Module 3

MAGNETIC CIRCUITS

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Magnetic field; Toroidal core: Flux density, Flux linkage; Magnetic circuit with airgap; Reluctance in series and parallel circuits; Self and mutual inductance; Transformer: turn ratio determination		

- Magnetic fields
- Flux density, Flux linkage Toroidal core:
- Magnetic circuit with airgap
- Reluctance in series and parallel circuits
- Self and mutual inductance
- Transformer: turn ratio determination

Magnetic fields

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- Magnetic fields exist in the space around permanent magnets and around wires that carry current, so basic source of magnetic field is electrical charge in motion.
- Fields in an iron permanent magnet are created by the spin of the electrons in atoms. These fields aid to one another producing the net external field. In most materials, the magnetic fields of the electrons tend to cancel one another.
- If a current-carrying wire is formed into a multi-turn coil, the magnetic field is greatly intensified, particularly if the coil is wound around an iron core.
- Magnetic fields are visualized as magnetic flux lines that form closed paths. The lines are close together where the field is strong and farther apart where the field is weak.
- Units of magnetic flux is webers (Wb).

Magnetic poles and field lines

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- A magnet has a north-seeking end (N) and a south-seeking end (S) and unlike ends of magnets are attracted.
- By convention, flux lines leave the north-seeking end (N) of a magnet and enter its south-seeking end (S).
- A compass acts exactly in the same manner and can be used to investigate the direction of the lines of flux.

Earth's field lines are directed from south to north. Thus, S would appear near the north geographic pole, because that is where the field lines enter the earth.

Magnetic flux density is represented by B and its unit is webers/sq meter (Wb/m^2) or Tesla

Right hand thumb rule

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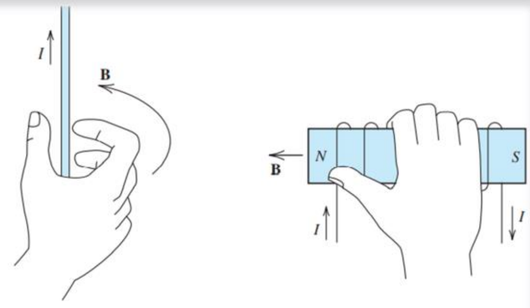
- The direction of the magnetic field produced by a current is determined by the right-hand rule.



(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field

(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

Flux Linkages

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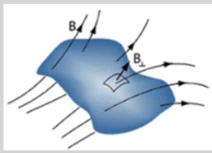
- The magnetic flux passing through a surface area A is determined by integrating the dot product of B and incremental area over the surface.



$$\phi = \int_A \mathbf{B} \cdot d\mathbf{A}$$

where, the direction of the vector dA is perpendicular to the surface.

and dA is an increment of area on the surface

- If the magnetic flux density is constant and perpendicular to the surface,

$$\phi = BA$$

(as BA Cos 0°=BA so max)

- Flux passing through the surface bounded by a coil links the coil. If the coil has N turns, then the total flux linkages

$$\lambda = N\phi$$

- The same flux links each turn of the coil, and this hold good when the turns are close together on an iron form, which is often the case in transformers and electrical machines.

Faraday’s Law

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- According to Faraday’s law of magnetic induction, a voltage is induced in a coil whenever its flux linkages are changing.

$$e = \frac{d\lambda}{dt}$$

- This can occur either because the magnetic field is changing with time or because the coil is moving relative to a magnetic field.
- The polarity of the induced voltage is such that the voltage would produce a current that opposes the original change in flux linkages; which is the Lenz’s law.

Magnetic Field Intensity and Ampère's Law

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- Magnetic flux density 'B' produces forces on moving charges and current-carrying conductors. It also induces voltage in a coil if the flux linkages are changing with time.
- Furthermore, voltage is induced across a moving conductor when it cuts through flux lines.
- Magnetic field intensity 'H' is another parameter to determine the establishment of magnetic fields. Magnetic fields are established by charges in motion or currents flowing in coils.
- H is determined by currents and the configuration of coils; and flux density 'B' depends on H, as well as the properties of the material filling the space around the coils.

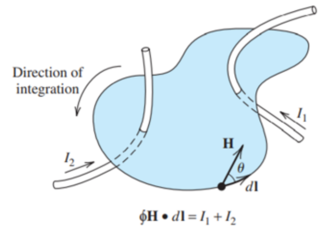
Magnetic field intensity 'H' & Magnetic flux density 'B' are related as ➡ $\mathbf{B} = \mu \mathbf{H}$

where **μ** is the **magnetic permeability of the material** with unit as **webers/ampere-meter (Wb/Am)**. Unit of H is **amperes/meter (A/m)**.

For free space, $\mu = \mu_0 = 4\pi \times 10^{-7}\ \mathrm{Wb/Am}$

Materials, most notably iron and certain rare-earth alloys, have a much higher magnetic permeability than free space. The relative permeability of a material is the ratio of its permeability to that of free space;

$$\mu_r = \frac{\mu}{\mu_0}$$



Ampère's law states that the line integral of the magnetic field intensity around a closed path is equal to the algebraic sum of the currents flowing through the area enclosed by the path.

$$\oint \mathbf{H} \cdot d\mathbf{l} = \sum i$$

in which dl is a vector element of length having its direction tangent to the path of integration and the vector dot product is given by

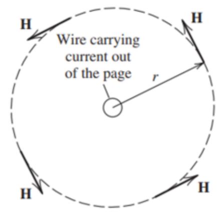
$$\mathbf{H} \cdot d\mathbf{l} = H\, dl\, \cos(\theta)$$ ⇒ where θ is the angle between H and dl

If the magnetic intensity has constant magnitude and points in the same direction as the incremental length dl everywhere along the path, Ampère's law reduces to

$$Hl = \sum i$$ ⇒ where l is the length of the path

Magnetic Field around a Long Straight Wire

- The magnetic field around a long straight wire carrying a current can be determined with Ampère's law aided by considerations of symmetry.
- By symmetry and the right-hand rule, B and H fall in a plane perpendicular to the wire and are tangent to circles having their centers at the wire.
- Magnitude of H is constant for a given radius r. Applying Ampère's law to the circular path,



$$Hl = H2\pi r = I$$

Magnetic field intensity $\Rightarrow$ $H = \dfrac{I}{2\pi r}$

Magnetic flux density $\Rightarrow$ $\boxed{B = \mu H = \dfrac{\mu I}{2\pi r}}$

Flux density and flux linkages in a Toroidal Core

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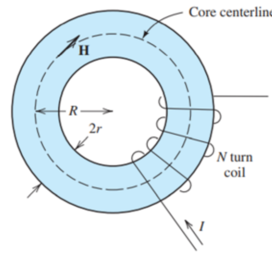
- By symmetry, field intensity is constant in magnitude along the dashed circular center line. i.e., it is assumed that the coil is wound in a symmetrical manner all the way around the toroidal core.
- Applying Ampère's law; the magnetic flux density 'B' on the center line of a toroidal core.



$$Hl = H2\pi R = NI \Rightarrow H = \frac{NI}{2\pi R} \Rightarrow \boxed{B = \frac{\mu NI}{2\pi R}}$$

As R >> r, the flux density is nearly constant over the cross section of the core.

Flux is equal to the product of the flux density and the area of the cross section

$$\phi = BA = \frac{\mu NI}{2\pi R}\pi r^2 = \frac{\mu NIr^2}{2R}$$

Total flux linkages $\Rightarrow$ $$\lambda = N\phi = \frac{\mu N^2 Ir^2}{2R}$$

Magnetic-Circuit Approach

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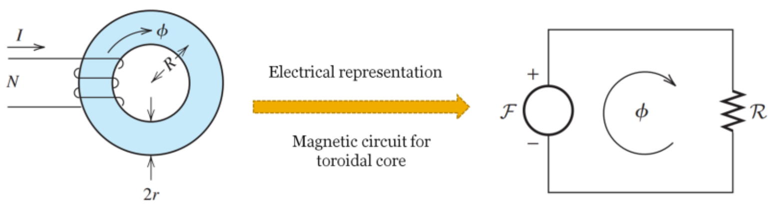
- Transformers, motors and generators contain coils wounded on iron cores and Magnetic-Circuit Approach is used to calculate magnetic fields in these devices.

The advantage of the magnetic-circuit approach is that it can be applied to unsymmetrical magnetic cores with multiple coils.

The magnetic-circuit approach is not an exact method for determining magnetic fields, but it is sufficiently accurate for many engineering applications.

- Coils are sources of magnetomotive forces (mmf) that are manipulated as voltage source in an electrical circuit.
- Reluctances in series or parallel are compared and combined as resistances.
- Fluxes are analogous to currents.



Magnetomotive force and Reluctance

- Due to symmetry of toroid, field intensity can be obtained using Ampère's law However, for complex configurations as cores without symmetry and having multiple coils, Ampère's law is not feasible and magnetic circuit concepts analogous to analyze electrical circuits are used.

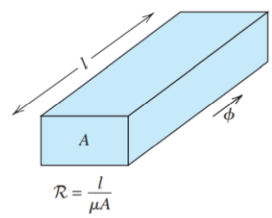
Magnetomotive force (mmf) of an N-turn current-carrying coil is ⇒ $\mathcal{F} = Ni$

- As a current-carrying coil in magnetic-circuit is the analog of a voltage source in an electrical circuit, magnetomotive force is analogous to source voltage and its units is given as **A·turns**; however, the number of turns is actually a pure number without physical units.

The reluctance of a path for magnetic flux of an iron bar is

$$\mathcal{R} = \frac{l}{\mu A}$$

⇒ where l is the length of the path (in the direction of the magnetic flux), A is the cross-sectional area, and μ is the permeability of the material.



When the bar is not straight, the length of the path is somewhat ambiguous, and then we estimate its value as the length of the centerline. Thus, l is sometimes called the mean length of the path.

Magnetic flux φ in a magnetic circuit is analogous to current in an electrical circuit. Magnetic flux, reluctance, and magnetomotive force are related by

$$\mathcal{F} = \mathcal{R}\phi$$

which is the counterpart of Ohm's law (V = Ri)
Units of reluctance are A·turns/Wb.

The magnetic circuit of a **toroidal coil** is analogous to an electrical circuit with a resistance connected across a voltage source. The mean length of the magnetic path is

$$l = 2\pi R$$

The cross section of the core is circular with radius r. Thus, the area of the cross section is

$$A = \pi r^2$$

Reluctance $\mathcal{R} = \frac{l}{\mu A} = \frac{2\pi R}{\mu \pi r^2} = \frac{2R}{\mu r^2}$

Magnetomotive force $\mathcal{F} = NI$

Flux $\boxed{\phi = \frac{\mu N r^2 I}{2R}}$

Problem

Determine the flux, flux linkages and the voltage induced in the coil for a toroidal core with μ_r = 5000, R = 10 cm, r = 2 cm, and N = 100. Consider the current as i(t) = 2 sin(200πt)

Permeability of the core material; $\mu = \mu_r \mu_0 = 5000 \times 4\pi \times 10^{-7}$

Flux; $$\phi = \frac{\mu N I r^2}{2R} = \frac{5000 \times 4\pi \times 10^{-7} \times 100 \times 2\sin(200\pi t) \times (2 \times 10^{-2})^2}{2 \times 10 \times 10^{-2}}$$

$$= (2.513 \times 10^{-3}) \sin(200\pi t) \text{ Wb}$$

Flux linkages; $$\lambda = N\phi$$
$$= 100 \times (2.513 \times 10^{-3}) \sin(200\pi t)$$
$$= 0.2513 \sin(200\pi t) \text{ weber turns}$$

Voltage induced in the coil; $$e = \frac{d\lambda}{dt} = 0.2513 \times 200\pi \cos(200\pi t)$$
$$= 157.9 \cos(200\pi t) \text{ V}$$

Toroidal core

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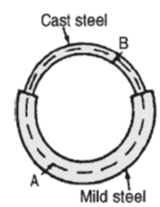
1. A closed magnetic circuit of cast steel contains a 6cm long path of cross-sectional area 1cm^2 and a 2cm path of cross-sectional area 0.5cm^2 . A coil of 200 turns is wounded around the 6cm length of the circuit and a current of 0.4 A flows. Determine the flux density in the 2cm path, if the relative permeability of the cast steel is 750. **[1.51 T]**

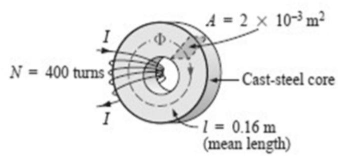
2. An iron ring of cross-sectional area 5cm^2 has a radial air gap of 2mm cut into it. If the mean length of the iron path is 40cm, calculate the magnetomotive force to produce a flux of 0.7mWb if *H* =1650At/m. **[2888 Aturns]**

3. Find the total mmf required to cause a flux of 500μWb in the magnetic circuit. Determine also the total circuit reluctance if mean length and the cross-sectional area of mild steel are 400mm and 500mm^2 and for cast steel is 300mm and 312.5mm^2 respectively. Relative permeability of cast steel and mild steel is 265 and 570. **[2888 Aturns, 4×10^6 Aturns/Wb]**



Problems

1. An iron ring of 400 cm mean circumference is made from iron of cross-section 20 cm^2. Its permeability is 500. If it is wound with 400 turns, what current would be required to produce a flux of 0.01 wb? [79.58A]
2. Find the value of I and μ_r for the material required to develop a magnetic flux of 4×10^{-4} wb if H = 170 AT/m.



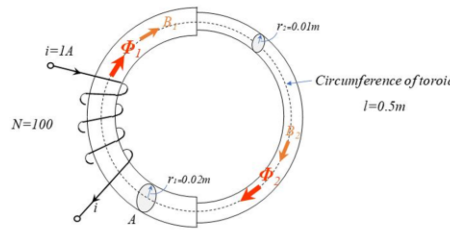
[68mA, 935.83]

Problems

1. A coil of 600 turns is wound uniformly on a ring of non-magnetic material. The ring has a uniform cross-sectional area of 200mm2 and a mean circumference of 500mm. If the current in the coil is 4A, determine (a) the magnetic field strength, (b) the flux density, and (c) the total magnetic flux in the ring.

2. A mild steel ring of cross-sectional area 4cm2 has a radial air-gap of 3mm cut into it. If the mean length of the mild steel path is 300mm, calculate the magnetomotive force to produce a flux of 0.48mWb.

3. Find the inductance and total reluctance of the core with a relative permeability ofb 1500. Also find B1,B2, $\Phi_1$and Φ_2



Magnetic Circuit with Air Gap

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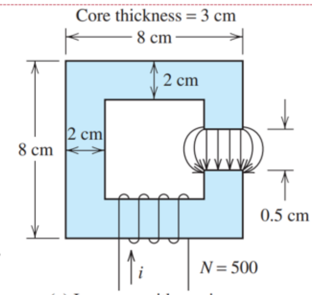
The core material has a relative permeability of 6000 and a rectangular cross section 2 cm by 3 cm. The coil has 500 turns. Determine the current required to establish a flux density of B_{gap} = 0.25 T in the air gap.



(a) Iron core with an air gap

The magnetic circuit is analogous to an electrical circuit with one voltage source and two resistances in series.

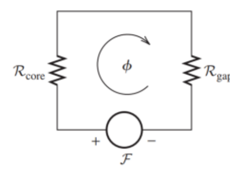
For the reluctance of the core, the centerline of the flux path is a square of 6 cm by 6 cm. So, the mean length of the iron core is $l_{core} = 4 \times 6 - 0.5 = 23.5$ cm

Cross-sectional area of core;

$$A_{core} = 2\text{ cm} \times 3\text{ cm} = 6 \times 10^{-4}\text{ m}^2$$

Permeability of the core;

$$\mu_{core} = \mu_r \mu_0 = 6000 \times 4\pi \times 10^{-7} = 7.540 \times 10^{-3}$$



(b) Magnetic circuit

Reluctance of the core; $\mathcal{R}_{core} = \dfrac{l_{core}}{\mu_{core} A_{core}} = \dfrac{23.5 \times 10^{-2}}{7.540 \times 10^{-3} \times 6 \times 10^{-4}}$

$$= 5.195 \times 10^4\text{ A·turns/Wb}$$

Reluctances in series

- For reluctance of the air gap; the flux lines tend to bow out which is called **fringing**. The effective fringing is calculated by adding the length of the gap to the depth and width for computing the effective gap area.
- Area of the air gap is larger than that of the iron core.
- So adding the length of the gap to each dimensions of the air-gap cross section; the effective area of the gap is

$$A_{\text{gap}} = (2\text{ cm } + 0.5\text{ cm}) \times (3\text{ cm } + 0.5\text{ cm}) = 8.75 \times 10^{-4}\text{ m}^2$$

Permeability of air is same as that of free space; $\mu_{\text{gap}} \cong \mu_0 = 4\pi \times 10^{-7}$

Reluctance of the gap; $\mathcal{R}_{\text{gap}} = \dfrac{l_{\text{gap}}}{\mu_{\text{gap}} A_{\text{gap}}} = \dfrac{0.5 \times 10^{-2}}{4\pi \times 10^{-7} \times 8.75 \times 10^{-4}}$

$$= 4.547 \times 10^6\text{ A}\cdot\text{turns/Wb}$$

Total reluctance is the sum of the reluctance of the core and the gap;

$$\mathcal{R} = \mathcal{R}_{\text{gap}} + \mathcal{R}_{\text{core}} = 4.547 \times 10^6 + 5.195 \times 10^4 = 4.600 \times 10^6$$

Flux, mmf & current estimation

- Even though the gap is much shorter than the iron core, the reluctance of the gap is higher than that of the core because of the much higher permeability of the iron and most of the magnetomotive force is dropped across the air gap.
- This is analogous to the fact that the largest fraction of the applied voltage is dropped across the largest resistance in a series electrical circuit.

Flux; $\phi = B_{\text{gap}} A_{\text{gap}} = 0.25 \times 8.75 \times 10^{-4} = 2.188 \times 10^{-4}$ Wb

The flux in the core is the same as that in the gap. However, the flux density is higher in the core, because the area is smaller.

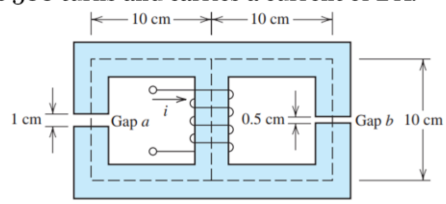
Magnetomotive force; $\mathcal{F} = \phi \mathcal{R} = 4.600 \times 10^{6} \times 2.188 \times 10^{-4} = 1006$ A·turns

Current; $\boxed{i = \dfrac{\mathcal{F}}{N} = \dfrac{1006}{500} = 2.012 \text{ A}}$ since; $\mathcal{F} = Ni$

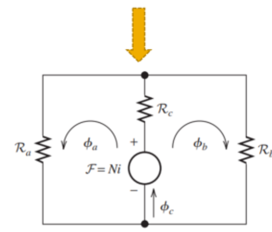
Reluctances in parallel

Flux density in each air gap for iron core having cross section of 2 cm by 2 cm and a relative permeability of 1000. The coil has 500 turns and carries a current of 2 A.

Like currents, flux gets distributed accordingly as they come across parallel paths. So there is more than one path available for the magnetic field lines (flux) to flow in the circuit. The coil sets up a total flux Φ in the circuit which gets divided into the parallel paths



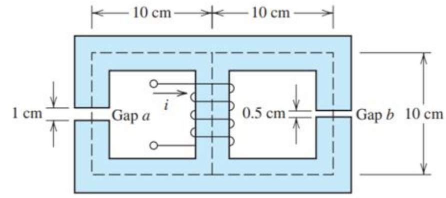
$$\varphi_c = \varphi_a + \varphi_b$$



So calculating reluctance of the three paths
For the center path,

$$\mathcal{R}_c = \frac{l_c}{\mu_r \mu_0 A_{\text{core}}} = \frac{10 \times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}}$$
$$= 1.989 \times 10^5 \text{ A·turns/Wb}$$

For the left-hand path, the total reluctance is the sum of the reluctance of the iron core plus the reluctance of gap a.

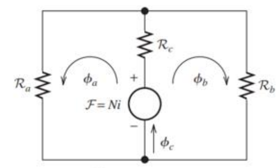


Taking fringing into account by adding the gap length to its width and depth in computing area of the gap.

Area of gap a is

A_a = 3 cm × 3 cm = 9 × 10−4 m²

Total reluctance of the left-hand path is



$$
\begin{aligned}
\mathcal{R}_a &= \mathcal{R}_{\text{gap}} + \mathcal{R}_{\text{core}} \\
&= \frac{l_{\text{gap}}}{\mu_0 A_a} + \frac{l_{\text{core}}}{\mu_r \mu_0 A_{\text{core}}} \\
&= \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 9 \times 10^{-4}} + \frac{29 \times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} \\
&= 8.842 \times 10^6 + 5.769 \times 10^5 \\
&= 9.420 \times 10^6 \text{ A·turns/Wb}
\end{aligned}
$$

Reluctance of the right-hand path is

$$\begin{aligned}\mathcal{R}_b &= \mathcal{R}_{\text{gap}} + \mathcal{R}_{\text{core}} \\ &= \frac{l_{\text{gap}}}{\mu_0 A_b} + \frac{l_{\text{core}}}{\mu_r \mu_0 A_{\text{core}}} \\ &= \frac{0.5 \times 10^{-2}}{4\pi \times 10^{-7} \times 6.25 \times 10^{-4}} + \frac{29.5 \times 10^{-2}}{1000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} \\ &= 6.366 \times 10^6 + 5.869 \times 10^5 \\ &= 6.953 \times 10^6 \text{ A·turns/Wb}\end{aligned}$$

Combining the reluctances R_a and R_b in parallel. Then, the total reluctance is the sum of R_c and this parallel combination:

$$\begin{aligned}\mathcal{R}_{\text{total}} &= \mathcal{R}_c + \frac{1}{1/\mathcal{R}_a + 1/\mathcal{R}_b} \\ &= 1.989 \times 10^5 + \frac{1}{1/(9.420 \times 10^6) + 1/(6.953 \times 10^6)} \\ &= 4.199 \times 10^6 \text{ A·turns/Wb}\end{aligned}$$

Flux in the center leg of the coil;

$$\phi_c = \frac{Ni}{\mathcal{R}_{\text{total}}} = \frac{500 \times 2}{4.199 \times 10^6} = 238.1\ \mu\text{Wb} \qquad (Since; F = NI = R\varphi)$$

Fluxes are analogous to currents, flux in the left-hand and right-hand paths are determine d using the current-division rule.

For gap a;

$$\phi_a = \phi_c \frac{\mathcal{R}_b}{\mathcal{R}_a + \mathcal{R}_b}$$

$$= 238.1 \times 10^{-6} \times \frac{6.953 \times 10^6}{6.953 \times 10^6 + 9.420 \times 10^6}$$

$$= 101.1\ \mu\text{Wb}$$

For gap b;

$$\phi_b = \phi_c \frac{\mathcal{R}_a}{\mathcal{R}_a + \mathcal{R}_b}$$

$$= 238.1 \times 10^{-6} \frac{9.420 \times 10^6}{6.953 \times 10^6 + 9.420 \times 10^6}$$

$$= 137.0\ \mu\text{Wb}$$

As; $\varphi_c = \varphi_a + \varphi_b$ the values for the fluxes satisfy.

So the flux densities in the gaps 'a' and 'b' are;

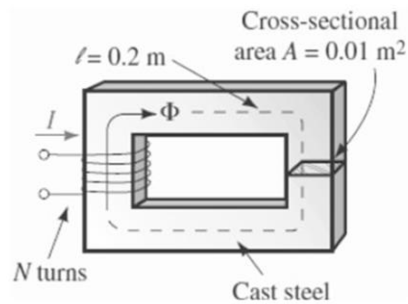
$$B_a = \frac{\phi_a}{A_a} = \frac{101.1\ \mu\text{Wb}}{9 \times 10^{-4}\ \text{m}^2} = 0.1123\ \text{T}$$

$$B_b = \frac{\phi_b}{A_b} = \frac{137.0\ \mu\text{Wb}}{6.25 \times 10^{-4}\ \text{m}^2} = 0.2192\ \text{T}$$

- Typically, for magnetic circuits consisting of iron cores with air gaps, the reluctance of the iron has a negligible effect on the results.
- Also as don't have a precise value of the permeability for the iron; it is often sufficiently accurate to assume zero reluctance for the iron cores.
- This is the counterpart of assuming zero resistance for the wires in an electrical circuit.

3. Determine H and φ, if B = 1.2 T, NI = 250 AT.

[1250AT/m, 1.24 x 10^{-4} wb]



4. The magnetic circuit has A_c = 16 cm^2 ; l=40 cm; l_g = 0.5mm; N = 350 turns; μ_r = 40000. For the flux density of 1 Tesla , find
   a) Flux, Φ, and total flux linkages, λ = NΦ [16x10^{-4} Wb,0.56Wb.turns]
   b) The required current to set the flux if there is no air-gap. [0.02A]
   c) The required current with the air-gap. [1.159 A]

5. A closed magnetic circuit of cast steel contains a 6 cm long path of cross sectional area 1 cm2 and a 2cm path of cross- sectional area 0.5 cm^2. A coil of 200 turns is wound around the 6cm length of the circuit and a current of 0.4A flows. Determine the flux density in the 2 cm path, if the relative permeability of the cast steel is 750.

   [R_1=6.366x105 AT/Wb ; R_2=4.244x105 AT/Wb; $\Phi=7.54\times10^{-5}$ Wb; B = 1.51T]

6. A cast steel core with uniform cross-sectional area 2 cm2 and mean length of 25 cm. Relative permeability of the stee1 is 48000. The air gap is 1 mm wide and the coil has 5000 turns. Determine the current in the coil to produce a flux density of 0.80 T in the air gap.

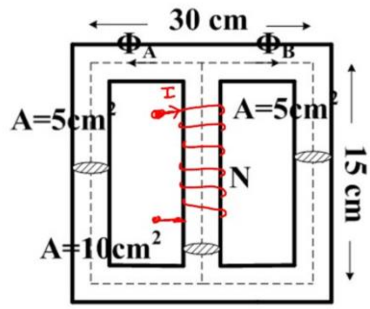
   [127.9 mA]

7. A mild steel ring has a radius of 50 mm and a cross-sectional area of 400 mm^2. A current of 0.5 A flows in a coil wound uniformly around the ring and the flux produced is 0.1 mWb. If the relative permeability at this value of current is 200, find (a) the reluctance of the mild steel and (b) the number of turns on the coil.

   [R=3.125×10^6 AT/Wb, N = 625 Turns]

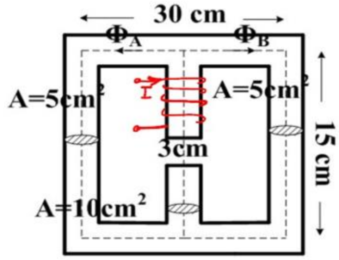
Parallel reluctance

1. Calculate the current required to flow through the winding to produce a flux of 1.4mwb in the central limb where a coil of 1000 turns is wounded. Assume the relative permeability of the core is 8000.



[0.0836A]

2. Calculate the current required to flow through the winding to produce a flux of 1.4mwb in the air gap present in the central limb where a coil of 1000 turns is wounded. Assume the relative permeability of the core is 8000.



[33.5A]

3. 2mwb is to be produced in the air gap of 0.1cm present in the central limb where the coil is wounded. How much ampere turns the coil must provide to achieve this if the dimensions of the core considering the mean length as 30cm by 20cm respectively? Relative permeability of the core material is 5000. All dimensions are in cm and the sectional area is 25cm^2 throughout.

[693.788228 A.turns]

Self inductance

- When the switch is closed (voltage is applied), the current does not immediately reach its maximum value; rather there is a gradual increase in current to its final equilibrium value. This effect is called self-inductance "L".
- Inductance is the measure of the opposition to the change in current.
- It is also defined as the ratio of the flux linkages in the circuit per unit current.

$$L = \frac{\lambda}{i}$$

Assuming that the flux is confined to the core so that all of the flux links all of the turns $\lambda = N\varphi$

So, $L = \frac{\lambda}{i}$ ➡ $L = \frac{N\phi}{i}$

Since, $\varphi = Ni/R$ ➡ $L = \frac{N^2}{\mathcal{R}}$

Hence, inductance depends on the number of turns and the core dimensions and core material. Inductance is proportional to the square of the number of turns.

As according to Faraday's law, voltage is induced in a coil when its flux linkages change

$e = \frac{d\lambda}{dt}$ ➡ $e = \frac{d(Li)}{dt}$ (since $\lambda = Li$)

For a coil wound on a stationary core, the inductance is constant with time.

So, $\boxed{e = L\frac{di}{dt}}$

Mutual inductance

- When two coils are wound on the same core, some of the flux produced by one coil links the other coil.
- The flux linkages of coil 2 caused by the current in coil 1 is $\boldsymbol{\lambda_{21}}$ and the flux linkages of coil 1 produced by its own current are denoted as $\boldsymbol{\lambda 11}$.
- Similarly, the current in coil 2 produces flux linkages $\lambda 22$ in coil 2 and $\lambda 12$ in coil 1
- The mutual inductance between the coils is

$$M = \frac{\lambda 21}{i_1} = \frac{\lambda 12}{i_2}$$

Coefficient of Coupling

$$L_1 = \frac{Flux\ linkages\ in\ a\ circuit}{Current\ in\ that\ circuit} = \frac{\lambda_{11}}{i_1} = \frac{N_1\varphi_1}{i_1}$$

$$L_2 = \frac{Flux\ linkages\ in\ a\ circuit}{Current\ in\ that\ circuit} = \frac{\lambda_{22}}{i_2} = \frac{N_2\varphi_2}{i_2}$$

$$M_{12} = \frac{N_1 K\varphi_2}{i_2} \qquad M_{21} = \frac{N_2 K\varphi_1}{i_1}$$ and $M_{12} = M_{21}$

Multiplying, $M^2 = \frac{N_1 K\varphi_2}{i_2} \text{x} \frac{N_2 K\varphi_1}{i_1}$ or $\frac{M^2}{\frac{N_2\varphi_2}{i_2} \text{x} \frac{N_1\varphi_1}{i_1}} = K^2$

$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$;where K is the coupling coefficient

Dot Convention

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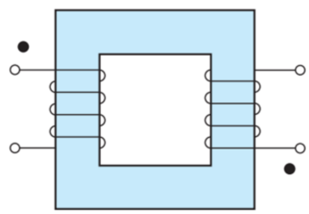
- It is standard practice to place a dot on one end of each coil in a circuit diagram to indicate how the fluxes interact.
- The dots are placed such that currents entering the dotted terminals produce aiding magnetic flux.
- According to the right-hand rule; a current entering either of the dotted terminals produces flux in a clockwise direction in the core.

If both currents enter or leave the dotted terminals, the mutual flux linkages add to the self flux linkages.



On the other hand, if one current enters a dotted terminal and the other leaves, the mutual flux linkages carry a minus sign.

Conductively Coupled Coils in series

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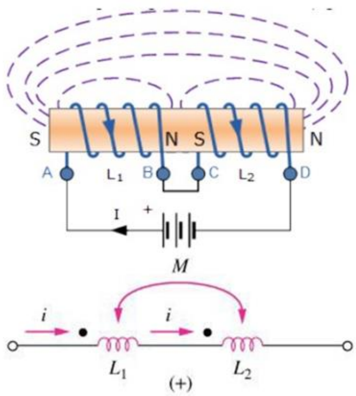
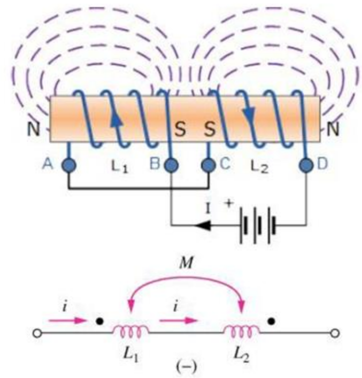
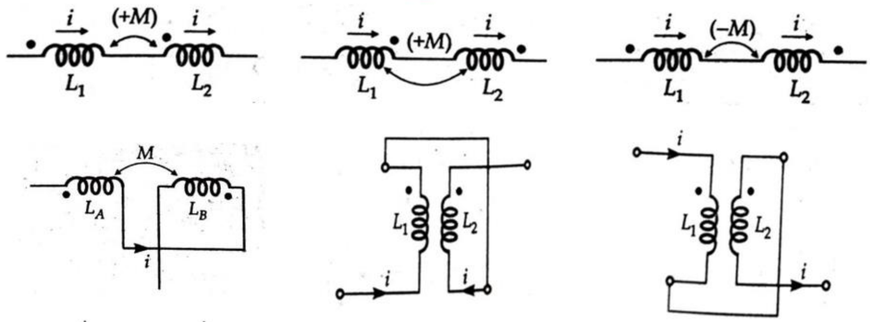


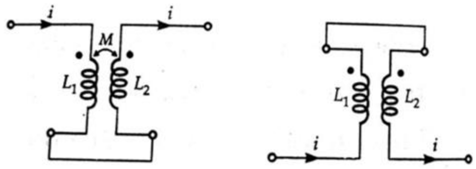


Series-aiding connection
$L=L_1+L_2+2M$

Series-opposing connection
$L=L_1+L_2-2M$

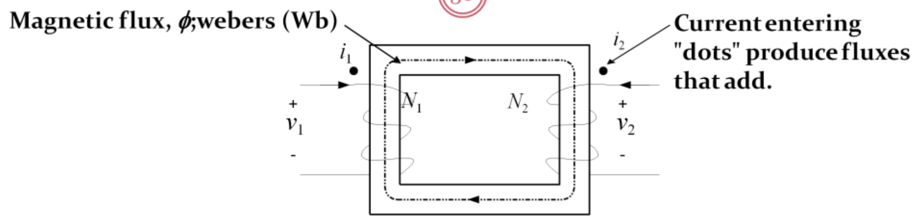
Conductively coupled coil configurations





The total inductance of two coupled coils in series depend on the placement of the dotted ends of the coils. The mutual inductances may add or subtract.

Inductively coupled coils



$\phi_{11} =$ flux in coil 1 produced by current in coil 1

$\phi_{21} =$ flux in coil 2 produced by current in coil 1

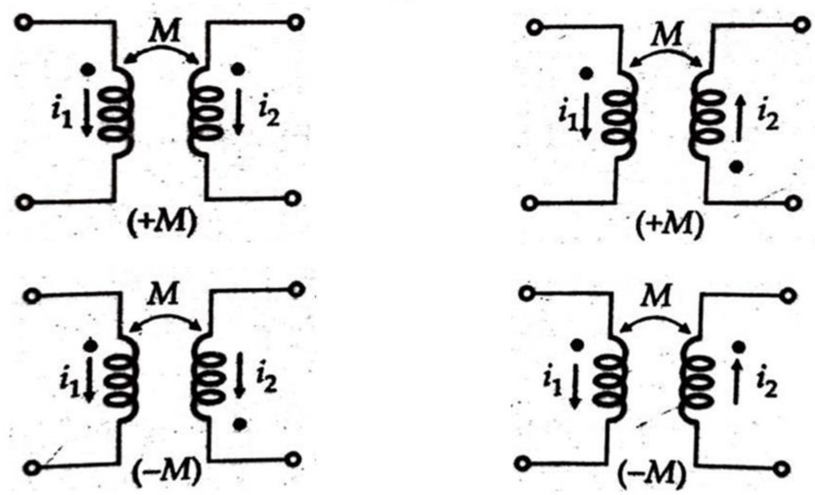
$\phi_{12} =$ flux in coil 1 produced by current in coil 2

$\phi_{22} =$ flux in coil 2 produced by current in coil 2

$\phi_1 =$ total flux in coil $1 = \phi_{11} + \phi_{12}$

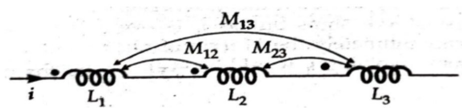
$\phi_2 =$ total flux in coil $2 = \phi_{21} + \phi_{22}$

Inductively coupled coil configurations



Problems

1. Find the total inductance of the three series connected coupled coils if L_1=1H, L_2=2H, L_3=5H,M_{12}=0.5H, M_{23}=1H, M_{13}=1H)



Coils are in series and current is entering through the dotted terminal in all the coils hence mutual inductance is positive.

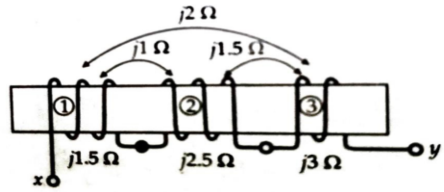
Inductance for Coil 1 = $L_1+M_{12}+M_{13}$

For Coil 2 = $L_2+M_{23}+M_{12}$

For Coil 3 = $L_3+M_{23}+M_{13}$

Total inductance = $L_1+L_2+L_3+2M_{12}+2M_{23}+2M_{13}$ = 13H

2. Find the equivalent circuit and the net inductance of the iron cored coupled coils in series connection if current is flowing from terminal x to y.



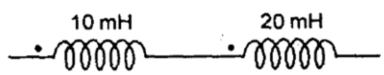
Solution: Through coil 1 current is in one direction and through coil 2 current is in opposite direction while in coil 3, the current direction is different from coil2 whereas same as in coil 1.

So $M_{12} = -\text{ve}$, $M_{23} = -\text{ve}$ whereas $M_{13} = +\text{ve}$

$L_1 = j1.5$, $L_2=j2.5$, $L_3=j3$, $M_{12}=j1$, $M_{23}=j1.5$, $M_{31}=j2$

Net inductance $= L_1+L_2+L_3-2M_{12}-2M_{23}+2M_{13} = j6$

3. If the resultant inductance of the inductors configuration is 40mH, then find the mutual inductance of coils.



4. The combined inductance of two coils connected in series is 0.6H or 0.1H depending on the relative directions of the currents in the coils. If one of the coils when isolated has a self-inductance of 0.2H, calculate
   a) mutual inductance
   b) coupling coefficient
   c) the two possible values of the induced emf in coil 2 when the current is increasing at 500A/s in series combination.

$L = L_1 + L_2 + 2M$ or $0.6 = L_1 + L_2 + 2M$ (*i*)
and $0.1 = L_1 + L_2 - 2M$ (*ii*)

(*a*) From (*i*) and (*ii*) we get, $M = \mathbf{0.125\ H}$

Let $L_1 = 0.2$ H, then substituting this value in (*i*) $L_2 = 0.15$ H

(*b*) Coupling coefficient $k = M\sqrt{L_1L_2} = 0.125/\sqrt{0.2 \times 0.15} = \mathbf{0.72}$

(c) $e_2 = L_2 \dfrac{di}{dt} \pm M \dfrac{di}{dt}$ ⟶ $e_2 = 0.2 \times 500 + 0.125 \times 500 = 162.5V$

$e_2 = 0.2 \times 500 - 0.125 \times 500 = 37.5V$

5. Two similar coils have a coupling coefficient of 0.25. When they are connected in series cumulatively, the total inductance is 80mH. Calculate the inductance of each coil. Also calculate the total inductance when the coils are connected in series differentially.

Solution. If each coil has an inductance of L henry, then $L_1 = L_2 = L$; $M = k\sqrt{L_1 L_2} = k\sqrt{L \times L} = kL$

When connected in series comulatively, the total inductance of the coils is

$$= L_1 + L_2 + 2M = 2L + 2M = 2L + 2kL = 2L\,(1 + 0.25) = 2.5L$$

$$\therefore \quad 2.5\,L = 80 \quad \text{or} \quad L = \mathbf{32\ mH}$$

When connected in series differentially, the total inductance of the coils is

$$= L_1 + L_2 - 2M = 2L - 2M = 2L - 2kL = 2L\,(1 - k) = 2L\,(1 - 0.25)$$

$$\therefore \quad 2L \times 0.75 = 2 \times 32 \times 0.75 = \mathbf{48\ mH}.$$

6. Two coils have a mutual inductance of 0.2H. If the current in one coil is changed from 10A to 4A in 10ms. Calculate (a) the induced emf in second coil (b) the change of flux linked with the second coil if it is wound with 500 turns.

7. Two coils with a coefficient of coupling of 0.5 between them, are connected in series so as to magnetise (a) in the same direction (b) in the opposite direction. The corresponding values of total inductances are for (a) 1.9H and for (b) 0.7H. Find the self inductances of the two coils and the mutual inductance between them.

$$L = L_1 + L_2 + 2M \quad \text{or} \quad 1.9 = L_1 + L_2 + 2M \quad ...(i)$$
$$L = L_1 + L_2 - 2M \quad \text{or} \quad 0.7 = L_1 + L_2 - 2M \quad ...(ii)$$

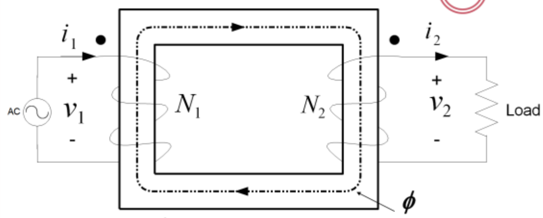
Subtracting (ii) from (i), $1.2 = 4M \quad \therefore \quad M = 0.3 \text{ H}$

Putting this value in (i) $L_1 + L_2 = 1.3 \text{ H} \quad ...(iii)$

$$M = k\sqrt{L_1 L_2} \quad \therefore \sqrt{L_1 L_2} = \frac{M}{k} = \frac{0.3}{0.5} = 0.6 \quad \therefore L_1 L_2 = 0.36$$

8. A 750 turns coil of inductance 3H carries a current of 2 A. Calculate the flux, linking with the coil, and the emf induced in the coil when the current collapses to zero in 20ms.

Transformer (voltage)



$$v_1(t) = N_1 \frac{d\phi}{dt}$$

$$v_2(t) = N_2 \frac{d\phi}{dt}$$

The changing flux through coil 2 induces a voltage, v_2 across coil 2

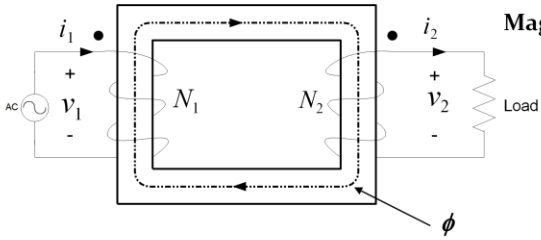
$$\phi = \frac{1}{N_1} \int v_1(t) dt$$

$$\frac{v_1}{v_2} = \frac{N_1 \frac{d\phi}{dt}}{N_2 \frac{d\phi}{dt}} = \frac{N_1}{N_2}$$

$$v_2 = \frac{N_2}{N_1} v_1$$

Turns ratio $n = \dfrac{N_2}{N_1}$

Transformer (current)



Magnetomotive force, mmf

$$F = Ni$$

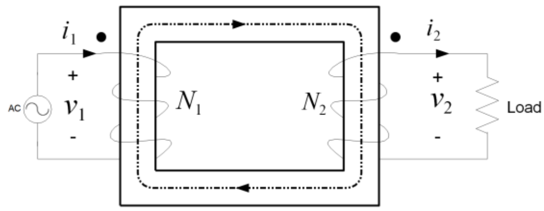
The total mmf applied to core is $F = N_1 i_1 - N_2 i_2 = R\phi$

For ideal transformer, the reluctance *R* is zero.

$$N_1 i_1 = N_2 i_2 \qquad i_2 = \frac{N_1}{N_2} i_1$$

Turns ratio $n = \dfrac{N_2}{N_1}$

Transformer (power)



Power delivered to an ideal transformer by the source is transferred to the load.

$$P = vi$$

Power delivered to primary

$$P_1 = v_1 i_1$$

Power delivered to load

$$P_2 = v_2 i_2$$

$$v_2 = \frac{N_2}{N_1} v_1 \qquad i_2 = \frac{N_1}{N_2} i_1 \qquad P_2 = v_2 i_2 = v_1 i_1 = P_1$$

Transformer basics

- Transformers work only with AC supply.
- There is no rotating parts, only the flux is changing (alternating) and the conductor is stationary. Hence it is statically induced emf.
- All of the flux links to all the windings of both coils. Thus, the voltage across each coil is proportional to the number of turns on the coil.

$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$

Transformer basics

- It is assumed that the reluctance of the core is negligible, so the total mmf of both coils is zero.

$$i_2(t) = \frac{N_1}{N_2} i_2(t)$$

- A consequence of the voltage and current relationships is that all of the power delivered to an ideal transformer by the source is transferred to the load.

$$P_1 = P_2$$

Problems

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1. Find the self and mutual inductance of the two windings 1 and 2 of an ideal transformer operating if N_1=500 turns, N_2=750 turns, I_1=2A, φ_1=10mwb, φ_{21}=6mwb , k = 0.6.

$$L_1 = \frac{N_1\varphi_1}{I_1} = \frac{500 \text{ X } 0.01}{2} = 2.5\text{H} \qquad M = \frac{N_2\varphi_{21}}{i_1} = \frac{750 \ X \ 0.006}{2} = 2.25H$$

$$M = K\sqrt{L_1 L_2} \qquad L_2 = \frac{M^2}{K^2 L_1} = \frac{2.25 \ X \ 2.25}{0.6 \ X \ 0.6 \ X \ 2.5} = 5.625H$$

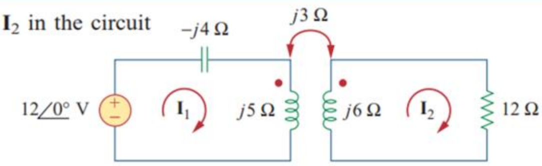
2. In an ideal transformer, the mutual inductance is 10H, number of turns in primary and secondary are 50 and 200 respectively. Find the value of primary current to produce 0.5wb flux to link the secondary coil. [10A]

Inductances in parallel

$$L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$ when mutual field assists the separate fields.

$$L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$ when the to fields oppose each other.

Calculate the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit



For coil 1, KVL gives $-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0$

$$j\mathbf{I}_1 - j3\mathbf{I}_2 = 12$$

For coil 2, KVL gives $-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0$

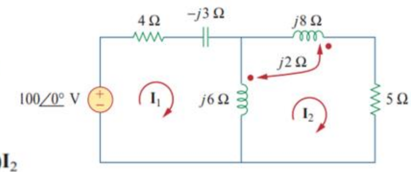
$$\mathbf{I}_1 = \frac{(12 + j6)\mathbf{I}_2}{j3} = (2 - j4)\mathbf{I}_2$$

Substituting, $(j2 + 4 - j3)\mathbf{I}_2 = (4 - j)\mathbf{I}_2 = 12$

$$\mathbf{I}_2 = \frac{12}{4 - j} = 2.91\underline{/14.04^\circ}\text{ A}$$

$$\mathbf{I}_1 = (2 - j4)\mathbf{I}_2 = (4.472\underline{/-63.43^\circ})(2.91\underline{/14.04^\circ})$$
$$= 13.01\underline{/-49.39^\circ}\text{ A}$$

Calculate the mesh currents in the circuit



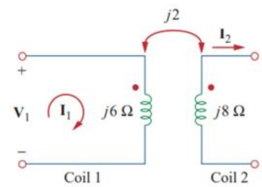
For mesh 1 $\quad -100 + \mathbf{I}_1(4 - j3 + j6) - j6\mathbf{I}_2 - j2\mathbf{I}_2 = 0$

$$100 = (4 + j3)\mathbf{I}_1 - j8\mathbf{I}_2$$

For mesh 2 $\quad 0 = -2j\mathbf{I}_1 - j6\mathbf{I}_1 + (j6 + j8 + j2 \times 2 + 5)\mathbf{I}_2$

$$0 = -j8\mathbf{I}_1 + (5 + j18)\mathbf{I}_2$$

matrix form, $\begin{bmatrix} 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + j3 & -j8 \\ -j8 & 5 + j18 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$



The determinants $\Delta = \begin{vmatrix} 4 + j3 & -j8 \\ -j8 & 5 + j18 \end{vmatrix} = 30 + j87$

$$\Delta_1 = \begin{vmatrix} 100 & -j8 \\ 0 & 5 + j18 \end{vmatrix} = 100(5 + j18)$$

$$\Delta_2 = \begin{vmatrix} 4 + j3 & 100 \\ -j8 & 0 \end{vmatrix} = j800$$

mesh currents $\mathbf{I}_1 = \dfrac{\Delta_1}{\Delta} = \dfrac{100(5 + j18)}{30 + j87} = \dfrac{1{,}868.2\underline{/74.5^\circ}}{92.03\underline{/71^\circ}} = 20.3\underline{/3.5^\circ}\text{ A}$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{j800}{30 + j87} = \frac{800\underline{/90^\circ}}{92.03\underline{/71^\circ}} = 8.693\underline{/19^\circ}\text{ A}$$