

Continuity at a point :

Definition : A function f is said to be Continuous at a point c , $a < c < b$, if

$$\lim_{x \rightarrow c} f(x) = f(c)$$

A function f is said to be Continuous from the left at c if $\lim_{x \rightarrow c^-} f(x) = f(c)$.

Also f is Continuous from the right at c if $\lim_{x \rightarrow c^+} f(x) = f(c)$. Clearly a function is Continuous at c if and only if it is Continuous from the left as well as from the right.

Continuity at an end point :

A function f defined on a closed interval $[a, b]$ is said to be Continuous at the end point a if it is Continuous from the right at a , that is,

$$\lim_{x \rightarrow a^+} f(x) = f(a).$$

Also the function is Continuous at the end point b of $[a, b]$ if

$$\lim_{x \rightarrow b^-} f(x) = f(b)$$

It is important to note that a function is Continuous at a point c if

(i) f is well-defined at $x=c$.

i.e. $f(c)$ exists.

(ii) $\lim_{x \rightarrow c} f(x)$ exists and $\lim_{x \rightarrow c} f(x) = f(c)$

Continuity in an interval.

A function f is said to be Continuous in an interval $[a, b]$ if it is Continuous at each and every point of the interval.

Discontinuous functions:

A function f is said to be discontinuous at a point c of its domain if it is not Continuous at c . The point c is then called a point of discontinuity of the function.

Theorem: If f, g be continuous functions at a point c then the functions $f+g, f-g, fg$ are also continuous at c and if $g(c) \neq 0$ then f/g is also continuous at c .

① Every constant function is continuous.

② The function $f(x) = x^n, x \in \mathbb{R}$ is continuous.

③ The function $f(x) = kx^n$ is continuous where $k \in \mathbb{R}$ and $k \neq 0$.

④ Every polynomial function of degree n is continuous.

⑤ Every rational function of the form $p(x)/q(x)$ where $p(x)$ and $q(x)$ are polynomials, is continuous ($q(x) \neq 0$).

⑥ The exponential function is continuous at all points of \mathbb{R} .
In particular the exponential function $f(x) = e^x$ is continuous.

⑦ The function $f(x) = \log x$, $x > 0$ is continuous at all points of \mathbb{R}^+ , where \mathbb{R}^+ is the set of positive real numbers.

⑧ The sine function $f(x) = \sin x$ is continuous at all points of \mathbb{R} .

⑨ The cosine function $f(x) = \cos x$ is continuous at all points of \mathbb{R} .