PCMMP: sistemul augmentat & MEGPP

Considerăm sistemul supradeterminat de ecuații liniare

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}}_{=A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{=\underline{x}} = \underbrace{\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}}_{=\underline{b}} \qquad (1)$$

Determinați soluția sistemului (1) în sensul celor mai mici pătrate folosind sistemul augmentat asociat și MEGPP.

În acest caz, $m = 3$ și $n = 2$.

Sistemul augmentat asociat lui (1) este dat de:

$$\begin{bmatrix} I_3 & A \\ A^T & O_2 \end{bmatrix} \begin{bmatrix} \underline{r} \\ \underline{x} \end{bmatrix} = \begin{bmatrix} \underline{b} \\ \underline{0}_2 \end{bmatrix} \implies$$

$$\underbrace{\left[\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{array}\right]}_{=B} \underbrace{\left[\begin{array}{c} r_1 \\ r_2 \\ r_3 \\ \hline x_4 \\ x_2 \end{array}\right]}_{=\underline{y}} = \underbrace{\left[\begin{array}{c} 2 \\ 1 \\ 1 \\ \hline 0 \\ 0 \end{array}\right]}_{=\underline{c}} \qquad (2)$$

Matricea extinsă a sistemului augmentat (2) este dată de:

$$\overline{B}^{(0)} = \overline{B} = [B|\underline{c}] = \left[\begin{array}{c|cccc|c} 1 & 0 & 0 & 1 & 1 & 2 \\ \hline 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{array}\right] \sim$$

$$\left.\begin{array}{l} (E_4 - E_1) \longrightarrow (E_4) \\ (E_5 - E_1) \longrightarrow (E_5) \end{array}\right\} \Longrightarrow$$

$$\sim \overline{B}^{(1)} = \left[B^{(1)} \,\middle|\, \underline{c}^{(1)} \right] = \left[\begin{array}{cc:ccc|c} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ \hdashline 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & -2 \\ 0 & 1 & 0 & -1 & -1 & -2 \end{array}\right]$$

$$(E_5 - E_2) \longrightarrow (E_5) \implies$$

$$\sim \overline{B}^{(2)} = \left[B^{(2)} \,\middle|\, \underline{c}^{(2)} \right] = \left[\begin{array}{ccc:cc|c} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ \hdashline 0 & 0 & 0 & -1 & -1 & -2 \\ 0 & 0 & 0 & -1 & -2 & -3 \end{array}\right]$$

$$\sim \overline{B}^{(3)} = \left[B^{(3)} \,\middle|\, \underline{c}^{(3)} \right] = \left[\begin{array}{cccc:c|c} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & -2 \\ \hdashline 0 & 0 & 0 & -1 & -2 & -3 \end{array}\right]$$

$$\sim \overline{B}^{(4)} \equiv \left[B^{(4)} \,|\, \underline{c}^{(4)}\right] = \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & -2 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{array}\right]$$

Am obținut următor sistem superior triunghiular echivalent:

$$\begin{cases} r_1 + x_1 + x_2 = 2 \\ r_2 + x_2 = 1 \\ r_3 = 1 \\ -x_1 - x_2 = -2 \\ -x_2 = -1 \Rightarrow \boxed{x_2 = 1} \Rightarrow \end{cases}$$

$$\Rightarrow x_1 = 2 - x_2 \Rightarrow \boxed{x_1 = 1}$$

$$\boxed{r_3 = 1} ; \quad r_2 = 1 - x_2 \Rightarrow \boxed{r_2 = 0}$$

$$r_1 = 2 - x_1 - x_2 \Rightarrow \boxed{r_1 = 0}$$

$\square$