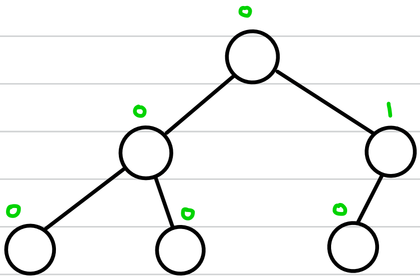
AVL TREES

↳ Height Balanced Binary Search Trees

No of edges

Balance factor = Height of left subtree − Height of right subtree

{-1, 0, 1} ⟶ Balanced Tree



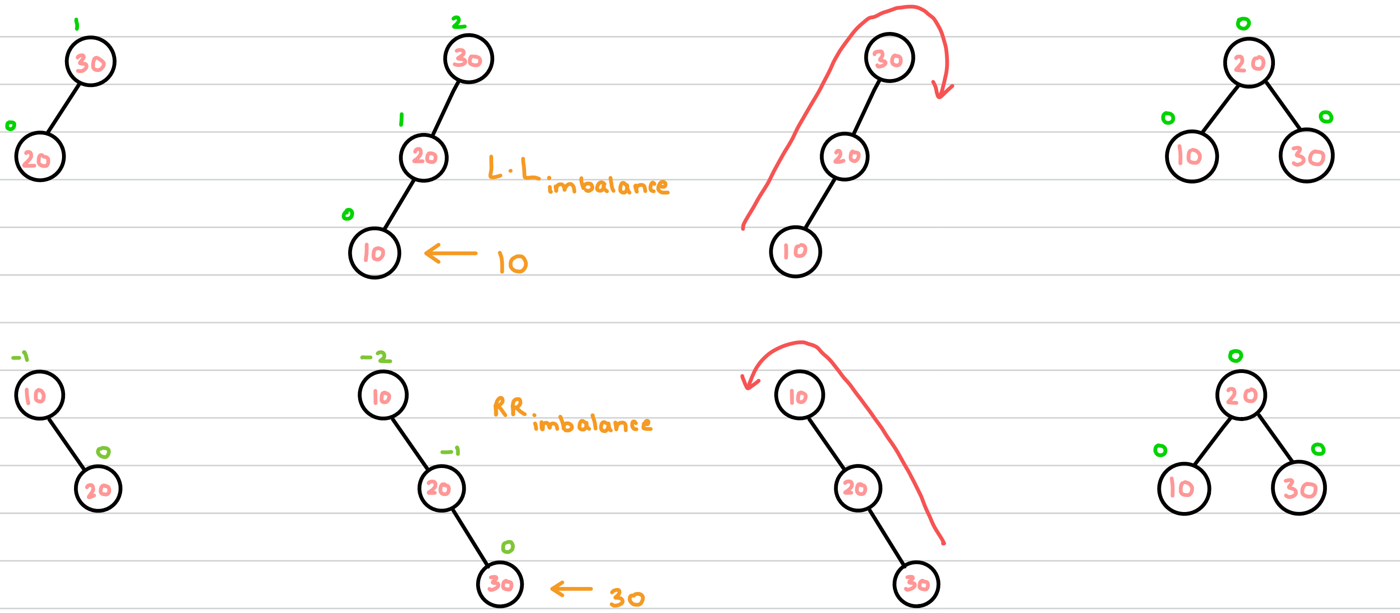
Balanced Tree

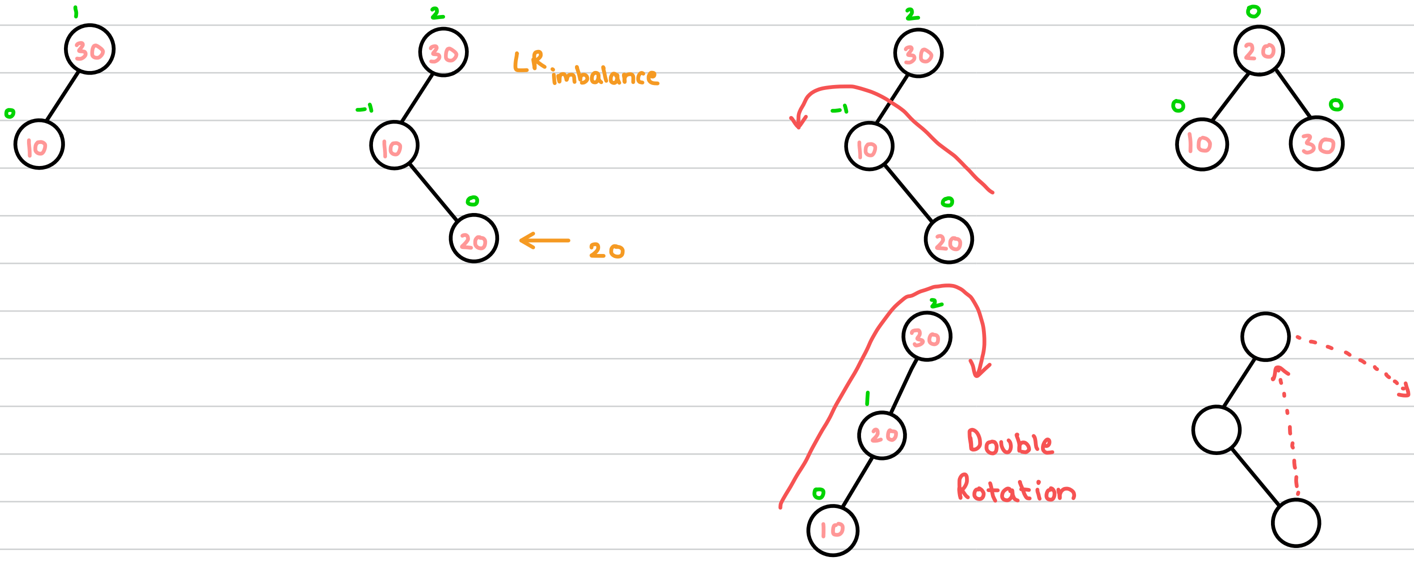
Rotations for insertion in AVL trees

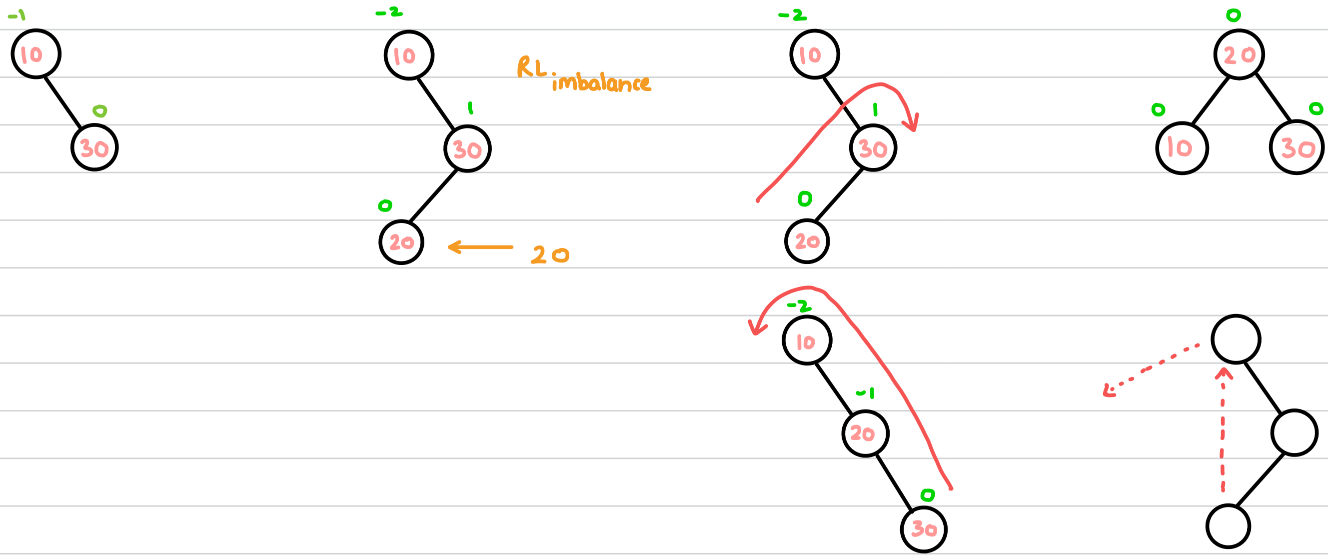
(1) LL Rotation] Single Rotation
(2) RR Rotation

(3) LR Rotation] Double Rotation
(4) RL Rotation

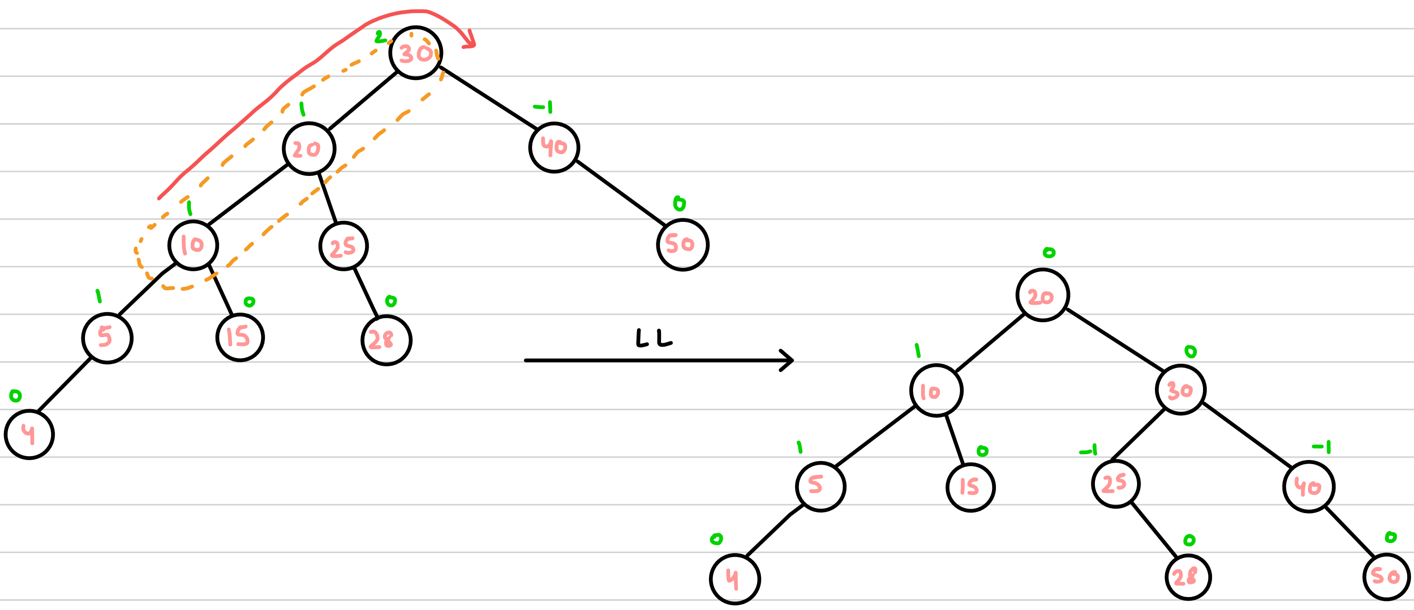
INITIAL	AFTER INSERTION	PERFORM ROTATION	AFTER ROTATION

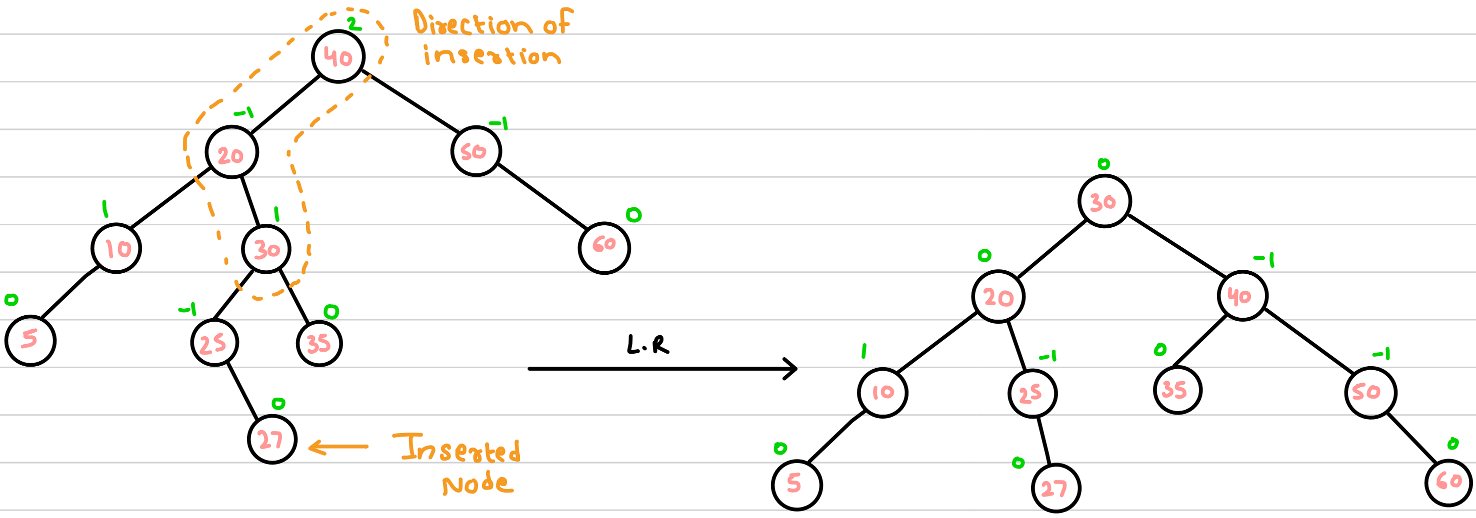






FORMULA OF ROTATION FOR INSERTION

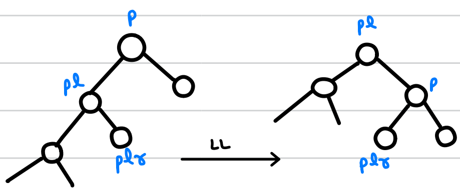




PROGRAM FOR LL ROTATION

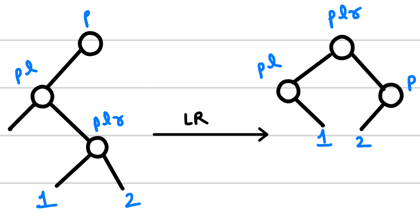
```
struct Node
{
      struct Node *lchild;
      int data;
      int height;      // we will set height for
      struct Node *rchild; each and every Node
} *root = NULL;

struct Node* Insert(Node *p, int key)
{
      if(p==NULL)
      {
            t = new Node;
            t -> data = key;
            t -> height = 1;
            t -> lchild = t -> rchild = NULL;
            return t;
      }
      if ( key < p -> data)
            p -> lchild = insert (p -> lchild, key);
      else if ( key > p -> data)
            p -> rchild = insert (p -> rchild, key);
      p -> height = NodeHeight(p);   // update height of each
                                     node at returning time
```

```
    if (BalanceFactor(p)==2 && BalanceFactor(p->lchild)==1)
            return LLRotation(p);
    else if (BalanceFactor(p)==2 && BalanceFactor(p->rchild)==-1)
            return LRRotation(p);
    else if (BalanceFactor(p)==-2 && BalanceFactor(r->child)==-1)
            return RRRotation(p);
    else if (BalanceFactor(p)==-2 && BalanceFactor(r->child)==1)
            return RLRotation(p);

    return p;
}

int NodeHeight(struct Node *p)
{
    int hl, hr;   // Height of left subtree (HL), height of right subtree (HR)
    hl = p && p->lchild ? p->lchild->height : 0;
    hr = p && p->rchild ? p->rchild->height : 0;
      NOT NULL
    return hl+hr ? hl+1 : hr+1;
}

int BalanceFactor(struct Node *p)
{
    int hl, hr;
    hl = p && p->lchild ? p->lchild->height : 0;
    hr = p && p->rchild ? p->rchild->height : 0;

    return hl - hr;
}

struct Node * LLRotation(struct Node *p)
{
    struct Node *pl = p->lchild;
    struct Node *plr = pl->lchild;

    pl->rchild = p;
    p->lchild = plr;
    p->height = NodeHeight(p);
    pl->height = NodeHeight(pl);
```



```
    if(root == p)      // If rotation was performed on
        root = pl;     //    root node, root needs to be
                       //    updated.
    return pl;
}
```

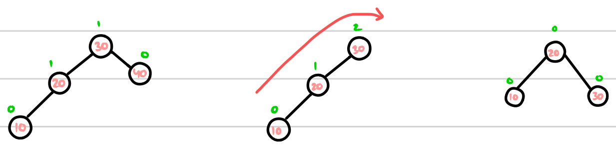
```
struct Node* LRRotation(struct Node *p)
{
    struct Node *pl = p->lchild;
    struct Node *plr = pl->rchild;

    pl->rchild = plr->lchild;
    p->lchild = plr->rchild;

    plr->lchild = pl;
    plr->rchild = p;

    pl->height = NodeHeight(pl);
    p->height = NodeHeight(p);
    plr->height = NodeHeight(plr);

    if (root == p)
        root = plr;
    return plr; // New root;
}
```



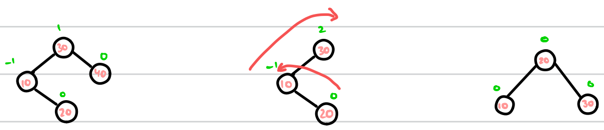
DELETION FROM AVL TREES WITH ROTATION

1. L 1 Rotation
2. L -1 Rotation
3. L 0 Rotation
4. R 1 Rotation
5. R -1 Rotation
6. R - 0 Rotation

L1

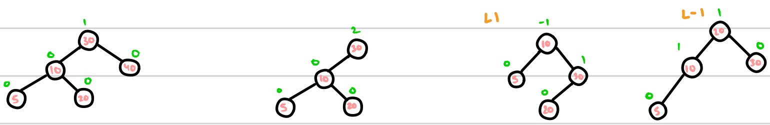


L-1



// Other three will be mirror images

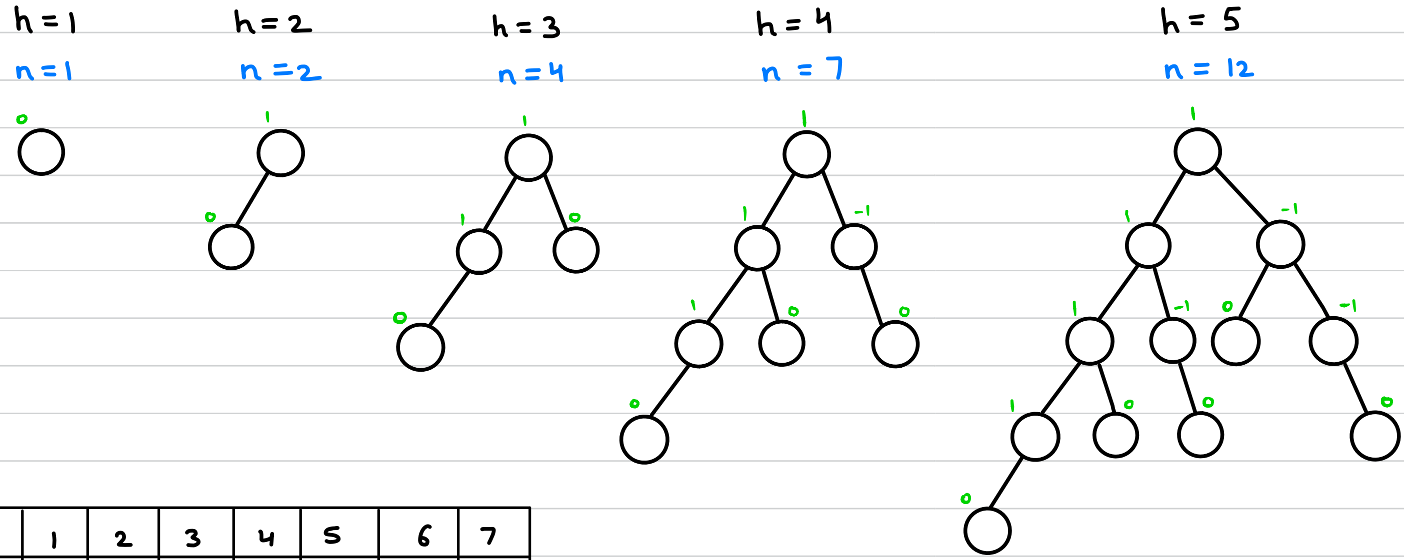
L-0



HEIGHT VS NODES OF AVL TREES

If height is given :

- Max nodes $n = 2^h - 1$ // Not $2^{h+1} - 1$ because height is starting from 1.
- Min nodes n = Look in table



h	1	2	3	4	5	6	7
n	1	2	4	7	12	20	33

+ +1

$$N(h) = \begin{cases} 0 & 0 \\ 1 & 1 \\ N(h-2) + N(h-1) + 1 & \text{otherwise} \end{cases}$$

// formula same as Fibonacci series

↳ balanced series

If 'N' Nodes are given find:

- Min height = $\log_2(n+1)$
- Max height = Look in table

Foreg for 13 nodes h = 5 // Look from node towards height

19 nodes h = 5