Tutorial of Orthogonal Matching Pursuit

A tutorial for Beginners and Dummies

Koredianto Usman

Sekolah Teknik Elektro dan Informatika

8 Mei 2014

Outline

1. Problem Statement
2. The Orthogonal Matching Pursuit
3. Koherensi Basis

Consider the following very simple example: Given the following sparse signals

$$x = \begin{pmatrix} -1.2 \\ 1 \\ 0 \end{pmatrix}$$

The following is the measurement matrix A (2 x 3):

$$A = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix}$$

Therefore $y = A \cdot x$ gives:

$$y = \begin{pmatrix} -1.65 \\ -0.25 \end{pmatrix}$$

Now, Given that : $y = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix}$ and $A = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix}$

How to find original **x**?

BASIS Previous example: Given : $y = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix}$ and
$A = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix}$
Columns in matrix A are called BASIS (CHEN and DONOHO : **ATOMS**). In the example, we have the following *atoms*:

$$b_1 = \begin{pmatrix} -0.707 \\ 0.707 \end{pmatrix} \quad b_2 = \begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix} \quad b_3 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

2. Orthogonal Matching Pursuit

Interpretation of equation Ax = y
Since A = $[b_1\ b_2\ b_3]$; and if we let : x = $[a\ b\ c]$, then
$A \cdot x = a \cdot b_1 + b \cdot b_2 + c \cdot b_3$ $A \cdot x$ is the linear combination of b_1, b_2, b_3.

$$A \cdot x = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1.2 \\ 1 \\ 0 \end{pmatrix}$$

$$= -1.2 \cdot \begin{pmatrix} -0.707 \\ 0.707 \end{pmatrix} + 1 \cdot \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} + 0 \cdot \begin{pmatrix} 0 \\ -1 \end{pmatrix} = y = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix}$$

2. Orthogonal Matching Pursuit

$$A \cdot x = \begin{pmatrix} -0.707 & 0.8 & 0 \\ 0.707 & 0.6 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1.2 \\ 0 \\ 1 \end{pmatrix}$$

$$= -1.2 \cdot \begin{pmatrix} -0.707 \\ 0.707 \end{pmatrix} + 1 \cdot \begin{pmatrix} -0.8 \\ 0.6 \end{pmatrix} + 0 \cdot \begin{pmatrix} 0 \\ -1 \end{pmatrix} = y = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix}$$

From the equation above, it is clear that atom b_1 contribute the biggest influence in y, next is b_2, dan last is b_3. ORTHOGONAL MATCHING PURSUIT works reversely: we start finding which of b_1, b_2, b_3 that will influence the STRONGEST to y. Then the SECOND STRONGEST from the residual, and so on.

2. Orthogonal Matching Pursuit

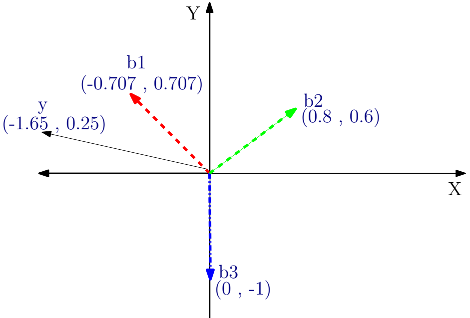
STRONGEST influence is measured using DOT PRODUCT / INNER PRODUCT OMP Algorithm:

1. find atom with has the biggest inner product with y

$$p_i = max_j < b_j, y >$$

2. calculate the residue $r_i = p_i - p_i \cdot < p_i, y >$
3. find atom with has the biggest inner product with r_i
4. repeat step 2 and 3 until residue achieve a certain threshold

Geometrically:



Here the dot product of y to any of b1, b2, b3:

$$< y, b_1 >= -1.34$$

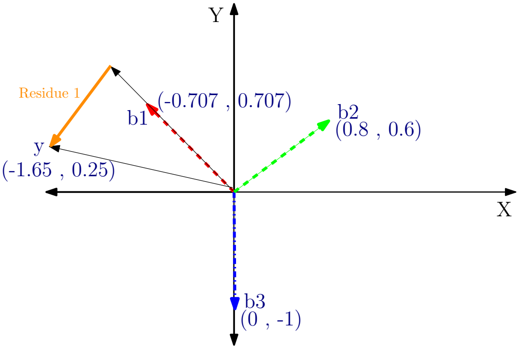
$$< y, b_2 >= 1.17$$

and

$$< y, b_3 >= 0.25$$

Taking the absolute value, we see b_1 gives the biggest inner product. Then, b_1 is chosen as the atom in first step, DOT PRODUCT -1.34. We next count the residue:

$$r_1 = y - b_1 \cdot < y, b_1 > = \begin{pmatrix} -1.65 \\ 0.25 \end{pmatrix} - (-1.34) \cdot \begin{pmatrix} 0.707 \\ 0.707 \end{pmatrix}$$



Next we count the DOT PRODUCT of this residue to b_2 and b_3 (no need to count with b_1, since this residue must perpendicular to b_1).

$$< r1, b1 >= 1$$

$$< r1, b3 >= -0.7$$

Taking the absolute value, we get b_2 as the next strongest influence.

2. Orthogonal Matching Pursuit

Next we count again the residue:

$$r2 = r1 - < r1, b2 > b2 = \begin{pmatrix} -0.7 \\ 0.7 \end{pmatrix} - (1) \cdot \begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix}$$

$$= \begin{pmatrix} -0.099 \\ 0.099 \end{pmatrix}$$

From residue r2, we finally count the final DOT PRODUCT, between r2 with the last b_3 :

$$< r2, b3 >= -0.099$$

The following base is in STRONGEST influence in order: b_1, b_2, b_3, with influence measure from dot product is : -1.34, 1, dan -0.099

Therefore, the reconstructed x is $\begin{pmatrix} -1.34 \\ 1 \\ -0.099 \end{pmatrix}$

Original was : $\begin{pmatrix} -1.2 \\ 1 \\ 0 \end{pmatrix}$

Base Coherency indicate how close one base to the others.

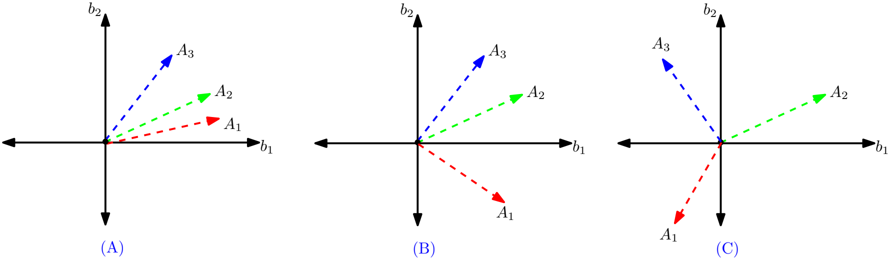


Figure : Ilustrasi Koherensi Basis. (A). Koherensi tinggi. (B). Koherensi Sedang. (C). Koherensi Rendah

3. Base Coherency

A simple measure for base coherency is INNER PRODUCT.

$$\mu = max_{(i,j),i \neq j}(< A_i, A_j >)$$

- Range of Coherency Value μ is : $0 \leq |\mu| \leq 1$
- 0 : least coherency
- 1 : full coherent

We expect least coherency in our base in order to success in MP.