TN1008
Advanced Simulation and Visualization of Fluids in Computer Graphics
Divergence-Free Smoothed Particle Hydrodynamics

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Abstract—

Index Terms—Divergence-free, SPH, divergence correction, density correction.

1 Introduction

Smoothed particle hydrodynamics, *SPH*, is a method that was first implemented in 1977 for astrophysical simulations by Gingold et al. [1]. Since then SPH has become a popular method for complex water simulations. SPH is a mesh-free Lagrangian method where the particles move in space and change physical properties as time progresses.

In this paper we are going to introduce the results from reproducing the divergence-free smoothed particle hydrodynamics method introduced by Bender et al. [2]. It is a method which corrects the divergence error, aiming for a divergence-free velocity field which is needed for an incompressible fluid. For the solution to be divergence-free the density has to be constant over time.

2 Background and Related Work

[3].

3 Method

3.1 Neighbourhood search

Since SPH only considers a finite amount of neighbouring particles, it is important to keep track of every particles neighbours. Searching through all particles for neighbours within the cutoff distance H for every particle is inefficient and takes $\mathcal{O}(N^2)$ time. The cutoff distance H is the kernel smoothing radius. To fasten this up a cell list was implemented. A cell list is a data structure that is divided into cells that have a length larger or equals to the cutoff distance H. When finding the neighbour of particle i, only the neighbouring cells have to be searched for particles within the cutoff distance, see figure 1.

The cell list is implemented by using a four dimensional vector where the first three dimensions are for the x, y and z coordinates for the cells and the fourth dimension is for storing the particles belonging to the cell. The amount of cells are decided by dividing the scene into cells of length H. The particles are then assigned to a cell according to Equation 1. If the particle moves out of its cell it is then assigned to the new cell.

$$sjdfkljsf \tag{1}$$

3.2 Kernel

A kernel function is used to simulate how particle-particle interactions decrease with the distance between the current particle and its neighbours. In SPH simulations this is an approximation of the Gaussian kernel function. Different kernels have been tested in previous works i.e. the poly6 kernel, the spiky kernel and the cubic spline kernel. According to Bender et. al. [2] the cubic spline kernel presented by Monaghan [4] was used. The kernel is described by Equation 2, where $q(x) = \frac{\|x\|}{h}$, x is the distance between the current particle and a neighbour particle and h is the support radius for the kernel. Particles further away than the support radius will not affect the current particle.

$$W_h(q(x)) = \frac{1}{\pi h^3}\begin{cases} 1-\frac{3}{2}q^2+\frac{3}{4}q^3 & 0 \leqslant q < 1 \\ \frac{1}{4}(2-q)^3 & 1 \leqslant q < 2 \\ 0 & q \geqslant 2 \end{cases} \tag{2}$$

The algorithm does also require the kernel gradient. To reduce the computational effort and memory requirements Bender et. al. [2] introduce a scalar function $g(q) = \frac{\partial W_h}{\partial q} \cdot \frac{1}{h\|x\|}$. The gradient kernel is then calculated by $\partial W_h(q(x)) = x \cdot g(x)$. The gradient kernel is described by Equation 3.

$$\partial W_h(q(x)) = x \cdot \frac{1}{h\,\|x\|} \cdot \frac{1}{\pi h^3}\begin{cases} -3q+\frac{9}{4}q^2 & 0 \leqslant q < 1 \\ -\frac{3}{4}(2-q)^2 & 1 \leqslant q < 2 \\ 0 & q \geqslant 2 \end{cases} \tag{3}$$

It is important to use the same kernel function for both W_h and ∇W_h to get the prediction and the correction step to be compatible to each other.

3.3 Divergence solver

3.4 Density solver

3.5 Navier-stokes

3.6 Adapted time step

3.7 Density and alpha factors

3.8 Screen space fluid rendering

4 Implementation

5 Results

6 Conclusions and Future Work

References

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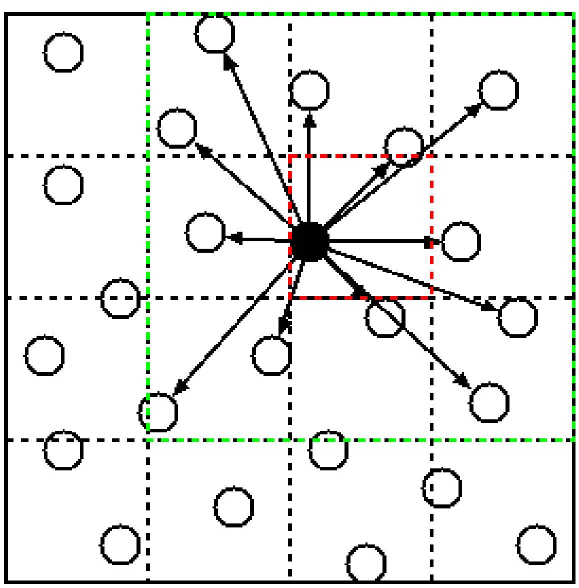

Fig. 1. Finding the neighbours for the filled in particle i by looking through all neighbouring cells, including its own cell, for particles within the cutoff distance H