

MEGPPS:

Rezolvati sistemul de ecuatii liniare:

$$\begin{cases} x_1 + x_2 - x_3 = 1 \\ x_1 + x_2 + 4x_3 = 2 \\ 2x_1 - x_2 + 2x_3 = 3 \end{cases}$$

folosind MEGPPS și metoda substituției descendente.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 4 \\ 2 & -1 & 2 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \underline{n=3}$$

k=1:

$$\overline{A} \equiv \overline{A}^{(1)} = \begin{bmatrix} A^{(1)} & \underline{b} \end{bmatrix} = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & 1 & 4 & 2 \\ 2 & -1 & 2 & 3 \end{array} \right]$$

$$\underline{i=1,3}: s_i := \max_{j=1,3} |a_{ij}^{(1)}|$$

$$s_1 = \max_{j=1,3} |a_{1j}^{(1)}| = 1$$

$$s_2 := \max_{j=1,3} |a_{2j}^{(1)}| = 4$$

$$s_3 := \max_{j=1,3} |a_{3j}^{(1)}| = 2$$

$$\underline{i=1,3}: \tilde{a}_{i1}^{(1)} := a_{i1}^{(1)} / s_i$$

$$\tilde{a}_{11}^{(1)} := a_{11}^{(1)} / s_1 = 1/1 = 1$$

$$\tilde{a}_{21}^{(1)} := a_{21}^{(1)} / s_2 = 1/4 = 1/4$$

$$\tilde{a}_{31}^{(1)} := a_{31}^{(1)} / s_3 = 2/2 = 1$$

$$\max_{i=1,3} |\tilde{a}_{i1}^{(1)}| = \max\{|1|, |1/4|, |1|\}$$

$$= 1 \Rightarrow \tilde{a}_{l1}^{(1)} = \begin{cases} \tilde{a}_{11}^{(1)} \\ \tilde{a}_{31}^{(1)} \end{cases} \Rightarrow l \in \{1,3\}$$

$l \in \{1,3\} \Rightarrow$ nu interschimbam
liniile lui $\bar{A}^{(1)} \Rightarrow$

$$P^{(1)} = I_3 \quad (1a)$$

$$P^{(1)} \bar{A}^{(1)} = P^{(1)} [A^{(1)} \quad \underline{b}^{(1)}] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & 1 & 4 & 2 \\ 2 & -1 & 2 & 3 \end{array} \right]$$

$$= [\tilde{A}^{(1)} \quad \tilde{\underline{b}}^{(1)}] = \tilde{A}^{(1)}$$

$$\tilde{a}_{11}^{(1)} = 1 \neq 0 \quad (\text{aplicăm MEGFP})$$

$$\underline{j=2,3}: \quad m_{ij}^{(1)} := \tilde{a}_{ij}^{(1)} / \tilde{a}_{11}^{(1)}$$

$$\bullet \quad m_{22}^{(1)} := \tilde{a}_{22}^{(1)} / \tilde{a}_{11}^{(1)} = 1 / 1 = 1$$

$$(E_2 - m_{22}^{(1)} E_1) \rightarrow (E_2):$$

$$\underline{j=2,3}: \quad \tilde{a}_{j2}^{(2)} := \tilde{a}_{j2}^{(1)} - m_{22}^{(1)} \tilde{a}_{j2}^{(1)}$$

$$\tilde{a}_{22}^{(2)} := \tilde{a}_{22}^{(1)} - m_{22}^{(1)} \tilde{a}_{22}^{(1)}$$

$$= 1 - 1 \cdot 1 = 0$$

$$\tilde{a}_{23}^{(2)} := \tilde{a}_{23}^{(1)} - m_{22}^{(1)} \tilde{a}_{23}^{(1)}$$

$$= 4 - 1 \cdot (-1) = 5$$

$$\tilde{a}_{21}^{(2)} = 0 \quad (\text{Nu mai trebuie calculat!})$$

$$\tilde{b}_2^{(2)} := \tilde{b}_2^{(1)} - m_{22}^{(1)} \tilde{b}_1^{(1)} = 2 - 1 \cdot 1 = 1$$

$$\bullet \quad m_{32}^{(1)} := \tilde{a}_{32}^{(1)} / \tilde{a}_{11}^{(1)} = 2 / 1 = 2$$

$$(E_3 - m_{32}^{(1)} E_1) \rightarrow (E_3):$$

$$\underline{j=2,3}: \quad \tilde{a}_{j3}^{(2)} := \tilde{a}_{j3}^{(1)} - m_{32}^{(1)} \tilde{a}_{j2}^{(1)}$$

$$\tilde{a}_{32}^{(2)} := \tilde{a}_{32}^{(1)} - u_3^{(1)} \tilde{a}_{12}^{(1)} \\ = -1 - 2 \cdot 1 = -3$$

$$\tilde{a}_{33}^{(2)} := \tilde{a}_{33}^{(1)} - u_3^{(1)} \tilde{a}_{13}^{(1)} \\ = 2 - 2 \cdot (-1) = 4$$

$$\tilde{a}_{31}^{(2)} = 0 \quad (\text{nu mai trebuie calculat!})$$

$$\tilde{b}_3^{(2)} := \tilde{b}_3^{(1)} - u_3^{(1)} \tilde{b}_1^{(1)} = 3 - 2 \cdot 1 = 1$$

Am obtinut:

$k=2$:

$$\bar{A}^{(2)} = [A^{(2)} \quad \underline{b}^{(2)}] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 0 & 5 & 1 \\ 0 & -3 & 4 & 1 \end{array} \right]$$

Obs: Matricea care transformă

$$\bar{A}^{(1)} = [A^{(1)} \quad \underline{b}^{(1)}] = P^{(1)} [A^{(1)} \quad \underline{b}^{(1)}] = P^{(1)} \bar{A}^{(1)} \\ \equiv P^{(1)} \bar{A} = P^{(1)} [A \quad \underline{b}]$$

în matricea $\bar{A}^{(2)} = [A^{(2)} \quad \underline{b}^{(2)}]$

este dată de :

$$M^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \quad (1b)$$

Mai exact, are loc relatia:

$$M^{(1)} P^{(1)} [A^{(1)} \quad \underline{b}^{(1)}] = [A^{(2)} \quad \underline{b}^{(2)}] \quad (1)$$

cu $P^{(1)}$ si $M^{(1)}$ date de (1a) si (1b).

$$\underline{i=2,3} : s_i := \max_{j=2,3} |a_{ij}^{(2)}|$$

$$s_2 := \max \{ |0|, |5| \} = 5$$

$$s_3 := \max \{ |-3|, |14| \} = 4$$

$$\underline{i=2,3} : \tilde{a}_{i2}^{(2)} := a_{i2}^{(2)} / s_i$$

$$\tilde{a}_{22}^{(2)} := a_{22}^{(2)} / s_2 = 0 / 5 = 0$$

$$\tilde{a}_{32}^{(2)} := a_{32}^{(2)} / s_3 = -3 / 4$$

$$\max_{i=2,3} |\tilde{a}_{i2}^{(2)}| = \max \{ |0|, |-3/4| \} = 3/4$$

$$\Rightarrow \tilde{a}_{l2}^{(2)} = \tilde{a}_{32}^{(2)} \Rightarrow l=3 > 2=i$$

\Rightarrow Interschimbă $(E_2) \leftrightarrow (E_3)$:

Matricea permutare simplă:

$$P^{(2)} = P_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad (2a)$$

$$P^{(2)} A^{(2)} = P^{(2)} [A^{(2)} \quad \underline{b}^{(2)}] = \underbrace{\begin{bmatrix} 1 & 1 & -1 & | & 1 \\ 0 & -3 & 4 & | & 1 \\ 0 & 0 & 5 & | & 1 \end{bmatrix}}$$

Obs: $\quad \quad \quad = A^{(3)} = [A^{(3)} \quad \underline{b}^{(3)}] = [U \quad \underline{\tilde{b}}]$

Matricea $M^{(2)} = I_3 \quad (2b)$

Din (2a) și (2b) rezultă:

$$M^{(2)} P^{(2)} [A^{(2)} \quad \underline{b}^{(2)}] = [A^{(3)} \quad \underline{b}^{(3)}] \quad (2)$$

Din relațiile (1) și (2) obținem:

$$\boxed{M^{(2)} P^{(2)} M^{(1)} P^{(1)} [A \quad \underline{b}] = [U \quad \underline{\tilde{b}}]}$$

i.e. $\begin{cases} x_1 + x_2 - x_3 = 1 \\ -3x_2 + 4x_3 = 1 \\ 5x_3 = 1 \end{cases} \Rightarrow \begin{cases} x_3 = 1/5 \\ x_2 = -1/15 \\ x_1 = 19/15 \end{cases}$