Test

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Gr. 113

1. $\Delta = \begin{vmatrix} 3 & -3 & 2 & 3 \\ -2 & 7 & -2 & 5 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 1 & 5 \end{vmatrix} = 3 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 7 & -2 & 5 \\ 0 & 3 & 7 \\ 0 & 1 & 5 \end{vmatrix} +$

$+ (-2) \cdot (-1)^{2+1} \cdot \begin{vmatrix} 3 & 2 & 3 \\ 0 & 3 & 7 \\ 0 & 1 & 5 \end{vmatrix} = 3(105-49) + 2(45-21) =$

$= 3 \cdot 56 + 2 \cdot 24 = 216 \Rightarrow \Delta = 216$

2. $\begin{vmatrix} 7 & 1 & 0 & 0 & \dots & 0 \\ 6 & 7 & 1 & 0 & \dots & 0 \\ 0 & 6 & 7 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \dots & 7 \end{vmatrix} = \det(A_n) = \Delta_n$

(a) $\Delta_2 = \begin{vmatrix} 7 & 1 \\ 6 & 7 \end{vmatrix} = 49 - 6 = 43$

$\Delta_3 = \begin{vmatrix} 7 & 1 & 0 \\ 6 & 7 & 1 \\ 0 & 6 & 7 \end{vmatrix} = 343 - 42 - 42 = 259$

$\Delta_4 = \begin{vmatrix} 7 & 1 & 0 & 0 \\ 6 & 7 & 1 & 0 \\ 0 & 6 & 7 & 1 \\ 0 & 0 & 6 & 7 \end{vmatrix} = 7 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 7 & 1 & 0 \\ 6 & 7 & 1 \\ 0 & 6 & 7 \end{vmatrix} + 1 \cdot (-1)^{1+2} \cdot \begin{vmatrix} 6 & 1 & 0 \\ 0 & 7 & 1 \\ 0 & 6 & 7 \end{vmatrix}$

$= 7 \cdot \Delta_3 - (294 - 36) = 7 \cdot 259 - 258 = 1813 - 258 = 1555$ (1)

$\Rightarrow \Delta_4 = 1555$

(b) $\Delta_n = \begin{vmatrix} 7 & 1 & 0 & 0 & \dots & 0 \\ 6 & 7 & 1 & 0 & \dots & 0 \\ 0 & 6 & 7 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \dots & 7 \end{vmatrix} = 7 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 7 & 1 & 0 & \dots & 0 \\ 6 & 7 & 1 & \dots & 0 \\ 0 & 6 & 7 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 7 \end{vmatrix} +$

$+ 1 \cdot (-1)^{1+2} \cdot \begin{vmatrix} 6 & 1 & 0 & 0 & \dots & 0 \\ 0 & 7 & 1 & 0 & \dots & 0 \\ 0 & 6 & 7 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \dots & \end{vmatrix} + 1 (-1)^{1+2} \cdot \begin{vmatrix} 6 & 1 & 0 & 0 & \dots & 0 \\ 0 & 7 & 1 & \dots & & \\ 0 & 0 & 6 & \dots & & 0 \end{vmatrix}$

$+ 6 \cdot (-1)^{2+1} \cdot \begin{vmatrix} 1 & 0 & 0 & \dots & 0 \\ 6 & 7 & 1 & \dots & \\ 0 & 6 & 7 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 7 \end{vmatrix} =$

$\Delta_n = 7 \cdot \Delta_{n-1} - 6 \cdot 1 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 7 & 1 & 0 & 0 & \dots & 0 \\ 6 & 7 & 1 & 0 & \dots & 0 \\ 0 & 6 & 7 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \dots & 7 \end{vmatrix} =$

$\Delta_n = 7 \Delta_{n-1} - 6 \cdot \Delta_{n-2}$

(c) Trebuie să arăt că $\Delta_n = \dfrac{6^{n+1} - 1}{5}$, $\forall n \geq 2$

Cf. a): $\Delta_2 = 43 = \dfrac{6^3 - 1}{5}$

$\Delta_3 = 259 = \dfrac{6^4 - 1}{5}$

$\Delta_4 = 1555 = \dfrac{6^5 - 1}{5}$

(c) P_p. adevărată $\Delta_n = \frac{6^{n+1}-1}{5}$

Cf. b):

$$\Delta_n = 7\Delta_{n-1} - 6\Delta_{n-2} = 7 \cdot \frac{6^n - 1}{5} - 6 \cdot \frac{6^{n-1}-1}{5} =$$

$$= \frac{7 \cdot 6^n - 7}{5} - \frac{6^n - 6}{5} = \frac{6^n(7-1) - 1}{5} = \frac{6^n \cdot 6 - 1}{5} = \frac{6^{n+1}-1}{5}$$

$\Rightarrow$ presupunerea e adevărată $\Rightarrow \Delta_n = \frac{6^{n+1}-1}{5}$