

Test

Gk. 113

$$1. \Delta = \begin{vmatrix} 3 & -3 & 2 & 3 \\ -2 & 7 & -2 & 5 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 1 & 5 \end{vmatrix} = 3 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 7 & -2 & 5 \\ 0 & 3 & 7 \\ 0 & 1 & 5 \end{vmatrix} +$$

$$+ (-2) \cdot (-1)^{2+1} \cdot \begin{vmatrix} 3 & 2 & 3 \\ 0 & 3 & 7 \\ 0 & 1 & 5 \end{vmatrix} = 3(105 - 49) + 2(45 - 21) =$$

$$= 3 \cdot 56 + 2 \cdot 24 = 216 \Rightarrow \Delta = 216$$

$$2. \begin{vmatrix} 7 & 1 & 0 & 0 & \dots & 0 \\ 6 & 7 & 1 & 0 & \dots & 0 \\ 0 & 6 & 7 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 7 \end{vmatrix} = \det(A_n) = \Delta_n$$

$$(a) \Delta_2 = \begin{vmatrix} 7 & 1 \\ 6 & 7 \end{vmatrix} = 49 - 6 = 43$$

$$\Delta_3 = \begin{vmatrix} 7 & 1 & 0 \\ 6 & 7 & 1 \\ 0 & 6 & 7 \end{vmatrix} = 343 - 42 - 42 = \cancel{259} 259$$

$$\Delta_4 = \begin{vmatrix} 7 & 1 & 0 & 0 \\ 6 & 7 & 1 & 0 \\ 0 & 6 & 7 & 1 \\ 0 & 0 & 6 & 7 \end{vmatrix} = 7 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 7 & 1 & 0 \\ 6 & 7 & 1 \\ 0 & 6 & 7 \end{vmatrix} + 1 \cdot (-1)^{1+2} \cdot \begin{vmatrix} 6 & 1 & 0 \\ 0 & 7 & 1 \\ 0 & 6 & 7 \end{vmatrix}$$

$$= 7 \cdot \Delta_3 - (294 - 36) = 7 \cdot 259 - 258 = \cancel{1813} 1813$$

$$\Rightarrow \Delta_4 = \cancel{1555} 1555$$

$$(b) \Delta_m = \begin{vmatrix} 7 & 1 & 0 & 0 & \dots & 0 \\ 6 & 7 & 1 & 0 & \dots & 0 \\ 0 & 6 & 7 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 7 \end{vmatrix} = 7 \cdot (-1)^{1+1} \begin{vmatrix} 7 & 1 & 0 & \dots & 0 \\ 6 & 7 & 1 & \dots & 0 \\ 0 & 6 & 7 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 7 \end{vmatrix} +$$

$$+ 1 \cdot (-1)^{1+2} \begin{vmatrix} 6 & 1 & 0 & 0 & \dots & 0 \\ 0 & 6 & 7 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 7 \end{vmatrix} + 1 \cdot (-1)^{1+2} \begin{vmatrix} 6 & 1 & 0 & 0 & \dots & 0 \\ 0 & 7 & 1 & \dots & 0 \\ 0 & 0 & 6 & \dots & 0 \end{vmatrix}$$

$$+ 6 \cdot (-1)^{2+1} \begin{vmatrix} 1 & 0 & 0 & \dots & 0 \\ 6 & 7 & 1 & \dots & 0 \\ 0 & 6 & 7 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 7 \end{vmatrix} =$$

$$\Delta_m = 7 \cdot \Delta_{m-1} - 6 \cdot 1 \cdot (-1)^{1+1} \begin{vmatrix} 7 & 1 & 0 & 0 & \dots & 0 \\ 6 & 7 & 1 & 0 & \dots & 0 \\ 0 & 6 & 7 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 7 \end{vmatrix} =$$

$$\Delta_m = 7 \Delta_{m-1} - 6 \cdot \Delta_{m-2}$$

(c) Trebuie să arătăm că $\Delta_m = \frac{6^{m+1} - 1}{5}$, $\forall m \geq 2$

Cf. a): $\Delta_2 = 43 = \frac{6^3 - 1}{5}$

$\Delta_3 = 259 = \frac{6^4 - 1}{5}$

$\Delta_4 = 1555 = \frac{6^5 - 1}{5}$

(2)

(c) Pp. adăvărată $\Delta_n = \frac{6^{n+1} - 1}{5}$

cf. b):

$$\Delta_n = 7\Delta_{n-1} - 6\Delta_{n-2} = 7 \cdot \frac{6^n - 1}{5} - 6 \cdot \frac{6^{n-1} - 1}{5} =$$

$$= \frac{7 \cdot 6^n - 7}{5} - \frac{6^n - 6}{5} = \frac{6^n(7-1) - 1}{5} = \frac{6^n \cdot 6 - 1}{5} = \frac{6^{n+1} - 1}{5}$$

\Rightarrow presupunerea e adăvărată $\Rightarrow \Delta_n = \frac{6^{n+1} - 1}{5}$