MEGFP :

Rezolvați sistemul de ecuații liniare

$$\begin{cases} 4x_1 - x_2 + x_3 = 8 \\ 2x_1 + 5x_2 + 2x_3 = 3 \\ x_1 + 2x_2 + 4x_3 = 11 \end{cases}$$

folosind MEGFP și metoda substituției descendente.

$$A = \begin{bmatrix} 4 & -1 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 4 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 8 \\ 3 \\ 11 \end{bmatrix} \Rightarrow \underline{n=3}$$

$\underline{k=1}$:

$$\overline{A} \equiv \overline{A}^{(1)} = \left[\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 2 & 5 & 2 & 3 \\ 1 & 2 & 4 & 11 \end{array}\right] = \left[A^{(1)} \ \underline{b}^{(1)}\right]$$

$a_{11}^{(1)} = 4 \neq 0$ (i.e. putem aplica MEGFP)

$\underline{i=\overline{2,3}}$: $m_i^{(1)} := a_{i1}^{(1)} / a_{11}^{(1)}$

$\bullet\ m_2^{(1)} = a_{21}^{(1)} / a_{11}^{(1)} = \frac{2}{4} = \frac{1}{2}$

$(E_2 - m_2^{(1)} E_1) \longrightarrow (E_2)$:

$\underline{j=\overline{2,3}}$: $a_{2j}^{(2)} = a_{2j}^{(1)} - m_2^{(1)} a_{1j}^{(1)}$

$a_{22}^{(2)} = a_{22}^{(1)} - m_2^{(1)} a_{12}^{(1)}$

$= 5 - \frac{1}{2}(-1) = 11/2$

$a_{23}^{(2)} = a_{23}^{(1)} - m_2^{(1)} a_{13}^{(1)}$

$= 2 - \frac{1}{2} 1 = 3/2$

$a_{21}^{(2)} = 0$ (Nu mai trebuie calculat!

$m_2^{(1)}$ se alege ai $a_{21}^{(2)} = 0$)

$b_2^{(2)} = b_2^{(1)} - m_2^{(1)} b_1^{(1)} = 3 - \frac{1}{2} 8 = -1$

$\bullet\ m_{31}^{(1)} = a_{31}^{(1)} / a_{11}^{(1)} = \frac{1}{4}$

$(E_3 - m_3^{(1)} E_1) \longrightarrow (E_3)$:

$\underline{j=\overline{2,3}}$: $a_{3j}^{(2)} = a_{3j}^{(1)} - m_3^{(1)} a_{1j}^{(1)}$

$a_{32}^{(2)} = a_{32}^{(1)} - m_3^{(1)} a_{12}^{(1)} =$

$= 2 - \frac{1}{4}(-1) = 9/4$

$$a_{33}^{(2)} = a_{33}^{(1)} - m_3^{(1)} a_{13}^{(1)}$$

$$= 4 - \frac{1}{4} 1 = 15/4$$

$a_{31}^{(2)} = 0$ (aceeași observație ca și cea pt $a_{21}^{(1)}$)

$$b_3^{(2)} = b_3^{(1)} - m_3^{(1)} b_1^{(1)} = 11 - \frac{1}{4} 8 = 9$$

Am obținut:

<u>$k=2$</u>:

$$\overline{A}^{(2)} = \left[\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 0 & 11/2 & 3/2 & -1 \\ 0 & 9/4 & 15/4 & 9 \end{array}\right] =: \left[A^{(2)} \ \underline{b}^{(2)}\right]$$

<u>Obs</u>: Matricea $M^{(1)}$ care transformă

$\overline{A} \equiv \overline{A}^{(1)} = \left[A^{(1)} \ \underline{b}^{(1)}\right]$ în

$\overline{A}^{(2)} = \left[A^{(2)} \ \underline{b}^{(2)}\right]$ este dată de:

$$M^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/4 & 0 & 1 \end{bmatrix}$$

Mai exact, are loc relația:

$$M^{(1)} \left[A^{(1)} \ \underline{b}^{(1)}\right] = \left[A^{(2)} \ \underline{b}^{(2)}\right] \qquad (1)$$

$a^{(2)}_{22} = 11/2 \neq 0$ (putem aplica MEGFP)

$\underline{i=\overline{3,3}:}$ $\quad m^{(2)}_{i} := a^{(2)}_{i2} / a^{(2)}_{22}$

- $m^{(2)}_{3} := a^{(2)}_{32} / a^{(2)}_{22} = \frac{9}{4}\,\frac{2}{11} = 9/22$

$(E_3 - m^{(2)}_{3} E_2) \longrightarrow (E_3):$

$\underline{j=\overline{3,3}}:$ $\quad a^{(3)}_{3j} := a^{(2)}_{3j} - m^{(2)}_{3}\, a^{(2)}_{2j}$

$$a^{(3)}_{33} = a^{(2)}_{33} - m^{(2)}_{3}\, a^{(2)}_{23}$$

$$= \frac{15}{4} - \frac{9}{22}\,\frac{3}{2} = \frac{15\cdot 11 - 27}{44} = \frac{138}{44}$$

$$= 69/22$$

$a^{(3)}_{32} = 0$

$b^{(3)}_{3} := b^{(2)}_{3} - m^{(2)}_{3}\, b^{(2)}_{2} = 9 - \frac{9}{22}(-1) = \frac{9\cdot 23}{22}$

$= 207/22$

Am obtinut:

$$\overline{A}^{(3)} = \left[\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 0 & 11/2 & 3/2 & -1 \\ 0 & 0 & 69/22 & 207/22 \end{array}\right] = [A^{(3)}\ \underline{b}^{(3)}]$$

$$\equiv [U\ \underline{\tilde{b}}]$$

<u>Obs</u>: Matricea $M^{(2)}$ care transformă

$\overline{A}^{(2)} = [\underline{A}^{(2)}\ \underline{b}^{(2)}]$ în $\overline{A}^{(3)} = [A^{(3)}\ \underline{b}^{(3)}] = [U\ \tilde{\underline{b}}]$

este dată de:

$$M^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 9/22 & 1 \end{bmatrix}$$

Mai exact, are loc relația:

$$M^{(2)}[A^{(2)}\ \underline{b}^{(2)}] = [A^{(3)}\ \underline{b}^{(3)}] = [U\ \tilde{\underline{b}}] \quad (2)$$

Din relațiile (1) și (2) obținem:

$$\boxed{M^{(2)} M^{(1)} [\underline{A}\ \underline{b}] = [\underline{U}\ \tilde{\underline{b}}]}$$

<u>Obs</u>: Sistemul $A\underline{x} = \underline{b}$ devine

$U\underline{x} = \tilde{\underline{b}}$, ie

$$\begin{cases} 4x_1 - x_2 + x_3 = 8 \\ \frac{11}{2}x_2 + \frac{3}{2}x_3 = -1 \\ \frac{69}{22}x_3 = \frac{207}{22} \end{cases}$$

și acestea se rezolvă prin metoda substituției descendente, i.e. de la ultima ecuație la prima ec:

$$\frac{69}{22}x_3 = \frac{207}{22} \iff x_3 = \frac{207}{22}\cdot\frac{22}{69} \iff$$

$$\boxed{x_3 = 3}$$

$$\frac{11}{2}x_2 + \frac{3}{2}x_3 = -1 \iff \frac{11}{2}x_2 = -1 - \frac{3}{2}x_3$$

$$\frac{11}{2}x_2 = -1 - \frac{3}{2}3 \iff \frac{11}{2}x_2 = -\frac{11}{2} \iff$$

$$\boxed{x_2 = -1}$$

$$4x_1 - x_2 + x_3 = 8 \iff$$

$$4x_1 = 8 + x_2 - x_3 \iff$$

$$4x_1 = 8 - 1 - 3 \iff 4x_1 = 4 \iff \boxed{x_1 = 1}$$