Differential Geometry

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This is an undergraduate course. Offered in Fall 2014 at Columbia University. Office hours: We 2:30-4:00.

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Course Overview

Lecture 1 (9/2/14)

We will not follow any particular text. Most undergrad texts on the subject only study surfaces, which will not be so easy to generalize to high dimension. We will do surface then move to manifold. Most grad level texts start from manifold, so we won't follow that either.

We will cover the following topics,

1. metric on surface (including high dimensional surfaces). This provides way of computing length, area, volume.
2. Connection (Christoffel symbols), which is equivalent to study covariant derivatives, i.e. differentiate vector fields on sub manifold.
3. Extrinsic v.s. intrinsic curvatures. The difference is whether curvature depends on how the surface is embedded. E.g. take a plane sheet, which has 0 curvature, curve it to be a cylinder; the intrinsic curvature will still be 0, because the paper is rigid in itself.
4. Gauss–Bonnet theorem
5. General relativity. To do that, we need to generalize to Lorentzian, which allows time and space to mix.

1 Riemannian Geometry

1.1 Stereographic Projection

1.2 Metric & Quandric Form

1.3 Area & Angle

Lecture 2 (9/4/14)

Lecture 3 (9/9/14)

Lecture 4 (9/11/14)