Jamalpur Science and Technology University
Department of CSE

2nd Year 2nd Semester Final Examination - 2023

Course Code: CSE 2232 Course title: Numerical Methods Sessional

Time: XXX Hours Marks: XXX

LAB SESSIONAL ASSIGNMENT

Answer any **all** questions. Figures in the right margin indicate marks.

General Instruction: Students are allowed to use any MATLAB environment. Ensure that each problem code is well-documented, includes appropriate comments and follow tasks. Every student is required to submit a Lab Sessional Report.

[1] Consider the following mathematical function: $f(x) = x^3 - 6x^2 + 11x - 6$
Write a Matlab/Python program to perform the following tasks:
   a) Accept user input for a range of x values (start, end, and step size).
   b) Calculate the corresponding y values using the given function.
   c) Plot a graph of the function over the specified range.

Ensure that the plot includes appropriate labels, title, and a legend. Additionally, provide the Matlab/Python code and the resulting graph.

[2] Consider the equation $f(x) = x^3 - 6x^2 + 11x - 6$
Write a Matlab/Python program to find one root of this equation using the Bisection Method within a specified interval.
Perform the following tasks:
   a) Define the function f(x) in Matlab/Python.
   b) Implement the Bisection Method to find a root within the interval [a,b].
   c) Accept user input for the interval [a,b],tolerance (error), and maximum number of iterations.
   d) Use the Bisection Method to find the root with the given tolerance or maximum number of iterations.
   e) Display the result along with the number of iterations taken.

[3] Consider the equation $f(x) = x^3 - 6x^2 + 11x - 6$
Write a Matlab/Python program to find one root of this equation using the Newton-Raphson Method.
Perform the following tasks:
   a) Define the function f(x) in Matlab/Python.
   b) Implement the Newton-Raphson Method to find a root starting from an initial guess.
   c) Accept user input for the initial guess, tolerance (error), and maximum number of iterations.
   d) Use the Newton-Raphson Method to find the root with the given tolerance or maximum number of iterations.
   e) Display the result along with the number of iterations taken.

[4] Consider the first-order ordinary differential equation (ODE):

$$\frac{dy}{dx} = -2xy$$

Write a Matlab/Python program to solve this ODE using the 4th order Runge-Kutta method.
Perform the following tasks:
   a) Define a Matlab/Python function for the ODE: $\frac{dy}{dx} = -2xy$.
   b) Implement the 4th order Runge-Kutta method to solve the ODE within a specified interval.
   c) Accept user input for the initial condition y0, the interval [a,b], step size h, and the number of steps.
   d) Use the Runge-Kutta method to solve the ODE and plot the solution curve.
   e) Display the resulting plot along with appropriate labels and title.

[5] Consider the following definite integral:

$$\int_a^b e^{-x^2} dx$$

Write a Matlab/Python program to approximate this integral using Simpson's 1/3 Rule.
Perform the following tasks:

a) Define a Matlab/Python function for the integrand: $f(x) = e - x^2$.
b) Implement the Simpson's 1/3 Rule to approximate the integral within a specified interval [a,b].
c) Accept user input for the interval [a,b] and the number of subintervals.
d) Use Simpson's 1/3 Rule to approximate the integral and display the result.
e) Additionally, compare the result with the actual value if known.

[6] Consider the following second-order linear boundary value problem:

$$y''(x) - 2y'(x) + y(x) = x, \qquad 0 < x < 1$$

with boundary conditions: $y(0) = 1, y(1) = 2$
Write a Matlab/Python program to solve this boundary value problem numerically using the Finite Difference Method.
Perform the following tasks:

a) Discretize the domain [0,1] into N+1 equally spaced points, where N is the number of internal nodes.
b) Formulate a system of linear equations based on the Finite Difference Method.
c) Implement a solver to solve the system of linear equations.
d) Display the numerical solution graphically.
e) Discuss the impact of changing the number of internal nodes on the accuracy of the solution.
f)

[7] Consider the following set of data points:

$$(x_0, y_0) = (1,2),\ (x_1, y_1) = (2,8),\ (x_2, y_2) = (3,18),\ (x_3, y_3) = (4,32)$$

Write a Matlab/Python program to perform Newton's Forward Interpolation to estimate the value of y at $x = 2.5$.
Perform the following tasks:

a) Implement the Newton Forward Interpolation method to construct the interpolation polynomial.
b) Use the interpolation polynomial to estimate y at x=2.5.
c) Display the coefficients of the interpolation polynomial.
d) Plot the given data points along with the interpolation polynomial.

[8] Consider the following set of data points:

$$(x_0, y_0) = (1,2),\ (x_1, y_1) = (2,8),\ (x_2, y_2) = (3,18),\ (x_3, y_3) = (4,32)$$

Write a Matlab/Python program to perform Lagrange Interpolation to estimate the value of y at x=2.5.
Perform the following tasks:

a) Implement the Lagrange Interpolation method to construct the interpolation polynomial.
b) Use the interpolation polynomial to estimate y at x=2.5.
c) Display the coefficients (Lagrange weights) of the interpolation polynomial.
d) Plot the given data points along with the interpolation polynomial.

[9] Solving Systems of Linear Equations: Given the system of linear equations:

$$3x + 2y - z = 1$$

$$2x - 2y + 4z = -2$$

$$-x + 0.5y - z = 0$$

a) Solve the system using **Gaussian elimination** and display the solution in tabular form in MATLAB.
b) Use MATLAB to solve the system using the inverse matrix method and compare results.
c) Represent the system graphically using a 3D plot (if applicable).
d) Discuss the error or difference (if any) between the methods.

[10] Matrix Inversion and Decomposition: Let

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix}$$

a) Compute the inverse of matrix A using MATLAB and verify the result by multiplying $A^{-1}A$. Display the results in tabular form.
b) Perform LU decomposition (without pivoting) of A, and verify the decomposition by reconstructing A.
c) Solve the system $Ax = b$ using LU decomposition where b=[3;12;6].
d) Generate a 3D plot for different values of x, y, z based on solutions obtained.

[11] Gaussian Elimination and Iterative Methods. Consider the system:

$$4x - y + z = 7$$

$$2x + 5y - 2z = -4$$

$$x - 3y + 6z = 6$$

a) Solve the system using Gaussian elimination and display the results in tabular form.
b) Implement the Gauss-Seidel method in MATLAB with an initial guess $x_0 = [0; 0; 0]$ and iterate until convergence (error tolerance 10^{-4}. Display results in a table.
c) Plot the error at each iteration using a convergence graph in MATLAB.
d) Compare the solutions obtained by Gaussian elimination and Gauss-Seidel and discuss their efficiency.

[12] Iterative Methods and Convergence Analysis. Given the coefficient matrix

$$A = \begin{bmatrix} 10 & -1 & 2 & 0 \\ -1 & 11 & -1 & 3 \\ 2 & -1 & 10 & -1 \\ 0 & 3 & -1 & 8 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 6 \\ 25 \\ -11 \\ 15 \end{bmatrix}$$

a) Solve the system using Jacobi iteration method in MATLAB. Display each iteration step in a table.
b) Solve the system using Gauss-Seidel iteration and compare convergence rates with Jacobi.
c) Plot the convergence of residual errors in both methods on the same graph.
d) Based on your results, analyze which method converges faster and explain why.