数学物理方程第12课

2023年12月20日 星期三 19:10

Green 函数法:
- 波动
- 导热
- Poisson
  - 第一类边界: $\begin{cases} \Delta u = f(x,y,z) & (x,y,z)\in\Omega \\ u(x,y,z) = \varphi(x,y,z) & (x,y,z)\in\partial\Omega \end{cases}$ 边界上的函数值.
  - 第二类…: 不考
- 一阶PDE

1. 标量f $\xrightarrow{\nabla}$ 矢量 ⇒ 梯度.

$$\nabla f = \frac{\partial f}{\partial x}\cdot\vec{i} + \frac{\partial f}{\partial y}\cdot\vec{j} + \frac{\partial f}{\partial z}\cdot\vec{k} = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

2. $$\nabla\cdot u = \nabla\cdot(u_x, u_y, u_z) = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}.$$

$$\Rightarrow \nabla\cdot\nabla f = \Delta f.$$

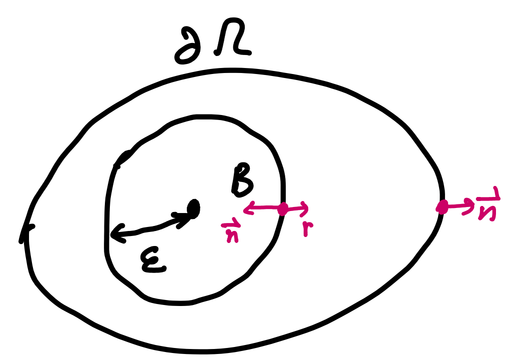
哈密尔顿算子　　Laplace 算子

3. $$\iiint_\Omega \nabla\cdot\vec{F}\,dV = \iint_{\partial\Omega} \vec{F}\cdot\vec{n}\,ds$$ [Gauss Principle]

4. 探寻 Δu, u, $\frac{\partial u}{\partial n}$ 之间的固有关系:

Green the I: $$\iiint_\Omega u\Delta v\,dV = \iint_{\partial\Omega} u\cdot\frac{\partial v}{\partial n}ds - \iiint_\Omega \nabla u\cdot\nabla v\,dV.$$

Green the II: $$\iiint_\Omega (u\Delta v - v\Delta u)\,dV = \iint_{\partial\Omega}\left(u\frac{\partial v}{\partial n} - v\frac{\partial u}{\partial n}\right)ds.$$



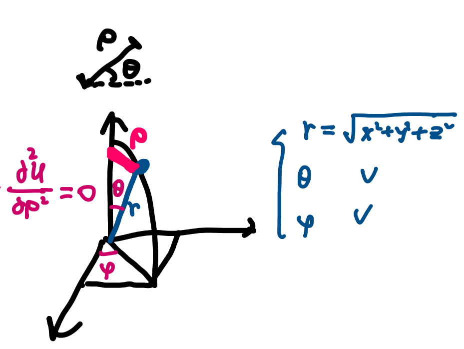
Green the III: $$-\iiint_\Omega \Gamma\cdot\Delta u\,dV = \iint_{\partial\Omega}\left(u\cdot\frac{\partial\Gamma}{\partial n} - \Gamma\cdot\frac{\partial u}{\partial n}\right)ds + u(\bar{x},\bar{y},\bar{z}) - \varepsilon\cdot\frac{\partial u}{\partial n}(\bar{x}_1,\bar{y}_1,\bar{z}_1)$$

$$\Rightarrow u(\xi,\eta,\zeta) = \iint_{\partial\Omega}\left(\Gamma\cdot\frac{\partial u}{\partial n} - u\cdot\frac{\partial\Gamma}{\partial n}\right)ds - \iiint_\Omega \Gamma\cdot\Delta u\cdot dV.$$

5. 直角系下: $\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ $\left(+\frac{\partial^2 u}{\partial z^2}\right)$

极坐标下: $\Delta u = \frac{\partial^2 u}{\partial \rho^2}$

球坐标下: $\Delta u = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2\cdot\frac{\partial u}{\partial r}\right) + \frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\cdot\frac{\partial u}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\cdot\frac{\partial^2 u}{\partial\varphi^2} = 0$



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