Factorizarea Doolittle

Determinați factorizarea Doolittle a matricei triunghiulare

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$A = \left[\begin{array}{c|cc} 2 & -1 & 0 \\ \hline -1 & 2 & -1 \\ 0 & -1 & 2 \end{array}\right] = \left[\begin{array}{c|cc} 1 & 0 & 0 \\ \hline l_{21} & 1 & 0 \\ 0 & l_{32} & 1 \end{array}\right] \left[\begin{array}{c|cc} u_{11} & u_{12} & 0 \\ \hline 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{array}\right]$$

$$\boxed{u_{11} = 2}$$

$$\boxed{u_{12} = -1}$$

$$l_{21} u_{11} = -1 \Rightarrow \boxed{l_{21} = -1/2}$$

$$A_{22} = L_{21} U_{12} + L_{22} U_{22} \Rightarrow$$

$$L_{22} U_{22} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} -1/2 \\ 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1/2 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 3/2 & -1 \\ -1 & 2 \end{bmatrix} \Rightarrow$$

$$\left[\begin{array}{c|c} 3/2 & -1 \\ \hline -1 & 2 \end{array}\right] = \left[\begin{array}{c|c} 1 & 0 \\ \hline l_{22} & 1 \end{array}\right] \left[\begin{array}{c|c} u_{22} & u_{23} \\ \hline 0 & u_{33} \end{array}\right] \Rightarrow$$

$$\boxed{u_{22} = 3/2}$$

$$\boxed{u_{23} = -1}$$

$$l_{32} u_{22} = -1 \Rightarrow \boxed{l_{32} = -2/3}$$

$$l_{32} u_{23} + u_{33} = 2 \Rightarrow u_{33} = 2 - l_{32} u_{23}$$

$$\Rightarrow u_{33} = 2 - \left(-\frac{2}{3}\right)(-1) \Rightarrow \boxed{u_{33} = 4/3}$$

Am obținut:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & -2/3 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 3/2 & -1 \\ 0 & 0 & 4/3 \end{bmatrix}$$