An Analysis of Ignite Hold Time

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Abstract

For damage dealing classes, total damage done is ultimately the metric of interest, and a complete simulation is the most readily available and accurate method to determine damage. However, due to the unique fire mage mechanic of ignite, an analytic solution to predict ignite hold times is still useful for both setting expectations and for building an intuition that can be leveraged "in the field" towards assigning rotations. Here, under several simplifying assumptions, an exact solution for ignite hold time is presented. This solution is applied to mage teams ranging from one to six mages with recommendations for ignite sustaining rotations in each case.

1. Ignite Mechanics

In Vanilla World of Warcraft, the fire mage specialization contains a Tier II talent called ignite. Its tooltip reads, "Your critical strikes from Fire damage spells cause the target to burn for an additional 40% of your spell's damage over 4 sec.". This simple description belies important details of a mechanic known as "rolling ignites", which functions as follows.

When a fire mage spell critically strikes (crits), a damage over time (DoT) is applied that inflicts an additional 40% of the spell damage in two 20% installments, or ticks – one tick two seconds after the spell hits and the other at four seconds. If another fire spell crits before the ignite expires, its damage adds to the DoT damage (again at 20% per tick), and the ignite duration is extended to four seconds after the second crit. The ticks continue at two second intervals[1] and crits continue to extend the ignite duration and stack damage up to five times total. After the full stack of five, further crits only extend the ignite duration, they do not modify ignite damage.

2. Assumptions

The analysis is simplified by making the following assumptions, which reasonably approximate most in-game scenarios.

- **Static conditions:** Ignite is fully stacked and none of the mages have remaining combustion charges. This condition is usually satisfied between 12 and 24 seconds from the start of rotation, faster with more mages or higher crit. When applying the results of this analysis to an encounter, the expected startup time (time until all combustion charges are expended) should be subtracted from the expected total encounter time.
- **Equal crit:** All mages have equal crit chance. While this assumption may not be close to matching some real world scenarios, it should be representative of minmax teams that are not carrying undergeared player(s). In extreme cases, it may be better overall boss

[1] As of March 2022, the tick timer is reset when a saving crit occurs after what would be the last ignite tick.

damage for undergeared players to spec frost. Regardless, the average crit of contributing fire mages is a safe and conservative[2] approximation.

- **Non-adaptive rotation:** Rotations are not reactive. Simulations have shown that a "saving" fire blast rotation is an overall damage increase for the case of a mage normally assigned to scorch spam. In the adaptive rotation, a scorch spamming mage casts fire blast if the ignite timer has less than two seconds remaining when a scorch cast is complete. This rotation is only advantageous if employed by one mage. Based on simulation output, this adaptive rotation extends the mean rotation hold time. However, the affect on hold time is small and it is much more difficult to model non-independent trials. So no adaptive rotation is considered in this treatment.

With these assumptions in hand, ignite hold times can be formulated and solved analytically.

3. General Form

According to the mechanics described in Section 1, an ignite will terminate when consecutive non-crits subtend a four second interval. Determining ignite hold time is thus directly related to finding the probability of a streak of non-crits over a series of opportunities. Given m chances to crit within every four second interval, and a total number of crit opportunities n, we define a probability function $\mathbb{P}(n,m)$ as the chance that a streak of non-crits with length $\geq m$ occurs within n independent trials. The de Moivre solution[1] for $\mathbb{P}(n,m)$ is given by

$$\mathbb{P}(n,m) = \sum_{j=1}^{\lfloor\frac{n+1}{m+1}\rfloor} (-1)^{j+1} \left[p + q\frac{n-jm+1}{j}\right] \binom{n-jm}{j-1} p^{jm} q^{j-1}, \tag{1}$$

where q is the crit chance and $p = 1 - q$ is the non-crit chance.

Solving for the number of trials $\bar{n}_m$ at which $\mathbb{P}(\bar{n}_m, m) = 0.5$ will produce the median number of casts over which ignite will hold. Since Equation 1 is not invertible, the solution process for $\bar{n}_m$ is to iterate over values for n, and evaluating $\mathbb{P}(n,m)$ until it is closest to 0.5. Code to perform this calculation is provided in Appendix A.

4. Solutions by Number of Mages

Throughout this section, the terms "crit value" and *effective crit* are used interchangeably and consistently in plots and tables. An individual's effective crit is their "raw crit" adjusted down to account for a hit rate lower than the cap. The effective crit is given by

$$\text{crit}_{\text{eff}} = \frac{\text{hit} \times \text{crit}_{\text{raw}}}{99\%}, \text{where} \tag{2}$$

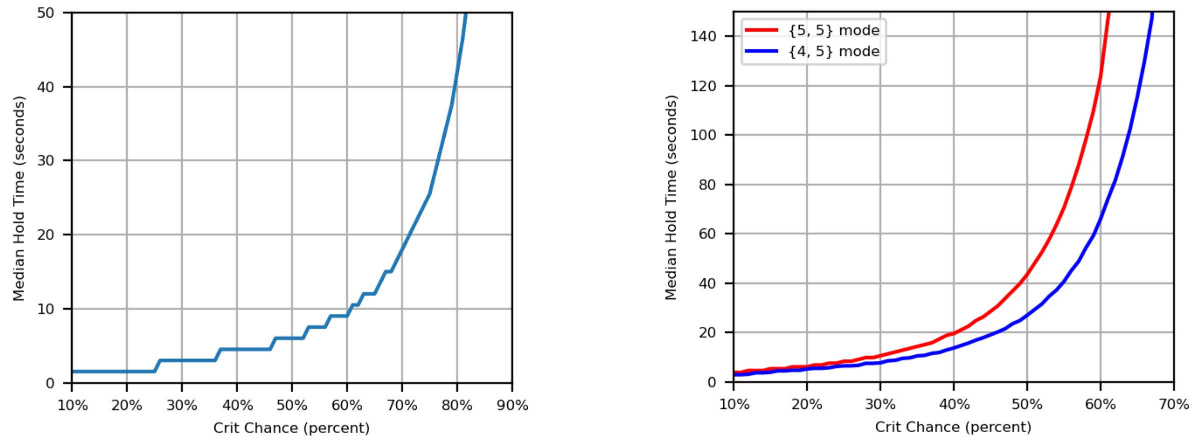
$$\text{crit}_{\text{raw}} \text{ is from base} + \text{gear} + \text{int} + \text{buffs} + \text{talents}, \tag{3}$$

where talents do not include incinerate.

4.1. One Mage

If only one mage is casting fire spells in an attempt to hold ignite, the only possible approach is to spam scorch. In this case, two successive scorch non-crits will result in ignite dropping. We can solve Equation 1 for $\bar{n}_2(q)$ as a function of crit chance q. With a cast time of 1.5 seconds, there will be 2/3 scorches cast per second. Therefore, the median number of casts must be divided by 2/3 to obtain the median duration in seconds.

[2]For example if two mages have an average crit $\bar{c}$, then the odds of either critting is lower under the average approximation than without it: $2\bar{c} - \bar{c}^2 < [1 - (1 - \bar{c} - \delta)(1 - \bar{c} + \delta)] = 2\bar{c} - \bar{c}^2 + \delta^2$.



(a) Median ignite hold time for a single mage spamming scorch as a function of the mage's crit chance. No interpolation is used so the curve is discretized in 1.5 second intervals.

(b) Median ignite hold time for two mages spamming scorch as a function of the mages' crit chance. Both the $\{5, 5\}$ and $\{4, 5\}$ modes (see Section 4.2) are shown. Note the median number of casts is divided by $\frac{4}{3}$, the number of casts per second, to obtain the median hold time in seconds.

Figure 1

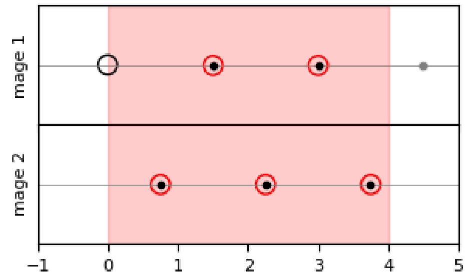
Figure 1a shows the median ignite hold time for one mage. The maximum theoretical effective crit chance (at hit cap) for a Gnome with full world buffs, raids buffs, consumes, the Fire Festival buff and maximum auras (3 x external mage Atiesh + Boomkin) is 68.6%. Even at these practically unachievable crit levels, ignite will only hold for 15 seconds on average with one mage sustaining it. Aside from the Loatheb encounter, sustaining ignite with a single mage is hopeless.

4.2. Two Mages

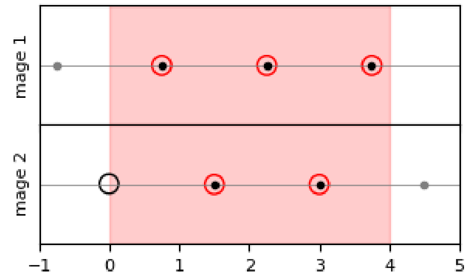
With two mages, there are 3 combinations of spammable fire spells: both fireball, both scorch, or one scorches and the other casts fireball. If both mages fireball, they will have the same average casts/second as one mage scorching, with lower crit due to incinerate. Therefore, based on the one mage results, we can eliminate both fireballing as a viable path to holding ignite. One mage scorching and the other fireballing is also not a viable rotation for sustaining ignite, as shown in Section 4.3. Both scorching will be the most likely rotation to hold ignite, and that is the rotation we will focus on for two mages.

With more than one mage casting, the termination criteria, m, depends on cast synchronization. As shown in Figure 2, for a given cast delay between mages, the criteria can be determined by setting a crit for the nth mage at t=0.0s, and counting the number of casts that occur before t=4.0s. For the case of two mages scorching, this exercise reveals two distinct modes: one in which there will always be 5 potential crits before the ignite refresh window ends, and another in which the stopping criteria alternates between 5 and 4. These modes are denoted $\vec{m} = \{5, 5\}$ and $\{4, 5\}$ respectively. If mage 1 starts casting at t=0.0s, the $\{4, 5\}$ mode is in effect if mage 2 starts casting between 0.0 and 0.5 seconds, or between 1.0 and 1.5 seconds. Otherwise the $\{5, 5\}$ mode is in effect. Since the mode dependency is cyclical (over the longest cast time), if the synchronization is drawn from a uniform distribution, one third of two mages scorching scenarios are $\{5, 5\}$ mode and two thirds are $\{4, 5\}$ mode.

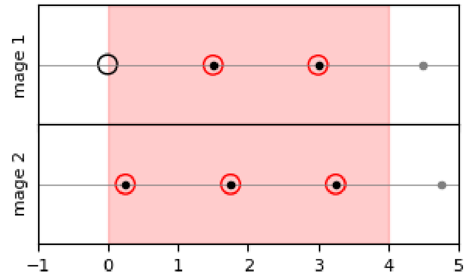
We already have a solution for the $\{5, 5\}$ mode: it is equivalent to $m = 5$ and Equation 1 can be directly applied to obtain $\bar{n}_5$. There is no obvious solution for the $\{4, 5\}$ mode however. Without derivation or proof, we posit a general solution for the median streak length with a



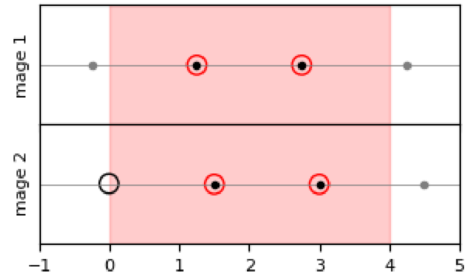
(a) A {5, 5} mode timeline when mage 1 crits.



(b) A {5, 5} mode timeline when mage 2 crits.



(c) A {4, 5} mode timeline when mage 1 crits.



(d) A {4, 5} mode timeline when mage 2 crits.

Figure 2: Casting timelines for two mages scorching. Black circles show a crit at t=0.0 seconds. Red circles show fire spell hits within the 4 second ignite refresh window. In the {5} mode case (a & b), mage 2 is 0.75 seconds behind mage 1. In the {4, 5} mode case (c & d), mage 2 is 0.25 seconds behind mage 1.

cyclical stopping criteria:

$$\bar{n}_{\vec{m}} \approxeq M \left(\sum_{i=1}^{M} \frac{1}{\bar{n}_{m_i}} \right)^{-1} . \tag{4}$$

The $\bar{n}_{\{4,5\}}(q)$ solution, along with $\bar{n}_5(q)$ is plotted in Figure 1b as a function of crit chance. There is a significant difference between the modes in ignite hold time, which implies that for a given rotation strategy, hold time outcome is not only subject to the random variation of crit occurrence, but also variation due to random synchronization.

4.3. Three or More Mages

The methods developed in Sections 4.1 and 4.2 can be applied to an arbitrary number of mages. For N mages, there are $N + 1$ possible combination of assignments consisting of some mages casting scorch and others fireball. For each combination, there are a number of modes, and the mode in any particular scenario is determined by the relative synchronization of casting. Table 1 lists the performance and frequency of all rotations and modes for two and three mages. To balance ignite hold time with individual damage, a N member mage team should use the rotation with modes that most closely matches their effective crit value. For example, if a three mage team has an effective crit value of 43%, crit values listed for modes in which two mages scorch and one fireballs are closest.

As the number of mages increases, the number of modes expands exponentially[3]. Inspecting the individual modes with a reference such as Table 1 is no longer practical for more than

[3]For {1-7} mages, the maximum mode rotations have {1, 3, 12, 43, 127, 407, 1361} modes respectively.

mages	scorch	fireball	norm	sequence • freq • crit		
2	2	0	3	(4, 5) • 2 • 55%	(5, 5) • 1 • 49%	
2	1	1	6	(3, 3, 3, 4) • 3 • 67%	(3, 3, 4, 4) • 2 • 64%	(3, 4, 4, 4) • 1 • 61%
2	0	2	3	(2, 2, 3, 3) • 1 • 78%	(2, 2, 2, 2) • 1 • 83%	(1, 2, 2, 3) • 1 • 92%
3	3	0	31	(7, 8, 8) • 11 • 36%	(6, 7, 8) • 11 • 40%	(7, 7, 8) • 9 • 38%
3	2	1	273	(5, 5, 6, 6, 6, 7) • 64 • 47%	(6, 6, 6, 6, 7, 7) • 51 • 43%	(5, 5, 5, 6, 6, 7) • 40 • 48%
				(6, 6, 6, 6, 6, 7) • 29 • 43%	(5, 6, 6, 6, 6, 7) • 27 • 45%	(5, 6, 6, 6, 7, 7) • 20 • 44%
				(5, 5, 6, 6, 7, 7) • 20 • 46%	(6, 6, 6, 7, 7, 7) • 11 • 42%	(5, 6, 6, 7, 7, 7) • 11 • 43%
3	1	2	366	(4, 4, 4, 5, 5, 5) • 62 • 57%	(4, 4, 4, 5, 6, 6) • 53 • 55%	(4, 5, 5, 5, 6, 6) • 49 • 51%
				(4, 4, 5, 5, 5, 6) • 40 • 54%	(4, 4, 4, 5, 5, 6) • 40 • 56%	(4, 5, 5, 5, 5, 5) • 29 • 53%
				(4, 4, 5, 5, 5, 5) • 29 • 55%	(5, 5, 5, 5, 5, 5) • 20 • 51%	(4, 5, 6, 6, 6, 6) • 11 • 49%
				(4, 5, 5, 5, 5, 6) • 11 • 52%	(4, 4, 5, 5, 6, 6) • 11 • 53%	(4, 4, 4, 4, 5, 6) • 11 • 58%
3	0	3	183	(3, 3, 3, 3, 4, 4) • 42 • 69%	(3, 3, 4, 4, 5, 5) • 21 • 63%	(2, 3, 3, 4, 4, 5) • 21 • 73%
				(2, 3, 3, 3, 3, 4) • 21 • 75%	(1, 2, 3, 3, 4, 5) • 21 • 92%	(3, 3, 4, 4, 4, 4) • 19 • 65%
				(2, 3, 3, 4, 4, 4) • 19 • 73%	(2, 3, 3, 3, 4, 4) • 19 • 74%	

Table 1: The sequence, frequency, and performance for all two and three mage modes. Each sequence is listed in parentheses in ascending order. Several permutations of the sequence may occur, but all are equivalent and combined. The sequence is followed by a frequency count which can be converted to a fraction by dividing by the normalization factor (norm) for the given rotation. Modes are listed in order of highest to lowest frequency. The performance is expressed as the effective crit value for which the mode sustains a median ignite of 40 seconds.

three mages. Instead, for a given rotation and effective crit value, we aggregate over all modes, weighted by mode frequency. Using this aggregation method, Figure 3 shows the hold times for each rotation for three to six mages.

These plots aid in finding a rotation that balances holding ignite with scorch, while casting the higher damage fireball spell when possible. The process is as follows:

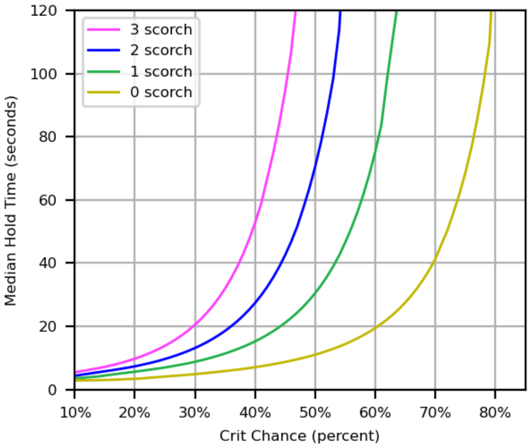
1. Determine the effective crit chance of the mage team. This will be the mean of the individual effective crit values (see Equation 2).

2. Estimate the fight length and subtract 12-24 seconds from that estimate.

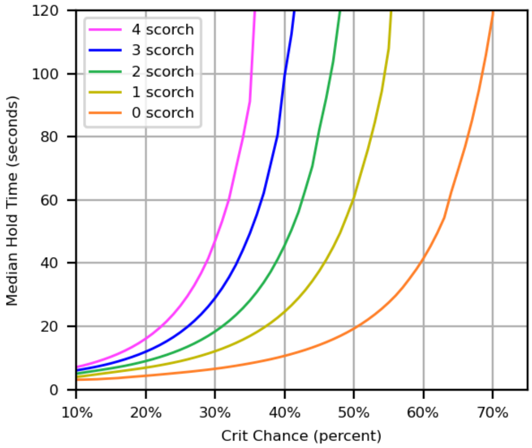
3. Consult the plot corresponding to the number of mages on the team. Find the crit value on the x-axis of the plot. The curve with a y-value closest to the modified estimated fight length is likely the optimal rotation.

References

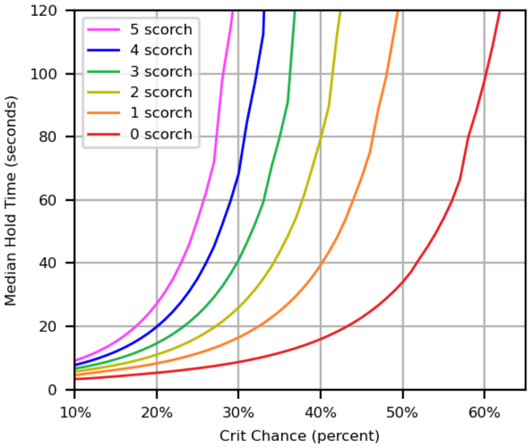
[1] Abraham de Moivre, *The Doctrine of Chances*, Second Edition, 1738.



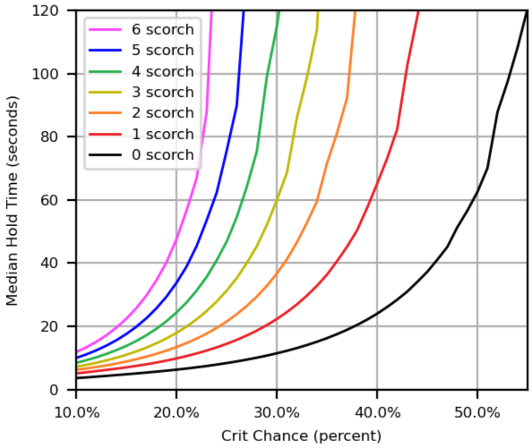
(a) Hold time as a function of crit rate for three mages.



(b) Hold time as a function of crit rate for four mages.



(c) Hold time as a function of crit rate for five mages.



(d) Hold time as a function of crit rate for six mages.

Figure 3: Hold time as a function of crit rate for three to six mages. For each rotation, the modes are aggregated in a weighted average.

A. Code to Calculate the Median Streak

```
import math

def binomial(n: float, k: float) -> float:
    """
    Calculate a binomial coefficient

    Args:
        n (float): Upper term
        k (float): Lower term

    Returns:
        float: Value for C(n, k).
    """
    gamma = math.gamma
    try:
        numerator = gamma(n + 1)
        return numerator / gamma(k + 1) / gamma(n - k + 1)
    except OverflowError:
        if k <= 0.0:
            return 1.0
        else:
            # large n, small k approximation
            factor_k = math.sqrt(2 * math.pi * k) * (k / math.e)**k
            return (n**k) / factor_k

def median_streak(m: int|float, q: float, interp: bool=False) -> float|int:
    """
    Calculate trial length with mediam streak m using brute force method

    Args:
        m (int | float): streak length
        q (float): Crit probability

    Returns:
        float: Median n_{1/2}
    """
    upper_limit = 500
    p = 1 - q
    prev = 0
    for n in range(1, upper_limit):
        summ = 0
        for j in range(1, int(n//m) + 1):
            if (n - j*m - j + 2) <= 0:
                break
            comp_a = (-1)**(j + 1)
            comp_b = (p + q*(n - j*m + 1)/j)
            comp_c = binomial(n - j*m, j - 1)
            comp_d = p**(j*m)*q**(j - 1)
            summ += comp_a*comp_b*comp_c*comp_d
```

```
    if summm >= 0.5:
        break
    prev = summm
else:
    return math.inf

if not interp:
    if summ - 0.5 > prev - 0.5:
        return n - 1
    else:
        return n
else:
    total = summ - prev
    return n*(0.5 - prev)/total + (n - 1)*(summ - 0.5)/total
```