



HASHING TECHNIQUE

• Why hashing?

There are three methods of searching

1. Linear Search - $O(n)$
2. Binary Search - $O(\log n)$ // But in binary search, elements must be arranged in sorted order, so we have to do some extra work
3. Hashing - $O(1)$

→ It is the fastest method, but requires very large space as it depends on maximum element

DRAWBACK

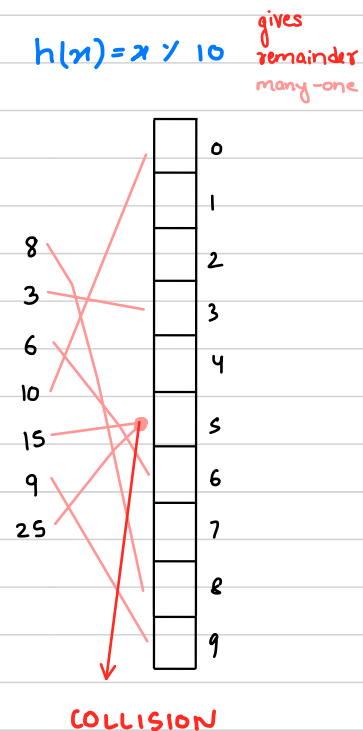
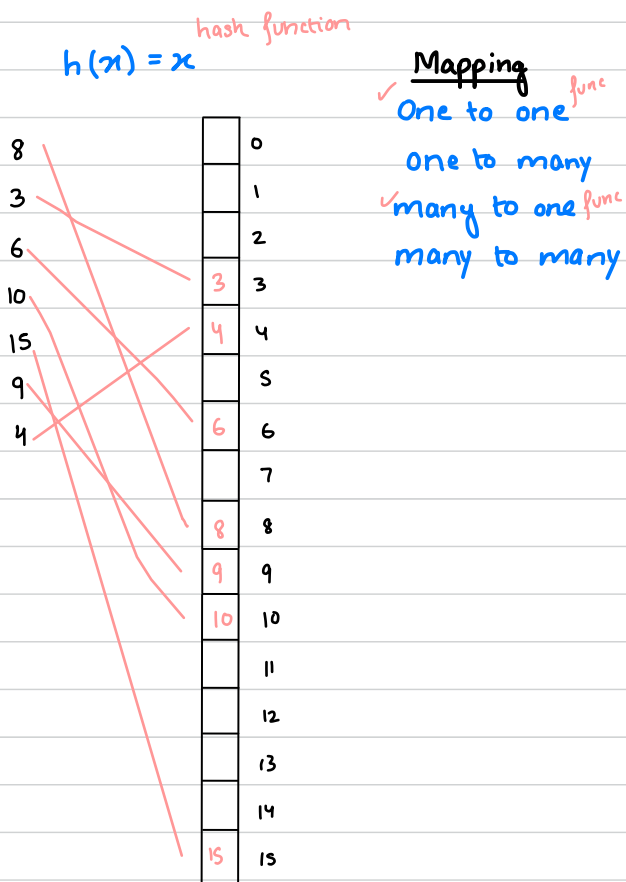
A

3	6	8	8	10	12	15	15	15	20
0	1	2	3	4	5	6	7	8	9

C

0	0	0	3	0	0	6	0	8	0	10	0	12	0	0	15	0	0	0	0	20
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

• Ideal Hashing



• How to avoid COLLISION?

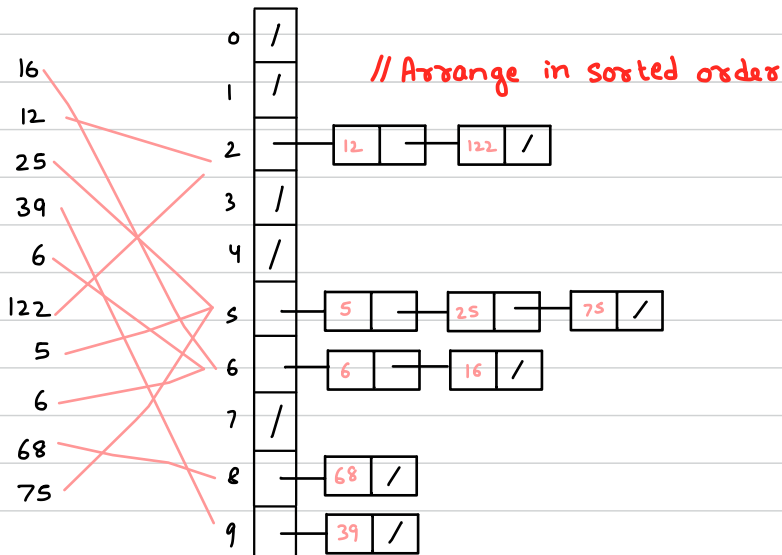
Open Hashing // NO space restriction

- Chaining

Closed Hashing // Limited Space

- Open Addressing // If collision, store element elsewhere in free space
 1. Linear Probing
 2. Quadratic Probing
 3. Double Hashing
- Methods for storing in free space

• CHAINING



Arrange on basis of first digit

Average Successful Search

$$h(x) = x \% 10 \leftarrow \text{Hash Function}$$

$$t = 1 + \frac{1}{2}$$

$n = 100$	No of keys
size = 10	No of index
$\lambda = \frac{n}{\text{size}}$	LOADING FACTOR

Average Unsuccessful Search

$$t = 1 + \lambda$$

If keys are : 5, 35, 95, 145, 175, 265, 845

Then modify your hash function

PROGRAM

```
void Insert ( struct Node *H[], int key)
{
    int index = hash(key);
    SortedInsert ( &H[index], key);
}
```

```
struct Node
{
    int data;
    struct Node *next;
};
```

```
int hash(int key)
{
    return key % 10;
}
```

```
void SortedInsert ( struct Node **H, int x)
{
    struct Node *t, *q = NULL, *p = *H;
    t = (struct Node *) malloc ( sizeof (struct Node));
    t->data = x;
    t->next = NULL;

    if (*H = NULL)
        *H = t;
    else
    {
        while ( p && p->data < x)
        {
            q = p;
            p = p->next;
        }

        if ( p == *H)
        {
            t->next = *H;
            *H = t;
        }
        else
        {
            t->next = q->next;
            q->next = t;
        }
    }
}
```

```

void main()
{
    struct Node * HashTable[10], * temp;
    int i;

    for (i = 0; i < 10; i++)
        HashTable[i] = NULL;

    Insert (HashTable, 12);
    Insert (HashTable, 22);
    Insert (HashTable, 42);

    temp = Search (HashTable[hash(21)], 21);
}

```

```

struct Node * Search (struct Node *p, int key)
{
    while (p != NULL)
    {
        if (key == p->data)
            return p;
        else
            p = p->next;
    }
    return NULL;
}

```

LINEAR PROBING

$$h(x) = x \% 10$$

	30	0
26	29	1
30		2
45	23	3
23	43	4
25	45	5
43	26	6
74	25	7
19	74	8
29	19	9

$$h'(x) = (h(x) + f(i)) \% 10$$

$$f(i) = i$$

where $i = 0, 1, 2, \dots$

// For probing

In case of collision,
insert at next free space

$$\begin{aligned} h'(25) &= (h(25) + f(0)) \% 10 \\ &= (5 + 0) \% 10 = 5 \end{aligned}$$

$$\begin{aligned} h'(25) &= (h(25) + f(1)) \% 10 \\ &= (5 + 1) \% 10 = 6 \end{aligned}$$

$$\begin{aligned} h'(25) &= (h(25) + f(2)) \% 10 \\ &= (5 + 2) \% 10 = 7 \end{aligned}$$

$$\begin{aligned} h'(29) &= (h(29) + f(0)) \% 10 \\ &= (9 + 0) \% 10 = 9 \\ &= (9 + 1) \% 10 = 0 \\ &= (9 + 2) \% 10 = 1 \end{aligned}$$

Cyclic behaviour

For searching: key : 45 ✓ $45 \% 10 = 5$ found at index 5

key : 74 ✓ $74 \% 10 = 4$

- Not found at index 4
- Search Next index until 74 is found or blank space is found
- Found at index 8

// Search takes more than constant time

Key : 40^x $40 \% 10 = 0$

- Not found at index 0
- At index 2, blank space found
- Element does not exist

ANALYSIS

$$\lambda = \frac{n}{\text{Size}} = \frac{9}{10} = 0.9$$

DRAWBACK

** $\lambda \leq 0.5$ // Table should be at most half filled so, there are blank spaces and searching can be made faster

Average Successful Search

$$t = \frac{1}{\lambda} \ln \left(\frac{1}{1-\lambda} \right)$$

Average Unsuccessful Search

$$t = \frac{1}{1-\lambda}$$

DELETION IS NOT EASY IN LINEAR PROBING

PROGRAM

```
int hash(int key)
{
    return key % 10;
}
```

```
int probe(int H[], int key)
{
    int index = hash(key);
    int i = 0;

    while (H[(index + i) % 10] != 0)
        i++;
    return (index + i) % 10;
}
```

```
void Insert(int H[], int key)
{
    int index = hash(key);
    if (H[index] != 0)
        index = probe(H, key);
    H[index] = key;
}
```

```
int Search(int H[], int key)
{
    int index = hash(key);
    int i = 0;

    while (H[(index + i) % 10] != key)
        i++;
    return (index + i) % 10;
}
```

```
void main()
{
```

```
    struct Node * HashTable[10], * temp;
    int i;
```

```
    for (i = 0; i < 10; i++)
        HashTable[i] = NULL;
```

```
    Insert(HashTable, 12);
    Insert(HashTable, 22);
    Insert(HashTable, 42);
```

```
    temp = Search(HashTable, 35);
}
```

QUADRATIC PROBING

The drawback of linear probing is that elements cluster together and form a group. To resolve this issue, quadratic probing is introduced.

	0
	1
	2
23	3
43	4
13	5
27	6
	7
	8
	9

$$h'(x) = (h(x) + f(i)) \cdot 10 \quad \text{where } f(i) = i^2$$

$i = 0, 1, 2, \dots$

$$\begin{aligned} h'(23) &= (h(23) + f(0)) \cdot 10 \\ &= (3 + 0) \cdot 10 = 3 \end{aligned}$$

Average Successful Search

$$\frac{-\log_e(1-\lambda)}{\lambda}$$

$$\begin{aligned} h'(43) &= (h(43) + f(0)) \cdot 10 \\ &= (3 + 0) \cdot 10 = 3 \\ &= (3 + 1) \cdot 10 = 4 \end{aligned}$$

Average Unsuccessful Search

$$\frac{1}{1-\lambda}$$

$$\begin{aligned} h'(13) &= (h(13) + f(2)) \cdot 10 \\ &= (3 + 4) \cdot 10 = 7 \end{aligned}$$

DOUBLE HASHING

	0
	1
	2
	3
	4
	5
	6
	7
	8
	9

$$h_1(x) = x \cdot 10$$

$$h_2(x) = R - (x \cdot R)$$

where R is prime num smaller than size of Hash Table

$$h'(x) = (h_1(x) + i \cdot h_2(x)) \cdot 10$$

where $i = 0, 1, 2, \dots$

$$\begin{aligned} h'(25) &= (5 + 1 \cdot 3) \cdot 10 = 8 \\ &7 - (25 \cdot 7) \\ &7 - 4 = 3 \end{aligned}$$

$$\begin{aligned} h'(15) &= (5 + 1 \cdot 6) \cdot 10 = 1 \\ &7 - (15 \cdot 7) \\ &7 - 1 = 6 \end{aligned}$$

$$\begin{aligned} h'(35) &= (5 + 1 \cdot 7) \cdot 10 = 2 \\ &7 - (35 \cdot 7) \\ &7 - 0 = 7 \end{aligned}$$

$$\begin{aligned} h'(95) &= (5 + 3 \cdot 3) \cdot 10 = 4 \\ &7 - (95 \cdot 7) \\ &7 - 4 = 3 \end{aligned}$$