Seminar geometrie 10 - 6 dec 2017

$Q: \mathbb{R}^3 \to \mathbb{R}$, $Q(x) = 2x_1x_2 + 2x_1x_3 + 2x_2x_3$

a) Să se arate că Q este f. pătratică

b) Să se det o bază în $\mathbb{R}^3/\mathbb{R}$ în care matr asociată lui Q are f-ormă diagonală, să se precizeze această matrice și să se det. sign lui Q în baza găsită

$$G = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Fie $g: \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ $g(x,y) = $ ~~[illegible]~~

$= x_1y_2 + x_1y_3 + x_2y_1 + x_2y_3 + x_3y_1 + x_3y_2$

g e biliniară și simetrică $\forall x \in \mathbb{R}^3$

$Q(x) = g(x,x) \Rightarrow Q$ forma pătratică

$x_1 = y_1 - y_2$
$x_2 = y_1 + y_2$
$x_3 = y_3$

$Q(x) = 2(y_1-y_2)(y_1+y_2) + 2(y_1-y_2)y_3 + 2(y_1+y_2)y_3 =$

$= 2y_1^2 - 2y_2^2 + 2y_1y_3 - \cancel{2y_2y_3} + 2y_1y_3 + \cancel{2y_2y_3} =$

$= 2y_1^2 - 2y_2^2 + 4y_1y_3 = 2(y_1^2 + 2y_1y_3 + y_3)^2 - 2y_3^2 - 2y_2^2 =$

$= 2(y_1+y_3)^2 - 2y_2^2 - 2y_3^2$

$z_1 = y_1 + y_3$
$z_2 = y_2$
$z_3 = y_3$

$Q(x) = 2z_1^2 - 2z_2^2 - 2z_3^2$

$B_c = \{e_1, e_2, e_3\} \xrightarrow{A} \{u_1, u_2, u_3\}$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$$

$y_1 = z_1 - z_3$ $\quad x_1 = z_1 - z_2 + z_3$ ~~[illegible]~~
$y_2 = z_2$ $\quad x_2 = z_1 - z_3 + z_2$
$y_3 = z_3$ $\quad x_3 = z_3$

$$A = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$u_1 = a_{11}e_1 + a_{21}e_2 + a_{31}e_3 = (1,1,0)$
$u_2 = (-1,1,0)$
$u_3 = (-1,-1,1)$

$$Q \xrightarrow{B} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

signatura lui $Q = (1, 2)$

Def. Fie $V/\mathbb{R}$ sp vect și $\langle , \rangle : V \times V \to \mathbb{R}$. Spunem că $\langle , \rangle$ este produs scalar dacă $\langle , \rangle$ este biliniară, [illegible] simetrică și pozitiv definită.

$(\forall x \in V \setminus \{0\} \quad \langle x, x \rangle > 0)$

Def. $(V/\mathbb{R}, \langle , \rangle)$ - sp vectorial euclidian

Def. $B = \{e_1, \dots, e_n\}$ se numește bază ortonormată dacă $\forall i, j = \overline{1,n}$ $\langle e_i, e_j \rangle = \delta_{ij}$

ex. $g: \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$, $g(x,y) = x_1y_1 - x_1y_2 - x_2y_1 + 6x_2y_2 + 3x_2y_3 + 3x_3y_2 + 3x_3y_3$

Să se arate că g este produs scalar și să se det o bază ortonormată în raport cu g a lui $\mathbb{R}^3/\mathbb{R}$

$\to$ produs scalar - Verificăm: biliniară, simetrică, pozitiv definită $< \begin{matrix} Q(x) \geq 0, \forall x \in \mathbb{R}^3 \\ Q(x) = 0 \Leftrightarrow x = 0_{\mathbb{R}^3} \end{matrix}$

$Q(x) = g(x,x) = x_1^2 - x_1x_2 - x_1x_2 + 6x_2^2 + 3x_2x_3 + 3x_2x_3 + 3x_3^2 =$

$= x_1^2 + 6x_2^2 + 3x_3^2 - 2x_1x_2 + 6x_2x_3 =$

$= x_1^2 - 2x_1x_2 + x_2^2 + 5x_2^2 + 6x_2x_3 + 3x_3^2 =$

~~$= (x_1 - x_2)^2 + 3(x_3^2 + 2x_2x_3 + x_2^2) + 2x_2^2$~~

$= (x_1 - x_2)^2 + 3(x_3^2 + 2x_2x_3) + 5x_2^2 =$

$= (x_1 - x_2)^2 + 3(x_3 + x_2)^2 + 2x_2^2 \geq 0 \quad \forall x \in \mathbb{R}^3$

$Q(x) = 0 \Rightarrow \begin{cases} x_1 - x_2 = 0 \\ x_2 + x_3 = 0 \\ x_2 = 0 \end{cases} \Rightarrow x = x_2 = x_3 = 0$

$\Rightarrow g$ pozitiv definită

$\begin{cases} y_1 = x_1 - x_2 \\ y_2 = x_2 \\ y_3 = x_2 + x_3 \end{cases} \Rightarrow \begin{cases} x_1 = y_1 + y_2 \\ x_2 = y_2 \\ x_3 = y_3 - y_2 \end{cases}$

$B_c \xrightarrow{A} B = \{u_1, u_2, u_3\}$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \quad A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \quad \begin{matrix} u_1 = (1,0,0) \\ u_2 = (1,1,-1) \\ u_3 = (0,-1,1) \end{matrix}$$

$Q(x) = y_1^2 + 2y_2^2 + 3y_3^2$

$$g, Q \xrightarrow{B} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$g(u_1, u_1) = 1$
$g(u_2, u_2) = 2$
$g(u_3, u_3) = 3$

$g(u_i, u_j) = 0 \quad i, j \in \overline{1,3} \quad i \neq j$

Bază ortogonală în raport cu g nu e ortonormată

$V_1 = u_1$

$V_2 = \dfrac{u_2}{\|u_2\|_g} = \dfrac{1}{\sqrt{g(u_2,u_2)}} \cdot u_2 = \dfrac{1}{\sqrt{2}}(1,1,1)$

$V_3 = \dfrac{u_3}{\|u_3\|_g} = \dfrac{1}{\sqrt{g(u_3,u_3)}} \cdot u_3 = \dfrac{1}{\sqrt{3}}(0,-1,1)$

$B' = \{V_1, V_2, V_3\}$ ortonormată în raport cu g

Procedeul de ortonormalizare Gram-Schmidt

Fie $(V/\mathbb{R}, \langle , \rangle)$ sp vectorial euclidian

$L = \{u_1, \dots, u_m\}$ sistem de vectori liniar independenți.

Atunci există $v_1, \dots, v_m \in V$ a.î. $\forall k \in \overline{1,m}$ $Sp\{u_1, \dots, u_k\}$ și $\forall i, j \in \overline{1,m}$ $\langle v_i, v_j \rangle = \delta_{ij}$

$v_1 = \dfrac{1}{\sqrt{\langle u_1, u_1 \rangle}} \cdot u_1 \quad g(v_1, v_1) = g\left(\dfrac{1}{\sqrt{\langle u_1, u_1 \rangle}} \cdot u_1, \dfrac{1}{\sqrt{\langle u_1, u_1 \rangle}} \cdot u_1\right)$

$= \dfrac{1}{\langle u_1, u_1 \rangle} \cdot \langle u_1, u_1 \rangle = 1$

$v_2' = u_2 + a v_1 \quad \langle v_2', v_1 \rangle = 0 \Rightarrow \langle u_2 + a v_1, v_1 \rangle = 0 \Rightarrow$

$\Rightarrow \langle u_2, v_1 \rangle + a \cdot 1 = 0 \Rightarrow a = -\langle u_2, v_1 \rangle$

$v_2' = u_2 - \langle u_2, v_1 \rangle v_1$

$v_2' \neq 0 \quad v_2 = \dfrac{1}{\sqrt{\langle v_2', v_2' \rangle}} \cdot v_2'$

②

$\langle \, , \, \rangle : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$

$\langle x, y \rangle = \sum_{i=1}^{n} x_i y_i$

bil, simetrică, poz def ⇒ produs scalar canonic

$\| \ \| : \mathbb{R}^n \to \mathbb{R}$

$\|x\| = \sqrt{\langle x, x\rangle} = \sqrt{\sum_{i=1}^{n} x_i^2}$ ← baza canonică e ortonormată $(I_n)$

ex. Să se ortonormalizeze în raport cu produsul scalar canonic sistemele de vectori

a) $L_1 = \{u_1, u_2\}$ $u_1 = (1,2,-1)$, $u_2 = (2,-1,1)$

b) $L_2 = \{u_1, u_2, u_3\}$ $u_1 = (0,1,1)$, $u_2 = (1,0,1)$, $u_3 = (1,1,0)$

a) $\exists\, v_1, v_2 \in V$

$v_1 = \frac{u_1}{\sqrt{\langle u_1, u_1\rangle}} = \frac{u_1}{\sqrt{\sum_{i=1}^{n} u_i^2}} = \frac{u_1}{\sqrt{6}} = \frac{1}{\sqrt{6}}(1,2,-1)$

$v_2' = u_2 + a v_1 =$

$\langle v_2', v_1\rangle = 0 \Rightarrow \langle u_2 + a v_1, v_1 \rangle = 0 \Rightarrow$

$\Rightarrow a = -\langle u_2, v_1\rangle = -\frac{1}{\sqrt{6}} \cdot \langle (2,-1,1)(1,2,-1)\rangle =$

$= -\frac{1}{\sqrt{6}}(2-2-1) = \frac{1}{\sqrt{6}}$

$v_2' = (2,-1,1) + \frac{1}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}}(1,2,-1) = (2,-1,1) + \frac{1}{6}(1,2,-1) =$

$= \left(2 + \frac{1}{6}, -1 + \frac{2}{6}, 1 - \frac{1}{6}\right) = \left(\frac{13}{6}, -\frac{4}{6}, \frac{5}{6}\right) = \frac{1}{6}(13,-4,5)$

$\|v_2'\| = \frac{1}{6}\sqrt{13^2 + (-4)^2 + 5^2} = \frac{1}{6}\sqrt{169+16+25} = \frac{1}{6}\sqrt{210}$

$v_2 = \frac{1}{\|v_2'\|} \cdot v_2' = \frac{6}{\sqrt{210}} \cdot \frac{1}{6}(13,-4,5) = \frac{1}{\sqrt{210}}(13,-4,5)$

b) $v_1 = \frac{u_1}{\|u_1\|} = \frac{u_1}{\sqrt{\sum_{i=1}^{n} u_i^2}} = \frac{1}{\sqrt{2}} \cdot u_1 = \frac{1}{\sqrt{2}} \cdot (0,1,1)$

$v_2' = u_2 + a v_1$ $\langle v_2', v_1\rangle = 0 \Leftrightarrow$

$\Leftrightarrow \langle u_2 + a v_1, v_1\rangle = 0 \Leftrightarrow a = -\langle u_2, v_1\rangle =$

$= -\frac{1}{\sqrt{2}}\langle (1,0,1), (0,1,1)\rangle = -\frac{1}{\sqrt{2}}(0+0+1) \Rightarrow a = -\frac{1}{\sqrt{2}}$

$v_2' = (1,0,1) + \left(-\frac{1}{\sqrt{2}}\right)(0,1,1) \cdot \frac{1}{\sqrt{2}} = (1,0,1) - \frac{1}{2}(0,1,1) =$

$= \left(1, -\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2}(2,-1,1)$

$\|v_2'\| = \frac{1}{2}\sqrt{6}$

$v_2 = \frac{1}{\|v_2'\|} \cdot v_2' = \frac{2}{\sqrt{6}} \cdot \frac{1}{2}(2,-1,1) = \frac{1}{\sqrt{6}}(2,-1,1)$

$v_3' = u_3 + a v_1 + b v_2$

$\langle v_3', v_1\rangle = 0 \Rightarrow a = -\langle u_3, v_1\rangle = -\langle (1,1,0), \frac{1}{\sqrt{2}}(0,1,1)\rangle$

$\langle v_3', v_2\rangle = 0 \Rightarrow b = -\langle u_3, v_2\rangle = -\langle (1,1,0), \frac{1}{\sqrt{6}}(2,-1,1)\rangle$

$a = -(0 + \frac{1}{\sqrt{2}} + 0) = -\frac{1}{\sqrt{2}}$

$b = -\frac{1}{\sqrt{6}}$

$v_3' = (1,1,0) - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}(0,1,1) - \frac{1}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}}(2,-1,1) =$

$= \left(1 - \frac{2}{6}, 1 - \frac{1}{2} + \frac{1}{6}, -\frac{1}{2} - \frac{1}{6}\right)$

$= \frac{1}{6}(4,4,-4) = \frac{4}{6}(1,1,-1)$

$\|v_3'\| = \frac{4}{6}\sqrt{1^2 + 1^2 + (-1)^2} = \frac{2}{3}\sqrt{3}$

$v_3 = \frac{1}{\|v_3'\|} \cdot v_3' = \frac{6}{4\sqrt{3}} \cdot \frac{4}{6} \cdot (1,1,-1) = \frac{1}{\sqrt{3}}(1,1,-1)$

* $Sp_{\mathbb{R}}\{u_1, u_2\} = Sp_{\mathbb{R}}\{v_1, v_2\}$ ← plan vect. (dim = 2)

ex. $S = \{x \in \mathbb{R}^3 \,/\, x_1 + x_2 - 2x_3 = 0\}$

Să se determine o bază ortonormată a lui $S/\mathbb{R}$

$x_1 + x_2 - 2x_3 = 0 \Rightarrow x_2 = -x_1 + 2x_3$

$S = \{(x_1, -x_1 + 2x_3, x_3) \,/\, x_1, x_3 \in \mathbb{R}\}$

$= \{x_1(1,-1,0) + x_3(0,2,1) \,/\, x_1, x_3 \in \mathbb{R}\}$

$= \langle \{\underbrace{(1,-1,0)}_{u_1}, \underbrace{(0,2,1)}_{u_2}\}\rangle$

* $v_1 = \frac{u_1}{\sqrt{\langle u_1, u_1\rangle}} = \frac{u_1}{\sqrt{1^2 + (-1)^2 + 0^2}} = \frac{1}{\sqrt{2}} \cdot (1,-1,0)$

$v_2' = u_2 + v_1 a$ $\langle v_2', v_1\rangle = 0 \Rightarrow$

$\Rightarrow \langle u_2 + a v_1, v_1\rangle = 0 \Rightarrow a = -\langle u_2, v_1\rangle =$

$= -\langle (0,2,1), \frac{1}{\sqrt{2}}(1,-1,0)\rangle = -\frac{1}{\sqrt{2}}(0-2+0)$

$\frac{2}{\sqrt{2}}$

$v_2' = (0,2,1) + \frac{2}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}(1,-1,0) = (1,1,1)$

$\|v_2'\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$v_2 = \frac{1}{\|v_2'\|} \cdot v_2' = \frac{1}{\sqrt{3}} \cdot (1,1,1)$

Propoziție Fie $(V/\mathbb{R}, \langle \, , \, \rangle)$ spațiu vectorial euclidian și $S \leq_{\mathbb{R}} V$ a.î. $\dim_{\mathbb{R}} V = n$, $n \in \mathbb{N}^*$

Atunci $S^{\perp} = \{y \in V \,/\, \langle x, y\rangle = 0 \ \forall x \in S\}$ este subspațiu vectorial în $V/\mathbb{R}$, $V = S \oplus S^{\perp}$

fie $u \in S \cap S^{\perp} \Rightarrow u \in S$ și $u \in S^{\perp}$

$\left.\begin{array}{l} u \in S^{\perp} \Rightarrow \langle x, u\rangle = 0, \forall x \in S \\ u \in S \end{array}\right| \Rightarrow \langle u, u\rangle = 0 \Rightarrow u = 0$

$S \cap S^{\perp} = \{0\}$

$\dim_{\mathbb{R}} S = m \Rightarrow \exists \{u_1, \dots, u_m\} \subset S$ bază ortonormată în $S/\mathbb{R}$

fie $z \in V$ Căutăm $x \in S$ și $y \in S^{\perp}$ a.î. $z = x + y$ $\Big| \Rightarrow$

$\Downarrow$ $\exists!\, \alpha_1, \dots, \alpha_n \in \mathbb{R}$ $x = \sum_{i=1}^{n} \alpha_i u_i$

$\Rightarrow z = \left(\sum_{i=1}^{m} \alpha_i u_i\right) + y$

fie $j = \overline{1, m}$

$\langle z, u_j\rangle = \langle y + \sum_{i=1}^{m} \alpha_i u_i, u_j\rangle = \langle y, u_j\rangle + \sum_{i=1}^{m} \alpha_i \langle u_i, u_j\rangle = \sum_{i=1}^{n} \alpha_i \delta_{ij} = \alpha_j \Rightarrow \alpha_j = \langle z, u_j\rangle$

$\left.\begin{array}{l} y \in S^{\perp} \\ u_j \in S \end{array}\right| \Rightarrow \langle y, u_j\rangle = 0$

$z = \sum_{i=1}^{m} \langle z, u_i \rangle u_i \in S$

$y = z - \sum_{i=1}^{m} \langle z, u_i \rangle u_i$

Fie $k = \overline{1,m}$ $\langle y, u_k \rangle = \langle z - \sum_{i=1}^{m} \langle z, u_i \rangle u_i, u_k \rangle =$

$= \langle z, u_k \rangle - \sum_{i=1}^{m} \langle z, u_i \rangle \langle u_i, u_k \rangle =$

$= \langle z, u_k \rangle - \sum_{i=1}^{m} \langle z, u_i \rangle \delta_{ik} = \langle z, u_k \rangle - \langle z, u_k \rangle = 0$

$\forall k \in \overline{1,m}$ $\langle y, u_k \rangle = 0$

$\{u_1, \dots, u_m\}$ bază în $S/\mathbb{R}$ $\Big| \Rightarrow \forall x \in S, \langle y, x \rangle = 0 \Rightarrow$

$\Rightarrow y \in S^{\perp}$

proiecția pe $S$

$\mathcal{P}_S : V \to V$ $\mathcal{P}_S(z) = \sum_{i=1}^{m} \langle z, u_i \rangle u_i$

EX. fie $S = \{x \in \mathbb{R}^3 / 2x_1 + x_2 - x_3 = 0\}$

Să se determine ~~o bază~~ $S^{\perp}$ și să se găsească proiecțiile ortogonale ale vectorului $u = (1,2,3)$ pe $S$ și pe $S^{\perp}$

Var1

$2x_1 + x_2 - x_3 = 0 \Rightarrow x_3 = +2x_1 + x_2$

$S = \{(x_1, x_2, -2x_1 - x_2) / x_1, x_2 \in \mathbb{R}\}$

$= \{x_1(1, 0, +2), x_2(0, 1, +1) / x_1, x_2 \in \mathbb{R}\}$

$= \langle \{ \underset{u_1}{(1, 0, -2)}, \underset{u_2}{(0, 1, -1)} \} \rangle$

$y \in S^{\perp} \Rightarrow \begin{cases} \langle y, u_1 \rangle = 0 \\ \langle y, u_2 \rangle = 0 \end{cases} \Leftrightarrow \begin{cases} y_1 + 2y_3 = 0 \Rightarrow y_1 = -2y_3 \\ y_2 + y_3 = 0 \Rightarrow y_2 = -y_3 \end{cases}$

$S^{\perp} = \{(-2y_3, -y_3, y_3) / y_3 \in \mathbb{R}\} =$

$= \{y_3(-2, -1, 1) / y_3 \in \mathbb{R}\} =$

$= \langle \{(-2, -1, 1)\} \rangle = \langle \{ \underbrace{(2, 1, -1)}_{v} \} \rangle$

coef din ec planului

$\dim S^{\perp} = 1$

! vectorii formați din coef din ec planului ~~dat~~ generează $S^{\perp}$

$\|v\| = \sqrt{6}$

$w = \frac{1}{\sqrt{6}}(2, 1, -1)$

$\mathcal{P}_{S^{\perp}}(u) = \langle u, w \rangle \cdot w =$

$= \langle (1,2,3), \frac{1}{\sqrt{6}}(2,1,-1) \rangle \cdot \frac{1}{\sqrt{6}}(2,1,-1) =$

$= \frac{1}{6}(2 + 2 - 3)(2,1,-1) = \frac{1}{6} \cdot 1 (2,1,-1) =$

$= \frac{1}{6}(2,1,-1)$

$\mathcal{P}_S(u) + \mathcal{P}_{S^{\perp}}(u) = u \Rightarrow \mathcal{P}_S(u) = u - \mathcal{P}_{S^{\perp}}(u) =$

$= (1,2,3) - \frac{1}{6}(2,1,-1) = (1 - \frac{2}{6}, 2 - \frac{1}{6}, 3 + \frac{1}{6}) =$

$= (\frac{4}{6}, \frac{11}{6}, \frac{19}{6}) = \frac{1}{6}(4, 11, 19)$

verifică ec.

$2 \cdot 4 + 11 - 19 = 0$

Var2 - mai puțin practică

$v_1 = (1,0,2)$

$v_2 = (0,1,1)$

$S = \langle \{v_1, v_2\} \rangle$

$u_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{5}} \cdot (1,0,2)$

$u_2' = v_2 + a u_1$

$a = -\langle v_2, u_1 \rangle = -\langle (0,1,1), \frac{1}{\sqrt{5}}(1,0,2) \rangle =$

$= -\frac{1}{\sqrt{5}}(0 + 0 + 2) = -\frac{2}{\sqrt{5}}$

$u_2' = v_2 + a u_1 = (0,1,1) - \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}}(1,0,2) =$

$= (0,1,1) - \frac{2}{5}(1,0,2) = (0 - \frac{2}{5}, 1, 1 - \frac{4}{5}) =$

$= (-\frac{2}{5}, 1, \frac{1}{5}) = \frac{1}{5}(-2, 5, 1)$

$\|u_2'\| = \frac{1}{5}\sqrt{4 + 25 + 1} = \frac{\sqrt{30}}{5}$

$u_2 = \frac{1}{\|u_2'\|} u_2' = \frac{5}{\sqrt{30}} \cdot \frac{1}{5}(-2, 5, 1) = \frac{1}{\sqrt{30}}(-2, 5, 1)$

$\mathcal{P}_S((1,2,3)) = \langle (1,2,3), u_1 \rangle u_1 + \langle (1,2,3), u_2 \rangle u_2 =$

$= \frac{1}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} \langle (1,2,3), (1,0,2) \rangle \cdot (1,0,2) +$

$+ \frac{1}{\sqrt{30}} \cdot \frac{1}{\sqrt{30}} \langle (1,2,3), (-2,5,1) \rangle (-2,5,1) =$

$= \frac{1}{5}(1 + 0 + 6)(1,0,2) + \frac{1}{30}(-2 + 10 + 3)(-2,5,1) =$

$= \frac{7}{5}(1,0,2) + \frac{11}{30}(-2,5,1) =$

$= \frac{1}{30}(42 - 22, 55, 84 + 11) = \frac{1}{30}(20, 55, 95)$

$= \frac{1}{6}(4, 11, 19)$