-x-x-x-

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1. Simulate the performance of Huffman coding for given numerical.
Consider a DMS, $S = (S1, S2, S3, S4)$ with probability $[0.5, 0.2, 0.2, 0.1]$. Encode the source using Huffman algorithm. Find entropy, average codeword length and efficiency Huffman.

→

Symbol	Probability	I	II	codeword	length
S1	0.5	0.5	0.5	1	1
S2	0.2	0.3	0.5	01	2
S3	0.2	0.2		000	3
S4	0.1			001	3

1. Average codeword length

$$L = \sum_{k=1}^{m} P_k \times \text{length of message}$$

$$= (0.5\times1) + (0.2\times2) + (0.2\times3) + (0.1\times3)$$

$$L = 0.5 + 0.4 + 0.6 + 0.3$$

$\boxed{L = 1.8}$ bits/msg

2. Entropy

$$H = \sum_{k=1}^{m} P_k \log_2\left(\frac{1}{P_k}\right)$$

$$= 0.5\log_2\frac{1}{0.5} + 0.2\log_2\frac{1}{0.2} + 0.2\log_2\frac{1}{0.2} + 0.1\log_2\frac{1}{0.1}$$

$$= 0.5 + 0.464 + 0.464 + 0.332$$

$\boxed{H = 1.76}$ bits/msg

3. Code efficiency

$$\eta = \frac{H}{L} = \frac{1.76}{1.8} = 0.977$$

$$= 97.7$$

4. Redundancy of code

$$\gamma = 1 - \eta$$

$$= 1 - 97.7$$

$$= 2.3\%$$

(10) LBC

- Simulate the linear block codes for given numerical. Obtain codeword for (6,3) LBC which has generator matrix $G = [100101; 010011; 001110]$. Find all possible codeword. Obtain corrected codeword if received codeword is 001110.

→ LBC :- (6,3). This mean

$n = 6 \quad q = 3$

$k = 3$

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

1 1 1 0 0 0

$G = [I_k : P_{k \times q}]$

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$c = [M][G]$

message m		Codeword [$c = m.G$]	
000	$0 \times r_1 + 0 \times r_2 + 0 \times r_3 = 0$	000000	c_1
001	r_3	001110	c_2
010	r_2	010011	c_3
011	$r_2 + r_3$ 010011 + 001110 = 011101	011101	
100	r_1	100101	
101	$r_1 + r_3$	101011	
110	$r_1 + r_2$	110110	
111	$r_1 + r_2 + r_3$	111000	

R = 001110

Hamming dis $d(R, C)$

$d(R, C_1 = 000000) = 001110 \text{ XOR } 000000$

$= 001110 \rightarrow \text{Distance} = 3$

$d(R\,C_2 = 001110) = 001110 \text{ XOR } 001110$

$= 000000 \rightarrow \text{Distance} = 0$

$d(R\,C_3 = 010011) = 001110 \text{ XOR } 010011$

$= 011101 \rightarrow \text{Distance} = 4$

The minimum hamming distance is 0, which occurs with codeword $c_2 = 00110$

(9)

2. Find all entropies, mutual information of channel when channel matrix is given as
$P(y/x) = (0.7, 0.3; 0.3, 0.7]$
given $P(x_1) = 0.6$, $p(x_2) = 0.4$

→

$$P(y/x) = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix} \qquad P(x_1) = 0.6 \quad P(x_2) = 0.4$$

① $H(x)$

$$= -P(x_1)\log_2(x_1) - P(x_2)\log_2(x_2)$$
$$= -0.6\log_2(0.6) - 0.4\log_2(0.4)$$
$$= 0.4421 + 0.5287$$
$$H(x) = \underline{\underline{0.97087}}$$

② $H(y)$, $H(x, y)$

$$P(y/x) = \begin{array}{c|cc} & y_1 & y_2 \\ x_1 & 0.7 & 0.3 \\ x_2 & 0.3 & 0.7 \end{array}$$

$P(x_1 y_1) = x_1 \cdot (y_1/x_1) = 0.6 \times 0.7 = 0.42$
$p(x_1 y_2) = x_1 \cdot (y_2/x_1) = 0.6 \times 0.3 = 0.18$
$P(x_2 y_1) = x_2 \cdot (y_1/x_2) = 0.4 \times 0.3 = 0.12$
$p(x_2 y_2) = x_2 \cdot (y_2/x_2) = 0.4 \times 0.7 = 0.28$

$$P[x\,y] = \begin{array}{c|cc} & y_1 & y_2 \\ x_1 & 0.42 & 0.18 \\ x_2 & 0.12 & 0.28 \end{array}$$

$P(y_1) = P(x_1 y_1) + P(x_2 y_1) = 0.42 + 0.12$
$= 0.54$

$P(y_2) = P(x_1 y_2) + P(x_2 y_2) = 0.18 + 0.28$
$= 0.46$

$\underline{H(y)} = -P(y_1)\log_2 P(y_1) - P(y_2)\log_2 P(y_2)$
$= -0.54\log_2(0.54) - 0.46\log_2(0.46)$
$= 0.4800 + 0.5153$
$= 0.9953 \approx 1$ bit/message

(iii) $H(x,y) = -\sum_{j=1}^{m} \sum_{k=1}^{n} P(x_j\, y_k) \log_2 P(x_j\, y_k)$

$= -0.42 \log_2 (0.42) - 0.18 \log_2 (0.18)$
$\quad - 0.12 \log_2 (0.12) - 0.28 \log_2 (0.28)$

$= 0.52564 + 0.4453 + 0.3670 + 0.5142$

$H(x,y) = \underline{\underline{1.85214}}$

(iv) $H(x/y) = H(xy) - H(y)$
$= 1.85214 - 1$
$= 0.85214$ bits/message

(v) $H(y/x) = H(xy) - H(x)$
$= 1.85214 - 0.9708$
$= 0.88134$ bits/message

$I(x:y) = H(x) - H(x/y)$
$= 0.9708 - 0.85214$
$= \underline{\underline{0.11866}}$