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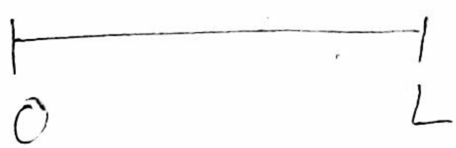
F.S application

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)$$

where
$$a_n = \frac{1}{L}\int^{L} f(u) \cos\left(\frac{n\pi u}{L}\right) du \quad \text{and}$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$



initial term $f(x)$

$t = 0$

~~Be~~ tial subssequence term of Scv

The boundary value problem of this is

$$\frac{\partial u(x,t)}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \quad \text{—— (I)}$$

$|u(x,t)| < M$, $u_x(0,t) = 0$, $u_x(L,t) = 0$ $u_x(L,t) = 0$

$u(x,0) = f(x)$

Its soln will be of the form

$$u(x,t) = XT \quad \text{—— (II)}$$

The (I) can be written as

$$XT' = kTX''$$

$$\Rightarrow \frac{T'}{kT} = \frac{X'}{X} = -\lambda^2 \text{ (say)}$$

when $\frac{T'}{kT} = -\lambda^2$

$$\Rightarrow T' + k\lambda^2 T = 0$$

$$T = e^{-k\lambda^2 t}$$

$dy + \alpha y = 0$

$y = e^{-\alpha x}$

$y = e^{mx}$

(a) $\frac{X''}{X} = -\lambda^2$

$\Rightarrow X'' + \lambda^2 X = 0$

$\Rightarrow X = a\cos\lambda x + b\sin\lambda x$

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$D^2 + \lambda^2 = 0$

$D = \pm \lambda i$

$y = a\cos\lambda x + b\sin$

Therefore soln of (1) because

$u(x,t) = XT$

$= \left(e^{-k\lambda^2 t}\right)\left(a\cos\lambda x + b\sin\lambda x\right)$

$= e^{-k\lambda^2 t}\left(A\cos\lambda x + B\sin\lambda x\right)$ — (3)

Now, we use bondury condition

$u_x(0,t) = 0$

$\Rightarrow \left(e^{-k\lambda^2 t}\right)B = 0$

$B = 0 \quad \left(e^{-k\lambda^2 t} \neq 0\right)$

Therfore (3) $\Rightarrow u(x,t) = A\left(e^{-k\lambda^2 t}\right)\cos\lambda x$

(ii) $U_x(L,t)=0$

$\Rightarrow A\left(e^{-k\lambda^2 t}\right)\sin(\lambda L)=0$

$\sin(\lambda L)=0 \quad \left[\begin{array}{l}A\neq 0\ \& \\ e^{-k\lambda^2 t}\neq 0\end{array}\right]$

$\Rightarrow \lambda L = n\pi$

$\lambda = \dfrac{n\pi}{L}$

Therefore (4) $\Rightarrow u(x,t)=Ae^{-kn^2\pi^2 t/L}\cos\left(\dfrac{n\pi x}{L}\right)$

Now to use Fourier series, we take (5) as the form (superposition principle)

$$u(x,t)=\frac{A_0}{2}+\sum_{m=1}^{\infty}A_m\left(e^{-km^2\pi^2 t/L^2}\right)\cos\left(\frac{m\pi x}{L}\right) \quad (6)$$

Now use $u(x,0)=f(x)$ in (6)

$$f(x)=\frac{A_0}{2}+\sum_{m=1}^{\infty}A_m\cdot\cos\left(\frac{m\pi x}{L}\right) \quad (7)$$

Using Furier series we have

$$A_m = \frac{2}{L}\int_0^L f(u) \cos\left(\frac{m\pi x}{L}\right) du$$

Therefore (6) $\Rightarrow$

$$u(x,t) = \frac{1}{L}\int_0^L f(u)\,du + \frac{2}{L}\sum_{n=1}^{\infty}\left(e^{-km^2\pi^2 t/L^2}\right)\int_0^L f(u)\cos\left(\frac{m\pi u}{L}\right) du$$

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$$f(x) = \int_0^{\infty}\left[A(\alpha)\cos\alpha x + B(\alpha)\sin\alpha x\right] d\alpha$$

when $A(\alpha) = \frac{1}{\pi}\int^{\infty} f(x) \cos\alpha u\, du$

$$B(\alpha) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin\alpha u\, du$$

The boundary value problem,

$$\frac{\delta u}{\delta t} = k \frac{\partial^2 u}{\delta x^2} \quad \text{—— (1)}$$

$$u(x,0) = f(x), \; u(0,t) = 0, \; |u(x,t)| < M$$

To solve: we find:

$$u(x,t) = e^{-k\lambda^2 t}(A\cos\lambda x + B\sin\lambda x) \quad \text{—— (2)}$$

Using initial condition

$$u(0,t) = 0$$

$$\Rightarrow A e^{-k\lambda^2 t} = 0$$

$$A = 0 \quad [e^{-k\lambda^2 t} \neq 0]$$

Therefore (2) $\Rightarrow$

$$u(x,t) = B(e^{-k\lambda^2 t})\sin\lambda x \quad \text{—— (3)}$$

To use fourier integrals, we reform the eqn (3) as

$$u(x,t) = \int_0^{\infty} B(\lambda)(e^{-k\lambda^2 t})\sin\lambda x \, d\lambda$$

Now by initial condition: $u(x,0) = f(x)$

$$\Rightarrow f(x) = \int_0^\infty B(\lambda) \sin \lambda x \, d\lambda \quad \cdots (5)$$

Then using Fourier integrals

$$B(\lambda) = \frac{2}{\pi} \int_0^\infty f(u) \sin \lambda u \, du$$

putting these eq^n (4) we have

$$u(x,t) = \frac{2}{\pi} \int_0^\infty \int_0^\infty f(u) \left(e^{-k\lambda^2 t}\right) \sin \lambda u \sin \lambda u \, du \, d\lambda$$

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Temp. $\frac{\delta u}{\delta t} = x \frac{\delta^2 u}{\delta x^2}$

Wave eq^n is differential form

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

With conditions: $y(x,0) = f(x)$, $y_t(x,0) = 0$ $|y(x,t)| < M$

Let, $y = XT$ be a sol^n of (1)

Then

$$XT'' = a^2 T X''$$

$$\Rightarrow \frac{T''}{a^2 T} = \frac{X''}{X} = -\lambda^2 \text{ (say)}$$

$\Rightarrow$ ① $\frac{T''}{a^2 T} = -\lambda^2$

$X'' + \lambda^2 X = 0$

$\Rightarrow X = m\cos\lambda x + n\sin\lambda x$

$D^2 = \pm a\lambda$

$\Rightarrow T'' + a^2\lambda^2 T = 0$

$\Rightarrow T = a\cos a\lambda t + b\sin a\lambda t$

Juge ② $\Rightarrow$

$y(x,t) = XT$

$= (m\cos\lambda x + n\sin\lambda x)(a\cos a\lambda t + b\cos a\lambda t)$

Using intian condition

$y_t(x,0) = 0$

$\Rightarrow 0 = (m\cos\lambda x + n\sin\lambda x)\, b$

$\Rightarrow b = 0$ $\qquad m\cos\lambda x + n\sin\lambda x \neq 0$

Using this (3) $\Rightarrow$

$$y(x,t) = (A\cos\lambda x + B\sin\lambda x)\cos a\lambda t$$

To use Fourier integral we form ~~for~~ (4). as

$$y(x,t) = A(\lambda)\cos\lambda x + B(\lambda)\sin\lambda x)\cos\lambda a t\, d\lambda \quad \text{--- (5)}$$

Using the condition, $u(x,0) = f(x)$ in (5)

$$f(x) = \int_0^{\alpha} A(\lambda)\cos\lambda x + B(\alpha)\sin\lambda x)\,d\lambda \quad \text{--- (6)}$$

Now use Fourier integrals

$$A(\lambda) = \frac{1}{\pi}\int^{\alpha} f(u)\cos\lambda u\,du$$

$$B(\lambda) = \frac{1}{\pi}\int_{-\alpha}^{\alpha} f(u)\sin\lambda u\,du$$

putting (5) we have

$$y(x,t) = \frac{1}{\pi}\int_0^{a}\int_{-\alpha}^{\alpha} f(u)(\cos\lambda x\cos\lambda u + \sin\lambda x\sin\lambda u)\cos\lambda a t\,du\,d\lambda$$

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