Reduction formula

$$\int_0^{\pi/2} \sin^m\theta \, d\theta = \int_0^{\pi/2} \cos^m\theta \, d\theta = \begin{cases} \dfrac{m-1}{m}, \dfrac{m-3}{m-2}, \dfrac{m-5}{m-4}, \dots \dfrac{6}{7} \dfrac{4}{5} \dfrac{2}{3} 1 & (m \text{ is odd}) \\[2ex] \dfrac{m-1}{m} \cdot \dfrac{m-3}{m-2}, \dfrac{m-5}{m-4} \cdot \dots \dfrac{5}{6} \dfrac{3}{4} \dfrac{1}{2} \dfrac{\pi}{2} & m \text{ is even} \end{cases}$$

If n is even & m is even

(1) $$\int_0^{\pi/2} \sin^m x \cos^n x \, dx = \frac{n-1}{m+n} \frac{n-3}{m+n-2} \frac{n-5}{m+n-4} \cdots \frac{1}{m+2} \frac{m-1}{m} \frac{m-3}{m-2} \frac{m-5}{m-4} \cdots \frac{1}{2} \frac{\pi}{2}$$

(2) If n is odd & m is any positive integer (even or odd)

then

$$\int_0^{\pi/2} \sin^m x \cos^n x \, dx = \frac{n-1}{m+n} \frac{n-3}{m+n-2} \frac{n-5}{m+n-4} \cdots \frac{2}{m+3} \frac{1}{m+1}$$

Note:

If one of m & n is odd then it is convenient to get the power of $\cos x$ as odd. For instance if m is odd & n is even then

$\int_0^{\pi/2} \sin^m x \cos^n x \, dx$

$= \int_0^{\pi/2} \sin^n x \cos^m x \, dx$

$= \frac{m-1}{n+m} \frac{(m-3)}{(n+m-2)} \frac{m-5}{n+m-4} \cdots \frac{2}{n+3} \frac{1}{n+1}$