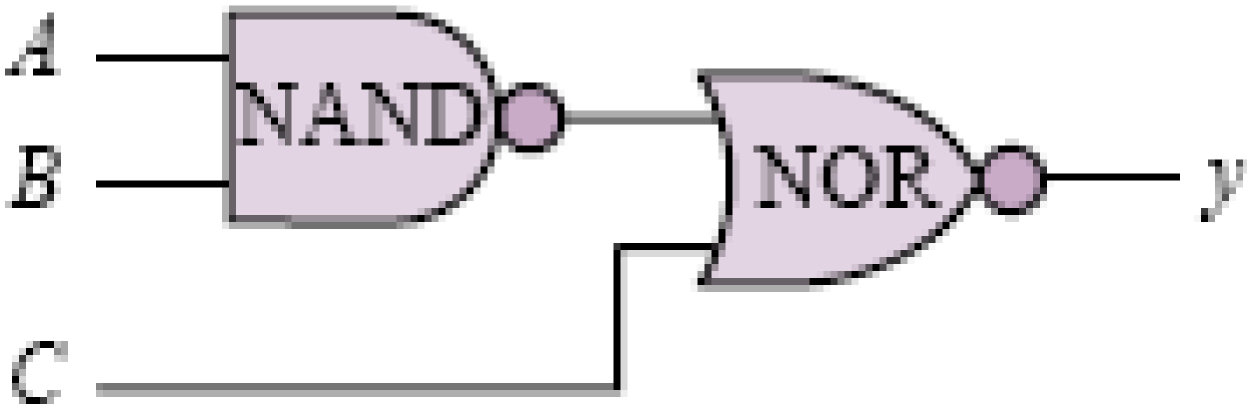
$$y = \overline{\overline{(A \cdot B)} + C}$$



Exercise:

Simplify

$$F = ABC' + AB'C + A'BC + ABC$$

Exercise:

Simplify

$$F = ABC' + AB'C + A'BC + ABC$$

Solution:

Since $X+X = X$, ABC can be repeated many times

$$= ABC' + ABC + AB'C + ABC + A'BC + ABC$$

$$= AB(C'+C) + AC(B'+B) + BC(A'+A)$$

$$= AB + AC + BC$$

Exercise:

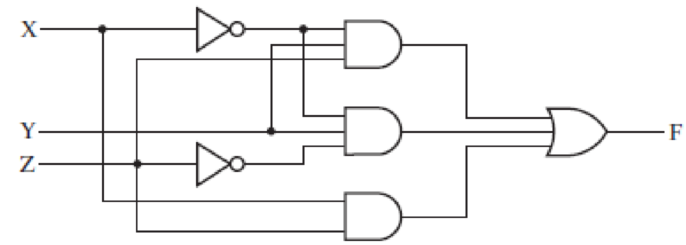
1. $F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$

2. $XY + \overline{X}Z + YZ$

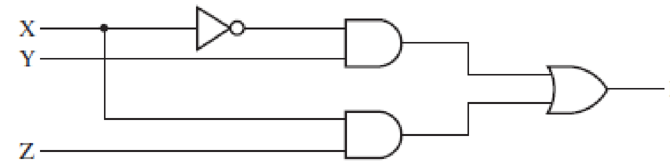
Exercise:

1. $F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$

$$
\begin{aligned}
F &= \overline{X}YZ + \overline{X}Y\overline{Z} + XZ \\
&= \overline{X}Y(Z + \overline{Z}) + XZ && \text{by identity 14} \\
&= \overline{X}Y \cdot 1 + XZ && \text{by identity 7} \\
&= \overline{X}Y + XZ && \text{by identity 2}
\end{aligned}
$$



(a) $F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$



(b) $F = \overline{X}Y + XZ$

2. $XY + \overline{X}Z + YZ$

$$
\begin{aligned}
XY + \overline{X}Z + YZ &= XY + \overline{X}Z + YZ(X + \overline{X}) \\
&= XY + \overline{X}Z + XYZ + \overline{X}YZ \\
&= XY + XYZ + \overline{X}Z + \overline{X}YZ \\
&= XY(1 + Z) + \overline{X}Z(1 + Y) \\
&= XY + \overline{X}Z
\end{aligned}
$$

Exercise:

$$f(A, B, C, D) = \overline{A} \cdot \overline{B} \cdot D + \overline{A} \cdot B \cdot D + B \cdot C \cdot D + A \cdot C \cdot D$$

Find: Simplified expression for logical function of four variables.

Solution:

Exercise:

$$f(A, B, C, D) = \overline{A} \cdot \overline{B} \cdot D + \overline{A} \cdot B \cdot D + B \cdot C \cdot D + A \cdot C \cdot D$$

Find: Simplified expression for logical function of four variables.

Solution:

$$f = \overline{A} \cdot \overline{B} \cdot D + \overline{A} \cdot B \cdot D + B \cdot C \cdot D + A \cdot C \cdot D$$

$$= \overline{A} \cdot D \cdot \left(\overline{B} + B\right) + B \cdot C \cdot D + A \cdot C \cdot D \qquad \text{Rule 14}$$

$$= \overline{A} \cdot D + B \cdot C \cdot D + A \cdot C \cdot D \qquad \text{Rule 4}$$

$$= \left(\overline{A} + A \cdot C\right) \cdot D + B \cdot C \cdot D \qquad \text{Rule 14}$$

$$= \left(\overline{A} + C\right) \cdot D + B \cdot C \cdot D \qquad \text{Rule 18}$$

$$= \overline{A} \cdot D + C \cdot D + B \cdot C \cdot D \qquad \text{Rule 14}$$

$$= \overline{A} \cdot D + C \cdot D \cdot (1 + B) \qquad \text{Rule 14}$$

$$= \overline{A} \cdot D + C \cdot D = \left(\overline{A} + C\right) \cdot D \qquad \text{Rules 2 and 6}$$

Exercise:

Simplify the following expression to show that it corresponds to the function $\overline{Z}$:

$$\overline{W}\cdot\overline{X}\cdot\overline{Y}\cdot\overline{Z}+\overline{W}\cdot X\cdot\overline{Y}\cdot\overline{Z}+W\cdot X\cdot\overline{Y}\cdot\overline{Z}+W\cdot\overline{X}\cdot\overline{Y}\cdot\overline{Z}+\overline{W}\cdot\overline{X}\cdot Y\cdot\overline{Z}$$
$$+\overline{W}\cdot X\cdot Y\cdot\overline{Z}+W\cdot X\cdot Y\cdot\overline{Z}+W\cdot\overline{X}\cdot Y\cdot\overline{Z}$$

Exercise:

Simplify the following expression to show that it corresponds to the function $\overline{Z}$:

$$\overline{W}\cdot\overline{X}\cdot\overline{Y}\cdot\overline{Z}+\overline{W}\cdot X\cdot\overline{Y}\cdot\overline{Z}+W\cdot X\cdot\overline{Y}\cdot\overline{Z}+W\cdot\overline{X}\cdot\overline{Y}\cdot\overline{Z}+\overline{W}\cdot\overline{X}\cdot Y\cdot\overline{Z}$$
$$+\overline{W}\cdot X\cdot Y\cdot\overline{Z}+W\cdot X\cdot Y\cdot\overline{Z}+W\cdot\overline{X}\cdot Y\cdot\overline{Z}$$

Solution:

1 and 5 $\quad \overline{W}\cdot\overline{X}\cdot\overline{Y}\cdot\overline{Z}+\overline{W}\cdot\overline{X}\cdot Y\cdot\overline{Z} = \overline{W}\cdot\overline{X}\cdot\overline{Z}$

$$= \overline{W}\cdot\overline{Z}$$

2 and 6 $\quad \overline{W}\cdot X\cdot\overline{Y}\cdot\overline{Z}+\overline{W}\cdot X\cdot Y\cdot\overline{Z} = \overline{W}\cdot X\cdot\overline{Z}$

$$= \overline{Z}$$

3 and 4 $\quad W\cdot X\cdot\overline{Y}\cdot\overline{Z}+W\cdot\overline{X}\cdot\overline{Y}\cdot\overline{Z} = W\cdot\overline{Y}\cdot\overline{Z}$

$$= W\cdot\overline{Z}$$

7 and 8 $\quad W\cdot X\cdot Y\cdot\overline{Z}+W\cdot\overline{X}\cdot Y\cdot\overline{Z} = W\cdot Y\cdot\overline{Z}$

Exercise:

Simplify

$$F = ((AB)' + (AC)')'$$

Exercise:

Simplify

$F= ((AB)' + (AC)')'$

Using De Morgan's Theorem

$F= ((AB)' + (AC)')'$

$= (AB)'' . (AC)''$

$= (AB).(AC)$

$= A.B.C$

Exercise:

Simplify

1) F= (A+B).(A+B)’ = 0

2) F= A.B + (A.B)’ = 1

3) F= (A+B.C’).(A+B.C’) =A+B

3. Simplify the following Boolean expression and draw the logic circuits for the simplified expressions.

(a) $(\overline{A}BC+A\overline{B}C+ABC+B\overline{C})$ (b) $\overline{B}(A+C)+C(\overline{A}+B)+AC$

(c) $(AB + C)(AB + D)$

a) $= \overline{A}BC + A\overline{B}C + ABC + B\overline{C}(A+\overline{A})$

$= \overline{A}BC + A\overline{B}C + ABC + AB\overline{C} + \overline{A}B\overline{C} = B + AC$

Answers:

a) B+AC

b) C+AB’

c) AB+CD

Example

Row No.	A B C	Minterms	Maxterms
0	0 0 0	$A'B'C' = m_0$	$A+B+C = M_0$
1	0 0 1	$A'B'C = m_1$	$A+B+C' = M_1$
2	0 1 0	$A'BC' = m_2$	$A+B'+C = M_2$
3	0 1 1	$A'BC = m_3$	$A+B'+C' = M_3$
4	1 0 0	$AB'C' = m_4$	$A'+B+C = M_4$
5	1 0 1	$AB'C = m_5$	$A'+B+C' = M_5$
6	1 1 0	$ABC' = m_6$	$A'+B'+C = M_6$
7	1 1 1	$ABC = m_7$	$A'+B'+C' = M_7$

Output can be 1or 0.

1 – is represented by product of inputs.

0 – is represented by sum of inputs.

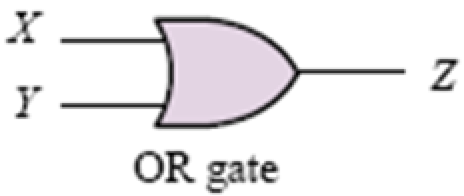
If the output is high for Row No. 3, 4, 5, 6, 7

What is the expression of logical function?

How to write a function

Minterms present in **f (output function)** correspond with the 1's of f in the truth table.

Maxterms present in **f (output function)** correspond with the 0's of f in the truth table.



output function

Z = X+Y

Or

Z = X'Y + XY' + X Y

X	Y	Z	Minterm	Maxterm
0	0	0	--	X+Y
0	1	1	X'Y	
1	0	1	XY'	
1	1	1	X Y	

Proof: Z = X'Y + X (Y+Y') = X'Y + X (1+Y)= X + Y(X+X') = X+Y

In SOP form

$$f = A'BC + AB'C' + AB'C + ABC'+ABC;$$

or

$$f(A,B,C) = m_3 + m_4 + m_5 + m_6 + m_7$$

or

$$f(A,B,C) = \sum m(3,4,5,6,7)$$

or

$$f(A,B,C) = 0.m_0 + 0.m_1 + 0.m_2 + 1.\ m_3 + 1.\ m_4 + 1.\ m_5 + 1.\ m_6 + 1.\ m_7$$

Sum of Products (SOP) and Product of Sums (POS)

A function can be written as a sum of **minterms**, which is referred to as a minterm expansion or a standard **sum of products**.

Min Term:

ANDed product of literals in which each variable appears exactly once, in true or complemented form.

A function can be written as a product of **maxterms**, which is referred to as a maxterm expansion or a standard **product of sums**.

Max term: ORed sum of literals in which each variable appears exactly once in either true or complemented form, but not both.

All possible minterms and maxterms of two variable truth table

X	Y	Minterm	Maxterm
0	0	$\overline{x}\,\overline{y}$	$x + y$
0	1	$\overline{x}\,y$	$x + \overline{y}$
1	0	$x\,\overline{y}$	$\overline{x} + y$
1	1	$x\,y$	$\overline{x} + \overline{y}$

Truth table