Model predictive control for horizontal bipedal locomotion

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MATHÉMATIQUES VISION APPRENTISSAGE

Zero Moment Point

At the Zero Moment Point (ZMP), the moments of inertial and external forces applied to the robot are null.

For a biped walking-robot, this point is the center of pressure (CoP) and must rely inside the convex hull of the "feet" to ensure the stability of the robot.

The ZMP approximation naturally decouples the forward and lateral motion and can be used in a Previous Control scheme.

ZMP equation

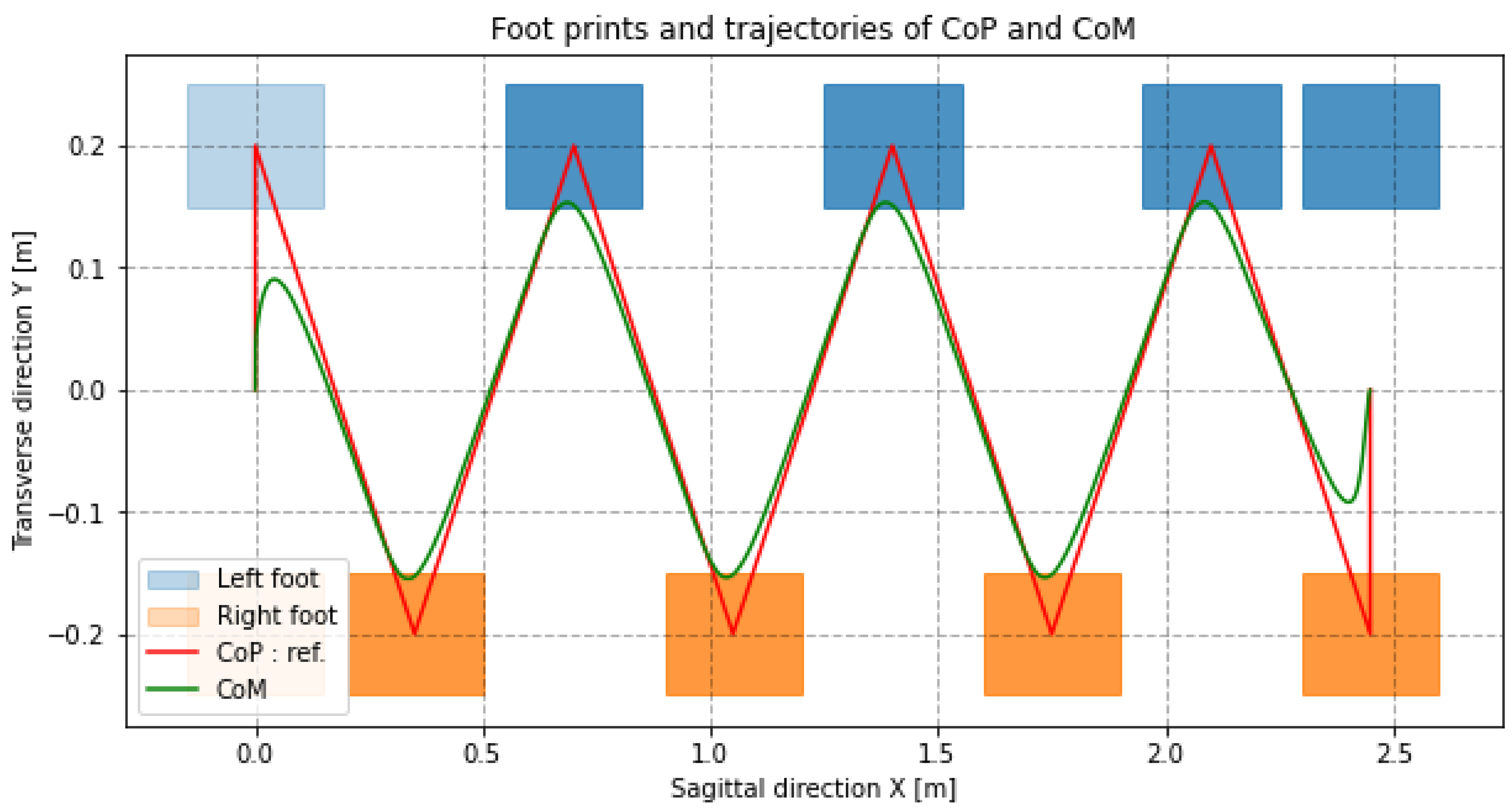
$$\vec{z} = \vec{x} - \frac{h_{COM}}{g}\ddot{\vec{x}}$$

$\vec{g}$: *Gravity force*

h_{COM} : *Height of the center of mass*

ZMP hypotheses

- The inertial effects of the moving parts of the robot are neglected
- The CoM doesn't move vertically
- No perturbation forces



Zero Moment Point and Previous Control

Model representation

$$\hat{x}_k = \begin{pmatrix} x(kT) \\ \dot{x}(kT) \\ \ddot{x}(kT) \end{pmatrix}$$

$$\hat{x}_{k+1} = \underbrace{\begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}}_{A} \hat{x}_k + \underbrace{\begin{bmatrix} T^3/6 \\ T^2/2 \\ T \end{bmatrix}}_{B} \dddot{x}_k$$

$$z_k = \begin{bmatrix} 1 & 0 & -\frac{h_{com}}{g} \end{bmatrix} \hat{x}_k$$

Parameters

$\vec{g}$	: *Gravity force*	*(9,81 m/s²)*
h_{COM}	: *Height of the center of mass*	*(0,8 m)*
T	: Time step for discretization	*(5 ms)*
N	: Number of planned steps	*(200)*
R/Q	: Regularization ratio	*(10⁻⁵)*

Analytical numerical scheme

$$Z_{k+1} = P_x\hat{x}_k + P_u\dddot{X}_k$$

$$\min_{\dddot{X}_k} \frac{1}{2}Q\left\|Z_{k+1} - Z_{k+1}^{ref}\right\|^2 + \frac{1}{2}R\left\|\dddot{X}_k\right\|^2$$

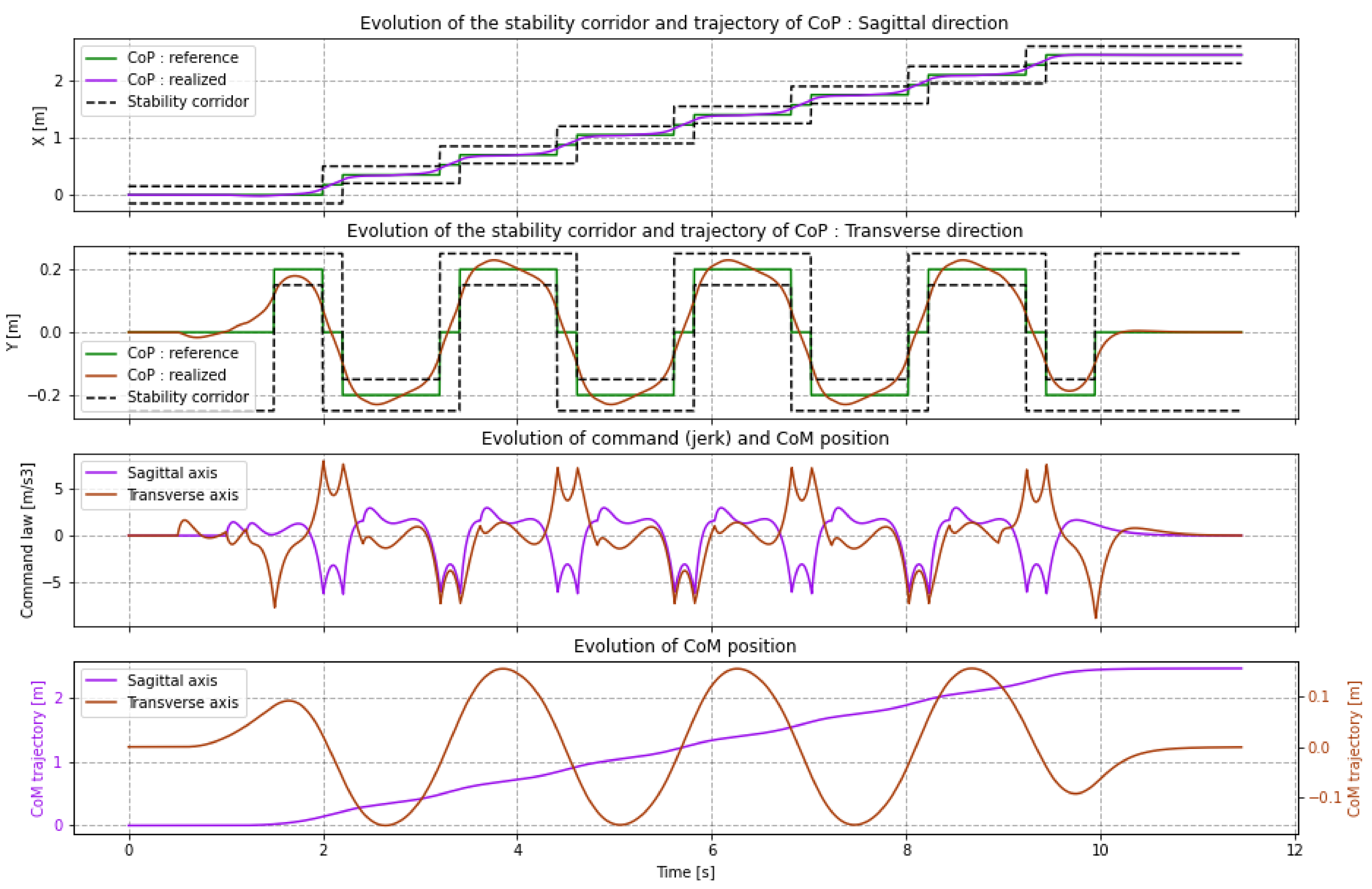
$$\dddot{\mathbf{X}}_k = -\left(\mathbf{P}_u^T\mathbf{P}_u + \frac{\mathbf{R}}{\mathbf{Q}}\mathbf{I}_{NxN}\right)^{-1}\mathbf{P}_u^T\left(\mathbf{P}_x\hat{\mathbf{x}}_k - \mathbf{Z}_k^{ref}\right)$$

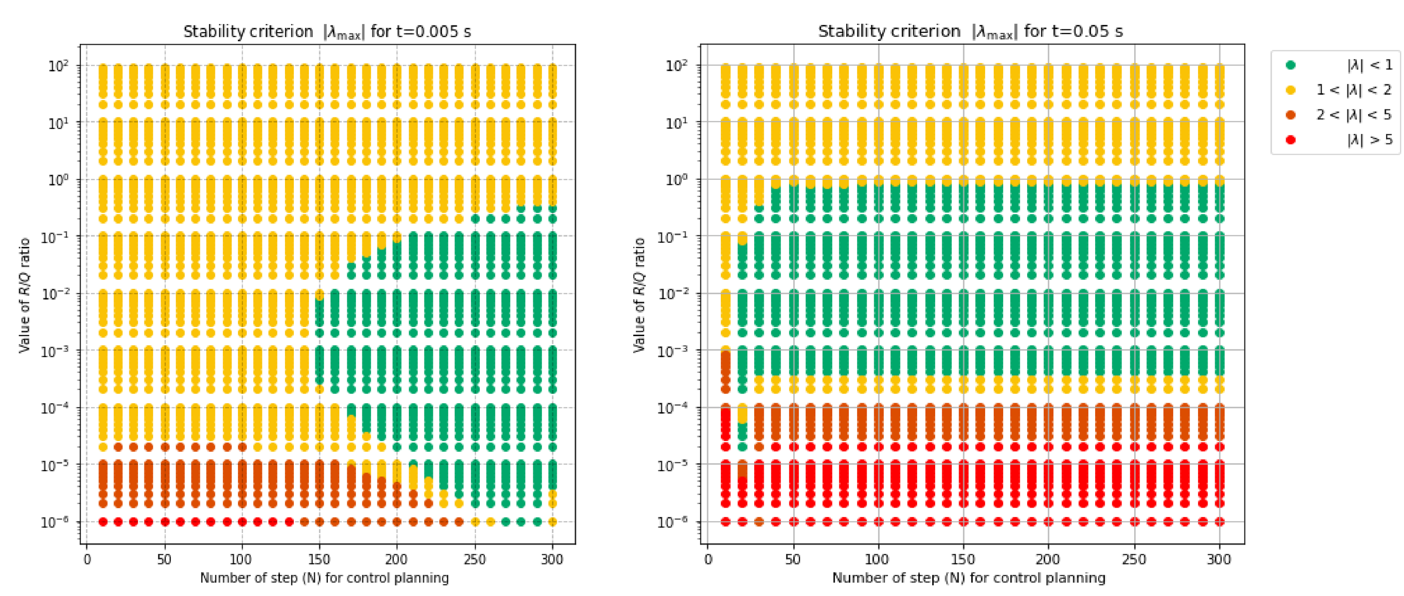
Algorithm

1. **"Off-line" computation** :
   P_u and P_x are computed after selection of N and T parameters.
   Z_k^{ref} is computed in accordance with a walking pattern.
2. **Previous control** :
   At step k, the command (jerk) is computed thanks to predictive control knowing Z_k^{ref} and $\hat{x}_k$ along N steps .
3. **Command**:
   The command applied is selected as the first value computed : $\dddot{x}_k = \dddot{X}_k[0]$
4. **Update the model** :
   The robot states are updated thanks to the relation $\hat{x}_{k+1} = A\hat{x}_k + B\dddot{x}_k$

Stability

1. **Control scheme** :
   It is tuned by selecting the regularization ratio R/Q and parameters N and T. The stability is evaluated by computing the eigen values of the following matrix : $A - Be^T\left(P_u^TP_u + \frac{R}{Q}I_{NxN}\right)^{-1}P_u^T P_x$
2. **Robot**:
   The constraints on the position of the CoP can only be checked a posteriori.





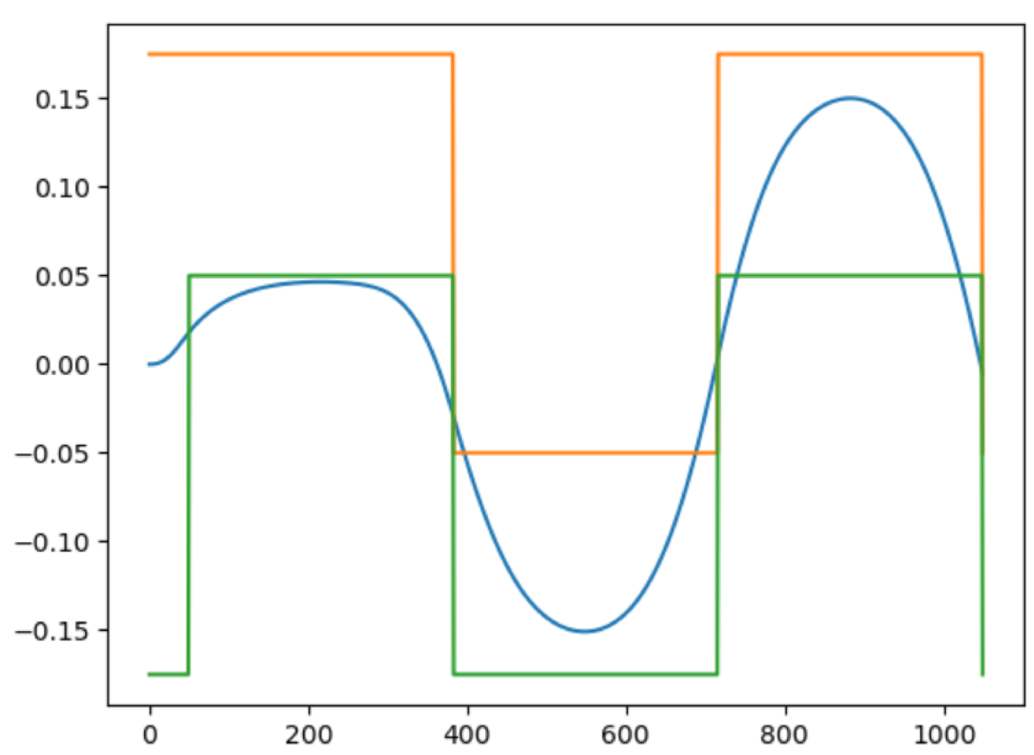
EXTENSION : Explicit constraints

The ZMP Preview Control scheme can be modified to explicitly take into account the stability constraints on the position of the CoP. By solving a Quadratic Program with inequality constraints, the stability conditions are directly used for the computation of the jerk command.

ZMP Preview Control scheme with QP

$$\min_{\dddot{X}_k} \frac{1}{2}\left\|\dddot{X}_k\right\|^2$$

$$\text{s.t} \quad Z_k^{min} \leq Z_k \leq Z_k^{max}$$



EXTENSION : Dealing with perturbations

Robot can be submitted to perturbations during the walking pattern. We compute the robot behavior under perturbations modeled by a Dirac delta function on the transverse velocity of the robot.

This model is consistent with the impact of a ball hitting the robot when considering an instantaneous elastic choc and a negligeable lateral velocity of the robot.

Dirac delta function on the lateral velocity

Kinetic energy theorem for the mechanical system $\Sigma = \{Robot + Ball\}$

$$\Delta E_{c,ball} + \Delta E_{c,robot} = W_{int} + W_{ext}$$

Isolated mechanical system:

$$W_{ext} = 0$$

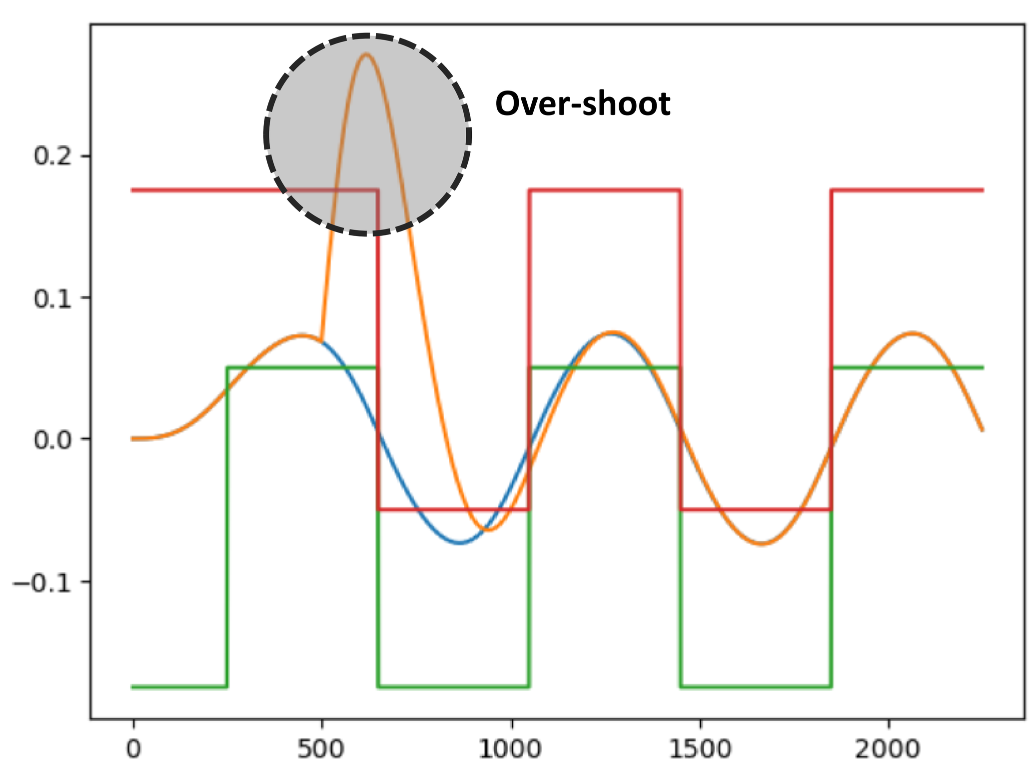
Elastic choc:

$$W_{int} = 0$$

$$\Delta E_{c,ball} = -\Delta E_{c,robot}$$

Numerical application:

$$\frac{M_{ball}}{M_{robot}} = 0{,}04 \quad ; \quad h = 0{,}5m \quad ; \quad \Delta v_{robot} = 0{,}6\ m\ s^{-1}$$



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