CURS II ANALIZA MATEHATICA I
-SPERANTA Critical law Cauchy

25.02.2022

bemous tratic.

Description ca f este integrabilà in seus queralizad \Rightarrow thui $\int_a^c f(x) dx$ Fi $\varepsilon > 0$, arteitrate, f(x) dx + Abunci: $\Rightarrow f c_{\varepsilon} \in (a, b)$ as tel ca $\int_a^c f(x) dx - \mathcal{L} = \varepsilon$, $\forall c \in (c_{\varepsilon}, b)$ and $d := \int_a^b f(x) dx \in \mathbb{R}$

 $\Rightarrow \forall c', c'' \in (c_{\varepsilon}, b) \text{ arew: } c' \in c''$ $\left| \int_{a}^{c''} f(x) dx - \int_{a}^{c} f(x) dx \right| \leq \left| \int_{a}^{c'} f(x) dx - \mathcal{L} \right|$

Asadar, $\forall \varepsilon > 0$, $\exists c_{\varepsilon} \in (a, b)$ aster theat $\forall c', c'' \in (c_{\varepsilon}, b)$, $\left| \int_{\alpha}^{c''} f(x) dx - \int_{\alpha}^{c'} f(x) dx \right| \leq \varepsilon$

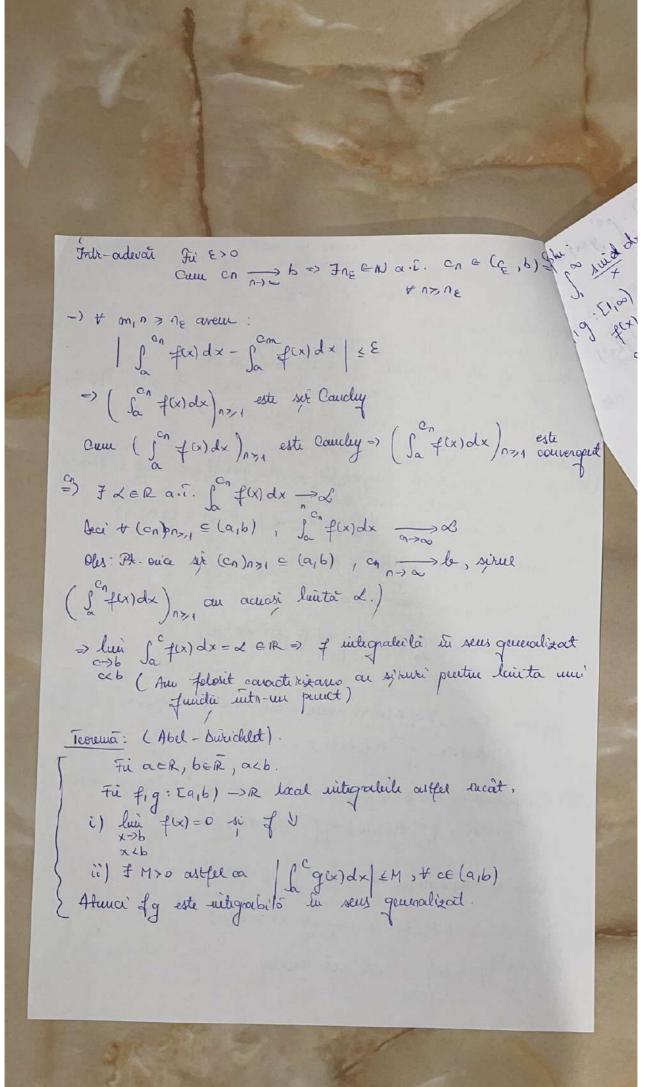
[] Presupureur ca: + & so J c & E(a,b) a.i. + c', c" & (c ,b) areur:

| Sa f(x)dx - Sa f(x)dx | = | Sa f(x)dx | < & O.

Vrem so demonstrain ca 7 lui ja fixide eR

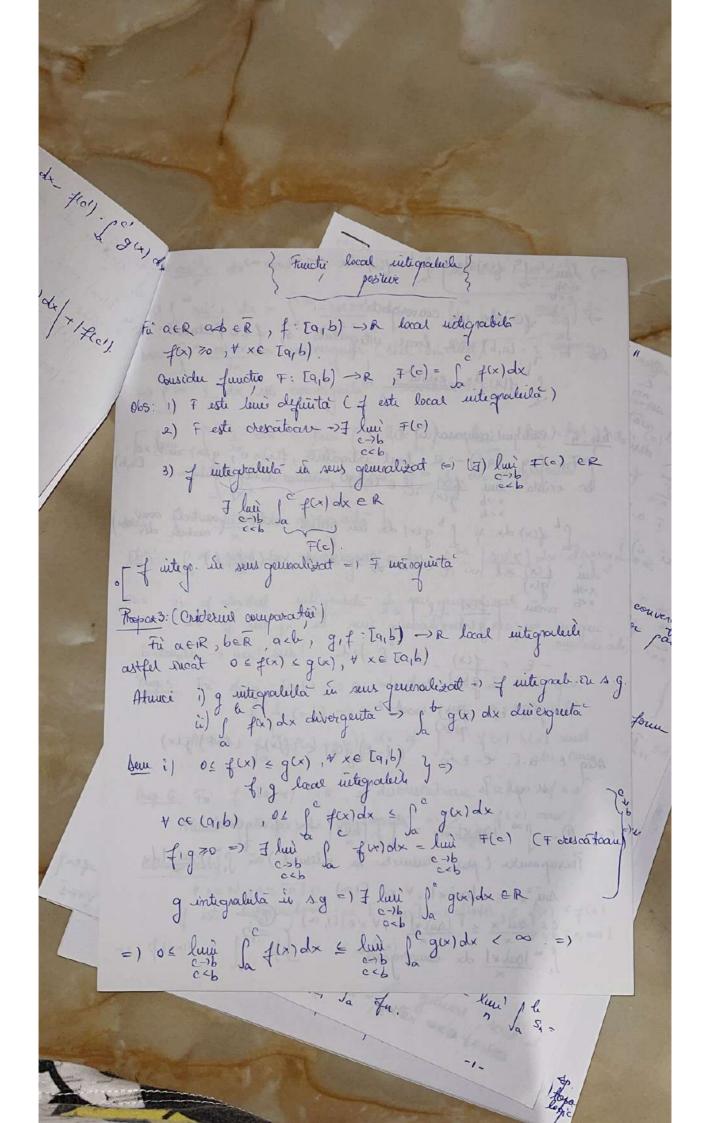
Fü (cn)n>1 \subseteq (a,b), cn $\xrightarrow{n\to\infty}$ b (arlettron)

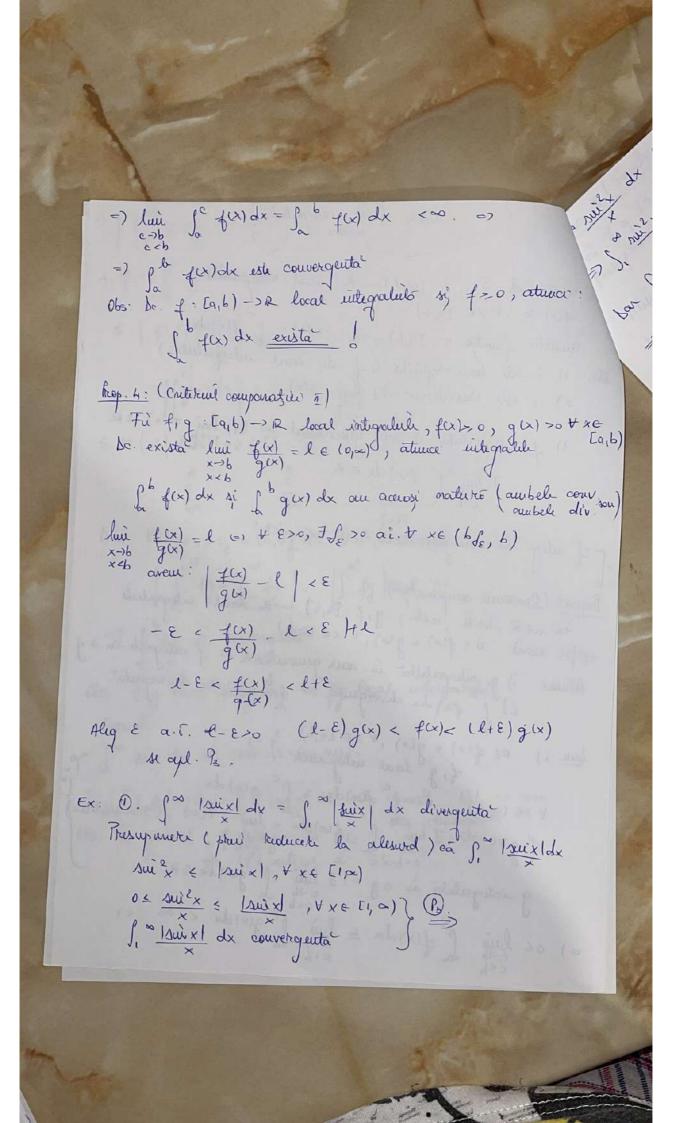
(Sa f(x)dx) est sir Cauchy

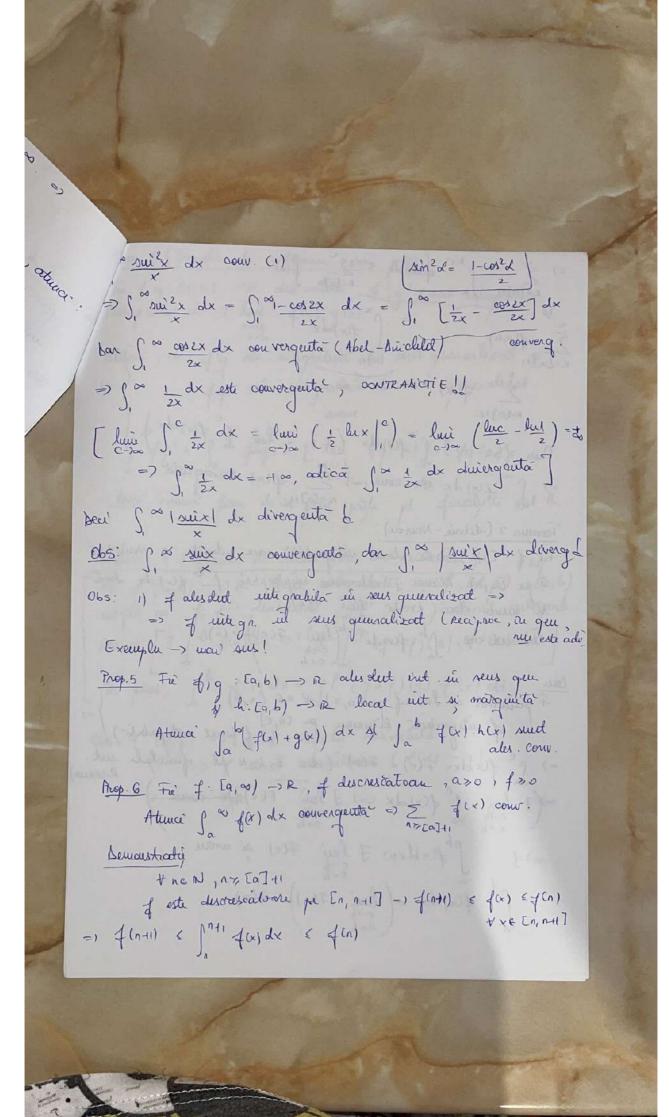


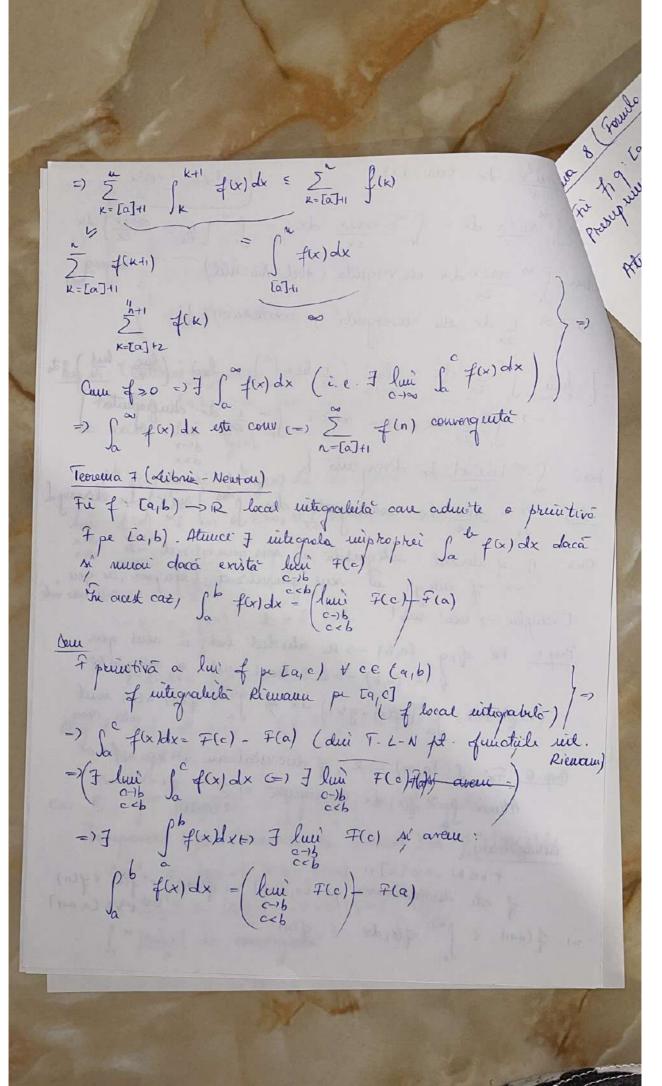
Co e CE D John . J. suid dx este convergenta \$19:[1,∞) →R $f(x) = \frac{1}{x}$ $\int_{0}^{x} f(x) dx = \int_{0}^{x} f(x) dx = \int_{0}^$ = | col 1 - col c | + 5 + cc (1,00) · frg continue => fig local integrabile =) f g integralisai en seus generalizat (1 suix dx esta couvergenta") obs In gund, fig integrabilà ru seus generalizat # fg intigrale in seus gen Obs: Jo suix dx este convergentat si Jo suix dx = 5/2 I' suix de convergentac $f: [0,1] \rightarrow \mathbb{R}$ $f(x) - \begin{cases} \frac{\sin x}{x}, & x \in [0,1] \\ 1, & x = 0 \end{cases}$ Souroustratie (Criterial lui Abel - Dirichlet) Cum lui f(x)=0. => + E>0, 7 GE (a,b) a.i. + xe(cE,b), 17(x) | EE Oles: of este descrescatoare, lui f(x)=0 => f(x)>0 + Atunci, fre d, c" & (CE, b), c'cc" Pe [d, e'], f v, f, g uitegrolule Rumann =>7 3 e (d, c") a ?. $\int_{c}^{c} f(x)g(x) dx = f(d) \int_{c}^{\epsilon} g(x) dx + f(c'') \cdot \int_{\epsilon}^{c''} g(x) dx$ done terminal

=) | [(" f(x)g(x) dx | = | f(c)) [g(x) dx - f(c)) . [c'g(x) d + f(c) p g(x)dx- f(c") p g(x)dx = < | f(d) |, | = g(x) dx |+ |f(c')|. | | = g(x) dx |+ |f(d). · || f(x) dx | + | f(d) | . | fa g(x) dx | < < M(2| f(c)) | +2 | f(c)) | = & M. XE = E Deci + E>0, 7 & e (a,b) a.t. + e', e' e (a,b), e'<011 | \int fg - \int fg |= | \int fg | \le \(\varepsilon \) =) | fg este convergenta Obs: f,g local integraleile -) fg local integrabila · Joseph dx este cour. (mai sus) Je | suix | dx = Je | suix | dx estr comp NU => +-Obs: In general, of nitign in SG. * If I witign in SG.









ua 8 (Formelo de mitigran prin parte) Fi f, q: Eq, b) -> R delivabeli, f', g' local integrabile Presupenin ca 7 lini f(x) g(x) Atunci, de 1/9 este integrabile in seus generalizat se ∫ fg' + ∫a f'g = [lui (fg)(-)] - fg(a) Deur Foloson form de vot-print pointe pt functione ent R Teoreuse 9 (Formulo de solui leone de romabilo) pluj.

Fri f: [a,b) -> R local integrabilo si f: [d, B) -> [a,b)
astful ca 1, f-1 divirabile ,/ f', (p-1)' local integrale. Atuci [7 jb fx)dx (=)] [(fof)(t) &'(t) dt] In plus, 1 f(x) dx = SP (fof) (t) f'(t) dt. Obs: f: [x,p) -> [a,b) sijecti dirabilà

=> f strict oresættore (f(x) =a)

Functile Gama of Beta Fruitzio Gamena (a lui Euler) by: $P:(0,\infty) \rightarrow (0,\infty)$ $P(\infty) = \int_{-\infty}^{\infty} e^{-t} dt, \forall <>0.$ M s n. functio Camuna (a lui Fuler) Prop 1) 17 este lui definité Prop 1) 17 (1) = 1 (lo e t dt = lui lo e t dt) d) M(1/2) = TT 3) p(a+1) = ap(a) + a>0 h) M(n) = (n-1) / + nen, n > 1 5) + de (0,1), M(2) M(1-x) = 1 sui rd Fructio Bota B: (0,00) -> (0,00) B(a,b) = 1 + a-1(1-t) off + a,beco,00) Oles: 1) B este leur définité d) Ac. a>1 si b>1 l' t a-! (1-t) b-1 dt proprent Prop: 1) B(a,b) = B(b,a), + a, b e lo, a) 2) \$ B (9,6) = \(\frac{17(a)}{17(a+b)} \), + a,b >0. $B(1/2, 1/2) = \frac{\Gamma(1/2) \Gamma(1/2)}{\Gamma(1)} = (\Gamma(1/2))^{\frac{1}{2}}$ B(1/2, 1/2) = 1 1 x 1/2-1 (1-t) 1/2-1 dt= $=\int_0^1 \frac{1}{\sqrt{t-t^2}} dt = \int_0^1 \frac{1}{\sqrt{t-t^2}} dt$