definition let G act on X. a block is a nonempty subset $B \subseteq X$ such that the different gB are disjoint. that is, $\forall g \in G$ either $gB = B$ or $gB \cap B = \varnothing$. the trivial blocks are the singletons as well as the whole X.

exercise if G acts doubly transitive on X then it is has no non-trivial blocks

exercise let G act on X with no non-trivial blocks, and let $N \trianglelefteq G$ be a normal subgroup. then either N acts trivially on X or N acts transitively on X.

theorem [Iwasawa] let G be a perfect group acting faithfully and transitively on X with no non-trivial blocks. suppose there exists a point x_0 with stabilizer H such that there exists a soluble normal subgroup $K \trianglelefteq H$ for which $\bigcup_{g \in G} gKg^{-1}$ generates G. then G is simple.

proof let $\{\mathrm{id}\} \neq N \trianglelefteq G$. we have $NK \trianglelefteq NH$, but the exercise tells us that N acts transitively on X, so that $NH = G$. it follows that the generating set $\bigcup_{g \in G} gKg^{-1}$ lies inside NK, meaning $NK = G$. finally, if $K^{(r)} = \{\mathrm{id}\}$ is trivial then $G = G^{(r)} = (NK)^{(r)} \subseteq NK^{(r)} = N$.