puzzle some pennies are placed on the infinite line $\mathbf{Z}$. the following moves are allowed:

i. removing two pennies at $n-1$ and at $n+1$ for a single penny at n.

ii. vice versa, removing a penny at n for two pennies at $n-1$ and at $n+1$.

show that from a starting state of a single penny at 0 we can reach an end state of a single penny at 6.

possible solution to puzzle

	1							
1		1						
1	1		1					
1	1	1		1				
1	1	1	1		1			
1	1	1	1	1		1		
1	1	1	1	1	1		1	
1	1	1	1	1	1	1		1
1	1	1	1	1	2		1	1
1	1	1	1	2	1	1	1	1
1	1		2	1	1	1	1	1
1		1	1	1	1	1	1	1
	1		1	1	1	1	1	1
		1		1	1	1	1	1
			1		1	1	1	1
				1		1	1	1
					1		1	1
						1		1
							1	

exercise how is the above related to the fact that $x^2 - x + 1 \mid x^6 - 1$?