*random realization:- inter - arrival time , service tim

2070 short note

System environment

The change occuring outside the system that affects the system is system environment.

Defining a system boundary is an important step:-

↳ If boundary is small, may not include necessary component.

↳ If large boundary, high degree of error propagation management difficulties.

Closed system

↳ that is not affected by exogenous events

↳ ~~pa~~ pratically unrealizable

Open System

affect by exogenous event

2070

Q no 13: Short note (2.5 marks)

Real time simulation

- some device and their function are so critical that any model of such device / sub-system could not achieve desired output.

The real device are used to provide input / feedback to the simulation program.

The output is deployed on the real device.

- Such type of simulation is known as real time simulation.

- The main reason behind real-time simulation is to simulate real world events on real time.

Simulation

- A simulation is the imition of the operation of the real world process or system over time.

- can be used to study simple models but should not use it if an analytical solution is available.

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2073
2071
5 marks
Q.no.4

Types of Simulation model

imp

1. Physical
2. Mathematical
3. Computer based

1 Physical model with example

Physical model is a model of actual system which has all the properties of the system or at least it is as close to the actual system as possible.

Nowadays, small model of car, helicopter and air-craft are avialable in market.

i) Static physical model

→ Static physical model is the model of the system which doesn't change with time.

example :-

An architecture before conducting a building makes a scaled down model of the building which reflects all its rooms outer-design and other important feature.

ii) Dynamic Physical model

→ Dynamic physical model are ones which change with time or which are function of time.

example

In wind tunnel, small air craft model (static mode are kept and air is blown over them with different velocities and pressure profile are measured with the help of transducer embedded in the mode.

2. mathematical model

=) Most of the system in general be transformed into mathematical equations. These equation are called mathematical model of that system.

=) If a mathematical model does not involve time i.e system doesnot change with time, it is called state mathematical model of the system.

example:- static market model

=) Generally there should be balance between the supply and demand of any product in market.

=) Supply increases if its price increases

=) on other hand demand decreaces with the increases of price.

=) Aim is to find the optimum price with which the demand can match the supply.

Lets donate mathematically.

Let P be price, S be supply and D be demand.

Then

$$D = a - bP$$
$$S = c + dP$$
$$S = D$$

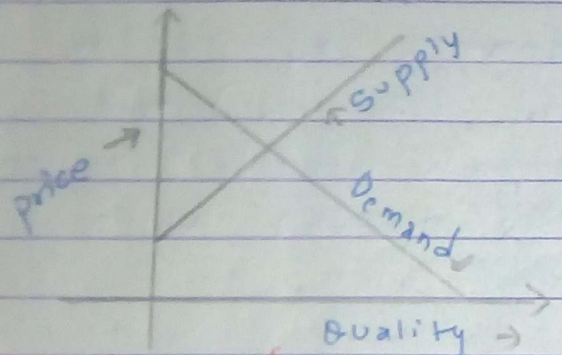

fig:- linear market model

Q. Describe lag model in brief with relevent example.

imp 2069 2072

i) **Distributed lag-model**

Models that have the properties of changing only at fixed interval of time and of basing current values of the variables on other current values and values that occured in previous interval are called distributed lag model.

example:-

Consider a simple dynamic mathematical model of the national economy.

Let C be consumption,

I be investment

T be taxes

G be Government expenditure

& Y be national income.

Then,

$C = 20 + 0.7\,(y - T)$

$I = 2 + 0.1y$

$T = 0 + 0.2y$

$y = C + I + G$

This is static model but it can be ~~model~~ made dynamic by pickinfo a fixed time interval (1 yr's) and expressing the current value of the variable in terms of the values of the previous value.

Any variable that appears in the form of its current values and one or more previous year. value is called lagged-variable

The static model can be made dynamic by lagging all the variables as

$C = 2.0 + 0.7\,(y - 1 - T - 1)$

$I = 2 + 0.1\,y\,1$

$T = 0 + 0.2\,y - 1$

$y = C - 1 + I - 1 + G - 1$

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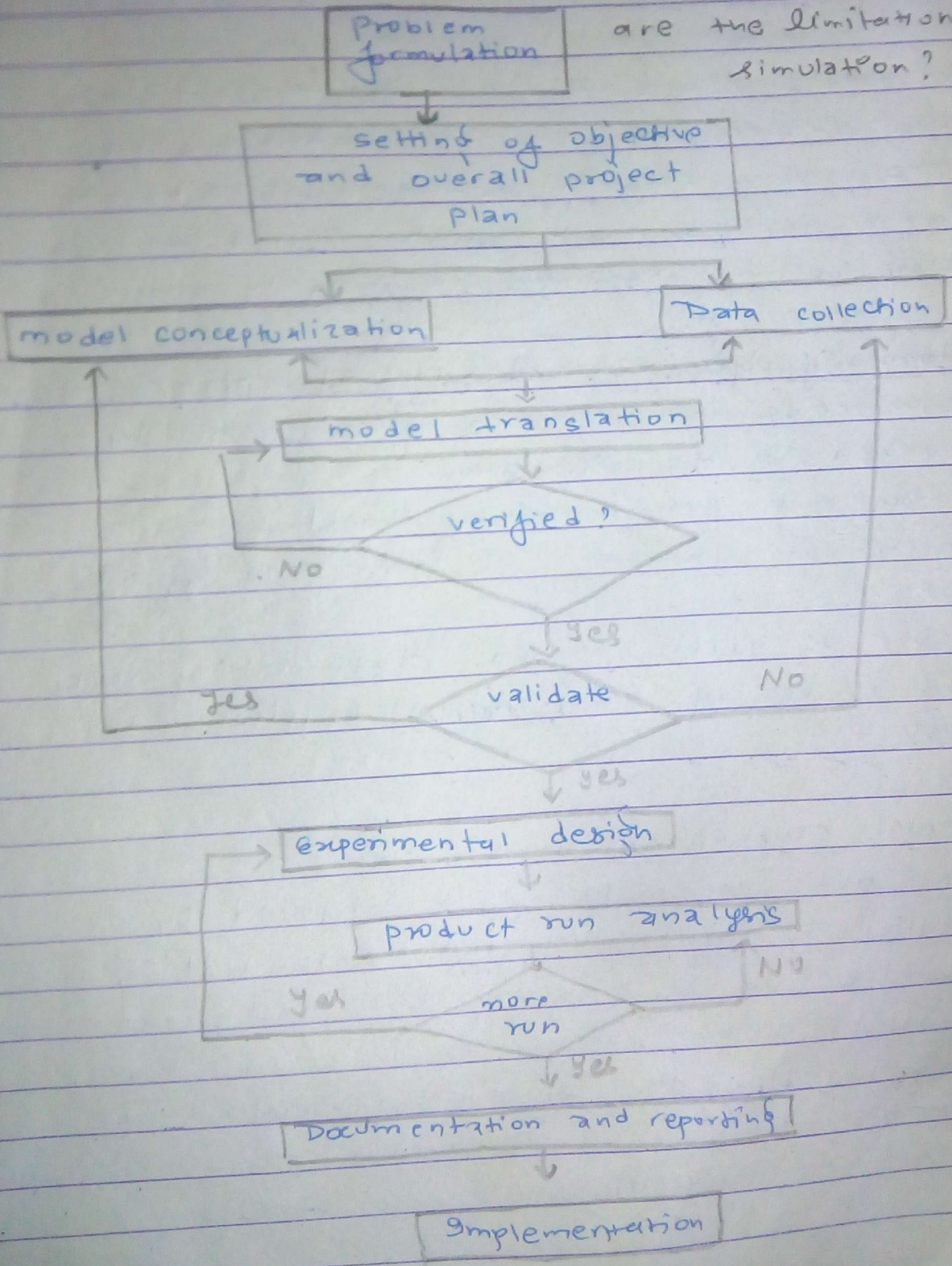
(Q5) what are the various steps in simulation study? Explain.

Steps in simulation :-

Q.1 explain the steps in simulation study. what are the limitation of simulation?



Problem Formulation

1. Every study should begin with the statement of the problem. If the statement is provided by the policy-maker or those who have the problem, the analyst must ensure that the problem being described is clearly understood.

2. Setting of objective and overall plan:

The objective indicates the question to by answered by the simulation.

→ At this point determination should be made concerning whether simulation is appropriate tool.

→ number of people involved - the cost of study and number of days to accomplish the task.

3. model conceptualization

→ The art of modeling is enhanced by an ability to abstract the essential feature of a problem to select and modify the basic assumption that characterize the system and then to ~~enti~~. enrich and elaborate the model until a useful experimental result.

#4 Data collection

→ There is a constant interplay between the construction of the model and the collection of the model input data.

→ Data collection takes large portion of the total time required to perform simulation so begin as early as possible.

#5 Model Translation

→ Most real world system results in model of information that requires a great deal of storage and computation so the model must be entered into computer recognizable format "called program"

→ Automod, Arena, SIMUL.

#6. Verified

→ is computer program performing properly?

→ for most of the pact common sense is used.

#7. validate:-

→ Validation is usually achieved through the calibaration of the model an iterative process of comparing model against actual system behaviour and using the difference between two and the insight is gained to improve the model.

#8. Experimental design

→ The alternative that are used to be simulated must be determined.

→ Concerned with the length of the initialization period, the length of simulation run and number of replication to be made on each run.

#9. Production run and analysis

Production runs and their sub sequent analysis are used to estimate measure of performance for the system design that are being simulated.

#10. More run

Given the analysis of runs that have been completed, the analyst determines whether additional runs are needed and what design those additional experiments should follow.

#11. Documentation and Reporting

Types → program
→ Progress

→ If the program is going to be used again by the same or different analysts it could be necessary to understand and have the program works.

→ Progress report gives the chronology of the work done and decision made.

#12. Implementation :-

→ The success of implementation depends on previous phases.

→ It is also contigent upon how throughly the analyst has involved the ultimate model user during the entire simulation process.

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Imp

Find 5 random number.
using multiplication congmential method
$a = 6507$
$c = 0$
$m = 1024$

using mixed congruential method
$a = 4951$
$c = 247$
$m = 256$.

Soln

Let $x_0 = 35$

Here given,
$x_0 = 35$, $a = 6507$ $c = 0$ & $m = 1024$

Now,

$x_{0+1} = (ax_0 + c) \bmod m$
$= (6507 \times 35 + 0) \bmod 1024$
$= 419$
$\therefore R_1 = 417$

$x_{1+1} = (ax_1 + 0) \bmod 1024$
$R_2 = 843$

$R_3 = 852$

$R_4 = 819$

$R_5 = 337$

using mixed congreuential method

$x_0 = 77$, $a = 4951$, $c = 247$, $m = 256$

wow,

$x_0 = (ax_0 + c) \mod m$

$R_1 = 34$

$R_2 = 133$

$R_3 = 42$

$R_4 = 61$

$R_5 = 178$

hence the random number are

34, 133, 42, 61, 178.

Q.6) Why an auto-correlation test is needed in random number?

2071 explain the process of testing for auto-correlation test ?

Test of auto correlation

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Tuesday

2069 / 2071

- The test of auto-correlation concerned with the dependence between number in sequence.
- The test compute the auto-correlation betn every n number (m is also known as log) starting with ith number.
- The auto-correlation ρ_{im} betwee n the following number would be of interest.

$R_i, R_{i+m}, R_{i+2m} \ldots\ldots R_{i+(M+1)m}$

- The value m is the largest integer such that $i + (M+1)m \le N$ where N is the total number of values in the sequence.

$N = 17$
$i = 3$
$m = 4$

~~R_3~~, ~~$R_{i+m} = 7$~~

~~R_7~~

A non-zero auto correlation implies a lack of independence so the following two tailed test is appropriate.

$H_0: \rho_{im} = 0$
$H_1: \rho_{im} \neq 0$

The test statistics can be calculated as,

$$Z_0 = \frac{\hat{\rho}_{im}}{\sigma_{\rho_{im}}}$$

where,

$$\hat{\rho}_{im} = \frac{1}{M+1}\left[\sum_{k=0}^{M} R_{i+km} \; R_{i+km} \; R_{i+(k+1)m}\right] - 0.25$$

$$\sigma_{\rho_{im}} = \sqrt{\frac{13M+7}{12(M+1)}}$$

after computing Z_0, do not reject null hypothesis of independence if $-Z_{\alpha/2} \leq Z \leq Z_{\alpha/2}$ where α is level of significance

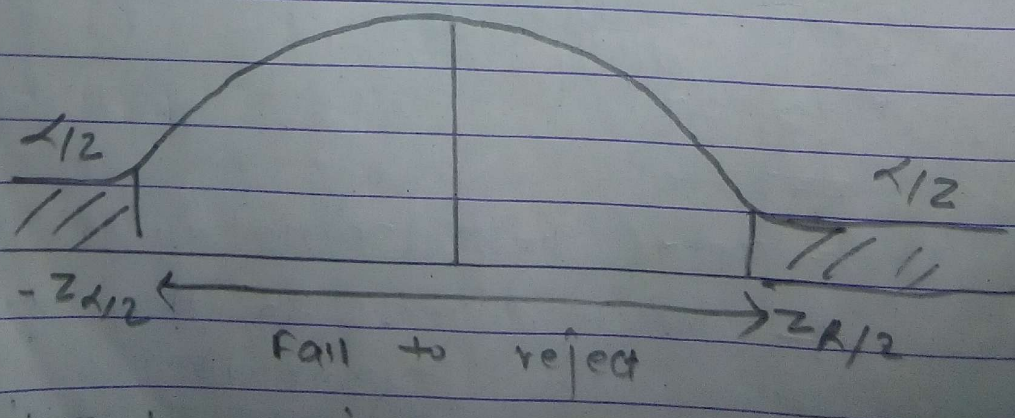
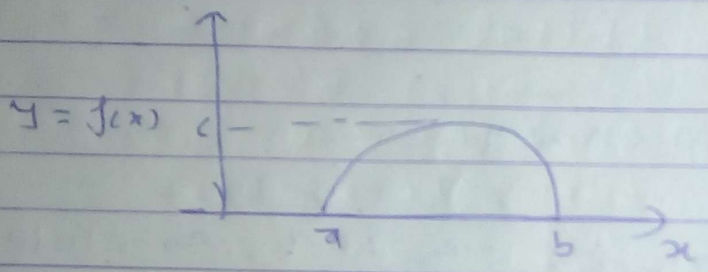


fig: failure to reject null hypothesis

number Generator.

1. Rejection method:-

The rejection method is applicable when the probability density function has the lower and upper limit to range 'a' and 'b' and upper bound c. The method can be specified as follow:-



a) Compute the value of two independent uniformly distributed variate v_1 and v_2

b) Compute $x_0 = a + u_1 (b-a)$

c) Compute $y_0 = cu_2$

d) If $y_0 \leq f(x_0)$ then accept x_0 as desired output otherwise repeat the process with two next uniform variate u_1 & u_2

Note:

u_1 and u_2 are random number between 0 and 1.

2070
2073 Define congestion in a queueing system, and describe its major characteristics.

Characteristics of queueing system

The key element of a queuing system are customers & servers.

- The term customer can refers to people, machines, trucks, patients, airlines, emails or dirty clothes; anything that arrives at a facility and require service.

- The term server might refers to a receptionists, repair persons, machines, medical personal, run ways at an airport, or CPU in computers - any resources (person, machine etc) that provide the requested service.

The characteristics of queueing system are as:

- Calling population
- The system capacity
- The arrival process
- Queue Behaviour & queue discipline
- Service Time & service Mechanism.

Calling Population

The population of potential customers referred to as calling population may be assumed to be finite or infinite.

For example:

consider the personal computers of employees of a small company that are supported by IT staff of there ~~technign~~ technicians. when a computer fails, need & new software etc. It is attented by one of the IT staff. The customer ~~and~~ are computers & the IT staff are server. In this case calling population is finite.

In system with large popn of potential customer, the calling popn is assumed infinite.

2. The System Capacity

- In many queueing system there is limit to the no. of customers that may be waiting in line or system.

- For example:-
an automatic car wash might have room for only 10 cars to ~~want~~ wait in line to enter the mechanism. so an arriving customer who find the system ~~is~~ is full doesnot enter but returns immediately to the calling popn.

when the system has the ~~limited~~ limited capacity, a distinction is made betn

Arrival rate and effective arrival rate.

3. The arrival Process

- The arrival process for infinite pop^n model is usually characterised in terms of inter-arrival time of the successive customer.

- Arrival may occurs at scheduled or at random time.
  - eg. arrival of telephone calls to a call center.
  - patient at a physician office
  - airline flights arrivals to an airport.

Short notes :-

4. Queue Behaviour & Queue Discipline

- Queue behaviour refers to the action of customer while in queue waiting for the service to begin.

- In some situation, there is possibility that incoming customer will balk (leave when they that the line is too long) renege (leave after being in the line when they see that line is moving too slowly) or Jocky (move from one line to another if they think they have choosen a slow line)

2072. Q8. Describe different type of queueing descipline & with example.

Queue discipline refers to the logical ordering of customer in a queue and determines which customers will be choosen for service when a server become available

Common queue discipline includes FIFO, LIFO, SIRO (service in random order) shortest processing time first (SPTF).

Service time & service mechanism

The service time of successive arrival are denoted by s_1, s_2 .... They may be constant or of random duration.

A queuing system consists of a number of service centers and interconnecting queues. Each service center consists of some number of server c, working in parallel that is upon getting to the ~~next~~ head of the line a customer taken the first available server.

Single Channel queue illustratio.

Simulate the single queue model.

2068 What are the kendall notations of Queuing system?

Queueing notation (kendall Notation)

Recognizing the diversity of queueing system kendall (1953) proposal a notational system for parallel server system which has been widely accepted.

- An abridge version of the convention is based on the format A|B|C|N|K. These letter represents the following system characteristics.

A represent the inter-arrival time duration.
B " " service time duration.
C " " number of parallel service.
N " " system capacity.
K " " the size of calling popl^n.

Common symbol for A and B include M (exponential or Markob), D (constant or

deterministic), PH (phase type), H (hyper expone
, G (Arbitary as general) & GI (General
independent.).

For example:

M | M | 1 | ∞ | ∞ indicates a single server system that has unlimited queue capacity and an infinite population of population arrival. The inter-arrival and service time are exponentially distributed when k and N are infinite and can be written as

M | M | 1.

2(b) What do you mean by server-utilization?

Server utilization ($\hat{\rho}$)

Server utilization is defined as the proportion of time that a server is busy. Observed server utilization denoted by $\hat{\rho}$ is defined over a specified time interval [0, T], long run server utilization is given by

$$\hat{\rho} = \frac{\text{total server busy time}}{\text{total time}}$$

eg:

total time = 20

total server busy time = 17

$$\hat{\rho} = \frac{17}{20} = 0.85$$

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Q.8 What do you mean by Multi-server Queue?

Multiserver queue

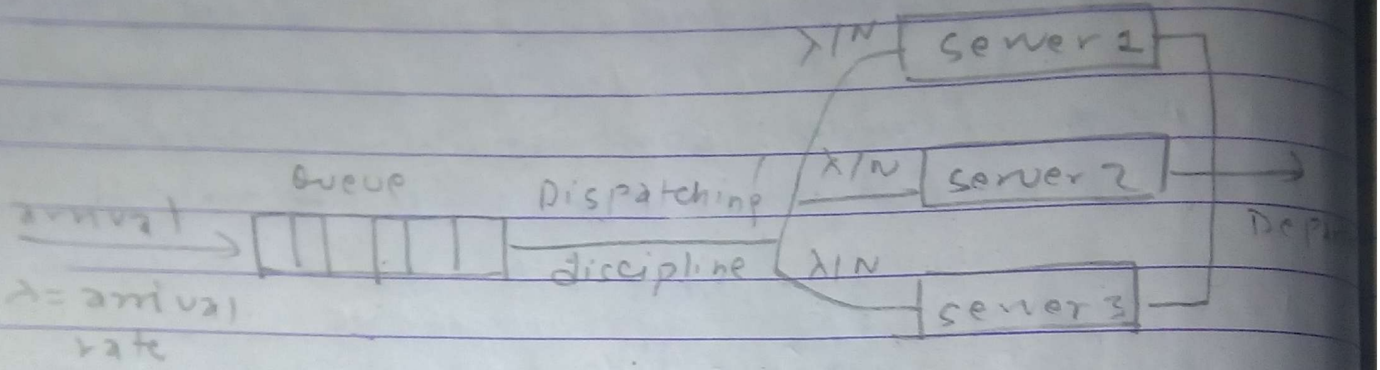

fig Multiserver queue

- Figure show a generalization of the simple model we have been discussing for multiple server, all sharing the common queue.
- If an item arrives and at ~~most~~ least one server is available then the item is immediately dispatched to that server.
- It is assumed that all server are identical, thus if more than one server is available it make no different which server is choosen for the item.
- If all server are busy a queue is begin to form.
- As soon as the one server becomes free an item is dispatched from the queue using the dispatching discipline in force.

gn is in infinite population & infinite queue size and dispatching discipline is FIFO.

The total server utilization in case of multi server queue for N server system is

$$\hat{S} = \frac{\lambda}{C\mu}$$

where, μ is the service rate
λ is the arrival rate.

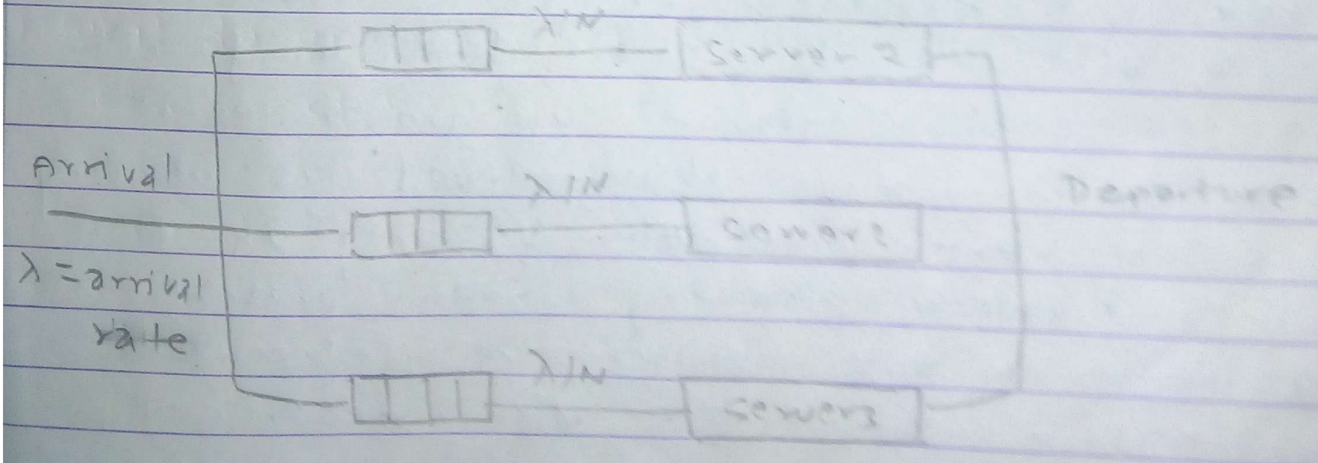

fig: Multiple single server queue

Explain the Markov chains with example and its applications.

Markov chain and its application

A markov chain named after Andrey Ma is a mathematical system that undergoes transition from one state to another between a finite or countable number of possible state.

- It is a random process characterized as memoryless that next state depend only on the current state and not on the sequence of events that proceed it.
- A markov chain can be defined formally as random process with Markov property. A markov chain, a sequence of random variable x_1, x_2 ... with the another betn a fininite or countable number of possible state.
- If a random process characterised as memoryless that next state depend only on the cur. markov property, given present state future and past states are independent.
- A markov chain are often described as directed graph where the edges are labelled by the probability of going ~~one~~ state to another state.

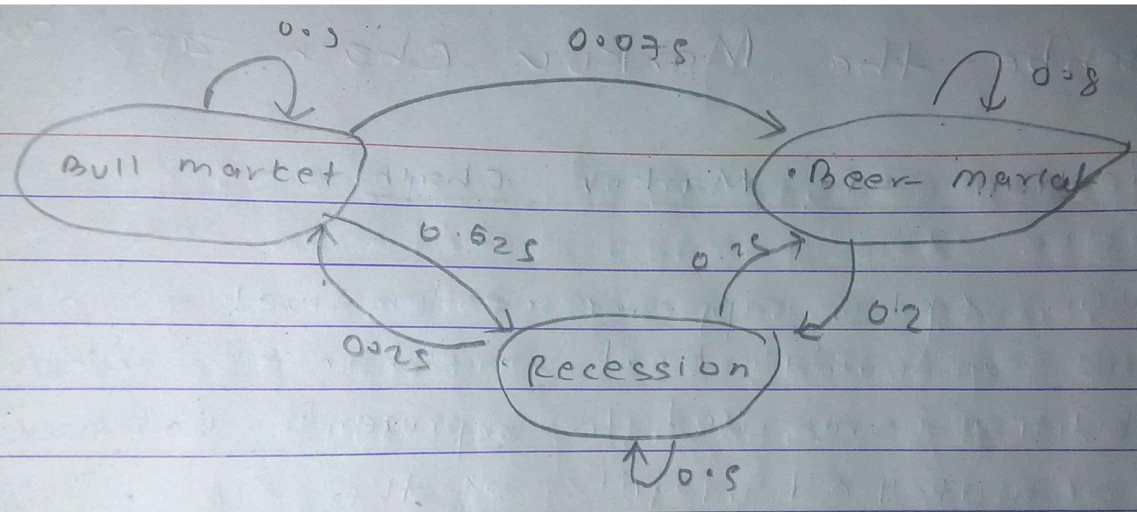

fig: example of Markov chain

eg: A simple weather model

The probability of wether condition modelled as either rainy or sunny.

$$P = \begin{array}{cc} \text{sunny} & \text{rainy} \\ \left[\begin{matrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{matrix}\right. & \left.\right] \end{array}$$

The weather on day 0 is known to be sunny. This is represented by a vector as,

$$x^{(0)} = [1 \quad 0]$$

The weather on day 1 can be represented as,

$$x^{(1)} = x^{(0)} P$$

$$= [1 \quad 0] \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix} = [0.9 \quad 0.1]$$

The weather on second day can be predicated the same way.

$$x^{(2)} = x^{(1)} P = [0.9 \quad 0.1] \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix}$$

$$=$$

In general the role can be written as

$$\boxed{x^n = x^{(n-1)} \cdot P}$$

Q13 Describe the Markov chain applica

Application of Markov Chain

1. Physics

Markovian system appears extensively in the dynamics and statistical mechanics whenever probabilities are used to represent unknown or unmodelled details of the system if is assumed that dynamics are real - Invariants and that not relevent history need to be considered which is not already in the state discipline.

2. Queueing Theory

Markov chain are the basis for the analytical treatment of queue optimizing the performance of ~~the~~ telecommunication network where message must often complete for limited resource (such as band width).

3. Internet application

The page rank of webpage as used by Google is defined by the markove

Q73: what do you mean by differential partial Equation. Describe importance of differential / partial eqn?

Differential Equation

A differential equation is a mathematical equation that relates some function with its derivatives. In applications, the function usually represents physical quantities, the derivatives represents their rate of change & the equation defines a relationship between the two

$$\frac{dx}{dy}$$

Partial Differential equation

A partial differential equation is a differential eqn that contains unknown multivariable functions and their partial derivatives. These are used to formulate problems involving function of several variables and are either solved by hand or used to create a relevent computer model.

Poision arrival Pattern

The arrival of a customer is completely random. This means that an arrival can occur at any time and the time of next arrival is a independent of the previous arrival.

- with this assumption it is possible to show that the ~~destination~~ distribution of the inter arrival time is exponential.
- This is equivalent to saying that the number of arrival per unit time is a variable with a poissions distribution. This distribution is used when chance of occurance of event out of large sample is small.

If x = number of arrival per unit time then the distribution function of arrival is given as,

$$f(n) = P_r(X = n) = \frac{e^{-\lambda}\lambda^n}{n!} \quad \left\{ n = 0, 1, 2 \right.$$

$p(x) = \lambda$.

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2071 , explain the example of verification of
2070 simulation model

- ~~It proceed by the comp~~
- It ask the question ! is the model implemented correctly in the simulation software ? Are the input parameter and logical structure of the model represented correctly.

- validation is concerned with building the corr model.
- It attempts to confirm that the model is an accurate representation of real system.
- Validation is usually achieved through the calibration of the model and the interactive process of comparing the model to actual behaviour .
- The process repeated until desired model accuracy is meet.

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Model building, verification & validation

Steps in model building:-

1. observing the real system and interacting among their various components and collecting data on their behaviour.
2. construction of conceptual models, collection of assumption about the component & the structure of the system plus hypothesis about the value of model.

3. The third step is to implement of the operational model usually by using simulation software and incorporating the assumption of the conceptual model into the world view and concept of simulation software.

- In reality, model building is not a linear process w their steps. Instead, model builder will return to each of three step many times while building, varifying & validating the model.

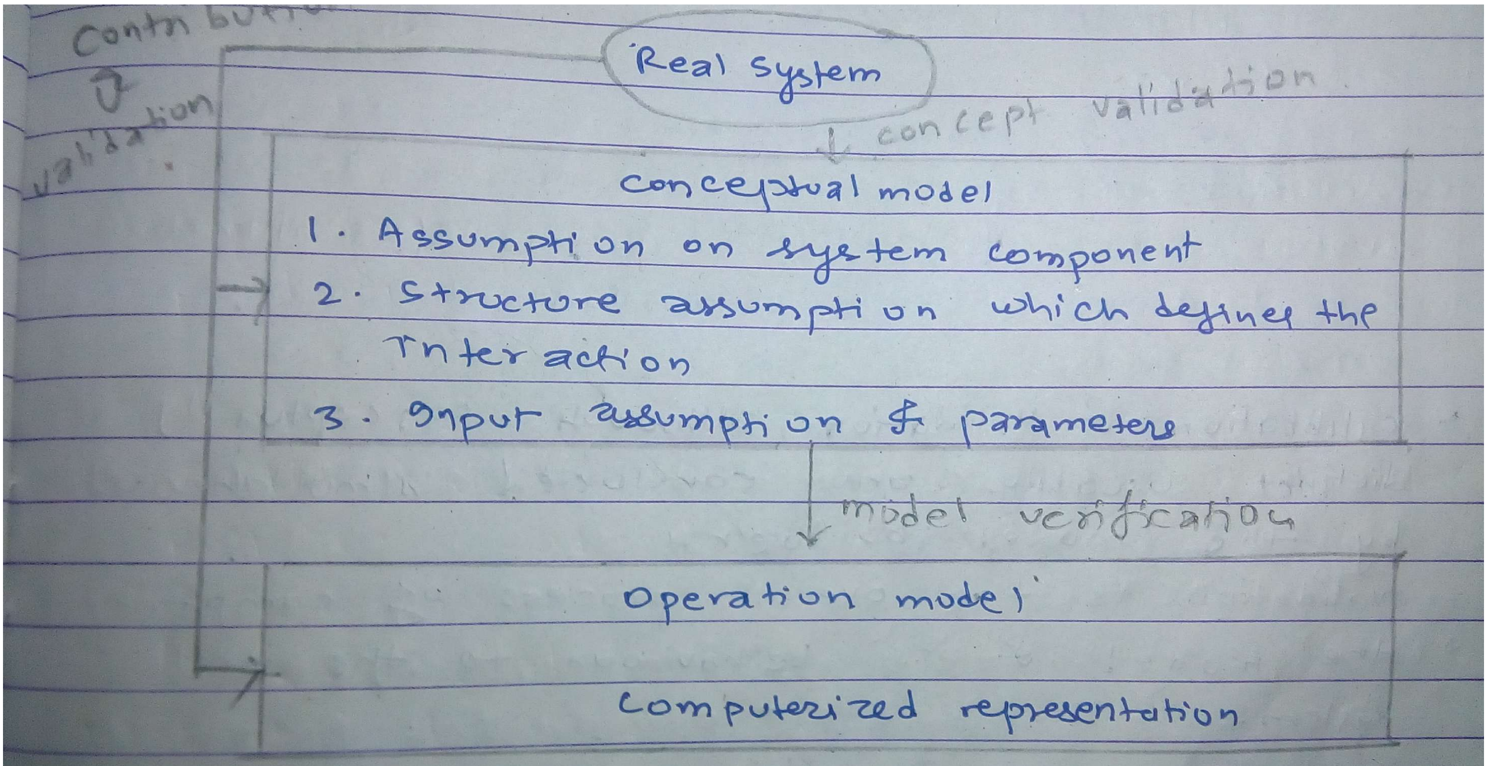

fig. Model building, verification & validation

2071(S): explain the example of calibration and validation.

2069 what do you mean by calibration & validation

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Wedensday

calibration & validation of model

2068

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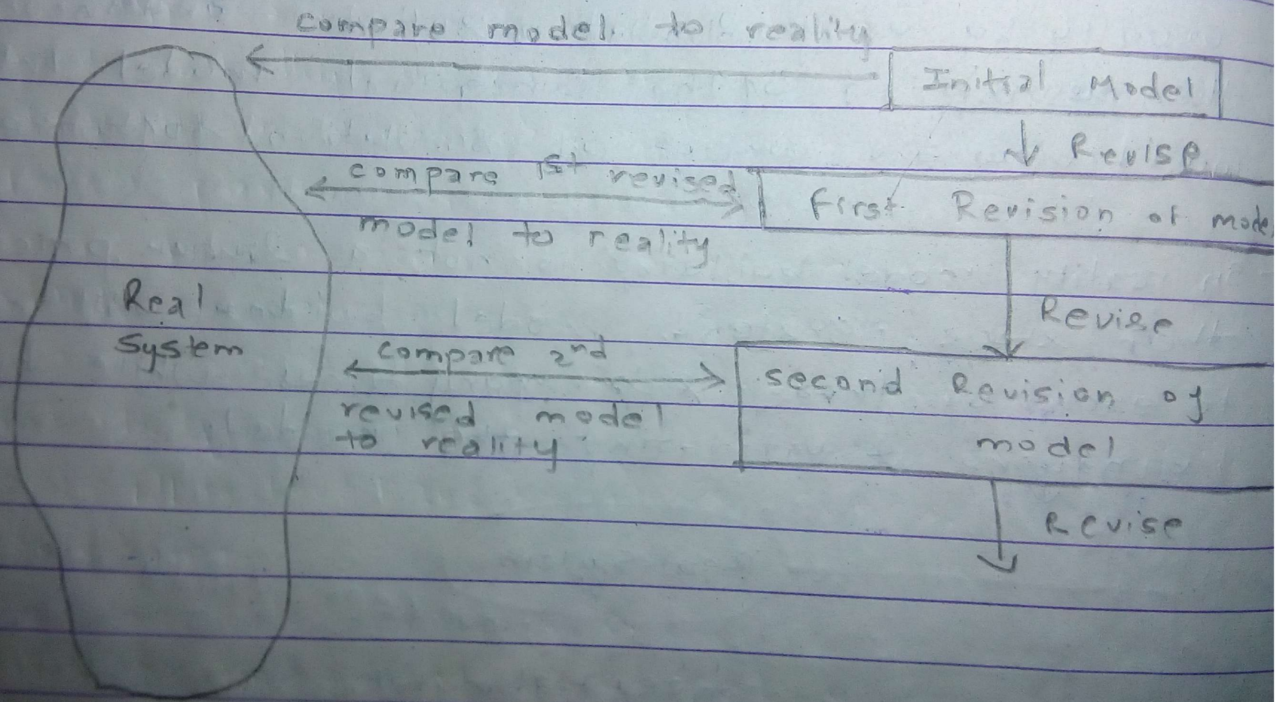


fig:- Iterative process of calibration & validation fig:- Iterative process of calibration & validation

- calibration & validation, although, conceptually distinct usually are conducted simultaneously by the model builder.
- validation is the overall process of comparing the model & its behaviour to the real system.
- Calibration is an iterative process of comparing the model to the real system making adjustment (or even major change) to the model comparing the revised model to the reality making additional adjustment,

comparing again & again.

- comparing of the model to reality is carried out by variety of test, some are subjective while other are objective.

- subjective test usually involves people who ~~can~~ are knowledge about one or more aspect of the system making judgement about the model & its output.

- objective test always require data on the system behaviour plus the corresposding data produced by the model. Then one or more statistical test are performed to compare some aspect of system data.

- The main drawback of calibration & validation of the model is that it difficult to decide when the revision loop is going to be finished, when the model is perfect.

2070: Different types of statements in CSMP.

CSS (continuous system simulation language)
- eg: CSMP (continuous system modelling program)

CSMP III

- IBM program product specially designed to satisfy the need of the engineer & scientist who wish to simulate the physical phenomena without having to spend time and resource.
- A CSMP III program is constructed from three type of statements

1. Structural statement
- They define the model - They consist of FORTAN like statement and functional block designed for a operation that frequently occur in model defn?

2. Data statement
- It assign numerical values to the parameter constants and initial condn.

3. Control statement
It specifies option in the assemblence and execution of the program and the choice of output.

structural statement.

make use of addition, subsbraction, multiplication, division and exponentation using same notation and rule as used in FORTANT.

for example.

$$x = \frac{64T(z-2)^2}{w}$$

can be expressed as

x = ((6.0 * 4 + (z - 20) ** 2.0) / w)

Data statement (remaining part for now)

2072: explain any four control statement that are used in GPSS.

2070, Diff type of statement is CSMP.

2069

Control statement.

1. Timer : to specify certain interval
2. PRINT : For printing output
3. PRTPLT : for print plotted output
4. TITLE : to place title
5. LABEL : label
6. END : end of the statement
7. STOP : stop the execution of the program.

2069 1. Explain hybrid simulation.
2068.

Hybrid Simulation:

For most of the studies the model is clearly either of continous or discrete nature, and that is the determining factor in deciding whether to use analog or digital computer for system simulation.

- However, there are times when an analog and digital computers are combind to provide a hybrid simulation.

- The hybrid simulation depends upon the application. One computer may be used to simulate the system being studied while the other is to provide the simulation of the environment in which the system is to operate.

The introduction of hybrid simulation require certain technological development for exploitation. High speed converter are needed to transform signal from one from of representation to other.

As a practical ~~matt~~. matter, the availability of mini computer have made hybrid simulation easier by lowering cost and allowing computer to be dedicated to an application.

2073.11 Why do we need the analysis of simulation output? How do you use simulation run statistics in output analysis? Explain.

=> Output analysis is the modeling stage concerned with designing replications, computing statistics from them and presenting them in textual or graphical format.

Output analysis focuses on the analysis of simulation results (output statistics).

It provides the system behaviour and generate prediction.

A good design of simulation ~~runs~~ runs in least cost.

Simulation Run Statistics

For central limit theorem, two more considerations exists;

1. The observation are manually independent
2. The distribution from which they are drawn is stationary

But most of the cases, both of these considerations do not ~~hold~~. hold.

(1) eg: (manually independent)

let's consider a system with 'n' successive entities. Let x_i represents wait time for ith entity. To represent mean wait time.

$$\bar{x}(n) = \frac{1}{n} \sum_{i=1}^{n} x_i$$

as wait time of ith entity affect wait time of remaining entities, they are not independent. Therefore, they are autocorrelated.

In most of the simulation run, the sample mean of autocorrelated data is approximated to a normal distribution as the sample size increases.

In such case, the following equation holds

$$\bar{x}(n) = \frac{1}{n} \sum_{i=1}^{n} x_i$$

however, varience of autocorrelated data will not exhibit same property as varience of autocorrelated data will not exhibit same property as varience of independent data.

Therefore, some adjustment is made for auto-correlated data. Failure to adjust will result into under-estimation or over-estimation

(2) eg: stationary:
- early arrivals get service quickly, later arrivals have wait.
- large number of simulation runs reduce the bias
- If distribution for particular sample size is taken, it might differ from another sample size
- This leads to dynamic distribution

eg: mean wait time

2073.10. Diff. betn fixed time step and the event to event model, with the help of suitable example.

→ Events means change in the state of system. for example the arrival of customer in queue is an event.

Discretes system can be of two types
1. Fixed time step
2. next - event model

1. ~~First~~ fixed time step
In a fixed time - step model a timer is simulated by the computer, this timer is updated by a fixed time interval τ, and the system is examined to see if any event has taken place during this time interval, all events that take place during that intervals are treated as if they occured simultaneously at the end of this interval.

2. next - event model
In next model event the computer advances time to the occurance of the ~~occuran~~ occurance of the next event, thus it shifts from one event to the next, the system state does

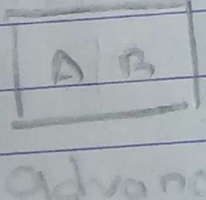
not change in between, when something of interest happens to the system, the current time is kept track of.

2073. 9.

Draw and discribe the different type of GPSS block that are used to gather statistics?

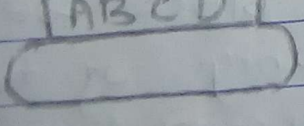
GPSS is highly structured, special purpose simulation programming language based on process interaction approach and oriented toward queueing system.

There are over (40 - 48) standard blocks in GPSS-



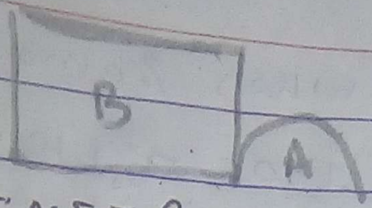
- An advance block delay the progress of transaction

advance



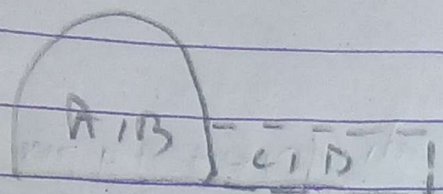
- Assign blocks are used to place or modify a value in a transaction parameter.

ASSIGN



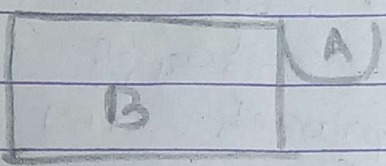
ENTER

An enter block, it either takes or waits for storage units.



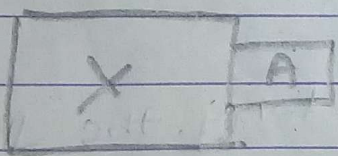
Generate

A generate block create transaction for future entry into the simulation



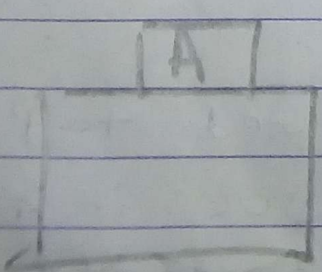
Leave

- A leave block increase the accessible storage units at a storage entity



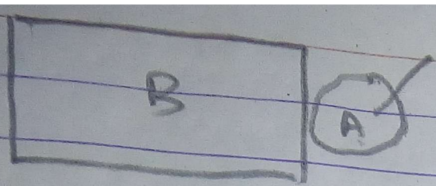
Logic

- A logic block changes the state of logic entity



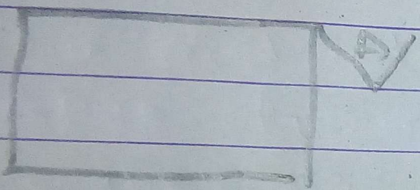
- A priority block sets the priority of the active transaction.

priority



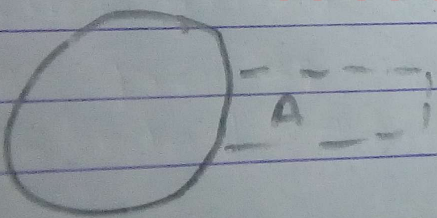
Queue

A queue block ~~re~~ updates Queue entity



Release

A release block release the transaction



terminate

A terminate removes the active transaction.

2077 Q.9 How do you estimate the effect of transient and initial bias in simulation output?

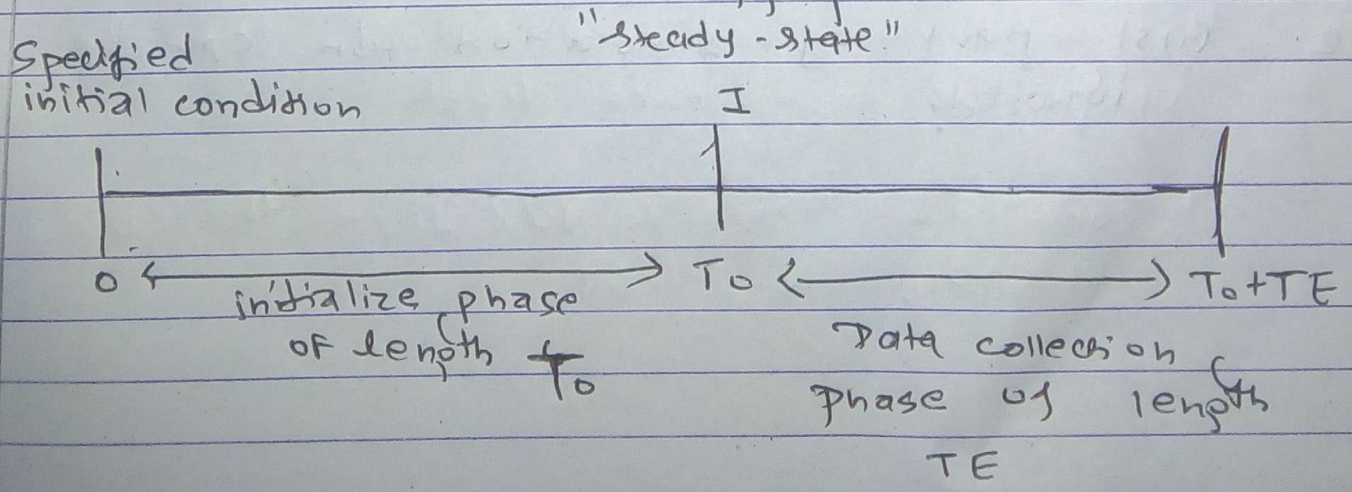
⇒ Initial system state representation is more representative of long-run conditions. This step is also known as intelligent - initialization

Two ways to specify the initial bias:

1. If the system exists, collect data on it and use these data to specify more typical initial conditions.
   - It require large data collection
   - Impossible to implement

2. Reduce the impact of initial conditions, by dividing each simulation run into two phases;

   - Initialize phase from time 0 to To
   - Data collection phase from time To to stopping time To + TE



- In such case, simulation begins at time 0 under specified initial conditions I_0, and runs for specified period of time T_0.

- Data collection on the response variables of interest starts from T_0 and until time $T_0 + T_E$.

- The length T_E of the data collection phase should be long enough to gurantee sufficiently precise estimates of steady state behavior.

elimination of inital bias

- Two approach

1. System can be started in a more respective state then the empty state
2. First part of simulation run can be ignored.

2072: Q.no 5 How do you use estimation method in the analysis of simulation output?

2069

Q.9 ~~it states~~

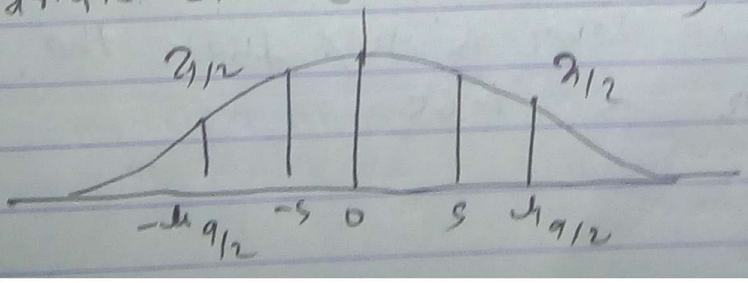
Central limit theorem states that the sum of n idependently and identically distributed variables, drawn from population that has a mean of M and a varience of s^2, is approximately distributed as a normal variable with a mean of nM and varience of ns^2.

$$X_{norm} = \frac{\sum_{i=1}^{N} x_i - \sum_{i=1}^{N} m_i}{\sqrt{\sum_{i=1}^{N} s_i^2}}$$

any normal distribution can be transformed into a standard normal distribution (mean = 0, varience = 1). Therefore, the normal variate will be

$$z = \frac{\sum_{i=1}^{n} x_i - nm}{\sqrt{n}.s}$$

The probability distribution of standard normal variate can be illustred as,



let for chousen value of u is satisfying

$$\phi(u) = 1 - a/2$$

- let the value of u be $u_{a/2}$
- The probability that z lies between $u_{a/2}$ and $u_{-a/2}$ is

$$\text{prob}\{-u_{a/2} < z < u_{a/2}\} = 1-a$$

- In terms of samble mean,

$$\text{Prob}\left\{\bar{x} - \frac{s}{\sqrt{n}} u_{a/2} \leq m \leq \bar{x} + \frac{s}{\sqrt{n}} u_{a/2}\right\} = 1-a$$

the constant $1-a$ is confidence level and

$$\bar{x} + \frac{s}{\sqrt{n}} \cdot u_{a/2}$$ is confidence interval

the size of confidence interval depends upon the confidence level.

If confidence level is 90% then the value of $u_{a/2}$ is 1.65.

In practicle population varience s^2 is not known, in such case, it is replaced by an estimate calculate using

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

2071 Q.9 explain the replication of runs.

- Repeated no of simulations generate independent results.
- For each simulation run, a different random numbers are used for same sample size 'n' and the simulation gives a set of independent determinations of the sample mean.

$\bar{x}(n)$

- even though the sample mean depends on degree of auto correlation, the independent determinations can be used to estimate the varience of the distribution.
- let the simulation is run for p times with independent random number series.
- let x_{ij} be the ith observation in the jth run and let the sample mean and varience for the jth run be,

$\bar{x}_j(n)$ $\quad$ $s_j^2(n)$

$$\bar{x}_j(n) = \frac{1}{n}\sum_{i=1}^{n} x_{ij} \qquad s^2(n) = \frac{1}{n-1}\sum_{i=1}^{n}[x_{ij} - \bar{x}_j(n)]^2$$

since there are 'p' number of runs,

$$x = \frac{1}{p}\sum_{j=1}^{p}\bar{x}_j(n) \qquad s^2\phi = \frac{1}{p}\sum_{j=1}^{p} s_j^2(n)$$

- It is suggested that number of repetation be as low as possible → reduction in the approximation of normal distribution with sample means.