*Canonical → complying to well established patterns/rules

# Canonical forms

⇒ There are ~~infinite~~ many number of expression for every Boolean function.

⇒ A Boolean expression in its canonical form is unique.

⇒ Two canonical forms → (i) Sum of products (SOP)
(ii) Product of sums (POS)

⇒ These two forms SOP & POS defines a unique way to represent a boolean function

⇒ By combining two boolean variable x and y with logical operator AND and To get 4 Boolean product terms. These product terms are called as min terms or standard product terms. These min terms are $x'y'$, $x'y$, $xy'$, $xy$.

⇒ Similarly by combining two variable x and y with logical OR operation. These Boolean sum terms are called as Max terms or standard sum terms. These max terms are $x'+y'$, $x'+y$, $x+y'$, $x+y$

| x | y | Min terms | Max terms |
|---|---|---|---|
| 0 | 0 | $m_0 = x'y'$ | $M_0 = x+y$ |
| 0 | 1 | $m_1 = x'y$ | $M_1 = x+y'$ |
| 1 | 0 | $m_2 = xy'$ | $M_2 = x'+y$ |
| 1 | 1 | $m_3 = xy$ | $M_3 = x'+y'$ |

☆ If the binary variable is zero, then it is represented as complement of variable in minterms and the variable itself in max term.

If variable is 1, it is represented as complement of the variable in Max terms and as the variable itself in minterms.

Minterms — standard product

$x = 1$

$x' = 0$

Max terms — standard sum

$x = 0$

$x' = 1$

## * Max & Min terms of three Binary variables *

| | | | Minterms | | Maxterms | |
|---|---|---|---|---|---|---|
| $x$ | $y$ | $z$ | Term | Designation | Term | Designation |
| 0 | 0 | 0 | $\bar{x}\bar{y}\bar{z}$ | $m_0$ | $x+y+z$ | $M_0$ |
| 0 | 0 | 1 | $\bar{x}\bar{y}z$ | $m_1$ | $x+y+\bar{z}$ | $M_1$ |
| 0 | 1 | 0 | $\bar{x}y\bar{z}$ | $m_2$ | $x+\bar{y}+z$ | $M_2$ |
| 0 | 1 | 1 | $\bar{x}yz$ | $m_3$ | $x+\bar{y}+\bar{z}$ | $M_3$ |
| 1 | 0 | 0 | $x\bar{y}\bar{z}$ | $m_4$ | $\bar{x}+y+z$ | $M_4$ |
| 1 | 0 | 1 | $x\bar{y}z$ | $m_5$ | $\bar{x}+y+\bar{z}$ | $M_5$ |
| 1 | 1 | 0 | $xy\bar{z}$ | $m_6$ | $\bar{x}+\bar{y}+z$ | $M_6$ |
| 1 | 1 | 1 | $xyz$ | $m_7$ | $\bar{x}+\bar{y}+\bar{z}$ | $M_7$ |

* Boolean exp functions expressed as a sum of minterms or product of maxterms are said to be in Canonical form.

(I) Sum of Product (SOP) Canonical form :-
(minterms)

$$xy = x(1) + y(1) = x(y+\bar{y}) + y(x+\bar{x})$$

$xy = xy + x\bar{y} + yx + y\bar{x}$

$xy = xy + x\bar{y} + xy + \bar{x}y$

$xy = xy + x\bar{y} + \bar{x}y + xy$ Min terms.

$F(x, y, z) = \Sigma m(3, 4, 5, 6, 7)$ funⁿ

$= m_3 + m_4 + m_5 + m_6 + m_7$

$= \bar{x}yz + x\bar{y}\bar{z} + x\bar{y}z + xy\bar{z} + xyz$

Minterms

(max-terms)

## (II) Product of sum (POS) Canonical form :-

$F(x, y, z) = \Pi M(0, 1, 2)$

$= M_0 M_1 M_2$

$= (x+y+z)(x+y+\bar{z})(x+\bar{y}+z)$

maxterms

Note :- the maxterms & minterms should contain all the variables.

★ Finding the minterms:-

1) Write down all the terms
2) Put X's where letters must be provided to convert the term to a minterm.
3) Use all combinations of the X's in each term to generate minterms, where an X is a 0 write a barred letter ; where it is 1 write an unbarred letter.
4) Droped the redundant terms.

Scanned by CamScanner

Exp find the minterms for $f = A+BC$

$f = A+BC$

$f = AXX + XBC$

$f = A00 + A01 + A10 + A11 + 0BC + 1BC$

$f = A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC + \bar{A}BC + ABC$

$f = A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC + \bar{A}BC$

$f = \Sigma m(100, 101, 110, 111, 011)$

$f = \Sigma m(m_4, m_5, m_6, m_7, m_3)$

$f = \Sigma m(4, 5, 6, 7, 3)$

Exp find minterm for $A\bar{B}\bar{C}\bar{D}$

$A\bar{B}\bar{C}\bar{D}$ minterm $\begin{cases} A = 0 \\ \bar{A} = 1 \end{cases}$

1000

m8 Ans

Exp.

| A B C | F |
|---|---|
| 0 0 0 | 0 |
| 0 0 1 | 1 |
| 0 1 0 | 0 |
| 0 1 1 | 0 |
| 1 0 0 | 1 |
| 1 0 1 | 1 |
| 1 1 0 | 0 |
| 1 1 1 | 1 |

SOP form

consider 1's

$\longrightarrow F = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + ABC$

$F = m_1 + m_4 + m_5 + m_7$

$F = \Sigma m(1, 4, 5, 7)$.

POS form

consider 0's

$F = (x+y+z)(x+\bar{y}+z)(x+\bar{y}+\bar{z})(\bar{x}+\bar{y}+z)$

$F = M_0 \, M_2 \, M_3 \, M_6$

$F = \Pi(0, 2, 3, 6)$.

Exp. Express the Boolean function $F = xy + \bar{x}z$ in a product of maxterms or (POS) form.

$F = xy + \bar{x}z$

Solution :- $F = xy + \bar{x}z$

for POS apply distributive law -

$F = (xy + \bar{x}) \cdot (xy + z)$

OR $F = (\bar{x} + xy)(z + xy)$

$F = (x + \bar{x})(\bar{x} + y)(z + x)(z + y)$

$F = (\bar{x} + y)(x + z)(y + z)$

This is incomplete

$F = (\bar{x} + y + x)(x + x + z)(x + y + z)$

$F = (\bar{x} + y + 0)(x + 0 + z)(0 + y + z)$

$(\bar{x} + y + 1)(x + 1 + z)(1 + y + z)$

$0 \rightarrow A \quad 1 \Rightarrow \bar{A}$

$F = (\bar{x} + y + z)(\bar{x} + y + \bar{z})(x + y + z)(x + \bar{y} + z)$
$(\bar{x} + y + z)(x + y + z)$

$F = (\bar{x} + y + z)(x + y + z)(x + \bar{y} + z)(\bar{x} + y + \bar{z})$

$F = (x + y + z)(x + \bar{y} + z)(\bar{x} + y + z)(\bar{x} + y + \bar{z})$

$F = M_0 \cdot M_2 \cdot M_4 M_5$

$F = \Pi M(0, 2, 4, 5) \rightarrow$ numbers are maxterms.

$\rightarrow$ Max terms

Product symbol

↳ ANDing