UNIVERSITY OF RWANDA

COLLEGE OF SCIENCE AND TECHNOLOGY

COLLEGE EXAMINATIONS – ACADEMIC YEAR 2017- 2018

SCHOOL OF SCIENCE

DEPARTMENT OF MATHEMATICS

FIRST YEAR SEMESTER II

FINAL EXAMINATION

MATHEMATICS FOR ENGINEEREERS II (MAT1264)

DATE: 14/MAY/2018 **TIME: 2 hours**

MAXIMUM MARKS = 50

INSTRUCTIONS:

1. This paper contains **TWO** sections.
2. Section A is compulsory, and Answer any **TWO** of the **THREE** questions in Section B.
3. No written materials allowed into the Examination Room.
4. Write all your answers in the answer booklet provided.
5. Do not forget to write your Registration Number.
6. Do not write any answers on this question paper.
7. Where appropriate draw large clearly labeled diagrams in your answers.

<u>SECTION A</u> (20 Marks)

Question1.

a) Solve the following differential equation:
$L\frac{di}{dt} + Ri = 0$, where L and R are none negativity constants. **(3 Marks)**

b) Find the Wronskian of the set $\{e^{-2x}, -e^{-2x}, 5\}$ and state if the set of solutions is independent or not. **(2 Marks)**

c) Calculate the Laplace transform of: $f(t) = te^{-t}$. **(3 Marks)**

d) If $\vec{A} = xz\vec{\imath} - y^2\vec{\jmath} + 2x^2y\vec{k}$, then find: $\vec{\nabla} \times \vec{A}$. **(3 Marks)**

e) Solve the following second order linear differential equation:
$y'' + 3y' - 4y = 0$, with $y(0) = 5$ and $y'(0) = -5$. **(3 Marks)**

f) Find the inverse Laplace transform of $F(s) = \dfrac{2s+3}{s^2+3s}$. **(2 Marks)**

g) Evaluate $\iint xydxdy$ over the area in the first quadrant bounded by the circle $x^2 + y^2 = a^2$. **(3 Marks)**

h) State the Green's Theorem in the xy plane. **(2 Marks)**

<u>SECTION B</u> /30 Marks

Question2:

a) Solve the following system of differential equations using Matrix method(eigenvalues method):

$$\begin{cases} \dfrac{dx}{dt} = -4x + y + z \\ \dfrac{dy}{dt} = x + 5y - z \\ \dfrac{dz}{dt} = y - 3z \end{cases}$$

(10 Marks)

b) Calculate the Laplace transform of $\int_0^t (x^2 + e^{-x})dx$ **(5 Marks)**

Question3:

a) Consider $u = x^2 - y^2$ and $v = 2xy$. Determine the Jacobian of transformation of u and v at point $(1,2)$. **(6Marks)**

b) Use the change of variables for evaluating the following double integral:

$\iint_D \frac{\sin(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}} dA$ where $D = \{(x,y)/\, x^2+y^2 = \frac{\pi^2}{9} \text{ and } x^2+y^2 = \pi^2\}$. **(9 Marks)**

Question 4:

a) Evaluate $\int_{(0,1)}^{(1,2)} (x^2-y)dx + (y^2+x)dy$ along the parabola $x = t$ and $y = t^2+1$. **(8Marks)**

b) Evaluate $\iint_S u(x,y,z)dS$ where $\boldsymbol{S}$ is the surface of the paraboloid $\boldsymbol{z} = \boldsymbol{2} - (\boldsymbol{x^2} + \boldsymbol{y^2})$ above the xy-plane and $\boldsymbol{u}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})$ equal to $\boldsymbol{x^2} + \boldsymbol{y^2}$. **(7 Marks)**

Question5:

a) Use the operator method described to find the general solution in the following linear systems:

$$\begin{cases} x' + y' - 2x - 4y = e^t \\ x' + y' - y = e^{4t} \end{cases}$$

b) Solve the following linear system of differential equation using the matrix method?

$$\begin{cases} x_1' = 3x_1 + x_2 - x_3 \\ x_2^{t\prime} = x_1 + 3x_2 - x_3 \\ x_3' = 3x_1 + 3x_2 - x_3 \end{cases}$$

Good luck!!!!!!