METODA GAUSS - JORDAN

Să se determine inversa matricii

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

folosind metoda Gauss - Jordan.

Folosim metoda Gauss - Jordan împreună cu MEGFP:

<u>$k=1$</u>:

$$\overline{A} := [A \quad I_3] = \overline{A}^{(1)} := [A^{(1)} \quad I_3] =$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$a_{11}^{(1)} = 1 \neq 0 \Rightarrow$ MEGFP

$\underline{i = \overline{2,3}}$: $m_i^{(1)} := a_{i1}^{(1)} / a_{11}^{(1)}$

$\bullet\ m_2^{(1)} := a_{21}^{(1)} / a_{11}^{(1)} = 2/1 = 2$

$\bullet\ m_3^{(1)} := a_{31}^{(1)} / a_{11}^{(1)} = 3/1 = 3$

$$\left.\begin{array}{l} (E_2 - m_2^{(1)} E_1) \longrightarrow (E_2) \\ (E_3 - m_3^{(1)} E_1) \longrightarrow (E_3) \end{array}\right\} \Longrightarrow$$

<u>$k=2$</u>:

$$\overline{A}^{(2)} = \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -3 & -1 & -2 & 1 & 0 \\ 0 & -5 & 1 & -3 & 0 & 1 \end{array}\right]$$

$a_{22}^{(2)} = -3 \neq 0 \Longrightarrow$ MEGFP

<u>$i=\overline{3,3}$</u>: $m_i^{(2)} := a_{i2}^{(2)} / a_{22}^{(2)}$

$\bullet\ m_3^{(2)} := a_{32}^{(2)} / a_{22}^{(2)} = -5/(-3) = 5/3$

$(E_3 - m_3^{(2)} E_2) \longrightarrow (E_3) \Longrightarrow$

$$\overline{A}^{(3)} = \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -3 & -1 & -2 & 1 & 0 \\ 0 & 0 & 8/3 & 1/3 & -5/3 & 1 \end{array}\right] \Longrightarrow$$

Astfel, obținem 3 sisteme liniare superior triunghiulare

$$\begin{cases} y_1 + 2y_2 = 1 \\ -3y_2 - y_3 = -2 \\ \frac{8}{3} y_3 = \frac{1}{3} \end{cases} \left| \begin{matrix} 0 \\ 1 \\ -\frac{5}{3} \end{matrix} \right| \begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$$

- Sistemul 1 dă coloana 1 a lui A^{-1}:

$$\begin{cases} x_{11} + 2x_{21} = 1 \\ -3x_{21} - x_{31} = -2 \\ \frac{8}{3} x_{31} = \frac{1}{3} \end{cases}$$

$$\boxed{x_{31}} = \frac{1}{3}\,\frac{3}{8} = \boxed{\frac{1}{8}}$$

$$\boxed{x_{21}} = -\frac{1}{3}\left(-2 + x_{31}\right) = -\frac{1}{3}\left(-2+\frac{1}{8}\right)$$

$$= -\frac{1}{3}\,\frac{-15}{8} = \boxed{\frac{5}{8}}$$

$$\boxed{x_{11}} = 1 - 2x_{21} = 1 - 2\,\frac{5}{8} = 1 - \frac{5}{4} = \boxed{-\frac{1}{4}}$$

• Sistemul 2 dă coloana 2 a lui A^{-1}:

$$\begin{cases} x_{12} + 2x_{22} = 0 \\ -3x_{22} - x_{32} = 1 \\ \frac{8}{3} x_{32} = -\frac{5}{3} \end{cases}$$

$$\boxed{x_{32} = \frac{5}{3}\,\frac{3}{8} = -\frac{5}{8}}$$

$$\boxed{x_{22} = -\frac{1}{3}(1 + x_{32}) = -\frac{1}{3}\left(1 - \frac{5}{8}\right) = -\frac{1}{8}}$$

$$\boxed{x_{12} = -2x_{22} = -2\,\frac{-1}{8} = \frac{1}{4}}$$

• Sistemul 3 dă coloane 3 a lui A^{-1}:

$$\begin{cases} x_{13} + 2x_{23} = 0 \\ -3x_{23} - x_{33} = 0 \\ \frac{8}{3} x_{33} = 1 \end{cases}$$

$$\boxed{x_{33} = 3/8} \qquad \boxed{x_{23} = -\frac{1}{3} x_{33} = -\frac{1}{8}}$$

$$\boxed{x_{13} = -2x_{23} = 1/4}$$

Am obținut:

$$A^{-1} = \begin{bmatrix} -1/4 & 1/4 & 1/4 \\ 5/8 & -1/8 & -1/8 \\ 1/8 & -5/8 & 3/8 \end{bmatrix}$$