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Examen CO

$\boxed{i = 42}$

$$\begin{cases} \inf(-x_1 - x_2) \\ -x_1 + x_2 + x_3 = 58 \\ x_1 + x_2 + x_4 = 42 \\ x_j \geq 0, \ j = \overline{1,4} \end{cases} \qquad A = \begin{pmatrix} -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} \qquad b = \begin{pmatrix} 58 \\ 42 \end{pmatrix}$$

$\inf(-x_1, -x_2)$ -> funcția obiectiv

B primal admisibilă?

$$B^{-1} = \frac{1}{\det B} \cdot B^*$$

$$B = (a_3, a_4) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = J_2$$

$$B^{-1} = J_2$$

$$B^{-1} \cdot b = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 58 \\ 42 \end{pmatrix} = \begin{pmatrix} 58 \\ 42 \end{pmatrix} \geq 0 \Rightarrow B \text{ primal admisibilă}$$

$x_1 = 0 \quad x_2 = 0$

$x_3 = 58 \quad x_4 = 42 \Rightarrow F = 0$

	c_B	VB	VVB	x_1	x_2	x_3	x_4
	0	x_3	58	-1	1	1	0
←	0	x_4	42	1	1	0	1
		Z	0	1	1	0	0

x_1: $J_2\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix}$

x_2: $J_2\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$

x_3: $J_2\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$

x_4: $J_2\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$

$Z_{VB} = 0$

Test de optim:

$Z_1^B - c_1 = 1 > 0$ => Testul de optim nu e indeplinit

Test de optim infinit:

$y_1^B = \begin{pmatrix}-1\\1\end{pmatrix} \not\leq 0$

$y_2^B = \begin{pmatrix}1\\1\end{pmatrix} \not\leq 0$ } => testul de optim infinit nu e indeplinit

VB	VVB	x_1 (-1)	x_2 (-1)	x_3	x_4
x_3	100	0	2	1	-1
x_1	42	1	1	0	1
Z	-42	0	-1	0	-1

$Z_2^B - c_2$

$(0\ -1)\begin{pmatrix}0\\1\end{pmatrix} = -1$

$(0\ -1)\begin{pmatrix}2\\1\end{pmatrix} = -1$

Test de optim

$Z_2^B - c_2 = -1 \leq 0$

↳ indeplinit ✓

$Z_4^B - c_4 = -1 \leq 0$

↳ indeplinit ✓

Caut sol. optime:

Iau $x_2^* = x_4^* = 0$

$$\begin{cases} -x_1 + x_3 = 58 \Rightarrow x_3 = 100 \\ \Rightarrow x_1 = 42 \end{cases}$$

Sol optimă: $x_1^* = 42$

$x_3^* = 100$

$x_2^* = x_4^* = 0$

Val optimă este $\boxed{-42}$

1. b) $\mathcal{R} = \{2, 4\}$

$z_2^B - c_2 = 1 - (-1) = 0$, $2 \in \mathcal{R}$ => sol optimă nu e unică

B primal admisibilă
$z_j^B - c_j \leq 0, \forall j \in \mathcal{R}$ } => x sol optimă

x sol admisibilă <=> $\begin{cases} -x_1 + x_2 + x_3 = 58 \\ x_1 + x_2 + x_4 = 42 \\ x_j \geq 0 \quad j = \overline{1,4} \end{cases}$

$x_j (z_j^B - c_j) = 0 \quad \forall j \in \mathcal{R} \iff \begin{cases} x_2 \cdot (-1) = 0 \\ x_4 \cdot (-1) = 0 \end{cases} \Big| \Rightarrow x_4 = 0$

$\Rightarrow \begin{cases} -x_1 + x_2 + x_3 = 58 \\ x_1 + x_2 = 42 \\ x_4 = 0 \\ x_j \geq 0, j = \overline{1,3} \end{cases}$

$x_1 = \lambda$ cu $\lambda \geq 0$ $\Rightarrow \begin{cases} x_2 = 42 - \lambda \geq 0 \Rightarrow \lambda \leq 42 \\ x_3 = 58 + \lambda - 42 + \lambda \geq 0 \Rightarrow \lambda \geq -8 \end{cases}$

$\lambda \geq -8$
$\lambda \leq 42$

Multimea sol: $\mathcal{P}^* = \{(\lambda, 42 - \lambda, 8 + \lambda, 0)^T \mid \lambda \in [0, 42]\}$.