Particle Physics I

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This is a graduate course. Offered in Fall 2014 at Columbia University. Required Course textbook: Donoghue, *Dynamics of the Standard Model*. Problem session: Friday 1-2pm

Contents

Overview

Lecture 1 (9/3/14)

The main focus of the course is on the standard model (applied field theory). It explains virtually all phenomena with few exceptions to calculate to extreme precisions.

Sketch of syllabus

1. Overview of SM
2. Symmetries in general and in SM
3. Effective field theory
4. Lagrangian of SM
5. Prediction in E&W, QCD sector, CP violation, flavor physics
6. Possible extension to SM (none is known for sure) e.g. dark matter, WIMPs, neutrino mass,...

The SM was put together by Weinberg in 1967-68. It describes strong, weak, EM forces with massless neutrino, but as we will see it is not that hard to put in neutrino masses. However the precise forms are not experimentally settled. We know e.g. Δm_{32}, Δm_{21} 2 masses splitting, we don't know the exact mass scales to determine their masses, nor do we know $m_3 > m_2$ or $m_3 < m_2$ and there could be more than 3 neutrinos. In addition, SM says nothing about gravity, no SUSY.

SM introduces 2 important things:

1) Nature is very well described by gauge theory interacting with matter field. This is sort of like Darwin evolution in Biology, a major conceptual realization.

2) It delivers extremely accurate predictions. The quantitative accuracy comes from sorting through details. How did we get here? Many brilliant theoretical ideas and extraordinary experiments. Unlike the elegant general relativity, no theorist would ever think up SM without the help of experimental data, because there are many detail that goes into.

Let's summary what we have learned from qft.

What is in SM?

Strong interaction between quarks (spin 1/2 elementary particles). There are 6 known quarks

	charge	mass
u	$2/3$	~ 3Mev
d	$-1/3$	~ 5Mev
s	$-1/3$	90Mev
c	$2/3$	~ 1270Mev
b	$-1/3$	~ 4200Mev
t	$2/3$	$173,000$Mev

Unanswered: why there are 6? putting 8 quarks will not change the formulation. What sets their masses?

u, d are constituents of proton, neutron. Free quarks are never seen.

3 quarks→hadron

quark, anti quark →meson.

Form of strong interaction corned by 8 gluons. That is the subject of QCD, a $SU(3)$ gauge theory of strong interaction.

QCD conserves quark flavor, to change quark flavors, we need weak interaction, which has much longer time scale. In SM the mass eigenstates for quarks and flavor eigenstates are not the same.

Leptons (spin 1/2)

	charge	mass
ν_e	0	$< 0.2 \times 10^{-5}$Mev
e	-1	.511Mev
ν_μ	0	$< 1.9 \times 10^{-1}$Mev
μ	-1	106Mev
ν_τ	0	< 18Mev
τ	-1	1770Mev

Note large range of masses, still open questions.

Electric charge quantized if quarks are bound (not required by SM). Two other gauge theories $SU(2)\times U(1)_Y$, $SU(2)$ gives weak quantum numbers that are responsible for mass decay and decays that change quark flavors.

In sum here are the total 28 SM parameters

6 quarks masses	1 Higgs mass
6 lepton masses	1 Higgs self coupling
3 coupling constants	4 quarks mixing (source of CP violation)
1 QCD vacuum angle	6 lepton mixing (not known precisely)

Generically SM masses are not explained. Higgs mass, 125GeV, is particularly troubling.

Where is E&M? $SU(2)\times U(1)_Y$ has $3+1=4$ gauge bosons as force carries.

$$\begin{array}{rl} \text{3 weak bosons} & W^{\pm}, Z \\ \text{1 massless photon} & \gamma \end{array}$$

In low energy, it becomes to E&M. This theory is known as symmetry breaking and is part of Higgs mechanism via Higgs fields (2 component complex scalar fields).

1 Scalar Field in SM

Lecture 2 (9/8/14)

Recall we started qft1 with $\lambda\phi^3$ in $d=6$ dimension. It is capable of describing relativistic physics & interaction of bosons. In such theory there is no internal spin dof, no fermion statistics, no gauge invariance, but it may be internal symmetry, also may be symmetry breaking.

1.1 Higgs

We know there is a Higgs particle. The data is not yet definitive about it is SM Higgs, but certainly strongly suggesting. SM does not predict Higgs mass, but does predict its couplings to all other known particles, spin, branching ratio...

Higgs in SM can be described by a scalar field theory, $\lambda\phi^4$.

$$\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \lambda\phi^4 \tag{1.1}$$

It gives spontaneous symmetry breaking. In particular it means at low energy physics doesn't exhibit the field symmetry of the theory. Higgs also gives masses to $W^\pm$,Z.

1.2 Effective Field Theory

This (1.1) also teaches us about renormalization. Look at free 2 point propagator

$$(\text{symm factor})\lambda\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-m^2+i\epsilon}$$

it is quadratic divergent. So we add a counter term $\delta m^2 \sim \Lambda^2$ (It requires free tuning of dimensional parameter, invoking Feynman trick) it is effectively cutting off the integral

$$\int^\Lambda\frac{d^4p}{(2\pi)^4}\frac{1}{p^2-m^2+i\epsilon}$$

Above is an example of EFT, which in its more modest view of goal: describe physics below some scale Λ_{eff}. EFT looks like renormalizable field theory at lowest order and higher dimensional interaction terms appear suppressed by dimensionful parameters. So (1.1) in full should be

$$\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \lambda\phi^4 + \underbrace{C_6\frac{\phi^6}{\Lambda_{eff}^2} + \ldots\ldots}_{\sum_{d=6}^{\infty} C_d\frac{\phi^d}{\Lambda_{eff}^{d-4}}}$$

for $d = 4$ theory ϕ has mass dim 1. Λ_{eff} is where EFT breaks down, new physics enters. Just like in classic physics, when we deal with harmonic oscillator we say m, k determine everything. Of course we ignore some energy effects, like thermal motion, atomic motion, because $m, k \gg T$.

1.3 Neutrinos

Fermi in 1930's postulated neutrinos to explain weak decay

$$n \to p + e^- + \bar{\nu}_e$$

$p = uud$, $n = udd$. So a d goes into u. Feynman diagram looks like

$$\begin{array}{ccccc} d & & & & u \\ & \searrow & & \nearrow & \\ & & \dashrightarrow & & \\ & \swarrow & W & \searrow & \\ e & & & & \nu_e \end{array} \tag{1.2}$$

The interaction term has dim $6 = \frac{3}{2} \times 4$,

$$\mathcal{L}_{int} \sim \frac{(du)(e\nu)}{M_W^2}$$

so it is suppressed by M_W^2. The propagator is

$$\frac{i}{p^2 - M_W^2}$$

$M_W \sim 100\text{GeV}$. In low energy the diagram looks like

$$\begin{array}{cccc} d & & & u \\ & \searrow & \nearrow & \\ & & & \\ & \swarrow & \searrow & \\ e & & & \nu_e \end{array} \tag{1.3}$$

All Fermi knew was that the coupling was very small, so he predicted W and it was discovered in 1980's.

1.4 Naturalness Problem

So SM Higgs sector is an EFT, we expect additional interaction $C_d \frac{\phi^d}{\Lambda_{eff}^{d-4}}$ to enter. This requires $m_H \ll \Lambda_{eff}$. In grand unified theorem (GUT), $\Lambda_{eff} \sim 10^{15}$GeV. But how comes a such large Λ_{eff} gives such small m_H? This is called technical naturalness. This suggests some kind of fine turning of parameter. SUSY solves the naturalness problem and allow light scalars. It uses a fact that boson loop give 1; fermion loop give -1.

1.5 Pion

Pion is another important SM scalar. The progress in particle physics would be very different if we did not discover Π in 1940's. It is not fundamental field, rather bound state of quarks, which is consequence of QCD $SU(3)_C$ gauge theory (1974).

$$\begin{array}{cc} \bar{u}d & \Pi^+ \\ \bar{d}u & \Pi^- \\ \bar{u}u + \bar{d}d & \Pi^0 \end{array}$$

Pion played vital role in understanding symmetry breaking. The scale of QCD $\Lambda_{QCD} \sim 500$MeV.

$$\begin{array}{lccc} & & & \text{spin} \\ \text{meson } q\bar{q} & \Pi & 140\text{MeV} & 0 \\ & K & 500\text{MeV} & 0 \\ & \rho & 770\text{MeV} & 1 \\ & & & \\ \text{baryon } qqq & & & \\ \text{nucleon=proton+neutron} & N & 1\text{GeV} & 1/2 \end{array} \tag{1.4}$$

Why is the Π so light? Pion is Goldstone boson of a dynamically broken symmetry, i.e. there is a symmetry of the QCD $\mathcal{L}$, but it is not a symmetry of the vacuum.

Chiral symmetry is broken. Chiral symmetry can rotate left & right handed massless fermion. The vacuum of QCD is the vacuum we live in. In this vacuum, left handed send into vacuum, by Lagrangian, will not change to right handed, but actually it will, so the vacuum is very complicated.

2 Symmetry Breaking

Lecture 3 (9/10/14)

Symmetry simplifies our understanding. In classical mechanics conservation of energy & momentum avoid micro details. Symmetry may not be manifest in the physical states. Generically due to the vacuum is not respecting symmetry. E.g. Ferromagnet $\mathcal{L}$ is invariant under $SO(3)$, while the ground state is not.

2.1 Scalar Field Example

Single component field, cf Peskin 11.1. Put $m^2 = -\mu^2$, $\mu^2 > 0$, with wrong sign of mass

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4!}\lambda\phi^4 \tag{2.1}$$

thus conjugate momentum is $\Pi = \frac{\partial \mathcal{L}}{\partial_0 \phi} = \partial_0 \phi \implies (\Pi \sim \dot{q})$, so

$$H = \int d^3x \mathcal{H} = \int d^3x [\frac{1}{2}\Pi^2 + \frac{(\nabla\phi)^2}{2} \underbrace{- \frac{\mu^2}{2}\phi^2 + \lambda\frac{\phi^4}{4!}}_{V(\phi)}]$$

minimum of H is a constant field and at the minimum of $V(\phi)$. So

$$\frac{\partial V(\phi)}{\partial \phi} = 0 \implies \phi^2 = v^2, \qquad v := \sqrt{\frac{6\mu^2}{\lambda}}$$

at $\phi = \pm v$, curvature is > 0. Later we will see non 0 curvature fluctuation gives massive particle; 0 curvature fluctuation gives massless particle.

From $V(\phi)$, if $\lambda < 0$, no lower bound, so $\lambda > 0$. Aside note: perturbation theory on $\frac{1}{4!}\lambda\phi^4$ is an expansion in power of λ that is not a convergent expansion, called asymptotic expansion. That is higher order in λ increases precision up to a point, then further orders reduce precision. The idea of the proof is that although λ^n is suppressed by $n!$ but the number of diagram grows faster. This is called Borel summability.

There are two vacuums

$$\langle\phi\rangle = \pm v$$

For ∞ volume system, ∞ energy to switch between $\langle\phi\rangle = v$ to $\langle\phi\rangle = -v$. As if there is a infinitely long wall (called domain wall) separate them. Domain wall is any ϕ that $E \neq E_{min}$. For finite volume system, you can do it in quantum tunneling.

Let's choose one vacuum, $\langle\phi\rangle = v$, to expand ϕ

$$\phi(x) = v + \sigma(x)$$

$\sigma(x)$ fluctuation, it has spactime dependence. Plugging in (2.1)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}(2\mu)^2\sigma^2 - \sqrt{\frac{\lambda}{6}}\mu\sigma^3 - \frac{1}{4!}\lambda\sigma^4$$

we get a mass term, $\sqrt{2}\mu$ for mass of σ. We see that now $\mathcal{L}$ is no longer respecting the $\phi \to -\phi$ symmetry, i.e. vacuum breaks that symmetry.

2.2 Linear Sigma Model

Suppose

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi^i)(\partial_\mu\phi_i) + \underbrace{\frac{1}{2}\mu^2\phi^i\phi^i - \frac{1}{4}\lambda(\phi^i\phi^i)^2}_{-V(\phi)} \tag{2.2}$$

ϕ is N real scalar fields, $\mathcal{L}$ has $SO(N)$ internal symmetry.

$$\phi_i \to R_{ij}\phi_j$$

For $SO(3)$ there are 3 generators, $R_{lm} = \left(e^{i\theta^{ij}T_{ij}}\right)_{lm}$

$$xy \text{ plane} \to \begin{pmatrix} \cos\theta & \sin\theta & \\ -\sin\theta & \cos\theta & \\ & & 1 \end{pmatrix} \implies T_{xy} = -i\begin{pmatrix} & 1 & \\ -1 & & \\ & & \end{pmatrix}$$

likewise

$$T_{yz} = \begin{pmatrix} & & \\ & & 1 \\ & -1 & \end{pmatrix}, \qquad T_{zx} = \begin{pmatrix} & & 1 \\ & & \\ -1 & & \end{pmatrix}$$

General $SO(N)$ group has

$$\frac{N(N-1)}{2} = \begin{pmatrix} N \\ 2 \end{pmatrix}$$

independent plane rotations.

Likewise we find minimum

$$\frac{\partial V}{\partial \phi} = 0 \implies |\phi|^2 = v^2, \quad v := \sqrt{\frac{\mu^2}{\lambda}}$$

Let

$$\langle \phi \rangle = (0, 0, ..., 0, v) \tag{2.3}$$

Define fluctuation, shift fields

$$\phi^i(x) = (\pi^k(x), v + \sigma(x))$$

$k = 1, ..., N-1$. Fluctuation in $1, ..., N-1$ space, no curvature. Fluctuation in radial σ has curvature. Thus (2.2) becomes

$$\begin{aligned} \mathcal{L} \;=\; & \frac{1}{2}(\partial_\mu \pi^k)(\partial_\mu \pi^k) + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma) - \frac{1}{2}(2\mu^2)\sigma^2 - \sqrt{\lambda}\mu\sigma^3 \\ & -\sqrt{\lambda}\mu\pi^k\pi^k\sigma - \frac{\lambda}{4}\sigma^4 - \frac{\lambda}{2}\pi^k\pi^k\sigma^2 - \frac{1}{4}\lambda(\pi^k\pi^k)^2 \end{aligned} \tag{2.4}$$

π^k has no mass, called $N-1$ massless Goldstone boson fields.

Theorem. *(Goldstone) for every spontaneously broken symmetry, they must contain a massless particle.*

By choosing (2.3), we have $\mathcal{L}$ (2.4) that is $SO(N-1)$ symmetry, so there are

$$\frac{N(N-1)}{2} - \underbrace{\frac{(N-1)(N-2)}{2}}_{\text{unbroken}} = \underbrace{N-1}_{\text{broken}}$$

broken symmetries, so there are $N-1$ massless particles.

Later we will do QCD, we will show $SO(3)$ is completely broken, And they are created from a different symmetry broken, gauge symmetry broken. There are 3 different Π^k with different masses, they are results of u, d are not exact symmetry, isospin symmetry.

2.3 Scalar Invariant Symmetry

Lecture 4 (9/15/14)

Because SUSY is not seen, people are looking for other explanation for why $m_{Higg} \ll \Lambda_{eff}$. Goldstone theorem may provide an answer. Current research topic: Could Higgs be Goldstone from broken scale invariant?

Scalar invariant symmetry is another internal symmetry, $\mathcal{L}$ is invariant under

$$x \to \lambda x$$

for any λ. This doesn't have to be exact symmetry. It can be corrected by m/Λ. For the naturalness problem, we find the symmetry is broken by dimensionful parameter, mass.

2.4 Isospin Symmetry

By 1960's physicists understood that strong interaction obeyed isospin $SU(2)$ global symmetry as a very good approximation (few percent level difference)

$$\frac{|m_u - m_d|}{\Lambda_{QCD}} \sim 1\%$$

$\Lambda_{QCD} \sim 300$MeV. One can calculate correction from isospin symmetry as expansion in $\frac{|m_u - m_d|}{\Lambda_{QCD}}$.

$$\Psi = \begin{pmatrix} p \\ n \end{pmatrix} \text{ has isospin } \frac{1}{2} \tag{2.5}$$

p =proton, n =neutron. Both are spin 1/2 Dirac field, so Ψ has 8 component anti-commuting (fermion) fields.

$$\Pi = \begin{pmatrix} \Pi^+ \\ \Pi^0 \\ \Pi^- \end{pmatrix} \text{ has isospin } 1 \tag{2.6}$$

Actually there are two sources why $m_n \neq m_p$. 1) $m_d - m_u \sim 3$MeV, with this alone $m_n - m_p \sim 3$MeV, 2) one is charged, the other is not. To calculate this effect, one first assume size of nuclear is 1fm. Then compute EM energy in vacuum space. With this source alone, $m_n - m_p \sim 1$MeV. So

$$m_n - m_p \sim 4\text{MeV}$$

2.5 Linear Sigma Model (cont.)

Let's modify $SO(N)$ real scale field theory (2.2) for Ψ and Π above, (2.5), (2.6), ignoring EM effects, so no gauge.

$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - m_\Psi)\Psi + \frac{1}{2}(\partial_\mu \Pi \partial^\mu \Pi - m_\Pi^2 \Pi \cdot \Pi) + i\underbrace{g\bar{\Psi}(\tau \cdot \Pi)\gamma_5}_{\text{Yukawa}}\Psi - \frac{1}{4}\lambda(\Pi \cdot \Pi)^2 \tag{2.7}$$

This doesn't describe all physics of Ψ and Π below 1GeV, e.g. not include ρ meson that decays to Π, cf (1.4). τ =Pauli matrices.

Why is there a γ_5? Because physical pions are pseudoscalar, under parity $\vec{x} \xrightarrow{P} -\vec{x}$

$$\Pi(t, \vec{x}) \xrightarrow{P} -\Pi(t, -\vec{x})$$

so for $\mathcal{L}$ to be parity invariant, we need

$$\bar{\Psi}\Pi\gamma_5\Psi(t, \vec{x}) \xrightarrow{P} -\bar{\Psi}\Pi\gamma_5\Psi(t, -\vec{x})$$

Let's check (2.7) is invariant under $SU(2)$ isospin

$$\Psi \to U\Psi, \qquad U = e^{-i\frac{\vec{\tau}}{2}\vec{\alpha}}$$

E.g. check, using $\mathrm{Tr}\tau_i\tau_j = 2\delta_{ij}$,

$$\begin{aligned}\Pi \cdot \Pi = \frac{1}{2}\mathrm{Tr}[(\tau \cdot \Pi)(\tau \cdot \Pi)] \quad &\to \quad \frac{1}{2}\mathrm{Tr}[(U\tau\Pi U^+)(U\tau\Pi U^+)] \\ &= \quad \frac{1}{2}\mathrm{Tr}[(\tau \cdot \Pi)(\tau \cdot \Pi)] = \Pi \cdot \Pi\end{aligned}$$

QED.

The Neother current associated with symmetry is

$$V_\mu^i = \bar{\Psi}\gamma_\mu \frac{\tau^i}{2}\Psi + \epsilon^{ijk}\Pi^j\partial_\mu\Pi^k \tag{2.8}$$

and current is conserved and the charge is defined,

$$\partial^\mu V_\mu^i = 0 \qquad Q^i = \int d^3x V_0^i$$

Now in the same spirit of modifying (2.2) to get (2.7). We modify the sigma model (2.4) to get

$$\begin{aligned}\mathcal{L} \quad = \quad & \bar{\Psi}(i\partial\!\!\!/)\Psi + \frac{1}{2}(\partial_\mu\Pi\partial^\mu\Pi + \partial_\mu\sigma\partial^\mu\sigma) - g\bar{\Psi}(\sigma - i\tau \cdot \Pi\gamma_5)\Psi \\ & + \frac{\mu^2}{2}(\sigma^2 + \Pi^2) - \frac{\lambda}{4}(\sigma^2 + \Pi^2)^2\end{aligned} \tag{2.9}$$

σ is a scalar field. Note the mass term $\sigma^2 + \Pi^2$ has wrong sign. To see what symmetry it has, we define

$$\Sigma = \sigma I + i(\tau \cdot \Pi) = \begin{pmatrix} \sigma + i\Pi_3 & \Pi_{1,2} \\ \Pi_{1,2} & \sigma - i\Pi_3 \end{pmatrix}$$

so under parity $\Sigma \to -\Sigma$, and

$$\frac{1}{2}\mathrm{Tr}(\Sigma^+\Sigma) = \sigma^2 + \Pi \cdot \Pi$$

and

$$\Psi_L = \frac{I+\gamma_5}{2}\Psi \qquad \Psi_R = \frac{I-\gamma_5}{2}\Psi$$

In this way

$$\begin{aligned}\mathcal{L} &= \bar{\Psi}_L(i\partial\!\!\!/)\Psi_L + \bar{\Psi}_R(i\partial\!\!\!/)\Psi_R + \frac{1}{4}\mathrm{Tr}(\partial_\mu\Sigma\partial^\mu\Sigma^+) \\ &\quad + \frac{\mu^2}{4}\mathrm{Tr}(\Sigma\Sigma^+) - \frac{\lambda}{16}(\mathrm{Tr}(\Sigma\Sigma^+))^2 - g(\bar{\Psi}_L\Sigma\Psi_R + \bar{\Psi}_R\Sigma^+\Psi_L)\end{aligned}$$

It is easy to check parity is still conserved. The particle Π EFT is commonly called the chiral $\mathcal{L}$.

Now we recognize $\mathcal{L}$ has more symmetry, it is invariant under $SU(2)_L \times SU(2)_R$, a much large symmetry than just $SU(2)$

$$U_{L,R} = e^{-i\frac{\vec{\tau}}{2}\alpha_{L,R}}$$

It has 6 parameters $\alpha_{L,R}$. Let us check that $\mathcal{L}$ is invariant under

$$\Psi_{L,R} \to U_{L,R}\Psi_{L,R} \qquad \Sigma \to U_L\Sigma U_R^+ \tag{2.10}$$

e.g.

$$\mathrm{Tr}(\Sigma\Sigma^+) \to \mathrm{Tr}(U_R\Sigma U_L^+ U_L\Sigma^+ U_R) = \mathrm{Tr}(\Sigma\Sigma^+)$$

QED.

We can also use (2.10) to figure out how σ, Π get transfer.

$$\sigma \to \frac{1}{2}\mathrm{Tr}(U_L U_R^+)\sigma + \frac{1}{2}\mathrm{Tr}(U_L\tau^k U_R^+)\Pi^k$$

if $\alpha_{L,R}$ are infinitely small,

$$\sigma \to \sigma + \frac{1}{2}(\vec{\alpha}_L - \vec{\alpha}_R)\cdot\Pi$$

and

$$\Pi^k \to \Pi^k - \frac{1}{2}(\alpha_L^k - \alpha_R^k)\sigma - \frac{1}{2}\epsilon^{klm}\Pi^l(\alpha_L^k + \alpha_R^k)$$

hence σ is unchanged if $\vec{\alpha}_L = \vec{\alpha}_R$, while Pion still mix amongst themselves even if

$\vec{\alpha}_L = \vec{\alpha}_R$.

The new symmetry group gives us more conserved currents

$$\begin{aligned} J^i_{L\mu} &= \bar{\Psi}_L\gamma_\mu\frac{\tau^i}{2}\Psi_L - \frac{1}{2}(\sigma\partial_\mu\Pi^i - \Pi^i\partial_\mu\sigma) + \frac{1}{2}\epsilon^{ijk}\Pi^j\partial_\mu\Pi^k \\ J^i_{R\mu} &= \bar{\Psi}_R\gamma_\mu\frac{\tau^i}{2}\Psi_R + \frac{1}{2}(\sigma\partial_\mu\Pi^i - \Pi^i\partial_\mu\sigma) + \frac{1}{2}\epsilon^{ijk}\Pi^j\partial_\mu\Pi^k \end{aligned}$$

Define vector current

$$V^i_\mu = J^i_{L\mu} + J^i_{R\mu}$$

which is invariant under parity. We get back (2.8). And axial vector current

$$A^i_\mu = J^i_{L\mu} - J^i_{R\mu} = \bar{\Psi}\gamma_\mu\gamma^5\frac{\tau^i}{2}\Psi - \sigma\partial_\mu\Pi^i + \Pi^i\partial_\mu\sigma$$

which changes sign under parity.

Lecture 5 (9/17/14)

One can show that

$$3\Pi\text{'s}+\sigma\text{under } SU(2)_L \times SU(2)_R \leftrightarrow 4 \text{ real scaler field under } SO(4)$$

which has $(4)(3)/2 = 6$ generators that agrees numbers of $\alpha_{L,R}$, and we should have 6 conserved currents. Indeed V^i_μ, A^i_μ.

2.6 Approximate Symmetry

Except gauge symmetry is exact, most symmetry in SM are broken. Particularly break symmetry in σ model by adding a term

$$\mathcal{L}' = a\sigma = \frac{a}{2}\mathrm{Tr}\Sigma$$

This breaks $SU(2)_L \times SU(2)_R$ symmetry, we have only $SU(2)$, and forces $\alpha_L = \alpha_R$. Thus A^i_μ is no longer conserved,

$$\partial_\mu A^{\mu i} = a\Pi^i$$

In the sense of EFT, λ, g are dimensionless, a has mass dimension 3. If λ, g

are $O(1)$ quantities, and if

$$\frac{a}{m^3} \ll 1$$

then symmetry breaking parameter is small.

2.7 Spontaneous Symmetry Breaking

Consider shift field

$$\sigma = v + \tilde{\sigma} \qquad v = \sqrt{\mu^2/\lambda}$$

then (2.9) becomes

$$\begin{aligned}\mathcal{L} &= \bar{\Psi}(i\partial\!\!\!/ - gv)\Psi + \frac{1}{2}(\partial_\mu \Pi \partial^\mu \Pi + \partial_\mu \tilde{\sigma} \partial^\mu \tilde{\sigma} - 2\mu^2 \tilde{\sigma}^2) \\ &\quad - g\bar{\Psi}(\tilde{\sigma} - i\tau \cdot \Pi\gamma_5)\Psi - \frac{\lambda v \tilde{\sigma}}{2}(\tilde{\sigma} + \Pi^2) - \frac{\lambda}{4}(\tilde{\sigma}^2 + \Pi^2)^2 \qquad (2.11)\end{aligned}$$

in this way $\tilde{\sigma}$ becomes massive $m_\sigma^2 = 2\mu^2$, Π is still massless, Ψ gains mass through Yukawa when $\langle \Psi \rangle \neq 0$.

3 Effective Field Theory

3.1 Nonlinear Sigma Model

We will explore linear σ model further (2.9) and develop and explicit form for an EFT. Imagine (2.9) was the true theory of the world, what does its low energy physics look like? How would we create a EFT of low energy physics without knowing the σ model at high energies?

Square Root Representation

An alternative representation to $\tilde{\sigma}$ & Π. Assume $v \gg 1$, let

$$\begin{aligned}S &= \sqrt{(\tilde{\sigma} + v)^2 + \Pi^2} - v \approx \tilde{\sigma} \\ \vec{\psi} &= \frac{v\vec{\Pi}}{\sqrt{(\tilde{\sigma} + v)^2 + \Pi^2}}\end{aligned}$$

Expand in v^{-1}, (Donoghue 1.9 on page 108)

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}[(\partial_\mu S)^2 - 2\mu^2 S^2] + \frac{1}{2}(\frac{v+S}{v})^2 \left((\partial_\mu \psi)^2 + \frac{(\psi\partial_\mu\psi)^2}{v^2-\psi^2}\right) \\ & -\lambda v S^3 - \frac{\lambda}{4}S^4 + \bar{\Psi}(i\not{\partial})\Psi - g\frac{v+S}{v}\bar{\Psi}\left(\sqrt{v^2-\psi^2} - i\psi\tau\gamma_5\right)\Psi\end{aligned}$$

Looks like a different theory, but implies same thing with different fields. We see no ψ in potential. The $\frac{(\psi\partial_\mu\psi)^2}{v^2-\psi^2}$ term can be thought of

$$\frac{(\psi\partial_\mu\psi)^2}{v^2-\psi^2} \to (\psi\partial_\mu\psi)^2 \underbrace{\sum_n \left(\frac{\psi^2}{v^2}\right)^n}_{\sum \frac{\psi^d}{\Lambda_{eff}^{d-4}}}$$

this looks like EFT. Imagine $S = 0$, possible because μ is large. Let's look at low energies $E \ll \mu$,

$$g\frac{v+S}{v}\bar{\Psi}\left(\sqrt{v^2-\psi^2} - i\psi\tau\gamma_5\right)\Psi \implies mass_\Psi = gv$$

the only thing left to explore is

$$(\partial_\mu\psi)^2 + \frac{(\psi\partial_\mu\psi)^2}{v^2-\psi^2}$$

this is called non-linear σ model. This is a model of 3 massless pions and their interactions. First term above tells us about $\Pi\Pi \to \Pi\Pi$ scattering. Weinberg calculated $\Pi\Pi \to \Pi\Pi$ scattering amplitude in 60's when at the time nothing could be calculated. According to him, this was the second thing he was proud of besides SM.

Exponential Representation

$$\Sigma = \sigma + i\tau\Pi = (v+S)U, \qquad U = e^{\frac{i\tau\Pi'(x)}{v}}$$

U is $SU(2)$ matrix. To leading order

$$\Pi'(x) = \Pi(x)$$

so

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu S)^2 - 2\mu^2 S^2] + \underbrace{\frac{1}{4}(v+S)^2 \mathrm{Tr}(\partial_\mu U \partial^\mu U^+)} \tag{3.1}$$

$$-\lambda v S^3 - \frac{\lambda}{4} S^4 + \bar{\Psi}(i\partial\!\!\!/)\Psi - g(v+S)\left(\bar{\Psi}_L U \Psi_R + \bar{\Psi}_R U^+ \Psi_L\right)$$

this has $U \to LUR^+$ symmetry, and the whole low energy pion physics is contained in

$$\frac{1}{4}(v+S)^2 \mathrm{Tr}(\partial_\mu U \partial^\mu U^+)$$

We now have 3 σ models, 1 linear and 2 non-linear. What happen if we want to calculate things, e.g.

$$\Pi^+\Pi^0 \to \Pi^+\Pi^0$$

Of course physics should be independent of representations, but perturbation will be different. For Feynman diagram, see Donoghue page 109, in linear model, we say the first figure is from the last term in (2.11), and the second figure is from the second term in (2.11), i.e.

$$\mathcal{L}_{int} = -\frac{\lambda}{4}(\Pi^2)^2 - \lambda v \tilde{\sigma} \Pi^2$$

$$i\mathcal{M}_{\Pi^+\Pi^0 \to \Pi^+\Pi^0} = -2i\lambda + (-2i\lambda v)^2 \frac{i}{q^2 - \underbrace{m_\sigma^2}_{2\mu^2}} = \frac{ig^2}{v^2} + \ldots$$

Lecture 6 (9/22/14)

However they give same results. There is a reason why the three models will give same answers.

Theorem. *(Haag theorem) If two fields are related non-linearly $\phi = \chi F(\chi)$ with $F(0) = 1$, then the same experiment results are found from $\mathcal{L}(\phi)$ & $\mathcal{L}(\chi F(\chi))$.*

3.2 Decoupling Theorem

There is another theorem used not only in EFT, but in understand of running coupling. It deals with the way that a heavy particle can appear in a low energy theory. It was discovered by Appelquist & Carazzone 1975.

Theorem. *(Decoupling theorem) If he remaining low energy theory is renormalizable, then all the effects of the heavy quark appear either as a renormalization of coupling constant of the theory or are suppressed by power of heavy particle mass.*

In our example, $E \ll \mu$, σ cannot be produced, but σ can enter in internal loops. An counter example unlike from (1.2) to (1.3) in the EFT $M_W \gg 1$, the propagator vanish

$$\begin{pmatrix} t \\ b \end{pmatrix}_L \quad SU(2) \text{ doublet}$$

as $m_t \to \infty$, M_W doesn't vanish. That is because $m_t \to \infty$ violates $SU(2)$ symmetry and the theory is not renormalizable. The mass of m_t is 170GeV much bigger than expected.

Proof of decoupling theorem.

Integral out heavy fields at tree level, l_i is light particle field, H is heavy

$$Z[l_i] = \int [dH] e^{i\int d^4x \mathcal{L}(H(x), l_i)}$$

so let's concentrate $\mathcal{L}$ involving H

$$\int d^4x \mathcal{L}_H = \int d^4x \frac{1}{2} (\underbrace{\partial_\mu H \partial^\mu H - m_H^2 H^2}_{-HDH}) + JH$$

J contains l_i. $D = \Box + m_H^2$

$$D^{-1}J = -\int d^4y \Delta_F(x-y) J(y)$$

or

$$D\Delta_F(x-y) = -\delta^4(x-y)$$

so

$$\int d^4x L_H = -\frac{1}{2}\int d^4x(\underbrace{H - D^{-1}J}_{H'})D(\underbrace{H - D^{-1}J}_{H'}) - JD^{-1}J$$

$H' = H + \int d^4y \Delta_F(x-y)J(y)$. Change variable from $H \to H'$ in path integral, $d[H] = d[H']$

$$\begin{aligned} Z[J] &= \int [dH'] e^{\frac{i}{2}\int d^4x(-H'DH' + JD^{-1}J)} \\ &= Z[0] e^{\frac{i}{2}\int d^4x JD^{-1}J} \end{aligned}$$

$H'DH'$ is Gaussian PI no J dependence. Or we put

$$\mathcal{L}_{eff}[J] = -\frac{1}{2}\int d^4x JD^{-1}J = -\frac{1}{2}\int d^4x d^4y J(x)\Delta_F(x-y)J(y)$$

If $k \ll m_H$,

$$\int d^4y \Delta_F(x-y) = -\frac{1}{m_H^2}$$

we expand to the leading order

$$J(y) = J(x) + (y-x)^\mu \left(\frac{\partial J}{\partial y^\mu}\right)_{y=x}$$

so

$$\begin{aligned} \mathcal{L}_{eff}[J] &= -\frac{1}{2}\int d^4x d^4y J(x)\Delta_F(x-y)\left[J(x) + (y-x)^\mu \left(\frac{\partial J}{\partial y^\mu}\right)_{y=x}\right] \\ &= \int d^4x \frac{J(x)J(y)}{2m_H^2} \end{aligned}$$

IBP cancel the first term. So additional interaction ala decoupling theorem $\frac{J(x)J(y)}{2m_H^2}$ is part of light quark J coupled to H. QED

Apply this to exponent representation σ model (3.1)

$$l_i \to U, H \to S$$

$$\mathcal{L}_{eff} = \underbrace{\frac{v^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^+)}_{\mathcal{L}_2} + \underbrace{\frac{v^2}{8m_S^2}[\mathrm{Tr}(\partial_\mu U \partial^\mu U^+)]^2 ...}_{\mathcal{L}_4} \tag{3.2}$$

Loops and divergences from $\mathcal{L}_2$ will generate all possible terms with symmetry of $\mathcal{L}_2$ provided the regularization method preserves the symmetry.

The $\mathcal{L}_2$ looks like non-renormalizable because v^2 is growing, but it is renormalizable. Because in the definition of U, there is $1/v^2$. The second term above is p^2/m_S^2 smaller than the first term for given p^2.

$$v^2(\partial_\mu U \partial^\mu U^+) = \partial_\mu \Pi \partial^\mu \Pi + \frac{(\Pi \partial_\mu \Pi)(\Pi \partial^\mu \Pi)}{v^2}$$

The equivalent

$$JH = \underbrace{\frac{v}{2}\mathrm{Tr}(\partial_\mu U \partial^\mu U^+)}_{J} \underbrace{S}_{H}$$

the fourth order term can give loops too. Loops require renormalization hence counter term.

3.3 Renormalization in EFT

If $\mathcal{L}_{exp}$ is taken as our complete theory then $\mathcal{L}_{eff}$ is the low energy theory to tree level with up to 4 derivatives. If we too include S^3, S^4 terms more interactions appear in $\mathcal{L}_{eff}$, then it is non-renormalizable. Apple what we learned to QCD. Suppose we have theory with $SU(2)_L \times SU(2)_R$ chiral symmetry. Still use exponent representation.

Write down all consistent $\mathcal{L}$ terms with the symmetry

$$U \to LUR^+$$

Possible terms

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + ...$$

2 derivative $\mathrm{Tr}(\partial_\mu U \partial^\mu U^+)$
4 derivative e.g. $\mathrm{Tr}(\partial_\mu U \partial^\mu U^+)\mathrm{Tr}(\partial_\mu U \partial^\mu U^+)$
6 derivative e.g. $\mathrm{Tr}(\partial_\mu U \partial^\mu U^+)\mathrm{Tr}(\partial_\mu U \Box \partial^\mu U^+)$

High derivative terms have dimensionful parameters in derivatives should be smaller for a given p^2 than low order term. Just like (3.2).

Think of QCD. $m_\Pi = 140\text{MeV} \ll m_\rho = 770\text{MeV}$,

$$\mathcal{L}_{eff} = \underbrace{\frac{F^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^+)}_{\mathcal{L}_2} + l_1(\mathrm{Tr}(\partial_\mu U \partial^\mu U^+))^2 + l_2 \mathrm{Tr}(\partial_\mu U \partial^\nu U^+)\mathrm{Tr}(\partial_\nu U \partial^\mu U^+) + ... \tag{3.3}$$

F is pion decay constant, $l_{1,2}$ low energy constants (LEC), they encapsulate the higher energy physics. E.g. $\alpha = 1/137$ is constant at low energy.

Lecture 7 (9/24/14)

As we know, renormalization e.g. $\overline{MS}$, has 3 parts:

1) Regulator

a) Hard Cutoff Λ, which is the shortest distance where solution can propagate.

b) Pauli-Villars

$$\int d^4p \left(\frac{i}{(p^2 - m^2)^2} - \frac{i}{(p^2 - m_H^2)^2} \right)$$

m_H large mass, it is there to cancel divergence.

c) Dimension regulation.

d) Lattice which is the only known non-perturbutive regulation.

2) Scheme, which divides contribution into finite and infinite parts as cutoff is removed. Choosing finite parameters and counter terms.

3) Renormalization Scale, at which physics scale are finite.

Loops in $\mathcal{L}_2$ and parameters in $\mathcal{L}_1$ define the theory to $O(p^4/v^4)$. $l_{1,2}$ are determined by high energy theory or by experiments. If by high energy theory calculation,

a) one can calculate using full theory, but possibly complicated.

b) consider a simple Greens function near E_{eff}, use the result in full theory to choose coefficients in EFT, e.g. vital part of precision SM calculation as quarks mas thresholds crossed.

3.4 Low Energy QED

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\not{D} - m)\Psi \tag{3.4}$$

The famous example of this is the Lamb shift. Bethe separated ∞ part from finite part. The problem was known to Dirac 1930's. The difficulties were the loops at vertex and propagators, e.g. electron self energy, photons self energy, called vacuum polarization. Bethe used quantization via path integral combined with dimension regulation and $\overline{MS}$.

What does QED look like up to order $q^2/m^2 \ll 1$. In this range we just have photons scattering, no $W^\pm$

$$\begin{aligned}\mathcal{L}_{eff}^{QED} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + c_1\frac{e^2}{m^2}F_\mu\partial_\lambda\partial^\lambda F^{\mu\nu} \\ &\quad \underbrace{+c_2\frac{e^4}{m^4}[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma})^2]}_{\text{light-light scattering}} + ... \end{aligned}\tag{3.5}$$

(Donoghue equation 8.5 page 137), high order terms like

$$F_{\mu\nu}\left(\frac{\partial_\lambda\partial^\lambda}{m^2}\right)^n F^{\mu\nu}, \quad F_{\mu\nu}F^{\mu\nu}\left(\frac{F_{\mu\nu}F^{\mu\nu}}{m^4}\right)^n$$

Full results to one loop order

$$c_1\frac{e^2}{m^2} = \frac{\alpha}{60\pi m^2}, \quad c_2\frac{e^4}{m^4} = \frac{\alpha^2}{90m^2}$$

$\alpha = e^2/4\pi = 1/137$.

3.5 Low Energy QCD

We have done the $\mathcal{L}_{eff}$ for pions, that is the non-linear σ model in exponent parametrization, the chiral perturbation theory. We now use it to solve why pions are light $m_\Pi = 140$MeV? Answers Goldstone mechanism.

Recall (3.3), let us introduce χ to represent a quark mass term, so $\mathcal{L}_2$ will give mass of pion and quark

$$\mathcal{L}_2 = \frac{F_\Pi^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^+) + \frac{F_\Pi^2}{4}\mathrm{Tr}(\chi U^+ + U\chi^+)$$

In fermion Lagrangian $m\bar{\Psi}\Psi$ invariant under $SU(2)_L \times SU(2)_R$

$$m\bar{\Psi}\Psi = m(\bar{\Psi}_L\Psi_R + m\bar{\Psi}_R\Psi_L)$$

$$\bar{\Psi}_L \to \bar{\Psi}U_L^+, \qquad \Psi_R \to U_R\Psi_R$$

lets consider

$$\Psi = \begin{pmatrix} u \\ d \end{pmatrix} M = \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}$$

then if $M \to LMR^+$,

$$\bar{\Psi}_L M\Psi_R \to \bar{\Psi}_L M\Psi_R, \qquad \bar{\Psi}_R M\Psi_L \to \bar{\Psi}_R M\Psi_L$$

If χ transforms the same as U, then $\mathcal{L}_2$ is $SU(2)_L \times SU(2)_R$. Most general form of χ is

$$\chi = 2B_0(S + iP)$$

B_0 arbitrary constant, S =scalar, P =pseudoscalar field.

Because M transfers like U, we can add

$$\mathrm{Tr}(MU^+ + M^+U)$$

to $\mathcal{L}_2$. If external agent rotates M, then total $\mathcal{L}_2$ is invariant. If M is fixed, we add a symmetry breaking term to $\mathcal{L}_2$. If $m_u = m_d$, then $L = R$ transformation is a symmetry.

$$\mathrm{Tr}(MU^+) \to \mathrm{Tr}(ML^+UR) = \mathrm{Tr}(MU\underbrace{RL^+}_{1})$$

4 QCD

4.1 QED

Lecture 8 (9/29/14)

Before we talk about QCD, let's recall QED, (3.4), where $D_\mu = \partial_\mu + ieA_\mu$ and the conserve gauge current

$$e(\bar{\Psi}\partial^\mu\Psi)A_\mu$$

The gauge transformation

$$\Psi(x) \to e^{-ie\alpha(x)}\Psi(x), \quad A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu\alpha(x)$$

then

$$m\bar{\Psi}\Psi \to m\bar{\Psi}e^{ie\alpha}e^{-ie\alpha}\Psi = m\bar{\Psi}\Psi$$

$$\bar{\Psi}\partial_\mu\Psi \to \bar{\Psi}\partial_\mu\Psi - i(\partial_\mu\alpha)\bar{\Psi}\Psi$$

$$\bar{\Psi}ieA_\mu\Psi \to \bar{\Psi}ieA_\mu\Psi + \bar{\Psi}i(\partial_\mu\alpha)\Psi$$

so $\mathcal{L}$ is gauge invariant.

Gauge invariant is vital, but leads to unphysical dof. Recall classical EM, commonly use coulomb gauge

$$\nabla \cdot \vec{A} = 0 \tag{4.1}$$

start with ϕ, $\vec{A}$, 4 dofs, where ϕ has no dynamical field, $\vec{A}$ has only 2 propagating dofs. Since ϕ satisfies

$$\nabla^2\phi = -4\pi\rho$$

locality appears to not manifest. So A_μ has 4 dofs only leads to 2 physical dof.

Two general ways to quantize EM

1) Quantize in a particular Lorentz frame, using (4.1), then it is not Lorentz invariant. Then like in non-relativistic quantum mechanism, quantize in box

$$\sum a_{\vec{k}}e^{i\vec{k}\vec{x}} + a^+_{\vec{k}}e^{-i\vec{k}\vec{x}}$$

2) Quantize in a Lorentz covariant form by choosing a gauge based on $\partial_\mu A^\mu = 0$. Quantizing 4 dofs Lorentz vector fields A^μ, appear to be too many dofs, then

show extra dofs don't contribute to physical amplitude. Consequence of underlying gauge symmetry, kind like

$$A_0, A_1, A_2, A_3, \qquad (1, -1, -1, -1)$$

one of the -1 cancel 1, so only 2 $1s$ left.

We then get Feynman rules (depend on gauge)

Fermion propagator

$$\frac{i}{\not{p} - m + i\epsilon}$$

Photon propagator

$$\frac{g_{\mu\nu}}{k^2 + i\epsilon}$$

Photon-fermion-fermion vertex

$$ie\gamma_\mu$$

Gauge parameter α can be chosen to be 0,1,∞.

Then is the renormalization. need regularization which preserves gauge invariant. Photon starts off massless (no $A_\mu A^\mu$ term, because it breaks gauge invariant.)

Recall in scalar field theory, one loop correction propagator is quadratic divergence

$$\int \frac{d^4p}{p^2}$$

p is the momentum of the loop, need $m_{scalar} \sim \Lambda^2_{cutoff}$.

Here vacuum polarization,

$$\int \frac{d^4p}{(k+p)(p)}$$

k is the momentum of the propagator and p is the momentum of the loop. Need photon mass term $m_\gamma A^\mu A_\mu$ to renormalize. But actually it is only logarithmic divergent, provided original symmetries preserved by regulator

$$ZF_{\mu\nu}F^{\mu\nu}$$

dimension regulator preserves gauge invariant.

What are the global symmetries of QED?

$U_V(1)$ electric charge. This symmetry is what photon couples to. If $m_{fermion} = 0$, there is a $U(1)_L \times U(1)_R$ symmetry of classical $\mathcal{L}_{classical}$. The low energy version of $SU(2)_L \times SU(2)_R$ in QCD. This symmetry is broken by m,

$$\mathcal{L}_{classical} = \bar{\Psi}_L i\not{D}\Psi_L + \bar{\Psi}_R i\not{D}\Psi_R$$

after adding $m_{fermion}$, $U_A(1)$ is broken, because axial charge is difference $L - R$ handed fermions, vector charge is $L + R$, total electron charge.

What about for quantum $\mathcal{L}_{quantum}$, if $m_{fermion} = 0$, would there be a $U(1)_L \times U(1)_R$ symmetry? No. because of anomaly. Anomaly means a summery of classical theory that is not present in quantum theory. No regularization that simultaneously preservers all classical symmetries.

We will talk about anomalies more later.

What are anomalies in SM?

1) there are anomalous currents where all masses are present to explicitly break the symmetry.

2) Anomalous gauge currents possible, but their coefficient is zero in SM just because of particular particle content.

In string theory all anomalies are cancel.

4.2 QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \sum_a^{N_f=\#\text{quarks}} \bar{\Psi}^{(a)}_j (i\not{D}_{jk} - m^{(a)}_{jk})\Psi^{(a)}_k \tag{4.2}$$

$a = 1, ..., 8$ labels gluons, also flavor (u, d, s), also related to 8 generators of $SU(3)$.

Ψ is 4 component Dirac spinor, 3 color vector, k is spin index. E.g.

$$\Psi^{(1)} = \underbrace{\begin{pmatrix} \psi_1^{(1)} \\ \psi_2^{(1)} \\ \psi_3^{(1)} \end{pmatrix}}_{3 \text{ colors}}, \qquad \psi_1^{(1)} = \underbrace{\begin{pmatrix} \psi_{1_1}^{(1)} \\ \psi_{1_2}^{(1)} \\ \psi_{1_3}^{(1)} \\ \psi_{1_4}^{(1)} \end{pmatrix}}_{4 \text{ comp Dirac}}$$

hence each a, $\Psi^{(a)}$ is 12 component complex vector space over spin and colors.

$$(D_\mu \Psi)_i = (\delta_{ik}\partial_\mu + ig_3 A_\mu^a \frac{\lambda_{ik}^a}{2})\Psi_k$$

λ^a Gell-Mann matrices

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_3 f^{abc} A_\mu^b A_\nu^c$$

g_3 is QCD coupling constant, f^{abc} is structure function for $SU(3)$ and $A_\mu^b A_\nu^c$ gives 3 or 4 gluons self interaction. Recall (3.5) light-light scattering has coupling $e^4/m^4 \sim \alpha^2$, here gluon-gluon scattering is g_3^2.

Lecture 9 (10/1/14)

Gauge invariance of $\mathcal{L}_{QCD}$ under $SU(3)_c$ gauge group

$$\Psi(x) \to e^{-i\alpha^a(x)\frac{\lambda^a}{2}}\Psi(x)$$

Gauge fields A_μ^a changes under these transformation and $F_{\mu\nu}^a$ also changes. Comparing to QED $F_{\mu\nu}$ is gauge invariant, $\vec{E}$,$\vec{B}$ are physical fields, In QCD E^a, B^a are not physical.

If $m^{(\alpha)} \propto I$, $\alpha = 1, 2$ in (4.2), then QCD has an $SU(2)_L \times SU(2)_R \to SU(2)_V$ global isospin symmetry.

If $m^{(\alpha)} = 0$, $\forall \alpha$ in (4.2), then QCD has an $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry. That is because

$$(\bar{\Psi}_L + \bar{\Psi}_R) i \not{D})(\Psi_L + \Psi_R) = \bar{\Psi}_L i \not{D} \Psi_L + \bar{\Psi}_R i \not{D} \Psi_R$$

no L, R coupling in kinetic term, but

$$\bar{\Psi} m \Psi = \bar{\Psi}_L m \Psi_R + \bar{\Psi}_R m \Psi_L$$

breaks $SU(N_f)_L \times SU(N_f)_R$.

$$\bar{\Psi}_j^{(a)} (i \not{D}_{jk} - m_{jk}^{(a)}) \Psi_k^{(a)}$$

Lorentz group Λ^μ_ν is representation for 4 vector, other irreducible representation are 2 $2D$ representation for L handed and R handed spinor. Dirac spinor representation contains L, R handed representations $\bar{\Lambda}^\mu_\nu$ (4×4 reducible representation) contains L handed & R handed 2D representation. They are accidentally have the same dimension, but they are completely different representations, one acts on spinor space, the other acts on spacetime.

However if we first do $m_u = m_d$ then let both go to 0, the $SU(N_f)_L \times SU(N_f)_R$ full symmetry will not be restored, i.e. same vacuum breaks chiral symmetry.

4.3 Symmetry Multiplet

Yang-Mills wrote down

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

1957. Glasgow worked on $SU(2)$ for weak interaction early 1960's, but vector bosons were massless, so Higgs mechanism was invented to give masses to vector bosons. from there Weinberg got EW and SM. 1960's Gell-Mann found that strongly interacting particle could be organized in groups, later 1960 early 1970's experiment revealed quarks, ˜1974 QCD was born.

Light hardronic states do group into multiplet of very similar masses.

$$\Pi^{\pm,0}$$

$$(n, p)$$

$$K^{\pm,0}, \bar{K}^0$$

see octant picture from particle data group.

Isospin multiplet seen for spin 0,1 mesons.

We mention that $\mathcal{L}_{QCD}$ has a vacuum state which breaks chiral symmetry. So we should set $SU(2)$ multiplet to high accuracy. If $m_u = m_d$ the exact isospin (except for EM). If $m_u \neq m_d$ this violation is

$$\frac{m_u - m_d}{\Lambda_{QCD}}$$

$\Lambda_{QCD} \sim 300$MeV. If $m_u = m_d = m_s$ then $SU(3)$ symmetry (known $m_s = 90$MeV, $m_u = 2 - 3$MeV, $m_s = 5$MeV)

4.4 Quarks

Quarks are confined into color neutral objects by very strong attractive force. The potential is linearly rising at smaller distance it is like EM potential $-1/r$, then it rises as $r \uparrow$, and it becomes position. So at large distance flux tube will break

$$q\bar{q} \to q\bar{u}\ u\bar{q}$$

4.5 QCD Perturbation Theory

Lecture 10 (10/6/14)

1974 asymptotic freedom discovered by Gross, etc. QCD effective coupling $\alpha(\mu)$ decreases as $\mu \uparrow$. Because $\langle \bar{\Psi}\Psi \rangle$ breaks chiral symmetry. Like for Ferromagnet, put in an external B field and let $B \to 0$, we will get non-magnetization as the $B \to 0$, so the magnetization becomes symmetry breaking measure. Here we put in a quark mass and let $m \to 0$, we will get non-zero $\langle \bar{\Psi}\Psi \rangle$ it is a measure of chiral symmetry breaking.

4.6 QCD at High Energy

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5 Appendix: Group Theory

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