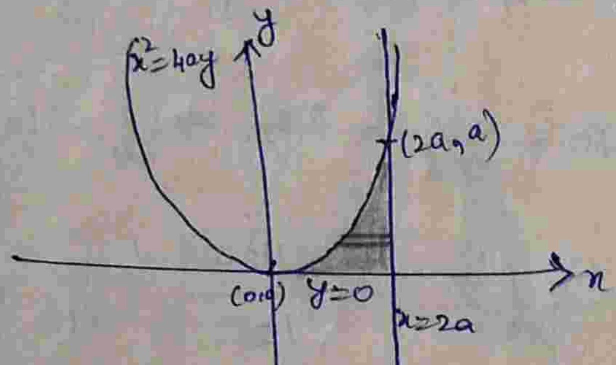
Area included between the curves $y = f_1(x)$ and $y = f_2(x)$ and the ordinates $x = a$ and $x = b$ is given by Area $= \int_a^b \int_{f_1(x)}^{f_2(x)} dy\,dx$ (or) $\int_a^b \int_{f_1(y)}^{f_2(y)} dx\,dy$

Area as a double integral [Cartesian Coordinates].

Evaluate $\iint_R xy\,dx\,dy$, where R is the domain bounded by X axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$. (AU may 96)

Sol:-



$$\iint_R xy\,dx\,dy = \int_0^a \int_{2\sqrt{ay}}^{2a} xy\,dx\,dy$$

$$= \int_0^a \left(\frac{x^2 y}{2}\right)_{2\sqrt{ay}}^{2a} dy$$

$$= \int_0^a \left(\frac{\cancel{4}^{2} a^2 y}{\cancel{2}} - \frac{\cancel{4}^{2} ay\,y}{\cancel{2}}\right) dy.$$

$$= \int_0^a \left(2a^2 y - 2ay^2\right) dy$$

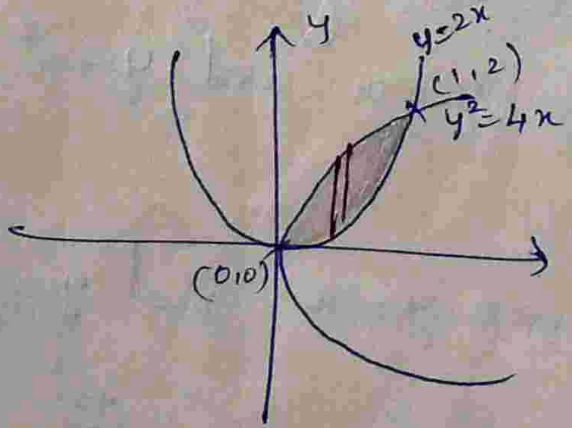
$$= \left(\cancel{2}a^2 \frac{y^2}{\cancel{2}} - 2a\frac{y^3}{3}\right)_0^a$$

$$= a^4 - \frac{2a^4}{3}$$

$$= \frac{a^4}{3} \,//.$$

Using double integration find the area enclosed by the curves $y = 2x^2$ and $y^2 = 4x$.
$\rightarrow (1)$ $\rightarrow (2)$
(A.U Nov 2001).

Sol:-



$y = 2x^2$

$y^2 = 4x$

$\cancel{4}x^4 = \cancel{4}x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x = 0,\ x = 1$

$y = 0,\ y = 2$

Therefore the point of intersection of (1) & (2) is $(0,0)$ and $(1,2)$

$$\therefore \text{The required area} = \int_0^1 \int_{2x^2}^{2\sqrt{x}} dy\, dx$$

$$\text{(or)} \quad = \int_0^2 \int_{y^2/4}^{\sqrt{y/2}} dx\, dy$$

$$= \int_0^1 (y)_{2x^2}^{2\sqrt{x}}\, dx$$

$$= \int_0^1 (2\sqrt{x} - 2x^2)\, dx$$

$$= \left(\frac{2x^{3/2}}{3/2} - 2x^3/3 \right)_0^1$$

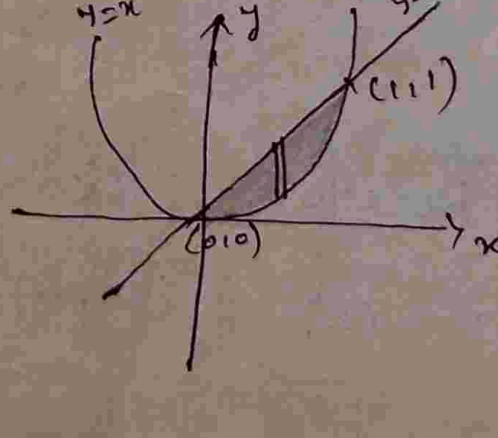
$$= 4/3 - 2/3$$

$$= 2/3 \,//$$

Using double integral, find the area bounded by $y = x$ and $y = x^2$. (AU may 1996)

Sol:- Given $y = x$ and $y = x^2$.

$y = x \rightarrow (1)$ $\quad y = x^2 \rightarrow (2)$



Sub (1) in (2), $x = x^2$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0,\ x = 1$

$\Rightarrow y = 0,\ y = 1$

$\therefore$ The point of intersection of (1) & (2) is

(0, 0) and (1, 1).

$\therefore$ The required area $= \int_0^1 \int_{x^2}^{x} dy\, dx$

$= \int_0^1 \int_y^{\sqrt{y}} dx\, dy$

$= \int_0^1 (y)_{x^2}^{x}\, dx$

$= \int_0^1 (x - x^2)\, dx$

$= \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1$

$= \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$

Find the smaller of the areas bounded by $y = 2 - x$ and $x^2 + y^2 = 4$ (AU - may 1996)

Sol:- Given $y = 2 - x$ & $x^2 + y^2 = 4$

$\Rightarrow x + y = 2$ —— (1) —— (2)

sub (1) in (2), $x^2 + (2-x)^2 = 4$

$x^2 + 4 + x^2 - 4x = 4$

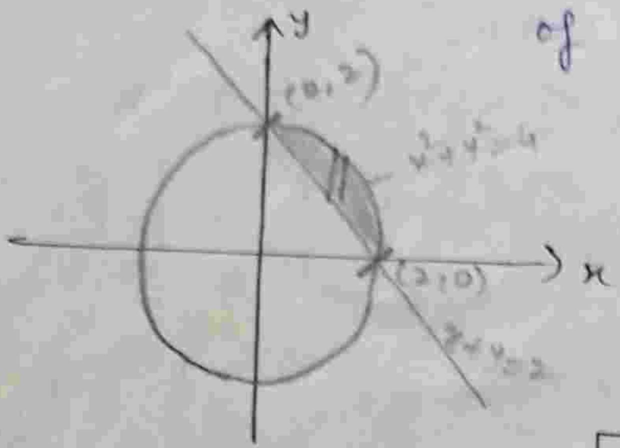
$2x^2 - 4x = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x=0,\ x=2$ $\quad\therefore$ Therefore the

$\Rightarrow y=2,\ y=0.$ point of intersection of (1) & (2) is $(0,2)$ and $(2,0)$



$\therefore$ The required area $= \displaystyle\int_0^2 \int_{2-x}^{\sqrt{4-x^2}} dy\,dx$

$= \displaystyle\int_0^2 \int_{2-y}^{\sqrt{1-y^2}} dx\,dy$

$= \displaystyle\int_0^2 (y)_{2-x}^{\sqrt{4-x^2}}\,dx$

$= \displaystyle\int_0^2 \left[\sqrt{4-x^2} - (2-x)\right] dx$

$= \displaystyle\int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (2-x)\,dx$

$= \left[\frac{4}{2} \sin^{-1}\frac{x}{2} + \frac{x}{2}\sqrt{4-x^2} \right]_0^2 - \left[2x - \frac{x^2}{2} \right]_0^2$

$= \left(2\frac{\pi}{2} + 0\right) - (0+0) - \left[(4-2)-(0-0)\right]$

$= \pi - 2$ Square units.

$$\left[\int \sqrt{a^2-x^2}\, dx = \frac{a^2}{2} \sin^{-1} \frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} \right]$$

Evaluate $\iint xy(x+y)\, dx\, dy$ over the region bounded by $x^2=y$ and $y=x$.

Sol:- Given $x^2=y \rightarrow (1)$ & $y=x \rightarrow (2)$

Sub (1) in (2)

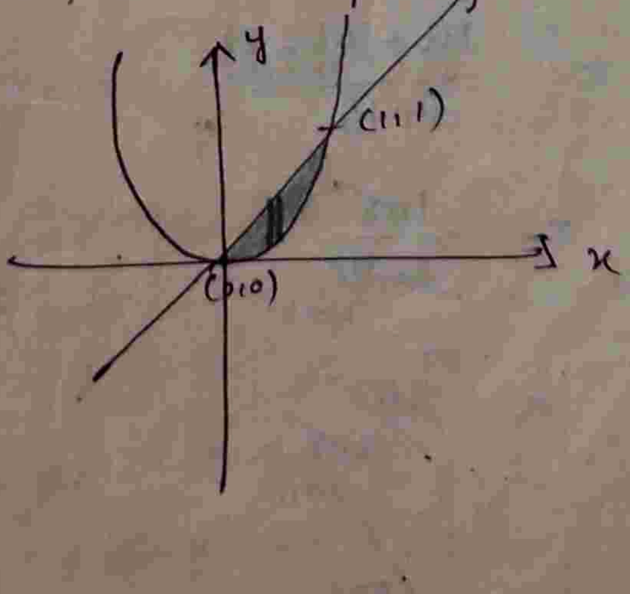
$$x^2=x$$

$$x^2-x=0$$

$$x(x-1)=0$$

$$x=0,\ x=1 \Rightarrow y=0,\ y=1.$$

$\therefore$ The point of intersection of (1) & (2) is $(0,0)$ & $(1,1)$.



$\therefore$ The required area $= \int_0^1 \int_{x^2}^{x} xy(x+y)\,dy\,dx$

$= \int_0^1 \int_{x^2}^{x} (x^2y + xy^2)\,dy\,dx$

$= \int_0^1 \left(x^2 y^2/2 + x y^3/3 \right)_{x^2}^{x} dx$

$= \int_0^1 \left(x^4/2 + x^4/3 - x^6/2 - x^7/3 \right) dx$

$= \int_0^1 \left(\frac{5x^4}{6} - x^6/2 - x^7/3 \right) dx$

$= \left(\frac{\cancel{5} x^5}{6 \times \cancel{5}} - x^7/14 - x^8/24 \right)_0^1$

$= 1/6 - 1/14 - 1/24$

$= \frac{336 - 144 - 84}{2016}$

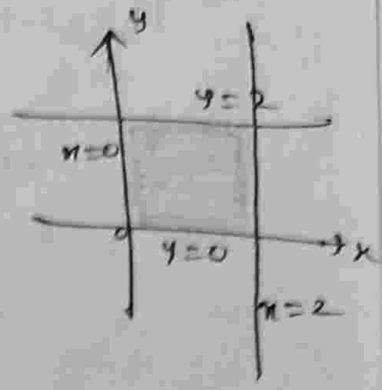
$= \frac{\cancel{108}\ 54}{\cancel{2016}\ 1008}$

$= \frac{\cancel{54}\ 27}{\cancel{1008}\ 504}$

$= 27/504$

$= 3/56$ //

Evaluate $\iint dx\,dy$ over the region bounded by $x=0,\ x=2,\ y=0,\ y=2$.



Sol:-

Given $x=0,\ x=2$

$y=0,\ y=2$.

$$\int_0^2\int_0^2 dx\,dy = \int_0^2 (x)_0^2\,dy$$

$$= \int_0^2 2\,dy$$

$$= 2\,(y)_0^2$$

$$= 4$$

Find the area bounded by the parabolas $y^2=4-x$ and $y^2=4-4x$ as a double integral and evaluate it. (AU Oct 2001)

Sol:-

Given $y^2 = 4-x \longrightarrow (1)$

$y^2 = 4-4x \longrightarrow (2)$

Sub (1) in (2) we get

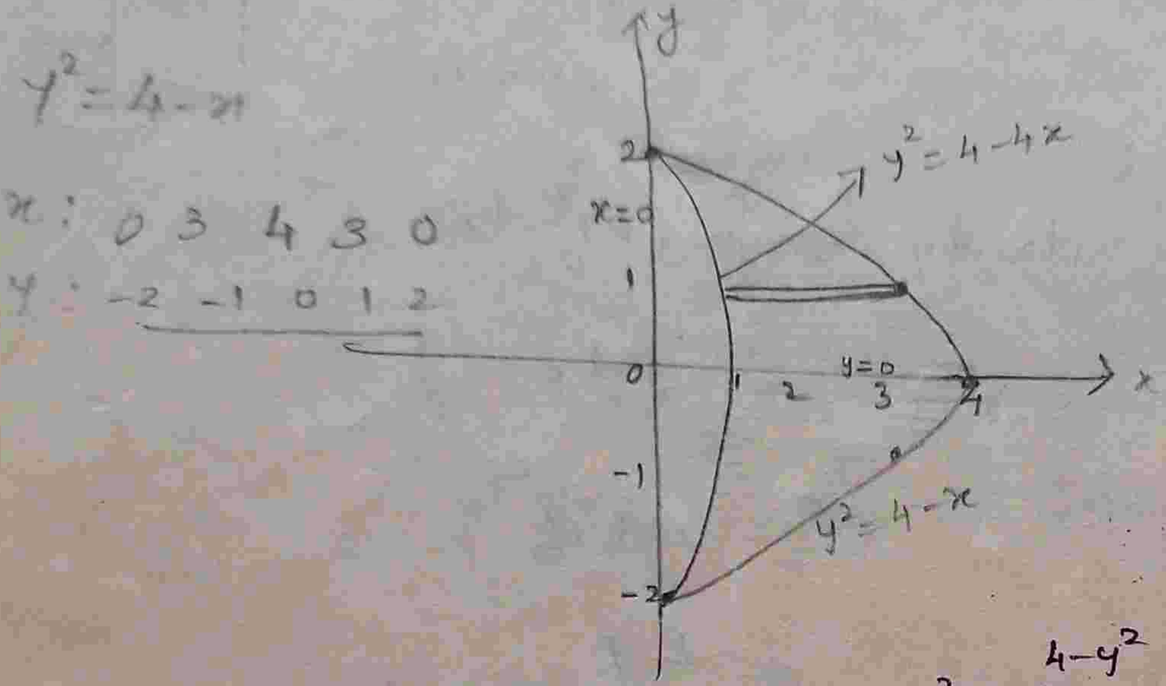
$$4-x = 4-4x$$

$$3x = 0$$

$$x = 0$$

$x=0 \Rightarrow y=\pm 2$

Therefore the point of intersection of (1) and (2) is $(0, 2)$ and $(0, -2)$.



$\therefore$ The required area $= \int_{-2}^{2}\int_{\frac{1}{4}(4-y^2)}^{4-y^2} dx\,dy$

$$= \int_{-2}^{2} \left[x\right]_{\frac{4-y^2}{4}}^{4-y^2} dy$$

$$= \int_{-2}^{2} \left[4-y^2 - \frac{4-y^2}{4}\right] dy$$

$$= \int_{-2}^{2} \left[4-y^2-1+\tfrac{1}{4}y^2\right] dy$$

$y^2 = 4 - 4x$

x:	0	1	0
y:	-2	0	2

$$= \int_{-2}^{2} \left[3 - \tfrac{3}{4} y^2\right] dy$$

$$= \left[3y - \tfrac{\cancel{3}}{4} \tfrac{y^3}{\cancel{3}}\right]_{-2}^{2}$$

$$= \left(3(2) - 8/4\right) - \left(3(-2) - \frac{(-2)^3}{4}\right)$$

$$= (6-2) - (-6 + 8/4)$$

$$= 4 - (-6+2)$$

$$= 4 - (-4) = 4 + 4 = 8 \,//.$$

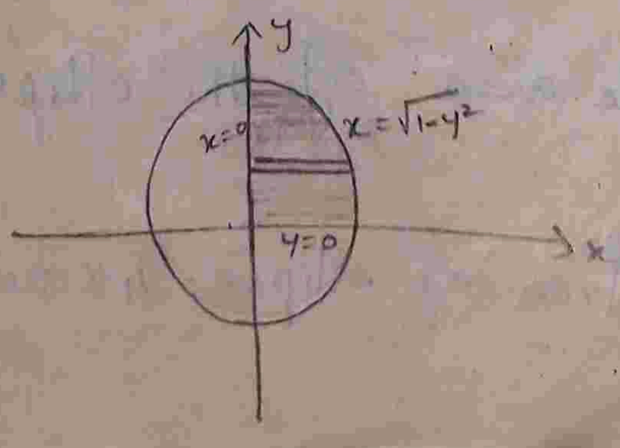
Evaluate $\iint xy \, dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = 1$.

Sol: Given $x^2 + y^2 = 1$

$$x^2 = 1 - y^2$$

$$x = \pm\sqrt{1-y^2}$$

positive quadrant therefore we take $x = \sqrt{1-y^2}$ only.



$\therefore$ The required area $= \int_0^1 \int_0^{\sqrt{1-y^2}} xy\,dx\,dy$

$$= \int_0^1 y \left[x^2/2 \right]_0^{\sqrt{1-y^2}} dy$$

$$= \int_0^1 y \frac{(1-y^2)}{2} dy$$

$$= 1/2 \int_0^1 (y - y^3)\,dy$$

$$= 1/2 \left[y^2/2 - y^4/4 \right]_0^1$$

$$= 1/2 \left[1/2 - 1/4 \right]$$

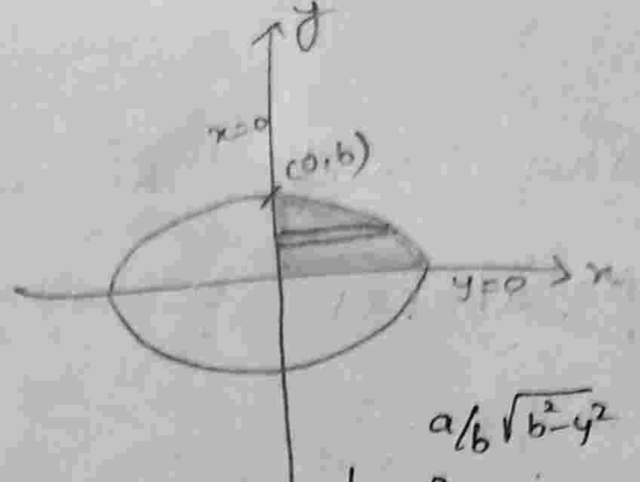
$$= 1/2 \left[\frac{2-1}{4} \right]$$

$$= 1/2 (1/4)$$

$$= 1/8 \,//.$$

find the area of the ellipse $x^2/a^2 + y^2/b^2 = 1$.

Sol:- Area of ellipse $= 4 \times$ area of quadrant,



$$\therefore \text{The required area} = 4\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} dx\, dy$$

$$= 4\int_0^b \left[x\right]_0^{\frac{a}{b}\sqrt{b^2-y^2}} dy$$

$$= 4\int_0^b \left[\frac{a}{b}\sqrt{b^2-y^2}\right] dy$$

$$= \frac{4a}{b}\int_0^b \sqrt{b^2-y^2}\, dy$$

$$= \frac{4a}{b}\left[\frac{b^2}{2}\sin^{-1}\frac{y}{b} + \frac{y}{2}\sqrt{b^2-y^2}\right]_0^b$$

$$= \frac{4a}{b}\left[\left(\frac{b^2}{2}\frac{\pi}{2}+0\right) - (0+0)\right]$$

$$= \frac{\cancel{4}a}{\cancel{b}}\frac{b^{\cancel{2}}}{\cancel{2}}\frac{\pi}{\cancel{2}}$$

$$= \pi ab$$

Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $16/3\, a^2$.

Sol:- Given $y^2 = 4ax \longrightarrow (1)$

$x^2 = 4ay \longrightarrow (2)$

Sub (1) in (2) we get

$$\left(\frac{y^2}{4a}\right)^2 = 4ay$$

$$\frac{y^4}{16a^2} = 4ay$$

$$y^4 = 64a^3y$$

$$y^4 - 64a^3y = 0$$

$$y(y^3 - 64a^3) = 0$$

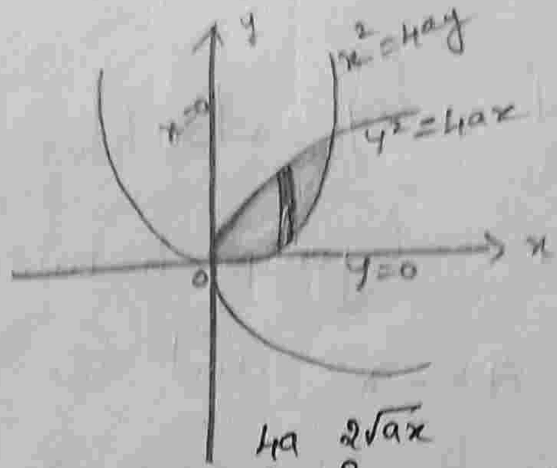
$$y = 0, \quad y^3 = 64a^3$$

$$y = 4a$$

$y = 0 \Rightarrow x = 0$

$y = 4a \Rightarrow x = 4a$

Therefore the point of intersection of (1) and (2) is $(0,0)$ and $(4a, 4a)$.



$\therefore$ The required area $= \displaystyle\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\, dx$

$$= \int_0^{4a} \left[y \right]_{x^2/4a}^{2\sqrt{ax}} dx$$

$$= \int_0^{4a} \left[2\sqrt{ax} - x^2/4a \right] dx$$

$$= \int_0^{4a} \left[2\sqrt{a}\, x^{1/2} - x^2/4a \right] dx$$

$$= \left[2\sqrt{a} \frac{x^{3/2}}{3/2} - 1/4a\; x^3/3 \right]_0^{4a}$$

$$= \left[\frac{4\sqrt{a}}{3} x^{3/2} - \frac{1}{12a} x^3 \right]_0^{4a}$$

$$= \frac{4\sqrt{a}}{3} (4a)^{3/2} - 1/12a\; (4a)^3$$

$$= \frac{4\sqrt{a}}{3} (2\sqrt{a})^3 - \frac{64a^3}{12a}$$

$$= \frac{32a^2}{3} - \frac{32a^2}{6} = \frac{32a^2}{3} - 16/3\, a^2$$

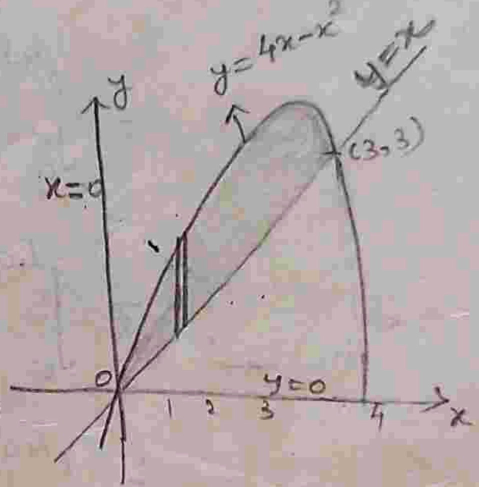
$$= 16/3\, a^2$$

Find by double integration, the area lying between the parabolas $y = 4x - x^2$ and the line $y = x$.

<u>Sol:-</u>

$$y = x \longrightarrow (1)$$

$$y = 4x - x^2 \longrightarrow (2)$$



Sub (1) in (2) we get

$$x = 4x - x^2$$

$$x^2 = 4x - x$$

$$x^2 = 3x$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$x = 0, \ x = 3$$

$$x = 0 \Rightarrow y = 0$$

$$x = 3 \Rightarrow y = 3$$

Therefore the point of intersection of (1) and (2) is $(0,0)$ and $(3,3)$.

$\therefore$ The required area $= \displaystyle\int_0^3 \int_x^{4x-x^2} dy\, dx$

$$= \int_0^3 (y)_x^{4x-x^2}\, dx$$

$y = 4x - x^2$

$x: -2 \ -1 \ 0 \ 1 \ 2$

$y:$

$$= \int_0^3 (4x - x^2 - x)\, dx$$

$$= \int_0^3 (3x - x^2)\, dx$$

$$= \left[3x^2/2 - x^3/3 \right]_0^3$$

$$= 27/2 - 27/3$$

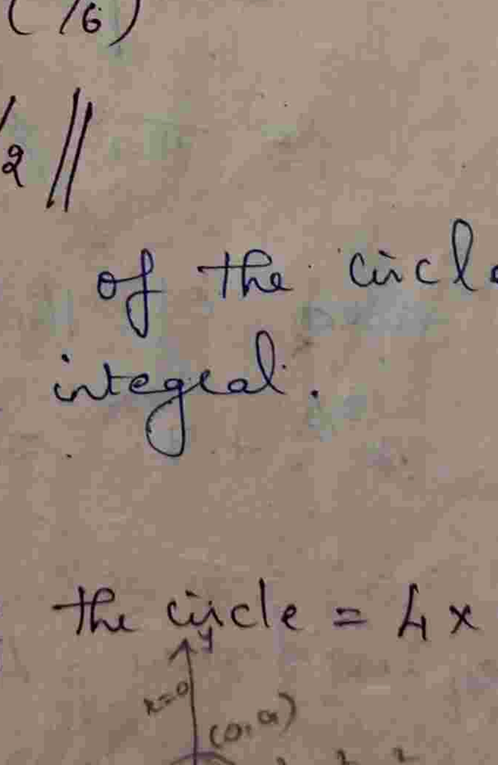
$$= 27(1/2 - 1/3)$$

$$= 27(1/6)$$

$$= 9/2 \,//$$

find the area of the circle $x^2 + y^2 = a^2$ using double integral.

Sol:- Area of the circle = 4 x area of quadrant



$$\text{Required area} = 4\int_0^a \int_0^{\sqrt{a^2-y^2}} dx\, dy$$

$$= 4\int_0^a [x]_0^{\sqrt{a^2-y^2}}\, dy$$

$$= 4\int_0^a \sqrt{a^2-y^2}\, dy$$

$$= 4\left[\frac{y}{2}\sqrt{a^2-y^2} + \frac{a^2}{2}\sin^{-1}\frac{y}{a}\right]_0^a$$

$$= 4\left[0 + \frac{a^2}{2}(\pi/2) - (0+0)\right]$$

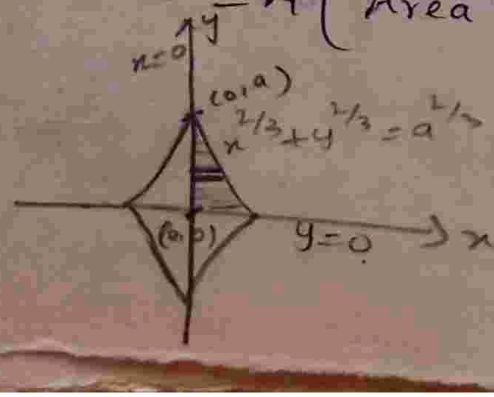
$$= \pi a^2$$

Find the area of the asteroid

$$x^{2/3} + y^{2/3} = a^{2/3}.$$

Sol:-

Required area $= 4$ (Area in I-quadrant)



$$\text{Required area} = 4\int_0^a \int_0^{(a^{2/3}-y^{2/3})^{3/2}} dx\, dy$$

$$= 4\int_0^a \left[x\right]_0^{(a^{2/3}-y^{2/3})^{3/2}} dy$$

$$= 4\int_0^a \left[a^{2/3}-y^{2/3}\right]^{3/2} dy$$

put $y = a\sin^3\theta$

$dy = 3a\sin^2\theta\cos\theta\, d\theta$

when $y=0,\ \theta=0$

when $y=a,\ \theta=\pi/2$

$$= 4\int_0^{\pi/2}\left[a^{2/3}-(a\sin^3\theta)^{2/3}\right]^{3/2} 3a\sin^2\theta\cos\theta\, d\theta$$

$$= 4\int_0^{\pi/2}\left(a^{2/3}-a^{2/3}\sin^2\theta\right)^{3/2} 3a\sin^2\theta\cos\theta\, d\theta$$

$$= 12a\int_0^{\pi/2}(a^{2/3})^{3/2}(1-\sin^2\theta)^{3/2}\sin^2\theta\cos\theta\, d\theta$$

$$= 12a\int_0^{\pi/2} a\cos^3\theta\sin^2\theta\cos\theta\, d\theta$$

$$= 12a^2 \int_0^{\pi/2} \cos^4\theta \sin^2\theta \, d\theta$$

$$= 12a^2 \int_0^{\pi/2} \cos^4\theta (1-\cos^2\theta) d\theta$$

$$= 12a^2 \left[\int_0^{\pi/2} \cos^4\theta \, d\theta - \int_0^{\pi/2} \cos^6\theta \, d\theta \right]$$

$$= 12a^2 \left[\frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} - \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} \right]$$

$$= \cancel{12}^{3} a^2 \cdot \frac{3}{\cancel{4}} \cdot \frac{1}{2} \cdot \frac{\pi}{2} \left[1 - \frac{5}{6} \right]$$

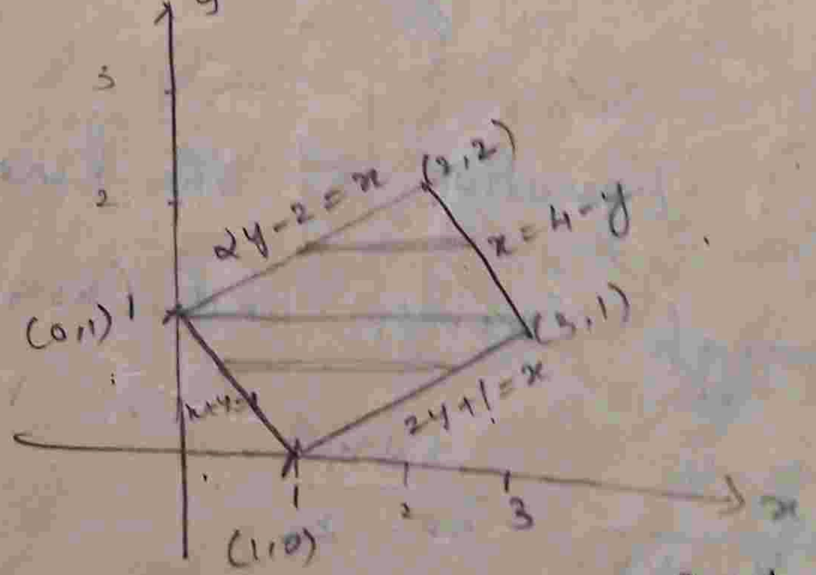
$$= \frac{9a^2\pi}{4} \left(\frac{1}{6} \right)$$

$$= \frac{3\pi a^2}{8} \quad //$$

Find the area of the parallelogram whose vertices are given as $A(1,0)$, $B(3,1)$, $C(2,2)$ and $D(0,1)$ using double integral.

If the area is found using a strip parallel to y-axis we need to consider 3* different regions; whereas if we use a strip parallel to x-axis, it is sufficient to consider two different regions namely DAB and DCB.

Equations of the lines DA, AB, DC and CB are respectively $x+y=1$, $2y+1=x$, $2y-2=x$ and $x=4-y$.



$$\text{Required area} = \int_0^1 \int_{1-y}^{1+2y} dx\,dy + \int_1^2 \int_{2y-2}^{4-y} dx\,dy$$

$$= \int_0^1 [(1+2y) - (1-y)]\,dy + \int_1^2 [(4-y) - (2y-2)]\,dy$$

$$= \int_0^1 3y\,dy + \int_1^2 (6-3y)\,dy$$

$$= 3\left(\frac{y^2}{2}\right)_0^1 + \left(6y - \frac{3y^2}{2}\right)_1^2$$

$$= \frac{3}{2} + \left(12 - \cancel{\frac{12}{2}}^{6} - 6 + \frac{3}{2}\right)$$

$$= \frac{3}{2} + \left(12 - 12 + \frac{3}{2}\right)$$

$$= \frac{3}{2} + \frac{3}{2}$$

$$= 3 \,//.$$

<u>Note</u>: Equation of the line passing through (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}.$$

* The 3 regions are $0 \le x \le 1$, $1 \le x \le 2$, $2 \le x \le 3$.

Evaluation of Double integrals in polar Co-ordinates :-

(1) Evaluate $\int_0^{\pi/2}\left[\int_0^{a\cos\theta} r\sqrt{a^2-r^2}\,dr\right]d\theta$.

Sol: $\int_0^{\pi/2}\left[\int_0^{a\cos\theta} r\sqrt{a^2-r^2}\,dr\right]d\theta = \int_0^{\pi/2}\left[\int_0^{a\cos\theta} -\frac{1}{2}(a^2-r^2)^{1/2}(-2r)dr\right]d\theta$

$$= \int_0^{\pi/2}\left[-\frac{1}{2}\frac{(a^2-r^2)^{3/2}}{3/2}\right]_0^{a\cos\theta} d\theta$$

$$= -\frac{1}{3}\int_0^{\pi/2}\left[(a^2-a^2\cos^2\theta)^{3/2}-(a^2-0)^{3/2}\right]d\theta$$

$$= -\frac{1}{3}\left[\int_0^{\pi/2}\left[a^3\sin^3\theta - a^3\right]d\theta\right]$$

$$= -\frac{1}{3}\left[\int_0^{\pi/2} a^3\sin^3\theta\, d\theta - \int_0^{\pi/2} a^3\, d\theta\right]$$

$$= -\frac{1}{3}\left[a^3\, 2/3 - a^3\,\pi/2\right]$$

$$= -\frac{a^3}{3}\left[2/3 - \pi/2\right] = \frac{a^3}{3}\left(\pi/2 - 2/3\right)$$

$$= \frac{a^3}{18}(3\pi - 4)$$

(7) Evaluate $\int_0^{\pi}\int_0^{a(1-\cos\theta)} r^2 \sin\theta \, dr \, d\theta$.

Sol:

$$\int_0^{\pi}\int_0^{a(1-\cos\theta)} r^2\sin\theta \, dr\, d\theta = \int_0^{\pi}\left[\frac{r^3}{3}\sin\theta\right]_0^{a(1-\cos\theta)} d\theta$$

$$= \int_0^{\pi} \frac{a^3(1-\cos\theta)^3}{3}\sin\theta \, d\theta$$

$$= \frac{a^3}{3}\int_0^{\pi}(1-\cos\theta)^3 \sin\theta \, d\theta.$$

put $t = 1-\cos\theta$

$dt = \sin\theta \, d\theta$

when $\theta = 0$, $t = 0$

when $\theta = \pi$, $t = 2$

$$= \frac{a^3}{3}\int_0^{2} t^3 \, dt$$

$$= \frac{a^3}{3}\left(\frac{t^4}{4}\right)_0^2$$

$$= \frac{a^3}{12}(16) = \frac{4a^3}{3}$$

③ Evaluate $\int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta$.

Sol:

$$\int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta = \int_0^{\pi}\left(\frac{r^2}{2}\right)_0^{a\sin\theta} d\theta$$

$$= \int_0^{\pi} \frac{a^2\sin^2\theta}{2}\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi} a^2\sin^2\theta\,d\theta$$

$$= \frac{2}{2}\int_0^{\pi/2} a^2\sin^2\theta\,d\theta$$

$$= a^2\int_0^{\pi/2}\sin^2\theta\,d\theta$$

$$= a^2\left(\frac{1}{2}\cdot\frac{\pi}{2}\right)$$

$$= \frac{\pi a^2}{4}$$

$$\left[\ \int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx, \text{ if } f(2a-x) = f(x)\right.$$

$$= 0, \text{ if } f(2a-x) = -f(x)$$

Here $\sin(\pi-\theta) = \sin\theta$

(4) Evaluate $I = 2\int_0^{\pi}\int_0^{a(1+\cos\theta)} r\,dr\,d\theta$.

Sol:.

$$I = 2\int_0^{\pi}\int_0^{a(1+\cos\theta)} r\,dr\,d\theta$$

$$= 2\int_0^{\pi}\left(\frac{r^2}{2}\right)_0^{a(1+\cos\theta)} d\theta$$

$$= 2\int_0^{\pi}\left(\frac{a^2(1+\cos\theta)^2}{2}\right)d\theta$$

$$= a^2\int_0^{\pi}(1+2\cos\theta+\cos^2\theta)\,d\theta$$

$$= a^2\int_0^{\pi}\left[1+2\cos\theta+\left(\frac{1+\cos 2\theta}{2}\right)\right]d\theta$$

$$= a^2\int_0^{\pi} \frac{3}{2}\,d\theta + 2\cos\theta\,d\theta + \frac{\cos 2\theta}{2}\,d\theta$$

$$= a^2\left(\frac{3}{2}\theta + 2\sin\theta + \frac{\sin 2\theta}{4}\right)_0^{\pi}$$

$$= a^2\left(\frac{3}{2}\pi + 0 + 0 - 0\right)$$

$$= \frac{3\pi a^2}{2}$$ //.

If the equation of a circle is given in the form $x^2+y^2+2gx+2fy+c=0$ then the following rules should be kept in mind.

(i) Centre $= \left(-\frac{1}{2}(\text{coefficient of } x), -\frac{1}{2}(\text{coefficient of } y)\right)$

(ii) Radius $= \sqrt{\left(\frac{1}{2}\text{ Coefficient of } x\right)^2+\left(\frac{1}{2}\text{ Coefficient of } y\right)^2-\text{Constant}}$.

For example, the centre and radius of the circle $x^2+y^2-8x-12y-48=0$ are $(4,6)$ and 10, respectively.

Reduction formula

$$\int_0^{\pi/2} \sin^m\theta \, d\theta = \int_0^{\pi/2} \cos^m\theta \, d\theta =$$

$$\begin{cases} \dfrac{m-1}{m} , \dfrac{m-3}{m-2} , \dfrac{m-5}{m-4} , \ldots \dfrac{6}{7} \dfrac{4}{5} \dfrac{2}{3} 1 & (m \text{ is odd}) \\[2ex] \dfrac{m-1}{m} . \dfrac{m-3}{m-2} \dfrac{m-5}{m-4} . \ldots \dfrac{5}{6} \dfrac{3}{4} \dfrac{1}{2} \dfrac{\pi}{2} & m \text{ is even} \end{cases}$$

Parametric form of Standard curves:

Curve	Parametric form
① parabola $y^2 = 4ax$ (Symmetric about x axis)	$x = at^2$, $y = 2at$
② circle $x^2 + y^2 = a^2$	$x = a\cos\theta$, $y = a\sin\theta$
③ Ellipse $x^2/a^2 + y^2/b^2 = 1$	$x = a\cos\theta$, $y = b\sin\theta$
④ hyperbola $x^2/a^2 - y^2/b^2 = 1$	$x = a\sec\theta$, $y = b\tan\theta$
⑤ Rectangular hyperbola $xy = c^2$	$x = ct$, $y = c/t$
⑥ parabola $x^2 = 4ay$ (Symmetric about y axis)	$x = 2at$, $y = at^2$
⑦ Ostrick: $x^{2/3} + y^{2/3} = a^{2/3}$	$x = a\cos^3\theta$, $y = a\sin^3\theta$