Lecture 15

Monday, March 7, 2022 5:50 AM

Linear Systems Modeling Part III and Non-Linear Modeling

Objectives

① Data compression using linear models

② Radial Basis funcn.

Admin

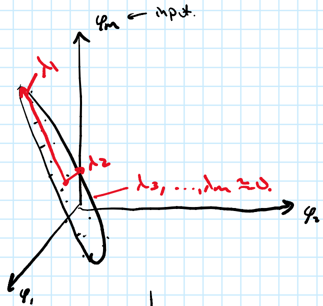
① HW02 – due.

② HW03 – next week.

③ Midterm through HW03
→ take-home.
Mar 30th.
→ open.

Last Time

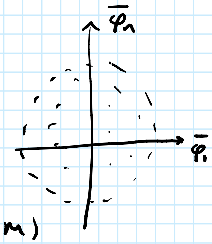
Let's consider a coord. transformation:

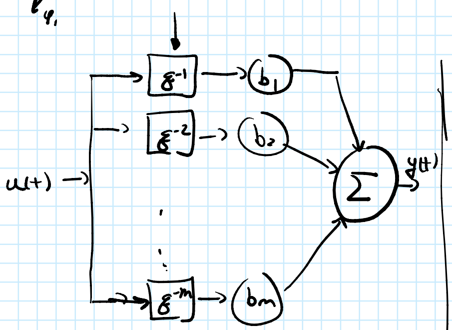


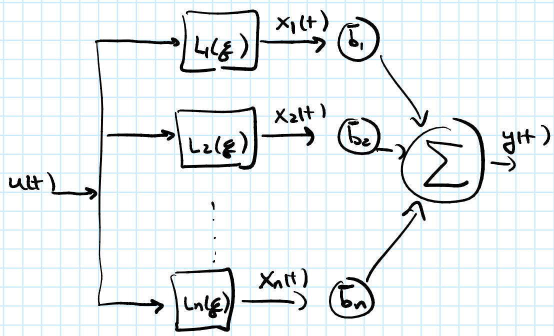
Transform the data ⇒ to a lower dimensional space

$\mathbb{R}^m \to \mathbb{R}^n \; (n < m)$

$(n << m)$







$$y(t) = \left[\sum_{i=1}^{m} b_i \xi^{-i}\right] u(t)$$

$$= [u(t-1) \;\; u(t-2) \cdots u(t-m)] \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$$

$$y(t) = \left[\sum_{i=1}^{n} \bar{b}_i L_i(\xi)\right] u(t)$$

$n << m$

Continuous Time Laguerre Series Expansion

$$L_k(s) = \underbrace{\frac{\sqrt{2\bar{a}}}{s+\bar{a}}}\underbrace{\left(\frac{s-\bar{a}}{s+\bar{a}}\right)^{k-1}} \quad \text{(continuous time)}$$

band-pass filter

$\Updownarrow$
$e^{-\bar{a}t}$

low-pass filter → slow pole

all pass filter

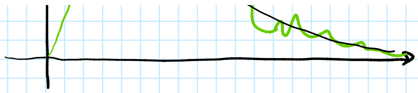
Impulse Response

Main point:

- Let p be a slow, real pole of a physical system, G(s)
- If Laguerre pole is chosen such that $\bar{a} \approx p$, then a truncated expansion

choose $\bar{a}$

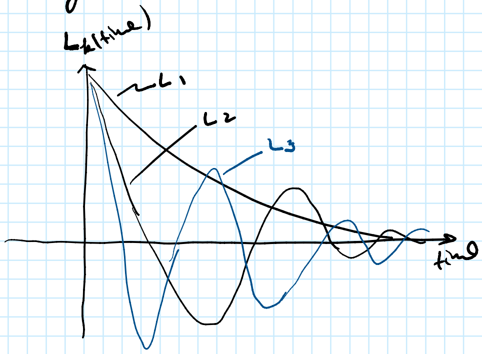
$$G_n(s) = \sum_{k=1}^{n} b_k \frac{\sqrt{2\bar{a}}}{s+\bar{a}} \left(\frac{s-\bar{a}}{s+\bar{a}}\right)^{k-1}$$



$$G_n(s) = \sum_{k=1}^{n} b_k \frac{\sqrt{2a}}{s+a}\left(\frac{s-a}{s+a}\right)^{k-1}$$

rapidly converges to $G(s)$ w/ small n.

- Laguerre series fucns in the domain:



capturing faster dynamics as $k\uparrow$

- Discrete time form

Time interval of data.

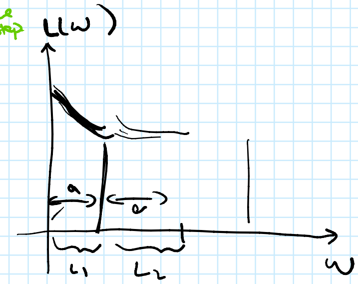
$$\underbrace{L_k(\xi) = \frac{K}{\xi-a}\left(\frac{1-a\xi}{\xi-a}\right)^{k-1}; \quad K = \sqrt{(1-a^2)T}}_{\text{subject to } a<1}$$

$\uparrow$ discrete time pole.

$\xi = z = e^{sT}$ $\Rightarrow$ $a = e^{-\bar{a}T}$ (continuous pole, time step)

$\Rightarrow 0 < |\xi| < 1$

Left hand plane maps to unit circle.



- How to implement in discrete time? ∈ in CODE.

$$x_1(\xi) = L_1(\xi)\, u(t) = \frac{K}{\xi-a}u(t) = \frac{K\xi^{-1}}{1-a\xi^{-1}}u(t)$$

$$\Rightarrow x_1(t)[1-a\xi^{-1}] = K\xi^{-1}(t)\,u(t)$$

$$\Rightarrow x_1(t) - a x_1(t-1) = K u(t-1)$$ depends on time history of x_1 & u.

$$\Rightarrow x_1(t) = a x_1(t-1) + K u(t-1)$$ set $x_1(0) = 0$.

- Next,

$$L_2 = \overbrace{\frac{K}{\xi-a}}^{L_1}\left(\frac{1-a\xi}{\xi-a}\right) \Rightarrow L_2 = L_1\left(\frac{1-a\xi}{\xi-a}\right) \Rightarrow x_2(t) = L_2 u(t)$$

$$\Rightarrow x_2(t) = L_1(\xi)\left(\frac{1-a\xi}{\xi-a}\right)u(t) = \overbrace{[L_1(\xi)u(t)]}^{x_1(t)}\frac{1-a\xi}{\xi-a}$$

$$\Rightarrow x_2(t) = x_1(t)\left(\frac{1-a\xi}{\xi-a}\right) = x_1(t)\frac{\xi^{-1}-a}{1-a\xi^{-1}}$$

$$\Rightarrow x_2(t)\,[1-a\xi^{-1}] = [\xi^{-1}-a]\,x_1(t)$$

$$\Rightarrow x_2(t) - a x_2(t-1) = x_1(t-1) - a x_1(t)$$

$$\Rightarrow x_2(t) = a x_2(t-1) + x_1(t-1) - a x_1(t)$$ ← depends on the history of x_1, x_2.

$\vdots$

$$x_k(t) = a x_k(t-1) + x_{k-1}(t-1) - a x_{k-1}(t)$$ ← recursive implimentation

what about $u(0)$, $x_1(0), x_1(1)$ } zero Initial conditions

$\Rightarrow$ can show convergence to

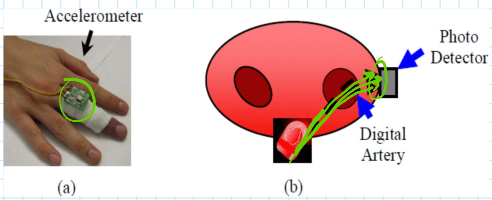
what about $u(0)$, $x_1(0), x_1(1)$, $x_2(0), x_2(1), x_3(2)$, ⋮ } zero Initial conditions ⇒ can show convergence to delta.

- Laguerre series is just one useful orthogonal basis funcn. for more compactly representing the underlying dynamics.

⇒ another useful funcn is Kautz:

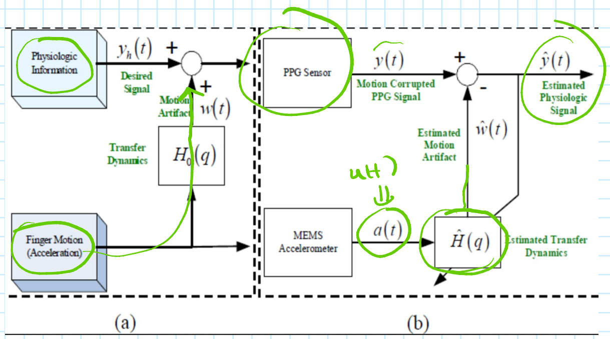
$$\sum_{k=1}^{\infty} g_k \frac{K}{s^2+bs+c}\left(\frac{s^2-bs+c}{s^2+bs+c}\right)^{k-1}$$

} particularly good for oscillatory systems.

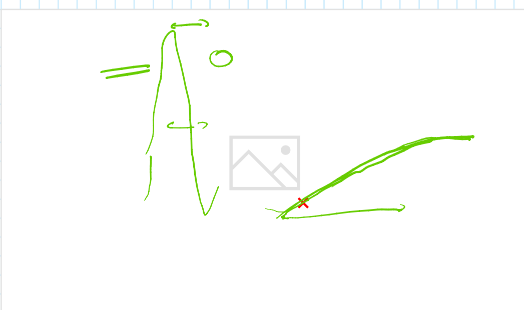


(a) (b)

a



(a) (b)



FIR

Laguerre