Laplace Transformation

$\mathcal{L}\{F(t)\}$ $\qquad F(t) \xrightarrow{\mathcal{L}} f(s)$

$= \int_0^\infty F(t) e^{-st} dt$ or $\int_0^\infty F(t) e^{-st} dt$

$= f(s)$ or $f(s)$

$F(t) = 1$

$\mathcal{L}\{1\} = \int_0^\infty 1 . e^{-st}$

$= \lim_{p \to \infty} \left[\frac{e^{-st}}{-s} \right]_0^p$

$= -\frac{1}{s} \lim_{p \to \infty} \left[\frac{1}{e^{st}} \right]_0^p$

$= \left(-\frac{1}{s}\right) \lim_{p \to \infty} \left[\frac{1}{e^{sp}} - 1 \right]$

$= \frac{1}{s} - \lim_{p \to \infty} \frac{1}{e^{sp}}$

$= \frac{1}{s}$ (when $s>0$)

$F(t) = 1 \xrightarrow{\infty} f(s) = \frac{1}{s}$

$F(t) = t^2$

$$L(t^2) = \int_0^\infty t^2 e^{-st}\,dt$$

$$= \lim_{p\to 0} \int_0^p t^2 e^{-st}\,dt$$

$$= \lim_{p\to\infty} t^2 \left(\frac{e^{-st}}{-s}\right)\Big|_0^p - \int_0^p 2t\left(\frac{e^{-st}}{-s}\right)$$

$$= \lim_{p\to 0}\left[\frac{-p^2}{se^{sp}} + \frac{2}{s}\left\{ t\frac{e^{-st}}{-s}\Big|_0^p - \int_0^p 1\left(\frac{e^{-st}}{-s}\right)dt\right.\right.$$

$$= \lim_{p\to 0}\left[\frac{-p^2}{se^{sp}} + \frac{2}{s} - \frac{2}{s^2}\frac{p}{e^{sp}} + \frac{2}{s^2} + \frac{e^{-st}}{-s}\Big|_0^p\right]$$

$= -\frac{1}{s} \lim_{p \to 0} \frac{p^2}{e^{sp}} - \frac{2}{s^2} \lim_{p \to \alpha} \frac{p}{e^{sp}} - \frac{2}{s^3} \lim_{p \to \alpha}$

$= -\frac{1}{s} \lim_{p \to \alpha} \frac{p^2}{e^{sp}} - \frac{2}{s^2} \lim_{p \to \alpha} \frac{p}{e^{sp}} - \frac{2}{s^3} \lim_{p \to} \frac{1}{e^{sp}} + \frac{2}{s^3}$

$= \frac{2}{s^3} \quad [s > 0]$

$1 \to \frac{1}{s} \qquad e^{at} \to \frac{1}{s-a} \quad (s > a)$

$t^2 \to \frac{2}{s^3}$

$t^n = \frac{n!}{s^{n+1}} \quad s > 0$

$\mathcal{L}\{e^{at}\} = ?$

$= \int_0^{\alpha} e^{at} \cdot e^{-st} \, dt$

$= \lim_{p \to \alpha} \int_0^{p} e^{-(s-a)t} \, dt$

$$= \frac{-1}{s-a} \lim_{p\to\alpha} \left[\frac{1}{e^{(s-a)t}}\right]_0^p$$

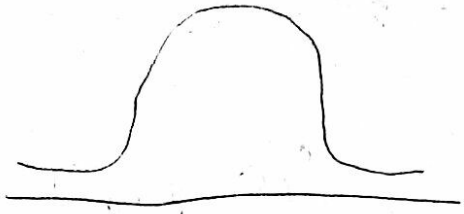
$$= -\frac{1}{(s-a)} \left[\lim_{p\to\alpha} \frac{1}{e^{(s-a)p}} - 1\right]$$

$$= \frac{1}{s-a} - \frac{1}{s-a} \lim_{p\to\alpha} \frac{1}{e^{(s-a)p}}$$

$$= \frac{1}{s-a} \quad (s>a)$$

statistic

* Non-parametric Test



ডেটার Normal distribute হবে না তখন - Non-parametric Test use করা হয়।

Sign Test

M	1	2	3	4	5	6
1	10	8	3	8	3	2
2	8	10	10	9	5	4
	+	-	-	+	-	-

ই

Run test

aa | bbb | aaa | b | a | Random

a | b | a | b | a | a | a — Randomly

ইউজারCT - Decision making theory

Laplace Transform

① $\mathcal{L}\{F(t)\} = f(s)$ [illegible]

ⓐ) $\mathcal{L}\{F'(t)\} = ?$ $\quad sf(s) - F(0)$

ⓑ) $\mathcal{L}\{F^{(n)}(t)\} = ?$

$\mathcal{L}\{F''(t)\} = s\,f'(s) - sF(0) - F'(0)$

$= s^2 f(s) - sF(0) - F'(0)$.

ⓐ) $\{F'(t)\} = \int_0^{\infty} e^{-st} F'(t)\,dt$

$= \lim_{p \to \infty} \int_0^{p} e^{-st} F'(t)\,dt$

$= \lim_{p \to \infty} \left[e^{-st} F(t) \Big|_0^p - \int_0^p (-s) e^{-st} F(t)\,dt \right]$

$= \lim_{p \to \infty} \frac{F(p)}{e^{sp}} - F(0) + \lim_{p \to 0} s \int_0^p F(t)\,dt$

$= s\mathcal{L}\{F(t)\} - F(0)$

$= sf(s) - F(0)$

$$F', F'', F''', F^{IV} \cdots F^{(n)}$$

$$\mathcal{L}\left\{F^{(n-1)}(t)\right\} = s^{n-1}f(s) - s^{n-2}F(0) - s^{n-3}F'(0) - \cdots + F^{(n-2)}(0)$$

$$\mathcal{L}\left\{F^{n}(t)\right\} = s^{n}f(s) - s^{n-1}F(0) \cdots\cdots F^{(n-1)}(0)$$

power এ যতো
ততগুলো derivative

Prove that

$$\mathcal{L}\left(F^{n}(t)\right\} = s^{n}f(s) - s^{n-1}F(0) \cdots\cdots F^{n-1}(0)$$

<u>Prove:</u> We first prove that

$$\mathcal{L}\left\{F'(t)\right) = sf(s) - F(0) \text{ ——— } (1) \quad (\text{done})$$

Using mathematical ~~ind~~ induction, suppose

$$\mathcal{L}\left\{F^{(n-1)}(t)\right\} = s^{n-1}f(s) - s^{n-2}F(0) \cdots\cdots F^{(n-2)}(0) \quad \cdots (2)$$

Then; $\mathcal{L}\left\{F^{(n)}(t)\right\} = \int_0^{\infty} e^{-st}F^{(n)}(t)\,dt$

$$= \lim_{p \to \infty} \int_0^{p} e^{-st}F^{(n)}(t)\,dt$$

$$= \lim_{p \to \infty} \left[e^{-st}F^{(n-1)}(t)\right]\Big|_0^p - \int_0^p (-s)e^{-st}F^{(n-1)}(t)\,dt$$

$$= \lim_{p\to\alpha} \frac{F^{(n-1)}(t)}{e^{st}} \Big|_0^p + s\int_0^p e^{-st} F^{(n-1)}(t)\, dt$$

$$= \lim_{p\to 0} \frac{F^{(n-1)}(t)}{e^{sp}} - F^{(n-1)}(0) + s\int_0^\alpha e^{-st} F^{(n-1)}(t)\, dt$$

$$= s\mathcal{L}\{F^{(n-1)}(t)\} - F^{(n-1)}(0)$$

$$= s\left[s^{n-1}f(s) - s^{n-2}F(0) \cdots - F^{(n-2)}(0)\right] - F^{n-1}(0)$$

$$\cancel{= s^n f}$$

$$= sf(s) - s^{n-1}F(0) \cdots - sF^{(n-2)}(0) - F^{(n-1)}(0)$$

Formula 2

$$\mathcal{L}\{F(t)\} = f(s)$$

$$\mathcal{L}\left\{\int_0^t F(u)\, du\right\} = ?$$

To find this

let $G(t) = \int_0^t F(u)\,du$

$\Rightarrow G'(t) = F(t)$ and $G(0) = 0$

Now taking laplace theorem,

$\mathcal{L}\{G'(t)\} = \mathcal{L}\{F(t)\}$

$\Rightarrow s\mathcal{L}\{G(t)\} - G(0) = f(s)$

$\Rightarrow s\,\mathcal{L}\{G(t)\} = f(s)$

$\Rightarrow \mathcal{L}\{G(t)\} = \frac{f(s)}{s}$

$\Rightarrow \mathcal{L}\left\{\int_0^t F(u)\,du\right\} = \frac{f(s)}{s}$

* $\sin at$ $\qquad\qquad$ $\cos at$

$$\mathcal{L}\{\sin at\} = \int_0^{\infty} e^{-st} \sin at \, dt$$

$$= \lim_{p \to \infty} \int_0^{p} e^{-st} \sin at \, dt$$

$$= \lim_{p \to \infty} \int \sin at \left(\frac{e^{-st}}{\quad} \right.$$

$$= \lim_{p \to \infty} \left[\frac{e^{-st} (s \sin at - a \cos at)}{s^2 + a^2} \right]_0^{p}$$

$$= \frac{1}{s^2 + a^2} \lim_{p \to \infty} \left[\frac{s \sin ap - a \cos ap}{e^{sp}} + a \right]$$

$$= \frac{a}{s^2 + a^2}$$

* $\mathcal{L}\{\cos at\} = \dfrac{s}{s^2+a^2}$

Formula-3

If $\mathcal{L}\{F(t)\} = f(s)$, then

a, $\mathcal{L}\{t\,F(t)\} = ?$ $\quad -f'(s)$

b. $\mathcal{L}\{t^n F(t)\} = ?$ $\quad (-1)^n f^{(n)}(s)$

We have

$$f(s) = \mathcal{L}\{F(t)\} = \int_0^\infty e^{-st} F(t)\,dt$$

Taking derivative on bothsides ~~with~~

$$f'(s) = \frac{d}{ds}\left(\int_0^\infty e^{-st} F(t)\,dt\right)$$

[Integral in differential]

$$= \int_0^\infty F(t)\,\frac{\partial}{\partial s}\left(e^{-st}\right)dt$$

$$= \int_0^\infty F(t)(-t)\,e^{-st}\,dt$$

$$= -\int_0^\infty e^{-st}\left(t\,F(t)\right)dt$$

$$= -\mathcal{L}\{t\,F(t)\} \quad \therefore -f'(s) = \mathcal{L}\{t\,F(t)\}$$

Formula-04

If $\mathcal{L}\{F(t)\} = f(s)$, then

$$\mathcal{L}\left\{\frac{F(t)}{t}\right\} = ?$$

Soln:

let, $G(t) = \frac{F(t)}{t}$

$\Rightarrow F(t) = t\,G(t)$

Taking laplace on both sides

$\Rightarrow f(s) = \mathcal{L}\{t\,G(t)\} = -g'(s)$

~~$\Rightarrow g(s)$~~

Taking derivative on both side w.r.to s with the limit s to ∞

$$g(s) = -\int_{\infty}^{s} f(s)\,ds = \int_{s}^{\infty} f(u)\,du$$

$$\Rightarrow \mathcal{L}\{G(t)\} = \int_{s}^{\alpha} f(s)\,ds$$

$$\Rightarrow \mathcal{L}\left\{\frac{F(t)}{t}\right\} = \int_{s}^{\alpha} f(u)\,du$$

Initial value theorem on laplace

$$\lim_{t \to 0} F(t) = \lim_{s \to \alpha} sf(s)$$

Final value Theorem on laplace

t^2

$$\lim_{t \to 0} F(t) = \mathcal{L}\left\{ e^{-st} F(t)\,dt \right\}$$

$$= \frac{1}{s}$$

$t^2 \to \frac{2!}{s^{2+1}} = \frac{2}{s^3}$

$f(t) \qquad f(s)$

$$\lim_{s \to \alpha} sf(s)$$

t^2

$$\lim_{t \to} F(t) = \lim_{s \to \alpha} sf(s)$$

$$\Rightarrow \lim_{t \to 0} t^2 = \lim_{s \to \alpha} s \frac{2}{s^3}$$

$$\Rightarrow 0 = \lim_{s \to \alpha} \frac{2}{s^2}$$

$$\Rightarrow 0 = 0$$

Application in Integration

(i) $\int_0^\infty t e^{-st} \cos at \, dt$ $\qquad \mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$

$= \mathcal{L}\{t \cos at\}$

$= -\frac{d}{ds}\left(\mathcal{L}\{\cos at\}\right)$

$= -\frac{d}{ds}\left(\frac{s}{s^2+a^2}\right)$

$= -\frac{(s^2+a^2)1 - 2s.s}{(s^2+a^2)^2}$

$= \frac{s^2-a^2}{(s^2+a^2)^2}$

(ii) $\int_0^\infty \left(\frac{e^{-3t} - e^{-6t}}{t}\right) dt$

$= \lim_{s \to 0} \left(\int_0^\infty e^{-st} \left(\frac{e^{-3t} - e^{-6t}}{t}\right) dt\right)$

$= \lim_{s \to 0} \mathcal{L}\left\{\frac{f(t)}{t}\right\}$

$$\mathcal{L}\{F(t)\} = \mathcal{L}\{e^{-3t} - e^{-6t}\}$$

$$= \frac{1}{s+3} - \frac{1}{s+6}$$

Now $\mathcal{L}\left\{\frac{F(t)}{t}\right\} = \int_s^{\infty} f(u)\,du$

$$= \int_s^{\infty} \left(\frac{1}{u+3} - \frac{1}{u+6}\right) du$$

$$= \lim_{p\to\infty} \left[\ln|u+3| - \ln|u+6|\right]_s^p$$

$$= \lim_{p\to\infty} \left[\ln\left|\frac{u+3}{u+6}\right|\right]_s^p$$

$$= \lim_{p\to\infty} \ln\left(\frac{p+3}{p+6}\right) - \ln\left(\frac{s+3}{s+6}\right)$$

$$= \ln\frac{s+6}{s+3}$$

Therefore, required result

$$\lim_{s\to 0} \ln\left(\frac{s+6}{s+3}\right) = \ln\frac{6}{3} = \ln 2$$

$\int_0^{\infty} \frac{\sin t}{t}\, dt$

$\lim_{s \to 0} \int_0^{\infty} e^{-st}\left(\frac{\sin t}{t}\right) dt$

$= \lim_{s \to 0} \mathcal{L}\left\{ \frac{\sin t}{t} \right\}$

prove that

#1 $\mathcal{L}\left\{\int_0^t \frac{\sin u}{u}\,du\right\} = \frac{1}{s}\tan^{-1}\left(\frac{1}{s}\right)$

sol $\mathcal{L}\{\sin t\} = \frac{1}{s^2+1^2}$

$\mathcal{L}\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \mathcal{L}\{\sin u\}\,du = \int_s^\infty \frac{1}{u^2+1}\,du$

$= \lim_{p\to\infty}\left[\tan^{-1} u\right]_s^p$

$= \lim_{p\to\infty}\left(\tan^{-1} p - \tan^{-1} s\right)$

$= \lim_{p\to\infty}\left\{\tan^{-1}\left(\frac{p-s}{1+ps}\right)\right\}$

$= \lim_{p\to\infty} \tan^{-1}\left(\frac{1-s/p}{\frac{1}{p}+s}\right)$

$= \tan^{-1}\left(\frac{1}{s}\right)$

$$\Rightarrow \left\{ \int_0^t \left(\frac{\sin u}{u} \right) du \right\}$$

$$\boxed{= \frac{f(s)}{s}}$$

$$= \frac{\mathcal{L}\left\{ \frac{\sin t}{t} \right\}}{s}$$

$$= \left(\frac{1}{s}\right) \tan^{-1}\left(\frac{1}{s}\right).$$

$1 \to \frac{1}{s}$

$t^n = \frac{n!}{s^{n+1}}$

$e^{at} \to \frac{1}{s-a}$

$\sin at \to \frac{a}{s^2+a^2}$

$\cos at \to \frac{s}{s^2+a^2}$

$\sinh at \to \frac{a}{s^2-a^2}$

$\cosh at \to \frac{s}{s^2-a^2}$

$\mathcal{L}^{-1}$

$1 \leftarrow \frac{1}{s}$

$\mathcal{L}\{1\} = \frac{1}{s}$

$\mathcal{L}\{e^{at} \cdot 1\} = \frac{1}{s-a}$

$\mathcal{L}\{e^{at}F(t)\} = f(s-a)$

Inverse laplace trasfoom

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-1)(s-2)}\right\}$$

$=$

$$\frac{1}{(s-1)(s-2)} = \frac{A}{s-1} + \frac{B}{s-2} = \frac{-1}{s-1} + \frac{1}{s-2}$$

$\Rightarrow 1 = A(s-2) + B(s-1)$

put $s = 1 \Rightarrow A = -1$

$s = 2 \Rightarrow B = 1$

$$= \mathcal{L}^{-1}\left\{\frac{-1}{s-1} + \frac{1}{s-2}\right\}$$

$$= -\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$$

$$= -e^{t} + e^{2t}$$

$$= f(t)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2(s-1)^2}\right\}$$

$$= \mathcal{L}^{-1}\{$$

$$\mathcal{L}^{-1}\left\{\frac{4s+12}{s^2+8s+16}\right\} \qquad \mathcal{L}^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{4s+12}{(s+4)(s+4)}\right\}$$

$$= \mathcal{L}^{-1}\{$$

1a)

Convolution theorem:

If $F(t)$ and $G(t)$ are two functions, then their convolution is defined by $F*G=\int_0^t F(t)G(t-u)\,du$

If $\mathcal{L}\{F(t)\}=f(s)$ and $\mathcal{L}\{G(t)\}=g(s)$, then

$$\mathcal{L}^{-1}\{f(s)g(s)\}=\int_0^t F(t)G(t-u)\,du=F*G$$

Example

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-2)(s^2+1)}\right\}$$

We know

$$\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}=e^{2t}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}=\sin t$$

$$\therefore \mathcal{L}^{-1}\left\{\frac{1}{s-2}\cdot\frac{1}{s^2+1}\right\}=\int_0^t(\sin u)\,e^{2(t-u)}\,du$$

$$=e^{2t}\left[\int_0^t e^{-2u}\sin u\,du\right]$$

$$=e^{2t}\left[\frac{e^{-2u}(-2\sin u-\cos u)}{2^2+1}\right]_0^t$$

$$=\frac{e^{2t}}{5}\left(-2\sin t-\cos t+1\right)$$

$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)^3}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)^2}\cdot\frac{1}{(s^2+4)}\right\} = \int_0^t \square \sin 2(t-u)\,du$

$= \int_0^t \left(\frac{u \sin 2u}{4}\right) \sin 2(t-4)\,du$

$=$

$\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\cdot\frac{1}{s^2+4}\right\}$

$= \int_0^t (\cos 2u)\sin(2(t-u))\,du$

$= \square$

$= \frac{t \sin 2t}{4}$

Sol

solving D.E using laplace and Inverse laplace Transform

Ex. $y''(t) + y(t)$, $y(0) = 1$, $y'(0) = -2$

Soln. Given D.E is

$$y''(t) + y(t) = t$$

Taking laplace transform on both sides

$$\mathcal{L}\{y''(t)\} + \mathcal{L}\{y(t)\} = \mathcal{L}\{t\}$$

$$\Rightarrow \left[s^2 Y(s) - sY(0) - Y'(0)\right] + \left[sY(s) - Y(0)\right] = \frac{1}{s^2}$$

$$\Rightarrow s^2 Y(s) - s + 2 + sY(s) - 1 = \frac{1}{s^2}$$

$$\Rightarrow Y(s)\{(s^2 + s)\} = \frac{1}{s^2} + s - 1 = \frac{1}{s^2(s^2+s)} + \frac{s}{s^2+1} \quad \frac{1}{s^2+s}$$

$$\Rightarrow Y(s) = \frac{\frac{1 + s^3 - s^2}{s^2}}{s^2 + s}$$

$$= \frac{1 + s^3 - s^2}{s^2(s^2 + s)}$$

$$= \frac{s - 1}{s(s^2 + 1)} + \frac{1}{s^3(s^2 + 1)}$$

To find $y(t)$ we take inverse Laplace transform on both sides, and get

$$y(t) = \mathcal{L}^{-1}\left\{ \frac{s-1}{s(s+1)} + \frac{1}{s^3(s+1)} \right\}$$

here,

$$\frac{s-1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$\frac{1}{s^3(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s+1}$$

$$s-1 = (s+1)A + Bs$$

$$\boxed{-1 = A}$$

(HW 1)

#1 $y''(t)-3y'(t)+2y=4e^{2t}$

$y(0)=-3$

$y'(0)=5$

(HW) $y''(t)-6y'(t)+9y=13e^{3t}$

$y(0)=2,\ y'(0)=6$

(HW 2)

$y''(t)+2y'(t)+y(t)=4\sin t$

$y(0)=2,\ y'(0)=1$

Given D.E $y''(t)+2y'(t)+y(t)=4\sin t$.

Taking laplace transform on both sides

$\mathcal{L}\{y''(t)\}+\mathcal{L}\{2y'(t)\}+\mathcal{L}\{y(t)\}=\mathcal{L}\{4\sin t\}$

$\Rightarrow [s^2y(s)-sy(0)-y'(0)]+2(sy(s)-y(0))+y(s)=4\frac{1}{s^2+1}$

$\Rightarrow (s^2y(s)-2s-1)+2[sy(s)-2]+y(s)=4\frac{1}{s^2+1}$

$\Rightarrow s^2y(s)+2sy(s)+y(s)=2s+1+4+4\frac{1}{s^2+1}$

$y(s)\{s^2+2s+1\}=2s+5+\frac{4}{s^2+1}$

$$Y(s) = \frac{2s}{s^2+2s+1} + \frac{5}{s^2+2s+1} + \frac{4}{(s^2+1)(s^2+2s+1)}$$

$$= \frac{2s+5}{s^2+2s+1} + \frac{4}{(s^2+1)(s^2+2s+1)}$$

$$= \frac{2s+5}{(s+1)^2} + \frac{4}{(s+1)^2(s^2+1)}$$

Here,

$$\frac{2s+5}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{(s+1)^2}$$

$$2s+5 = A(s+1) + B$$

$$\Rightarrow -2+5 = A(-1+1) + B \quad \text{when } s=-1$$

$$\boxed{3 = B}$$

Again $s=0$ then

$$5 = A$$

Again $\dfrac{4}{(s+1)^2(s^2+1)} = \dfrac{A}{s+1} + \dfrac{B}{(s+1)^2} + \dfrac{cs+D}{s^2+1}$

$$4 = A(s+1)(s^2+1) + B(s^2+1) + (cs+D)(s+1)^2$$

$$4 = (s^3+s+s^2+1)A + (s^2+1)B + (cs+D)(s^2+2s+1)$$

when $s=-1$ $\quad 4 = (s^3+s+s^2+1)A + (s^2+1)B + (cs^3+2cs^2+cs+s^2D+2sD+D)$

$$4 = 2B \qquad B = 2$$

এখন x^3, s^2, s এর সহগ সমীকৃত করে পাই

~~$A=0$, $B=0$, $D=0$~~ $\qquad A = 0$

$A+c=0$ $\qquad A+c=0$

$A+c=0$,

$A+B+2c+D=0$ | $A-2+2c+D=0$, $2-2+2c=0$ $\quad c=0$

$D=0$

$A+c+2D=0$

$A+B+D=4$ | $A+2+D=4$

$A+D=2$

$A=2$

$B=2$

$D=0$

$c=-2$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{2s+5}{(s+1)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{4}{(s+1)^2(s^2+1)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{2(s+1)+3}{(s+1)^2}\right\} + \left[\mathcal{L}^{-1}\left\{\frac{2}{s+1}\right\} + \left\{\frac{2}{(s+1)^2}\right\} + \left\{\frac{-2s}{s^2+1}\right\}\right]$$

$$= \mathcal{L}^{-1}\left\{\frac{2}{s+1}\right\} + \left\{\frac{+3}{(s+1)^2}\right\} + 2e^{-t} - 2e^{-t}t + 2\cos t$$

$$= 2e^{-t} + 3e^{-t}t$$

$$f(s) = \frac{1}{s^2(s^2+1)^2}$$

$$\frac{1}{s^2(s^2+1)^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^2+1} + \frac{D}{(s^2+1)^2} + \frac{Es+F}{s^2+1}$$

$$1 = As(s+1)^2 + B(s+1)^2 + Cs^2(s+1) + Ds^2$$

When $s = 0$ হলে

$$B = 1$$

when $s = -1$

$$D = 1$$

$$1 = A(s^3 + 2s^2 + s) + B(s^2 + 2s + 1) + C(s^3 + s^2) + Ds^2$$

s^3, s^2, s and s^0 এর সহগ সমীকৃত করে

$A + C = 0$ $\qquad C = 2$ $\qquad \cancel{4 + 4 + 2}$

$2A + B + C + D = 0$

$A + 2B = 0$ $\qquad A = -2$

$B = 1$

$\mathcal{L}^{-1}\left\{\frac{-2}{s}\right\}+\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}+\mathcal{L}^{-1}\left\{\frac{2}{s+1}\right\}+\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\}$

$2 \cancel{-2f(t)+t+2}e^{-t}+$

$2 \cdot 2 + t + 2e^{-t} + e^{-t}t$

$y''(t) - 3y'(t) + 2y = 4e^{2t}$ $\quad y(0) = -3$, $y'(0) = 5$

Given D.E $y''(t) - 3y'(t) + 2y = 4e^{2t}$

Taking Laplace

$\mathcal{L}\{y''(t)\} - \mathcal{L}\{3y'(t)\} + 2\mathcal{L}\{y\} = \mathcal{L}\{4e^{2t}\}$

$\Rightarrow [s^2Y(s) - sY(0) - Y'(0)] - 3[sY(s) - Y(0)] + \frac{2}{s^2} = 4\frac{1}{s-2}$

$\Rightarrow s^2Y(s) + 3s - 5 - 3sY(s) - 3 + \frac{2}{s^2} = \frac{4}{s-2}$

$\Rightarrow Y(s)\{s^2 - 3s\} = \frac{4}{s-2} - \frac{2}{s^2} + 8 - 3s$

$\Rightarrow Y(s) = \frac{4}{(s-2)(s^2-3s)} - \frac{2}{s^2(s^2-3s)} + \frac{8}{s^2-3s} - \frac{3s}{s^2-3s}$

cover-up rule

$$\Rightarrow y(s) = \frac{4}{s(s-2)(s-3)} - \frac{2}{s^3(s-3)} + \frac{8}{s(s-3)} - \frac{3s}{s(s-3)}$$

$$= \frac{4}{s(s-2)(s-3)} - \frac{2}{s^3(s-3)} + \frac{8-3s}{s(s-3)}$$

$$= \frac{4}{6s} - \frac{4}{2(s-2)} + \frac{4}{3(s-3)} - \frac{8}{3s} - \frac{1}{3(s-3)} - \frac{1}{s}\left\{\frac{2}{s(s-3)}\right\}$$

$$= \frac{2}{3s} - \frac{2}{s-2} + \frac{4}{3(s-3)} - \frac{8}{3s} - \frac{1}{3(s-3)} - \frac{2}{s^2}\left\{\frac{1}{-3s} - \frac{1}{3(s-3)}\right\}$$

$$= \frac{2}{3s} - \frac{2}{s-2} + \frac{4}{3(s-3)} - \frac{8}{3s} - \frac{1}{3(s-3)} + \frac{2}{3s^3} + \frac{2}{3s^2(s-3)}$$

$$= \frac{2}{3s} - \frac{2}{s-2} + \frac{4}{3(s-3)} - \frac{8}{3s} - \frac{1}{3(s-3)} + \frac{2}{3s^3} + \frac{2}{3s}\left\{\frac{1}{s(s-3)}\right\}$$

$$= \frac{2}{3s} - \frac{2}{s-2} + \frac{4}{3(s-3)} - \frac{8}{3s} - \frac{1}{3(s-3)} + \frac{2}{3s^3} - \frac{2}{9s^2} - \frac{2}{9s(s-3)}$$

$$= \frac{2}{3s} - \frac{2}{s-2} + \frac{4}{3(s-3)} - \frac{8}{3s} - \frac{1}{3(s-3)} + \frac{2}{3s^3} - \frac{2}{9s^2} + \frac{2}{27s} + \frac{2}{27(s-3)}$$

HW 3

$$y''(t) - 6y'(t) + 9y = t^3 e^{2t}$$

$y(0) = 2$

$y'(0) = 6$

Taking laplace

$$\Rightarrow \mathcal{L}\{y''(t)\} - 6\mathcal{L}\{y'(t)\} + 9\mathcal{L}\{y\} = \mathcal{L}\{t^3 e^{2t}\}$$

$$\Rightarrow [s^2 Y(s) - sy(0) - y'(0)] - 6[sy(0) - y(0)] + \frac{9}{s^2} =$$