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# Lab Report 3
# AC Circuits

Experiment conducted by: Gurraunak Singh Bedi

Lab Partner: Thomas Cesare Bolton

## Abstract

This experiment was conducted in order to comprehend the oscilloscope's Sinusoidal plots. Understanding the phase shift between the voltage and the current was the secondary goal. This was done by measuring the peak-to-peak AC voltage and deriving the current and phase-shift in three different kinds of circuits for different values of AC frequency. These circuits were RC, RL and only Resistance circuits respectively. The current (I) flowing in the RC circuit was 0.52±0.03mA and the phase-shift showed a exponential and then a gradual increase with decrease in reactance. On the other hand, the current (I) flowing in the RL circuit was 0.803±0.013mA and the phase-shift showed a decrease with increase in reactance. For the resistance only circuits there was no phase-shift as there was no reactance

## Introduction

The key concepts involved in this experiment were Sinusoidal motion, AC voltage (peak to peak), Capacitance, Inductance, Reactance, Phase-shift, Oscilloscope (Vertical Gain, Horizontal time base)

A periodic wave with the waveform or shape of the trigonometric sine function is called a sine wave, sinusoidal wave, or sinusoid. A characteristic that makes periodic waves different is that any two sine waves with the same frequency (but arbitrary phase) can be joined linearly to produce another sine wave with the same frequency. On the other hand, a sine wave of any phase can be expressed as the linear combination of two sine waves, the sine and cosine components, having phases of zero and a quarter cycle, respectively, if a certain phase is selected as the zero reference. The general form of a sine wave of any arbitrary phase is given by -:

$$y(t) = Asin(\omega t + \varphi) \tag{1}$$

Where t represents time, $\omega$ represents angular frequency, A represents amplitude and $\varphi$ is the phase shift

This principle is observed in AC circuits as well. The magnitude of the current and voltage vary sinusoidally. The phase shift depends on the type of circuit used. In a RC circuit, the voltage leads the current by an angle of -:

$$\varphi = \tan^{-1}\left(\frac{X_c}{R}\right) \tag{2}$$

Where $X_c$ is the Reactance Capacitance and R is the external resistance. $X_c$ is the resistance offered to the flow of AC current through the capacitor. This is given by -:

$$X_c = \frac{1}{2\pi\nu C} \tag{3}$$

Where $\nu$ is the frequency and C is the capacitance of the capacitor. Capacitance is the capability of a material object or device to store electric charge. It is measured by the charge in response to a difference in electric potential, expressed as the ratio of those quantities.

In a RL circuit, the current leads the voltage by an angle of -:

$$\varphi = \tan^{-1}\left(\frac{X_l}{R}\right) \tag{4}$$

Where $X_l$ is the Reactance Inductance and R is the external resistance. $X_l$ is the resistance offered to the flow of AC current through the Inductor. This is given by -:

$$X_l = 2\pi\nu L \tag{5}$$

Where $\nu$ is the frequency and L is the inductance of the inductor. Inductance is the tendency of an electrical conductor to oppose a change in the electric current flowing through it. It is defined as the ratio of the induced voltage to the rate of change of current causing it. It is a proportionality constant that depends on the geometry of circuit conductors and the magnetic permeability of the conductor and nearby materials

The key instrument used in this experiment, to measure the AC voltage (peak-to-peak), was the oscilloscope. An oscilloscope is a kind of electrical test device that shows the voltage variations of one or more signals graphically over time. After the waveform is shown, many characteristics can be examined, including distortion, amplitude, frequency, rise time, and time interval. For this experiment the vertical gain (voltage base) was measured for the voltage and the horizontal time base was manipulated to measure the frequency.

The other principles used in the calculation for this experiment were uncertainties and errors. There were three types of errors involved –

Instrumental error – Errors that occur due to the minimum scaling of the non–digital instrument where, it may be possible to estimate additional figures. Hence, the instrumental error can be taken as ± half of the smallest unit.

Systematic error – Common mistakes constant throughout the entire experiment, such as incorrect calibration of instrument or human error such as reaction time, etc.

Random error – Small and unpredicted fluctuations in either the technique or the environment that alter the results of the experiment.

To express the error of data sets, two methods were used, described in detail in subsequent pages of this report -
Statistical analysis of large data sets
Linest function in excel

The final principle used was error propagation. The error of the quantities, which were combined in a formula to get the result, gave rise to new errors. Therefore, to work out these new errors, the formulas, listed below, were used –

| | | | |
|---|---|---|---|
| Addition and Subtraction | z = x ± y | $\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ | (6) |
| Multiplication and Division | z = xy or $\frac{x}{y}$ | $\Delta z = \lvert z\rvert\sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$ | (7) |
| Powers | z = $x^n$ | $\Delta z = \lvert n\rvert x^{n-1}\Delta x$ | (8) |
| Constant | z = Cx | $\Delta z = \lvert C\rvert\Delta x$ | (9) |

To minimize the error on the readings, repeated readings were taken. This minimized the systematic and random errors

## Method

### Calibration of Frequency

For the first step, the frequency dial of the AC generator needed to be calibrated. The diagram given below was followed to setup the AC function generator to produce a sinusoidal wave of frequency 100 Hz -:

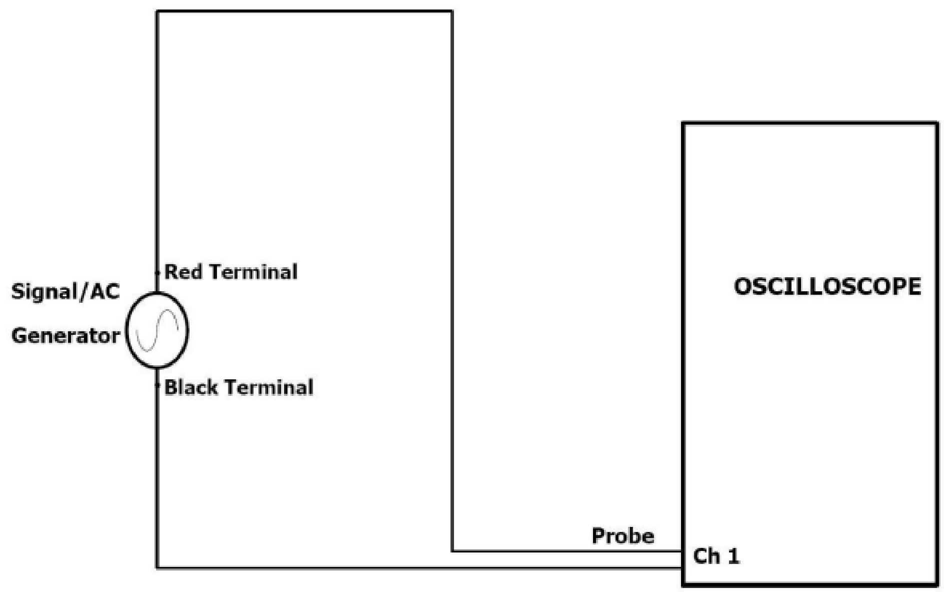


*Figure 1 circuit for calibrating the frequency*

The output voltage was set to 5V. The time division dial was set to 10ms and the vertical gain was adjusted till the amplitude of the wave fill the screen. Repeated readings were taken for a range of frequencies between 100 Hz and 5000 Hz

**The RC circuit**

The circuit, for this part of the experiment, was setup according to the diagram given below -:

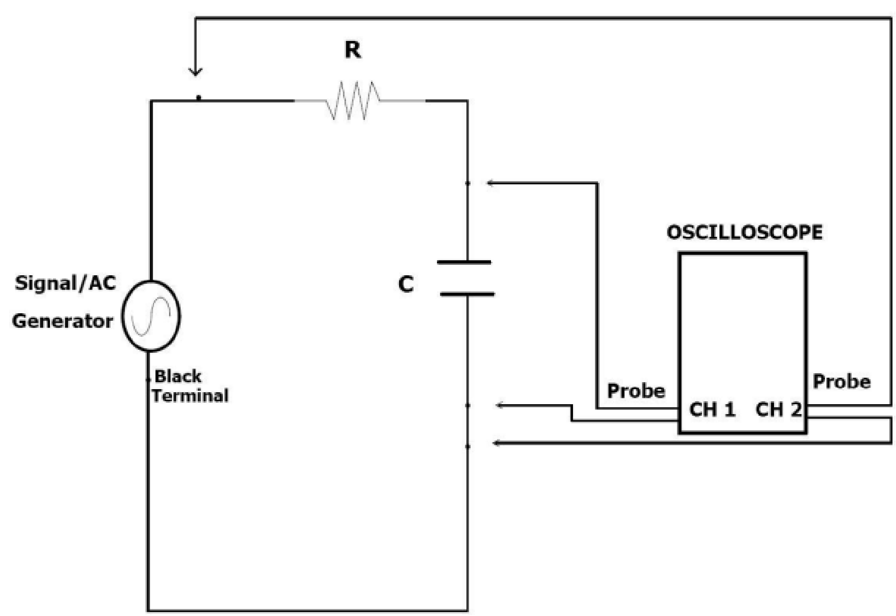


*Figure 2 RC circuit*

The value of the capacitor was set to be 0.2μF and the resistor used was a 5kΩ resistor. The peak-to-peak voltage was set, similar to the earlier part, as 5V. The voltage across ch1 ($V_c$) was measured by the oscilloscope. Repeated readings were taken for a range of frequencies between 100 Hz and 5000 Hz. Also, the phase difference between the voltage across reactance and the impedance was measured

### The RL circuit

A similar set up, as the previous part, was used in this part of the experiment except, the capacitor was replaced by an inductor of inductance of 100mH. Otherwise, the circuit and the value of all the components remained same.

Similar as before, repeated readings were taken for a range of frequencies between 100 Hz and 5000 Hz and the phase difference was measured

### Resistors in series

Same as before, the inductor was replaced by a resistor of 5kΩ. Now the circuit became a resistance-based series circuit.

For this part, only three readings were taken for 100Hz, 1000Hz and 5000Hz respectively.

## Results

### Calibration of the frequency

The results for this part of the experiment are as follows -:

| calculated f | Given f |
|---|---|
| 100 | 100 |
| 500 | 500 |
| 1000 | 1000 |
| 1515 | 1500 |
| 2000 | 2000 |
| 2500 | 2500 |
| 3030 | 3000 |
| 3472 | 3500 |
| 4032 | 4000 |
| 4545 | 4500 |
| 5000 | 5000 |

*Table 1 calculated frequency*

The calibration error for the oscilloscope was estimated to be 0.002%, which was negligible. Therefore, it was ignored and the AC generator and the oscilloscope were correctly calibrated.

### The RC Circuit

The measured $V_c$ and the calculated $X_c$, using formula (3), are given as follows -:

| frequency | voltage | reactance |
|---|---|---|
| 100.000 | 4.240 | 7957.747 |
| 500.000 | 1.600 | 1591.549 |
| 1000.000 | 0.840 | 795.775 |
| 1500.000 | 0.580 | 530.516 |
| 2000.000 | 0.420 | 397.887 |
| 2500.000 | 0.340 | 318.310 |
| 3000.000 | 0.290 | 265.258 |
| 3500.000 | 0.240 | 227.364 |
| 4000.000 | 0.220 | 198.944 |
| 4500.000 | 0.190 | 176.839 |
| 5000.000 | 0.180 | 159.155 |

*Table 2 results for RC circuit*

The error for the above calculation was calculated using formulas (8) and (9). They are given as follows -:

| $\Delta V$ | frequency | voltage | reactance | $\Delta f$ | $\Delta R_c$ |
|---|---|---|---|---|---|
| 0.005 | 100.000 | 4.240 | 7957.747 | 0.001 | 0.200 |
| 0.005 | 500.000 | 1.600 | 1591.549 | 0.001 | 1.000 |
| 0.005 | 1000.000 | 0.840 | 795.775 | 0.001 | 2.000 |
| 0.005 | 1500.000 | 0.580 | 530.516 | 0.001 | 3.000 |
| 0.005 | 2000.000 | 0.420 | 397.887 | 0.001 | 4.000 |
| 0.005 | 2500.000 | 0.340 | 318.310 | 0.001 | 5.000 |
| 0.005 | 3000.000 | 0.290 | 265.258 | 0.001 | 6.000 |
| 0.005 | 3500.000 | 0.240 | 227.364 | 0.001 | 7.000 |
| 0.005 | 4000.000 | 0.220 | 198.944 | 0.001 | 8.000 |
| 0.005 | 4500.000 | 0.190 | 176.839 | 0.001 | 9.000 |
| 0.005 | 5000.000 | 0.180 | 159.155 | 0.001 | 10.000 |

*Table 3 results with uncertainties*

Two graphs were formulated out of the above given data. One was for Voltage vs Reactance capacitance which is given below -:

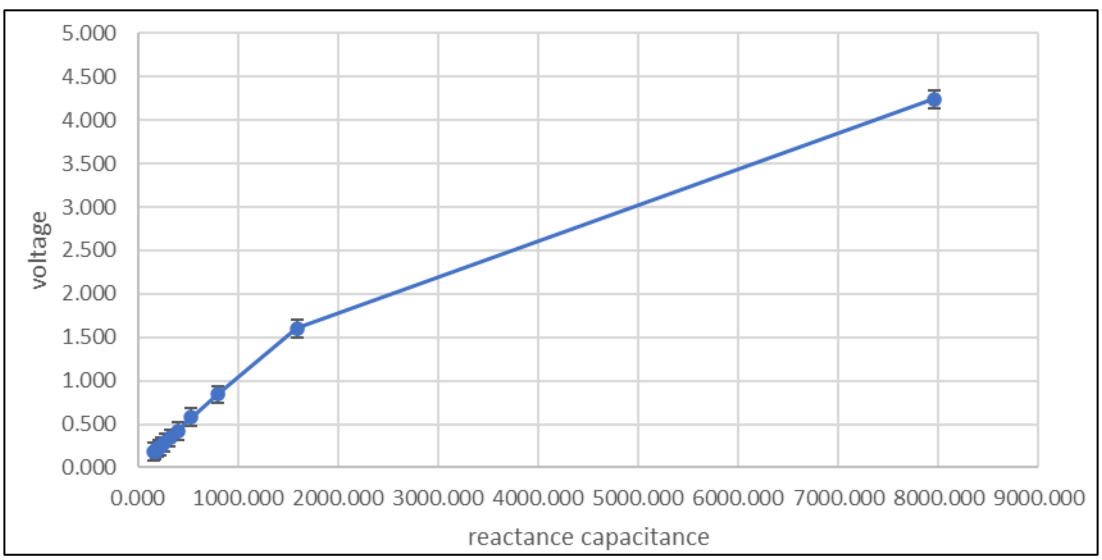


*Figure 3 graph of voltage vs reactance*

Form the graph, it was worked out that $V_c \propto X_c$ and the gradient for this graph represented the current (I) flowing through the circuit. This was derived using the formula -:

$$V_c = \frac{I}{2\pi f C} = IX_c \tag{10}$$

The value of the gradient I derived using the LINEST function was found out to be 0.52±0.03mA

The other graph made was for voltage vs frequency. This is given below -:

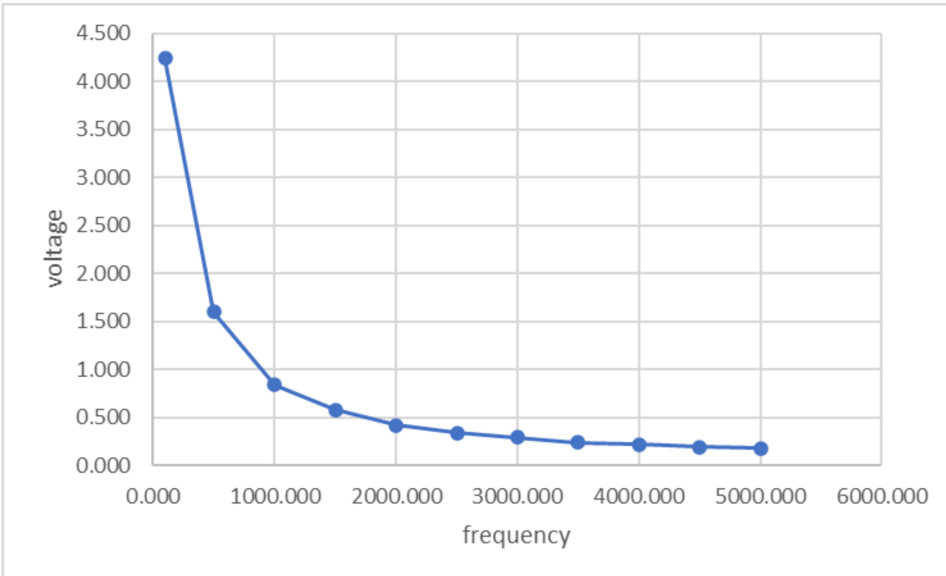


*Figure 4 graph of voltage vs frequency*

This graph showed that as frequency increases, voltage decreases. Therefore, the gradient was found out to be negative

∴ $$V_c \propto \frac{1}{f}$$

The phase for the capacitor only is I leads V by $\frac{\pi}{2}$. Phase for the reactance derived by formula (2) is given below -;

| frequency | reactance | impedance phase | phase shift |
|---|---|---|---|
| 100.000 | 7957.747 | 57.858 | 32.142 |
| 500.000 | 1591.549 | 17.657 | 72.343 |
| 1000.000 | 795.775 | 9.043 | 80.957 |
| 1500.000 | 530.516 | 6.057 | 83.943 |
| 2000.000 | 397.887 | 4.550 | 85.450 |
| 2500.000 | 318.310 | 3.643 | 86.357 |
| 3000.000 | 265.258 | 3.037 | 86.963 |
| 3500.000 | 227.364 | 2.604 | 87.396 |
| 4000.000 | 198.944 | 2.279 | 87.721 |
| 4500.000 | 176.839 | 2.026 | 87.974 |
| 5000.000 | 159.155 | 1.823 | 88.177 |

*Table 2 phase shift for RC circuit*

A shot taken for the observed phase shift for 100Hz is given below -:

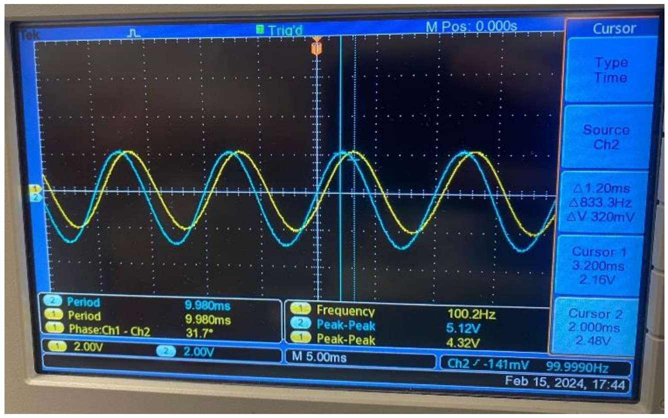


*Figure 5 graph for phase shift of RC circuit*

**The RL Circuit**

Similar to the previous part, the measured $V_l$ and the calculated $X_l$, using formula (5), are given as follows -:

| frequency | voltage | reactance |
|---|---|---|
| 100.000 | 0.220 | 62.832 |
| 500.000 | 0.360 | 314.159 |
| 1000.000 | 0.620 | 628.319 |
| 1500.000 | 0.900 | 942.478 |
| 2000.000 | 1.220 | 1256.637 |
| 2500.000 | 1.460 | 1570.796 |
| 3000.000 | 1.720 | 1884.956 |
| 3500.000 | 1.960 | 2199.115 |
| 4000.000 | 2.200 | 2513.274 |
| 4500.000 | 2.400 | 2827.433 |
| 5000.000 | 2.620 | 3141.593 |

*Table 5 results for RL circuit*

The error for the above calculation was calculated using formulas (8) and (9). They are given as follows -:

| frequency | voltage | reactance | delta f | delta $R_l$ |
|---|---|---|---|---|
| 100.000 | 0.220 | 62.832 | 0.001 | 0.079 |
| 500.000 | 0.360 | 314.159 | 0.001 | 0.395 |
| 1000.000 | 0.620 | 628.319 | 0.001 | 0.790 |
| 1500.000 | 0.900 | 942.478 | 0.001 | 1.184 |
| 2000.000 | 1.220 | 1256.637 | 0.001 | 1.579 |
| 2500.000 | 1.460 | 1570.796 | 0.001 | 1.974 |
| 3000.000 | 1.720 | 1884.956 | 0.001 | 2.369 |
| 3500.000 | 1.960 | 2199.115 | 0.001 | 2.763 |
| 4000.000 | 2.200 | 2513.274 | 0.001 | 3.158 |
| 4500.000 | 2.400 | 2827.433 | 0.001 | 3.553 |
| 5000.000 | 2.620 | 3141.593 | 0.001 | 3.948 |

*Table 6 results with uncertainties*

Two graphs were formulated out of the above given data. One was for Voltage vs Reactance inductance which is given below -:

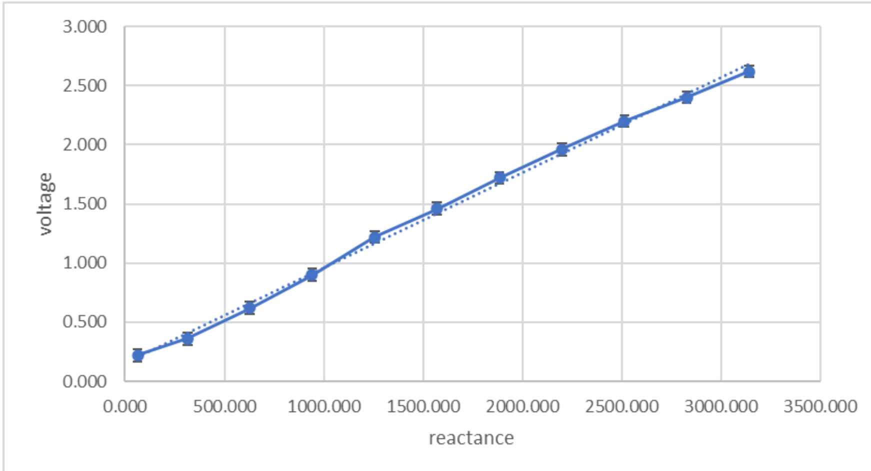


*Figure 6 graph of voltage vs reactance*

Form the graph, it was worked out that $V_l \propto X_l$ and the gradient for this graph represented the current (I) flowing through the circuit. This was derived using the formula -:

$$V_c = I2\pi f L = IX_l \tag{11}$$

The value of the gradient I derived using the LINEST function was found out to be 0.803±0.013mA

The other graph made was for voltage vs frequency. This is given below -:

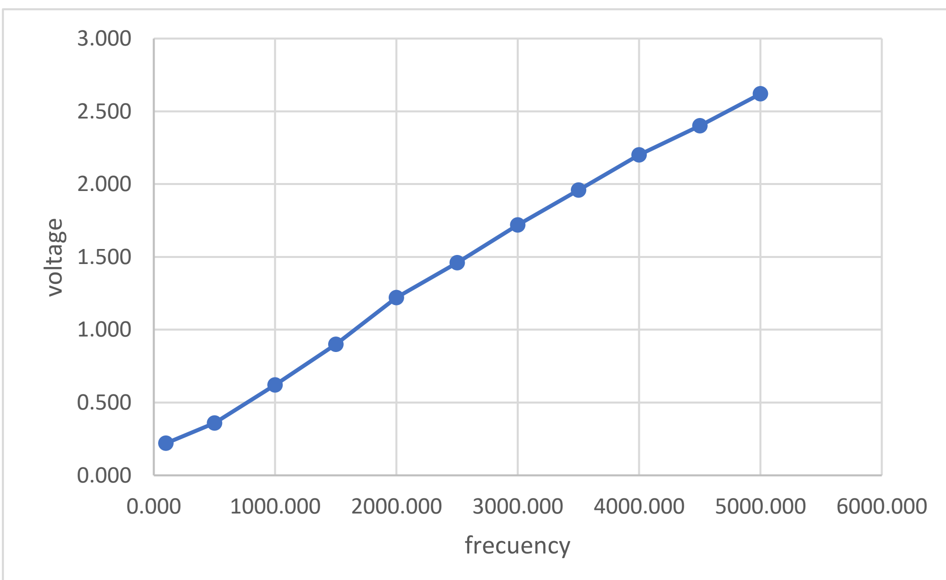


*Figure 6 graph of voltage vs frequency*

This graph showed that as frequency increases, voltage increases. Therefore, the gradient was found out to be positive

∴
$$V_l \propto f$$

The phase for the inductors only is V leads I by $\frac{\pi}{2}$. Phase for the reactance derived by formula (4) is given below -;

| frequency | reactance | impedance phase | phase shift |
|---|---|---|---|
| 100.000 | 62.832 | 0.720 | -89.280 |
| 500.000 | 314.159 | 3.595 | -86.405 |
| 1000.000 | 628.319 | 7.162 | -82.838 |
| 1500.000 | 942.478 | 10.675 | -79.325 |
| 2000.000 | 1256.637 | 14.108 | -75.892 |
| 2500.000 | 1570.796 | 17.441 | -72.559 |
| 3000.000 | 1884.956 | 20.656 | -69.344 |
| 3500.000 | 2199.115 | 23.741 | -66.259 |
| 4000.000 | 2513.274 | 26.687 | -63.313 |
| 4500.000 | 2827.433 | 29.488 | -60.512 |
| 5000.000 | 3141.593 | 32.142 | -57.858 |

*Table 7 phase shift for RL circuit*

A shot taken for the observed phase shift for 1500Hz is given below -:

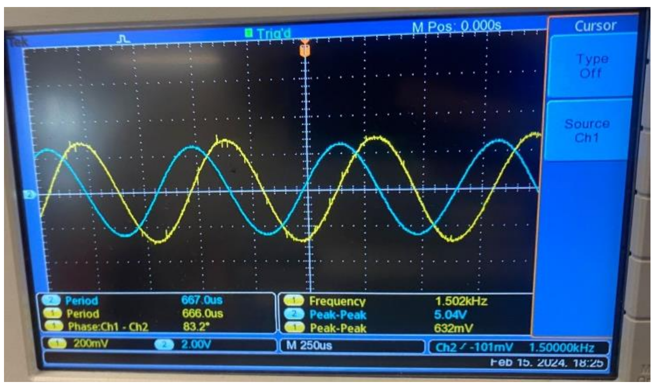


*Figure 8 graph for phase shift of RL circuit*

**Resistors in series**

For this part, only three readings were taken for frequencies 100Hz, 1000Hz and 5000Hz. For all three frequencies the V was 2.5V and the total resistance was 10000Ω. There was also, no phase-shift as there was no reactance in play.

## Discussion

For the first part of the experiment, the calibration of the frequency had negligible error of 0.001% deviation from the input value. This error could be due to the heating up of equipment during the process. This error was classified as a systematic error that was minimized by taking multiple readings.

For RC and RL circuits both, the voltage showed an increase with increase in reactance, displaying that V is directly proportional to reactance. However, RC circuits showed an exponential growth, while RL circuits displayed a more linear growth. While systematic error like heating up of equipment, the parallax of the eye while adjusting the horizontal and vertical knobs and other errors, were present, the errors of the oscilloscope were also taken into consideration. These errors were minimized by taking multiple readings for different values of frequency.

In RC circuits, the voltage vs frequency graph showed that $V_c \propto \frac{1}{f}$, but on the other hand the same graph in RL circuit, displayed that $V_l \propto f$. However, in the resistance only circuit the reactance is zero, therefore, for every frequency the voltage remains the same.

For the phase-shift, both the circuits showed that φ is directly proportional to the reactance in the circuit. The resistance only circuit did not display any phase-shift as there is no reactance present in the circuit.

## Conclusion

The purpose of this experiment was to understand the Sinusoidal graphs on the oscilloscope. The secondary objective was to comprehend the phase shift that occurs between the voltage and the current. In order to do this, the peak-to-peak AC voltage in three distinct circuit types was measured, and the current and phase-shift were calculated for various AC frequency values. These circuits were merely resistance, RL, and RC circuits, in that order. The RC circuit's current flowed at $0.52 \pm 0.03$ mA, and the phase-shift revealed an exponential increase at first, followed by a steady increase as the reactance decreased. In contrast, the RL circuit's current (I) was $0.803 \pm 0.013$ mA, and as reactance increased, the phase-shift decreased. Since there was no reactance in the resistance-only circuits, there was no phase shift.

## References

Young, Hugh, and Roger Freedman. University Physics with Modern Physics, Global Edition, Pearson Education, Limited, 2019.

Hughes, Ifan, and Thomas Hase. Measurements and Their Uncertainties: & Practical Guide to Modern Error Analysis, Oxford University Press, Incorporated, 2010.

https://en.wikipedia.org/wiki/Oscilloscope accessed 23/02/24

https://en.wikipedia.org/wiki/Sine_wave accessed 23/02/24