Facultatea de Matematica si Informatica
Universitatea din Bucuresti

12 Mai 2020

Restanta EDP I (online)

Disciplina: Ecuatii cu derivate partiale
Tipul examinarii: Restanta (scris)
Nume student: ______________________________
Seriile 30, 31, 32
Timp de lucru : 2 ore si 30 min (incluzand atasarea rezolvarilor pe Moodle)

Acest examen contine 4 probleme (toate obligatorii).

Examenul este individual. Nu uitati sa va salvati foile cu rezolvarile subiectelor intr-un singur fisier de tip PDF in timp util astfel incat sa va incadrati in cele 2 ore si 30 minute pentru incarcarea fisierului pe platforma Moodle.

Salvati fisierul PDF creat cu numele vostru (Nume_Prenume_Grupa.pdf).

Pentru elaborarea lucrarii scrise puteti folosi orice materiale ajutatoare.

Pentru redactare tineti cont de urmatoarele sugestii:

- Daca folositi o teorema fundamentala, rezultat cunoscut, etc **indicati** acest lucru si explicati cum se poate aplica rezultatul respectiv.
- **Organizati-va munca** intr-un mod coerent pentru a avea toti de castigat ! Incercati ca la crearea fisierului PDF fiecare problema sa fie redactata in ordinea aparitiei pe foaia cu subiecte. Ideal ar fi ca si subpunctele sa fie redactate in ordine. Daca nu stiti a rezolva un anumit subpunct scrieti numarul subpunctului si lasati liber.
- Raspunsurile corecte dar argumentate incomplet (din punct de vedere al calculelor/explicatiilor) vor primi punctaj partial.

Barem: P1 (2.5p) + P2 (2.5p)+ P3 (2.5p) +P4 (2.5p) + 1p oficiu= **11p** (Se pleaca din nota 11).

Rezultatele finale vor fi postate pe Moodle in cel mai scurt timp posibil, dar dupa proba orala. Pentru orice nelamuriri scrieti-mi la adresa cristian.cazacu@fmi.unibuc.ro, sau lasati un mesaj pe chat-ul grupului "Restanta EDP I" creat pe Microsoft Teams.

Problema 1. *(2.5p). Consideram functia* $u : \mathbb{R}^4 \setminus \{0\} \to \mathbb{R}$ *data de*

$$u(x) = |x|^{-\frac{1}{3}}, \quad x = (x_1, \dots, x_4),$$

1). *Calculati Laplacianul lui* u *folosind eventual formula Laplacianului pentru functii radiale si evaluati apoi* $\Delta u(1,1,1,1)$.

2). *Gasiti* $\lambda \in \mathbb{R}$ *astfel incat*

$$\operatorname{div}(|x|^2 \nabla u(x)) = \lambda u, \quad \forall x \in \mathbb{R}^4 \setminus \{0\}.$$

3). *Sa se determine pentru ce valori* $p \geq 1$ *functia* $w : \mathbb{R}^4 \setminus \{0\} \to \mathbb{R}$, *definita prin* $w(x) := \frac{|u(x)|^p}{1+|x|}$, *apartine lui* $L^1(B_1(0))$, *unde* $B_1(0)$ *este bila unitate din* $\mathbb{R}^4$ *centrata in origine.*

4). *Sa se determine pentru ce valori* $p \geq 1$ *are loc* $w \in L^1(\mathbb{R}^4 \setminus \overline{B_1(0)})$.

5). *Aratati ca*

$$\operatorname{div}\left(\frac{x}{|x|^2}\right) = \frac{2}{|x|^2}, \quad \forall x \in \mathbb{R}^4 \setminus \{0\}.$$

Problema 2. *(2.5p). Fie* $\Omega := \{(x,y) \in \mathbb{R}^2; \quad x^2 + y^2 < 4\}$ *si* $\partial\Omega$ *frontiera lui* Ω. *Fie problema*

$$(1) \qquad \begin{cases} -\Delta u(x,y) = \frac{2}{1+\sin^2 x}, & (x,y) \in \Omega \\ u(x,y) = 0, & (x,y) \in \partial\Omega \end{cases}$$

1). *Aratati ca problema* (1) *are cel mult o solutie* $u \in C^2(\Omega) \cap C(\overline{\Omega})$.

2). *Aratati ca solutia* u *a problemei* (1) *este functie para in raport cu* x *si calculalti* $u_x(0,0)$.

3). *Gasiti constanta* C *astfel incat functia* $v(x,y) = C(x^2+y^2)$ *sa verifice* $-\Delta v = 2$ *in* Ω.

4). *Folosind eventual principiul de maxim pentru functii armonice sa se determine solutia problemei*

$$(2) \qquad \begin{cases} -\Delta w(x,y) = 2, & (x,y) \in \Omega \\ w(x,y) = 0, & (x,y) \in \partial\Omega. \end{cases}$$

5). *Folosind eventual principiul de maxim pentru functii sub/super armonice sa se arate ca solutia problemei* (1) *verifica*

$$|u(x,y)| \leq 2, \quad \forall (x,y) \in \overline{\Omega}.$$

Problema 3. *(2.5p). Consideram urmatoarea problema de tip "unde"*

$$(3) \qquad \begin{cases} u_{tt}(x,t) + 3u_{tx}(x,t) - 4u_{xx}(x,t) = 0, & x \in \mathbb{R}, t > 0, \\ u(x,0) = f(x), & x \in \mathbb{R}, \\ u_t(x,0) = g(x), & x \in \mathbb{R}, \end{cases}$$

unde $f, g \in C^2(\mathbb{R})$ *sunt functii date.*

1). *Aratati ca daca* $v = v(x,t)$ *este o functie de clasa* C^2 *atunci*

$$(4) \qquad (\partial_t - \partial_x)(v_t(x,t) + 4v_x(x,t)) = v_{tt}(x,t) + 3v_{tx}(x,t) - 4v_{xx}(x,t), \quad \forall x, \forall t.$$

2). Rezolvati problema cu valori initiale (3) *satisfacuta de u (scrieti forma generala a lui u) reducand-o la rezolvarea a doua ecuatii de transport (una omogena si alta neomogena).*

3). Folosind conditiile la $t = 0$ *deduceti solutia u a problemei* (3) *in cazul particular* $f(x) = \cos x$ *si* $g(x) = e^{-x}$.

Problema 4. *(2.5p). Consideram problema Cauchy*

$$\begin{cases} u_t(x,t) - u_{xx}(x,t) + \frac{t^2}{t^3+1}u(x,t) = 0, & x \in \mathbb{R},\ t > 0, \\ u(x,0) = e^{-x}, & x \in \mathbb{R}. \end{cases} \tag{5}$$

1). Gasiti o functie $\phi : \mathbb{R} \to \mathbb{R}$ *astfel incat functia* $v(x,t) := u(x,t)\phi(t)$ *sa verifice ecuatia caldurii*

$$v_t(x,t) - v_{xx}(x,t) = 0, \quad \forall x \in \mathbb{R},\ \forall t > 0. \tag{6}$$

2). Scrieti problema Cauchy verificata de v si calculati $v(0,1)$.

3). Determinati explicit solutia problemei (5).

Neagu Georgiana - Liliana
Grupa 301

12.05.2020

Restanță EDP

① $u: \mathbb{R}^4 \setminus \{0\} \to \mathbb{R}$

$u(x) = |x|^{-\frac{1}{3}}$, $x = (x_1, \dots, x_4)$

1) $\Delta u(1,1,1,1) = ?$

$$\Delta(u(x)) = \Delta\left(|x|^{-\frac{1}{3}}\right) = -\frac{1}{3}|x|^{-\frac{1}{3}-2} \cdot \left(-\frac{1}{3}+4-2\right)$$

$$= -\frac{1}{3}|x|^{-\frac{7}{3}} \cdot \left(-\frac{1}{3}+2\right) = -\frac{1}{3}\cdot\frac{5}{3}|x|^{-\frac{7}{3}} = -\frac{5}{9}|x|^{-\frac{7}{3}}$$

$$\Delta u(1,1,1,1) = -\frac{5}{9}\left(\sqrt{1^2+1^2+1^2+1^2}\right)^{-\frac{7}{3}} =$$

$$= -\frac{5}{9}\, 2^{-\frac{7}{3}}$$

2) $\operatorname{div}\left(|x|^2 \nabla u(x)\right) = \lambda u$

$$\nabla(u(x)) = \nabla\left(|x|^{-\frac{1}{3}}\right) = -\frac{1}{3}\cdot x \cdot |x|^{-\frac{1}{3}-2} =$$

$$= -\frac{1}{3}\cdot x\cdot |x|^{-\frac{7}{3}}$$

4)

$$(2)\begin{cases} -\Delta w(x,y) = 2 & ,(x,y)\in\Omega \\ w(x,y) = 0 & ,(x,y)\in\partial\Omega \end{cases}$$

Fie w soluția problemei (2).

Definim $z = w - v$

$\Rightarrow -\Delta z = -\Delta(w-v) = \Delta v - \Delta w = -2+2=0$

$\Rightarrow$ z armonică

$$z\big|_{\partial\Omega} = w\big|_{\partial\Omega} - v\big|_{\partial\Omega} = 0-(-2) = 2$$

Din Principiul de maxim avem:

z armonică $\Rightarrow$ z își atinge maximul și minimul pe frontieră

$$2 = \min_{\partial\Omega} z \le z(x,y) \le \max_{\partial\Omega} z = 2 \Rightarrow z \equiv 2$$

$\Rightarrow w - v = 2 \Rightarrow w = 2+v \Rightarrow$

$w(x,y) = 2 - \frac{1}{2}(x^2+y^2)$

3) $c = ?$ $v(x,y) = c(x^2+y^2)$

$-\Delta v = 2$ στο Ω

$$\frac{\partial v}{\partial x} = 2c \cdot x \Rightarrow \frac{\partial^2 v}{\partial x^2} = 2c$$

$$\frac{\partial v}{\partial y} = 2cy \Rightarrow \frac{\partial^2 v}{\partial y^2} = 2c$$

$$\Rightarrow \Delta v = 2c + 2c = 4c$$

$$\Rightarrow -\Delta v = 2$$

$$-4c = 2$$

$$c = -\frac{1}{2}$$

$$\Rightarrow v(x,y) = -\frac{1}{2}(x^2+y^2)$$

② (1) $\begin{cases} -\Delta u(x,y) = \frac{2}{1+\sin^2 x} & , (x,y) \in \Omega \\ u(x,y) = 0 & , (x,y) \in \partial\Omega \end{cases}$

1) (1) are cel mult o soluție $u \in \mathcal{C}^2(\Omega) \cap \mathcal{C}(\bar{\Omega})$

Presupunem u_1, u_2 soluții ale lui (1).

$\begin{cases} \Delta u_1 = \Delta u_2 \\ u_1|_{\partial\Omega} = u_2|_{\partial\Omega} = 0 \end{cases} \iff \begin{cases} -\Delta(u_1 - u_2) = 0 \\ (u_1 - u_2)|_{\partial\Omega} = 0 \end{cases}$

$\Rightarrow \int_\Omega \Delta U \cdot v = \int_{\partial\Omega} \frac{\partial U}{\partial \nu} \cdot v \, d\sigma - \int_\Omega \nabla U \cdot \nabla v \, dx$

Fie $U = v = u_1 - u_2 \Rightarrow \Delta U = 0$ și $v|_{\partial\Omega} = 0$

$\Rightarrow 0 = 0 - \int_\Omega |\nabla(u_1 - u_2)|^2 dx \Rightarrow \int_\Omega |\nabla(u_1 - u_2)|^2 dx = 0$

$\Rightarrow \nabla(u_1 - u_2) = 0 \Rightarrow u_1 - u_2$ este constantă pe Ω

③ Dar $u_1 - u_2|_{\partial\Omega} = 0 \xrightarrow[\text{până la } \partial\Omega]{\text{continuă}} u_1 - u_2 = 0 \Rightarrow$

Fie $\omega = u + w \Rightarrow -\Delta\omega = -\Delta u + (-\Delta w) =$

$$= \frac{2}{1+\sin^2 x} + 2 \geq 0 \Rightarrow \omega \text{ super armonică}$$

$x \in [0,2)$

$$\xrightarrow[\text{pt. super-armonice}]{\text{Principiul de maxim}} \inf_{\overline{\Omega}} \omega = \inf_{\partial\Omega} \omega = 0$$

$$\Rightarrow \omega(x,y) \geq 0, \ \forall (x,y) \in \overline{\Omega}$$

$$\Rightarrow u(x,y) \geq -w(x,y) = -2 + \frac{1}{2}(x^2+y^2) \geq -2$$

$$\forall (x,y) \in \overline{\Omega}$$

$$\Rightarrow u(x,y) \geq -2, \ \forall (x,y) \in \overline{\Omega} \quad (2)$$

Din (1) & (2) $\Rightarrow |u(x,y)| \leq 2$

5) $|u(x,y)| \le 2, \ \forall (x,y) \in \overline{\Omega}$

$|u(x,y)| \le 2 \iff -2 \le u(x,y) \le 2$

Fie w ca la 4) și $\varphi = w - u$

$$-\Delta\varphi = -\Delta w - (-\Delta u) = 2 - \frac{2}{1+\sin^2(x)} \geq 0$$

$x \in [0,2), \ \forall (x,y) \in \Omega$

$\Rightarrow \varphi$ e super armonică $\xrightarrow[\text{pentru funcții super-armonice}]{\text{Principiul de maxim}}$

$$\inf_{\overline{\Omega}} \varphi = \inf_{\partial\Omega} \varphi = 0$$

$\Rightarrow \varphi(x,y) \geq 0, \ \forall (x,y) \in \overline{\Omega} \Rightarrow$

$w - u \geq 0, \ \forall (x,y) \in \overline{\Omega}$ (Din 1) $u = v$)

$\Rightarrow \underbrace{2 - \frac{1}{2}(x^2+y^2)}_{\geq 2} \geq u(x,y), \ \forall (x,y) \in \overline{\Omega}$

$\Rightarrow 2 \geq u(x,y), \ \forall (x,y) \in \overline{\Omega}$ (1)

$$\begin{cases} u(x,0) = f(x) \\ u_t(x,t) + 4u_x(x,t) = g(x+t) + 4f'(x,t) \end{cases}$$

$$w(s) = u(x+4s, t+s)$$

$$w'(s) = u_x(x+4s, t+s)\cdot 4 + u_t(x+4s, t+s)\cdot 1$$

$$= g(x+t+5s) + 4f'(x+t+5s)$$

$$w'(\tau) = g(x+t+5\tau) + 4f'(x+t+5\tau) \xrightarrow{\int_0^s d\tau}$$

$$\int_0^s w'(\tau)\,d\tau = \int_0^s \left[g(x+t+5\tau) + 4f'(x+t+5\tau)\right] d\tau$$

$$w(s) - w(0) = \int_0^s \left[g(x+t+5\tau) + 4f'(x+t+5\tau)\right] d\tau$$

$$\|$$

$$u(x,t) = w(s) - \int_0^s \left[g(x+t+5\tau) + 4f'(x+t+5\tau)\right] d\tau$$

$$= u(x+4s, t+s) - \int_0^s g(x+t+5\tau)\,d\tau -$$

$$- 4\int_0^s f'(x+t+5\tau)\,d\tau$$

$$\overset{s=-t}{=} \underbrace{u(x-4t, 0)}_{f((x-4t))} - \int_0^{-t} g(x+t+5\tau)\,d\tau - 4\int_0^{-t} f'(x+t+5\tau)\,d\tau$$

12

Notăm $v = u_t + 4u_x \Rightarrow \begin{cases} v_t - v_x = 0 \\ v(x,0) = u_t(x,0) + \\ \quad + 4u_x(x,0) \\ \quad = g(x) + 4f'(x) \end{cases}$

$$-v_x + v_t = 0 \Leftrightarrow \underbrace{(-1,1)\cdot \nabla v}_{\substack{\parallel \\ (v_x, v_t)}} = 0$$

$\Rightarrow v$ e constantă pe direcția $(-1,1)$

$$v(x,t) = v((x,t)) = v((x+t-t,t)) =$$

$$= v((x+t,0) + \underbrace{(-t,t)}_{\substack{\parallel \\ t(-1,1)}}) = v(x+t,0) =$$

$$= g(x+t) + 4f'(x+t)$$

$$\Rightarrow \underbrace{u_t(x,t) + 4u_x(x,t) = g(x+t) + 4f'(x,t)}_{\text{Prima ecuație de transport}}$$

$$= \frac{e^{\frac{4t(t-x)}{4t}}}{2\sqrt{t}} = \frac{e^{t-x}}{2\sqrt{t}}$$

$$\Rightarrow v(x,t) = \frac{e^{t-x}}{2\sqrt{t}}$$

2) $v(0,1) = ?$

$$v(0,1) = \frac{e^{1-0}}{2} = \frac{e}{2}.$$

Continuare (1)

4) $w \in L^p(\mathbb{R}^4 \setminus \overline{B_1(0)}) \Leftrightarrow$

$$\int_{\mathbb{R}^4 \setminus B_1(0)} |w(x)|^p \, dx < \infty \Leftrightarrow \int_{\mathbb{R}^4 \setminus \overline{B_1(0)}} \frac{|u(x)|^p}{1+|x|} \, dx < \infty$$

$$\int_{\mathbb{R}^4 \setminus \overline{B_1(0)}} \frac{|x|^{-\frac{p}{3}}}{1+|x|} \, dx \overset{\text{Formula}}{\underset{\text{Co-Ariei}}{=}} \int_1^\infty \left(\int_{\partial B_s(0)} \frac{|x|^{-p/3}}{1+|x|} \, d\sigma \right) ds =$$

$$= \int_1^\infty \frac{s^{-p/3}}{1+s} \, |\partial B_s(0)| \, ds = W_4 \int_1^\infty \frac{s^{-p/3+4}}{1+s} < \infty$$

$$\Leftrightarrow -\frac{p}{3} + \overset{3}{\cancel{4}} < 0 \Leftrightarrow -p < -12 \Leftrightarrow p > 12$$

(17)

$$\frac{y = 5\tau + x + t}{dy = 5\, d\tau} \quad f((x-4t)) - \frac{1}{5}\int_{x+t}^{x-4t} g(y)\,dy - \frac{4}{5}\int_{x+t}^{x-4t} f'(y)\,dy$$

$$= f((x-4t)) - \frac{4}{5} f((x-4t)) + \frac{4}{5} f((x+t)) - \frac{1}{5}\int_{x+t}^{x-4t} g(y)\,dy$$

$$= \frac{4f(x+t) + f(x-4t)}{5} - \frac{1}{5}\int_{x+t}^{x-4t} g(y)\,dy$$

a doua ecuație

3) $f(x) = \cos x$

$g(x) = e^{-x}$

$$\int_{x-4t}^{x+t} g(y)\,dy = \int_{x-4t}^{x+t} e^{-y}\,dy = -e^{-y}\Big|_{x-4t}^{x+t} =$$

$$= -e^{-x-t} + e^{4t-x}$$

$$\Rightarrow u(x,t) = \frac{4\cos(x+t) + \cos(x-4t)}{5} - \frac{1}{5}\left(-e^{-x-t} + e^{4t-x}\right)$$

13

$$\ln|\Phi(t)| = \int \frac{t^2}{t^3+1}\,dt$$

$$\underset{dz=3t^2dt}{\overset{z=t^3+1}{=\!=}} \frac{1}{3}\int \frac{1}{z}\,dz = \frac{1}{3}\ln|z| + C =$$

$$= \frac{\ln(t^3+1)}{3} + C$$

$$\Rightarrow |\Phi(t)| = e^{\frac{\ln(t^3+1)}{3} + C} \text{, Alég } C=0$$

$$\Rightarrow \Phi(t) = e^{\frac{\ln(t^3+1)}{3}} = e^{\ln(t^3+1)^{1/3}} = (t^3+1)^{\frac{1}{3}}$$

3) $v(x,t) = u(x,t)\cdot e^{\frac{\ln(t^3+1)}{3}}$

$$v(x,0) = u(x,0)\cdot e^{0} = u(x,0) = e^{-x}$$

$$\begin{cases} v_t(x,t) - v_{xx}(x,t) = 0 \\ v(x,0) = e^{-x} = v_0(x) \end{cases}$$

$$v(x,t) = \frac{1}{\sqrt{4\pi t}} \int_{\mathbb{R}} e^{-\frac{|x-y|^2}{4t}}\, e^{-y}\, dy$$

④ (5) $\begin{cases} u_t(x,t) - u_{xx}(x,t) + \dfrac{t^2}{t^3+1} u(x,t) = 0 \\ u(x,0) = e^{-x} \end{cases}$ $x \in \mathbb{R}, t > 0$

1) $\Phi : \mathbb{R} \to \mathbb{R}$, $v(x,t) = u(x,t) \cdot \Phi(t)$

(6) $v_t(x,t) - v_{xx}(x,t) = 0$

$$v_t(x,t) = u_t(x,t) \cdot \Phi(t) + u(x,t) \cdot \Phi'(t)$$

$$v_{xx}(x,t) = u_{xx}(x,t) \cdot \Phi(t)$$

$$u_t(x,t) \cdot \Phi(t) + u(x,t) \cdot \Phi'(t) - u_{xx}(x,t) \cdot \Phi(t) = 0$$

$$\Phi(t) \left(-\frac{t^2}{t^3+1}\right) \cdot u(x,t) + u(x,t) \cdot \Phi'(t) = 0 \; \Big| : u(x,t)$$

$$\Phi(t) \left(-\frac{t^2}{t^3+1}\right) + \Phi'(t) = 0$$

$$\Phi'(t) = \frac{t^2}{t^3+1} \Phi(t)$$

$$\frac{\Phi'(t)}{\Phi(t)} = \frac{t^2}{t^3+1} \quad \Big| \int dt$$

14

$$|x|^2 \cdot \nabla(u(x)) = |x|^2 \cdot \left(-\frac{1}{3}\right) \cdot x \cdot |x|^{-\frac{4}{3} - \frac{3}{2}}$$

$$= -\frac{1}{3} x |x|^{-\frac{7}{3} + \frac{3}{2}}$$

$$= -\frac{1}{3} |x|^{-\frac{1}{3}} \cdot x$$

$$\Rightarrow \operatorname{div}\left(|x|^2 \cdot \nabla u\right) = \operatorname{div}\left(-\frac{1}{3} \cdot x \cdot |x|^{-\frac{1}{3}}\right) =$$

$$= -\frac{1}{3} \operatorname{div}\left(x \cdot |x|^{-\frac{1}{3}}\right) =$$

$$= -\frac{1}{3}\left(\overset{3}{4} - \frac{1}{3}\right) |x|^{-\frac{1}{3}} = -\frac{1}{3} \cdot \frac{11}{3} |x|^{-\frac{1}{3}} =$$

$$= -\frac{11}{9} |x|^{-\frac{1}{3}}$$

Cum $\operatorname{div}\left(|x|^2 \nabla u(x)\right) = \lambda u$

$$\Rightarrow \lambda = -\frac{11}{9}$$

5) $\operatorname{div}\left(\frac{x}{|x|^2}\right) = \frac{2}{|x|^2}$, $\forall x \in \mathbb{R}^4 \setminus \{0\}$

$$\operatorname{div}\left(x \cdot |x|^{-2}\right) = (4-2)|x|^{-2} = 2|x|^{-2} =$$

$$= \frac{2}{|x|^2} \quad \text{A}$$

$\Rightarrow u_1 = u_2 \Rightarrow$ (1) are cel mult o soluție

2) (1) are soluție pară

$u_x(0,0) = ?$

$u(-x,y) = u(x,y)$

Fie $v(x,y) = u(-x,y)$

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial x}(-x,y) \Rightarrow$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 u}{\partial x^2}(-x,y)$$

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial y}(-x,y) \Rightarrow$$

$$\frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 u}{\partial y^2}(-x,y)$$

$$\Rightarrow \Delta v(x,y) = \Delta u(-x,y)$$

$$\Delta u(-x,y) = -\frac{2}{1+\sin^2(-x)} = -\frac{2}{1+\sin^2 x} = \Delta u(x,y)$$

$$\Rightarrow \Delta v(x,y) = -\frac{2}{1+\sin^2 x}$$

$$v(x,y)\Big|_{\partial\Omega} = u\Big|_{\partial\Omega}(-x,y) = 0$$

$\Rightarrow$ v soluție pt. (1)

$$\overset{1)}{\Longrightarrow} u = v \Rightarrow u(-x,y) = u(x,y)$$

$\Rightarrow$ este pară.

$$u_x(0,0) = ?$$

Știm că $u(x,y) = u(-x,y) \Rightarrow$

$$\frac{\partial u}{\partial x}(x,y) = -\frac{\partial u}{\partial x}(-x,y)$$

$$u_x(0,0) = -u_x(0,0) \Rightarrow 2u_x(0,0) = 0 \mid :2$$

$$\Rightarrow u_x(0,0) = 0$$

5

(3)

$$(3)\begin{cases} u_{tt}(x,t)+3u_{tx}(x,t)-4u_{xx}(x,t)=0 \\ u(x,0)=f(x) \\ u_t(x,0)=g(x) \end{cases} \quad x\in\mathbb{R},\ t>0$$

1) $v=v(x,t)$

(4) $(\partial_t-\partial_x)(v_t(x,t)+4v_x(x,t)) =$

$= v_{tt}(x,t)+3v_{tx}(x,t)-4v_{xx}(x,t)$

$\partial_t(v_t(x,t))+\partial_t(4v_x(x,t))-\partial_x(v_t(x,t))-$

$-\partial_x(4v_x(x,t))=$

$= v_{tt}(x,t)+4v_{tx}(x,t)-v_{tx}(x,t)-4v_{xx}(x,t)$

$= v_{tt}(x,t)+3v_{tx}(x,t)-4v_{xx}(x,t)$ (A)

2) 2 ecuații de transport

Fie u soluția problemei (3) $\Rightarrow (\partial_t-\partial_x)(u_t+4u_x)=0$

$$\Rightarrow \int_{\mathbb{R}} e^{-(x-y)^2} e^{-y} \, dy =$$

$$= \int_{\mathbb{R}} e^{\frac{-x^2+2xy-y^2}{4t}} \cdot e^{-y} \, dy =$$

$$= e^{-\frac{x^2}{4t}} \int_{\mathbb{R}} e^{-\left(\frac{y+(2t-x)}{2\sqrt{t}}\right)^2} e^{\frac{(2t-x)^2}{4t}} \, dy \overset{*}{=}$$

$$\tau = \frac{y+(2t-x)}{2\sqrt{t}}$$

$$d\tau = \frac{1}{2\sqrt{t}} \, dy$$

$$\overset{*}{=} e^{\frac{-x^2+(2t-x)^2}{4t}} \int_{\mathbb{R}} e^{-\tau^2} \, d\tau = e^{\frac{-x^2+(2t-x)^2}{4t}} \cdot \sqrt{\pi}$$

$$\Rightarrow v(x,t) = \frac{1}{\sqrt{4\pi t}} \, e^{\frac{-x^2+(2t-x)^2}{4t}} \cdot \sqrt{\pi}$$

$$= \frac{e^{\frac{-\cancel{x^2}+4t^2-4tx+\cancel{x^2}}{4t}} \cdot \cancel{\sqrt{\pi}}}{2\cancel{\sqrt{\pi}} \cdot \sqrt{t}}$$