Course Code: CSE 4205

Course Title: Digital Signal Processing

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Data vs. Signal

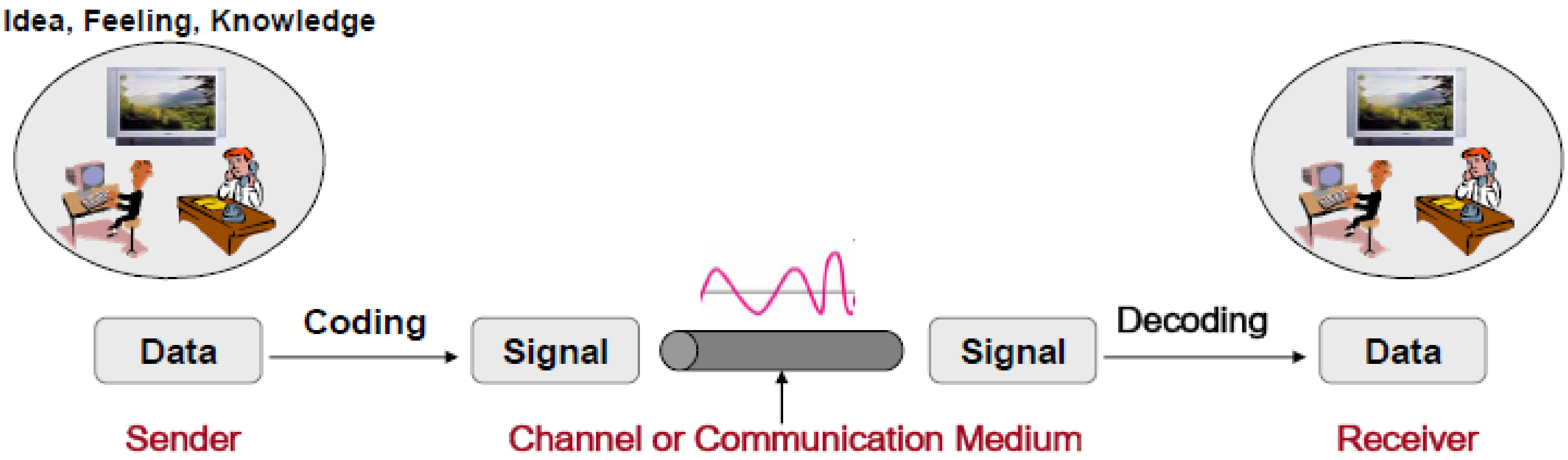
Data – information formatted in human/machine readable form

- examples: voice, music, image, file

Signal – electric or electromagnetic representation of data

- transmission media work by conducting energy along a physical path; thus, to be transmitted, data must be turned into energy in the form of electro-magnetic signals

Transmission – communication of <u>data</u> through propagation and processing of <u>signals</u>



What is Signal?

A signal is defined as a function of one or more variables which conveys information.

A signal is a physical quantity that varies with time in general or any other independent variable.

Noise: Any signal which does not convey information is called noise.

Dimension of Signal

Dimension of a signal is the number of independent variables.

One Dimensional Signal:

When a function depends on a single independent variable to represent the signal, it is said to be one-dimensional signal.

The ECG signal and speech signal shown in Fig. 2.1(a) and 2.1(b) respectively are examples of one-dimensional signals where the independent variable is time. The magnitude of the signals is dependent variable.

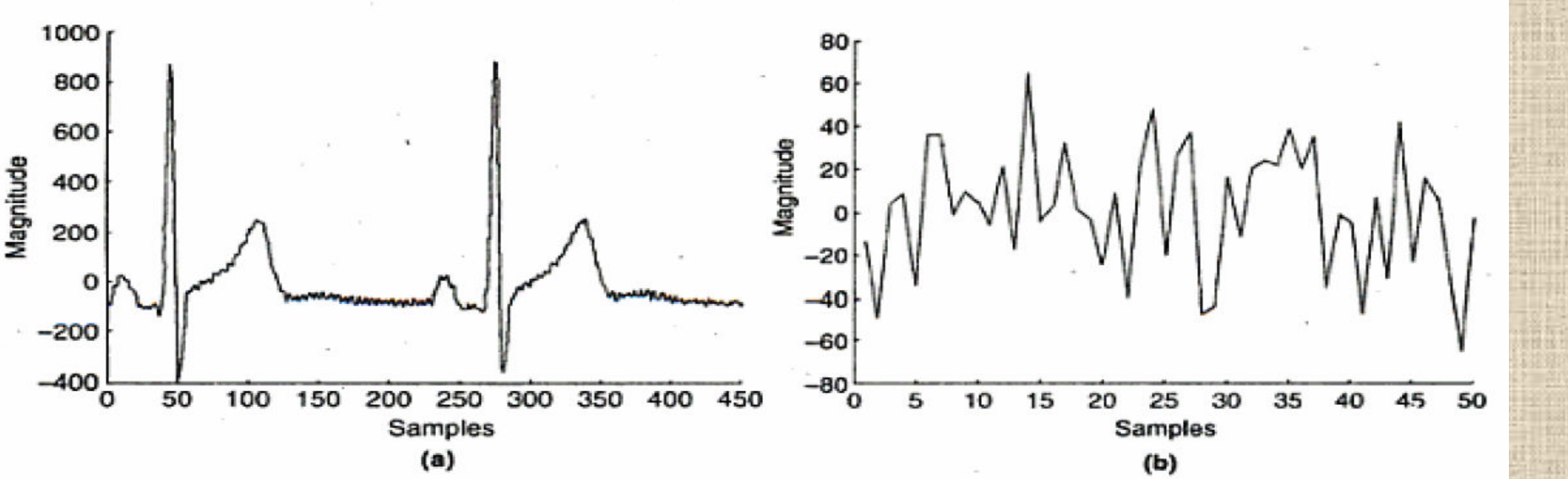


Fig. 2.1 One-dimensional Signal

Dimension of Signal contd.

Two- dimensional signal: When a function depends on two independent variables to represent the signal, it is said to be two-dimensional signal.

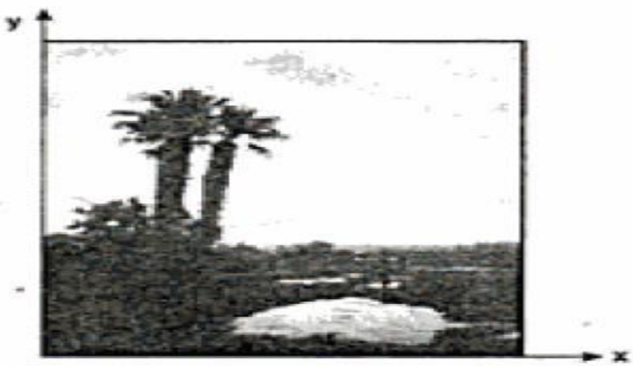


Fig. 2.2 Two-dimensional Photograph

Here two independent variables are coordinates which is denoted by x and y, the pixel value is dependent variable with respect to coordinate.

Dimension of Signal contd.

Multi-dimensional signal: When a function depends on more than one (two, three or more) independent variables to represent the signal, it is said to be multi-dimensional signal.

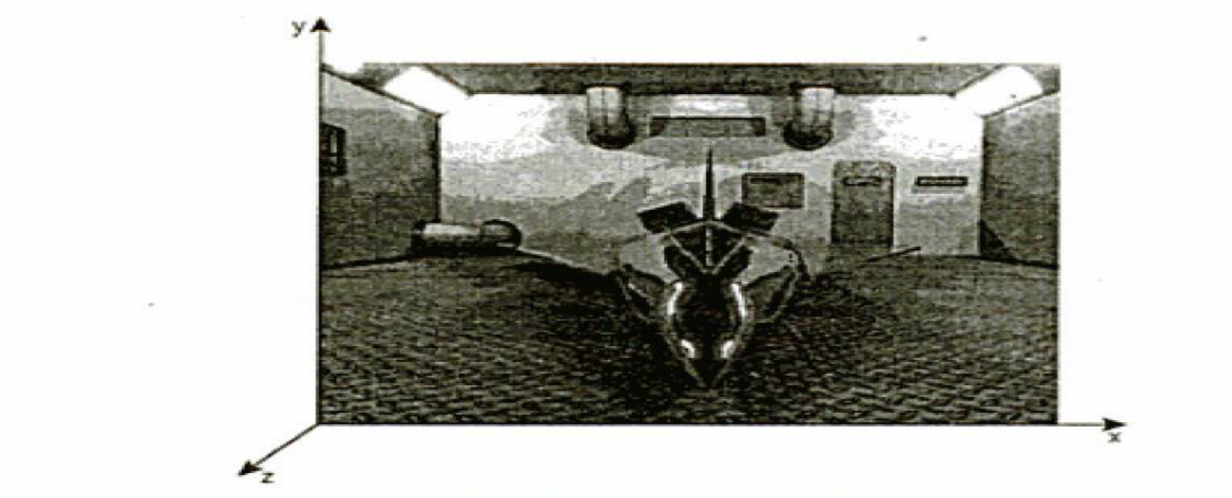


Fig. 2.3 3D-Space Missile

Here three independent variables are 3D coordinates which is denoted by x, y and z the pixel value is dependent variable with respect to coordinate.

Input and Output Signals

Input Signal: A signal that enters a system from an external source is referred to as an input signal.

Example: Voltage from function generator, electrocardiogram from heart etc

Output Signal: A signal produced by the system after calculation and modification of the input signal is called output signal.

Example: Output voltage from an amplifier, Modulated signal from a modulator etc.

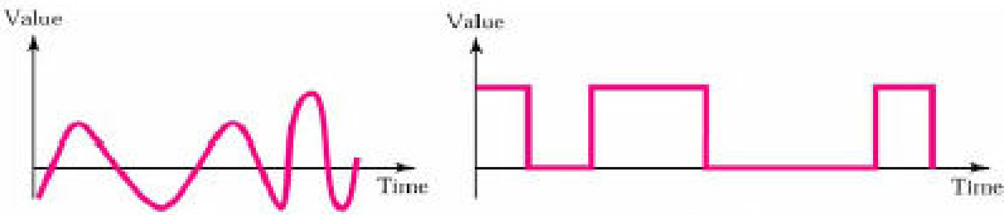
Analog vs. Digital

Analog vs. Digital Data

- **analog data** – representation variable takes on continuous values in some interval, e.g. voice, temperature, etc.
- **digital data** – representation variable takes on discrete (a finite & countable number of) values in a given interval, e.g. text, digitized images, etc.

Analog vs. Digital Signal

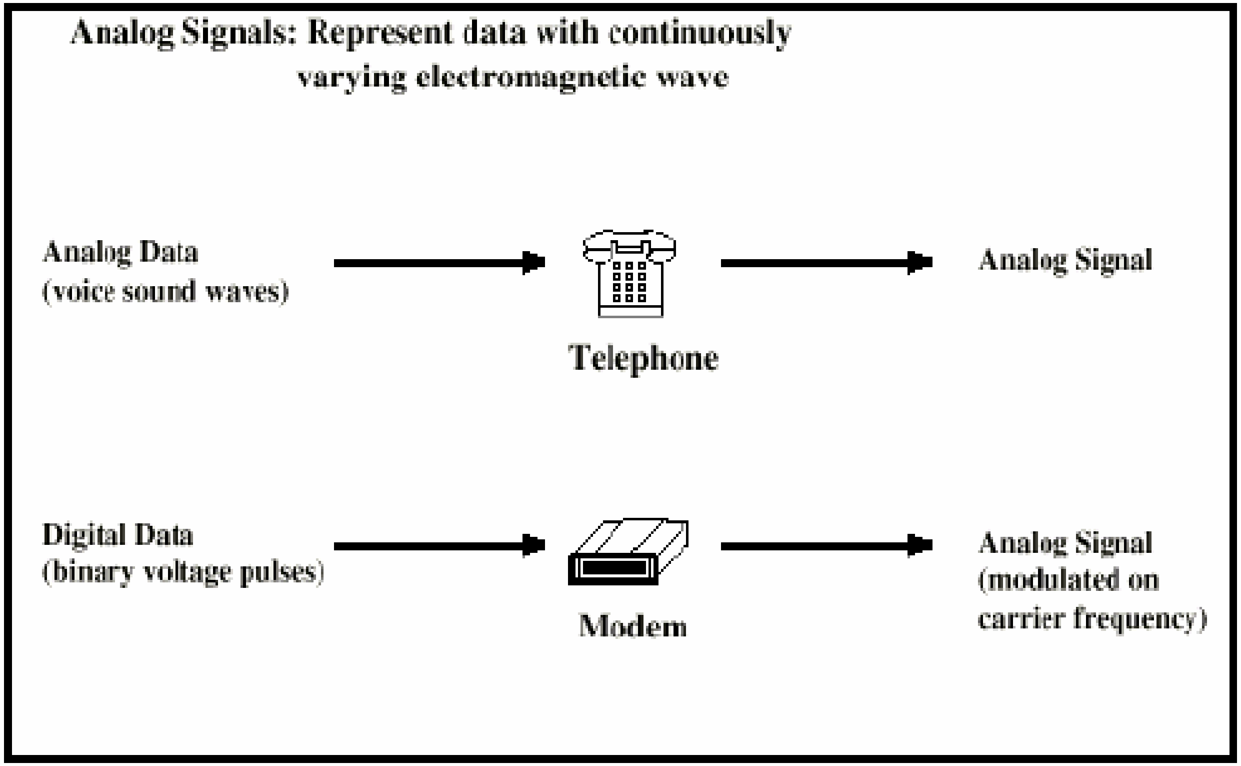
- **analog signal** – signal that is continuous in time and can assume an infinite number of values in a given range (continuous in time and value)
- **discrete (digital) signal** – signal that is continuous in time and assumes only a limited number of values (maintains a constant level and then changes to another constant level)

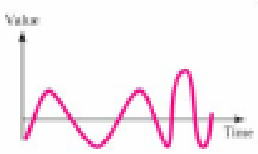


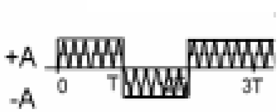
a. Analog signal b. Digital signal

Analog vs. Digital (cont.)

Both analog and digital data can be transmitted using either analog or digital signals.







example: analog signaling of analog and digital data

Analog Signals

Classification of Analog Signals

(1) **Simple Analog Signal** – cannot be decomposed into simpler signals

- sinewave – most fundamental form of periodic analog signal – mathematically described with 3 parameters

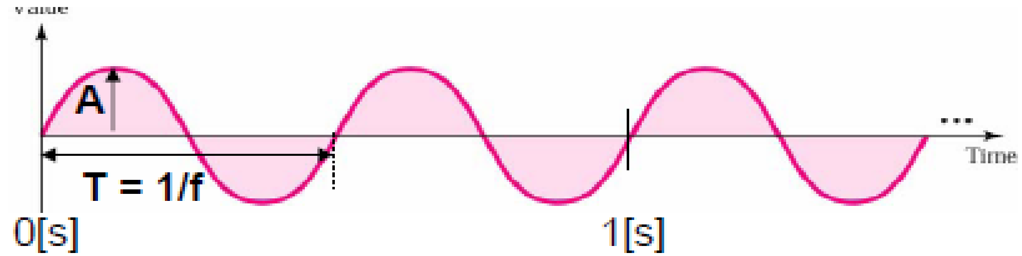
$$s(t) = A \cdot \sin(2\pi f t + \varphi)$$

(1.1) **peak amplitude** (A) – absolute value of signal's highest intensity – unit: volts [V]

(1.2) **frequency** (f) – number of periods in one second – unit: hertz [Hz] = [1/s] – inverse of period (T)!

(1.3) **phase** (φ) – absolute position of the waveform relative to an arbitrary origin – unit: degrees [º] or radians [rad]

The origin is usually taken as the last previous passage through zero from the negative to the positive direction.

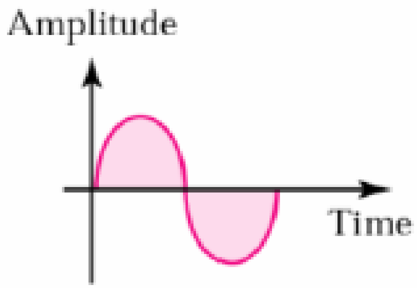


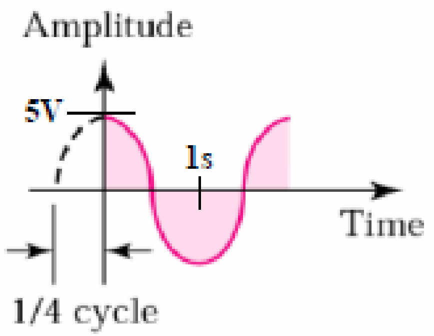
(2) **Composite Analog Signal** – composed of multiple sinewaves

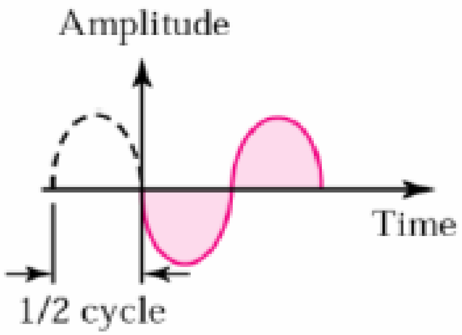
Simple Analog Signals

Phase in Simple Analog Signals – measured in degrees or radians

- $360^\circ = 2\pi$ rad
- $1^\circ = 2\pi/360$ rad
- 1 rad = $(360/2\pi)^\circ = 57.29578^\circ$
- phase shift of 360° = shift of 1 complete period
- phase shift of 180° = shift of 1/2 period
- phase shift of 90° = shift of 1/4 period







$\varphi = 0^\circ$ or 360°

Simple Analog Signals

Frequency in Simple Analog Signals – rate of signal change with respect to time

- change in a short span of time ⇒ high freq.
- change over a long span of time ⇒ low freq.
- signal does not change at all ⇒ zero freq.

??

- signal changes instantaneously ⇒ ∞ freq.

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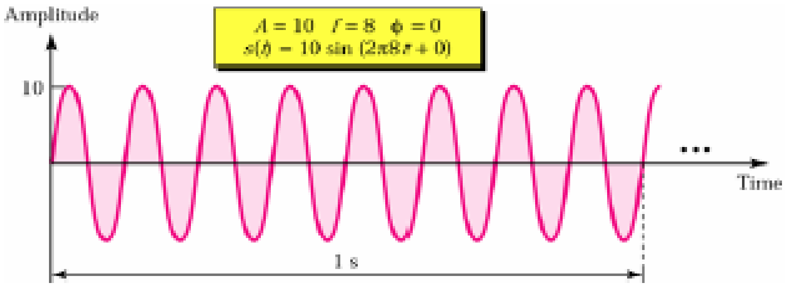
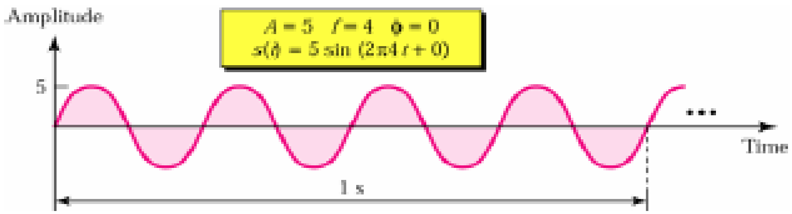




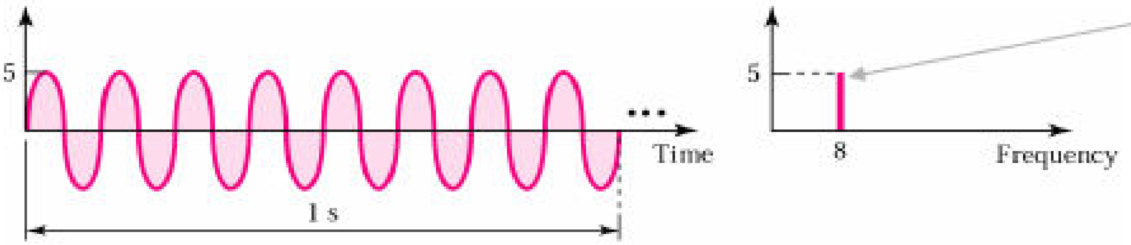
Time Domain Plot – specifies signal amplitude at each instant of time

- does NOT express explicitly signal's phase and frequency

Frequency Domain Plot – specifies peak amplitude with respect to freq.

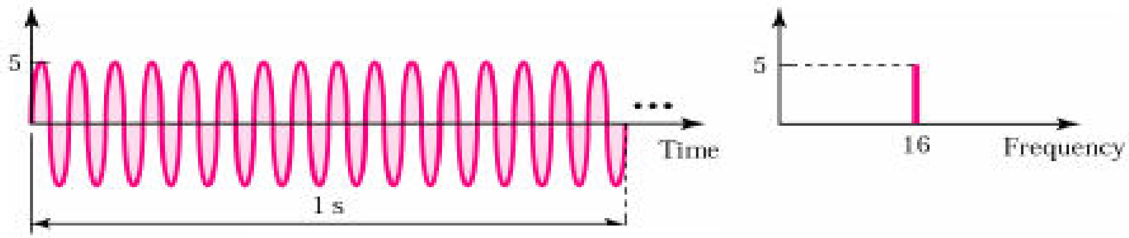
- phase CANNOT be shown in the frequency domain

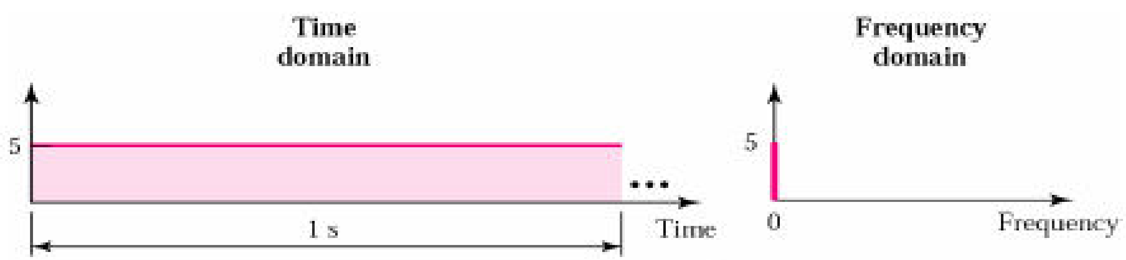
Simple Analog Signals



b. A signal with frequency 8

One 'spike' in frequency domain shows two characteristics of the signal:
spike position = signal frequency,
spike height = peak amplitude.

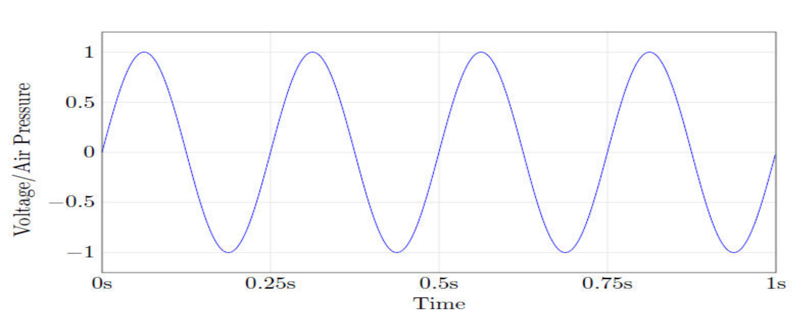




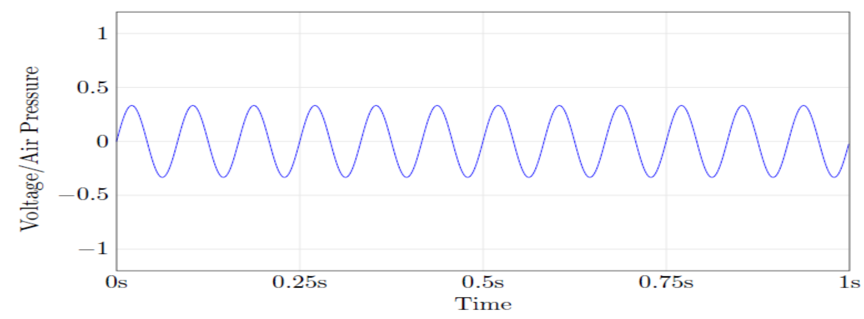
Analog signals are best represented in the frequency domain.

Time Domain

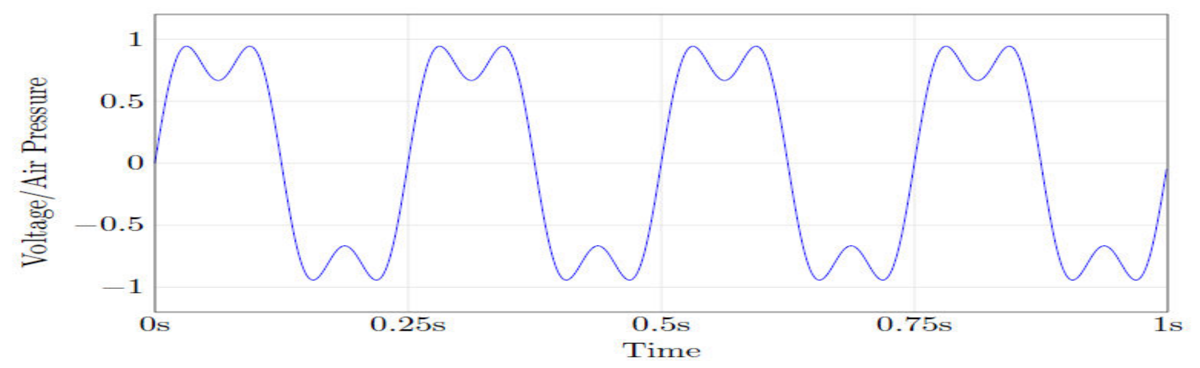
Frequency=4Hz



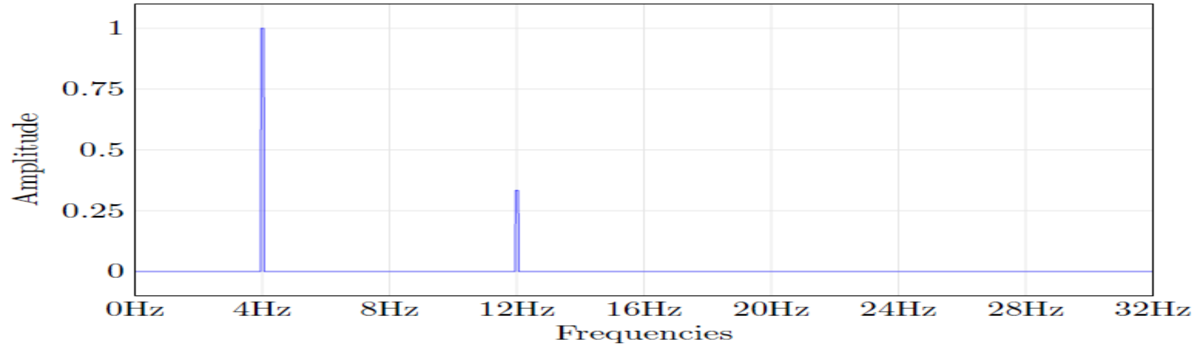
Frequency=12Hz



Composite Signal



Frequency Domain



Composite Analog Signals

Fourier Analysis – any composite signal can be represented as a combination of simple sine waves with different frequencies, phases and amplitudes

$$s(t) = A_1 \sin(2\pi f_1 t + \varphi_1) + A_2 \sin(2\pi f_2 t + \varphi_2) + \ldots$$

- periodic composite signal (period=T, freq. = f_0=1/T) can be represented as a sum of simple sines and/or cosines known as *Fourier series*:

$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} [A_n \cos(2\pi n f_0 t) + B_n \sin(2\pi n f_0 t)]$$

With the aid of good table of integrals, it is easy to determine the frequency-domain nature of many signals.

$$A_n = \frac{2}{T}\int_0^T s(t)\cos(2\pi n f_0 t)dt, \quad n = 0,1,2, \ldots$$

$$B_n = \frac{2}{T}\int_0^T s(t)\sin(2\pi n f_0 t)dt, \quad n = 1,2,3, \ldots$$

- f_0 is referred to as ‘fundamental frequency’
- integer multiples of f_0 are referred to as harmonics

Composite Analog Signals (cont.)

Angular Frequency – aka radian frequency – number of 2π revolutions during a single period of a given signal

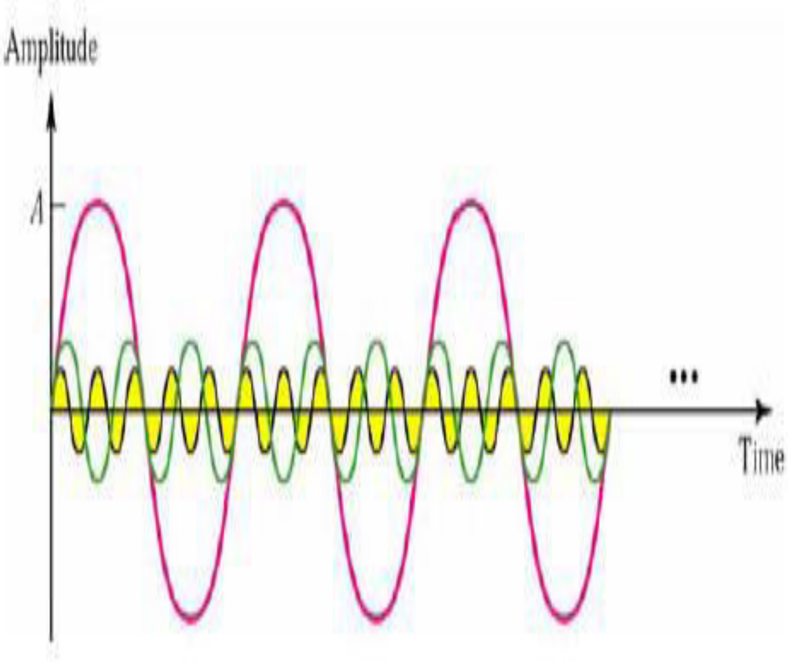
$$\omega = \frac{2\pi}{T} = 2\pi \cdot f$$

- simple multiple of ordinary frequency

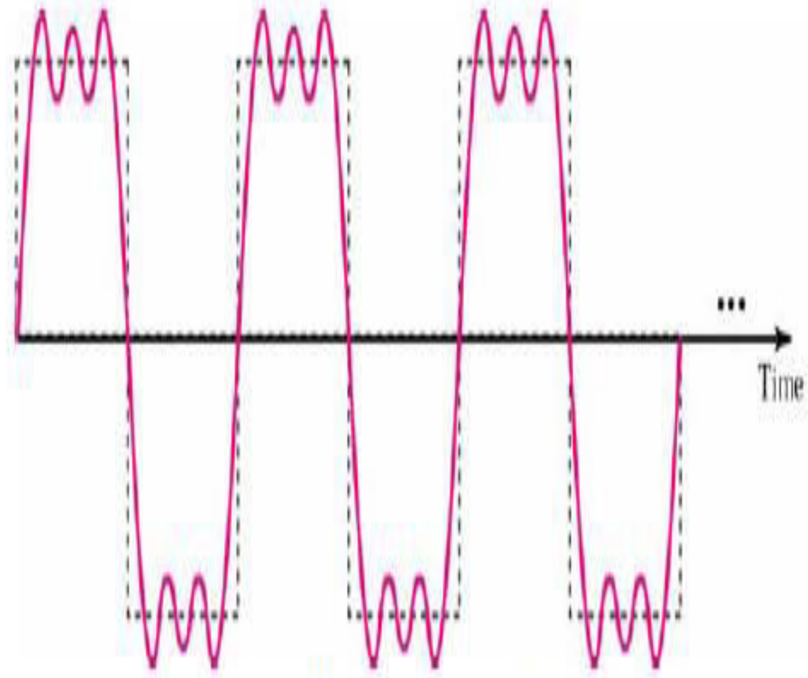
$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t) \right]$$

$$A_n = \frac{2}{T} \int_0^T s(t)\cos(n\omega_0 t)dt, \qquad n = 0,1,2,\ldots$$

$$B_n = \frac{2}{T} \int_0^T s(t)\sin(n\omega_0 t)dt, \qquad n = 1,2,\ldots$$



three harmonics



adding three harmonics

$$s(t) = A_1 \sin(2\pi f t) + A_2 \sin(2\pi(3f)t) + A_3 \sin(2\pi(5f)t)$$

Composite Analog Signals (cont.)

Example **[frequency spectrum and bandwidth of analog signal]**

A periodic signal is composed of five sinewaves with frequencies of 100, 300, 500, 700 and 900 Hz.

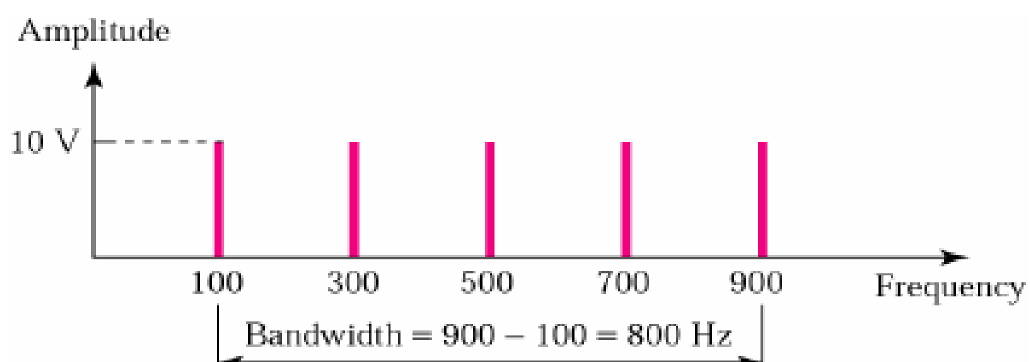
What is the **bandwidth** of this signal?

Draw the **frequency spectrum**, assuming all components have a max amplitude of 10V.

Solution:

$B = f_{highest} - f_{lowest} = 900 - 100 = 800$ Hz

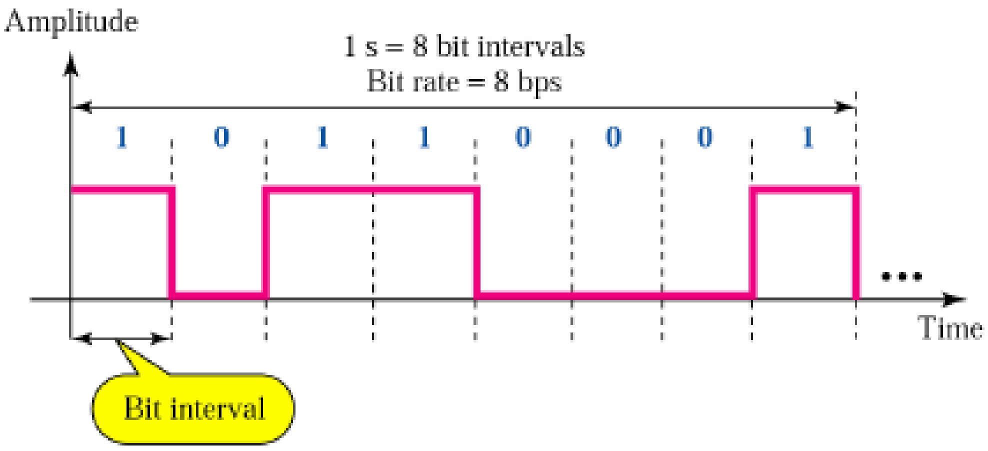
The spectrum has only five spikes, at 100, 300, 500, 700, and 900.



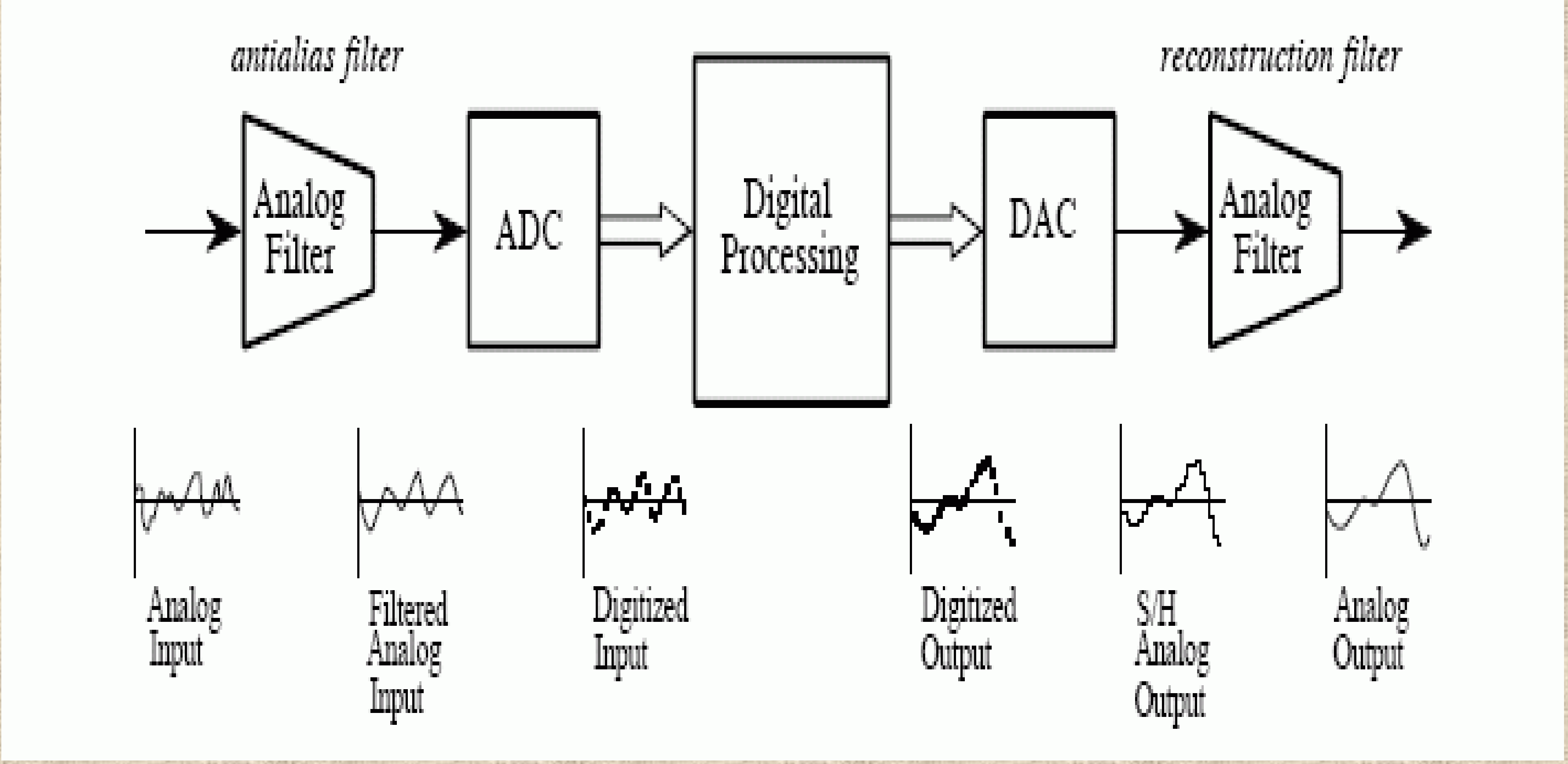
Digital Signals

Digital Signals – sequence of voltage pulses (DC levels) – each pulse represents a *signal element*

- binary data are transmitted using only 2 types of signal elements (1 = positive voltage, 0 = negative voltage)
- key digital-signals terms:
  - bit interval – time required to send a single bit, unit: [sec]
  - bit rate – number of bit intervals per second – unit: [bps]



Communication Overview



Analog to Digital Conversion

- Sampling
- Quantization
- Coding

Sampling

Sampling is a process by which a continuous-time signal is converted into a sequence of discrete samples, with each sample representing the amplitude of the signal at a particular instant of time.

Two types of sampling

1. Uniform sampling
2. Non-uniform sampling

<u>Uniform sampling:</u> The space between any two samples is fixed throughout the signal under consideration.

<u>Non-uniform sampling:</u> The space between any two samples varies throughout the signal under consideration.

Uniform sampling is preferred over non-uniform sampling since it is simple to analyze and easy to implement.

Sampling

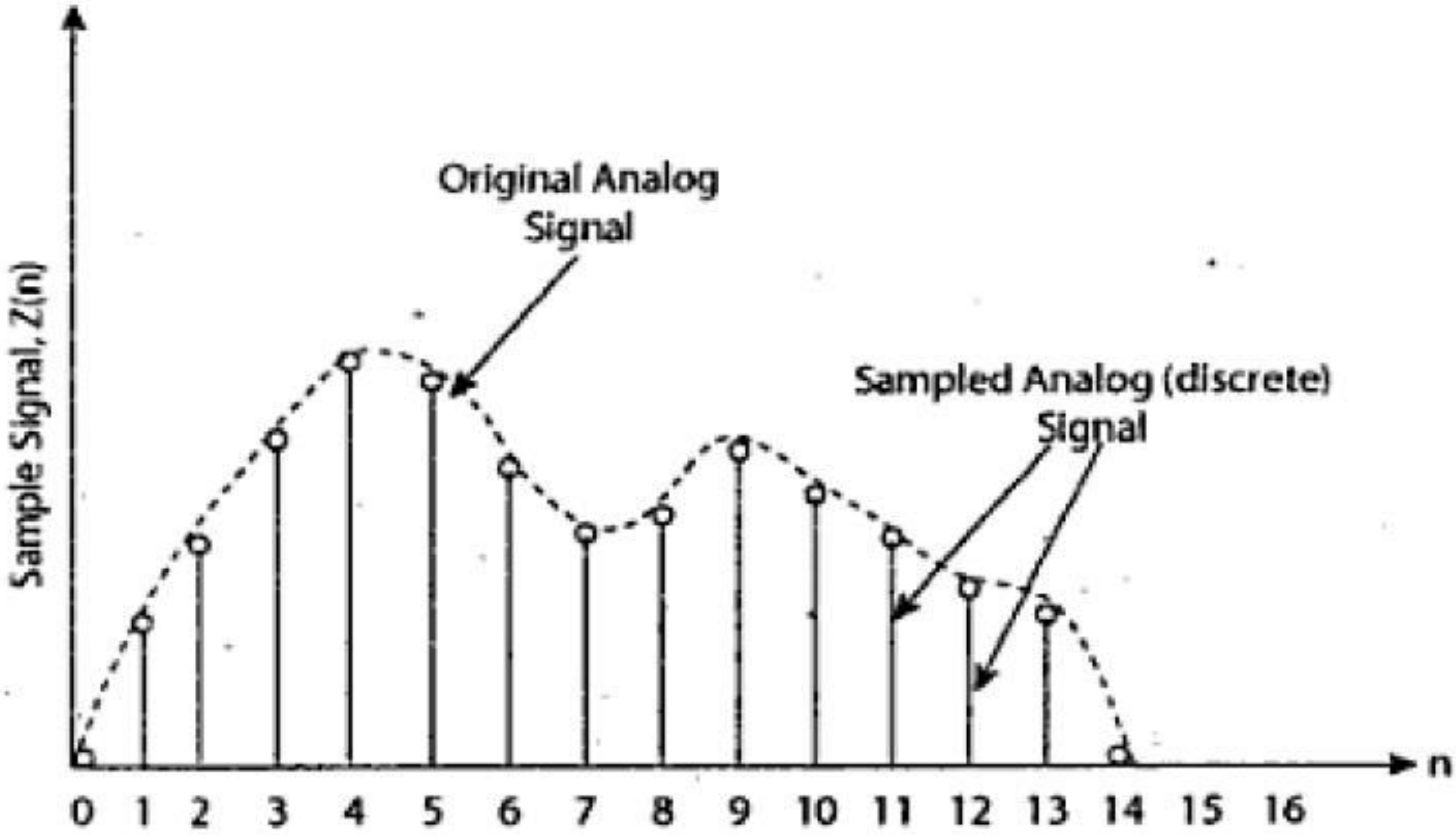


Fig. 2.4 Uniform Sampling of signal

Quantization

Quantization is a process by which each sample produced by sampling circuit to the nearest level selected from finite number of discrete amplitude levels.

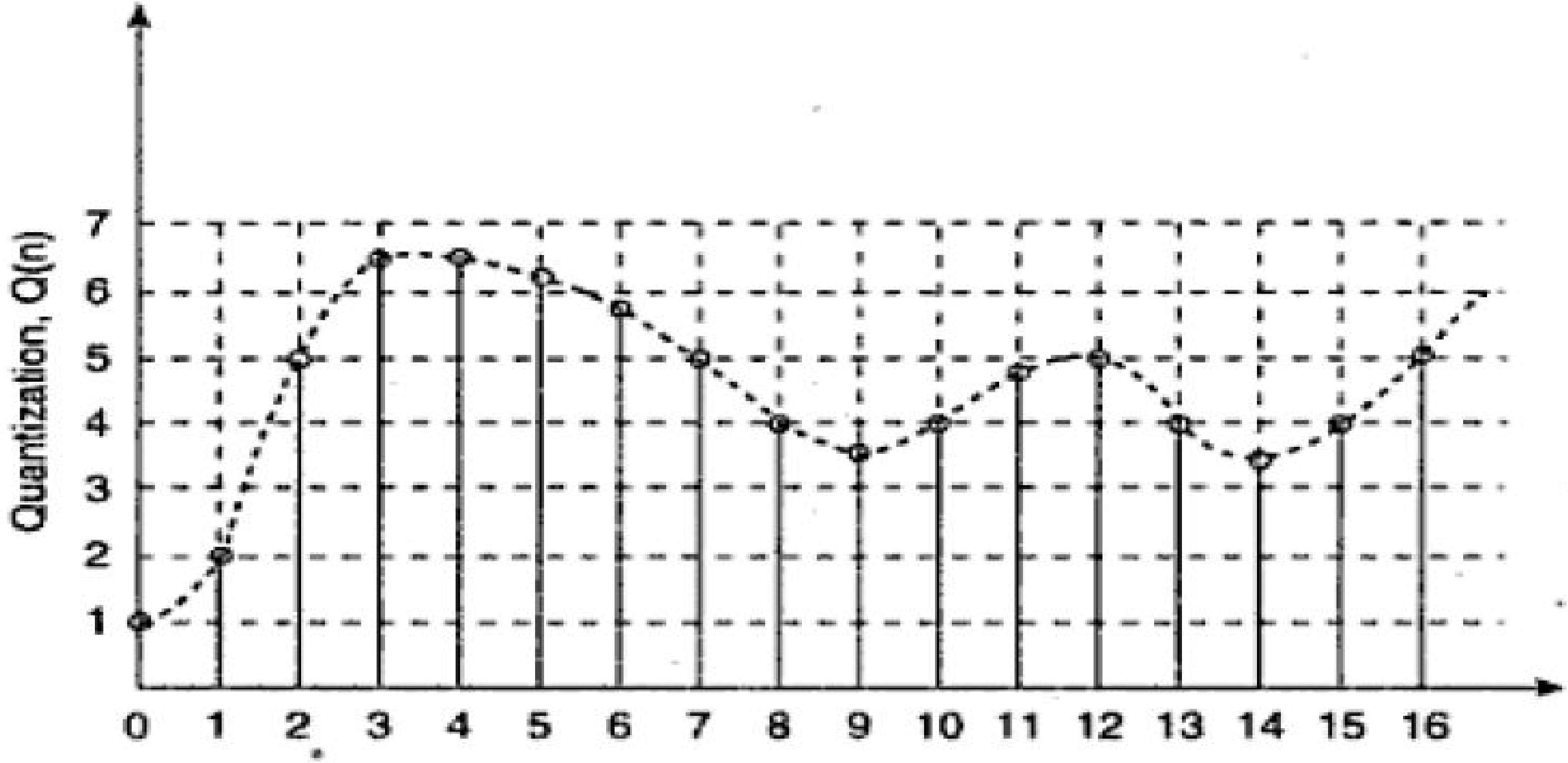


Fig. 2.5 Quantization of Signal

Coding

Coding is needed in order to represent each quantized sample by a binary number ‘0’ and ‘1’. The ‘0’ represents the “low” state and ‘1’ represents the “high” state.

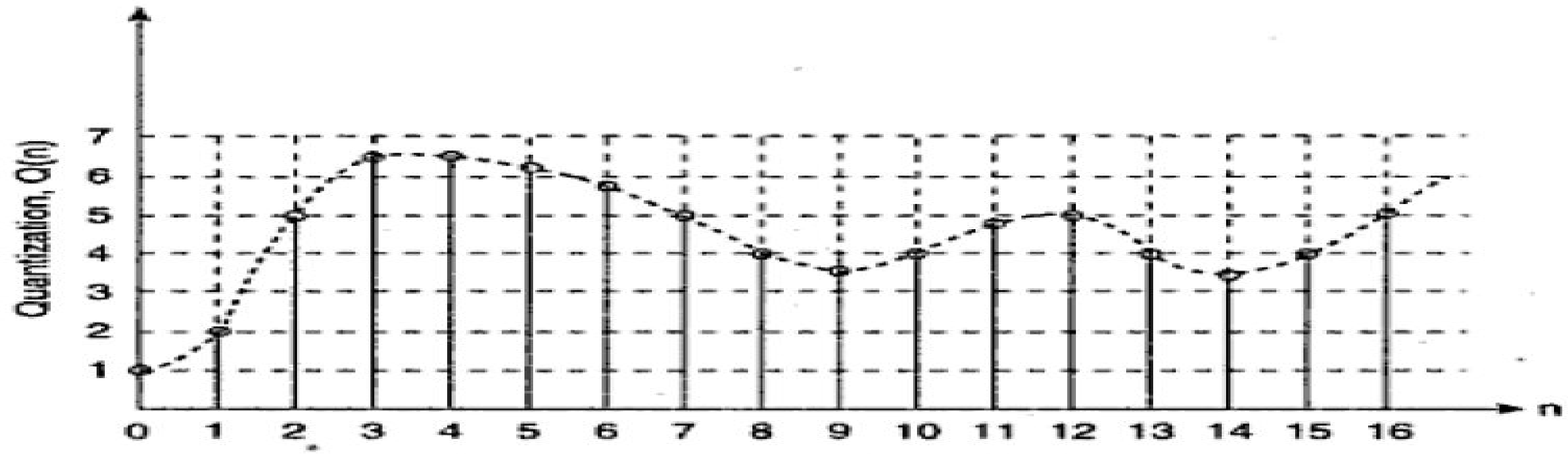
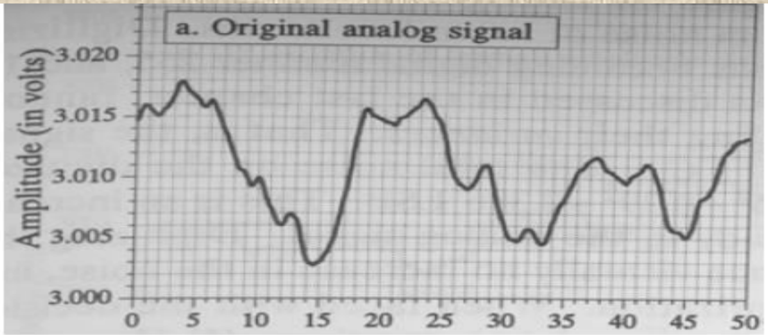


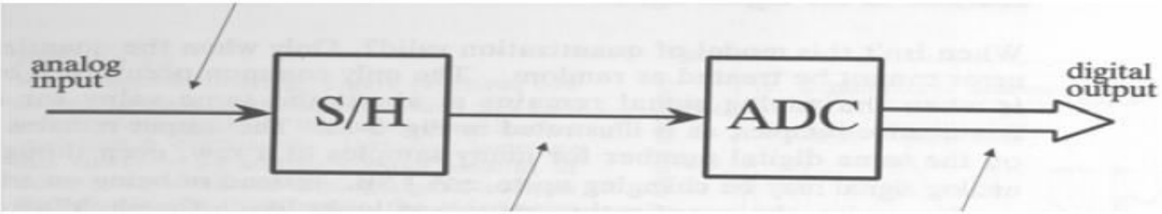
Fig. 2.5 Quantization of Signal

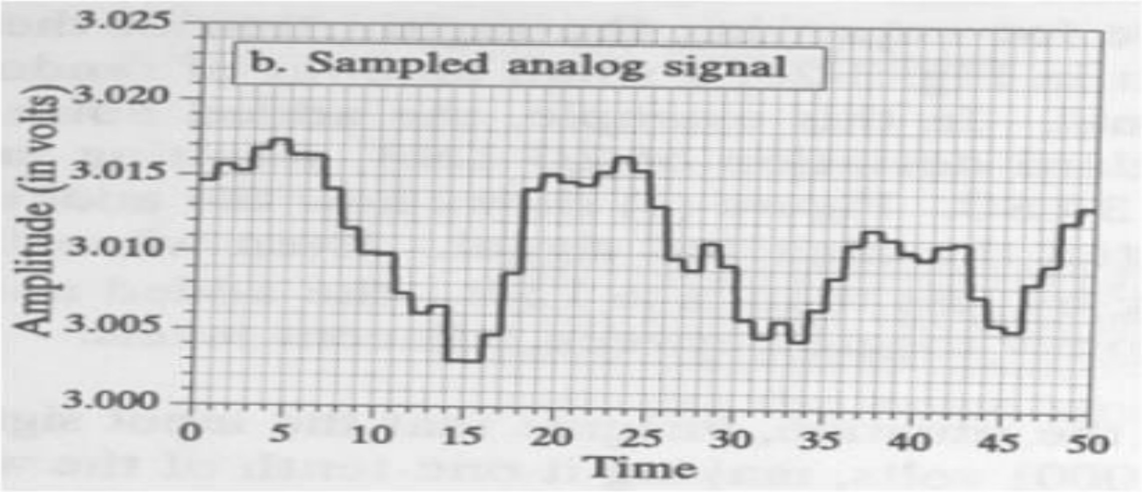
Table 2.1 3-bit Quantization and its Binary Representation

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Q(n)	1	2	5	7	7	7	6	5	4	4	4	5	5	4	4	4	5
Binary	001	010	101	111	111	111	110	101	100	100	100	101	101	100	100	100	101

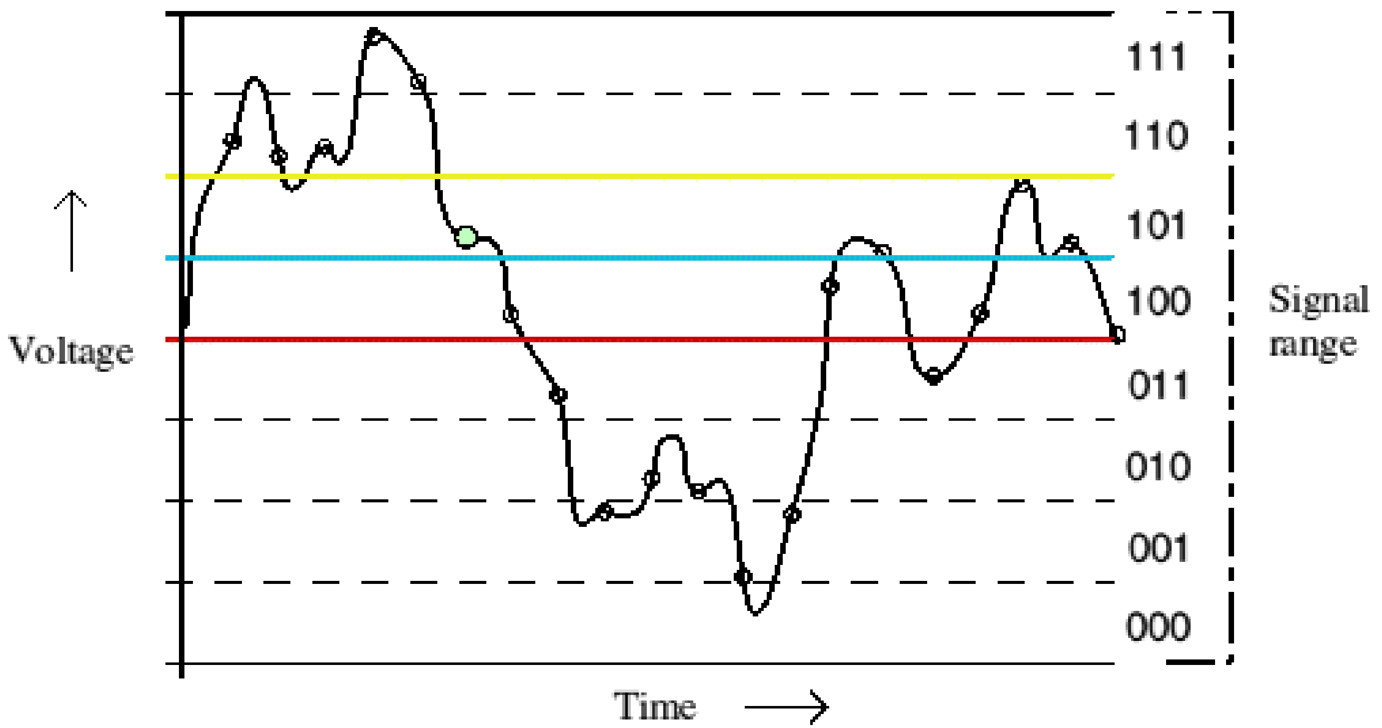
Analog to Digital Conversion contd..







Analog to Digital Conversion contd..



Turning analog waves into digital signals.

110110110111111101100011001010010001001100101011100101101100..

Digital to Analog Conversion

Reverse process of ADC.

Classification of Signals

Signals are classified based on their fundamental properties.

- Continuous-time signal and Discrete-time signal.
- Periodic signal and Aperiodic signal
- Even signal and Odd signal
- Deterministic and Random signal

Continuous-time signal

A signal x(t) is said to be a continuous signal if it is defined for all time t.

The amplitude of the signal varies continuously with time.

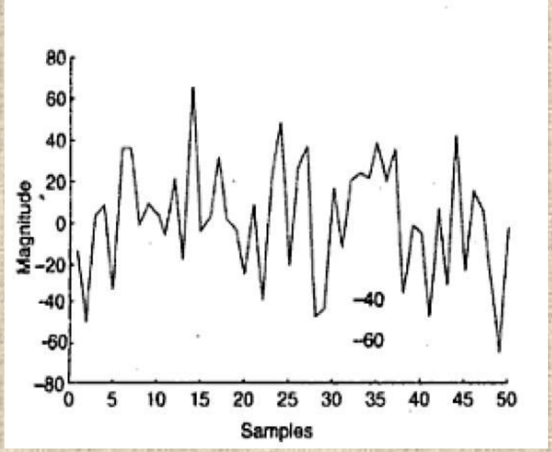


Fig- a(Speech signal)

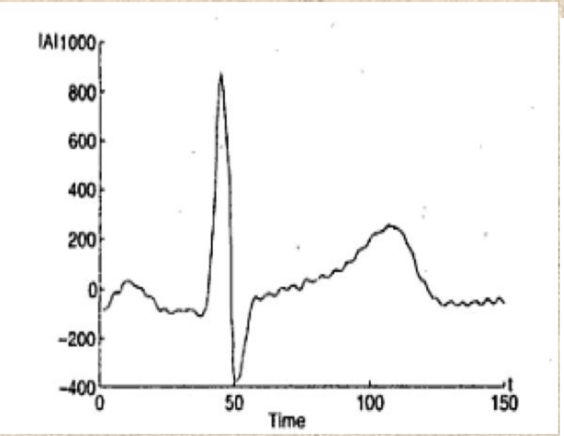


Fig-b (Electrocardiogram)

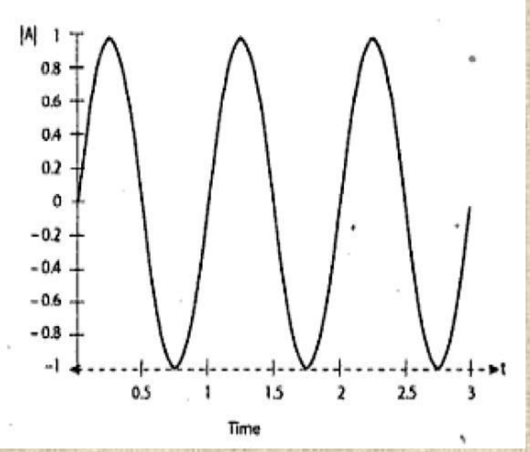


Fig-c (Sinusoidal Signal)

Discrete-time signal

Most signals that are obtained from sources are continuous in time. Such signals have to be discredited since the processing done on the digital computer is digital in nature.

A signal x(n) is said to be discrete-time signal if it can be defined for a discrete instant of time(say n)

A continuous-time signal x(t) can be converted to discrete-time signal x(n) by substituting t=nT.

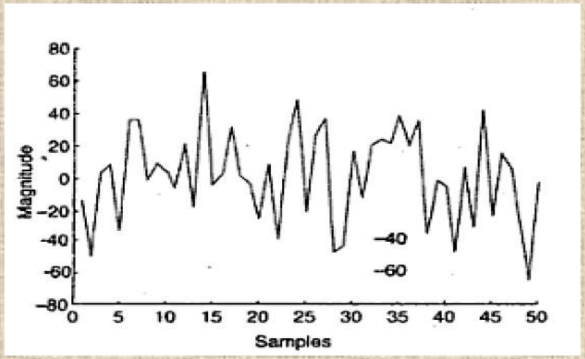
$$x(t) = x(nT)\big|_{t=nT} = x(n)$$

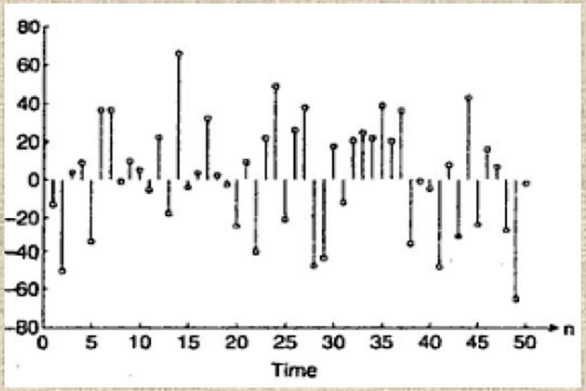
$$n = 0,\ \pm 1,\ \pm 2,\ +3, \ldots$$

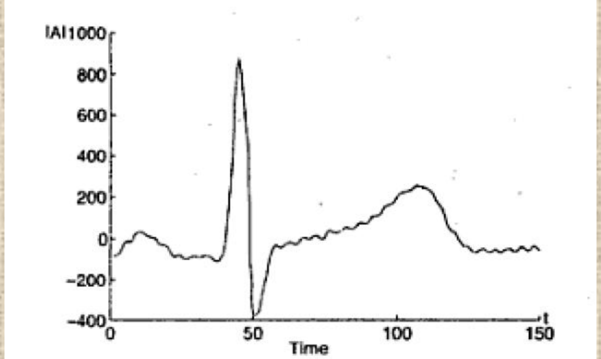
where n = Constant integer, which can take positive or/and negative values

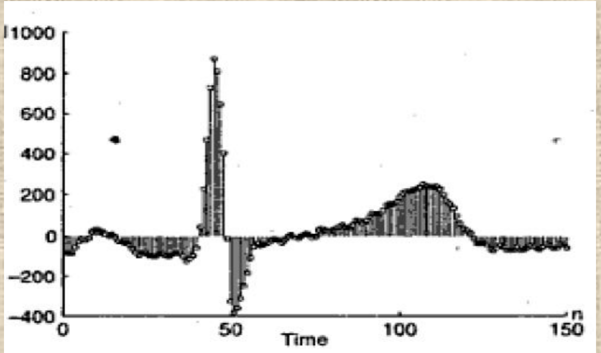
T = Sampling period, is an integer (normally T is assume to be unity)

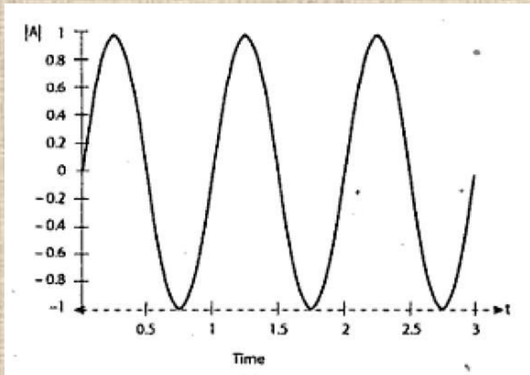
Discrete-time signal contd..

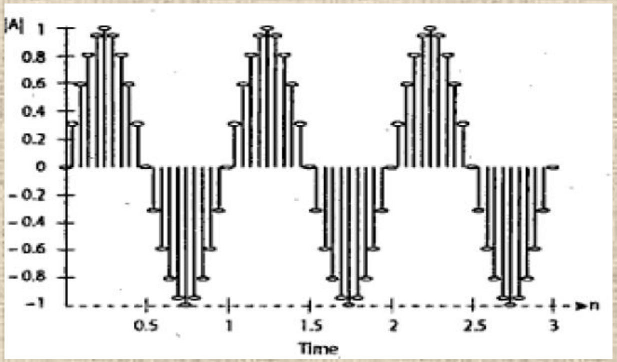












Some Exercises

Problem 2.1

Problem 2.2

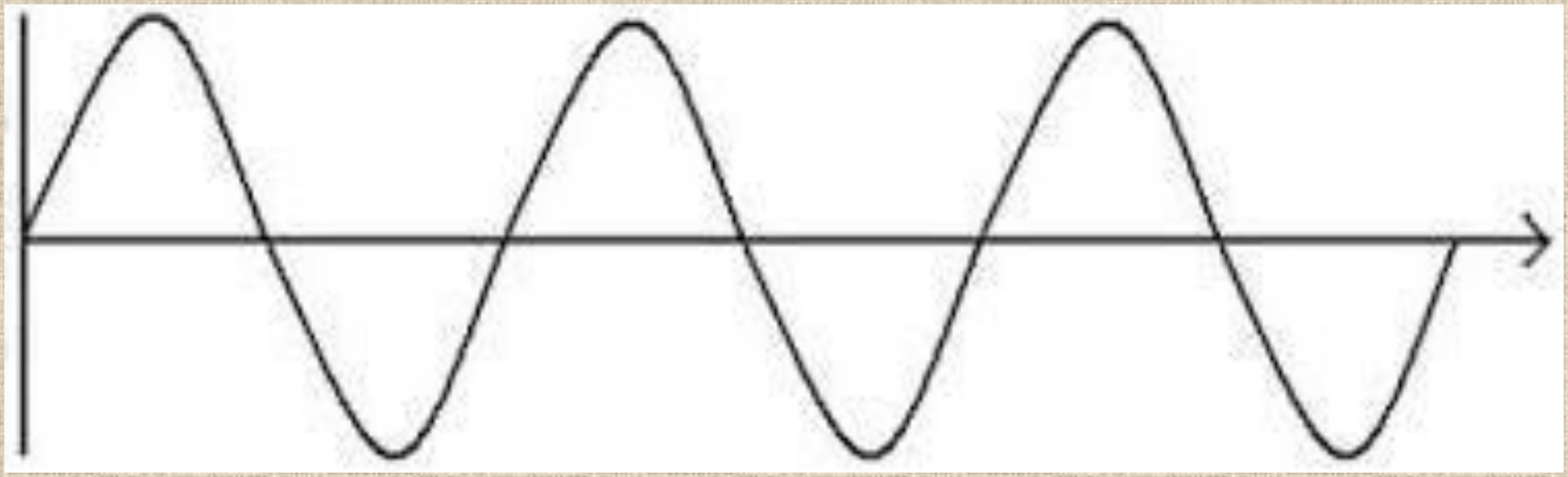
Periodic signal and Aperiodic Continuous-time signal

A Continuous-time signal x(t) is said to be periodic if

$$x(t)=x(t+T)=x(t+2T)=x(t+3T)=.....=x(t+nT) \quad , \quad T>0$$

T=period of a cycle

n= any integer



Periodic signal and Aperiodic Continuous-time signal contd…

- Prove that the cosine signal is periodic with periodicity T.

- Solve problem 2.3

Discrete-time periodic signal

A discrete-time signal is said to be periodic with , period N, if $x(n)=x(n+N)$.

Even and Odd Continuous-time signal

A continuous-time signal is said to be even, if it satisfies the condition

$$x(t)=x(-t), \text{ for all } t$$

Example: cos x

Even signals are symmetric about the vertical axis.

A continuous-time signal is said to be odd, if it satisfies the condition

$$x(t)=-x(-t), \text{ for all } t$$

Example: sin x

Odd signals are antisymmetric about the vertical axis.

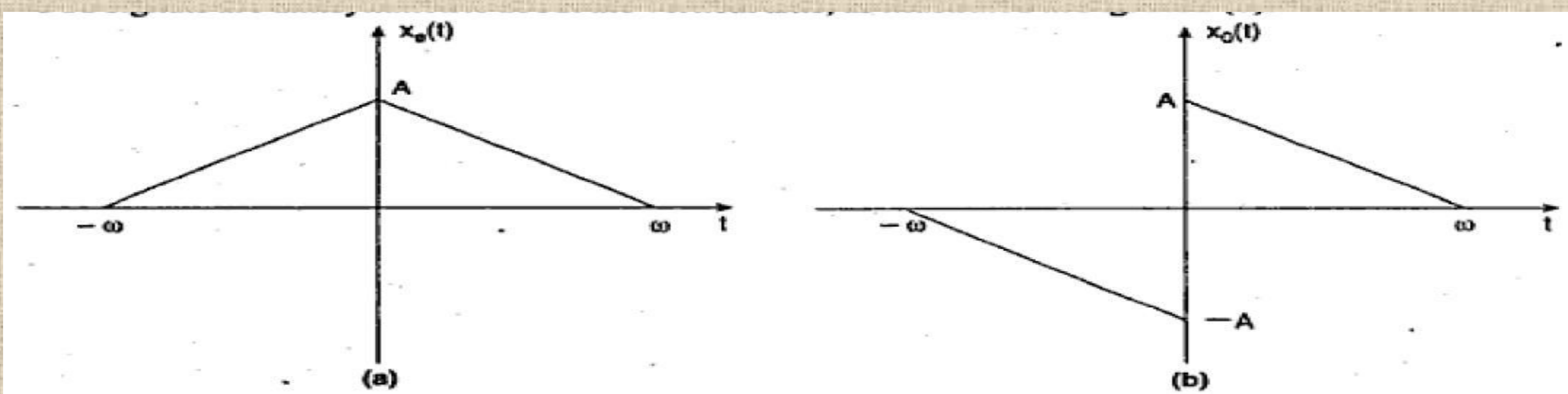


Fig. 2.12 (a) Even Signal (b) Odd Signal

Let us consider a signal, $x(t)$ which can be decomposed as odd and even signals, i.e.

$$x(t) = x_o(t) + x_e(t) \tag{2.7}$$

where $x_o(t)$ represents the odd signal and $x_e(t)$ represents the even signal.

From the definition of odd and even signal,

$$-x_o(t) = x_o(-t) \tag{2.8a}$$

$$x_e(t) = x_e(-t) \tag{2.8b}$$

Replace $t = -t$ in equation (2.7),

$$x(-t) = x_o(-t) + x_e(-t) \tag{2.9}$$

Substituting equation (2.8) in (2.9),

$$x(-t) = -x_o(t) + x_e(t) \tag{2.10}$$

Solve equations (2.7) and (2.10).

Adding (2.7) and (2.10), $\quad x(t) + x(-t) = 2\,x_e(t)$

Subtracting (2.7) from (2.10),

$$x(t) - x(-t) = 2\,x_o(t)$$

Solve for $x_e(t)$ and $x_o(t)$,

$$x_e(t) = \frac{x(t) + x(-t)}{2} \tag{2.11}$$

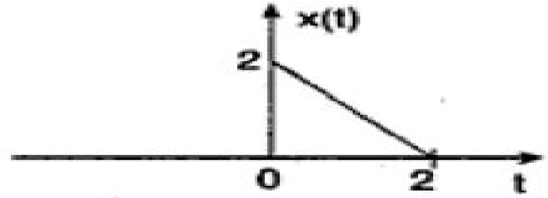
$$x_o(t) = \frac{x(t) - x(-t)}{2} \tag{2.12}$$

Equations (2.11) and (2.12) gives the relation between odd and even signals.

Even and Odd Continuous-time signal contd..

Problem 2.18

Draw the odd and even representations for the given signal.

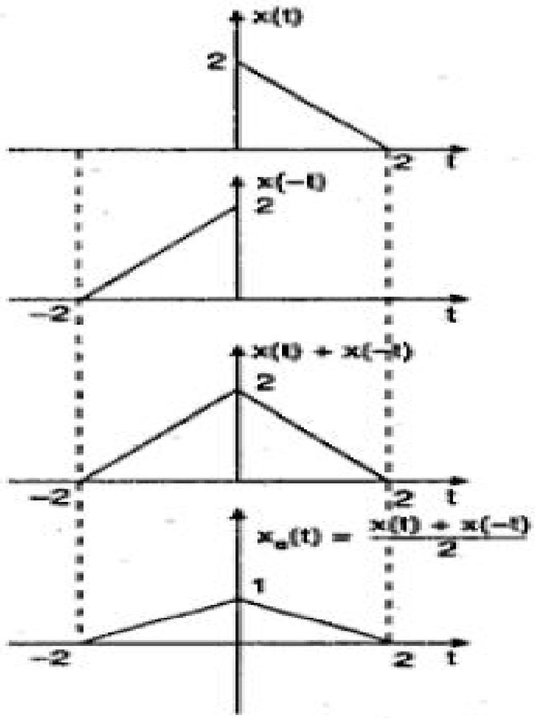


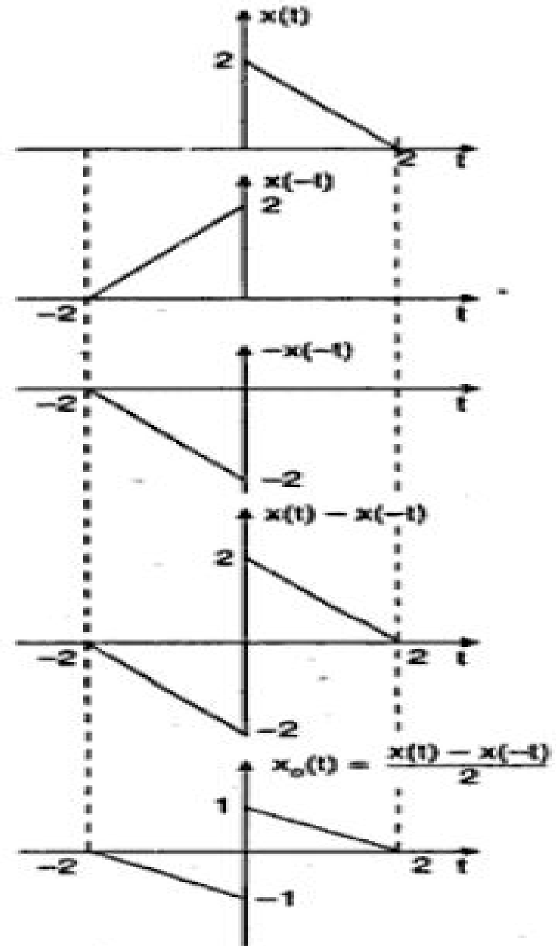
For even signal,

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

For odd signal,

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$



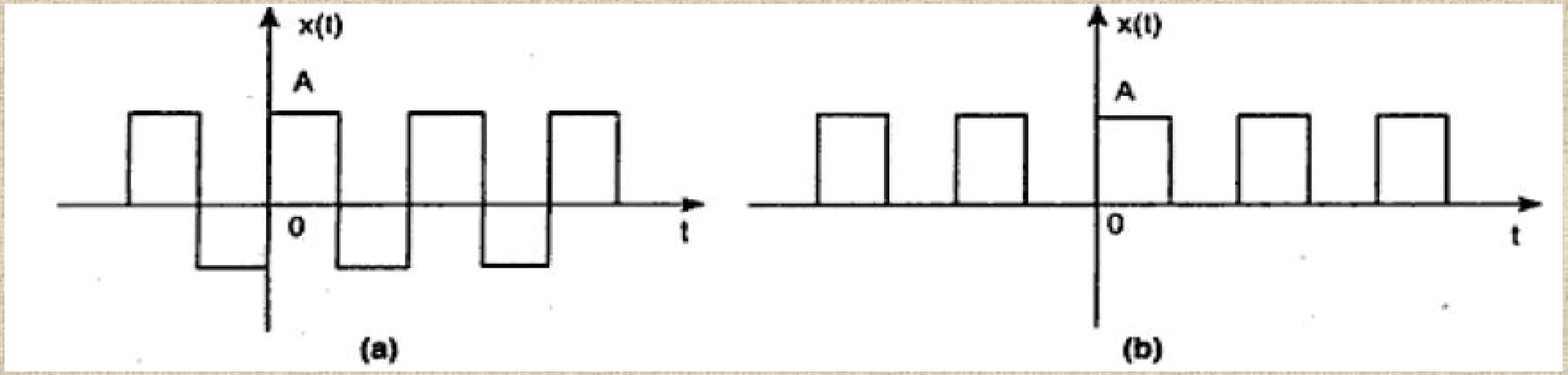


Deterministic and Random Signal

Deterministic Signal: A signal about which there is certainty with respect to its values at any time.

A kind of signal where future values of the signal are predictable

Example: sinusoidal signal, square wave etc.



Random Signal

A signal about which there is uncertainty with respect to its values at any time.

The future values of a random signal are unpredictable.

Example: speech signal, noise etc.

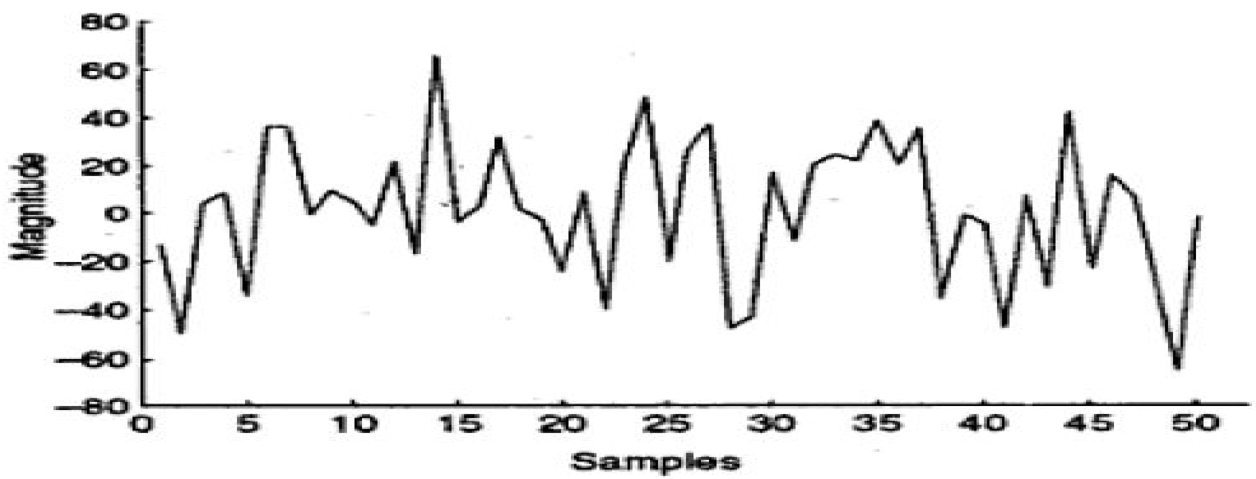


Fig. 2.21 Speech Signal

Basic operations on signal

- Amplitude scaling of signals
- Addition of signals
- Multiplication of signals
- Differentiation on signals
- Integration on signals
- Time scaling of signals

Basic operations on signal

Amplitude Scaling of Signals:

$$y(t)=\alpha\, x(t)$$

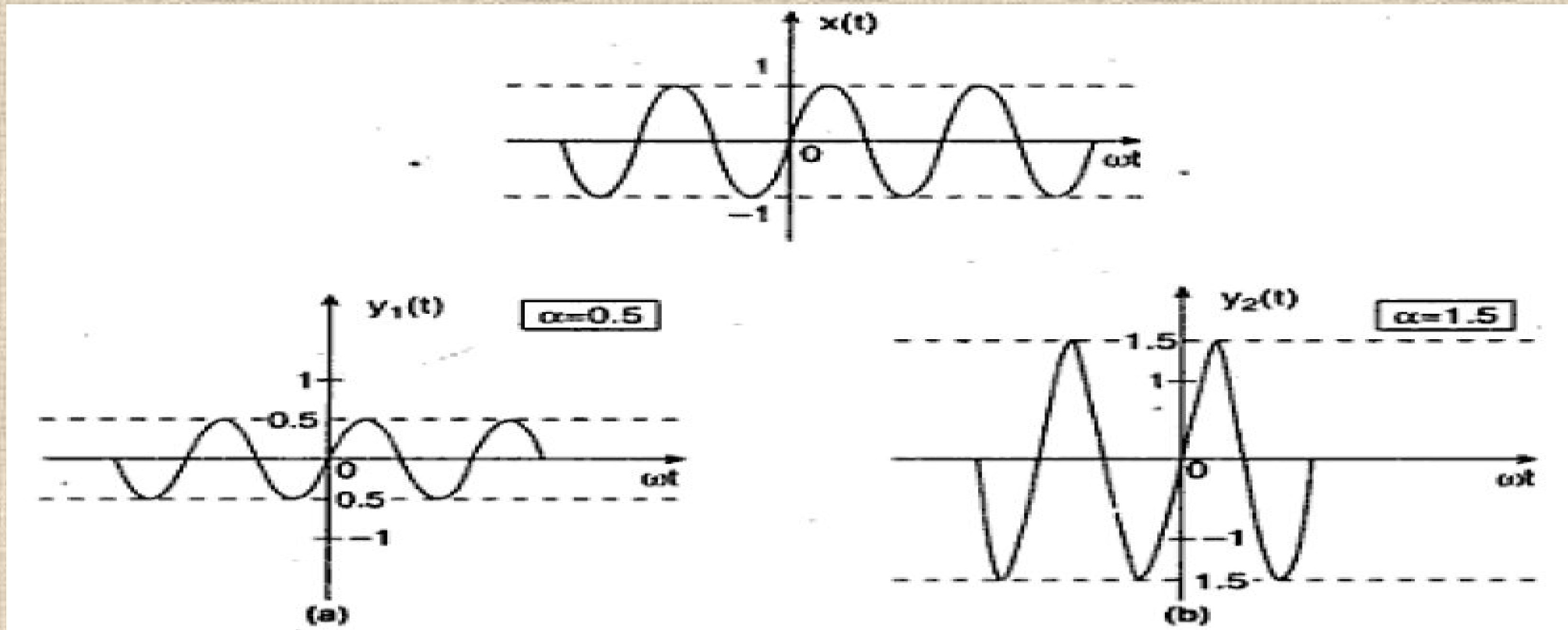
Where α =scaling factor (if α <1, then the signal attenuates, if α >1, then the signal amplifies).

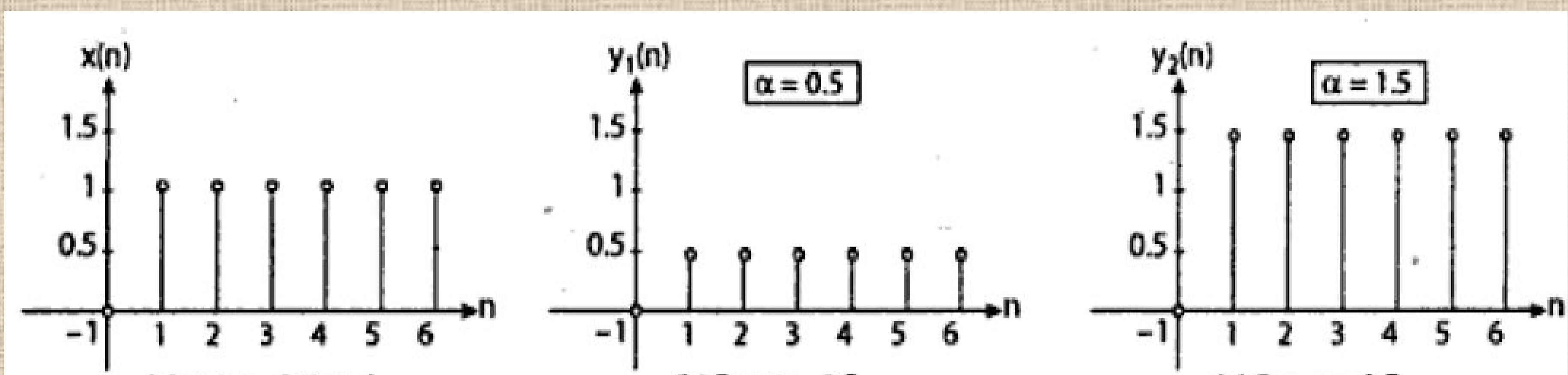
Let us consider the original signal x(t)= sin ωt .

If the scaling factor is α , then scaled signal is

x(t)= α sin ωt

Amplitude scaling





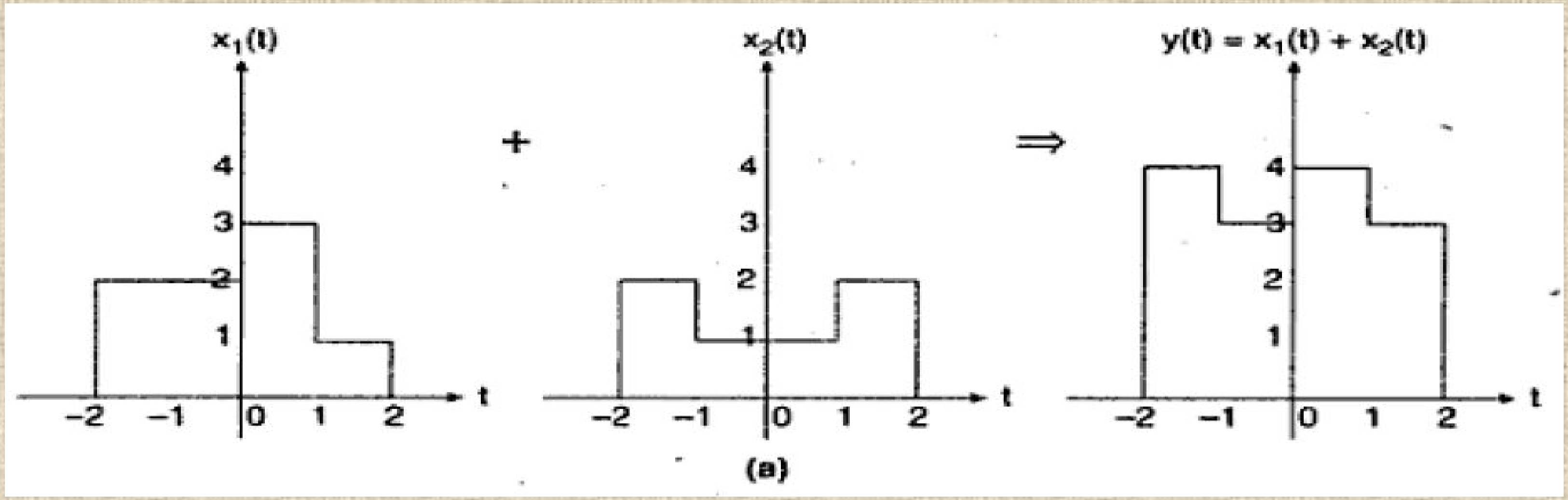
(a) Original Signal (b) For α = 0.5 (c) For α = 1.5

Addition of signals

Continuous-time signal:

Let two signals, $x_1(t)$ and $x_2(t)$. Then the result signal y(t) is

$$y(t) = x_1(t) + x_2(t)$$

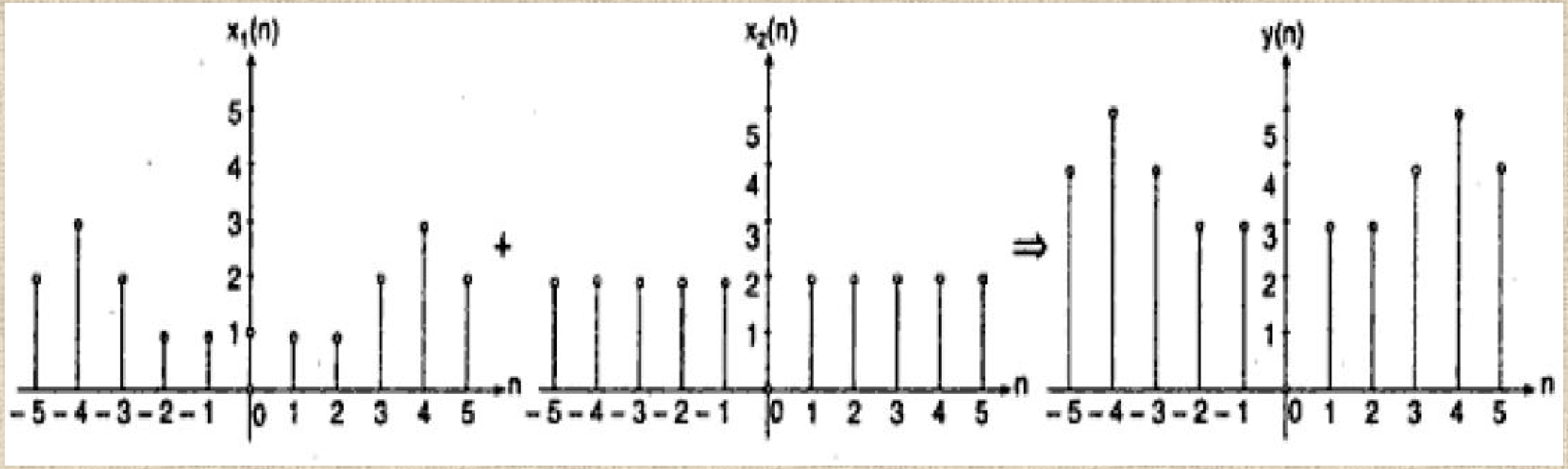


Addition of signals contd..

Discrete- time signals:

Let two signals $x_1(n)$ and $x_2\,(n)$ .Then the result signal y(t) is

$$y(n) = x_1(n) + x_2\,(n)$$



Multiplication of signals

Continuous-time signal:

Let two signals, $x_1(t)$ and $x_2(t)$. Then the result signal y(t) is

$$y(t) = x_1(t) \times x_2(t)$$

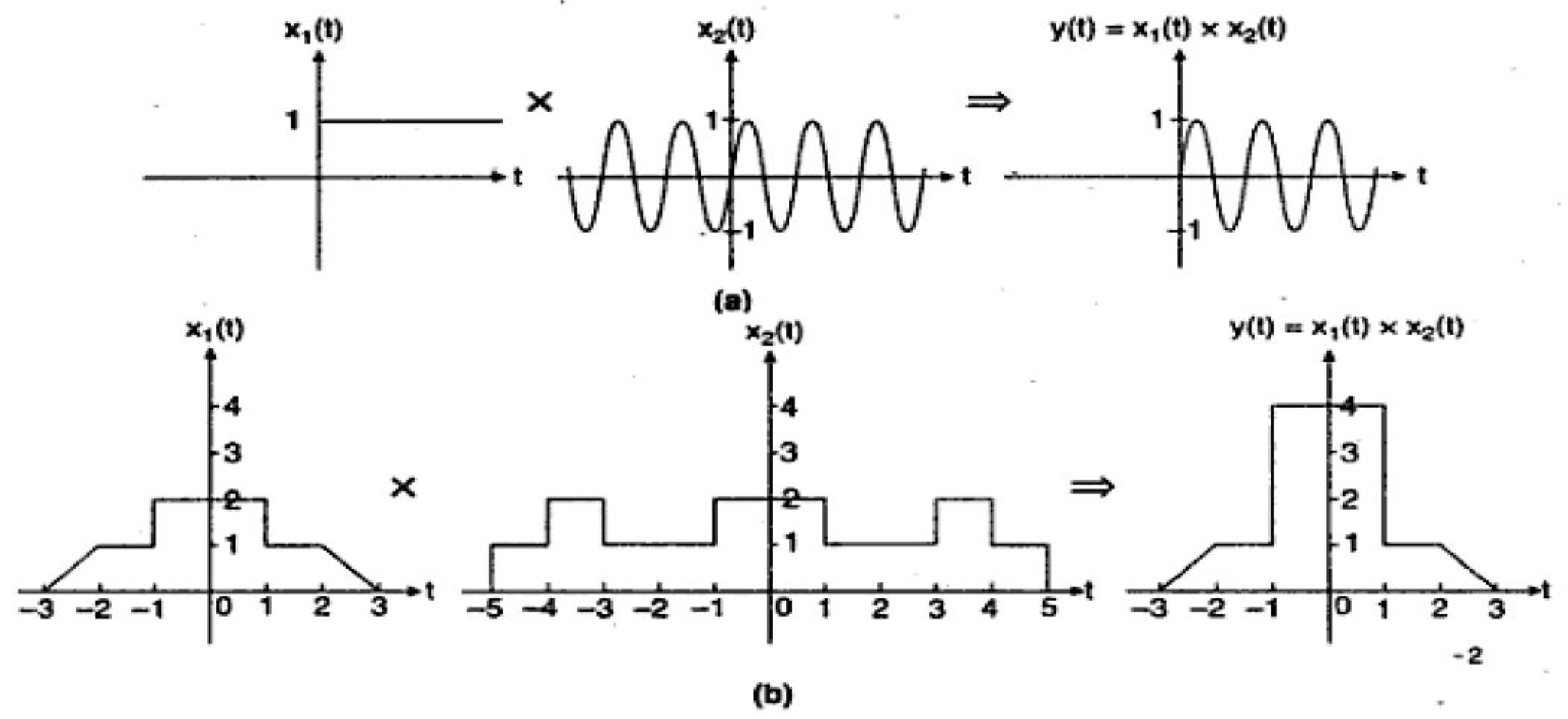


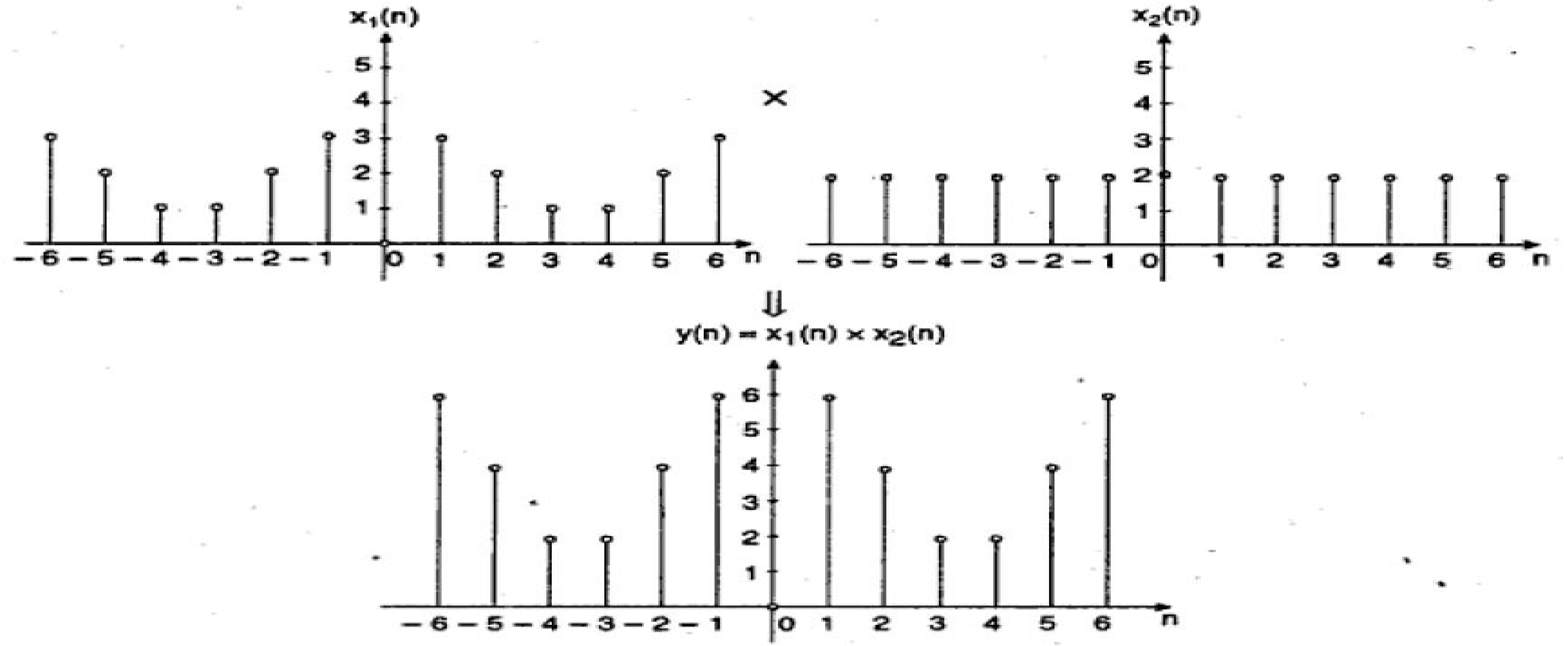
Fig. 2.26 Multiplication of Signals

Multiplication of signals contd..

Discrete- time signals:

Let two signals $x_1(n)$ and $x_2(n)$ .Then the result signal y(t) is

$$y(n) = x_1(n) \times x_2(n)$$

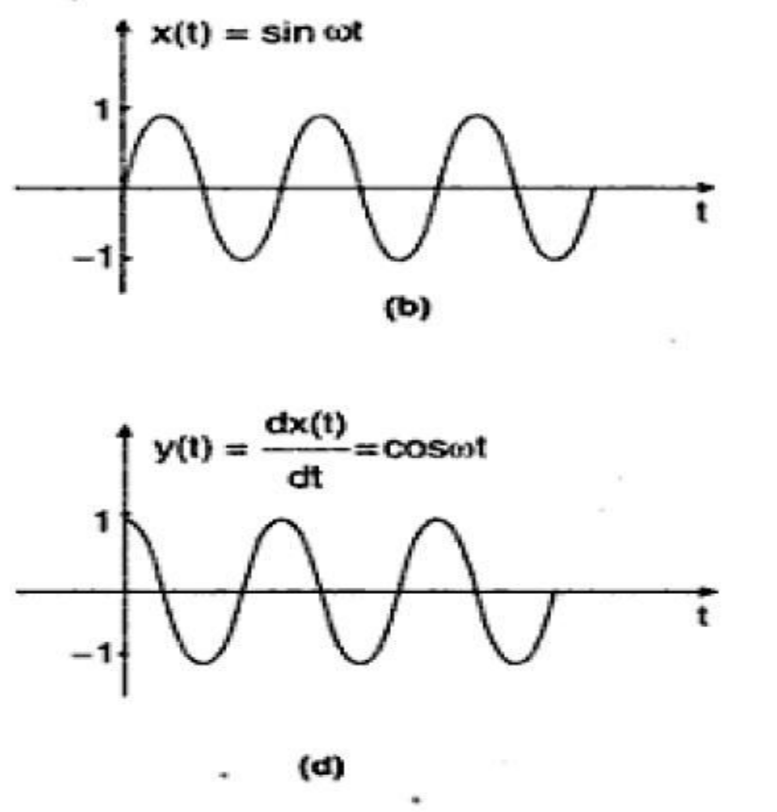


(a)

Differentiation of signals

The derivative of an input signal $x(t)$ with respect to time is defined by

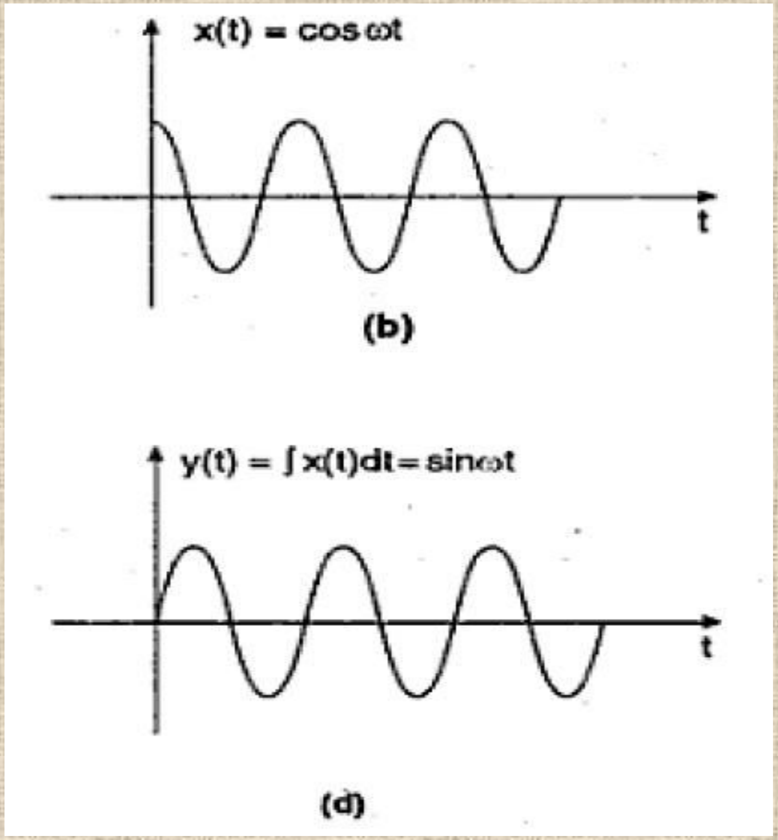
$$y(t) = \frac{d\,x(t)}{dt}$$



Integration of signals

The integration of an input signal $x(t)$ with respect to time $y(t)$ is defined by

$$y(t) = \int_{-\infty}^{t} x(t)\, dt$$



Time scaling of signals

If the original signal is $x(t)$, then time scaled signal is:

$$y(t) = x(\beta t)$$

Where β = scaling factor (if β <1, then the signal expands; if β >1, then the signal compresses).

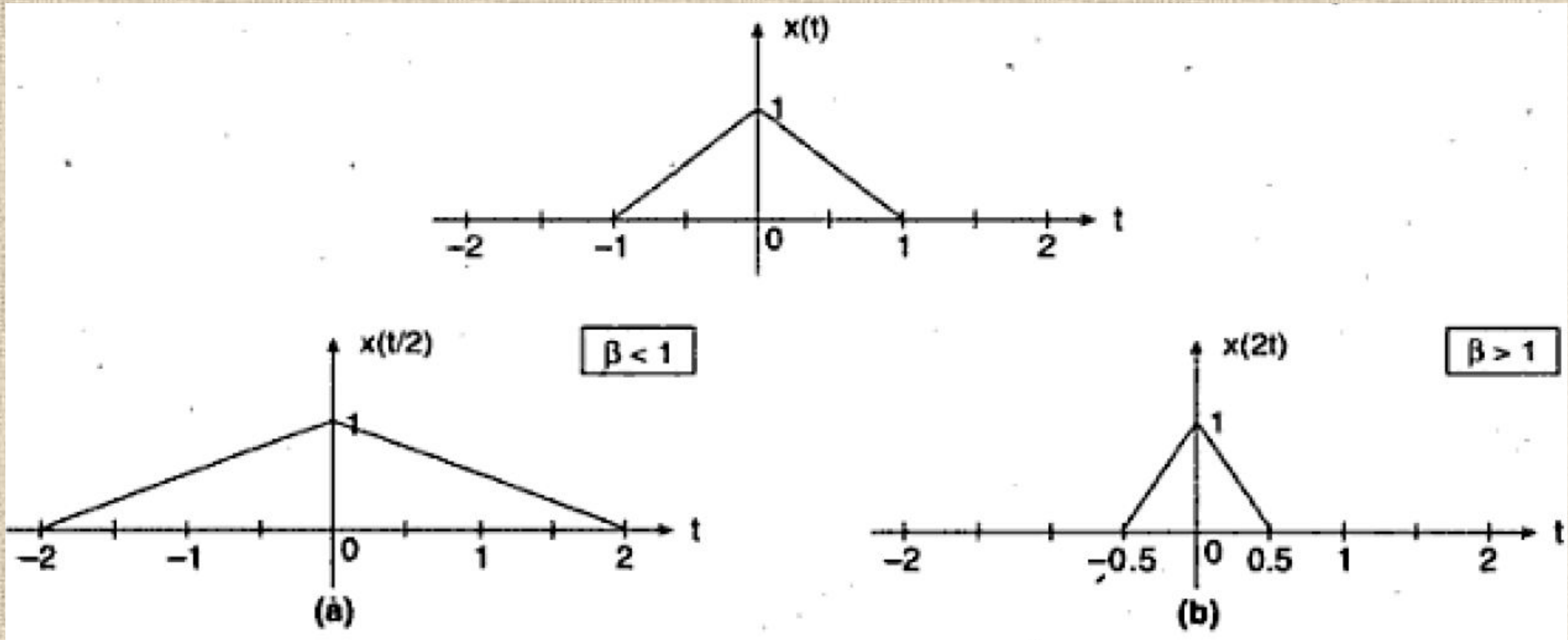


Fig. 2.30 Time Scaling (a) $\beta < 1$ (b) $\beta > 1$

Types of signals

- Exponential signal
- Sinusoidal signal
- Step signal
- Impulse signal
- Ramp signal

Exponential signal

1. Real exponential signal
2. Complex exponential signal

Real exponential signal(continuous-time): $x(t) = B\,e^{\alpha t}$

Where B is scaling factor and α is a real parameter.

For α <0, the magnitude of a real exponential signal decays exponentially.

For α >0, the magnitude of a real exponential signal rises exponentially.

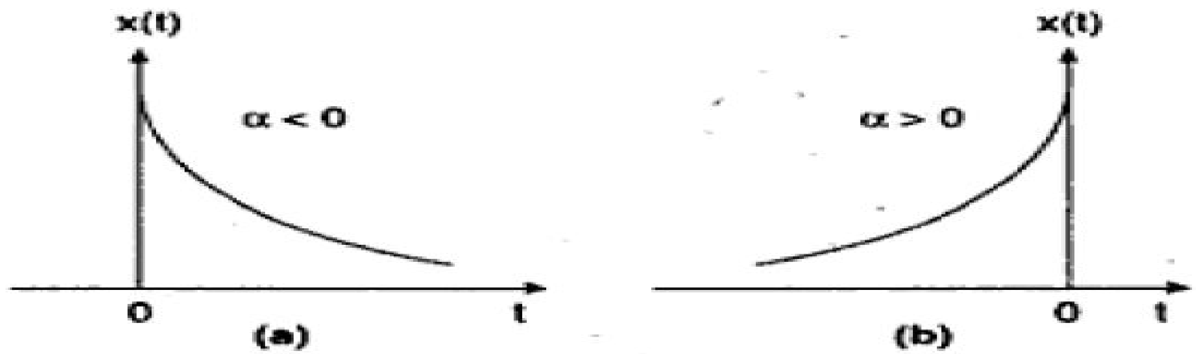


Fig. 2.37 Continuous-time Exponential Signal (a) α < 0, (b) α > 0

Exponential signal

Real exponential signal(discrete-time): $x(t) = B\,\alpha^{n}$

Where B is scaling factor and α is a real parameter.

For α >1, the magnitude of a real exponential signal rises exponentially.

For 1> α >0, the magnitude of a real exponential signal decays exponentially.

For 0> α >-1, the magnitude of a real exponential signal decays exponentially. For each integer value of n, the signal is represented alternatively.

For α <-1, the magnitude of a real exponential signal grows exponentially. For each integer value n, the signal is represented alternatively.

Real exponential signal(discrete-time) contd..

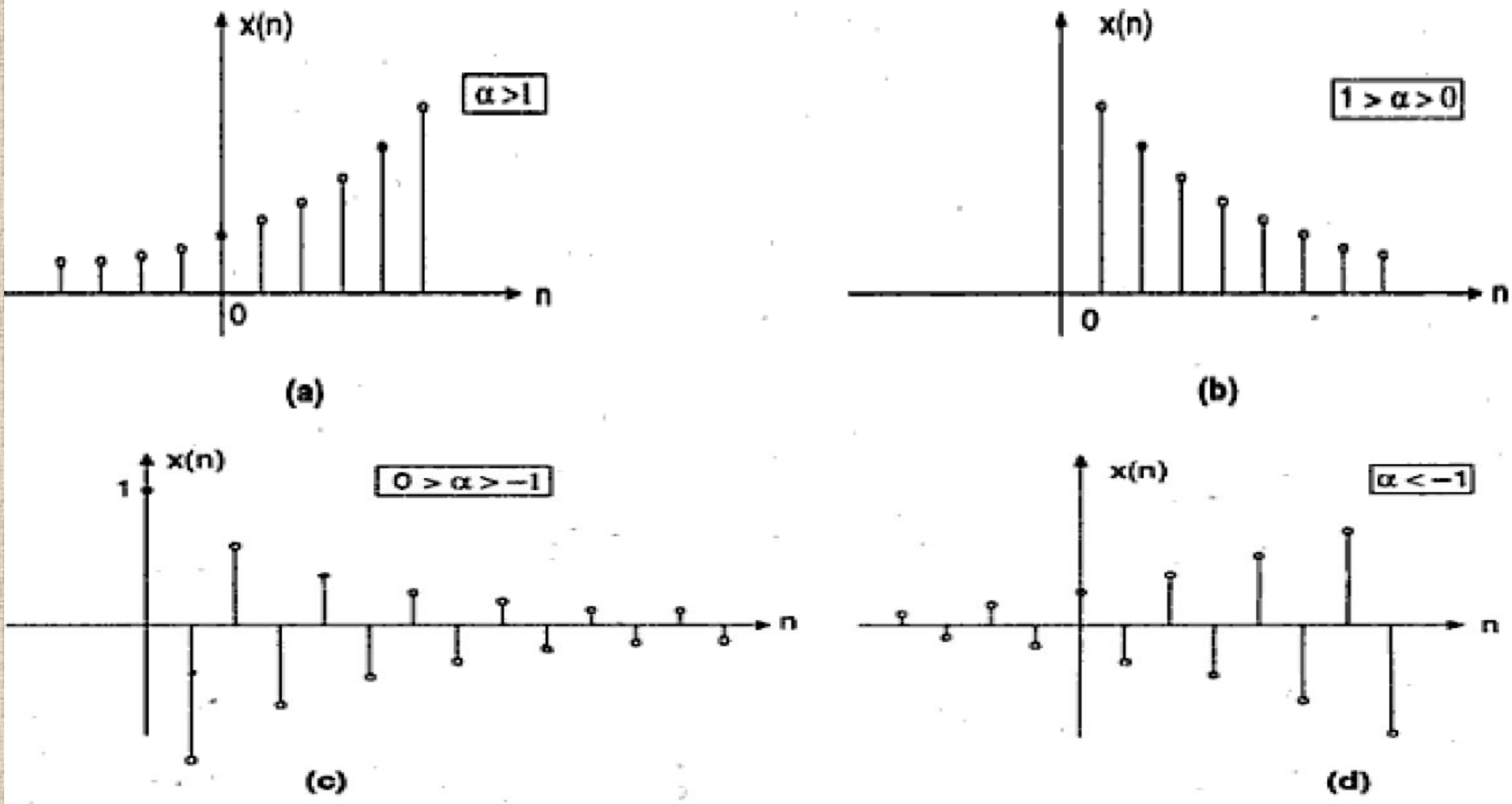


Fig. 2.38 Discrete-time Exponential Signal
(a) α > 1, (b) 1 > α > 0, (c) 0 > α > −1, (d) α < −1

Complex exponential signal

$$x(t) = e^{j\omega_0 t}$$

An important property of this complex exponential signal is its periodicity, i.e.

$$e^{j\omega_0 t} = e^{j\omega_0 (t+T)}$$

$$e^{j\omega_0 t} = e^{j\omega_0 t} e^{j\omega_0 T}$$

We know that,

$$e^{j\omega_0 T} = \cos(\omega_0 T) + j\sin(\omega_0 T) = 1$$

Sinusoidal signal

The continuous-time version of a sinusoidal signal $x(t)$ in its general form may be written as

$$X(t) = A\cos(\omega t + \phi)$$

where A = Amplitude of the signal $x(t)$

ω = Frequency of the signal $x(t)$ (radian/s)

ϕ = Phase angle of the signal $x(t)$ (radians)

Complex exponential representation of sinusoidal signal

$$e^{j\omega T} = \cos(\omega t) + j\sin(\omega t)$$

$$e^{-j\omega T} = \cos(\omega t) - j\sin(\omega t)$$

$$A\cos(\omega t + \phi) = \frac{A}{2}\left(e^{j(\omega t + \phi)} + e^{-j(\omega t + \phi)}\right)$$

$$A\cos(\omega t + \phi) = \frac{A}{2}\left(e^{j\omega t}e^{j\phi} + e^{-j\omega t}e^{-j\phi}\right)$$

$$A\cos(\omega t + \phi) = \mathrm{Re}\{Ae^{j(\omega t + \phi)}\}$$

$$A\sin(\omega t + \phi) = \mathrm{Im}\{Ae^{j(\omega t + \phi)}\}$$

Step Function

The continuous-time step function is commonly denoted by u(t) and defined as

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

The discrete-time step function is commonly denoted by u(n) and defined as

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

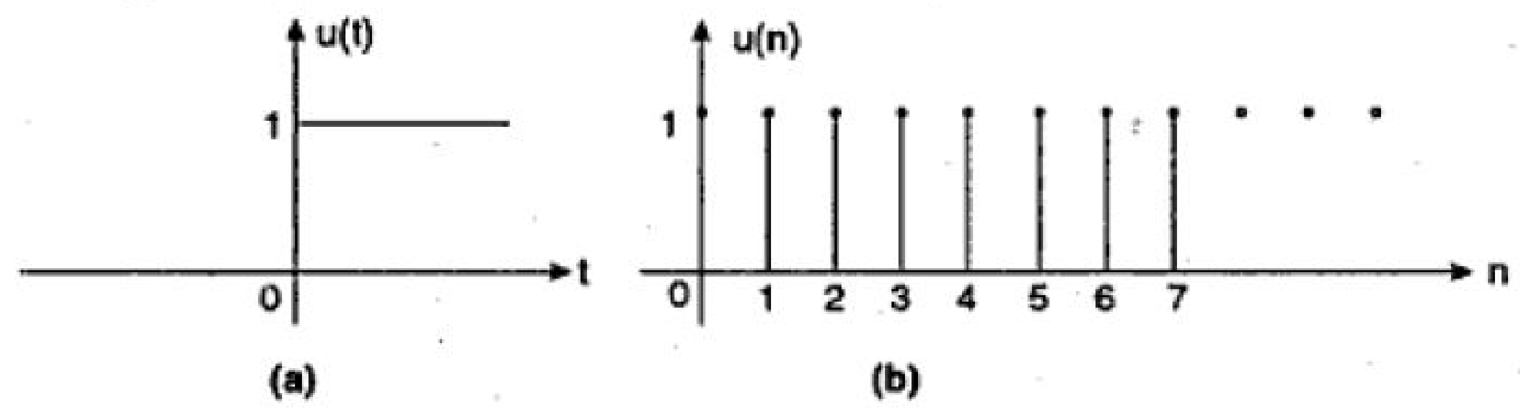


Fig. 2.41 Step Signal
(a) Continuous-time Signal (b) Discrete-time Signal

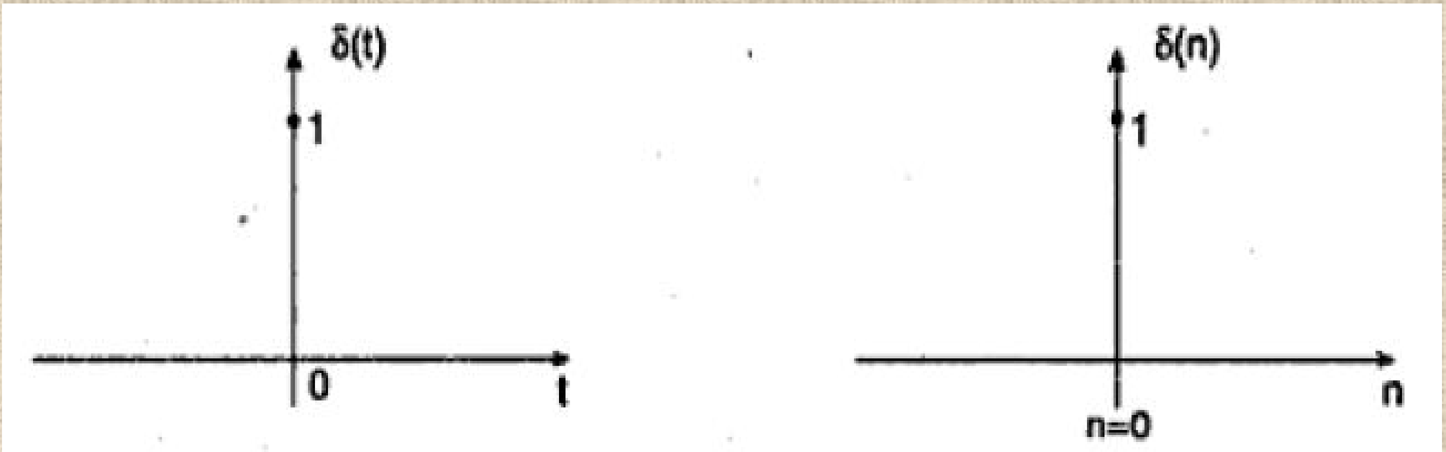
Impulse Function

The Impulse function is a derivative of the step function u(t) with respect to time. It is denoted by $\delta(t)$

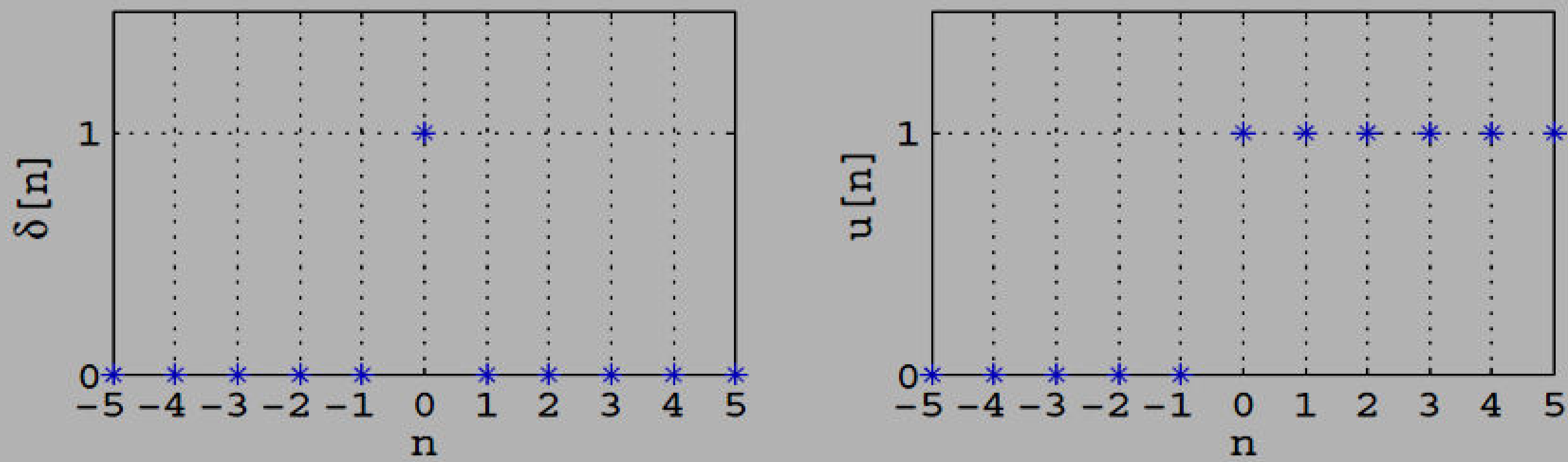
$$\delta(t) = \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

For discrete-time signal

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



Impulse Function contd..



$$\delta[n] = u[n] - u[n-1].$$

Impulse Function contd..

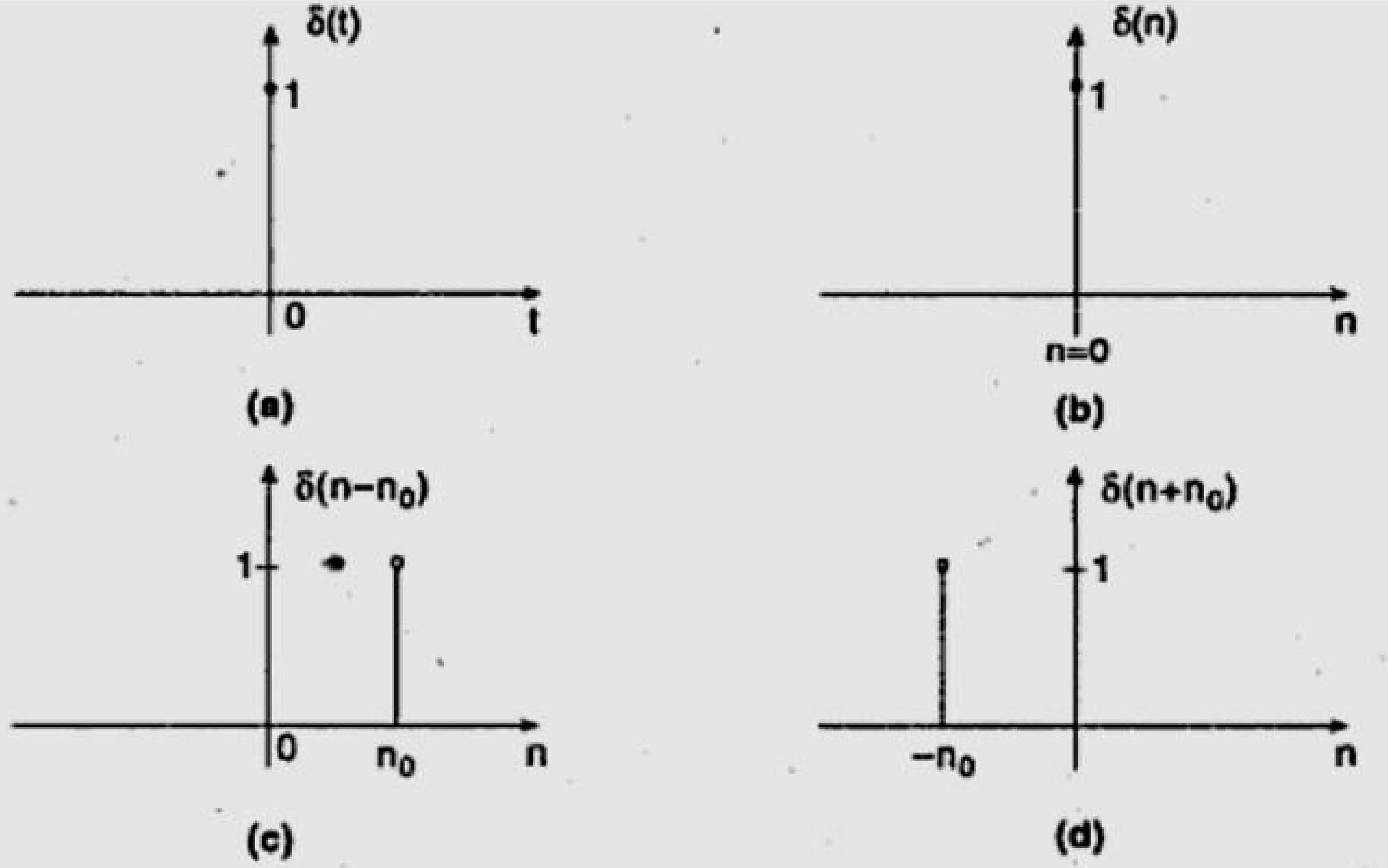


Fig. 2.42 Impulse Signal
(a) Continuous-time Impulse (b) Discrete-time Impulse
(c) Discrete-time Input Shift by $+n_o$ ***(d) Discrete-time Impulse Shift by*** $-n_o$

Ramp Function

The integral of the step function is a ramp function of unit slope.

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

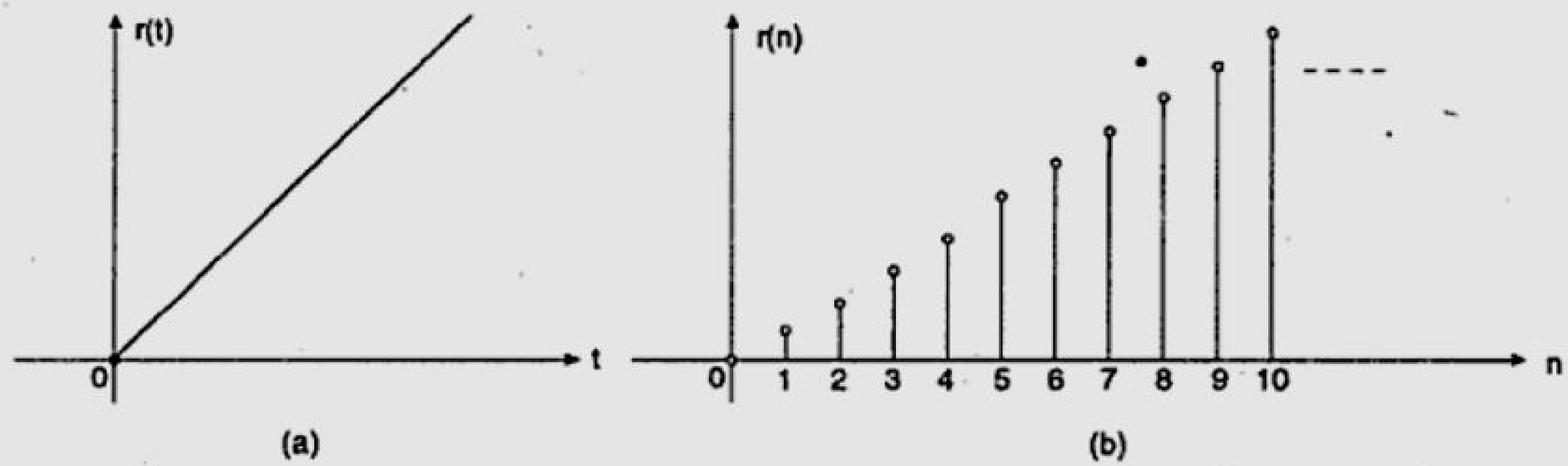
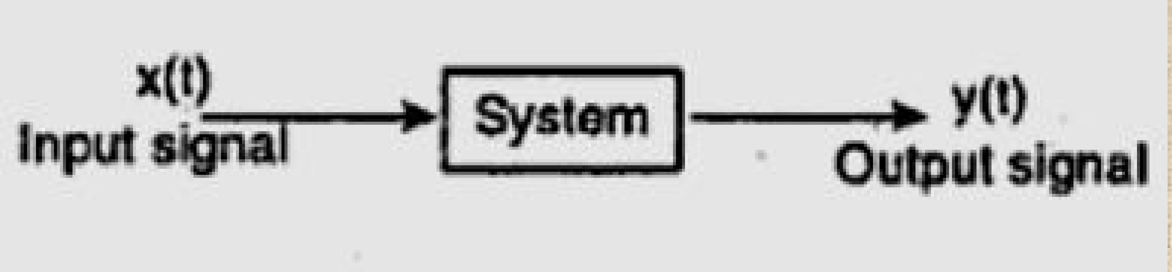


Fig. 2.43 Ramp Signal
(a) Continuous-time Signal (b) Discrete-time Signal

SYSTEM

A system is an entity that manipulates one or more signals to perform a function, which results in a new output signal.



Time-invariant and Time-variant System

A system is said to be time-invariant if the input signal is delayed or advanced any factor that leads the output signal to delay or advancement by the same factor.

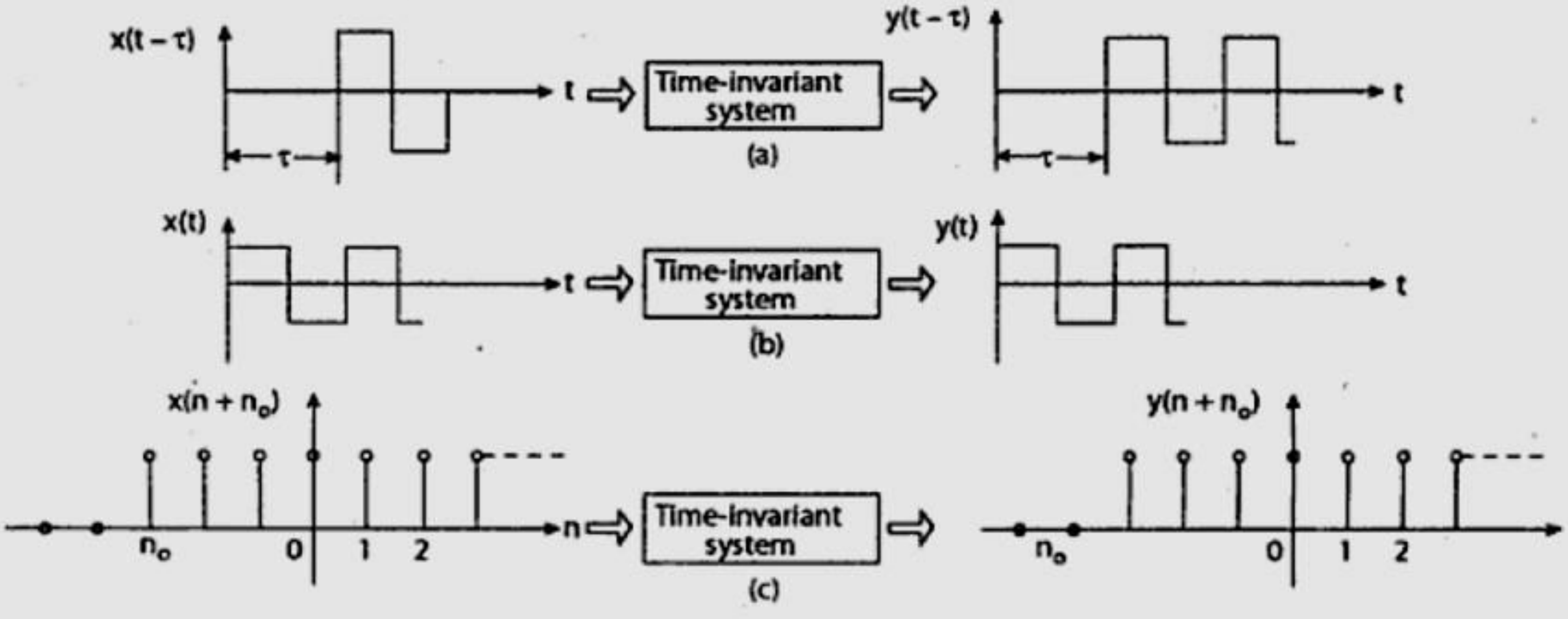


***Fig. 2.53* Basic Time-invariant Systems**
(a) Delayed Time-invariant System (b) Time-invariant System
(c) Advanced Time-invariant System

Linear and Nonlinear System

Let us consider a system, an input signal x(t) which responds with y(t). Another input signal x(t) which responds with y(t). Then the system is said to be linear if

1. The response to $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$ (additive property).
2. The response to $ax_1(t) + bx_2(t)$ is $ay_1(t) + by_2(t)$ (scaling property).