definitions

a nonempty family $\mathcal{D} \subseteq 2^M$ closed under complements and under countable disjoint unions is called a λ-system or a Dynkin system.

a nonempty family $\mathcal{P} \subseteq 2^M$ closed under finite intersections is called a π-system.

exercise $\mathcal{F} \subseteq 2^M$ is a σ-algebra iff it is both a π-system and a λ-system.

exercise a nonempty family $\mathcal{D} \subseteq 2^M$ is a λ-system iff it is closed under countable increasing unions and is closed under complements of subsets in supersets.

exercise for $\mathcal{F} \subseteq 2^M$, define the Dynkin system generated by $\mathcal{F}$.

proposition (Dynkin) for a π-system $\mathcal{P}$. the Dynkin system generated by $\mathcal{P}$ equals the σ-algebra generated by $\mathcal{P}$.

proof we'll show that $\sigma(\mathcal{P}) = \lambda(\mathcal{P})$ by demonstrating that $\lambda(\mathcal{P})$ is a π-system.
let $\mathcal{F} = \{Q \in 2^M : Q \cap P \in \lambda(\mathcal{P}) \ \forall P \in \mathcal{P}\}$. then clearly $\mathcal{P} \subseteq \mathcal{F}$. on the other hand, $\mathcal{F}$ is a λ-system, and thus $\lambda(\mathcal{P}) \subseteq \mathcal{F}$. that is, $\forall P \in \mathcal{P} \ \forall Q \in \lambda(\mathcal{P}) \quad P \cap Q \in \lambda(\mathcal{P})$. this is almost what we want. fix an arbitrary $Q \in \lambda(\mathcal{P})$ and let $\mathcal{S} = \{A \in 2^M : Q \cap A \in \lambda(\mathcal{P})\}$. then $\mathcal{P} \subseteq \mathcal{S}$. on the other hand, again, $\mathcal{S}$ is a λ-system, and, again, $\lambda(\mathcal{P}) \subseteq \mathcal{S}$. this means $\forall Q, R \in \lambda(\mathcal{P}) \quad Q \cap R \in \lambda(\mathcal{P})$, namely $\lambda(\mathcal{P})$ is in fact a π-system.

corollary let $\mathcal{D}$ be a λ-system containing the π-system $\mathcal{P}$. then $\sigma(\mathcal{P}) \subseteq \mathcal{D}$.

corollary let μ_1, μ_2 be finite measures on (M, Σ) agreeing on a π-system $\mathcal{P}$. if $\sigma(\mathcal{P}) = \Sigma$ then $\mu_1 = \mu_2$.