Q.1) $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n^2)$ $d = 2$

$\log_3 2 \quad \log_2 3 = 1.5$ $a > b^s$ hence $O(n^a)$

$c = \log_b a \cdot$ $n^1 = n^{1.5} < n^2$

$f(n) > n^c$

$T(n) = O(n^2)$

2) $T(n) = 4T(n/2) + n^2$

$\log_2 4 = 2$ $d = 2$

$2 = 2$

$O(n^d \log n) = O(n^2 \log n)$

3) $T(n) = T(n/2) + 2b$

$\log_2 1 = 0$ $0 < b$

$c = \log_b a = \log_2 c = 0$

$n^c = n^0 = 1$

$f(n) > n^c$ $T(n) = O(2^n)$

$\log_n 2^n > 0$

$2^n = b \cdot \log_n 2^n$

4) $T(n) = 2^b T(n/2) + b^b$

$a = 2^b$

$b = 2$ $d = 2^b b^b$

$c = b$

n^b hence $n^n \log_2 n$

Q5) $T(n) = 16\,T(n/4) + n$

$a = 16, b = 4$

$f(n) = \log_4 16 = 2$

$c = 2$

$d = 1$

$\circ \quad c > d$

$O(n^2) \qquad O(n^{\log_b a}) = O(n^2) \qquad O(n^2 \log_2 n)$

$O(n^d \log n^2)$

Q2 $2T(n/2) + n\log n \qquad T(n) = 2T(n/2) + n\log n$

$a = 2, b = 2$

$f(n) = n\log n$

$c = \log 2 = 1$

$n^c = n^1 = n$

$n\log n > n$

$f(n) > n^c$

$T(n) = O(n\log n)$

$n^{\log n} \qquad n^{\log n}$

$\log_n n\log n$

$\log_n n + \log_n \log n$

$1 + \log_n \log n$

Q7) $T(n) = 2T(n/2) + n/\log b$

$a = 2, b = 2 \quad f(n) = n/\log b$

$c = 1$

$n^c = n^1 = n$

$\dfrac{n}{\log n} < n$

$f(n) < n^c$

$T(n) = O(n)$

Q8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, b = 4, \quad f(n) = n^{0.31}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.11}$

$f(n) > n^c$

$T(n) = O(n^{0.51})$

Q10) $T(n) = 16\,T(n/4) + n! \qquad a = 16, b = 4 \quad f(n) = n!$

$\therefore \quad c = \log_b a = \log_4 16 = 2$

$n^c = n^2 \qquad As \quad n \to \infty$

(11) $4T(n/2) + \log n$

$a=4, b=2, f(n)=\log n$

$c = \log_b a = \log_2 4 = 2$

$n^2 = n^c$

$f(n) = \log n$

$\therefore \log n < n^2 \quad f(n) < n^c$

$T(n) = \Theta(n^c) = \Theta(n^2)$

(12) $T(n) = \sqrt{n}\, T(n/2) + \log n$

$a = \sqrt{n}, b=2$

$c = \log_b a = \log_2 \sqrt{n} = \pm \frac{1}{2} \log_2 n$

$\therefore \dfrac{\log_2 n}{2} < \log n$

$f(n) > n^?$

$T(n) = \Theta(f(n)) = \Theta(\log(n))$

(13) $T(n) = 3T(n/2) + n$

$a=3\ b=2\ f(n)=n$

$n^c = n^{1.54}$

$n < n^{1.58}$

$f(n) < n^c$

$T(n) = \Theta(n^{1.5})$

(14) a) $T(n) = 3T(n/3) + \sqrt{n} f(n)$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^c = n$

$\sqrt{n} < n$

$f(n) < n^c$

$T(n) = \Theta(n)$

(18) $T(n) = 3T(n/4) + n\log b$

$a>2\ b=4,\ f(n) = n\log b$

$c = \log_b a = \log_4 3 = 0.79$

$n^c = n^{0.79} \qquad n^{0.79} < n\log n$

$T(n) = \Theta(n\log n)$

(27) $T(n) = 3T(n/3) + n/2$

$a=3; b=3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$n^c = n^c = n \qquad$ As $n/2 < n$

$f(n) < n^c$

$T(n) = \Theta(n)$

(19) $T(n) = 4T(n/2) + n(\log n)$

$a= 4\ b=2\ f(n) = \dfrac{n}{\log n}$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\dfrac{n}{\log n} < n^2$

$T(n) = \Theta(n^2)$

(20) $T(n) = 7T(n/3) + n^2$

$a=7, b=3\ f(n)=n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$T(n) = \Theta(n^2)$

(21) $T(n) = T(n/2) + n(2-\cos n)$

$a=1, b=2$

$c = \log_b a = \log_2 1 = 0$