Lecture 24

Monday, April 25, 2022 8:53 AM

The Art of System Identification - Part II: Input Design and Map of Estimation/Modeling Techniques

Admin

① HWs 4,5 due ←

② Survey posted. ←

③ please submit consultation slide decks.

Last time

using many parameters $\theta \in \mathbb{R}^d$ (large d) may provide an accurate model with small bias ($\theta_0 \in \mathcal{D}_M$), but a large d tends to worsen the variance and convergence speed.

in terms of # of data collected

-> particularly if some parameters have small sensitivity values ← ψ_i

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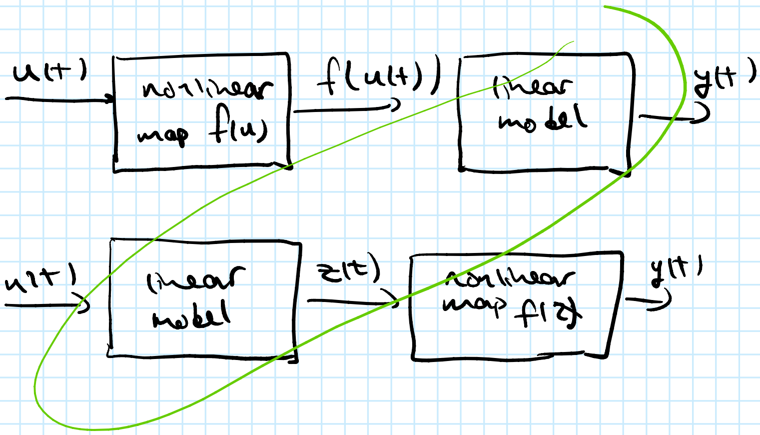
Design Space for Parameter Estimation Experiments

1) Data Acquisition

- Sensors – what to measure
  - Input & output variables
  - Disturbances. (If feasible, measure).
- Actuation: what to manipulate.
  - Inputs -> physical limitations.
  - -> complexity of input.
- Sampling: High sampling rate means:
  - more data
  - Capture noise, rather than signal!
  - => pre-filter design.

2) Model structure selection

- Linear / non-linear
  - a) linear parameterization of nonlinear system.
  - b) Hammerstein & Weiner models



Hammerstein model

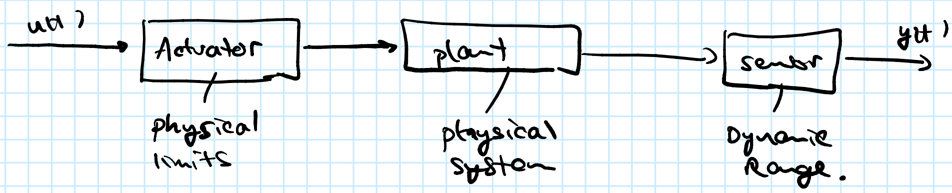
Weiner model

FIR, ARX, ARMAX, Laguerre,

3) Input signal design.

⇒ How do we design the best input sequence to improve parameter estimation covariance.

<u>Practical Requirements for Input Design</u>



- when conducting experiments, various constraints due to physical limitations

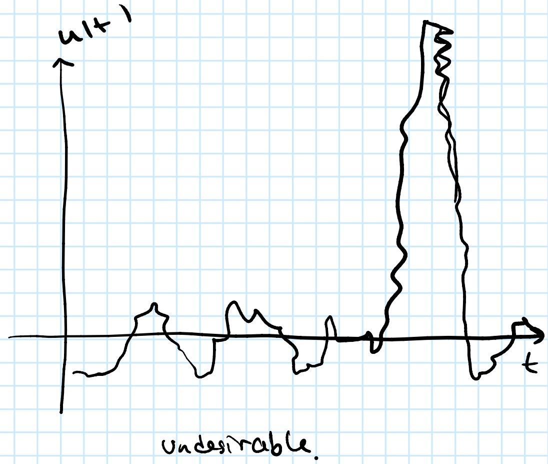
→ Among others, the input aplitude is a common constraints.

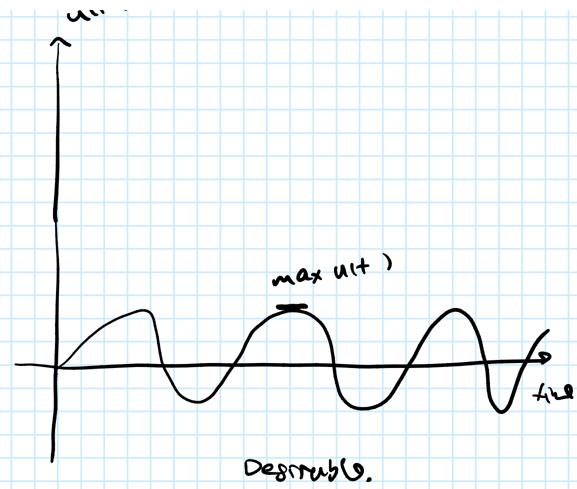
$$\underline{u} \le u(t) \le \bar{u}$$

lower limit — upper limit

- In general, the larger the input magnitude, $|u(t)|$ becomes, smaller the parameter covariance becomes

⇒ The input sequence should have large amplitude <u>most of the time</u>.





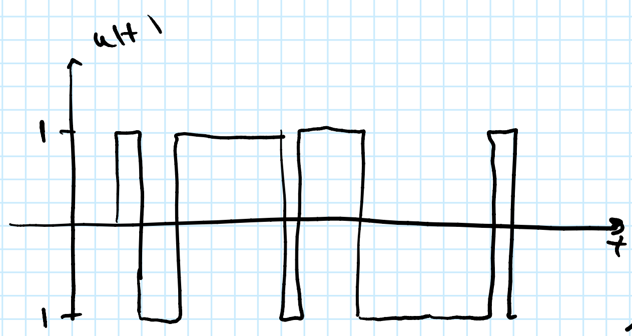
- To evaluate this aspect of the input signal, the following crest factor is often used for signals w/ zero mean:

$$Cr^2 = \frac{\max_{1 \le t \le N} u^2(t)}{\frac{1}{N}\sum_{t=1}^{N} u^2(t)}$$

Note: $Cr \geq 1$ & smaller is better.

- <u>Parameter Estimation Using Random Signals</u>
    - The best crest factor (1) is achieved w/ zero mean binary signals.

$$u(t) = \pm \bar{u}$$



An example of a "pseudo random binary input"

$$R_u(\tau) = E[u(t)u(t-\tau)] = \begin{cases} 1, & \tau = 0 \\ 0, & \tau \neq 0 \end{cases}$$

- In system identification, random inputs are often used because of their noise reduction properties.

→ Consider a model defined in terms of an impulse response.

∞

→ consider a model defined in terms of an impulse response.

$$y(t) = \sum_{k=1}^{\infty} g(k)\,u(t-k) + v(t)$$

↑ impulse response model. ↑ noise

- where $v(t)$ is uncorrelated w/ random input $u(t)$.

$$E[u(t)\,v(t-\tau)] \equiv 0 \quad \text{for all } \tau.$$

- evaluate the correlation between input & output

$$R_{yu}(\tau) = E[y(t)\,u(t-\tau)] = E\left[\left[\sum_{k=1}^{\infty} g(k)\,u(t-k) + v(t)\right]u(t-\tau)\right]$$

$= 0.$

$$= \sum_{k=1}^{\infty} g(k)\,\underbrace{E[u(t-k)\,u(t-\tau)]}_{R_u(k-\tau)} \Rightarrow \begin{cases} = 0 & k \neq \tau \\ = 1 & k = \tau \end{cases}$$

$$R_{yu}(\tau) = g(\tau)$$

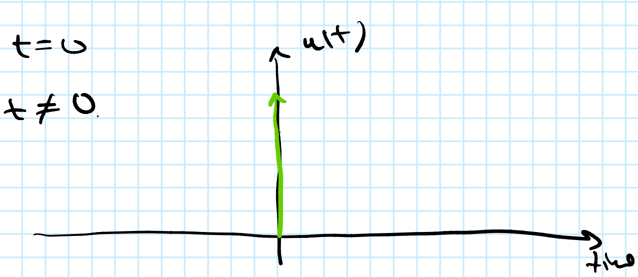
- The noise term is averaged out when the input-output correlation is computed.

 -) particularly useful for noisy systems.

 → If the input is a random sequence w/ unit variance, $R_{yu}(\tau)$ gives impulse response $g(k)$.

 → The advantage is clear when compared w/ a standard impulse response.

$$u(t) = \begin{cases} \alpha, & t = 0 \\ 0, & t \neq 0 \end{cases}$$



$$\Rightarrow y(t) = \alpha g(t) + \cancel{v(t)}$$

• Estimate the impulse response,

$$\hat{g}(t) = \frac{y(t)}{\alpha} = g(t) + \frac{v(t)}{\alpha}$$

↑

$$\hat{g}(t) = \frac{y(t)}{\alpha} = g(t) + \underbrace{\frac{v(t)}{\alpha}}_{\text{noise}}$$

- The noise $v(t)$ shows up in the output.
  $\Rightarrow$ To eliminate, the magnitude of impulse must be large.

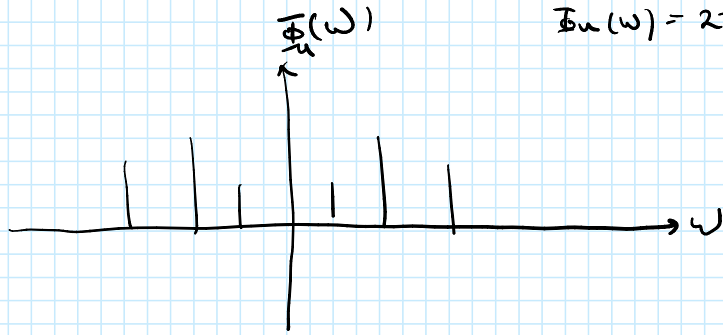
- pseudo-Random Binary signals (PRBSs)
  $\rightarrow$ PRBSs are periodic, deterministic signals w/ approx. white-noise properties

- Sinusoidal inputs.

  - A natural choice of input is a sum of sinusoids.

$$u(t) = \sum_{k=1}^{p} a_k \cos(\omega_k t + \phi_k)$$

  - At steady state, multi-sine funcn has following power spectrum:



$$\Phi_u(\omega) = 2\pi \sum_{k=1}^{p} \frac{a_k^2}{4}\left[\delta(\omega-\omega_k) + \delta(\omega+\omega_k)\right]$$

- If $G(q,\theta)$ has at most n unknown parameters, $p \geq n$ frequencies will satisfy persistence of excitation