

The point counts below are in moduli space, where each point is counted as the reciprocal of its stabilizer (in PGL_4). To get counts in parameter space, multiply by $\#\mathrm{PGL}_4(\mathbb{F}_q) = q^6(q^4 - 1)(q^3 - 1)(q^2 - 1)$. To get averages, divide by q^4 .

Alternatively, the coefficient of $(-q)^i$ is the Betti number β_{4-i} of the associated cover of moduli space. To get the Betti numbers for the cover of parameter space, tensor with $H^*(\mathrm{PGL}_4)$.

In some identification with the blow up of 6 points (so defined up to $W(E_6)$), the exceptional divisors are named E_1, \dots, E_6 , the conics are named F_1, \dots, F_6 , and the lines through pairs of points are named G_{12}, \dots, G_{56} .

Marking	Example	Unordered	Ordered
Nothing		q^4	Same as unordered
One line	E_1	q^4	Same as unordered
Two skew lines	E_1, E_2	$q^4 - q^3 + 1$	$q^4 - q^3 + q^2 - q + 2$
Two intersecting lines	E_1, G_{12}	q^4	$q^4 - q + 1$
Three skew lines	E_1, E_2, E_3	$q^4 - 2q^3 + q^2 - q + 4$	$q^4 - 4q^3 + 9q^2 - 15q + 14$
Tritangent	E_1, F_2, G_{12}	q^4	$q^4 - q + 1$
Four skew lines	E_1, E_2, E_3, E_4	$q^4 - 2q^3 + 2q^2 - 3q + 4$	$q^4 - 10q^3 + 45q^2 - 95q + 75$
Five skew lines	E_1, E_2, \dots, E_5	$q^4 - q^3 + q^2 - q + 2$	$q^4 - 15q^3 + 81q^2 - 185q + 150$
Six skew lines	E_1, E_2, \dots, E_6	$q^4 - q^3 + 1$	$q^4 - 15q^3 + 81q^2 - 185q + 150$
Double six	$E_1, \dots, E_6, F_1, \dots, F_6$	$q^4 - q^3$	$q^4 - 15q^3 + 81q^2 - 185q + 150$
Twenty-seven lines		q^4	$q^4 - 15q^3 + 81q^2 - 185q + 150$