Digital Signal Processing

CHAPTER 1

Discrete Time Signals and Systems

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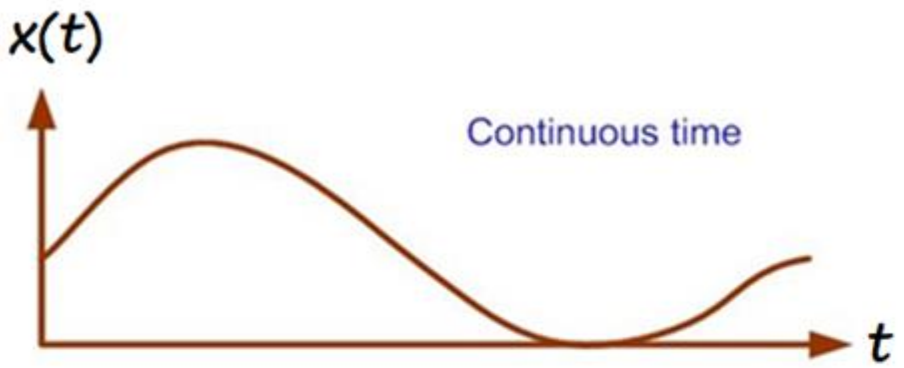
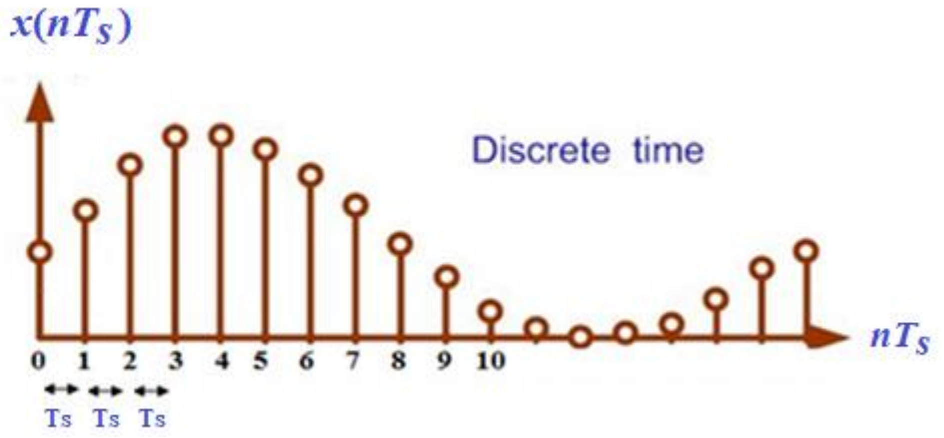
Discrete Time Signals

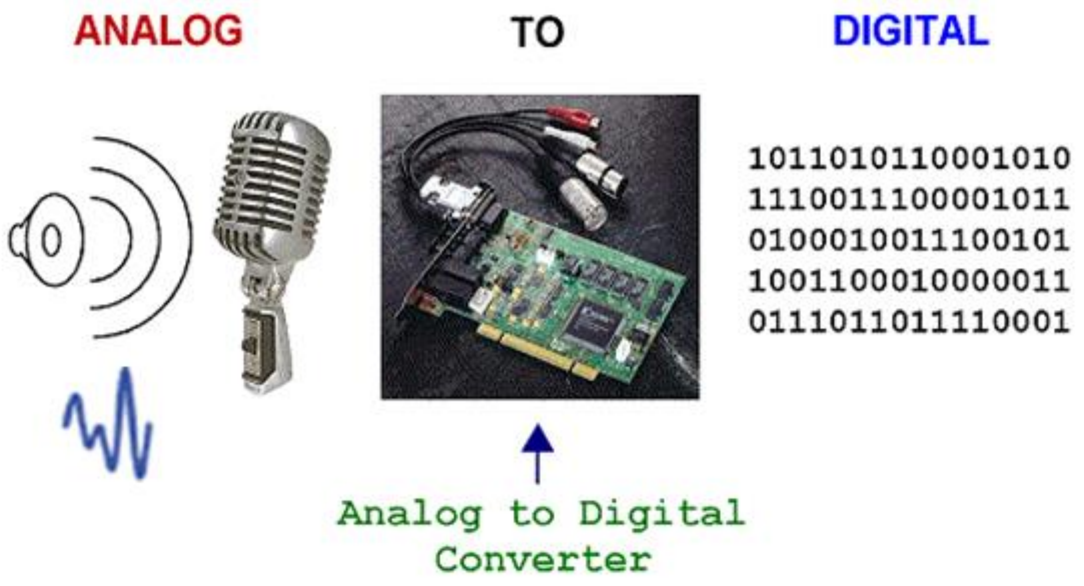
- In the Digital Signal Processing (DSP), the continuous time signal x(t) should be converted into a discrete form x[n] and then to a digital signal.
- The Discrete Time Signal is represented as sequence of values called "Samples".
- The index n in x[n] is called time index and it should be integer number.





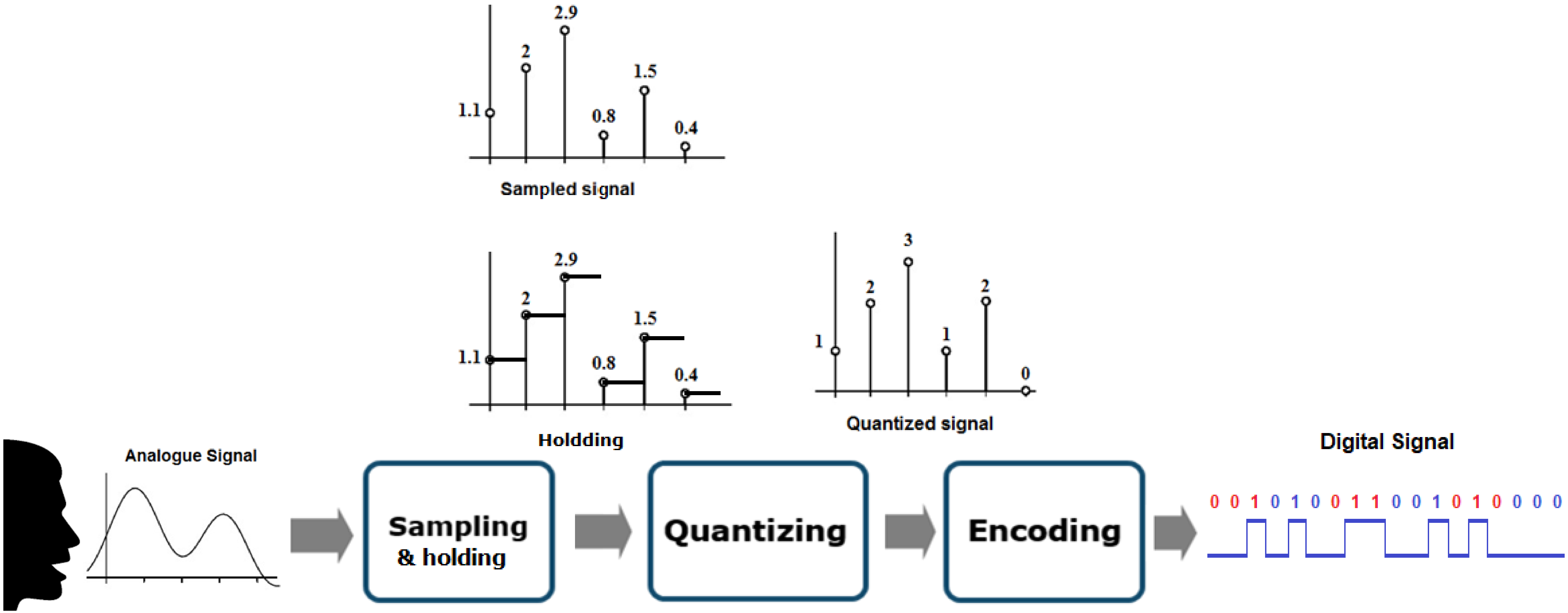
Analogue to Digital Conversion (ADC or A/D)

- It is a process that coverts Analogue Continuous signal x(t) to Digital codes. It is called "Digitization" as well.
- ADC Provides a link between the analogue world of transducers and the digital signal processing world.

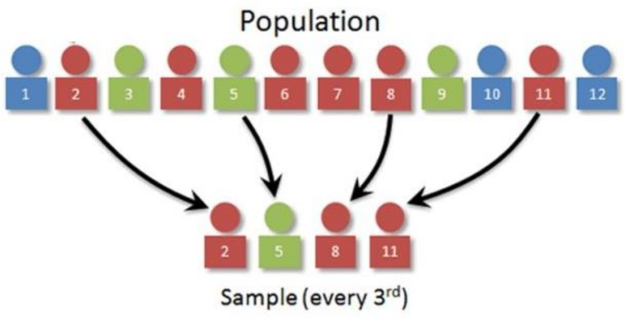


ADC can be done through 3 <u>successive</u> steps: (Digitization steps)

1. **Sampling:** Converting Continuous time function to Discrete (taking samples).
2. **Quantizing:** Converting the values of discrete time samples to the <u>nearest</u> available levels in the A/D.
3. **Encoding:** Assigning a unique digital code to each sample.



Sampling and Hold (S/ H):



- Sampling is the process of converting the continuous-time signal x(t) into discrete -time signal (sequence) $x(nT_s)$, shortly x(n).
- Taking a snapshot (sample) for x(t) at every T_s seconds.
- T_s is called sampling interval, i.e. the time interval between two adjacent samples.
- Preferring regularly spaced samples (constant T_s).
- f_s is called sampling frequency or sampling rate measured by (sample/sec) or Hz.

$$x(t) \xrightarrow{\quad sampling \quad} x(nT_s) \quad or \quad x(n)$$

Sampling Process Techniques:

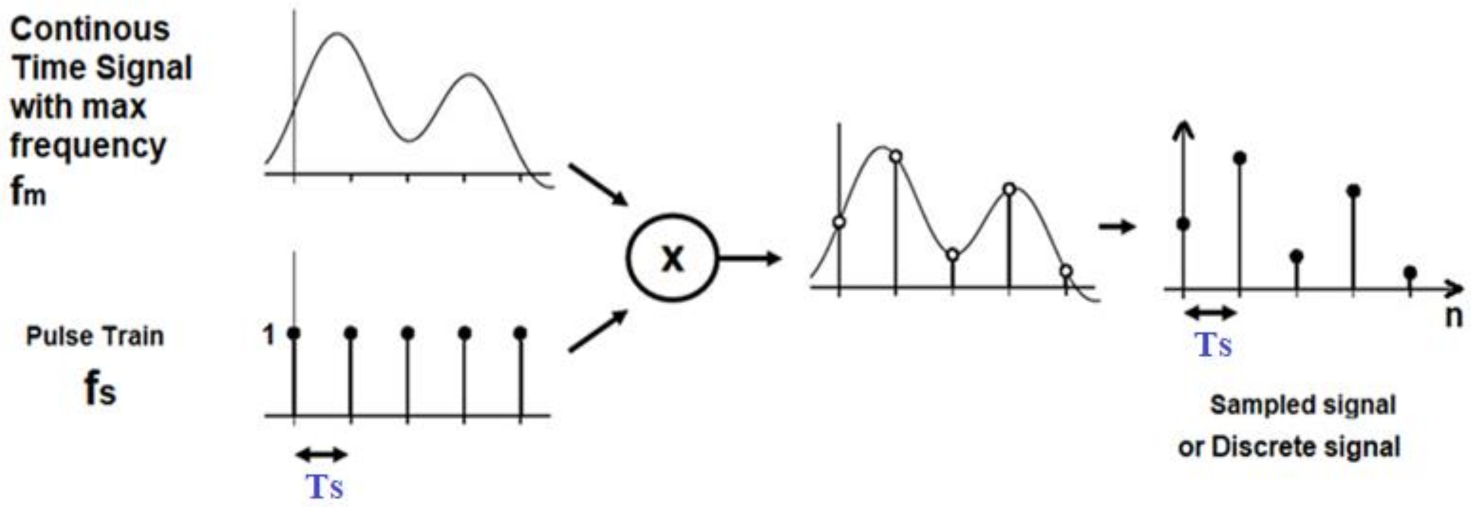
1. Multiplication Technique: Multiplying the continuous signal with pulse train.

$$\boxed{T_S = \frac{1}{f_s}}$$

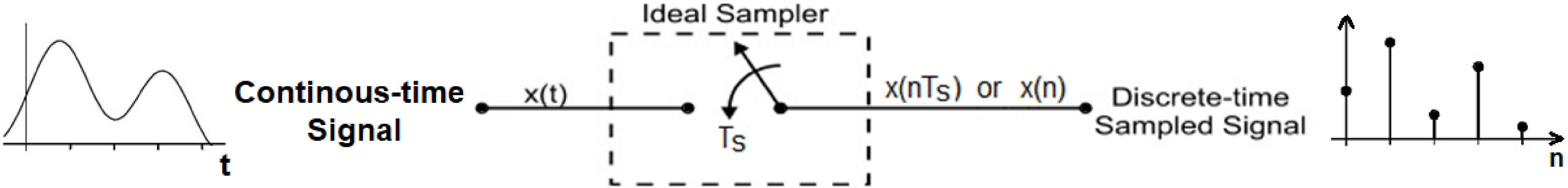
Where:

f_s: Sampling frequency, or (sampling rate), or Number of samples per second (Hz).

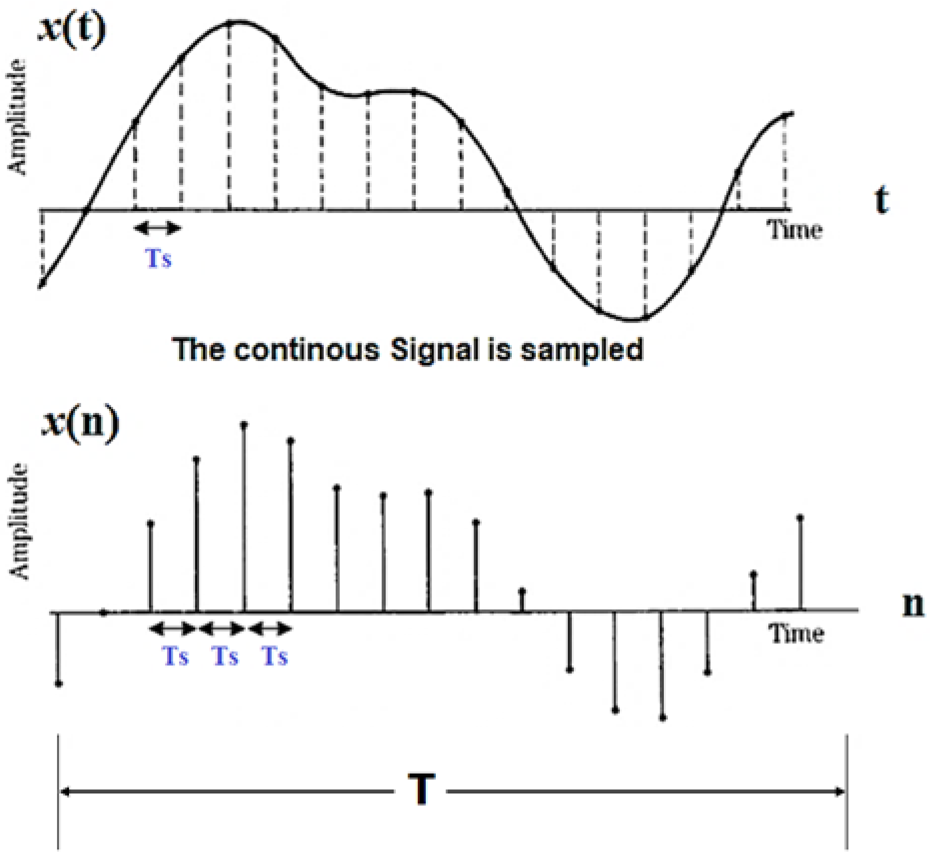
T_s: Sampling Time or sampling interval (sec)



2. Switching Technique:



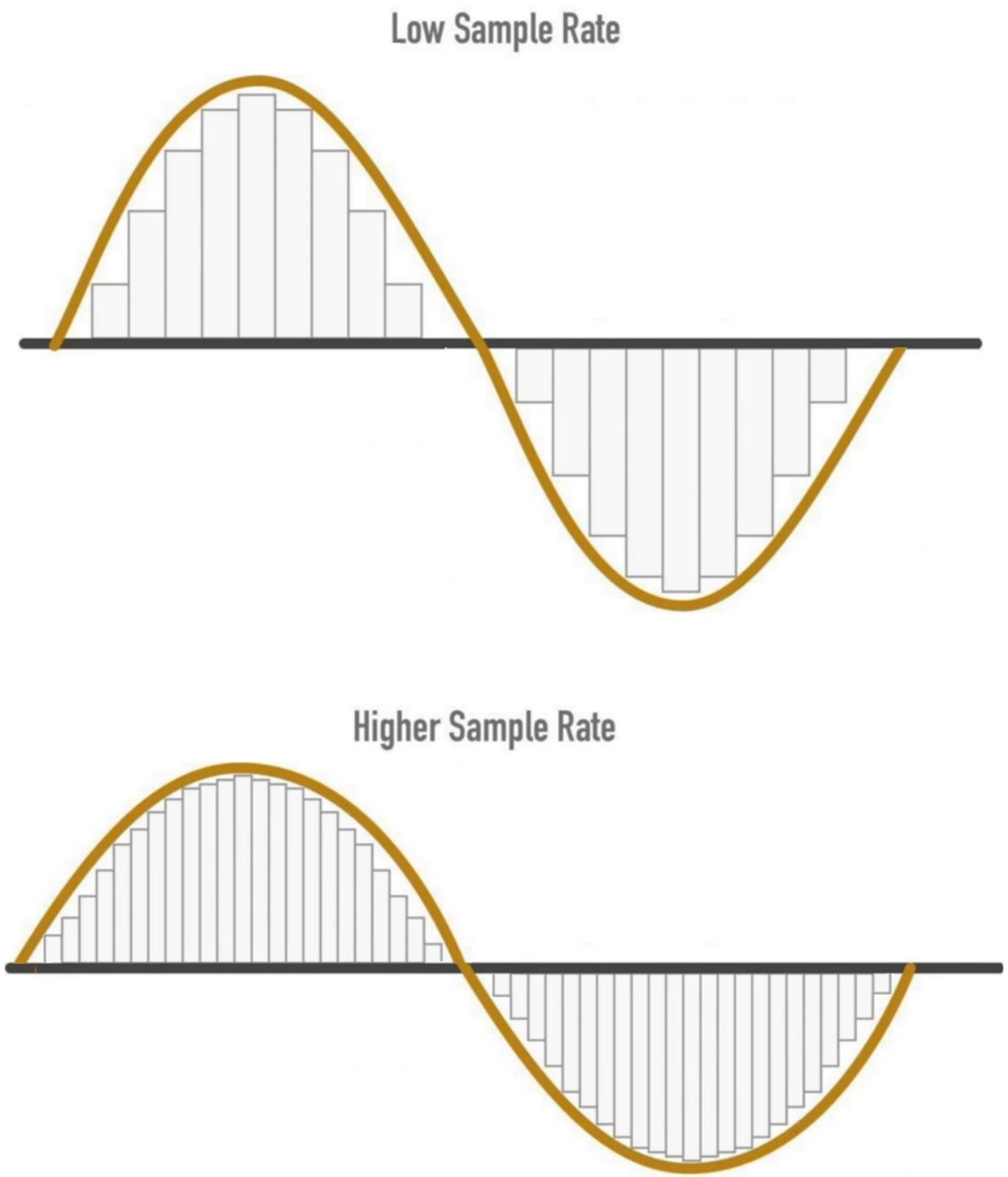
Sampling Process



For finite duration time signals $\boldsymbol{T = NT_s}$

where N: *the total number of samples for the finite duration discrete signal*

T: *the total time duration of the finite duration signal.*


Low Sample Rate
Higher Sample Rate

Nyquist Condition


Harry Nyquist
Swedish electronic engineer

- The Nyquist rate, named after Harry Nyquist, specifies a sampling rate fs. In units of samples per second its value is twice the highest frequency f_m in (Hz) of a signal to be sampled.
- With an equal or higher sampling rate, the resulting discrete-time sequence is said to be free of the distortion known as aliasing.

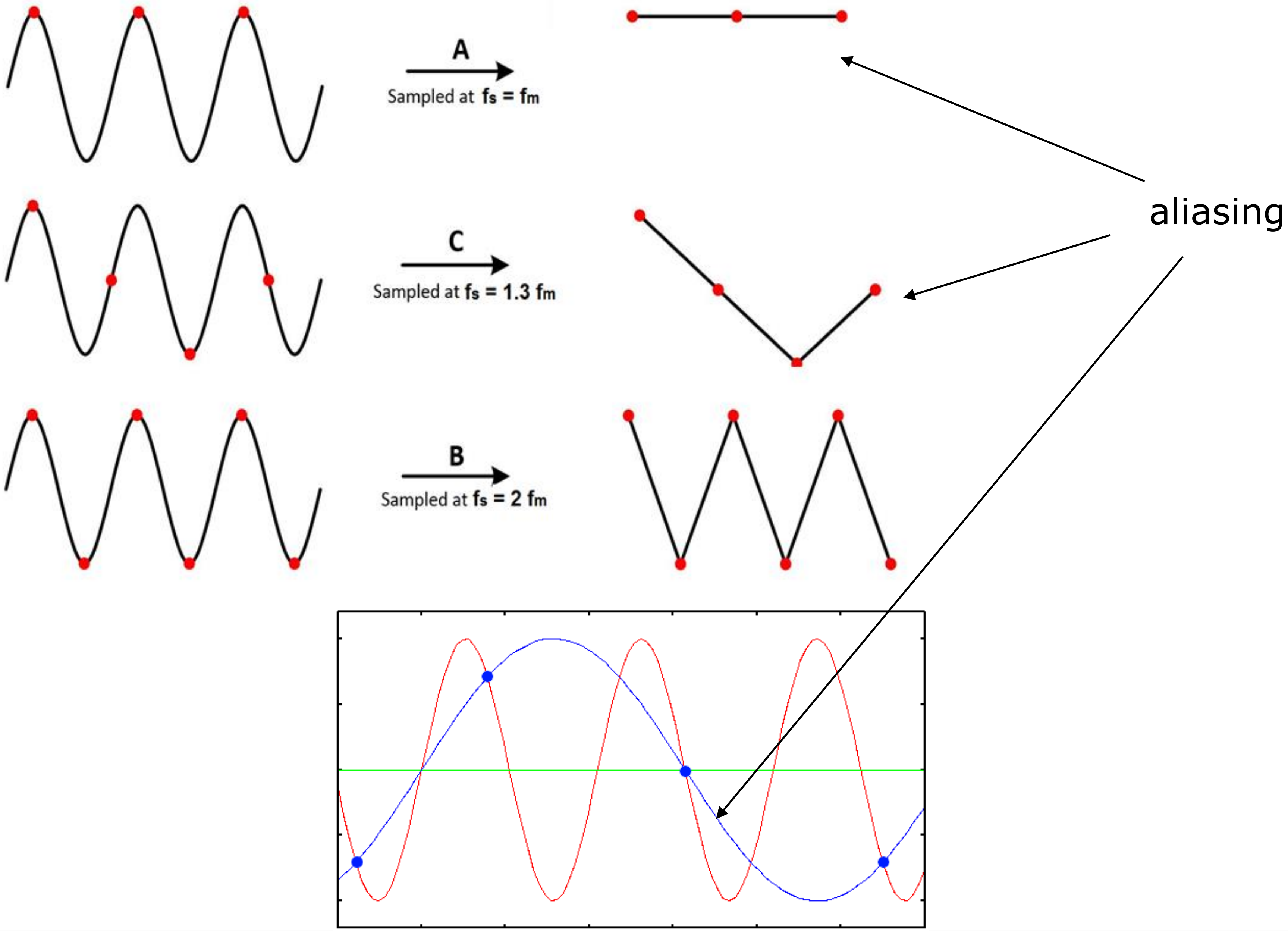
$$\boxed{f_s \geq 2f_m}$$ Nyquist's Condition

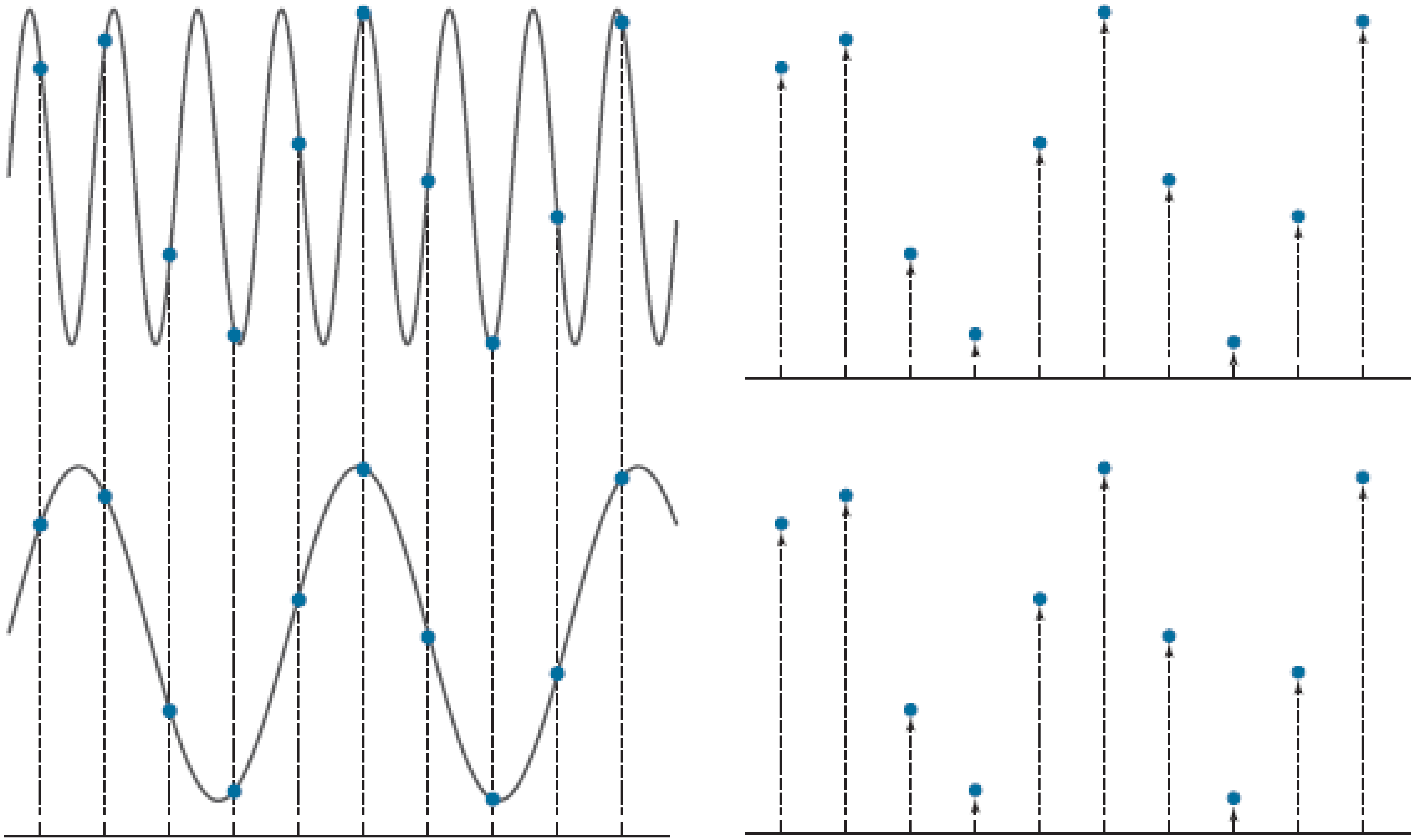
$$\boxed{or, \quad \frac{f_m}{f_s} \leq 0.5}$$

$f_m = f_{max}$ *maximum frequency component in the signal to be sampled*

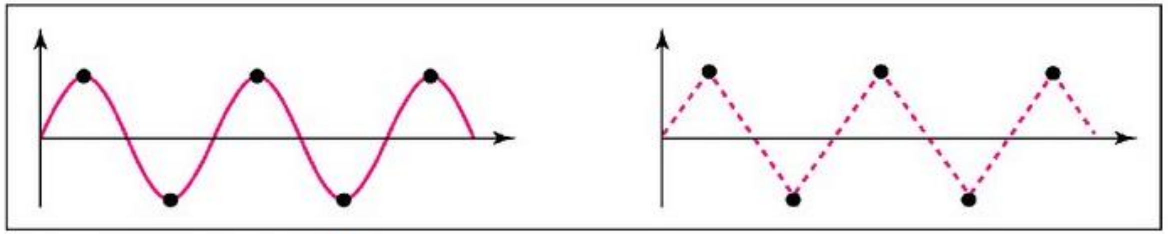
- The minimum acceptable sampling frequency ($2f_m$) value is called Nyquist Frequency or Nyquist Rate "F_N". Then: $\boxed{F_N = 2f_m}$

- Aliasing: is the generation of a false (alias) frequency along with the correct one when doing frequency sampling.


A
Sampled at fs = fm
C
Sampled at fs = 1.3 fm
B
Sampled at fs = 2 fm
aliasing

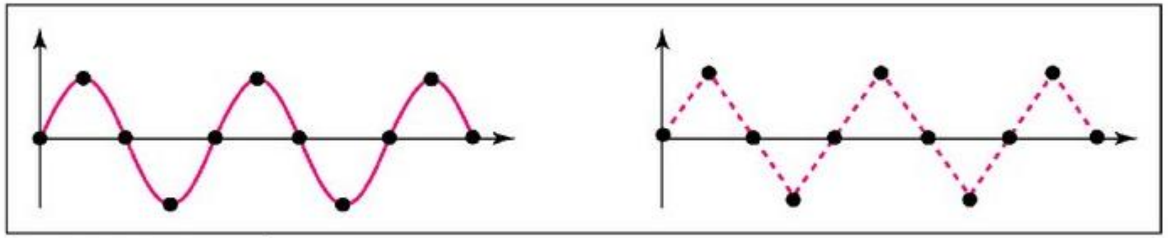


Recovery of a sampled sine wave for different sampling rates



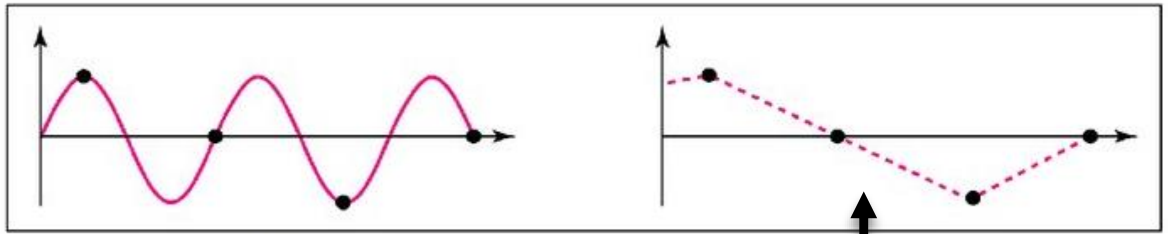
a. Nyquist rate sampling: $f_s = 2$ **fm**

Sampling at the Nyquist rate can create a good approximation of the original sine wave.



b. Oversampling: $f_s = 4$ **fm**

Oversampling can also create the same approximation, but is redundant and unnecessary.



c. Undersampling: $f_s =$ **fm**

Sampling below the Nyquist rate does not produce a signal that looks like the original sine wave.

aliasing

Example: Convert the continuous signal x(t) below to x(n), then write it in terms of frequency ratio $x(t) = cos(\omega t)$

$$x(t) \xrightarrow{\text{sampling}} x(nT_s) \rightarrow x(n)$$

$$x(nT_s) = cos(\omega nT_s)$$

$$x(nT_s) = cos(2\pi f nT_s)$$

$$x(n) = cos\left(2\pi f n \frac{1}{f_s}\right)$$

$$x(n) = cos\left(2\pi n \frac{f}{f_s}\right)$$

$$\frac{2f}{f_s} = S \ (\text{is called Sampling Ratio})$$

$$x(n) = cos\left(2\frac{f}{f_s}\pi n\right)$$

$$x(n) = cos(S\,\pi n)$$

for acceptable sampling without aliasing, $\frac{f}{f_s} \leq 0.5$

or, for acceptable sampling without aliasing, $\frac{2f}{f_s} \leq 1$ or $S \leq 1$

Example: Consider the following analogue signal: $x(t) = 3\cos 100\pi t$

1. Determine the minimum sampling frequency required (sampling rate) to avoid aliasing.
2. Convert the continuous signal x(t) to x(n) by using a suitable sampling rate.

Assume it

Sol:

$t \rightarrow nT_s$

1. $f_{max} = 50$ Hz, Then, $f_s \geq 2f_{max}$

   $f_s \geq 100\ Hz$ then, the minimum sampling frequency $F_N = 100$ Hz

S

2. Assume $f_s = 400$ Hz

$$x(nT_s) = 3cos(100\pi nT_s) = 3cos\left(100\pi n\frac{1}{f_s}\right) = 3cos\left(\pi n\frac{100}{400}\right) = 3cos(0.25\pi n)$$

Example: Consider the following analogue signal:

$$x(t) = 3\cos 100\pi t + \cos 400\pi t + 2\cos 300\pi t$$

1. Determine the sampling frequency required for ADC that avoids aliasing.

2. Convert the continuous signal x(t) to x(n) by using the selected f_s. (Inclass)

Sol:

$f_1 = 50$ Hz, $f_2 = 200$ Hz , $f_3 = 150$ Hz

f_{max} is the highest frequency = f_2 = 200 Hz

Then, $f_s\ must\ be\ \geq 2f_{max}$

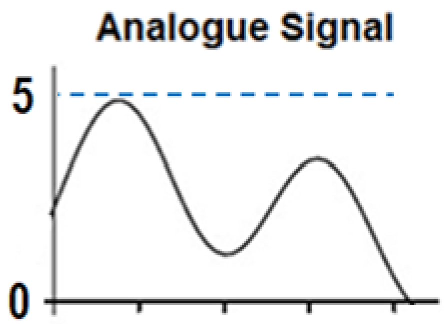
$f_s \geq 400\ Hz$

Quantization:

- Is the process of converting the values of discrete time samples to the nearest value from a finite set of possible values.
- Or, is the process of mapping a large set of input values to a (countable and available) smaller set.
- Rounding and truncation are typical examples of quantization processes.
- Partitioning the reference signal range into a number of discrete quanta (levels), then matching the input signal to the correct quantum.

k = number of bits in one digital code word (number of bits that represent one sample)

L = number of quantization levels, $L = 2^{k}$



If we assume a continuous signal should be represented by a dynamic range that varies from 0 to 5 volt, and then has to be digitized.

k =1 bit , L=2	
Voltage (Volt)	Digital code
0	0
5	1

k =2 bit , L= 4	
Voltage (Volt)	Digital code
0	00
1.66	01
3.333	10
5	11

Δ

$\Delta = \textit{Quantization Step (Resolution)},$

$$\Delta = \frac{\textit{Dynamic Range}}{L-1} = \frac{5}{4-1} = \frac{5}{3} = 1.66$$

k =3 bit , L= 8		Level
Voltage (volt)	Digital code	
0	000	0
0.714	001	1
1.43	010	2
2.14	011	3
2.86	100	4
3.57	101	5
4.29	110	6
5.00	111	7

Δ

Δ

$\Delta = \mathit{Quantization\ Step}\ (\mathit{Resolution}),$

$$\Delta = \frac{\mathit{Dynamic\ Range}}{L-1} = \frac{5}{8-1} = \frac{5}{7} = 0.714$$

k=4 bit , L=16	
Voltage volt	Digital code
0	0000
0.333333	0001
0.666666	0010
0.999999	0011
1.333332	0100
1.666665	0101
1.999998	0110
2.333331	0111
2.666664	1000
2.999997	1001
3.33333	1010
3.666663	1011
3.999996	1100
4.333329	1101
4.666662	1110
5	1111

$$\Delta = \frac{Dynamic\ Range}{L-1} = \frac{5}{15} = 0.3333$$

k = 5 bit , L = 32	
Voltage volt	Digital code
0	00000
0.1613	00001
0.3226	00010
0.4839	00011
0.6452	00100
0.8065	00101
0.9678	00110
1.1291	00111
1.2904	01000
1.4517	01001
1.613	01010
1.7743	01011
1.9356	01100
2.0969	01101
2.2582	01110
2.4195	01111
2.5808	10000
2.7421	10001
2.9034	10010
3.0647	10011
3.226	10100
3.3873	10101
3.5486	10110
3.7099	10111
3.8712	11000
4.0325	11001
4.1938	11010
4.3551	11011
4.5164	11100
4.6777	11101
4.839	11110
5	11111

$$\Delta = \frac{Dynamic\ Range}{L-1} = \frac{5}{31} = 0.1613$$

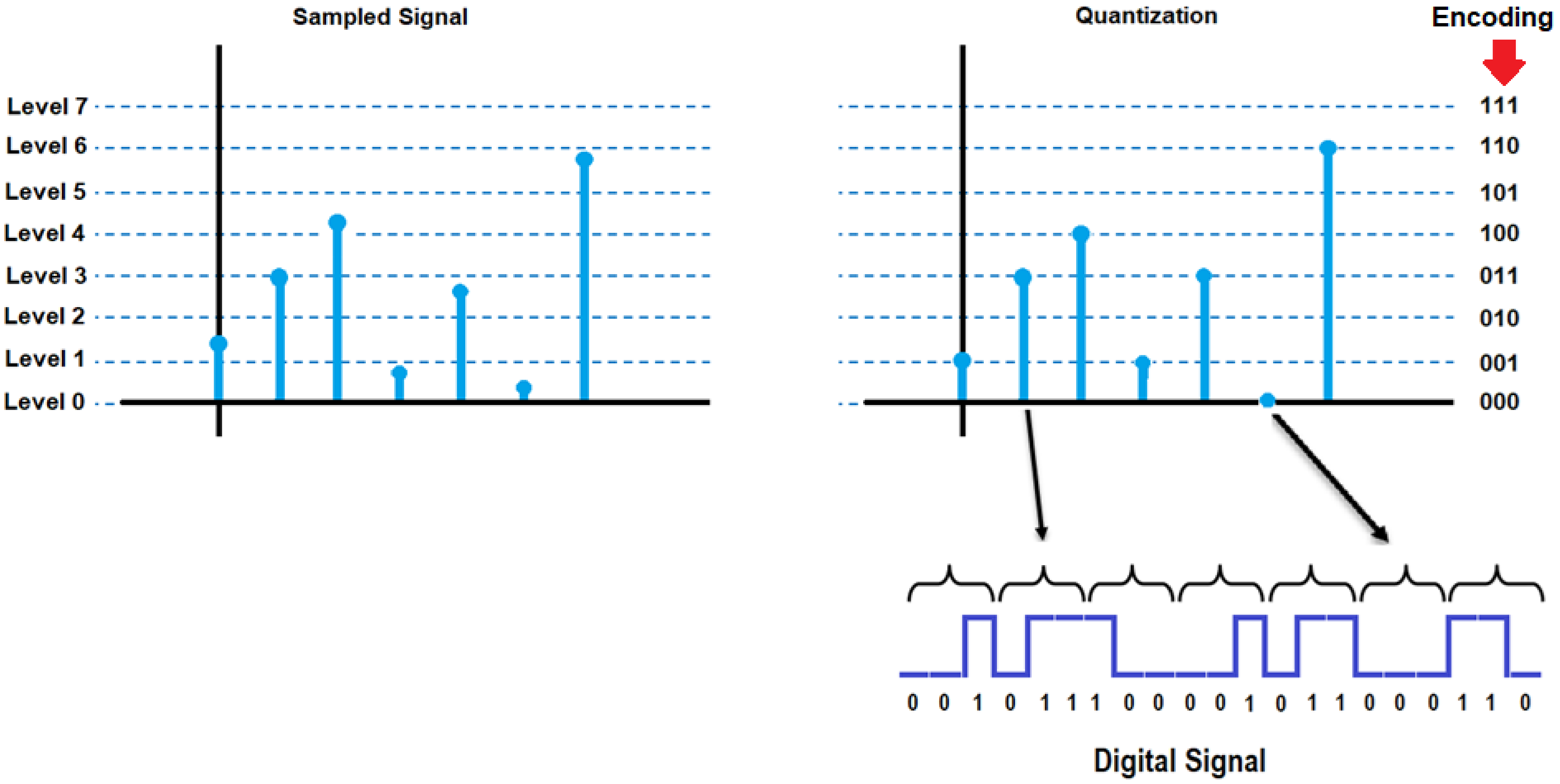
$$\Delta = \frac{\boldsymbol{Dynamic\ Range}}{\boldsymbol{L-1}} = \frac{\mathbf{5}}{\mathbf{1023}} = \mathbf{0.00488}$$

$$\boldsymbol{Max.\,Quantization\ error\ (MQ_e) = \pm\frac{\Delta}{2}}$$

n =10 bit , L=1024		
Voltage volt	Digital code	Level
0	**0000000000**	**0**
0.00488	**0000000001**	**1**
0.00977	**0000000010**	**2**
0.01465	**0000000011**	**3**
4.99024	**1111111101**	**1021**
4.99512	**1111111110**	**1022**
5	**1111111111**	**1023**

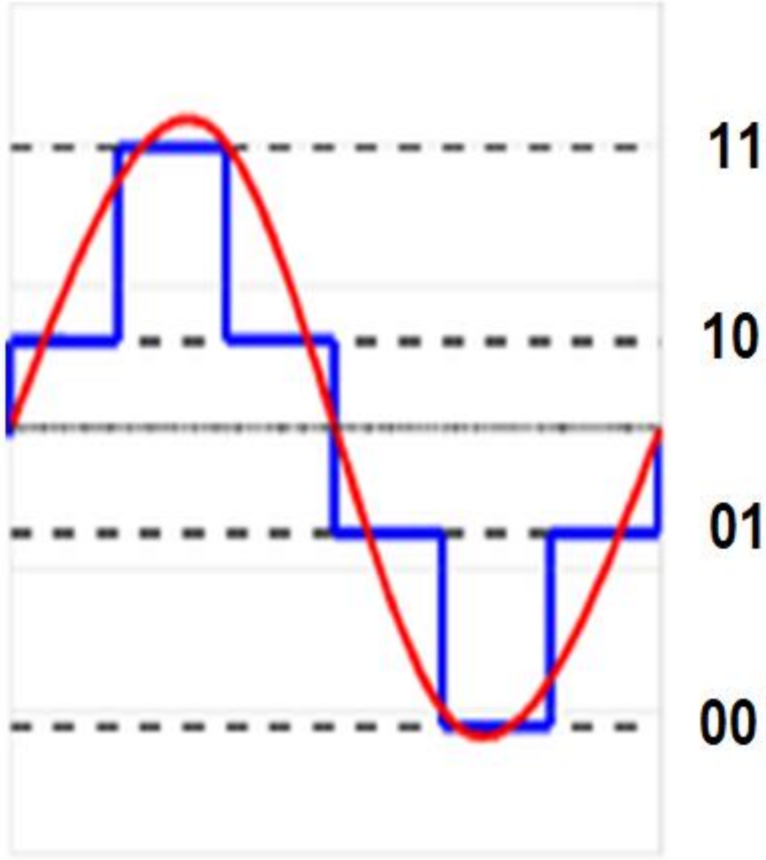
Digitization (ADC)

Sampling , Quantization and Encoding using 3 bit ADC

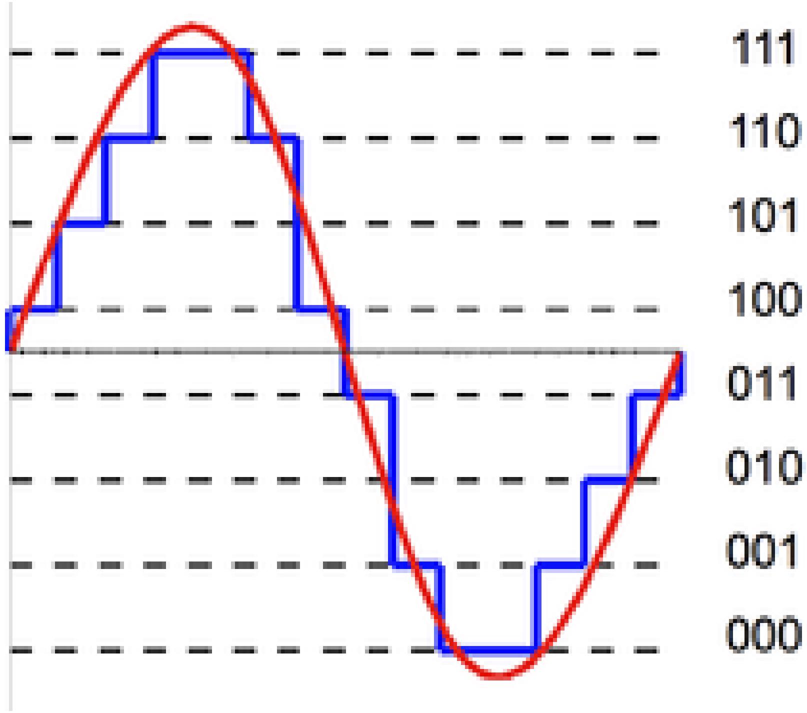


Encoding:

Assigning a unique digital code to each quantum, then allocating the digital code to the input signal. different coding systems can be used.

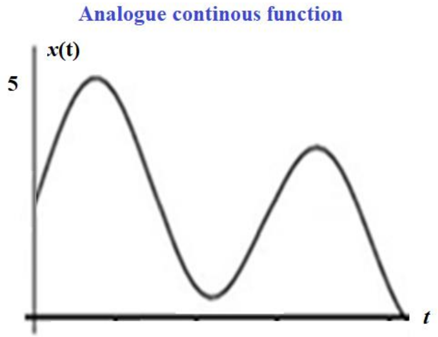


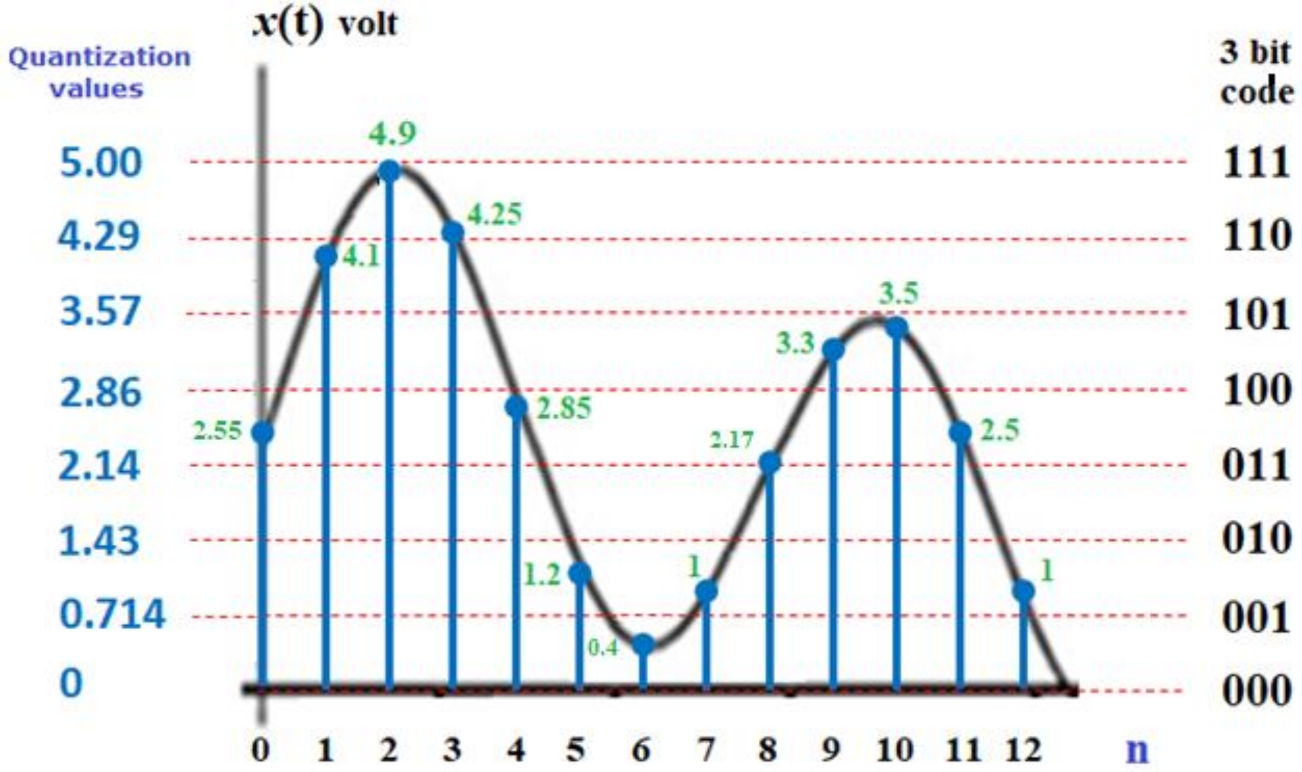
2-bit resolution with **Four** levels



3-bit resolution with **Eight** levels.

Example: 3 bit ADC → L= 8 quantization levels → $\Delta = 0.714$

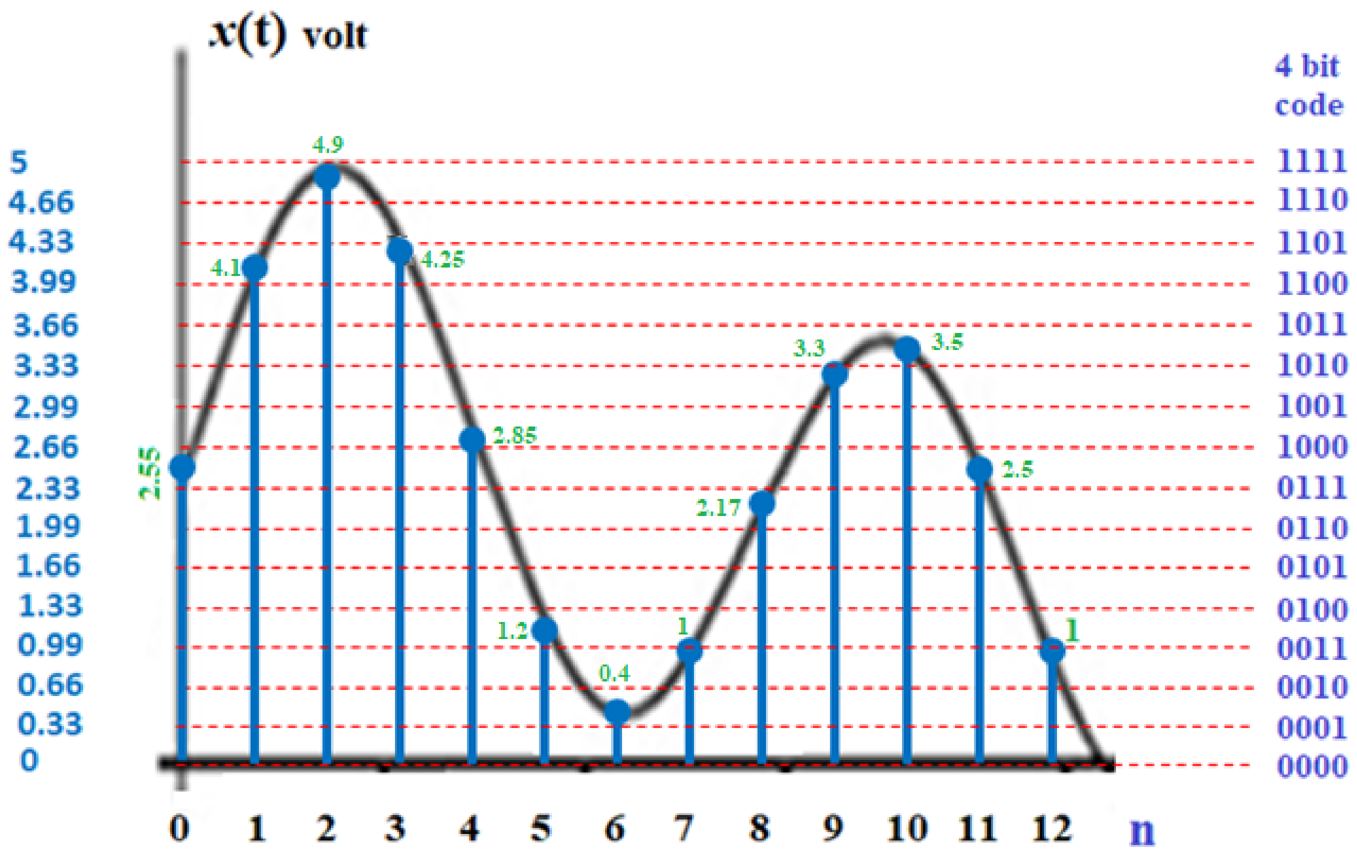




x(n)	Sample value	Assigned code	Error (Qe)
x(0)	2.55	100	0.31
x(1)	4.1	110	0.19
x(2)	4.9	111	0.1
x(3)	4.25	110	0.04
x(4)	2.85	100	0.01
x(5)	1.2	010	0.23
x(6)	0.4	001	0.314
x(7)	1	001	0.286
x(8)	2.17	011	0.03
x(9)	3.3	101	0.27
x(10)	3.5	101	0.07
x(11)	2.5	100	0.36
x(12)	1	001	0286

Sampling , Quantization and Encoding using 4 bit ADC

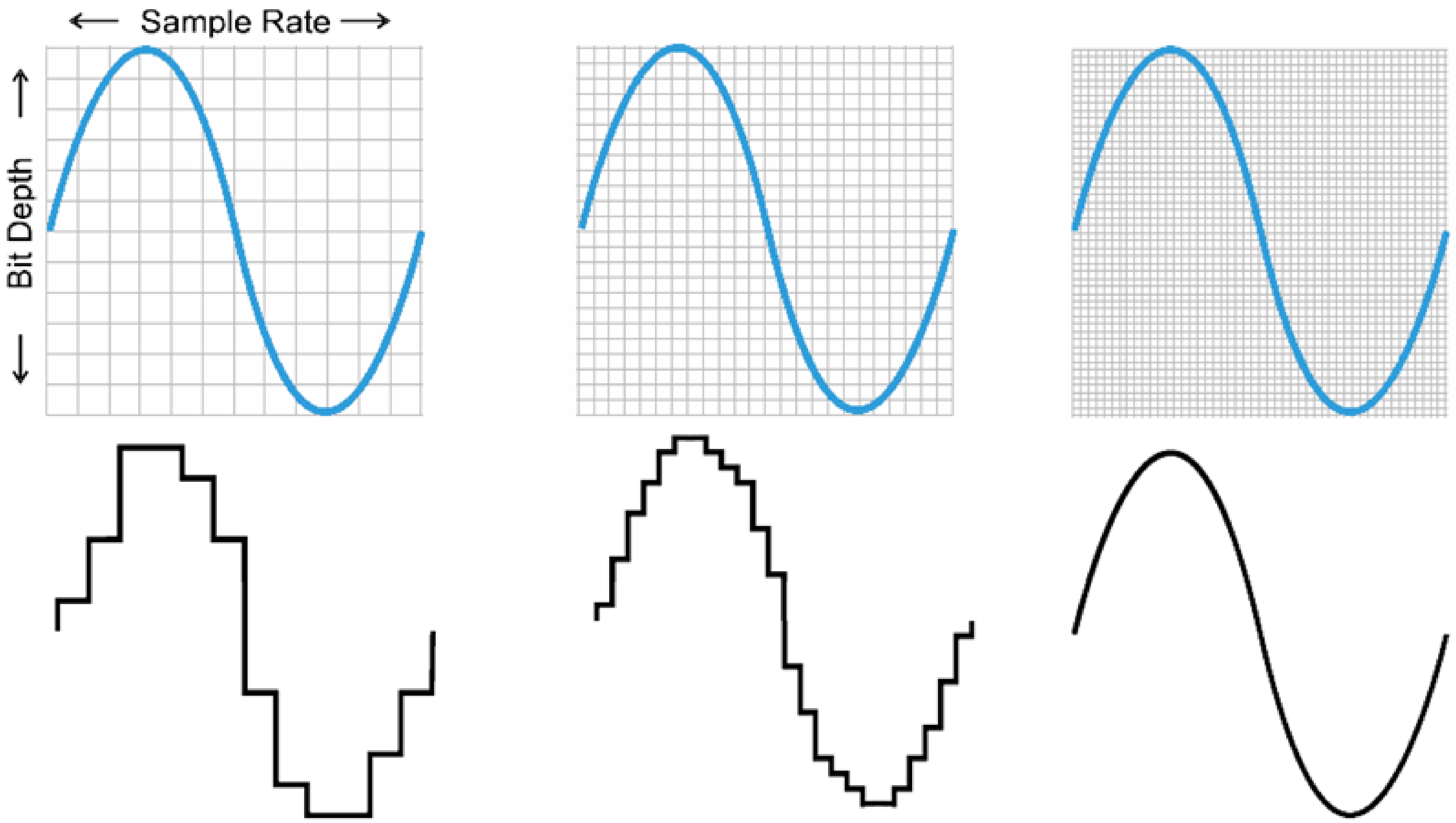
Example: 4 bit ADC → L= 16 quantization levels → $\Delta = 0.333$



x(n)	Actual Sample value	Assigned code	Error (Qe)
x(0)	2.55	1000	0.11
x(1)	4.1	1100	0.23
x(2)	4.9	1111	0.1
x(3)	4.25	1101	0.08
x(4)	2.85	1000	0.19
x(5)	1.2	0100	0.13
x(6)	0.4	0001	0.07
x(7)	1	0011	0.01
x(8)	2.17	0111	0.16
x(9)	3.3	1010	0.03
x(10)	3.5	1010	0.17
x(11)	2.5	1000	0.16
x(12)	1	0011	0.01

2.55 → 2.66 , then the error here = 0.11

1 → 0.999 , then the error here = 0.001



- Lower resolution (higher value of Δ)
- Lower fs

- Higher resolution (lower value of Δ)
- Higher fs

Bit Rate (R):

- It is the number of bits that are conveyed or processed per unit of time.
- The bit rate is expressed in bit per second unit (bit/s or bps), often in conjunction with an SI prefix such as kilo (1 kbit/s = 1000 bit/s), 1 Mega (1 Mbit/s = 1000 kbit/s),

$$\boxed{R = f_s \times k} \qquad bit/sec$$

Example: Determine the Bit Rate, resolution and max quantization error for ADC of continuous time signal with a dynamic range of 8 volt and the sampling frequency is 2 kHz (sample/sec) using 3 bit ADC.

sol:

Bit Rate (R)= f_s x k = 2000 x 3 = 6000 bit/sec

$$\boldsymbol{Resolution\ (\Delta) = \frac{Dynamic\ Range}{L-1} = \frac{8}{8-1} = 1.142\ volt\quad (quantization\ step)}$$

$$\boldsymbol{Max.Quantization\ error\ (MQ_e) = \pm\frac{\Delta}{2} = \pm\frac{1.142}{2} = \pm 0.571\ volt}$$

Example: a discrete signal x(n)=3 cos(0.1πn),

a. How many bits per sample were required in the A/D with a dynamic range of 6 volt. The continuous signal is quantized with resolutions 0.1
b. Is there any aliasing case? (Is the sampling process satisfying Nyquist Condition?)
c. What is the sampling rate that is used during the A/D if x(t) has frequency = 400 Hz.

Sol: a. Dynamic range = 6, For Δ =0.1

$$\Delta = \frac{\boldsymbol{Dynamic\ Range}}{\boldsymbol{L-1}} = \frac{\boldsymbol{6}}{\boldsymbol{L-1}} = \boldsymbol{0.1\ volt}$$

$$\therefore \quad L = 61$$

$$But \quad L = 2^k$$

$$logL = log2^k$$

$$logL = klog2$$

$$k = \frac{log\ L}{log2} = \frac{log61}{log2} = 5.9 \ \cong 6\ bit\ per\ sample$$

HW: Repeat for Δ =0.02

b. x(n)=3 cos(**0.1**πn)

S =0.1 < 1 ,

then no aliasing occured

c.

$$S = \frac{2\ f}{fs} = \frac{2x400}{fs} = \frac{800}{fs} = 0.1$$

fs=8 kHz

Example: **Consider the continuous time signal $x(t) = 4cos(100\pi t)$**

a. Determine the minimum sampling rate required to avoid aliasing.
b. Suppose that the signal is sampled at the rate f_s=300 Hz. What is the discrete time signal x(n) obtained after the sampling process?
c. Suppose that the signal is sampled at f_s=75 Hz. What is the discrete time signal x(n) obtained after the sampling process?
d. Find the sampling ratio in (b) and (c) and discuss the aliasing status for both cases.

Sol:

a. $f = 50\ Hz = f_{max}$, **then,** $f_s \geq 2f_{max}$ $then\ F_s = 2f_{max} = 100\ Hz$

b. $x(n) = 4cos(100\pi n T_s) = 4cos\left(100\pi n \frac{1}{f_s}\right) = 4cos\left(\pi n \frac{100}{300}\right) = 4cos(0.333\pi n)$

c. $x(n) = 4cos(100\pi n T_s) = 4cos\left(100\pi n \frac{1}{f_s}\right) = 4cos\left(\pi n \frac{100}{75}\right) = 4cos(1.333\pi n)$

d. In (b), S=0.333 < 1 , then no aliasing occurs.
In (c), S=1.333 > 1 , then an aliasing occurs

Example: **If x(t)=6 sin(5000t) , find its discrete version without aliasing.**

Sol:

$$\omega = 5000 = 2\pi f \;\; \rightarrow \;\; f = \frac{\omega}{2\pi} = \frac{5000}{2\pi} = 795.7\ Hz \;\cong 796\ Hz = \; f_m$$

Then to do sampling without aliasing, f_s must be ≥ $2f_m$,   i.e. $f_s \geq 1592$

Choosing f_s = 3000 Hz

$$x(n) = 6sin(2\pi 796 nT_s) = 6sin\left(2\pi 796 n\frac{1}{f_s}\right) = 6sin\left(2\pi 796 n\frac{1}{3000}\right) = 6sin(0.53\pi n)$$

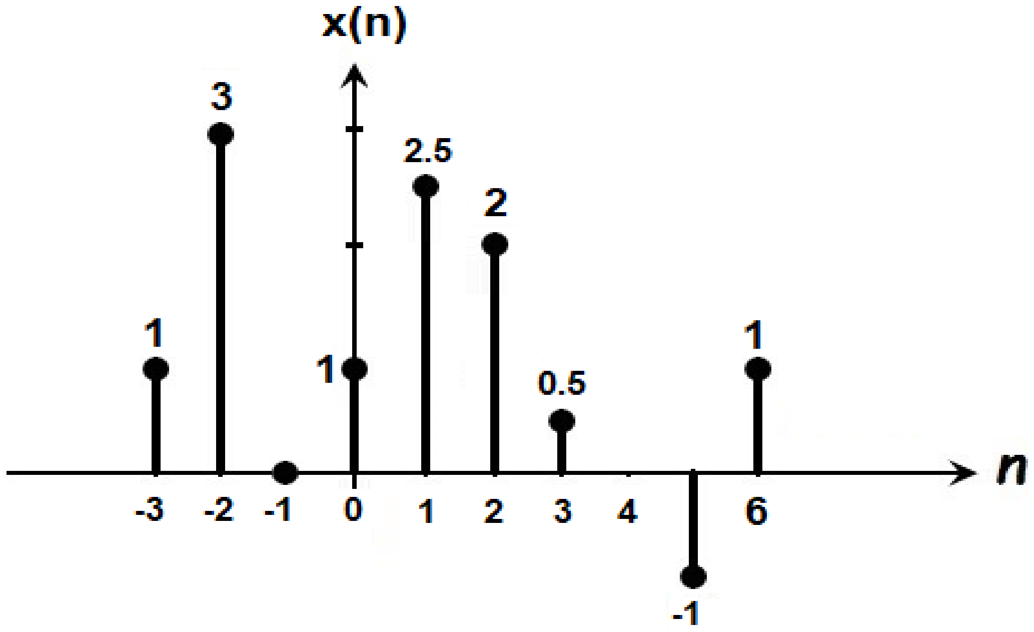
Inclass Problem: **Which of the following discrete time signals satisfies the Nyquist's Condition?**

$a.\;\; \cos(0.01\pi n)$ $b.\;\; \cos(3\pi n)$ $c.\;\; \cos(2n)$

Inclass Problem: **Determine the resolution and the bit rate of a sampling process that uses 8 bit A/D converter and the sampling rate is 20 sample/sec to sample the continuous time signal. The dynamic range is 12 volt.**

Time domain analysis of discrete time signals:

The discrete time signal x[n] shown can be written as the following sequence styles, the arrow ↑ under the sample value that is at n= 0 i.e. at x[0].



$$x[n] = [\ 1\ ,\ 3\ ,\ 0\ ,\ \underset{\uparrow}{1}\ ,\ 2.5\ ,\ 2\ ,\ 0.5\ ,\ 0\ ,\ -1\ ,\ 1\ ]$$, or

$$x[n] = [\ 1\ ,\ 3\ ,\ 0\ ,\ 1\ ,\ 2.5\ ,\ 2\ ,\ 0.5\ ,\ 0\ ,\ -1\ ,\ 1\ ]$$ defined in $-3 \le n \le 6$

$x[-3]= 1$, $x[-1]= 0$, $x[3]= 0.5$, $x[4]= 0$, $x[5]= -1$ $x[8]=0$.... etc

➢ The basic discrete signals can be reviewed in CH1 of Eng. Analysis

Drawing the discret time signals:

Example: Draw (sketch) the following discrete time signals:

(a) $x(n) = e^{0.2n}$

(b) $x(n) = e^{0.2n}\, u(n)$ Time Window

(c) $x(n) = \cos\dfrac{\pi n}{4}$

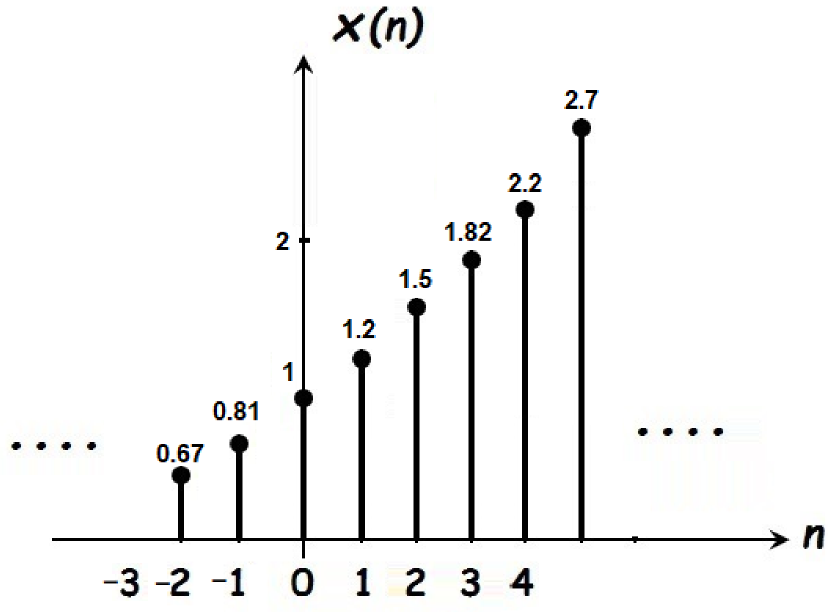
(d) $x(n) = e^{-\frac{n}{5}} \cos n$

Time Window

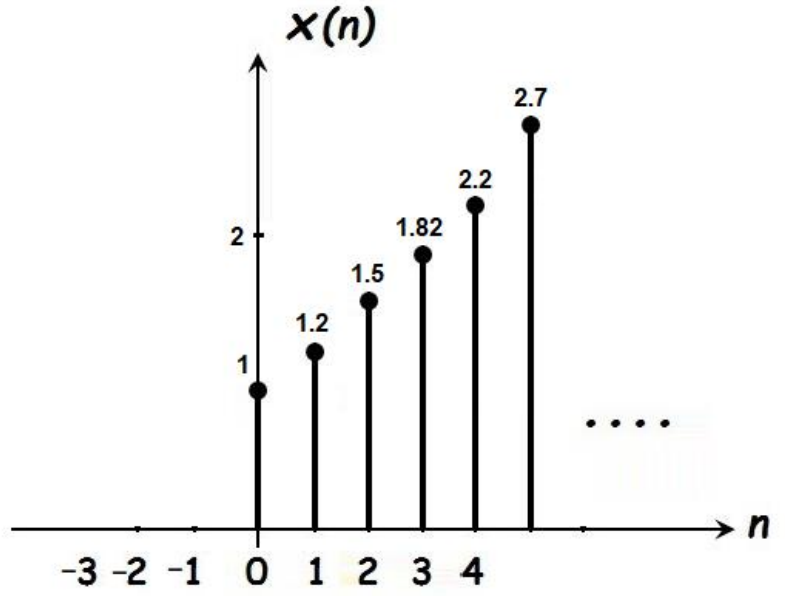
(e) $x(n) = 20(0.9)^n\, \overbrace{[u(n+2) - u(n-4)]}$

$(a) \quad x(n) = e^{0.2n}$

n	$x(n)$
-2	0.67
-1	0.81
0	1
1	1.2
2	1.5
3	1.82
4	2.22
5	2.7
⋮	

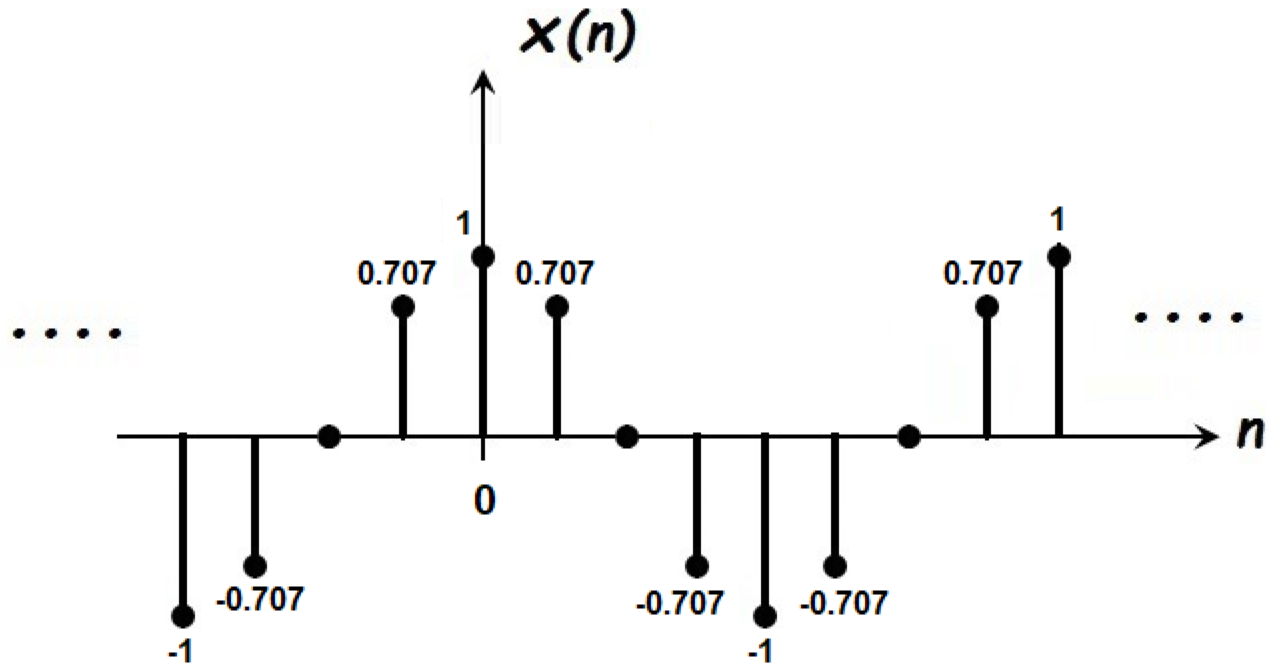


$(b) \quad x(n) = e^{0.2n}\, u(n)$



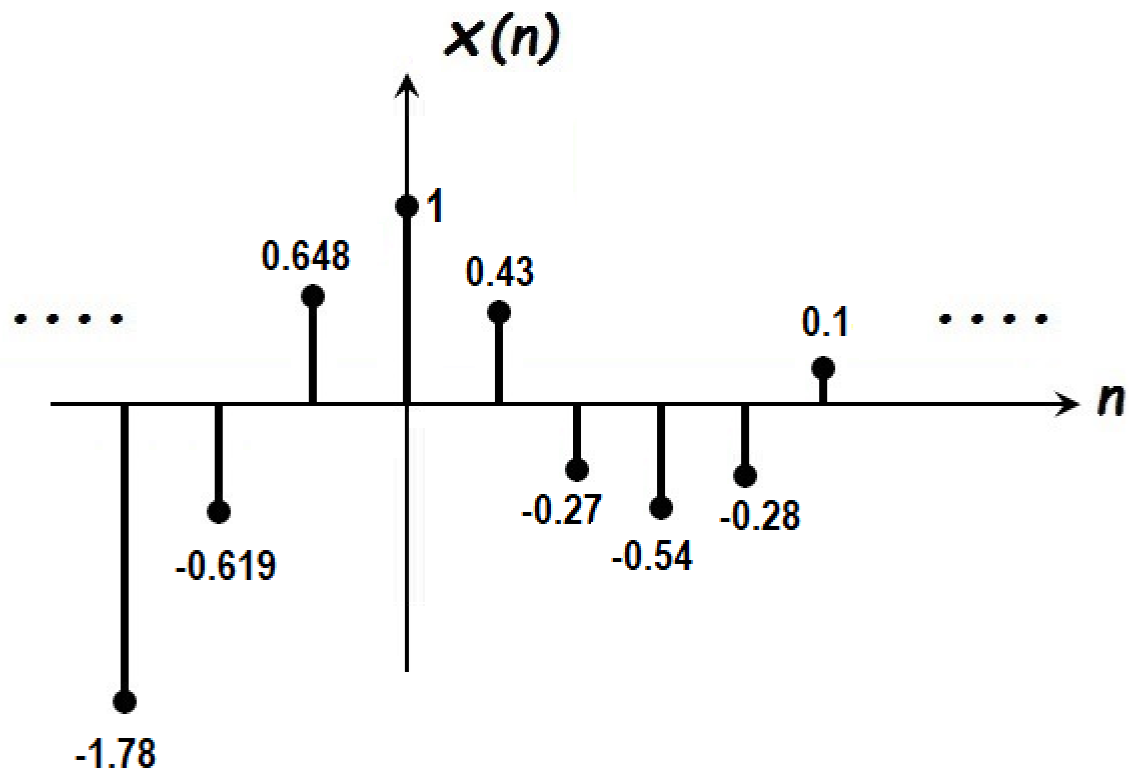
$$(c) \quad x(n) = \cos\frac{\pi n}{4}$$

n	$x(n)$
-4	-1
-3	-0.707
-2	0
-1	0.707
0	1
1	0.707
2	0
3	-0.707
4	-1
5	-0.707
⋮	



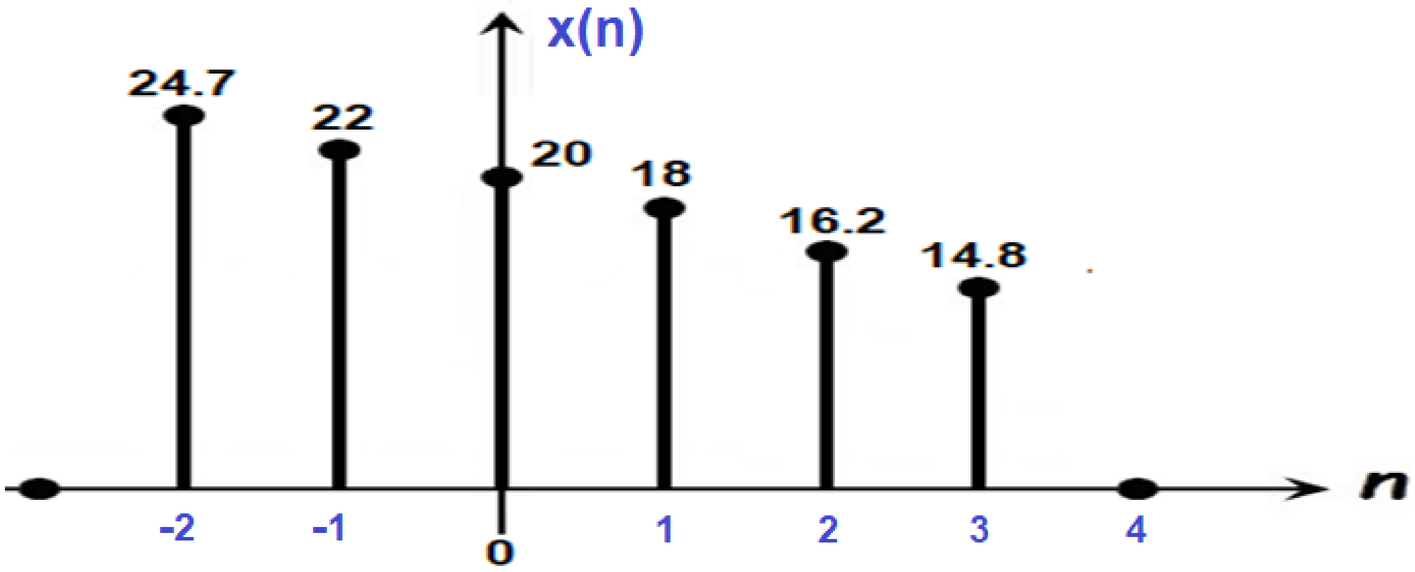
$(d) \quad x(n) = e^{-\frac{n}{5}} \cos n$

n	$e^{-0.2n}$	$\cos n$	$x(n)$
-3	1.8	-0.99	-1.78
-2	1.49	-0.416	-0.61984
-1	1.2	0.54	0.648
0	1	1	1
1	0.81	0.54	0.4374
2	0.67	-0.416	-0.27872
3	0.54	-1	-0.54
4	0.44	-0.65	-0.286
5	0.36	0.28	0.1008
⋮			



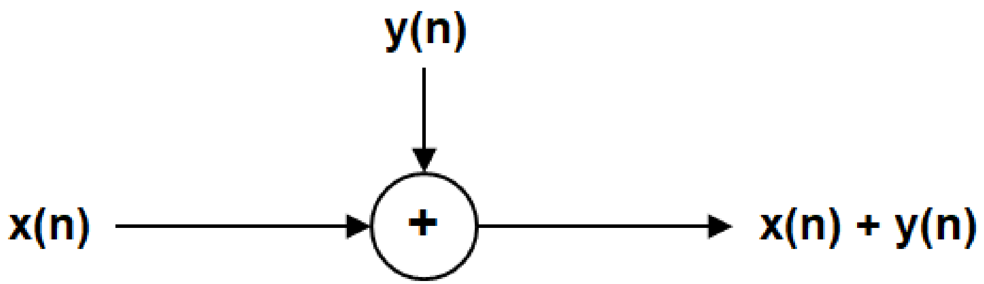
$$(e)\quad x(n) = 20(0.9)^n\ \underbrace{[u(n+2) - u(n-4)]}_{\text{Time window}}$$

n	…	-4	-3	-2	-1	0	1	2	3	4	5	…
$20(0.9)^n$		30.48	27.43	24.7	22	20	18	16.2	14.8	13.122	11.8	
window	0	0	0	1	1	1	1	1	1	0	0	0
x(n)	0	0	0	24.7	22	20	18	16.2	14.8	0	0	0

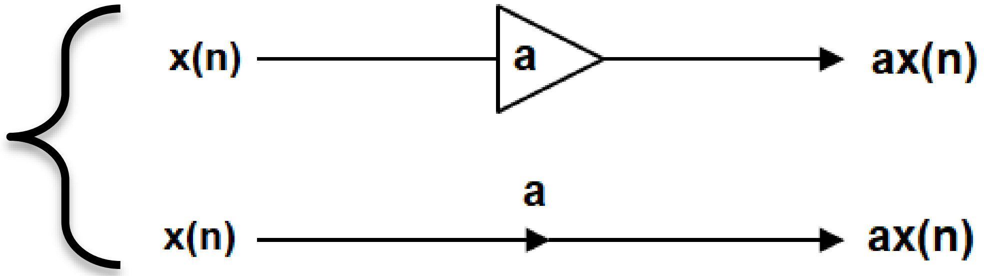


Three elementary operations:

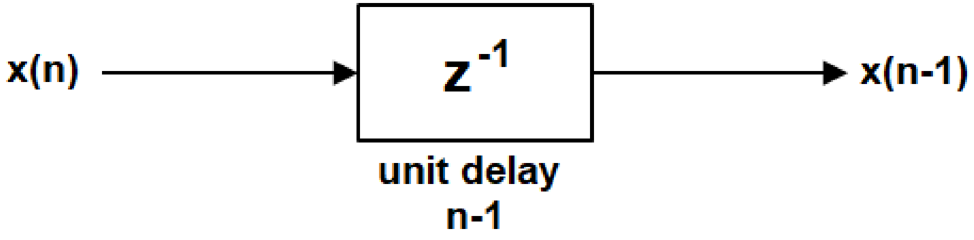
Addition



Scalar Multiplication



Unit Delay



Example: The following two sequences are of length 5 defined over $0 \le n \le 4$

$$x[n] = [\ 1\ ,\ 2.5\ ,\ 0\ ,\ 3\ ,\ -4]$$

$$g[n] = [\ -2\ ,\ 0\ ,\ 2\ ,\ 1.5\ ,\ -2]$$

Find the following:

$$h_1(n) = x(n)\ .\ g(n) = [\ -2\ ,\ 0\ ,\ 0\ ,\ 4.5\ ,\ 8\ ]$$

$$h_2(n) = x(n) + g(n) = [\ -1\ ,\ 2.5\ ,\ 2\ ,\ 4.5\ ,\ -6]$$

$$h_3(n) = 3x(n) = [\ 3\ ,\ 7.5\ ,\ 0\ ,\ 9\ ,\ -12]$$

$$h_4(n) = 3x(n) - g(n) = [\ 5\ ,\ 7.5\ ,\ -2\ ,\ 7.5\ ,\ -14\ ]$$

If $y(n) = [\ 2\ ,\ -1\ ,\ 3\ ]$ define over $-1 \leq n \leq 1$

Find $h_5(n) = y(n) + g(n)$

$$g[n] = [\ -2\ ,\ 0\ ,\ 2\ ,\ 1.5\ ,\ -2]$$
$$\uparrow$$

$$y(n) = [\ 2\ ,\ -1\ ,\ 3\ ]$$
$$\uparrow$$

Since both sequences are not with same lengths, then they should be with same length. This can be done by adding zeros in the suitable locations, this is called " Zero Padding "

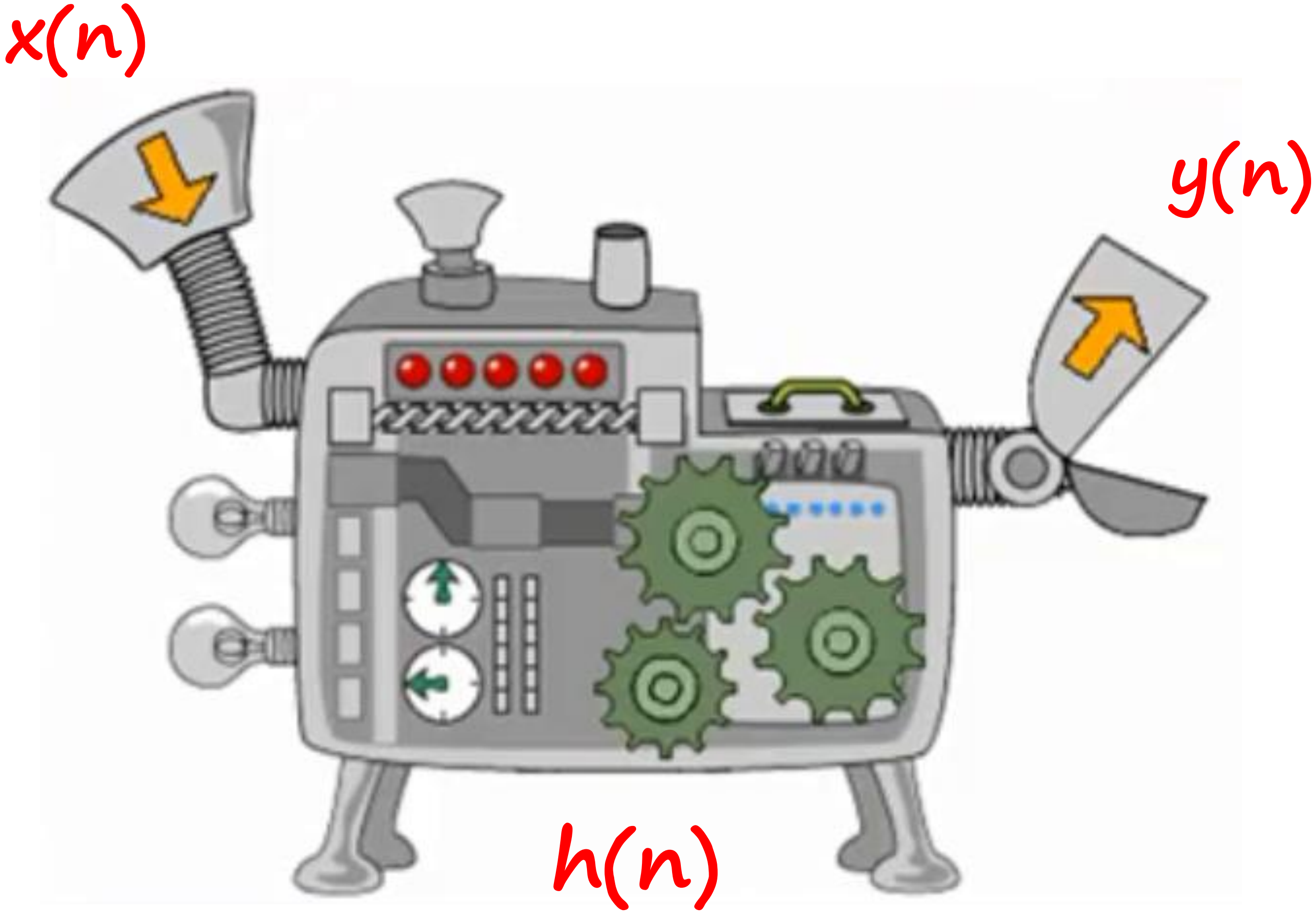
$$g[n] = [\ 0\ ,\ -2\ ,\ 0\ ,\ 2\ ,\ 1.5\ ,\ -2]$$

$$y(n) = [\ 2\ ,\ -1\ ,\ 3\ ,\ 0\ ,\ 0\ ,\ 0\ ]$$

$$h_5(n) = [\ 2\ ,\ -3\ ,\ 3\ ,\ 2\ ,\ 1.5\ ,\ -2\ ]$$

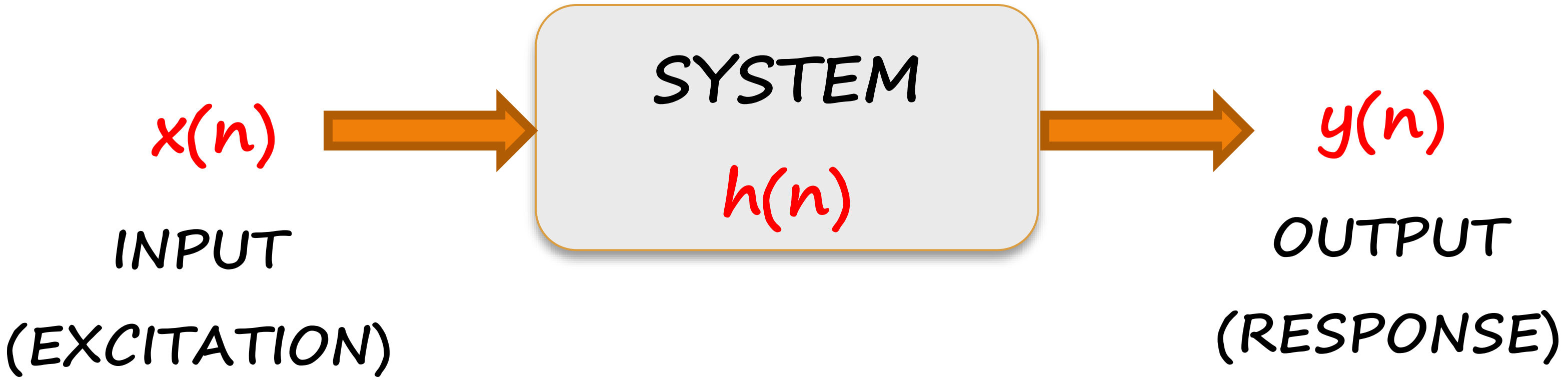
Inclass Problem: The following sequence represents one period of discrete sinusoidal sequence of $x(n)=A\cos(\omega n)$ =[3 , 0 , -3 , 0 , 3]. Find the values of A and ω if the sampling frequency is 100 Hz.

Discrete Time System



Discrete Time System:

- The discrete time system is as the continuous time system relating to the system classification (can be reviewed in chapter 3) .

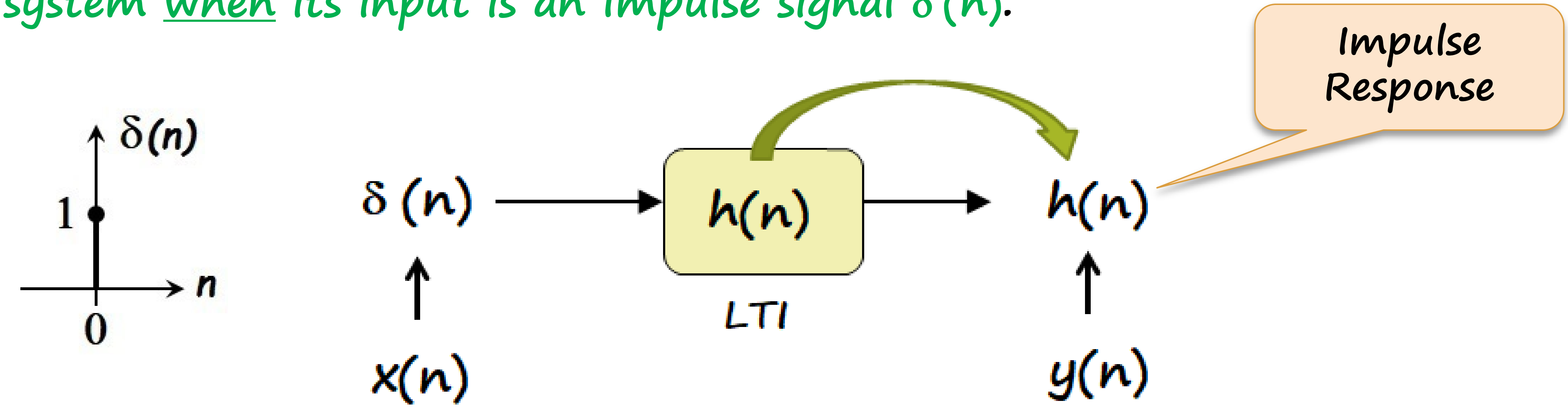


The Time Domain Analysis of the LTI discrete time system:

- ❑ Impulse Response.
- ❑ Discrete Convolution.

The <u>Impulse Response</u> of the discrete time systems can be into two types FIR and IIR:

As explained in Ch3, the Impulse Response means: the Response (output) of a LTI system <u>when</u> its input is an Impulse signal $\delta(n)$.



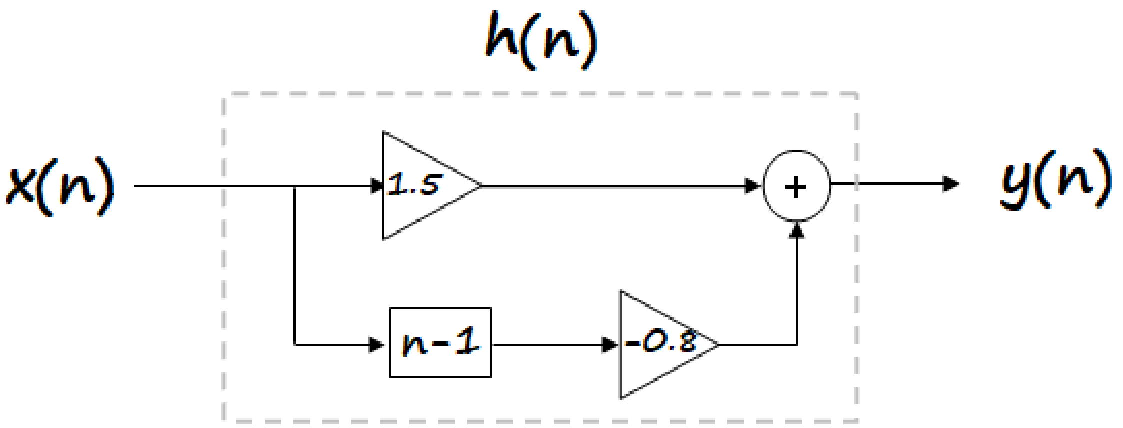
- The Finite Impulse Response FIR (forward, Non-Recursive System that has No Feedback). Its present output depends on the input only.

Discrete systems described by the Difference Equation (DE)

1. Discrete systems with Finite Duration Impulse response (FIR)

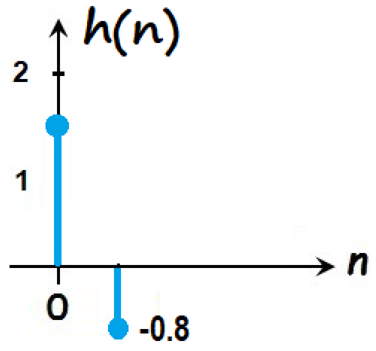
$$y(n) = b_o x(n) + b_1 x(n-1) + b_2 x(n-2) + \ldots$$

Example: Find and draw the Impulse response h(n) of the FIR system shown:



Sol:

- Finding system difference equation: $y(n) = 1.5\ x(n) - 0.8\ x(n-1)$
- Apply impulse signal $\delta(t)$ at the input to get the impulse response at the output, $x(n)=\delta(n)$, simply, replace each x by δ, then $y(n) \rightarrow h(n)$.
  $h(n) = 1.5\ \delta(n) - 0.8\ \delta(n-1)$
- Draw h(n), note it is with Finite length so it is FIR (non recursive system).



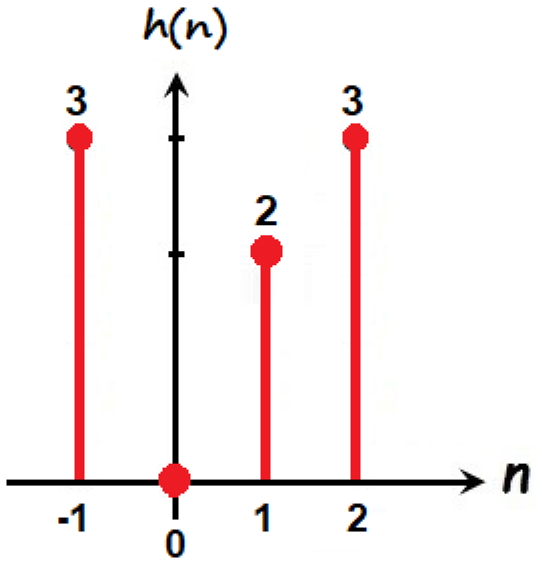
Example: Find and draw the Impulse response h(n) of the FIR system

$$y(n)= 3x(n+1)+2x(n-1)+3x(n-2)$$

Sol:

By making the input $x(n) = \delta(n)$, then $y(n) = h(n)$

$$h(n)= 3\delta(n+1)+2\delta(n-1)+3\delta(n-2)$$

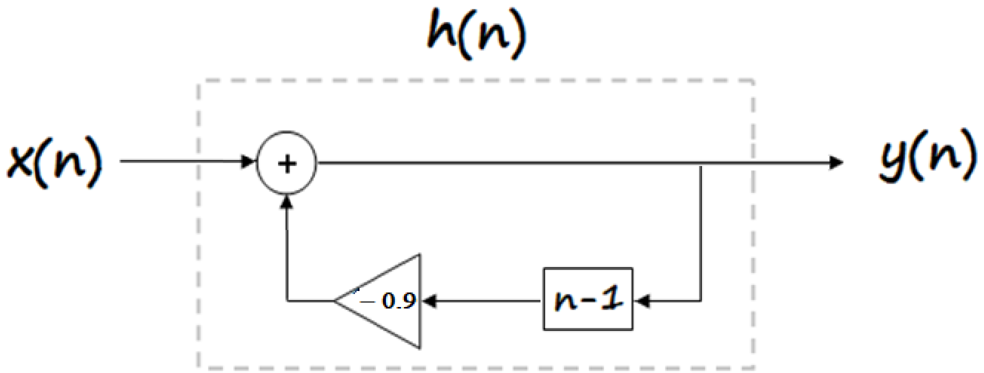


2. Discrete systems with Infinite Duration Impulse Response (IIR)

- The Infinite Impulse Response IIR (Recursive System that has Feedback).
- Its present output depends on the present inputs and on the past outputs

$$y(n) = b_o x(n) + b_1 x(n-1) + b_2 x(n-2) + \dots\ a_1 y(n-1) + a_2 y(n-2) + a_3 y(n-3) + \dots$$

Example: Find and plot the Impulse response h(n) of the IIR system shown for n= -1, 0, 1, …, 5, assume the initial condition is zero:



Sol:

- Finding system difference equation: $y(n) = x(n) - 0.9\ y(n-1)$
- Apply impulse signal $\delta(t)$ at the input to get the impulse response at the output, simply, replace each x by δ, then $y(t) \rightarrow h(n)$.
- $h(n) = \delta(n) - 0.9\ h(n-1)$

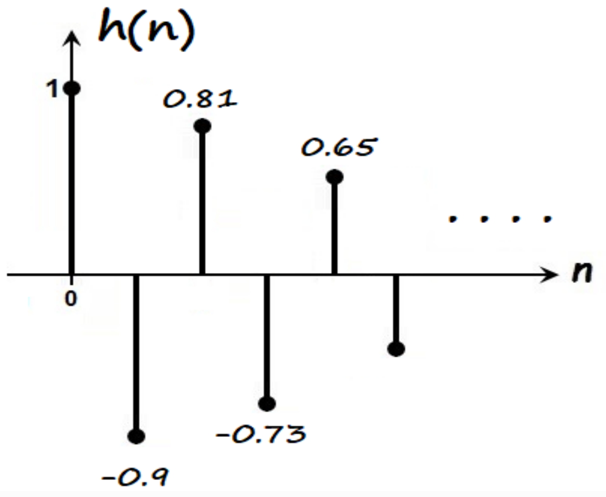
Finding $h(n)$ should be in steps by apply $h(n) = \delta(n) - 0.9\, h(n-1)$ using different n as following:

initial condition

- At $n=-1$, $h(-1) = \delta(-1) - 0.9\, h(-1-1) = 0 - 0.9h(-2) = -0.9(0) = 0$
- At $n=0$, $h(0) = \delta(0) - 0.9\, h(-1) = 1 - 0.9(0) = 1 - 0 = 1$
- At $n=1$, $h(1) = \delta(1) - 0.9\, h(0) = 0 - 0.9(1) = 0 - 0.9 = -0.9$
- At $n=2$, $h(2) = \delta(2) - 0.9\, h(1) = 0 - 0.9(-0.9) = 0 + 0.81 = 0.81$
- At $n=3$, $h(3) = \delta(3) - 0.9\, h(2) = 0 - 0.9(0.81) = 0 - 0.73 = -0.73$

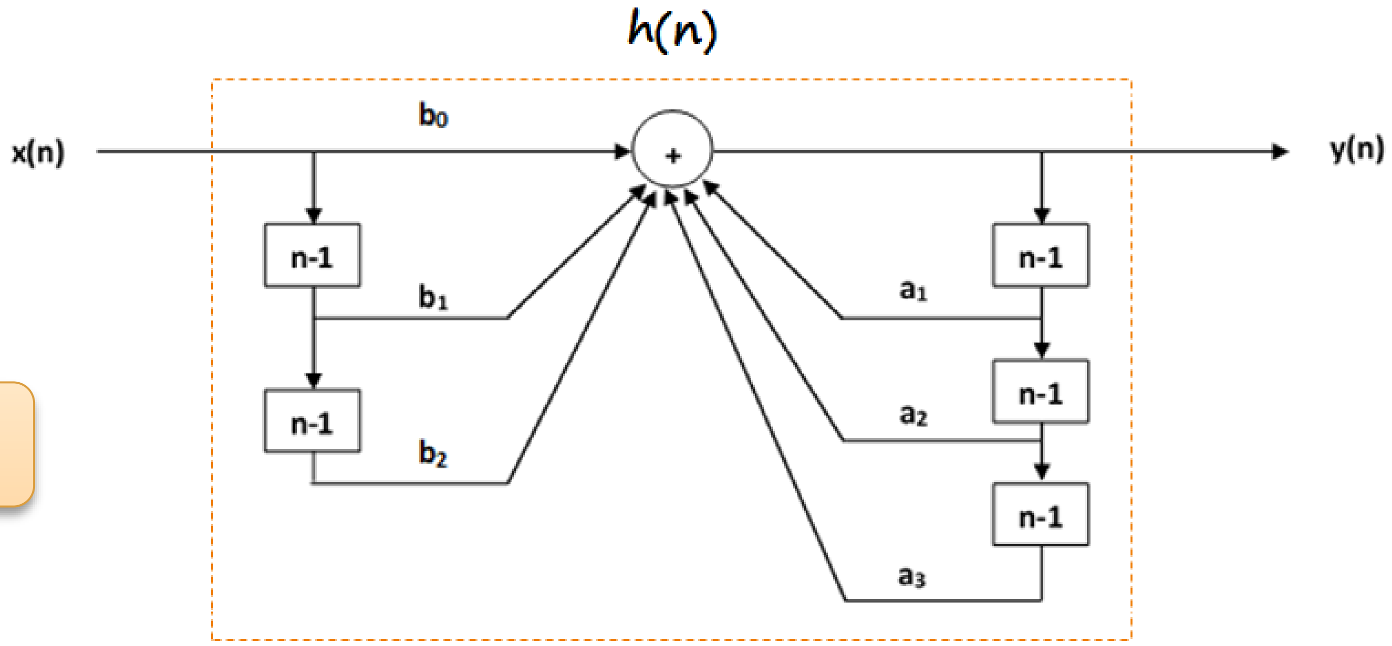
.
.

Draw $h(n)$,



note: the impulse response $h(n)$ is with Infinite length, so it is IIR.

Discrete IIR system realization Direct Form I



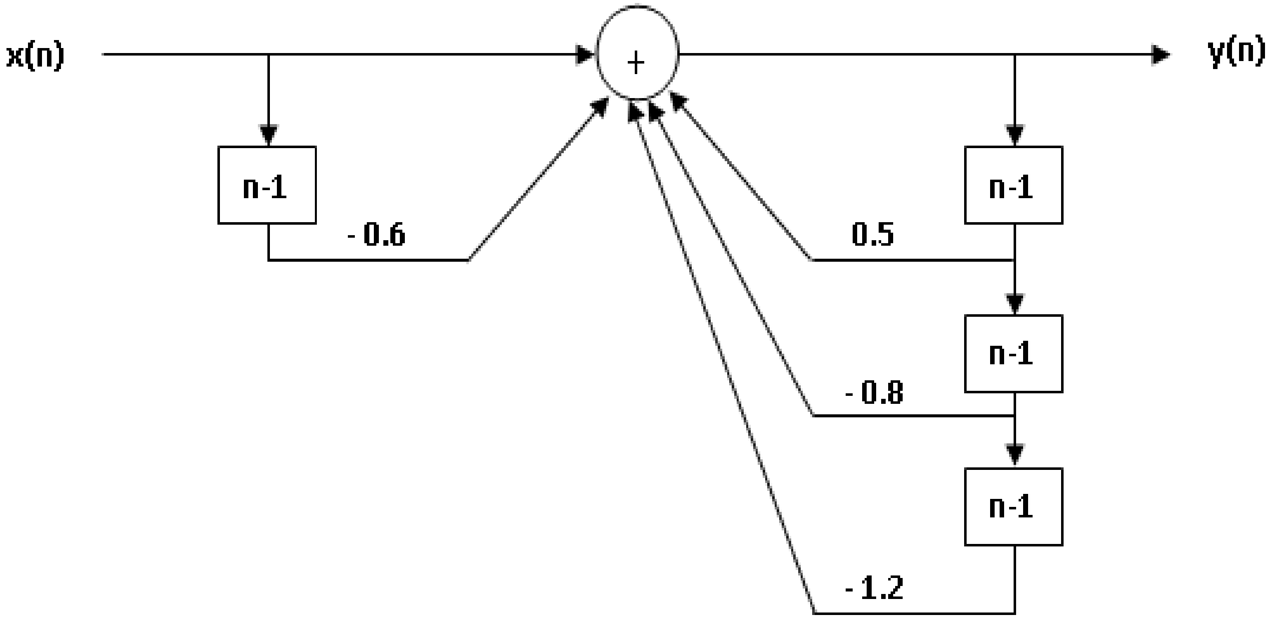
Difference Equation

Sol:

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \dots\ a_1 y(n-1) + a_2 y(n-2) + a_3 y(n-3) + \dots$$

b and a are Real numbers and called system Coefficients that determine system c/cs like the gain, bandwidth, quality factor ... etc.

Example: Find the system difference equation of the following system that is realized using direct form I.



Sol:

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + a_1 y(n-1) + a_2 y(n-2) + a_3 y(n-3)$$

$$y(n) = x(n) - 0.6x(n-1) + 0.5y(n-1) - 0.8y(n-2) + -1.2y(n-3)$$

Discrete Convolution: is a mathematical way of combining two signals to form a third one.

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow$$

$$y(n) = x(n) * h(n)$$

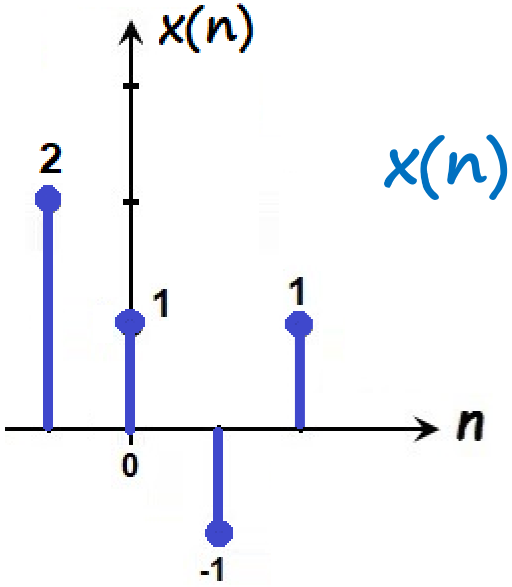
$$y(n) = h(n) * x(n)$$

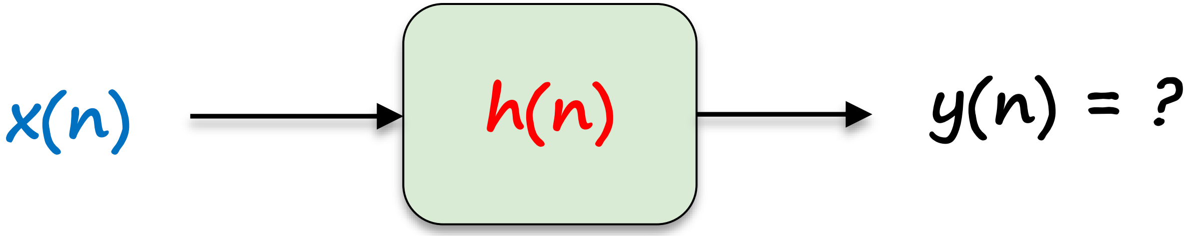
The Discrete Convolution Definition can be written as following:

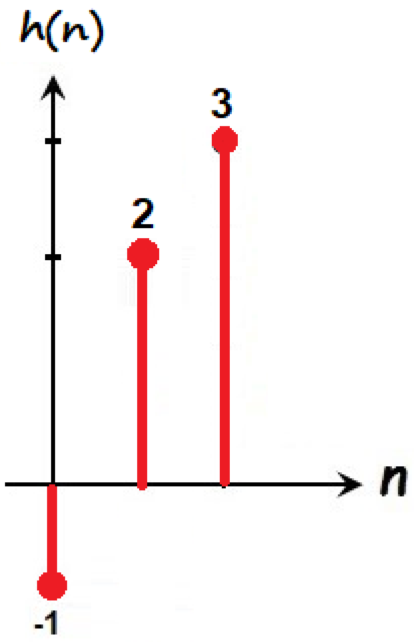
$$y(n) = \sum_{k=-\infty}^{\infty} x(n-k) \times h(k)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \times h(n-k)$$

Example: Find the output of the following system by using the discrete convolution.







- No. of samples of x(n) is M = 4
- No. of samples of h(n) is N = 3
- Then the No. of samples of y(n) = M + N − 1

$$= 4 + 3 - 1 = 6$$

Using Tabular method.

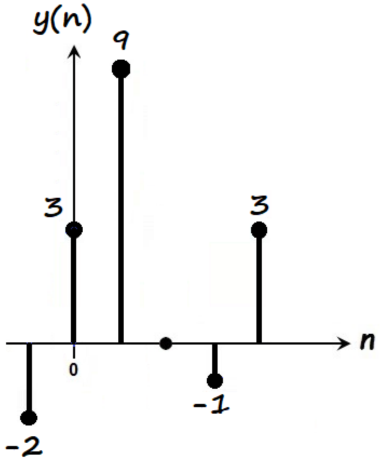
		x(n)			
		2	1	-1	1
	-1	-2	-1	1	-1
h(n)	2	4	2	-2	2
	3	6	3	-3	3

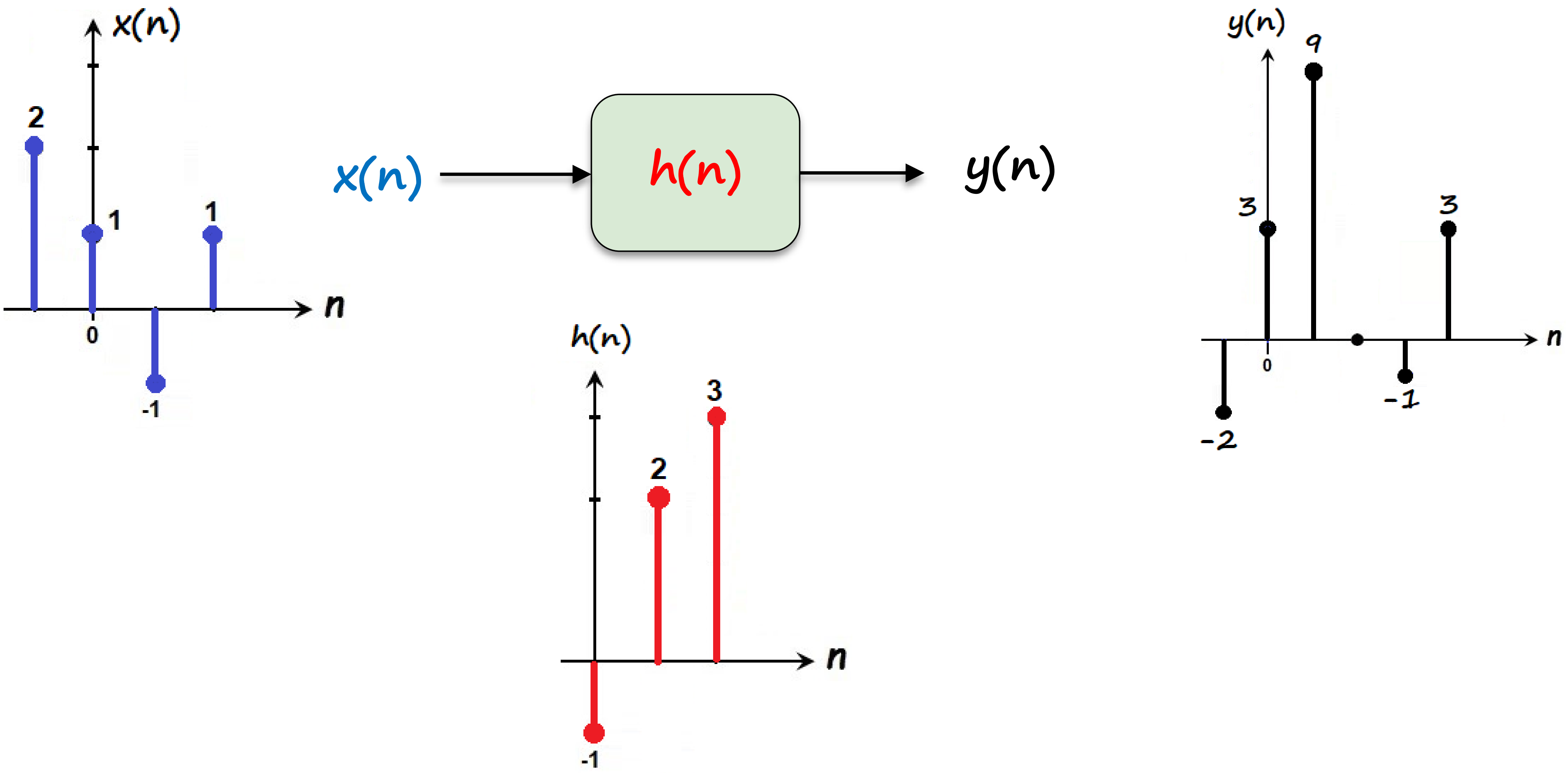
$$y(n) = [\underset{\uparrow}{-2} \quad 3 \quad 9 \quad 0 \quad -1 \quad 3]$$

$$P_{y0} = P_{x0} + P_{h0} - 1$$

$$P_{y0} = 2^{nd} + 1^{st} - 1^{st} = 2^{nd}$$

The 2^{nd} sample of y is at n = 0





x(n)
2
1
1
0
-1
n
x(n)
h(n)
y(n)
h(n)
3
2
-1
n
y(n)
9
3
3
0
-1
-2
n

Discrete Convolution Properties:

1. Associative

$$x_1(n) * x_2(n) * x_3(n) = [\ x_1(n) * x_2(n)\ ] * x_3(n)$$

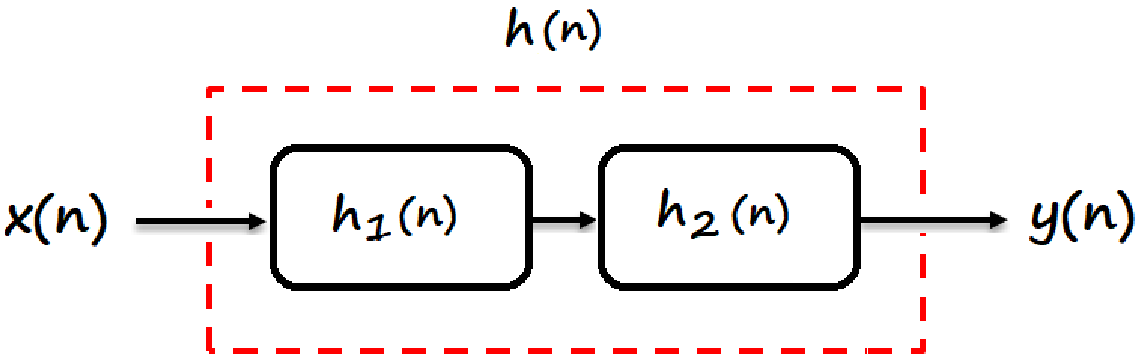
$$= x_1(n) * [\ x_2(n) * x_3(n)\ ]$$

2. Distributive

$$x_1(n) * [\ x_2(n) + x_3(n)\ ] = x_1(n) * x_2(n) + x_1(n) * x_3(n)\ ]$$

Systems Interconnections types:

1. Series (Cascade) Connection



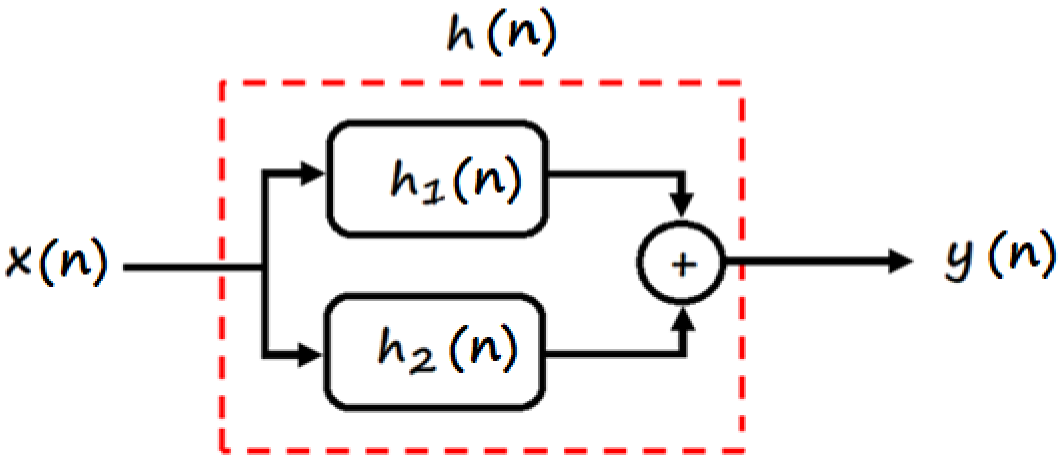
$$h(n) = h_1(n) * h_2(n)$$

$h_1(n)$ is impulse response of subsystem 1

$h_2(n)$ is impulse response of subsystem 2

$h(n)$ is the overall Impulse response (of all system

2. Parallel Connection



$$h(n) = h_1(n) + h_2(n)$$

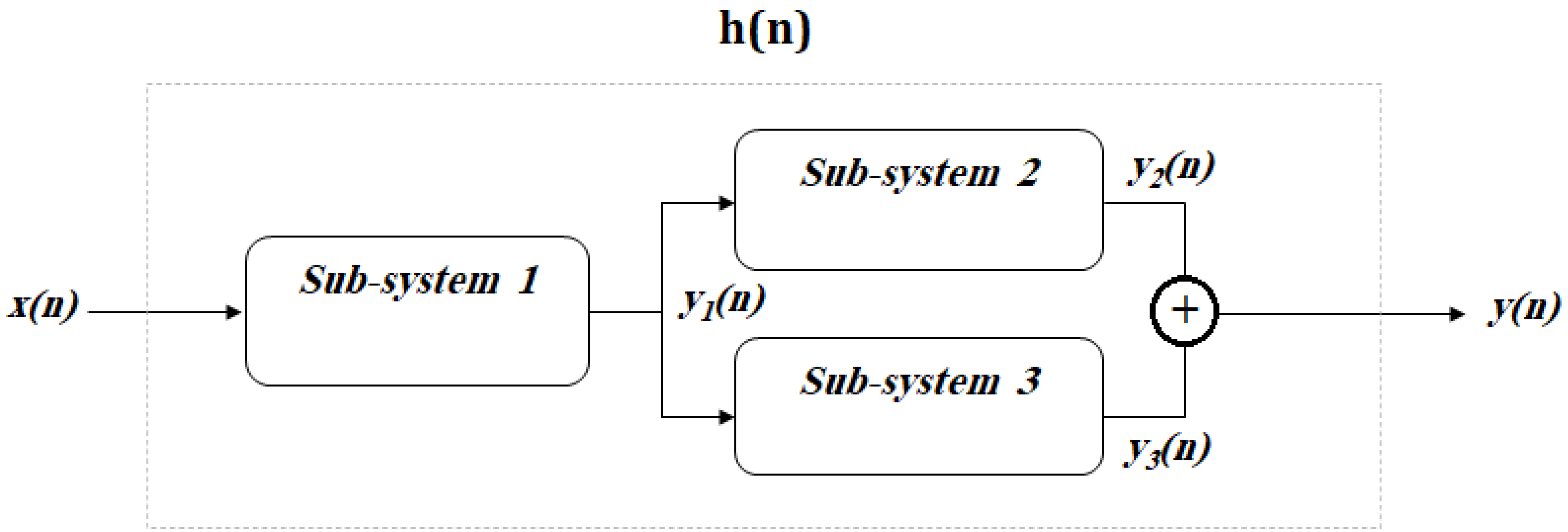
InClass Problem: Find the output of the following system by using the discrete convolution.

If its input $x(n)= \delta(n+1) + \delta(n-1)$, Given that the sub-systems have the following impulse responses:

$$h_1(n)= 2\delta(n)+\delta(n-1) - 3\delta(n-2)$$

$$h_2(n)= 3\delta(n+1)+2\delta(n-1)+3\delta(n-2)$$

$$h_3(n)= - \delta(n)+\delta(n-1) - \delta(n-2)$$



Example: Find and draw the Impulse response h(n) of the cascade FIR system shown: