

HW #1

1. Prove that  $\sqrt{3}$  is irrational.

Bonus (not required): prove that for all primes  $p \geq 2$ ,  $\sqrt{p}$  is irrational (among other things, this shows us that there exist an infinite number of irrational numbers, due to the infinitude of primes)

Assume  $\sqrt{3}$  is rational

then  $\sqrt{3} = \frac{a}{b}$  ( $a, b$  is integer, and have no common factors)

$$\rightarrow 3 = \frac{a^2}{b^2}$$

$$\rightarrow 3b^2 = a^2$$

So  $a^2$  is divisible by 3, so as  $a$ .

$$3b^2 = (3k)^2 \text{ (where } 3k = a \text{)}$$

$$\rightarrow b^2 = 3k^2$$

So  $b^2$  is also divisible by 3, so as  $b$ .

$\rightarrow a$  and  $b$  have common factor 3, which is a contradiction. Hence  $\sqrt{3}$  is irrational.

2. Recall that the power set of a set  $A$  is the set of all subsets (not necessarily proper) of  $A$ .

Prove using induction, that for any finite set  $A$  with  $|A| = n$ ,  $|P(A)| = 2^n \forall n \geq 1$ , where  $P(A)$  denotes the power set of  $A$ .

Proof:

Base case:  $n = 0$

$$A = \{\} , P(A) = \{\emptyset\} \Rightarrow 2^0 = 1$$

Ind Step:

prove  $p(k) \rightarrow p(k+1)$

Assume  $p(k) = 2^k$  where  $k \in \mathbb{N}$

$$p(k) + 2^k = 2^k + 2^k$$

$$= 2 \cdot 2^k$$

$$= 2^{k+1}$$

□