Basic Electrical and Electronics Engineering

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LECTURE 2.5

Steady State AC analysis of a RL, RC, RLC Series circuits

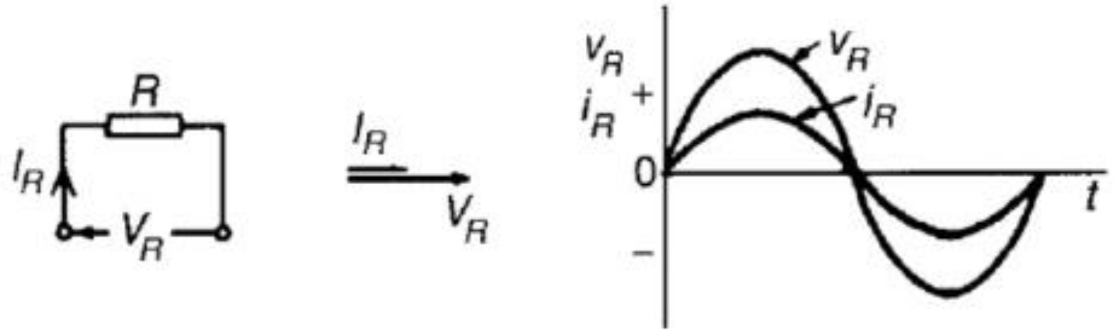
Single Phase Series AC Circuits

R - Resistance
L - Inductance
C - Capacitance
RL - Series Resistance and Inductance
RC - Series Resistance and Capacitance
RLC - Series Resistance, Inductance and Capacitance

AC Circuit consisting Resistance only

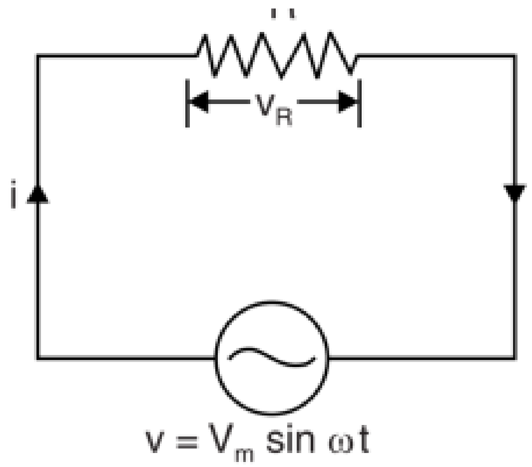
R - Resistance (Circuit with Resistor)

In a purely resistive a.c. circuit, the current i_R and applied voltage V_R are in phase.



Circuit diagram | Phasor diagram | Current and voltage waveforms

AC Circuit consisting Resistance only



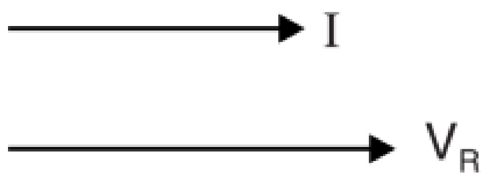
Consider a circuit containing a pure resistance of R Ω connected across an alternating voltage source

$$v = V_m \sin \omega t$$

$$v = iR \qquad i = \frac{v}{R} \qquad i = \frac{V_m}{R} \sin \omega t \qquad I_m = \frac{V_m}{R}$$

$$i = I_m \sin \omega t$$

$$\frac{V_m}{\sqrt{2}} = \frac{I_m}{\sqrt{2}} R \qquad V = V_R = IR$$



$$v = V_m \sin \omega t \qquad i = I_m \sin \omega t$$

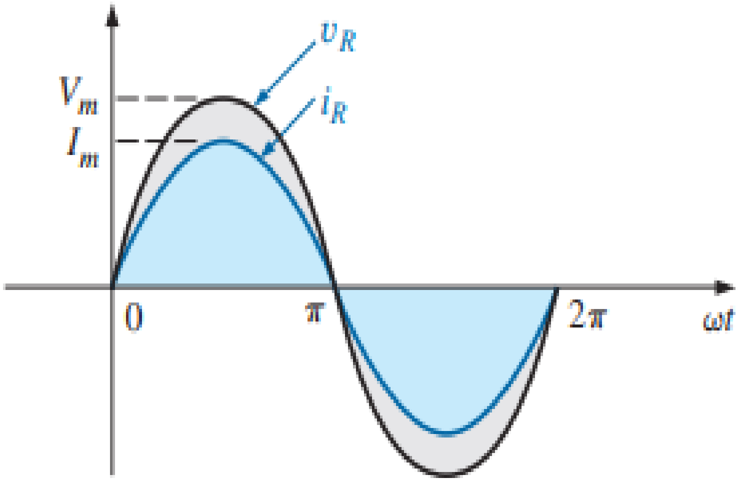
The applied voltage and the circuit current are in phase with each other

AC Circuit consisting Resistance only

$$v = iR = (I_m \sin \omega t)R = I_m R \sin \omega t = V_m \sin \omega t$$

where $$\boxed{V_m = I_m R}$$

for a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.



The voltage and current of a resistive element are in phase.

AC Circuit consisting Inductance only

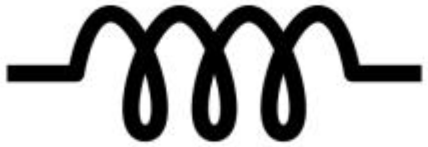
- An inductor is a passive element designed to store energy in its magnetic field.

- Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

The current-voltage relationship is

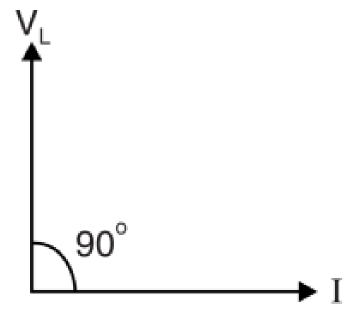
$$v = L\frac{di}{dt} \qquad i = \frac{1}{L}\int v\,dt$$

where L -inductance



AC Circuit consisting Inductance only

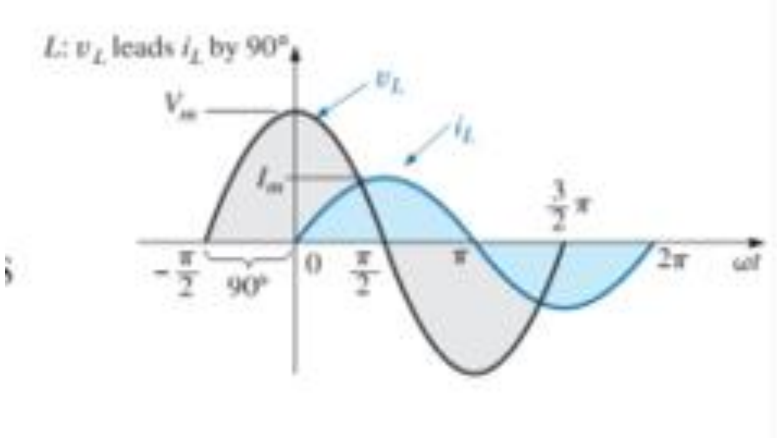
With, $i_L = I_m \sin(\omega t)$ (1) $\quad v_l = L\dfrac{di_L}{dt}$



$$v_l = L\frac{d}{dt}(I_m \sin(\omega t)) = \omega L I_m \cos \omega t$$

$$v_l = V_m \sin(\omega t + 90°) \ (2)$$

Voltage Leads the current by an angle 90° or Current lags behind the voltage by π/2 radians or 90º. *Hence in a pure inductance, current lags the voltage by 90º.*

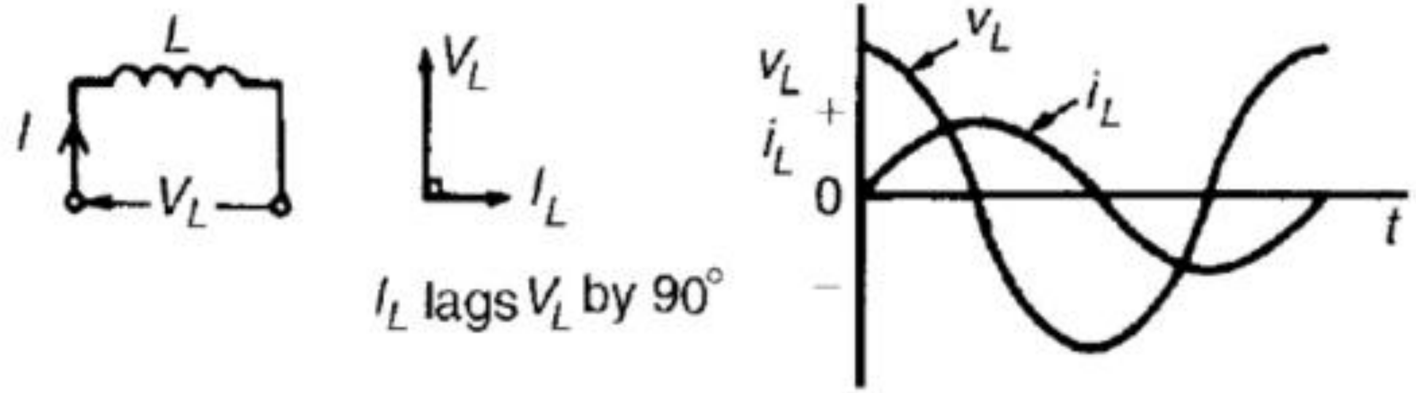


For a pure inductor, the voltage across the coil leads the current through the coil by 90°.

Inductive reactance. $\quad I_m = \dfrac{V_m}{\omega L} \quad \dfrac{V_m}{I_m} = \omega L \quad X_L = \omega L = 2\pi f L$

L - Inductance (a.c circuit with inductor)

In a purely inductive a.c. circuit, the current I_L lags the applied voltage V_L by 90^0 ($\pi/2$) rads



Circuit diagram

Phasor diagram

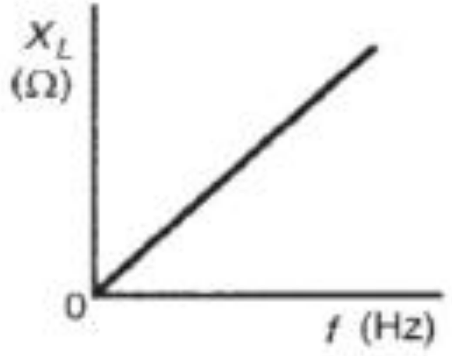
Current and voltage waveforms

In a purely inductive circuit the opposition to the flow of alternating current is called the inductive reactance (X_L)

L - Inductance (a.c circuit with inductor)

$$X_L = \frac{V_L}{I_L} = 2\pi f L \quad \Omega$$

f is the supply frequency, in hertz.
L is the inductance, in henrys.
X_L is proportional to frequency.



$$v = V\angle 0°$$

$$i = I\angle -90°$$

$$X_L = \frac{v}{i} = \frac{V\angle 0°}{I\angle -90°} = X_L\angle 90°$$

$Convert\ X_L\angle 90°$ in rectangular coordinate

$$X_L\angle 90° = jX_L$$

Problem 1

(a) Calculate the reactance of a coil of inductance 0.32 H when it is connected to a 50Hz supply.
(b) A coil has a reactance of 124 Ω in a circuit with a supply of frequency 5kHz. Determine the inductance of the coil.

(a) Inductive reactance, $X_L = 2\pi f L$

$$= 2\pi(50)(0.32)$$

$$= \mathbf{100.5\ \Omega}$$

(b) Since $X_L = 2\pi f L$, inductance

$$L = \frac{X_L}{2\pi f}$$

$$= \frac{124}{2\pi(5000)}\text{H}$$

$$= \mathbf{3.95\ mH}$$

Problem 2

A coil has an inductance of 40mH and negligible resistance. Calculate its inductive reactance and the resulting current if connected to (a) a 240V, 50Hz supply, and (b) a 100V, 1kHz supply.

(a) Inductive reactance, $X_L = 2\pi fL$

$$= 2\pi(50)(40 \times 10^{-3})$$

$$= \mathbf{12.57\ \Omega}$$

$$\text{Current, } I = \frac{V}{X_L}$$

$$= \frac{240}{12.57} = \mathbf{19.09\ A}$$

(b) Inductive reactance, $X_L = 2\pi(1000)(40 \times 10^{-3})$

$$= \mathbf{251.3\ \Omega}$$

$$\text{Current, } I = \frac{V}{X_L}$$

$$= \frac{100}{251.3} = \mathbf{0.398\ A}$$

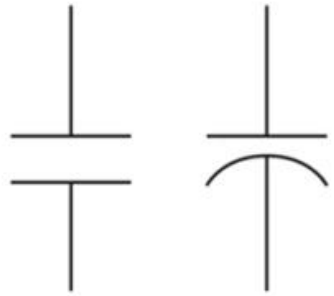
AC Circuit consisting Capacitance only

- A capacitor consists of two metallic surfaces or conducting surfaces separated by a dielectric medium.
- Capacitance is the proportionality constant relating the charge on the conducting plates to the potential. measured in farads (F)

The current-voltage relationship is

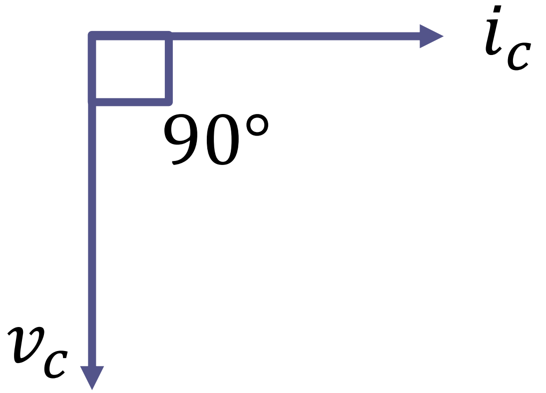
$$i = C\frac{dv}{dt} \qquad v = \frac{1}{C}\int_{-\infty}^{t} i\, dt$$

where, C –Capacitance



AC Circuit consisting Capacitance only

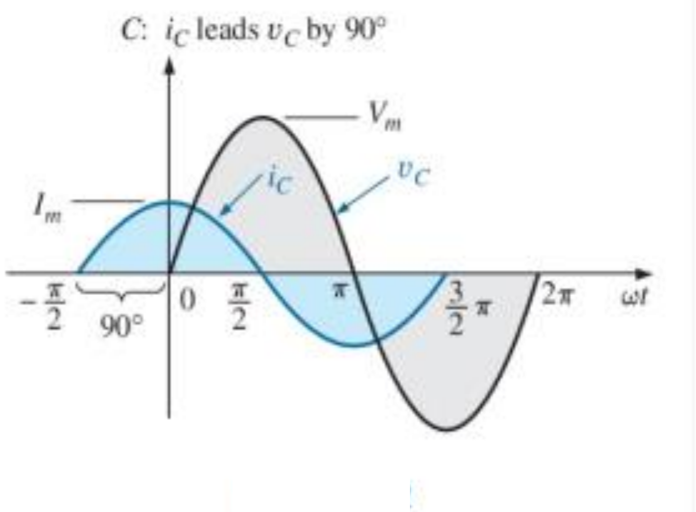
With, $v_C = V_m \sin(\omega t)$ (1) $\quad i_C = C\dfrac{dv_C}{dt}$



$$i_C = C\frac{d}{dt}(V_m \sin(\omega t)) = \omega C V_m \cos \omega t$$

$$i_C = I_m \sin(\omega t + 90°) \quad (2)$$

Current Leads the Voltage by an angle 90° or Voltage lags the current by π/2 radians or 90º. *Hence in a pure Capacitive circuit, the current leads the voltage by 90º.*



The current of a purely capacitive element leads the voltage across the element by 90°.

Capacitive reactance

$$I_m = \omega C V_m \qquad \frac{V_m}{I_m} = \frac{1}{\omega C} = \frac{V_C}{I} \qquad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

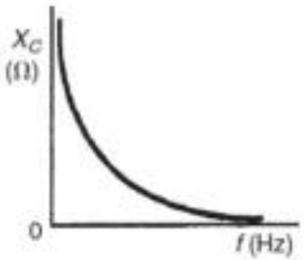
AC Circuit consisting Capacitance only

C - Capatiance (a.c circuit with capacitor)

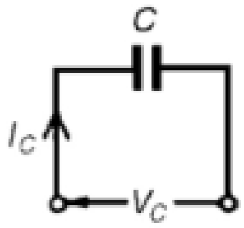
In a purely capacitive a.c. circuit, the current IC leads the applied voltage VC by 90◦ (i.e. π/2 rads)

In a purely capacitive circuit the opposition to the flow of alternating current is called the capacitive reactance, X_C

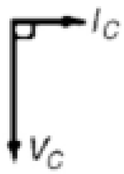
$$X_C = \frac{V_C}{I_C} = \frac{1}{2\pi f C}\,\Omega$$



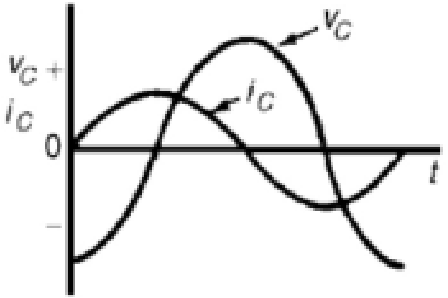
C is the capacitance in farads.
X_C varies with frequency f.



Circuit diagram



Phasor diagram



Current and voltage waveforms

Problem 3

Determine the capacitive reactance of a capacitor of 10μF when connected to a circuit of frequency (a) 50Hz (b) 20kHz.

(a) Capacitive reactance $X_C = \frac{1}{2\pi f C}$

$$= \frac{1}{2\pi(50)(10 \times 10^{-6})}$$

$$= \frac{10^6}{2\pi(50)(10)}$$

$$= \mathbf{318.3\,\Omega}$$

(b) $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(20 \times 10^3)(10 \times 10^{-6})}$

$$= \frac{10^6}{2\pi(20 \times 10^3)(10)}$$

$$= \mathbf{0.796\,\Omega}$$

Problem 4

A capacitor has a reactance of 40Ω when operated on a 50Hz supply. Determine the value of its capacitance.

Since $X_C = \frac{1}{2\pi f C}$, capacitance $C = \frac{1}{2\pi f X_C}$

$$= \frac{1}{2\pi(50)(40)} \text{F}$$

$$= \frac{10^6}{2\pi(50)(40)} \mu\text{F}$$

$$= \mathbf{79.58\,\mu F}$$

Problem 5

Calculate the current taken by a 23μF capacitor when connected to a 240V, 50Hz supply.

$$\text{Current } I = \frac{V}{X_C} = \frac{V}{\left(\frac{1}{2\pi f C}\right)}$$

$$= 2\pi f C V$$

$$= 2\pi(50)(23 \times 10^{-6})(240)$$

$$= \mathbf{1.73\,A}$$

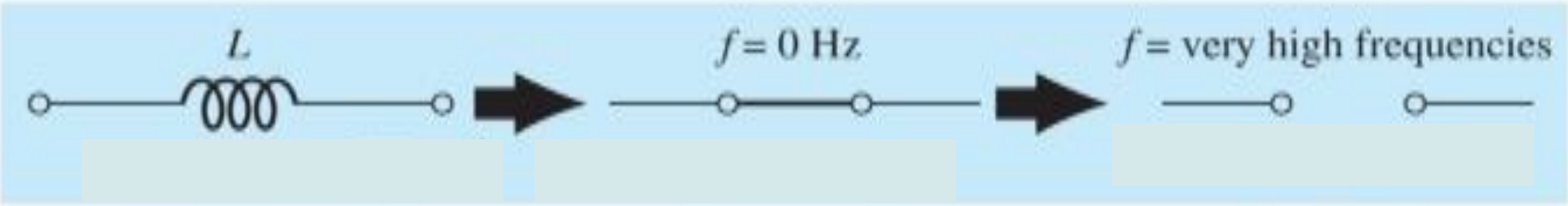
Inductor behavior at low and high frequency

In particular, note that at $f = 0$ Hz, the reactance of each plot is zero ohms as determined by substituting $f = 0$ Hz into the basic equation for the reactance of an inductor:

$$X_L = 2\pi f L = 2\pi(0\text{ Hz})L = 0\ \Omega$$

Since a reactance of zero ohms corresponds with the characteristics of a short circuit, we can conclude that

at a frequency of 0 Hz, an inductor takes on the characteristics of a short circuit, as shown in Fig. 14.21.



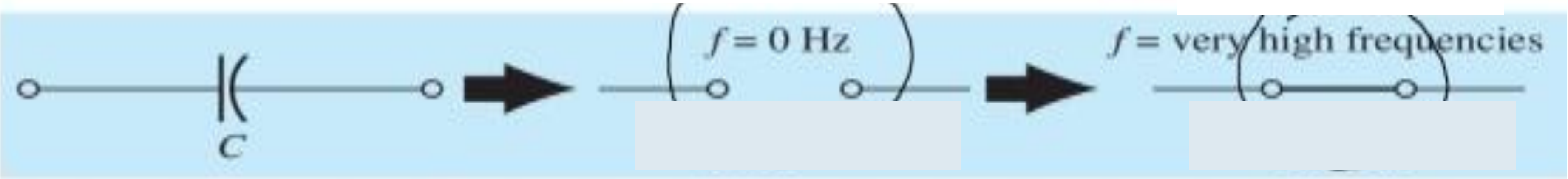
Capacitor behavior at low and high frequency

At or near 0 Hz, the reactance of any capacitor is extremely high, as determined by the basic equation for capacitance:

$$X_C = \frac{1}{2\pi f c} = \frac{1}{2\pi(0\ \text{Hz})C} = \infty\Omega$$

The result is that

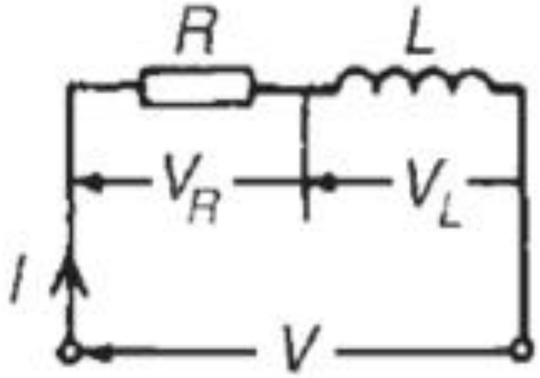
at or near 0 Hz, the characteristics of a capacitor approach those of an open circuit,

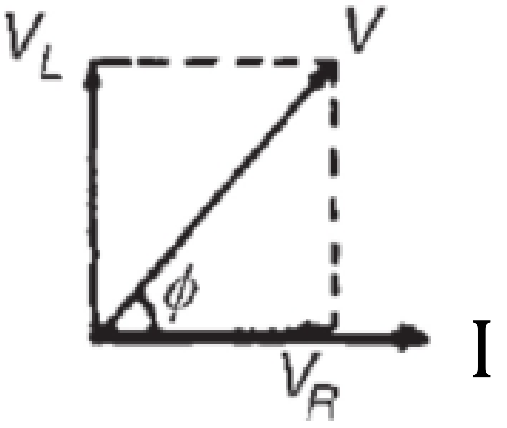


Single Phase Series AC Circuits

RL Series Circuit

In an a.c. circuit containing inductance L and resistance R, the applied voltage V is the phasor sum of V_R and V_L





Current I lags the applied voltage V by an angle lying between 0◦ and 90◦ (depending on the values of VR and VL)

Single Phase Series RL Circuits

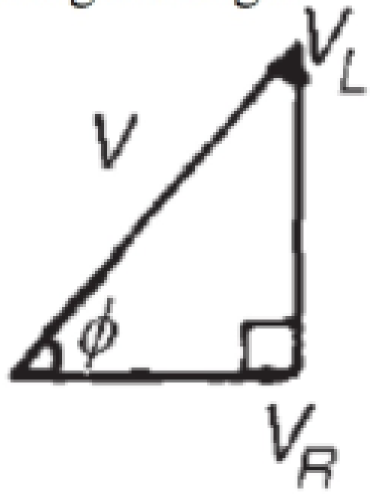
$V = \sqrt{(V_R^2 + V_L^2)}$ (by Pythagoras' theorem)

$\tan\phi = \dfrac{V_L}{V_R}$ (by trigonometric ratios)

In an a.c. circuit, the ratio $\dfrac{\text{applied voltage } V}{\text{current } I}$ is called the **impedance Z**, i.e.

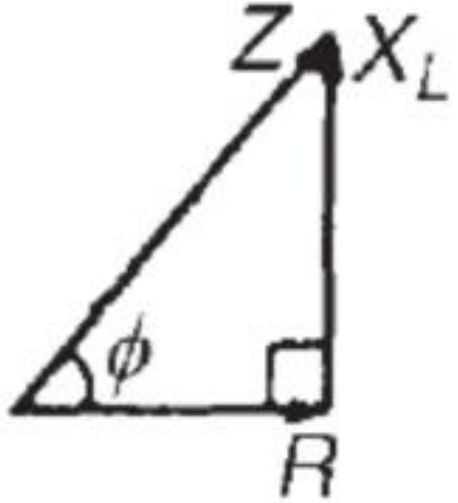
$$Z = \frac{V}{I}\ \Omega$$

Voltage triangle

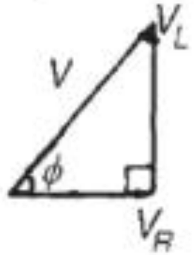


Impedance Triangle for Series RL Circuits

Each side of the voltage triangle is divided by current I then the ‘impedance triangle’ is derived.



Voltage triangle



For the R–L circuit: $Z = \sqrt{(R^2 + X_L^2)}$ (by Pythagoras’ theorem)

$\tan\phi = \dfrac{X_L}{R}$, $\sin\phi = \dfrac{X_L}{Z}$ and $\cos\phi = \dfrac{R}{Z}$ (by trigonometric ratios)

Problem 6

In a series R–L circuit the p.d. across the resistance R is 12V and the p.d. across the inductance L is 5V. Find the supply voltage and the phase angle between current and voltage.

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Solution

From the voltage triangle of Figure 15.6,

supply voltage $V = \sqrt{(12^2 + 5^2)}$ i.e. $\boldsymbol{V = 13\,V}$

(Note that in a.c. circuits, the supply voltage is **not** the arithmetic sum of the p.d.'s across components. It is, in fact, the **phasor sum**.)

$$\tan\phi = \frac{V_L}{V_R} = \frac{5}{12}, \text{ from which } \phi = \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \mathbf{22.62° \ lagging}$$

('Lagging' infers that the current is 'behind' the voltage, since phasors revolve anticlockwise.)

Problem 7

A coil has a resistance of 4 ohms and an inductance of 9.55mH. Calculate (a) the reactance, (b) the impedance, and (c) the current taken from a 240V, 50Hz supply. Determine also the phase angle between the supply voltage and current.

$R = 4\,\Omega$; $L = 9.55\,\text{mH} = 9.55 \times 10^{-3}\,\text{H}$; $f = 50\,\text{Hz}$; $V = 240\,\text{V}$

Solution

(a) Inductive reactance, $X_L = 2\pi fL$

$$= 2\pi(50)(9.55 \times 10^{-3})$$

$$= \mathbf{3\ \Omega}$$

(b) Impedance, $Z = \sqrt{(R^2 + X_L^2)} = \sqrt{(4^3 + 3^2)} = \mathbf{5\ \Omega}$

(c) Current, $I = \dfrac{V}{Z} = \dfrac{240}{5} = \mathbf{48\ A}$

$$\text{Since } \tan\phi = \frac{X_L}{R},\ \phi = \tan^{-1}$$

$$\frac{X_L}{R} = \tan^{-1}\frac{3}{4}$$

$$= \mathbf{36.87^\circ\ lagging}$$

Problem 8

A coil takes a current of 2A from a 12V d.c. supply.
When connected to a 240V, 50Hz supply the current is 20A.
Calculate the resistance, impedance, inductive reactance and inductance ofthe coil.

Problem 8

A coil takes a current of 2A from a 12V d.c. supply. When connected to a 240V, 50Hz supply the current is 20A. Calculate the resistance, impedance, inductive reactance and inductance ofthe coil.

Solution

$$\text{Resistance } R = \frac{\text{d.c. voltage}}{\text{d.c. current}} = \frac{12}{2} = 6\,\Omega$$

$$\text{Impedance } Z = \frac{\text{a.c. voltage}}{\text{a.c. current}} = \frac{240}{20} = 12\,\Omega$$

Since $Z = \sqrt{(R^2 + X_L^2)}$, inductive reactance,

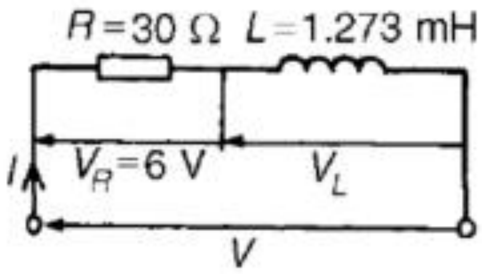
$$X_L = \sqrt{(Z^2 - R^2)} = \sqrt{(12^2 - 6^2)} = 10.39\,\Omega$$

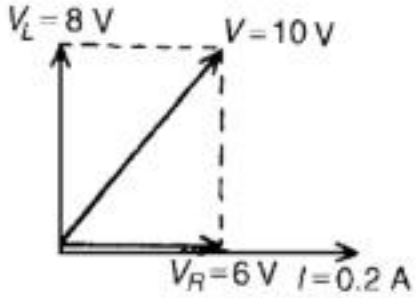
$$\text{Since } X_L = 2\pi f L, \text{ inductance } L = \frac{X_L}{2\pi f} = \frac{10.39}{2\pi(50)} = \mathbf{33.1\ mH}$$

Problem 9

A pure inductance of 1.273mH is connected in series with a pure resistance of 30 Ohms. If the frequency of the sinusoidal supply is 5kHz and the p.d. across the 30 Ohms resistor is 6V, determine the value of the supply voltage and the voltage across the 1.273mH inductance. Draw the phasor diagram.

Solution





Supply voltage, $V = IZ$

Current $I = \dfrac{V_R}{R} = \dfrac{6}{30} = 0.20\,\text{A}$

Inductive reactance $X_L = 2\pi fL$

$$= 2\pi(5\times10^3)(1.273\times10^{-3})$$

$$= 40\,\Omega$$

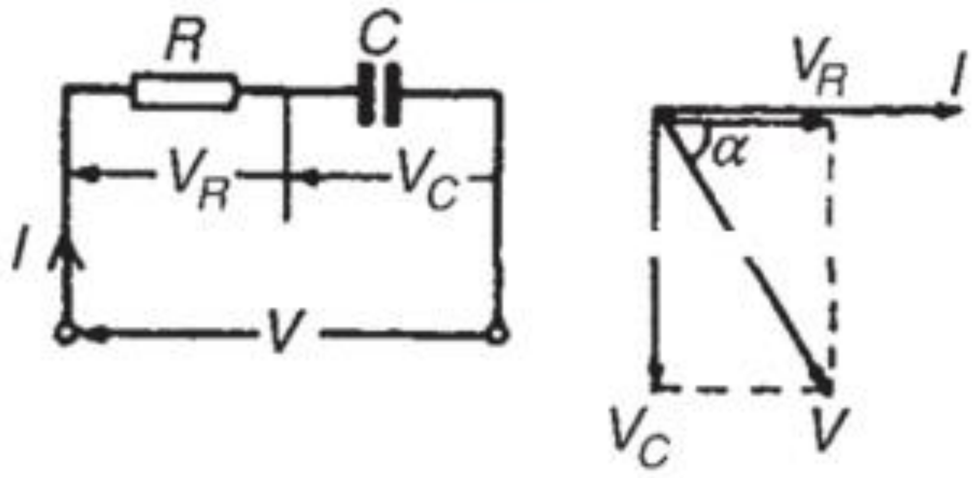
Impedance, $Z = \sqrt{(R^2 + X_L^2)} = \sqrt{(30^2 + 40^2)} = 50\,\Omega$

Supply voltage $V = IZ = (0.20)(50) = \mathbf{10\,V}$

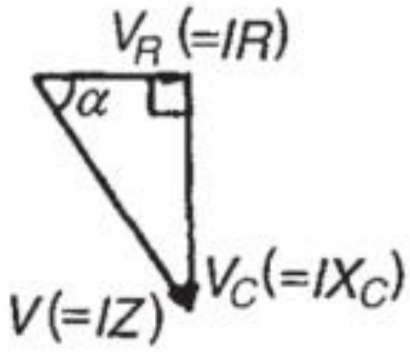
Voltage across the 1.273 mH inductance, $V_L = IX_L$

Single Phase Series RC AC Circuits

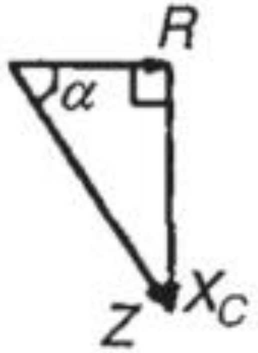
In an a.c. series circuit containing capacitance C and resistance R, the applied voltage V is the phasor sum of V_R and V_C and thus the current I leads the applied voltage V by an angle lying between 0° and 90° (depending on the values of V_R and V_C), shown as angle α.



Single Phase Series RC AC Circuits



voltage triangle



$$Z = \frac{V}{I}\ \Omega$$

impedance triangle

$$V = \sqrt{(V_R^2 + V_C^2)} \quad \text{(by Pythagoras' theorem)}$$

$$Z = \sqrt{(R^2 + X_C^2)}$$

$$\tan\alpha = \frac{X_C}{R}, \quad \sin\alpha = \frac{X_C}{Z} \quad \text{and} \quad \cos\alpha = \frac{R}{Z}$$

Problem 9

A resistor of 25Ohms is connected in series with a capacitor of 45μF. Calculate (a) the impedance, and (b) the current taken from a 240V, 50Hz supply. Find also the phase angle between the supply voltage and the current.

$$R = 25\,\Omega;\ C = 45\,\mu\text{F} = 45 \times 10^{-6}\,\text{F};\ V = 240\,\text{V};\ f = 50\,\text{Hz}$$

Solution

Capacitive reactance, $X_C = \frac{1}{2\pi f C}$

$$= \frac{1}{2\pi(50)(45 \times 10^{-6})}$$

$$= 70.74\,\Omega$$

(a) Impedance $Z = \sqrt{(R^2 + X_C^2)} = \sqrt{[(25)^2 + (70.74)^2]}$

$$= \mathbf{75.03\,\Omega}$$

(b) Current $I = \frac{V}{Z} = \frac{240}{75.03} = \mathbf{3.20\,A}$

Phase angle between the supply voltage and current,

$$\alpha = \tan^{-1}\left(\frac{X_C}{R}\right)$$

hence $\alpha = \tan^{-1}\left(\frac{70.74}{25}\right) = \mathbf{70.54° \text{ leading}}$

Problem 10

A capacitor C is connected in series with a 40 Ohms resistor across a supply of frequency 60Hz. A current of 3 A flows and the circuit impedance is 50 Ohms. Calculate: (a) the value of capacitance, C, (b) the supply voltage, (c) the phase angle between the supply voltage and current, (d) the p.d. across the resistor, and (e) the p.d. across the capacitor. Draw the phasor diagram.

Solution

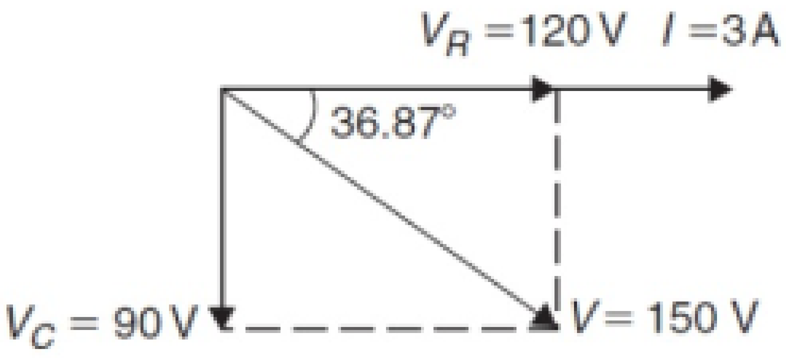
(a) Impedance $Z = \sqrt{(R^2 + X_C^2)}$

Hence $X_C = \sqrt{(Z^2 - R^2)} = \sqrt{(50^2 - 40^2)} = 30\,\Omega$

$$X_C = \frac{1}{2\pi f C}$$

$$\text{hence } C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60)30}\,\text{F}$$

$$= \mathbf{88.42\,\mu F}$$



Phasor diagram

(b) Since $Z = \dfrac{V}{I}$ then $V = IZ = (3)(50) = \mathbf{150\,V}$

(c) Phase angle, $\alpha = \tan^{-1}\dfrac{X_C}{R} = \tan^{-1}\left(\dfrac{30}{40}\right)$

$$= \mathbf{36.87^\circ\ leading}$$

(d) P.d. across resistor, $V_R = IR = (3)(40) = \mathbf{120\,V}$

(e) P.d. across capacitor, $V_C = IX_C = (3)(30) = \mathbf{90\,V}$

Problem 10

A capacitor C is connected in series with a 40 Ohms resistor across a supply of frequency 60Hz. A current of 3 A flows and the circuit impedance is 50 Ohms. Calculate: (a) the value of capacitance, C, (b) the supply voltage, (c) the phase angle between the supply voltage and current, (d) the p.d. across the resistor, and (e) the p.d. across the capacitor. Draw the phasor diagram.

A capacitor of capacitance 79·5 μ F is connected in series with a non-inductive resistance of 30 Ω across 100 V, 50 Hz supply. Find (i) impedance (ii) current (iii) phase angle and (iv) equation for the instantaneous value of current.

Solution. (*i*) Capacitive reactance, $X_C = \dfrac{1}{2\pi f C} = \dfrac{10^6}{2\pi \times 50 \times 79\cdot 5} = 40\ \Omega$

Circuit impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = \mathbf{50\ \Omega}$

(*ii*) Circuit current, $I = V/Z = 100/50 = \mathbf{2\ A}$

(*iii*) $\tan\phi = X_C/R = 40/30 = 1.33$

$\therefore$ Phase angle, $\phi = \tan^{-1} 1.33 = \mathbf{53°}$ ***lead***

(*iv*) $I_m = 2 \times \sqrt{2} = 2.828\ \text{A}$

$\omega = 2\pi f = 2\pi \times 50 = 314$ rad/sec.

$\therefore$ $i = \mathbf{2.828 \sin (314\, t + 53°)}$

The current in a 2·2 kΩ resistor is $i = 5\sin(2\pi \times 100t + 45°)mA$

(i) Write the mathematical expression for the voltage across the resistor.

(ii) What is the r.m.s. value of the resistor voltage ?

(iii) What is the instantaneous value of resistor voltage at t = 0·4 ms ?

Solution. $I_m = 5 \text{ mA} = 5 \times 10^{-3} \text{ A}; \ R = 2.2 \text{ k}\Omega = 2.2 \times 10^3 \ \Omega$

(*i*) $V_m = I_m R = 5 \times 10^{-3} \times 2.2 \times 10^3 = 11 \text{ V}$

Since voltage across the resistor R is in phase with current,

$\therefore \quad v = V_m \sin(2\pi \times 100\,t + 45°)$

or $\quad v = 11 \sin(2\pi \times 100\,t + 45°)\text{V}$ **Ans.**

(*ii*) R.M.S. value of resistor voltage, $V_{r.m.s.} \ \dfrac{V_m}{\sqrt{2}} = \dfrac{11}{\sqrt{2}} = \ = \mathbf{7.78 \ V}$

(*iii*) The instantaneous value of resistor voltage at t = 0·4 ms is

$$v(0{\cdot}4 \text{ ms}) = 11\sin[2\pi \times 100 \times 0{\cdot}4 \times 10^{-3} + 45°]\text{V}$$
$$= 11\sin[0{\cdot}2513 \text{ rad} + 45°]\text{V}$$
$$= 11\sin[*14{\cdot}4° + 45°]\text{ V} = 11\sin 59{\cdot}4° = \mathbf{9{\cdot}47 \ V}$$

Impedance

The impedance **Z** of a circuit is the ratio of the phasor voltage **V** to the phasor current **I**, measured in ohms (Ω).

$$Z = \frac{V}{I}$$

Resistive Circuit

$$v = V_m \sin \omega t$$

$$i = I_m \sin \omega t$$

$$v = iR$$

$$V_m \angle 0^o = I_m \angle 0^0 R$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \boldsymbol{R}$$

$$Z = R$$

Inductive Circuit

$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t - \pi/2)$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V_m \angle 0^o}{I_m \angle -90^0}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\omega L\, I_m \angle 0^o}{I_m \angle -90^0}$$

$$Z = j\omega L$$

$$Z = jX_L$$

Capacitive Circuit

$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t + \pi/2)$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V_m \angle 0^o}{I_m \angle 90^0}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{(1/\omega C)\, I_m \angle 0^o}{I_m \angle 90^0}$$

$$Z = \frac{1}{j\omega C}$$

$$Z = -jX_C$$

In General,

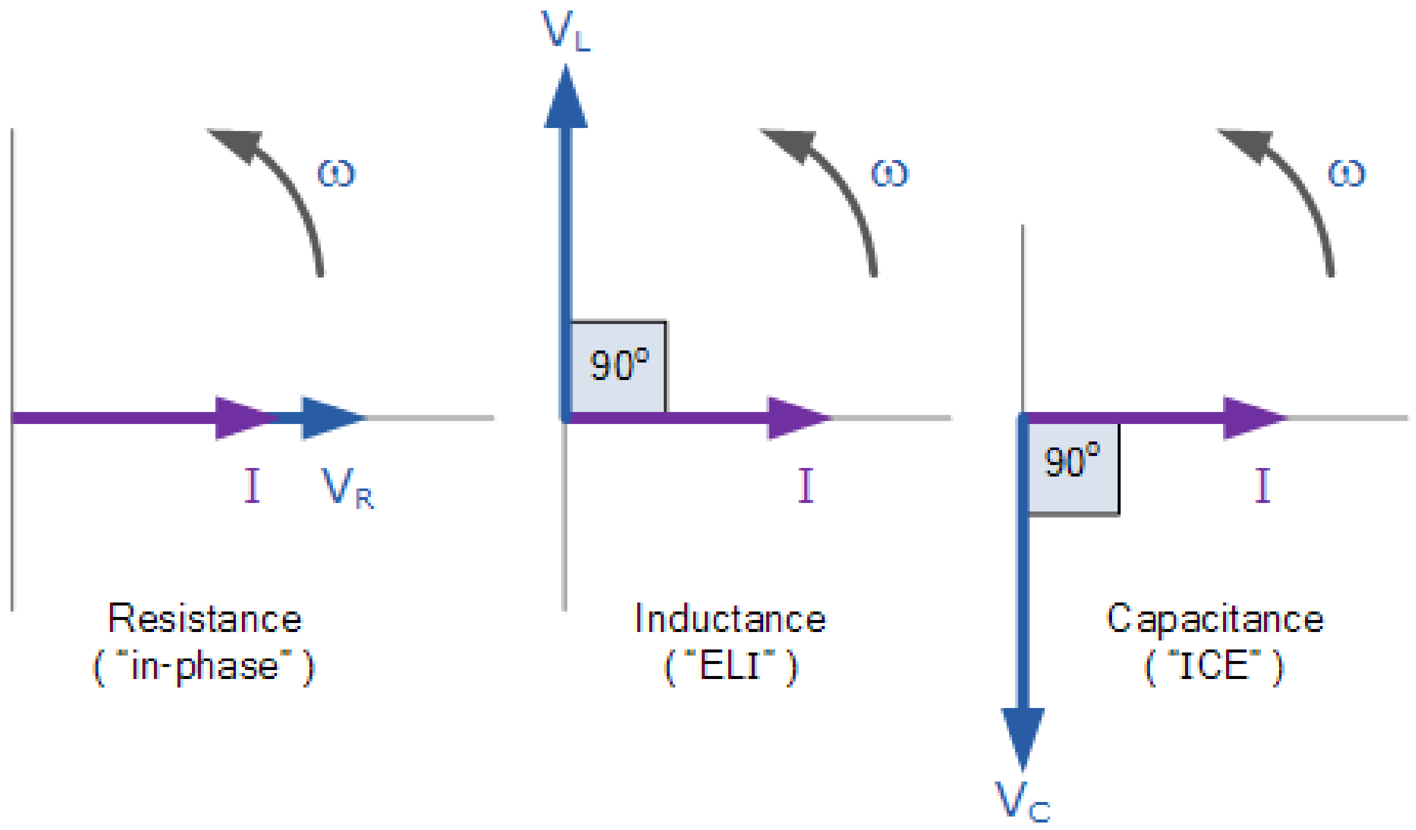
$$Z = R + jX$$

$$\mathbf{Z} = R + jX = |\mathbf{Z}| \angle \theta$$

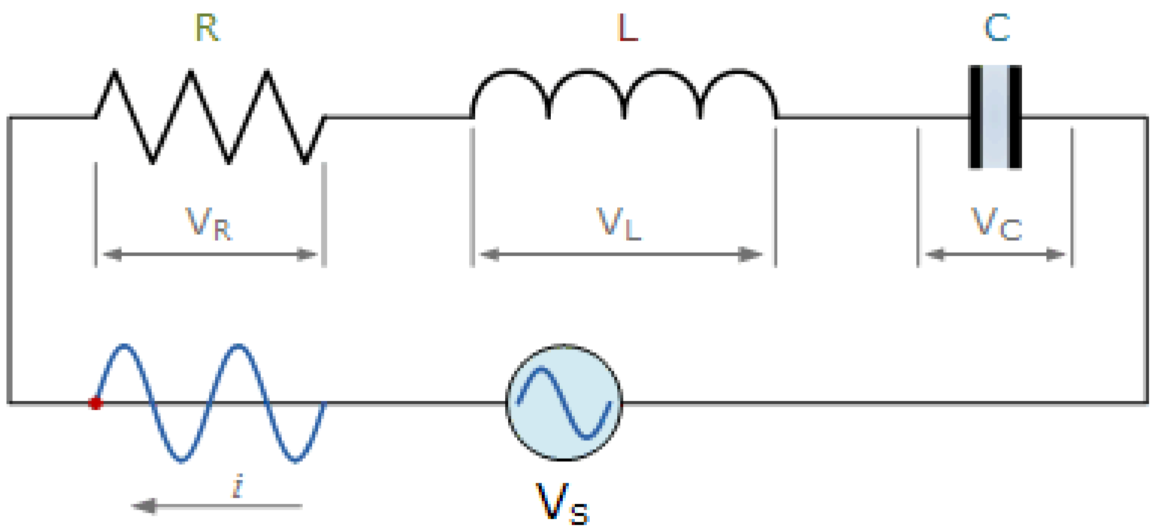
$$|Z| = \sqrt{R^2 + X^2}$$

$$\theta = \tan^{-1} \frac{X}{R}$$

Phasor Summary



Series RLC Circuit



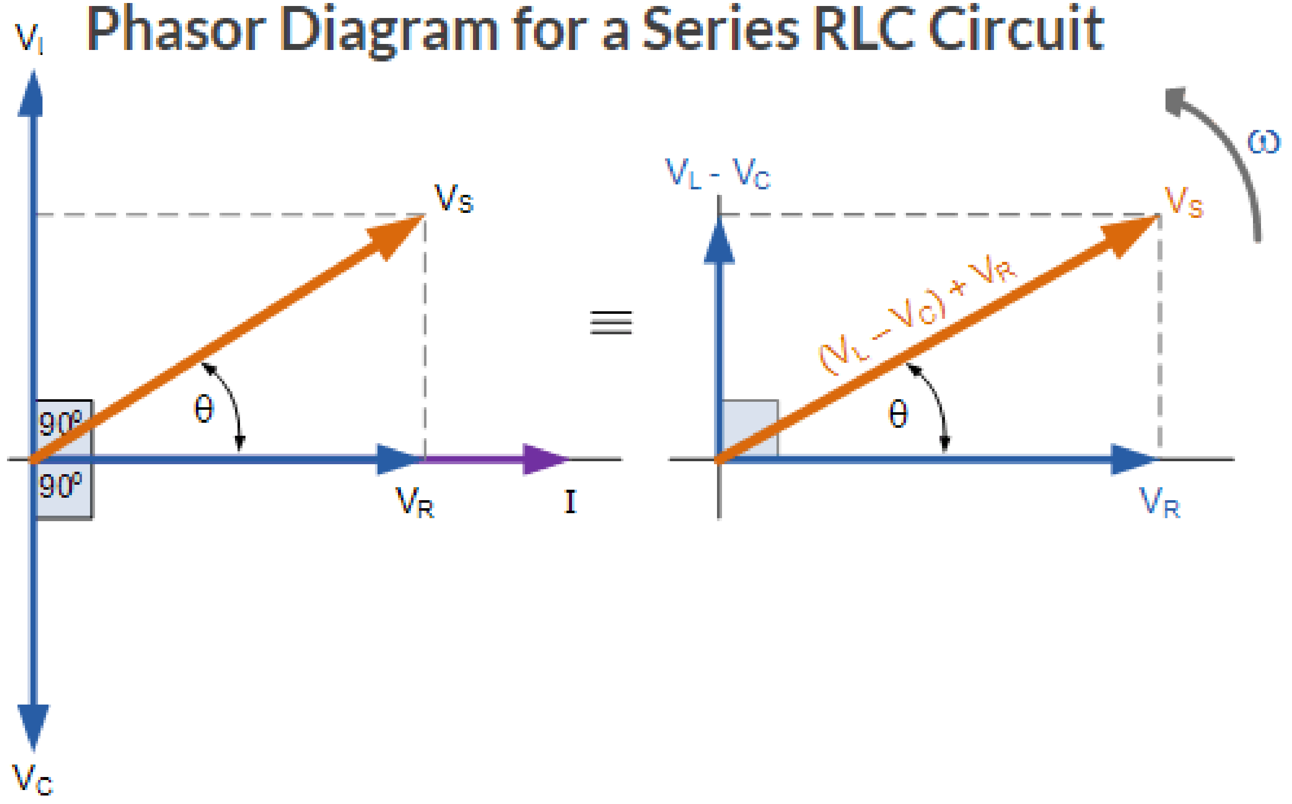
KVL: $V_S - V_R - V_L - V_C = 0$

$$V_S - IR - L\frac{di}{dt} - \frac{Q}{C} = 0$$

$$\therefore V_S = IR + L\frac{di}{dt} + \frac{Q}{C}$$

Series RLC Circuit

Phasor Diagram for a Series RLC Circuit



Voltage Triangle for a Series RLC Circuit

$$V_S^2 = V_R^2 + (V_L - V_C)^2$$

$$V_S = \sqrt{V_R^2 + (V_L - V_C)^2}$$

The Impedance of a Series RLC Circuit

$$V_R = I.R \qquad V_L = I.X_L \qquad V_C = I.X_C$$

$$V_S = \sqrt{(I.R)^2 + (I.X_L - I.X_C)^2}$$

$$V_S = I.\sqrt{R^2 + (X_L - X_C)^2}$$

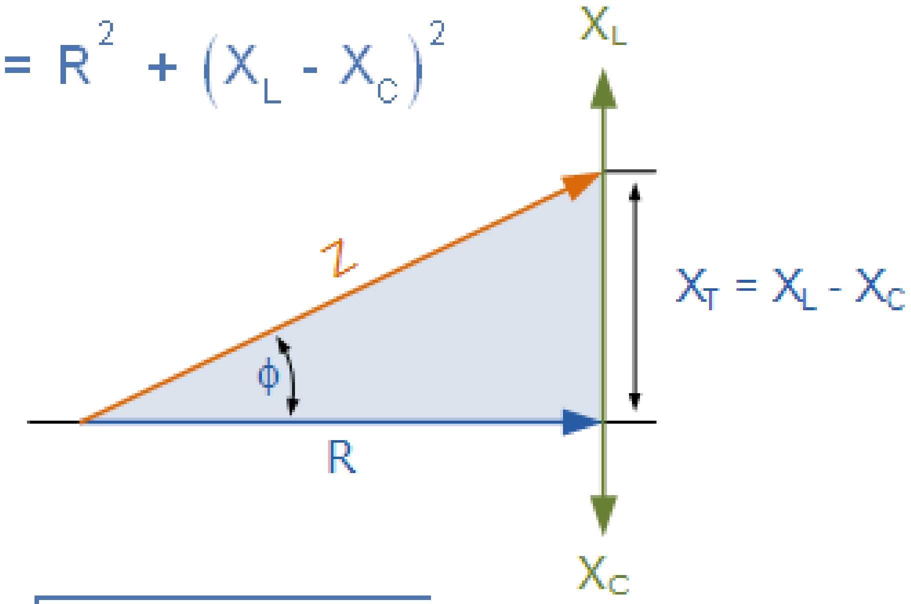
$$\therefore V_S = I \times Z \qquad \text{where: } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The impedance Z of a series RLC circuit depends upon the angular frequency, ω as do X_L and X_C If the capacitive reactance is greater than the inductive reactance, $X_C > X_L$ then the overall circuit reactance is capacitive giving a leading phase angle.

Likewise, if the inductive reactance is greater than the capacitive reactance, $X_L > X_C$ then the overall circuit reactance is inductive giving the series circuit a lagging phase angle. If

The Impedance Triangle for a Series RLC Circuit

$$Z^2 = R^2 + (X_L - X_C)^2$$



$$\text{Impedance, } Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\cos\phi = \frac{R}{Z} \qquad \sin\phi = \frac{X_L - X_C}{Z} \qquad \tan\phi = \frac{X_L - X_C}{R}$$

Series RLC Resonance

In an R–L–C series circuit, when XL =XC, the applied voltage V and the current I are in phase. This effect is called series resonance.

At resonance

$V_L = V_C$

$Z = R$ (minimum circuit impedance possible in an R-L-C circuit)

$I = V/R$ (maximum current possible in an R-L-C circuit)

Since, $X_L = X_C$

When $X_L = X_C$

The applied voltage V and the current I are in phase.
This effect is called series resonance

Series RLC Resonance

Since, $X_L = X_C$

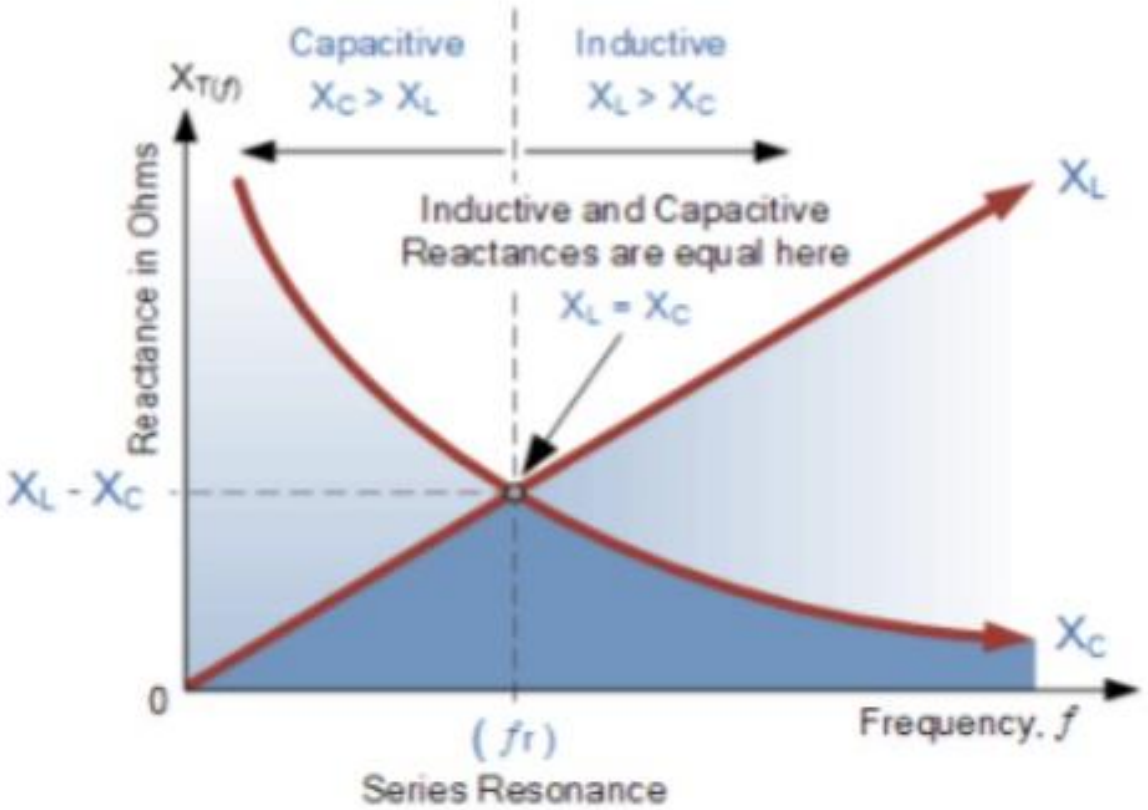
$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{(2\pi)^2 LC}$$

$$f_r = \frac{1}{2\pi\sqrt{(LC)}} \text{Hz}$$

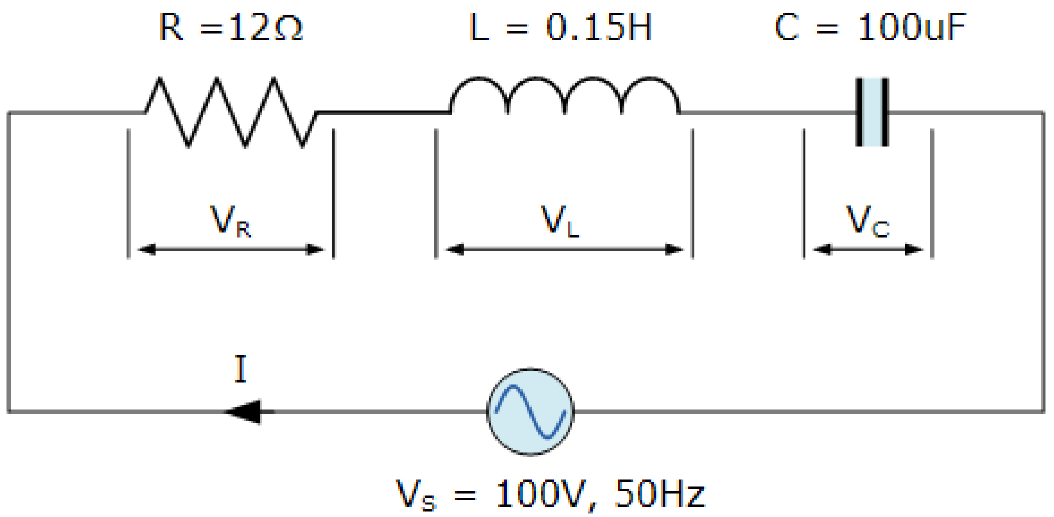
fr is the resonant frequency

The series resonant circuit is often described as an acceptor circuit since it has its minimum impedance, and thus maximum current, at theresonant frequency



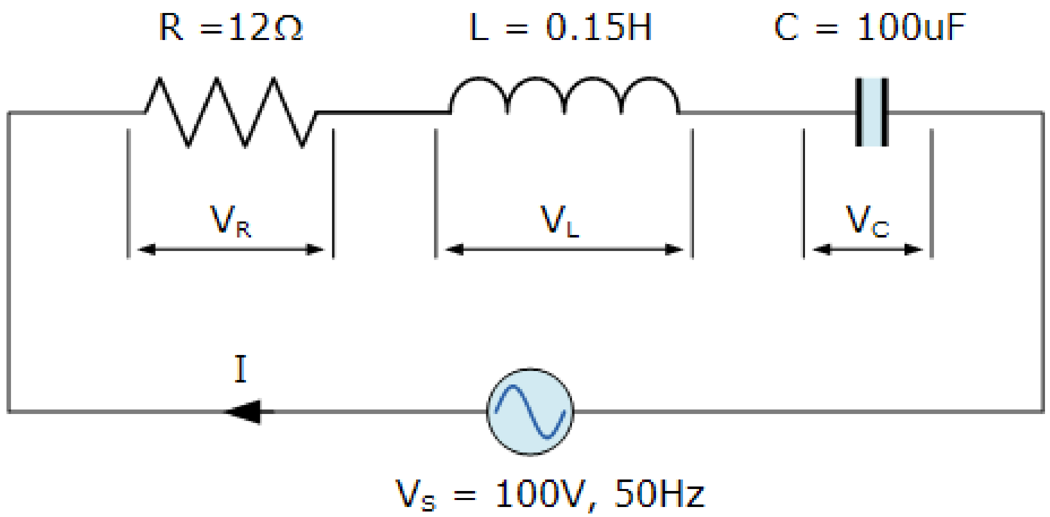
Series Resonance Frequency

Problem



Calculate impedance and current

Problem



Calculate impedance and current

Inductive Reactance, X_L.

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.15 = 47.13\Omega$$

Capacitive Reactance, X_C.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\,\Omega$$

Circuit Impedance, Z.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.13 - 31.83)^2}$$

$$Z = \sqrt{144 + 234} = 19.4\Omega$$

Circuits Current, I.

$$I = \frac{V_S}{Z} = \frac{100}{19.4} = 5.14\,\text{Amps}$$

Phase angle

$$\cos\phi = \frac{R}{Z} = \frac{12}{19.4} = 0.619$$

$$\therefore \cos^{-1} 0.619 = 51.8^0 \text{ lagging}$$

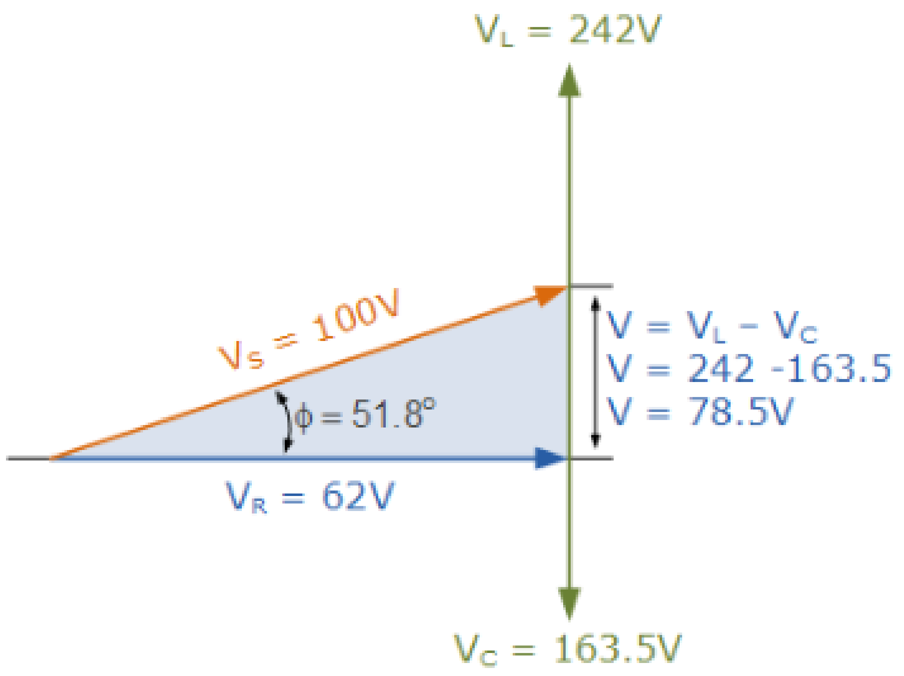
Voltages across the Series RLC Circuit, V_R, V_L, V_C.

$$V_R = I \times R = 5.14 \times 12 = 61.7 \text{ volts}$$

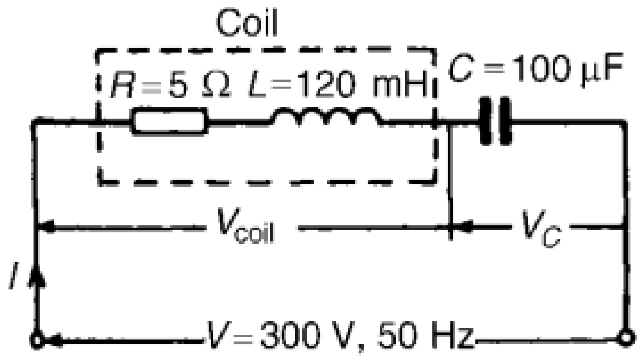
$$V_L = I \times X_L = 5.14 \times 47.13 = 242.2 \text{ volts}$$

$$V_C = I \times X_C = 5.14 \times 31.8 = 163.5 \text{ volts}$$

Phasor Diagram.

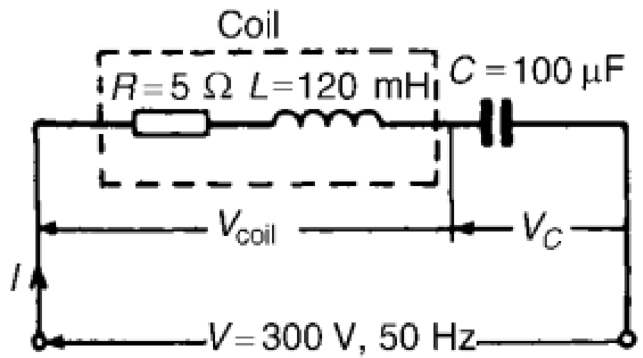


Problem 15. A coil of resistance 5 Ω and inductance 120 mH in series with a 100 μF capacitor, is connected to a 300 V, 50 Hz supply. Calculate (a) the current flowing, (b) the phase difference between the supply voltage and current, (c) the voltage across the coil and (d) the voltage across the capacitor.



Problem 15. A coil of resistance 5 Ω and inductance 120 mH in series with a 100 μF capacitor, is connected to a 300 V, 50 Hz supply. Calculate (a) the current flowing, (b) the phase difference between the supply voltage and current, (c) the voltage across the coil and (d) the voltage across the capacitor.

The circuit diagram is shown in Figure 15.13



$$X_L = 2\pi f L = 2\pi(50)(120 \times 10^{-3}) = \mathbf{37.70\ \Omega}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(100 \times 10^{-6})} = \mathbf{31.83\ \Omega}$$

Since X_L is greater than X_C the circuit is inductive.

$X_L - X_C = 37.70 - 31.83 = 5.87\ \Omega$

$$\text{Impedance } Z = \sqrt{[R^2 + (X_L - X_C)^2]} = \sqrt{[(5)^2 + (5.87)^2]} = 7.71\ \Omega$$

(a) $\text{Current } I = \dfrac{V}{Z} = \dfrac{300}{7.71} = \mathbf{38.91\ A}$

(b) $$\text{Phase angle } \phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\frac{5.87}{5} = 49.58^\circ$$
$$= \mathbf{49^\circ 35'}$$

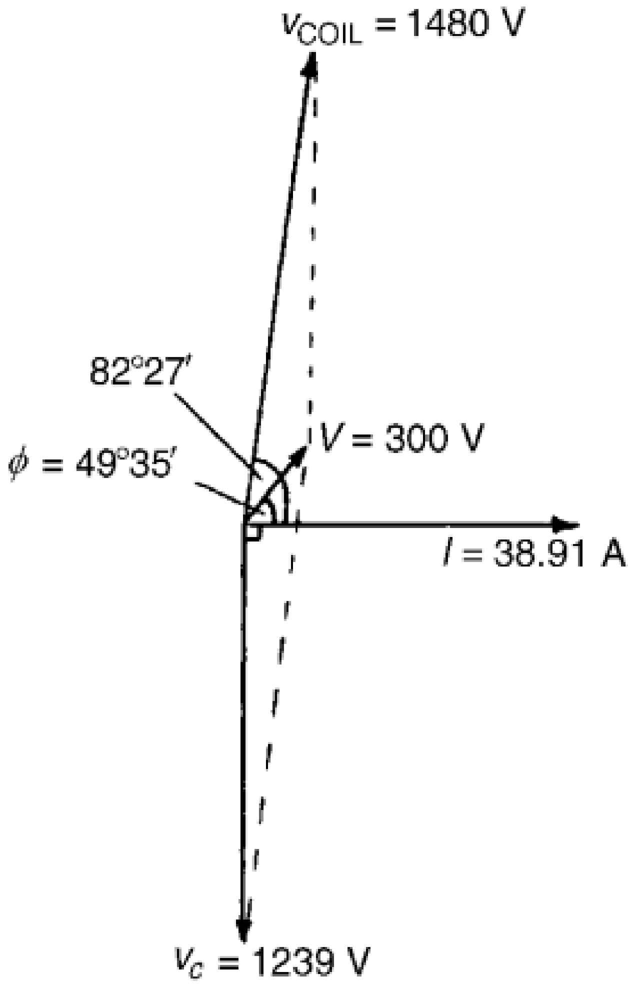
(c) $$\text{Impedance of coil } Z_{\text{COIL}} = \sqrt{(R^2 + X_L^2)} = \sqrt{[(5)^2 + (37.70)^2]}$$
$$= 38.03\ \Omega$$

$$\text{Voltage across coil } V_{\text{COIL}} = IZ_{\text{COIL}} = (38.91)(38.03) = \mathbf{1480\ V}$$

$$\text{Phase angle of coil} = \arctan\frac{X_L}{R} = \arctan\left(\frac{37.70}{5}\right) = 82.45^\circ$$
$$= 82^\circ 27' \text{ lagging}$$

(d) $\text{Voltage across capacitor } V_C = IX_C = (38.91)(31.83) = \mathbf{1239\ V}$

Phasor diagram



A 240 V, 50 Hz AC supply is applied a coil of 0.08 H inductance and 4 Ω resistance connected in series with a capacitor of 8 μF. Calculate the following:

a) Impedance,

b) Circuit current,

c) Phase angle between voltage and current,

d) Power factor,

e) Real and Reactive Power.

a) Impedance, = 372.99∠-89.38 Ohms = 4-j372.089 Ohms -
b) Circuit current, = 0.643 ∠89.38 Amps – [2Marks]
c) Phase angle between voltage and current, = 89.38º – [2M
d) Power factor, = 0.0108(Lead) – [2Marks]
e) Real and Reactive Power. = 1.669 Watts & 154.31 VAr