Example 2.31. *Minimize the following expressions using 4-variable K–map (see Fig. 2.14).*

(a) $f_a = \Sigma m\ (2, 5, 6, 9, 12, 13)$

(b) $f_b = \Sigma m\ (0, 1, 2, 3, 8, 9, 10, 11)$

(c) $f_c = \Sigma m\ (4, 5, 6, 7, 12, 13, 14, 15)$

(d) $f_d = \Sigma m\ (2, 6, 8, 9, 10, 11, 14)$

(e) $f_e = \Sigma m\ (0, 1, 4, 5, 8, 9, 10, 11, 14, 15)$

(f) $f_f = \Sigma m\ (0, 2, 5, 7, 8, 10, 13, 15)$

(g) $f_g = \Sigma m\ (1, 3, 4, 6, 9, 11, 12, 14)$

Solution. (a) $f_a = \Sigma m\ (2, 5, 6, 9, 12, 13)$ The K-map is shown in Fig. 2.14.

$\therefore \quad f_a = \overline{A}\,\overline{B}C\overline{D} + B\overline{C} + A\overline{C}D$

AB \ CD	00	01	11	10
00				1
01	1	1		
11	1	1		
10		1		

Fig. 2.14

(b) $f_b = \Sigma m\ (0, 1, 2, 3, 8, 9, 10, 11)$

The K–map for the problem is shown in Fig. 2.15.

It requires a 4–variable K–map.

AB \ CD	00	01	11	10
00	1	1	1	1
01				
11				
10	1	1	1	1

Fig. 2.15

$f_b = \overline{B}$

(c) $f_c = \Sigma m\ (4, 5, 6, 7, 12, 13, 14, 15)$

The above problem requires a 4-variable K–map as shown in Fig. 2.16.

The above problem requires a 4-variable K–map as shown in Fig. 2.16.

AB \ CD	00	01	11	10
00				
01	1	1	1	1
11	1	1	1	1
10				

Fig. 2.16

$f_c = B$

(*d*) $f_d = \Sigma m\ (2, 6, 8, 9, 10, 11, 14)$

For this problem, we need a 4-variable K–map as shown in Fig. 2.17.

AB \ CD	00	01	11	10
00				1
01				1
11				1
10	1	1	1	1

Fig. 2.17

$f_d = C\overline{D} + A\overline{B}$

(*e*) $f_e = \Sigma m\ (0, 1, 4, 5, 8, 9, 10, 11, 14, 15)$

For the above problem, we need a 4-variable K–map as shown in Fig. 2.18 (*a*).

AB \ CD	00	01	11	10
00	1	1		
01	1	1		
11			1	1
10	1	1	1	1

Fig. 2.18 (*a*)

$f_e = \overline{A}\,\overline{C} + A\overline{B} + AC$

(*f*) $f_f = \Sigma m\ (0, 2, 5, 7, 8, 10, 13, 15)$

For the given problem, we need a 4-variable K–map as shown in Fig. 2.18 (*b*).

AB \ CD	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			1

Fig. 2.18 (*b*)

$f_f = BD + \overline{B}\,\overline{D}$

(*g*) $f_g = \Sigma m\ (1, 3, 4, 6, 9, 11, 12, 14)$

The K–map is shown in Fig. 2.18 (*c*).

AB \ CD	00	01	11	10
00		1	1	
01	1			1
11	1			1
10		1	1	

Fig. 2.18 (*c*)

$f_g = \overline{B}D + B\overline{D}$

Example 2.32. *Simplify the truth table shown in Fig. 2.19 (a) and 2.19 (b) using K–map method.*

Solution.

A	B	C	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Fig. 2.19 (*a*)

A	B	C	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Fig. 2.19 (b)

Example 2.33. *Simplify the following expressions using K–map.*

(a) $\bar{A}B + ABD + A\bar{B}C\bar{D} + BC$

(b) $BC + A\bar{C} + AB + BCD$

(c) $ABC + \bar{A}BC + \bar{A}BC + AB\bar{C} + \bar{A}\,\bar{B}\,\bar{C}$

(d) $f = A\bar{B}\bar{C}D + \bar{A}\,\bar{B}\,\bar{C}\,\bar{D} + AB\bar{C}D + \bar{A}BCD + ABD + \bar{B}C\bar{D} + \bar{A}B\bar{C}D$

(e) $f = \bar{A}\,\bar{C} + A\bar{B}\bar{C} + ABC + A\bar{B}C$

(f) $f = AB + A\bar{C} + AD + A\bar{B}C + ABC$

(g) *Find the complement of the function* $(B\bar{C}+\bar{A}D)\,(A\bar{B}+C\bar{D})$ *and reduce it using K–map.*

Solution. (*a*)

$$
\begin{aligned}
f &= \bar{A}B + ABD + A\bar{B}C\bar{D} + BC \\
&= \bar{A}B\,(C+\bar{C})\,(D+\bar{D}) + ABD\,(C+\bar{C}) + A\bar{B}C\bar{D} \\
&\quad + (A+\bar{A})\,(D+\bar{D})\,BC \\
&= (\bar{A}BC+\bar{A}B\bar{C})\,(D+\bar{D}) + ABCD + AB\bar{C}D + A\bar{B}C\bar{D} \\
&\quad + (ABC+\bar{A}BC)\,(D+\bar{D}) \\
&= \bar{A}BCD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + \bar{A}B\bar{C}\,\bar{D} + A\bar{B}C\bar{D} + ABCD + \bar{A}BC\bar{D} \\
&\qquad + ABCD + AB\bar{C}D \\
&= \bar{A}BCD + \bar{A}B\bar{C}D + ABCD + AB\bar{C}D + \bar{A}BC\bar{D} + \bar{A}B\bar{C}D \\
&= \Sigma m\ (4,\ 5,\ 6,\ 7,\ 15,\ 13) \\
&= BD + \bar{A}BC
\end{aligned}
$$

For simplification using K-map (see Fig. 2.20).

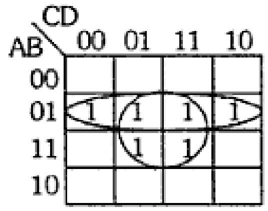

Fig. 2.20

$$f = \overline{A}B + BD$$

(*b*) $f = BC + A\overline{C} + AB + BCD$

$= (A+\overline{A})\,BC\,(D+\overline{D}) + A\,(B+\overline{B})\,\overline{C}\,(D+\overline{D})$

$+ AB\,(C+\overline{C})\,(D+\overline{D}) + (A+\overline{A})\,BCD$

$= ABCD + \overline{A}BCD + ABC\overline{D} + \overline{A}BC\overline{D} + AB\overline{C}D + AB\overline{C}\,\overline{D} + A\overline{B}\,\overline{C}D$

$+ A\overline{B}\,\overline{C}\,\overline{D} + ABCD + ABC\overline{D} + AB\overline{C}D + AB\overline{C}\,\overline{D} + ABCD + \overline{A}BCD$

$= ABCD + \overline{A}BCD + ABC\overline{D} + \overline{A}BC\overline{D} + AB\overline{C}D$

$+ AB\overline{C}\,\overline{D} + A\overline{B}\,\overline{C}D + A\overline{B}\,\overline{C}\,\overline{D}$

$= \Sigma m\ (15, 7, 14, 6, 13, 12, 9, 8)$

$= \Sigma m\ (6, 7, 8, 9, 12, 13, 14, 15)$

$= A\overline{C} + BC$

For simplification using K-map see Fig. 2.21.

For simplification using K-map see Fig. 2.21.

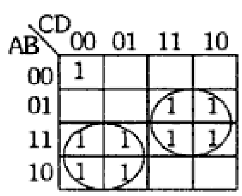

Fig. 2.21

(*c*) $f = ABC + \overline{A}\,\overline{B}C + \overline{A}BC + AB\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

$f = \Sigma\ (7, 1, 3, 6, 0)$

For simplification see Fig. 2.22.

$= \overline{A}\,\overline{B} + BC + AB$

$= B\,(A+C) + \overline{A}\,\overline{B}$

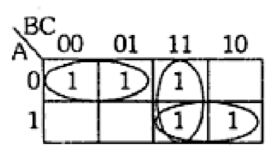

Fig. 2.22

(*d*) $f = A\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D + \overline{A}BCD + ABD\,(C+\overline{C})$

$(A+\overline{A})\,\overline{B}C\overline{D} + \overline{A}B\overline{C}D$

$$= A\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D + \overline{A}BCD + ABCD + AB\overline{C}D$$

$$+ A\overline{B}C\overline{D} + \overline{A}\,\overline{B}C\overline{D} + \overline{A}B\overline{C}D$$

$$= \Sigma m\ (9, 0, 13, 7, 15, 10, 2, 5)$$

$$= \Sigma m\ (0, 2, 5, 7, 9, 10, 13, 15)$$

For simplification (see Fig. 2.23)

AB \ CD	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10				1

Fig. 2.23

(e)

$$f = \overline{A}\,\overline{C} + A\overline{B}\,\overline{C} + ABC + A\overline{B}C$$

$$= \overline{A}\,\overline{C}\,(B+\overline{B}) + A\overline{B}\,\overline{C} + ABC + A\overline{B}C$$

$$= \overline{A}B\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + ABC + A\overline{B}C$$

$$= \Sigma f\ (2, 0, 4, 7, 5)$$

$$= \Sigma f\ (0, 2, 4, 5, 7) \text{ (see Fig. 2.24)}$$

$$= \overline{B}\,\overline{C} + AC + \overline{A}\,\overline{C}$$

$$= AC + \overline{C}\,(A+\overline{B})$$

A \ BC	00	01	11	10
0	1			1
1	1	1	1	

Fig. 2.24

(*f*)

$$f = AB + A\overline{C} + AD + A\overline{B}C + ABC$$

$$= AB\,(C+\overline{C})\,(D+\overline{D}) + A\,(B+\overline{B})\,\overline{C}\,(D+\overline{D}) + A\,(B+\overline{B})$$

$$(C+\overline{C})\,D + A\overline{B}C\,(D+\overline{D}) + ABC\,(D+\overline{D})$$

$$= ABCD + ABC\overline{D} + AB\overline{C}D + AB\overline{C}\,\overline{D} + AB\overline{C}D + A\overline{B}\,\overline{C}D$$

$$+ A\overline{B}\,\overline{C}D + A\overline{B}\,\overline{C}\,\overline{D} + ABCD + A\overline{B}CD + AB\overline{C}D + A\overline{B}\,\overline{C}D$$

$$+ A\overline{B}CD + A\overline{B}C\overline{D} + ABCD + ABC\overline{D}$$

$$= ABCD + ABC\overline{D} + AB\overline{C}D + AB\overline{C}\,\overline{D}$$

$$+ A\overline{B}\,\overline{C}D + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}CD + A\overline{B}C\overline{D}$$

$$= \Sigma m\ (15, 14, 13, 12, 9, 8, 11, 10)$$

$$= \Sigma\ 8, 9, 10, 11, 12, 13, 14, 15$$

AB \ CD	00	01	11	10
00				
01				
11	1	1	1	1
10	1	1	1	1

Fig. 2.25

For simplification using K–map see Fig. 2.25

$$f = A$$

(g) $$f = (B\bar{C} + \bar{A}D)(A\bar{B} + C\bar{D})$$
$$= B\bar{C}A\bar{B} + \bar{A}DA\bar{B} + B\bar{C}C\bar{D} + \bar{A}D\bar{C}\bar{D} = 0$$
$\therefore$ $$\bar{f} = (\bar{B} + C)(A + \bar{D}) + (\bar{A} + B)(\bar{C} + D)$$
$$= 1$$

Example 2.34. *Plot the K–map for EX– OR and EX– NOR functions of 2, 3 and 4 variables.*

Solution. *For 2–variables*

f = EX–OR of 2 variables = $A \oplus B = \bar{A}B + A\bar{B}$ the K–map is shown in Fig. 2.26 (*a*).

EX–NOR of 2 variables = $A \odot B = \bar{A}\bar{B} + AB$ the K–map is shown in Fig. 2.26 (*b*).

A \ B	0	1
0		1
1	1	

Fig. 2.26 (*a*)

A \ B	0	1
0	1	
1		1

Fig. 2.26 (*b*)

For 3–variables

EX–OR of 3 variables = $A \oplus B \oplus C = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$ See Fig. 2.27 (*a*)

EX–NOR 3 variables = $A \odot B \odot C = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C}$ See Fig. 2.27 (*b*)

A \ BC	00	01	11	10
0		1		1
1	1		1	

Fig. 2.27 (*a*)

A \ BC	00	01	11	10
0	1		1	
1		1		1

Fig. 2.27 (*b*)

(g) $$f = (B\bar{C} + \bar{A}D)(A\bar{B} + C\bar{D})$$
$$= B\bar{C}A\bar{B} + \bar{A}DA\bar{B} + B\bar{C}C\bar{D} + \bar{A}D\bar{C}\bar{D} = 0$$
$\therefore$ $$\bar{f} = (\bar{B} + C)(A + \bar{D}) + (\bar{A} + B)(\bar{C} + D)$$
$$= 1$$

Example 2.35. *Minimize the following expression in POS form*

(*a*) $f = \Pi M\ (0, 1, 3, 4, 7)$

(*b*) $f = \Pi M\ (0, 4, 6, 7, 8, 12, 13, 14, 15)$

Solution. (*a*) Step 1. Fig. 2.29 shows the 3 variable K–map and it is plotted according to the given expression.

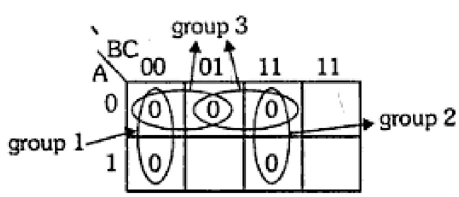

Fig. 2.29

Step 2. There are no isolated 0's.

Step 3. The cell 4 is adjacent to cell 0, which give a group 1. Cell 3 and cell 7 are adjacents and are combined and give a group 2.

Step 4. There are no quads or octets.

Step 5. The 0 in the cell 1 can be combined with 0 in the cell 3 or cell 0 to form a pair. This pair is referred to as group 3.

Step 6. All 0's are covered.

f in SOP = SOP of group 1 + group 2 + group 3

$$= \overline{B}\,\overline{C} + \overline{A}\,\overline{B} + BC$$

$$f \text{ in POS} = \overline{\overline{B}\,\overline{C} + \overline{A}\,\overline{B} + BC}$$

$$= (\overline{B}+\overline{C})\,(B+C)\,(A+B)$$

(*b*) See Fig. 2.30 $\quad f = \overline{C}\,\overline{D} + AB + BC$

$$f \text{ in POS} = \overline{\overline{C}\,\overline{D} + AB + BC}$$

$$= (C+D)\left(\overline{A}+\overline{B}\right)\left(\overline{B}+\overline{C}\right)$$

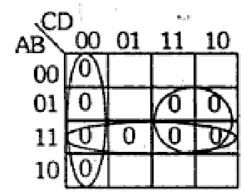

Fig. 2.30

Example 2.38. *Simplify the following Boolean functions using K–map technique.*

(a) $f\,(A, B, C, D, E) = \Sigma\, m\,(0, 2, 3, 4, 6, 7, 9, 11, 16, 18, 19, 20, 22, 23, 25, 27)$

(b) $f\,(A, B, C, D, E) = \Sigma\, m\,(0, 4, 7, 8, 9, 10, 11, 16, 24, 25, 26, 27, 29, 31)$

(c) $f\,(A, B, C, D, E) = \Sigma\, m\,(0, 4, 8, 12, 16, 18, 20, 22) + \Sigma d\,(24, 26, 28, 30, 31)$

Solution. (*a*)

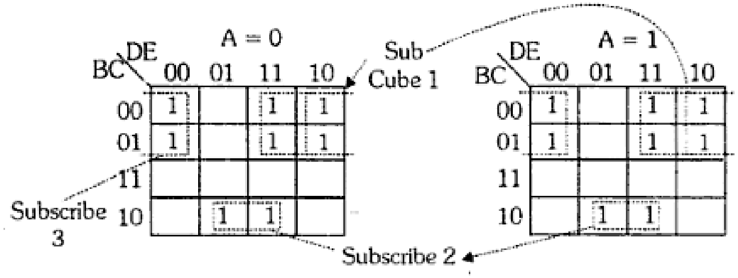

Fig. 2.38

Totally, we got 3 subcubes (see Fig. 2.38)

Subcube 1 is an octet, the resultant is = $\overline{B}D$

Subcube 2 is a quad, the resultant is = $B\overline{C}E$

Subcube 3 is a quad, the resultant is = $\overline{B}\,\overline{D}\,\overline{E}$

$\therefore$ $\quad f = \overline{B}D + B\overline{C}E + \overline{B}\,\overline{D}\,\overline{E}$

(*b*)

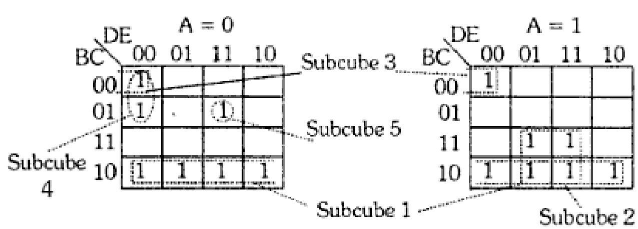

Fig. 2.39

Totally, we have 5–subcubes after grouping the adjacents (see Fig. 2.39)

Subcube 1 is an octet which gives = $B\overline{C}$

Subcube 2 is a quad which gives = ABE

Subcube 3 is a pair which gives = $\overline{B}\,\overline{C}\,\overline{D}\,\overline{E}$

Subcube 4 is a pair which gives = $\overline{A}\,\overline{B}\,\overline{D}\,\overline{E}$

Subcube 5 is single which gives = $\overline{A}\,\overline{B}CDE$

$$f = B\overline{C} + ABE + \overline{B}\,\overline{C}\,\overline{D}\,\overline{E} + \overline{A}\,\overline{B}\,\overline{D}\,\overline{E} + \overline{A}\,\overline{B}CDE$$

(c)

A = 0

BC \ DE	00	01	11	10
00	1			
01	1			
11	1			
10	1			

Sub cube 1

A = 1

BC \ DE	00	01	11	10
00	1			1
01	1			1
11	×		×	×
10	×			×

Sub cube 2

Fig. 2.40

Here, the don't care cells 24, 26, 28 and 30 are treated as 1's to form a subcube 1 of octet size, which results (see Fig. 2.40)

$$f = \overline{D}\overline{E}$$

The don't care cells 26 and 30 are taken as 1 to form a subcube 2 of octet size, which results = $A\overline{E}$. The remaining don't care cell 31 is treated as zero.

$$\therefore \quad f = \overline{D}\overline{E} + A\overline{E} = \overline{E}(A + \overline{D})$$

Example 2.42. *Use a K-map to convert the $F = (A + \overline{B} + \overline{C})(\overline{A} + B + \overline{C})$ into its SOP form.*

Solution. The K-map with the given maxterms is shown in Fig 2.48.

A \ BC	00	01	11	10
0	1	1	0	1
1	1	0	1	1

Fig. 2.48

Looping of 1 cells gives 3 subcubes, the resultant in SOP form is

$$f = \overline{C} + \overline{A}\overline{B} + AB$$
$$= \overline{C} + \overline{A}\overline{B} + AB$$

Example 2.43. *Convert the given Boolean functions into the SOP form*

(a) $f = (A + B)(\overline{B} + C)$

(b) $f = (A + B)(A + \overline{C})(\overline{A} + \overline{B})(\overline{A} + C)$

Solution. (a) The K-map for the given Boolean function is shown in Fig. 2.49

The looping of 1 cells gives

$$f = A\overline{B} + BC$$

A \ BC	00	01	11	10
0	0	0	1	0
1	1	1	1	0

Fig. 2.49

(b) The K-map for the given Boolean function is shown in Fig. 2.50

From the 1 cells,

$$f = \overline{A}B\overline{C} + A\overline{B}C$$

A \ BC	00	01	11	10
0	0	0	0	1
1	0	1	0	0

Fig. 2.50