Práctica para II Parcial

CDI - IIS 2023

1) $2y = 3t - 4$

$y = \frac{3t}{2} - \frac{4}{2}$

$m = \frac{3}{2}$

La pendiente de la tangente a $y = t + 2\sqrt{t}$ la hallo con la derivada

$y' = 1 + \frac{1}{\sqrt{t}}$ luego igualo a $m = \frac{3}{2}$

$$1 + \frac{1}{\sqrt{t}} = \frac{3}{2} \Rightarrow \frac{1}{\sqrt{t}} = \frac{3}{2} - 1 \Rightarrow \frac{1}{\sqrt{t}} = \frac{1}{2} \Rightarrow$$

$\sqrt{t} = 2 \Rightarrow t = 4$

$\therefore t = 4$ es donde la tangente es paralela a $2y = 3t - 4$

2) $r^{-2} - qr + \tan(rq) = 5q + 1$

$$-2r^{-3}r' - r - qr' + \sec^2(rq)[r'q + r] = 5$$

$$-2r^{-3}r' - r - qr' + r'\sec^2(rq) \cdot q + r\sec^2(rq) = 5$$

$$r'(-2r^{-3} - q + q\sec^2(rq)) = 5 + r - r\sec^2(rq)$$

$$r' = \frac{5 + r - r\sec^2(rq)}{-2r^{-3} - q + q\sec^2(rq)}$$

$$r'(0,-1) = \frac{5 + (-1) - (-1)\sec^2 0}{-2(-1)^{-3} - 0 + 0} = \frac{5}{2} \qquad \boxed{m = \frac{5}{2}}$$

3) a) $$w'(m) = \frac{\left[3\tan^2(2\sqrt{m}) \cdot \sec^2(2\sqrt{m}) \cdot \frac{1}{\sqrt{m}} \csc\frac{\pi}{3} - \frac{1}{1+m^2}\right] \cdot B - A \cdot \cos((k+1)^m) \cdot (k+1)^m \ln(k+1)}{\operatorname{sen}^2((k+1)^m)}$$

con $A = \tan^3(2\sqrt{m})\csc\frac{\pi}{3} - \arctan m$ y $B = \operatorname{sen}[(k+1)^m]$

b) $\ln y = \ln\left[x(\cos x)^{x^2}\right]$

$\ln y = \ln x + x^2 \ln\cos x$

$\frac{y'}{y} = \frac{1}{x} + 2x\ln\cos x + x^2 \cdot \frac{-\operatorname{sen} x}{\cos x}$

$y' = y\left[\frac{1}{x} + 2x\ln\cos x - x^2\tan x\right]$

$$\boxed{y' = x(\cos x)^{x^2}\left[\frac{1}{x} + 2x\ln\cos x - x^2\tan x\right]}$$

4) tangente a la curva

$$y^2 = 3xy + 5$$
$$2yy' = 3y + 3xy'$$
$$2yy' - 3xy' = 3y$$
$$y'(2y - 3x) = 3y$$
$$y' = \frac{3y}{2y - 3x}$$

Recta

$$2x + 3y + 1 = 0$$
$$3y = -2x - 1$$
$$y = -\frac{2x}{3} - \frac{1}{3}$$
$$m = -\frac{2}{3}$$

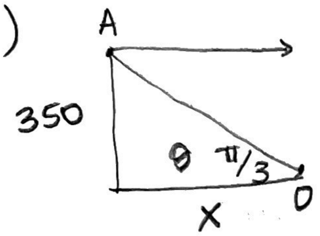
Como son perpendiculares entonces $m_1 \cdot m_2 = -1$

$$\frac{\cancel{3}y}{2y-3x} \cdot -\frac{2}{\cancel{3}} = -1 \Rightarrow 2y = 2y - 3x \Rightarrow x = 0$$

Ocurre cuando $x=0$ entonces

$$y^2 = 3 \cdot 0 \cdot y + 5 \Rightarrow y^2 = 5 \Rightarrow y = \pm\sqrt{5}$$

Así que los puntos son $(0, \sqrt{5})$ y $(0, -\sqrt{5})$

5) 

$\theta' = \frac{1}{40}$ rad/seg

x = distancia horizontal

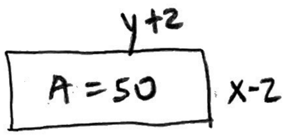
$x' = ?$

$$x = \frac{350}{\tan\frac{\pi}{3}}$$
$$x = \frac{350}{\sqrt{3}}$$

$$\tan\theta = \frac{350}{x} = 350x^{-1}$$
$$\sec^2\theta \; \theta' = -350x^{-2}x'$$
$$\frac{\sec^2\theta \cdot \theta'}{-350 \cdot x^{-2}} = x'$$

$$x' = \frac{\sec^2(\pi/3) \cdot \frac{1}{40}}{-350 \cdot \left(\frac{\sqrt{3}}{350}\right)^2} = \frac{4 \cdot \frac{1}{40}}{-\frac{3}{350}} = -\frac{350}{30} = -\frac{35}{3} = -11{,}66$$

R/ El avión se acerca al punto del observador a 11,66 m/s.

6) 

$$50 = (y+2)(x-2)$$
$$\frac{50}{y+2} + 2 = x$$

$$P = 2(y+2) + 2(x-2)$$
$$P = 2(y+2) + 2 \cdot \frac{5}{y+2}$$
$$P' = 2y' + 10 \cdot -(y+2)^{-2} \cdot y'$$
$$P' = 2 \cdot 2 - 10 \cdot (7)^{-2} \cdot 2$$
$$P' = 4 - \frac{20}{49} \Rightarrow P' = 3{,}59$$

10(y+2

R/ El perímetro aumenta a razón de 3,59 cm por segundo

7) $X_0 = 2$

$$X_n = X_{n-1} - \frac{X_{n-1}^3 - 4}{3X_{n-1}^2}$$

$$X_1 = 2 - \frac{2^3 - 4}{3 \cdot 2^2} = 1,666667$$

$X_2 = 1,591111$

$X_3 = 1,5874097$

8) $e^{-x} - \cos x = 0 \qquad X_0 = 1$

$$X_n = X_{n-1} - \frac{e^{-X_{n-1}} - \cos X_{n-1}}{-e^{-X_{n-1}} + \operatorname{sen} X_{n-1}} \Rightarrow X_1 = 1 - \frac{(e^{-1} - \cos 1)}{-e^{-1} + \operatorname{sen} 1} = 1,364075$$

$X_2 = 1,29442292$

9) • $\lim\limits_{x \to 0} \dfrac{\sec^2 x - 1}{x^2 \sec^2 x} = \text{"}\dfrac{0}{0}\text{"}$ aplico L'Hopital

$$= \lim_{x \to 0} \frac{2\sec x \sec x \tan x}{2x\sec^2 x + x^2 \cdot 2\sec x \sec x \tan x}$$

$$= \lim_{x \to 0} \frac{2\sec^2 x \tan x}{2x \sec^2 x + 2x^2 \sec^2 x \tan x} = \text{"}\frac{0}{0}\text{"} \quad \text{simplifico}$$

$$= \lim_{x \to 0} \frac{\tan x}{x + x^2 \tan x} \quad \text{Aplico L'Hopital}$$

$$= \lim_{x \to 0} \frac{\sec^2 x}{1 + 2x\tan x + x^2 \sec^2 x} = \frac{1}{1} = 1 \qquad \boxed{\therefore \lim_{x \to 0} \frac{\sec^2 x - 1}{x^2 \sec^2 x} = 1}$$

$$\left(\ln(1 - 5x^{-1})\right)' = \frac{5x^{-2}}{1 - 5x^{-1}}$$

• $y = \lim\limits_{x \to 0} \left(1 - \dfrac{5}{x}\right)^x \quad \infty^0$

$$\ln y = \lim_{x \to 0} \ln\left(1 - \frac{5}{x}\right)^x$$

$$\ln y = \lim_{x \to 0} x \ln\left(1 - \frac{5}{x}\right)$$

$$\ln y = \lim_{x \to 0} \frac{\ln\left(1 - \frac{5}{x}\right)}{x^{-1}} \quad \text{"}\frac{\infty}{\infty}\text{"}$$

$$\ln y = \lim_{x \to 0} \frac{\frac{5\cancel{x^{-2}}}{1 - 5x^{-1}}}{-x^{-2}}$$

$$\ln y = \lim_{x \to 0} -\frac{5}{1 - 5x^{-1}}$$

$\ln y = -0$

$e^0 = y$

$1 = y$

$$\boxed{\therefore \lim_{x \to 0} \left(1 - \frac{5}{x}\right)^x = 1}$$

- $\lim\limits_{a\to 1} \dfrac{\ln a - a + 1}{(a-1)\ln a} = "\dfrac{0}{0}"$ ↓ aplico L'Hôpital

$\to \lim\limits_{a\to 1} \dfrac{1/a - 1}{\ln a + \frac{a-1}{a}}$ $"\dfrac{0}{0}"$ ↓ multiplico el uno conveniente $"\dfrac{a}{a}"$

$\Rightarrow \lim\limits_{a\to 1} \dfrac{1 - a}{a\ln a + a - 1}$ $"\dfrac{0}{0}"$ ↓ aplico L'Hôpital

$\Rightarrow \lim\limits_{a\to 1} \dfrac{-1}{\ln a + 1 + 1} = \boxed{-\dfrac{1}{2}}$

$\boxed{\therefore \lim\limits_{a\to 1} \dfrac{1}{a-1} - \dfrac{1}{\ln a} = \dfrac{1}{2}}$

- $\lim\limits_{x\to 0} \cos(2x)^{3/x^2} = "1^{\infty}"$

$\ln y = \lim\limits_{x\to 0} \dfrac{3}{x^2}\ln\cos(2x)$ $"\dfrac{0}{0}"$ ↓ L'Hôpital

$\ln y = \lim\limits_{x\to 0} 3 \cdot \dfrac{\frac{-\operatorname{sen}(2x)\cdot \cancel{2}}{\cos(2x)}}{\cancel{2}x}$

$\ln y = \lim\limits_{x\to 0} -3\,\dfrac{\tan(2x)}{x}$ $"\dfrac{0}{0}"$ ↓ L'Hôpital

$\ln y = \lim\limits_{x\to 0} \dfrac{-3\sec^2(2x)\cdot 2}{1}$

$\ln y = -6 \Rightarrow e^{-6} = y$

$\boxed{\therefore \lim\limits_{x\to 0} \cos(2x)^{3/x^2} = e^{-6}}$

10) a) $D = \mathbb{R} - \{2\} =]-\infty, 2[\cup]2, \infty[$, Interseca en $(0,0)$

b) $x = 2$

c)

	$-\infty$	2	6	∞
f'	+	−	+	
f	↗	↘	↗	

f crece en $]-\infty, 2[\cup]6, \infty[$
f decrece en $]2, 6[$

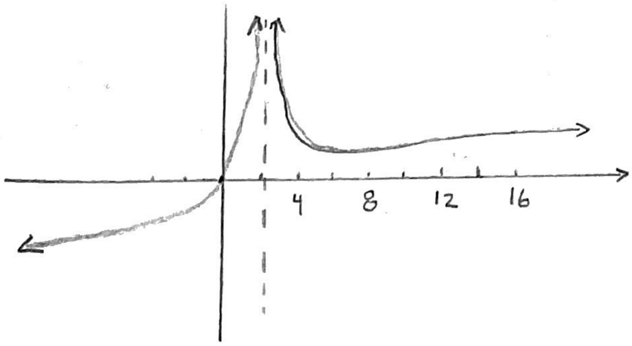
Pto mínimo $\left(6, \dfrac{6}{\sqrt[3]{16}}\right)$

d)

	$-\infty$	2	12	∞
f''	+	+	−	
f	∪	∪	∩	

Punto de inflexión $\left(12, \dfrac{12}{\sqrt[3]{100}}\right)$

e)

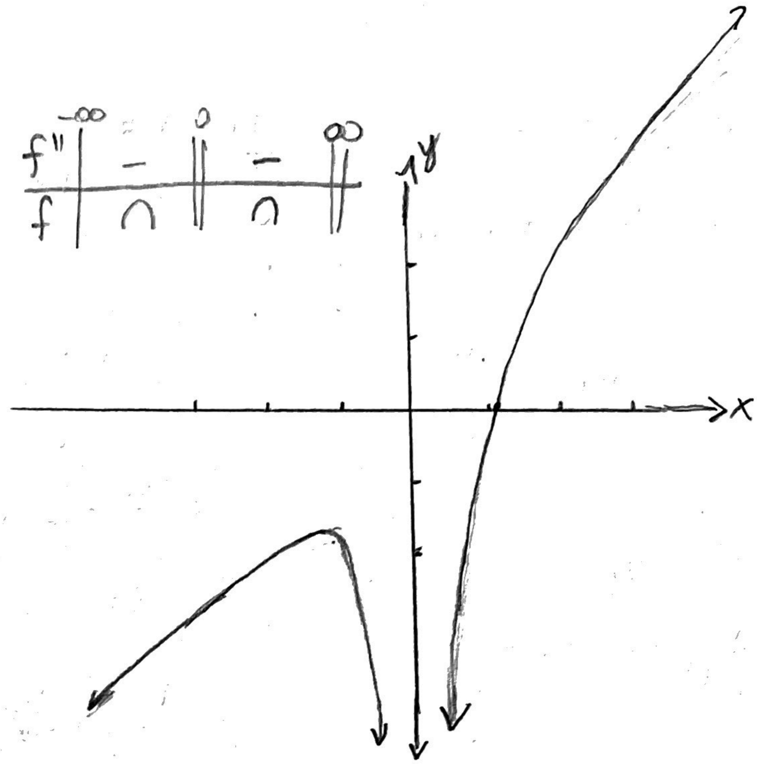


11)

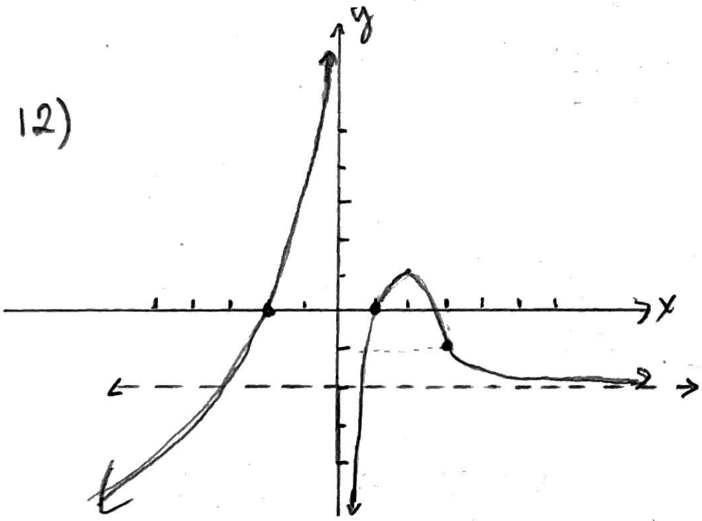
	$-\infty$	$-\sqrt[3]{2}$	0	∞
f'	$+$	$-$	$+$	
f	$\nearrow$	$\searrow$	$\nearrow$	

Pto máximo $(-1{,}26\ ,\ -1{,}89)$

	$-\infty$	0	∞
f''	$-$	$-$	
f	$\cap$	$\cap$	



12)



13) $f(x) = 5x^{2/3} - x^{5/3}$

$f'(x) = \frac{10}{3}x^{-1/3} - \frac{5}{3}x^{2/3} = \frac{5}{3}x^{-1/3}(2-x)$

$x=-1$

$f(-1) = 5-(-1)$
$= 6$

$x=0$

$f(0)=0$

$x=2$

$f(2) = 5\cdot 4^{1/3} - 32^{1/3} \approx 4{,}762$

Pto máximo $(-1,6)$, Pto mínimo $(0,0)$

14) Horizontales no hay

Verticales $x=0$ y $x=1$

Oblicua $y=-x+1$

$$\begin{array}{r|l} x^3+x-2 & -x^2+x \\ \cline{2-2} -x^3-x^2 & -x+1 \\ \hline -x^2+x-2 & \\ \hline -2 & \end{array}$$

15) $V=12$ $\qquad A = 2\pi r h + 4\pi r^2$

$V = \frac{4\pi}{3} r^3 + \pi r^2 h$

$\dfrac{12 - \frac{4\pi}{3} r^3}{\pi r^2} = h$

$A = 2\cancel{\pi r} \cdot \dfrac{12 - \frac{4\pi}{3} r^3}{\cancel{\pi} r^{\cancel{2}}} + 4\pi r^2$

$A = \frac{2}{r}\left(12 - \frac{4\pi}{3} r^3\right) + 4\pi r^2$

$= \frac{24}{r} - \frac{8\pi}{3} r^2 + \frac{12}{3}\pi r^2 = 24 r^{-1} + \frac{4}{3}\pi r^2$

$A' = -24 r^{-2} + \frac{8\pi r}{3}$

$-24 r^{-2} + \frac{8\pi}{3} r = 0 \Rightarrow -24 + \frac{8\pi}{3} r^3 = 0 \Rightarrow$

$r = \sqrt[3]{\frac{9}{\pi}}$

$\frac{8\pi}{3} r^3 = 24$

$\frac{3\cdot 3}{8}$

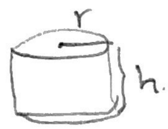
$A'' = 24 r^{-3} + \frac{8\pi}{3}$

$A''\left(\sqrt[3]{\frac{9}{\pi}}\right) = 24\left(\sqrt[3]{\frac{9}{\pi}}\right)^{-3} + \frac{8\pi}{3} > 0$

$\boxed{\text{En } r = \sqrt[3]{\tfrac{9}{\pi}} \text{ hay un mínimo}}$

16) costo $= 15 \cdot 2 \cdot \pi r^2 + 2\pi r \cdot h \cdot 10$

$C = 30\pi r^2 + 20\pi r h$



$V = \pi r^2 h \qquad \frac{225}{\pi r^2} = h$

$C = 30\pi r^2 + 20\pi r \cdot \frac{225}{\pi r^2}$

$C = 30\pi r^2 + 4500 r^{-1}$

$C' = 60\pi r - 4500 r^{-2}$

$60\pi r - 4500 r^{-2} = 0$

$60\pi r = 4500 r^{-2}$

$r^3 = \frac{4500}{60\pi}$

$r = \sqrt[3]{\frac{450}{6\pi}}$

$r = \sqrt[3]{\frac{75}{\pi}}$

$C'' = 60\pi + 4500 r^{-3}$

$C''\left(\sqrt[3]{\frac{75}{\pi}}\right) = 60\pi + 4500\left(\sqrt[3]{\frac{75}{\pi}}\right)^{-3} > 0$

en $r = \sqrt[3]{\frac{75}{\pi}}$ hay un mínimo

$$\boxed{\begin{array}{l} h \approx 8{,}64 \qquad r = 2{,}879 \\ C = 2\,344{,}23 \end{array}}$$