## SEMIHAR 6

Dérivate partiale ale functiiler compuse. Diférentiabilitate de ordin superior. Formula lui Taylor

Exercitive 1 Calculati derivatele partiale all functiiler compuse m'urmatoarele casuri:

a) g(x,y)= f(\$\frac{4}{2},24y)

b) g(R,d) = f(Rcod, Raind)

PEZOLVARE a) Se considerci functia f: R2 > 1R

de clasa c² pe R2 of functia f: {cx,y} & R2 y + of > R2

definita prin f(x,y) = (x, 2xy) = (f,1x,y), b(x,y)

Functia g: {b,y} & R2 | y + of = D este definita

prin g(x,y) = (f of)(x,y). + (x,y) & D.

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b)  $f: \mathbb{R}^2 \to \mathbb{R}$  functje de clasa c<sup>1</sup>  $P: \mathbb{R}^2 \to \mathbb{R}$ ,  $f(\mathbb{R}, d) = (\mathbb{R}\cos d, \mathbb{R}\sin d) = (\mathbb{R}(\mathbb{R}, d), \mathbb{R}\cos d)$  $g: \mathbb{R}^2 \to \mathbb{R}$ ,  $g(\mathbb{R}, d) = (f \circ f)(\mathbb{R}, d)$ 

OR (P,d) = Of (PRODE, Packed). Of (P,d+ Or (PCOSK, Packet))

DE [P,d] = Of [RCOX, Round] (RCOX) + Of [RCOX, ROUND]

[Round] = Of [RCOX, Round]. COX + Of [RCOX, Round]

[Round] = Of [RCOX, Round]. COX + Of [RCOX, Round]

Dod (2,d) = Of (PRODE IPANA). Of (P.d) + Of (PRODE, P.Dine)

Ob (Pid) = Of (PRODE, PANA). (PRODE) + Of (PRODE, PShe)

(Paind) = Of (PRODE, PANA) (- PANA) +

+ Of (PRODE, PANA). (PRODE).

DEFINITIE Fie D=DCP omultime desolisat si f: D > R o functil de clasa C2 pe D. Se numeste laplacianal functili & functia Af: D > R definita prin Af (xt) = 2 th) 2 (x) + . + 2 (xt) & x & D.

Re seved, Df= = = 3 34,2 EXERCITION 2 Feet: P2 s Robernetie de clasa : c2 à function g: P2 > P2 definitai prini g(P, X) = = f(RCO)d, Roind) F(RodeR2. Calculate Ag. REBOLNARE Stim den exercitul, pot 16), ca OR ERIX) = Of IRCON, ROUND, COOK + Of IRCON, R sind. 02 (R,d) = - Of (R, cood, Rsond). Psind+ Of (Rsond, Rsind) Prood. FR (Rid) = D ( DQ (PR)) = rosd DR ( Du ( Prosd, Pahal) trand of (of (prosd, paind))= rosd (of (prosd, paind) rose + It (Prost, Prind). mind) + mind (It (Prost, Prind) con + of (Rand, Round). sind) To (Rid) = cos2 x 2 f (Rcos x, Paind) + 2containd 3 f (Rcos), Paind) + 2containd 3 f (Rcos), Paind) + 12 containd 3 f (Rcos), Paind) + (Rco) & (Rcos), Paind) + (Rcos), Paind) + (Rcos) & (Rcos), Paind) + (Rcos), Paind) + (Rcos) & (Rcos), Paind) + (Rcos), Of (P,d) = of ( Of (P,d)) = - Proof. of (Prood, Prind) - Prind of ( of ( Prind, Prind)). + Prind of (Prod, Prim)
+ Prood. of ( of ( Priod, Prind))

Od2 (P.d) = - Roosd Of (Pross, Round) - R sind total Production Pr - Prind of (Prood, 2 mind) + 12 cood [ of prood, 12 mind).
(-12 mind) + of (Prood, 12 mind). Prood ] 02 (PLL)=-PLOSE (PRODE, PRINT)-RRIVE. OF (PROSE, PRINT) + P2 m2 L II (Prod, Prind) - 2P2 wholeon of Prod, Prind) + P2 cos L St (Prod, Prind), HP, L) ete2 (2) Adunam relative onit si obtenem  $\Delta g(R, \chi) = \frac{\partial g}{\partial \chi^2} (R, \chi) + \frac{\partial g}{\partial R^2} (R, \chi) = \kappa \omega^2 \chi \left( \frac{\partial^2 f}{\partial \mu^2} (R \kappa \omega) + \frac{\partial g}{\partial \mu^2} (R \kappa \omega) \right)$ + P234 (Rand) + sin2 (B4 (Rand) + + P2 22 Lexost, Raind) - Pauld Of Lexost, Raind -- Proof of (Proof, Pmhd) + 2 cost and 34 (Proof, Print).

EXERCITION 3 File f: R2 > R o functil definita

frun flags = g2ln(1+32), y = 0

0, 4 = 0

a) Calculate derivatele partiale ale functie f si araitate sa feste functie de dasa c' pe R<sup>2</sup>

b) Calculate derivatele partiale de ordinal doi matte function of si verificate continuitare acestora.

\*REZOLVARE

a) 
$$\frac{1}{1+1}(x_1y) = (y^2 \ln(1+x^2))^{\frac{1}{2}} = y^2 \cdot \frac{(1+x^2)^{\frac{1}{2}}}{1+x^2} = \frac{2y^2x}{1+x^2} + (x_1y) \in \mathbb{R}^2 \text{ an } y \neq 0$$
 $\frac{1}{1+1}(x_1y) = \frac{1}{1+1}(x_1y) = \frac$ 

of (x,y) = (y2ln(1+ x2)) y= 2y ln(1+ x2)+y2 (+x2) y = 2y ln (+'x2) - 2x2y + (x,y)ex2 ouy+0

of (d, 0) = line flag) - flago = line y² lm(1+d²) - 0

y d, 0) = line flag) - flago = line y² lm(1+d²) - 0

= lime y ln (1+d²) = 0

of (x,y)= \ \frac{24^2x}{x^2+4^2} : y = 0

Functiile of ni of sunt continue pe 12° {(x,y)|

lem of (24,4) = line 242x (+14) - (210) 2442

 $\left|\frac{2y^2x}{x^2+y^2}\right| = \frac{2y^2|x|}{x^2+y^2} \le \frac{2y^2|x|}{x^2+y^2} = \frac{2|x|}{y^2} = 2|x| = 3$ Thim of  $(x,y) = 0 = \frac{2y}{x}(x,0) = \frac{2y}{x} = xx$   $(x,y) \to (x,y) = 0 = \frac{2y}{x}(x,0) = \frac{2y}{x} = xx$ 

functie continua mi (d, 0)

lim of (xiy)= lim [2yln (1+x2)-2x2y]
(014) -> (xi) > (xiy)= lim (xiy) -> (x

|24 m(1+ x²) - 2x²y | = 214 lm(1+ x²) + 2x² 141 = 2x² 141 + 4141 lm | 1+x² = 2141+4141 | 1+x² = = 2141+4172 = lim Of (x;y) = 0= 24(x0) = 34 lmothe continua m² (x;0).

ASADAR, fordmite toate derentille partiale si sunt functii continue? adica feste functie de clasa c' pe per

(2) 34 (x/y)= 3y (34)(x/y)= (24/32) のましょう)= のましましば、の)= lim がん(以り)- がは(人の) 2以え (以り) のましばしい)  $=\lim_{N\to\infty}\frac{2y^2d}{d^2+y^2}-0=\lim_{N\to\infty}\frac{2yd}{x^2+y^2}=0 \; \forall \; d \in \mathbb{R}.$ 3+ 24 (x,y)= 0x ( of (x,y))= (2yln(1+x2)-2xy/x = 2y. \frac{(1+\frac{1}{1+\frac{1}{2}})^2}{1+\frac{1}{1+\frac{1}{2}}} \frac{(2\frac{1}{2}\frac{1}{2})^2}{(1+\frac{1}{2}\frac{1}{2})} \frac{(2\frac{1}{2}\frac{1}{2})^2}{(1+\frac{1}{2}\frac{1}{2})} \frac{(2\frac{1}{2}\frac{1}{2})^2}{(1+\frac{1}{2}\frac{1}{2})^2} \frac{(2\frac{1}{2}\frac{1 34 (x,0) = line ox ( 24) (x,0) = line oy (x,0) - of (x,0) - of (x,0) = lim 0-0 x-3d x-d = 0 + der 2f (x,y) = 3f (x,y) = { (x,y) = { (x,y) = 0 } (x,y) = 0 } (x,y) = 0 0, 4=0 Le observa ca derivatelle partiale mixte de ordinul doi sunt egale in orice punt din 122. Intrebarea este daça trebuie sa fierantique inorice pund pentru a verifica daca aceasta conditie este necesar în cadrul teoremei lui Schwarz.

Vom verifica continuitatealor de (0,0) lim 34 (254)=line (254)=line (254)2)2 (254)-10,0) (254)2)2 lim 34 (1, 1)= lim 4.1, 1/2 = lim 4/4 = 1

(1, 2+1, 2) = lim 4/4 = 1 ein  $\frac{34}{000}$   $(\frac{1}{n},0) = \lim_{n\to\infty} \frac{4\cdot\frac{1}{n}\cdot0}{\left(\frac{1}{n^2}+0\right)^2} = 0$ 1+0=) # lem of (xy), # of (xy) =)

24 of (xy) + (qo) or ay (xy), # oyor (x,y) =)

0xoy oyox nu sunt continue & (qo). CONCLUZIE: Continuidates lui of este o conditie suficienta, dar nux necesara pentru egalitatea off off of a cadrul teoremei lui Schwarz offy EXERCITIVE 4 se considerá functia 4: P2-3P, f(xy)= xy3+2xy-2x2+3x+y-2 10 (24 y) e122. a) Calculate derivatele partiale de ordinel doi alefunciei f b) Calculati df (1,2) si df (1,2) c) saise sorie polinomul Taylor de ordin 2

assaint functie f in punctul (1,2)

## PEZOLVARE

a) of (xy)= (xy)+2xy-2x2+3x+y-2) x= = 43+24-4x+3 4(2,4)=122 04 (x14)= (x43+2x4-2x2+3x+4-2)4= = 32/18 + 202 +1 4 (24/4) 0122 of of function continue pe 122 plunatie de 122 multime deschisa flunctie de classe c' pe 122 of (x,y) = 0 ( of (x,y)=(43+2y-4x+3)=-4 040y (x,y)= (3xy2+2x+1) = 3y2+2 oyor (x,4) = (43+24-4x+3) = 342+2 of (x,y)= (3xy2+2x+1)y=6xy or orday ogox ogg functii continue per 1 => f functie de clasai c'e R2 b) df(1,2): R2 -> R df(112)(年,水)=鉄(1,2).火+銭(1/2).火=

= - 2×+1519 + (x, y) E 12

$$d^{2}f(1,2): \mathbb{R}^{2} \longrightarrow \mathbb{R} \cdot \text{Remtm.} \quad (U,V) \in \mathbb{R}^{2},$$

$$d^{2}f(1,2)(U,V) = \frac{\partial^{2}f}{\partial x^{2}}(1,2)U^{2} + 2\frac{\partial^{2}f}{\partial x\partial y}(1,2)U \cdot V +$$

$$+ \frac{\partial^{2}f}{\partial y^{2}}(1,2)V^{2} = -4U^{2} + 2\theta \cdot UV + 12V^{2}$$

$$c) T_{2}(x,y) = f(1,2) + \frac{1}{1!} df(1,2)(x,y) - (x,2)$$

$$+ \frac{1}{2!} d^{2}f(1,2)(x,y) - (x,2)$$

$$= f(1,2) + \frac{1}{1!} df(1,2)(x-1,y-2) + \frac{1}{2!} d^{2}f(1,2)(x-1,y-2)$$

$$= f(1,2) + \frac{\partial^{2}f}{\partial x^{2}}(1,2)(x-1)^{2} + 2\frac{\partial^{2}f}{\partial x^{2}}(1,2)(y-2) +$$

$$+ \frac{1}{2!} \left[ \frac{\partial^{2}f}{\partial x^{2}}(1,2)(y-1)^{2} + 2\frac{\partial^{2}f}{\partial x^{2}}(1,2)(x-1)y-2) +$$

$$+ \frac{\partial^{2}f}{\partial y^{2}}(1,2)(y-2)^{2} \right]$$
Deci, pentul orice  $(x,y) \in \mathbb{R}^{2}$ ,
$$T_{2}(x,y) = 13 + 11(x-1) + 15(y-2) +$$

$$+ \frac{1}{2!} \left[ -4(x-1)^{2} + 28(x-1)y-2) + 12(y-2)^{2} \right]$$

 $+\frac{1}{2}\left[-4(x-1)^{2}+28(x-1)M-2)+12(y-2)^{2}\right]$ 

TEMA

Ex. 1 Calculati derivatele partiale ale functile of (x,y) = f(xy, x²+y²) + (x,y) e r².

Ex. 2 Le considera functia f: r² > R definita prin f(x,y) = x³ + 3xy² + y³ + 2x - 3y + 1 + (xy) e r².

a) Calculate derivatele partiale de ordinul doi ale functile f

b) Calculati df(1,0) sidf(40). c) sà se scrie polinomed Eylor de orden doi assciat functiei f au punctul (40)