Quantum Mechanics cont’d…

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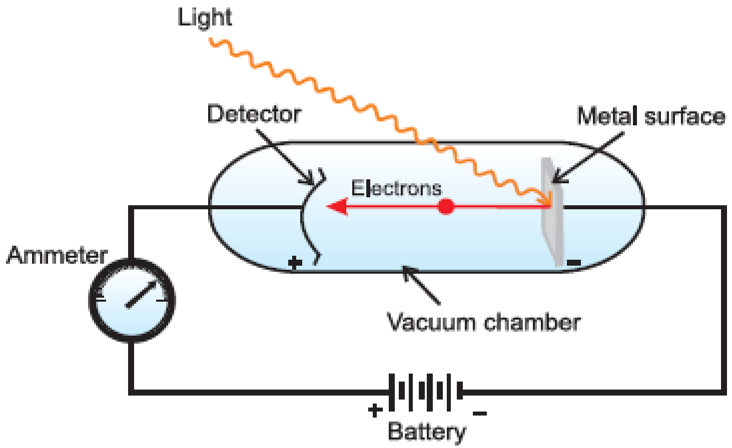
Photoelectric Effect

- Einstein explained the photoelectric effect using the basis of quantum ideas.
- Assumed EM radiations travels through space in discrete quanta called photons as during the emission and absorption processes.
- The energy of a photon of frequency ν is $h\nu$.

Using Conservation of energy

$$\mathrm{h}\nu = \underbrace{\boxed{h\nu_0}}_{\text{Work Function}} + \frac{1}{2}mv^2$$

ν_0 is called the **threshold frequency**

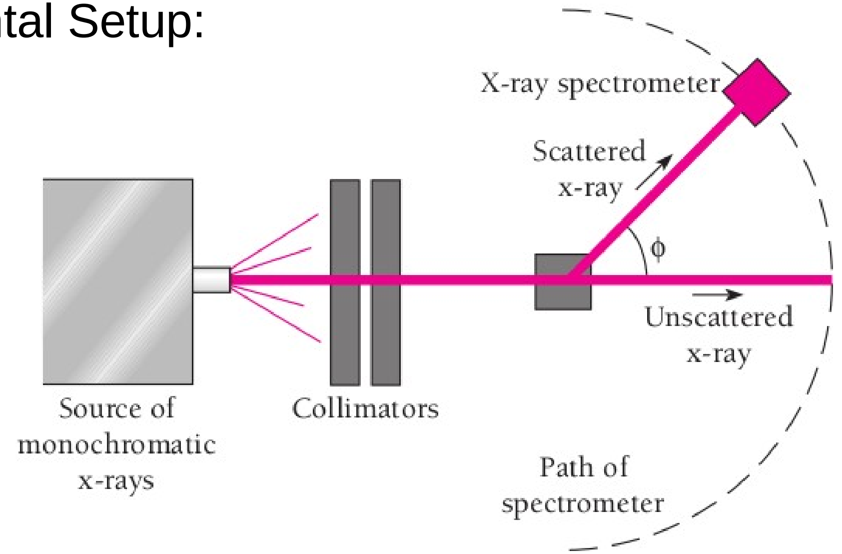


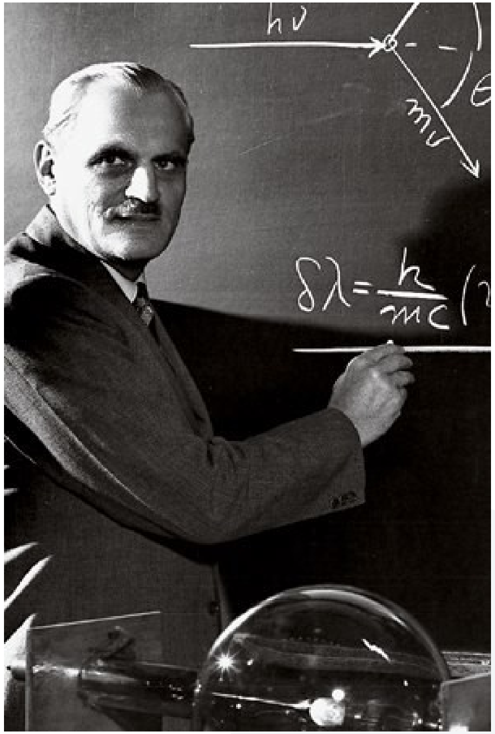
Einstein was awarded the **1921 Nobel Prize in Physics** for "his discovery of the law of the **photoelectric effect**".

Compton Effect

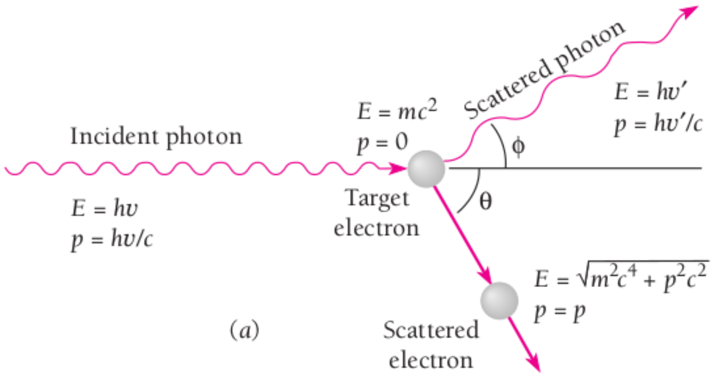
The aim of the experiment conducted by Arthur Holly Compton in 1923 was to confirm the quantum theory of light.

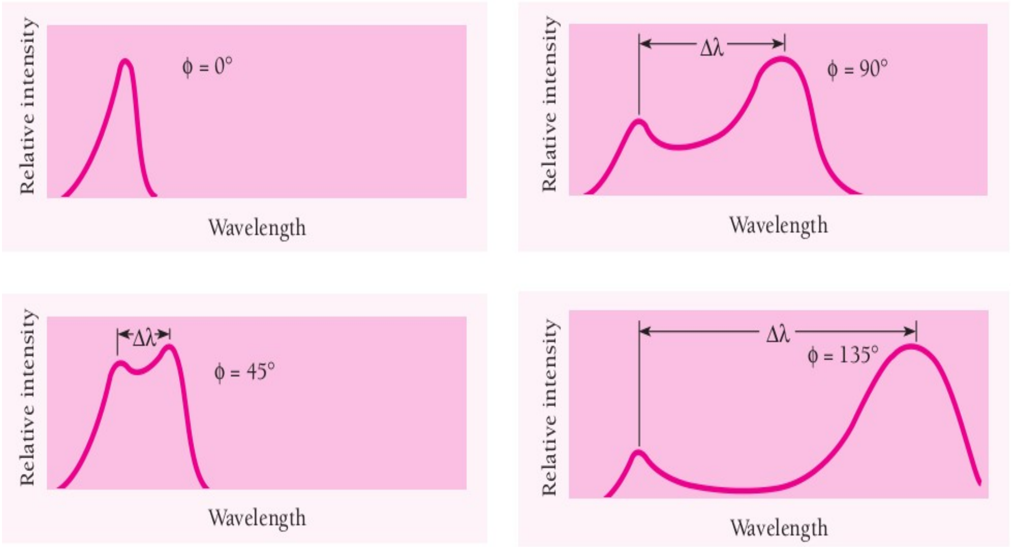
Experimental Setup:





Nobel Prize for Physics in 1927





Compton effect

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

Compton wavelength

$$\lambda_C = \frac{h}{mc}$$

$$\lambda_c = 0.0243 \mathring{A}$$

For electron

$$\boxed{\lambda' - \lambda = \lambda_C(1 - \cos\phi)}$$

Compton Effect in terms of Compton wavelength

$$Case I : \phi = 0°, \lambda' = \lambda$$

$$Case II : \phi = 90°, \lambda' = \lambda + \lambda_c$$

$$Case III : \phi = 180°, \lambda' = \lambda + 2\lambda_c$$

Compton Effect: Conclusions

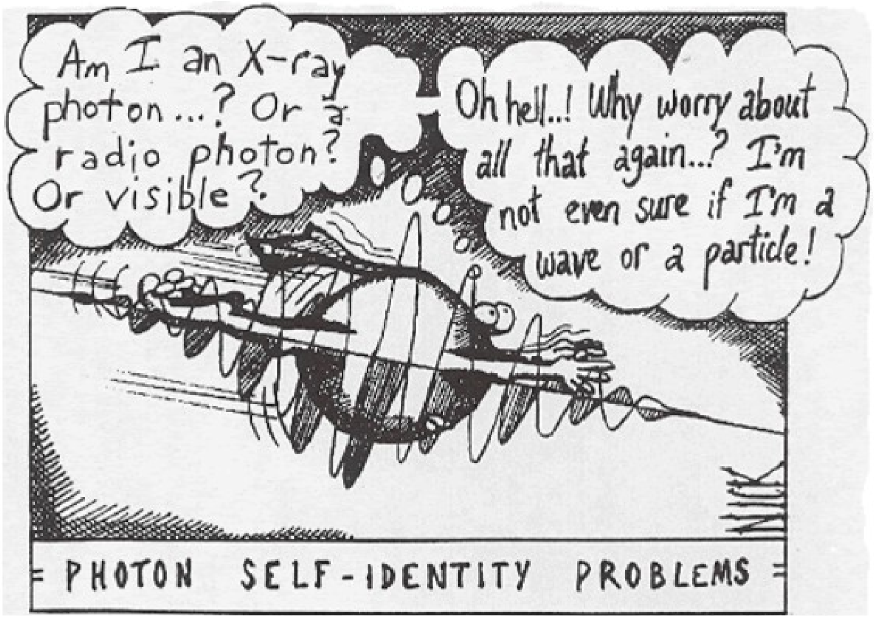
What wave theory predicts: (Classical view)

- The wave theory predicts that no wavelength change should take place.
- The Incoming EM wave causes the electron to oscillate with the same frequency as the wave.
- Therefore, the oscillating electron should reemit the EM waves with the same frequency (Thomson scattering)

Confirmation of quantum theory:

- Incoming photon collides with the electron and transfers some of the energy to the electron.
- The scattered photons now have less energy than before and so decrease in frequency by $\Delta \nu$ and an increase wavelength by $\Delta \lambda$
- This violates classical Thomson scattering
- This transfer of energy during collisions tells about the particle nature of photons

Wave-Particle Duality



Do particles also behave like waves, and if yes then what kind of wave ?

de Broglie Hypothesis

in 1924, de Broglie's hypothesis stated that for any moving particle/object is associated with wave properties. These waves are known as matter waves

Nobel Prize for Physics in 1929

If an object of mass **m** is moving with velocity **v** and has energy **E**, the wavelength and frequency of the matter wave associated with the object is given as:

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

where h is Planck's constant.

The waves associated with material particles are called the **matter waves** or **de-Broglie waves.**

Calculate the De Broglie wavelength of the (a) electron moving at 2×10^6 m/s and a cricket ball of mass 200gm moving at 20 m/s. Which of this entity particle behaves more like a wave and which of the entity behaves more like a particle?

Electron

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63\times10^{-34}}{9.1\times10^{-32}\times2\times10^{6}}$$

$$\lambda = 3.64\times10^{-10}m$$

$$\lambda = 3.64A^{o}$$

Cricket ball

$$\lambda = \frac{h}{p}$$

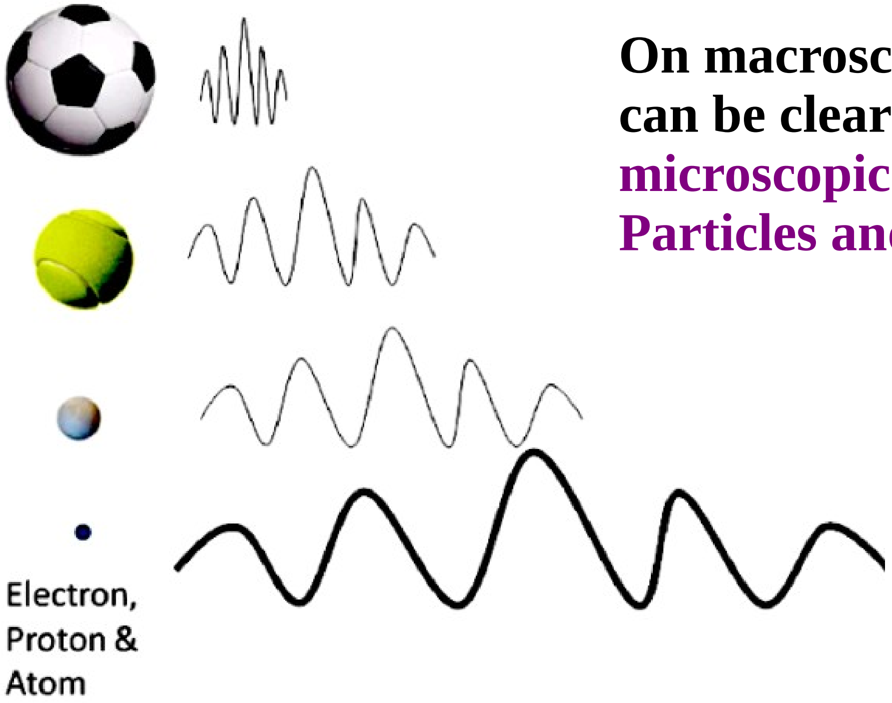
$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63\times10^{-34}}{200\times10^{-3}\times20}$$

$$\lambda = 1.6575\times10^{-10}m$$

$$\lambda = 1.6575\times10^{-34}m$$

De Broglie suggested that Eq. $\lambda = \frac{h}{p} = \frac{h}{mv}$ is a completely general one that **applies to material particles as well as to photons**.



On macroscopic scales Waves and Particles can be clearly distinguished. However on the microscopic scale, Waves behaves like Particles and Particles behaves like Waves.

Take home Points

- Photons carry both energy & momentum.

$$\boxed{E = hc/\lambda} \quad \boxed{p = E/c = h/\lambda}$$

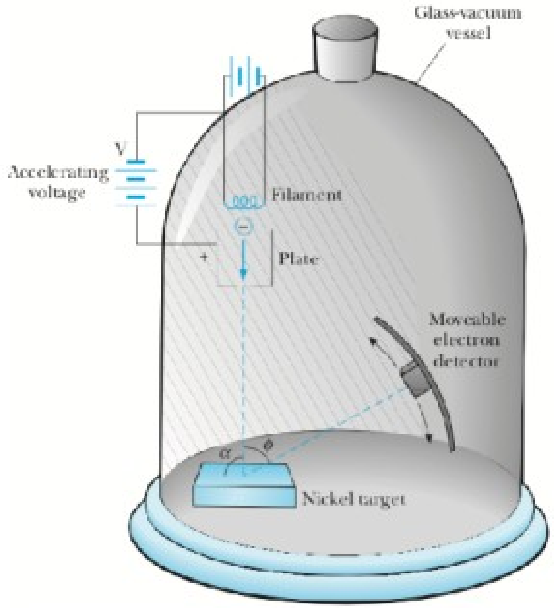
- Matter also exhibits wave properties. For an object of mass m, and velocity, v, the object has a wavelength, $\lambda = h / mv$
- One can probe 'see' the fine details of matter by using high energy particles (they have a small wavelength !)

Davisson-Germer Experiment (Proof of Matter Wave)

The Davisson–Germer experiment gives the first-ever evidence for the wave nature of matter. Direct experimental proof that electrons possess a wavelength $\lambda = \dfrac{h}{p}$ was provided by the diffraction experiments of American physicists Clinton J. Davisson and Lester H. Germer at the Bell Laboratories in New York City in 1927

Nobel Prize for Physics in 1937

Davisson-Germer Experiment (Proof of Matter Wave)



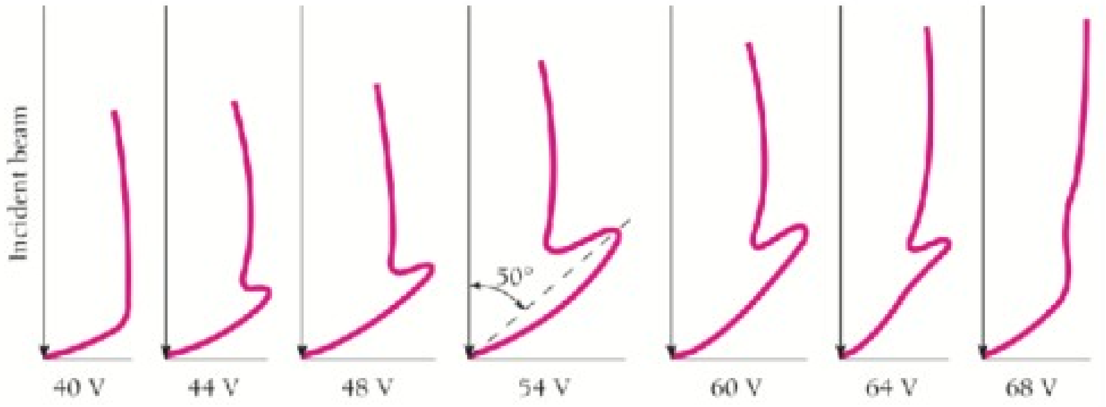
Schematic of the Davisson-Germer experiments

Experiment:

- Electrons emitted by the filament are accelerated to get the desired velocity by applying a suitable voltage.
- The electrons are scattered in all directions from the nickel crystal.
- The intensity of the scattered electron beam is measured for different values of scattered angle, ϕ, and for different voltages

Davisson-Germer Experiment: Results

Polar plot of electron distribution at different electron energies



From these experimental curves, the following inferences can be drawn :

(i) The intensity of scattered electrons depends upon the angle of scattering ϕ.

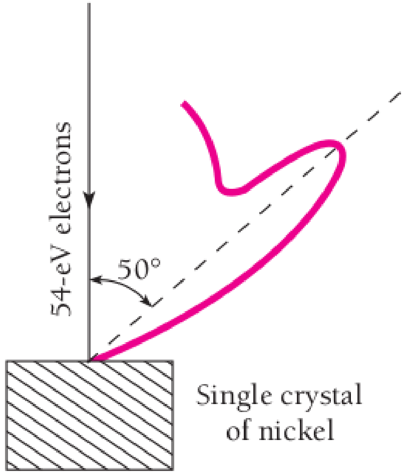
(ii) Always a 'bump' or a kink occurs in the curve at $\phi = 50°$, the angle which the scattered beam makes with the incident beam.

(iii) The size of the bump goes on increasing as the accelerating voltage is increased.

(iv) The size of the bump becomes maximum when the accelerating voltage is 54 volts.

(v) The size of the bump starts decreasing with a further increase in the accelerating voltage.

Let us see whether we can verify that de Broglie waves are responsible for the findings of Davisson and Germer. In a particular case, a beam of 54-eV electrons was directed perpendicularly at the nickel target and a sharp maximum in the electron distribution occurred at an angle of 50° with the original beam. The angles of incidence and scattering relative to the family of Bragg planes shown in Fig. 3.8 are both 65°. The spacing of the planes in this family, which can be measured by x-ray diffraction, is 0.091 nm. The Bragg equation for maxima in the diffraction pattern is



$$n\lambda = 2d \sin \theta$$

Here $d = 0.091$ nm and $\theta = 65°$. For $n = 1$ the de Broglie wavelength λ of the diffracted electrons is

$$\lambda = 2d \sin \theta = (2)(0.091 \text{ nm})(\sin 65°) = 0.165 \text{ nm}$$

Using de-Broglie's hypothesis

$$\mathrm{KE} = \tfrac{1}{2}mv^2$$

the electron momentum mv is

$$\begin{aligned} mv &= \sqrt{2m\mathrm{KE}} \\ &= \sqrt{(2)(9.1 \times 10^{-31}\ \mathrm{kg})(54\ \mathrm{eV})(1.6 \times 10^{-19}\ \mathrm{J/eV})} \\ &= 4.0 \times 10^{-24}\ \mathrm{kg \cdot m/s} \end{aligned}$$

The electron wavelength is therefore

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ \mathrm{J \cdot s}}{4.0 \times 10^{-24}\ \mathrm{kg \cdot m/s}} = 1.66 \times 10^{-10}\ \mathrm{m} = 0.166\ \mathrm{nm}$$

which agrees well with the observed wavelength of 0.165 nm. The Davisson-Germer experiment thus directly verifies de Broglie's hypothesis of the wave nature of moving bodies.

Davisson-Germer Experiment: Results Analysis

- According to classical physics, there should be very little variation in the intensity of the electron beam with the angle of scattering voltages
- The appearance of the bump in a particular direction is due to constructive interference of electrons scattered from different layers of regularly spaced atoms of the crystal.
- The selective reflection of the 54-volt electrons at an angle of 50° between the incident and the scattered beam can be termed the diffraction of electrons from the regularly spaced electrons of nickel crystal by virtue of their wave nature.
- Establish the wave nature of the particle.