Seminar geometrie 5

V/K sp vect

$B_1 = \{f_1, \dots, f_n\}$
$B_2 = \{g_1, \dots, g_n\}$ baze în V/K

$A = (a_{ij})_{i,j \in \overline{1,n}}$ este matricea de trecere de la B_1 la B_2

dacă $g_1 = a_{11} f_1 + a_{21} f_2 + \dots + a_{n1} f_n$

$g_2 = a_{12} f_1 + a_{22} f_2 + \dots + a_{n2} f_n \Leftrightarrow$

$\vdots$

$g_n = a_{1n} f_1 + a_{2n} f_2 + \dots + a_{nn} f_n$

$\forall i = \overline{1,n} \quad g_i = \sum_{j=1}^{n} a_{ji} f_j$

$B_1 \xrightarrow{A} B_2 \xrightarrow{C} B_3$, $\underbrace{}_{D}$ fie $i = \overline{1,n}$ $f_i = \sum_{j=1}^{n} d_{ji} f_j$ (1)

$B_3 = \{f_1, \dots, f_n\}$ $f_i = \sum_{k=1}^{n} c_{ki} g_k = \sum_{k=1}^{n} c_{ki} \sum_{j=1}^{n} a_{jk} f_j =$

$= \sum_{j=1}^{n} \left(\sum_{k=1}^{n} c_{ki} a_{jk} \right) f_j = \sum_{j=1}^{n} \left(\sum_{k=1}^{n} a_{jk} c_{ki} \right) f_j$ (2)

Din (1) și (2) $\Rightarrow \forall j \in \overline{1,n}$ $d_{ji} = \sum_{k=1}^{n} a_{jk} c_{ki}$

$D = AC$

OBS. 1) $B_1 \xrightarrow{A} B_2 \xrightarrow{A'} B_1$, $\underbrace{}_{I_n}$

$\left.\begin{matrix} A \cdot A' = I_n \\ A' \cdot A = I_n \end{matrix}\right\} \Rightarrow A$ este inversabilă și $A^{-1} = A'$

2) $B_1 \xrightarrow{A} B_2$, $x \in V \Rightarrow (\exists)!\ x_1, \dots, x_n \in K$,

$x_1', \dots, x_n' \in K$ a.î.

$x = \sum_{i=1}^{n} x_i f_i = \sum_{j=1}^{n} x_j' g_j$

$\left.\begin{matrix} x = \sum_{j=1}^{n} x_j' \cdot g_j = \sum_{j=1}^{n} x_j' \cdot \sum_{i=1}^{n} a_{ij} f_i = \sum_{i=1}^{n} \left(\sum_{j=1}^{n} a_{ij} x_j' \right) f_i \\ x = \sum_{i=1}^{n} x_i f_i \end{matrix}\right\} \Rightarrow$

$\Rightarrow \forall i = \overline{1,n}$, $x_i = \sum_{j=1}^{n} a_{ij} x_j'$

$\mathbb{R}^3/\mathbb{R}$ $B_1 = \{f_1, f_2, f_3\}$ $f_1 = (0,1,1)$, $f_2 = (1,0,1)$, $f_3 = (1,1,0)$

$B_2 = \{g_1, g_2, g_3\}$ $g_1 = (1,1,1)$, $g_2 = (0,1,1)$, $g_3 = (0,0,1)$

a) Să se det matricea de trecere de la B_1 la B_2

b) Să se scrie vectorul $x = f_1 + 2f_2 + 3f_3$ baza B_2

$B_1 \xrightarrow{A} B_C \xrightarrow{C} B_2$, $\underbrace{}_{D = AC}$

$f_1 = e_2 + e_3 \Rightarrow e_2 = f_1 - e_3$

$f_2 = e_1 + e_3 \Rightarrow e_1 = f_2 - e_3$

$f_3 = e_1 + e_2 = f_1 + f_2 - 2e_3 \Rightarrow -2e_3 = f_3 - f_1 - f_2 \Rightarrow$

$e_3 = \frac{-f_3 + f_2 + f_1}{2}$

$e_1 = f_2 - \frac{f_3 - f_2 + f_1}{2} = \frac{f_2 + f_3 - f_1}{2}$

$e_2 = \frac{f_1 + f_3 + (-f_2)}{2}$

$\left.\begin{matrix} e_1 = -\frac{1}{2} f_1 + \frac{1}{2} f_2 + \frac{1}{2} f_3 \\ e_2 = \frac{1}{2} f_1 - \frac{1}{2} f_2 + \frac{1}{2} f_3 \\ e_3 = \frac{1}{2} f_1 + \frac{1}{2} f_2 - \frac{1}{2} f_3 \end{matrix}\right\} \Rightarrow A = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

b) $g_1 = e_1 + e_2 + e_3$

$g_2 = e_2 + e_3$

$g_3 = e_3$

$C = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow D = AC$

$D = \frac{1}{2} \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{pmatrix}$

$B_1 \xrightarrow{D} B_2$

$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = D \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$\frac{1}{2} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \Big/ 2$

$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \Leftrightarrow$

$\begin{cases} a + 2b + c = 2 \\ a + c = 4 \\ a - c = 6 \quad (+) \end{cases}$ $\Rightarrow 5 + c = 4 \Rightarrow c = -1$

$2a = 10$

$a = 5$ $\quad 5 + 2b - 1 = 2 \Rightarrow b = -1$

$= 5g_1 - g_2 - g_3$

Dacă $n \in \mathbb{N}^*$, K corp $S \leq_K K^n$, atunci $\exists\, m \in \mathbb{N}^*$, $A \in \mathcal{M}_{m,n}/K$ a.î. $S = \{x \in K^n / A \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} \text{u.o.i}\}$

$\dim_K S = p \quad p < n$

$\exists \{f_1, \dots, f_p\}$ bază în $S/K \Rightarrow S/K \Rightarrow \{f_1, \dots, f_p\}$ s.l.i. $\Rightarrow \exists f_{p+1}, \dots, f_n \in \mathbb{R}^n$ a.î. $\{f_1, \dots, f_p, f_{p+1}, \dots, f_n\}$ bază în K^n/K

$B_C = \{e_1, \dots, e_n\}$ baza canonică a lui K^n/K

$B_C \xrightarrow{C} B_1$ fie $x = (x_1, \dots, x_n) \in K^n$

$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = C \begin{pmatrix} x_1' \\ \vdots \\ x_n' \end{pmatrix}$ $\quad x = \sum_{i=1}^{n} x_i' f_i = x_1' f_1 + \dots + x_p' f_p + x_{p+1}' f_{p+1} + \dots + x_n' f_n$

$B_1 \xrightarrow{D} B_C$

$\begin{pmatrix} x_1' \\ \vdots \\ x_n' \end{pmatrix} = D \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$

$x \in S \Leftrightarrow \begin{cases} x_{p+1}' = 0 \\ \vdots \\ x_n' = 0 \end{cases} \Rightarrow \forall i \in \overline{p+1,n}$,

$\sum_{j=1}^{n} d_{ij} x_j = 0 \Leftrightarrow d_{p+1,1} x_1 + \dots + d_{p+1,n} x_n = 0$

$\vdots$

$d_{n1} x_1 + \dots + d_{nn} = 0$

Def Fie V/K, W/K sp vect și $f: V \to W$. Spunem că f este aplicație liniară (morfism) de sp vectoriale dacă:

1) $\forall x, y \in V$ $f(x+y) = f(x) + f(y)$

2) $\forall \alpha \in K$ $\forall x \in V$ $f(\alpha x) = \alpha f(x)$

EX: 1) $f: \mathbb{R}^2 \to \mathbb{R}^3$ $f(x) = (x_1, x_2, 0)$

2) $f: \mathbb{R}^2 \to \mathbb{R}^3$ $f(x) = (x_1 + x_2, 2x_1 - x_2, 3x_2)$

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