Q1) $j = 1$ $i = 1$
$j = 2$ $i = 1+2$
$i = 1+2+3$] m-level

For (i) $:- 1 + 2 + 3 + \cdots n$

$< n$

$\frac{m(m+1)}{2} < n$

$m \approx \sqrt{n}$ Hence $T(n) = \sqrt{n}$

Q2) Reccorence relation for function that prints Fibonacci series.

$f(n) = f(n-1) + f(n-2)$ $f(0) = 0$, $f(1) = 1$

$f(n)$

$f(n-1)$ $f(n-2)$

$f(n-2)$ $f(n-3)$ $f(n-3)$ $f(n-4)$

For every function call we get 2 function calls for n levels

so $2 \times 2 \times 2 \cdots$ n times

$T(n) = 2^n$

Max space : Considering recursive stack: no of calls $= n$

Each call has space complexity $O(1)$

$\therefore T(n) = O(n)$

without considering recursive stack

$T(n) = O(1)$

write programs with complexity $n(\log n)$, n^3, $\log(\log n)$

1)

```
for (int i=0 ; i<n : ++i)
{
  for( int j=1 ; j<=n ; j*=2)
  {   sum += 1;
  }
}
```

$T(n) = O(n \log n)$

2) n^3

```
for (int i=0 ; i<n ; ++i)
for (int j=0 ; j<n ; ++j)
for (int k=0 ; k<n ; ++k)
{ sum++; }
```

(3) $\log(\log n)$

```
for ( int i = 1 ; i<=n ; i*=2)
{
    sum++;
}
```

4) solve.

$$T(n) = T(n/4) + T(n/2) + (n^2)$$

n — 0

$T(n/4)$ $T(n/2)$ — 1

$T(n/3)$ $T(n/16)$ $T(n/4)$ $T(n/8)$ — 2

$0 \to cn^2$ $1 \to \frac{n^2}{4^2} + \frac{n^2}{2^2} = \frac{c5n^2}{16}$

$2 \to \frac{n^2}{8^2} + \frac{n^2}{16^2} + \frac{n^2}{4^2} + \frac{n^2}{8^2} = \left(\frac{5}{16}\right)^2 n^2 c$

max level $= \frac{n}{2^k} = 1$ $= k = \log_2 n$

$$T(n) = c\,n^2 + \left(\frac{5}{16}\right)n^2 + \left(\frac{5}{16}\right)^2 n^2 + \dots + \left(\frac{5}{16}\right)^{\log n} n^2)$$

$$T(n) = cn^2\left[1 + \left(\frac{5}{16}\right) + \left(\frac{5}{16}\right)^2 + \dots + \left(\frac{5}{16}\right)^{\log n}\right]$$

$$T(n) = cn^2 \times + \times \left(\frac{1-(5/16)^{\log n}}{1-5/16}\right)$$

$$T(n) = cn^2 \times \frac{11}{5} \times \left(1-\left(\frac{5}{16}\right)^{\log n}\right)$$

$$T(n) = O(n^2 c)$$

$O(cn^2)$ ans

i $\quad$ j
1 $\quad$ 1
2 $\quad$ 2+3+5 $\qquad j = (n-1)/i$ time
3 $\quad$ 3+5+7
$\quad$ 2+5+7

$\sum_{i=1}^{n} \frac{(n-1)}{i} \quad \because T(n) = \frac{(n-1)}{1} + \frac{(n-1)}{2} + \frac{(n-2)}{3} + \frac{(n-1)}{n}$

$T(n) = n\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n}\right] - 1\cdot[1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}] = n\log n - \log$

$T(n) = O(n\log n)$

(Q 8) $\quad$ i
= 1
2 $\qquad 2^{2^k}$
$2^k \qquad (2^k)^k = 2k^2$
2^{2k}
$2^{2^{2k}} \qquad 2km$

$2^{k^m} <= n$

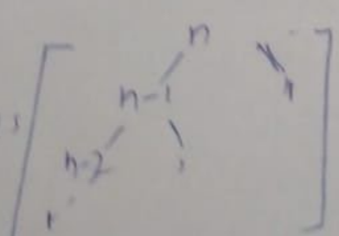
$\sum_{i=1}^{m} \qquad k^m = \log_2 n \qquad 1+1+1 \cdots m$ time

$m = \log_k \log_2 n \qquad T(n) = O(\log_k \log n)$

2^{k^2}
$(2^k)^{k}$

7) Write a recurrence relation when quick sort repeatedly divides array into two parts of 99% & 1%. Derive time complexity in this case.

$T(n) = T(n-1) + O(1)$

n levels


$T(n) \left(T(n-1) + T(n-2) \right) \cdots T(1) + O(1) \qquad = n \times n$

$T(n) = O(n^2)$

Lowest height = 2
highest " " n

diff = $n-2 \quad n>1$

Hence linear result.

a) $100 < \log\log n < \log n < (\log n)^2 < \sqrt{n} < n < n\log n < \log(n!) < n^{2} <$
$2^n < 4^n < 2^{2^n}$

b) $2^{(2^n)}$, 4 $\quad 1 < \log\log n < \sqrt{\log n} < \log n < \log 2n < 2\log n < n < n\log n <$
$2n < 4n < \log(n!) < n^2 < n! < 2^{2^n}$

$\log_3 n < \log 2n < 5n < n\log(n) < n\log_2 n < \log(n!) < 8n^3 < 7n^3 < n! < 8^{2n}$