puzzle some pennies are placed on the infinite line ${\bf Z}$. the following moves are allowed:

- i. removing two pennies at n-1 and at n+1 for a single penny at n.
- ii. vice versa, removing a penny at n for two pennies at n-1 and at n+1.

show that from a starting state of a single penny at 0 we can reach an end state of a single penny at 6.

exercise how is the above related to the fact that $x^2 - x + 1 \mid x^6 - 1$?

possible solution to puzzle

```
1
1 1
1 1 1
1 1 1 1
1 1 1 1 1
1 1 1 1 1 1
1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 2 1 1
1 \quad 1 \quad 1 \quad 1 \quad 2 \quad 1 \quad 1 \quad 1 \quad 1
      2 1 1 1 1 1
1 1
1 1 1 1 1 1 1 1
  1
      1 1 1 1 1 1
    1
         1 1 1 1 1
       1 1 1 1 1
         1 1 1 1
           1 1 1
             1 1
               1
```