proposition $\sum_{p \text{ prime}} \frac{1}{p} = \infty$

proof (Erdős) suppose the sum converges. fix k so that $\sum_{n>k} \frac{1}{p_n} \leq \frac{1}{2}$. then at least half of the elements of $[N]$ have all prime factors in $\{p_1, \ldots, p_k\}$. for such a number, its squarefree part has 2^k possibilities, and its perfect square part has at most $\sqrt{N}$ possibilities. if N is large enough, $N/2 \leq 2^k\sqrt{N}$ cannot hold.