Special topics in Logic and Security I

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References

- Cremers, C. J. F. (2006). Scyther : semantics and verification of security protocols Eindhoven: Technische Universiteit Eindhoven DOI: 10.6100/IR614943
- Cremers C. and Mauw S. Operational Semantics and Verification of Security Protocols. Springer, 2012.

Protocol specification

$$
\begin{aligned}
\mathit{RoleTerm} \quad ::= \quad & \mathit{Var} \mid \mathit{Fresh} \mid \mathit{Role} \mid \mathit{Func}\,(\mathit{RoleTerm}^*) \\
& \mid (\mathit{RoleTerm}, \mathit{RoleTerm}) \mid \{\!|\ \mathit{RoleTerm}\ |\!\}_{\mathit{RoleTerm}} \\
& \mid sk(\mathit{RoleTerm}) \mid pk(\mathit{RoleTerm}) \mid k(\mathit{RoleTerm}, \mathit{RoleTerm})
\end{aligned}
$$

$$
\begin{aligned}
\mathit{RoleEvent}_R \quad ::= \quad & \mathit{send}_{\mathit{Label}}(R, \mathit{Role}, \mathit{RoleTerm}) \\
& \mid \mathit{recv}_{\mathit{Label}}(\mathit{Role}, R, \mathit{RoleTerm}) \\
& \mid \mathit{claim}_{\mathit{Label}}(R, \mathit{Claim}[, \mathit{RoleTerm}])
\end{aligned}
$$

$$
\mathit{RoleEvent} = \bigcup_{R \in \mathit{Role}} \mathit{RoleEvent}_R
$$

Protocol specification

- $P(R)$ is the specification of the role R for $P \in \mathit{Protocol}$ and $R \in \mathit{Role}$.

  $P(R) = (KN_0(R), s) \in \mathcal{P}(\mathit{RoleTerm}) \times \mathit{RoleEvent}_R^*$
- Role specification

$$\mathit{RoleSpec} = \{(kn, s) \mid kn \in \mathcal{P}(\mathit{RoleTerm}) \wedge \forall rt(rt \in kn \rightarrow \mathit{vars}(rt) = \emptyset) \\ \wedge\, s \in \mathit{RoleEvent}^* \wedge\ \mathit{wellformed}(s)\}$$

- Protocol specification

$$\mathit{Protocol} = \mathit{Role} \rightharpoonup \mathit{RoleSpec}$$

A protocol specification is a partial function from roles to role specifications.

Protocol specification

$$Protocol = Role \rightharpoonup RoleSpec$$

The roles i and r of NSPK are specified as follows:

$$\begin{aligned}
NS(i) = \quad & (\{i, r, ni, sk(i), pk(i), pk(r)\}, \\
& [send_1(i, r, \{\!|\, ni, i \,|\!\}_{pk(r)}), \\
& recv_2(r, i, \{\!|\, ni, V \,|\!\}_{pk(i)}), \\
& send_3(i, r, \{\!|\, V \,|\!\}_{pk(r)}), \\
& claim_4(i, synch)])
\end{aligned}
\qquad
\begin{aligned}
NS(r) = \quad & (\{i, r, nr, sk(r), pk(r), pk(i)\}, \\
& [recv_1(i, r, \{\!|\, W, i \,|\!\}_{pk(r)}), \\
& send_2(r, i, \{\!|\, W, nr \,|\!\}_{pk(i)}), \\
& recv_3(i, r, \{\!|\, nr \,|\!\}_{pk(r)}), \\
& claim_5(r, synch)])
\end{aligned}$$

RoleEvent Order

For a protocol P we define:

- the *RoleEvent Order* is the total order $\epsilon_1 <_R \cdots <_R \epsilon_n$ where R is a role with $P(R) = (kn, [\epsilon_1, \ldots, \epsilon_n])$.
- the *Communication Relation* $\dashrightarrow \subseteq RoleEvent \times RoleEvent$ is defined by

  $\epsilon_1 \dashrightarrow \epsilon_2$ if and only if
  there exist $\mathbf{l} \in Label$, $R, R' \in Role$, $rt_1, rt_2 \in RoleTerm$
  such that $\epsilon_1 = send_{\mathbf{l}}(R, R', rt_1)$ and $\epsilon_2 = recv_{\mathbf{l}}(R', R, rt_2)$
- the *Protocol Order* is

$$\prec_P = \left(\dashrightarrow \cup \bigcup_{R \in Role} <_R \right)^+$$

Protocol Order $\prec_P \subseteq \mathit{RoleEvent} \times \mathit{RoleEvent}$

For the NSPK protocol

$$
\begin{array}{ll}
NS(i) = & (\{i, r, ni, sk(i), pk(i), pk(r)\}, \\
& [\mathit{send}_1(i, r, \{\!|\, ni, i \,|\!\}_{pk(r)}), \\
& \mathit{recv}_2(r, i, \{\!|\, ni, V \,|\!\}_{pk(i)}), \\
& \mathit{send}_3(i, r, \{\!|\, V \,|\!\}_{pk(r)}), \\
& \mathit{claim}_4(i, \mathit{synch})])
\end{array}
\qquad
\begin{array}{ll}
NS(r) = & (\{i, r, nr, sk(r), pk(r), pk(i)\}, \\
& [\mathit{recv}_1(i, r, \{\!|\, W, i \,|\!\}_{pk(r)}), \\
& \mathit{send}_2(r, i, \{\!|\, W, nr \,|\!\}_{pk(i)}), \\
& \mathit{recv}_3(i, r, \{\!|\, nr \,|\!\}_{pk(r)}), \\
& \mathit{claim}_5(r, \mathit{synch})])
\end{array}
$$

the event order is:

$$
\begin{array}{ccc}
\mathit{send}_1(i, r, \{\!|\, ni, i \,|\!\}_{pk(r)}) & \prec_{NS} & \mathit{recv}_1(i, r, \{\!|\, W, i \,|\!\}_{pk(r)}) \\
\curlywedge_{NS} & & \curlywedge_{NS} \\
\mathit{recv}_2(r, i, \{\!|\, ni, V \,|\!\}_{pk(i)}) & \succ_{NS} & \mathit{send}_2(r, i, \{\!|\, W, nr \,|\!\}_{pk(i)}) \\
\curlywedge_{NS} & & \curlywedge_{NS} \\
\mathit{send}_3(i, r, \{\!|\, V \,|\!\}_{pk(r)}) & \prec_{NS} & \mathit{recv}_3(i, r, \{\!|\, nr \,|\!\}_{pk(r)}) \\
\curlywedge_{NS} & & \curlywedge_{NS} \\
\mathit{claim}_4(i, \mathit{synch}) & & \mathit{claim}_5(r, \mathit{synch})
\end{array}
$$

Formalizing protocol execution

Protocol execution

- The specification of a protocol describes each role.
- When a protocol is executed, an agent can play any role, one or more times, sequentially or in parallel (concurrently).
- A single execution of a role is called a *run*. Different runs are described using *RunIdentifiers* (RID). The concrete execution of a protocol is described using *RunTerms*.
- Turning a role description into a run with the help of run identifiers is called *instantiation*. Note that the fresh values should be uniquely identified in each run.

The description of a run: *RunTerm*

$$
\begin{aligned}
\mathit{RunTerm} \quad ::= \quad & \mathit{Var}^{\#\mathit{RID}} \mid \mathit{Fresh}^{\#\mathit{RID}} \mid \mathit{Role}^{\#\mathit{RID}} \\
& \mid \mathit{Agent} \\
& \mid \mathit{Func}\,(\mathit{RunTerm}^*) \\
& \mid (\mathit{RunTerm}, \mathit{RunTerm}) \\
& \mid \{\!|\, \mathit{RunTerm} \,|\!\}_{\mathit{RunTerm}} \\
& \mid \mathit{AdversaryFresh} \\
& \mid \mathit{sk}(\mathit{RunTerm}) \mid \mathit{pk}(\mathit{RunTerm}) \mid k(\mathit{RunTerm}, \mathit{RunTerm})
\end{aligned}
$$

- $^{-1} : \mathit{RunTerm} \rightarrow \mathit{RunTerm}$
- *AdversaryFresh* are run terms generated by an adversary

Deduction system on $Term = RoleTerm \cup RunTerm$

We extend the deduction to

$$Term = RoleTerm \cup RunTerm$$

$$\vdash \subseteq \mathcal{P}(Term) \times Term$$

$M \vdash t$ means that t can be deduced knowing M

$\vdash$ is the least relation with the following properties:

$$\begin{array}{lrll}
\text{if} & t \in M & \text{then} & M \vdash t \\
\text{if} & M \vdash t_1 \text{ and } M \vdash t_2 & \text{then} & M \vdash (t_1, t_2) \\
\text{if} & M \vdash (t_1, t_2) & \text{then} & M \vdash t1 \text{ and } M \vdash t_2 \\
\text{if} & M \vdash t \text{ and } M \vdash k & \text{then} & M \vdash \{\!|\, t \,|\!\}_k \\
\text{if} & M \vdash \{\!|\, t \,|\!\}_k \text{ and } M \vdash k^{-1} & \text{then} & M \vdash t \\
\text{if} & M \vdash t_1 \text{ and } \ldots \text{ and } M \vdash t_n & \text{then} & M \vdash f\,(t_1, \ldots, t_n)
\end{array}$$

Deduction system on $\mathit{Term} = \mathit{RoleTerm} \cup \mathit{RunTerm}$

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\text{if} & M \vdash (t_1, t_2) & \text{then} & M \vdash t1 \text{ and } M \vdash t_2 \\
\text{if} & M \vdash t \text{ and } M \vdash k & \text{then} & M \vdash \{\!| \, t \, |\!\}_k \\
\text{if} & M \vdash \{\!| \, t \, |\!\}_k \text{ and } M \vdash k^{-1} & \text{then} & M \vdash t \\
\text{if} & M \vdash t_1 \text{ and } \ldots \text{ and } M \vdash t_n & \text{then} & M \vdash f\,(t_1, \ldots, t_n)
\end{array}
$$

Exercise: Prove that $\{\{\!| \, n^{\#1} \, |\!\}_k, \{\!| \, k^{-1} \, |\!\}_{pk(r^{\#3})}, sk(r^{\#3})\} \vdash \{\!| \, n^{\#1} \, |\!\}_{sk(r^{\#3})}$.

The description of a run: *RunTerms*

- In order to specify a protocol we used generic terms from *RoleTerm*, which will be instantiated when we describe a concrete run.

  For example:
  the generic term i that designates the initiator role will be instantiated with A (Alice) which designates a concrete agent;

  the generic fresh value ni will be instantiated with $ni^{\#1}$, $ni^{\#2}$, ... which are the concrete values generated in the first run, the second run.

- An *instantiation* is a triplet

$$(\theta, \rho, \sigma) \in \mathit{RID} \times (\mathit{Role} \rightharpoonup \mathit{Agent}) \times (\mathit{Var} \rightharpoonup \mathit{RunTerm})$$

Term instantiation

Let *Inst* be the set of all instantiations:

$Inst = RID \times (Role \rightharpoonup Agent) \times (Var \rightharpoonup RunTerm)$

To any $inst = (\theta, \rho, \sigma) \in Inst$ we associate a function

$$inst : RoleTerm \to RunTerm$$

defined as follows:

- $inst(n) = n^{\#\theta}$ pentru $n \in Fresh$
- $inst(R) = \rho(R)$ for $R \in Role \cap dom(\rho)$
  $inst(R) = R^{\#\theta}$ for $R \in Role \setminus dom(\rho)$
- $inst(V) = \sigma(V)$ for $V \in Var \cap dom(\sigma)$
  $inst(V) = V^{\#\theta}$ for $V \in Var \setminus dom(\sigma)$

Term instantiation

$Inst = RID \times (Role \rightharpoonup Agent) \times (Var \rightharpoonup RunTerm)$

$inst = (\theta, \rho, \sigma) \in Inst$

- $inst(f(t_1, \ldots, t_n)) = f(inst(t_1), \ldots, inst(t_n))$
- $inst(t_1, t_2) = (inst(t_1), inst(t_2))$
- $inst(\{\!|\ t_1\ |\!\}_{t_2}) = \{\!|\ inst(t_1)\ |\!\}_{inst(t_2)}$
- $inst(sk(t)) = sk(inst(t))$, $inst(pk(t)) = pk(inst(t))$
- $inst(k(t_1, t_2)) = k(inst(t_1), inst(t_2))$

Term instantiation

$inst = (\theta, \rho, \sigma) \in RID \times (Role \rightharpoonup Agent) \times (Var \rightharpoonup RunTerm)$

Example:
$t = \{\!|\, W, nr, r \,|\!\}_{pk(i)} \in RoleTerm$

- $inst = (2, \{i \mapsto B, r \mapsto A\}, \{W \mapsto ni^{\#1}\})$

  $inst(\{\!|\, W, nr, r \,|\!\}_{pk(i)}) = \{\!|\, ni^{\#1}, nr^{\#2}, A \,|\!\}_{pk(B)} \in RunTerm$

- $inst = (2, \{i \mapsto B\}, \emptyset)$

  $inst(\{\!|\, W, nr, r \,|\!\}_{pk(i)}) = \{\!|\, W^{\#2}, nr^{\#2}, r^{\#2} \,|\!\}_{pk(B)} \in RunTerm$

Operational semantics

Recall operational semantics for programming languages

- Language

```
E ::= n | x | E + E
C ::= x=E;|C C|
      { C } | {}
P ::= int x = n ; P | C
```

- Rules for transitions:

$$\langle int\ x = i; p, \sigma\rangle \rightarrow \langle int\ \ p, \sigma_{x\leftarrow i}\rangle$$

$$\frac{\langle e_1, \sigma\rangle \rightarrow \langle e_1', \sigma\rangle}{\langle e_1 + e_2, \sigma\rangle \rightarrow \langle e_1' + e_2, \sigma\rangle}$$

- Transition system: a program execution is a sequence of transitions

$$\begin{array}{lcl} \langle x = 0; x = x + 1;\ ,\ \perp\rangle & \rightarrow & \langle x = x + 1;\ ,\ x \mapsto 0\rangle \\ & \rightarrow & \langle x = 0 + 1;\ ,\ x \mapsto 0\rangle \\ & \rightarrow & \langle x = 1;\ ,\ x \mapsto 0\rangle \\ & \rightarrow & \langle \{\}\ ,\ x \mapsto 1\rangle \end{array}$$

Labelled transition system (LTS)

We recall that a *labelled transition system* is a tuple $(St, L, \rightarrow, st_0)$ where:

- St is the set of states
- L is the set of labels
- $\rightarrow \subseteq S \times L \times S$ is the transition relation
- $st_0 \in St$ is the initial state

An *execution*: $[st_0, \alpha_1, st_1, \alpha_2, \ldots, \alpha_n, st_n]$ such that $st_i \stackrel{\alpha_{i+1}}{\rightarrow} st_{i+1}$

A *trace*: $[\alpha_1, \alpha_2, \ldots, \alpha_n]$

The operational semantics of a security protocol P is defined using a labelled transition system

$$(State, RunEvent, \rightarrow, st_0(P))$$

Possible executions

The set of all possible executions is

$$Run = Inst \times RoleEvent^*$$

- The runs that can be created by a protocol P are defined by

  $runsof : Protocol \times Roles \rightarrow \mathcal{P}(Run)$

  $$\begin{aligned} runsof(P, R) = \{(inst, s) \mid \text{ there exists } kn \text{ such that } P(R) = (kn, s) \\ inst = (\theta, \rho, \sigma) \text{ with } dom(\rho) = roles(s)\} \end{aligned}$$

  where $R \in dom(P)$ and $roles(s)$ are the roles from s.

- For $F \subseteq Run$ we set

  $runIds(F) = \{\theta \mid ((\theta, \rho, \sigma), s) \in F \text{ for some } \rho, \sigma, s\}$

States

$Run = Inst \times RoleEvent^*$

$$State = \mathcal{P}(RunTerm) \times \mathcal{P}(Run)$$

$st = \langle\!\langle AKN, F \rangle\!\rangle \in State$

- AKN is the adversary knowledge,
- $F \subseteq Run$ are the runs that has to be executed.

Exemple:

- $st_1 = \langle\!\langle \{A, B, pk(A), pk(B), \{\!|\, ni^{\#2} \,|\!\}_{pk(A)}\}, \emptyset \rangle\!\rangle$
- $st_2 = \langle\!\langle AKN, F \rangle\!\rangle$ where

  $AKN = \{A, B, pk(A), pk(B)\}$ and
  $F = \{(\,(2, \{i \mapsto A, r \mapsto B\}, \emptyset), [send_1(i, r, \{\!|\, ni \,|\!\}_{pk(r)})]\,)\}$

RunEvent

In order to specify a protocol we use:

$$\begin{aligned} \mathit{RoleEvent}_R \quad ::= \quad & \mathit{send}_{\mathit{Label}}(R, \mathit{Role}, \mathit{RoleTerm}) \\ & \mid \mathit{recv}_{\mathit{Label}}(\mathit{Role}, R, \mathit{RoleTerm}) \\ & \mid \mathit{claim}_{\mathit{Label}}(R, \mathit{Claim}[, \mathit{RoleTerm}]) \end{aligned}$$

$$\mathit{RoleEvent} = \bigcup_{R \in \mathit{Role}} \mathit{RoleEvent}_R$$

In order to describe the *concrete execution* of a protocol we define:

$$\mathit{RunEvent} = \mathit{Inst} \times (\mathit{RoleEvent} \cup \{\mathit{create}(R) \mid R \in \mathit{Role}\})$$

- $\mathit{create}(R)$ is used to mark a new run of a role.

Operational semantics

We are now able to define the operational semantics of a security protocol P using the labelled transition system

$$(State, RunEvent, \rightarrow, st_0(P))$$

$State = \mathcal{P}(RunTerm) \times \mathcal{P}(Run)$

$st_0(P) = \langle\langle AKN_0(P), \emptyset \rangle\rangle$ where $AKN_0(P)$ is the initial adversary knowledge.

In order to model the adversary knowledge, the set of agents is partitioned in *honest agents* and *corrupted agents*: $Agent = Agent_H \cup Agent_C$. The (Dolev-Yao) adversary controls the network, (s)he creates fresh terms and (s)he knows the initial knowledge of the compromised agents.

For example, in the Needham-Schroeder protocol

$$AKN_0(NS) = AdversaryFresh \ \cup \ Agent \cup \{pk(A) \mid A \in Agent\} \cup \{sk(A) \mid A \in Agent_C\}$$

Operational semantics

$$(State, RunEvent, \rightarrow, st_0(P))$$

- $State = \mathcal{P}(RunTerm) \times \mathcal{P}(Run)$ where $Run = Inst \times RoleEvent^*$
- $st_0(P) = \langle\langle AKN_0(P), \emptyset \rangle\rangle$ where $AKN_0(P)$ is the initial adversary knowledge
- $Inst = RID \times (Role \rightharpoonup Agent) \times (Var \rightharpoonup RunTerm)$
- $RunEvent = Inst \times (RoleEvent \cup \{create(R) \mid R \in Role\}$
- The *transition system* has four rules, one for each of the events:

$$create, send, recv, claim$$

Operational semantics: transitions

$$(\mathit{State}, \mathit{RunEvent}, \rightarrow, \mathit{st}_0(P))$$

$$[\mathit{create}_P]\ \frac{R \in \mathit{dom}(P) \quad ((\theta,\rho,\emptyset),s) \in \mathit{runsof}(P,R) \quad \theta \notin \mathit{runsIDs}(F)}{\langle\!\langle \mathit{AKN}, F \rangle\!\rangle \xrightarrow{((\theta,\rho,\emptyset),\mathit{create}(R))} \langle\!\langle \mathit{AKN}, F \cup \{((\theta,\rho,\emptyset),s)\} \rangle\!\rangle}$$

Recall that

$\mathit{runIds}(F) = \{\theta \mid ((\theta,\rho,\sigma),s) \in F$ for some $\rho,\sigma,s\}$ and

$F \subseteq \mathit{Run} = \mathit{Inst} \times \mathit{RoleEvent}^*$.

Operational semantics : transitions

$$(State, RunEvent, \rightarrow, st_0(P))$$

$$[send]\ \frac{e = send_l(R_1, R_2, m) \quad (inst, [e] \cdot s) \in F}{\langle\langle AKN, F \rangle\rangle \xrightarrow{(inst, e)} \langle\langle AKN \cup \{inst(m)\}, F \setminus \{(inst, [e] \cdot s)\} \cup \{(inst, s)\} \rangle\rangle}$$

$$[claim]\ \frac{e = claim_l(R, c, t) \quad (inst, [e] \cdot s) \in F}{\langle\langle AKN, F \rangle\rangle \xrightarrow{(inst, e)} \langle\langle AKN, F \setminus \{(inst, [e] \cdot s)\} \cup \{(inst, s)\} \rangle\rangle}$$

Matching

We define a predicate *Match* that matches an incoming message (from *RunTerm*) with a given pattern (from *RoleTerm*) and extends the instantiation:

$$Match \subseteq Inst \times RoleTerm \times RunTerm \times Inst$$

$Match(inst, pt, m, inst')$ holds if

- $inst = (\theta, \rho, \sigma)$,
  $inst' = (\theta, \rho, \sigma')$
- $inst'(pt) = m$, $pt \in RoleTerm$, $m \in RunTerm$
- $dom(\sigma') = dom(\sigma) \cup vars(pt)$
- $\sigma \subseteq \sigma'$
- $\sigma'(v) \in type(v)$ for any $v \in dom(\sigma')$,

where $vars(pt)$ is the set of variables from *Var* which appear in *pt*, and $type(v)$ is a function that depends on the agent model.

Matching

Example:

We consider $type(V) \in \{S_1, S_2, S_3, S_4, S_5\}$ such that

$$
\begin{array}{lll}
S_1 & ::= & Agent \\
S_2 & ::= & Func\,(RunTerm^*) \\
S_3 & ::= & Fresh|\ AdversaryFresh \\
S_4 & ::= & sk(RunTerm)\,|\,pk(RunTerm) \\
S_5 & ::= & k(RunTerm, RunTerm)
\end{array}
$$

If $type(X) = S_3$ then

- $Match((1, \rho, \emptyset), X, nr^{\#2}, (1, \rho, \{X \mapsto nr^{\#2}\})$
- $\neg\ Match((1, \rho, \emptyset), nr, nr^{\#2}, inst')$ wrong instantiation
- $\neg\ Match((1, \rho, \emptyset), X, (nr^{\#1}, nr^{\#2}), inst')$ wrong type

Operational semantics : transitions

$$(State, RunEvent, \rightarrow, st_0(P))$$

$$[recv]\ \frac{e = recv_l(R_1, R_2, pt) \;\; AKN \vdash m \;\; (inst, [e] \cdot s) \in F \;\; Match(inst, pt, m, inst')}{\langle\langle AKN, F \rangle\rangle \xrightarrow{(inst', e)} \langle\langle AKN, F \setminus \{(inst, [e] \cdot s)\} \cup \{(inst', s)\}\rangle\rangle}$$

- Recall that $Match(inst, pt, m, inst')$ holds if the incoming message m is matched with the pattern pt and the instantiation $inst'$ is $inst$ extended with the new assignments.
- Note that m is inferred from AKN from using the previously defined deduction system on terms. Clearly m might be added to AKN in a *send* transtion, but also m can be defined using adversary capabilities: for example, $AKN \vdash \{\!|\, na, A \,|\!\}_{pk(A)}$, where $A \in Agent$ and $na \in AdversaryFresh$

Rules for $(State, RunEvent, \rightarrow, st_0(P))$

$$[create_P]\ \frac{R \in dom(P) \quad ((\theta, \rho, \emptyset), s) \in runsof(P, R) \quad \theta \notin runsIDs(F)}{\langle\!\langle AKN, F \rangle\!\rangle \xrightarrow{((\theta,\rho,\emptyset),create(R))} \langle\!\langle AKN, F \cup \{((\theta, \rho, \emptyset), s)\} \rangle\!\rangle}$$

$$[send]\ \frac{e = send_l(R_1, R_2, m) \quad (inst, [e] \cdot s) \in F}{\langle\!\langle AKN, F \rangle\!\rangle \xrightarrow{(inst,e)} \langle\!\langle AKN \cup \{inst(m)\}, F \setminus \{(inst, [e] \cdot s)\} \cup \{(inst, s)\} \rangle\!\rangle}$$

$$[recv]\ \frac{e = recv_l(R_1, R_2, pt) \quad AKN \vdash m \quad (inst, [e] \cdot s) \in F \quad Match(inst, pt, m, inst')}{\langle\!\langle AKN, F \rangle\!\rangle \xrightarrow{(inst',e)} \langle\!\langle AKN, F \setminus \{(inst, [e] \cdot s)\} \cup \{(inst', s)\} \rangle\!\rangle}$$

$$[claim]\ \frac{e = claim_l(R, c, t) \quad (inst, [e] \cdot s) \in F}{\langle\!\langle AKN, F \rangle\!\rangle \xrightarrow{(inst,e)} \langle\!\langle AKN, F \setminus \{(inst, [e] \cdot s)\} \cup \{(inst, s)\} \rangle\!\rangle}$$

- $[send]$ is the only rule that **changes the adversary knowledge**

Operational semantics: traces

$$(State, RunEvent, \rightarrow, st_0(P))$$

- Execution:
  $[st_0, \alpha_1, st_1, \alpha_2, \ldots, \alpha_n, st_n]$ where $\alpha_i \in RunEvent$ și $st_i = \langle\langle AKN_i, F_i \rangle\rangle$
- Knowing the initial state we define the execution using traces $[\alpha_1, \alpha_2, \ldots, \alpha_n]$.

  Given a protocol P, we define $traces(P)$ as the set of the finite traces of the labelled transition system $(State, RunEvent, \rightarrow, st_0(P))$ associated to P.

Example: trace for the Needham-Schroeder protocol

$((1, \rho, \emptyset), create(i))$

$((1, \rho, \emptyset), send_1(i, r, \{\!| ni, i |\!\}_{pk(r)}))$

$((2, \rho, \emptyset), create(r))$

$((2, \rho, \{W \mapsto ni^{\#1}\}), recv_1(i, r, \{\!| W, i |\!\}_{pk(r)}))$

$((2, \rho, \{W \mapsto ni^{\#1}\}), send_2(r, i, \{\!| W, nr |\!\}_{pk(i)}))$

$((1, \rho, \{V \mapsto nr^{\#2}\}), recv_2(r, i, \{\!| ni, V |\!\}_{pk(i)}))$

$((1, \rho, \{V \mapsto nr^{\#2}\}), send_3(i, r, \{\!| V |\!\}_{pk(r)}))$

$((1, \rho, \{V \mapsto nr^{\#2}\}), claim_4(i, synch))$

$((2, \rho, \emptyset), recv_3(i, r, \{\!| nr |\!\}_{pk(r)}))$

$((2, \rho, \emptyset), claim_5(r, synch))$

$$NS : Role \rightharpoonup RoleSpec$$

$$\begin{aligned}
NS(i) = \quad & (\{i, r, ni, sk(i), pk(i), pk(r)\}, \\
& [send_1(i, r, \{\!| ni, i |\!\}_{pk(r)}), \\
& recv_2(r, i, \{\!| ni, V |\!\}_{pk(i)}), \\
& send_3(i, r, \{\!| V |\!\}_{pk(r)}), \\
& claim_4(i, synch)])
\end{aligned}$$

$$\begin{aligned}
NS(r) = \quad & (\{i, r, nr, sk(r), pk(r), pk(i)\}, \\
& [recv_1(i, r, \{\!| W, i |\!\}_{pk(r)}), \\
& send_2(r, i, \{\!| W, nr |\!\}_{pk(i)}), \\
& recv_3(i, r, \{\!| nr |\!\}_{pk(r)}), \\
& claim_5(r, synch)])
\end{aligned}$$

Example: trace for the Needham-Schroeder protocol

$$[create_P]\ \frac{R \in dom(P) \quad ((\theta,\rho,\emptyset),s) \in runsof(P,R) \quad \theta \notin runsIDs(F)}{\langle\!\langle AKN, F\rangle\!\rangle \xrightarrow{((\theta,\rho,\emptyset),create(R))} \langle\!\langle AKN, F \cup \{((\theta,\rho,\emptyset),s)\}\rangle\!\rangle}$$

The initial state is
$st_0(NS) = \langle\!\langle AKN_0(NS), \emptyset\rangle\!\rangle$ where
$AKN_0(NS) = AdversaryFresh \ \cup \ Agent \cup$
$\{pk(A) \mid A \in Agent\} \cup \{sk(A) \mid A \in Agent_C\}$

For $\rho = \{i \mapsto A, r \mapsto B\}$, the first transition on t is

$$st_0(NS,t) = \langle\!\langle AKN_0(NS), \emptyset\rangle\!\rangle \xrightarrow{((1,\rho,\emptyset),create(i))} st_1(NS,t)$$

where

$st_1(NS,t) = \langle\!\langle AKN_0(NS), \{((1,\rho,\emptyset), s_1)\}\rangle\!\rangle$

$$\begin{aligned} s_1 = \quad & [send_1(i,r,\{\!|\ ni, i\ |\!\}_{pk(r)}), recv_2(r,i,\{\!|\ ni, V\ |\!\}_{pk(i)}), \\ & send_3(i,r,\{\!|\ V\ |\!\}_{pk(r)}), claim_4(i, synch)] \end{aligned}$$

Example: trace for the Needham-Schroeder protocol

$$[send] \frac{e = send_l(R_1, R_2, m) \quad (inst, [e] \cdot s) \in F}{\langle\!\langle AKN, F \rangle\!\rangle \xrightarrow{(inst, e)} \langle\!\langle AKN \cup \{inst(m)\}, F \setminus \{(inst, [e] \cdot s)\} \cup \{(inst, s)\} \rangle\!\rangle}$$

For $\rho = \{i \mapsto A, r \mapsto B\}$, the second transition on t is

$$st_1(NS, t) = \langle\!\langle AKN_0(NS), \{((1, \rho, \emptyset), s_1)\} \rangle\!\rangle \xrightarrow{((1,\rho,\emptyset), send_1(i, r, \{\!|\, ni, i \,|\!\}_{pk(r)}))} st_2(NS, t)$$

where

$st_2(NS, t) = \langle\!\langle AKN_0(NS) \cup \{\{\!|\, ni^{\#1}, A \,|\!\}_{pk(B)}\}, \{((1, \rho, \emptyset), s_2)\} \rangle\!\rangle$ and

$s_2 = [recv_2(r, i, \{\!|\, ni, V \,|\!\}_{pk(i)}), send_3(i, r, \{\!|\, V \,|\!\}_{pk(r)}), claim_4(i, synch)]$

Thank you!