PCMMP & MEGFP

Considerăm sistemul supradeterminat de ecuații liniare

$$\begin{cases} x_1 + 2x_2 = 1 \\ 3x_1 - x_2 + x_3 = 0 \\ -x_1 + 2x_2 + x_3 = -1 \\ x_1 - x_2 - 2x_3 = 2 \\ 2x_1 + x_2 - x_3 = 2 \end{cases} \quad (1)$$

Determinați soluția sistemului (1) cu sensul celor mai mici pătrate folosind sistemul de ecuații normale asociat lui (1) și MEGFP

$$A^T = \begin{bmatrix} 1 & 3 & -1 & 1 & 2 \\ 2 & -1 & 2 & -1 & 1 \\ 0 & 1 & 1 & -2 & -1 \end{bmatrix}; \quad \underline{b} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \\ 2 \end{bmatrix};$$

Sistemul de ecuații normale:

$$(A^TA)\,\underline{x} = A^T\underline{b} =: \underline{\tilde{b}}$$

$$A^TA = \begin{bmatrix} 16 & -2 & -2 \\ -2 & 11 & 2 \\ -2 & 2 & 7 \end{bmatrix}, \quad \underline{\tilde{b}} = \begin{bmatrix} 8 \\ 0 \\ -7 \end{bmatrix}$$

$$\left[\begin{array}{c|cc|c} 16 & -2 & -2 & 8 \\ \hline -2 & 11 & 2 & 0 \\ -2 & 2 & 7 & -7 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c|c} 16 & -2 & -2 & 8 \\ 0 & 43/4 & 7/4 & 1 \\ \hline 0 & 7/4 & 27/4 & -6 \end{array}\right]$$

$$\left(E_2 + \tfrac{1}{8}E_1\right) \to (E_2)$$

$$\left(E_3 + \tfrac{1}{8}E_1\right) \to (E_3)$$

$$\longrightarrow \left[\begin{array}{ccc|c} 16 & -2 & -2 & 8 \\ 0 & 43/4 & 7/4 & 1 \\ 0 & 0 & 278/43 & -265/43 \end{array}\right]$$

$$\left(E_3 - \tfrac{7}{43}E_2\right) \to (E_3)$$

$$\begin{cases} 16x_1 - 2x_2 - 2x_3 = 8 \\ \frac{43}{4}x_2 + \frac{7}{4}x_3 = 1 \\ \frac{278}{43}x_3 = -\frac{265}{43} \end{cases} \Rightarrow$$

$$x_3 = -\frac{265}{278} \simeq -0{,}9532$$

$$x_2 = \frac{1}{43}\left(4 - 7x_3\right) = \frac{1}{43}\,\frac{4\times 278 + 7\times 265}{278}$$

$$= \frac{1112 + 1855}{11954} = \frac{2967}{11954} \simeq 0{,}2482$$

$$x_1 = \frac{1}{8}\left(4 + x_2 + x_3\right) \simeq -0{,}0917$$

$$\underline{x} = \left(-0{,}0917 \quad 0{,}2482 \quad -0{,}9532\right)^T$$