Factorizarea Crout

Determinați factorizarea Crout a matricei triunghiulare

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$A = \left[\begin{array}{c|cc} 2 & -1 & 0 \\ \hline -1 & 2 & -1 \\ 0 & -1 & 2 \end{array}\right] = \left[\begin{array}{c|cc} l_{11} & 0 & 0 \\ \hline l_{21} & l_{22} & 0 \\ 0 & l_{32} & l_{33} \end{array}\right] \left[\begin{array}{c|cc} 1 & u_{12} & 0 \\ \hline 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{array}\right]$$

$$\boxed{l_{11} = 2}$$

$$\boxed{l_{21} = -1}$$

$$l_{11} u_{12} = -1 \Rightarrow \boxed{u_{12} = -1/2}$$

$$A_{22} = L_{21} U_{12} + L_{22} U_{22} \Rightarrow$$

$$L_{22} U_{22} = A_{22} - L_{21} U_{12} =$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} \begin{bmatrix} -1/2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1/2 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 3/2 & -1 \\ -1 & 2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} l_{22} & 0 \\ l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{23} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/2 & -1 \\ -1 & 2 \end{bmatrix} \Rightarrow$$

$$\boxed{l_{22} = 3/2} \qquad \boxed{l_{32} = -1}$$

$$l_{22} u_{23} = -1 \Rightarrow \frac{3}{2} u_{23} = -1 \Rightarrow \boxed{u_{23} = -2/3}$$

$$l_{32} u_{23} + l_{33} = 2 \Rightarrow l_{33} = 2 - (-1)\left(\frac{-2}{3}\right)$$

$$\Rightarrow \boxed{l_{33} = 4/3}$$

Am obtinut:

$$L = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 3/2 & 0 \\ 0 & -1 & 4/3 \end{bmatrix}, \qquad U = \begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 1 & -2/3 \\ 0 & 0 & 1 \end{bmatrix}$$