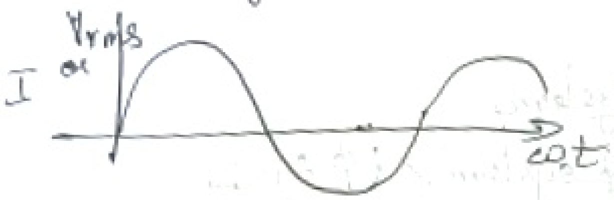
3 Phase Circuits

A system which generates single alternating voltage & current is termed as 'single phase system (1ϕ). It utilizes only one winding

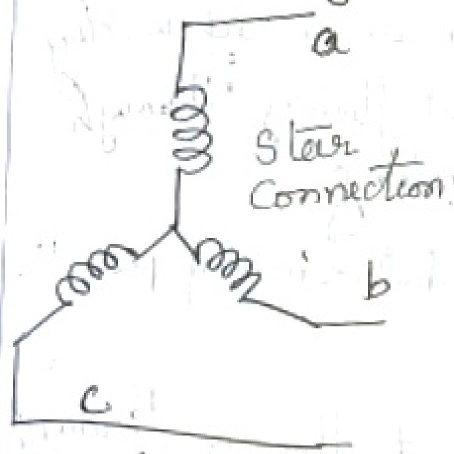


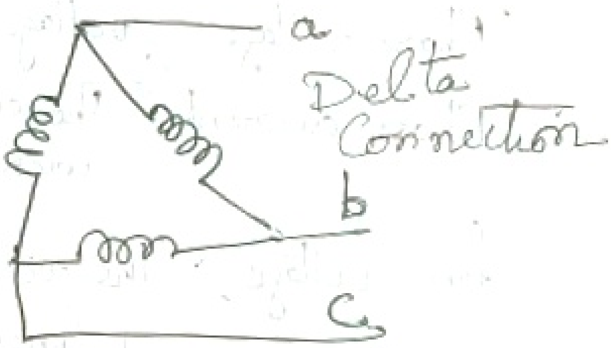
Phasor diagram

$V = V_m \sin \omega t$

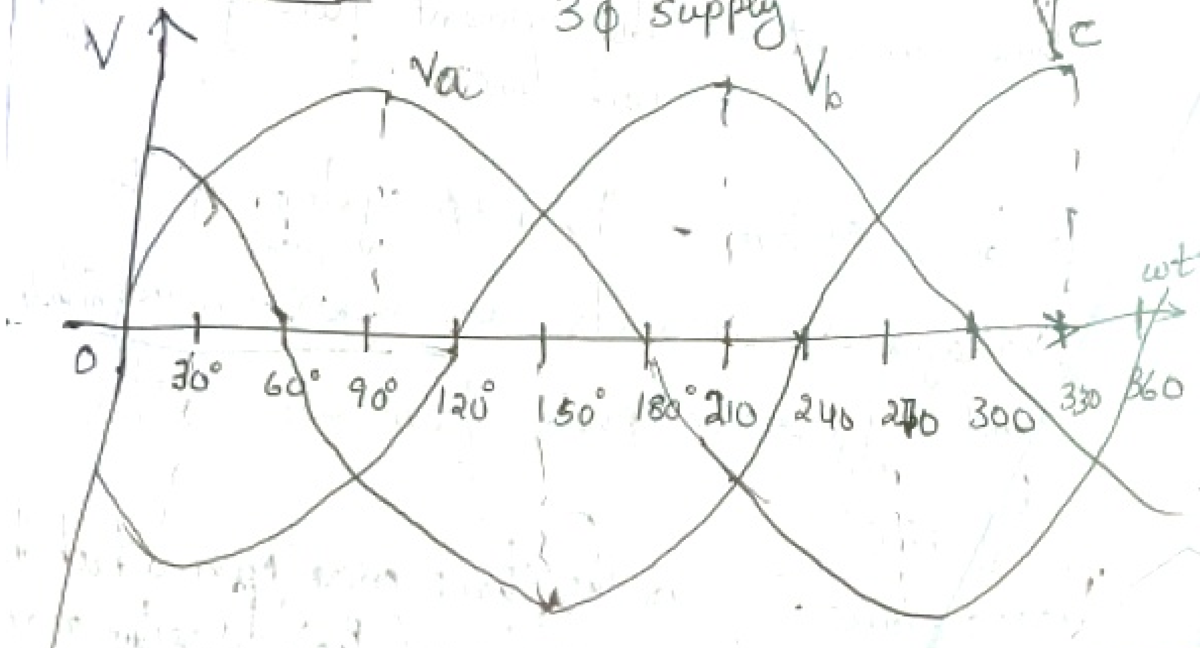
A three phase system (3ϕ) consists of three phases ~~which~~ where seperate but identical windings are used. These windings are displaced electrically with an 120° phase shift.

These windings are shown below

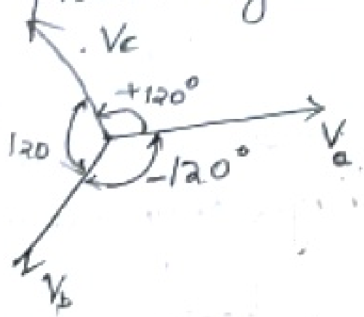




3ϕ supply



Phasor diagram



3ϕ

Advantages of 3ϕ system

* The output of the 3ϕ system > 1ϕ system
* Transmission & distribution costs are low

Disadvantages

* 6 conductors required

Basic Terminology

Phase voltage - voltage induced in each winding

Phase current - The current flowing through each winding

line voltage - The voltage between any pair of lines

line current - The current flowing through each line

Balanced system - In a 3ϕ system if the voltages & currents are equal in magnitude and differs in phase from one another by $120°$.

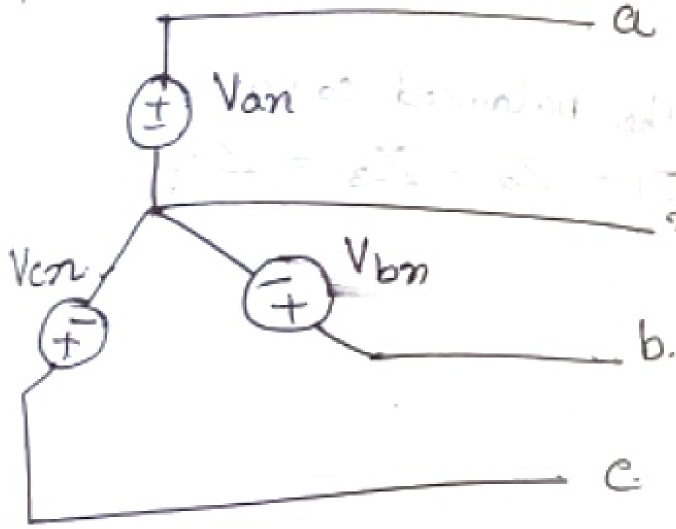
Phase sequence - It is the time order in which the voltages pass through their respective max values

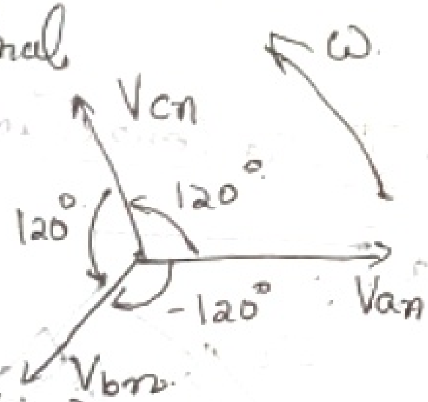
Types of connections of 3ϕ: $\angle 120°$ $\quad V_{cn} = V_p \angle +120°$

Star connection
Delta connection

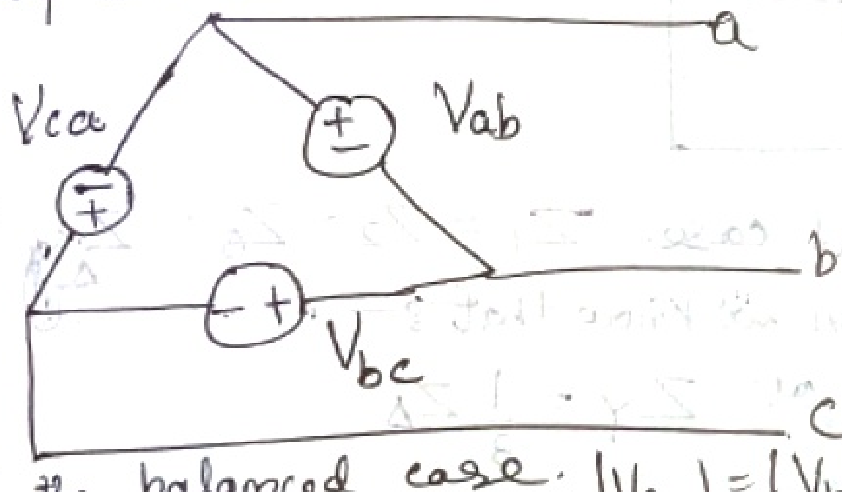
Balanced → (only case considered here in class)
Unbalanced

3ϕ Source (star connected)





3ϕ source (Delta Connected)

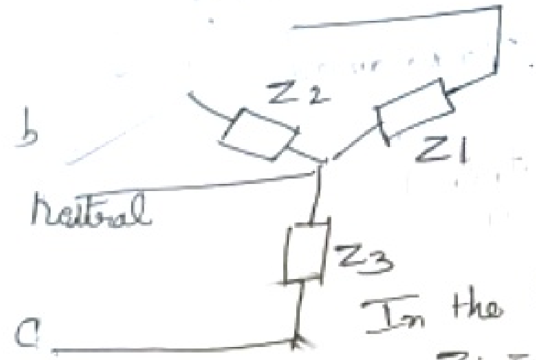


In the balanced case. $|V_{an}| = |V_{bn}| = |V_{cn}| = V_P$

$$V_{an} = V_p \angle 0°$$
$$V_{bn} = V_p \angle -120°$$
$$V_{cn} = V_p \angle -240° = V_p \angle +120°$$

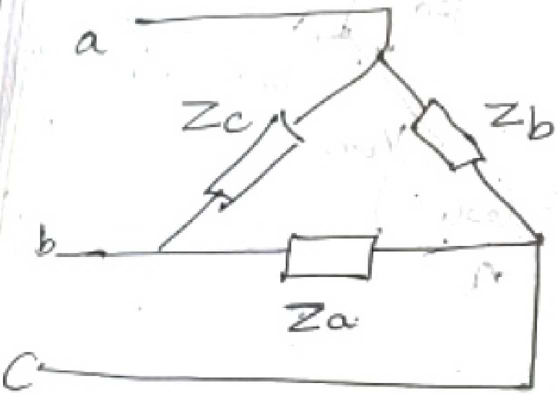
Van leads Vbn leads Vcn

Phasor diagram



In the balanced case:
$Z_1 = Z_2 = Z_3 = Z_Y$

Delta Connection



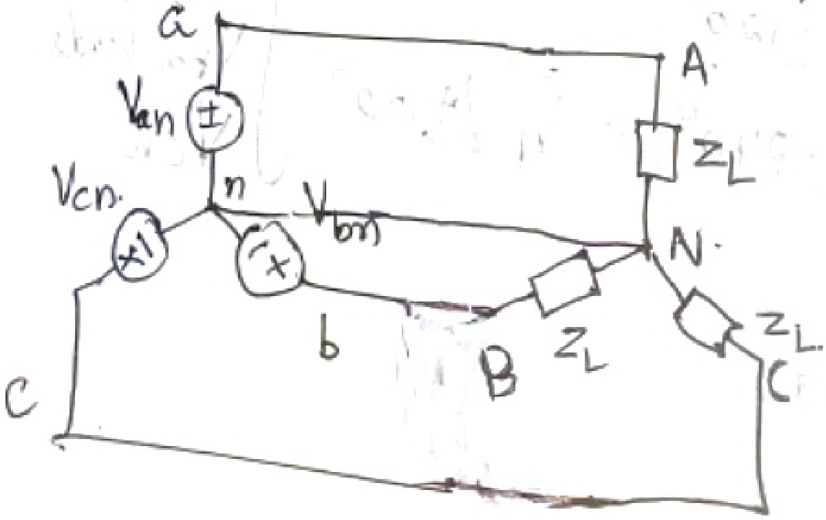
In the balanced case. $Z_1 = Z_2 = Z_3 = Z_\Delta$

Also, by conversion we know that :—

$Z_\Delta = 3Z_Y$ or $Z_Y = \frac{1}{3} Z_\Delta$

Star Connection

Consider a balanced 4 wire Y Y system



(mention Z_S & Z_L are small)

Phase Voltages (V_p)

$V_{an} = V_p \angle 0^\circ$. $V_{bn} = V_p \angle -120^\circ$ $V_{cn} = V_p \angle +120^\circ$

Line Voltages (V_L)

$$V_{ab} = V_{an} + V_{nb} = V_{an} - V_{bn} = V_p \angle 0^\circ - V_p \angle -120^\circ$$

$$= V_p \left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}\, V_p \angle 30^\circ$$

Similarly, $V_{bc} = V_{bn} - V_{cn} = \sqrt{3}\, V_p \angle -90^\circ$

V $V_{ca} = V_{cn} - V_{an} = \sqrt{3}\, V_p \angle -210^\circ$

$$\boxed{|V_L| = \sqrt{3}\, V_p}$$

Applying KVL to each phase, we obtain line currents

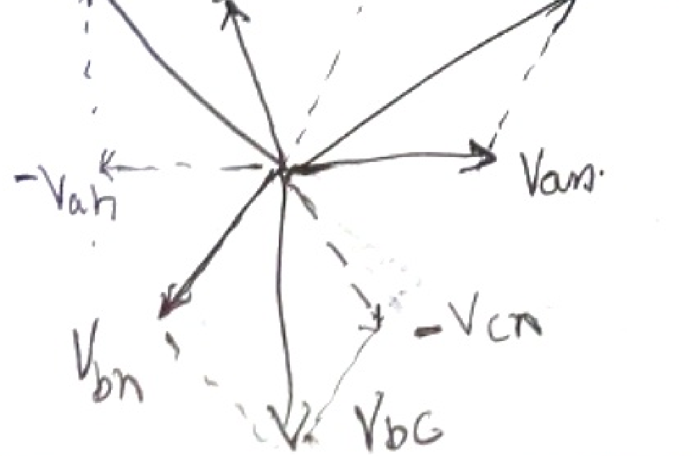
$$\boxed{I_a = \frac{V_{an} \angle 0^\circ}{Z_Y}} \quad I_b = \frac{V_{bn}}{Z_Y} = \frac{V_{an} \angle -120^\circ}{Z_Y} = I_a \angle -120^\circ$$

$$I_c = \frac{V_{cn}}{Z_Y} = \frac{V_{an} \angle +120^\circ}{Z_Y} = I_a \angle +120^\circ$$

$$I_n = I_a + I_b + I_c = I_a \left(1 + 1\angle -120^\circ + 1\angle +120^\circ\right)$$

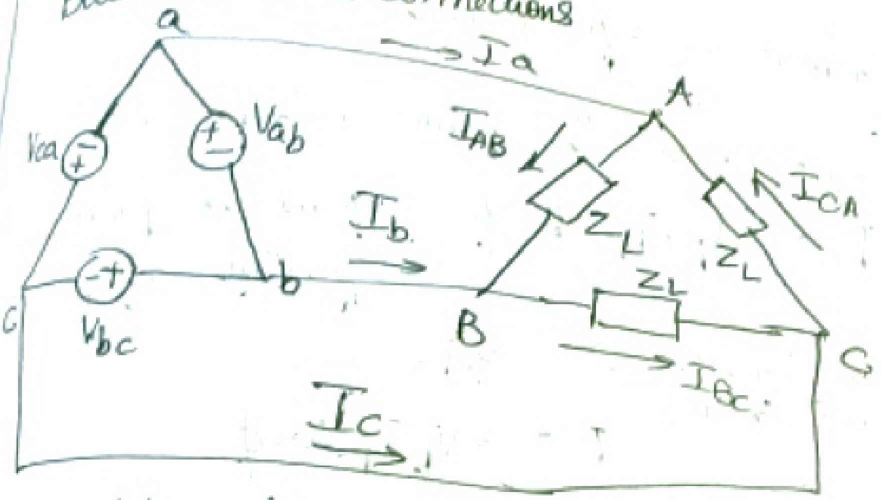
$$\boxed{I_n = I_a(0) = 0}$$

Line current is the same as phase current, and presence of neutral wire is not mandatory



Phasor diagram of Voltages (line & phase).

Balanced Δ-Δ connections



$Z_L = Z_\Delta$ (delta balanced load).

$V_{ab} = V_p\angle 0^\circ$; $V_{bc} = V_p\angle -120^\circ$ $V_{ca} = V_p\angle +120^\circ$

The line voltages are the same as phase voltages

Phase currents are

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{V_{ab}}{Z_\Delta} \qquad I_{BC} = \frac{V_{BC}}{Z_\Delta} = \frac{V_{bc}}{Z_\Delta}$$

$$I_{CA} = \frac{V_{CA}}{Z_\Delta} = \frac{V_{ca}}{Z_\Delta}$$

KCL at node A, B, C respectively gives us

$$I_a = +I_{AB} - I_{CA} = I_{AB}(1 - 1\angle +120) = \sqrt{3}\, I_{AB}\angle -30$$

$$I_b = I_{BC} - I_{AB} = I_{AB}(1\angle -120 - 1) = \sqrt{3}\, I_{AB}\angle -150$$

$$I_c = I_{CA} - I_{BC} = I_{AB}(1\angle +120 - 1\angle -120) = \sqrt{3}\, I_{AB}\angle 90^\circ$$

So $I_a = I_b = I_c = I_L$. (line current)

$$I_L = \sqrt{3}\, I_P$$