MEGPP:

Rezolvați sistemul de ecuații liniare:

$$\begin{cases} x_1 + x_2 - x_3 = 1 \\ x_1 + x_2 + 4x_3 = 2 \\ 2x_1 - x_2 + 2x_3 = 3 \end{cases}$$

folosind MEGPP și metoda substituției descendente.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 4 \\ 2 & -1 & 2 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow \underline{n=3}$$

$\underline{k=1}$:

$$\overline{A} \equiv \overline{A}^{(1)} = \left[A^{(1)} \ \underline{b}^{(1)} \right] = \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & 1 & 4 & 2 \\ 2 & -1 & 2 & 3 \end{array}\right]$$

$$\max_{i=\overline{1,3}} |a_{i1}^{(1)}| = \max\{|1|, |1|, |2|\}$$

$$= 2 = |a_{31}^{(1)}| =: |a_{\ell 1}^{(1)}| \Rightarrow \underline{\ell = 3 > 1}$$

$(E_3) \longleftrightarrow (E_1)$:

Matricea permutare simplă:

$$P^{(1)} = P_{13} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad (1a)$$

$$P^{(1)} \overline{A}^{(1)} = P^{(1)} \left[A^{(1)} \ \underline{b}^{(1)} \right] = \left[\begin{array}{ccc|c} 2 & -1 & 2 & 3 \\ 1 & 1 & 4 & 2 \\ 1 & 1 & -1 & 1 \end{array} \right]$$

$$=: \left[\widetilde{A}^{(1)} \ \underline{\widetilde{b}}^{(1)} \right] = \overline{\widetilde{A}}^{(1)}$$

$\widetilde{a}^{(1)}_{11} = 2 \neq 0$ (aplicăm MEGFP)

$\underline{i = \overline{2,3}}$: $m^{(1)}_i := \widetilde{a}^{(1)}_{i1} / \widetilde{a}^{(1)}_{11}$

- $m^{(1)}_2 := \widetilde{a}^{(1)}_{21} / \widetilde{a}^{(1)}_{11} = 1/2$

$(E_2 - m^{(1)}_2 E_1) \longrightarrow (E_2)$:

$\underline{j = \overline{2,3}}$: $a^{(2)}_{2j} := \widetilde{a}^{(1)}_{2j} - m^{(1)}_2 \widetilde{a}^{(1)}_{1j}$

$$a^{(2)}_{22} := \widetilde{a}^{(1)}_{22} - m^{(1)}_2 \widetilde{a}^{(1)}_{12}$$

$$= 1 - \tfrac{1}{2}(-1) = 3/2$$

$$a^{(2)}_{23} := \tilde{a}^{(1)}_{23} - m^{(1)}_2 \tilde{a}^{(1)}_{13}$$
$$= 4 - \frac{1}{2} 2 = 3$$

$a^{(2)}_{21} = 0$ (Nu mai trebuie calculat! $m^{(1)}_2$ se alege a.î. $a^{(2)}_{21} = 0$)

$$b^{(2)}_2 := \tilde{b}^{(1)}_2 - m^{(1)}_2 \tilde{b}^{(1)}_1$$
$$= 2 - \frac{1}{2} 3 = 1/2$$

- $m^{(1)}_3 := \tilde{a}^{(1)}_{31} / \tilde{a}^{(1)}_{11} = 1/2$

$(E_3 - m^{(1)}_3 E_1) \longrightarrow (E_3)$:

$\underline{j = \overline{2,3}}$: $a^{(2)}_{3j} := \tilde{a}^{(1)}_{3j} - m^{(1)}_3 \tilde{a}^{(1)}_{1j}$

$$a^{(2)}_{32} := \tilde{a}^{(1)}_{32} - m^{(1)}_3 \tilde{a}^{(1)}_{12}$$
$$= 1 - \frac{1}{2}(-1) = 3/2$$
$$a^{(2)}_{33} := \tilde{a}^{(1)}_{33} - m^{(1)}_3 \tilde{a}^{(1)}_{13}$$
$$= -1 - \frac{1}{2} 2 = -2$$

$a^{(2)}_{31} = 0$ (aceeași observație ca cea pt $\tilde{a}^{(2)}_{21}$)

$$b^{(2)}_3 := \tilde{b}^{(1)}_3 - m^{(1)}_3 \tilde{b}^{(1)}_1 = 1 - \frac{1}{2} 3 = -1/2$$

Am obținut:

<u>k=2</u>:

$$\overline{A}^{(2)} = [A^{(2)}\ \underline{b}^{(2)}] = \left[\begin{array}{ccc|c} 2 & -1 & 2 & 3 \\ 0 & 3/2 & 3 & 1/2 \\ 0 & 3/2 & -2 & -1/2 \end{array}\right]$$

Obs: Matricea care transformă

$$\overline{A}^{(1)} = [\tilde{A}^{(1)}\ \tilde{\underline{b}}^{(1)}] = P^{(1)}[A^{(1)}\ \underline{b}^{(1)}]$$
$$= P^{(1)}\overline{A}^{(1)} \equiv P^{(1)}\overline{A} = P^{(1)}[A\ \underline{b}]$$

în matricea $\overline{A}^{(2)} = [A^{(2)}\ \underline{b}^{(2)}]$ este

$$M^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/2 & 0 & 1 \end{bmatrix} \quad (1b)$$

Mai exact, are loc relația:

$$M^{(1)} P^{(1)} [A^{(1)}\ \underline{b}^{(1)}] = [A^{(2)}\ \underline{b}^{(2)}] \quad (1)$$

cu $P^{(1)}$ și $M^{(1)}$ date de (1a) și (1b).

$$\max_{i=\overline{2,3}} |a^{(2)}_{i2}| = \max\{|3/2|, |3/2|\}$$

$$= 3/2 = a^{(2)}_{22} = a^{(2)}_{32} \Rightarrow l \in \{2,3\}$$

$$\Rightarrow 2 \in \{2,3\}$$

$\Rightarrow$ Nu trebuie interschimbate liniile matricei $\overline{A}^{(2)} \Rightarrow$

Matricea permutare simplă:

$$P^{(2)} = I_3 \quad (2a)$$

$$P^{(2)}\overline{A}^{(2)} = P^{(2)}[A^{(2)}\ \underline{b}^{(2)}] = \left[\begin{array}{ccc|c} 2 & -1 & 2 & 3 \\ 0 & 3/2 & 3 & 1/2 \\ 0 & 3/2 & -2 & -1/2 \end{array}\right]$$

$$= [\tilde{A}^{(2)}\ \tilde{\underline{b}}^{(2)}] = \overline{\tilde{A}}^{(2)}$$

$\tilde{a}^{(2)}_{22} = 3/2 \neq 0$ (aplicăm MEGFP)

$\underline{i=\overline{3,3}}$: $\quad m^{(2)}_i := \tilde{a}^{(2)}_{i2} / \tilde{a}^{(2)}_{22}$

- $m^{(2)}_3 := \tilde{a}^{(2)}_{32} / \tilde{a}^{(2)}_{22} = \frac{3}{2}\,\frac{2}{3} = 1$

$(E_3 - m^{(2)}_3 E_2) \longrightarrow (E_3)$:

$\underline{j=\overline{3,3}}$: $\quad a^{(3)}_{3j} := \tilde{a}^{(2)}_{3j} - m_3\,\tilde{a}^{(2)}_{2j}$

$$a_{33}^{(3)} := -2-1\cdot 3 = -5$$

$$a_{23}^{(3)} = 0 \quad (\text{nu mai trebuie calculat!})$$

$$b_3^{(3)} := \tilde{b}_3^{(2)} - m_3^{(2)}\tilde{b}_2^{(2)} = -\frac{1}{2} - 1\frac{1}{2} = -1$$

Am obținut:

$$\overline{A}^{(3)} = [A^{(3)}\ \underline{b}^{(3)}] = \left[\begin{array}{ccc|c} 2 & -1 & 2 & 3 \\ 0 & 3/2 & 3 & 1/2 \\ 0 & 0 & -5 & -1 \end{array}\right] = [U\ \underline{\tilde{b}}]$$

<u>Obs</u>: Matricea care transformă

$$\overline{\tilde{A}}^{(2)} = [\tilde{A}^{(2)}\ \underline{\tilde{b}}^{(2)}] = P^{(2)}[A^{(2)}\ \underline{b}^{(2)}] = P^{(2)}\overline{A}^{(2)}$$

în matricea $\overline{A}^{(3)} = [A^{(3)}\ \underline{b}^{(3)}] = [U\ \underline{\tilde{b}}]$

este dată de

$$M^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \qquad (2b)$$

Mai exact, are loc relația:

$$M^{(2)}P^{(2)}[A^{(2)}\ \underline{b}^{(2)}] = [A^{(3)}\ \underline{b}^{(3)}] \qquad (2)$$

cu $M^{(2)}$ și $P^{(2)}$ date de (2a) și (2b).

Din relațiile (1) și (2) obținem:

$$\boxed{M^{(2)} P^{(2)} M^{(1)} P^{(1)} [\underline{A}\ \underline{b}] = [U\ \underline{\tilde{b}}]}$$

Obs: Sistemul $A\underline{x} = \underline{b}$ a devenit de forma $U\underline{x} = \underline{b}$, ie

$$\begin{cases} 2x_1 - x_2 + 2x_3 = 3 \\ \frac{3}{2}x_2 + 3x_3 = \frac{1}{2} \\ -5x_3 = -1 \end{cases}$$

și acesta se rezolvă prin metoda substituției descendente, ie de la ultima ecuație la prima:

$-5x_3 = -1 \Rightarrow \boxed{x_3 = 1/5}$

$\frac{3}{2}x_2 = \frac{1}{2} - 3x_3 \Leftrightarrow \frac{3}{2}x_2 = \frac{1}{2} - \frac{3}{5} \Leftrightarrow$

$x_2 = \frac{2}{3}\ \frac{5-6}{10} \Rightarrow \boxed{x_2 = -1/15}$

$2x_1 = 3 + x_2 - 2x_3 \Rightarrow \ldots \Rightarrow \boxed{x_1 = 19/15}$