Miloșteanu Andreea
Mitroi Valentina

10.11.2021

Tutoriat 4
Geometrie I
(exerciții)

1. a) Dați exemple de două drepte care verifică $R_{O,\frac{\pi}{3}} = S_{d_2} \circ S_{d_1}$.

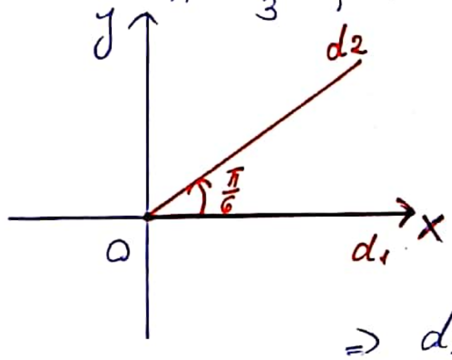
b) Să se arate că $R_{N,\frac{\pi}{3}} \circ R_{M,\frac{\pi}{6}} = R_{P,\frac{\pi}{2}}$, unde $M=(1,2)$, $N=(-1,2)$, $P(x,y)=?$

SOL.

a) Alegem pentru dreapta d_1 axa Ox care are ecuația $d_1: y=0$.

Din teorie știm că orice rotație $R_{M,\beta}$ se poate scrie $S_{d_2} \circ S_{d_1}$, $d_1 \cap d_2 = \{M\}$, $m(\widehat{d_1,d_2}) = \frac{1}{2}\beta$.

La noi, $\beta = \frac{\pi}{3}$ și $M = O = (0,0)$. Deci: $\begin{cases} d_1 \cap d_2 = \{O\} \\ m(\widehat{d_1,d_2}) = \frac{\pi}{6} \end{cases}$



Deoarece $\left.\begin{array}{l} m(\widehat{d_1,d_2}) = \frac{\pi}{6} \\ d_1: y=0 \end{array}\right| \Rightarrow \left.\begin{array}{l} m_{d_2} = tg\frac{\pi}{6} = \frac{\sqrt{3}}{3} \\ O(0,0) \in d_2 \end{array}\right| \Rightarrow$

$\Rightarrow d_2: y-0 = m_{d_2}(x-0)$

$d_2: y = \frac{\sqrt{3}}{3}x \Leftrightarrow d_2: \sqrt{3}x - 3y = 0.$

b) $R_{N,\frac{\pi}{3}} \circ R_{M,\frac{\pi}{6}} = R_{P,\frac{\pi}{3}+\frac{\pi}{6}} = R_{P,\frac{\pi}{2} + 2k\pi}$

$$R_{N,\frac{\pi}{3}}: \begin{pmatrix} x' \\ y' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{pmatrix}}_{A(\frac{\pi}{3})} \begin{pmatrix} x+1 \\ y-2 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x+1 \\ y-2 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$R_{M,\frac{\pi}{6}}: \begin{pmatrix} x' \\ y' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{pmatrix}}_{A(\frac{\pi}{6})} \begin{pmatrix} x-1 \\ y-2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} x-1 \\ y-2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$R_{P,\frac{\pi}{2}}: \begin{pmatrix} x' \\ y' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{pmatrix}}_{A(\frac{\pi}{2})} \begin{pmatrix} x-a \\ y-b \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x-a \\ y-b \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

$R_{P,\frac{\pi}{2}}: X' = A\left(\frac{\pi}{2}\right)\begin{pmatrix}x-a\\y-b\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} = A\left(\frac{\pi}{2}\right)\begin{pmatrix}x\\y\end{pmatrix} - A\left(\frac{\pi}{2}\right)\begin{pmatrix}a\\b\end{pmatrix} + \begin{pmatrix}a\\b\end{pmatrix} =$

$= A\left(\frac{\pi}{2}\right)\begin{pmatrix}x\\y\end{pmatrix} + \left(I_2 - A\left(\frac{\pi}{2}\right)\right)\begin{pmatrix}a\\b\end{pmatrix}$

$R_{N,\frac{\pi}{3}} \circ R_{M,\frac{\pi}{6}}: X \xrightarrow{R_{M,\frac{\pi}{6}}} \underbrace{A\left(\frac{\pi}{6}\right)\begin{pmatrix}x-1\\y-2\end{pmatrix} + \begin{pmatrix}1\\2\end{pmatrix}}_{\text{nod } X=\binom{x}{y}} \xrightarrow{R_{N,\frac{\pi}{3}}} A\left(\frac{\pi}{3}\right)\left[A\left(\frac{\pi}{6}\right)\begin{pmatrix}x-1\\y-2\end{pmatrix} +\right.$

$\left. + \begin{pmatrix}1 & \overset{N}{\boxed{+1}}\\ 2 & \boxed{-2}\end{pmatrix}\right] + \overset{N}{\begin{pmatrix}-1\\2\end{pmatrix}} = A\left(\frac{\pi}{2}\right)\begin{pmatrix}x\\y\end{pmatrix} + A\left(\frac{\pi}{2}\right)\begin{pmatrix}-1\\-2\end{pmatrix} + A\left(\frac{\pi}{3}\right)\begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}-1\\2\end{pmatrix}$

Prin asemănare: $\left(I_2 - A\left(\frac{\pi}{2}\right)\right)\begin{pmatrix}a\\b\end{pmatrix} = A\left(\frac{\pi}{2}\right)\begin{pmatrix}-1\\-2\end{pmatrix} + A\left(\frac{\pi}{3}\right)\begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}-1\\2\end{pmatrix}$

$$\begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}-1\\-2\end{pmatrix} + \begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}-1\\2\end{pmatrix}$$

$$\begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}2\\-1\end{pmatrix} + \begin{pmatrix}1\\\sqrt{3}\end{pmatrix} + \begin{pmatrix}-1\\2\end{pmatrix}$$

$$\begin{pmatrix}a+b\\-a+b\end{pmatrix} = \begin{pmatrix}2\\1+\sqrt{3}\end{pmatrix} \Rightarrow \begin{cases}a+b=2\\-a+b=1+\sqrt{3}\end{cases} \oplus$$

$$2b = 3+\sqrt{3} \Rightarrow b = \frac{3+\sqrt{3}}{2}$$

$$a = 2-b = \frac{2}{2} - \frac{3+\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$$

Așadar, $P = \left(\frac{1+\sqrt{3}}{2}, \frac{3+\sqrt{3}}{2}\right)$

2. a) Să se scrie ecuația omotetiei $\mathcal{H}_{O,3}$, unde $O=(0,0)$, $K=3$.

b) Să se scrie ecuația omotetiei $\mathcal{H}_{A,2}$, unde $A=(1,2)$, $K=2$.

c) Să se determine $\mathcal{H}_{A,2}(d)=d'$, unde $d: 2x+y-1=0$.

SOL. a) $\mathcal{H}_{O,K}: \begin{cases} x'=Kx \\ y'=Ky \end{cases}$ Deci $\mathcal{H}_{O,3}: \begin{cases} x'=3x \\ y'=3y \end{cases}$

b) $\mathcal{H}_{A,K}: \begin{cases} x'=Kx+a(1-K) \\ y'=Ky+b(1-K) \end{cases}$ Deci $\mathcal{H}_{A,2}: \begin{cases} x'=2x-1 \\ y'=2y-2 \end{cases}$

$$\boxed{\begin{aligned} x &= \tfrac{1}{2}(x'+1)=\tfrac{1}{2}x'+\tfrac{1}{2} \\ y &= \tfrac{1}{2}(y'+2)=\tfrac{1}{2}y'+1 \end{aligned}}$$

c) $\mathcal{H}_{A,K}^{-1}=\mathcal{H}_{A,\frac{1}{K}}$

Deci $\mathcal{H}_{A,2}^{-1}=\mathcal{H}_{A,\frac{1}{2}}: \begin{cases} x=\frac{1}{2}x'+\frac{1}{2} \\ y=\frac{1}{2}y'+1 \end{cases}$

$d: 2x+y-1=0$

$d': 2\left(\frac{1}{2}x'+\frac{1}{2}\right)+\frac{1}{2}y'+1-1=0 \;/\cdot 2$

$d': 2x'+y'+2=0$

Sau $A(1,2)\notin d \Rightarrow d \parallel d' \Rightarrow \vec{m_d}=\vec{m_{d'}}=(2,1)$

Fie $\left.\begin{array}{l} d': ax'+by'+c=0 \\ \vec{m_{d'}}=(2,1) \end{array}\right| \Rightarrow d': 2x'+y'+c=0$

Fie $P(0,1)\in d \Rightarrow P'(x',y')\in d'$, $P'=\mathcal{H}_{A,2}(P)=(2\cdot 0-1, 2\cdot 1-2) \Rightarrow$

$\Rightarrow P'=(-1,0)\in d' \Rightarrow$

$\Rightarrow 2\cdot(-1)+0+c=0 \Rightarrow c=2$

$d': 2x'+y'+2=0$

3. Fie $f: E_2 \to E_2$, $f(x,y) = (3x-2, 3y+2)$ și $d: x+3y-2=0$.

a) Arătați că f este o omotetie și calculați centrul și raportul ei.

b) Aflați $f(d)=d'$ și calculați distanța de la d la d'.

Sol. a) $f: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3x-2 \\ 3y+2 \end{pmatrix} = 3\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

O omotetie de centru M și raport K are ecuația:

$$\mathcal{H}_{M,K}: \begin{pmatrix} x' \\ y' \end{pmatrix} = K\begin{pmatrix} x \\ y \end{pmatrix} + (1-K)\begin{pmatrix} a \\ b \end{pmatrix}, \quad M=(a,b)$$

Prin asemănare, $K=3$, deci $1-K=-2$

$$-2\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \Leftrightarrow -2\begin{pmatrix} a \\ b \end{pmatrix} = 2\begin{pmatrix} -1 \\ 1 \end{pmatrix} \Leftrightarrow \begin{cases} -a=-1 \\ -b=1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} a=1 \\ b=-1 \end{cases} \Rightarrow M=(1,-1)$$

Așadar, am ~~obținut~~ obținut o omotetie $\mathcal{H}_{M,3}$, $M=(1,-1)$ și $K=3$.

b) Obs. că $M(1,-1) \notin d \Rightarrow d \parallel d'$.

$$\mathcal{H}_{M,3}: \begin{cases} x'=3x-2 \\ y'=3y+2 \end{cases} \Rightarrow \mathcal{H}_{M,3}^{-1} = \mathcal{H}_{M,\frac{1}{3}}: \begin{cases} x = \frac{1}{3}x' + \frac{2}{3} \\ y = \frac{1}{3}y' - \frac{2}{3} \end{cases}.$$

$d: x+3y-2=0$

$d': \frac{1}{3}x' + \frac{2}{3} + 3\left(\frac{1}{3}y' - \frac{2}{3}\right) - 2 = 0 \,/\cdot 3$

$d': x'+2+3y'-6-6=0 \Rightarrow d': x'+3y'-10=0.$

$\vec{m}_d = \vec{m}_{d'} = (1,3)$, deci e adv. că $d \parallel d'$.

sau Alegem $\begin{cases} A(-1,1) \in d, \text{ deci } \mathcal{H}_{M,3}(A) = A' = (3\cdot(-1)-2,\ 3\cdot 1+2) = (-5,5) \in d' \\ B(2,0) \in d, \text{ deci } \mathcal{H}_{M,3}(B) = B' = (3\cdot 2-2,\ 3\cdot 0+2) = (4,2) \in d' \end{cases}$

$$d': \frac{x'-x_{A'}}{x_{B'}-x_{A'}} = \frac{y'-y_{A'}}{y_{B'}-y_{A'}} \Leftrightarrow d': \frac{x'+5}{9} = \frac{y'-5}{-3} \Leftrightarrow d': -3x'-15 = 9y'-45 \,/:(-3)$$

$$x'+5 = -3y'+15$$

$$\Rightarrow d': x'+3y'-10=0$$

$d: x+3y-2=0$ și $d': x'+3y'-10=0$

Vrem să aflăm $dist(d,d')$.

Avem că $B(2,0)\in d$, deci $dist(d,d') = dist(B,d') = \frac{|2\cdot 1+3\cdot 0-10|}{\sqrt{1^2+3^2}} = \frac{8}{\sqrt{10}} =$

$= \frac{8\sqrt{10}}{10} = \frac{4\sqrt{10}}{5}$.

4. Fie punctele $A(1,3)$, $B(2,4)$ și $K=3$.

a) Să se arate că $T_{\overrightarrow{AB}} \circ \mathcal{H}_{A,K} = \mathcal{H}_{B,K} \circ T_{\overrightarrow{AB}} = \mathcal{H}_{C,K}$.

b) Determinați C și demonstrați că punctele A, B, C sunt coliniare.

Sol. $\overrightarrow{AB} = (x_B-x_A, y_B-y_A) = (1,1)$

$T_{\overrightarrow{AB}}: \begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}1\\1\end{pmatrix} \Leftrightarrow T_{\overrightarrow{AB}}: \begin{cases}x'=x+1\\y'=y+1\end{cases}$

$\mathcal{H}_{A,K}: \begin{pmatrix}x'\\y'\end{pmatrix} = 3\begin{pmatrix}x\\y\end{pmatrix} - 2\begin{pmatrix}1\\3\end{pmatrix} \Leftrightarrow \mathcal{H}_{A,K}: \begin{cases}x'=3x-2\\y'=3y-6\end{cases}$

$\mathcal{H}_{B,K}: \begin{pmatrix}x'\\y'\end{pmatrix} = 3\begin{pmatrix}x\\y\end{pmatrix} - 2\begin{pmatrix}2\\4\end{pmatrix} \Leftrightarrow \mathcal{H}_{B,K}: \begin{cases}x'=3x-4\\y'=3y-8\end{cases}$

$T_{\overrightarrow{AB}} \circ \mathcal{H}_{A,K}: (x,y) \xrightarrow{\mathcal{H}_{A,K}} (3x-2, 3y-6) \xrightarrow{T_{\overrightarrow{AB}}} ((3x-2)+1, (3y-6)+1) \rightarrow$

$\rightarrow (3x-1, 3y-5) \rightarrow 3(x,y)-(1,5)$ (1)

$\mathcal{H}_{B,K} \circ T_{\overrightarrow{AB}}: (x,y) \xrightarrow{T_{\overrightarrow{AB}}} (x+1, y+1) \xrightarrow{\mathcal{H}_{B,K}} (3(x+1)-4, 3(y+1)-8) \rightarrow$

$\rightarrow (3x-1, 3y-5) \rightarrow 3(x,y)-(1,5)$ (2)

Din (1) și (2) $\Rightarrow T_{\overrightarrow{AB}} \circ \mathcal{H}_{A,K} = \mathcal{H}_{B,K} \circ T_{\overrightarrow{AB}}$.

b) $\mathcal{H}_{C,K}: \begin{pmatrix}x'\\y'\end{pmatrix} = 3\begin{pmatrix}x\\y\end{pmatrix} - 2\begin{pmatrix}a\\b\end{pmatrix}$

$C=(a,b)$

Deci $-2\begin{pmatrix}a\\b\end{pmatrix} = -\begin{pmatrix}1\\5\end{pmatrix} \Leftrightarrow \begin{cases}2a=1\\2b=5\end{cases} \Rightarrow a=\frac{1}{2}, b=\frac{5}{2} \Rightarrow C\left(\frac{1}{2}, \frac{5}{2}\right)$

Testez dacă $A(1,3)$, $B(2,4)$, $C\left(\frac{1}{2},\frac{5}{2}\right)$ sunt coliniare.

$\begin{vmatrix} 1 & 3 & 1 \\ 2 & 4 & 1 \\ \frac{1}{2} & \frac{5}{2} & 1 \end{vmatrix} = 4+5+\frac{3}{2}-2-\frac{5}{2}-6 = 3-2-\frac{2}{2} = 1-1=0 \Rightarrow A, B, C$ coliniare

5. Fie $\mathcal{H}_{M,2}$ omotetia de centru $M(1,1)$ şi raport 2 şi $R_{M,\frac{\pi}{3}}$ rotaţia de centru M şi unghi orientat $\frac{\pi}{3}$.

a) Să se determine $\mathcal{H}_{M,2}(d)$, unde $d: x+y+1=0$.

b) Fie $f=\mathcal{H}_{M,2}\circ R_{M,\frac{\pi}{3}}$. Fie punctele $A(1,3)$, $B(5,6)$ şi $A'=f(A)$; $B'=f(B)$. Aflaţi $dist(A',B')$.

SOL. a) $\mathcal{H}_{M,2}: \begin{pmatrix}x'\\y'\end{pmatrix} = 2\begin{pmatrix}x\\y\end{pmatrix} - \begin{pmatrix}1\\1\end{pmatrix} \Leftrightarrow \begin{cases}x'=2x-1\\y'=2y-1\end{cases}$ $\boxed{\mathcal{H}_{M,2}}$

$$\mathcal{H}_{M,2}^{-1} = \mathcal{H}_{M,\frac{1}{2}}: \begin{cases} x=\frac{1}{2}x'+\frac{1}{2}\\ y=\frac{1}{2}y'+\frac{1}{2}\end{cases}$$

$d: x+y+1=0$, $\mathcal{H}_{M,2}(d)=d'$

Obs. că $M(1,1)\notin d \Rightarrow d \parallel d'$.

$d': \frac{1}{2}x'+\frac{1}{2}+\frac{1}{2}y'+\frac{1}{2}+1=0 \,/\cdot 2 \Rightarrow d': x'+y'+4=0$

(Se obs. că $\vec{m_d} = \vec{m_{d'}} = (1,1)$, deci $d\parallel d'$.)

b) $R_{M,\frac{\pi}{3}}: \begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}\cos\frac{\pi}{3} & -\sin\frac{\pi}{3}\\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3}\end{pmatrix}\begin{pmatrix}x-1\\y-1\end{pmatrix}+\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}x-1\\y-1\end{pmatrix}+\begin{pmatrix}1\\1\end{pmatrix}$

$\Rightarrow R_{M,\frac{\pi}{3}}: \begin{cases}x'=\frac{1}{2}(x-1)-\frac{\sqrt{3}}{2}(y-1)+\overset{2}{1}\\ y'=\frac{\sqrt{3}}{2}(x-1)+\frac{1}{2}(y-1)+\overset{2}{1}\end{cases} \Rightarrow R_{M,\frac{\pi}{3}}: \begin{cases}x'=\frac{1}{2}x-\frac{\sqrt{3}}{2}y+\frac{\sqrt{3}+1}{2}\\ y'=\frac{\sqrt{3}}{2}x+\frac{1}{2}y+\frac{1-\sqrt{3}}{2}\end{cases}$

$f=\mathcal{H}_{M,2}\circ R_{M,\frac{\pi}{3}}: (x,y) \xrightarrow{R_{M,\frac{\pi}{3}}} \left(\frac{1}{2}x-\frac{\sqrt{3}}{2}y+\frac{\sqrt{3}+1}{2}, \frac{\sqrt{3}}{2}x+\frac{1}{2}y+\frac{1-\sqrt{3}}{2}\right) \xrightarrow{\mathcal{H}_{M,2}}$

$\rightarrow \left(2\left(\frac{1}{2}x-\frac{\sqrt{3}}{2}y+\frac{\sqrt{3}+1}{2}\right)-1, 2\left(\frac{\sqrt{3}}{2}x+\frac{1}{2}y+\frac{1-\sqrt{3}}{2}\right)-1\right)\rightarrow$

$\rightarrow \left(x-\sqrt{3}y+\sqrt{3}+1-1, \sqrt{3}x+y+1-\sqrt{3}-1\right) \rightarrow \left(x-\sqrt{3}y+\sqrt{3}, \sqrt{3}x+y-\sqrt{3}\right)$

$f(A)=A' = \left(1-\sqrt{3}\cdot 3+\sqrt{3}, \sqrt{3}\cdot 1+3-\sqrt{3}\right) \Rightarrow A'=\left(1-2\sqrt{3}, 3\right)$

$[A=(1,3)]$

$f(B)=B'=\left(5-6\sqrt{3}+\sqrt{3}, 5\sqrt{3}+6-\sqrt{3}\right) \Rightarrow B'=\left(5-5\sqrt{3}, 4\sqrt{3}+6\right)$

$[B=(5,6)]$

$A' = (1-2\sqrt{3}, 3)$ și $B' = (5-5\sqrt{3}, 4\sqrt{3}+6)$

$$dist(A', B') = \sqrt{(x_{B'}-x_{A'})^2 + (y_{B'}-y_{A'})^2} = \sqrt{(5-5\sqrt{3}-1+2\sqrt{3})^2 + (4\sqrt{3}+6-3)^2} =$$

$$= \sqrt{(4-3\sqrt{3})^2 + (4\sqrt{3}+3)^2} = \sqrt{16 - \cancel{24\sqrt{3}} + 27 + 48 + \cancel{24\sqrt{3}} + 9} =$$

$$= \sqrt{25+27+48} = \sqrt{52+48} = \sqrt{100} = 10.$$

6. Fie $a_{0,2,\frac{\pi}{4}}$ asemănare directă de centru origine, raport 2 și unghi $\frac{\pi}{4}$.

a) Să se scrie ecuația asemănării directe.

b) Să se determine $d' = a_{0,2,\frac{\pi}{4}}(d)$, unde $d: x+y+2=0$.

SOL. a) $a_{0,2,\frac{\pi}{4}} = \mathcal{H}_{0,2} \circ \mathcal{R}_{0,\frac{\pi}{4}}$

$$\mathcal{R}_{0,\frac{\pi}{4}}: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \mathcal{R}_{0,\frac{\pi}{4}}: \begin{cases} x' = \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y \\ y' = \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y. \end{cases}$$

$$\mathcal{H}_{0,2}: \begin{pmatrix} x' \\ y' \end{pmatrix} = 2\begin{pmatrix} x \\ y \end{pmatrix} \Leftrightarrow \mathcal{H}_{0,2}: \begin{cases} x' = 2x \\ y' = 2y \end{cases}$$

$$a_{0,2,\frac{\pi}{4}} = \mathcal{H}_{0,2} \circ \mathcal{R}_{0,\frac{\pi}{4}}: (x,y) \xrightarrow{\mathcal{R}_{0,\frac{\pi}{4}}} \left(\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y, \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right) \xrightarrow{\mathcal{H}_{0,2}}$$

$$\rightarrow \left(2\left(\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y\right), 2\left(\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y\right)\right) \rightarrow (\sqrt{2}x - \sqrt{2}y, \sqrt{2}x + \sqrt{2}y)$$

$$\Rightarrow a_{0,2,\frac{\pi}{4}}: \begin{cases} x' = \sqrt{2}x - \sqrt{2}y \\ y' = \sqrt{2}x + \sqrt{2}y \end{cases}$$

b) $$a_{0,2,\frac{\pi}{4}}: \boxed{X = \frac{1}{K} A^T(\alpha) \cdot X} = \frac{1}{2}\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix} \Rightarrow \begin{cases} x = \frac{1}{4}(\sqrt{2}x' + \sqrt{2}y') \\ y = \frac{1}{4}(-\sqrt{2}x' + \sqrt{2}y') \end{cases}$$

$$d: x+y+2=0 \Rightarrow d': \frac{1}{4}(\sqrt{2}x' + \sqrt{2}y') + \frac{1}{4}(-\sqrt{2}x' + \sqrt{2}y') + 2 = 0 \,/\cdot 4 \Rightarrow$$

$$\Rightarrow d': 2\sqrt{2}y' + 8 = 0 \Rightarrow d': \sqrt{2}y' + 4 = 0.$$