Digital Assignment-①

Course Name: Engineering Physics

Course Code: 5310

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Q1) Length of rope = 5 m , weight of rope = 1.45 gm $= 1.45 \times 10^{-3}$ Kg

Frequency = 120 Hz , wavelength = 60 cm = 0.6 m

Tension = ?? , mass = ??

Ans) We know that,

$$v = f\lambda$$

$$v = 120 \times 0.6 \text{ m/sec}$$

$$\boxed{v = 72 \text{ m/s}}$$

We also know that,

$$\mu = \frac{\text{mass of rope}}{\text{length of rope}} = \frac{1.45 \times 10^{-3}}{5}$$

$$\boxed{\mu = 2.9 \times 10^{-4} \text{ Kg/m}}$$

we know that,

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$v^2 = \frac{F_T}{\mu}$$

$$F_T = v^2 \mu$$

$$F_T = (72)^2 \times (2.9 \times 10^{-4})$$

$$\boxed{F_T = 1.503\ N}$$

Mass required to produce tension is given by:

$$F_T = mg$$

$$m = \frac{F_T}{g} \qquad \{g = 9.8\ m/s\}$$

$$m = \frac{1.503}{9.8}$$

$$\boxed{m = 0.153\ Kg}$$

* Tension in the rope is <u>1.503 N</u>.

* Mass to produce tension is <u>0.153 Kg</u>

Q2) Tension in string = 88.2N, Mass of String = 500 mgm = ~~0.5 m~~ 0.5×10^{-3} Kg

Length of string = 50cm = 0.5 m

a) Wave Speed

b) Fundamental Frequency

c) First & Second overtones

Ans) Mass density of rope:

$$\mu = \frac{\text{mass of rope}}{\text{length of rope}}$$

$$\mu = \frac{0.5 \times 10^{-3}}{0.5} = 1 \times 10^{-3} \text{ Kg/m}$$

$$\boxed{\mu = 1 \times 10^{-3} \text{ Kg/m}}$$

We know,

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$= \sqrt{\frac{88.2}{10^{-3}}}$$

$$\boxed{v = 297.153 \text{ m/s}}$$

we

v

v^2

F_T

F_T

$\boxed{F_T}$

Mass
given

* Te

* Ma

Fundamental Frequency:

$$f_1 = \frac{v}{2L}$$

$$f_1 = \frac{297.153}{2 \times 0.5}$$

$$\boxed{f_1 = 297.153\ Hz}$$

<u>Overtones</u>

i) First Overtone:

$$f_2 = 2 f_1$$

$$\boxed{f_2 = 594.306\ Hz}$$

ii) Second Overtone:

$$f_3 = 3 f_1$$

$$\boxed{f_3 = 891.459\ Hz}$$

Q3) Length of string = 30 cm, Fundamental Frequency = 256 Hz

Length$_2$ of string = 80 cm, mass = 0.75 g

Tension in string = ? ?

Ans) Linear mass density:

$$\mu = \frac{mass}{length} = \frac{0.00075}{0.8}$$

$$\boxed{\mu = 9.375 \times 10^{-4}}$$

Wave speed:

$$v = 2Lf$$

$$v = 2 \times 0.3 \times 256$$

$$\boxed{v = 153.6 \text{ m/s}}$$

We know,

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$v^2 = \frac{F_T}{\mu}$$

$$F_T = v^2 \mu$$

$$= (153.6)^2 \times 9.375 \times 10^{-4}$$

$$\boxed{F_T = 22.1\ N}$$

Q4) Fundamental Frequency = 196 Hz
Where should finger be placed to make it 440 Hz.

Ans) We know that,

$$f \propto \frac{1}{L}$$

$$\frac{f_1}{f_2} = \frac{L_2}{L_1}$$

Ratio of frequency:

$$\frac{440}{196} = \frac{L_1}{L_2}$$

$$L_2 = \frac{L_1}{2.2449}$$

Let say total length of string be L_1 & the vibrating portion be L_2

$$d = L_1 - L_2$$

$$d = L_1 - \frac{L_1}{2.2449}$$

$$d = L_1(1 - 0.4456)$$

$$d = 0.5544\,L_1$$

So, the finger should be placed at approx. 55.44% of the string's length from it's orignal length.

Q5) mass = 2 Kg, Height of mine shaft = 80 m, mass of rock = 20 Kg

a) Speed of wave

b) No. of wavelengths if $f = 2Hz$ & $L = 80m$.

Ans) We know that $v = \sqrt{\frac{F_T}{\mu}}$

$$\mu = \frac{\text{mass of rope}}{\text{length of rope}} = \frac{2}{80}$$

$$\boxed{\mu = 0.025\ Kg/m}$$

$$F_T = m \cdot g$$

$$= 20 \times 9.8$$

$$\boxed{F_T = 196\ N}$$

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$= \sqrt{\frac{196}{0.025}}$$

$$\boxed{v = 88.5 \text{ m/s}}$$

We know,

$$v = f\lambda$$

$$\lambda = \frac{v}{f} = \frac{88.5}{2} = 44.25 \text{ m}$$

$$n = \frac{L}{\lambda}$$

$$n = \frac{80}{44.25} \simeq 1$$

So full wavelength is 1.

Q6) $y(x,t) = (2.75\,cm)\cos(0.410\,rad/cm \cdot x + 6.20\,rad/s \cdot t)$

a) Time & Horizontal distance

b) Wave number & no. of waves

c) Wave Speed & max. speed of cork

Ans) $A = 2.75\,cm = 2.75 \times 10^{-2}\,m$

$K = 0.410, \quad \omega = 6.20$

$T = \frac{2\pi}{\omega}$

$T = \frac{2\pi}{6.20} = 1.01$

$\boxed{T = 1.01\,sec}$

$v = \frac{\omega}{K}$

$v = \frac{6.20}{0.41}$

$\boxed{v = 15.121\,m/s}$

$$\text{Distance} = v \cdot T$$

$$= 0.151 \cdot 1.01$$

$$\boxed{\text{Distance} = 0.153\,m}$$

Wave number:

$$K = 0.410 \text{ rad/cm} = 41.0 \text{ rad/m}$$

$$= 41.0 \text{ rad/m}$$

Frequency:

$$f = \frac{1}{T}$$

$$f = \frac{1}{1.01} = 0.99$$

$$\boxed{f = 0.99\,Hz}$$

Wave speed:

Calculated earlier:

$$\boxed{v = 0.151\,m/s}$$

Maximum speed of cork

$$v_{max} = \omega \cdot A$$

$$v_{max} = 6.20 \times 0.0275$$

$$\boxed{v_{max} = 0.171 \, m/s}$$

Q7) Given eq^n

$$y(x,t) = A \cos\left[\omega\left(\frac{x}{v} - t\right)\right]$$

We know,

$\omega = 2\pi f$, $K = \frac{2\pi}{K}$, $v = f\lambda$, $f = \frac{v}{\lambda}$

$$= \omega\left(\frac{x}{v} - t\right)$$

$$= 2\pi f\left(\frac{x}{v} - t\right)$$

$$= 2\pi\left(\frac{v}{\lambda}\right)\left(\frac{x}{v} - t\right)$$

$$= 2\pi\left(\frac{x}{\lambda} - \frac{vt}{\lambda}\right)$$

$$= 2\pi \, \frac{x - vt}{\lambda}$$

$$\boxed{y(x,t) = A\cos\left(2\pi \frac{x - vt}{\lambda}\right)}$$

b) Velocity of wave:

$$v_y = \frac{dy}{dt}$$

$$\frac{d}{dt}\left(A\cos\left(\frac{\omega x}{v} - \omega t\right)\right)$$

$$v_y = -A \cancel{\omega} \sin\left(\omega \frac{x}{v} - \omega t\right) \cdot (-\omega)$$

$$\boxed{v_y = A\omega \sin\left(\omega \frac{x}{v} - \omega t\right)}$$

c) Maximum speed of a particle:

$$v_{max} = A\omega$$

$$v = \frac{\omega}{K} \quad , \quad K = \frac{2\pi}{\lambda}$$

$$A \cancel{\omega} = \frac{\cancel{\omega}\lambda}{2\pi}$$

$$\boxed{A = \frac{\lambda}{2\pi}}$$

Here,

we can conclude that

$$\boxed{A \propto \lambda}$$

If

$A \uparrow$ so, $\lambda \uparrow$

When v_{max} is less than v

$$A\omega < \frac{\omega}{K}$$

$$A < \frac{1}{K} = \frac{\lambda}{2\pi}$$

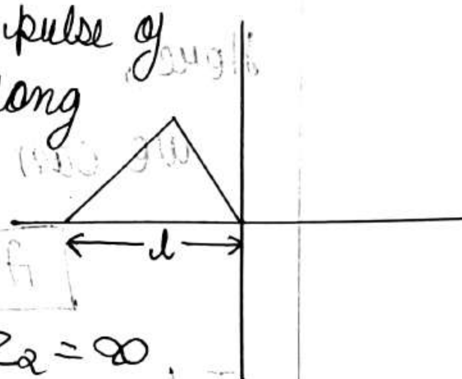
$$A < \frac{\lambda}{2\pi}$$

So, v_{max} is less than v when $A < \frac{\lambda}{2\pi}$

Similarly,

v_{max} is greater than v when $A > \frac{\lambda}{2\pi}$

Q8) Consider a triangle pulse of length l traveling along a string at a fixed at one end, with a reflection co-efficient $Z_2 = \infty$



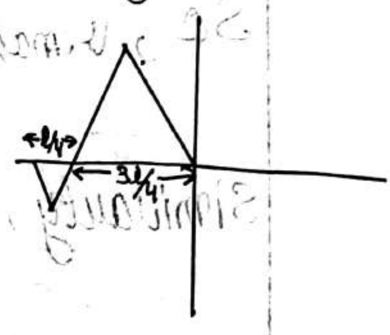
Some points that can be drawn are:

1) Wave moves in a triangular shape towards the fixed end.

2) It has reflection co-efficient $Z_2 = \infty$, which means that the wave ~~is~~ will be reflected completely.

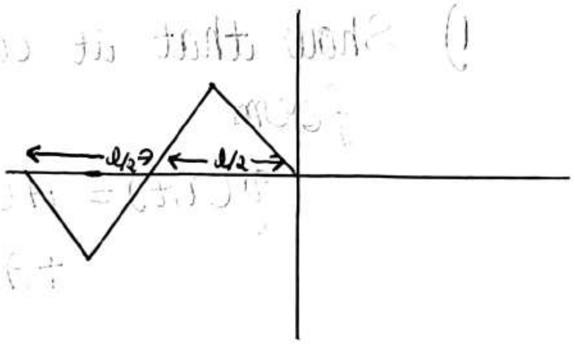
a) $1/4$ of the pulse is reflected

This means that $1/4$ of the wave is inverted & the rest $3/4$ is still moving without being inverted



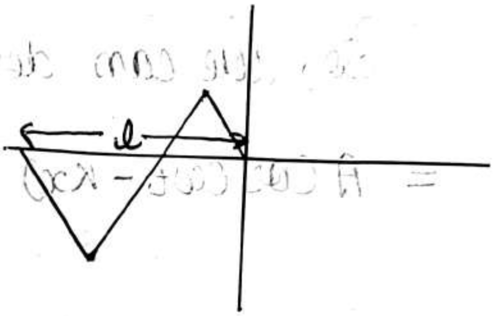
b) $\frac{1}{2}$ of pulse is reflected

Half of the wave is reflected backwards while the other half still moves towards the fixed end.



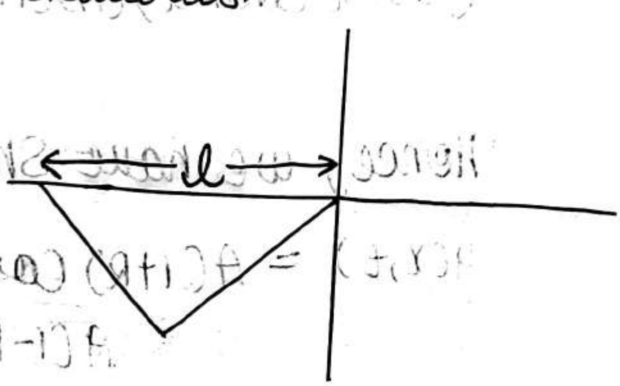
c) $\frac{3}{4}$ of pulse is reflected.

This means that $\frac{3}{4}$ of the wave is reflected backwards while the other $\frac{1}{4}$ still moves towards the fixed end



d) when entire pulse is reflected

The entire pulse is inverted or reflected backwards.



Q9) Displacement of a wave is given as

$$y(x,t) = A\cos(\omega t - Kx) + RA\cos(\omega t + Kx)$$

1) Show that it can be expressed in the form

$$y(x,t) = A(1+R)\cos\omega t\cos Kx + A(1-R)\sin\omega t\sin Kx$$

We know that,

$$\cos(a+b) = \cos a\cos b - \sin a\sin b$$

&

$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$

So, we can derive that,

$$= A\cos(\omega t - Kx) = A(\cos\omega t\cos Kx) + A(\cos\omega t\cos Kx)$$

$$= AR(\cos\omega t\cos Kx) - AR(\sin\omega t\sin Kx)$$

$$= (\cos\omega t\cos Kx)(A)(1+R) + (\sin\omega t\sin Kx)(A)(1-R)$$

Hence, ~~we have~~ shown that,

$$y(x,t) = A(1+R)\cos\omega t\cos Kx + A(1-R)\sin\omega t\sin Kx$$

2) Verify that it satisfies the wave equation :

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

Given,

~~y(x,t) = A(1+R) cA~~

$y(x,t) = A(1+R)\cos\omega t \cos Kx + A(1-R)\sin\omega t \sin Kx$

$$\frac{\partial y}{\partial t} = -A(1+R)\omega \sin\omega t \cos Kx + A(1-R)\omega\cos\omega t \sin Kx$$

$$\frac{\partial^2 y}{\partial t^2} = -A(1+R)\omega^2 \cos\omega t \cos Kx - A(1-R)\omega^2 \sin\omega t \sin Kx$$

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 \left[A(1+R)\cos(\omega t)\cos(Kx) + A(1-R)\sin(Kx)\sin(\omega t) \right]$$

$y(x,t) = A(1+R)\cos\omega t \cos Kx + A(1-R)\sin\omega t \sin Kx$

$$\frac{\partial y}{\partial x} = -A(1+R)K\cos\omega t \sin Kx + A(1-R)K\sin\omega t \cos Kx$$

$$\frac{\partial^2 y}{\partial x^2} = -A(1+R)K^2\cos(\omega t)\cos(Kx) - A(1-R)K^2 \sin(\omega t)\sin(Kx)$$

$$\frac{\partial^2 y}{\partial x^2} = -K^2 \left[A(1+R)\cos\omega t \cos Kx + A(1-R)\sin Kx \sin\omega t \right]$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{+\omega^2}{+K^2} \frac{\partial^2 y}{\partial x^2}$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}}$$

Hence, the wave equation is satisfied.

Q10) Given there are two strings connected to one another. The density of string 2 is 4 times that of string 1.

Given,

$$y_{incident} = A_1 \cos(K_1 x - \omega_1 t)$$

$$y_{reflected} = A_r \cos(-K_1 x - \omega_1 t)$$

$$y_{ref} = A_r \cos(K_1 x + \omega t)$$

Given,

$$y_{trans} = A_t \cos(K_2 x - \omega_1 t)$$

where,

$$K_2 = \frac{\omega_1}{v_2}, \quad v_2 = \sqrt{\frac{T}{\mu_2}}$$

Given,

$$\mu_2 = 4\mu_1 \Rightarrow v_2 = \frac{v_1}{2}, \; k_2 = 2k_1$$

$$\boxed{y_{\substack{trans \\ mitted}} = A_1 \cos(2k_1 x - \omega_1 t)}$$

<u>Reflection & Transmission Co-efficients</u>

We know,

$$y_{incident} + y_{reflected} = y_{transmitted}$$

$$R = \frac{A_r}{A_1}$$

$$T = \frac{A_t}{A_1}$$

We know that,

$$1 + R = T$$

&

$$1 - R = \frac{T}{2}$$

Solving :

$$R = \frac{1-2}{1+2} = \frac{-1}{3}, \; T = \frac{2}{3}$$

Also,

$$\boxed{R = -1/3}$$

$$\boxed{T = 2/3}$$

<u>Here, we can conclude that,</u>

- Since, $\boxed{R = -1/3}$ indicates that the reflected wave has an inverted amplitude due to denser medium in string 2.

- Since, $\boxed{T = 2/3}$ means that two-thirds of the wave's amplitude is transmitted into string 2. The reduction accounts for the energy sharing b/w reflected & transmitted waves.