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Tutoriat 2 - Geometrie I

$\boxed{\text{Ex 1}}$. Fie $M(2,4)$ și S_M simetria centrală.

a) Să se scrie ecuația simetriei centrale.

b) Să se determine $S_M(d)$, unde $d: x-y+3=0$.

$\boxed{\text{SOL}}$

a) $S_M: X'=-X+2X_0 \Rightarrow S_M: \begin{pmatrix} x' \\ y' \end{pmatrix} = -\begin{pmatrix} x \\ y \end{pmatrix} + 2\cdot\begin{pmatrix} 2 \\ 4 \end{pmatrix} \Leftrightarrow$

$\Leftrightarrow S_M: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -x+4 \\ -y+8 \end{pmatrix} \Leftrightarrow S_M: \begin{cases} x' = -x+4 \\ y' = -y+8 \end{cases}$

b) $S_M(d) = ?$

Inversa lui S_M este: $(S_M)^{-1} \begin{cases} x = -x'+4 \\ y = -y'+8 \end{cases}$ $(S_M = S_M^{-1})$

$(d'): (-x'+4)-(-y'+8)+3=0$

$(d'): -x'-y'-1=0 \Leftrightarrow (d'): x'-y'+1=0.$

$\boxed{\text{SAU}}$

$M(2,4)\in d? \Rightarrow 2-4+3=1\neq 0 \Rightarrow M\notin d \Rightarrow d \parallel d'$, unde $S_M(d)=d'$.

Avem: $(d'): x-y+c=0$

Alegem $P(0,3)\in d \Rightarrow S_M(P)=P'(4,5)\in d'$

$P'(4,5)\in d' \Rightarrow 4-5+c=0 \Rightarrow \boxed{c=1}$

$d': x-y+1=0$

$\boxed{Ex\,2}$. Fie $d_1: x+2y+1=0$ și $M=(1,1)$. Să se determine $S_d(M)=M'$.

$\boxed{SOL}$: $(d_1): x+2y+1=0 \Rightarrow \begin{cases} a=1 \\ b=2 \\ c=1 \end{cases}$

$$S_d: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \frac{b^2-a^2}{a^2+b^2} & \frac{-2ab}{a^2+b^2} \\ \frac{-2ab}{a^2+b^2} & \frac{a^2-b^2}{a^2+b^2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \frac{-2ac}{a^2+b^2} \\ \frac{-2bc}{a^2+b^2} \end{pmatrix}$$

$$S_d: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & -\frac{3}{5} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -\frac{2}{5} \\ -\frac{4}{5} \end{pmatrix} \Rightarrow$$

$$\Rightarrow S_d: \begin{cases} x' = \frac{3}{5}x - \frac{4}{5}y - \frac{2}{5} \\ y' = -\frac{4}{5}x - \frac{3}{5}y - \frac{4}{5} \end{cases}$$

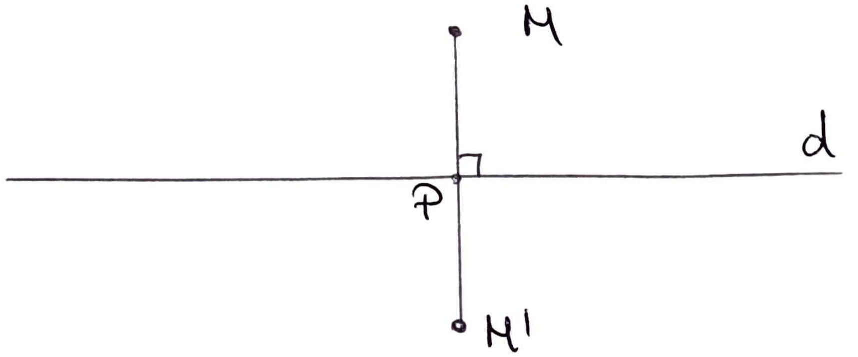
$$S_d(M) = M'\left(\frac{3}{5}\cdot 1 - \frac{4}{5}\cdot 1 - \frac{2}{5} ; -\frac{4}{5}\cdot 1 - \frac{3}{5}\cdot 1 - \frac{4}{5}\right) =$$

$$= M'\left(\frac{3}{5} - \frac{4}{5} - \frac{2}{5} ; -\frac{4}{5} - \frac{3}{5} - \frac{4}{5}\right) =$$

$$= M'\left(-\frac{3}{5}, -\frac{11}{5}\right)$$

$\boxed{sau}$

$M(1,1)$, $(d_1): x+2y+1=0$



$\vec{n}_{d_1} = (1,2)$, dar $MM' \perp d_1 \Rightarrow \vec{u}_{MM'} = (1,2)$

$(MM'): \frac{x-1}{1} = \frac{y-1}{2} \Leftrightarrow 2x-2 = y-1$

$2x - y - 1 = 0$

$(MM') \cap (d_1) = \{P\} \Rightarrow \begin{cases} 2x-y-1=0 \,/\cdot 2 \\ x+2y+1=0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} 4x-2y-2=0 \\ x+2y+1=0 \end{cases}$ $(+)$

$5x - 1 = 0 \Rightarrow x = \frac{1}{5} \Rightarrow y = 2x-1 = \frac{2}{5} - 1 \Rightarrow y = \frac{-3}{5}$

$P(1/5, -3/5)$ este mijlocul lui $[MM'] \Rightarrow$

$\Rightarrow \begin{cases} \frac{1}{5} = \frac{x+1}{2} \\ -\frac{3}{5} = \frac{y+1}{2} \end{cases} \Rightarrow \begin{cases} 2 = 5x+5 \\ -6 = 5y+5 \end{cases} \Rightarrow \begin{cases} x = -\frac{3}{5} \\ y = -\frac{11}{5} \end{cases} \Rightarrow$

$\Rightarrow M'\left(-3/5\,;\, -11/5\right)$

$\boxed{Ex\,3}$. Se dă vectorul $\vec{v} = (3,1)$. Se cere:

a) ecuația translației

b) $T_{\vec{v}}(A) = A'$, unde $A = (1,-3)$

c) $T_{\vec{v}}(d) = d'$, unde $d: 3x+4y-1=0$

$\boxed{SOL}$

a) $T_{\vec{v}}: X' = X + X_0 \Leftrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} \Leftrightarrow$

$\Leftrightarrow T_{\vec{v}}: \begin{cases} x' = x+3 \\ y' = y+1. \end{cases}$

b) $T_{\vec{v}}(A) = A'(1+3; -3+1) = A'(4, -2)$

c) $T_{\vec{v}}: \begin{cases} x' = x+3 \\ y' = y+1 \end{cases} \Rightarrow \begin{cases} x = x'-3 \\ y = y'-1 \end{cases}$

$(d'): 3(x'-3) + 4(y'-1) - 1 = 0$

$(d'): 3x' - 9 + 4y' - 4 - 1 = 0$

$(d'): 3x' + 4y' - 14 = 0.$

$\boxed{\text{Ex 4}}$ Fie $(d): 2x+y+4=0$, $(d_1): 2x+y-5=0$ și $M(1,1)$, $(d_2): x+y+1=0$. Se cere: $S_d(M)$, $S_d(d_1)$, $S_d(d_2)$.

$\boxed{\text{SOL}}$: Scriem mai întâi S_d (ecuațiile).

$(d): 2x+y+4=0 \Rightarrow \begin{cases} a=2 \\ b=1 \\ c=4 \end{cases}$

$$S_d: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -\frac{16}{5} \\ -\frac{8}{5} \end{pmatrix}$$

$$S_d: \begin{cases} x' = -\frac{3}{5}x - \frac{4}{5}y - \frac{16}{5} \\ y' = -\frac{4}{5}x + \frac{3}{5}y - \frac{8}{5} \end{cases}$$

Acum putem afla ce ni se cere.

$M(1,1) \Rightarrow S_d(M) = M'\left(-\frac{3}{5} - \frac{4}{5} - \frac{16}{5}; -\frac{4}{5} + \frac{3}{5} - \frac{8}{5}\right) =$

$= M'\left(-\frac{23}{5}; -\frac{9}{5}\right)$

$(d_1): 2x+y-5=0.$

Se observă faptul că $d_1 \parallel d \Rightarrow d \parallel S_d(d_1)=d_1'$

$S_d^{-1}=S_d$: (involuție) $\begin{cases} x=-\frac{3}{5}x'-\frac{4}{5}y'-\frac{16}{5} \\ y=-\frac{4}{5}x'+\frac{3}{5}y'-\frac{8}{5} \end{cases}$

$(d_1'): 2\left(-\frac{3}{5}x'-\frac{4}{5}y'-\frac{16}{5}\right)+\left(-\frac{4}{5}x'+\frac{3}{5}y'-\frac{8}{5}\right)-5=0 \,/\cdot 5$

$(d_1'): 2(-3x'-4y'-16)+(-4x'+3y'-8)-25=0$

$(d_1'): -6x'-8y'-32+(-4x')+3y'-8-25=0$

$(d_1'): -10x'-5y'-65=0 \,/:(-5)$

$(d_1'): 2x'+y'+13=0 \quad (d_1'=S_d(d_1) \parallel d)$

$(d_2): x+y+1=0.$

$$\begin{cases} x+y+1=0 \\ 2x+y+4=0 \end{cases} \quad (-)$$

$-x-3=0 \Rightarrow x=-3 \Rightarrow y=+2 \Rightarrow A(-3,2)=d_2\cap d.$

$S_d(d_2)=d_2'$

$(d_2'): -\frac{1}{5}(3x'+4y'+16)-\frac{1}{5}(4x'-3y'+8)+1=0 \,/\cdot 5$

$(d_2'): -3x'-4y'-16-4x'+3y'-8+5=0$

$(d_2'): -7x'-y'-19=0$

$(d_2'): 7x'+y'+19=0.$

$\boxed{Ex5}$. Fie $M(1,1)$ și $M'(2,3)$

a) Să se arate că $S_{M'} \circ S_M = T_{\vec{u}}$. Aflați $\vec{u}$.

b) Fie $T_{\vec{u}}(d) = d'$, unde $d: x + 2y - 4 = 0$

Calculați aria triunghiului determinat de dreptele d' și de axele de coordonate.

$\boxed{SOL}$

a) $S_M: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix} + 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} \Leftrightarrow \begin{cases} x' = -x + 2 \\ y' = -y + 2 \end{cases}$

$S_{M'}: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix} + 2\begin{pmatrix} 2 \\ 3 \end{pmatrix} \Leftrightarrow \begin{cases} x' = -x + 4 \\ y' = -y + 6 \end{cases}$

$S_{M'} \circ S_M: (x, y) \xrightarrow{S_M} (-x+2, -y+2) \xrightarrow{S_{M'}}$

$(-(-x+2)+4; -(-y+2)+6) = (x-2+4; y-2+6) =$

$= (x+2; y+4) \Rightarrow T_{\vec{u}}: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix} \Rightarrow \vec{u} = (2, 4)$

b) $T_{\vec{u}}: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix} \Leftrightarrow T_{\vec{u}}: \begin{cases} x' = x + 2 \\ y' = y + 4 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x = x' - 2 \\ y = y' - 4 \end{cases}$

$d: x + 2y - 4 = 0$

$(d'): x' - 2 + 2y' - 8 - 4 = 0$

$(d'): x' + 2y' - 14 = 0$.

Două metode pentru a afla intersecția cu axele de coordonate:

$\boxed{\text{Metoda 1}}$

$$\begin{cases} d \cap Ox \Rightarrow A(x, 0) \\ d \cap Oy \Rightarrow B(0, y) \end{cases}, \text{ unde } \begin{cases} (Ox): y = 0 \\ (Oy): x = 0 \end{cases}$$

$\boxed{\text{Metoda 2}}$

Ecuația prin tăieturi a lui d':

$$x' + 2y' - 14 = 0 \Rightarrow x' + 2y' = 14 \quad | :14$$

$$\frac{x'}{14} + \frac{y'}{7} = 1 \Rightarrow \begin{cases} A(14, 0) \\ B(0, 7) \end{cases}$$

Acum, pentru $A_{\Delta AOB}$ avem tot 2 metode:

$\boxed{\text{Metoda 1}}$ $A_{\Delta AOB} = \dfrac{c_1 \cdot c_2}{2} = \dfrac{14 \cdot 7}{2} = 49$

$\boxed{\text{Metoda 2}}$ $A_{\Delta AOB} = \dfrac{1}{2} \cdot |\Delta| = \dfrac{1}{2} \cdot 14 \cdot 7 = 49.$

$$|\Delta| = \left|\begin{vmatrix} 14 & 0 & 1 \\ 0 & 7 & 1 \\ 0 & 0 & 1 \end{vmatrix}\right| = 14 \cdot 7$$