Basic Electrical and Electronics Engineering

Dr. Sonam Shrivastava/ Assistant Professor (Sr.) /SELECT

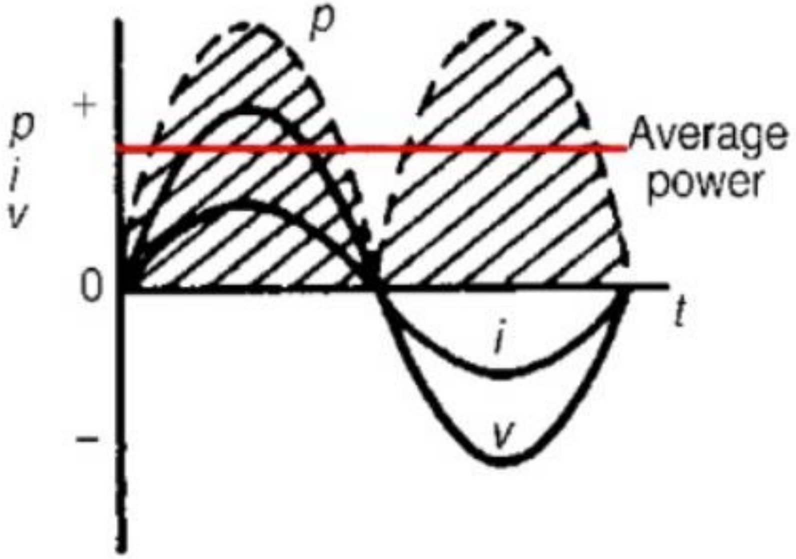
LECTURE 2.6

AC Power Calculations

The value of power at any instant is given by the product of the voltage and current at that instant, i.e. the instantaneous power, p=vi, as shown in red

For a purely resistive a.c. circuit, the average power dissipated, P, is given by:

$$P = VI = I^2R = \frac{V^2}{R} \text{ watts}$$



Power in Resistive Components

- Suppose a voltage $v = V_p \sin \omega t$ is applied across a resistance R. The resultant current i will be

$$i = \frac{v}{R} = \frac{V_P \sin \omega t}{R} = I_P \sin \omega t$$

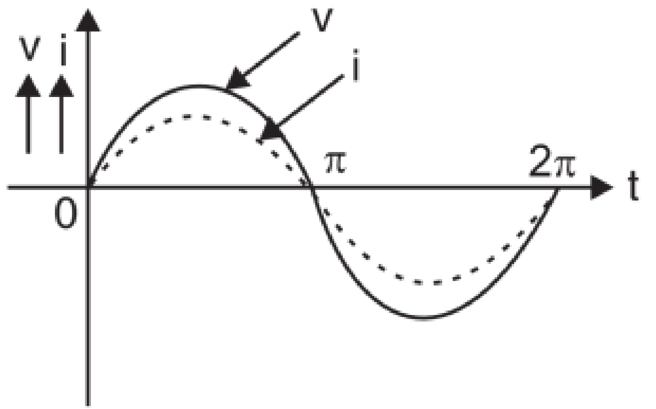
- The result power p will be

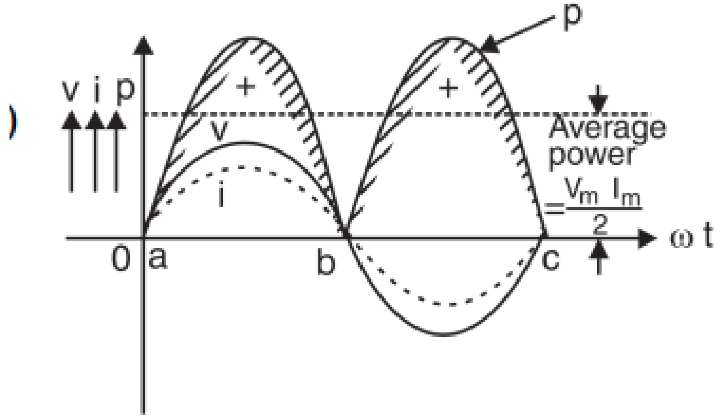
$$p = vi = V_P \sin \omega t \times I_P \sin \omega t = V_P I_P (\sin^2 \omega t) = V_P I_P \left(\frac{1 - \cos 2\omega t}{2}\right)$$

- The average value of $(1 - \cos 2\omega t)$ is 1, so

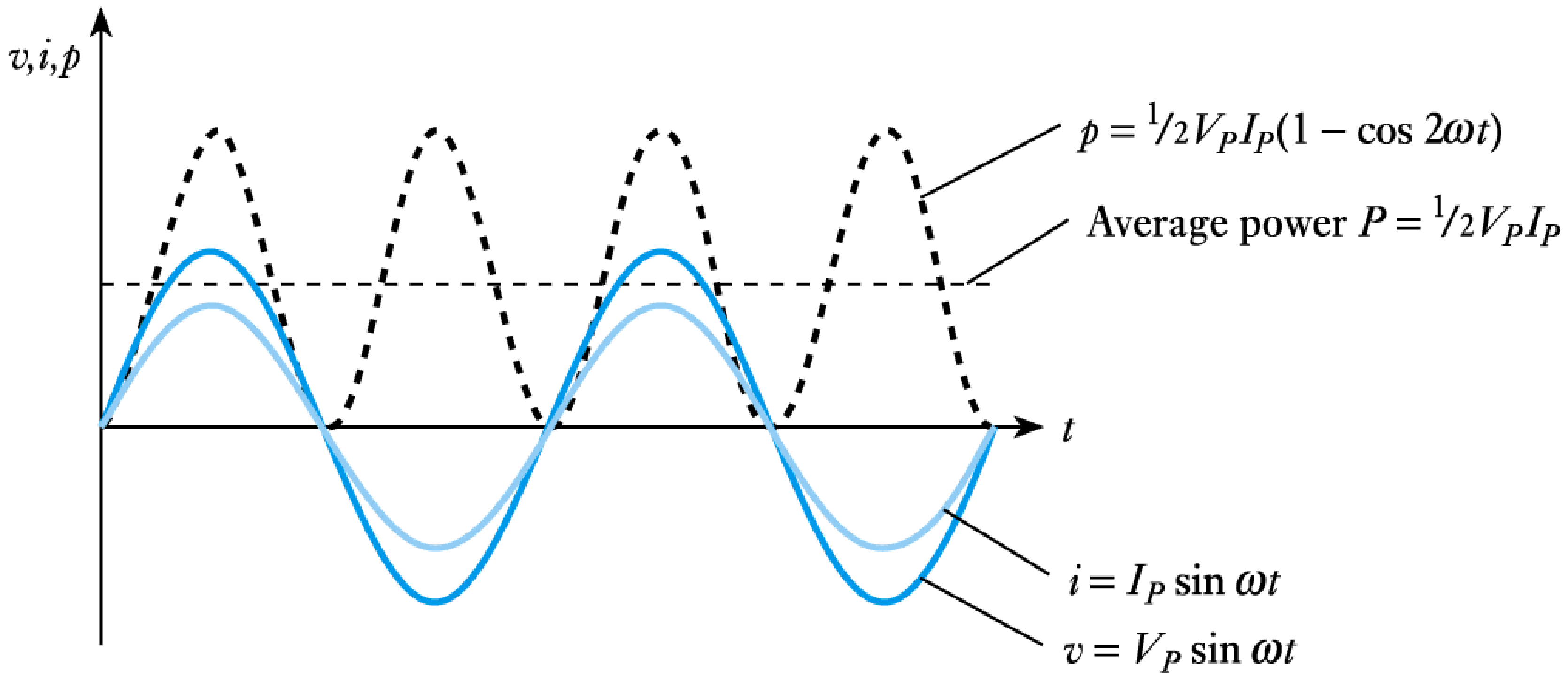
$$\text{Average Power } P = \frac{1}{2} V_P I_P = \frac{V_P}{\sqrt{2}} \times \frac{I_P}{\sqrt{2}} = VI$$

where V and I are the **RMS voltage and current**





- **Relationship between *v*, *i* and *p* in a resistor**



Power in Inductors

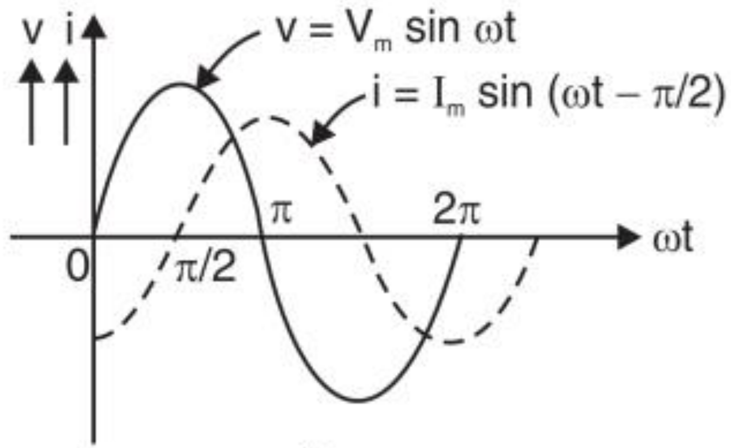
- From our discussion of inductors we know that the current lags the voltage by 90°. Therefore, if a voltage $v = V_m \sin \omega t$ is applied across an inductance L, the current will be given by
- $i = -I_m \cos \omega t$
- Therefore $p = vi$

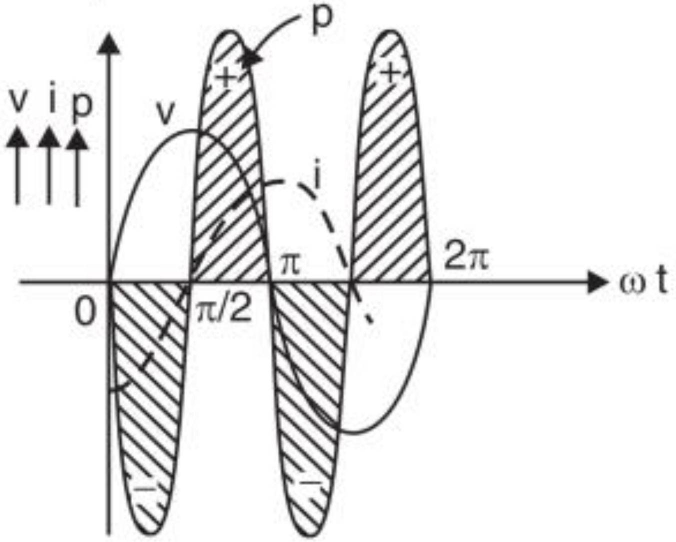
$$= V_m \sin \omega t \times -I_m \cos \omega t$$

$$= -V_m I_m (\sin \omega t \times \cos \omega t)$$

$$= -V_m I_m \left(\frac{\sin 2\omega t}{2}\right)$$

- Again the average power is zero

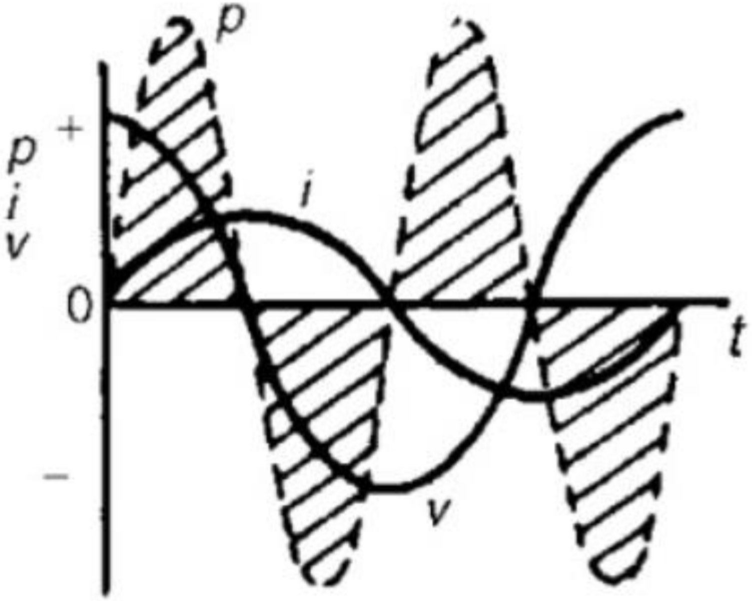




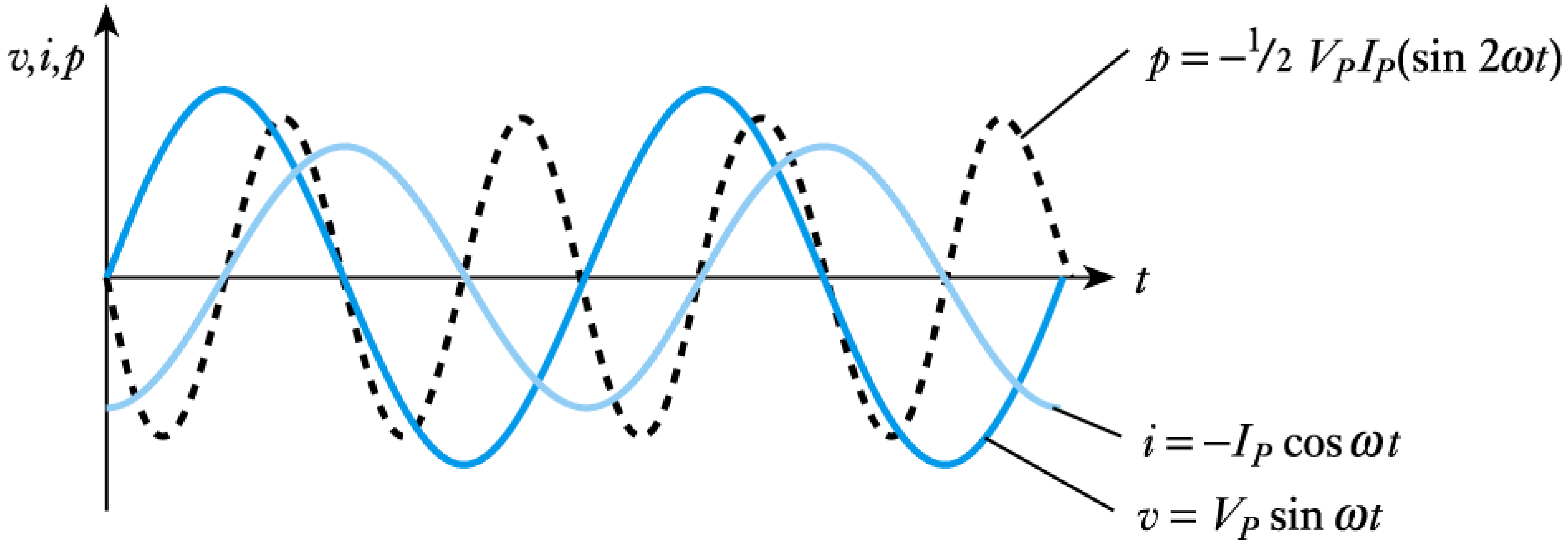
Power in Inductors

For a purely inductive a.c. circuit, the average power is zero.

$$Average\ Power = 0$$



- **Relationship between *v*, *i* and *p* in an inductor**

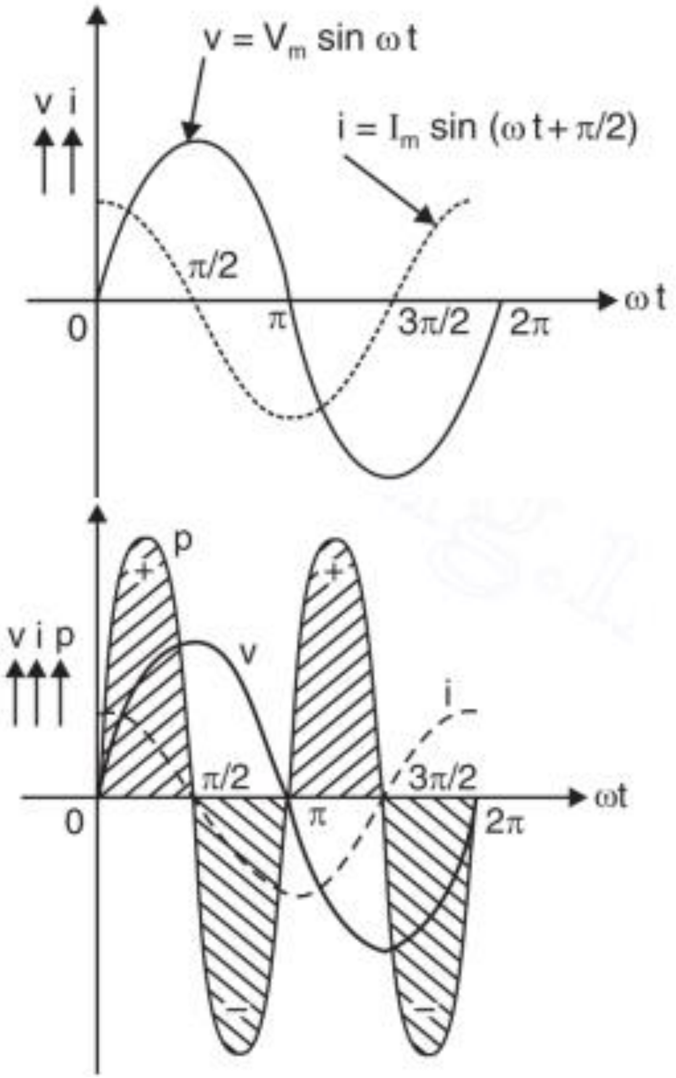


Power in Capacitors

- From our discussion of capacitors we know that the current leads the voltage by 90°. Therefore, if a voltage $v = V_m \sin \omega t$ is applied across a capacitance C, the current will be given by
- $i = I_m \cos \omega t$
- Then

$$p = vi$$
$$= V_m \sin \omega t \times I_m \cos \omega t$$
$$= V_m I_m (\sin \omega t \times \cos \omega t)$$
$$= V_m I_m \left(\frac{\sin 2\omega t}{2}\right)$$

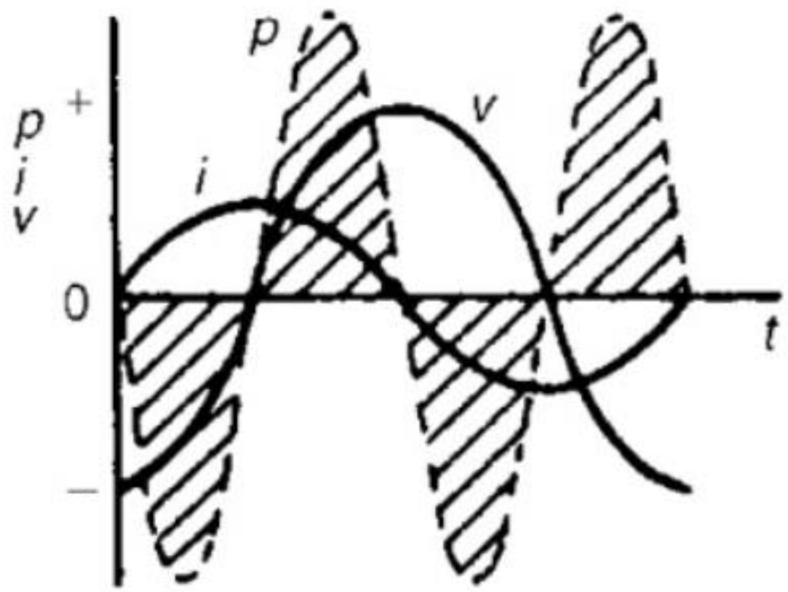
- The average power is zero



Power in Capacitors

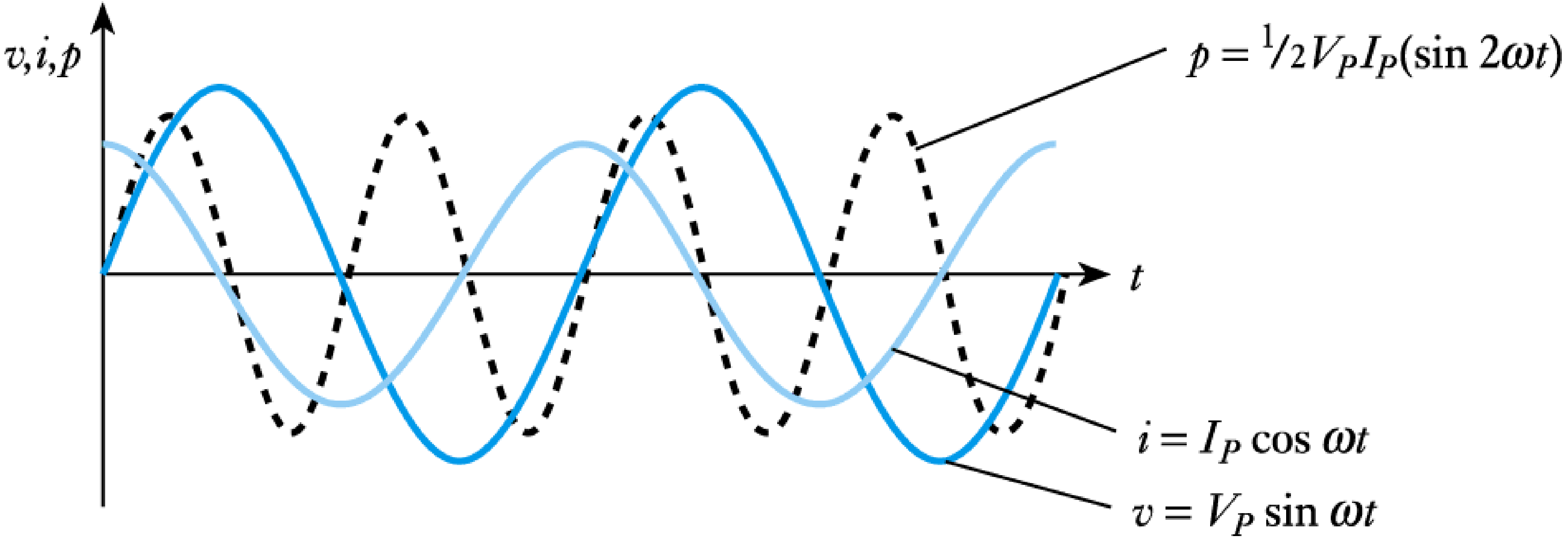
For a purely capacitive a.c. circuit, the average power is zero.

Average Power = 0



Power in Capacitors

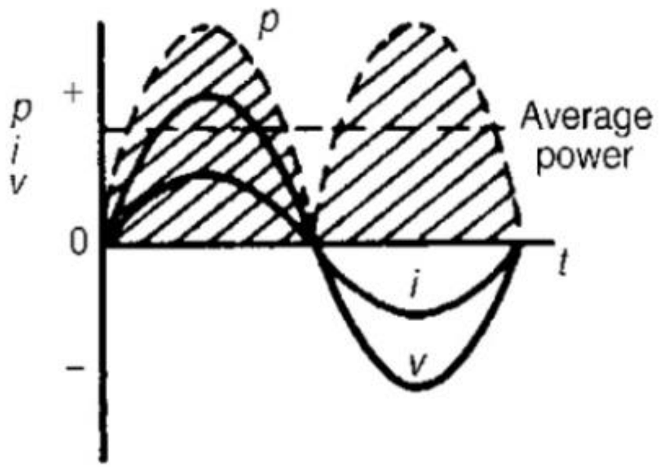
- **Relationship between *v*, *i* and *p* in a capacitor**



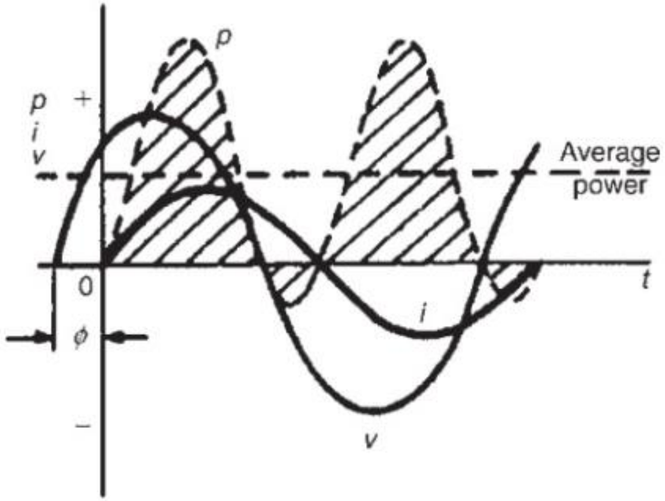
Power in RL Circuits

R–L circuit where the current lags the voltage by angle φ. The waveform for power (where p=vi) is shown by the broken line, and its shape, and hence average power, depends on the value of angle φ.

Pure R Circuit



RL Circuit



Power in Circuit with Resistance and Reactance

- When a sinusoidal voltage $\upsilon = V_m \sin \omega t$ is applied across a circuit with resistance *and* reactance, the current will be of the general form $i = I_m \sin(\omega t - \phi)$
- Therefore, the instantaneous power, p is given by

$$
\begin{aligned}
p &= vi \\
&= V_m \sin \omega t \times I_m \sin(\omega t - \varphi) \\
&= \frac{1}{2} V_m I_m \{\cos \varphi - \cos(2\omega t - \varphi)\}
\end{aligned}
$$

$$
p = \frac{1}{2} V_m I_m \cos \varphi - \frac{1}{2} V_m I_m \cos(2\omega t - \varphi)
$$

Power in RL Circuits

$$p = \frac{1}{2} V_m I_m \cos\varphi - \frac{1}{2} V_m I_m \cos(2\omega t - \varphi)$$

- The expression for p has two components
- **The second part oscillates at 2ω and has an average value of zero over a complete cycle**
  - this is the power that is stored in the reactive elements and then returned to the circuit within each cycle
- The first part represents the power dissipated in resistive components. **Average power dissipation** is

$$P = \frac{1}{2} V_m I_m (\cos\varphi) = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times (\cos\varphi) = VI\cos\varphi$$

Power in RL Circuits

- The average power dissipation given by

$$P = \frac{1}{2} V_m I_m (\cos \varphi) = VI \cos \varphi$$

is termed the **active power** in the circuit and is measured in watts (W)

- The product of the RMS voltage and current *VI* is termed the **apparent power**, ***S***. To avoid confusion this is given the units of volt amperes (VA)

Power Factor

- From the above discussion it is clear that

$$P = VI\cos\phi$$
$$= S\cos\phi$$

- In other words, the active power is the apparent power times the cosine of the phase angle.
- This cosine is referred to as the **power factor**

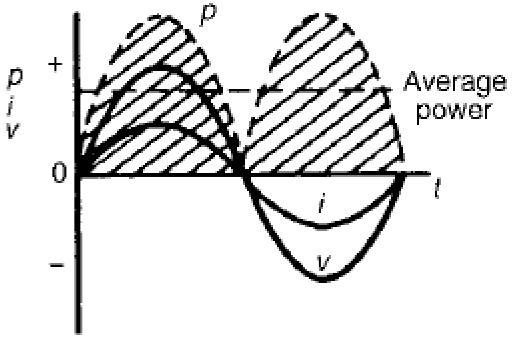
$$\frac{\text{Active power (in watts)}}{\text{Apparent power (in volt amperes)}} = \text{Power factor}$$

$$\text{Power factor} = \frac{P}{S} = \cos\phi$$

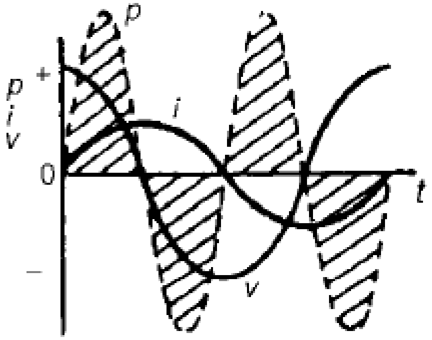
Power in Resistive, Inductive and Capacitive Circuit

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

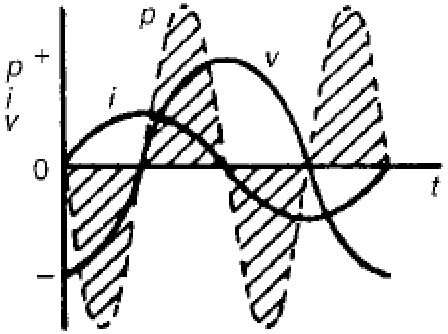
$\theta_v = 0, and\ \theta_i = 0$	$\theta_v = 0, and\ \theta_i = -90°$	$\theta_v = 0, and\ \theta_i = 90°$



PURE RESISTANCE-AVERAGE POWER = VI



PURE INDUCTANCE-AVERAGE POWER = 0



PURE CAPACITANCE-AVERAGE POWER = 0

Active and Reactive Power

- When a circuit has resistive and reactive parts, the resultant power has 2 parts:
  - The first is *dissipated* in the resistive element. This is the **active power, *P***
  - The second is *stored* and *returned* by the reactive element. This is the **reactive power**, ***Q*** , **volt amperes reactive** or **VAR**

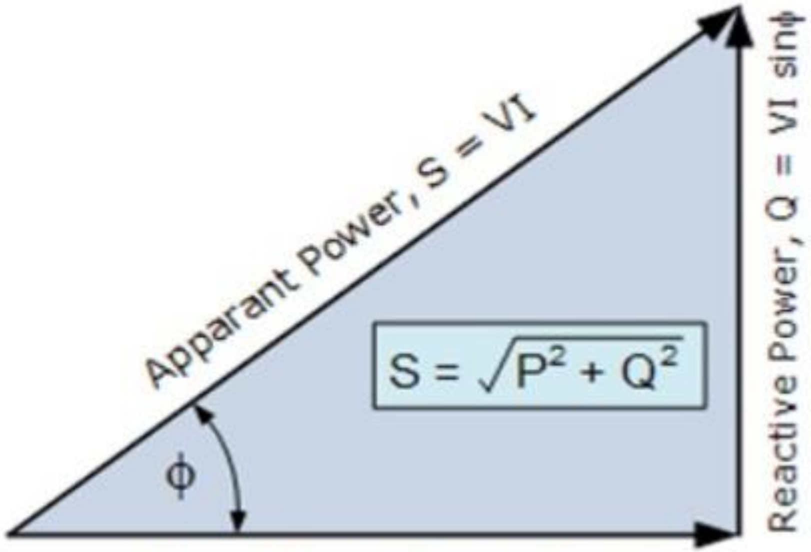
- **Real power results from energy being used for work or dissipated as heat, reactive power is the result of energy being stored, to establish electric (capacitors) or magnetic (inductors) fields, and later being released back to the source.**

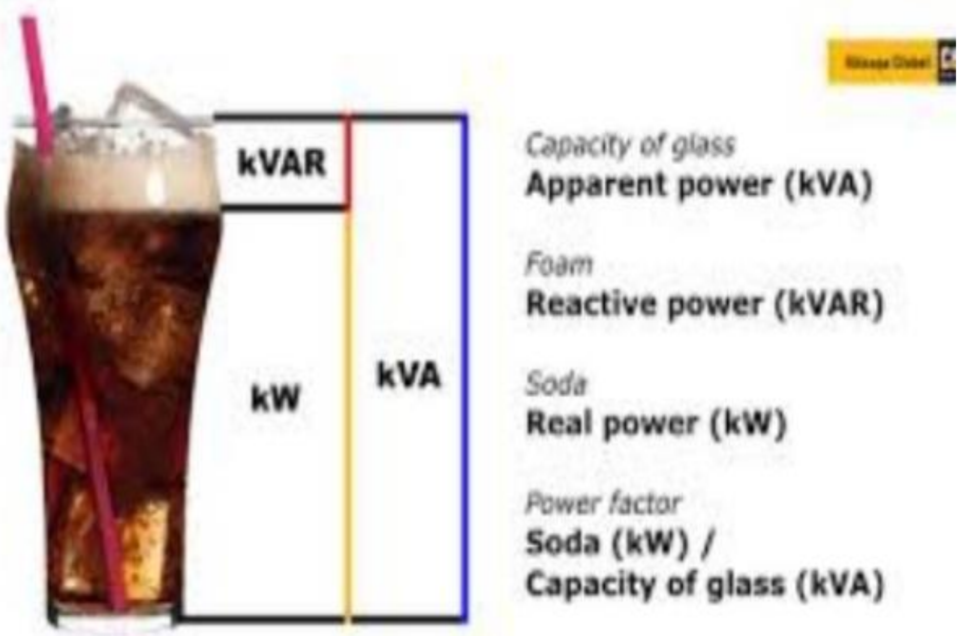
- **While reactive power is not dissipated it does have an effect on the system for example, it increases the current that must be supplied and increases losses with cables**

For an R–L, R–C or R–L–C series a.c. circuit, the average power P is given by:

$$P = VI\cos\varphi \text{ watts}$$

Power triangle





Power Factor

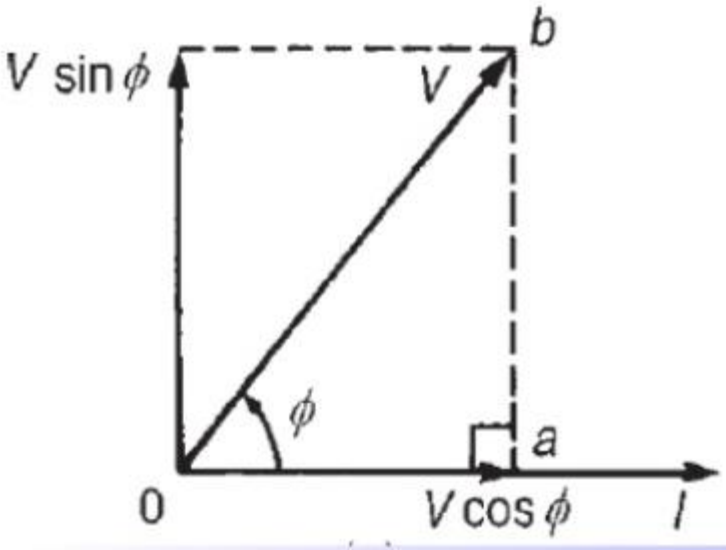
$$\text{power factor} = \frac{\text{active power } P}{\text{apparent power } S}$$

For sinusoidal voltages and currents,

$$\text{power factor} = \frac{P}{S} = \frac{VI\cos\phi}{VI}$$

$$= \cos\phi = \frac{R}{Z} \text{ (from the impedance triangle)}$$

$\sin(\theta) = \cos(90° - \theta)$

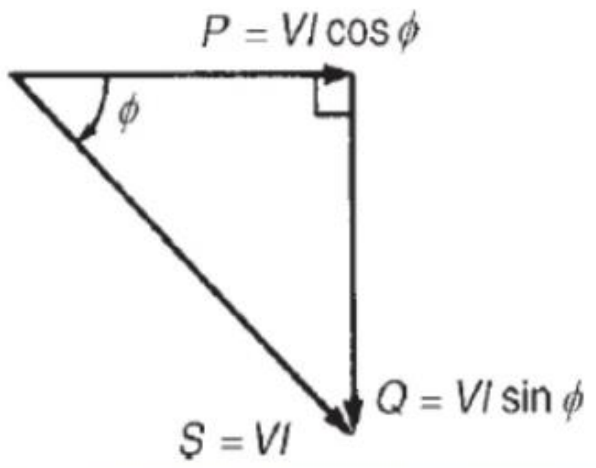


True or active power $P = VI\cos\phi$ watts (W)

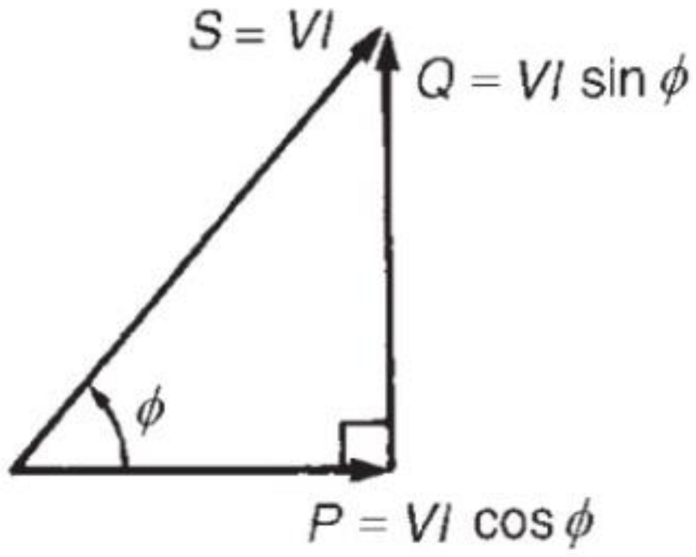
Apparent power $S = VI$ voltamperes (VA)

Reactive power $Q = VI\sin\phi$ vars (var)

A circuit in which current lags voltage (i.e. an inductive circuit) is said to have a lagging power factor, and indicates a lagging reactive power Q.
A circuit in which current leads voltage (i.e. a capacitive circuit) is said to have a leading power factor, and indicates a leading reactive power Q.

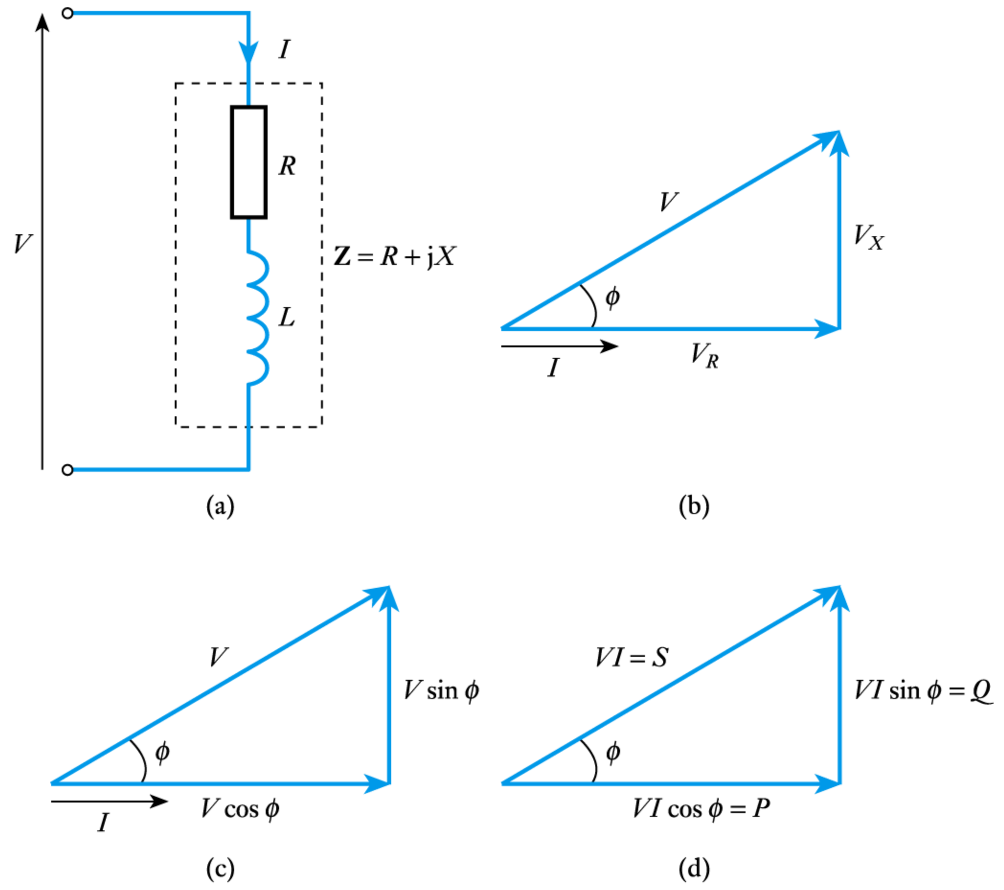


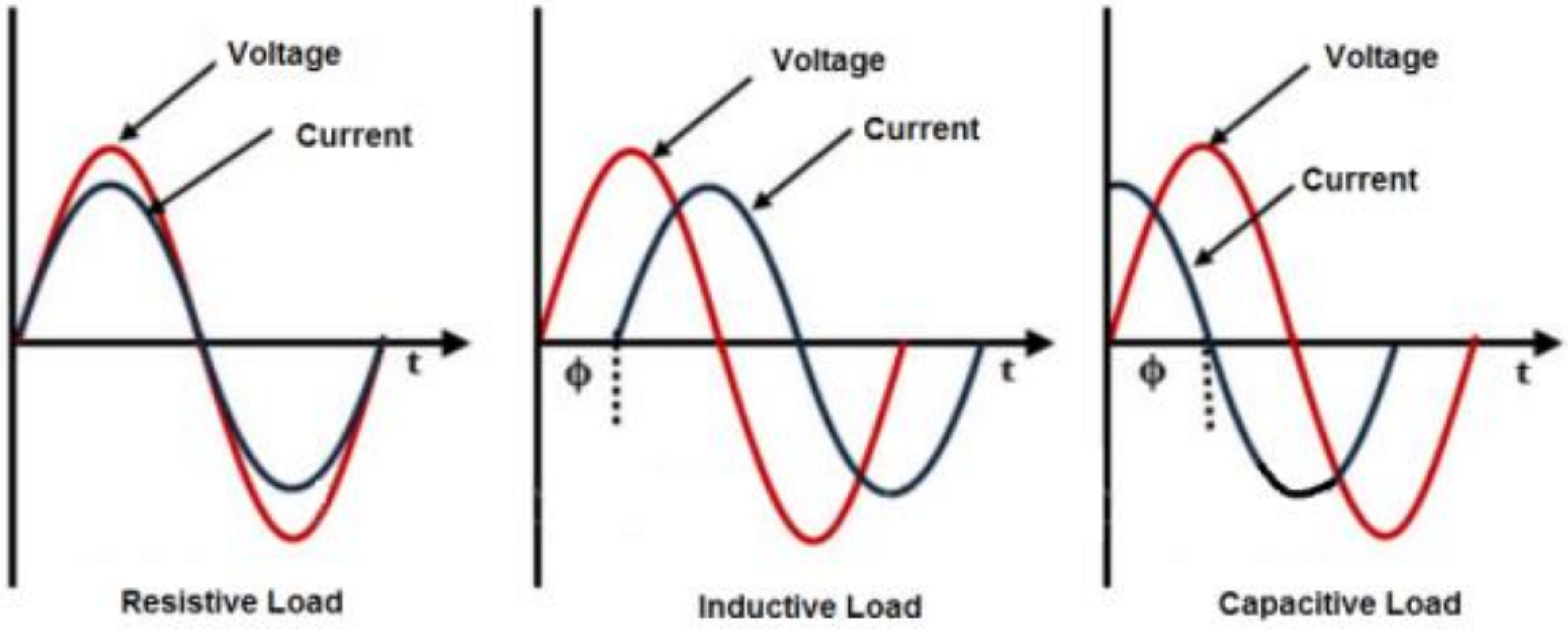
Capacitive



Inductive

- Consider an RL circuit
    - the relationship between the various forms of power can be illustrated using a power triangle





Voltage
Current
t
Resistive Load
Voltage
Current
ϕ
t
Inductive Load
Voltage
Current
ϕ
t
Capacitive Load

Instantaneous Power, p(t)

The instantaneous power (in watts) is the power at any instant of time.

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$p(t) = v(t)i(t)$$

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Example

A resistive load (R) absorbs power at all times, while a reactive load (L or C) absorbs zero average power.

$$v(t) = 120\cos(377t + 45°)\ \text{V} \quad \text{and} \quad i(t) = 10\cos(377t - 10°)\ \text{A}$$

Find Instantaneous value of power and power consumed by load

Example

A resistive load (R) absorbs power at all times, while a reactive load (L or C) absorbs zero average power.

$$v(t) = 120\cos(377t + 45^\circ)\ \text{V} \qquad \text{and} \qquad i(t) = 10\cos(377t - 10^\circ)\ \text{A}$$

$$p = vi = 1200\cos(377t + 45^\circ)\cos(377t - 10^\circ)$$

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

$$p = 600[\cos(754t + 35^\circ) + \cos 55^\circ]$$

$$p(t) = 344.2 + 600\cos(754t + 35^\circ)\ \text{W}$$

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}120(10)\cos[45^\circ - (-10^\circ)] \quad = 600\cos 55^\circ = 344.2\ \text{W}$$

- A coil having a resistance of 7 Ω and an inductance of 31·8 mH is connected to 230 V, 50 Hz supply. Calculate (i) the circuit current (ii) phase angle (iii) power factor (iv) power consumed and (v) voltage drop across resistor and inductor.

Solution. (*i*) Inductive reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\ \Omega$

$$\text{Coil impedance, } Z = \sqrt{R^2 + X_L^2} = \sqrt{7^2 + 10^2} = 12.2\ \Omega$$

$\therefore$ Circuit current, $I = V/Z = 230/12.2 =$ **18·85 A**

(*ii*) $\tan\phi = X_L/R = 10/7$

$\therefore$ Phase angle, $\phi = \tan^{-1}(10/7) =$ **55° lag**

(*iii*) Power factor $= \cos\phi = \cos 55° =$ **0.573 lag**

(*iv*) Power consumed, $P = VI\cos\phi = 230 \times 18.85 \times 0.573 =$ **2484·24 W**

(*v*) Voltage drop across $R = IR = 18.85 \times 7 =$ **131.95V**

Voltage drop across $L = IX_L = 18.85 \times 10 =$ **188.5 V**

A pure inductive coil allows a current of 10 A to flow from a 230 V, 50 Hz supply. Find (i) inductive reactance (ii) inductance of the coil (iii) power absorbed. Write down the equations for voltage and current

Solution. (*i*) Circuit current, $I = V/X_L$ $(V_L = V)$

$\therefore$ Inductive reactance, $X_L = V/I = 230/10 =$ **23 Ω**

(*ii*) Now, $X_L = 2\pi f L \quad \therefore \quad L = \dfrac{X_L}{2\pi f} = \dfrac{23}{2\pi \times 50} =$ **0.073 H**

(*iii*) Power absorbed = **Zero**

$V_m = 230 \times \sqrt{2} = 325.27$ V ; $I_m = 10 \times \sqrt{2} = 14.14$ A ; $\omega = 2\pi \times 50 = 314$ rad/s

Since in a pure inductive circuit, current lags behind the voltage by π/2 radians, the equations are :

$$\boldsymbol{v = 325.27 \sin 314\, t \quad ; \quad i = 14.14 \sin (314\, t - \pi/2)}$$

A 230 V, 50 Hz a.c. supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected in series with a 6·8 μF capacitor. Calculate (i) impedance (ii) current (iii) phase angle between current and voltage (iv) power factor and (v) power consumed

A 230 V, 50 Hz a.c. supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected in series with a 6·8 μF capacitor. Calculate (i) impedance (ii) current (iii) phase angle between current and voltage (iv) power factor and (v) power consumed

Solution

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.06 = 18.85\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 6.8} = 468\ \Omega$$

(*i*) Circuit impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(2.5)^2 + (18.85 - 468)^2} = \mathbf{449.2\ \Omega}$

(*ii*) Circuit current, $I = V/Z = 230/449.2 = \mathbf{0.512\ A}$

(*iii*) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{18.85 - 468}{2.5} = -179.66$

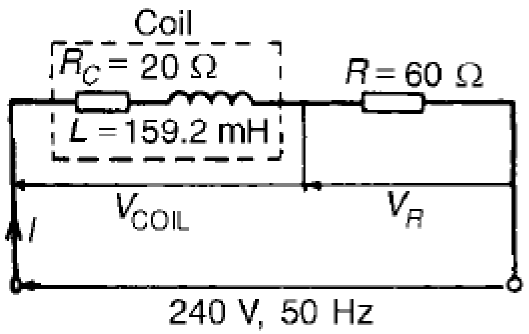
$\therefore$ Phase angle, $\phi = \tan^{-1} - 179.66 = -89.7° = \mathbf{89.7°}$ ***lead***

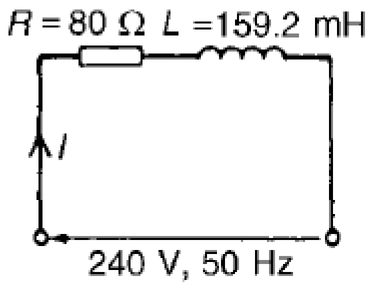
(*iv*) Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{2.5}{449.2} = \mathbf{0.00557}$ ***lead***

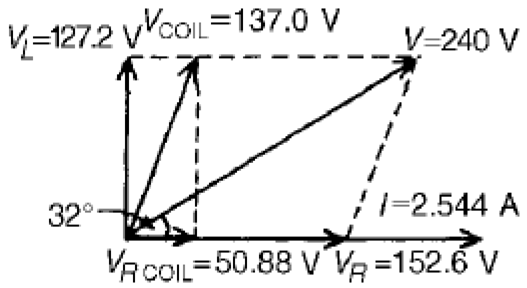
(*v*) Power consumed, $P = VI\cos\phi = 230 \times 0.512 \times 0.00557 = \mathbf{0.656\ W}$

Example

Problem 12. A coil of inductance 159.2 mH and resistance 20 Ω is connected in series with a 60 Ω resistor to a 240 V, 50 Hz supply. Determine (a) the impedance of the circuit, (b) the current in the circuit, (c) the circuit phase angle, (d) the p.d. across the 60 Ω resistor and (e) the p.d. across the coil. (f) Draw the circuit phasor diagram showing all voltages. (g) Calculate Active and reactive power consumed by the load







Complex power (S)

Complex power (in VA) is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. As a complex quantity, its real part is real power P and its imaginary part is reactive power Q.

$$\boldsymbol{S} = \boldsymbol{P} + \boldsymbol{jQ} = \boldsymbol{VI}^*$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i), \qquad Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

$$\boldsymbol{V} = V\angle\theta_V \qquad \boldsymbol{I} = I\angle\theta_I \qquad \boldsymbol{I}^* = I\angle -\theta_I$$

$$\boldsymbol{S} = \boldsymbol{VI}^* = VI\ \angle(\theta_V - \theta_I)$$

$$\boldsymbol{S} = VI\cos(\theta_V - \theta_I) + j\ VI\sin(\theta_V - \theta_I)$$

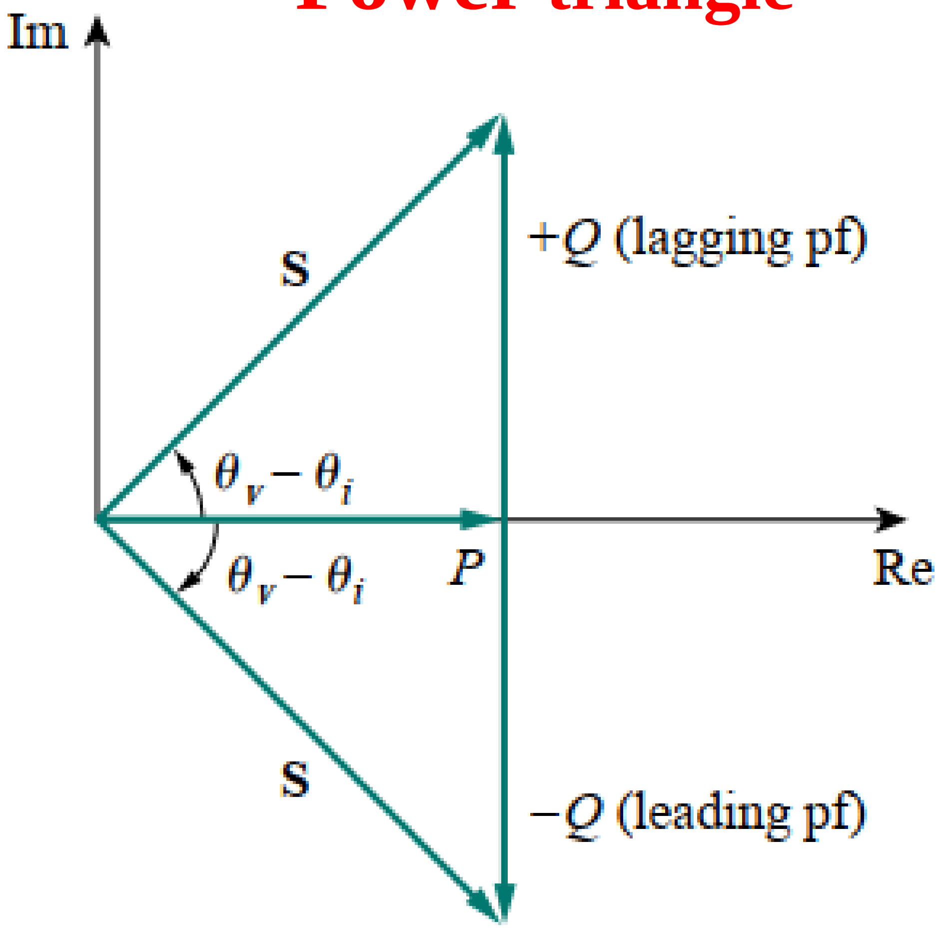
Complex Power

Introducing the complex power enables us to obtain the real and reactive powers directly from voltage and current phasors.

$$\text{Complex Power} = \mathbf{S} = P + jQ = \frac{1}{2}\mathbf{V}\mathbf{I}^*$$
$$= V_{\text{rms}} I_{\text{rms}} \underline{/\theta_v - \theta_i}$$
$$\text{Apparent Power} = S = |\mathbf{S}| = V_{\text{rms}} I_{\text{rms}} = \sqrt{P^2 + Q^2}$$
$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$
$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$
$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

This shows that how complex power contain all the power information in a given load.

Power triangle



- ❑ $Q = 0$ for resistive loads (unity PF)
- ❑ $Q < 0$ for capacitive loads (leading PF)
- ❑ $Q > 0$ for inductive loads (lagging PF)

Various power terms

Complex power	$\mathbf{S} = P + jQ = \frac{1}{2}\mathbf{V}\mathbf{I}^* = VI\angle(\phi_v - \phi_i)$
Apparent Power	$S = \lvert\mathbf{S}\rvert = VI = \sqrt{P^2 + Q^2}$
Real Power	$P = Re(\mathbf{S}) = S\cos(\phi_v - \phi_i)$
Reactive Power	$Q = Im(\mathbf{S}) = S\sin(\phi_v - \phi_i)$
Power Factor	$\frac{P}{S} = \cos(\phi_v - \phi_i)$

Problem

The voltage across a load is $v(t) = 60\cos(\omega t - 10°)$ V and the current through the element in the direction of the voltage drop is $i(t) = 1.5\cos(\omega t + 50°)$ A. Find: (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

Problem

The voltage across a load is $v(t) = 60\cos(\omega t - 10^\circ)$ V and the current through the element in the direction of the voltage drop is $i(t) = 1.5\cos(\omega t + 50^\circ)$ A. Find: (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

Solution

(a) For the rms values of the voltage and current, we write

$$\mathbf{V}_{\text{rms}} = \frac{60}{\sqrt{2}}\underline{/-10^\circ}, \qquad \mathbf{I}_{\text{rms}} = \frac{1.5}{\sqrt{2}}\underline{/+50^\circ}$$

The complex power is

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}_{\text{rms}}^* = \left(\frac{60}{\sqrt{2}}\underline{/-10^\circ}\right)\left(\frac{1.5}{\sqrt{2}}\underline{/-50^\circ}\right) = 45\underline{/-60^\circ} \text{ VA}$$

The apparent power is

$$S = |\mathbf{S}| = 45 \text{ VA}$$

Solution

(b) We can express the complex power in rectangular form as

$$\mathbf{S} = 45\underline{/-60^\circ} = 45[\cos(-60^\circ) + j\sin(-60^\circ)] = 22.5 - j38.97$$

Since $\mathbf{S} = P + jQ$, the real power is

$$P = 22.5\ \text{W}$$

while the reactive power is

$$Q = -38.97\ \text{VAR}$$

Solution

(c) The power factor is

$$\text{pf} = \cos(-60^\circ) = 0.5 \text{ (leading)}$$

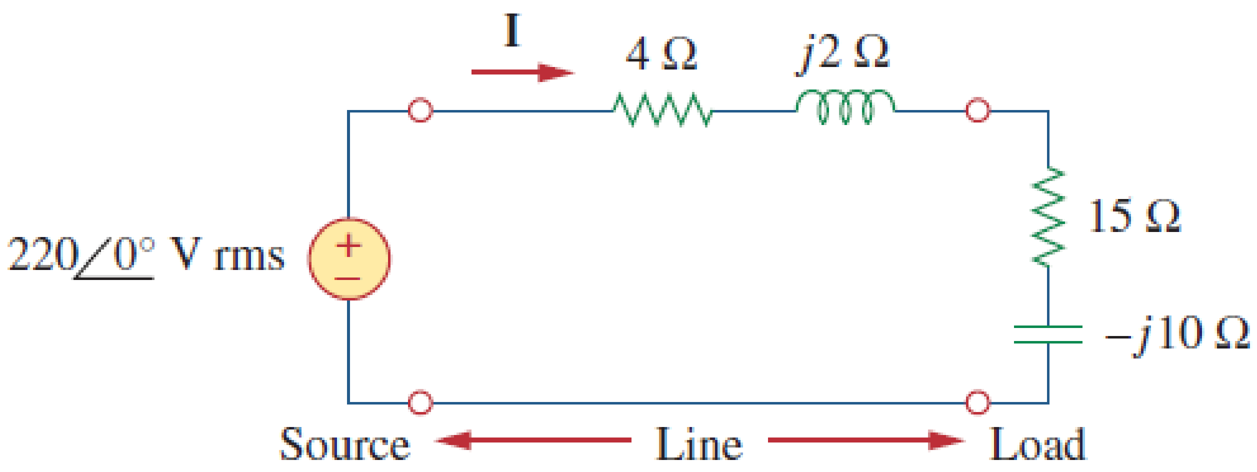
It is leading, because the reactive power is negative. The load impedance is

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{60\underline{/-10^\circ}}{1.5\underline{/+50^\circ}} = 40\underline{/-60^\circ}\ \Omega$$

which is a capacitive impedance.

Example 1 – AC Series Circuit

Figure 11.24 shows a load being fed by a voltage source through a transmission line. The impedance of the line is represented by the $(4 + j2)\ \Omega$ impedance and a return path. Find the real power and reactive power absorbed by: (a) the source, (b) the line, and (c) the load.



The total impedance is

$$\mathbf{Z} = (4 + j2) + (15 - j10) = 19 - j8 = 20.62\underline{/-22.83^\circ}\ \Omega$$

The current through the circuit is

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{220\underline{/0^\circ}}{20.62\underline{/-22.83^\circ}} = 10.67\underline{/22.83^\circ}\text{ A rms}$$

(a) For the source, the complex power is

$$\mathbf{S}_s = \mathbf{V}_s\mathbf{I}^* = (220\underline{/0^\circ})(10.67\underline{/-22.83^\circ})$$
$$= 2347.4\underline{/-22.83^\circ} = (2163.5 - j910.8)\text{ VA}$$

From this, we obtain the real power as 2163.5 W and the reactive power as 910.8 VAR (leading).

(b) For the line, the voltage is

$$\mathbf{V}_{\text{line}} = (4 + j2)\mathbf{I} = (4.472\underline{/26.57^\circ})(10.67\underline{/22.83^\circ})$$
$$= 47.72\underline{/49.4^\circ}\text{ V rms}$$

The complex power absorbed by the line is

$$\mathbf{S}_{\text{line}} = \mathbf{V}_{\text{line}}\mathbf{I}^* = (47.72\underline{/49.4^\circ})(10.67\underline{/-22.83^\circ})$$
$$= 509.2\underline{/26.57^\circ} = 455.4 + j227.7 \text{ VA}$$

or

$$\mathbf{S}_{\text{line}} = |\mathbf{I}|^2\mathbf{Z}_{\text{line}} = (10.67)^2(4 + j2) = 455.4 + j227.7 \text{ VA}$$

That is, the real power is 455.4 W and the reactive power is 227.76 VAR (lagging).

(c) For the load, the voltage is

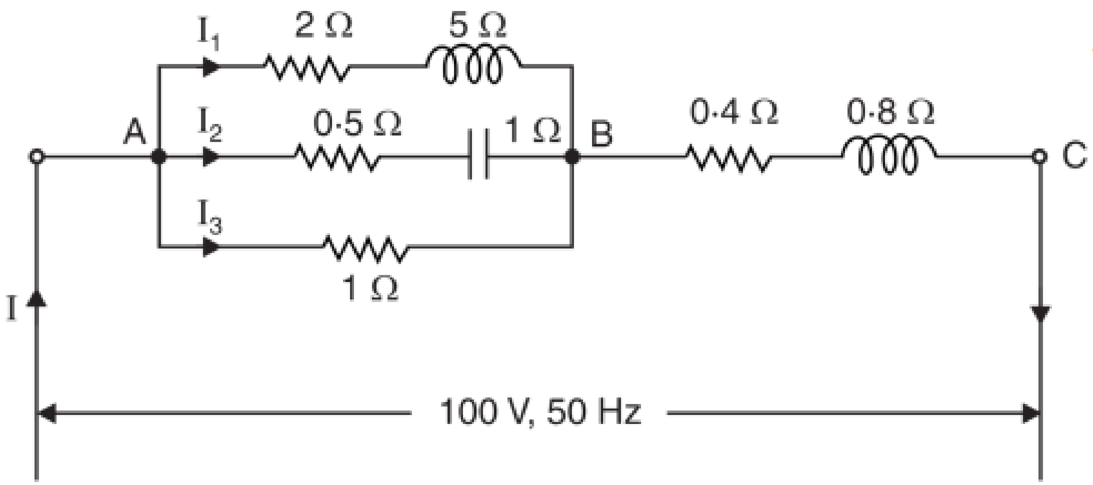
$$\mathbf{V}_L = (15 - j10)\mathbf{I} = (18.03\underline{/-33.7^\circ})(10.67\underline{/22.83^\circ})$$
$$= 192.38\underline{/-10.87^\circ} \text{ V rms}$$

The complex power absorbed by the load is

$$\mathbf{S}_L = \mathbf{V}_L\mathbf{I}^* = (192.38\underline{/-10.87^\circ})(10.67\underline{/-22.83^\circ})$$
$$= 2053\underline{/-33.7^\circ} = (1708 - j1139) \text{ VA}$$

Example For the circuit shown in figure, determine (i) The circuit impedance (ii) The source current

Solution



$$Z_1 = 1\Omega \qquad Z_2 = (0.5 - j)\Omega \qquad Z_3 = (2 + j5)\Omega$$

$$\frac{1}{Z_{AB}} = \frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{1} + \frac{1}{(0.5 - j)} + \frac{1}{(2 + j5)}$$

$$= 1 + (0.4 + j0.8) + (0.068 - j0.172) = 1.468 + j0.628$$

$$Z_{AB} = \frac{1}{1.468 + j0.628} = 0.576 - j0.246$$

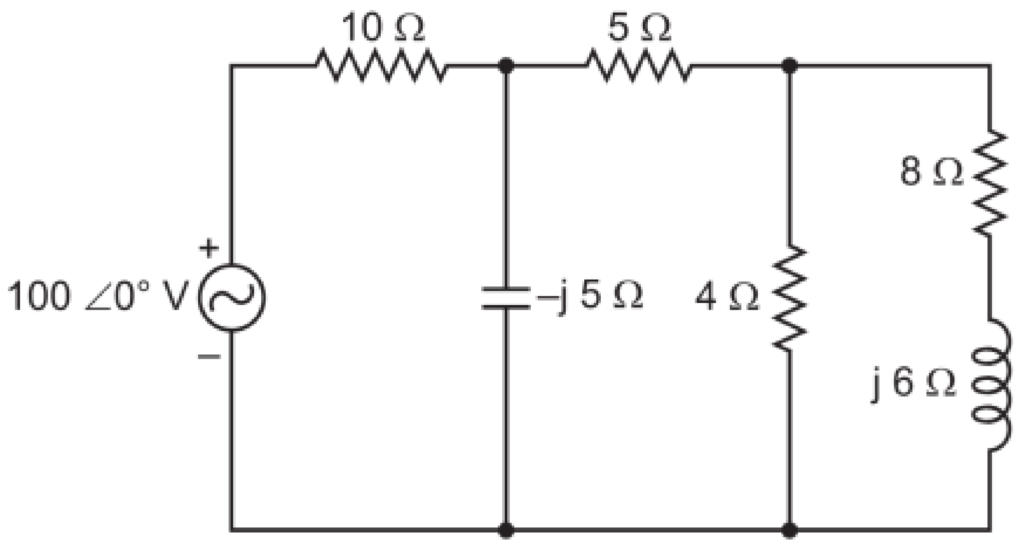
$$Z_{BC} = 0.4 + j0.8$$

$$Z_{AC} = Z_{AB} + Z_{AC} = (0.975 + j0.554)\Omega = 1.12\angle 29.5°\Omega$$

$$I = \frac{V}{Z_{AC}} = \frac{100\angle 0°}{1.12\angle 29.5°} = 89.3\angle - 29.5°$$

Example Determine the current flowing through the branch containing 4 Ω resistance

Solution



$4\Omega||(8+j6)\Omega \qquad \dfrac{4\times(8+j6)}{4+(8+j6)} = 2.93 + j0.53\Omega$

$$5 + 2.93 + j0.53 = 7.93 + j0.53\Omega$$

$$(-j5)\Omega||(7.93 + j0.53)\Omega$$

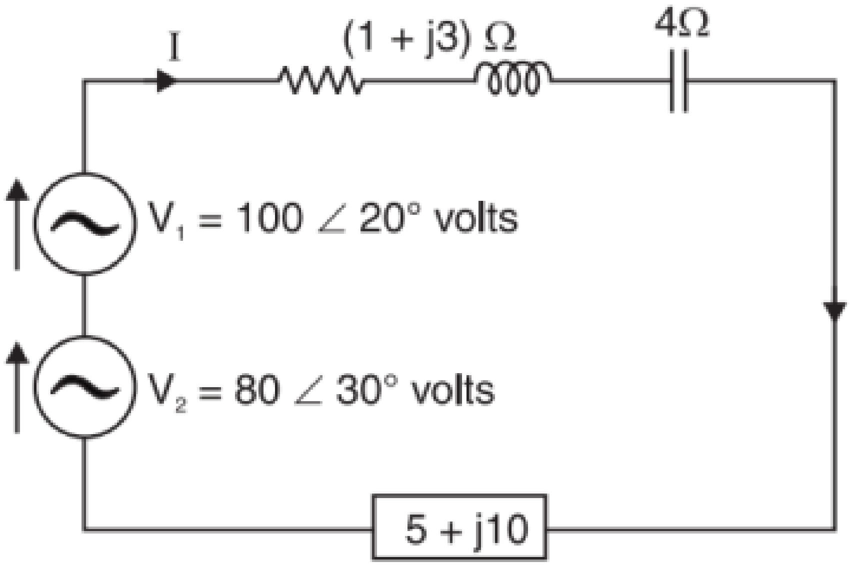
$$\frac{(-j5)\times(7.93+j0.53)}{(-j5)+(7.93+j0.53)} = 2.39 - j3.65\Omega$$

$$I = \frac{V}{Z} = \frac{100}{(12.39 - j3.65)} = (7.426 + j2.188)A = 7.742\angle 16.41°A$$

$$I_1 = (7.426 + j2.188)\frac{(-j5)}{(-j5)+(7.93+j0.53)} = (3.05 - j2.96)A$$

$$I_{4\Omega} = (3.05 - j2.96)\frac{(8+j6)}{12+j6} = 2.63 - j1.764 = 3.167\angle -33.85°$$

Example For the circuit shown, determine the (i) magnitude of current (ii) power factor of the circuit



Solution. (*i*) Resultant voltage, $\mathbf{V} = \mathbf{V_1} + \mathbf{V_2} = 100\angle 20° + 80\angle 30°$

$$= (93.97 + j\,34.2) + (69.28 + j\,40)$$

$$= (163.25 + j\,74.2)\text{volts} = 179.32\angle 24.44° \text{ volts}$$

Total impedance, $\mathbf{Z} = (1 + j\,3) + (-j\,4) + 5 + j\,10$

$$= (6 + j\,9)\,\Omega = 10.82\angle 56.31°\,\Omega$$

$\therefore$ Circuit current, $\mathbf{I} = \dfrac{\mathbf{V}}{\mathbf{Z}} = \dfrac{179.32\angle 24.44°}{10.82\angle 56.31°} = 16.57\angle -31.87° \text{ A}$

$\therefore$ $I =$ **16.57 A**

(*ii*) Phase angle, $\phi = 24.44° + 31.87° = 56.31°$

$\therefore$ Power factor $= \cos\phi = \cos 56.31° =$ ***0.554 lag***

Example

Example 13.31. *A load having impedance of (1 + j1) Ω is connected to an a.c. voltage represented as* $v = 20\sqrt{2}\ \cos(\omega t + 10°)$ *V.*

(i) Find the current in load, expressed in the form of $i = I_m \sin(\omega t + \phi) A$.

(ii) Find real power consumed by the load.

Solution. (*i*) Load impedance, $\mathbf{Z} = (1 + j\,1)\,\Omega = \sqrt{2}\angle 45°\,\Omega$

Now $\quad v = 20\sqrt{2}\ \cos(\omega t + 10°) = 20\sqrt{2}\ \sin(\omega t + 90° + 10°) = 20\sqrt{2}\ \sin(\omega t + 100°)$

Also $\quad V_m = 20\sqrt{2}$ volts ; $V(\text{r.m.s.}) = \dfrac{20\sqrt{2}}{\sqrt{2}} = 20$ volts

$$\therefore \qquad \text{Load current, } \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{20\angle 100°}{\sqrt{2}\angle 45°} = 10\sqrt{2}\angle 55°$$

Peak value of load current is given by ;

$$I_m = I \times \sqrt{2} = 10\sqrt{2} \times \sqrt{2} = 20\text{ A}$$

$$\therefore \qquad i = I_m \sin(\omega t + \phi) = \mathbf{20\ sin\ (\omega t + 55°)}$$

(*ii*) Real power consumed, $P = VI\cos\phi = 20 \times 10\sqrt{2} \times \cos 45° = \mathbf{200\ W}$

Example

Example 12.41. *A resistance R, an inductance L = 0.01 H and a capacitance C are connected in series. When an alternating voltage v = 400 sin (3000 t – 20°) is applied to the series combination, the current flowing is $10\sqrt{2}$ sin (3000t – 65°). Find the values of R and C.*

Solution.

$$\phi = 65° - 20° = 45° \text{ lag } i.e., \text{ circuit is inductive}$$

$$X_L = \omega L = 3000 \times 0.01 = 30\ \Omega$$

$$\tan 45° = X/R \quad \therefore \quad X = R$$

$$Z = V_m/I_m = 400/10\sqrt{2} = 28.3\ \Omega$$

$$Z^2 = R^2 + X^2 = R^2 + R^2 = 2R^2$$

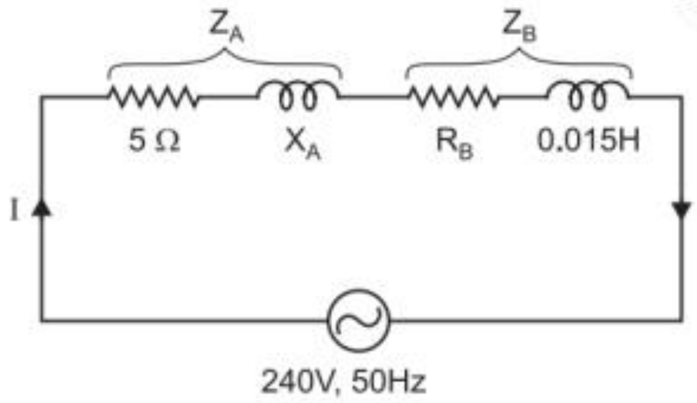
$$\therefore \quad R = Z/\sqrt{2} = 28.3/\sqrt{2} = \mathbf{20\ \Omega}$$

Now $$X = X_L - X_C \quad \therefore \quad X_C = X_L - X = 30 - 20 = 10\ \Omega$$

$$\therefore \quad C = \frac{1}{\omega X_C} = \frac{1}{3000 \times 10} = \mathbf{33.3 \times 10^{-6}\ F}$$

Example

Example 12.5. *Two coils A and B are connected in series across a 240 V, 50 Hz supply. The resistance of A is 5 Ω and the inductance of B is 0.015H. If the input from the supply is 3 kW and 2 kVAR, find the inductance of A and the resistance of B. Calculate the voltage across each coil.*



Solution. Fig. 12.9 (*i*) shows the circuit diagram while Fig. 12.9 (*ii*) shows the power traingle of the circuit.

$$kVA \text{ drawn from supply} = \sqrt{(kW)^2 + (kVAR)^2} = \sqrt{3^2 + 2^2} = 3.606 \text{ kVA}$$

$$\text{Circuit current, } I = \frac{kVA \times 10^3}{V} = \frac{3.606 \times 10^3}{240} = 15.02 \text{ A}$$

$$\text{Circuit impedance, } Z = \frac{V}{I} = \frac{240}{15.02} = 15.975\ \Omega$$

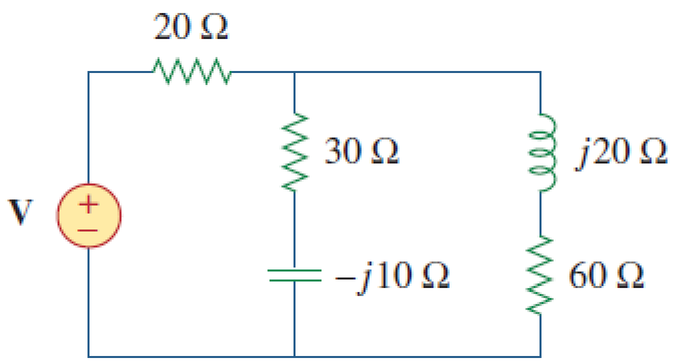
Now, $$I^2(R_A + R_B) = \text{Active power} = 3 \times 10^3$$

or $$R_A + R_B = \frac{3 \times 10^3}{(15.02)^2} = 13.3\ \Omega$$

$\therefore$ $$R_B = 13.3 - R_A = 13.3 - 5 = \mathbf{8.3\ \Omega}$$

Example 2 – Parallel AC Circuit

In the circuit in Fig. 11.25, the 60-Ω resistor absorbs an average power of 240 W. Find **V** and the complex power of each branch of the circuit. What is the overall complex power of the circuit? (Assume the current through the 60-Ω resistor has no phase shift.)



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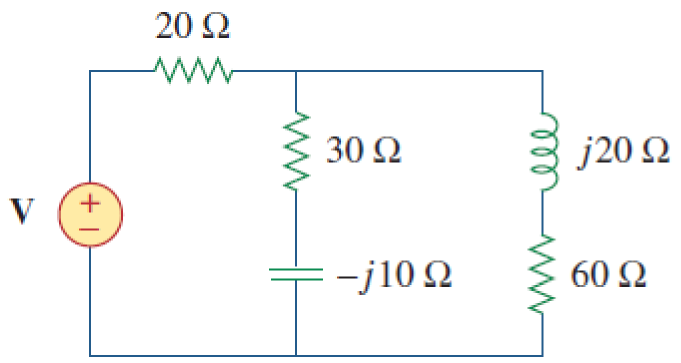
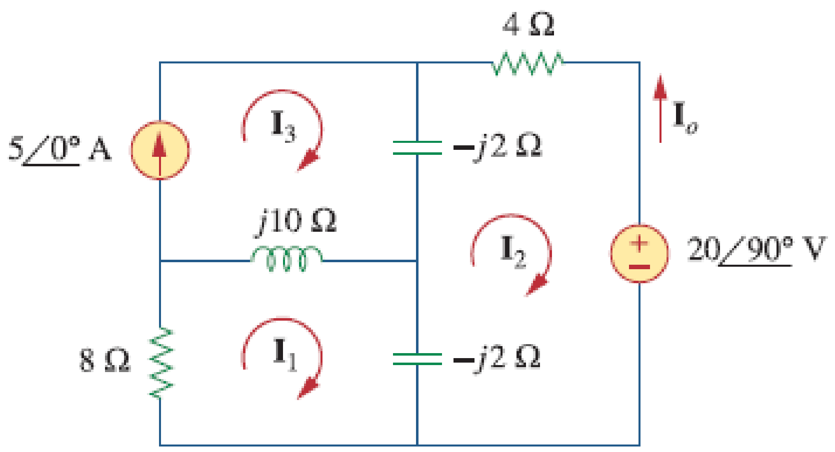


Answer: $240.7\angle 21.45^\circ$ V (rms); the 20-Ω resistor: 656 VA; the $(30 - j10)$ Ω impedance: $480 - j160$ VA; the $(60 + j20)$ Ω impedance: $240 + j80$ VA; overall: $1376 - j80$ VA.

Example 3 - MESH ANALYSIS in AC Circuits

Determine current $\mathbf{I}_o$ in the circuit of Fig. 10.7 using mesh analysis.



Applying KVL to mesh 1, we obtain

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$$

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\underline{/90^\circ} = 0$$

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50$$
$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10$$

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\underline{/-35.22^\circ}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\underline{/-35.22^\circ}}{68} = 6.12\underline{/-35.22^\circ}\text{ A}$$