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Structural vector autoregressions with smooth transition in variances

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ABSTRACT

In structural vector autoregressive analysis identifying the shocks of interest via heteroskedasticity has become a standard tool. Unfortunately, the approaches currently used for modeling heteroskedasticity all have drawbacks. For instance, assuming known dates for variance changes is often unrealistic while more flexible models based on GARCH or Markov switching residuals are difficult to handle from a statistical and computational point of view. Therefore we propose a model based on a smooth change in variance that is flexible as well as relatively easy to estimate and illustrate its use by analysis of the interaction between monetary policy and the stock market based on a five-dimensional system of U.S. variables. For the benchmark setup it is found that previously used conventional identification schemes in this context are rejected by the data if heteroskedasticity is allowed for. We also illustrate the implications of using different transition variables and varying the sample period.

1. Introduction

Structural vector autoregressions are popular tools for empirical macroeconomic analysis. The underlying model is a basic reduced form linear vector autoregression (VAR) as advocated by Sims (1980). The standard structural VAR (SVAR) approach derives identifying restrictions for the structural shocks and imposes them on the reduced form of the model. Such restrictions usually come from economic theory related to the variables involved. However, there is a growing strand of literature that proposes to use features of the data to help with the identification of structural shocks. More specifically, distributional assumptions (Lanne and Lütkepohl, 2010) as well as heteroskedasticity (Rigobon, 2003; Lanne et al., 2010) may be useful for identification purposes. In this paper we use heteroskedasticity for the identification of shocks. This approach is attractive because changes in the volatility of different macroeconomic time series are broadly documented and discussed by Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Blanchard and Simon (2001), and Stock and Watson (2003) among others.

Alternative approaches for modeling changes in volatility have been used in this context. For example, Rigobon (2003), Rigobon and Sack (2003), Lanne and Lütkepohl (2008) and Bacchiocchi and Fanelli (2015) use simply a deterministic shift in the variances while Normandin and Phaneuf (2004) and Bouakez and Normandin (2010) model the changes in volatility by a vector GARCH process and Lanne et al. (2010) propose a Markov switching (MS) mechanism for changes in volatility.

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The GARCH and MS approaches have the disadvantage that estimation of the models is very involved and so far reliable estimation methods are available only for small models with three or four variables and a moderate number of lags and volatility states at best. On the other hand, assuming an exogenous sudden shift in variance as in Rigobon (2003) and others is also not very attractive because, in practice, gradual changes in volatility seem more plausible in many situations. Therefore, in this study we propose an intermediate approach. More precisely, we consider the SVAR with heteroskedastic residuals modeled by a smooth transition function.

In the smooth transition literature it is more common to model non-linearity in the mean equation (e.g., Hubrich and Teräsvirta, 2013). However we use this idea to take into account heterskedasticity present in the data. There are some studies that consider smooth transition in the second moments of multiple time series models. Examples are Silvennoinen and Teräsvirta (2005; 2009), Carvalho (2007) and Yang (2014). Most of these papers impose smooth transition in a multivariate GARCH framework which complicates the identification and estimation issues for our purposes. Therefore we focus on a simpler smooth transition mechanism in the covariance structure. Of course, more complicated models may be needed to fully capture the volatility structure of a given set of data. However, the scarcity of macroeconomic data often calls for rather simple models. Simple models are also useful in the present context to study the issues related to structural identification in VAR models.

Our current setup has a number of advantages compared to other volatility models used in this context. If the transition function is parameterized parsimoniously, the parameters are relatively easy to estimate. A well developed toolkit for the statistical analysis of smooth transition regression models is available and can potentially be adopted for the purposes of identification of structural shocks. The estimation of the model as set up in the present paper benefits from currently available computational power and can be performed with reasonable computation time. The timing of the change in volatility is determined by the data and does not have to be imposed exogenously by the analyst. Depending on the parametrization of the transition function, the timing of the transition to a new state may be estimated. By a suitable choice of the transition variable, the change in volatility regimes may be endogenized, that is, it may be linked to relevant economic variables. Given these advantages of models with smooth transition in the variances, they may be strong competitors of MS- and GARCH-SVAR models or models with exogenously imposed heteroskedasticity regimes.

We illustrate the use of the smooth transition VAR model by investigating the interaction between monetary policy and the stock market based on a five-dimensional system of U.S. variables. The relation between monetary policy and the stock market has been analyzed with VAR models by a number of authors. Our benchmark study is Bjørnland and Leitemo (2009). Previous studies vary in the identifying assumptions for shocks of interest and the data they use. Generally they find some interdependence between monetary policy and the stock market. However, the magnitude of the effects of monetary policy shocks on the stock market differ widely in the various studies. A direct comparison of alternative identifying assumptions is usually difficult because the authors typically use just-identifying restrictions that cannot be tested with statistical tools in a conventional SVAR framework. In that situation, using heteroskedasticity as an additional identification device seems plausible and we apply that tool to check the identifying assumptions used by Bjørnland and Leitemo (2009). Since their system is five-dimensional, using MS- or GARCH-SVAR models is challenging while our smooth transition model works well and we can use it to test the identifying restrictions of Bjørnland and Leitemo (2009). It turns out that they are rejected in our benchmark setup. While we also find a strong interaction between monetary policy and the stock market, we argue that the data suggest a different interpretation of monetary shocks than that used by other authors in a conventional SVAR setting.

The remainder of the paper is organized as follows. Section 2 briefly discusses conventional VAR and SVAR models as well as identification of structural shocks. Section 3 sets up the smooth transition SVAR model and explains how it can be used for identification purposes. A suitable estimation procedure is discussed as well. The empirical example analyzing the relation between U.S. monetary policy and the stock market is discussed in Section 4. The last section summarizes the conclusions from our study.

2. The baseline model

The baseline model is a VAR of order p (VAR(p)) of the form

$$y_t = \nu + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t, \tag{1}$$

where $y_t = (y_{1t}, \ldots, y_{Kt})'$ is a vector of observable variables, the A_i are $(K \times K)$ coefficient matrices, ν is a $(K \times 1)$ constant term and the u_t are K-dimensional serially uncorrelated reduced form residuals with mean zero and covariance matrix Σ_u.

The structural residuals are denoted by ε_t. They have zero mean and are serially uncorrelated. Typically they are also assumed to be instantaneously uncorrelated, that is, $\varepsilon_t \sim (0, \Sigma_\varepsilon)$, where Σ_ε is a diagonal matrix. Sometimes it is actually assumed that the variances of the structural shocks are normalized to one so that Σ_ε is an identity matrix.

The structural residuals are typically obtained from the reduced form residuals, u_t, by a linear transformation,

$$u_t = B\varepsilon_t \quad \text{or} \quad \varepsilon_t = B^{-1} u_t. \tag{2}$$

The matrix B contains the instantaneous effects of the structural shocks on the observed variables. Given the relation between the reduced form residuals and the structural residuals, the matrix B has to satisfy $\Sigma_u = B\Sigma_\varepsilon B'$. In other words, in principle B can be any matrix satisfying $\Sigma_u = B\Sigma_\varepsilon B'$. The relation between the reduced form and structural residuals does not uniquely determine the matrix B and, hence, the structural innovations are not uniquely determined without further assumptions.

The conventional approach is to impose further restrictions on B directly to make it unique. These restrictions may be zero restrictions indicating that a certain shock does not have an instantaneous effect on one of the variables. Alternatively, they may be implied by restrictions on the long-run effects of a structural shock or by other kinds of information. The matrix of long-run effects of structural shocks is given by

$$\Xi_\infty = (I_K - A_1 - \cdots - A_p)^{-1}B,$$

assuming that the inverse exists. That condition is satisfied for stable, stationary processes without unit roots. For integrated and cointegrated processes the long-run effects matrix is related to the cointegration structure of the model (see, e.g., Lütkepohl, 2005). For our purposes it is sufficient to know that the matrix of long-run effects can be computed from the reduced form and structural parameters and imposing restrictions on that matrix implies restrictions on B.

Typically the restrictions on B just-identify the structural model and, hence, the structural shocks. In other words, there are just enough restrictions for uniqueness of B and no more. If there are two competing sets of just-identifying assumptions or theories implying just-identifying restrictions, they lead to identical reduced forms and cannot be tested against the data. Hence, the conventional setup is often uninformative regarding the validity of specific economic theories. In the next section it is discussed how heteroskedasticity can be used to improve the situation in this case.

3. SVAR model with heteroskedastic residuals

3.1. Smooth transition in variances

Suppose u_t is a heteroskedastic error term with smoothly changing covariances,

$$\mathbb{E}(u_t u_t') = \Omega_t = (1 - G(s_t))\Sigma_1 + G(s_t)\Sigma_2, \tag{3}$$

where Σ_1 and Σ_2 are distinct covariance matrices and $G(s_t)$ is a (typically parametric) transition function. It depends on an exogenous or predetermined transition variable s_t. This quantity can be a deterministic or stochastic variable which determines the transition to another volatility state. If s_t is stochastic, it is assumed to be exogenous or a lagged dependent variable to ensure standard properties of related inference procedures. Alternatively, the transition variable may be a deterministic variable such as $s_t = t$. In the latter case the unconditional residual covariance matrix Ω_t is time varying and, hence, the VAR process is heteroskedastic and not stationary anymore because it has time-varying second moments. Thus, the model is very flexible because it can capture unconditional as well as conditional heteroskedasticity, depending on the choice of transition variable.

In the application discussed in Section 4, the logistic transition function proposed by Maddala (1977),

$$G(\gamma, c, s_t) = (1 + \exp[-\exp(\gamma)(s_t - c)])^{-1}, \tag{4}$$

is used. It contains the slope parameter γ and the location parameter c. The slope parameter determines the speed of the transition. If γ is very large and $s_t < c$, $G(\gamma, c, s_t)$ will be close to zero and, hence, Ω_t is roughly equal to Σ_1, while for $s_t > c$ and large γ, the transition function $G(\gamma, c, s_t)$ will be about one such that $\Omega_t \approx \Sigma_2$. In fact, for $\gamma \to \infty$ the model approaches a threshold model, where $\Omega_t = \Sigma_1$ for $s_t < c$ and $\Omega_t = \Sigma_2$ for $s_t > c$.

Notice that $0 < G(\gamma, c, s_t) < 1$. Thus, assuming that Σ_1 and Σ_2 are positive definite, Ω_t is a convex combination of two positive definite matrices and, hence, it is also a positive definite matrix. Depending on the transition variable, the sample can be divided in more than two volatility regimes as long as they can be represented as convex combinations of the two covariance matrices Σ_1 and Σ_2 (see the example in Section 4). Thus, the model is quite flexible and can describe quite diverse heteroskedasticity patterns.

We could have parametrized the logistic transition function in (4) by replacing $\exp(\gamma)$ by a parameter $\gamma^* > 0$. Our parametrization using $\exp(\gamma)$ has practical reasons because $\exp(\gamma)$ is strictly positive for all $\gamma \in \mathbb{R}$ while for $\gamma^* = 0$ the transition function would be a constant which implies that the parameters Σ_1 and Σ_2 are no longer separately identified. In the smooth transition literature this feature is sometimes used to test for time-varying coefficients (e.g., Teräsvirta et al., 2010 [Section 16.3]). We do not use this device and prefer the parametrization in Eq. (4) because it is convenient for identifying structural shocks in a SVAR analysis.

In the smooth transition literature alternative transition functions have also been considered which would be feasible choices in the present framework as well. For example, the exponential smooth transition model uses a transition function $G(\gamma^*, c^*, s_t) = 1 - \exp[\gamma^*(s_t - c^*)^2]$ (see Teräsvirta et al., 2010 [Section 16.3]). While such alternative transition functions may have merit occasionally because they may describe the transition from one regime to another more accurately in specific situations, the basic logistic transition function in (4) offers sufficient flexibility for our purposes. In the current framework the choice of a suitable transition variable may be the more crucial problem. Ideally it should be based on subject matter theory.

To gain further flexibility for the ST-SVAR model one could consider more than one transition function. We are not pursuing this possibility in the current study because such an extension would complicate the volatility model and the identification of structural shocks through heteroskedasticity. Our main objective is to describe the volatility changes in a way which is easy to work with in a SVAR framework. Macroeconomic data are often not rich enough to be informative about sophisticated higher order moment structures. We emphasize, however, that with a suitable choice of transition variable quite flexible volatility changes are possible, as we will illustrate in our empirical example in Section 4.

3.2. Identification

The two covariance matrices Σ_1 and Σ_2 can be used for the identification of structural shocks. There exists a decomposition

$$\Sigma_1 = BB' \text{ and } \Sigma_2 = B\Lambda B', \tag{5}$$

where $\Lambda = \text{diag}(\lambda_1, \ldots, \lambda_K)$ is a diagonal matrix with nonnegative diagonal elements (see Lütkepohl, 1996 [Section 6.1.2 (10)] or Theorem 7.6.4 in Horn and Johnson, 2013). Apart from changes in signs of the columns of B, this decomposition is in fact unique for a given ordering of the λ_i if these quantities are all distinct (Lanne and Lütkepohl, 2008). Thus, if the B matrix from (5) is used to obtain the structural errors from the reduced form residuals, the covariance matrices of the structural shocks are I_K and Λ for regimes associated with Σ_1 and Σ_2, respectively. The positive diagonal elements of the Λ matrix can thus be interpreted as variances of structural shocks in the Σ_2 regime relative to the Σ_1 regime. For Ω_t to be invertible it is actually enough that one of the Σ_1 and Σ_2 matrices is positive definite and the other one is positive semidefinite. In that case it is implicitly assumed that Σ_1 is invertible to ensure that B is nonsingular. If Σ_2 is singular, at least one of the λ_i is zero and cannot be interpreted as a regular variance. If more than one λ_i is zero, the corresponding columns of B are not identified through heteroskedasticity. In that case identification of the relevant columns of B has to come from other sources such as exclusion restrictions on the impact effects.

An important advantage of using the transformation matrix B is that the data are in principle informative on the uniqueness condition and, hence, the identification condition for the shocks can be investigated with statistical methods. Unfortunately, so far no formal statistical tests are available for this purpose. Note that such tests would involve testing equality of the diagonal elements of Λ. If two of the diagonal elements are identical, then the corresponding columns of B are not uniquely identified and, hence, there would be unidentified parameters under the null hypothesis. This complication is known to invalidate standard χ^2 asymptotics (see Davies, 1987). Constructing suitable statistical tests for identification in this framework is an interesting topic for future research.

Using B as transformation matrix to obtain $\varepsilon_t = B^{-1}u_t$ throughout the sample implies that the structural shocks have the same instantaneous effects regardless of the residual volatility and are normalized such that they have unit variance when $\mathbb{E}(u_t u_t') = \Sigma_1$. If the uniqueness conditions for B are satisfied, any restrictions imposed on B in a conventional SVAR framework become over-identifying and can be tested against the data. Lanne et al. (2010) use likelihood ratio tests for this purpose. Of course, if heteroskedasticity does not fully identify B, then such tests may have no power against invalid restrictions imposed on B.

If structural shocks are obtained through heteroskedasticity, they may be quite distinct from the economic shocks of interest. Still the potentially unique decomposition of covariance matrices provides additional information that can help identifying structural shocks of interest. If the shocks happen to be the same as the economically identified shocks, such shocks are over-identified in our framework and, hence, different sets of restrictions become testable. In addition, identification through heteroskedasticity can be used to identify only some of the shocks by using data properties and imposing those restrictions that are not rejected by the data for identifying economically interesting shocks. In the application section this point will be illustrated and further explained.

3.3. Estimation

We refer to the model specified in (1), (3), (4) and (5) as a smooth transition structural VAR (ST-SVAR) model. It can be estimated via Gaussian maximum likelihood (ML). Under the assumption of normality of the residuals, the log-likelihood function for the model is

$$\log L = \text{constant} - \frac{1}{2}\sum_{t=1}^{T} \log\det(\Omega_t) - \frac{1}{2}\sum_{t=1}^{T} u_t'\Omega_t^{-1}u_t, \tag{6}$$

where $u_t = y_t - \nu - A_1 y_{t-1} - \cdots - A_p y_{t-p}$ and Ω_t is given by (3) and (5).

Note, however, that for consistent estimation the parameters have to be identified. Although the reduced-form VAR slope and deterministic parameters are automatically identified, the same is not necessarily true for the structural parameters B, Λ and the parameters γ and c of the transition function. For example, if $\Sigma_1 = \Sigma_2$ and, hence, Ω_t is time-invariant, γ and c are not identified. Also, if the transition variable is such that it cannot capture the volatility changes in the residuals, the transition parameters may be poorly identified. In that case, Gaussian ML does not work and iterative estimation algorithms may not converge. In other words, for Gaussian ML to work, it is important to choose a suitable transition variable.

The model is nonlinear and has many parameters to be estimated. Therefore our maximization algorithm for the log likelihood uses a grid search over γ and c. For a given pair (γ, c), estimation proceeds in the following two steps.

Step 1: For given starting values of the VAR parameters $\nu, A_1, \dots, A_p$, the structural parameters B, Λ are estimated by maximizing the log likelihood function using nonlinear optimization. This step may be done subject to economic restrictions on the B or Ξ_∞ matrices.

Step 2: For the updated structural parameters the VAR part of the model is reestimated. Note that given the transition parameters (γ, c) and structural parameters B, Λ the model is linear in the VAR part. For that reason the vectorized VAR coefficients $b = \text{vec}(\nu, A_1, \dots, A_p)$ can be estimated with a weighted least squares procedure with the weights given by Ω_t^{-1}, that is,

$$\hat{b} = \left[(Z' \otimes I_K)\, W_T\, (Z \otimes I_K)\right]^{-1} (Z' \otimes I_K) W_T y,$$

where

$$W_T = \begin{bmatrix} \Omega_1^{-1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \Omega_T^{-1} \end{bmatrix}$$

is a $(KT \times KT)$ block-diagonal weighting matrix. Moreover, $y = (y_1', \dots, y_T')'$ is a $(KT \times 1)$ data vector, and each row of the $(T \times (1 + Kp))$ data matrix Z contains a leading one for the constant as well as lagged observations: $(1, y_{t-1}', \dots, y_{t-p}')$.

These two steps are iterated until there is no improvement in the log-likelihood value and convergence is ensured for properly identified models.

As mentioned earlier, a grid search is performed for the parameters of the transition function (γ, c) and for each pair of values the previously described estimation is carried out. The range of γ is chosen such that the full range of transition functions from very flat to very steep functions is covered. For the location parameter, c, a range of

$$\left[\min_{t=1,\dots,T}(s_t), \max_{t=1,\dots,T}(s_t)\right]$$

may be considered. Thereby the ML estimators of all parameters are found.

This procedure is computationally demanding but not infeasible. One may consider other numerical optimization algorithms to improve the computational efficiency. For example, simulated annealing algorithms have been proposed for nonlinear optimization problems (see, e.g., Goffe et al., 1994 or Brooks and Morgan, 1995). They may also be useful for optimizing the log-likelihood function of the ST-SVAR model.

Once the ML estimates are available, the standard errors of the estimates are obtained as square roots of the inverted information matrix. The information matrix is estimated using the outer product of the numerical first order derivatives (see Hamilton, 1994 [p. 143]).

In structural VAR analysis impulse responses are usually used for investigating the transmission process of the shocks. We construct confidence intervals around the impulse responses using a fixed design wild bootstrap procedure. The method preserves the pattern of heteroskedasticity and contemporaneous dependence of the data as noted by Gonçalves and Kilian (2004). In the context of structural VAR models identified via heteroskedasticity the method was proposed by Herwartz and Lütkepohl (2014) and used by Lütkepohl and Netšunajev (2014) and Netšunajev (2013) among others. In that procedure the bootstrap samples are constructed conditionally on the ML estimates as

$$y_t^* = \hat{\nu} + \hat{A}_1 y_{t-1} + \dots + \hat{A}_p y_{t-p} + u_t^*,$$

where $u_t^* = \eta_t \hat{u}_t$ and η_t is a Rademacher distributed random variable that assumes values -1 and 1 with probability 0.5 each. We bootstrap parameter estimates conditionally on the initially estimated transition parameters (γ, c) to preserve the volatility pattern. The ML estimates are used as starting values in optimizing the log-likelihood in the bootstrap replications.

At this point there are no analytical or simulation results that confirm the reliability of this procedure in the present framework. In a related context, Brüggemann et al. (2016) show that the wild bootstrap does not properly capture the higher-order moments of the distribution of interest and, hence, may not produce reliable confidence intervals for impulse responses. They propose an alternative bootstrap method with more appealing theoretical properties. Unfortunately, in a simulation study they find that it may be very unreliable in small samples. Therefore we use the wild bootstrap in the example section.

4. Empirical illustration

To illustrate the ST-SVAR methodology we reconsider a study of Bjørnland and Leitemo (2009). The authors are interested in the interdependence between U.S. monetary policy and stock prices. The topic has been of considerable interest in the literature. The main objective of this section is to illustrate some important issues to be aware of in applying the ST-SVAR model. We first present our benchmark model and then discuss an alternative sample period.

4.1. Benchmark setup

As in Bjørnland and Leitemo (2009) a five-dimensional VAR with the vector of variables $y_t = (q_t, \pi_t, c_t, \Delta sp_t, r_t)'$ is considered, where

Table 1
Comparison of VAR(3) models for $y_t = (q_t, \pi_t, c_t, \Delta sp_t, r_t)'$, sample period:1970M1 - 2007M6.

Model	$\log L_T$	AIC	SC
VAR(3)	−3159.344	6508.689	6898.432
ST-SVAR(3) ($s_t = t$)	−2878.255	5980.510	6439.997
ST-SVAR(3) ($s_t = \pi_{t-1}$)	−2879.677	5983.353	6442.840
ST-SVAR(3) ($s_t = \pi_{t-2}$)	−2872.879	**5969.757**	**6429.993**
ST-SVAR(3) ($s_t = \pi_{t-3}$)	−2876.725	5977.450	6436.936

Note: L_T – likelihood function, AIC $= -2\log L_T + 2\times$no of free parameters, SC $= -2\log L_T + \log T \times$no of free parameters.

- q_t is the linearly detrended log of an industrial production index;
- π_t denotes the annual change in the log of consumer prices (CPI index) ($\times 100$);
- c_t is the annual change in the log of the World Bank (non energy) commodity price index ($\times 100$);
- sp_t is the log of the real S&P500 stock price index deflated by the consumer price index to measure the real stock prices; the series is in first differences to represent monthly returns (Δsp_t);
- r_t denotes the Federal Funds rate.

We use monthly data for the period 1970M1 - 2007M6 which is longer than that of Bjørnland and Leitemo (2009) who exclude the 1970s and start their analysis in 1983M1. With the exception of the commodity price index the data is downloaded from the Federal Reserve Bank of St. Louis database FRED. The commodity price index is from the World Bank.

Some authors have argued that monetary policy has changed during our sample period (see, e.g., Huizinga and Mishkin, 1986). Hence, the assumption that only the variance of the structural shocks but not the responses of the variables have changed may be regarded as unrealistic. Therefore we later consider a reduced sample period starting in 1983 as in Bjørnland and Leitemo (2009). However, some authors have in fact used a sample period that covers the 1970s and 1980s and have assumed that only the volatility of the shocks changes (e.g., Lanne and Lütkepohl, 2008). Therefore we first illustrate the potential of our model by considering the longer sample period.

Two alternative types of transition variables s_t are considered. The first one is just time, i.e., $s_t = t$. Such a deterministic transition variable is plausible if the first and last parts of the sample have a different volatility pattern and there is possibly a transition period from the initial to the final volatility state. For our sample period a decline of macroeconomic volatility starting in the middle of the 1980s and known as the Great Moderation period has been diagnosed in a number of articles (e.g., Kim and Nelson, 1999, Blanchard and Simon, 2001, Stock and Watson, 2003). Also a shift in variance due to changes in monetary policy may have occurred in the first half of the 1980s (e.g., Huizinga and Mishkin, 1986). Some such changes in volatility may be captured with time as a transition variable. As discussed earlier, such a model implies unconditional heteroskedasticity and, hence, time-varying second moments.

On the other hand, the volatility patterns of financial variables such as stock prices may be more sophisticated so that a stochastic transition variable may be better suited for describing the changes in volatility. While a transition variable $s_t = t$ allows a shift from one volatility state to another but no return to the initial state, a stochastic variable s_t may also capture various moves back and forth between two volatility states. Since there is potentially some interaction between monetary policy and the stock market and since monetary policy is accounting for inflation, we use lags of that variable as alternative transition variable. Of course, this raises the question which lag to use. We try different lags and then use the one that leads to the largest value of the log-likelihood function.

For the transition variable $s_t = t$, in a first round the grid for c is over a subset of the integers $\{1, \dots, T\}$ and for γ a grid in steps of 0.1 in the interval $[-3.5, 3.5]$ is used. For $s_t = \pi_{t-j}$, the grid for c is in the interval [1, 5] with steps 0.2 and for γ a grid in the interval $[-1, 1]$ with steps 0.1 is used. The choice of the interval [1, 5] for c is motivated by the fact that the transition variable (annual inflation) lies most of the time in this interval. In a final step the grid is refined in the neighborhood of the values maximizing the likelihood function in the first round grid search.

The VAR lag order is chosen by information criteria applied to the standard VAR model not accounting for changes in residual volatility. The more generous Akaike Information Criterion (AIC) suggests a VAR(3) for our sample period 1970M1 - 2007M6. We have applied the three GARCH tests described in Lütkepohl and Milunovich (2016) to the residuals of the VAR(3) model and obtained p-values of less than 0.1% and hence strong indication of time-varying volatility in the residuals. Notice that these tests also have power against unconditional heteroskedasticity. Therefore it makes sense to account for heteroskedasticity by fitting the smooth transition models. Estimation of the unrestricted ST-SVAR(3) models is done with the relative variances λ_i ordered from smallest to largest.

Some statistics for the estimated models without and with smooth transition in variance and using alternative transition variables are presented in Table 1. It is obvious that the ST-SVAR models allowing for heteroskedasticity are clearly preferred by AIC and the Schwarz criterion (SC). The smallest log-likelihood, AIC and SC values are obtained for a transition variable $s_t = \pi_{t-2}$. Thus, we use two lags of inflation when lagged inflation is considered as the transition variable.

We have also experimented with alternative transition variables. For example, we have used lagged interest rates as transition variables. None of the alternatives worked very well for our purposes and partly resulted in poorly identified or

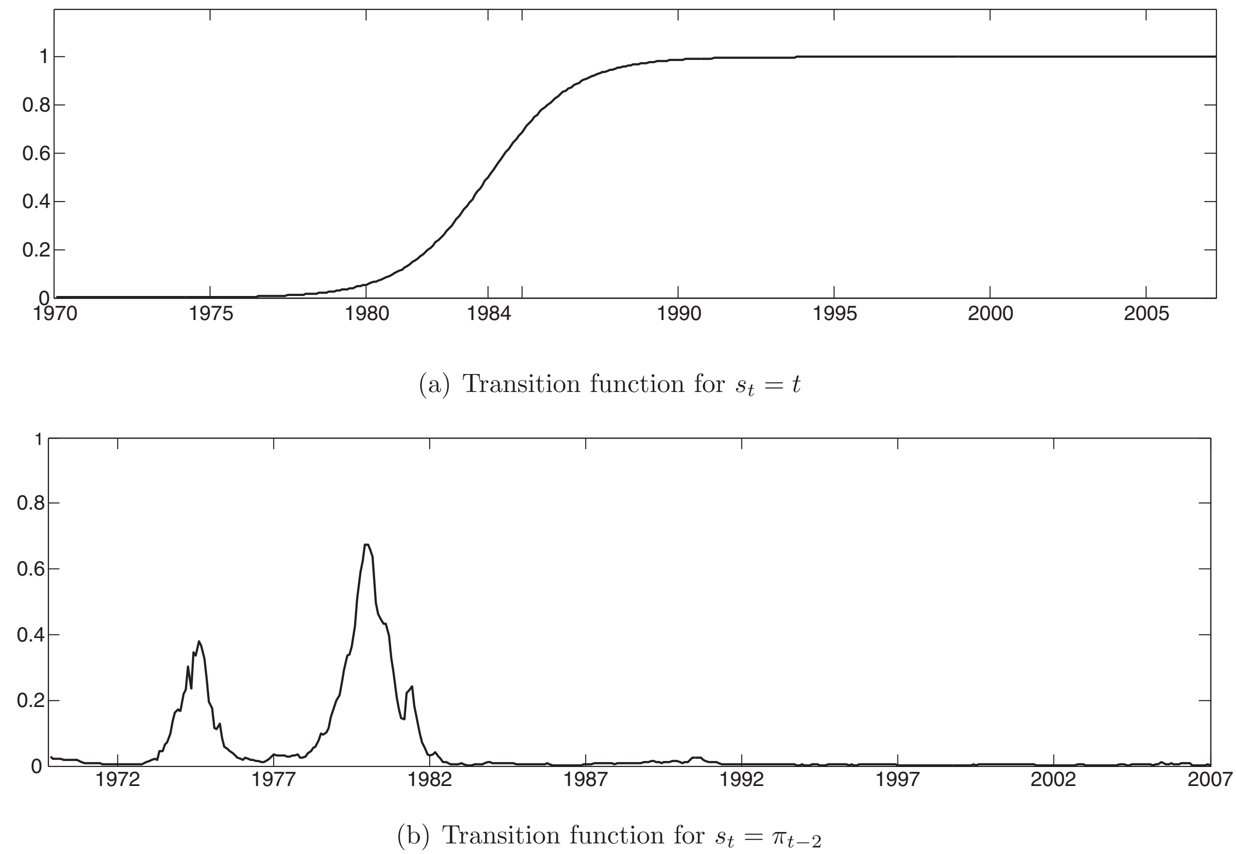


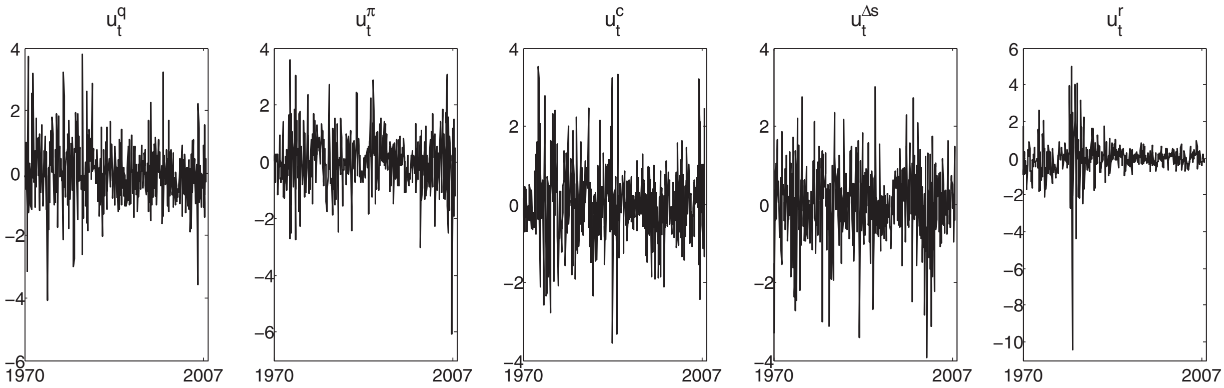
Fig. 1. Transition functions for ST-SVAR models for sample period 1970M1 - 2007M6.

even unidentified models. Therefore we do not present detailed results here. We will illustrate the implications of working with a poorly identified model in Section 4.3.

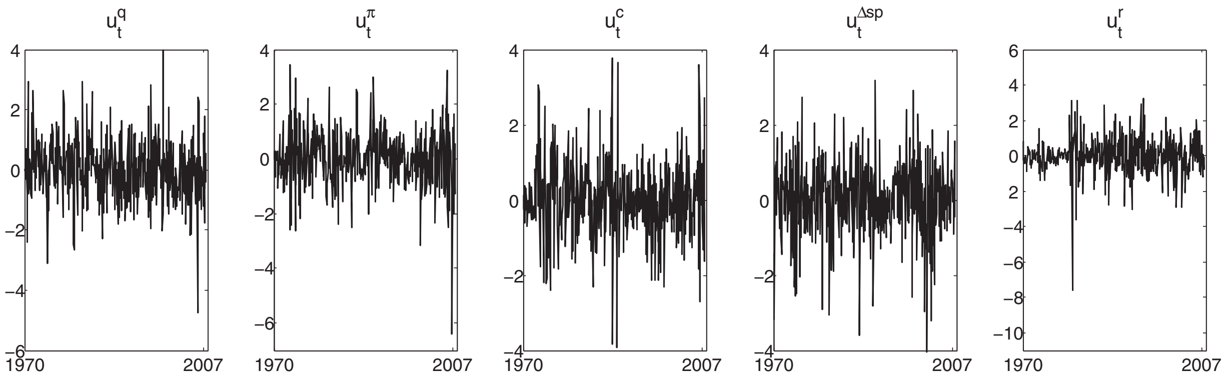
The transition functions of the ST-SVAR(3) models with transition variables $s_t = t$ and $s_t = \pi_{t-2}$ are presented in Fig. 1. For $s_t = t$ there is a gradual change in variance from the end of the 1970s to the middle of the 1980s. This corresponds well to the beginning of the Great Moderation period and possibly a change in monetary policy after the term of the Fed chairman Volcker. If the transition variable $s_t = \pi_{t-2}$ is used, a more sophisticated volatility pattern is found. In that case the recession periods in the middle 1970s and around 1980 are associated with different volatility states which are a combination of Σ_1 and Σ_2. In fact, the transition function has a higher peak in the second period around 1980 meaning that the Σ_2 covariance is weighted by a larger number and hence the volatility is higher than in the period around the middle of the 1970s. Note that the smoothed state probabilities of the model used by Chen and Velinov (2013) resemble the shifts to the more volatile state in panel (b) of Fig. 1 (see Chen and Velinov, 2013 [Figure 4.2]). This lends further support to the ST-SVAR model with transition variable $s_t = \pi_{t-2}$ for our sample. It also shows that the ST-SVAR model with a single transition term can capture more than two volatility regimes by assigning covariance matrices that are convex combinations of Σ_1 and Σ_2 of different magnitude.

To see how well our models capture the residual volatility, we plot standardized residuals for the conventional VAR model that does not allow for heteroskedasticity and two ST-SVAR models in Fig. 2. The standardization is done by dividing each estimated reduced-form residual $\hat{u}_{kt}$ by its corresponding estimated standard error, say $\hat{\omega}_{kk,t}$, where the latter quantity is the square root of the kth diagonal element of $\hat{\Omega}_t$ for the ST-SVAR residuals. If our volatility models capture the changes in volatility well, then the standardized residuals ideally should not display changes in the volatility pattern throughout our sample period. Clearly, the VAR(3) residuals show more changes in volatility than the ST-SVAR(3) models. It is also obvious from Fig. 2 that a transition variable $s_t = t$ is not able to fully capture the volatility changes notably in the residuals of the interest rate equation. Even when lagged inflation is used as a transition variable, the volatility changes are not captured perfectly.[1] Still, based on the statistics in Table 1 the ST-SVAR model with $s_t = \pi_{t-2}$ is clearly better than its competitors.

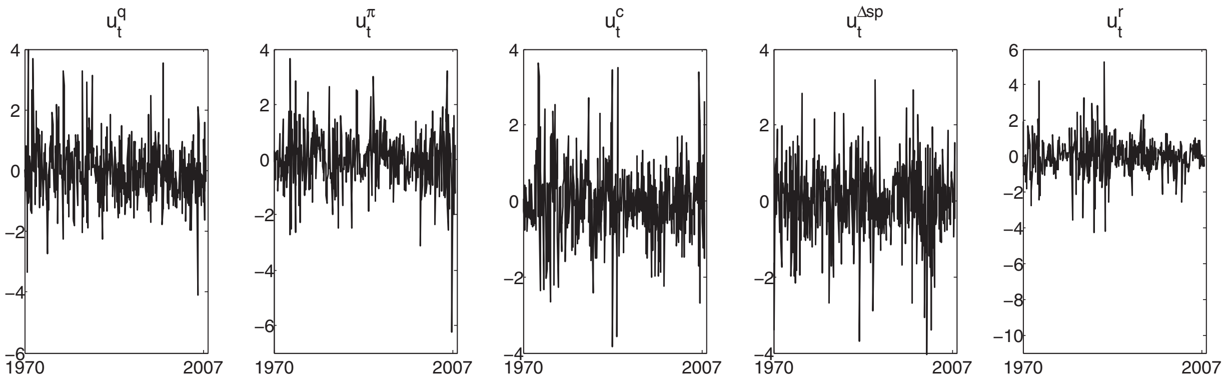
[1] In fact, we have applied the GARCH tests considered in Lütkepohl and Milunovich (2016) to the standardized residuals and found that they reject the no-GARCH null hypothesis at conventional significance levels.



(a) Standardized Residuals of VAR(3) Model



(b) Standardized Residuals of ST-SVAR(3) Model, $s_t = t$



(c) Standardized Residuals of ST-SVAR(3) Model, $s_t = \pi_{t-2}$

Fig. 2. Residuals of the VAR(3) and ST-VAR(3) models for sample period 1970M1 - 2007M6.

Since we are interested in using changing variances of structural shocks for identification purposes, a central question of interest is whether we have sufficient heterogeneity in the volatility changes to get identification. The estimated λ_i of the ST-SVAR models with both alternative transition variables, $s_t = t$ and $s_t = \pi_{t-2}$, are shown in Table 2 together with estimated standard errors. The estimated relative variances of the model with $s_t = t$ are all below one meaning that the transition occurs from a high volatility state to a low volatility state. This is consistent with the economic narratives on the U.S. Great Moderation starting in the mid 1980s. It may also reflect changes in monetary policy after the Volcker period. For the ST-SVAR model with $s_t = \pi_{t-2}$ all estimated λ_i except the first one are greater than one. Hence, for this model the Σ_2 state is the high volatility state. Again this corresponds well to the interpretation of the transition function given earlier.

Table 2
Estimates of relative variances of ST-SVAR(3) models for unrestricted B and Ξ_∞, sample period: 1970M1 - 2007M6.

	$s_t = t$		$s_t = \pi_{t-2}$	
Parameter	Estimate	Std. dev.	Estimate	Std. dev.
λ_1	0.019	0.002	0.899	0.957
λ_2	0.315	0.057	2.739	1.608
λ_3	0.548	0.088	4.176	2.357
λ_4	0.867	0.154	8.091	3.108
λ_5	0.927	0.172	299.562	72.414

Although there is obvious heterogeneity in the estimated relative variances in Table 2 a formal statistical test of the identification conditions would be useful. For that purpose, null hypotheses $H_0 : \lambda_i = \lambda_j$ have to be tested. Rejecting them for all pairs (i, j), $i \neq j$, would support full identification of the structural matrix B. Unfortunately, we are not aware of any formal statistical tests that are suitable in the present situation, as explained earlier. Given the heterogeneity in the λ_i, we conclude from the results in Table 2 that there is some additional identifying information in the heteroskedastic structure of the error terms. Full identification is difficult to claim, however, without formal tests.

Not having a full set of identified shocks is not necessarily a problem for our approach because it is enough if there is some extra identifying information that allows us to test conventional identifying restrictions. For example, we may be able to check a recursively identified model with a triangular B matrix. Such a set of restrictions could be of interest because recursively identified shocks have been used by a number of authors in the present context (e.g., Millard and Wells, 2003, Cheng and Jin, 2013, Park and Ratti, 2000 and others).

4.2. Structural analysis

Of course, from the point of view of our analysis it is of particular interest to test the restrictions specified by Bjørnland and Leitemo (2009) and others. Their restrictions can be visualized in the following way:

$$B = \begin{bmatrix} * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & * \\ * & * & * & * & * \end{bmatrix} \quad \text{and} \quad \Xi_\infty = \begin{bmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & 0 \\ * & * & * & * & * \end{bmatrix}, \tag{7}$$

where the asterisks indicate unrestricted elements and zeros denote elements restricted to zero. The shocks of particular interest are the shocks ordered fourth and fifth (corresponding to columns four and five in the matrices B and Ξ_∞). This identification suggests that the last shock is the monetary policy shock and it has no immediate impact on industrial production, inflation and commodity prices as well as no long-run effect on stock prices. The shock ordered fourth is viewed as the stock market shock and it has no contemporaneous effect on the real side of the economy. Note that the first three shocks are not of interest for the current analysis. They can be identified arbitrarily to perform a conventional SVAR analysis.

In the following we consider alternative sets of restrictions for the initial effects matrix B and the long-run effects matrix Ξ_∞ and check their compatibility with the data.

- M1: B is lower triangular (recursive identification);
- M2: B and Ξ_∞ are restricted as in (7) (Bjørnland–Leitemo identification);
- M3: only the two last columns of B and Ξ_∞ are restricted as in (7);
- M4: only B is restricted as in (7).

We have estimated ST-SVAR(3) models with these types of restrictions. No ordering is imposed on the λ_i when restricted structural models with constraints on B and/or Ξ_∞ are estimated. Assuming that there is enough heterogeneity in the variances of the structural shocks to obtain curvature in the likelihood function we can test the restrictions by LR tests. Some results are shown in Table 3, where most models are tested against the unrestricted ST-SVAR(3). In each case the test is done against a model with the same transition variable.

The p-values of all tests are smaller than 5% and, hence, the tests reject at conventional significance levels regardless of the transition variable used. Thus, as a first conclusion it is clear that heteroskedasticity provides sufficient information to check the restrictions. Since the p-values are based on the standard χ^2 distributions of the LR statistics under H_0, they are based on the validity of standard asymptotics. Note that models M1 and M2 are fully identified by the zero restrictions imposed on B and Ξ_∞, meaning that the order of the structural shocks and the λ_i is determined by the restrictions on B and Ξ_∞. Thus, under H_0 we have full identification when the first, second and fifth null hypotheses are tested. So we can rely on standard χ^2 asymptotics for these cases. For testing M3 and M4, χ^2 asymptotics for the LR tests relies on the assumption of full identification through heteroskedasticity which is not obviously satisfied for our models. Despite the fact that full identification may not be obtained via heteroskedasticity, the large values of the LR statistics show that there is some curvature in the likelihood function that suggests rejecting the restrictions.

Table 3
Tests for identifying restrictions in ST-SVAR models, sample period: 1970M1 - 2007M6.

			$s_t = t$		$s_t = \pi_{t-2}$	
H_0	H_1	df	LR statistic	p-value	LR statistic	p-value
M1	Unrestricted B, Ξ_∞	10	23.395	0.009	18.678	0.045
M2	Unrestricted B, Ξ_∞	10	35.845	8.9×10^{-5}	25.811	0.004
M3	Unrestricted B, Ξ_∞	7	30.909	6.4×10^{-5}	21.654	0.002
M4	Unrestricted B, Ξ_∞	9	22.491	0.0074	18.317	0.031
M2	M4	1	13.354	2.5×10^{-4}	7.494	0.006

Table 4
Comparison of VAR(3) models for $y_t = (q_t, \pi_t, c_t, \Delta sp_t, r_t)'$, sample period: 1983M1 - 2007M6.

Model	$\log L_T$	AIC	SC
VAR(3)	−1617.098	3424.196	3773.161
ST-SVAR(3) ($s_t = t$)	−1538.396	**3300.792**	**3712.204**
ST-SVAR(3) ($s_t = \pi_{t-2}$)	−1584.674	3393.348	3804.760

Note: L_T – likelihood function, AIC $= -2\log L_T + 2\times$no of free parameters, SC $= -2\log L_T + \log T\times$no of free parameters.

The first test shows that in our framework a recursively identified model has no support from the data and the second test makes the same point for the restrictions used by Bjørnland and Leitemo (2009). The last test in Table 3 indicates that their long-run restriction is clearly rejected as an additional restriction in a model with the same impact restrictions as in their model. Note that in all these tests conventional restrictions identify the model under H_0. Thus, standard asymptotic theory implies that the tests have χ^2 distributions under H_0. The degrees-of-freedom and, hence, the p-values may be smaller if B is not fully identified via heteroskedasticity.

The third and fourth tests in the table show that rejection of the first and second null hypotheses is not just due to the restrictions imposed to identify shocks that are not of interest in the present context. The statistics take large values which indicates that the restrictions in the fourth and fifth columns of B and Ξ_∞ contribute to the rejection of the models M1 and M2. Thus, there is evidence against the identifying restrictions imposed in a conventional analysis of the interaction between monetary policy and the stock market. Overall the restrictions are clearly rejected by the data at least for our extended sample period once heteroskedasticity is taken into account. This point can be made on the basis of the first, second and fifth test in Table 3 even if heteroskedasticity does not fully identify B and, hence, the p-values of the third and fourth test may not be reliable.

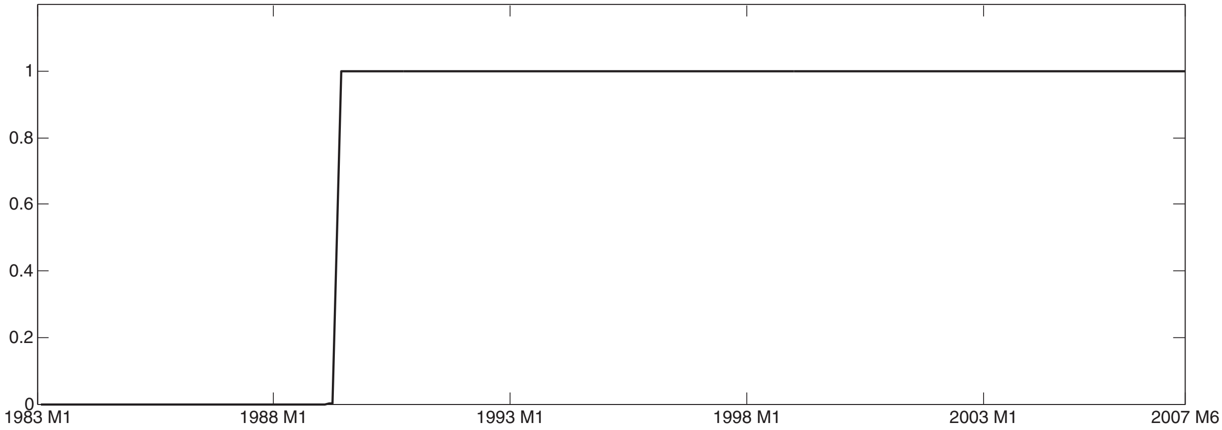
4.3. Reduced sample period

The fact that monetary policy in the U.S. has changed during our sample period may have been a reason for Bjørnland and Leitemo (2009) to consider a shorter sample period starting in 1983. The Bjørnland–Leitemo restrictions may have been rejected because they are not valid for the extended sample period. They may still hold for a sample starting in the mid 1980s. Clearly, if the change in monetary policy has caused changes in the responses of the variables to monetary shocks, then a model that only allows the error variances to change is too restrictive. Therefore we have reduced the sample period and we have fitted ST-SVAR models to a shorter sample starting in 1983M1, as in Bjørnland and Leitemo (2009). More precisely, we use a sample period 1983M1 - 2007M6.

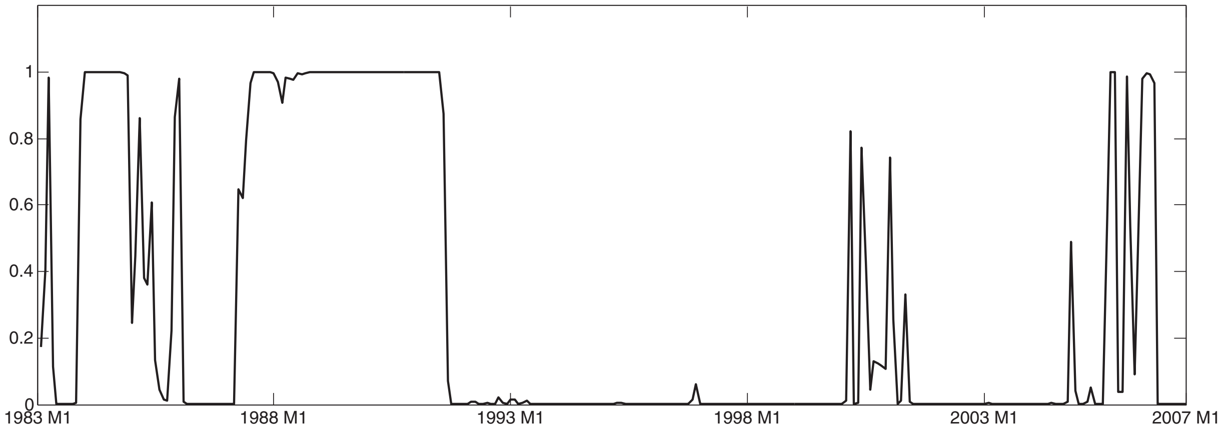
Since the AIC favors again a VAR order 3 for the reduced sample, we fit the preferred models used for the longer sample, that is, we consider ST-SVAR(3) models with transition variables $s_t = t$ and $s_t = \pi_{t-2}$. The log likelihood values and corresponding values of the AIC and SC model selection criteria are presented in Table 4. Both criteria are minimized for ST-SVAR models with transition variable $s_t = t$. In fact, the SC value of a VAR(3) without allowing for heteroskedasticity is smaller than the SC value of the ST-SVAR(3) model with lagged inflation as transition variable. This result indicates that the latter model may be poorly identified in the sense that the two covariance matrices Σ_1 and Σ_2 may be quite similar when $s_t = \pi_{t-2}$. In other words, a transition variable $s_t = \pi_{t-2}$ may not capture possible changes in volatility well for the reduced sample. The situation is different when time is used as transition variable.[2]

The transition functions for the two ST-SVAR models are shown in Fig. 3. The first one, based on $s_t = t$, is just a sudden shift function in 1989M4 and the second transition function, based on $s_t = \pi_{t-2}$, indicates a number of sudden shifts from one volatility regime to the other. In other words, the transition function corresponding to $s_t = \pi_{t-2}$ also indicates that no clear separation of two regimes is found if this transition variable is used.

[2] We note that the multivariate GARCH tests used for the extended sample period indicate that there is also heteroskedasticity in the reduced sample period but that the heteroskedasticity is captured reasonably well by the model using $s_t = t$. For the standardized residuals of the ST-SVAR(3) model with transition variable $s_t = t$ the no-GARCH null hypothesis is not rejected at the 1% significance level.



(a) Transition function for $s_t = t$



(b) Transition function for $s_t = \pi_{t-2}$

Fig. 3. Transition functions for ST-SVAR models for sample period 1983M1 - 2007M6.

Table 5
Estimates of relative variances of ST-SVAR(3) models for unrestricted B and Ξ_∞, sample period: 1983M1 - 2007M6.

	$s_t = t$		$s_t = \pi_{t-2}$	
Parameter	Estimate	Std. dev.	Estimate	Std. dev.
λ_1	0.144	0.029	0.812	0.274
λ_2	0.406	0.104	1.299	0.296
λ_3	0.742	0.208	1.980	0.432
λ_4	0.979	0.265	2.130	0.583
λ_5	2.430	0.609	3.033	0.925

The estimated relative variances corresponding to the two transition variables are shown in Table 5. Taking into account the sampling variability as reflected in the estimated standard errors, it is clear that distinct λ_i are not supported in this case. Some estimated standard errors are relatively large and a number of the estimated λ_i are not far from one in particular for transition variable $s_t = \pi_{t-2}$. Recall that $\Sigma_1 = \Sigma_2$ if $\lambda_1 = \cdots = \lambda_K = 1$. This situation may occur when lagged inflation is used as transition variable whereas for $s_t = t$ a couple of the relative variances appear to be clearly different from 1. Hence, for $s_t = \pi_{t-2}$ the two covariance matrices Σ_1 and Σ_2 may be quite similar or even identical and we may be in a poorly identified or unidentified situation whereas for $s_t = t$ a clear separation in two volatility regimes is still found.

It is instructive to look at the LR test results for the restrictions on B based on these models. We report the tests in Table 6. While for $s_t = t$ all null hypotheses are still rejected at conventional significance levels, the same is not true for the transition variable $s_t = \pi_{t-2}$. Again, this result indicates that the latter transition variable does not pick up changes in

Table 6
Tests for identifying restrictions in ST-SVAR models, sample period: 1983M1 - 2007M6.

			$s_t = t$		$s_t = \pi_{t-2}$	
H_0	H_1	df	LR statistic	p-value	LR statistic	p-value
M1	Unrestricted B, Ξ_∞	10	18.873	0.026	5.961	0.744
M2	Unrestricted B, Ξ_∞	10	35.478	1.035×10^{-4}	10.277	0.417
M3	Unrestricted B, Ξ_∞	7	20.791	0.004	9.147	0.242
M4	Unrestricted B, Ξ_∞	9	22.540	0.013	7.506	0.677
M2	M4	1	12.933	3.228×10^{-4}	2.771	0.096

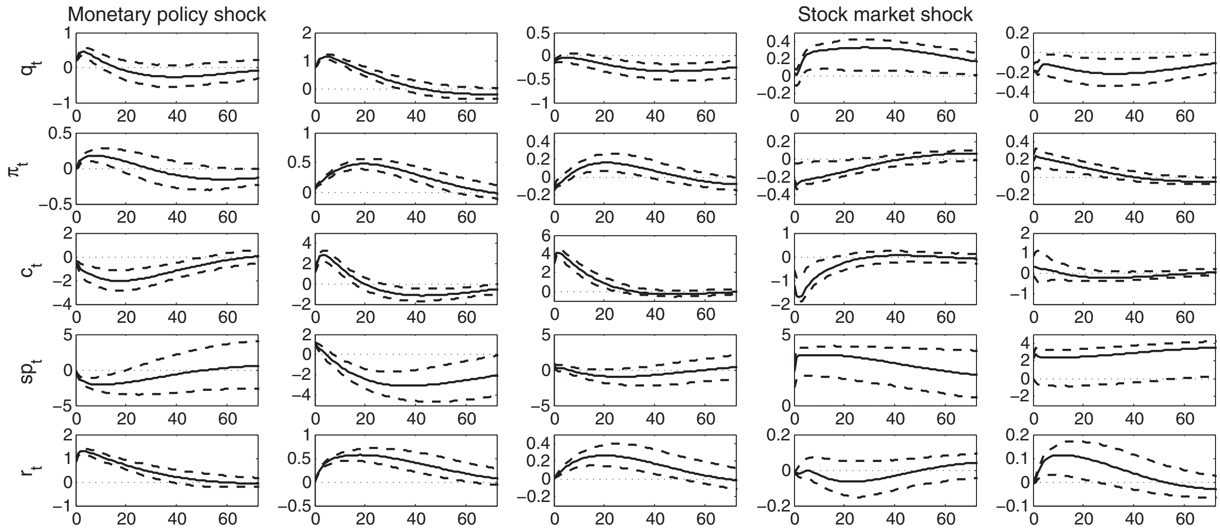


Fig. 4. Impulse response functions for fully unrestricted ST-SVAR model with transition variable $s_t = t$. Solid line - point estimate of the response, dashed line - 68% confidence intervals based on 1000 bootstrap replications.

volatility and results in a poorly identified model. Clearly, for such a model one would not expect the LR tests to have power because heteroskedasticity is not captured by the model. Hence, one would not expect the restrictions to be rejected even if they were false. Thus, the results for the reduced sample period illustrate potential problems that may arise if the transition variable is poorly selected and the estimated volatility model is not well identified. On the other hand, the simpler volatility model based on $s_t = t$ picks up a variance pattern with sufficient heterogeneity in the second moments to enable the LR tests to reject the restrictions. Apparently, for generating identifying information, it is not necessary that the heteroskedasticity is described accurately. It is also interesting to see that the ST-SVAR model is flexible enough to estimate a sudden shift if that describes essential features of the volatility structure.

4.4. Impulse response analysis

To illustrate issues related to impulse response analysis in the context of identification via heteroskedasticity, we now return to the longer sample from 1970M1 - 2007M6 for which the identifying restrictions are unanimously rejected. In this case it is of interest to see the impact of the different identifying restrictions on the impulse responses and compare them to impulse responses obtained without imposing the restrictions. Of course, there is one problem with impulse responses computed from our ST-SVAR models identified via heteroskedasticity. The shocks obtained by this kind of identification do not have a natural labelling and may not be interpretable as economic shocks. The estimation results in Table 2 indicate that only the last shock may be well identified in the ST-SVAR model with lagged inflation as transition variable and the fourth and fifth shocks may not be clearly separated in the model with $s_t = t$.

For the ST-SVAR model with $s_t = \pi_{t-2}$ we have generated impulse responses corresponding to the last shock. They indicate that the shock in principle qualifies as a monetary policy shock because it induces a clear instantaneous response of the interest rate. Viewing the interest rate as the policy instrument, an immediate response of that variable to the monetary policy shock is a minimum requirement for a shock to qualify as a monetary policy shock. Therefore we use this shock as a candidate for a monetary policy shock when we compare its responses to those from other identification schemes.

It may also be instructive to take a look at the impulse responses associated with the model using $s_t = t$ as transition variable. We present the impulse responses of the model with $s_t = t$ in Fig. 4. They are plotted with 68% confidence intervals based on 1000 bootstrap replications. It turns out that there is again only one shock that can be interpreted as monetary policy shock. In this case the first shock is the only one with a significant impact effect on the interest rate. Likewise, the

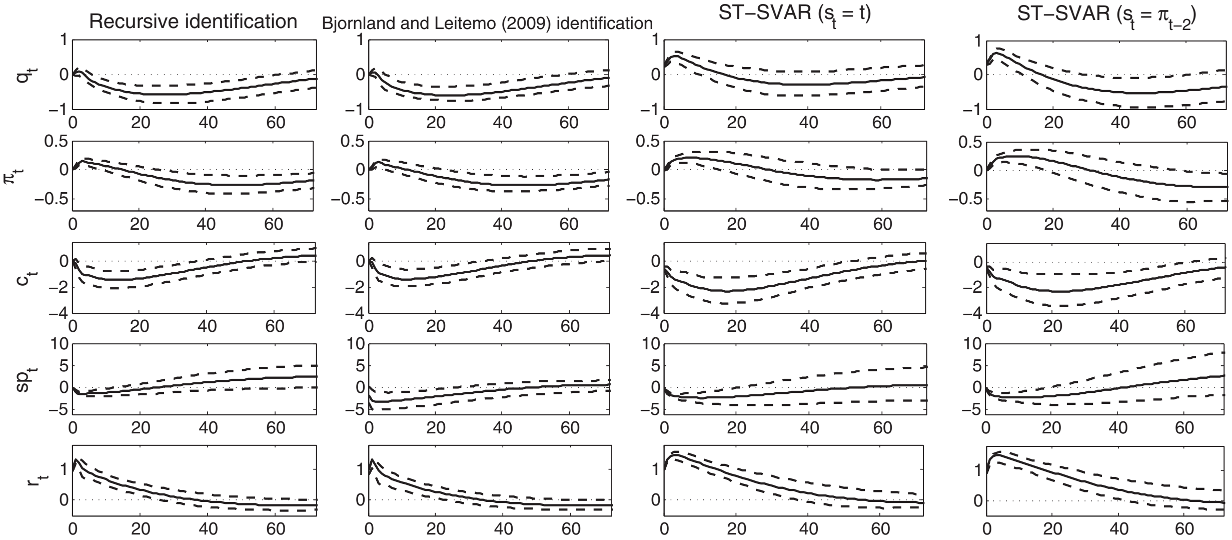


Fig. 5. Responses to candidate monetary policy shocks using different identification schemes. Solid line - point estimate of the response, dashed line - 68% confidence intervals based on 1000 bootstrap replications.

fourth shock is the only one that qualifies as a stock market shock because it is the only one that has a clear effect on the stock index on impact. Therefore we label the first and fourth shocks as monetary policy and stock market shocks, respectively. In other words, the monetary policy shock is the one with the smallest relative variance (λ_1) in the second volatility regime and the stock market shock is the fourth shock with the second largest relative variance (λ_4). An obvious problem with the labelling of the stock market shock is, however, that the shock may not be clearly separated from the last shock (see the estimated relative variances in Table 2). Indeed, the last shock also has a positive impact on the stock index. It is not significant, however, when judged by the confidence interval which includes zero. Thus, we conclude that in the ST-SVAR models we do not find a clearly identified stock market shock.

Although the conventional identifying restrictions are rejected in our framework, it is of interest to compare the responses to a monetary policy shock to those obtained from the ST-SVAR models. One obstacle for comparing impulse responses obtained from these models to impulse responses from a conventional analysis is the variation in the volatility of the shocks. In a conventional analysis the shocks are occasionally scaled to be of size one standard deviation. Since in our framework the standard deviation changes across the sample this scaling is problematic. Therefore, to compare shocks from a conventional analysis and from our approach, we look at shocks of one unit. For example, a monetary policy shock may be scaled to lead to a 25 or 100 basis points increase in the interest rate.

In Fig. 5 we compare impulse responses identified in four different ways:

1. A recursive ordering based on a VAR(3);
2. The Bjørnland and Leitemo (2009) identification based on a VAR(3);
3. Identification via heteroskedasticity based on the ST-SVAR(3) model with transition variable $s_t = t$.
4. Identification via heteroskedasticity based on the ST-SVAR(3) model with transition variable $s_t = \pi_{t-2}$.

In Fig. 5 the impulse responses of q_t, π_t, c_t, sp_t, and r_t to a candidate monetary policy shock of 100 basis points are compared.

The response of the output variable q_t to a monetary policy shock is rather standard for the conventional SVAR models. There is a lagged negative effect reaching its lowest point at around the second year after the shock. On the contrary, the ST-SVAR models produce a significantly positive reaction on impact that dies out after several months. Even though the point estimates of the impulse responses are negative after nearly two years, the reaction is insignificant at the given confidence level. Looking at the response of inflation π_t, one can observe a small initial increase for both conventional identification schemes, whereas the initial dynamics is more pronounced in both models identified via heteroskedasticity. The situation is different for commodity prices. They fall initially in all four models and the decrease is more pronounced in the ST-SVAR models compared to standard SVAR models.

The reaction of stock prices is of primary interest for us. Recursive identification yields a small drop that vanishes quickly and may even turn into an increase that is not significant, however. The stock prices are more responsive in the model identified in the Bjørnland and Leitemo (2009) fashion. We observe a more pronounced decrease of about 3% with the effect lasting for nearly two years. Even though we do not get the same magnitude of decline of the stock index as Bjørnland and Leitemo (2009), their observation of a more pronounced reaction than in the recursively identified scheme is confirmed when identification is done via heteroskedasticity. The reaction depicted in the last two columns of Fig. 5 reveals a very small initial impact effect of the monetary policy shock on stock prices. However, the stock prices drop by 2.3% after a year. The persistence of the reaction is similar to what we observed in the second column. Even though we reject the long-run identifying restriction of Bjørnland and Leitemo (2009), the observed effect on the stock prices appears to be transitory. Thus, this reaction of stock returns to a monetary policy shock identified via heteroskedasticity is closer to the reaction obtained with the Bjørnland and Leitemo (2009) identification scheme than with recursive identification.

However, the monetary policy shocks identified via heteroskedasticity have some economically counter intuitive properties. Namely there is a very pronounced output puzzle and a rather pronounced price puzzle. From the macroeconomic perspective one would not expect these two features for the dynamic responses to a monetary policy shock. On the one hand, the price puzzle for the monetary policy shock is observed by Park and Ratti (2000) and Bjørnland and Leitemo (2009), although the latter use detrended output as suggested by Giordani (2004). On the other hand, no study finds a positive reaction of output. Comparing the monetary policy shock identified via heteroskedasticity to a variety of shocks studied in the literature one can find a sibling to our shock. To be precise, it very much resembles the money market equilibrium shock defined by Li et al. (2010) as an exogenous change in the velocity of money. This shock also leads to an increase in output, prices and interest rates on impact and a moderate decrease in stock returns (see Li et al., 2010 [Figure 1]). Lastrapes (1998) studies a money supply shock and also finds a tendency for output to respond positively on impact. This leads us to think that the monetary policy shock identified via heteroskedasticity captures rather the demand side than the supply side of the money market (compare to the reaction of variables after a money demand shock in Favara and Giordani, 2009). Hence, labelling this shock as a 'monetary policy shock' in the conventional sense may be misleading. However, as discussed before, there is no other shock in the ST-SVAR(3) models that leads to an immediate increase in the Federal Funds rate and qualifies as a monetary policy shock. Therefore in our system and in the system analyzed by Bjørnland and Leitemo (2009) there is hardly any shock that captures the supply side of the money market. This finding sheds some light on why the identifying restrictions are not supported by the data: the restrictions do not identify the desired shock in our extended sample period. This finding shows the importance of the identifying assumptions and that it makes sense to take into account as much information as possible. In particular, it is worth taking advantage of identifying information in the volatility of the shocks.

5. Conclusions

In the present paper we set up a SVAR model with smooth transition in the variances of the residuals. The model is an alternative to other ways of modeling changes in variance in VAR models such as Markov switching or multivariate GARCH models. Our ST-SVAR model has the advantage of being reasonably easy to estimate. Moreover, a well developed toolkit for the statistical analysis of smooth transition regression models is available that may be adoptable for the present models in future work.

We show how the model can be used for identifying shocks in SVAR analysis and for testing conventional identifying restrictions. Although we utilize only one transition function between volatility states, it is possible to extend the model by adding further transition terms. Such an extension seems technically feasible if there are enough data in each volatility state. Note, however, that even our setup can capture various volatility states if a suitable transition variable is chosen. This is, for example, seen in the application when lagged inflation is used as a transition variable. It is also possible to allow a level term to change during the sample period by attaching the transition function also to that term. As long as the VAR coefficients are time invariant, impulse response analysis can be performed as discussed in the present paper.

As an illustration of the ST-SVAR approach we analyze the relation between monetary policy and the stock market using a system considered by Bjørnland and Leitemo (2009) but with an extended sample period. The estimated model suggests a rather smooth transition from a high volatility state to a low volatility state at around 1984 when time is used as a transition variable. This is consistent with the economic narratives on the Great Moderation in the U.S. being in place from the mid 1980s. A more sophisticated volatility pattern is obtained when lagged inflation is used as transition variable. In that case some changes in residual volatility are also better captured by the smooth transition model.

The main question of interest, however, is the interaction between the U.S. stock market and monetary policy. Bjørnland and Leitemo (2009) propose a combination of short- and long-run identifying restrictions on the impulse responses and contrast their results with a recursive identification scheme. Our models allow us to test the identification of Bjørnland and Leitemo (2009) as well as a competing Cholesky based recursive identification scheme. We reject both the identification based on recursive ordering of the variables and on a combination of short- and long-run restrictions for our extended sample period. Of course, this result is conditional on our ST-SVAR models which, as we have emphasized, may not fully capture all heteroskedasticity in the data. The analysis illustrates, however, how our model can be used in principle for investigating the validity of conventional identifying restrictions in a SVAR analysis.

Conditional on our preferred model, our approach can also be used to study the impulse responses to one of the shocks which is clearly identified via heteroskedasticity in our model. The analysis reveals why the data do not support the conventional restrictions. We find that in a conventional SVAR the monetary policy shock represents the supply side of the market. Using our model we find that there is a monetary shock representing the demand side of the money market which has pronounced short-term effects on the variables. Unfortunately, we do not find support for a clearly identified stock market shock in our models. Thus, when identification via heteroskedasticity is used, the interaction between monetary policy and the stock market is seen in a quite different light than in a conventional analysis. As stressed earlier, these results are obviously conditional on our ST-SVAR model which captures some of the heteroskedasticity in the data but still leaves room for further refinements in future research.

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