Project Part 2

Unsupervised Learning (K-means)

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K-MEANS ALGORITHM

Introduction

K-means algorithm is an unsupervised learning algorithm used for clustering. It divides the data points into k-clusters of similar objects. A cluster is a collection of data objects that are similar to one another within the same cluster and are dissimilar to the objects in other clusters. Similarity is measured by calculating the distance (Euclidean Distance).

Objective

To implement the k-means algorithm on the given dataset, which contains a set of 2-D points, using to different strategies for choosing the initial cluster centres.

- Strategy 1: randomly pick the initial centres from the given samples.
- Strategy 2: pick the first centre randomly; for the ith centre (i>1), choose a sample (among all possible samples) such that the average distance of this chosen one to all previous (i-1) centres is maximal.
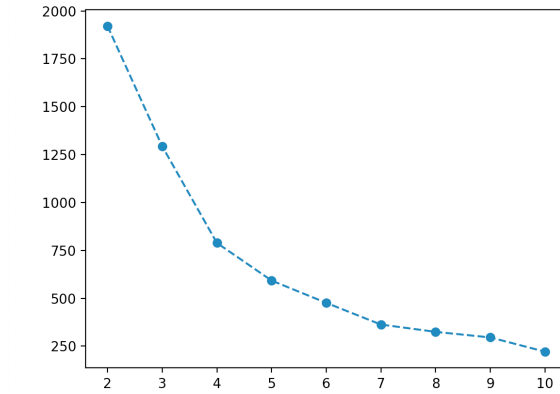
Formula:

1) Euclidean distance $(D) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2) Strategy 2 = $argmax_i \frac{1}{n} \sum_j D(C_i, C_j)$ where 'i' represents the point index of the new center, j ∈ n represents the point index of the existing centres
3) Objective function = $\sum_{i=1}^{k} \sum_{x \in D_i} ||x - \mu_i||^2$ [D represents the data set and μ_i is the centroid]

Pseudo Code (Algorithm)

- Given: dataset and a number of k (ranging from 2 to 10)
- Initialize centroids (either strategy 1 or strategy 2)
  - <u>Do</u> classify data points according to the nearest centroid (μ_i)
  - Recompute μ_i
  - <u>Until</u> they converge (no change)
- Return centroid list

Result

STRATEGY 1

-- K = 2 --
Initial Centroid List : [(3.1308866885623967, 6.301357109231977), (3.7500464678370093, 4.900701139147971)]
Final Centroid List : [(4.8526119302871174, 7.271641711202775), (5.000562341588762, 2.4854274751531933)]
******* Objective Function Value : 1921.0334858562055

-- K = 3 --
Initial Centroid List : [(8.757548451892736, 8.817454409709498), (3.0999940868022384, 0.8385498984419097), (3.1834039230512237, 5.421840131568017)]
Final Centroid List : [(6.497249620849709, 7.522972929895171), (5.477400388679034, 2.254981027984723), (2.5614644894663545, 6.088613382895479)]
******* Objective Function Value : 1293.7774523911348

-- K = 4 --
Initial Centroid List : [(2.641451406258703, 2.6220682177926107), (2.3643033531884674, 1.0520971253700746), (3.252246409117296, 2.4788534037773373), (7.529630088974836, 8.796171120440778)]
Final Centroid List : [(2.8337661044723124, 6.918956898933875), (3.2428534716041297, 2.551979054939448), (7.252626831256577, 2.4001582635520533), (6.579576431887845, 7.57333594820388)]
******* Objective Function Value : 788.9645806635216

-- K = 5 --
Initial Centroid List : [(6.856532248221895, 7.724688252918387), (4.807540932988571, 3.0346495391326473), (4.783632112114197, 7.106442879583119), (6.633523323424615, 0.9802070492997959), (3.009328298642023, 1.4506571693366839)]
Final Centroid List : [(7.299749694765005, 8.413318379491713), (5.339072124427115, 4.465511754739297), (3.1507276114491836, 7.121929064073764), (7.556167822397726, 2.235167959857534), (2.681986334188929, 2.094615867800809)]
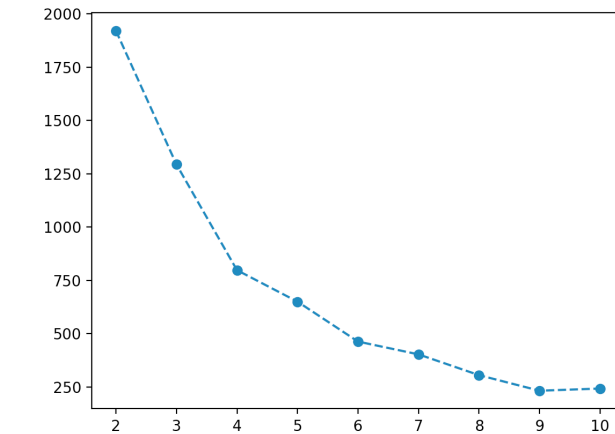******* Objective Function Value : 476.1187516763532

-- K = 6 --
Initial Centroid List : [(8.031502051193474, 8.883813536823052), (7.888286940839 8, 8.410931245447076), (3.245166109622 2475, 0.8218365040472543), (2.8744890711352955, 2.6575989995970533), (6.111068508792309, 6.234975548756099), (4.660059312873503, 7.060595553422103)]
Final Centroid List : [(7.756483249146484, 8.556689279063415), (5.464277356727894, 6.837713536435891), (3.145061482959145, 0.9077065486588153), (3.4955665791995627, 3.5661123157286907), (7.414192434680615, 2.3216911383868664), (2.5633381461259046, 6.978224800606624)]
******* Objective Function Value : 362.8660888144436

-- K = 7 --
Initial Centroid List : [(3.2492998038878316, 5.591251713423602), (6.58072120112474, -0.07668239503630714), (1.69565649157431, 7.680824581467082), (3.852121462721742, -1.0871522588579658), (4.660059312873503, 7.060595553422103), (5.609442417811811, 2.9132703195678546), (3.1308866885623967, 6.301357109231977)]
Final Centroid List : [(2.214068054222364, 3.286445548570339), (7.556167822397726, 2.235167959857534), (2.541652523310345, 7.002678323536489), (3.1660397272577874, 0.8616886118323305), (7.914309977818314, 8.519909807700076), (5.2305366674047375, 4.279342496049099), (5.2402829638043125, 7.531310293267821)]
******* Objective Function Value : 362.8660888144436

-- K = 8 --
Initial Centroid List : [(3.0410170219628694, -0.3613848735372547), (1.966339226546456, 7.308450380099507), (2.2351836490990076, 3.772182518528192), (6.470118291290549, 5.540355428924968), (2.2579084512260508, 7.447780032003513), (1.0521742722626426, 3.8894374123577706), (6.099526960462512, 9.017861398130798), (2.4817741956340473, 1.6740254700990456)]
Final Centroid List : [(7.2602424750027055, 1.6505279666227712), (2.541652523310345, 7.002678323536489), (4.915140555577336, 4.029578974958319), (7.594369019401092, 4.021929553923636), (5.229435980984225, 7.52711249580731), (2.214068054222364, 3.286445548570339), (7.914309977818314, 8.519909807700076), (3.1660397272577874, 0.8616886118323305)]
******* Objective Function Value : 324.7101186384023

-- K = 9 --
Initial Centroid List : [(6.404831490693028, 5.605780843211222), (3.3540983769674098, 5.796037225992769), (2.1664174325210266, 2.9941463698877517), (1.7261440794317036, 6.8181940721178975), (1.73949419226726, 7.460858443985074), (1.69565649157431, 7.680824581467082), (5.36626615294954, 6.514342305587313), (1.9256185314039267, 2.7385763244050594), (5.0725075355427425, 7.898340476116463)]
Final Centroid List : [(7.556167822397726, 2.235167959857534), (4.96399532561623, 3.5936668284791367), (2.242047519125402, 3.251007486318421), (3.138347677015053, 5.933723217937643), (1.9859396300104464, 7.283217354058579), (2.5448854407360666, 8.474169510238502), (5.464277356727894, 6.837713536435891), (3.1690614508664035, 0.8143251472991676), (7.756483249146484, 8.556689279063415)]
******* Objective Function Value : 295.55978170822897

-- K = 10 --
Initial Centroid List : [(8.221446282273055, 8.60551337360542), (6.678126202986436, 1.1080156972424497), (3.1308866885623967, 6.301357109231977), (4.957286957556451, 6.908979840290732), (4.951859575817064, 4.117566936783895), (3.8427898922439203, 5.535466947744628), (5.7714422285783105, 9.040753944871518), (3.7261084438667154, 5.2043243906921255), (3.5446126706973637, 0.9426188179429711), (2.5366923982985132, 2.2422267166268712)]
Final Centroid List : [(7.914309977818314, 8.519909807700076), (7.556167822397726, 2.235167959857534), (2.183214622031253, 7.703553409180906), (6.15468228405522, 5.701407205679413), (5.183123899476708, 3.2140910620351697), (3.0604227545573677, 6.029718158624873), (5.021776597933489, 7.8240125818092165), (4.331939478230939, 4.336266052296369), (3.1690614508664035, 0.8143251472991676), (2.242047519125402, 3.251007486318421)]
******* Objective Function Value : 221.304452877737

-- K = 2 --

Initial Centroid List : [(2.4898969303766347, 8.400478632861432), (3.021056869801129, 9.26213795798542)]
Final Centroid List : [(5.000562341588762, 2.4854274751531933), (4.852611930287174, 7.271641711202775)]
******* Objective Function Value : 1921.0334858562055

-- K = 3 --

Initial Centroid List : [(2.106061618875275, 8.231837694909952), (2.951474415631231, 7.766156048982289), (3.2492998038878316, 5.591251713423602)]
Final Centroid List : [(2.5614644894663545, 6.088613382895479), (6.497249620849709, 7.522972929895171), (5.477400388679034, 2.254981027984723)]
******* Objective Function Value : 1293.7774523911357

-- K = 4 --

Initial Centroid List : [(8.092090169466674, 3.3906505867562244), (5.044700929419418, 8.490601185961385), (2.6587575050634498, 1.7541119002706955), (4.843063281899375, 7.507578949013776)]
Final Centroid List : [(7.2270767273807826, 2.5223436136880415), (7.135607267157955, 7.916517263892731), (3.2125746077046626, 2.496580865799525), (3.392621139896188, 6.892881496955304)]
******* Objective Function Value : 797.9601840789948

-- K = 5 --

Initial Centroid List : [(3.9872431110331834, 4.042547799500241), (2.389526061607529, 7.221955635454229), (4.783632112114197, 7.106442879583119), (3.120736964994129, 0.4897907901976055), (6.792518315497752, 2.5620809523410912)]
Final Centroid List : [(3.4955665791995627, 3.5661123157286907), (2.6756987971688004, 6.982927498982885), (6.489843945177897, 7.558852441314683), (3.145061482959145, 0.9077065486588153), (7.414192434680615, 2.3216911383868664)]
******* Objective Function Value : 650.1928893162575

-- K = 6 --

Initial Centroid List : [(8.007064409166201, 2.7753199731436236), (8.076416518034618, 9.271620015802068), (6.470118291290549, 5.540355428924968), (5.481219654489373, 6.551717768804221), (2.9100822099635435, 7.519439837064963), (4.502364450590697, 2.928880404940474)]
Final Centroid List : [(7.556167822397726, 2.235167959857534), (7.914309977818314, 8.519909807700076), (5.2305366674047375, 4.279342496049099), (5.2402829638043125, 7.531310293267821), (2.541652523310345, 7.002678323536489), (2.681986334188929, 2.094615867800809)]
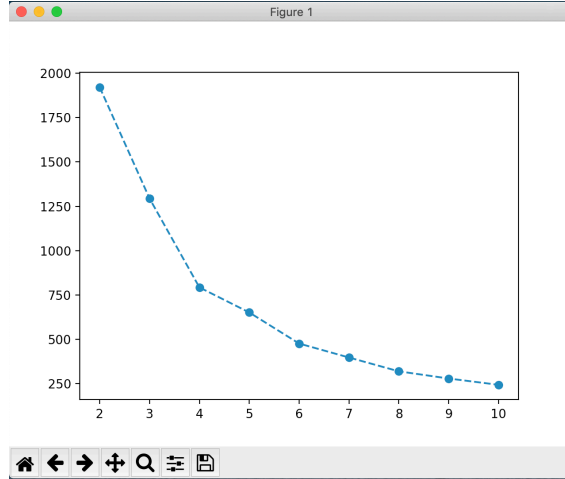******* Objective Function Value : 462.9263558248375

-- K = 7 --

Initial Centroid List : [(3.7500464678370093, 4.900701139147971), (2.5804690660280762, 6.530235492096196), (2.8162902922231297, 3.199972495099669), (5.044700929419418, 8.490601185961385), (6.215390304639243, 6.261392254423516), (7.444728022754429, 2.41849318404437), (2.4099848886739594, 7.991749448627895)]
Final Centroid List : [(4.96399532561623, 3.5936668284791367), (3.00060229054155, 5.782957863293782), (2.736250054522389, 1.9751010338399708), (7.756483249146484, 8.556689279063415), (5.464277356727864, 6.837135361435891), (7.556167822397726, 2.235167959857534), (2.183214622031253, 7.703553409180906)]
******* Objective Function Value : 402.91835862050726

-- K = 8 --

Initial Centroid List : [(6.976905728972921, 7.965091986724786), (2.7845243023650736, 6.618471581818432), (4.807540932988571, 3.0346495391326473), (2.515552088741975, 6.41071773765037), (8.226274849535631, 2.260487012503675), (1.9256185314039267, 2.7385763244050594), (5.7714422285783105, 9.040753944871518), (3.047435882855882, 1.5580763520111716)]
Final Centroid List : [(7.914309977818314, 8.519909807700076), (2.183214622031253, 7.703553409180906), (5.345603315617595, 4.203354778508424), (3.138347677015053, 5.933723217937643), (7.556167822397726, 2.235167959857534), (2.214068054222364, 3.286445548570339), (5.2402829638043125, 7.531310293267821), (3.1660397272577874, 0.8616886118323305)]
******* Objective Function Value : 305.3870732847836

-- K = 9 --

Initial Centroid List : [(8.226274849535631, 2.260487012503675), (4.91330965187404, 3.2577242509859516), (3.5446126706973637, 0.9426188179429711), (8.201293857837541, 9.212915410514553), (2.200114959631794, 1.5386322076504713), (8.094693454952871, 7.791998464329252), (4.758510501654859, 8.243174587119283), (2.078985690973731, 7.167393130690192), (4.729351540291086, 3.808390445364688)]
Final Centroid List : [(7.556167822397726, 2.235167959857534), (4.912514973376666, 3.5631409634126254), (3.1690614508664035, 0.8143251472991676), (7.914309977818314, 8.519909807700076), (2.242047519125402, 3.251007486318421), (6.15468228405522, 5.701407205679413), (5.021776597933489, 7.8240125818092165), (2.183214622031253, 7.703553409180906), (3.138347677015053, 5.933723217937643)]
******* Objective Function Value : 232.34550801111072

-- K = 10 --

Initial Centroid List : [(2.3765062376641364, 8.152417782663814), (3.7500464678370093, 4.900701139147971), (4.05095773912432, 4.052127672114385), (3.0104761241001885, 6.542864553074541), (2.0592490214343924, 7.20598797993189), (4.403047337930657, 4.814343539366011), (7.962800901371823, 2.817612745033582), (7.771269867371587, 8.91428052368416), (8.061602434575416, 4.044232618396686), (6.945115609070664, 8.305179450953418)]
Final Centroid List : [(2.477565190085822, 8.447636655827047), (2.214068054222364, 3.286445548570339), (3.1660397272577874, 0.8616886118323305), (3.2311125632348365, 5.848817638525821), (2.090208729220501, 7.118362869737812), (5.345603315617595, 4.203354778508424), (7.056682932407747, 1.333196794745837), (7.914309977818314, 8.519909807700076), (7.986758244802879, 3.012729309091755), (5.2402829638043125, 7.531310293267821)]
******* Objective Function Value : 242.6388741904175

STRATEGY 2

--- K = 2 --- ----------
Initial Centroid List : [(2.3605714528864787, 6.007966227729477), (8.875780722296865, 8.960923612139595)]
Final Centroid List : [(5.000562341588762, 2.4854274751531933), (4.852611930287174, 7.271641711202775)]
******* **Objective Function Value : 1921.0334858562055**

--- K = 3 --- ----------
Initial Centroid List : [(8.678052765337032, 9.087579163606126), (3.0410170219628694, -0.3613848735372547), (3.021056869801129, 9.26213795798542)]
Final Centroid List : [(6.497249620849709, 7.522972929895171), (5.477400388679034, 2.254981027984723), (2.5614644894663545, 6.088613382895479)]
******* **Objective Function Value : 1293.7774523911348**

--- K = 4 --- ----------
Initial Centroid List : [(1.0521742722626426, 3.8894374123577706), (8.875780722296865, 8.960923612139595), (3.0410170219628694, -0.3613848735372547), (3.021056869801129, 9.26213795798542)]
Final Centroid List : [(3.2285300905383707, 2.5240486292057867), (6.856583330228901, 7.661434199806484), (7.252626831256577, 2.4001582635520533), (3.153427001856, 6.91292070093253)]
******* **Objective Function Value : 792.7110095863353**

--- K = 5 --- ----------
Initial Centroid List : [(8.619479454228491, 2.985983190047101), (3.021056869801129, 9.26213795798542), (3.0410170219628694, -0.3613848735372547), (8.875780722296865, 8.960923612139595), (1.3483715993964611, 3.963796377807688)]
Final Centroid List : [(7.414192434680615, 2.3216911383868664), (2.981943628234257, 7.123400347453532), (2.9587703023677445, 1.432764367064417), (6.881665083281389, 7.756030672686048), (3.91512030145731, 4.0416668866073175)]
******* **Objective Function Value : 651.4033302031506**

--- K = 6 --- ----------
Initial Centroid List : [(3.01047612410011885, 6.542864553074541), (7.597313421987085, 1.1650474267989992), (8.875780722296865, 8.960923612139595), (3.0410170219628694, -0.3613848735372547), (3.021056869801129, 9.26213795798542), (2.200114959631794, 1.5386322076504713)]
Final Centroid List : [(5.464277356727894, 6.837713536435891), (7.414192434680615, 2.3216911383868664), (7.756483249146484, 8.556689279063415), (3.145061482959145, 0.9077065486588153), (2.5633381461259046, 6.9782248006060624), (3.4955665791995627, 3.5661123157286907)]
******* **Objective Function Value : 476.1187516763528**

--- K = 7 --- ----------
Initial Centroid List : [(2.4898969303766347, 8.400478632861432), (7.597313421987085, 1.1650474267989992), (8.875780722296865, 8.960923612139595), (3.0410170219628694, -0.3613848735372547), (3.021056869801129, 9.26213795798542), (2.200114959631794, 1.5386322076504713), (8.619479454228491, 2.985983190047101)]
Final Centroid List : [(2.5633381461259046, 6.9782248006060624), (7.428034259463345, 2.282918666452028), (7.914309977818314, 8.519909807700076), (3.145061482959145, 0.9077065486588153), (5.021776597933489, 7.8240125818092165), (3.4955665791995627, 3.5661123157286907), (6.15468228405522, 5.701407205679413)]
******* **Objective Function Value : 397.44812244413686**

--- K = 8 --- ----------
Initial Centroid List : [(3.7500464678370093, 4.900701139147971), (8.875780722296865, 8.960923612139595), (3.0410170219628694, -0.3613848735372547), (3.021056869801129, 9.26213795798542), (7.597313421987085, 1.1650474267989992), (1.69565649157431, 7.680824581467082), (8.619479454228491, 2.985983190047101), (2.200114959631794, 1.5386322076504713)]
Final Centroid List : [(5.3321424077429915, 4.444481111506235), (7.914309977818314, 8.519909807700076), (3.1690614508664035, 0.8143251472991676), (4.923731090150018, 7.833731542772698), (7.056682932407747, 1.333196794745837), (2.5131744928356072, 6.883061799423911), (7.986758244802879, 3.012729309091755), (2.242047519125402, 3.251007486318421)]
******* **Objective Function Value : 319.47442215977026**

--- K = 9 --- ----------
Initial Centroid List : [(7.578050252952152, 3.8248701734522355), (3.021056869801129, 9.26213795798542), (3.0410170219628694, -0.3613848735372547), (8.875780722296865, 8.960923612139595), (2.200114959631794, 1.5386322076504713), (1.69565649157431, 7.680824581467082), (7.597313421987085, 1.1650474267989992), (7.5976350478966745, 7.9392400048313085), (1.3483715993964611, 3.963796377807688)]
Final Centroid List : [(4.912514973376666, 3.5631409634126254), (4.859398750231154, 7.941638208906054), (3.392397796044764, 0.5065576443944665), (7.914309977818314, 8.519909807700076), (2.5755081335118746, 2.3149966532137327), (2.4631734791294, 6.989095694190998), (7.556167822397726, 2.235167959857534), (6.15468228405522, 5.701407205679413), (2.46204926319718, 4.294396379247468)]
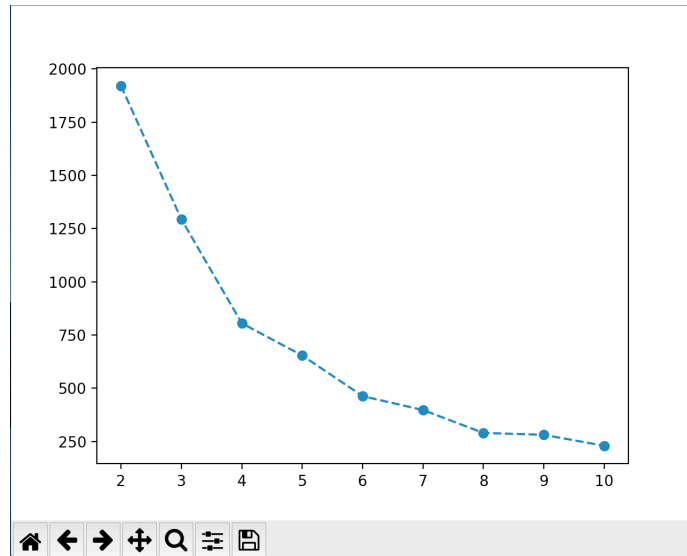******* **Objective Function Value : 278.9837722493407**

--- K = 10 --- ----------
Initial Centroid List : [(6.055098891417727, 7.23007607727676), (3.0410170219628694, -0.3613848735372547), (8.875780722296865, 8.960923612139595), (1.69565649157431, 7.680824581467082), (7.597313421987085, 1.1650474267989992), (2.200114959631794, 1.5386322076504713), (3.021056869801129, 9.26213795798542), (8.619479454228491, 2.985983190047101), (1.3483715993964611, 3.963796377807688), (7.5976350478966745, 7.9392400048313085)]
Final Centroid List : [(5.432070680361331, 6.869308835910069), (3.1690614508664035, 0.8143251472991676), (8.398707529316267, 8.925497059743181), (3.138347677015053, 5.933723217937643), (7.071897430868479, 1.3691741521440337), (2.242047519125402, 3.251007486318421), (2.183214622031253, 7.703553409180906), (7.931544233611159, 3.161139968382462), (4.912514973376666, 3.5631409634126254), (7.267169511874265, 8.275692874735975)]
******* **Objective Function Value : 243.45056737269005**

--- K = 2 --- ---------
Initial Centroid List : [(8.218975256689072, 8.951050496546863), (3.0410170219628694, -0.3613848735372547)]
Final Centroid List : [(4.852611930287174, 7.271641711202775), (5.000562341588762, 2.4854274751531933)]
******* Objective Function Value : 1921.0334858562055

--- K = 3 --- ---------
Initial Centroid List : [(3.5335073736461786, 0.3319889386265751), (8.875780722296865, 8.960923612139595), (1.69565649157431, 7.680824581467082)]
Final Centroid List : [(5.477400388679034, 2.254981027984723), (6.497249620849709, 7.522972929895471), (2.5614644894663545, 6.088613382895479)]
******* Objective Function Value : 1293.7774523911346

--- K = 4 --- ---------
Initial Centroid List : [(7.748670736549092, 1.718123240122596), (3.021056869801129, 9.26213795798542), (3.0410170219628694, -0.3613848735372547), (8.875780722296865, 8.960923612139595)]
Final Centroid List : [(6.783746087903545, 2.8501999866465444), (3.3426476922451784, 6.926028030100171), (2.852351493110535, 2.2818648297203246), (7.179286211861507, 8.05207909972969)]
******* Objective Function Value : 805.1166457472607

--- K = 5 --- ---------
Initial Centroid List : [(2.100548909914467, 1.4414401856186492), (8.875780722296865, 8.960923612139595), (3.021056869801129, 9.26213795798542), (7.597313421987085, 1.1650474267989992), (3.0410170219628694, -0.3613848735372547)]
Final Centroid List : [(3.5545080959230115, 3.628130619771138), (6.8042102875897745, 7.656222203211083), (3.00668800078913, 7.034815026483998), (7.414192434680615, 2.3216911383868664), (3.145061482959145, 0.9077065486588153)]
******* Objective Function Value : 653.9567908403943

--- K = 6 --- ---------
Initial Centroid List : [(4.949560744079336, 3.22624796965257), (8.875780722296865, 8.960923612139595), (3.0410170219628694, -0.3613848735372547), (3.021056869801129, 9.26213795798542), (7.597313421987085, 1.1650474267989992), (1.69565649157431, 7.680824581467082)]
Final Centroid List : [(5.2305366674047375, 4.279342496049099), (7.914309977818314, 8.519909807700076), (2.681986334188929, 2.094615867800809), (5.2402829638043125, 7.531310293267821), (7.556167822397726, 2.235167959857534), (2.541652523310345, 7.002678323536489)]
******* Objective Function Value : 462.9263558248375

--- K = 7 --- ---------
Initial Centroid List : [(2.4898969303766347, 8.400478632861432), (7.597313421987085, 1.1650474267989992), (8.875780722296865, 8.960923612139595), (3.0410170219628694, -0.3613848735372547), (3.021056869801129, 9.26213795798542), (2.200114959631794, 1.5386322076504713), (8.619479454228491, 2.98598319047101)]
Final Centroid List : [(2.5633381461259046, 6.9782248006606624), (7.428034259463345, 2.282918666452028), (7.914309977818314, 8.519909807700076), (3.145061482959145, 0.9077065486588153), (5.021776597933489, 7.8240125818092165), (3.4955665791995627, 3.5661123157286907), (6.15468228405522, 5.701407205679413)]
******* Objective Function Value : 397.44812244413686

--- K = 8 --- ---------
Initial Centroid List : [(2.695113024232428, 5.939673524983461), (8.875780722296865, 8.960923612139595), (3.0410170219628694, -0.3613848735372547), (7.597313421987085, 1.1650474267989992), (3.021056869801129, 9.26213795798542), (2.200114959631794, 1.5386322076504713), (7.5976350478966745, 7.9392400048313085), (8.619479454228491, 2.98598319047101)]
Final Centroid List : [(2.536501078790139, 6.85941978448268168), (7.914309977818314, 8.519909807700076), (3.1690614508664035, 0.8143251472991676), (4.818330576085663, 3.69502320071854), (4.859398750231154, 7.941638208906054), (2.242047519125402, 3.251007486318421), (6.15468228405522, 5.701407205679413), (7.556167822397726, 2.235167959857534)]
******* Objective Function Value : 289.93272604483815

--- K = 9 --- ---------
Initial Centroid List : [(2.3605714528864787, 6.007966227729477), (8.875780722296865, 8.960923612139595), (3.0410170219628694, -0.3613848735372547), (7.597313421987085, 1.1650474267989992), (3.021056869801129, 9.26213795798542), (8.619479454228491, 2.98598319047101), (1.69565649157431, 7.680824581467082), (2.200114959631794, 1.5386322076504713), (7.5976350478966745, 7.9392400048313085)]
Final Centroid List : [(3.89497885598108813, 4.917516281920117), (7.914309977818314, 8.519909807700076), (3.1690614508664035, 0.8143251472991676), (6.182966504540978, 2.083050197023721), (4.986279019961613, 7.797378117091519), (7.941713964744382, 2.87966135379709), (2.297647316142252, 7.394026355610831), (2.242047519125402, 3.251007486318421), (6.15468228405522, 5.701407205679413)]
******* Objective Function Value : 281.0199554193879

--- K = 10 --- ---------
Initial Centroid List : [(6.633523323424615, 0.9802070492997959), (3.021056869801129, 9.26213795798542), (8.875780722296865, 8.960923612139595), (3.0410170219628694, -0.3613848735372547), (1.69565649157431, 7.680824581467082), (8.619479454228491, 2.98598319047101), (2.200114959631794, 1.5386322076504713), (7.5976350478966745, 7.9392400048313085), (1.3483715993964611, 3.963796377807688), (7.597313421987085, 1.1650474267989992)]
Final Centroid List : [(4.912514973376666, 3.56314096341262254), (4.859398750231154, 7.941638208906054), (7.914309977818314, 8.519909807700076), (3.392397796044764, 0.5065576443944665), (2.4631734791294, 6.9890956694190998), (7.986758244802879, 3.012729309091755), (2.5755081335118746, 2.3149966532137327), (6.15468228405522, 5.701407205679413), (2.46204926319718, 4.294396379247468), (7.056682932407747, 1.333196794745837)]
******* Objective Function Value : 229.4975606517998