CS 581: Theory of Computation
Fall 2019
Ronnie Song

HW #1

1. Prove that $\sqrt{3}$ is irrational.

Bonus (not required): prove that for all primes $p \geq 2, \sqrt{p}$ is irrational (among other things, this shows us that there exist an infinite number of irrational numbers, due to the infinitude of primes)

Assume $\sqrt{3}$ is rational

then $\sqrt{3} = \frac{a}{b}$ (a, b is integer, and have no common factors)

$\rightarrow 3 = \frac{a^2}{b^2}$

$\rightarrow 3b^2 = a^2$

So a^2 is divisible by 3, so as a.

$3b^2 = (3k)^2$ (where $3k = a$)

$\rightarrow b^2 = 3k^2$

So b^2 is also divisible by 3, so as b.

$\rightarrow$ a and b have common factor 3, which is a contradiction. Hence $\sqrt{3}$ is

2. Recall that the **power set** of a set A is the set of all subsets (not necessarily proper) of A.

Prove using induction, that for any finite set A with $|A| = n$, $|P(A)| = 2^n \ \forall n \geq 1$, where $P(A)$ denotes the power set of A.

Proof:

Base case: $n = 0$

$A = \{\}$, $P(A) = \{\emptyset\} \Rightarrow 2^0 = 1$

Ind Step: prove $p(k) \rightarrow p(k+1)$

Assume $p(k) = 2^k$ where $k \in \mathbb{N}$

$$p(k) + 2^k = 2^k + 2^k$$
$$= 2 \cdot 2^k$$
$$= 2^{k+1}$$

$\square$