

Tutoriat 9

1) Fie $\psi, \varphi \in \text{Form}$. Demonstrați că:

$$\{\neg(\psi \rightarrow \varphi)\} \vdash (\psi \rightarrow \varphi) \rightarrow \neg(\psi \rightarrow (\varphi \rightarrow \psi))$$

Duc:
Fie $\Gamma = \{\neg(\psi \rightarrow \varphi), \psi \rightarrow \varphi, \neg\neg(\psi \rightarrow (\varphi \rightarrow \psi))\}$

Dir 3.34.ii) \Rightarrow

$$\begin{aligned} \Gamma &\vdash \neg(\psi \rightarrow \varphi) & (1) \\ \Gamma &\vdash \psi \rightarrow \varphi & (2) \\ \Gamma &\vdash \neg\neg(\psi \rightarrow (\varphi \rightarrow \psi)) & (3) \end{aligned}$$

Consider formulele:

$$\begin{aligned} \varphi_1 &= \neg(\psi \rightarrow \varphi) \\ \varphi_2 &= \psi \rightarrow \varphi \\ \varphi_3 &= \neg(\psi \rightarrow \psi) \end{aligned}$$

3.51 (4) $\varphi, \psi \in \text{Form}$ avem:

$$\{\psi, \neg\psi\} \vdash \neg\psi \rightarrow (\psi \rightarrow \varphi) \quad (38)$$

\forall 3.39 i) $\Delta \subseteq \Gamma \Rightarrow \text{Thm}(\Delta) \subseteq \text{Thm}(\Gamma)$

În 3.51. considerăm $\psi = \varphi_2$ și $\neg\psi = \varphi_1$, iar $\varphi = \varphi_3$.
Deci avem:

$$\Gamma \vdash \neg(\psi \rightarrow \varphi) \rightarrow ((\psi \rightarrow \varphi) \rightarrow \neg(\psi \rightarrow \psi)) \quad (4)$$

Aplicăm MP((4), (3)) \Rightarrow

$$\Rightarrow \Gamma \vdash (\psi \rightarrow \varphi) \rightarrow \neg(\psi \rightarrow \psi) \quad (5)$$

($\Gamma \cup \{\psi \rightarrow \varphi\} = \Gamma$)

Aplicăm Th. Deductiei: \Rightarrow

$$\Rightarrow \Gamma \vdash \neg(\psi \rightarrow \psi). \quad (6)$$

Aplicăm Prop. 3.50. : $\Gamma \cup \{\neg\psi\} \vdash \neg(\psi \rightarrow \psi) \Rightarrow \Gamma \vdash \psi$
Considerăm la noi $\neg\psi = \neg\neg(\psi \rightarrow (\varphi \rightarrow \psi))$

Obținem

$$\{\neg(\psi \rightarrow \varphi), \psi \rightarrow \varphi\} \vdash \neg(\psi \rightarrow (\varphi \rightarrow \psi)) \quad (7)$$

Aplicăm Th. Deductiei \Rightarrow

$$\Rightarrow \{\neg(\psi \rightarrow \varphi)\} \vdash (\psi \rightarrow \varphi) \rightarrow \neg(\psi \rightarrow (\varphi \rightarrow \psi))$$

Correct

Method 2: Solving axioms

$$A_1: \vdash \theta_1 \rightarrow (\theta_2 \rightarrow \theta_1).$$

Consider $\theta_1 = \neg(\psi \rightarrow \phi)$ & $\theta_2 = \neg(\neg\psi \rightarrow (\phi \rightarrow \psi))$

$$(A_1): \vdash \neg(\psi \rightarrow \phi) \rightarrow (\neg(\neg\psi \rightarrow (\phi \rightarrow \psi)) \rightarrow \neg(\psi \rightarrow \phi)) \quad (1)$$

$$(K.D): \{\neg(\psi \rightarrow \phi)\} \vdash \neg(\neg\psi \rightarrow (\phi \rightarrow \psi)) \rightarrow \neg(\psi \rightarrow \phi) \quad (2)$$

$$A_3: \vdash (\neg\theta_1 \rightarrow \neg\theta_2) \rightarrow (\theta_2 \rightarrow \theta_1)$$

Consider $\theta_3 = \psi \rightarrow \phi$ & $\theta_4 = \neg\psi \rightarrow (\phi \rightarrow \psi)$

$$(A_3): \vdash (\neg(\neg\psi \rightarrow (\phi \rightarrow \psi)) \rightarrow \neg(\psi \rightarrow \phi)) \rightarrow ((\psi \rightarrow \phi) \rightarrow \neg(\neg\psi \rightarrow (\phi \rightarrow \psi))) \quad (3)$$

Obs! φ axiom $\Rightarrow \Gamma \vdash \varphi \rightarrow \varphi, (\forall) \Gamma \subset \text{form.}$

$$\text{Deci } \{\neg(\psi \rightarrow \phi)\} \vdash (\neg(\neg\psi \rightarrow (\phi \rightarrow \psi)) \rightarrow \neg(\psi \rightarrow \phi)) \rightarrow ((\psi \rightarrow \phi) \rightarrow \neg(\neg\psi \rightarrow (\phi \rightarrow \psi))) \quad (4)$$

$$\text{MP } (2), (4) \Rightarrow \{\neg(\psi \rightarrow \phi)\} \vdash (\psi \rightarrow \phi) \rightarrow \neg(\psi \rightarrow (\phi \rightarrow \psi))$$