# Vector Calculus

## Scalar quantity:

A Scalar quantity has only magnitude.

**Ex:** The units for time (minutes, days, hours etc)

length, area, volume, Speed, mass, density, pressure, temperature, energy, entropy, work, power.

## Vector quantity:

A vector quantity has both magnitude and direction.

**Ex:** displacement, direction velocity, acceleration, momentum, force, lift, drag, thrust, weight.

Note:

In vector algebra we mostly deal with Constant vectors, viz. vectors which are Constant in magnitude and fixed in direction.

In vector Calculus we deal with variable vectors ie vectors which are varying in magnitude or direction or both.

A variable quantity whose value at any point in a region of space depends upon the position of the point is Called a point function. There are two types of point functions.

Scalar point Function:

Let R be a region of space at each point of which a scalar $\varphi = \varphi(x, y, z)$ is given, then $\varphi$ is called a scalar function and R is called a scalar field.

Examples: The temperature distribution is a medium, the distribution of atmospheric pressure in space are examples of scalar point functions.

**Vector point Function:**

Let R be a region of space at each point of which a vector $\vec{v} = \vec{v}(x,y,z)$ is given, then $\vec{v}$ is called a vector point function and R is called a vector field.

Examples: The velocity of a moving fluid at any instant, the gravitational force are examples of vector point functions.

## Vector differential operator $\nabla$ (del)

The vector differential operator $\nabla$ is defined as $\nabla = \vec{i}\,\partial/\partial x + \vec{j}\,\partial/\partial y + \vec{k}\,\partial/\partial z$

Where $\vec{i}, \vec{j}, \vec{k}$ are unit vectors along the three rectangular axes OX, OY and OZ.

## Gradient of a Scalar point Function:

If $\varphi(x, y, z)$ be a scalar point function and continuously differen[tiable]

then the vector

$$\nabla\varphi = \left(\vec{i}\,\partial/\partial x + \vec{j}\,\partial/\partial y + \vec{k}\,\partial/\partial z\right)\varphi$$

$$\nabla\varphi = \vec{i}\,\frac{\partial\varphi}{\partial x} + \vec{j}\,\frac{\partial\varphi}{\partial y} + \vec{k}\,\frac{\partial\varphi}{\partial z}$$

is called gradient of $\varphi$ and also be written as grad $\varphi$.

**Note:** $\nabla\varphi$ defines a vector field.

## Properties of Gradient :-

1. If $f$ and $g$ are two Scalar point function than $\nabla(f \pm g) = \nabla f \pm \nabla g$ (or) grad $(f \pm g)$ = grad $f$ $\pm$ grad $g$.

**Solution:**

$$\nabla(f \pm g) = \left(\vec{i}\,\partial/\partial x + \vec{j}\,\partial/\partial y + \vec{k}\,\partial/\partial z\right)(f \pm g)$$

$$= \vec{i}\,\partial/\partial x\,(f \pm g) + \vec{j}\,\partial/\partial y\,(f \pm g) + \vec{k}\,\partial/\partial z\,(f \pm g)$$

$$= \vec{i}\,\frac{\partial f}{\partial x} \pm \vec{i}\,\frac{\partial g}{\partial x} + \vec{j}\,\frac{\partial f}{\partial y} \pm \vec{j}\,\frac{\partial g}{\partial y} + \vec{k}\,\frac{\partial f}{\partial z} \pm \vec{k}\,\frac{\partial g}{\partial z}$$

$$\nabla(f \pm g) = \left(\vec{i}\,\frac{\partial f}{\partial x} + \vec{j}\,\frac{\partial f}{\partial y} + \vec{k}\,\frac{\partial f}{\partial z}\right) \pm \left(\vec{i}\,\frac{\partial g}{\partial x} + \vec{j}\,\frac{\partial g}{\partial y} + \vec{k}\,\frac{\partial g}{\partial z}\right)$$

$$\nabla(f \pm g) = \nabla f \pm \nabla g.$$

2) If $f$ and $g$ are two Scalar point functions then $\nabla(fg) = f\nabla g + g\nabla f$ (or) grad$(fg)$ = $f$ grad $g$ + $g$ grad $f$.

**Solution:**

$$\nabla(fg) = \left(\vec{i}\,\partial/\partial x + \vec{j}\,\partial/\partial y + \vec{k}\,\partial/\partial z\right)(fg)$$

$$= \vec{i}\,\partial/\partial x\,(fg) + \vec{j}\,\partial/\partial y\,(fg) + \vec{k}\,\partial/\partial z\,(fg)$$

$$= \vec{i}\left(f\frac{\partial g}{\partial x} + g\frac{\partial f}{\partial x}\right) + \vec{j}\left(f\frac{\partial g}{\partial y} + g\,\partial f/\partial y\right) + \vec{k}\left(f\frac{\partial g}{\partial z} + g\frac{\partial f}{\partial z}\right)$$

$$= f\left(\vec{i}\frac{\partial g}{\partial x} + \vec{j}\frac{\partial g}{\partial y} + \vec{k}\frac{\partial g}{\partial z}\right) + g\left(\vec{i}\frac{\partial f}{\partial x} + \vec{j}\,\partial f/\partial y + \vec{k}\frac{\partial f}{\partial z}\right)$$

$$= f\nabla g + g\nabla f$$

3. If $f$ and $g$ are two Scalar point functions then $\nabla(f/g) = \frac{g\nabla f - f\nabla g}{g^2}$ where $g \neq 0$ (or)

$$\text{grad}(f/g) = \frac{g(\text{grad} f) - f(\text{grad} g)}{g^2}$$

Solution:

$$\nabla(f/g) = \left(\vec{i}\,\partial/\partial x + \vec{j}\,\partial/\partial y + \vec{k}\,\partial/\partial z\right)(f/g)$$

$$= \Sigma \vec{i}\,\partial/\partial x\,(f/g)$$

$$= \Sigma \vec{i}\left(\frac{g\,\partial f/\partial x - f\,\partial g/\partial x}{g^2}\right)$$

$$= 1/g^2\left[g\,\Sigma \vec{i}\,\partial f/\partial x - f\,\Sigma \vec{i}\,\partial g/\partial x\right]$$

$$\nabla(f/g) = 1/g^2\,(g\nabla f - f\nabla g)$$

4. Prove that Gradient of a Constant is zero.

Proof: If $\varphi(x,y,z)$ be a Constant, then $\frac{\partial\varphi}{\partial x}$, $\frac{\partial\varphi}{\partial y}$ and $\frac{\partial\varphi}{\partial z}$ are zero.

$$\therefore \nabla\varphi = \vec{i}\frac{\partial\varphi}{\partial x} + \vec{j}\frac{\partial\varphi}{\partial y} + \vec{k}\frac{\partial\varphi}{\partial z}$$

$$\nabla\varphi = \vec{i}(0) + \vec{j}(0) + \vec{k}(0) = 0$$