Ex I $\sup A = ?$ $\inf A = ?$
$\min A = ?$ $\max A = ?$ ptr. mulț. din $\mathbb{R}$:

1.1. $A = \left\{ (-1)^n \cdot \frac{1}{am + bn + c} \;\middle|\; m, n \in \mathbb{N} \right\}$

1.2. $B = \left\{ (-1)^n \cdot \frac{am + bn + 1}{m + c} \;\middle|\; m, n \in \mathbb{N} \right\}$

1.3. $C = \left\{ (-1)^{n+1} \cdot \frac{m + n}{am + 1} \;\middle|\; m, n \in \mathbb{N} \cup \{0\} \right\}$

Ex II ① Dem cu def că șirurile urm. sunt șiruri Cauchy:

1.1) $x_n = \frac{(\cos 1)^a}{4^2} + \frac{(\cos 2)^a}{4^4} + \dots + \frac{(\cos n)^a}{4^{2n}}$

1.2) $x_n = \frac{(\sin 1)^a}{3^2} + \frac{(\sin 2)^a}{3^4} + \dots \frac{(\sin n)^a}{3^{2n}}$

② Dem cu def. că șirurile urm. au limită:

2.1) $x_n = 2^n - \frac{1}{n^2} + 4$

2.2) $x_n = 3^n - \frac{2}{n+1} + 1$.

2.3) $x_n = \sqrt{n^2 + 1} - n$

2.4) $x_n = \sqrt{n^2 + 2} - n$

(3) $\lim x_n = ?$ pentru șirurile:

a) $x_1 = 1$, $x_{n+1} = 3x_n + a$, $n \geq 1$

b) $x_1 = 1$, $x_{n+1} = 4x_n + a$, $n \geq 1$

c) $x_1 = 2$ $x_{n+1} = 2 \cdot x_n + a$ $n \geq 1$.

III) $\underline{\lim}\, x_n = ?$ $\overline{\lim}\, x_n = ?$ pentru șirurile:

a) $x_n = \left(\frac{n+a}{n+5}\right)^n \cdot (-1)^n + (-1)^{n+1} + \sin \frac{n\pi}{2}$, $n \in \mathbb{N}$

b) $x_n = \left(\frac{n+a}{n-5}\right)^{2n} \cdot (-1)^{\frac{n(n+1)}{2}} + \cos \frac{n\pi}{2}$, $n \in \mathbb{N}$

c) $x_n = (-1)^{\frac{n(n+1)(n+2)}{3}} + \left(\frac{-1}{a}\right)^n + \cos n\pi$, $n \in \mathbb{N}$

d) $x_n = (-1)^{\frac{n(n+1)(n+2)}{3}} \cdot \left(\frac{1}{a}\right)^{n+1} + \sin n\pi$, $n \in \mathbb{N}$.

SUBIECT

III

① Studiați uniform continuitatea fct:

a) $f:(0,\infty)\to\mathbb{R}$, $f(x)=\frac{1}{x^2+x}$

b) $f:(0,\infty)\to\mathbb{R}$, $f(x)=\frac{2}{x^2+3x}$.

c) $f:[0,\infty)\to\mathbb{R}$, $f(x)=e^{-2x}+2$

d) $f:(-\infty,0]\to\mathbb{R}$, $f(x)=2e^{x+1}+x+1$.

e) $f:(-\infty,1)\to\mathbb{R}$ $f(x)=\frac{1}{x-1}$

f) $f:(-\infty,2)\to\mathbb{R}$, $f(x)=\frac{x}{x-2}$.

② Fie $f:\mathbb{R}\to\mathbb{R}$, $f(x)=\sqrt{x^2+a}$

Dem că: $f(x+1)-f(x)\le 1$, $\forall x\in\mathbb{R}$

SUBIECT

I 1. Natura seriei: $\sum\limits_{n\geq 1} \dfrac{\sqrt{a^n+3^n+1}}{6^{n+1}+n^2+n+1}$

2. Absolut convergenta seriei $\sum\limits_{n\geq 1} \dfrac{2^n+3}{(-a)^n+1}$

3. Absolut convergenta seriei $\sum\limits_{n\geq 1} \dfrac{(-a)^n-1}{8^n+5}$

4. Stabiliți în funcție de a și b natura seriei: $\sum\limits_{n\geq 1} \left(\dfrac{\alpha n+a}{\alpha n+b}\right)^{\alpha n}$

SUBIECT

II 1. $A=\left\{(-1)^n \dfrac{1}{a\cdot n} \mid n\in\mathbb{N}\right\}$;

2. $A=\left\{\left(-\dfrac{1}{a}\right)^n \mid n\in\mathbb{N}\right\}$

3. $A=\left\{\left(-\dfrac{1}{a+1}\right)^n \mid n\in\mathbb{N}\right\}$

4. $A=\left\{\dfrac{(-1)^n}{na} \mid n\in\mathbb{N}\right\}$

5. $A=\left\{\dfrac{(-1)^n}{n+a} \mid n\in\mathbb{N}\right\}$

Stabiliți: $\mathring{A}, \bar{A}, A', Fr A$, dacă:

- A mărginită
- A compactă
- A conexă.