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Model-based estimation methods, summer semester 2020

Bonus Sheet 3 (submit until 14:30 on Wed May 27, 2020)

Note: Submit your solution as Matlab script (.m file) till 14:30 on Wed May 27, 2020 via Moodle! Be prepared for a short Q&A session of your team via Zoom on Thu May 28, 2020, taking place from 14:30 on. Details on the Q&A session will be announced in Moodle.

Bonus Problem 3: Iterative regularization, discrepancy principle, L-curve

(2 Points)

We reconsider the backward heat conduction problem from Problem 11, with $n = 100$ unknowns. Using the usual notation, we have to solve the linear problem

$$A\tilde{\xi} = \tilde{y} \qquad \text{with} \quad A = \exp(-TC), \quad \tilde{\xi} = \vec{u}_0^{\text{est}}, \quad \tilde{y} = \vec{u}_T^{\text{meas}}$$

where the measured final temperature $\vec{u}_T^{\text{meas}}$ is generated by $u(T)$ adding a perturbation provided in the file `error.mat` with normal distribution and variance $\sigma = 10^{-2}$. Note that the measurement error has variance $\sigma = 10^{-2}$, leading to $\|\tilde{y} - y\|_2 \approx \sigma\sqrt{n} = 10^{-1}$.

Different from Problem 11, this time we consider another initial temperature distribution u(0) given by

$$u(x, 0) = \exp(\frac{(x - L/2)^2}{0.02}), \qquad x \in [0, L].$$

To simplify the setup of the inverse problem, we provide a file `BP3_start.m` to start with.

To solve the problem iteratively and for the purpose of regularization, we apply 250 iterations of the Landweber method

$$\tilde{\xi}_{k+1}^{LW} = \tilde{\xi}_k^{LW} - \beta A^T (A\tilde{\xi}_k^{LW} - \tilde{y}), \qquad k = 0, 1, 2, \dots$$

with $\tilde{\xi}_0^{LW} = 0$ and $\beta = 1$. Additionally, we apply 100 iterations of the CGNE method (cf. Lecture notes) obtaining the iterates

$$\tilde{\xi}_{k+1}^{CG}, \qquad k = 0, 1, 2, \dots$$

with $\tilde{\xi}_0^{CG} = 0$.

By choosing a suitable stopping index k^* (which plays the role of a regularization parameter) for each of the methods, we will obtain regularized solutions $\tilde{\xi}_{k^*}^{LW}$, $\tilde{\xi}_{k^*}^{CG}$. Note that k^* will be different for Landweber and CGNE, respectively.

a) Use the discrepancy principle with $\varepsilon_a = 10^{-1}$ to obtain a good stopping index k^* for the Landweber iterates, respectively. Plot the exact and estimated initial temperature ξ^*, $\tilde{\xi}_{k^*}^{LW}$ as well as the final temperature data y, $\tilde{y}$, $\bar{y} = A\tilde{\xi}_{k^*}^{LW}$ in respective figures. Discuss your results.

b) Repeat step a) for the CGNE method. Which iterative method gives better results?

c) Now we want to choose a stopping index k^* by means of the L-curve. To this end, plot the values $(\|A\tilde{\xi}_k^{LW} - \tilde{y}\|_2, \|\tilde{\xi}_k^{LW}\|_2)$ for $k = 1, \ldots, 250$ in a `loglog` plot. What do you observe?

d) Repeat step c) for the CGNE method. For the obtained index k^*, plot initial and final temperatures in respective plots as in b), and discuss the results.
Note: Pick a suitable k^* from the appropriate range of the L-curve. You don't have to precisely determine the index with maximal curvature.