ECON-122
Introduction to Econometrics

Agnieszka Postepska

July 20th, 2011

Interpretation of the coefficients

Partial effects

- knowing the computation behind the $\hat{\beta}_{OLS}$ is very important
- however, as the goal is to be able make inference about the real life relationships, we need to know how to interpret the coefficients
- as in the simple regression model, the intercept $\hat{\beta}_0$ tells us what the average value of y would be if ALL the $x's$ were 0
- now we can better understand the **ceteris paribus** condition - consider the following model with two independent variables:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u_i \\ \text{then } \Delta y &= \beta_1 \Delta x_1 + \beta_2 \Delta x_2 \end{aligned}$$

- now, lets say we want to know what is the effect of x_1 on y - then we hold x_2 fixed, so that $\Delta x_2 = 0$ and the equation becomes:

$$\Delta y = \beta_1 \Delta x_1 \Rightarrow \beta_1 = \frac{\Delta y}{\Delta x}$$

Therefore, β_1 tells us what is the effect on y if we vary x_1 holding everything else constant $\Rightarrow$ thus the name, *partial effects*

Changing more than one variable

- you can get the effect of changes of as many $x's$ as you want in any combination you want
- just keep fixed whichever independent variable(s) you want to hold fixed, decide how big of change in other $x's$ you are interested in and do the math
- example with three independent variables:

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u_i \\ \text{then } \Delta y &= \hat{\beta}_1 \Delta x_1 + \hat{\beta}_2 \Delta x_2 + \hat{\beta}_3 \Delta x_3 \end{aligned}$$

- now, lets say we want to know what is the effect of 1 unit change in x_1 and x_3 on y - then we hold x_2 fixed, so that $\Delta x_2 = 0$ and the equation becomes:

$$\Delta y = \hat{\beta}_1 + \hat{\beta}_3$$

Therefore, $\hat{\beta}_1 + \hat{\beta}_3$ tells us the total effect on y if we vary x_1 and x_2 by 1 unit holding everything else constant $\Delta x_2 = 0$

Algebraic properties oh OLS estimator in the MLRM

- as in the simple model, let's first define the *residuals* and *fitted values* in the MLRM
  For each observation we have:

$$\begin{aligned} \hat{y}_i &= \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + ... + \hat{\beta}_k x_{ik} = x_i \hat{\beta} \\ \hat{u}_i &= y_i - \hat{y}_i = y_i - x_i \hat{\beta} \end{aligned}$$

- and similarly as in the simple regression model, the first two properties are:

$$\begin{aligned} \text{(i) } \sum_i \hat{u}_i &= 0 \\ \text{(ii) } \sum_i x_{ij} \hat{u}_i &= 0 \quad \text{where } j = 1, 2, ..., k \end{aligned}$$

  Notice that (ii) is really k restrictions - one for each independent variable x_j
  And, the third property: (iii) the point $(\bar{x}_1, \bar{x}_2, ..., \bar{x}_k, \bar{y})$ is always on the OLS regression line

Partialling out interpretation of multiple regression

- now, we will derive explicit formulas for each $\hat{\beta}_j^{OLS}$
- one of the most important thing to take from this part of the class is that when estimating particular coefficient we use information from all $x's$, not only the x_j in which effect we are interested in
- Consider the following scenario:

$$\begin{aligned} \hat{y} &= \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 \\ \text{then } \hat{\beta}_1 &= \frac{\sum_i \hat{r}_{i1} y_i}{\sum_i \hat{r}_{i1}^2} \end{aligned}$$

where $\hat{r}_{i1}$ are the OLS residuals from a regression of x_1 on x_2

- $\hat{r}_{i1}$ is the part of x_{i1} that is uncorrelated with x_{i2} - so we have partialled out the effect of x_{i2} on x_{i1}
- this formula illustrates that $\hat{\beta}_1$ shows the effect of x_1 on y after x_2 has been partialled out

Concept of degrees of freedom

- notice the resemblance of the variance estimator in the simple regression model - in the denominator in both cases we have the **degrees of freedom** (*df*)

Definition

The number of degrees of freedom is the number of values in the final calculation of a statistic that are free to vary. In a regression model with $k+1$ parameters to estimate we have:

$$\begin{aligned} df &= n-(k+1) \\ &= \text{number of observations} - \text{number of parameters to estimate} \end{aligned}$$

- the number of independent pieces of information that go into the estimate of a parameter is called the *degrees of freedom*
- we need $k+1$ equations to exactly solve for the parameters -so in principle we need exactly $n=k+1$ observations to solve this system of equations

Model specification

Structural estimation vs. reduced form models

- ideally we would like to estimate general equilibrium models directly as theory tells us
- *structural form* begin from deductive theories of the economy
- in practice *structural estimation* is very rare (difficult from both data and estimators perspective)
- most empirical work is based on *reduced form* models (partial equilibrium models)
- *reduced form* models begin by identifying particular relationships between variables
- **selection of variables is crucial for quality of estimates**
- model specification is often data driven which is unfortunate as it is then likely to deliver numbers that are meaningless...

Irrelevant variables in a regression model

- consider the following model:

$$\begin{aligned} y &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u \\ \text{and assume: } \beta_3 &= 0 \\ \Rightarrow E(y|x_1, x_2, x_3) &= E(y|x_1, x_2) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \end{aligned}$$

- the above equation describes the population - because we do know that $\beta_3 = 0$ we estimate the following equation:

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{\beta}_3 x_3$$

- what is the effect of x_3 on our estimates?
  - on unbiasedness of β_1 and β_2?
  - on the variance of the estimates?

Excluding relevant variables from the regression model

- consider the following model:

$$wage = \beta_0 + \beta_1 education + \beta_2 ability + u$$

- we are primarily interested in th effect of β_1 on y
- what is the equation we will estimate (is education observable? is ability observable?)
- suppose this is the equation we estimate:

$$\tilde{wage} = \tilde{\beta}_0 + \tilde{\beta}_1 education$$

- now we will show when is it the case that simple regression model produces the same estimates as the multiple regression model and when it is not the case and excluding a variable from the regression biases estimates (as it requires a lot of algebra we turn back to the blackboard)