Soluţia problemei Cauchy

$$x' = t^2 + x^2, \quad x(0) = 1$$

este:

a. Alt răspuns

b. $x(t) = \pm e^{t^2-1}\sqrt{t^2+1}$

c. $x(t) = \pm\sqrt{t^2+1}$

d. $x(t) = \pm e^{t^2-1}$

e. $x(t) = \pm\frac{e^{t^2-1}}{\sqrt{t^2+1}}$

Se consideră sistemul

$$\begin{cases} x' = 3t^2x + y \\ y' = 3t^2y + x \end{cases}$$

Ştiind că acest sistem admite ca integrală primă funcţia definită prin $F(t,(x,y)) = (x-y)e^{t-t^3}$, soluţia generală a acestui sistem este:

a. $x(t) = c_1e^{t^3} + c_2e^t, \quad y(t) = -c_1e^{t^3} + c_2e^t, \quad c_1, c_2 \in \mathbf{R}$

b. $x(t) = c_1e^{3t} + c_2e^t, \quad y(t) = -c_1e^{3t} + c_2e^t, \quad c_1, c_2 \in \mathbf{R}$

c. $x(t) = c_1e^{t-t^3} + c_2e^{t+t^3}, \quad y(t) = -c_1e^{t-t^3} + c_2e^{t+t^3}, \quad c_1, c_2 \in \mathbf{R}$

d. $x(t) = c_1e^{t^3-t} + c_2e^{t^3+t}, \quad y(t) = -c_1e^{t^3-t} + c_2e^{t^3+t}, \quad c_1, c_2 \in \mathbf{R}$

e. Alt răspuns

Fie $\varphi(.,\lambda) : I(\lambda) \subset \mathbf{R} \to \mathbf{R}$, $\lambda \in \mathbf{R}$, soluţia maximală a problemei Cauchy

$$x' = x^2 + \lambda tx - \lambda, \quad x(0) = 0.$$

Atunci $D_2\varphi(t,0) = \frac{\partial\varphi}{\partial\lambda}(t,0)$ este:

a. Alt răspuns

b. $e^t - 2t + 5$

c. e^{t^2+5}

d. $\frac{1}{t^2}$

e. $-t$

Soluţia parametrizată a problemei la limită

$$q(y-p) + xy - z = 0, \quad x = 1, z = y$$

este:

a. $x(t,\sigma) = e^{-t},\ y(t,\sigma) = \sigma,\ z(t,\sigma) = \sigma e^{-t}$

b. $x(t,\sigma) = \sigma e^{4t}\sin t,\ y(t,\sigma) = \sigma e^{4t}\cos t,\ z(t,\sigma) = \sigma^2e^{8t}$

c. $x(t,\sigma) = e^{2t},\ y(t,\sigma) = \sigma,\ z(t,\sigma) = \sigma e^{-2t}$

d. Alt răspuns

e. $x(t,\sigma) = e^{-t},\ y(t,\sigma) = \sigma e^{-t},\ z(t,\sigma) = \sigma e^{-2t}$

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5. $q(y-p)+xy-z=0 \quad x=1 \quad z=y$

$p=\frac{\partial z}{\partial x}(x,y)$

$q=\frac{\partial z}{\partial y}(x,y)$

$F((x,y),z,(p,q))=qy-qp+xy-z=0$

$$\begin{cases} x=1 \\ y=\sigma \\ z=\sigma \end{cases} \quad \begin{array}{l} \alpha(\sigma)=\begin{pmatrix}1\\ \sigma\end{pmatrix} \\ \beta(\sigma)=\sigma \end{array}$$

$$\left.\begin{cases} q(\sigma-p)+1\cdot\sigma-\sigma=0 \Rightarrow q(\sigma-p)=0 \\ (p,q)\cdot\begin{pmatrix}0\\1\end{pmatrix}=1 \Rightarrow q=1 \end{cases}\right\} \Rightarrow \sigma-p=0 \Rightarrow \Rightarrow p=\sigma$$

$$\delta(\sigma)=\begin{pmatrix}p\\q\end{pmatrix}=\begin{pmatrix}\sigma\\1\end{pmatrix}$$

$$\begin{cases} x'=-q & x(0)=1 \\ y'=y-p & y(0)=\sigma \\ z'=p\cdot x'+q\cdot y'=p\cdot(-q)+qy-qp & z(0)=\sigma \\ p'=-y+p=p-y & p(0)=\sigma \\ q'=-q-x+q=-x & q(0)=1 \end{cases}$$

$$\begin{cases} y' = y - p \\ p' = p - y \end{cases} \Rightarrow y = p - p'$$

$$p' - p'' = p - p' - p$$

$$\left.\begin{array}{l} p'' - 2p' = 0 \Rightarrow p(t) = c_1 e^{2t} + c_2 \\ p(0) = \sigma \end{array}\right\} \Rightarrow c_1 + c_2 = \sigma$$

$$\left.\begin{array}{l} y = p - p' = c_1 e^{2t} + c_2 - 2c_1 e^{2t} \\ y(0) = \sigma \end{array}\right\} \Rightarrow -c_1 + c_2 = \sigma$$

$$\left.\begin{array}{l} c_1 + c_2 = \sigma \\ -c_1 + c_2 = \sigma \end{array}\right\} \Rightarrow c_2 = \sigma, \quad \Rightarrow c_1 = 0$$

$$y(t) = \sigma$$

$$p(t) = \sigma$$

$$\left.\begin{cases} x' = -q \\ q' = -x \Rightarrow -x' = q'' \end{cases}\right\} \Rightarrow q'' = q \Rightarrow q(t) = c_1 e^{t} + c_2 e^{-t}$$

$$\left.\begin{array}{l} q(0) = 1 \Rightarrow c_1 + c_2 = 1 \\ x(t) = -q' = -c_1 e^{t} + c_2 e^{-t} \\ x(0) = 1 \Rightarrow -c_1 + c_2 = 1 \end{array}\right\} \overset{+}{\Rightarrow} c_2 = 1 \Rightarrow c_1 = 0$$

$$\Rightarrow \begin{cases} q(t) = e^{-t} \\ x(t) = e^{-t} \end{cases}$$

$$\left.\begin{array}{l} z' = q(y - p) - qp = e^{-t}(\sigma - \sigma) - e^{-t} \cdot \sigma = -\sigma e^{-t} \\ z(0) = \sigma \end{array}\right\} \Rightarrow z(t) = \sigma e^{-t}$$

$$q(y - p) + xy - z = 0$$

$$e^{-t}(\underbrace{\sigma - \sigma}_{0}) + \cancel{e^{-t} \cdot \sigma} - \sigma \cancel{e^{-t}} = 0$$

$$\begin{cases} x' = 3t^2 x + y \\ y' = 3t^2 y + x \end{cases}$$

$$F(t, (x, y)) = (x-y)e^t - t^3$$

$$x - y$$

$$yp - zp + xq - zq - 2z = 0$$

$$-p - q - 2$$

$$-p^2 - pq - 2p$$

$$-qp + q^2 + 2q$$

$$p' = -q + p^2 + pq + 2p$$

$$q' = -p + q^2 + qp + 2q$$

$$x' = x^2 + \lambda t x - \lambda$$

$$x x' - (x')^2 = 0$$

$$x(0) = 1 \qquad x'(0) = 1$$