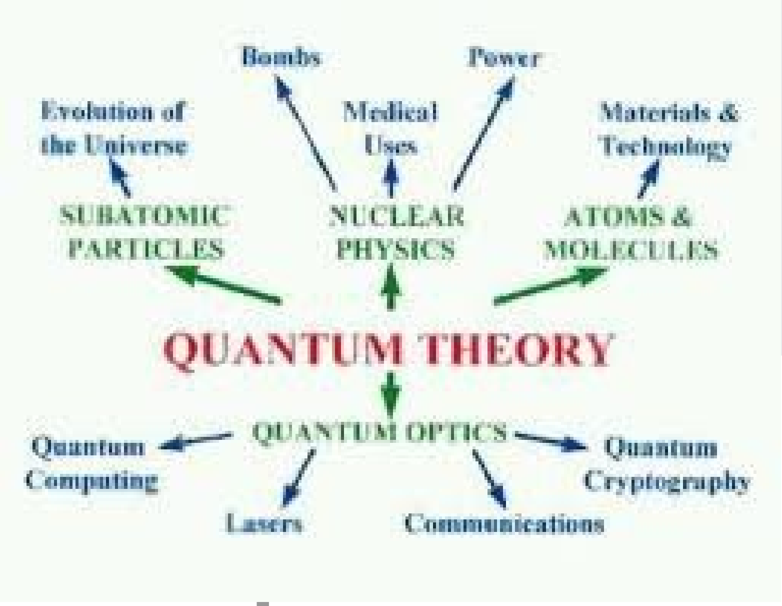
Quantum Mechanics

Module:3	Elements of quantum mechanics	6 hours
Need for Quantum Mechanics: Idea of Quantization (Planck and Einstein) - Compton effect (Qualitative) – de Broglie hypothesis - - Davisson-Germer experiment - Wave function and probability interpretation - Heisenberg uncertainty principle - Schrödinger wave equation (time dependent and time independent).		



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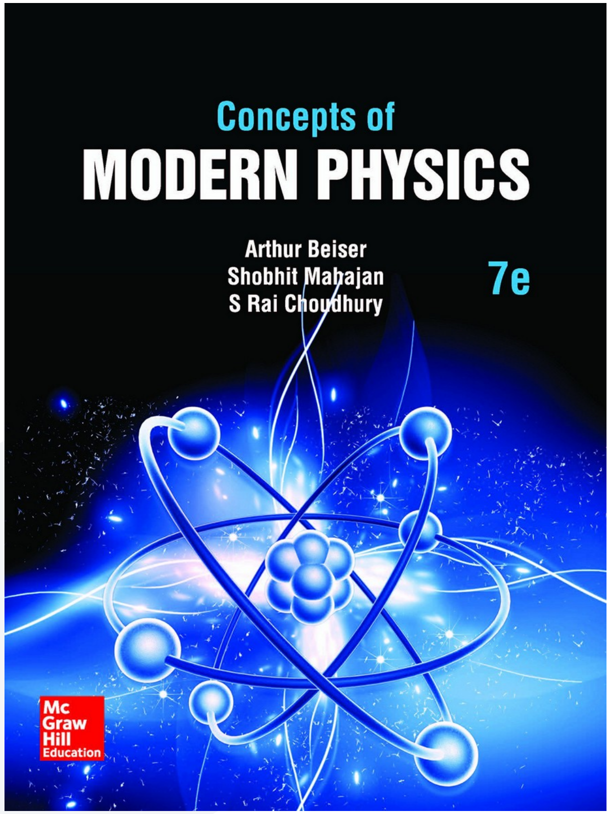
Books: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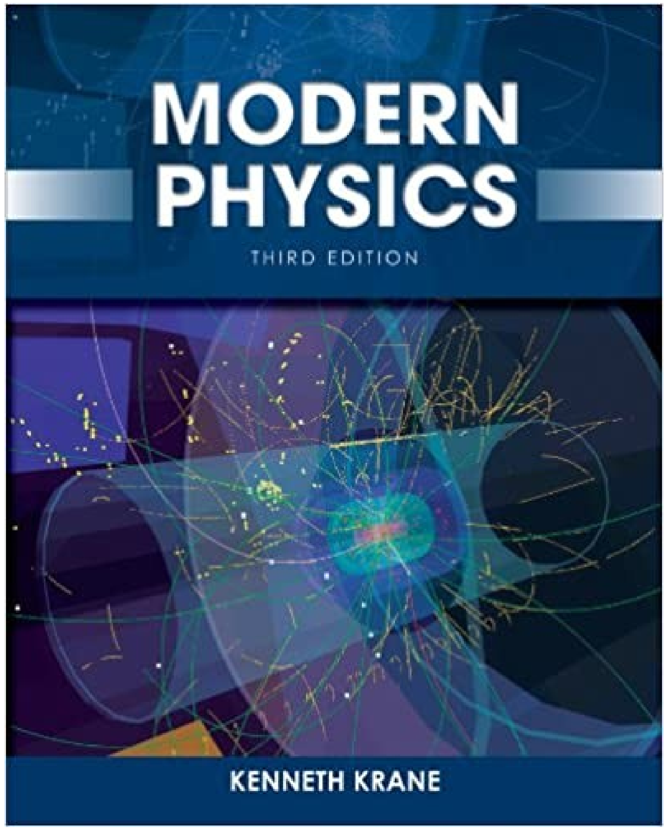
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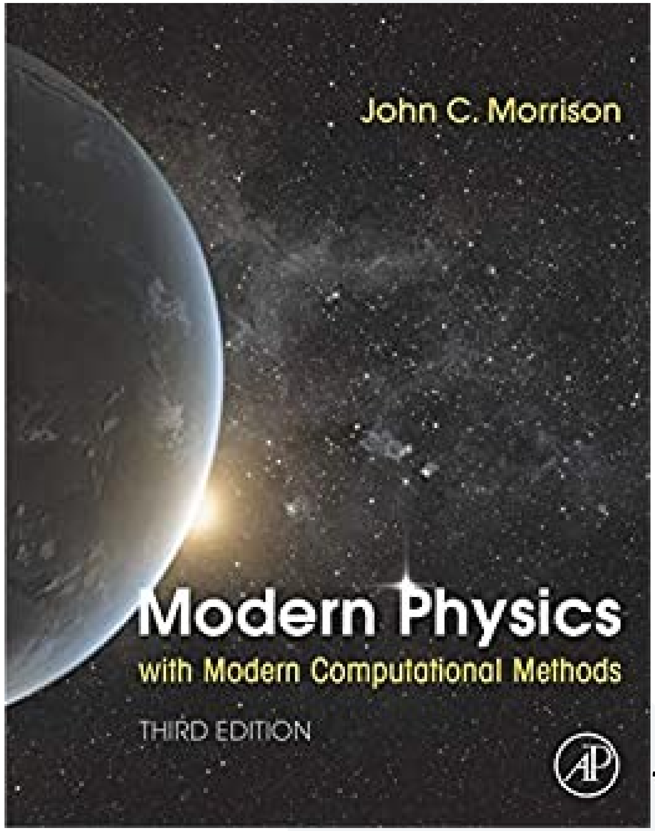
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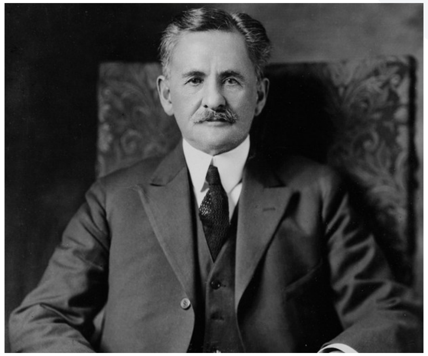
The Classical Attitude

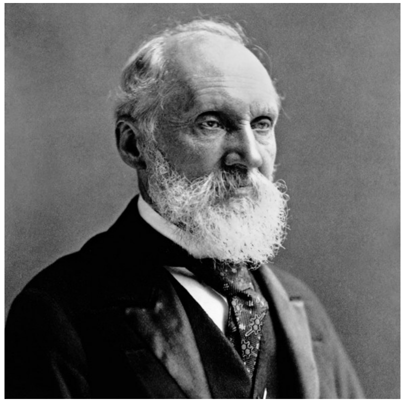
"The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote.... Our future discoveries must be looked for in the sixth place of decimals."

Albert A. Michelson
Speech at the dedication of Ryerson Lab
University of Chicago, 1894

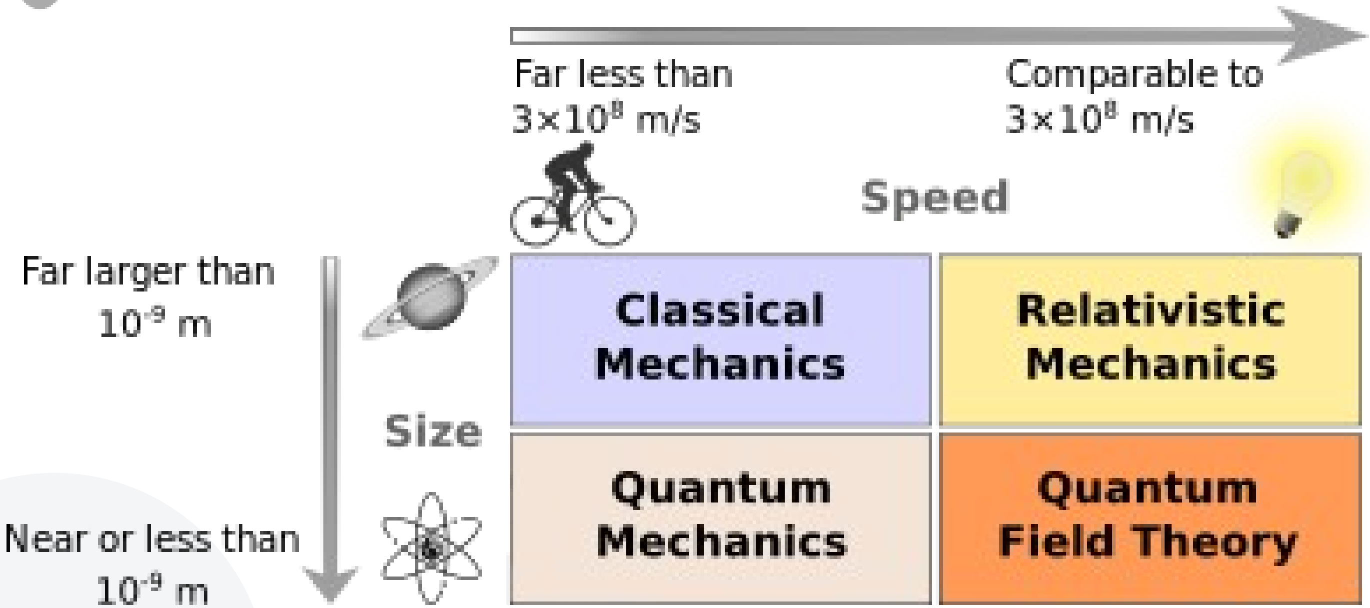


"There is nothing new to be discovered in physics now. All that remains is more and more precise measurement."

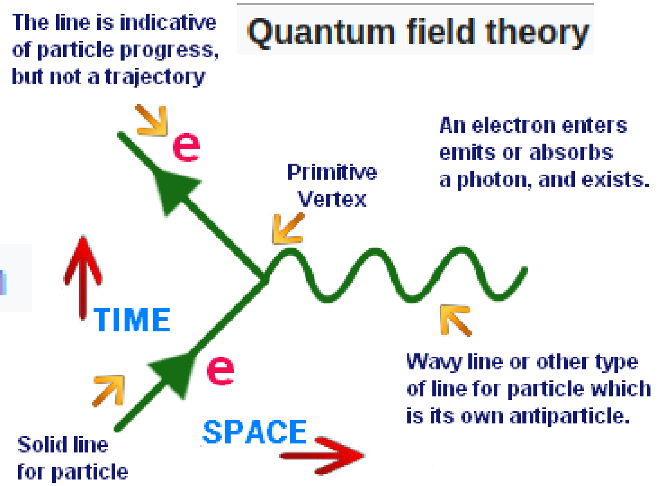
Lord Kelvin, 1897



Classical vs Quantum Mechanics



Feynman diagram

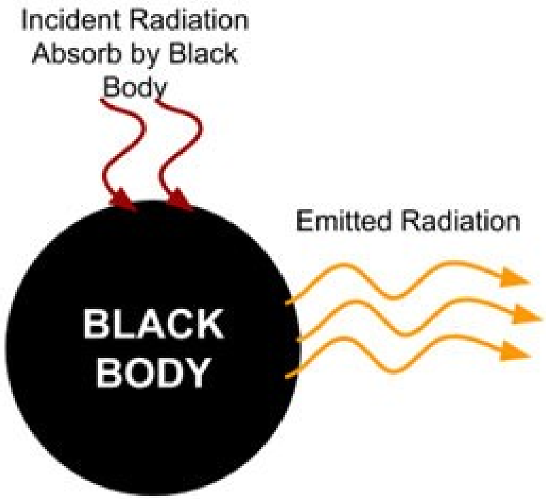


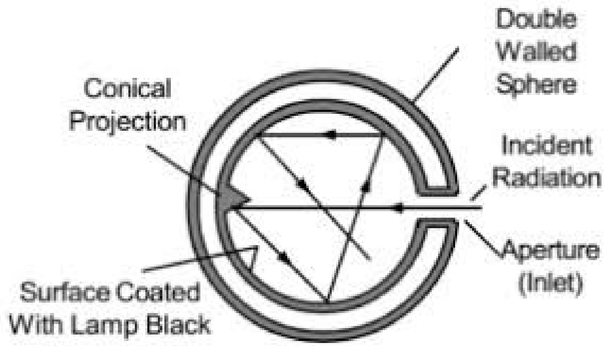
Need for Quantum Mechanics

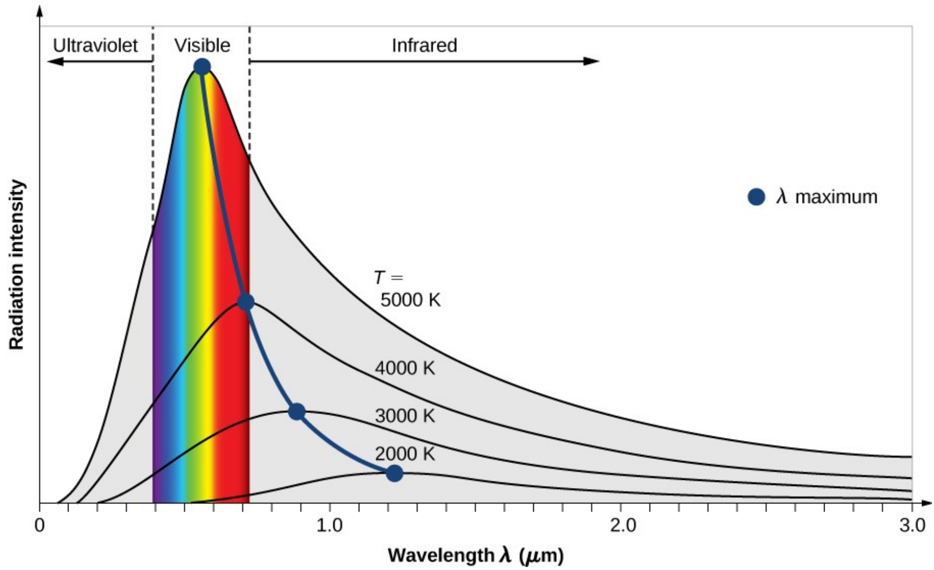
(Inadequacy of Classical Mechanics)

- It doesn’t hold in the region of atomic dimensions.
- It could not explain stability of atoms.
- **It could not explain observed spectrum of black body radiations.**
- It could not explain the origin of discrete spectra of atoms.
- **It could not explain the observed phenomena like photoelectric effect, Compton effect**, Raman effect etc.

Black-Body Radiation

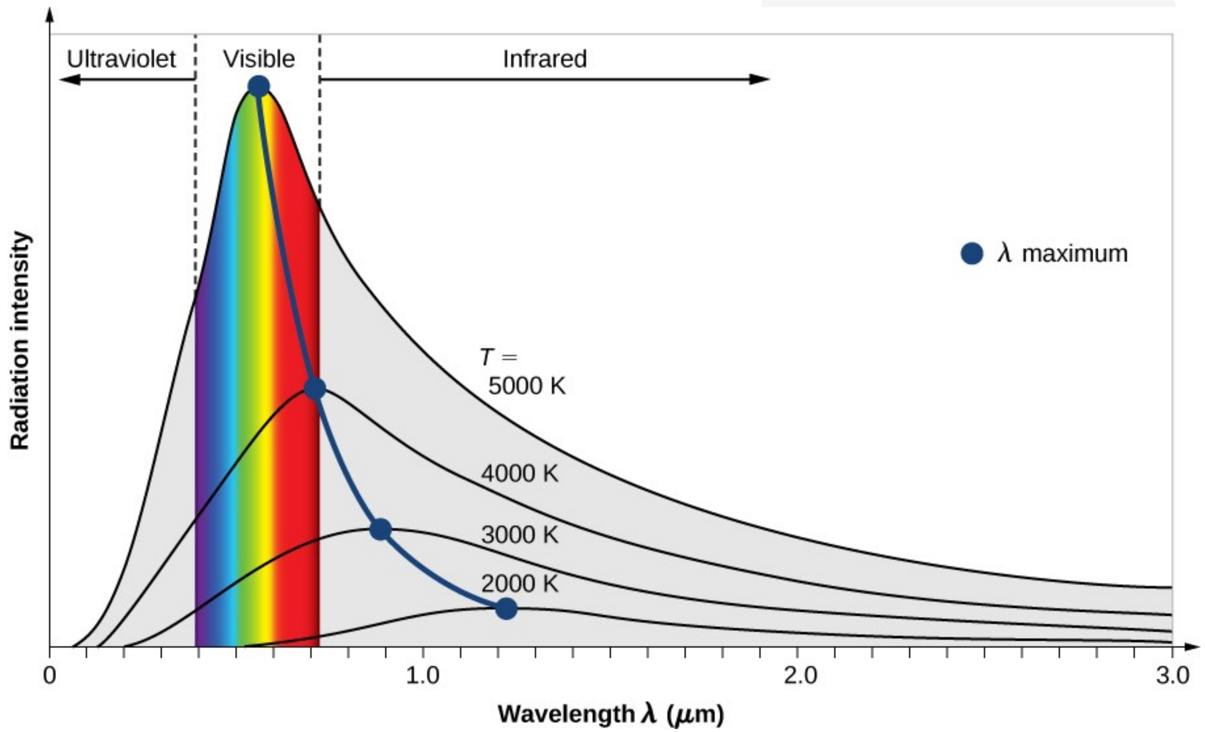






Any body that absorbs all radiation incident upon it, regardless of frequency. Such a body is called a **blackbody**.

Total Power radiated by a Blackbody



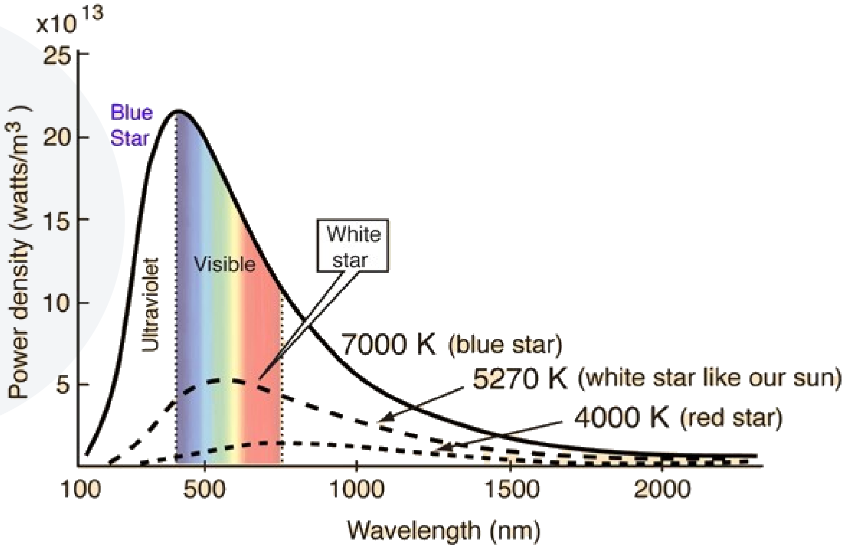
The second experimental relation is **Stefan's law,** which concerns the total power of blackbody radiation emitted across the entire spectrum of wavelengths at a given temperature. In $6.2.2$, this total power is represented by the area under the blackbody radiation curve for a given **T**. As the temperature of a blackbody increases, the total emitted power also increases. Quantitatively, Stefan's law expresses this relation as

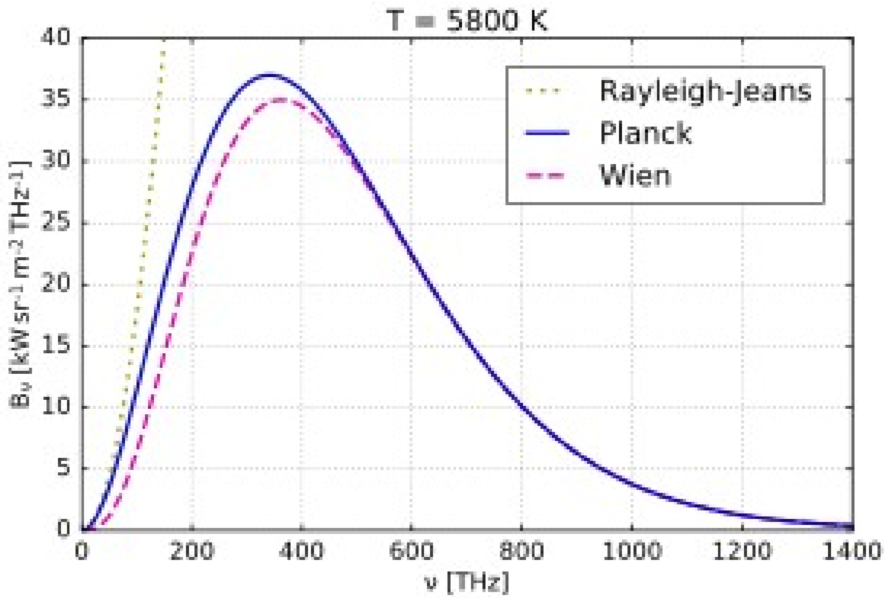
$$P(T) = \sigma A T^4$$

where A is the surface area of a blackbody, T is its temperature (in kelvins), and σ is the **Stefan–Boltzmann constant**, $\sigma = 5.670 \times 10^{-8} W/(m^2 \cdot K^4)$. Stefan's law enables us to estimate how much energy a star is radiating by remotely measuring its temperature.

Drawbacks of Wein’s theory

- Wein’s arguments **purely** based on **thermodynamic**.
- The argument with experimental results **was good only in high frequency region**.





lambda_max=b/T

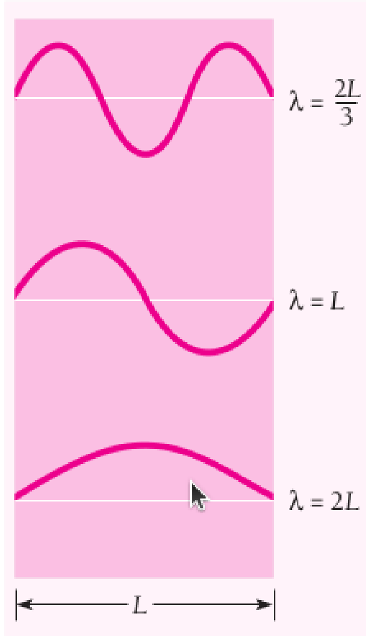
Rayleigh and Jeans

- They treat radiation inside a black body as standing EM waves and estimated the number of modes of vibration per unit volume in the frequen and $\nu + d\,\nu$.

The number of standing waves in the cavity in the frequency ran

Density of standing waves in cavity

$$G(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{c^3}$$



- This formula is independent of shape of the cavity.
- And higher the frequency, shorter is the wavelength and hence more is the number of possible standing waves.

Next step is to find the average energy per standing wave.

According to the theorem of equipartition of energy.
The average energy per standing wave is

$$\bar{\epsilon} = kT$$

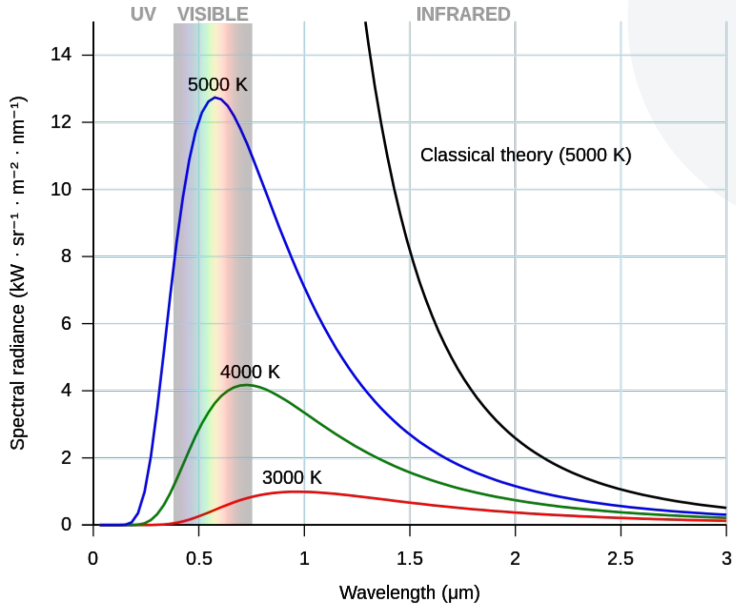
Here k is **Boltzmann's constant:**

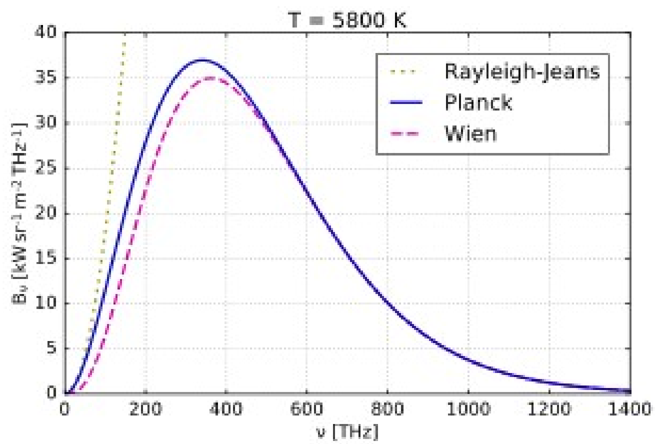
Boltzmann's constant $$k = 1.381 \times 10^{-23} \text{ J/K}$$

The total energy $u(\nu)\, d\nu$ per unit volume in the cavity in the frequency interval from ν to $\nu + d\nu$ is therefore

Rayleigh-Jeans formula $$u(\nu)\, d\nu = \bar{\epsilon} G(\nu)\, d\nu = \frac{8\pi kT}{c^3} \nu^2\, d\nu$$

The **ultraviolet catastrophe** (**also called the Rayleigh–Jeans catastrophe**) refers to the fact that the Rayleigh–Jeans law accurately predicts experimental results at radiative frequencies below 100 THz, but begins to diverge from empirical observations as these frequencies reach the ultraviolet region of the electromagnetic spectrum.





Planck Radiation Formula

In 1900 the German physicist Max Planck used "lucky guesswork" (as he later called it) to come up with a formula for the spectral energy density of blackbody radiation:

Planck radiation formula

$$u(\nu)\,d\nu = \frac{8\pi h}{c^3}\frac{\nu^3\,d\nu}{e^{h\nu/kT}-1}$$

Here h is a constant whose value is

Planck's constant

$$h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$$

Planck’s Hypothesis

The problem that challenged Max Plack was a theoretical explanation for the blackbody radiation curves. In 1900, Planck modified the Wein’s formula in such a way that it fitted precisely with the experimental curves and then looked for a theoretical basis for the formula.

He assumed that the atoms of the walls of the blackbody behave like tiny electromagnetic oscillators, each with a characterstic frequency of oscillation. The oscillators emit electromagnetic energy into the cavity and absorb electromagnetic energy from it.

Planck's Hypothesis cont'd...

Planck then boldly put forward the following suggestions:

1) An oscilltor can only have energies given by

$$\text{Oscillator energies} \qquad \epsilon_n = nh\nu \qquad n = 0, 1, 2, \ldots$$

In other words, the oscillator energy is quantized.

2) Oscillators can absorb or emit energy only in discrete units called **quanta**. That is,

$$\Delta E_n = \Delta n h\nu = h\nu$$

An oscilltor, in a quantized state, neither emits nor absorbs energy.

At high frequencies, $h\nu \gg kT$ and $e^{h\nu/kT} \to \infty$, which means that $u(\nu)\, d\nu \to 0$ as observed. No more ultraviolet catastrophe. At low frequencies, where the Rayleigh-Jeans formula is a good observation to the data, $h\nu \ll kT$ and $h\nu/kT \ll 1$

In general,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

If x is small, $e^x \approx 1 + x$, and so for $h\nu/kT \ll 1$ we have

$$\frac{1}{e^{h\nu/kT}-1} \approx \frac{1}{1 + \frac{h\nu}{kT} - 1} \approx \frac{kT}{h\nu} \qquad h\nu \ll kT$$

Thus at low frequencies Planck's formula becomes

$$u(\nu)\, d\nu \approx \frac{8\pi h}{c^3}\nu^3 \left(\frac{kT}{h\nu}\right) d\nu \approx \frac{8\pi kT}{c^3}\nu^2\, d\nu$$

which is the Rayleigh-Jeans formula. Planck's formula is clearly at least on the right track; in fact, it has turned out to be completely correct.