4COSCOO2W MATHEMATICS FOR COMPUTING

Week 7 Seminar Tasks

Graph Theory

READING

Lecture 5 Notes (available on Blackboard)

Chapters 10-11. *Grossman, P., 2017. Discrete mathematics for computing. Bloomsbury Publishing.*

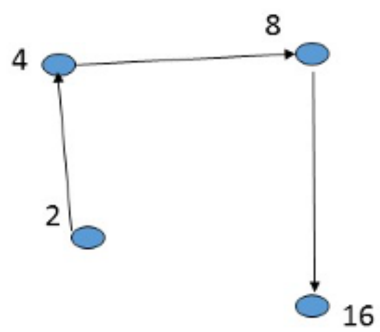
TASK 1. GRAPH NOTATION. GRAPH TYPES

See Lectures 5 Notes -Slides 5-11

Task 1.1. For each of the following graphs represented in the diagrams below do the following:

- form set V of all its nodes and set E of all its edges
- determine if it is a directed or undirected graph
- determine if it is a cyclic or acyclic graph, for a cyclic graph give an example of a cyclic path in it
- is there a pattern a graph can represent?

a.



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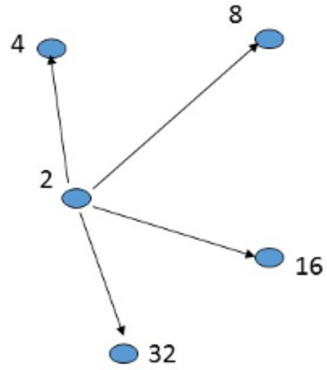

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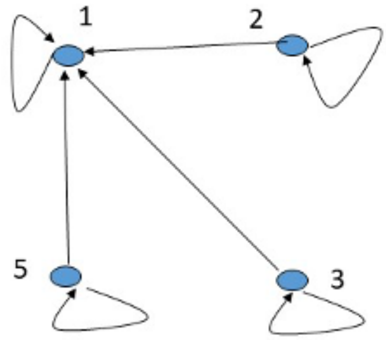

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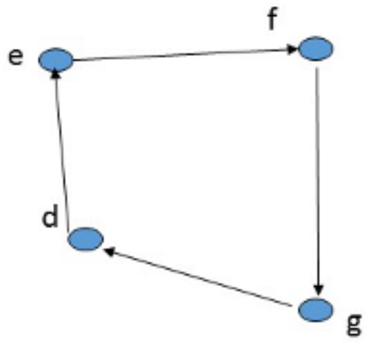
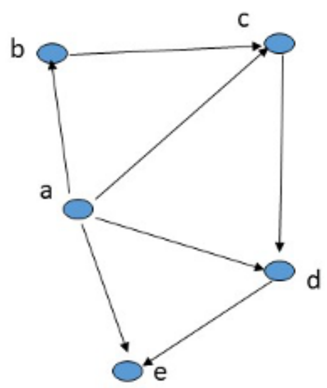


e.



Task 1.2. For each of the following graphs defined by the given set of nodes, **V**, and set of edges, **E**, do the following:

- draw its representation corresponding to the definition of V and E
- determine if it is a directed or undirected graph
- determine if it is a cyclic or acyclic graph, for a cyclic graph give an example of a cyclic path in it.

V={d,e,f,g}; E={(d,e), (e,f), (f,g),(g,d)}

V={a,b,c,d,e}; E={(a,b), (a,c), (a,d), (a,e), (b,c), (c,d), (d,e)}

V={a,b,c}; E={(a,a), (a, b), (b,b), (b,a), (b, c), (c,c), (c,b), (a,c),(c,a)}

TASK 2. WEIGHTED GRAPHS

See Lectures 5 Notes -Slide 13

You are part of a network optimisation team responsible for analysing the efficiency of data packet transmission from a central server located in London to various data centres spread out across the United Kingdom. Your primary focus is to study the network latency, which is a critical factor for network performance.

This task aims to construct a weighted directed graph representing the network latency from the central server to each data centre. You will use the data recorded on Monday, 30th October, 2023, between 8 am and 10 am.

The latencies recorded are as follows:

- Manchester: 50 milliseconds at 8 am
- Birmingham: 35 milliseconds at 9:30 am
- Bristol: 45 milliseconds at 9:30 am
- Southampton: 40 milliseconds at 8:30 am
- Glasgow: 65 milliseconds at 8 am
- Edinburgh: 70 milliseconds at 8 am

1. Define the Set of Nodes **V**:
Create a set V representing each data centre as a node within your graph. This set does not include the London server, as it is the origin and not a destination node.

2. Construct the Set of Edges **E** :
For each pair of nodes x and y in V, you will create a directed edge (x, y, w) if and only if the latency from London to x is less than the latency to y. The weight w of this edge is the difference in latency from London to y minus the latency from London to x. This represents the relative latency increase from node x to node y.

3. Graph Creation:
Utilise the sets **V** and **E** to draw a weighted directed graph. Your graph should visually display the nodes and the directed edges with their respective weights, indicating the direction of higher latency.

4. Analysis:
Provide a brief analysis of the graph. This may include observations about the distribution of latencies, potential implications for data packet transmission routes, and any patterns that might influence network optimisation strategies.

TASK 3. TREES.

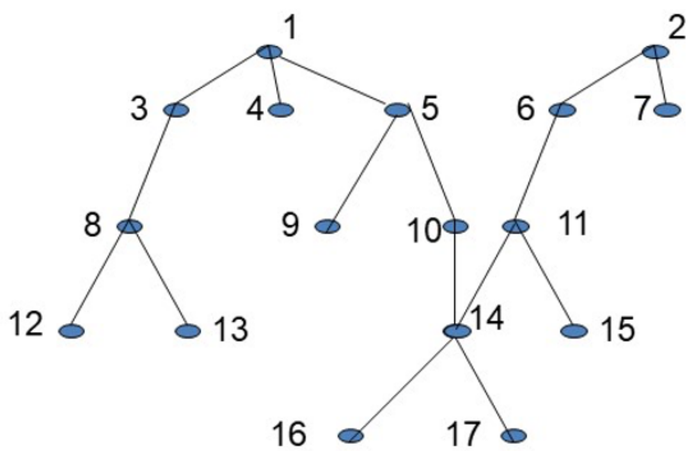
See Lectures 5 Notes -Slides 21-26

In the figures below, you will see attempts to draw rooted trees. Justify if the graphs drawn are trees.

For those cases where you established trees, identify:

- All the leaves of a tree
- Depth of a tree
- If a tree is a binary tree

a.



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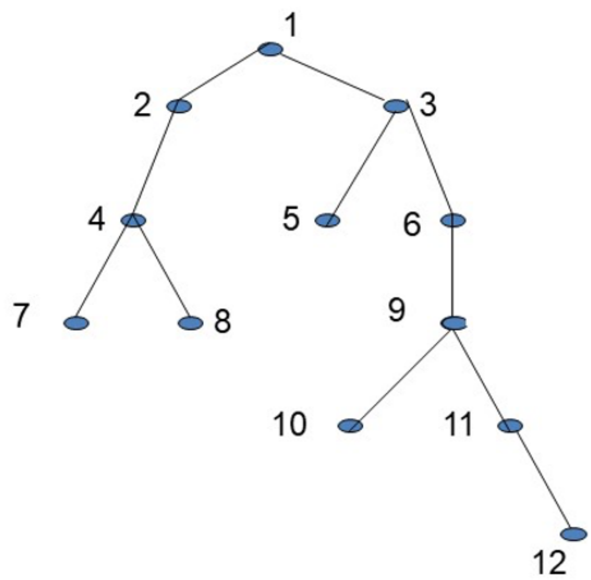

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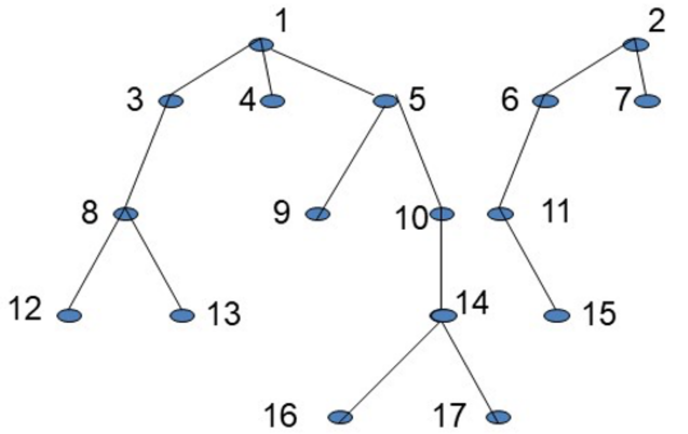
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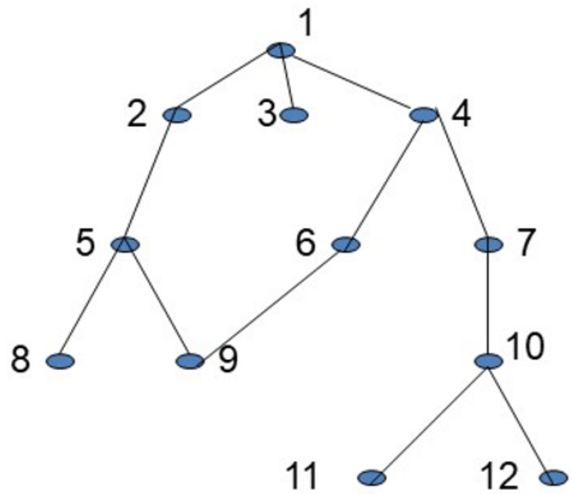
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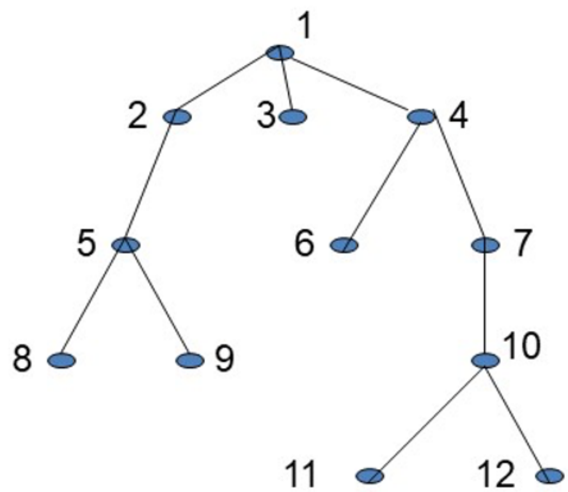
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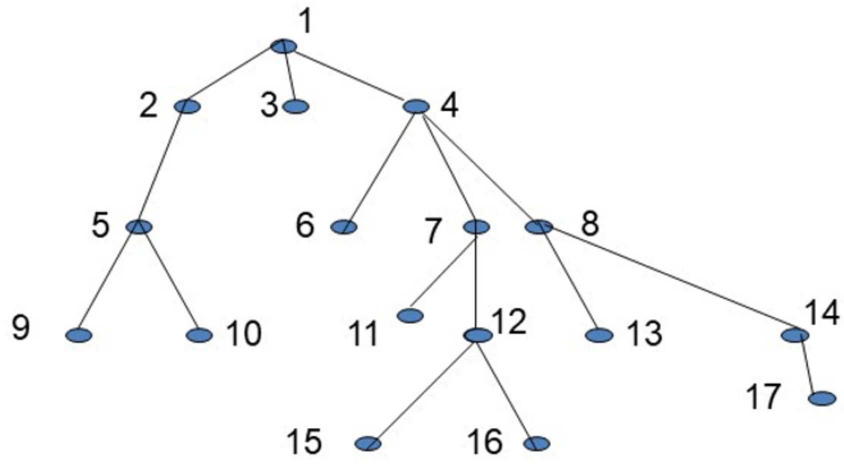
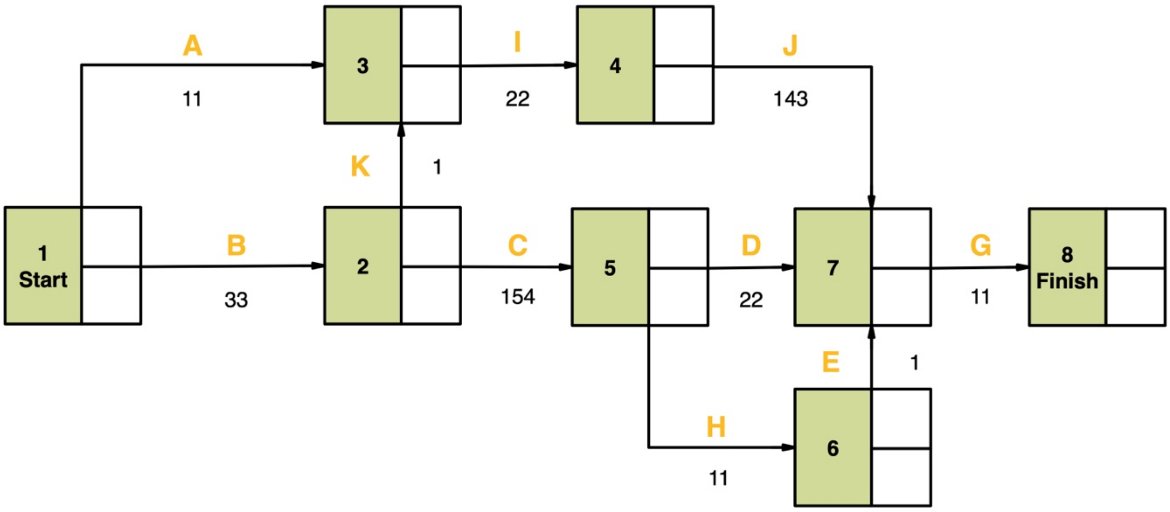


TASK 4: CRITICAL PATH ANALYSIS IN PROJECT SCHEDULING

See Lectures 5 Notes -Slides 14-19



The provided diagram represents a project schedule using a network graph, where each node (or box) represents a task within the project. The edges represent activity and are labelled with letters, and the duration of each task is given below. The arrows indicate the sequence of tasks, demonstrating which tasks must be completed before others can begin.

Using the provided network graph, your objective is to perform a critical path analysis. This will involve identifying the longest path through the network, which determines the shortest time in which the project can be completed.

NB: The critical path is the sequence of stages with *no slack time*; if any of these stages are delayed, the entire project will be delayed.

For your analysis, follow these steps:

1. Calculate the Earliest Start Time (EST) and Latest Start Time (LST) for each task.

2. Determine the Total Float (or Slack) for each task, which is the difference between the LST and EST.

3. Identify the critical path, which is the path from Start to Finish that has zero slack for all tasks.