Test

ONUȚU RADU-CONSTANTIN
Gr. 113

1. $$\Delta = \begin{vmatrix} 3 & -3 & 2 & 3 \\ -2 & 7 & -2 & 5 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 1 & 5 \end{vmatrix} = 3\cdot(-1)^{1+1}\cdot\begin{vmatrix} 7 & -2 & 5 \\ 0 & 3 & 7 \\ 0 & 1 & 5 \end{vmatrix} +$$

$$+ (-2)\cdot(-1)^{2+1}\cdot\begin{vmatrix} 3 & 2 & 3 \\ 0 & 3 & 7 \\ 0 & 1 & 5 \end{vmatrix} = 3\left(105-49\right)+2\left(45-21\right) =$$

$$= 3\cdot 56 + 2\cdot 24 = 216 \Rightarrow \Delta = 216$$

2. $$\begin{vmatrix} 7 & 1 & 0 & 0 & \dots & 0 \\ 6 & 7 & 1 & 0 & \dots & 0 \\ 0 & 6 & 7 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \dots & 7 \end{vmatrix} = \det(A_n) = \Delta_n$$

(a) $$\Delta_2 = \begin{vmatrix} 7 & 1 \\ 6 & 7 \end{vmatrix} = 49 - 6 = 43$$

$$\Delta_3 = \begin{vmatrix} 7 & 1 & 0 \\ 6 & 7 & 1 \\ 0 & 6 & 7 \end{vmatrix} = 343 - 42 - 42 = 259$$

$$\Delta_4 = \begin{vmatrix} 7 & 1 & 0 & 0 \\ 6 & 7 & 1 & 0 \\ 0 & 6 & 7 & 1 \\ 0 & 0 & 6 & 7 \end{vmatrix} = 7\cdot(-1)^{1+1}\cdot\begin{vmatrix} 7 & 1 & 0 \\ 6 & 7 & 1 \\ 0 & 6 & 7 \end{vmatrix} + 1\cdot(-1)^{1+2}\cdot\begin{vmatrix} 6 & 1 & 0 \\ 0 & 7 & 1 \\ 0 & 6 & 7 \end{vmatrix}$$

$$= 7\cdot\Delta_3 - \left(294 - 36\right) = 7\cdot 259 - 258 = 1813 - 2$$
$$= 1555$$

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$\Rightarrow \Delta_4 = \cancel{666}\ 1555$

(b) $$\Delta_n = \begin{vmatrix} 7 & 1 & 0 & 0 & \dots & 0 \\ 6 & 7 & 1 & 0 & \dots & 0 \\ 0 & 6 & 7 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \dots & 7 \end{vmatrix} = 7\cdot(-1)^{1+1}\cdot \begin{vmatrix} 7 & 1 & 0 & \dots & 0 \\ 6 & 7 & 1 & \dots & 0 \\ 0 & 6 & 7 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 7 \end{vmatrix} +$$

$$\cancel{+ 1\cdot(-1)^{1+2}\cdot \begin{vmatrix} 6 & 1 & 0 & 0 & \dots & 0 \\ 0 & 6 & 7 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \dots & \end{vmatrix}} \quad \cancel{+ 1(-1)^{1+2}\cdot \begin{vmatrix} 6 & 1 & 0 & 0 & \dots & 0 \\ 0 & 7 & 1 & \dots & & \\ 0 & 0 & 6 & \dots & & 0 \end{vmatrix}}$$

$$+ 6\cdot(-1)^{2+1}\cdot \begin{vmatrix} 1 & 0 & 0 & \dots & 0 \\ 6 & 7 & 1 & \dots & \\ 0 & 6 & 7 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 7 \end{vmatrix} =$$

$$\Delta_n = 7\cdot\Delta_{n-1} - 6\cdot 1\cdot(-1)^{1+1}\cdot \begin{vmatrix} 7 & 1 & 0 & 0 & \dots & 0 \\ 6 & 7 & 1 & 0 & \dots & 0 \\ 0 & 6 & 7 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \dots & 7 \end{vmatrix} =$$

$$\Delta_n = 7\Delta_{n-1} - 6\cdot\Delta_{n-2}$$

(c) Trebuie să arăt că $\Delta_n = \cancel{6^n}\ \dfrac{6^{n+1}-1}{5}$, $\forall n \geq 2$

(cf. a):

$\Delta_2 = 43 = \dfrac{6^3-1}{5}$

$\Delta_3 = \cancel{437}\ 259 = \dfrac{6^4-1}{5}$

$\Delta_4 = 1555 = \dfrac{6^5-1}{5}$

(c) Pp. adevărată $\Delta_n = \frac{6^{n+1}-1}{5}$

(cf. b):

$$\Delta_n = 7\Delta_{n-1} - 6\Delta_{n-2} = 7 \cdot \frac{6^n - 1}{5} - 6 \cdot \frac{6^{n-1}-1}{5} =$$

$$= \frac{7 \cdot 6^n - 7}{5} - \frac{6^n - 6}{5} = \frac{6^n(7-1)-1}{5} = \frac{6^n \cdot 6 - 1}{5} = \frac{6^{n+1}-1}{5}$$

$\Rightarrow$ presupunerea e adevărată $\Rightarrow \Delta_n = \frac{6^{n+1}-1}{5}$