Q.2 Let T_1 and T_2 be two distinct common tangents to the ellipse $E: \frac{x^2}{6}+\frac{y^2}{3}=1$ and the parabola $P: y^2=12x$. Suppose that the tangent T_1 touches P and E at the points A_1 and A_2, respectively and the tangent T_2 touches P and E at the points A_4 and A_3, respectively. Then which of the following statements is(are) true?

(A) The area of the quadrilateral $A_1A_2A_3A_4$ is 35 square units

(B) The area of the quadrilateral $A_1A_2A_3A_4$ is 36 square units

(C) The tangents T_1 and T_2 meet the x-axis at the point $(-3,0)$

(D) The tangents T_1 and T_2 meet the x-axis at the point $(-6,0)$

Q.3 Let $f:[0,1]\to[0,1]$ be the function defined by $f(x)=\frac{x^3}{3}-x^2+\frac{5}{9}x+\frac{17}{36}$. Consider the square region $S=[0,1]\times[0,1]$. Let $G=\{(x,y)\in S: y>f(x)\}$ be called the green region and $R=\{(x,y)\in S: y<f(x)\}$ be called the red region. Let $L_h=\{(x,h)\in S: x\in[0,1]\}$ be the horizontal line drawn at a height $h\in[0,1]$. Then which of the following statements is(are) true?

(A) There exists an $h\in\left[\frac{1}{4},\frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the green region below the line L_h

(B) There exists an $h\in\left[\frac{1}{4},\frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the red region below the line L_h

(C) There exists an $h\in\left[\frac{1}{4},\frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the red region below the line L_h

(D) There exists an $h\in\left[\frac{1}{4},\frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the green region below the line L_h

SECTION 2 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

Q.4 Let $f:(0,1)\to\mathbb{R}$ be the function defined as $f(x)=\sqrt{n}$ if $x\in\left[\frac{1}{n+1},\frac{1}{n}\right)$ where $n\in\mathbb{N}$. Let $g:(0,1)\to\mathbb{R}$ be a function such that $\int_{x^2}^{x}\sqrt{\frac{1-t}{t}}dt<g(x)<2\sqrt{x}$ for all $x\in(0,1)$. Then $\lim_{x\to 0} f(x)g(x)$

(A) does **NOT** exist
(B) is equal to 1
(C) is equal to 2
(D) is equal to 3

Q.5 Let Q be the cube with the set of vertices $\{(x_1,x_2,x_3)\in\mathbb{R}^3 : x_1,x_2,x_3\in\{0,1\}\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices $(0,0,0)$ and $(1,1,1)$ is in S. For lines ℓ_1 and ℓ_2, let $d(\ell_1,\ell_2)$ denote the shortest distance between them. Then the maximum value of $d(\ell_1,\ell_2)$, as ℓ_1 varies over F and ℓ_2 varies over S, is

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{\sqrt{8}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{12}}$

Q.6 Let $X=\left\{(x,y)\in\mathbb{Z}\times\mathbb{Z} : \frac{x^2}{8}+\frac{y^2}{20}<1 \text{ and } y^2<5x\right\}$. Three distinct points P, Q and R are randomly chosen from X. Then the probability that P, Q and R form a triangle whose area is a positive integer, is

(A) $\frac{71}{220}$ (B) $\frac{73}{220}$ (C) $\frac{79}{220}$ (D) $\frac{83}{220}$

Q.7 Let P be a point on the parabola $y^2 = 4ax$, where $a > 0$. The normal to the parabola at P meets the x-axis at a point Q. The area of the triangle PFQ, where F is the focus of the parabola, is 120. If the slope m of the normal and a are both positive integers, then the pair (a, m) is

(A) $(2,3)$ (B) $(1,3)$ (C) $(2,4)$ (D) $(3,4)$