Basic Electrical and Electronics Engineering

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LECTURE 3

Module 4

Digital Systems

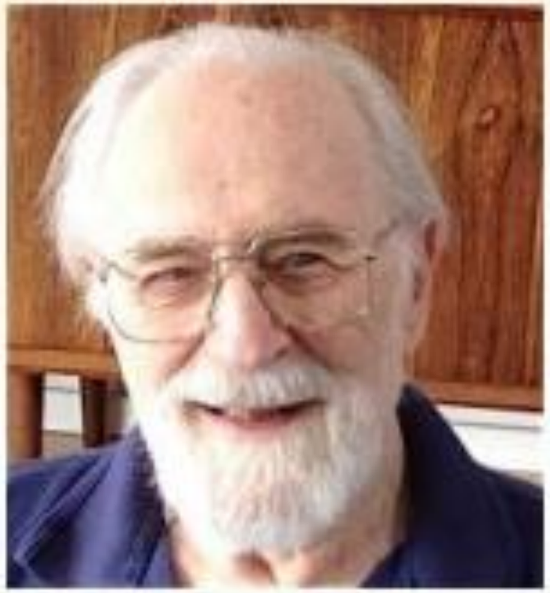
Lecture 3

Topics to be covered

- **K-Map Rules**
- **Minimization using K map**

Karnaugh map

- The Karnaugh map (K–map), introduced by Maurice Karnaugh in 1953, is a grid like representation of a truth table which is used to simplify Boolean algebra expressions.
- A Karnaugh map has zero and one entries at different positions. It provides grouping together Boolean expressions with common factors and eliminates unwanted variables from the expression.



- In a K-map, crossing a vertical or horizontal cell boundary is always a change of only one variable.

- **The K-map is an array of squares (or cells) in which each square represents a binary value of the input variables.**

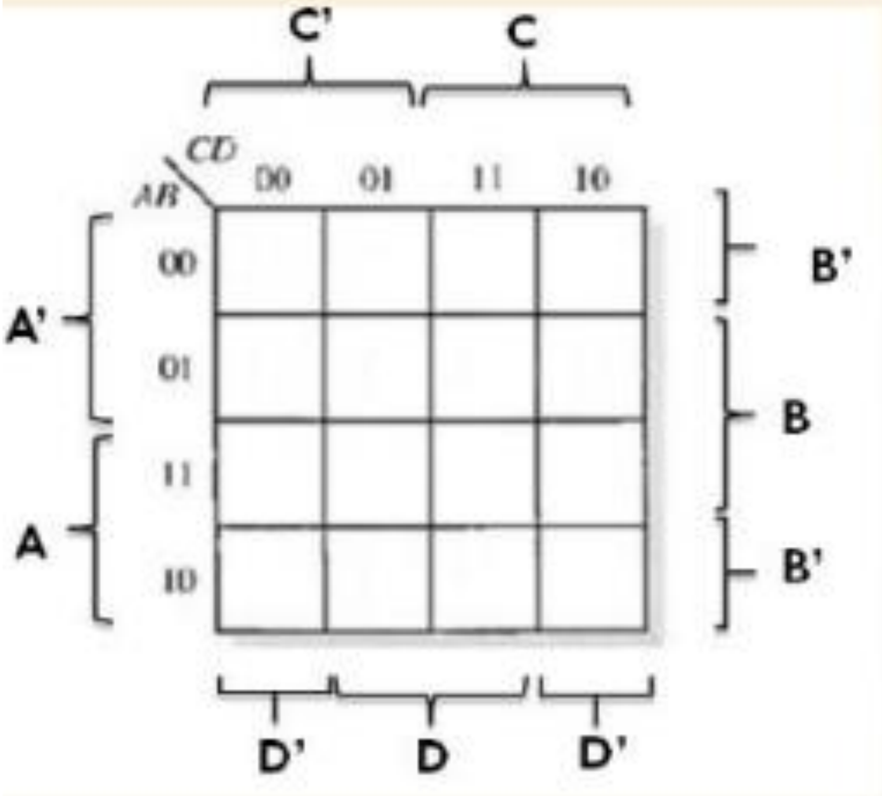
- The **number of squares** in a Karnaugh map is equal to **(2^n)** the total number of possible input variable combinations (i.e number of squares is equal to the number of rows in a truth table).

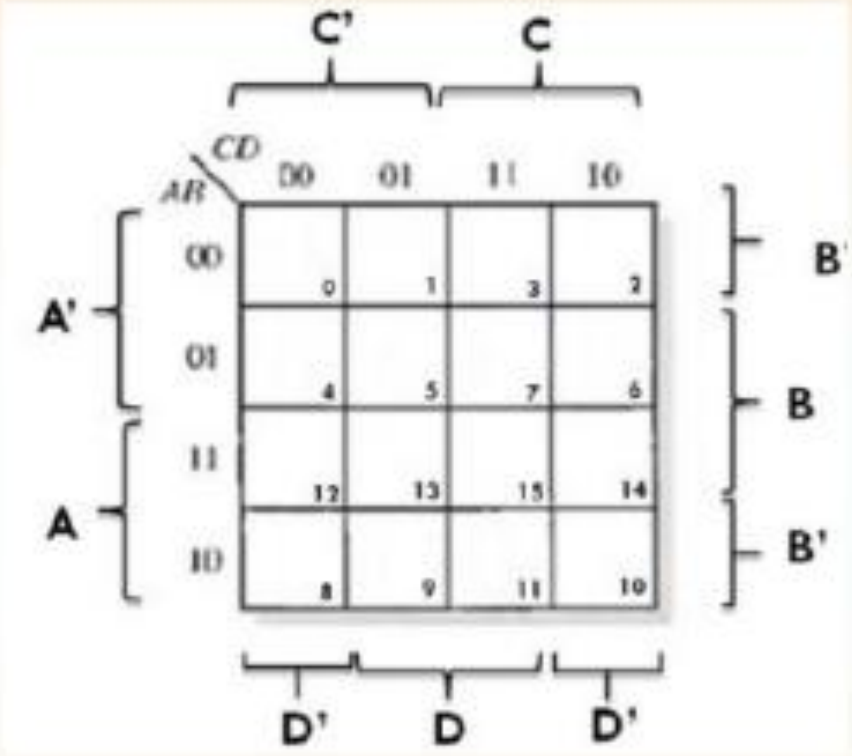
- **Ex.** for two variables, the number of square is $2^2 = 4$, for three variables, the number of squares is $2^3 = 8$ and for four variables, the number of squares is $2^4 = 16$.

A \ B	$\overline{B}$ 0	B 1
$\overline{A}$ 0	$\overline{A}.\overline{B}$	$\overline{A}.B$
A 1	$A.\overline{B}$	A.B

A \ BC	00	01	11	10
0	0 A'B'C'	1 A'B'C	3 A'BC	2 A'BC'
1	4 AB'C'	5 AB'C	7 ABC	6 ABC'

AB \ CD	00	01	11	10
00	0 A' B' C' D'	1 A' B' C' D	3 A' B' C D	2 A' B' C D'
01	4 A' B C' D'	5 A' B C' D	7 A' B C D	6 A' B C D'
11	12 A B C' D'	13 A B C' D	15 A B C D	14 A B C D'
10	8 A B' C' D'	9 A B' C' D	11 A B' C D	10 A B' C D'


C'
C
CD
AB
00
01
11
10
A'
A
B'
B
B'
D'
D
D'


C'
C
CD
AB
00
01
11
10
0
1
3
2
4
5
7
6
12
13
15
14
8
9
11
10
A'
A
B'
B
B'
D'
D
D'

Procedure to build the Karnaugh map

- Find number of squares (or cells), and draw the array
- Assign decimal value and binary equivalent to each square (follow gray code)
- Enter 1's in the cells corresponding to the minterms of the given expression
- Group 1's in pairs, quads, octets,… 2^n cells (follow rules)
- Write the reduced term corresponding to each group, which gives expression with minimum number of terms

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Two, Three and Four variable Karnaugh map

2-variable K-map

A \ B	0	1
0	0	1
1	2	3

3-variable K-map

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

4-variable K-map

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

AB \ C	0	1
00	0	1
01	2	3
11	6	7
10	4	5

Two, Three and Four variable Karnaugh map and Minterms

	$\overline{B}$	B
$\overline{A}$	$\overline{A}\,\overline{B}$	$\overline{A}B$
A	$A\overline{B}$	AB

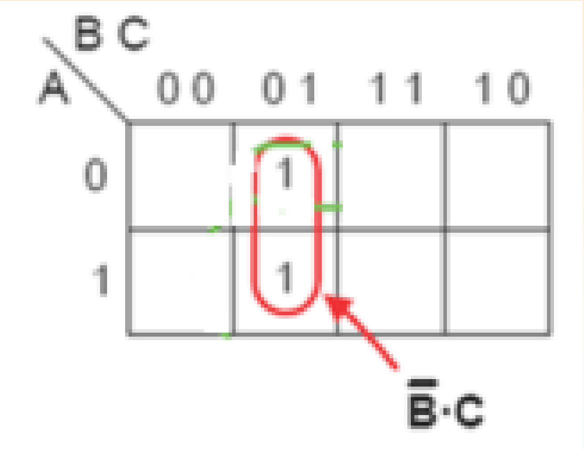
A \ BC	B'C' 00	B'C 01	BC 11	BC' 10
A' 0	A'B'C' 0	A'B'C 1	A'BC 3	A'BC' 2
A 1	AB'C' 4	AB'C 5	ABC 7	ABC' 6

	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	$\overline{A}\,\overline{B}\,\overline{C}D$	$\overline{A}\,\overline{B}CD$	$\overline{A}\,\overline{B}C\overline{D}$
$\overline{A}B$	$\overline{A}B\overline{C}\,\overline{D}$	$\overline{A}B\overline{C}D$	$\overline{A}BCD$	$\overline{A}BC\overline{D}$
AB	$AB\overline{C}\,\overline{D}$	$AB\overline{C}D$	$ABCD$	$ABC\overline{D}$
$A\overline{B}$	$A\overline{B}\,\overline{C}\,\overline{D}$	$A\overline{B}\,\overline{C}D$	$A\overline{B}CD$	$A\overline{B}C\overline{D}$

Manual Logic reduction

$$= \overline{A}\,\overline{B}C + A\overline{B}C = \overline{B}C\,(\overline{A} + A)$$

K-map



Disadvantages:

- Finding the suitable pair of terms is difficult.
- If the suitable terms are not paired, it will lead to long reduction process.

Advantages:

- In every two adjacent cells, one of the variables changes (1 and 0).
- All minterms which differ by one variable are in adjacent cells.
- Through grouping, the changing variable mapped by the loop can be eliminated.

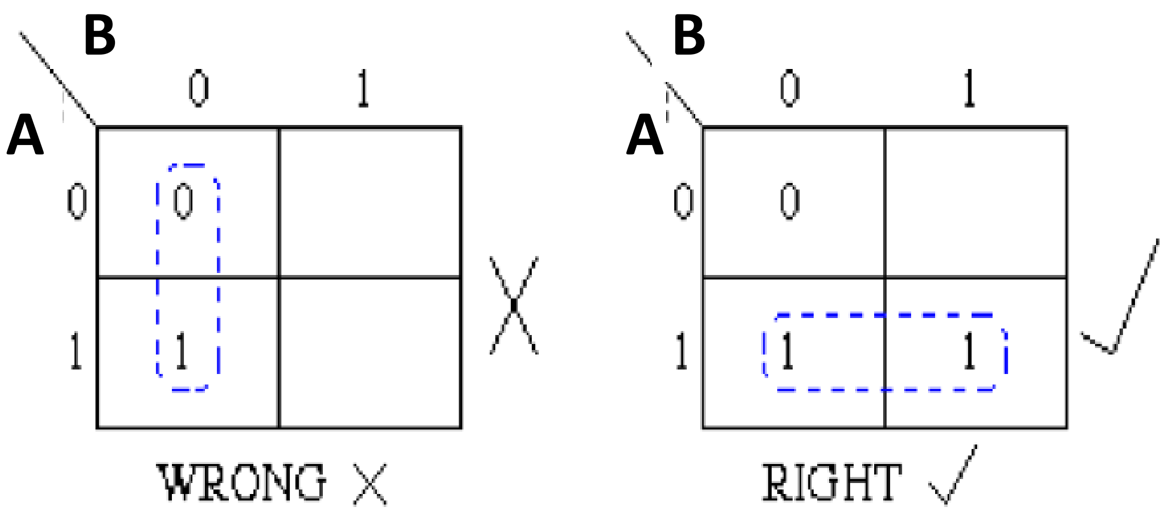
Rules for Grouping adjacent cells containing ones

1. No zeros allowed.
2. No diagonal groupings.
3. Only power of 2 number of cells in each group.
4. Form larger group possible.
5. Each group should contain at least one unique one.
6. Every one must be in at least one group.
7. Overlapping allowed.
8. Wrap around allowed.
9. Number of groups must be as fewer as possible.

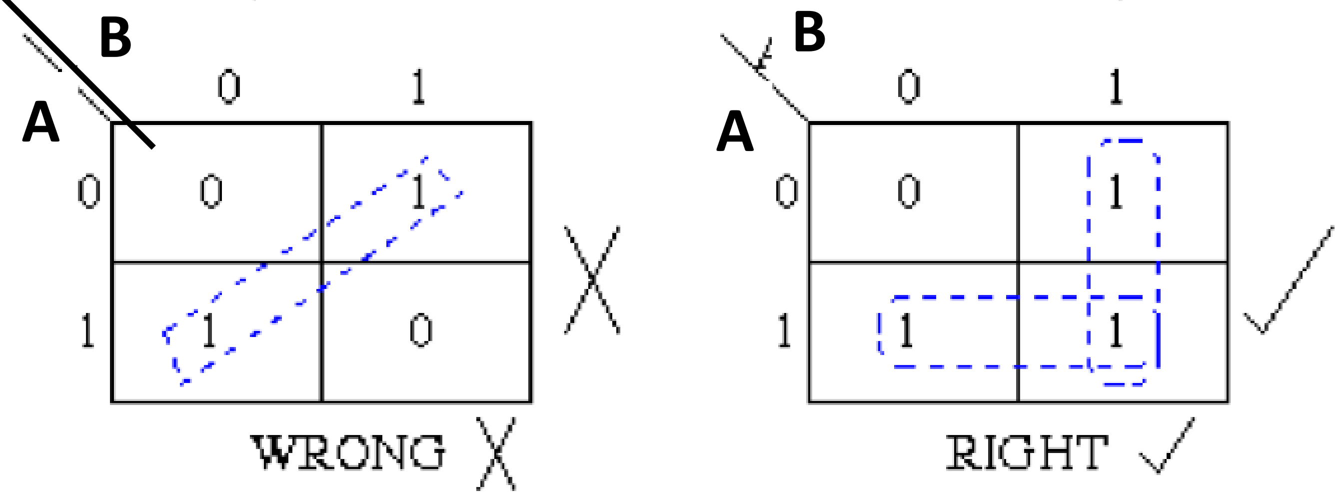
Karnaugh Maps - Rules of Simplification

The Karnaugh map uses the following rules for the simplification of expressions by *grouping* together adjacent cells containing *ones*

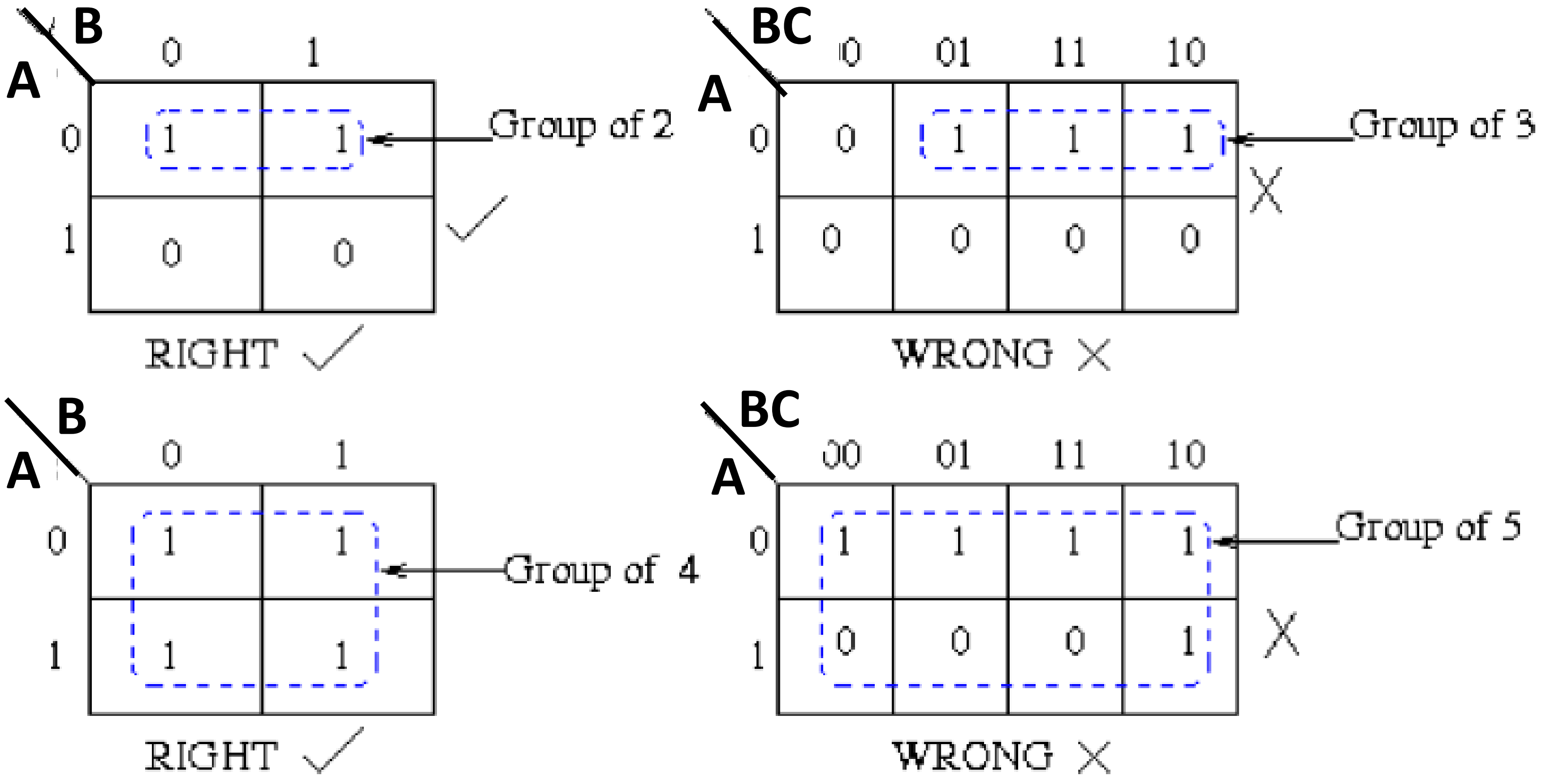
- **Groups may not include any cell containing a zero**



- Groups may be horizontal or vertical, but not diagonal.

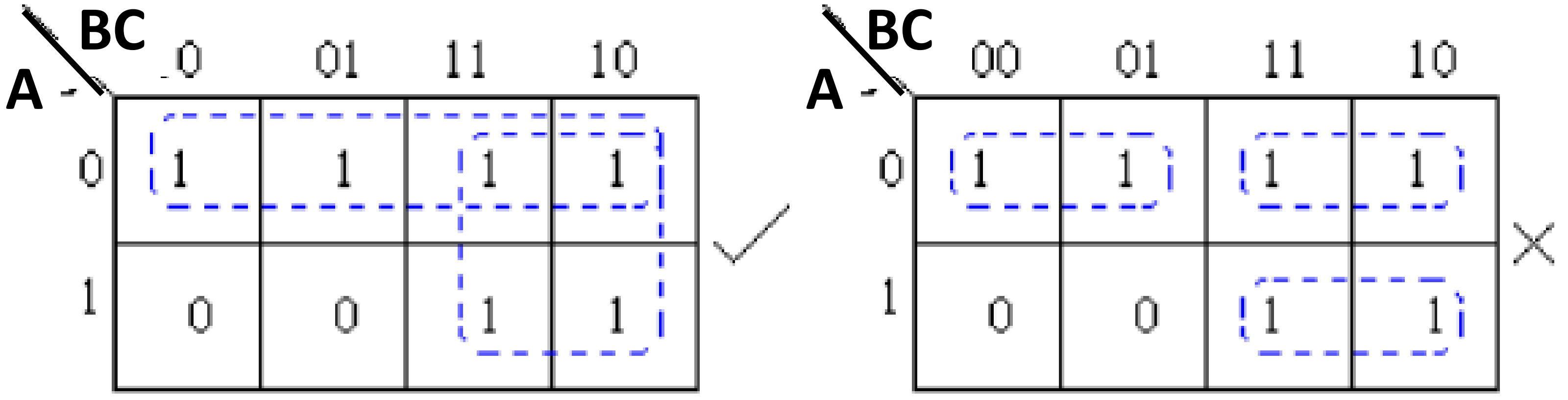


- Groups must contain 1, 2, 4, 8, or in general 2^n cells.
  That is if n = 1, a group will contain two 1's since $2^1 = 2$.
  If n = 2, a group will contain four 1's since $2^2 = 4$.



- **Each group should be as large as possible.**

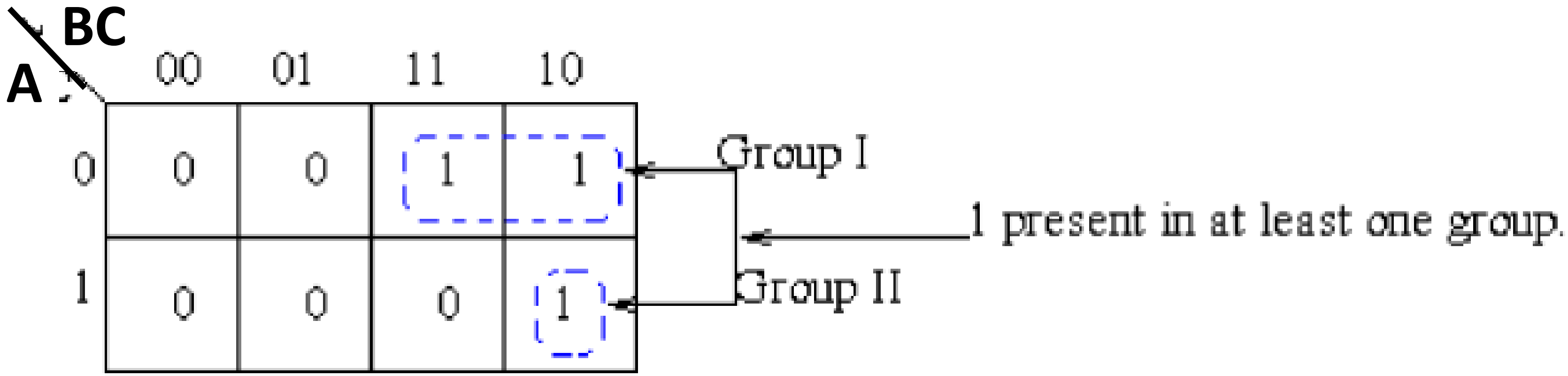
Karnaugh Maps - Rules of Simplification



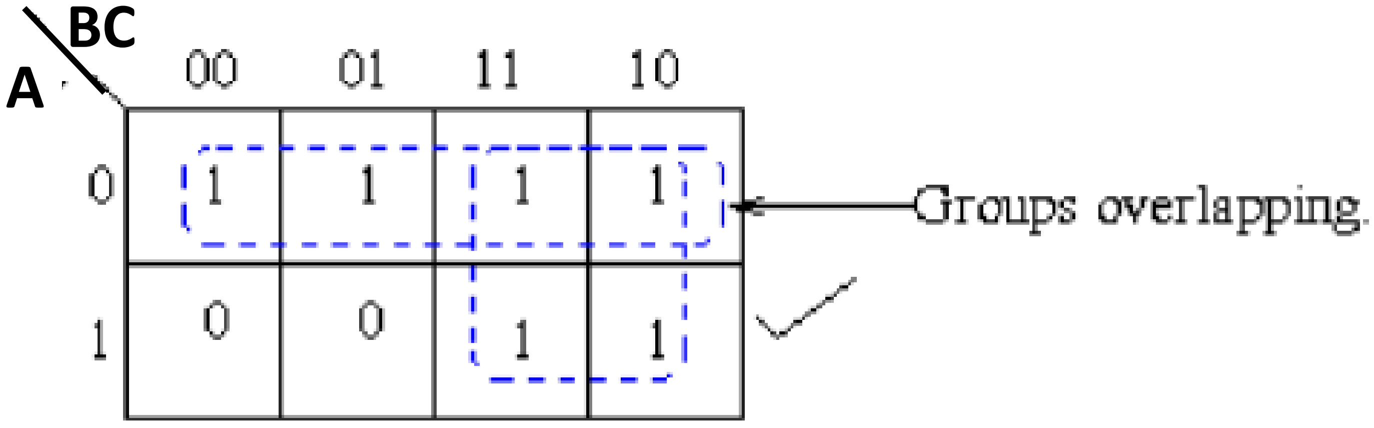
RIGHT ✓ WRONG ✗

(Note that no Boolean laws broken, but not sufficiently minimal)

- **Each cell containing a *one* must be in at least one group.**



- **Groups may overlap.**



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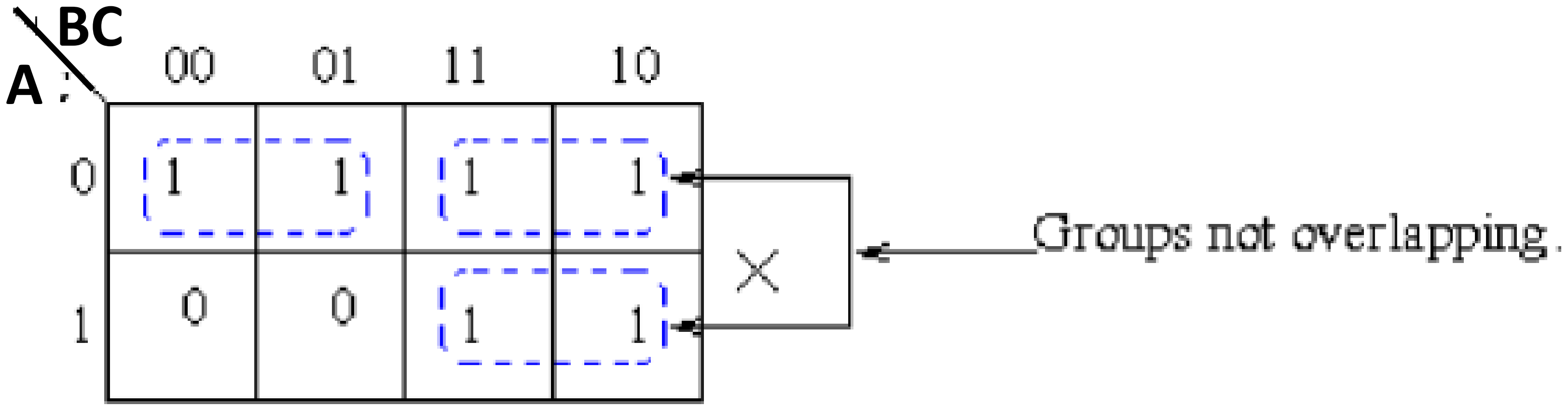
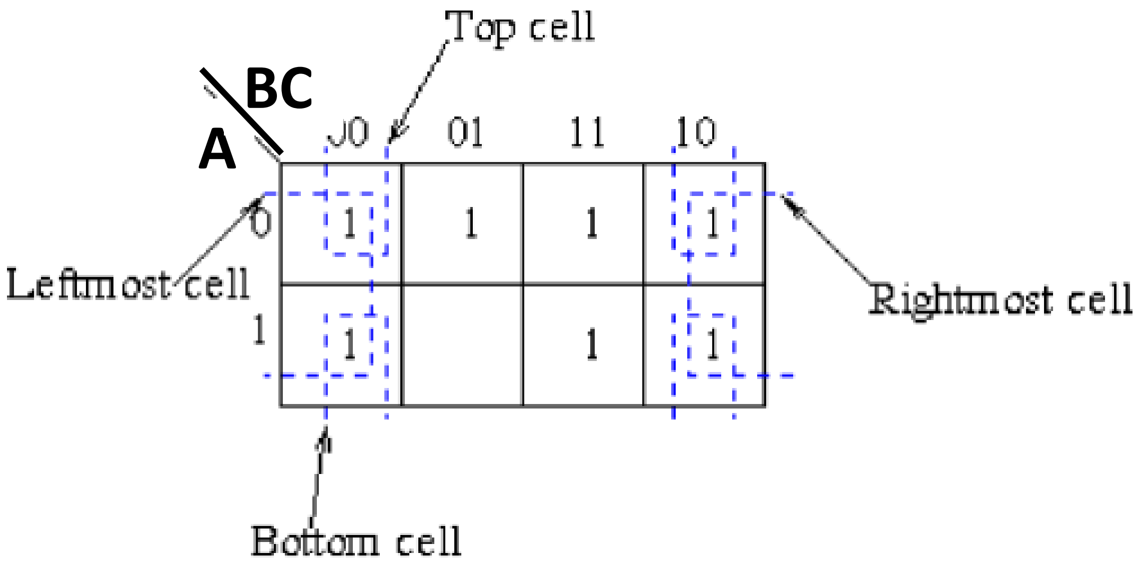
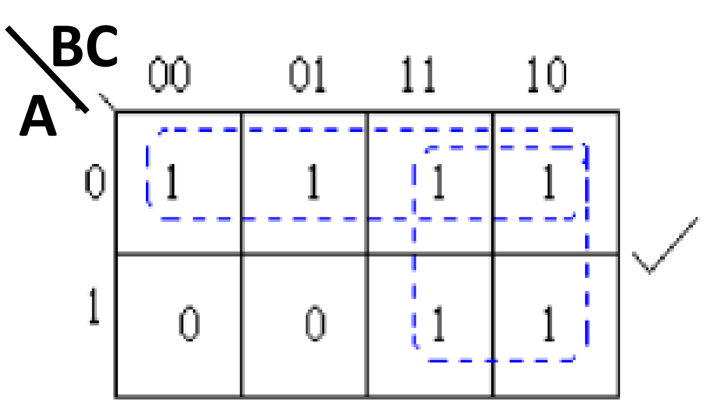


WRONG ×

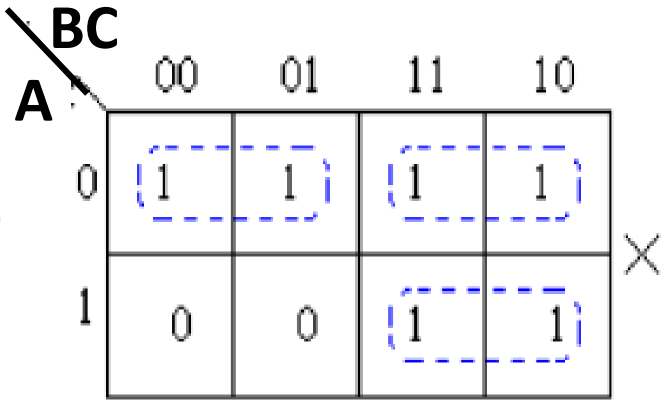
- **Groups may wrap around the table. The leftmost cell in a row may be grouped with the rightmost cell and the top cell in a column may be grouped with the bottom cell.**



- There should be as few groups as possible, as long as this does not contradict any of the previous rules.



RIGHT ✓

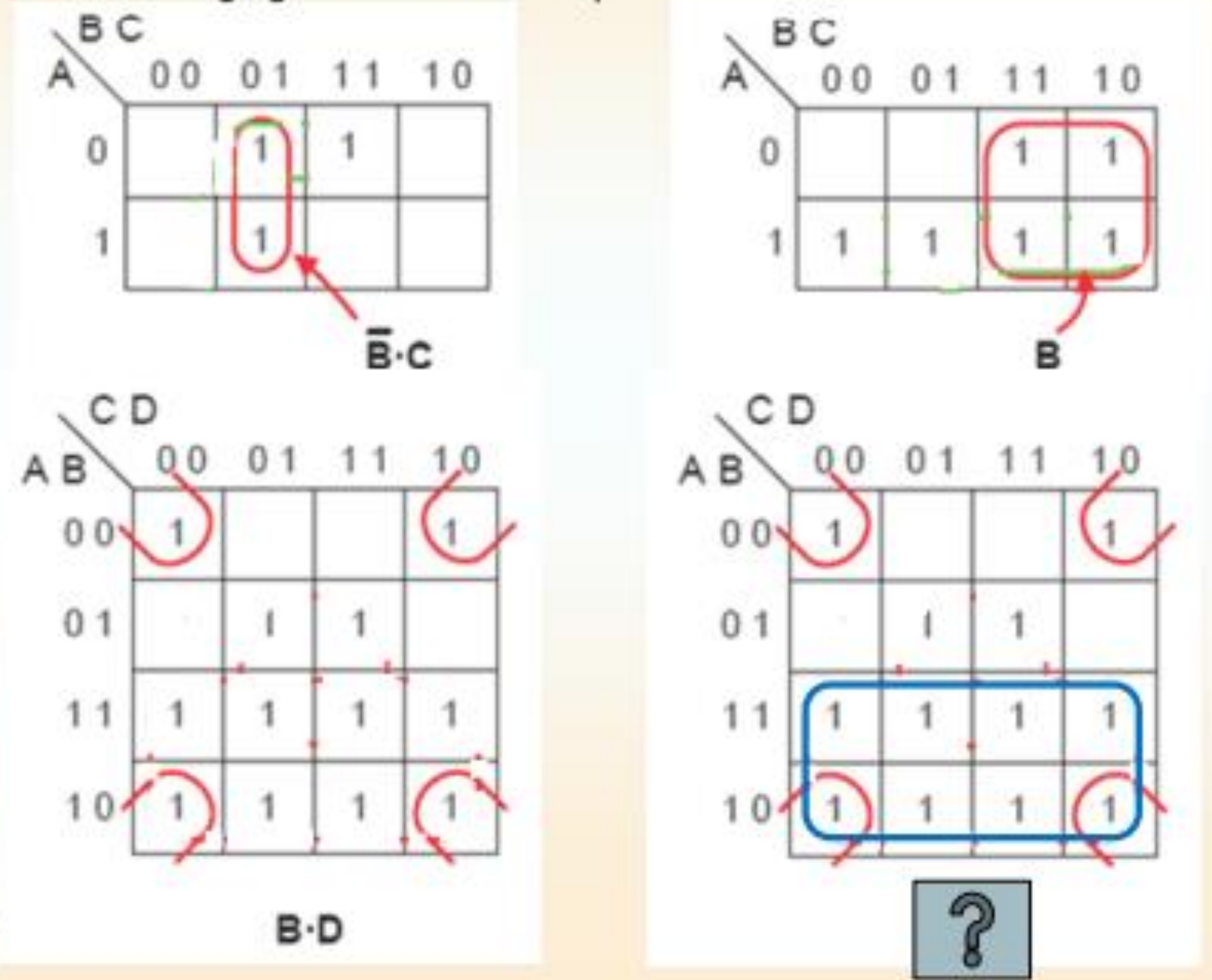


WRONG ✗

Writing the logic function after grouping

Write the mintems corresponding to each loop by omitting the changing variables.

for example **A** is a changing variable in red loop



Procedure in K-map (continued)

Step-1: Find the minterms

Ex. $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + A\overline{B}C + A\,\overline{B}\,\overline{C}$

Step-2: Find the cells in K-map corresponding to the minterms. Enter 1's in that cells.

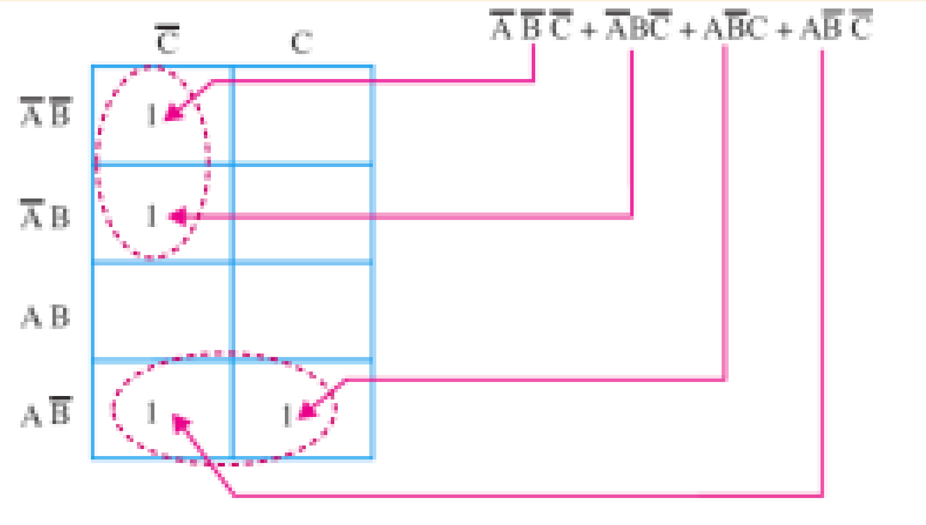
000	**010**	**101**	**100**	Binary value of input variables corresponding to the **Minterms**
0	**2**	**5**	**4**	**Cell number**

Step-3: Group 1's in the adjacent cells. Look for largest possible loop. (8 cell, 4 cell, 2 cell).

Step-4: Form minimum number of loops.

Step-5: Write the reduced function from the K-map.

Mapping a Standard SOP Expression on the Karnaugh Map



$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + A\overline{B}C + A\,\overline{B}\,\overline{C}$$

$$= \overline{A}\,\overline{C} + A\overline{B}$$

Mapping a Nonstandard SOP Expression on the Karnaugh Map

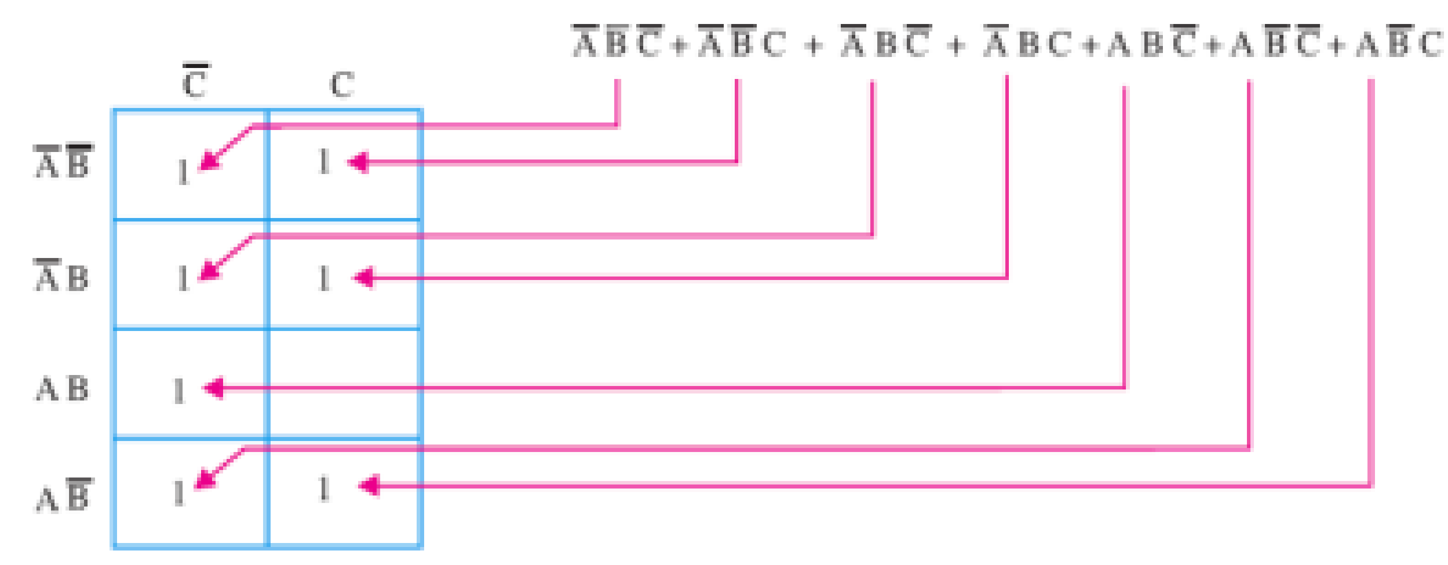
Suppose we have the *SOP* expression :

$$\overline{A} + A\overline{B} + AB\overline{C}$$

Expanding the resulting expression, we get,

$$\overline{A}\,(B + \overline{B})\,(C + \overline{C}) + A\overline{B}\,(C + \overline{C}) + AB\overline{C}$$

$$= \overline{A}BC + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + A\,\overline{B}\,\overline{C} + AB\overline{C}$$



(A′+B′+C′)

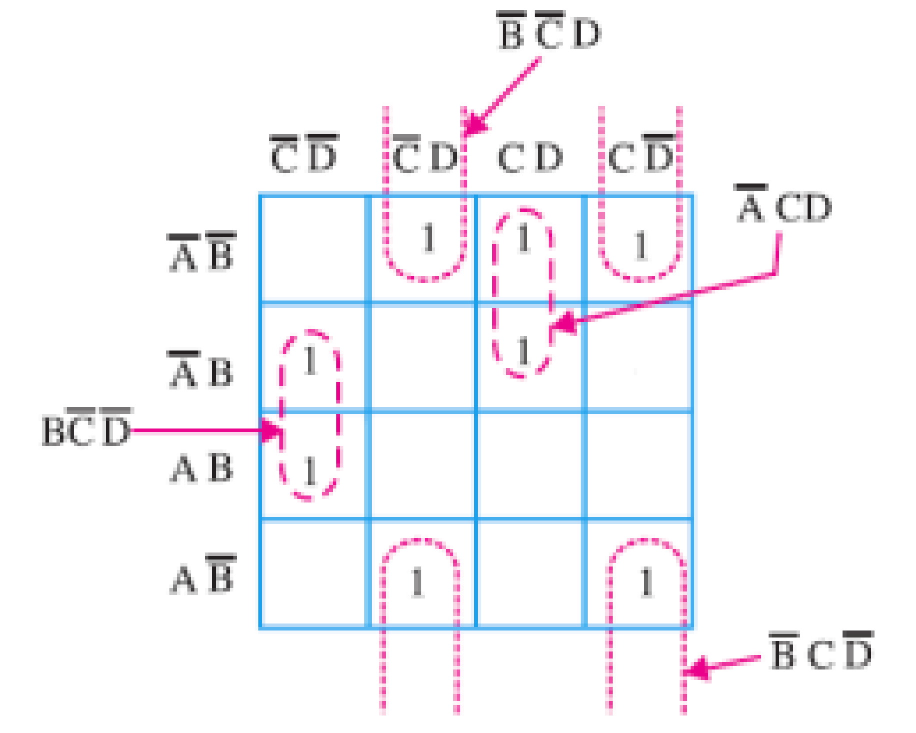
Mapping Directly on Karnaugh Map from a Truth Table

P 50 **Exercise:**

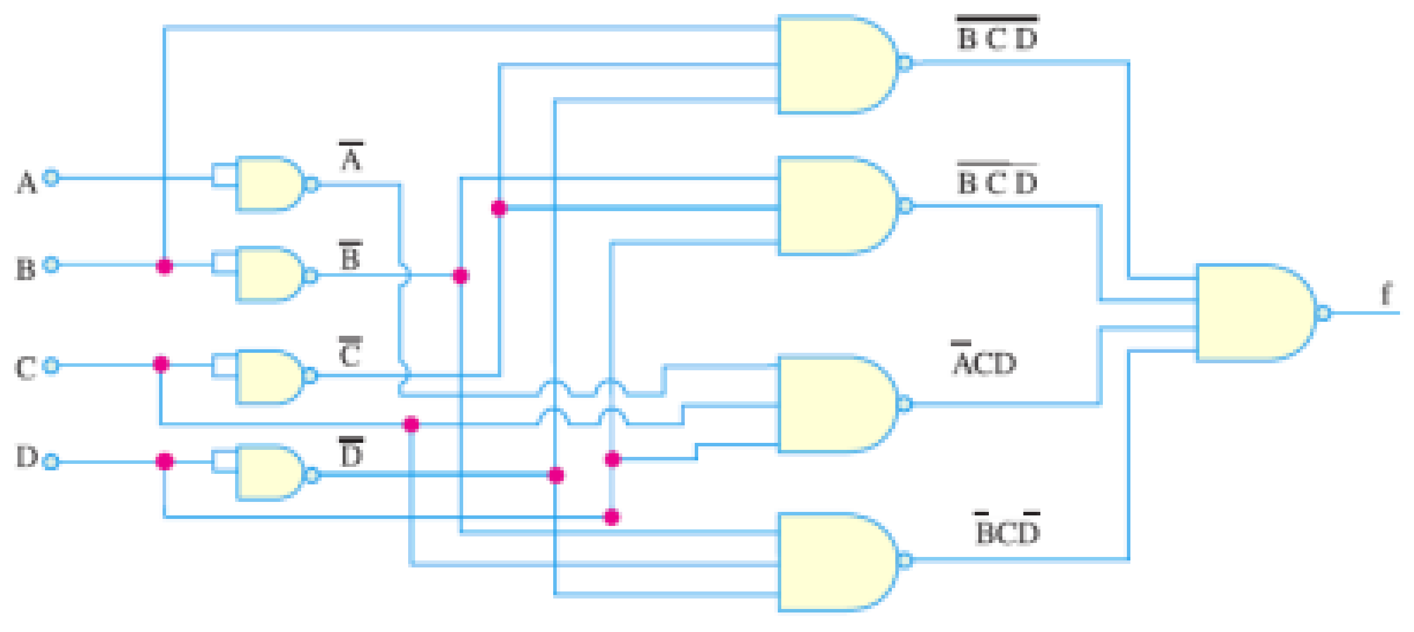
Implement the following Boolean expression using minimum number of 3-input NAND gates.

$$f(A, B, C, D) = \Sigma(1, 2, 3, 4, 7, 9, 10, 12)$$

Decimal Number	Inputs A	B	C	D	Output f	
0	0	0	0	0	0	
1	0	0	0	1	1	$\overline{A}\,\overline{B}\,\overline{C}\,D$
2	0	0	1	0	1	$\overline{A}\,\overline{B}\,C\,\overline{D}$
3	0	0	1	1	1	$\overline{A}\,\overline{B}\,C\,D$
4	0	1	0	0	1	$\overline{A}\,B\,\overline{C}\,\overline{D}$
5	0	1	0	1	0	
6	0	1	1	0	0	
7	0	1	1	1	1	$\overline{A}\,B\,C\,D$
8	1	0	0	0	0	
9	1	0	0	1	1	$A\,\overline{B}\,\overline{C}\,D$
10	1	0	1	0	1	$A\,\overline{B}\,C\,\overline{D}$
11	1	0	1	1	0	
12	1	1	0	0	1	$A\,B\,\overline{C}\,\overline{D}$
13	1	1	0	1	0	
14	1	1	1	0	0	
15	1	1	1	1	0	



$$f(A, B, C, D) = B\overline{C}\,\overline{D} + \overline{B}\,\overline{C}\,D + \overline{A}\,C\,D + \overline{B}\,C\,\overline{D}$$



$$f(A,B,C,D) = B\overline{C}\,\overline{D} + \overline{B}\,\overline{C}D + \overline{A}CD + \overline{B}C\overline{D}$$

P 52 Exercise: Reduce the expression $F = \Sigma m(0,1,2,3,6,7,13,15)$ by mapping and implement in NAND logic.

- Enter 1 for given minterms in corresponding location and 0 for others.
- Group the maximum number of 1s in the order of 2^n.
- Ensure that all the 1s must come under at least once in a group.
- Simplify the resultant minterms using basic laws and OR the Result.
- Implement the hardware.

AB \ CD	00	01	11	10
00	1_{0}	1_{1}	1_{3}	1_{2}
01	0_{4}	0_{5}	1_{7}	1_{6}
11	0_{12}	1_{13}	1_{15}	0_{14}
10	0_{8}	0_{9}	0_{11}	0_{10}

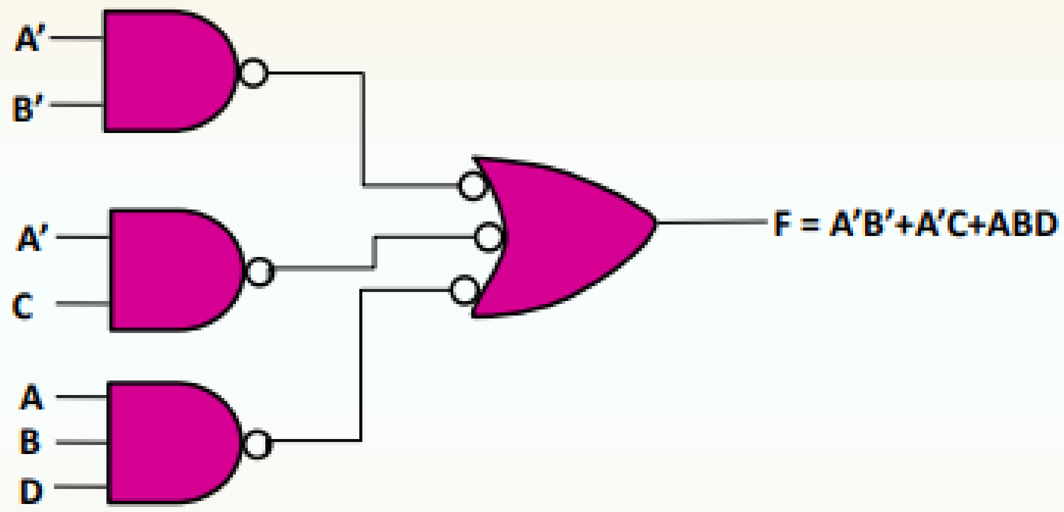
Group I = A'B'C'D' A'B'C'D A'B'CD A'B'CD'
= A'B'

Group II = A'BCD A'BCD' A'B'CD A'B'CD'
= A'C

Group III = ABC'D ABCD
= ABD

F = A'B'+A'C+ABD

NAND Implementation

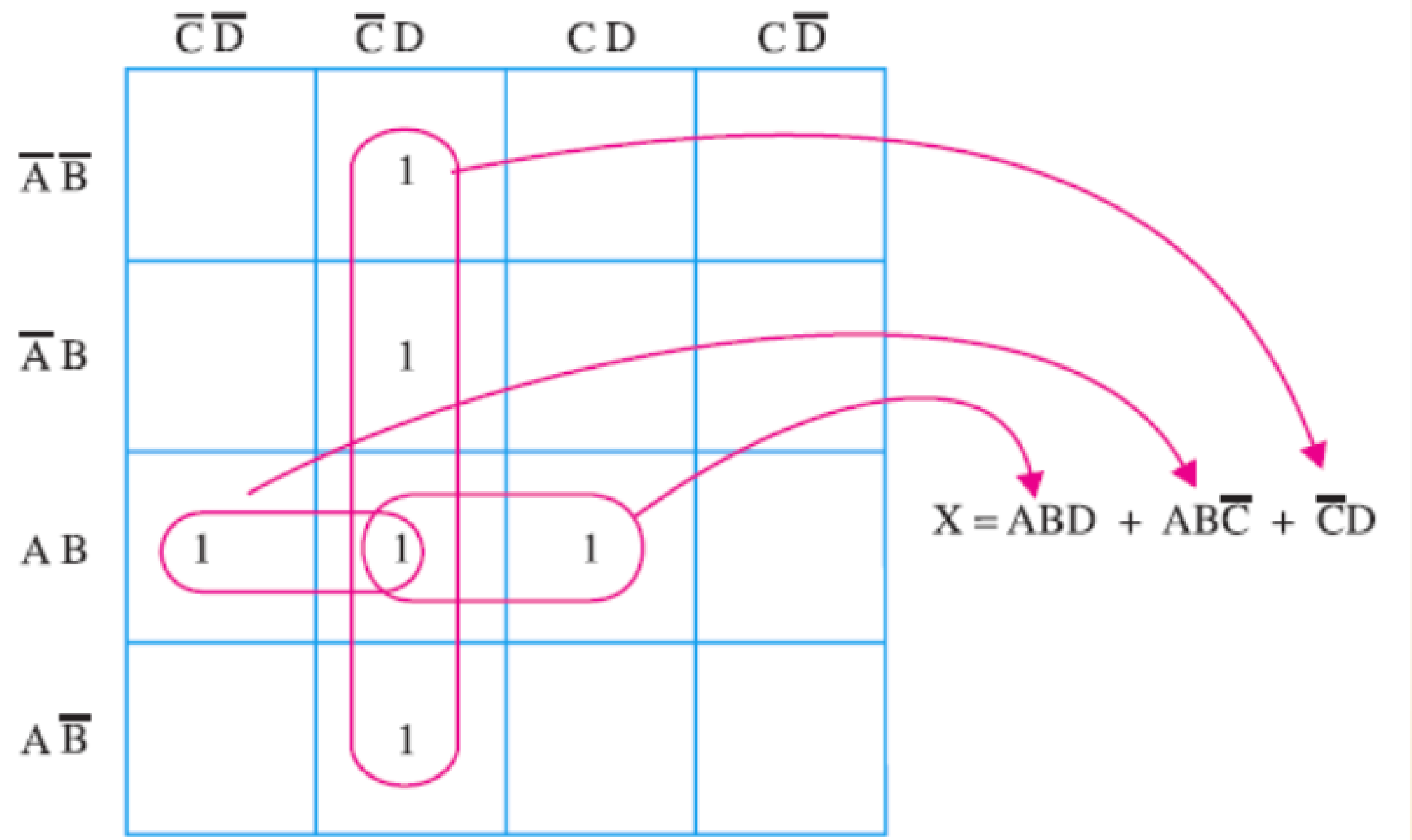


Question: Simplify the following SOP expression using the Karnaugh mapping procedure.

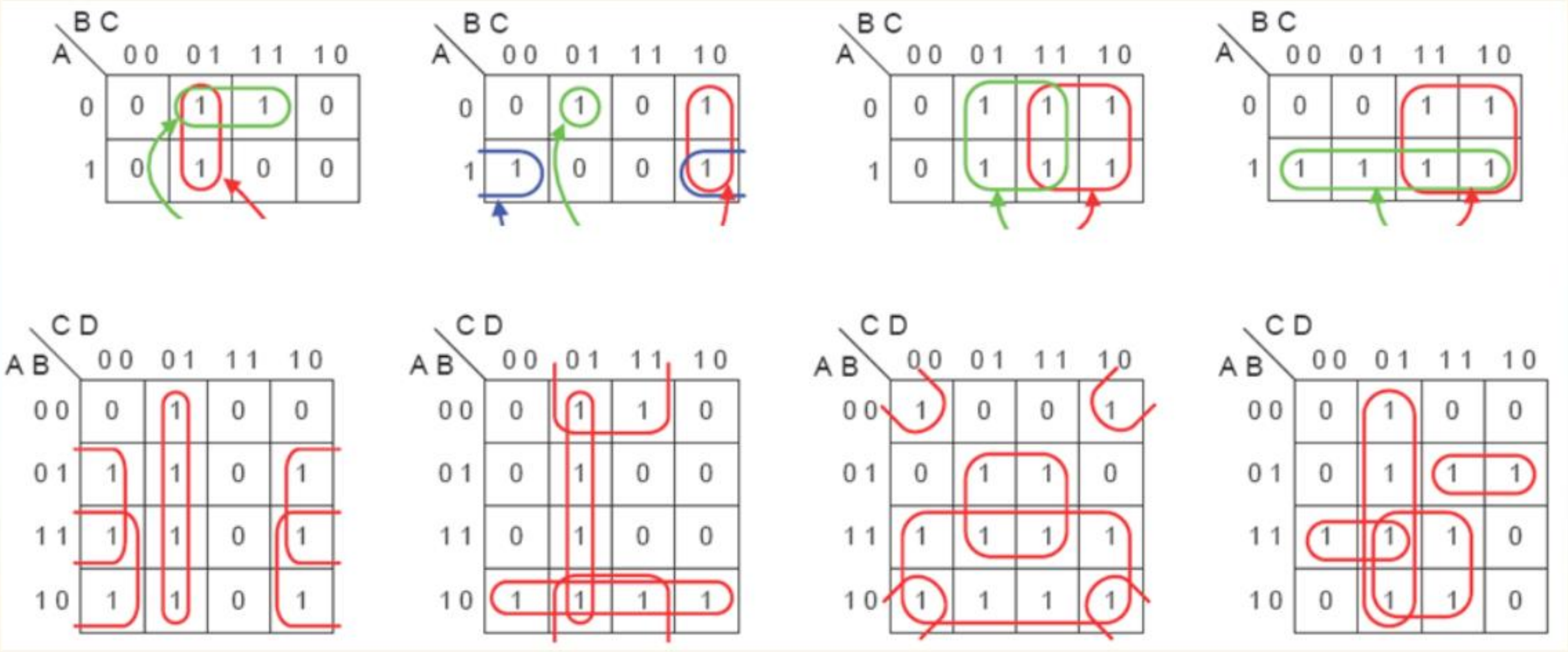
$$X = \overline{A}B\overline{C}D + A\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}D + AB\overline{C}D + AB\overline{C}\,\overline{D} + ABCD$$

Question: Simplify the following SOP expression using the Karnaugh mapping procedure.

$$X = \bar{A}B\bar{C}D + A\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}D + AB\bar{C}D + AB\bar{C}\bar{D} + ABCD$$



Write the logic functions from the K-map

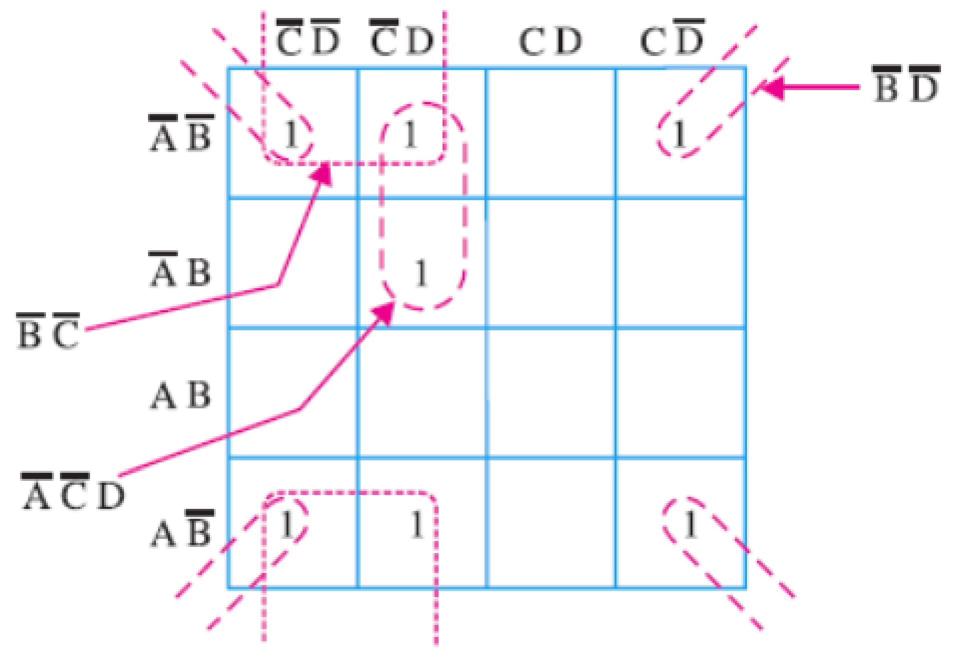


Question: For the given K-MAP determine the simplified SOP function

	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	1	1		1
$\overline{A}B$		1		
AB				
$A\overline{B}$	1	1		1

Question: For the given K-MAP determine the simplified SOP function

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1		1
$\bar{A}B$		1		
AB				
$A\bar{B}$	1	1		1



$$X = \bar{B}\bar{D} + \bar{B}\bar{C} + \bar{A}\bar{C}D$$

Question:
Consider a logic circuit with 4 input variables in which the output is high when at least 3 inputs are high. Reduce the logic function using K-map.

Example 2.31. *Minimize the following expressions using 4-variable K–map*

(a) $f_a = \Sigma m\ (2, 5, 6, 9, 12, 13)$

(b) $f_b = \Sigma m\ (0, 1, 2, 3, 8, 9, 10, 11)$

(c) $f_c = \Sigma m\ (4, 5, 6, 7, 12, 13, 14, 15)$

(d) $f_d = \Sigma m\ (2, 6, 8, 9, 10, 11, 14)$

(e) $f_e = \Sigma m\ (0, 1, 4, 5, 8, 9, 10, 11, 14, 15)$

(f) $f_f = \Sigma m\ (0, 2, 5, 7, 8, 10, 13, 15)$

(g) $f_g = \Sigma m\ (1, 3, 4, 6, 9, 11, 12, 14)$