

1. 弦振动/固有频率: "弦固有属性"  $\Rightarrow$  齐次PDE.

①  $U_{tt} - a^2 U_{xx} = 0$ . 分离变量  $u(x,t) = X(x) \cdot T(t)$

$\Rightarrow \frac{\ddot{T}(t)}{T(t)} = \frac{\ddot{X}(x)}{X(x)} = \lambda$

$\Rightarrow \begin{cases} \lambda_n = (\frac{n\pi}{l})^2 \\ X_n(x) = \sin(\frac{n\pi}{l}x) \end{cases} \quad n=1, 2, 3, \dots$

$\Rightarrow T_n(t) = C_n \cos(a\sqrt{\lambda_n}t) + C_{2n} \sin(a\sqrt{\lambda_n}t).$

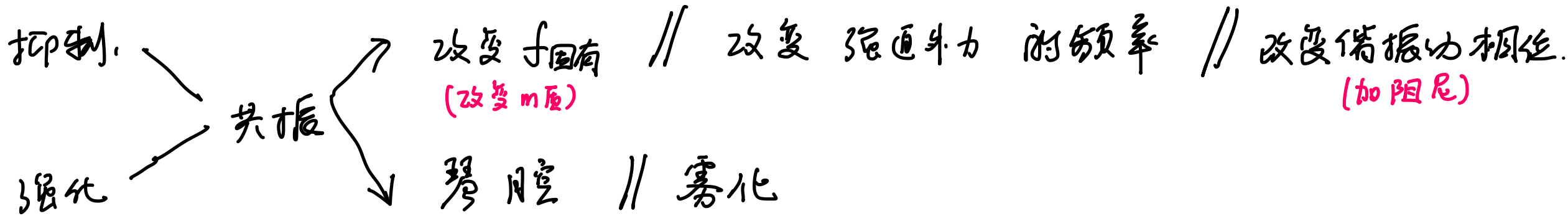
②  $\omega_n = a\sqrt{\lambda_n} = \frac{n\pi a}{l} \quad (n=1, 2, 3, \dots)$  固定频率与特征值一一对应.

$\uparrow$

$\lambda = -\frac{1}{X\omega} \cdot \frac{d^2 X\omega}{dx^2}$

$\uparrow$

$-\frac{d^2 X(x)}{dx^2} = \lambda \cdot X(x).$



2. 一维波动定解问题:

$\begin{cases} U_{tt} - a^2 U_{xx} = A. \\ \underline{u(0,t) = 0.} \quad \underline{u(l,t) = F \sin \omega t.} \\ u(x,0) = f(x). \quad u_t(x,0) = 0. \end{cases}$

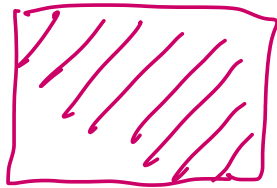
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经验:  $\begin{cases} \text{多用中间变量.} \\ \text{记得用量纲反代验证.} \end{cases}$

step1: 边界条件齐次化

设  $u(x,t) = v(x,t) + w(x,t).$



step2: 特征值 problem:

step3: 正交分解

step4. 建立初值问题

step5: 求解初值问题