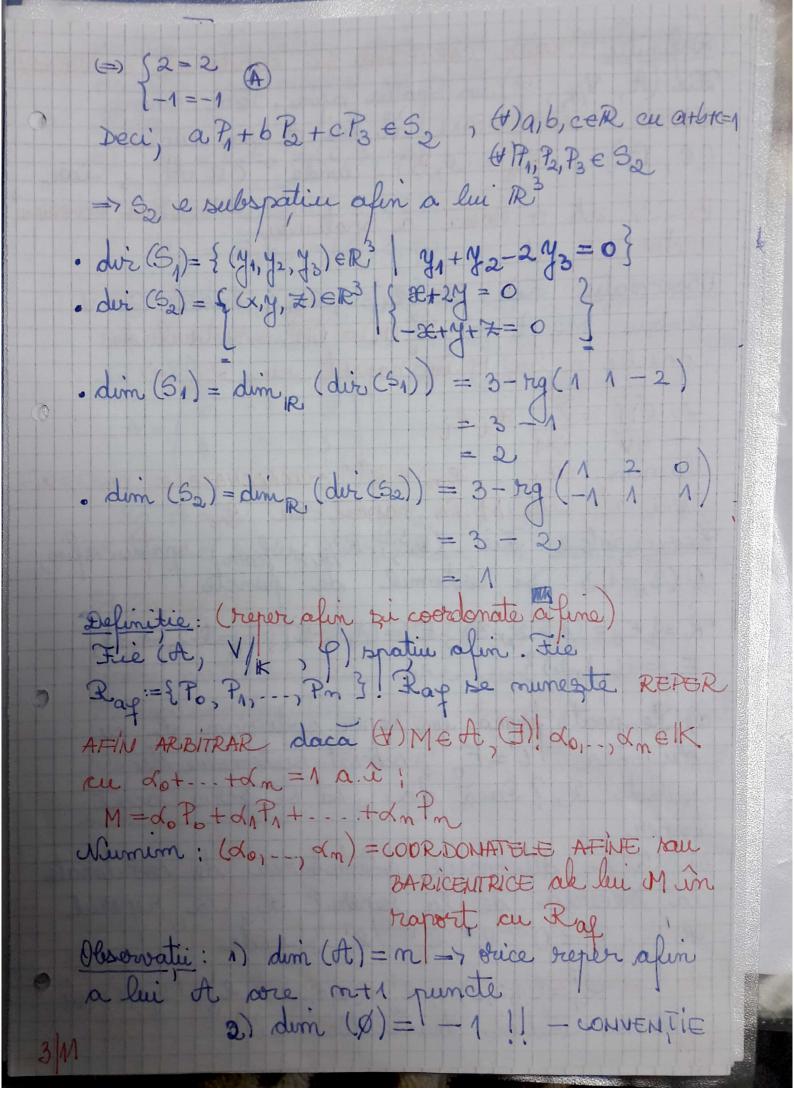
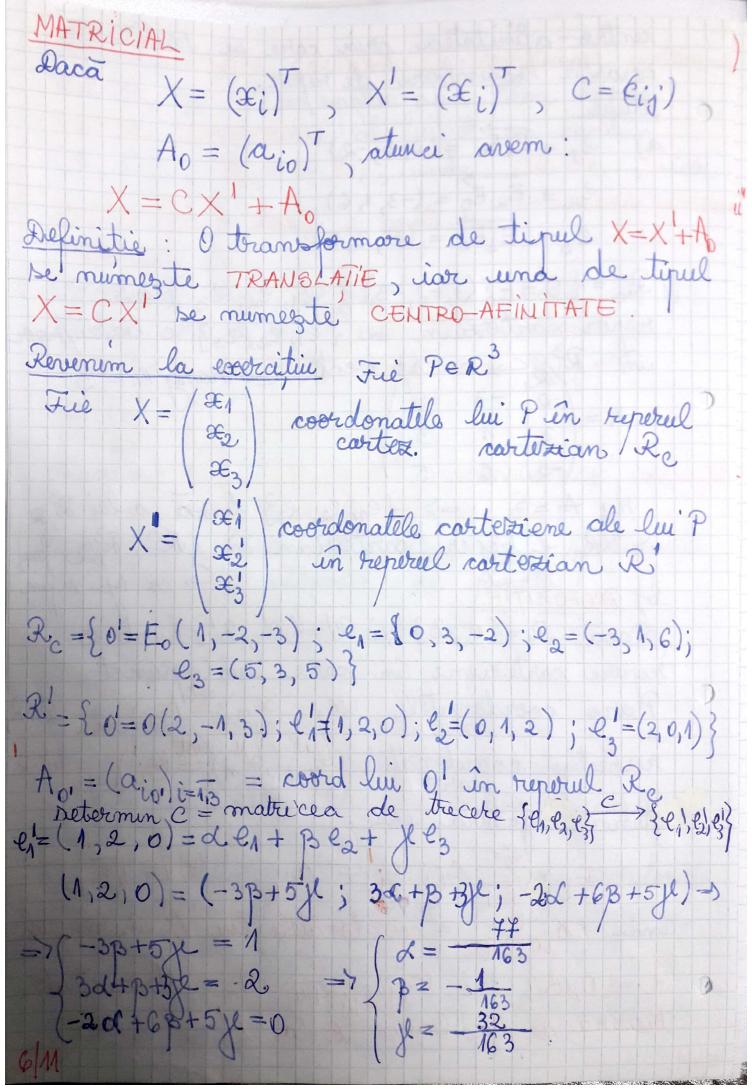
TAFALAN ALEXANDRU-MARIAN TUTORIAT GEOMETRIE - Nº 2 (SAPTAMANA 3) Subspatii a line. Operatii cu subspatii afine Definitie: (subspatin alin) File (At, V/K, 9) spatie afin, file A, C. A submultime. Ay se numeste SUBSPAJIU AFIN al lui A daca (3) 0/EA, au proposietatea ca multimea dir (A') = { 01p | Pe A1 } este SUBSPATIU VECTORIAL a lui V/1K Teorema de coractorizore a subspatiilor afine Fie (A- 1/K; f) sp. afin si A/ = A. Alanci the substation AFING (+) men, (+) of, -, of the ru ∑ «i = 1 si (+) Pr,..., Pro€ Ay avem ra Z LiAiE A1 ( inchis la combinatio aline arbitrare au "elem" dui An) Exercitive 1: Fie S1={xeR3 | x1+x2-2x3=1} si  $S_2 = \{ x \in \mathbb{R}^3 | \{ x_1 + 2x_2 = 2 \} \}$ se avate cà 31 si 52 sunt substratie aline ûn (R³, R³/R) fran) zi rou se/ determine dir (S1), dir (S2), dim (S1) zi dim (S2). Tolorim definition File 0= (2,1,0) & 51

during (6n) = 6101 = { 01P | PEB13 = { (2-&,1-4, 1- 2) = R3 |x+y-2x=1)  $P = (x, y, \pm)$   $O'P' = (2-x, 1-y, 1-\pm) \in \mathbb{R}^3$ =>dur<sub>0</sub>, (51) = { (a, b, c)  $\in \mathbb{R}^3$  | -a+2+1-b+2c-2=13=>  $\Rightarrow dwi_{0}(S_{0})=\{(a,b,c)\in\mathbb{R}^{3} | -a-b+2c=0\} \leq \mathbb{R}^{3}$ Deci, Sque subspatin afin a lui R3. Bentru S2, Folosim. Tetrema imentionata mau ses: Tree  $P_1 \in S_2 \Rightarrow \int \mathfrak{X}_1 + 2\mathfrak{X}_2 = 2$   $(\mathfrak{X}_1, \mathfrak{X}_2, \mathfrak{X}_3)$   $\{-\mathfrak{X}_1 + \mathfrak{X}_2 + \mathfrak{X}_3 = -1\}$ P2 e52 >> { y1+242=2 (y1, y2, y3) {-y1+ y2+ y3=-1 (2) (\*1, 2, 2, 3) => S \*1+2 \*2 = 2 (\*1, 2, 2, 2) {-7++ \*2+2=-1 3 Fie a, b, ceR, cu a+b+c=1 a7,+672+c73 = (a)e,+64,+02,; a)e,+642+042+02; ax3+043+cx3)  $a7_1+b7_2+c7_3 \in 5_2 \in 5$   $ax_1+by_1+cx_1 = 2ax_2+2by_2+2cx_2=2$   $ex_1-by_1-cx_1+ax_2+by_2+cx_3+ax_3+by_3+cx_2=2$  $(a(x_1+2x_2)+b(y_1+2y_2)+c(x_1+2x_2)=2)$   $(a(x_1+2x_2)+b(y_1+y_2+y_3)+c(-x_1+x_2+x_3)=-1$  $3)\{2a+2b+2c=2$  (=>) $\{2(a+b+c)=2$  (=) $\{-a-b-c=-1\}$ 

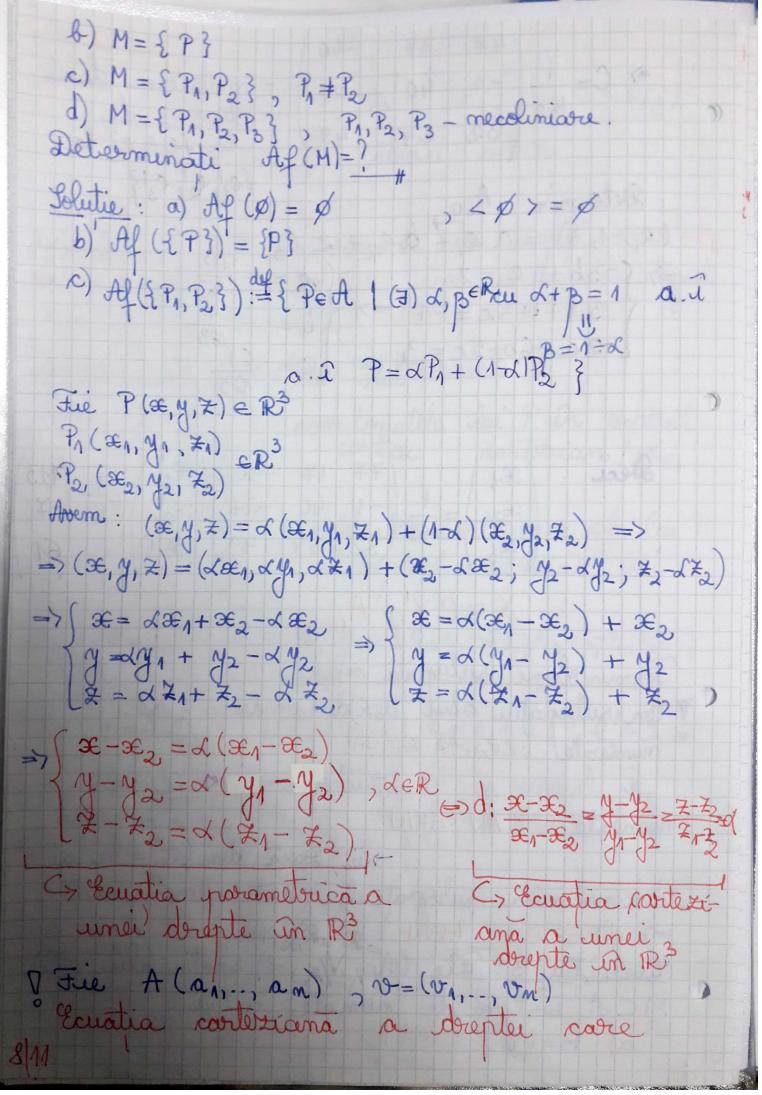


Definitie: (reper CARTEZIAN pie coordonate contexiene) File (A, V/K, 9) un spatiu afin. Vom numi REPER CARTEZIAN al lui A un cuplu (0, B), Rc = (0, B), unde 00 A, iar BCV este o loza a spatiului Ly originea REPERULUI rectorial V Observatie File B={e,-, en} boxa ûn V Bentru ME A = 7 72" (M) = OM = vectoral de positie a lui Mûn raport cu Rc =7 -> OM = 7 æ; e; Mumin: (261, -, 26m) EKM -COORDONATELE CARTEXI-ENE ale lui Mûn raport ou Re. Exercitiul 2: File (R3; R3/R; Yearn) spatiu afin, O(2;-1;3) si sistemul de puncte  $R = \{E_0 = (1, -2, -3), E_1 = (1, 1, -5), E_2 = (-2, -1, 3),$  $E_4 = (6, 1, 2)$ a) Le poate forma un reper cortezian le cu originea 10'= E, assciat sistemului de purate R? Daca da sa se socie acest brener e) la se determine schimbarea de coordonate la trecerea de la reperul Rc la reperul R'= 60"=0, e, e, e, e, e, 3 3 unde e,=(1,2,0); e;=(0,1,2); e;=(2,0,1) si sa se indice translatia si

centro-afinitatea prin care se realizeaxa aceasta schimbare de repet. Tolutie a)  $e_1 = E_1 E_0 = (0, 3, -2)$ ez = Ez Eo = (-3, 1, 6) l3 = E3 = (5, 3, 5) Rc = {0 = Eo (1,-2,-3); en, e2, e3 } este reper carterian => {e1, e2, e3} e laxa/repor in R3 R so ca sp. vect (=> rang A = 3 A=3 => {e1, e2, e3} baza a lui R3/2 Re e reper cartezian un (R3; R3/12) fay & PROPOZITIE Daca R= 40, B3 si R'= 40', B'3 sunt 2 repere cortezione în core un punct Pare coordonatele (2,, , , , , ): (Ei) i= nu mik respective coordonatile (£,,-, &m) = (£;) = un R atunci legatura este E = = Cij Xj + aio , (+) i= 1, 12, unde (aio):= 1.m = coordonatelo lui 0 in reporul R={0,B} si (cij)ij=1/1 este matricea de trecere de la baza Bla B

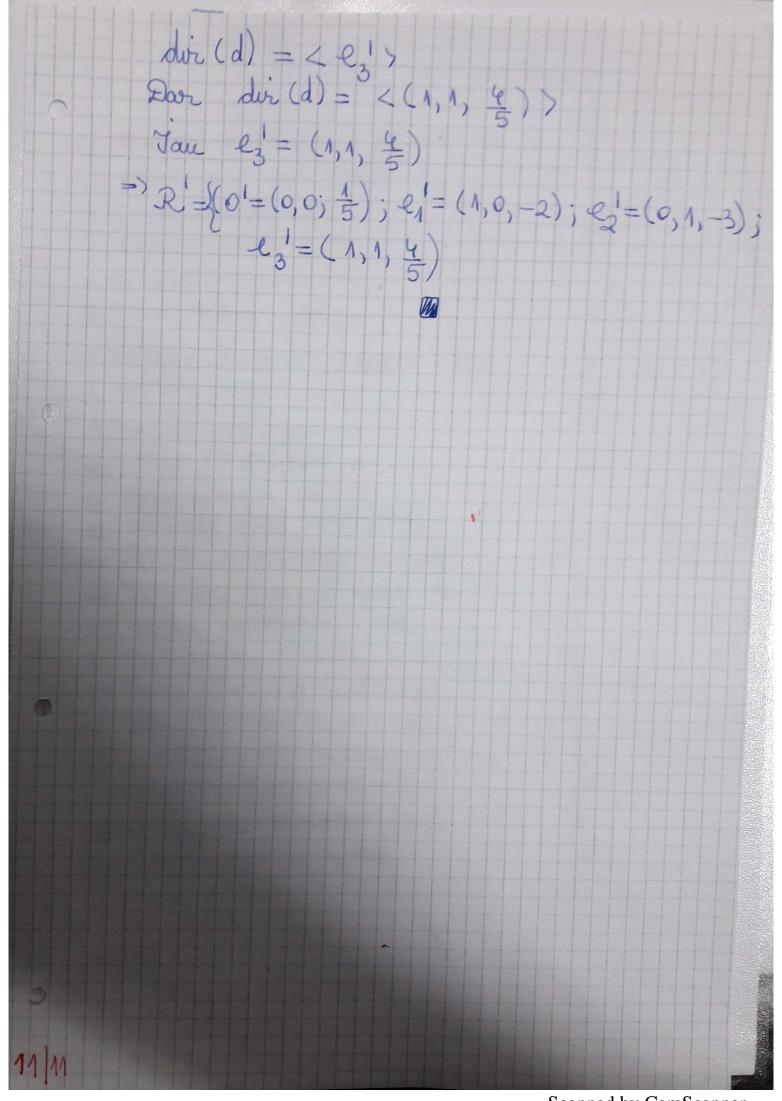


=> \ -3b +5 \chi 3 \alpha + b + 3 \\ -2 \alpha + 6 \\ \hline \ \frac{2}{3} \\		
+5C +b+6c /XXXX ration PL mea PLA 3 suste	Un subs numezte umezte	Detoin (2, -1, 3) => \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \
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= n si n = 7	= n si 1 = n si	, eq, ez} la



trace pour punctul A su are directia v d: 3 - A1 = 3 - A2 = Vectoral ve dir (d) -> < v> = dir (d) Af((P1,P23)=d c>o duanta = { t.o IteR } d) Af ({P1, P2, P3}) = {PEA ( (3) d, B, KER cud+B+)=1 Pa, Pa, By mecolinicone a. a P= XPa+BP2+Jet3 0+B+K=1=> 0=1-6-B P= & (x1, y1, 71) + B(x2, y2, 72) + ((x3, y3, 73) => ( x-x3 = d(x1-x3)+ B(x2-x3) 1d, BER ) y-y3 = x(y1-y3)+B(y2-y3) しまーキョ=め(ギューキョ)+ち(チューキョ) > Ecuatia parametrica a senui plan in R V Ecuatia cartesiana a planului det de pundul A (X1, X2, X3) si avand directia generata de zu, v 3  $\mathfrak{X}-\mathfrak{X}_1$   $\mathfrak{X}-\mathfrak{X}_2$   $\mathfrak{X}-\mathfrak{X}_3$ 43 =0 · len le 2 V due (T) = < x1, 00 > E{ Lu+BV | d, BER} de peintelle A (a1, a2, A3) 1, B(b1, b2, b3)

X-A2 X-A3 x-01 b2-a2 b3-a3 =0 01-01 C2-A2 C3-A3 (ABC) C1-A1 dim (II) = 2 => Af(2P1, P2, P33) = TT Grun plan. Le considera reperul cartesian canonic Rc = {0(0,0,0); le, = (1,0,0), e, = (0,1,0), e, = (0,0,1)} La se construiasca un reper R = {0'; e1, e2, e3} a. û e, si e, e dur (TT), unde TI: 2x+3y+Z-1=0 0'=(0,0; 1), iar dreopta ce trèce prin 0' si are ca vector director pe e3 contine punctul P=(1,1,1). Solutie: 11: 200+34+7-1=0 dir (11): 2 x+3y+ x = 0 cf7 sp. vectorial => dur (II)={(x,y, -2x-3y)| x,yeR} = { \( \pi(1,0,-2) + \( \frac{1}{2} \) (0,1,-3) \( \pi\_1, \frac{1}{2} \) \( \pi\_2, \frac{1}{2} \) \( \pi\_3, \frac{1}{2} \) \( \pi\_1, \frac{1}{2} \) \( \pi\_1, \frac{1}{2} \) \( \pi\_2, \frac{1}{2} \) \( \pi\_3, \frac{1}{2} \) \( \pi\_1, \frac{1}{2} \) \( \pi\_1, \frac{1}{2} \) \( \pi\_1, \frac{1}{2} \) \( \pi\_2, \frac{1}{2} \) \( \pi\_1, \pi\_1, \frac{1}{2} \) \( \pi\_1, \frac{1}{2} \) \( \pi\_1, \ = Sp {(1,0,-2); (0,1,-3)} Bum e, , e c dir (11) => Pot lua: e,= (1,0,-2) Brin ecuatia drapter d' care trèce prin , d: 1 = 7 = 7 = 5



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