ECON-122
Introduction to Econometrics

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The five assumptions revisited

The five assumptions in the multiple regression model

- A1: LINEARITY IN PARAMETERS

$$\begin{aligned} y_i &= \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + ... + \beta_k x_{ki} + u_i \\ \mathbf{y} &= \mathbf{X}\beta + \mathbf{u} \end{aligned}$$

- A2 RANDOM SAMPLING: we have a random sample of n observations $x_{i1}, x_{i2}, ..., x_{ik}, y_i$, where $i = 1, 2, ..., n$ from the population following the model in A1
- A3: NO PERFECT COLLINEARITY: equivalent of the variation in x in simple regression model - here it means that no regressor is constant and there are no exact linear relationships among the independent variables
- A4: ZERO CONDITIONAL MEAN

$$\begin{aligned} E(u_i|\mathbf{X}) &= 0 \qquad \forall_{i=1,2,...,n} \\ E(u|x_1, x_2, ..., x_k) &= 0 \end{aligned}$$

The five assumptions in the multiple regression model cont.

- A5: HOMOSKEDASTICITY and NO SERIAL CORRELATION

$$\begin{array}{rrcll} (i) & Var(u_i|\mathbf{X}) & = & \sigma^2 & \forall_{i \neq j} \\ (ii) & Cov(u_i, u_j) & = & 0 & \forall_{i \neq j} \end{array}$$

and (i) and (ii) imply:

$Var(\mathbf{u}|\mathbf{X}) = \sigma^2 \mathbf{I}_n$ where $\mathbf{I}_n$ is a n x n identity matrix

Unbiasedness and variance of OLS in multiple regression

Under A1, A2 and A4, $\hat{\beta}_{OLS}$ is unbiased for β:

$$E(\hat{\beta}_{OLS}|\mathbf{X}) = \beta$$

Under A1-A4,:

$$Var(\hat{\beta}_{OLS}|\mathbf{X}) = \sigma^2(\mathbf{X}'\mathbf{X})^{-1}$$

Proofs and derivations on the blackboard (everything should look analogous to derivations in SLRM)

The error term variance σ^2

- once again we have an useless expression for the variance for our estimator as we don't know σ^2
- just as before we will estimate using the residuals

The unbiased estimator of the error variance σ^2 can be written as:

$$\hat{\sigma}^2 = \frac{\hat{\mathbf{u}}'\hat{\mathbf{u}}}{n-k-1}$$

- and then, finally, we have a meaningful expression to calculate σ^2

Under A1-A5,

$$E(\hat{\sigma}^2) = \sigma^2$$

The Gauss-Markov Theorem

- We have established that under some assumptions, $\hat{\beta}_{OLS}$ is an unbiased estimator for β
- We have also found an expression for the variance of our OLS estimator
- recall the desired properties of an estimator that we talked about - unbiasedness and efficiency
- now we are going to show that, under some assumptions, $\hat{\beta}_{OLS}$ is the best linear unbiased estimator (BLUE) , meaning that in a class of all linear estimators, it has the smallest variance (efficiency)
- Formally this fact is known as Gauss-Markov Theorem (proof in Appendix E)

Gauss-Markov Theorem

Under A1-A5, $\hat{\beta}_{OLS}$ is the best linear unbiased estimator (BLUE) of β.