数学物理方程第十一课

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PDE的解法:

- 正交分解法:
  - 分离变量法：未知函数分解为自变量独立函数乘积 ⇐ 特征值问题 + 初值ODE定解问题
  - 积分变换法：Fourier变换 / Laplace变换 $\boxed{\text{不学}}$
- 特征线法：自变量沿特征线整合. ⇒ 消元降维 towards 一/二阶波动方程.
- Green函数法: poisson方程 → Green函数 towards 圆域/半空间法.

Chapter 6.1 一阶偏微分方程特征线法

$$u_t + 2u_x + u = t, \quad u(x,0) = 2-x$$

$$\frac{dx}{dt} = 2 \Rightarrow x = 2t + \tau$$

$$\begin{cases} \eta = x \\ \xi = \tau = x - 2t \end{cases} \Rightarrow \begin{cases} x = \eta \\ t = \frac{\eta - \xi}{2} \end{cases}$$

$$U_t = \frac{du}{d\eta} \cdot \frac{d\eta}{dt} = 2U_\eta$$

$$U_x = \frac{du}{d\eta} \cdot 1 = U_\eta$$

$$4U_\eta + U = \frac{\eta - \xi}{2}$$

齐次:

$$4 \cdot \frac{du}{d\eta} = -u$$

$$\frac{1}{u} du = -\frac{1}{4} \cdot d\eta$$

$$\ln u = -\frac{1}{4}\eta + C$$

$$u = C \cdot e^{-\frac{1}{4}\eta}$$

$$\frac{dx}{dt} = 2 \Rightarrow x = 2t + \tau.$$

$$\begin{cases} \eta = x \\ \xi = \tau = x - 2t \end{cases} \Rightarrow \begin{cases} x = \eta \\ t = \frac{x - \xi}{2} = \frac{\eta - \xi}{2} \end{cases}$$

$$U_t = U_\xi \cdot \frac{d\xi}{dt} = U_\xi \cdot (-2)$$

$$U_x = U_\eta \cdot \frac{d\eta}{dx} = U_\eta \cdot 1 \quad (\times) \quad U_x = U_\xi \cdot \frac{d\xi}{dx} = U_\xi$$

$$\Rightarrow -2U_\xi + 2U_\eta + u = \frac{\eta - \xi}{2}$$

$\Rightarrow u =$