problem

i. in a few hours our king will place his n sages in a line and assign each of them a different number in $\{0, 1, \ldots, n\}$.
ii. each sage will only be able to see the numbers of the sages in front of him, and in particular not his own number.
iii. starting from the sage at the back (who can see all numbers but his own), each sage will, in turn, guess his number, and this guess will be heard by all the sages.
iv. a sage must not guess a number that has previously been guessed. (and his guess has to be in $\{0, 1, \ldots, n\}$).
devise a method to guarantee a maximal number of sages guess correctly.

solution

the last sage (the first to guess) has two viable options for his number and has no way to guarantee a correct guess. our method will guarantee all the other sages, however, will succeed. they'll do so as follows. first, they invent an imaginary sage standing after the last sage, that has the remaining number in $\{0, 1, \ldots, n\}$ as his number. the last stage makes his guess so that the two numbers unknown to him are placed so that the permutation of all the $n+1$ numbers is even. in fact, any sage will have two viable options as to their number, and only one will make the permutation even. the first sage to guess might be wrong, but all the others will guess correctly.

example suppose $n = 5$ and the sages have numbers $3, 0, 4, 2, 6$. then the fifth sage sees $3, 0, 4, 2$ and must guess 1 or 6. he guesses 1, since $3, 0, 4, 2, 1, 6$ is even, and $3, 0, 4, 2, 6, 1$ is odd. hearing the guess of 1, the fourth sage sees $3, 0, 4$ and must decide between $3, 0, 4, 2, 1, 6$ or $3, 0, 4, 6, 1, 2$. the even permutation is, of course, the first, so the fourth sage deduces his number is 2. and so on, they all come to understand the total permutation as $3, 0, 4, 2, 1, 6$.