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Lab Report
Series LCR Circuits

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Abstract

This experiment was conducted in order to study the behavior of damped harmonic oscillations in a resonant circuit consisting of an inductor L, a capacitor C and a resistor R connected in series. The secondary aim of this experiment was to deduce values for the resistance(R), capacitance(C) and inductance(L) of the circuit elements. This was done using three different methods. First one being measuring and deriving a relation of resonant frequency with varying capacitance giving the values of L and C as 9.37±0.12 mH and 101.07±7.37 nF respectively. The second method involved the variation of the quality factor with resistance which provided the experimental values of L and internal resistance of the inductor r_L. The calculated values of L and r_L using this method were 10.986±0.297mH and 0.351±0.01Ω, respectively. The last part involved understanding the transient response of the LCR circuit and recording the logarithmic decay of oscillations in the circuit which also provided the values of L, r_L and C. These values were 10.62±0.74 mH, 0.38±0.06 Ω and 87.09±6.01 nF, respectively.

Introduction

One of the main principles followed in this experiment was harmonic oscillation. The combination of an inductor, a capacitor and a resistor connected in series with an alternating voltage source has a direct analogy with a mechanical system undergoing forced or damped harmonic vibrations. The motion of a point-mass m attracted to the origin by a force $= kx$ and subjected to a resistive force $= b\frac{dx}{dt}$ can be written as the differential equation -:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \tag{1}$$

The corresponding equation for a LCR series circuit driven by an alternating signal $V_0 \sin \omega t$ is –:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + Cq = 0 \tag{2}$$

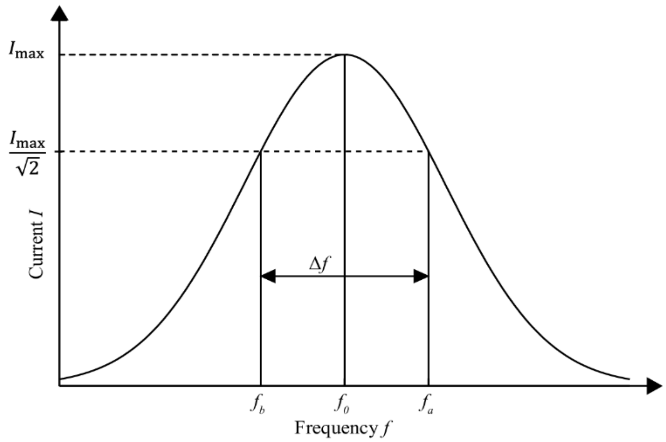
where q is the charge on the capacitor C at time t. Comparing eq (1) and (2) it was seen that the inductance is the electrical analogue of mass, resistance is the analogue of a velocity-dependent friction, and the reciprocal of the capacitance is analogous to the spring constant k. Another important principle used was the resonant frequency of a LCR series circuit. The resonant frequency is value of frequency of the alternating voltage at which the current in the series LCR circuit reaches a maximum value. This can be described by the relation -:

$$f_0{}^2 = \frac{1}{4\pi^2 LC} \tag{3}$$

Where f_0 is the resonant frequency of the circuit. The measure of how sharply-defined the current maximum is around f_0 is called the Quality factor(Q). If the current has a value I_{max} at f_0, and it falls to a value $\frac{I_{max}}{\sqrt{2}}$ at frequencies f_a and f_b, the Q value can then be determined by -:

$$Q = \frac{f_0}{f_b - f_a} = \frac{f_0}{df} = \frac{2\pi f_0 L}{R} \quad (4)$$

This can also be understood by the given graph -:



Graph 1 - current vs frequency in a LCR series circuit

Another key principle applied was the exponential decay of damped oscillations in the LCR series circuit. The amplitude of a damped harmonic oscillator that is not driven by an external force will decay exponentially with time and as seen in eq (2) LCR series circuit works as a damped harmonic oscillator. Therefore, the logarithmic decrement λ is given by -:

$$\lambda = \frac{1}{m} \ln \frac{V_j}{V_{j+m}} \quad (5)$$

Where V_j represents the amplitude of the j^{th} oscillation. Given $R^2 \ll \frac{4L}{C}$, λ can also be defined as the following -:

$$\lambda = \frac{R}{4Lf_0} \quad (6)$$

The source for alternating voltage used was a waveform generator, which is an electronic instrument that produces a variety of signal shapes, such as sine, square, triangle, and sawtooth waves, over a wide range of frequencies. It can be adjusted in parameters like frequency, amplitude, and waveform type to simulate different conditions in a circuit.

The key instrument used in this experiment, to measure the AC voltage (peak-to-peak) and frequency, was the oscilloscope. An oscilloscope is a kind of electrical test device that shows the voltage variations of one or more signals graphically over time. After the waveform is shown, many characteristics can be examined, including distortion, amplitude, frequency, rise time, and time interval. For this experiment the vertical gain (voltage base) was measured for the voltage and the horizontal time base was manipulated to measure the frequency.

Another important instrument used was Buffer Amplifier. A voltage buffer amplifier is used to convert a voltage signal with high output impedance from one circuit into an identical voltage signal with low output impedance for another circuit. By interposing the buffer amplifier, the second circuit is prevented from exerting an excessive load on the first circuit, which could otherwise disrupt its intended operation. Without the voltage buffer, the voltage in the second circuit would be affected by the high output impedance of the first circuit, as it exceeds the input impedance of the second circuit.

The other principles used in the calculation for this experiment were uncertainties and errors. There were three types of errors involved –

Instrumental error – Errors that occur due to the minimum scaling of the non–digital instrument, where, it may be possible to estimate additional figures. Hence, the instrumental error can be taken as ± half of the smallest unit.

Systematic error – Common mistakes constant throughout the entire experiment, such as incorrect calibration of instrument or human error such as reaction time, etc.

Random error – Small and unpredicted fluctuations in either the technique or the environment that alter the results of the experiment.

To express the error of data sets, two methods were used, described in detail in subsequent pages of this report -
Statistical analysis of small data sets
LINEST function in excel

The final principle used was error propagation. The error of the quantities, which were combined in a formula to get the result, gave rise to new errors. Therefore, to work out these new errors, the formulas, listed below, were used –

Addition and Subtraction	z = x ± y	$\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$	(7)
Multiplication and Division	z = xy or $\frac{x}{y}$	$\Delta z = \lvert z\rvert\sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$	(8)
Powers	z = x^n	$\Delta z = \lvert n\rvert x^{n-1}\Delta x$	(9)
Constant	z = Cx	$\Delta z = \lvert C\rvert\Delta x$	(10)
Function	z = f(x,y)	$\Delta z = \sqrt{\left(\frac{\delta f}{\delta x}\right)^2 (\Delta x)^2 + \left(\frac{\delta f}{\delta y}\right)^2 (\Delta y)^2}$	(11)

To minimize the error on the readings, repeated readings were taken. This minimized the systematic and random errors

Method

The variation of resonant frequency with capacitance

For this initial experiment, the circuit was set up using the below given figure -:

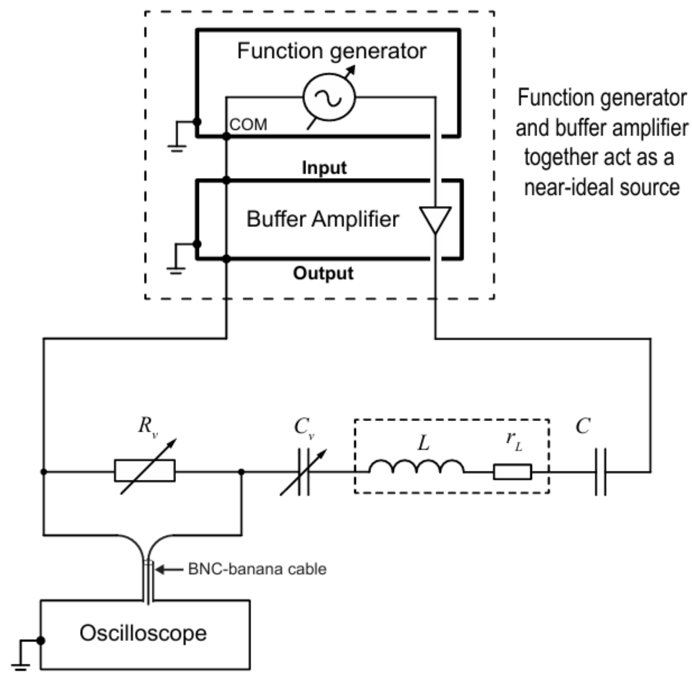


Figure 1 - setup for the first experiment

It consisted of an unknown inductance L (including its internal resistance r_L) connected in series with an unknown fixed capacitance C. Additionally, a variable capacitor C_V and a variable resistor R_V were also part connected in series in the circuit. The circuit was powered by a signal generator, which was connected to a buffer amplifier with an extremely low output impedance (much less than 1Ω), which made it function as an almost ideal voltage source. The generator's frequency was monitored using a digital frequency meter. The signal generator was adjusted to give a 1V peak-to-peak sine signal and R_V was set to a fixed value of about 100 **Ω**. C_V was varied from 0.01 to 1 µF but the step size was chosen such that the steps for $\frac{1}{C_v}$ were equally spaced. For each setting of C_V, the frequency of the signal generator was adjusted until a maximum voltage is observed across R_v. this was the resonant frequency f_0. This frequency was measured and noted.

The variation of the quality factor with resistance

For this part the C_v was removed from the same circuit as in fig 1 and new resonant frequency was found for R_V over a range of 10 to 100 **Ω**. For each case the width of frequency df = $f_b - f_a$ was also measured. This was done by measuring f_b and f_a at $\frac{V_{max}}{\sqrt{2}}$ (higher and lower than f_0 respectively)

The transient response of the LCR circuit

The same circuit, as used in previous part, was used in this part and the R_V was fixed to a value around 10 **Ω**. The signal generator was set to produce a square signal with a frequency of around 70 Hz and a peak-to-peak output voltage from the buffer amplifier of 10 V. The oscilloscope settings were adjusted so that the voltage signal across R_V could be examined closely as the square input signal jumps from one value to the other. A damped harmonic oscillation of the signal across R_V was visible. The variable range of R_V was kept the same as before (10-100Ω). For this range of R_V, the value of V_{pp} (peak to peak) for 4 different damped oscillations were recorded which were then used to calculate multiple values of the logarithmic decay (λ) of the circuit

Results

The variation of resonant frequency with capacitance

The data collected for this experiment is as follows -:

C_V(nf)	f_0(khz)
10	17.4
11.23595506	16.3
12.82051282	15.3
14.92537313	14.3
17.85714286	13.3
22.22222222	12.3
29.41176471	10.8
43.47826087	9.6
83.33333333	7.7
1000	5.4

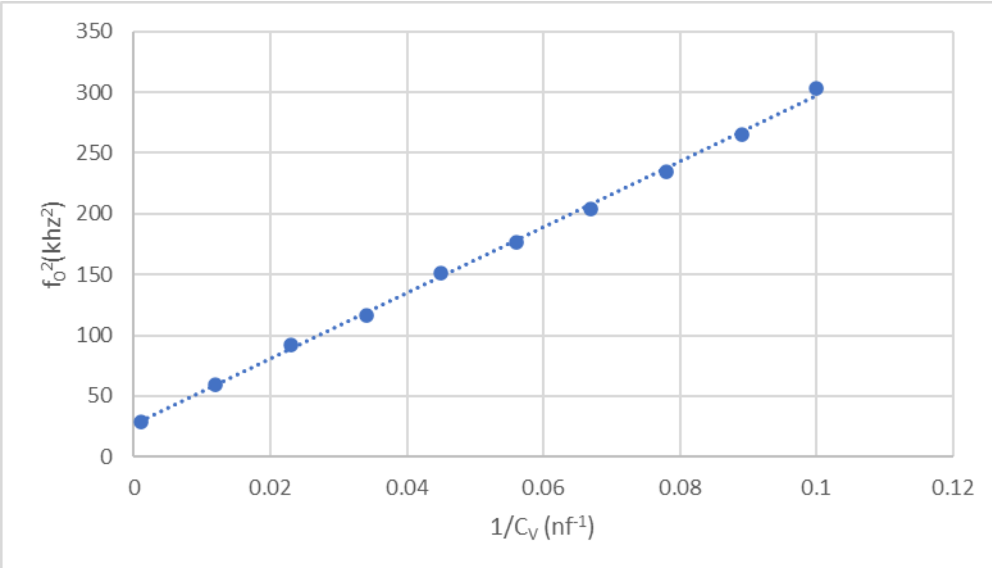
Table 1 - data for variation of resonant frequency with capacitance

By manipulating this data, values of f_0^2 and $\frac{1}{C_V}$ were obtained. The following are the errors on these values which were calculated using eq (9) -:

Δf_0(khz)	ΔC_V(nf)	Δf_0^2(khz^2)	$\Delta 1/C_V$ (nf^{-1})
0.005	0.005	0.174	0.00005
0.005	0.005	0.163	3.9605E-05
0.005	0.005	0.153	0.00003042
0.005	0.005	0.143	2.2445E-05
0.005	0.005	0.133	0.00001568
0.005	0.005	0.123	1.0125E-05
0.005	0.005	0.108	5.78E-06
0.005	0.005	0.096	2.645E-06
0.005	0.005	0.077	7.2E-07
0.005	0.005	0.054	5E-09

Table 2 - error on variation of frequency with variable capacitance

The following is a plot of the values of f_0^2 and $\frac{1}{C_V}$ -:



Graph 2 - plot of the values of f0^2 and 1/Cv (error bars are too small to be seen)

The total capacitance of the circuit C_t can be written as $\frac{1}{C_t} = \frac{1}{C} + \frac{1}{C_v}$. Inputting this in eq (3), the following relation was derived -:

$$f_0{}^2 = \frac{1}{4\pi^2 L C_t} = \frac{1}{4\pi^2 LC} + \frac{1}{4\pi^2 L C_v} \tag{12}$$

Using the equation deduced from the graph of f_0^2 vs $\frac{1}{C_V}$, it was observed that the gradient of the straight line was equal to $\frac{1}{4\pi^2 L}$ and the y-intercept equaled $\frac{1}{4\pi^2 LC}$. The values of gradient and y-intercept of the line were calculated using the LINEST function in Excel. These values were then substituted in the below given equations -:

$$L = \frac{1}{4\pi^2 grad} \tag{13}$$

$$C = \frac{1}{4\pi^2 L(intercept)} \tag{14}$$

Therefore, the values of L and C were found out to be 9.37mH and 101.07nF respectively. The error on these values were calculated using equations derived by eq (9), (10) and (11). The formulae are as follows -:

$$\Delta L = \frac{L \Delta grad}{grad} \tag{15}$$

$$\Delta C = C\sqrt{\left(\frac{\Delta intercept}{intercept}\right)^2 + \left(\frac{\Delta L}{L}\right)^2} \tag{16}$$

Thus, the final values of L and C were 9.37±0.12 mH and 101.07±7.37 nF respectively.

The variation of the quality factor with resistance

The data recorded for this part is as follows -:

$R_v(\Omega)$	f_a(khz)	f_b(khz)	V_{pp} at f_0 (mv)
10	4.027	6.252	813
20	3.976	6.306	831
30	3.91	6.43	839
40	3.84	6.51	855
50	3.801	6.601	867
60	3.75	6.68	873
70	3.69	6.8	877
80	3.62	6.91	883
90	3.61	6.92	891
100	3.54	7.07	894

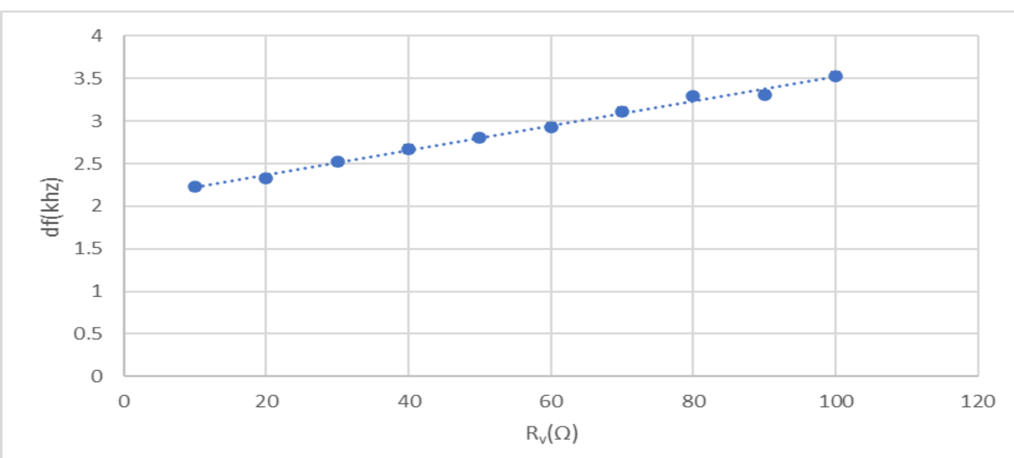
Table 3 - recorded data for variation of Q value with Rv

This data, was then used to calculate df using df = $f_b - f_a$. The errors in the above table were deduced using eq (7) and some were instrumental errors. They are given as follows -:

$\Delta R_v(\Omega)$	Δf(khz)	Δdf(khz)
0.5	0.005	0.00707
0.5	0.005	0.00707
0.5	0.005	0.00707
0.5	0.005	0.00707
0.5	0.005	0.00707
0.5	0.005	0.00707
0.5	0.005	0.00707
0.5	0.005	0.00707
0.5	0.005	0.00707
0.5	0.005	0.00707

Table 4 - errors on the recorded data

The following graph of R_V vs df was then plotted -:



Graph 3 - graph for Rv vs df (error bars are too small to be seen)

Using the total resistance in a series circuit R = R_V + r_L and putting it in eq (4), the following relation was derived -:

$$df = \frac{R}{2\pi L} = \frac{R_v}{2\pi L} + \frac{r_L}{2\pi L} \tag{17}$$

Similar to the previous part, from the graph of R_V vs df, it was determined that the gradient of the straight line corresponds to $\frac{1}{2\pi L}$, while the y-intercept corresponds to $\frac{r_L}{2\pi L}$. The gradient and y-intercept were calculated using the LINEST function in Excel, and these values were then substituted into the following equations -:

$$L = \frac{1}{2\pi grad} \tag{18}$$

$$r_L = (intercept)2\pi L \tag{19}$$

As a result, the values of L and r_L were determined to be 10.986 mH and 0.351Ω, respectively. The uncertainties associated with this value of L was calculated using eq (15). For r_L the equation for error was derived eq (11). The corresponding formula is as follows:

$$\Delta r_L = r_L \sqrt{\left(\frac{\Delta intercept}{intercept}\right)^2 + \left(\frac{\Delta L}{L}\right)^2} \tag{20}$$

Therefore, the reported values of L and r_L are 10.986±0.297 mH and 0.351±0.01 Ω, respectively

The variation of the quality factor with resistance

For this part of the experiment, multiple values of V_{pp} (peak to peak) were recorded, each for a different oscillation and R_V. These are as follows -:

R_v(Ω)	V1(mV)	V2(mV)	V3(mV)	V4(mV)
10	262	220	192	158
20	472	380	328	256
30	670	470	400	270
40	840	550	440	280
50	960	640	470	310
60	1070	660	490	320
70	1130	710	510	320
80	1250	740	490	260
90	1360	740	540	260
100	1420	720	520	260

Table 5 - values of Vpp for variable Rv

These values were then used in eq (5) to get six different values of λ which were then averaged out. This was done to reduce the random and systematic errors. The following are the different value of λ before averaging out -:

λ1	λ2	λ3	λ4	λ5	λ6
0.175	0.1361	0.195	0.166	0.155	0.1686
0.217	0.1472	0.248	0.197	0.182	0.2039
0.355	0.1613	0.393	0.277	0.258	0.303
0.423	0.2231	0.452	0.338	0.323	0.3662
0.405	0.3087	0.416	0.362	0.357	0.3768
0.483	0.2978	0.426	0.362	0.391	0.4024
0.465	0.3309	0.466	0.398	0.398	0.4206
0.524	0.4122	0.634	0.523	0.468	0.5234
0.609	0.3151	0.731	0.523	0.462	0.5515
0.679	0.3254	0.693	0.509	0.502	0.5659

Table 6 - different value of logarithmic decay

The error on these values were calculated using eq (7) and the below given formula derived using eq (11) -:

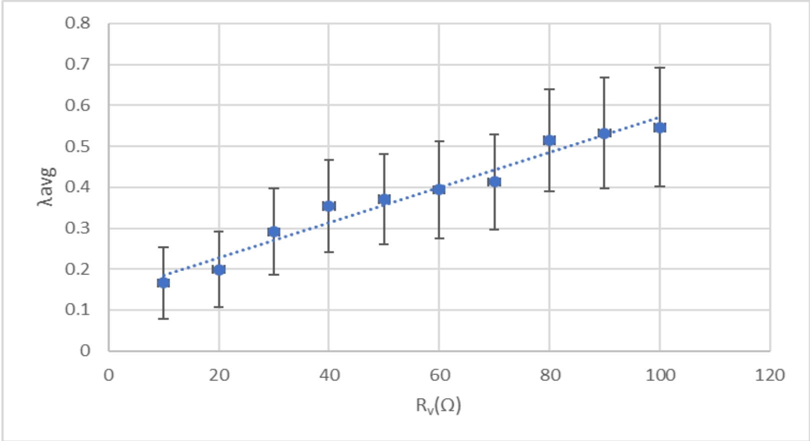
$$\Delta\lambda = \Delta V_{pp}\sqrt{\left(\frac{1}{mV_jV_{j+m}}\right)^2 + \left(\frac{V_j}{mV_{j+m}}\right)^2} \tag{21}$$

The error are as follows -:

ΔV	Δλ	Δλavg
0.05	0.036	0.088
0.05	0.037	0.091
0.05	0.043	0.105
0.05	0.046	0.112
0.05	0.045	0.11
0.05	0.049	0.119
0.05	0.048	0.117
0.05	0.051	0.124
0.05	0.055	0.135
0.05	0.059	0.145

Table 7 - error on λ and Vpp

Using the recorded data, a graph between R_V and λ_{avg}. The plot is as follows -:



Graph 4 - graph for Rv vs logarithmic decay

Similar to the previous part, substituting the total resistance in a series circuit, R = $R_V + r_L$, into Equation (6), the resulting relationship was derived as follows -:

$$\lambda = \frac{R}{4Lf_0} = \frac{R_V}{4Lf_0} + \frac{r_L}{4Lf_0} \tag{22}$$

As in the previous section, the graph of R_V versus λ_{avg} was analyzed to determine that the gradient of the straight line corresponds to $\frac{1}{4Lf_0}$, and the y-intercept corresponds to $\frac{r_L}{4Lf_0}$. The gradient and y-intercept were calculated using the LINEST function in Excel and subsequently substituted into the following equations -:

$$L = \frac{1}{4f_0 grad} \tag{23}$$

$$r_L = (intercept)4Lf_0 \tag{24}$$

For the value of f_0, the period of the decaying oscillations (τ) was recorded as 1/60 sec and as $f_0 = \frac{1}{\tau}$, therefore, f_0 was reported as 60 Hz. For the value of C, eq (3) was used. Thus, the values of L, r_L and C were found out to be 10.62 mH, 0.38 Ω and 87.09 nF, respectively. The errors on these values were calculated using equations derived from eq (8), (9) and (11). The equations are as follows -:

$$\Delta L = L\sqrt{\left(\frac{\Delta f_0}{f_0}\right)^2 + \left(\frac{\Delta grad}{grad}\right)^2} \tag{25}$$

$$\Delta r_L = r_L\sqrt{\left(\frac{\Delta intercept}{intercept}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta f_0}{f_0}\right)^2} \tag{26}$$

$$\Delta C = C\sqrt{2\left(\frac{\Delta f_0}{f_0}\right)^2 + \left(\frac{\Delta L}{L}\right)^2} \tag{27}$$

Therefore, the reported values of L, r_L and C are 10.62±0.74 mH, 0.38±0.06 Ω and 87.09±6.01 nF, respectively

Discussion

The literature values of L, C and r_L were 10 mH, 100 nF and 0.45Ω, respectively

The first part provided the calculated values of L and C as 9.37±0.12 mH and 101.07±7.37 nF respectively. No value of r_L was provided. These values of L and C were 6.3% and 1.07% deviated from the literature values of L and C, respectively. Part two, however provided lesser accurate value of L as 10.986±0.297 mH, which was 9.86% deviated, but resulted in a new value of r_L as 0.351±0.01 Ω, which was 22% deviated from the literature value. The last part provided a value for each of the three variables L, r_L and C as 10.62±0.74 mH, 0.38±0.06 Ω and 87.09±6.01 nF, respectively. The results for L and r_L were more accurate, than the ones provided in the earlier parts, as they were 6.2% and 15.56% deviated, respectively. But a less accurate calculation was observed for the value of C as it was 12.91% deviated. These deviations from the literature values were due to a multitude of errors.

A major source of errors in this entire experiment was non ideal equipment. Errors related to imperfect calibration of the signal generator or frequency meter led to slight fluctuations and inaccuracies in the frequency readings, may have been prevalent. Not accounting for the parasitic (internal resistance that was not defined) in the inductor or capacitor affected the true resonance behavior. Moreover, the internal resistance may not be constant over all frequency ranges. Buffer amplifier output impedance may not be exactly zero thus making the voltage source not entirely ideal, therefore, affecting the voltage signal. For the second part, the precision of the oscilloscope might not be great enough to accurately detect $\frac{V_{max}}{\sqrt{2}}$, which may have led to inaccurate readings. The variable resistance used, may have also been imperfectly calibrated, resulting in inaccurate recordings. For the last part, the square waves generated by the voltage source, might have been non-ideal, thus leading to having imperfect sharp transitions. Other systematic error might have arisen due to the oscilloscope calibration errors in measuring the voltage decay amplitude and time intervals. To overcome these error, rigorous statistical analysis was introduced for both large and small data sets. Every instrumental error that could have been found out or recorded, was propagated in the calculation to account for a precise error analysis. The line of best fit method and regression statistics used in graphs to calculate gradients and slopes of line, were done on a high degree of precision to minimize errors.

Some human errors might have also been prevalent in this experiment. Some of those include, having misread the peak resonance voltage due to fluctuations in the oscilloscope or personal bias. Setting the capacitor incorrectly when adjusting C_V values might have led to non-uniform step sizes for 1/ C_V. For the second part, human eye error also had to taken into account as the difficulty in pinpointing f_a and f_b was high due to fluctuating values of $\frac{V_{max}}{\sqrt{2}}$ in the oscilloscope. Moreover, inconsistent adjustments made to the signal generator, to adjust for the fluctuations, may have caused slight deviations in the recorded resonance frequency. For the final part, misjudging the damping envelope when determining the amplitudes V_j and V_{j+m}, introduced subjectivity. Other human errors introduced might have been due to manual set up of most equipment. Other error like heating up of equipment, the parallax of the eye while adjusting the horizontal and vertical knobs and other errors, random instabilities caused by loose wires and contacts, fluctuations in voltage readings caused by noise in the signal generator or environmental electromagnetic interference, were also present. These errors were minimized by taking multiple readings for different values of frequency, resistance, voltage (peak to peak) and logarithmic decay.

Conclusion

This experiment aimed to study damped harmonic oscillations in a resonant LCR series circuit and determine the resistance R, capacitance C, and inductance L of the circuit components using three methods. In the first method, the resonant frequency was analyzed by varying the capacitance, yielding $L=9.37\pm0.12$ mH and $C=101.07\pm7.37$ nF, with deviations of 6.3% and 1.07%, respectively, from the literature values. No value for r_L (internal resistance of the inductor) was obtained in this part. The second method involved studying the variation of the quality factor with resistance. This provided $L=10.986\pm0.297$ mH and $r_L=0.351\pm0.01\ \Omega$, with deviations of 9.86% and 22%, respectively. The final method examined the transient response and logarithmic decay of oscillations, yielding $L=10.62\pm0.74$ mH, $r_L=0.38\pm0.06\ \Omega$ and $C=87.09\pm6.01$ nF. These results had deviations of 6.2% for L, 15.56% for r_L, and 12.91% for C. Overall, deviations from the literature values arose from systematic, random, and human errors, with each method offering varying levels of accuracy for the circuit parameters. The transient response method provided more accurate values for L and r_L, while the resonant frequency method was the most reliable for C.

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