ME8873 – Statistical Model Estimation
Homework Set No. 5
Due April 22nd, 2022

Complete the following two problems for full credit. ***Note***: All problems in this assignment should be worked algebraically/by hand.

Problem 1 (10 points):
Obtain the power spectrum of a sinusoid:

$$u(t) = A\cos\omega_0 t$$

and sketch a plot of the power spectrum against frequency ω, $\ -\infty < \omega < +\infty$.

Problem 2 (10 pts):
A common autocorrelation function encountered in physical problems is

$$R(\tau) = \sigma^2 e^{-\beta|\tau|}\cos\omega_0\tau$$

a) Find the power spectrum of $R(\tau)$.

b) $R(\tau)$ will be recognized as a damped cosine function. Sketch both the autocorrelation and power spectral functions for the lightly damped case.

Problem 3 (10 pts):

Shown below is the cardiovascular system with input *u(t)* being the blood flow rate from the left ventricle and output *y(t)* the blood pressure at the aorta. To identify the system, input-output data {*u(t)*} and {*y(t)*}, were recorded for a long period of time, and the auto-covariance of the input sequence: $R_u(\tau) = E[u(t)u(t-\tau)]$ was computed for the first several τ ;

$$R_u(0) = 2,\ R_u(1) = 1,\ R_u(2) = -1,\ R_u(3) = -2,\ R_u(4) = -1,\ R_u(5) = 1, \cdots$$

For this system, two model structures for the transfer function from *u(t)* to *y(t)* are considered:

$$M_A(\theta_A) = \{\theta_A = (a, b)^T \mid G_A(q) = \frac{bq^{-1}}{1 + aq^{-1}}\},$$

$$M_B(\theta_B) = \{\theta_B = (a_1, a_2, b)^T \mid G_B(q) = \frac{bq^{-1}}{1 + a_1 q^{-1} + a_2 q^{-2}}\}$$

Given the auto-covariance, is the data set informative enough with respect to model structure M_A? Is it informative enough with respect to M_B? Explain why.
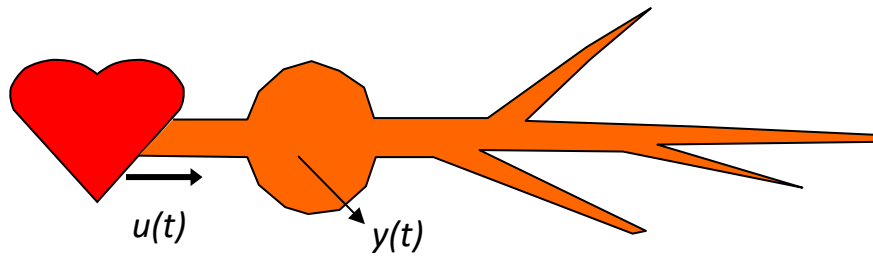
Figure 1: Schematic diagram of cardiovascular system