definitions given $t_1, \ldots, t_m$ and $k \geq 0$, we'll write $f_k = t_1^k + \cdots + t_m^k$ in addition to $e_k = \sum_{i_1 < \cdots < i_k} t_{i_1} \ldots t_{i_k}$.

example if $m = 3$ then $e_0 = 1$, $e_1 = t_1 + t_2 + t_3$, $e_2 = t_1 t_2 + t_2 t_3 + t_1 t_3$, $e_3 = t_1 t_2 t_3$, and $e_k = 0$ for $k \geq 4$.

observation e_k is the coefficient of t^{m-k} in $(t + t_1) \ldots (t + t_m)$.

identity (Newton-Girard) for $k \geq 1$ we have $f_k = (-1)^{k-1} \left[k e_k + \sum_{\ell=1}^{k-1} (-1)^\ell e_{k-\ell} f_\ell \right]$.

examples $f_1 = \sum t_i = e_1$, and $\sum t_i^2 = -\left[2 \sum_{i<j} t_i t_j - \left(\sum t_i\right)^2 \right]$.

exercise verify the case $k = 3$ by hand.

<u>proof</u> (Euler) let $P(t) = (t+t_1)\dots(t+t_m)$. then $\dfrac{P'(t)}{P(t)} = \dfrac{1}{t+t_1} + \dots + \dfrac{1}{t+t_m}$. applying $\dfrac{1}{x-\alpha} = \dfrac{1}{x} + \dfrac{\alpha}{x^2} + \dfrac{\alpha^2}{x^3} + \dots$ we have $P'(t) = P(t) \sum_{n\geq 0} \dfrac{(-1)^n f_n}{t^{n+1}}$. we finish by plugging $P(t) = \sum e_{m-j} t^j$ and equating the t^{m-k-1} coefficients.