

task given a finite set X , a function $f : X \rightarrow X$ and an element $x \in X$ compute the periodicity parameters of the sequence $x, f(x), f(f(x)), f(f(f(x))), \dots$

clarification one must compute the minimal $s \geq 0$ such that $f^s(x)$ appears infinitely often in the sequence (the number of steps to periodicity), as well as the minimal $\ell \geq 1$ such that $f^{s+\ell}(x) = f^s(x)$ (the period).

algorithm (Floyd) first find the minimum $k \geq 1$ for which $f^k(x) = f^{2k}(x)$. then, the first $r \geq 0$ occurrence of $f^r(x) = f^{k+r}(x)$ is the number of steps to enter the infinite loop. finally, the first $p \geq 1$ such that $f^r(x) = f^{r+p}(x)$ is the length of the loop.

pseudo code

```
def compute-periodicity-parameters(f,x):
    slow = f(x)
    fast = f(f(x))
    while slow != fast:
        slow = f(slow)
        fast = f(f(fast))

    searcher = x
    no-steps-to-cycle = 0
    while searcher != slow:
        no-steps-to-cycle += 1
        searcher = f(searcher)
        slow = f(slow)

    loop-entry = searcher
    searcher = f(searcher)
    cycle-length = 1
    while searcher != loop-entry:
        cycle-length += 1
        searcher = f(searcher)

    return no-steps-to-cycle, cycle-length
```

exercise verify the algorithm is correct and of linear time complexity $O(\text{no-steps-to-cycle} + \text{cycle-length})$.