1. $A^3(\mathbb{R})$ $P_0=(1,2,1)$, $P_1=(2,3,2)$, $P_2=(1,0,1)$ $P_3=(1,1,3)$
Să se arate că $R=\{P_0, P_1, P_2, P_3\}$ reper afin și să se determine coordonatele lui $M=(2,3,3)$ în raport cu R.

R - reper afin $\Leftrightarrow B=\{\overrightarrow{P_0P_1}, \overrightarrow{P_0P_2}, \overrightarrow{P_0P_3}\}$ bază în $\mathbb{R}^3$

$B=\{(1,1,1), (0,-2,0), (0,-1,2)\}$

$A=\begin{pmatrix} 1 & 0 & 0 \\ 1 & -2 & -1 \\ 1 & 0 & 2 \end{pmatrix}$ $\det A=-4\neq 0 \Rightarrow B$ bază $\Rightarrow R$ reper afin

$\overrightarrow{P_0M}=(1,1,2)$

$\overrightarrow{P_0M}=\cancel{A}\cdot x_1\cdot\overrightarrow{P_0P_1}+x_2\cdot\overrightarrow{P_0P_2}+x_3\cdot\overrightarrow{P_0P_3}=x_1\cdot(1,1,1)+x_2(0,-2,0)+x_3(0,-1,2)$

$\begin{cases} x_1=1 \\ x_1-2x_2-x_3=1 \\ x_1+2x_3=2 \end{cases}$ $x_3=1/2$

$\cancel{1}-2x_2-\frac{1}{2}=\cancel{1} \Rightarrow -2x_2=\frac{1}{2} \Rightarrow x_2=-\frac{1}{4}$

$\overrightarrow{P_0P_1}-\frac{1}{4}\overrightarrow{P_0P_2}+\frac{1}{2}\overrightarrow{P_0P_3}$

$M=y_0P_0+y_1P_1+y_2P_2+y_3P_3$

$\overrightarrow{P_0M}=y_1\overrightarrow{P_0P_1}+y_2\overrightarrow{P_0P_2}+y_3\overrightarrow{P_0P_3}$

$y_1=1$

$y_2=-1/4$ $y_0=1-1+1/4-1/2=-1/4$

$y_3=1/2$

$M=-1/4P_0+P_1-1/4P_2+1/2P_3$.

3. $\mathcal{A} = A^n(\mathbb{F}_p)$

a) Câte puncte are un spațiu afin de dimensiune k?

b) Câte subpuncte afine de dimensiune k are $\mathcal{A}$?

$A, B \in \mathcal{A}$ $\quad d = AB$

$d = AB \quad d = \{tA + (1-t)B \mid t \in \mathbb{F}_p\} \qquad P \in d \Leftrightarrow \vec{PA} = \lambda \cdot \vec{PB}$

$f : \mathbb{F}_p \to d \quad f(t) = tA + (1-t)B$

f surjectivă - evident

f injectivă pt. că $t_1 \neq t_2 \Rightarrow t_1 A + (1-t_1)B \neq t_2 A + (1-t_2)B$

f bijecție $\Rightarrow |d| = p$

$k = 1 \rightsquigarrow p$

$k = 2 \qquad \Pi = (ABC)$ $\;A, B, C$ necoliniare $\;\Pi = \{\alpha A + \beta B + (1-\alpha-\beta)C \mid \alpha, \beta \in \mathbb{F}_p\}$

$f : \mathbb{F}_p^2 \to \Pi \quad f(\alpha, \beta) = \alpha A + \beta B + (1-\alpha-\beta)C$ la fel bijectivă $\Rightarrow |\Pi| = p^2$

b) $k = 1 \to V = \{\alpha \cdot v \mid \alpha \in \mathbb{F}_p\}$ $\; v \neq 0$

$|\{v \in \mathbb{F}_p \mid v \neq 0\}| = p - 1$

$N = \dfrac{p^n - 1}{p - 1} = p^{n-1} + p - 1$

$k = 2 \quad V = \langle \{v_1, v_2\} \rangle \quad \{v_1, v_2\}$ liniar independenți

$M = \{(v_1, v_2) \mid v_1, v_2 \in \mathbb{F}_p^n, v_1, v_2 \text{ liniar indep}\}$

$|M| = \underset{v \neq 0}{(p^n - 1)}(p^n - 1 - p + 1) = (p^n - 1)(p^n - p)$

$N_{n,2} = \dfrac{(p^n-1)(p^n-p)}{p^2-1} = \dfrac{p(p^n-1)(p^{n-1}-1)}{(p-1)(p+1)}$

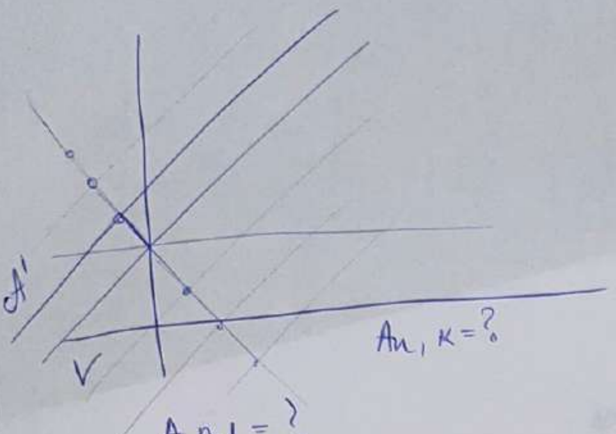
$k \quad \underset{[^n_k]_p}{N_{n,k}} = \dfrac{(p^n-1)(p^n-p)(p^n-p^2)\cdots(p^n-p^k)}{(p-1)(p^2-p)\cdots(p^k-p^{k-1})}$

Nr de subspații afine

$A_{n,k} = ?$

$\mathcal{A}' \subset A^n(\mathbb{F}_p)$ subsp. $\mathcal{A}' = \{a + v \mid v \in V'\}$

$V' \subset \mathbb{F}_p^n$ subsp. vect. de dim k



$A_{n,1} = ?$

$\cancel{A_{n,1} = (p-1)\, N_{n,1}}$ $A_{n,1} = (p^n - p - p + 1) = (p^n - 2p + 1)\, N_{n,1}$

$\cancel{A_{n,2} =}$

$\{a + v \mid v \in V\} = \{a' + v \mid v \in V\}$

$A_{n,2} = (p^n - p^2 - p^2 + 1)\, N_{n,2}$

$$\boxed{A_{n,k} = (p^n - 2p^{k^2} + 1)\, N_{n,k}.}$$

2. (A, V, φ) sp. afin $M \subset A$ submulțime

$$Af(M) = \bigcap_{\substack{A' \supset M \\ A' \text{ subsp.}}} A'$$

$Af(M) =$ dc. $M \subset A'$, A' subsp. afin $\Rightarrow Af(M) \subset A'$

$$Af(M) \subset \bigcap_{M \in A'} A'$$

$$P \in \bigcap_{M \in A'} A'$$

$P \notin Af(M)$

$M \cup \{P\}$ afin indep.

$Af(M) \cup \{P\} \neq Af(M)$

$M \subset Af(M \cup \{P\})$

$Af(M) \subset \bigcap_{M \in A'} A'$ subsp. $\Rightarrow$ Unul dintre $A' - Af(M) \Rightarrow P \in Af(M)$

$\Rightarrow P \in Af(M)$

$$\bigcap_{M \in A'} A' = Af(M)$$

(A, V, φ), $A', A'' \subset A$

$Dir(A') = \{\overrightarrow{OP} \mid P \in A'\}$ pt. un $O \in A'$

$Dir(A'') = \{\overrightarrow{O_1P} \mid P \in A''\}$ pt. un $O_1 \in A''$

$A' \parallel A'' \Leftrightarrow Dir(A') \subset Dir(A'')$ sau $Dir(A'') \subset Dir(A')$

4) Dacă $\mathcal{A}' \parallel \mathcal{A}''$ $\quad \mathcal{A}' \cap \mathcal{A}'' \neq \emptyset \Rightarrow \mathcal{A}' \subset \mathcal{A}''$ sau $\mathcal{A}'' \subset \mathcal{A}'$ $\quad O \in \mathcal{A}' \cap \mathcal{A}''$

$Dir(\mathcal{A}') \{\overrightarrow{OP} \mid P \in \mathcal{A}'\}$

$Dir(\mathcal{A}'') - \{\overrightarrow{OQ} \mid Q \in \mathcal{A}''\}$

$Dir(\mathcal{A}') \subset Dir(\mathcal{A}'') \Rightarrow \overrightarrow{OP} \in Dir(\mathcal{A}'') \ \forall P \in \mathcal{A}'$

$\overrightarrow{OP} = \overrightarrow{OQ}$ cu $Q \in \mathcal{A}' \Rightarrow P = Q \in \mathcal{A}''$

Deci $A' \subset A''$

5) (A, V, φ) spațiu afin $\mathcal{A}' \subset \mathcal{A}$ subsp.

$\mathcal{A}' \parallel \mathcal{H} \Leftrightarrow \mathcal{A}' \subset \mathcal{H}$ sau $\mathcal{A}' \cap \mathcal{H} = \emptyset$

$\dim \mathcal{H} = \dim \mathcal{A} - 1$

"$\Rightarrow$" $\mathcal{A}' \parallel \mathcal{H}$ dacă $\mathcal{A}' \cap \mathcal{H} \neq \emptyset \Rightarrow \mathcal{A}' \subset \mathcal{H}$

sau $\mathcal{A}' \cap \mathcal{H} = \emptyset$

"$\Leftarrow$" $\mathcal{A}' \subset \mathcal{H} \Rightarrow Dir(\mathcal{A}') \subset Dir(\mathcal{H}) \Rightarrow \mathcal{A}' \parallel \mathcal{H}$

Pt. că $\mathcal{A}' \cap \mathcal{H} = \emptyset \Rightarrow Dir(\mathcal{A}') \subset Dir(\mathcal{H})$

$\dim Dir(\mathcal{H}) = \dim(\mathcal{A})^{-n}$

$Dir(\mathcal{H}) = \{\overrightarrow{OP} \mid P \in \mathcal{H}\} - \{\overrightarrow{OP_1}, \dots \overrightarrow{OP_{n-1}}\}$

$Q \in \mathcal{A}' \setminus \mathcal{H}$

$\overrightarrow{OQ} \notin Dir(\mathcal{H})$

$\{\overrightarrow{OP_1}, \dots, \overrightarrow{OP_{n-1}}, \overrightarrow{OQ}\}$ bază în V

$Dir(\mathcal{A}') = \{\overrightarrow{QT} \mid T \in \mathcal{A}'\} = \{\overrightarrow{QO} + \overrightarrow{OT} \mid T \in \mathcal{A}'\} \{\overrightarrow{QO} + a_1 \overrightarrow{OP_1} + \dots + a_n \overrightarrow{OP_n} + b\overrightarrow{Q_0}\}$

$\overrightarrow{QT} = (1+b)\overrightarrow{QO} + a_1\overrightarrow{OP_1} + \dots + a_n\overrightarrow{OP_{n1}} - \alpha\overrightarrow{OQ} + v, \ v \in Di(\mathcal{H})$

~~din~~ P. că $Dir(\mathcal{A}') \not\subset Dir(\mathcal{H})$ $\quad \exists \overrightarrow{QT} \in Dir(\mathcal{A}')$

Unul dintre $\mathcal{A}' = Af(M)$ $\quad QT \notin Dir(\mathcal{H})$

$P \in Af(M)$ $\quad \Rightarrow \overrightarrow{QT}$

$T = \alpha Q + (1-\alpha) Q$

$P \in A'$, $V = \{\overrightarrow{PQ} \mid Q \in A\}$, $O \in \mathcal{H} \Rightarrow Dir(\mathcal{H}) - \{\overrightarrow{OT} \mid T \in \mathcal{H}\}$

$\dim di(\mathcal{H}) = \dim V - 1$

$\exists T' \in \mathcal{H}$ a.î. $V = din(\mathcal{H}) \oplus \langle \overrightarrow{OT} \rangle$

$Dir(A') = \{PP' \mid P' \in A'\} \Rightarrow \{\overrightarrow{PO} + \overrightarrow{OP'} \mid P' \in A'\}$

$\dim(di(\mathcal{H}) \cap di(\mathcal{A})) = (n-1) + \dim(\mathcal{A}') - n = \dim(\mathcal{A}') - 1$

Dacă $di(\mathcal{H}) \not\subset din(A) \Rightarrow di(\mathcal{H}) + di(\mathcal{A}') = V$

$n = (n-1) + \dim(\mathcal{A}') - \dim(H \cap A)$!?

7. A_0, A_1 $k \in K$

$A_k = \{(1-k) A_0 + kA_1 \mid A_1 \in A_1\}$

A_k ca subsp., dim $A_k = ?$

$P, Q \in A_k \Rightarrow tP + (1-t) Q \in A_k$, $\forall t$

$P = (1-k) A_0 + k A_1$

$Q = (1-k) B_0 + k B_1$

$tP + (1-t) Q = t(1-k) A_0 + tk A_1 + (1-t)(1-k) B_0 + (1-t) k B_1$

$= (1-k)(t A_0 + (1-t) B_0) + k \cdot (t A_1 + (1-t) B_1) \in A_k.$

$\uparrow_{A_0}$ $\uparrow_{A_1}$

Dim $(A_k) = ?$

$O_0 \in A_0$, $O_1 \in A_1$, $O = (1-k) O_0 + k O_1$

Dim$(A_k) = \{\overrightarrow{OA} \mid A \in A_k\} = \{(1-k)\overrightarrow{OA_0} + k\overrightarrow{OA_1}, A \in A_1\}$

$(1-k)\overrightarrow{OA_0} + k\overrightarrow{OA_1} = (1-k)^2 \overrightarrow{O_0A_0} + k(1-k)\overrightarrow{O_1A_0} + k(1-k)\overrightarrow{O_0A_1} +$

$+ k^2 \overrightarrow{O_1A_1}$

$k(1-k)(\overrightarrow{O_1A_0} + \overrightarrow{O_0A_1}) = k(1-k)(\overrightarrow{O_1A_1} + \overrightarrow{A_1A_0} + \overrightarrow{O_0A_0} + \overrightarrow{A_0A_1})$

$= k(1-k)(\overrightarrow{O_1A_1} + \overrightarrow{O_0A_0}) \Rightarrow (1-k)\overrightarrow{O_0A_0} + k\overrightarrow{O_1A_1}$

Dim$(A_k) =$ Dim$(A_0) +$ Dim(A_1) $\Rightarrow$ dim $A_k =$ dim $A_0 +$ dim $A_1 -$
$-$ dim(dir$(A_0) \cap$ dir(A_1))

8. ABCD tetraedru. Să se arate că centrul său de greutate este mijlocul segmentelor ce unesc mijlocul laturilor și. A' este centrul de greutate al $\triangle BCD$, $G \in AA'$ și se află la 3/4 de A.

$\overrightarrow{AG} = 3/4\, \overrightarrow{AA'}$

$M_1 = 1/2\, A + 1/2\, B$

$M_2 = 1/2\, B + 1/2\, C$

segm opuse

AB, CD
AC, BD
AD, BC

mijl segm $M_1M_2 = 1/2 M_1 + 1/2 M_2 =$
$= 1/4 A + 1/4 B + 1/4 C + 1/4 D$
$= G.$

$M_3 = 1/2 A + 1/2 C$

$M_4 = 1/2 B + 1/2 D$

$A = 1/3 B + 1/3 D + 1/3 C$

$AA' = t A + \frac{(1-t)}{3} B + \frac{(1-t)}{3} C + (\frac{1-t}{3}) D.$

$t = 1/4 \Rightarrow G \in AA'$

$\vec{AG} = 1/4 \vec{AB} + 1/4 \vec{AC} + 1/4 \vec{AD}$

$\vec{AA'} = 1/3 \vec{AB} + 1/3 \vec{AC} + 1/3 \vec{AD} \Rightarrow \vec{AG} = 3/4 \vec{AA'}$

9. ABCD tetraedru P, Q, R, S a.î. $\vec{AP} = k\vec{AB}$,
$\vec{AQ} = k\vec{AD}$,
$\vec{CR} = k \vec{CB}$
$\vec{CS} = k \cdot \vec{CD}$

$I =$ mijl (AC)
$J =$ mijl (BD)

PS, QR, IJ concurente

$P = k \cdot B + (1-k) A$
$Q = k \cdot D + (1-k) A$
$R = k \cdot B + (1-k) \cdot C$
$S = k \cdot D + (1-k) C$

$PS: t k B + t(1-k) A + k \cdot D + t(1-k) C$

$IJ: t/2 A + t/2 C + (1-t)/2$

$\begin{cases} t(1-k) = \frac{s}{2} \\ tk = 1 - \frac{s}{2} \end{cases}$

$tk = k(1-t) \Rightarrow t = 1-t \Rightarrow t = 1/2$

$s = 2t(1-k) \Rightarrow \frac{1-k}{2}$

$(1-t)(1-k) = \frac{s}{2}$

$k(1-t) = \frac{1-s}{2}.$

$H = \frac{1-k}{2} A + \frac{k}{2} B + \frac{1-k}{2} C + \frac{k}{2} D$

GEOMETRIE II 07.03.2022

SEMINAR 4

1. Nr. de subsp. afini de dimensiune k în $A^k(\mathbb{F}_p)$

$\begin{bmatrix} n \\ k \end{bmatrix}_p$ = nr. de subspații în $(\mathbb{F}_p)^2$

$$\begin{bmatrix} n \\ k \end{bmatrix}_p = \frac{(p^n-1)\cdots(p^n-p^{k-1})}{(p^k-1)\cdots(p^k-p^{k-1})}$$

$\mathcal{A}_0 = x_0 + A$

$\mathcal{A}_1 = x_1 + A$

$\mathcal{A}_0 \cap \mathcal{A}_1 - ?$

$\mathcal{A}$ este subsp. afin în $A^n(\mathbb{F}_p)$ $\mathcal{A} = \{x_0 + v \mid v \in A\}$ $A \subset A^n(\mathbb{F}_p)$

$\dim A = \dim \mathcal{A} = k$

$|\mathcal{A}| = |A| = p^k$

$\mathcal{A}_0 \cap \mathcal{A}_1 \neq \emptyset$ $x \in \mathcal{A}_0 \cap \mathcal{A}_1$

$x = x_0 + v_0 = x_1 + v_1$ $v_0, v_1 \in A$

$x_0 + v_0 = x_1 + v_1 \to x_0 = \underset{\mathcal{A}_1}{\underset{\cap}{x_1}} + \underset{A}{\underset{\cap}{(v_1 - v_0)}} \to x_0 \in \mathcal{A}_1$ $\Big| \Rightarrow \mathcal{A}_0 = \mathcal{A}_1$

La fel, $x_1 \in \mathcal{A}_0$

$\mathcal{A}_0 \cap \mathcal{A}_1 = \emptyset$ sau $\mathcal{A}_0 = \mathcal{A}_1$ $A^n(\mathbb{F}_p) = \bigcup_{j=1}^{l} A_j$ $A_i \cap A_j = \emptyset$

$|A^n(\mathbb{F}_p)| = p^n$

$|A_j| = p^k$

$l = p^{n-k}$

nr. de subsp. afini de dim $k = \begin{bmatrix} n \\ k \end{bmatrix}_p p^{n-k}$

Ex: $p=2$ $n=2$

$2\begin{bmatrix} 2 \\ 1 \end{bmatrix}_2 = 2 \cdot \frac{2^2-1}{2-1} = 6 = 4+2$ drepte $A^2(\mathbb{Z}/2)$

$p=3$, $n=2$ $3\begin{bmatrix} 2 \\ 1 \end{bmatrix}_3 = \frac{3^2-1}{3-1} \cdot 3 = 12$ $(9+3)$ drepte în $A^2(\mathbb{Z}_3)$

În general, în $A^2(\mathbb{Z}/p)$ avem:

$p \frac{p^2-1}{p-1} = p(p-1)$ drepte.

3. $\mathcal{A} = \mathbb{R}^4$

$A = (1, 0, 1, 2) \quad B = (0, 1, 2, 3) \quad C = (0, 0, 1, -1)$

2. $\mathcal{H}$ - hiperplan

$\mathcal{A}$ subspațiu

$\mathcal{A} \cap \mathcal{H} = \emptyset \Rightarrow \mathcal{A} \parallel \mathcal{H}$

(*) $\mathcal{A}_0 \cap \mathcal{B}_0 \neq \emptyset \Leftrightarrow \overrightarrow{AB} \in A_0 + B_0$, unde $A_0 = \operatorname{dir}(\mathcal{A}_0)$ $\quad A_0 \subset \mathcal{A}_0$
$B_0 = \operatorname{dir}(\mathcal{B}_0) \quad B_0 \subset \mathcal{B}_0$

DEMONSTRAM

$\mathcal{A}_0 \cap \mathcal{B}_0 = \emptyset \Leftrightarrow \overrightarrow{AB} \notin A_0 + B_0, \ \forall A \in \mathcal{A}, \ \forall B \in \mathcal{B}$.

Presup. $\mathcal{A} \nparallel \mathcal{H} \Leftrightarrow \exists\, v \in A$ a.î. $v \notin H$. $\Rightarrow A + H = V$ întregul sp. vectorial

$\forall P \in \mathcal{A}, Q \in \mathcal{H} \quad \overrightarrow{PQ} \in A + H \Rightarrow \mathcal{A} \cap \mathcal{H} \neq \emptyset$ ⚡ Deci $\mathcal{A} \parallel \mathcal{H}$.

(*) $\mathcal{A} \cap \mathcal{B} \neq \emptyset$

$\overrightarrow{PQ} \in A + B$.

$O \in \mathcal{A} \cap \mathcal{B}$.

$\overrightarrow{PQ} = \underset{A}{\overrightarrow{PO}} + \underset{B}{\overrightarrow{OQ}} \in A + B$

$A = \{ \overrightarrow{OP_1} \mid P_1 \in \mathcal{A} \}$

$B = \{ \overrightarrow{OP_2} \mid P_2 \in \mathcal{B} \}$

"$\Leftarrow$" $\overrightarrow{PQ} \in A + B$

$\overrightarrow{PQ} = u - v$, $u \in A$, $v \in B$.

$u = \overrightarrow{PM} \quad M \in \mathcal{A}$

~~$\overrightarrow{PM} = \overrightarrow{PQ}$~~ $\quad \underbrace{\overrightarrow{PM}}_{= u} + \overrightarrow{MQ} = \overrightarrow{PQ} \Rightarrow \underbrace{\overrightarrow{QM}}_{= -v} \in B$

~~$\overrightarrow{MB}$~~

$M - Q = v \in B$. Deci $M \in \mathcal{A} \cap \mathcal{B}$.

3. $\mathcal{A} = \mathbb{R}^4$

$A = (1, 0, 1, 2)$ $\quad B = (0, 1, 2, 3)$ $\quad C = (0, 0, 1, -1)$

$\mathcal{A}' = \langle \{A, B, C\} \rangle$

Det. un sistem de ecuații pt. $\mathcal{A}'$ și dim $\mathcal{A}'$.

$P \in \langle \{A, B, C\} \rangle \in$

$AP \in \langle \{\overrightarrow{AB}, \overrightarrow{AC}\} \rangle = V$.

$P = (x, y, z, t)$ $\quad \overrightarrow{AB} = (-1, 1, 1, 1)$ $\quad \overrightarrow{AC} = (-1, 0, 0, -3)$.

$(x_1, x_2, x_3, x_4) \in V \Leftrightarrow \begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = 0. \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = 0. \end{cases}$

$(a_{11}, a_{12}, a_{13}, a_{14}), (a_{21}, a_{22}, a_{23}, a_{24})$. bază în $V^{\perp}$

$\begin{cases} -a+b+c+d=0. \\ -a-3d=0. \end{cases} \Rightarrow \begin{cases} a=-3d \\ b+c=-4d \end{cases} \Leftrightarrow \begin{cases} a=-3d \\ b=-c-4d \end{cases} \Leftrightarrow \{(-3d, -c-4d, c, d) \mid c, d \in \mathbb{R}\}$

$= \{c(0,-1,1,0) + d(-3,-4,0,1) \mid c, d \in \mathbb{R}\}$

$\underbrace{\begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix}}_{\text{Minor princ.}} \begin{matrix} 1 & 1 \\ 0 & -3 \end{matrix}$

$V: \begin{cases} -x_2 + x_3 = 0. \\ -3x_1 + 4x_2 + x_4 = 0 \end{cases}$

$(x-1, y, z-1, t-2) \in V$.

$\begin{cases} -y + z - 1 = 0 \\ -3(x-1) - 4y + t - 2 = 0 \end{cases} \Leftrightarrow$

$\mathcal{A}: \begin{cases} -y + z = 1 \\ -3x - 4y + t = -1 \end{cases}$

4. $\mathcal{A}: \begin{cases} z_1 - i z_2 = 0 \\ 2z_1 + z_3 + 1 = 0. \end{cases}$

$d \subset A^3(\mathbb{C})$

Ec. parametrice ale lui d. $\operatorname{Dir}(d) = ?$

$\operatorname{Dir}(d): \begin{cases} z_1 - i z_2 = 0 \\ 2z_1 + z_3 = 0. \end{cases}$ $A = \begin{pmatrix} 1 & -i & 0 \\ 2 & 0 & 1 \end{pmatrix}$

$\begin{cases} z_1 - i z_2 = 0. \\ z_1 = -\frac{1}{2} z_3 \end{cases}$ $\begin{cases} z_1 = \underline{-1/2}\, z_3 \\ z_2 = \underline{\underline{1/2}}\, z_3 \end{cases}$

$z_2 = \frac{1}{i} z_1 = -i z_1.$

Bază în $\operatorname{Dir}(d) = \{ (\underline{-1/2}, \underline{\underline{1/2}}, 1) \}$.

$(0, 0, -1) \in d$

$d: \begin{cases} z_1 = -t/2 \\ z_2 = \frac{it}{2} \\ z_3 = -1 + t \end{cases}$ $t \in \mathbb{C}.$

5. $d_1: \frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{0} = \frac{w}{2}$

$d_2: \frac{x}{1} = \frac{y}{1} = \frac{z-3}{0} = \frac{w-1}{2}$

$d_3: \frac{x}{1} = \frac{y}{1} = \frac{z-3}{1} = \frac{w-1}{1}$

$d_1 \vee d_2 = ?$ $d_1 \vee d_3 = ?$

$d_1 \parallel d_2 = \dim (d_1 \vee d_2) = 2.$

$d_1: \begin{cases} x - y = 0 \\ z = 2 \\ 2x - w = 2 \end{cases}$ $d_2: \begin{cases} x - y = 0. \\ z = 3 \\ 2x - w = 1 \end{cases}$

$d_1 \vee d_2 \subset H_1$ $x - y = 0.$

$A \in d_1$ $A = (1, 1, 2, 0)$

$B \in d_2$ $B = (0, 0, 3, 1)$

$C \in d_2$ $C = (1, 1, 3, 3)$.

$\Leftrightarrow \left| \; x \quad y \quad z \quad t \quad 1 \; \right| = 0.$

$d_1 \vee d_2 = \langle \{A, B, C\} \rangle$

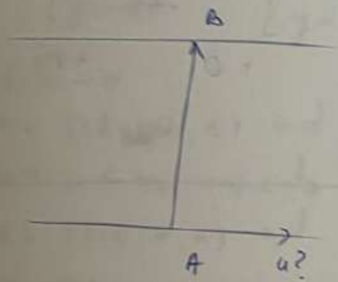
$\text{Dir}(d_2) = \langle \{(1,1,0,2)\} \rangle$

$u = (1,1,0,2)$

$\text{Dir}(d_1 \vee d_2)$ $\quad \vec{AB} = (-1,-1,1,1)$

$\text{Dir}(d_1 \vee d_2) = \langle \{(1,1,0,2), (-1,-1,1,1)\} \rangle$

$$\begin{cases} a+b+2d = 0 \\ -a-b+c+d = 0 \end{cases} \Leftrightarrow \begin{cases} b = a-2d \\ -b+c = a-d \end{cases}$$



$$\text{Dir}(d_1 \vee d_2): \begin{cases} x-y=0 \\ -2y-3z+w=0. \end{cases}$$

$$d_1 \vee d_2: \begin{cases} x-y=b_1 \\ 2y+3z-w=b_2 \end{cases}$$

$a \Rightarrow d$

$$B \in d_1 \vee d_2: \begin{cases} b_1 = 0 \\ b_2 = 8 \end{cases}$$

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6. $\mathcal{A} = \mathbb{R}^n$ Orice hiperplan a lui $\mathcal{A}$ separă spațiul.
Î $\mathbb{C}^n$ nu mai este adevărat.

Putem presupune că $\mathcal{H}: x_1 = 0$. $\mathcal{A} \setminus \mathcal{H} = \{(x_1, \dots x_n) \in \mathbb{R}^n \mid x_1 \neq 0\} = \Delta_1 \cup \Delta_2$

$\Delta_1 = \{(x_1, \dots x_n) \in \mathbb{R}^n, x_1 < 0\}$

$\Delta_2 = \{(x_1, \dots, x_n) \in \mathbb{R}^n, x_1 > 0\}$

$f: \mathcal{A} \to \mathcal{H}$

$f(x_1, \dots, x_n) = x_1 \quad \mathcal{A} \setminus \mathcal{H} = f^{-1}((-\infty, 0) \cup (0, \infty))$

$\Rightarrow \mathcal{A} \setminus \mathcal{H}$ nu e convexă

Are 2 componente convexe Δ_1, Δ_2

În $\mathbb{C}^n$ $\mathcal{H}: z_1 = 0$ $\quad \mathcal{A} \setminus \mathcal{H} = \{(z_1, \dots, z_n) \mid z_1 \neq 0\}.$

$A = (z_1, \dots, z_n)$

$B = (w_1, \dots, w_n)$

1) $\cancel{z_1} = a_1 + ib_1 \qquad \cancel{z_1} = (a_1, b_1)$

$w_1 = c_1 + d_1 \qquad w_1 = (c_1, d_1)$

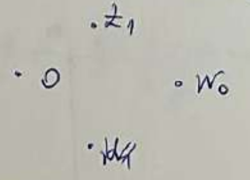
$\cancel{w_1 \neq \lambda z_1 \quad , \lambda \in \mathbb{R}^*} \quad 0$

nu sunt colinbre $w_1 \neq \lambda z_1, \lambda \in \mathbb{R}^*$

2) $z_1 = \lambda \cdot w_2$, (cu $\lambda \in \mathbb{R}^*$ fix)

Vom lua w_0 a.î. $w_0 \neq \lambda z, \forall \lambda \in \mathbb{R}$

$[z_1, w_0] \cup [w_0, w_1]$ drum în $\mathbb{C}$. care nu unește z_1 cu w_2 și nu-l conține pe $\mathbb{O}$.



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5) $d_1: \frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{0} = \frac{w}{2}$

$d_3: \frac{x}{1} = \frac{y}{1} = \frac{z-3}{1} = \frac{w-1}{1}$

$d_1 \vee d_3$: ?

$d_1: \begin{cases} x-y=0 \\ z=2 \\ 2x-w=2 \end{cases} \qquad d_3: \begin{cases} x-y=0 \\ y-z=-3 \\ x-w=-1 \end{cases} \qquad d_1 \cap d_3: \begin{cases} x-y=0 \\ z=2 \\ y-z=-3 \\ 2x-w=2 \\ x-w=-1 \end{cases}$ $\quad x=3$, $w=4$

$\left.\begin{cases} y=3 \\ z=2 \\ 3-2 \neq -3 \end{cases}\right\}$ sistem incompatibil

$\dim (d_1 \vee d_3) = 3 \qquad d_1 \vee d_3 \subset \mathbb{R}^4$

$\dim (d_3) = \langle\{(1,1,0,2), (1,1,1,1)\}\rangle$

$O_1 = (1,1,2,1)$

$O_2 = (0,0,3,1)$

$\overrightarrow{O_1O_3} = (-1,-1,1,0)$.

$\dim(d_3) = \begin{vmatrix} x & y & z & t \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 0 \end{vmatrix} = 0 \Rightarrow$

$$\Leftrightarrow \begin{vmatrix} x & y & z & t \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \\ -1 & -1 & 1 & 0 \end{vmatrix} = 0.$$

$$5x - 5y = 0 \Rightarrow x - y = 0.$$

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7. Găsiți dacă există dreptele care taie simultan dreptele:

$d_1: \begin{cases} x = 3z \\ y = -3/2 \end{cases}$ $\quad d_2: \begin{cases} x = z = 0 \\ y = 3/2 \end{cases}$ $\quad d_3: \begin{cases} x - z = 3 \\ y = z \end{cases}$ $\quad d_4: \begin{cases} x - z = 0 \\ y = z \end{cases}$

$d \cap d_1 - \emptyset$

$P = (3t, -3/2, t) \in d$

$Q = (-1, 3/2, +s) \in d_2 = (3, 3/2, -3)$

$R = (3+\theta, \theta, \theta) \in d$

$\{\overrightarrow{RP}, \overrightarrow{RQ}\}$ liniar dep. $\qquad R' = (\sigma, \sigma, \sigma)$

$(3t - \theta - 3, -3/2 - \theta, t - \theta) = \lambda(-s - \theta - 3, 3/2 - \theta, s - \theta).$

$-3/2 - \theta = \lambda(3/2 - \theta) \Rightarrow 3\lambda/2 + 3/2 = \theta(1 - \lambda)$

$\theta = \frac{1}{1-\lambda}\left(\frac{3\lambda}{2} + \frac{3}{2}\right)$

$t - \theta = \lambda \cdot s - \lambda \cdot \theta$

$t - \lambda s = (1 - \lambda)\theta = 3/2(\lambda + 1)$

$t = 3/2(\lambda + 1) + \lambda \cdot s$

$3t - \theta - 3 = \lambda(-s - \theta - 3)$

$\frac{9}{2}(\lambda+1) + \lambda \cdot s - 3 = -\lambda s + (1 - \lambda)\theta - 3\lambda$

$\frac{9}{2}(\lambda + 1) + 2\lambda s = 3 + \frac{3\lambda}{2} + \frac{3}{2} - 3\lambda$

$*\ s = -3$

$*\ t = \frac{3}{2}(1 - \lambda)$

$*\ \theta = \frac{1}{(1-\lambda)}\left(\frac{3\lambda}{2} + \frac{3}{2}\right)$

$\overrightarrow{RP} = \left(9/2(1-\lambda) - 3/2 \cdot \frac{1+\lambda}{1-\lambda} - 3, -\frac{3}{2}\left(1 + \frac{3}{2} \cdot \frac{1+\lambda}{1-\lambda}\right), 3/2\left(1 - \lambda - \frac{1+\lambda}{1-\lambda}\right)\right)$