

Basic Electrical and Electronics Engineering (BEEE102L)

3 phase systems

Dr. Sonam Shrivastava/ Assistant Professor (Sr.) /SELECT

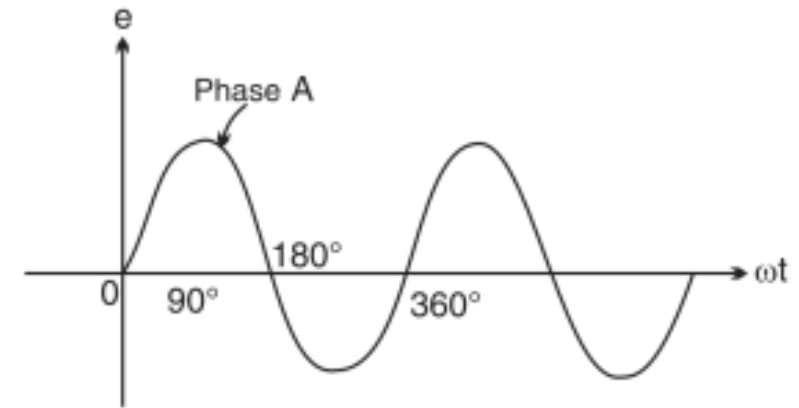
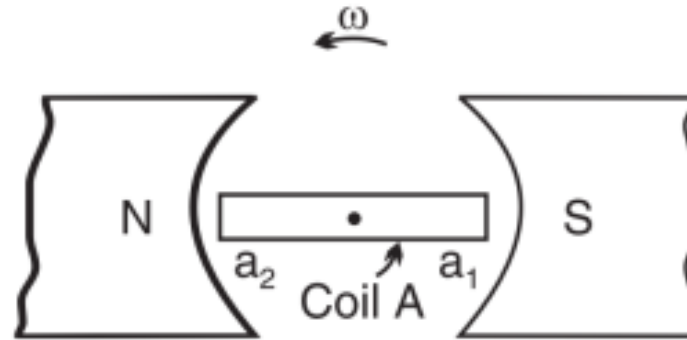
LECTURE 2.10

TOPICS

- **Three Phase Systems**
- **Star & Delta Connections**
- Three Phase Power Measurement
- Electrical Safety
- Fuses and Earthing
- Residential Wiring

Single Phase System

$$e_{a_1a_2} = E_m \sin \omega t$$

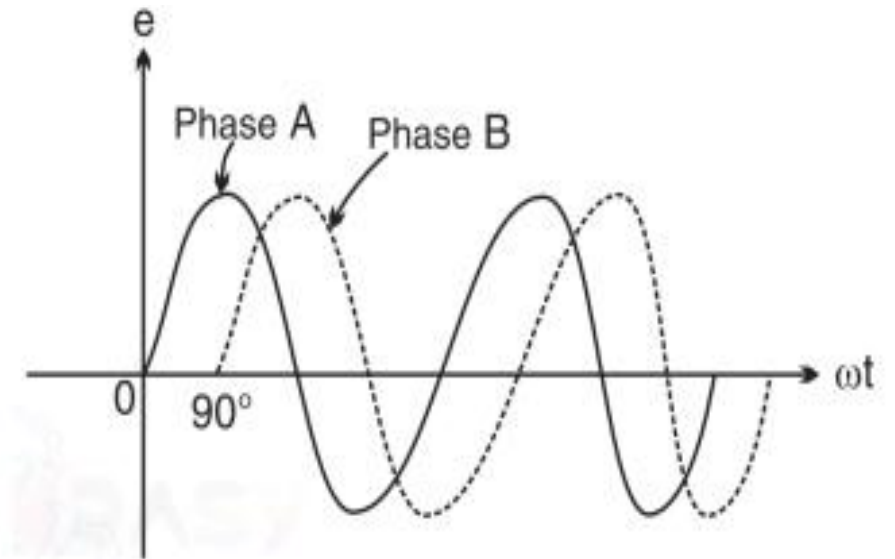
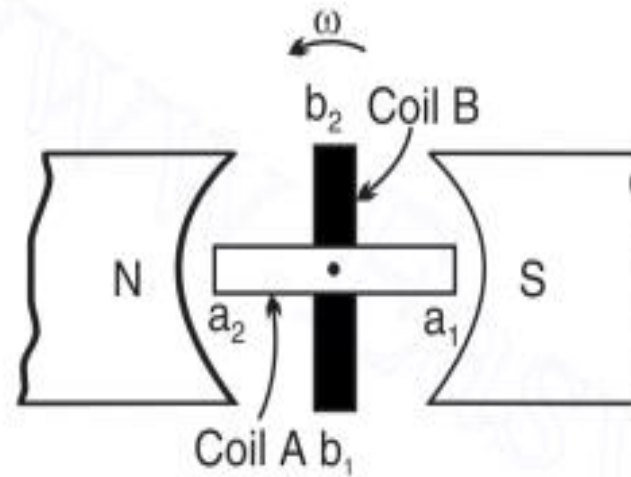


Circuits or systems in which the ac sources operate at the same frequency but different phases are known as *polyphase*

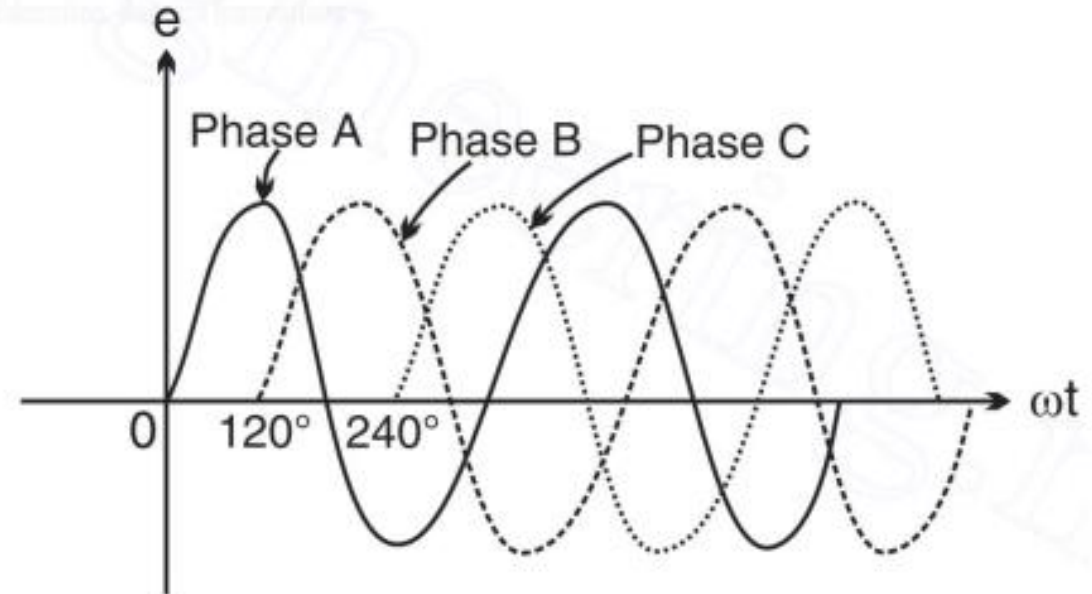
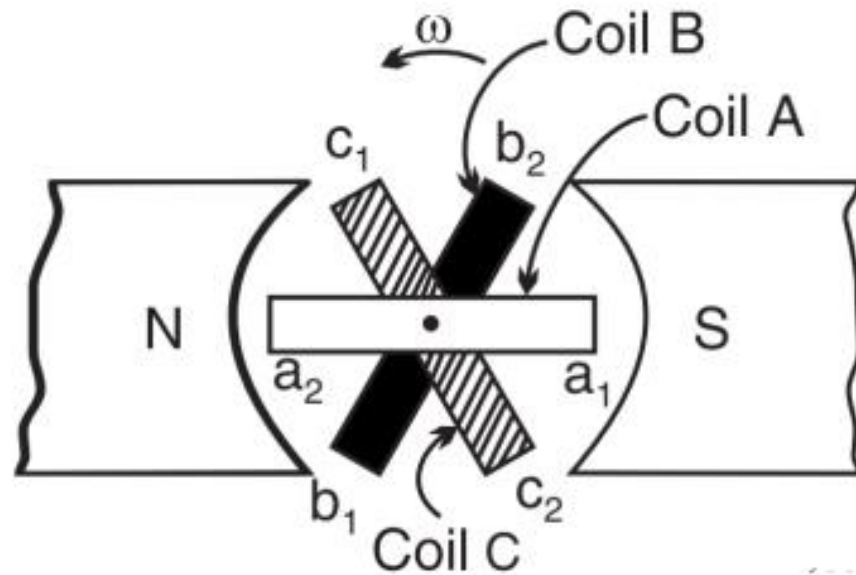
Two Phase System

$$e_{a_1a_2} = E_m \sin \omega t$$

$$e_{b_1b_2} = E_m \sin(\omega t - 90^\circ)$$



Three Phase System



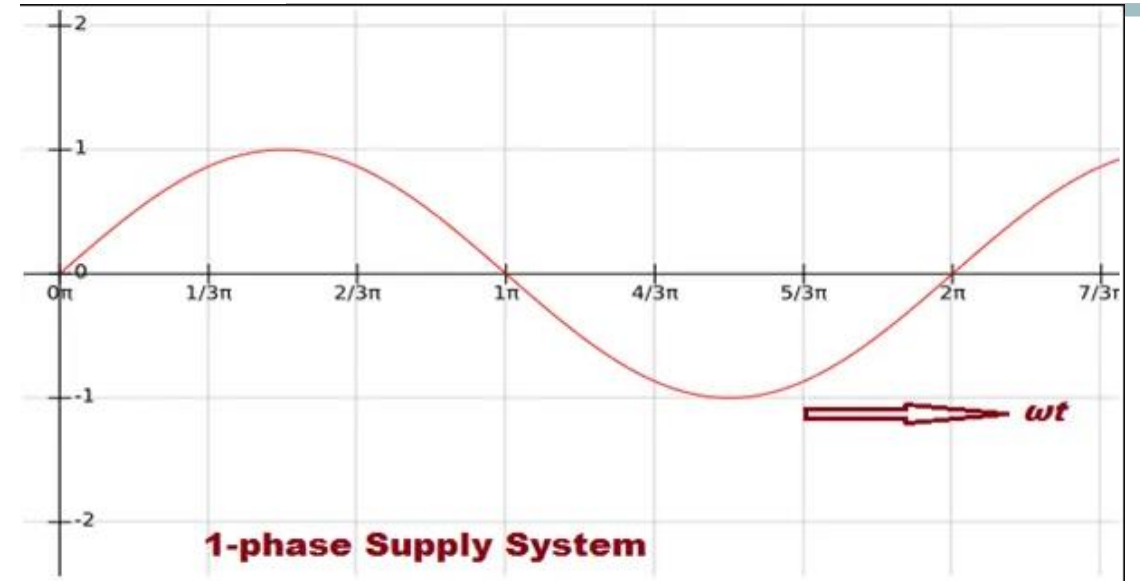
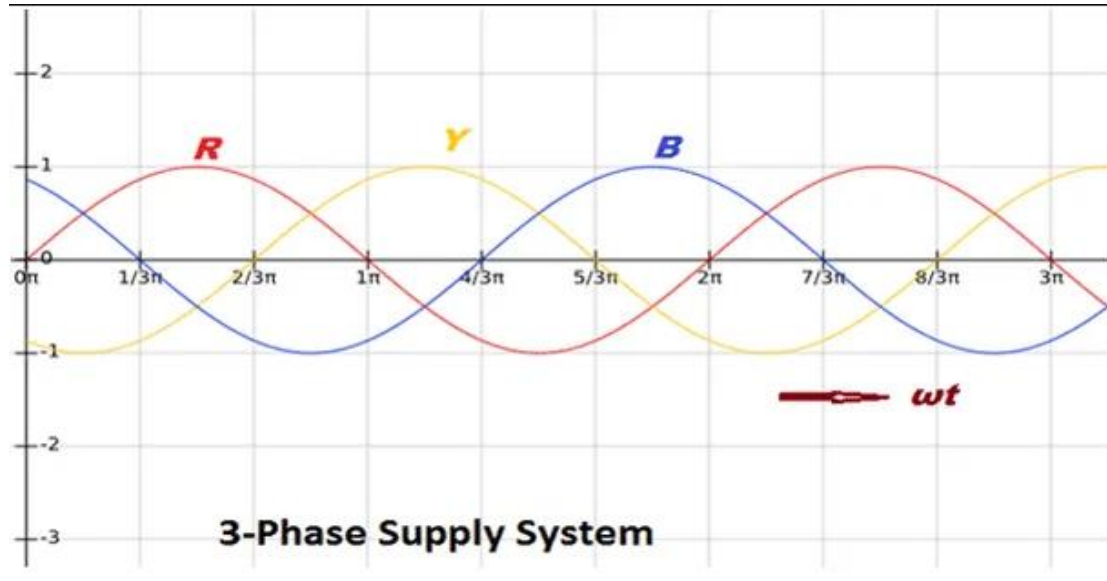
$$e_{a_1a_2} = E_m \sin \omega t$$

$$e_{b_1b_2} = E_m \sin(\omega t - 120^\circ)$$

$$e_{c_1c_2} = E_m \sin(\omega t - 240^\circ)$$

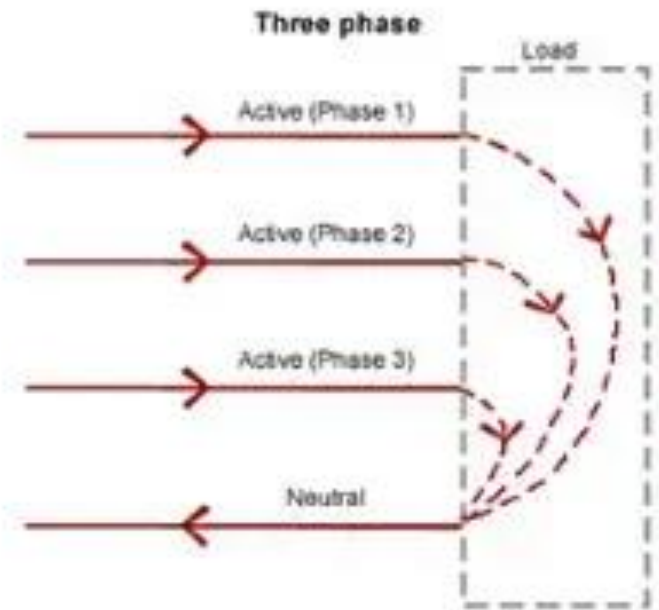
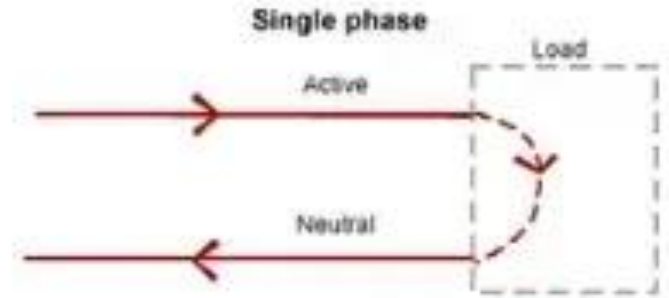
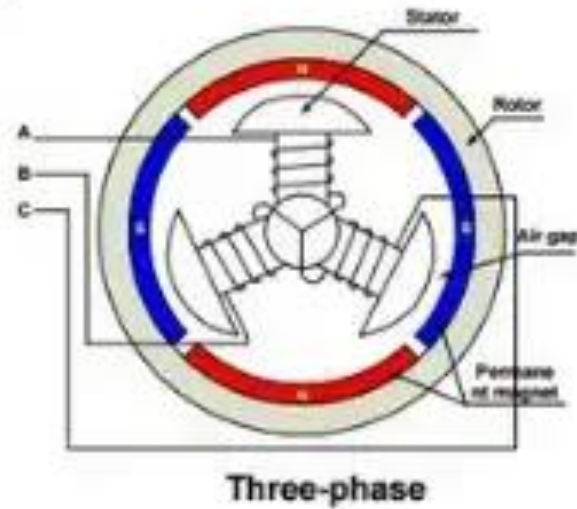
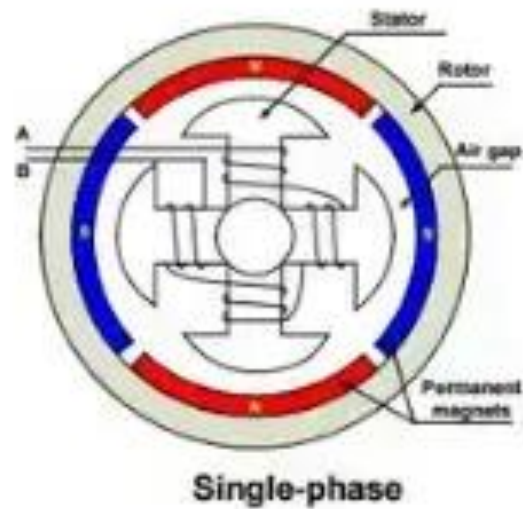
Why Three Phase?

- ❖ **Constant power.** In a single-phase circuit, the instantaneous power varies sinusoidally from zero to a peak value at twice the supply frequency. However, in a balanced 3-phase system, the power supplied at all instants of time is constant.
- ❖ **Greater output.** The output of a 3-phase machine is greater than that of a single-phase machine for a given volume and weight of the machine.
- ❖ **Cheaper.** The three-phase motors are much smaller and less expensive than single-phase motors because less material (copper, iron, insulation) is required.
- ❖ **Power transmission economics.** Transmission of electric power by 3-phase system is cheaper than that of single-phase system, even though three conductors are required instead of two.
- ❖ **Rotating Magnetic Field** A 3-phase system can set-up a rotating uniform magnetic field in stationary windings. This cannot be done with a single-phase current.



Presently 3- ϕ AC system is very popular and being used worldwide for power generation, power transmission, distribution and for electric motors.

3 Phase vs Single Phase Power Systems



Advantages

Three phase system has the following advantages as compare to single phase system:

- Power to weight ratio of 3- ϕ alternator is high as compared to 1- ϕ alternator.
- Thus, for generation for same amount of Electric Power, the size of 3- ϕ alternator is small as compare to 1- ϕ alternator.
- Hence, the overall cost of alternator is reduced for generation of same amount of power.
- Moreover, due to reduction in weight, transportation and installation of alternator become convenient and less space is required to accommodate the alternator in power houses.
- A 3-phase system can be used to feed a 1- ϕ load, whereas vice-versa is not possible.

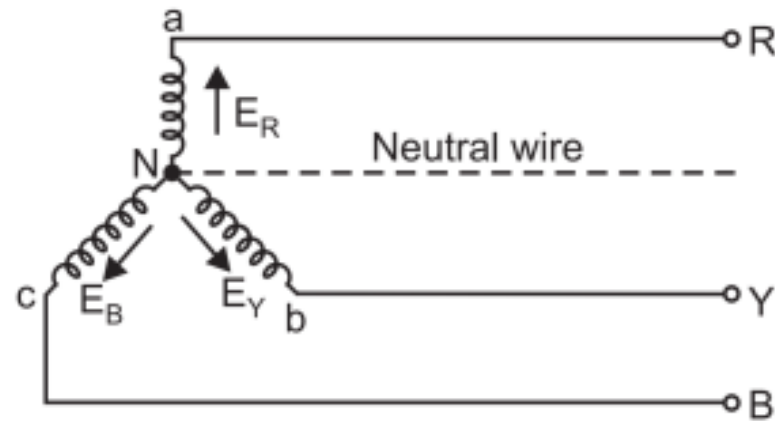
Advantages

- For electric power transmission and distribution of same amount of power, the requirement of conductor material is less in 3- ϕ system as compare to 1- ϕ system.
- Hence, the 3- ϕ transmission and distribution system is economical as compare 1- ϕ system.
- 3-phase motor is having better power factor
- 3-phase induction motor is self-started as the magnetic flux produced by 3-phase supply is rotating in nature with a constant magnitude.
- Whereas 1- ϕ induction motor is not self-started as the magnetic flux produced by 1- ϕ supply is pulsating in nature.
- Hence, we have to make some arrangement to make the 1- ϕ induction motor self-started which further increases the cost of 1- ϕ induction motor.

Three Phase Configurations

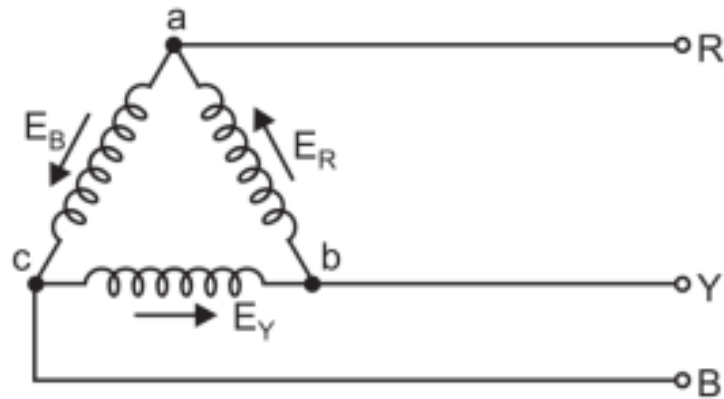
- (i) Star or Wye (Y) connection
- (ii) Delta (D) connection

Y-connection



If a neutral conductor exists, the system is called *3-phase, 4 wire system*. If there is no neutral conductor, it is called *3-phase, 3-wire system*.

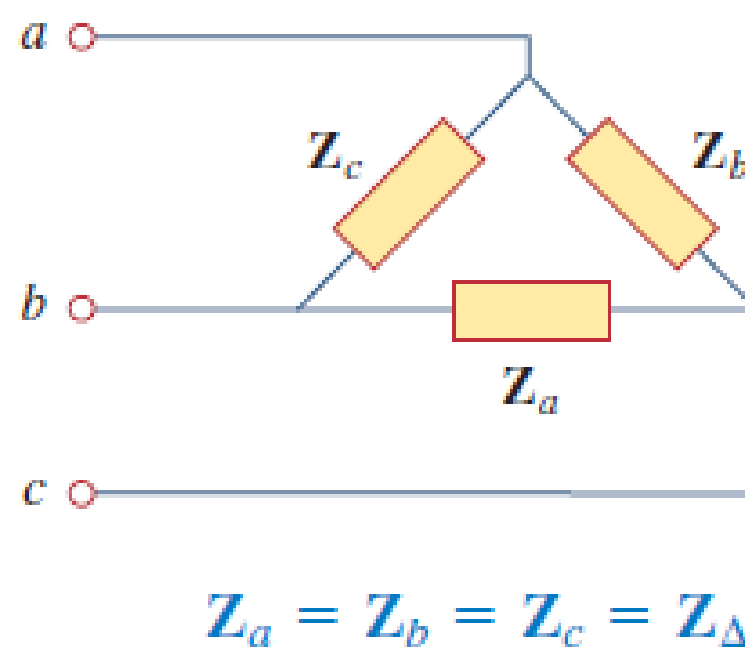
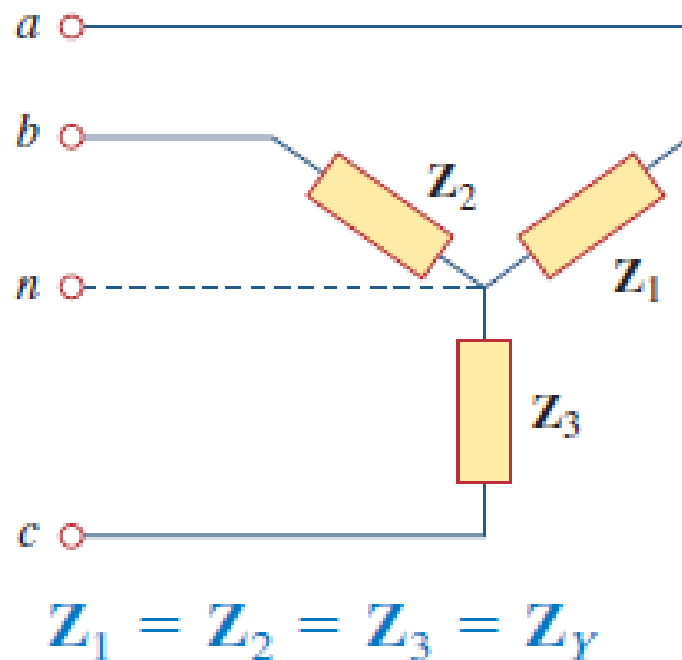
D-connection



In a D-connection, no neutral point exists and only 3-phase, 3-wire system can be formed.

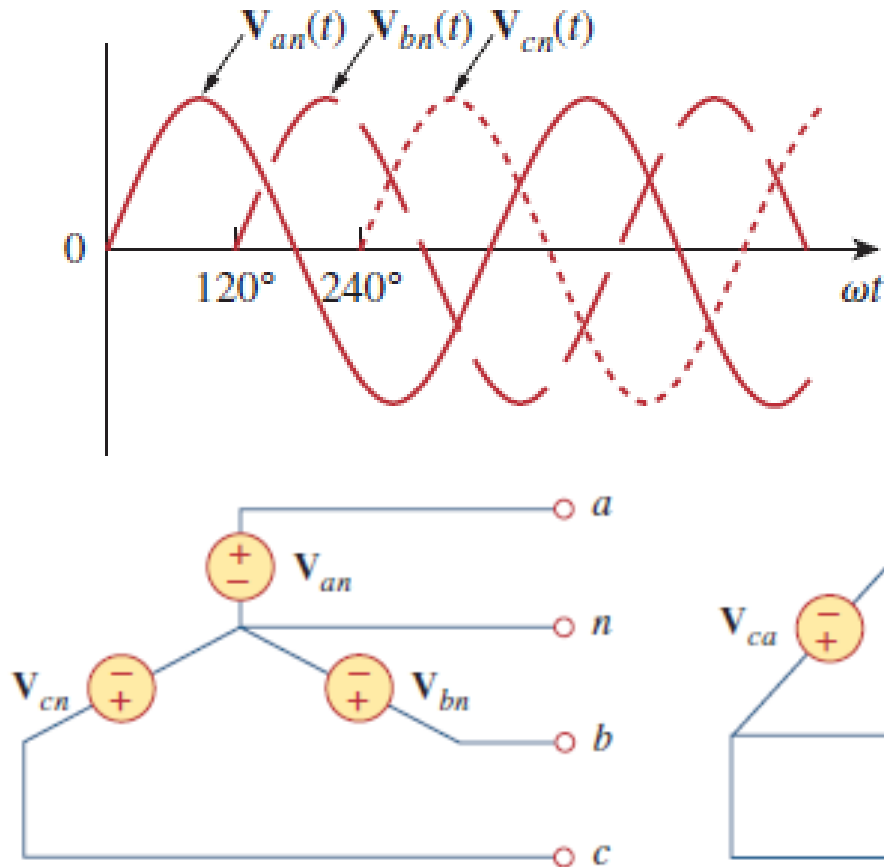
Star & Delta Connections

The **phase sequence** is the time order in which the voltages pass through their respective maximum values.

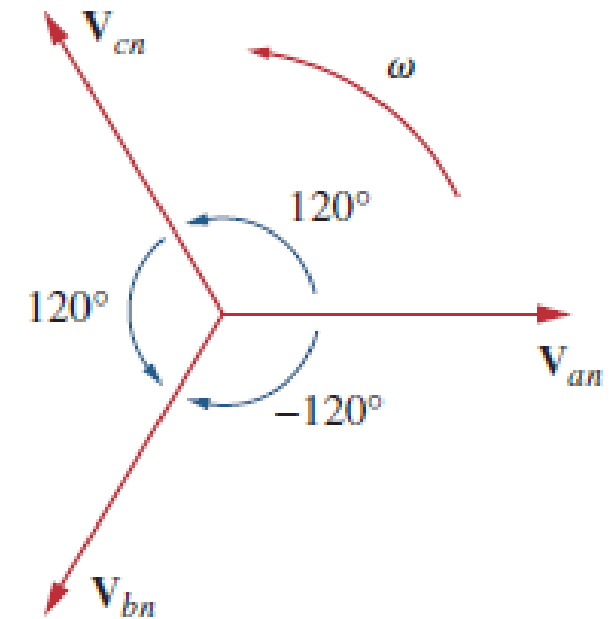


A **balanced load** is one in which the phase impedances are equal in magnitude and in phase.

Three Phase System



Balanced phase voltages are equal in magnitude and are out of phase with each other by 120° .



$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

$$V_{an} + V_{bn} + V_{cn} = 0$$

$$|V_{an}| = |V_{bn}| = |V_{cn}|$$

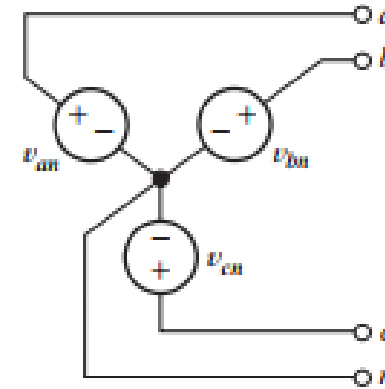
Balanced three-PHASE Circuits

- We consider the most common case: three equal-amplitude ac voltages having phases that are 120° apart.
- This is known as a balanced three-phase source.
- The source shown in Figure is said to be **wye connected (Y connected)**.
- The three voltages shown in Figure

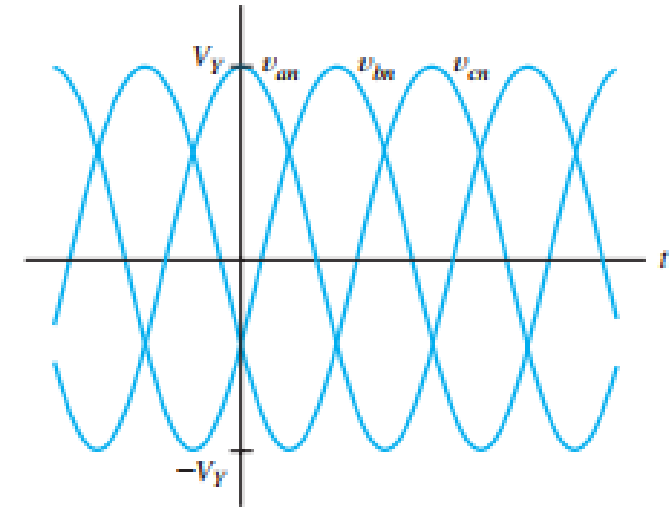
$$v_{an}(t) = V_Y \cos(\omega t)$$

$$v_{bn}(t) = V_Y \cos(\omega t - 120^\circ)$$

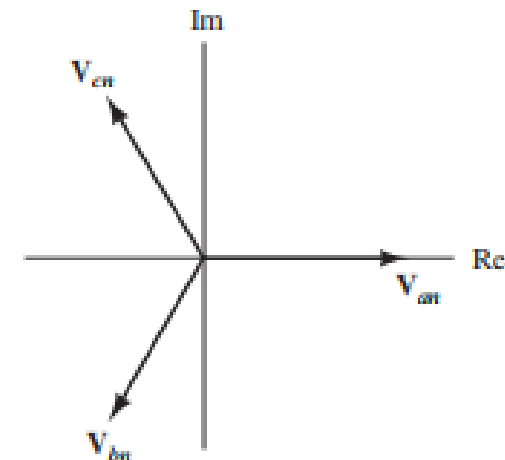
$$v_{cn}(t) = V_Y \cos(\omega t + 120^\circ)$$



(a) Three-phase source

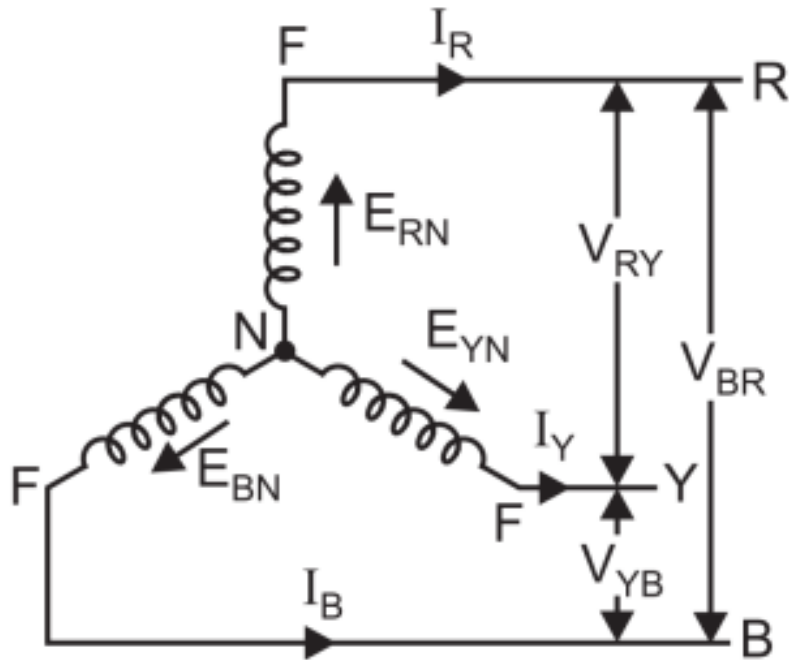


(b) Voltages versus time



(c) Phasor diagram

Star or Wye Connected System



The voltages E_{RN} , E_{YN} , and E_{BN} are respectively between lines R , Y , and B , and the neutral line N . These voltages are called *phase voltages*. If the voltage sources have the same amplitude and frequency ω and are out of phase with each other by 120° , the voltages are said to be *balanced*.

Phase Voltage

$$E_{RN} = V_{ph} \angle 0^\circ \quad E_{YN} = V_{ph} \angle -120^\circ$$

$$E_{BN} = V_{ph} \angle -240^\circ$$

When voltages are balanced $E_{RN} + E_{YN} + E_{BN} = 0$

$$|E_{RN}| = |E_{YN}| = |E_{BN}| = V_{ph}$$

Line Voltage

$$V_{RY} = V_L \angle 0^\circ \quad V_{YB} = V_L \angle -120^\circ$$

$$V_{BR} = V_L \angle -240^\circ$$

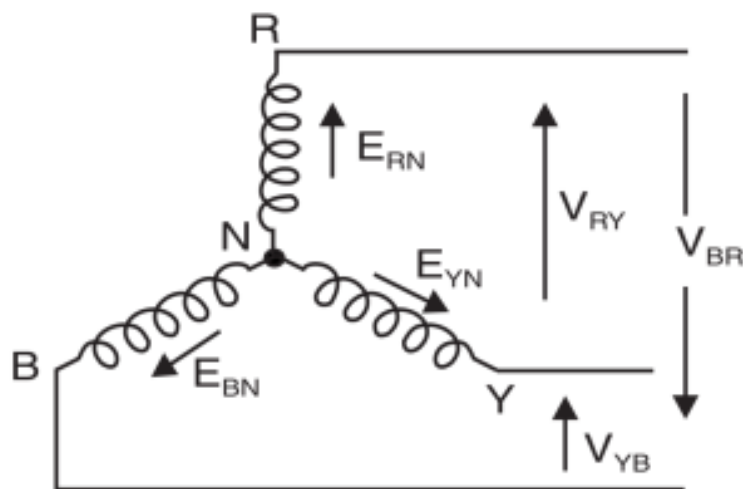
Line voltage = $\sqrt{3} \times$ Phase voltage

$$V_L = \sqrt{3} V_{ph}$$

Line current = Phase current

$$I_L = I_{ph}$$

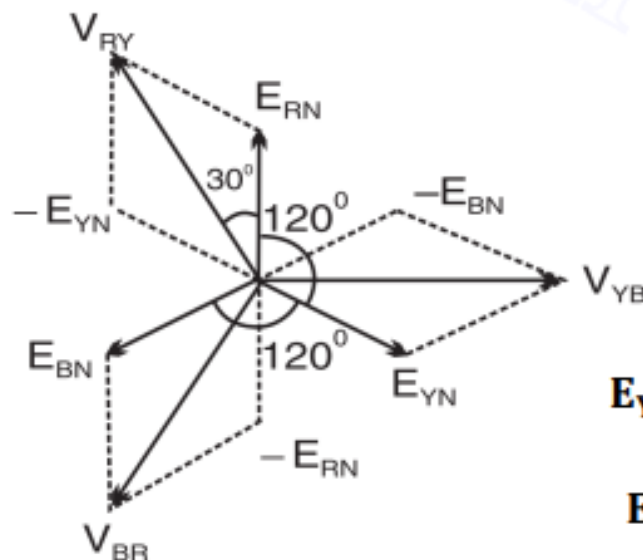
Star or Wye Connected System



line voltage leads phase voltage by 30°

$$V_L = \sqrt{3}V_P \angle 30^\circ$$

$$I_P = I_L$$



$$V_{RY} = E_{RN} + E_{NY} = E_{RN} - E_{YN}$$

$$V_{RY} = 2V_{Ph} \cos(60^\circ/2) = 2V_{Ph} \cos 30^\circ = \sqrt{3}V_{Ph}$$

$$V_{YB} = E_{YN} - E_{BN} = \sqrt{3}V_{Ph}$$

$$V_{BR} = E_{BN} - E_{RN} = \sqrt{3}V_{Ph}$$

$$E_{RN} = V_{Ph} \angle 0^\circ = V_{Ph}(1 + j0)$$

$$E_{YN} = V_{Ph} \angle -120^\circ = V_{Ph}(-0.5 - j0.866)$$

$$E_{BN} = V_{Ph} \angle -240^\circ = V_{Ph}(-0.5 + j0.866)$$

$$V_{RY} = E_{RN} - E_{YN} = V_{Ph}(1 + j0) - V_{Ph}(-0.5 - j0.866)$$

$$V_{RY} = V_{Ph}(1.5 + j0.866) = \sqrt{3}V_{Ph} \angle 30^\circ$$

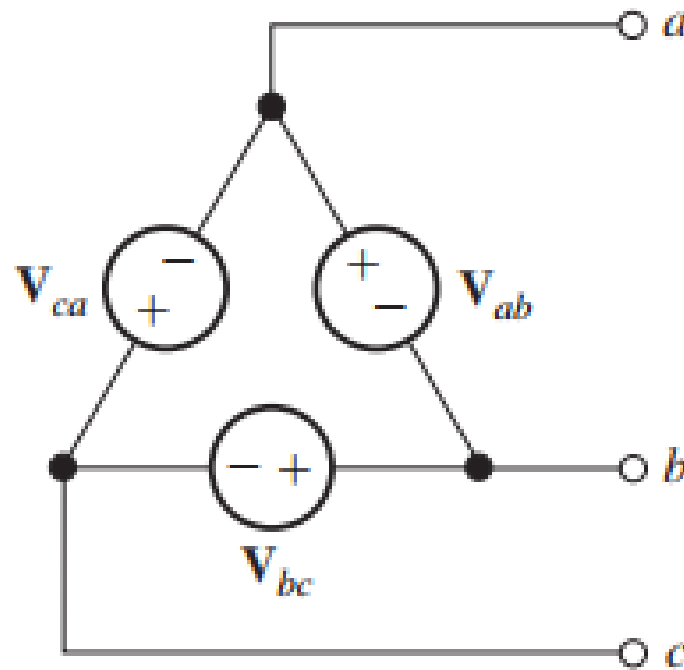
$$V_{RY} = V_{ph} \angle 0^\circ - V_{ph} \angle -120^\circ$$

$$V_{RY} = V_{ph}((\cos 0^\circ + j \sin 0^\circ) - (\cos(-120^\circ) + j \sin(-120^\circ)))$$

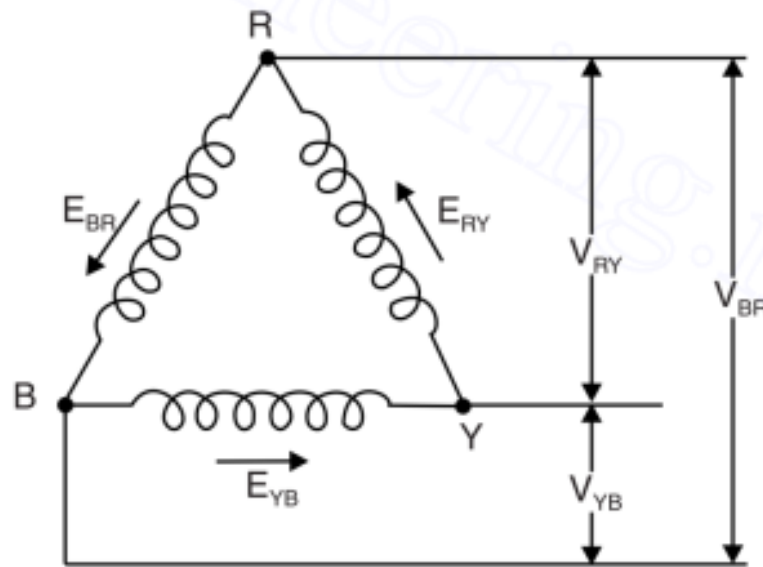
$$V_{RY} = V_L = V_{ph} \left(\frac{3}{2} + \frac{\sqrt{3}}{2}j \right)$$

Delta-Connected Sources

- A set of balanced three-phase voltage sources can be connected in the form of a delta, as shown in Figure



Delta Connected System



When voltages are balanced

$$\mathbf{E}_{RY} = V_{Ph} \angle 0^\circ = V_{Ph}(1 + j0)$$

$$\mathbf{E}_{YB} = V_{Ph} \angle -120^\circ = V_{Ph}(-0.5 - j0.866)$$

$$\mathbf{E}_{BR} = V_{Ph} \angle -240^\circ = V_{Ph}(-0.5 + j0.866)$$

$$\mathbf{E}_{RY} + \mathbf{E}_{YB} + \mathbf{E}_{BR} = V_{Ph}(1 + j0) + V_{Ph}(-0.5 - j0.866) + V_{Ph}(-0.5 + j0.866) = 0$$

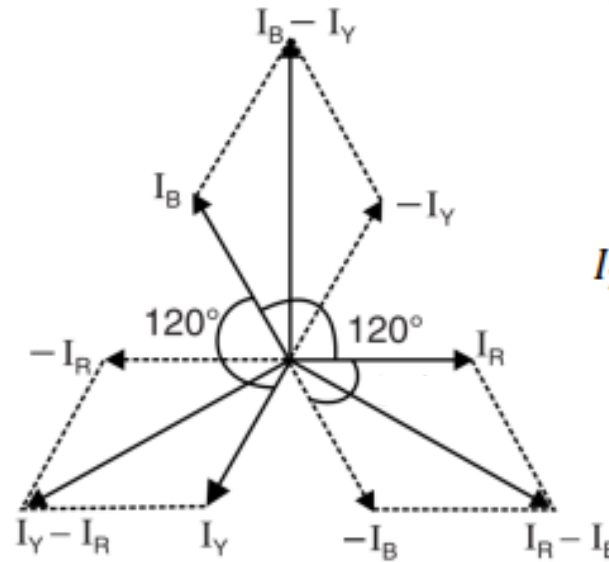
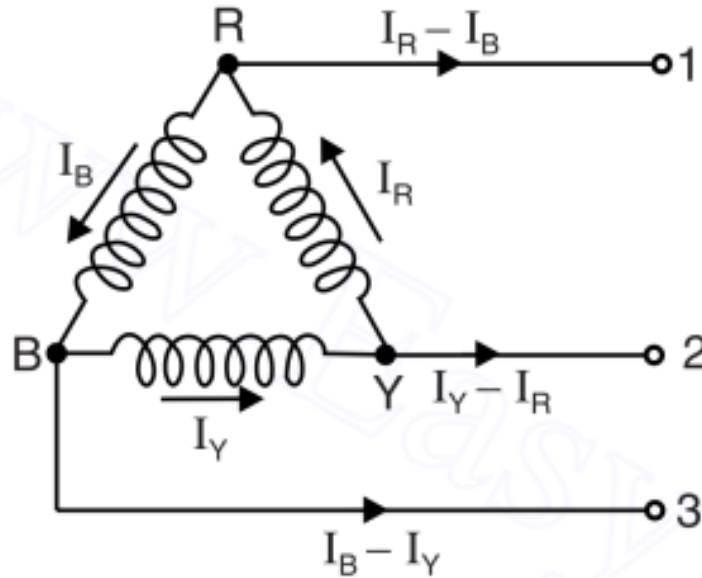
Line voltage = Phase voltage

$$V_L = V_{Ph}$$

Line current = $\sqrt{3} \times$ Phase current

$$I_L = \sqrt{3} I_{Ph}$$

Delta Connected System



$$I_1 = I_R - I_B \quad I_2 = I_Y - I_R \quad I_3 = I_B - I_Y$$

$$I_1 = 2I_{ph} \cos(60^\circ/2) = 2I_{ph} \cos 30^\circ = \sqrt{3}I_{ph}$$

$$I_2 = I_Y - I_R = \sqrt{3}I_{ph} \quad I_3 = I_B - I_Y = \sqrt{3}I_{ph}$$

$$I_R = I_{ph} \angle 0^\circ = I_{ph}(1 + j0)$$

$$I_Y = I_{ph} \angle -120^\circ = I_{ph}(-0.5 - j0.866)$$

$$I_B = I_{ph} \angle -240^\circ = I_{ph}(-0.5 + j0.866)$$

$$I_1 = I_R - I_B = I_{ph}(1 + j0) - I_{ph}(-0.5 + j0.866)$$

$$I_1 = I_{ph} \angle 0^\circ - I_{ph} \angle -240^\circ$$

$$I_1 = I_{ph}((\cos 0^\circ + j \sin 0^\circ) - (\cos(-240^\circ) + j \sin(-240^\circ)))$$

$$I_1 = I_{ph}\left(\frac{3}{2} - \frac{\sqrt{3}}{2}j\right)$$

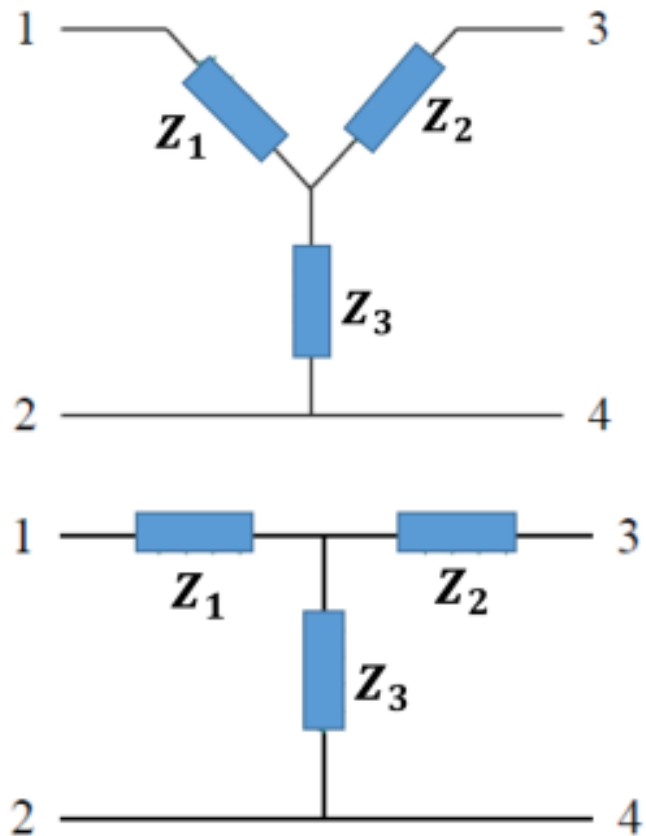
line Current lags phase current by 30°

$$I_L = \sqrt{3}I_P \angle -30^\circ$$

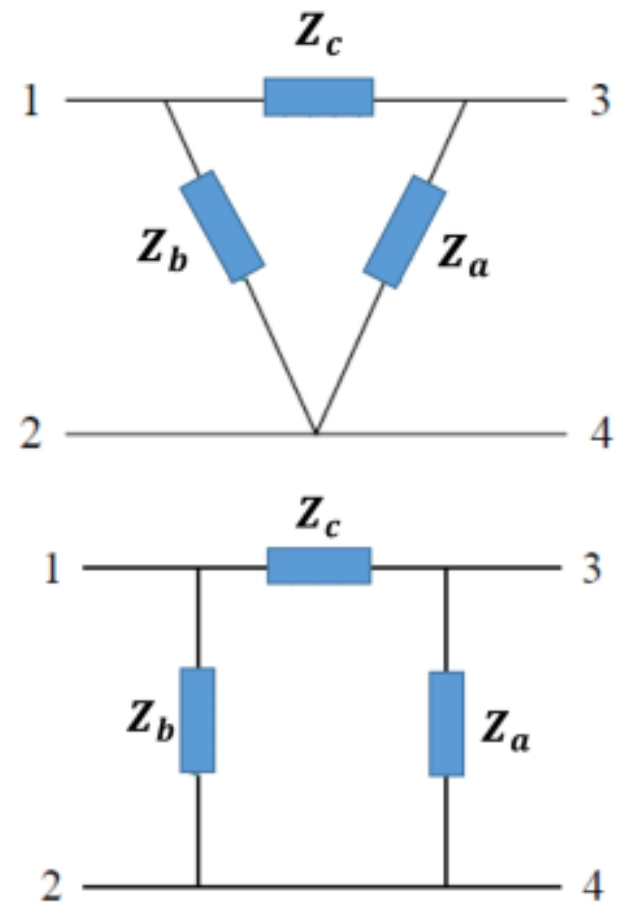
$$V_P = V_L$$

Three Phase Load

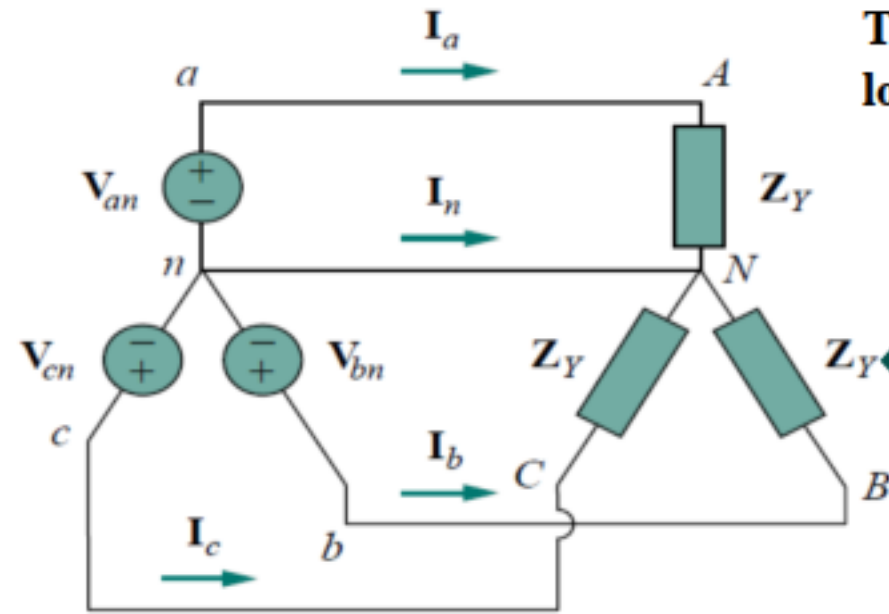
Star or Y-connected Load



Δ -connected Load

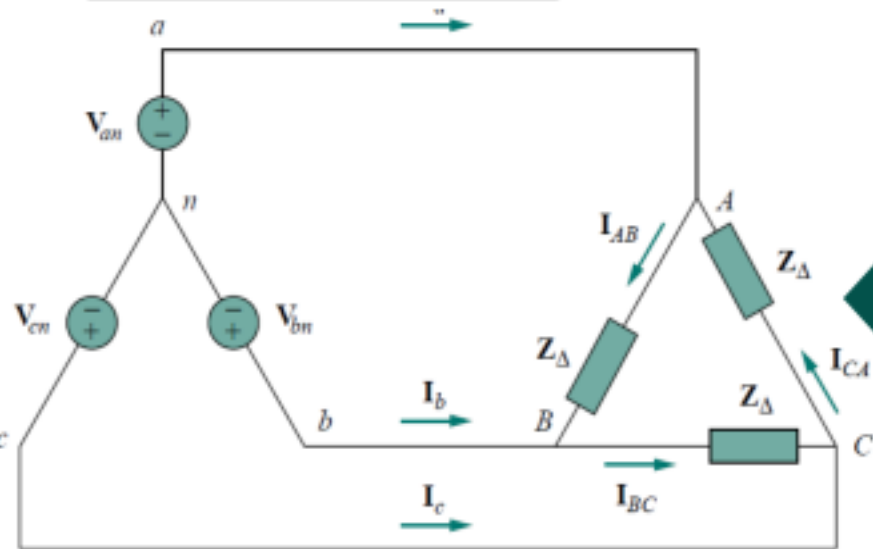
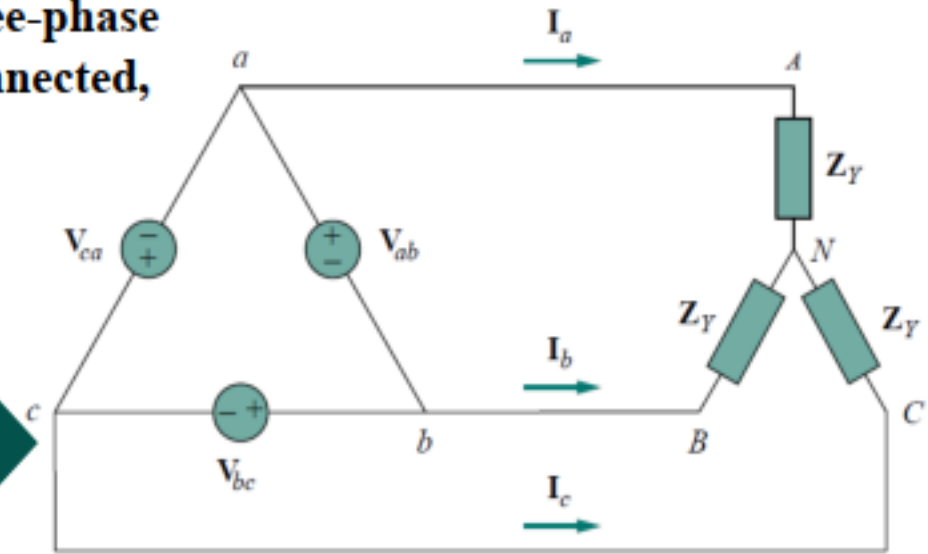


Three-phase source and the three-phase load can be either wye- or delta-connected,



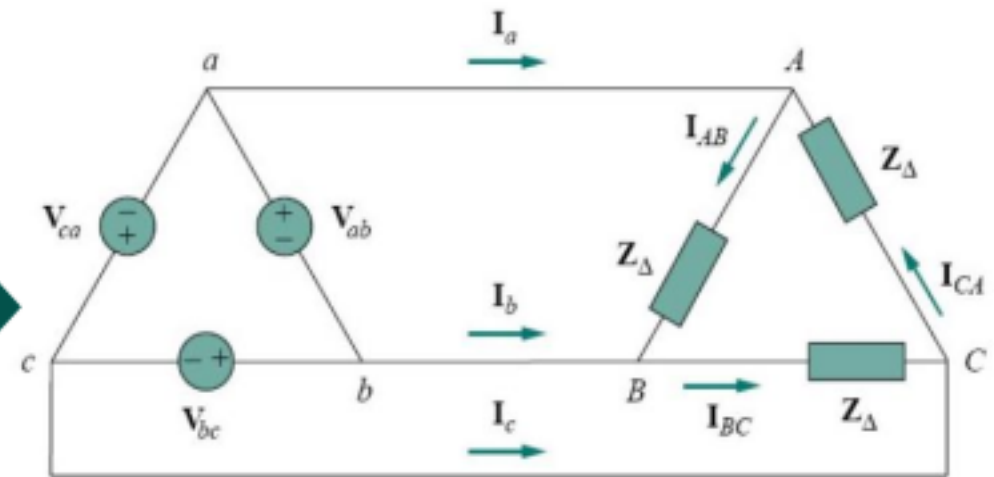
Y - Y Connection

Δ - Y Connection



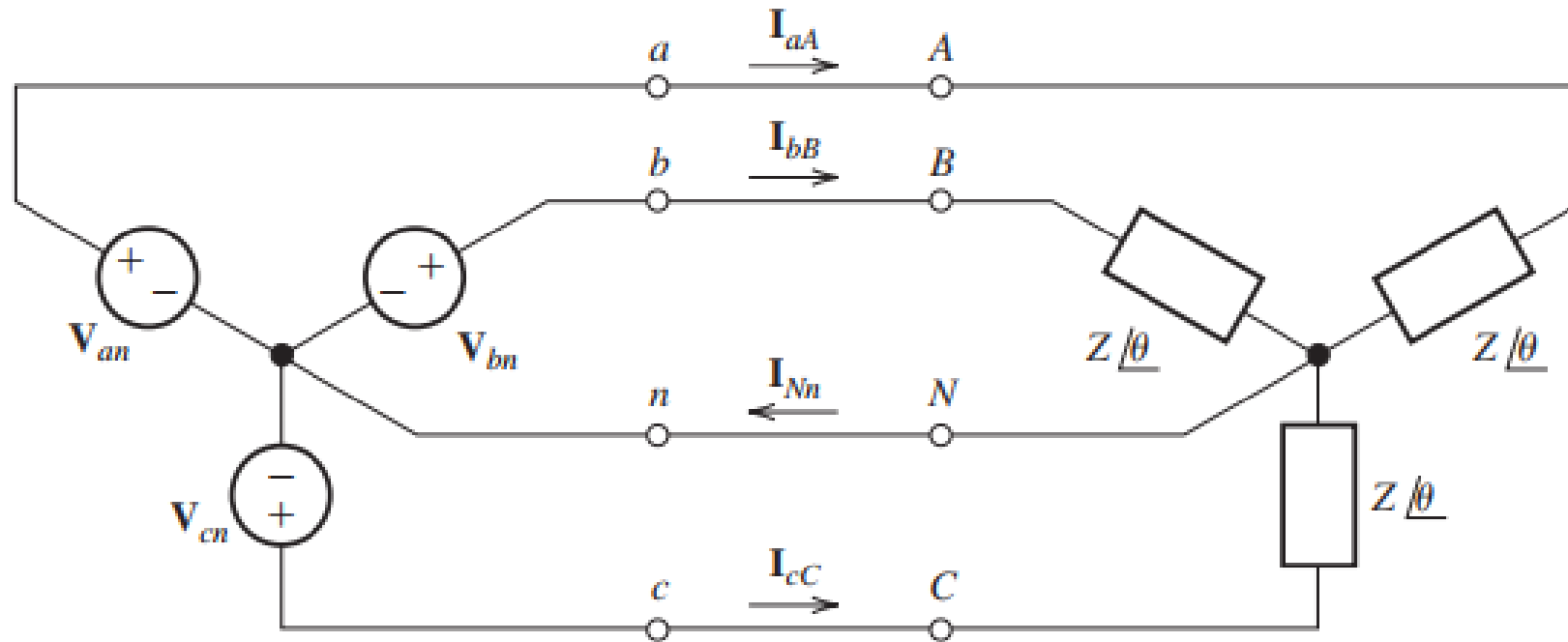
Y - Δ Connection

Δ - Δ Connection



Wye - Wye Connection

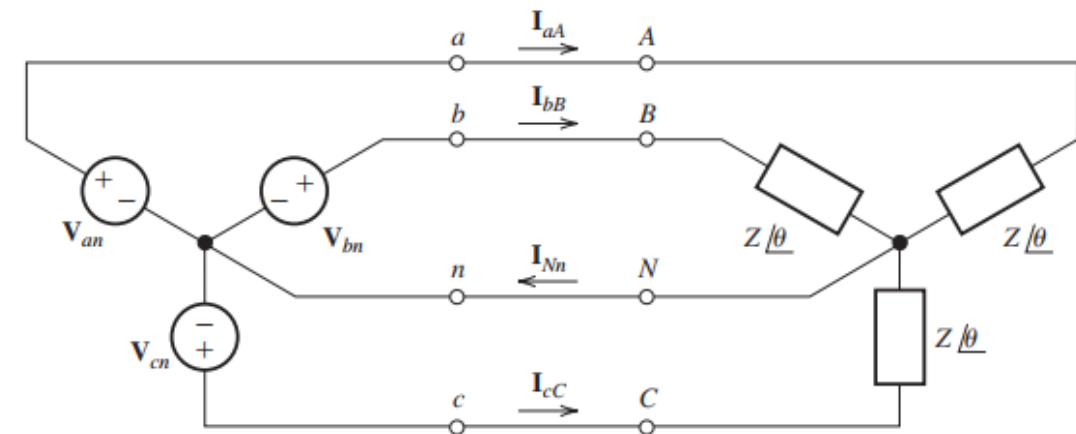
- Consider the three-phase source connected to a balanced three-phase load shown in Figure.



Wye - Wye Connection

- The wires a-A, b-B, and c-C are called lines, and the wire n-N is called the neutral.
- This configuration is called a wye-wye (Y-Y) or star-star connection with neutral.
- By the term balanced load, means that the three load impedances are equal.
- phase A of the source is $v_{an}(t)$, and phase A of the load is the impedance connected between A and N.
- V_Y is the line-to-neutral voltage of the wye-connected source.
- I_{aA} , I_{bB} , and I_{cC} are called line currents.

line-to-neutral voltage is phase voltage



Wye - Wye Connection

- The current in phase A of the load is given by

$$I_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z} \angle \theta} = \frac{V_Y \angle 0^\circ}{\mathbf{Z} \angle \theta} = I_L \angle -\theta$$

- Where $I_L = V_Y/Z$
- Because the load impedances are equal, all of the line currents are the having the same magnitude, except for phase.
- Thus, the currents are given by

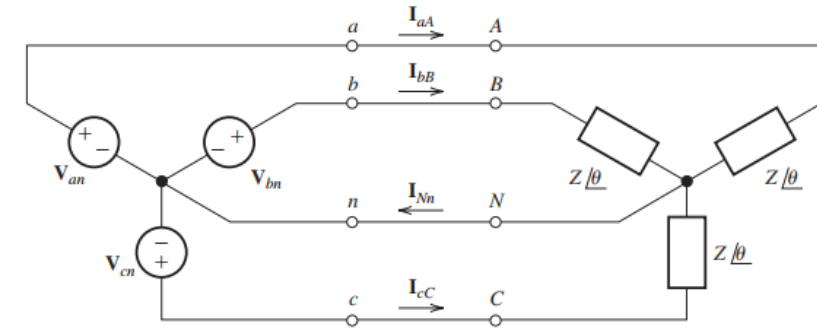
$$i_{aA}(t) = I_L \cos(\omega t - \theta)$$

$$i_{bB}(t) = I_L \cos(\omega t - 120^\circ - \theta)$$

$$i_{cC}(t) = I_L \cos(\omega t + 120^\circ - \theta)$$

Wye - Wye Connection

- The neutral current in Figure is given by



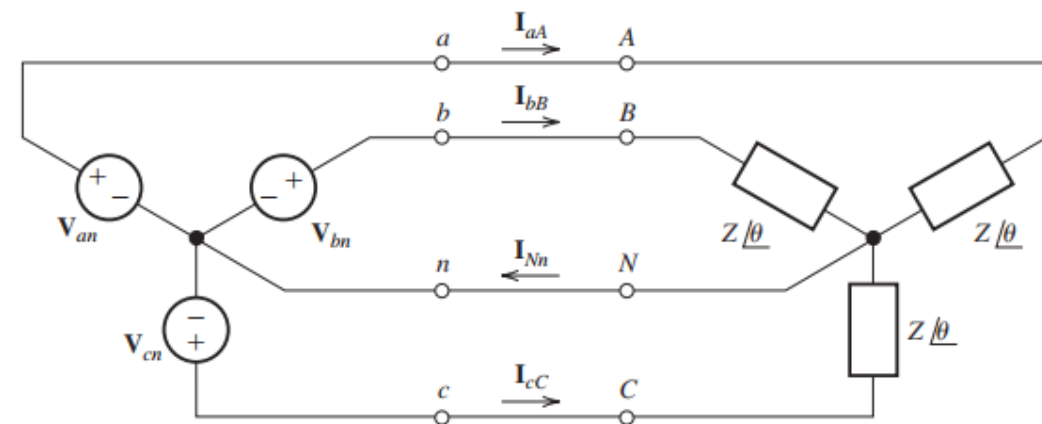
- In terms of phasors, this is $i_{Nn}(t) = i_{aA}(t) + i_{bB}(t) + i_{cC}(t)$

$$\begin{aligned}
 \mathbf{I}_{Nn} &= \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC} \\
 &= I_L \angle -\theta + I_L \angle -120^\circ - \theta + I_L \angle 120^\circ - \theta \\
 &= I_L \angle -\theta \times (1 + 1 \angle -120^\circ + 1 \angle 120^\circ) \\
 &= I_L \angle -\theta \times (1 - 0.5 - j0.866 - 0.5 + j0.866) \\
 &= 0
 \end{aligned}$$

- Thus, the neutral current is zero in a balanced three-phase system.

Line-to-Line Voltages

- The voltages between terminals a, b, or c and the neutral point n are called line-to-neutral voltages / **phase voltages**.
- On the other hand, voltages between a and b, b and c, or a and c are called line-to-line voltages or, more simply, **line voltages**.
- Thus V_{an} , V_{bn} , and V_{cn} are line-to-neutral voltages (phase voltages, whereas V_{ab} , V_{bc} , and V_{ca} are line-to-line voltages **Line voltages**.
- Let us consider the relationships between line-to-line voltages and line-to-neutral voltages.



Line-to-Line Voltages

- We can obtain the following relationship by applying KVL to Figure:

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn}$$

$$\mathbf{V}_{ab} = V_Y \angle 0^\circ - V_Y \angle -120^\circ$$

$$\mathbf{V}_{ab} = \sqrt{3}V_Y \angle 30^\circ$$

$$V_L = \sqrt{3}V_Y$$

- We denote the magnitude of the line-to-line voltage as V_L .

Line-to-Line Voltages

- Thus, the relationship between the line-to-line voltage V_{ab} and the line-to-neutral voltage V_{an} is

$$V_{ab} = V_{an} \times \sqrt{3} \angle 30^\circ$$

Similarly, it can be shown that

$$V_{bc} = V_{bn} \times \sqrt{3} \angle 30^\circ$$

and

$$V_{ca} = V_{cn} \times \sqrt{3} \angle 30^\circ$$

Example:

Analysis of a Wye-Wye System

- A balanced positive-sequence wye-connected 60-Hz three-phase source has line-to neutral voltages of $V_Y = 1000 \text{ V}$.
- This source is connected to a balanced wye-connected load.
- Each phase of the load consists of a 0.1H inductance in series with a 50Ω resistance.
- Find the line currents, the line-to-line voltages, the power, and the reactive power delivered to the load.

Three Phase Power

Total Power, $P = 3 \times \text{Power in each phase}$

Y-System

$$P = 3 \times V_{Ph} I_{Ph} \cos \phi$$

$$V_{Ph} = \frac{V_L}{\sqrt{3}} \quad I_{Ph} = I_L$$

$$P = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \times \cos \phi$$

$$\text{Real Power } P = \sqrt{3} V_L I_L \cos \phi$$

$$\text{Reactive Power, } Q = \sqrt{3} V_L I_L \sin \phi$$

$$\text{Apparent Power, } S = \sqrt{3} V_L I_L = \sqrt{P^2 + Q^2}$$

$$\text{Power Factor} = \cos \phi = \frac{P}{S}$$

$$\text{For balanced condition} \quad \mathbf{I}_N = \mathbf{I}_R + \mathbf{I}_Y + \mathbf{I}_B = 0$$

Δ -System

$$P = 3 \times V_{Ph} I_{Ph} \cos \phi$$

$$V_{Ph} = V_L \quad I_{Ph} = \frac{I_L}{\sqrt{3}}$$

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \times \cos \phi$$

$$\text{Real Power } P = \sqrt{3} V_L I_L \cos \phi$$

$$\text{Reactive Power, } Q = \sqrt{3} V_L I_L \sin \phi$$

$$\text{Apparent Power, } S = \sqrt{3} V_L I_L = \sqrt{P^2 + Q^2}$$

$$\text{Power Factor} = \cos \phi = \frac{P}{S}$$

- Y Connection

Three similar coils each of resistance of $20\ \Omega$ and inductance of $0.17\ \text{H}$ are connected across a 3 phase, $440\ \text{V}$, $50\ \text{Hz}$ supply. Find the line and phase values of the current, real power and apparent power when they are connected in (a) Star and (b) Delta

$$Z_{ph} = 20 + j53.38 \Omega$$

STAR

$$V_L = 440 \angle 0^\circ \text{ V}$$

$$V_{ph} = \frac{440}{\sqrt{3}} \angle -30^\circ = 254.04 \angle -30^\circ \text{ V}$$

$$I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.04 \angle -30^\circ}{20 + j53.38}$$

$$I_{L1} = I_{ph1} = 4.45 \angle -99.46^\circ \text{ A}$$

$$I_{L2} = I_{ph2} = 4.45 \angle -219.46^\circ \text{ A}$$

$$I_{L3} = I_{ph3} = 4.45 \angle 30.54^\circ \text{ A}$$

DELTA

$$V_L = V_{ph} = 440 \angle 0^\circ \text{ V}$$

$$I_{ph1} = \frac{440 \angle 0^\circ}{20 + j53.38} = 7.72 \angle -69.4^\circ$$

$$I_{ph2} = 7.72 \angle -189.44^\circ \text{ A}$$

$$I_{ph3} = 7.72 \angle 50.44^\circ \text{ A}$$

$$I_{L1} = 4.45 \angle -39.44^\circ \text{ A}$$

$$I_{L2} = 4.45 \angle -159.44^\circ \text{ A}$$

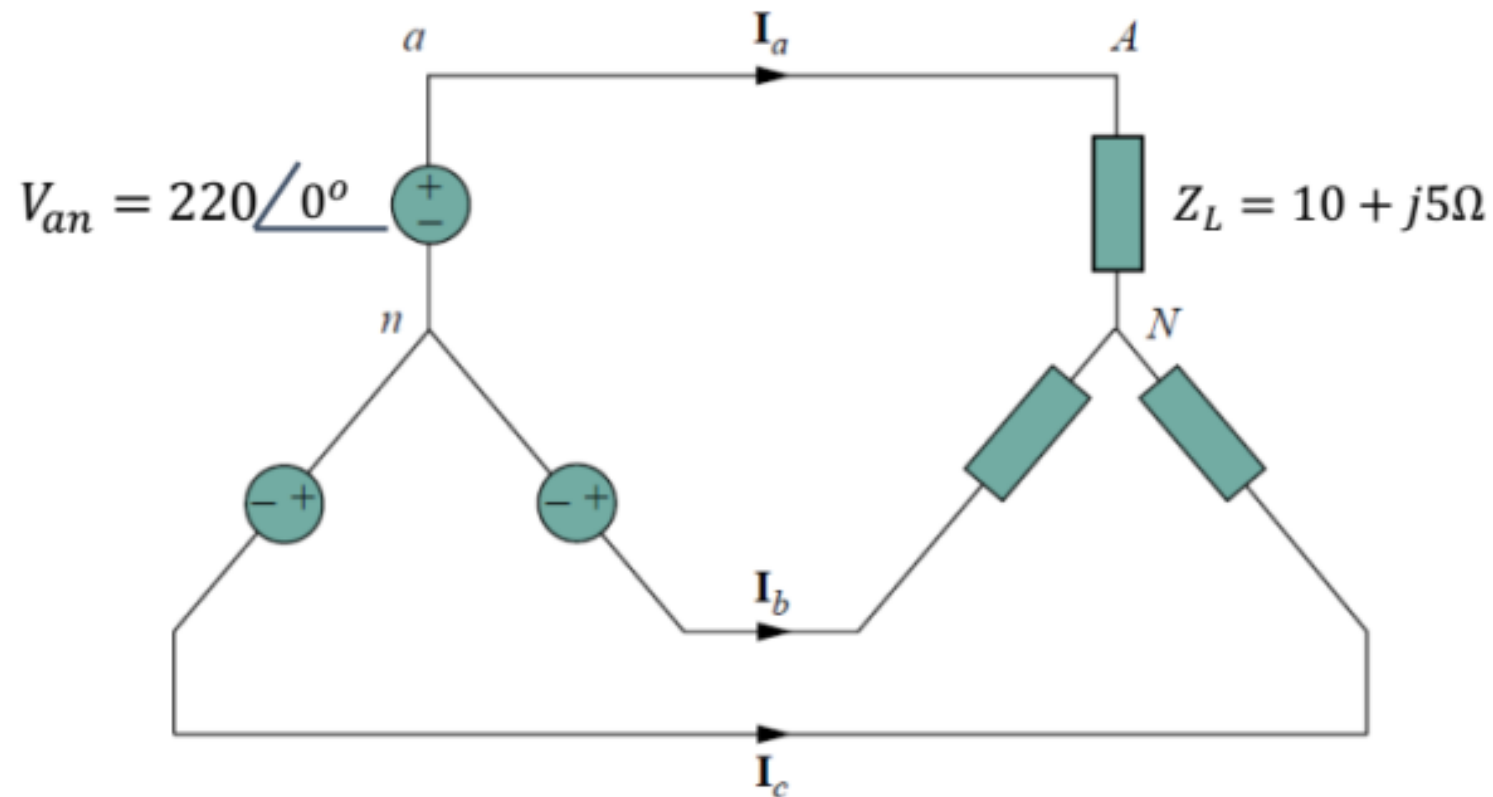
$$I_{L3} = 4.45 \angle 81.44^\circ \text{ A}$$

A Y-connected balanced 3-phase generator load with the phase voltage of 220 V is connected to a balanced Y-connected load with an impedance per phase of $10+j5 \Omega$. Calculate the (i) line voltages, (ii) phase currents, (iii) line currents and (iv) complex power at the source and load.

One line voltage of a balanced Y-connected source is $180 \angle -20^\circ$. If the source is connected to a balanced Δ -connected load of $20 \angle 40^\circ$. Calculate the (i) phase voltages, (ii) phase currents, (ii) line currents and (iii) complex power at the source and load.

A balanced Y-connected load with a phase resistance of 40Ω and a reactance of 25Ω is supplied by a balanced Δ -connected source with a line voltage of 210 V. Calculate the (i) phase voltages, (ii) phase currents, (ii) line currents and (iii) complex power at the source and load.

- A Y-connected balanced 3-phase generator load with the phase voltage of 220 V is connected to a balanced Y-connected load with an impedance per phase of $10 + j5 \Omega$. Calculate the (i) line voltages, (ii) phase currents, (iii) line currents and (iv) complex power at the source and load.



At Source (Y)

Given

$$V_P = 220 \angle 0^\circ \text{ V}$$

Relationship between phase and line, voltage and current in Y connection

$$\begin{aligned} V_L &= \sqrt{3}V_P \angle 30^\circ \\ I_L &= I_P \end{aligned}$$

(i) line voltage

$$V_L = \sqrt{3}(220) \angle 0^\circ \angle 30^\circ$$

$$V_L = 381.05 \angle 30^\circ \text{ V}$$

At Load (Y)

Given

$$Z = 10 + j5 \Omega$$

Relationship between phase and line, voltage and current in Y connection

$$\begin{aligned} V_L &= \sqrt{3}V_P \angle 30^\circ \\ I_L &= I_P \end{aligned}$$

(i) line voltage

Line voltage of load is equal to line voltage of the source

$$V_{AB} = V_{ab} \quad V_L = 381.05 \angle 30^\circ \text{ V}$$

(ii) Phase current

$$\begin{aligned} I_P &= \frac{V_P}{Z_L} = \frac{220}{10 + j5 \Omega} = \frac{220}{11.18 \angle 26.56} \\ &= 19.678 \angle -26.56 \text{ A} \end{aligned}$$

At Source (Y)

(iii) line current

Line current of source is equal to line current of the load

$$I_L = 19.678 \angle -26.56^\circ \text{ A}$$

(ii) Phase current

$$I_L = I_P = 19.678 \angle -26.56^\circ \text{ A}$$

(iii) Complex power

$$S = -(P + jQ)$$

$$S = -(3V_P I_P \cos(\theta_V - \theta_I) + j3V_P I_P \sin(\theta_V - \theta_I))$$

$$= -(3(220)(19.678) \cos(0^\circ + 26.56^\circ) + j3(220)(19.678) \sin(0^\circ + 26.56^\circ))$$

$$S = -(11616.86 + j5807.15) \text{ VA}$$

Source power should be negative

At Load (Y)

(iii) line current

$$I_L = I_P = 19.678 \angle -26.56^\circ \text{ A}$$

(iii) Complex power

$$S = P + jQ$$

$$S = 3V_P I_P \cos(\theta_V - \theta_I) + j3V_P I_P \sin(\theta_V - \theta_I)$$

$$= 3(220)(19.678) \cos(0^\circ + 26.56^\circ) + j3(220)(19.678) \sin(0^\circ + 26.56^\circ)$$

$$S = 11616.86 + j5807.15 \text{ VA}$$

Load power should be positive

Calculate the line currents for the circuit given in Fig. 1. Hence, calculate the average power across the load in each phase.

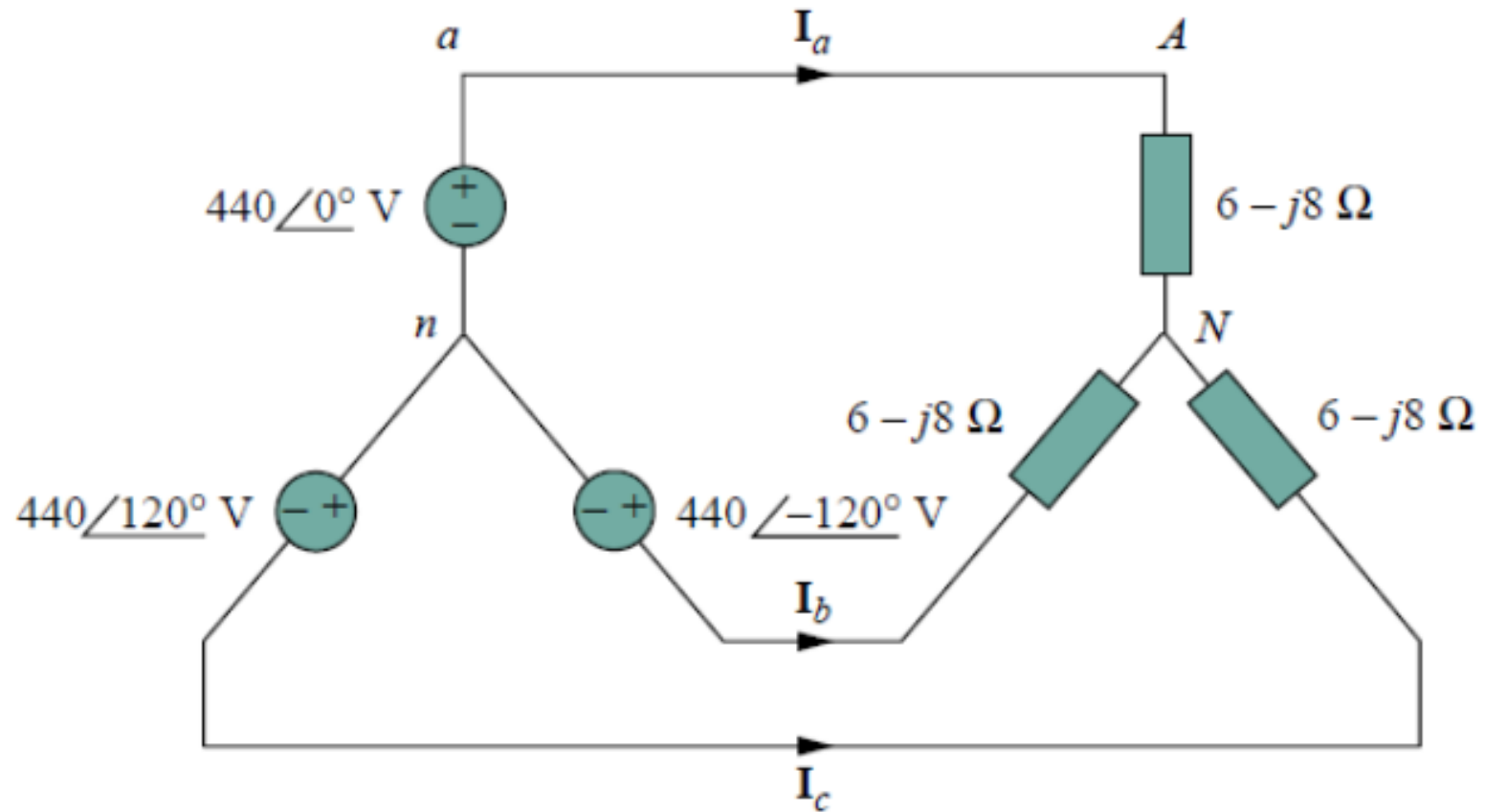


Fig. 1

This is a balanced Y-Y system.



Using the per-phase circuit shown above,

$$I_a = \frac{440\angle 0^\circ}{6 - j8} = \underline{44\angle 53.13^\circ \text{ A}}$$

$$I_b = I_a \angle -120^\circ = \underline{44\angle -66.87^\circ \text{ A}}$$

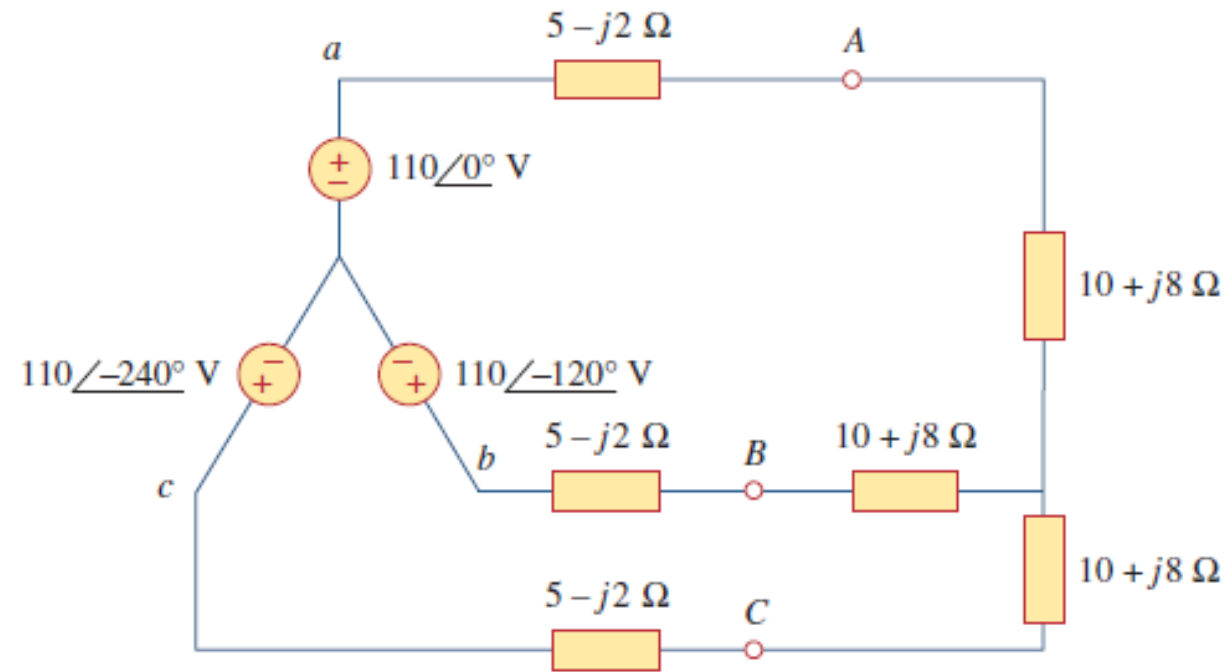
$$I_c = I_a \angle 120^\circ = \underline{44\angle 173.13^\circ \text{ A}}$$

Average power calculations:

$$P_a = P_b = P_c = 11.616 \text{ kW}$$

$$P_{\text{avg}} = 11.616 \text{ kW} \times 3 = 34.85 \text{ kW}$$

EXAMPLE



$$I_a = \frac{V_{an}}{Z_Y}$$

where $Z_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155\angle 21.8^\circ$. Hence,

$$I_a = \frac{110\angle 0^\circ}{16.155\angle 21.8^\circ} = 6.81\angle -21.8^\circ \text{ A}$$

A balanced delta-connected load has a phase current $I_{AC} = 10\angle 30^\circ \text{ A}$.

(a) Determine the three line currents assuming that the circuit operates in the positive phase sequence.

(b) Calculate the load impedance if the line voltage is $V_{AB} = 110\angle 0^\circ \text{ V}$

$$(a) \quad \mathbf{I}_{CA} = -\mathbf{I}_{AC} = 10 \angle (-30^\circ + 180^\circ) = 10 \angle 150^\circ$$

This implies that

$$\mathbf{I}_{AB} = 10 \angle 30^\circ$$

$$\mathbf{I}_{BC} = 10 \angle -90^\circ$$

$$\mathbf{I}_a = \mathbf{I}_{AB} \sqrt{3} \angle -30^\circ = \underline{\mathbf{17.32 \angle 0^\circ A}}$$

$$\mathbf{I}_b = \underline{\mathbf{17.32 \angle -120^\circ A}}$$

$$\mathbf{I}_c = \underline{\mathbf{17.32 \angle 120^\circ A}}$$

$$(b) \quad \mathbf{Z}_\Delta = \frac{\mathbf{V}_{AB}}{\mathbf{I}_{AB}} = \frac{110 \angle 0^\circ}{10 \angle 30^\circ} = \underline{\mathbf{11 \angle -30^\circ \Omega}}$$