Tutoriat 6

1) a) $\vDash v_0 \to v_0$

b) $\neg(v_0 \to v_0)$ nesatisfiabilă

Sol: a) Vrem că $e^+(v_0 \to v_0) = 1$ $(\forall)$ e o evaluare

Avem $e^+(v_0 \to v_0) = e^+(v_0) \to e^+(v_0) = e(v_0) \to e(v_0)$

Met I:

v_0	$v_0 \to v_0$
0	1
1	1

Met II: $\bullet\ e(v_0) = 1 \Rightarrow e^+(v_0 \to v_0) = 1$

$\bullet\ e(v_0) = 0 \Rightarrow e^+(v_0 \to v_0) = 1$

b) Vrem că $e^+(\neg(v_0 \to v_0)) = 0$ $(\forall)$ e o evaluare

Avem $e^+(\neg(v_0 \to v_0)) = \neg e^+(v_0 \to v_0)$.

Dar $e^+(v_0 \to v_0) = 1$ $(\forall)$ e o evaluare (din a)) $\Rightarrow$

$\Rightarrow e^+(\neg(v_0 \to v_0)) = 0$ $(\forall)$ e o evaluare

Notăm $v_0 \to v_0$ cu $\top$ (adevărul)

$\neg(v_0 \to v_0)$ cu $\bot$ (falsul)

Obs! $\vDash \varphi \iff \varphi \sim \top$

φ nesatisfiabilă $\iff \varphi \sim \bot$

Def: $(\forall)$ $\varphi, \chi, \chi' \in Form$, definim

$\varphi_\chi(\chi')$ = expresia obț. din φ prin înlocuirea lui χ cu χ'.

Obs! $\cdot\ \varphi_\chi(\chi') \in Form$

$\cdot\ \varphi_\varphi(\chi') = \chi'$

$\cdot$ dacă $\chi \notin SubForm(\varphi) \Rightarrow \varphi_\chi(\chi') = \varphi$

Prop: $(\forall)$ $\varphi, \chi, \chi' \in Form$ și $v \in V$ avem: $\bullet\ \chi \sim \chi' \Rightarrow \varphi \sim \varphi_\chi(\chi')$

$\bullet\ \varphi$ tautologie/nesatisfiabilă $\Rightarrow \varphi_v(\chi)$ taut/nesat.

Not: $((\dots(\varphi_1 \wedge \varphi_2) \wedge \varphi_3) \wedge \dots \wedge \varphi_{n-1}) \wedge \varphi_n = \varphi_1 \wedge \dots \wedge \varphi_n = \bigwedge_{i=1}^{n} \varphi_i$

$((\dots(\varphi_1 \vee \varphi_2) \vee \varphi_3) \vee \dots \vee \varphi_{n-1}) \vee \varphi_n = \varphi_1 \vee \dots \vee \varphi_n = \bigvee_{i=1}^{n} \varphi_i$

pentru niște formule φ_i, $i = \overline{1,n}$.

Prop. 3.22.

2) $(\forall)\, e: V \longrightarrow \{0,1\}$ o evaluare, avem:

a) $e^+(\varphi_1 \wedge \ldots \wedge \varphi_n) = 1 \iff e^+(\varphi_i) = 1 \quad (\forall)\, i \in \overline{1,n}$.

b) $e^+(\varphi_1 \vee \ldots \vee \varphi_n) = 1 \iff e^+(\varphi_i) = 1$ pentru un $i \in \overline{1,n}$.

Sol: a) $e^+(\varphi_1 \wedge \ldots \wedge \varphi_n) = 1$

$e^+(\varphi_1) \wedge \ldots \wedge e^+(\varphi_n) = 1$.

Pentru $n=2$ avem:

$e^+(\varphi_1) \wedge e^+(\varphi_2) = 1 \iff e^+(\varphi_1) = e^+(\varphi_2) = 1$.

$e^+(\varphi_1)$	$e^+(\varphi_2)$	$e^+(\varphi_1) \wedge e^+(\varphi_2)$
0	0	0
0	1	0
1	0	0
1	1	1

Raționăm inductiv:

P.p. $e^+(\varphi_1) \wedge \ldots \wedge e^+(\varphi_{n-1}) = 1 \iff$

$\iff e^+(\varphi_i) = 1, \ i = \overline{1,n-1}$

$e^+(\varphi_1) \wedge \ldots \wedge e^+(\varphi_n) = (e^+(\varphi_1) \wedge \ldots \wedge e^+(\varphi_{n-1})) \wedge e^+(\varphi_n)$

$= e^+(\underbrace{\varphi_1 \wedge \ldots \wedge \varphi_{n-1}}_{\overset{not}{=} \psi}) \wedge e^+(\varphi_n)$ și obținem același tabel de adevăr,

deci $e^+(\varphi_1) \wedge \ldots \wedge e^+(\varphi_n) = 1 \iff e^+(\varphi_1 \wedge \ldots \wedge \varphi_{n-1}) = 1 = e^+(\varphi_n) \iff$

$\iff e^+(\varphi_1) \wedge \ldots \wedge e^+(\varphi_{n-1}) = 1 = e^+(\varphi_n) \underset{\text{ip. de inducție}}{\iff}$

$\iff e^+(\varphi_1) = \ldots = e^+(\varphi_{n-1}) = 1 = e^+(\varphi_n). \iff e^+(\varphi_i) = 1 \quad (\forall)\, i = \overline{1,n}$

b) Temă!

Prop. 3.23

3) $(\forall)\, \varphi_i \in Form$, $i = \overline{1,n}$ avem

a) $\neg(\varphi_1 \vee \ldots \vee \varphi_n) \sim \neg\varphi_1 \wedge \ldots \wedge \neg\varphi_n$

b) $\neg(\varphi_1 \wedge \ldots \wedge \varphi_n) \sim \neg\varphi_1 \vee \ldots \vee \neg\varphi_n$

Sol: a) Vrem că $(\forall)$ e o evaluare cu $e \in Mod(\neg(\varphi_1 \vee \ldots \vee \varphi_n)) \iff$

$\iff e \in Mod(\neg\varphi_1 \wedge \ldots \wedge \neg\varphi_n)$.

Fie $e \in Mod(\neg(\varphi_1 \vee \ldots \vee \varphi_n)) \iff e^+(\neg(\varphi_1 \vee \ldots \vee \varphi_n)) = 1 \iff$

$\iff \neg e^+(\varphi_1 \vee \ldots \vee \varphi_n) = 1 \iff \neg(e^+(\varphi_1) \vee \ldots \vee e^+(\varphi_n)) = 1$

Pentru $n=2$ avem: $\neg(e^+(\varphi_1) \vee e^+(\varphi_2)) = \neg e^+(\varphi_1) \wedge \neg e^+(\varphi_2)$

$e^+(\varphi_1)$	$e^+(\varphi_2)$	$\neg e^+(\varphi_1)$	$\neg e^+(\varphi_2)$	$e^+(\varphi_1) \vee e^+(\varphi_2)$	$\neg(e^+(\varphi_1) \vee e^+(\varphi_2))$	$\neg e^+(\varphi_1) \wedge \neg e^+(\varphi_2)$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

Raționând inductiv (ca la ex. 2)) obținem că:

$\neg(e^+(\varphi_1) \vee \ldots \vee e^+(\varphi_n)) = 1 \iff \neg e^+(\varphi_1) \wedge \neg e^+(\varphi_2) \wedge \ldots \wedge \neg e^+(\varphi_n) = 1 \iff$

$\iff e^+(\neg \varphi_1) \wedge \ldots \wedge e^+(\neg \varphi_n) = 1 \iff e^+(\neg \varphi_1 \wedge \ldots \wedge \neg \varphi_n) = 1.$

b) Temă.

Def: Fie Γ o mulțime de formule.

- $e : V \longrightarrow \{0,1\}$ model pt. Γ dacă $e \vDash \gamma$ $(\forall)$ $\gamma \in \Gamma$
  Not. $e \vDash \Gamma$
- Γ satisfiabilă dacă are un model
- Γ finit satisfiabilă dacă orice submulțime finită a sa e satisfiabilă

Not.: $Mod(\Gamma) = Mod(\varphi_1, \ldots, \varphi_n) = \bigcap_{\varphi \in \Gamma} Mod(\varphi)$

Def: Fie Γ, Δ mulțimi de formule.

- $\varphi \in Form$ e consecință semantică a lui Γ dacă $Mod(\Gamma) \subseteq Mod(\varphi)$
  Not. $\Gamma \vDash \varphi$
  $Cn(\Gamma) = \{\varphi \in Form \mid \Gamma \vDash \varphi\}$
- Δ consecință semantică a lui Γ dacă $Mod(\Gamma) \subseteq Mod(\Delta)$
  Not. $\Gamma \vDash \Delta$
- Γ și Δ logic echivalente dacă $Mod(\Gamma) = Mod(\Delta)$
  Not. $\Gamma \sim \Delta$.