Laborator (Online) 08.12.2022

① Metoda inversă

$X \sim F$, $U \sim \mathcal{U}([0,1])$ atunci

X și $F^{-1}(U)$ erau repartizate la fel

$\underline{F^{-1}}: (0,1) \to \mathbb{R}$

$F^{-1}(u) = \inf\{x \in \mathbb{R} \mid F(x) \geq u\}$ fct cuantilă

v.a discrete

X v.a $X: \Omega \to \mathbb{R}$

$X(\Omega) = \{x_1, x_2, \dots x_n\}$ $x_1 \leq x_2 \leq \dots \leq x_n$

discrete → cantitative : vârsta, kg

discrete → calitative → nominale

calitative → ordinale

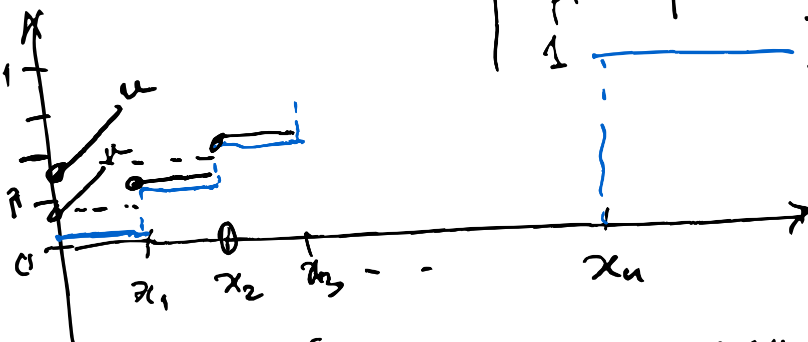
naționalitate, sex, culoarea ochilor

rating filme / hotel (scăzut, mediu, ridicat)

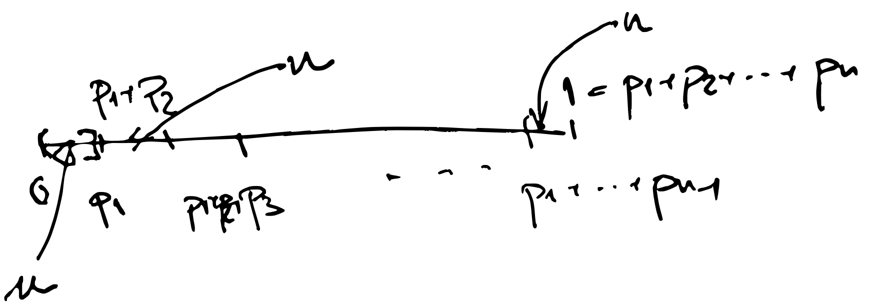
$X(\Omega) = \{\omega_1, \omega_2, \dots\} \xrightarrow{h} \{1, 2, \dots n\}$

$$\mathbb{P}(X=k)=p_k$$

$$F(x)=\mathbb{P}(X\le x)=\begin{cases} 0 & , x<x_1 \\ p_1 & , x_1\le x<x_2 \\ p_1+p_2 & , x_2\le x<x_3 \\ \dots & \dots \\ p_1+\dots+p_{k-1} & , x_{k-1}\le x<x_k \\ \dots & \\ 1 & , x\ge x_n \end{cases}$$



$$F^{-1}(u)=\begin{cases} x_1 & , 0<u\le p_1 \\ x_2 & , p_1<u\le p_1+p_2 \\ \dots & \\ x_n & , p_1+\dots+p_{n-1}<u\le 1 \end{cases}$$



$$F^{-1}(u)=\sum_{i=1}^{n} x_i\,\mathbb{1}_{(p_1+\dots+p_{i-1},\,p_1+\dots+p_i]}(u)$$

$U \sim \mathcal{U}(0,1)$ $\quad X = F^{-1}(U) \longrightarrow U(\omega)$

- găsesc indicele i pt care

$$\underbrace{p_1 + \dots + p_{i-1}} < U(\omega) \leq \overline{p_1 + \dots + p_i}$$

$$X(\omega) = x_i$$

- reordonăm suma $(p_1 \geq p_2 \geq \dots)$

Ex1

$$X \sim \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0.2 & 0.5 & 0.1 & 0.2 \end{pmatrix}$$

$p_2 \quad p_3$

$F \to 0$

$T \to 1$

$$X \sim \begin{pmatrix} 1 & 2 \\ 0.2 & 0.8 \end{pmatrix}$$

$U \sim \mathcal{U}([0,1])$

$$X \leftarrow 1 + \underline{(U > 0.2)}$$

$$\mathbb{P}(X = 1) = \mathbb{P}(U \leq 0.2) = 0.2$$

$B(p)$ $\quad \mathbb{1}_{\{U > 1-p\}}$

$$\mathbb{P}(X = 2) = \mathbb{P}(U > 0.2) = 0.8$$

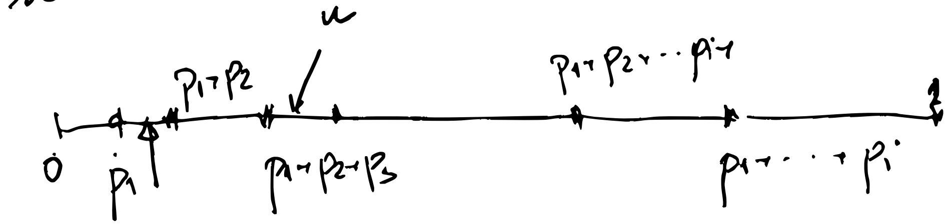
$$X \leftarrow 1 + (U > 0.2) + (U > 0.7) + (U > 0.8)$$

$$\mathbb{P}(0.2 < U \leq 0.7) = 0.5$$

$$\mathbb{P}(0.7 < U \leq 0.8) = 0.1$$

$$\mathbb{P}(U \geq 0.8) = 0.2$$

u



$u <=$ pOrdCum

Sum (F F F T T T T -- T)

$u >$ pOrdCum

Sum (T T T F F - - - F) => 3

Geometrică

$X \sim \text{Geom}(p)$ $\qquad$ $\mathbb{P}(X=k) = p(1-p)^{k-1}$

$k \in \{1,2,\dots\}$

$$F(k) = \mathbb{P}(X \le k) = \sum_{i=1}^{k} \mathbb{P}(X=i)$$

$$= \sum_{i=1}^{k} p \underbrace{(1-p)}_{q}{}^{i-1} = p\left(1+q+\dots+q^{k-1}\right)$$

$$= p \cdot \frac{1-q^k}{1-q} = 1-(1-p)^k$$

$U \sim \mathcal{U}[0,1)$

$$\underbrace{p_1+\dots+p_{i-1}}_{F(i-1)} < U < \underbrace{p_1+p_2+\dots+p_i}_{F(i)}$$

$$1-(1-p)^{i-1} < U < 1-(1-p)^{i}$$

$$\Leftrightarrow (1-p)^{i} < 1-U < (1-p)^{i-1} \Leftrightarrow i\log(1-p) < \log(1-U) < (i-1)\log(1-p)$$

$$\Leftrightarrow i-1 < \frac{\log(1-U)}{\log(1-p)} < i$$

$$i = \left[\frac{\log(1-U)}{\log(1-p)}\right] + 1$$

$$X = \left[\frac{\log(1-U)}{\log(1-p)}\right] + 1$$

$U \sim \mathcal{U}[0,1] \Rightarrow 1-U \sim \mathcal{U}[0,1]$

$$X = \left[\frac{\log U}{\log(1-p)}\right] + 1$$

<u>3) Uniforme</u>

$X \sim \mathcal{U}(\{1,2,\dots N\})$ $\quad \mathbb{P}(X=k) = \frac{1}{N}$

$$F(i) = \sum_{j=1}^{i} \underbrace{\mathbb{P}(X=j)}_{1/N} = i/N$$

$F(i-1) \leq U < F(i)$

$i-1 \leq NU < i$

$$\boxed{i = [NU] + 1}$$

$u \sim \mathcal{U}[0,1] \Rightarrow Nu \sim \mathcal{U}[0,N]$

$\lfloor Nu \rfloor \sim \mathcal{U}(\{0,1,\dots,N-1\})$

$\lfloor Nu \rfloor + 1 \sim \mathcal{U}(\{1,2,\dots,N\})$

4) Binomial

$X \sim B(n,p)$

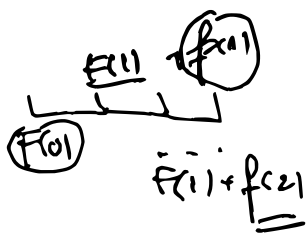
$\mathbb{P}(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

$f(k)$

$F(i) = \mathbb{P}(X \le i)$

$f(k+1) \quad f(k)$

$F(i-1) \le u < F(i)$



$$f(k+1) = \mathbb{P}(X=k+1)$$

$$= \binom{n}{k+1} p^{k+1} (1-p)^{n-k-1}$$

$$= \frac{n-k}{k+1} \cdot \frac{p}{1-p} \cdot f(k)$$

$$f(0) = (1-p)^n$$