ELEC4605 ASSIGNMENT 1 – TERM 3, 2020

NOISY QUANTUM SYSTEMS

In this assignment, you will be applying the filter function formalism presented in the lectures, along with a density matrix and rotation operator analysis to study the effect of noise on quantum systems. As in the lectures, we will consider an ensemble of spins as our prototypical quantum system. It is advised you review the section on "controlling quantum systems" in your lecture notes before attempting this assignment.

You are to investigate a spin ensemble in a noisy environment. You can assume that the noise is stochastic (a random process) and both stationary and ergodic. Whilst the origin of the noise is uncertain, you can assume that it couples to the spin magnetically (i.e. it appears as an effective magnetic field noise to the spins). The noise causes dephasing of the spins during pulsed experiments, resulting in a decay of their average coherence over time. The gyromagnetic ratio of the spins is given by the "isolated electron" value $\gamma_e = 28$ GHz/T. We have recorded some sample traces of the effective magnetic field noise, which are provided to you in two different formats: the first as a matrix in a MAT-file for importing into MATLAB (should you choose to use this for your analysis), and the second as a comma separated variable file (for importing into other numerical software packages). Both files contain the same noise samples. You may access this data through the following link: https://www.dropbox.com/sh/khds16l8jxvlrdm/AACN_egz1Szm2a48YQLNo281a?dl=0.
The traces were sampled with a time resolution of 2.5 µs and are in units of magnetic field (Tesla). The individual time traces run along each row, with 100 sample traces provided.

(a) Use the filter-function theory to plot the decay of coherence of the spin ensemble due to this noise for the following experimental pulse sequences:
(i) Ramsey (free induction decay). [*10 marks*]
(ii) Hahn echo. [*10 marks*]
(iii) A multi-pulse dynamical decoupling sequence with an increasing number of pulses and a fixed duration between them. It's up to you to select an appropriate pulse separation time (or perhaps you might wish to consider a few different cases). [*10 marks*]

In each part above, you should plot the decay of the coherence as a function of the total pulse sequence length (τ) and fit each curve <u>with an appropriate function</u> to extract the coherence/dephasing time. Note, you will need to calculate the noise power spectral density (PSD) to complete this task, you may either do this directly by taking the Fourier transform of the autocorrelation function of the noise, or you can use a built-in function from your numerical software package (e.g. `pwelch();` in MATLAB). Be sure to include a discussion of your results.

(b) Use the same noise signals to perform a "brute-force" simulation of the decoherence by calculating density matrices and applying rotation and time-evolution operators (i.e. do not use the filter-function formalism). It is <u>not sufficient</u> to use the simplified analytical form of the expectation of the density operator (for example, as presented on slide 70 of your lecture notes) – you are expected to perform matrix multiplications here. Each noise trace can be viewed as the noise experienced by individual spins in the ensemble. You may also break the traces up to smaller segments to increase the number of spins, if need be. You can assume that all pulses are instantaneous.

(i) Why are you able to break individual noise traces into smaller segments to increase the effective number of spins in your simulation? [*5 marks*]

(ii) Simulate the decay of the coherence for each of the pulse sequences listed in part a (plot the decay curves and extract the coherence times) and compare the results with those obtained from the filter-function theory approach above. [*25 marks*]

(iii) List the pros and cons of the filter function and brute-force methods. [*5 marks*]

(c) In this final part you are to utilise the spins to measure the spectral density of the noise. To do this, you will perform brute-force simulations (as you did in part b) for various pulse sequences and use the resulting coherence decay parameter(s) to estimate the noise power spectral density. Review slide 92 of your lecture notes before completing this section.

(i) Use filter-function theory to express the noise PSD in terms of the measurement filter function spectrum and parameter(s) you can extract from a coherence decay experiment. Assume that the bandwidth of the measurement filter is sufficiently narrow that the noise power spectral density can be treated as a constant. [5 marks]

(ii) To extract the noise power spectral density, one strategy is to measure the spin coherence decay for pulse sequences with spectrums centred on different frequencies, in conjunction with the expression derived in part c(i) above. With this in-mind, describe the steps to reconstruct the noise power spectral density through brute-force simulations of the coherence decay. Make sure you describe the pulse sequences you would use and discuss any limitations of this method as well as assumptions made. [*10 marks*]

(iii) Use the brute-force simulations to reconstruct the PSD of the noise. You can only utilise information extracted from the coherence decay measurements together with your knowledge of the pulse sequence applied. Be as quantitative in extracting the noise PSD as possible. [*20 marks*]

Submission: You are expected to complete this assignment and write a report summarising your results <u>by yourself</u>. The report is due on Friday, week 5 (16/10/2020) at 6pm. Submission will be via the Moodle course page. This assignment is worth 12.5% of your total grade for the course and will be marked out of 100.