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Tutoriat 1

MEGFP

1. Rezolvați sist de ec. liniare, folosind MEGFP și metoda substituției descendente.

$$\begin{cases} 4x_1 - x_2 + x_3 = 8 \\ 2x_1 + 5x_2 + 2x_3 = 3 \\ x_1 + 2x_2 + 4x_3 = 11 \end{cases}$$

Sol

$$A = \begin{bmatrix} 4 & -1 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 4 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 8 \\ 3 \\ 11 \end{bmatrix} \Rightarrow n = 3 \Rightarrow k = \overline{1,2}$$

$k = 1$: $$A \equiv \bar{A} = \left[\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 2 & 5 & 2 & 3 \\ 1 & 2 & 4 & 11 \end{array}\right] = [A^{(1)}\ \underline{b}^{(1)}]$$

→ $a_{11}^{(1)} = 4 \neq 0$ (i.e putem aplica MEGFP)

$i = \overline{2,3}$: $m_i^{(1)} = a_{i1}^{(1)} / a_{11}^{(1)}$

$m_2^{(1)} = a_{21}^{(1)} / a_{11}^{(1)} = 2/4 = 1/2 \rightsquigarrow E_2 \leftarrow E_2 - m_2^{(1)} E_1$

$j = \overline{2,3}$: $a_{2j}^{(2)} = a_{2j}^{(1)} - m_2^{(1)} a_{1j}^{(1)}$

$\boxed{a_{22}^{(2)}} = a_{22}^{(1)} - m_2^{(1)} a_{12}^{(1)} = 5 - \frac{1}{2} \cdot (-1) = 5 + \frac{1}{2} = 11/2$

$\boxed{a_{23}^{(2)}} = a_{23}^{(1)} - m_2^{(1)} a_{13}^{(1)} = 2 - \frac{1}{2} \cdot 1 = 3/2$

$a_{21}^{(2)} = 0$ (am ales $m_2^{(1)}$ a.î. $a_{21}^{(2)} = 0$)

$\boxed{b_2^{(2)}} = b_2 - m_2^{(1)} b_1 = 3 - 1/2 \cdot 8 = -1$

$m_3^{(1)} = a_{31}^{(1)} / a_{11}^{(1)} = 1/4 \rightsquigarrow E_3 \leftarrow E_3 - m_3^{(1)} E_1$

$j = \overline{2,3}$: $a_{3j}^{(2)} = a_{3j}^{(1)} - m_3^{(1)} a_{1j}^{(1)}$

$\boxed{a_{32}^{(2)}} = a_{32}^{(1)} - m_3^{(1)} a_{12}^{(1)} = 2 - 1/4 \cdot (-1) = 9/4$

$\boxed{a_{33}^{(2)}} = a_{33}^{(1)} - m_3^{(1)} a_{13}^{(1)} = 4 - 1/4 \cdot 1 = 15/4$

$a_{31}^{(2)} = 0$ (am ales $m_3^{(1)}$ a.î. $a_{31}^{(2)} = 0$)

$\boxed{b_3^{(2)}} = b_3^{(1)} - m_3^{(1)} b_1^{(1)} = 11 - 1/4 \cdot 8 = 9$

Am obținut $k = 2$: $$\bar{A}^{(2)} = \left[\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 0 & \boxed{11/2} & \boxed{3/2} & \boxed{-1} \\ 0 & \boxed{9/4} & \boxed{15/4} & \boxed{9} \end{array}\right] = [A^{(2)}\ \underline{b}^{(2)}]$$

Matricea $M^{(1)}$ care transformă $\bar{A} \equiv \bar{A}^{(1)} = [A^{(1)}\ \underline{b}^{(1)}]$ în $\bar{A}^{(2)} = [A^{(2)}\ \underline{b}^{(2)}]$ este dată de

$$M^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/4 & 0 & 1 \end{bmatrix}$$

Mai exact, are loc relația $M^{(1)}[A^{(1)}\ \underline{b}^{(1)}] = [A^{(2)}\ \underline{b}^{(2)}]$ (1)

$\rightarrow$ $a_{22}^{(2)} = 11/2 \neq 0$ (i.e putem aplica MEGFP)

$i = \overline{3,3}$: $m_i^{(2)} = a_{i2}^{(2)} / a_{22}^{(2)}$

$$m_3^{(2)} = a_{32}^{(2)} / a_{22}^{(2)} = 9/4 \,/\, 11/2 = \frac{9}{\cancel{4}_2} \cdot \frac{\cancel{2}}{11} = 9/22 \rightsquigarrow E_3 \leftarrow E_3 - m_3^{(2)} E_2$$

$j = \overline{3,3}$: $a_{3j}^{(3)} = a_{3j}^{(2)} - m_3^{(2)} a_{2j}^{(2)}$

$$\boxed{a_{33}^{(3)}} = a_{33}^{(2)} - m_3^{(2)} \cdot a_{23}^{(2)} = \frac{15}{4} - \frac{9}{22} \cdot \frac{3}{2} = \frac{138}{44} = \frac{69}{22}$$

$a_{32}^{(3)} = 0$

$$\boxed{b_3^{(3)}} = b_3^{(2)} - m_3^{(2)} b_2^{(2)} = 9 - \frac{9}{22} \cdot (-1) = \frac{9 \cdot 23}{22} = \frac{207}{22}$$

Am obținut $\bar{A}^{(3)} = \left[\begin{array}{ccc|c} 4 & -1 & 1 & 8 \\ 0 & 11/2 & 3/2 & -1 \\ 0 & 0 & \boxed{69/22} & \boxed{207/22} \end{array}\right] = [A^{(3)}\ \underline{b}^{(3)}] = [U\ \tilde{\underline{b}}]$

Matricea $M^{(2)}$ care transformă $\bar{A}^{(2)} = [A^{(2)}\ \underline{b}^{(2)}]$ în $\bar{A}^{(3)} = [A^{(3)}\ \underline{b}^{(3)}] = [U\ \tilde{\underline{b}}]$ este dată de:

$$M^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -9/22 & 1 \end{bmatrix}$$

Mai exact, are loc relația $M^{(2)}[A^{(2)}\ \underline{b}^{(2)}] = [A^{(3)}\ \underline{b}^{(3)}] = [U\ \tilde{\underline{b}}]$ (2)

Din (1) și (2) $\Rightarrow M^{(2)} M^{(1)} [A^{(1)}\ \underline{b}^{(1)}] = [U\ \tilde{\underline{b}}]$

Sistemul $A\underline{x} = \underline{b}$ devine $U\underline{x} = \tilde{\underline{b}}$, i.e:

$$\begin{cases} 4x_1 - x_2 + x_3 = 8 \\ \frac{11}{2} x_2 + \frac{3}{2} x_3 = -1 \\ \frac{69}{22} x_3 = \frac{207}{22} \end{cases}$$

și rezolvăm prin met. subst. desc:

$\frac{69}{22} x_3 = \frac{207}{22} \Rightarrow x_3 = \frac{207}{69} = 3$

$\frac{11}{2} x_2 = -1 - \frac{9}{2} \Rightarrow x_2 = -1$

$4x_1 = 8 + (-1) - 3 = 4 \Rightarrow x_1 = 1$

Așadar $\underline{x} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$