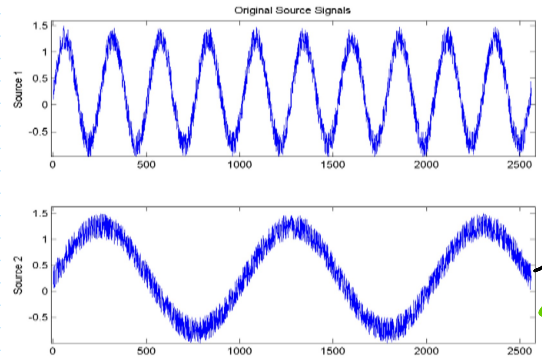
Lecture 9

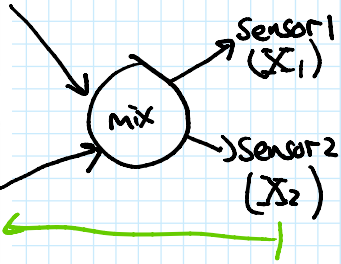
Tuesday, February 8, 2022 8:59 PM

Independent Component Analysis and Partial Least Squares Regression

Admin
① HW #1 due Thurs.
② OH Today: 3-4pm.



"Blind Source Separation"

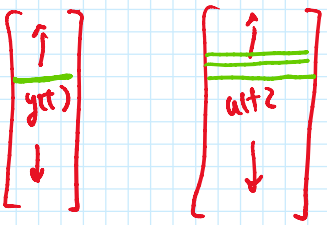


Problem 6:

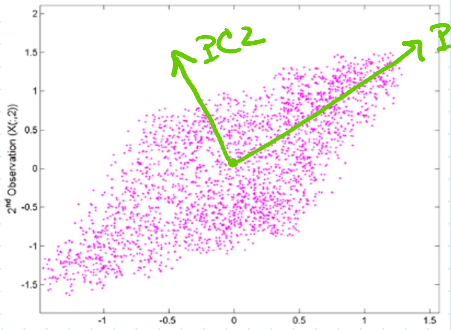
$y = \varphi^T \theta$

$y(t) = \varphi(t)^T \theta$

$y(t) = [u(t-1) \quad u(t-2) \quad u(t-3)]\,\theta$



Plot X_1 vs. X_2

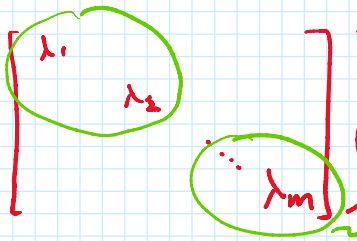


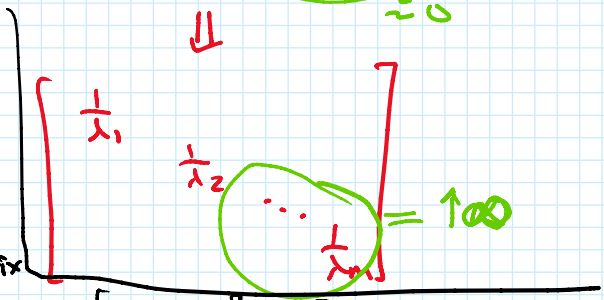
X_2

PCA

$\text{Cov}(T_1, T_2) = 0$

- one criterion for finding directions in the data that may reflect the underlying processes





X_1

data collected over time. $\left[\begin{matrix} X_1 & X_2 \end{matrix}\right]$ $\Rightarrow X = T P^T$

"observations" "source data" "mixing" matrix

$\begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$

$x_1 = w_{11} T_1 + w_{12} T_2$

$x_2 = w_{21} T_1 + w_{22} T_2$

- Each column of T represents a "source"
  - biological signal
  - sensor noise
  - biological noise.
- If we want to remove "noise" from the data, set that column in T to zero, then transform the data back to X.

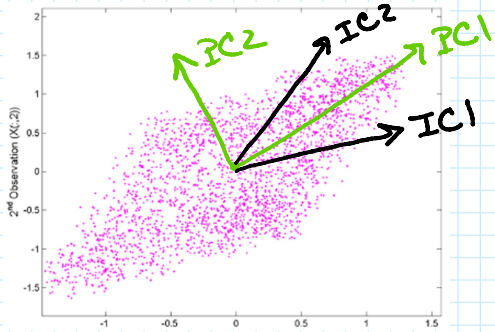
Independent Component Analysis (ICA)

- How do we know the sources are uncorrelated?
  - → Two people speaking w/ similar cadences
  - → Human gait synchronizes w/ heart rate.
- PCA → $\text{Cov}(T_1, T_2) = 0$ ← one metric of "independence"

→ Human gait synchronizes w/ heart rate.

- PCA → $cov(T_1, T_2) = 0$ ← one metric of "independence"

Plot X_1 vs. X_2



X_2

X_1

[PCA: $X = T P^T$]

- Independent sources don't need to be orthogonal.
- Goal of is to project observed data onto maximally independent axes.
- Instead of using covariance as our independence measure, we use a measure of how statistically independent the sources are.

- In ICA $X = \hat{Z} A^T$ ← Blind source separation.
  - $(A^T)^{-1}$
  - $\hat{Z}$: sources

$\Rightarrow \hat{Z} = X W^T$

- procedure: Initialize W^T, then iteratively update to maximize a func. that measures independence of of columns of $\hat{Z}$.
- If the colums are independent $p(\hat{z}) = \prod_{i=1}^{m} p(\hat{z}_i)$
  - $p(\hat{z}_i)$: prob. dens. func. of i^{th} source signal.
- cost funcns:
  - maximum likelihood
  - Higher order moments.
  - max information transfer
  - Negentropy.
- packages.
  - rica, fastica (MATLAB)
  - ica (R).

HW: Blind source separation: PCA, ICA.

Reading: "Blind Source Separation".