Cryptol: A Domain Specific Language for Verification of Cryptographic Algorithms

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July 23, 2016

Abstract

Contents

Chapter 1

Introduction

The aim of this project is to investigate domain specific languages that enhance reliability of cryptographic algorithms. The focus of this work is on Cryptol, however other languages with similar facilities will also be covered as related works.

May 25th - June 2nd:	Surveying Cyptol and similar languages
June 2nd - June 9th:	In-depth investigation of Cryptol's design and theory
June 9th - June 23rd:	Implementing AES algorithm in Cryptol
June 23rd - July 30th:	Verification of implementaion using Cryptol and SAW.
July 30th - July 7th:	Finishing up evaluation and writing report
July 7th - July 14th:	Preparing for presentation

Figure 1.1: Time Line of The Project

Therefore the first part of the project will be a brief survey of Cryptol and similar languages. This part will also cover high-level design of the Cryptol language and theory behind its functions for formal verification.

The second part of the project will be empirical study of the Cryptol language with an implementation. For the implementation part, the AES symmetric-key algorithm [1, **?**] is to be implemented and properties related to the algorithm will be defined and checked in Cryptol to evaluate whether the language can efficiently verify the implementation. AES is chosen because it is the modern standard for symmetric-key encryption is widely used. The focus in this part is the evaluation of the language, rather than actual implementation of the algorithm, therefore other algorithms written in Cryptol, if available, will also be

used to conduct empirical evaluations.

Another tool provided as a part of the Cryptol project is called The Software Analysis Workbench (SAW). SAW also provides formal verification for properties of programs written in Cryptol. SAW utilizes symbolic execution to translate programs into formal models. This tool will also be used to verify the AES implementation in Cryptol in order to see if it provides better functionalities for verification.

Figure 1.1 provides a time frame for the project. This is just a rough estimation, but the project will follow the steps specified. As mentioned before, because the focus is investigation of the Cryptol language, more time will be spent on evaluating the language functionalities.

1.1 Organizations

Chapter 2

Literature Survey

In this chapter we present previous works that uses programming language as a means to improve reliability of cryptographic applications. In Section 2.1, we give broad overview of several different works, whereas in Section 2.2 we dig into the details of the Cryptol programming language and the guarantees it provides.

2.1 Languages for Cryptographic Applications

The idea of using language features to enhance cryptogprahic applications have been investigated for over a decade. Some works, such as [2] builds upon existing languages, and provides extensions, e.g. libraries and frameworks, for efficient implementation of cryptographic protocols. Other studies [3] create brand-new domain specific programming languages dedicated to cryptographic applications. These works also focus on different aspects of implementation. Some focus on relaibility and correctness guarantees, while others emphasize ease of use and performance.

Charm [2] is an extensible framework in Python designed for rapid prototyping of cryptographic schemes. Charm promotes modularity and reusability of cryptographic primitives, and successfully increases inter-operability of existing numeric libraries such Sage and the Stanford Pairing-Based Crypto (PBC). It also provides benchmarking and profiling utilities for determining the performance of cryptographic algorithms.

NaCl [4] is a C/C++ library for implementing cryptographic protocols that provides security guarantee through features such as no data flow from secrets to load address, and no padding oracles.

ZKPDL [5] is an interpreted description language for specifying zero-knowledge

protocols, motivated by applications such as electronic cash. Although the language is designed specifically for implementing prover and verifer of zero-knowledge, the language itself also have potentials for specifying other types of privacy-preserving systems. The ZKPDL interpreter also performs optimizations for protocols.

Similar to ZKPDL, TASTY [6] is a novel compiler designed specifically for generating efficient two-party computation protocols. TASTY provides a high-level domain specific language in which the user can specify the computation to be performed on encrypted data, and the compiler would translate that directly to a secure protocol. Moreover, TASTY uses the FairPlay [7] system to evaluate the protocol generated.

The Ceritified computer-aided cryptography [8] project provides a computer-aided framework for proving concrete security for cryptogprahic implementations. It extends EasyCrypt, an interative framework for verifying the security of cryptographic applications, to provide formal verification for cryptographic applications implemented in a C-like language. The framework also supports generation of optimized machine code based on the high-level language while retaining the security properties.

One of the study that is perhaps closest to Cryptol is CAO [9], a language designed to facilitate high-level, performant implementation of the AES algorithm. The CAO compiler utilises advanced techniques to improve performance of the implementation, but provides no specific functions for verifying the correctness of the algorithm. Agosta et al. [10] also proposed a domain specific language for cryptography based on Python. The major benefit that this work provides, however, is syntactic.

cPLC [11] is a more recent attempt at providing a domain specific languages for cryptogprahy. Instead of borrowing the syntax of existing programming languages, cPLC provides a language that is closed to the mathematical notations used in the cryptography community to describe protocols. Moreover cPLC provides native support for mathematical entities and operations such as groups that are often used in cryptography.

μCryptol [12, 13] is a language derived from Cryptol. The study focuses providing a verifying compiler that proves the correctness of the code tranformation process.

Cryptol applies techniques for formal verification of cryptographic protocols, a subject that have been studied for decades. We refer to [14] for these works.

2.2 The Cryptol Language

As has been mentioned before, Cryptol is a high-level programming language designed for cryptogpraphy. It provides a formal methods-based approach to cryptographic developments.

The Cryptol language offers several benefits to development of cryptographic protocols:

- Cryptol is designed to be the standard language of cryptography
- The Cryptol language provides high assurance of the correctness of the implementation
- The Cryptol source program can be used as source for code generation to multiple target platforms

We introduce the features that bring about these benefits in detail below.

2.2.1 The Standard Language of Cryptography

Cryptol is ambitiously designed to become the standard language of cryptography. Implementation of cryptographic algorithms in Cryptol are expected to serve as a high-level formal specification or at least reference implementation of the algorithms. This means that the language allows for algorithms specified in academic papers and standards to be translated into Cryptol source code in a manner that is straightforward and readable. Moreover, Cryptol frees developers from machine level detail so that they could focus on developing new algorithms.

To achieve this goal, Cryptol is designed to be a pure functional programming language similar to Haskell. The Cryptol syntax, just like Haskell's, is heavily inspired mathematical notations, and therefore can express computations in cryptographical protocols easily. It is also argued that functional programs are usually shorter and easier to understand. The persistent data structures provided in Cryptol allows for easier analysis and optimization of computation. Furthermore, the functional style naturally helps to create simple and clean abstractions so that the program is well-structured.

2.2.2 High Assurance Programming

To serve as authoritative specifications, Cryptol programs need to be correct first. Cryptol comes with several features that provides strong guarantees of correctness of functions:

- Type System
- Formal Verification
- Automated Testing

Type System

```
1 encrypt: {n} (0 < n, n < 3) => [8*n] -> [8*n]
```

Figure 2.1: Cryptol Type Signature

Cryptol uses a type system based on the Hindley-Milner type system [15], extend with size-polymorphism and arithmetic type predicates [3]. The type system is designed capture constraints that naturally arise from cryptographic algorithms such as fixed-size keys and input blocks.

See Figure 2.1 for an example of type signatures in Cryptol. The signature specifies a function *encrypt* that takes as input an 8-bit or 16-bit integer and output a number of the same size. The quantified type variable n in the type signature are bounded by the predicates $0 < n$ and $n < 3$. The predicates put a limit on the values of the type variable. Any arithmetic operations can be used in such predicates. Type signatures of these kinds rule out a large number of illegal input statically and provides strong guarantees to the correctness of the program. The AES algorithm [1] for example operates on 128, 192 or 256-bit keys. Such a constraint can precisely specified and checked in Cryptol type system.

Formal Verification and Automated Testing

The Cryptol language provides native formal verification utilities designed for equivalence and safety-checking. Correctness properties can be specified in Cryptol as part of the source code accompanying the algorithm. The Cryptol checker then checks for correctness of property with the SAT/SMT solvers. The Z3 prover is used by default.

Figure 2.2 shows a correctness property defined in Cryptol. The property states simply that decryption after encryption with the same key should result in the original message. We can see that a correctness property in Cryptol is but another function that returns a Bit (boolean) type value.

The Cryptol language provides an interactive mode similar to Haskell's with which developers can test functions interactively. Cryptol can conduct verification of the above property automatically with a simple *:prove* command issued in its interative prompt, as shown in Figure 2.3.

In cases where the property is invalid, the Cryptol checker provides an input for which the property does not hold true as counterexample. This gives developers a concrete scenario to look into and debug.

Because Cryptol properties are just functions, it is possible to write conditions in the property to construct proofs efficiently for more restricted scenarios. For polymorphic functions, we can also restrict its type signatures when proving properties.

Of course it is possible that external theorem prover used by Cryptol could not finish the proof within a reasonable amount of time. Therefore Cryptol also

```
1   encrypt: {n} [8] -> String n -> String n
2   // implementation of encryption
3
4   decrypt: {n} [8] -> String n -> String n
5   // implementation of decryption
6
7   encryptCorrect: {n} [8] -> String n -> Bit
8   property encryptCorrect =
9            decrypt key (encrypt key msg) == msg
```

Figure 2.2: Correctness Property in Cryptol

```
1   >> :prove encryptCorrect
2   >> Q.E.D
```

Figure 2.3: Proving Correctness Properties in Cryptol

provides another command *:check* that can be used in the same way as *:prove* to conducted automated testing on the property. Figure 2.4 demonstrates how this utility can be used in Cryptol.

Similar to proving correctness, we can check in Cryptol whether a property is satisfiable through the *:sat* command. The command finds a satisfying input for the property using an off-the-shelf SAT solver. Finding satisfying assignments are interesting in cryptography because it can be used to formulate attacks to the cryptographic protocol. Figure 2.5 provides an example of known plaintext attacks specified in Cryptol. Such a property would certainly have a satisfying assignment. The point, however, is that Cryptol should never be able to find that assignment in a feasible amount of time in order for the encryption function to be safe.

2.3 How Formal Verification Works in Cryptol

Under the hood, the Cryptol source is translated to a *symbolic bit-vector* language, for which there are existing methods that can the decide if the code segments satisfies a certain property. Translation is done through symbolic evaluation, by executing the function in question with symbolic variables.

```
1   >> :set tests=10000
2      // setting number of test cases generated
3   >> :check encryptCorrect
```

Figure 2.4: Testing Correctness Properties in Cryptol

```
1    >> :sat (\key -> encrypt key msg == cipher)
```

Figure 2.5: Known Plaintext Attack in Cryptol

Language	Programming Paradigm	Features
Cryptol	Purely Functional	high-level specification, formal verification
CAO [9]	Imperative	high-level specification, performance
cPLC [11]	Imperative	high-level specification, mathematical syntax, numerical libraries

Figure 2.6: Comparing Cryptographic Programming Languages

The symbolic bit-vector program can be reduced to a SAT (boolean satisfiability) instance and solved with an off-the-shelf SAT solver such as *lingeling*. For checking equivalence property such as the one in Figure 2.2, we ask the SAT solver if there exists an assignment that would make the property return 0 (false). If there is, then the property does not hold, and part of the assignment will be translated to an input and returned as counterexample for the property. The SAT-based property checking approach has been used to prove equivalence of Cryptol program before and after compiler optimization [16].

Cryptol also supports using SMT (satisfiability modulo theory) solvers in place of SAT solvers to decide the bit-vector programs. The reason why SMT solvers might be better than SAT solvers in this case is that they tend to natively support higher-level structural information and arithmetic operations better than SAT solvers. Many SMT solvers come with dedicated mode for bit-vector programs. In practice, it was found that SMT solvers work better when proving properties that involve algebraic equalities.

2.4 Comparing Cryptol with Other Cryptographic Languages

Figure 2.6 compares Cryptol with CAO and cPLC, the languages we have covered in Section 2.1. These two languages are chosen because, like Cryptol, they focus on cryptographic applications in general. As shown in the table, Cryptol is the only when that employs a functional programming style, and also the only one that faciliates formal verification. The other two languages, although both high-level languages, put a strong focus on performance of the implementations.

Chapter 3

Implementation and Evaluation

This section presents the implementation part of the project. For the purpose of empirically evaluating the Cryptol language in practice, we created an implementation of the AES algorithm and devised properties to verify the implementation. The AES algorithm is the current standard of symmetric key encryption and is well specified in [1]. A brief introduction to the algorithm and the implementation is provided in Section 3.1. The correctness properties and results of verification is dicussed in Section 3.3. In Section 3.4 we summarize the observations of the language from the implementation process.

3.1 The AES Algorithm

We start by providing a brief introduction to the AES algorithm in the section to faciliate our discuss of the implementation.

As a block cipher, the AES algorithm runs on a 4×4 column-major order matrix of bytes. The matrix is formally called a *state* and it has a fixed size of 128 bits. See Figure 3.1 for an example of states.

The other input that the algorithm takes is the encryption key, which can be

$$\begin{bmatrix} b_0 & b_4 & b_8 & b_{12} \\ b_1 & b_5 & b_9 & b_{13} \\ b_2 & b_6 & b_{10} & b_{14} \\ b_3 & b_7 & b_{11} & b_{15} \end{bmatrix} \tag{3.1}$$

Figure 3.1: A State in AES

128, 192, or 256-bit long. In AES, a sequence of substitution and permutation steps are repeated for a number of rounds, and the number of rounds is decided by the key size. For 128-bit keys, 10 rounds of repetition is required, whereas 12 and 14 rounds are needed for 192-bit and 256-bit keys respectively. The number of rounds are needed to use the encryption key efficiently. More specifically, the algorithm does not simply reuse the encryption key for every round of operations. Instead, the key is expanded into (number of rounds + 1) round keys, which of 128 bits. The expansion is done using Rijndael's key schedule. This way a different round key is used in each round and there is no repetition. Note that because each entry in the state matrix is one byte, most arithmetic operations in Cryptol are defined over a finite field $GF(2^8)$ (GF stands for Galois Field).

In brief there are 5 major functions used in AES:

- *KeyExpansion*: for expanding encryption key to round keys, as we have discussed before.
- *AddRoundKey*: Applying round key to the state using bitwise *xor*.
- *SubBytes*: Substituting each byte in the state according to a lookup table called the S-box.
- *ShiftRows*: Shifting the last three rows of the state cyclically each by different number of bytes.
- *MixColumns*: Transform each column (treated as a polynomial on the field $GF(2^8)$) through matrix multiplication.

We refer to [1] for more detailed description of these operations. Algorithm 1 shows the pseudo-code for the algorithm using the operations introduced. The decryption function is simply the reverse of the encryption function, and is made up of operations and rounds that reverse the operations and rounds of the encryption function.

3.2 Implementation Details

We present below a few language features that are prominently used in our implementation.

Type Synonyms

Remember that size of input is specified in the type signature. In Cryptol, we can define type synonyms to improve reusability and readability, just like in Haskell. The major data structures in AES are defined as type synonyms in Cryptol in our implementation. Remember that a State in Cryptol is a 4×4 matrix of bytes. An example of the State type is shown in Figure 3.2.

Algorithm 1 The AES Algorithm

```
1: roundKeys = expandKey(key)
2:              ▷ roundKeys is an n + 1 array where n is the number of rounds.
3: state = addRoundkey(state, roundKeys[0])
4: for i = 1 to n − 1 where n is the number of rounds do
5:     state = subBytes(state)
6:     state = shiftRows(state)
7:     state = mixColumns(state)
8:     state = addRoundKey(state, roundKeys[i])
9: end for
10: state = subBytes(state)
11: state = shiftRows(state)
12: state = addRoundKey(state, roundKeys[n])
13:                ▷ The last round does not involve the MixColumns operation.
14: return state
```

```
1   type State = [4][4][8]
```

Figure 3.2: Defining the State type in Cryptol

Fold

As a functional programming language, Cryptol provides no supports for writing for-loop or while-loop. The only natural way of writing a loop in Cryptol, besides recursive functions is *fold*. In functional programming, fold is a way to recursively analyze a data structure such as a list, combine the results of each iteration and accumulatively build up a return value. See Figure 3.3 for an example of loops written with *fold* in Cryptol. This is a function that computes the sum of the list *xs*. Note that the variable *ys* is used in its in own definition. This is similar to a *foldr* function in Haskell [17]. The result, however, in this case is a list that includes the value of each intermediate step.

Folds are the recommended way of writing loops in Cryptol, and therefore are frequently used in our implementation.

```
1   xs = [1 .. 10] // a list of numbers
2   ys = [0] # [ y + x | x <- xs
3                      | y <- ys ]
4   // ys = [0, 1, 3, ..., 55]
```

Figure 3.3: A Loop Written in Fold in Cryptol

```
1     type State = [4][4][8]
2     type RoundKey = State
3     type KeySize = 128    // 192, 256
4
5     // encryption functions
6     subBytes : State -> State
7     shiftRows: State -> State
8     mixColumns: State -> State
9     addRoundKey: RoundKey -> State -> State
10    encrypt: [128] -> [KeySize] -> [128]
11
12    // decryption functions
13    reverseSubBytes: State -> State
14    reverseShiftRows: State -> State
15    reverseMixColumns: State -> State
16    decrypt: [128] -> [KeySize] -> [128]
```

Figure 3.4: Function Signatures of Major AES Operations

3.3 Implementation and Verification

Using a pure functional language such as Cryptol to implement the AES algorithm, fortunately does not result in any complications more than necessary. Each major operation can be implemented as a pure function, i.e. one that is free of side effects, and of course the eventual encryption and decryption functions are also side-effect free.

In Figure 3.4 we list type signatures of the major functions in AES. This provides a high-level overview of our implementation. Note how well this corresponds to the description of AES in Section 3.1. All the major operations transform *State* type. The encryption and decryption function works on 128-bit plain text, and takes as input a key that is either 128, 192, or 256-bit long. Of course these are not the only functions and types in the implementation. There are many more helper functions such as those for computing multiplicative inverse on a finite field and for matrix multiplication. We also implement multiple versions of the same function where possible, so that these different versions can be used to verify each other. This is conducted under the assumption the under the correct specification, the same mistake is unlikely to be made in two different versions.

The next step after implementation is naturally verifying our implementation. This is however not as straight-forward as the example shown in Figure 2.2, because high-level properties defined directly over the encryption and decryption functions generally take too long to prove. This is likely because the AES algorithm has an intentionally large state space, and the purpose of the algorithm is to obfuscate and make it computationally impossible to analyze the relationship between the input and the output. Therefore instead of verifying

the encryption and decryption functions as a whole, we adopt a different strategy: verify each component of the algorithm separately. As the composition of components is pretty straight-forward for AES, this approach would provide strong guarantee that the implementation is correct.

A set of correctness properties are defined in order to verify the implementation. These properties for functions are designed using the three strategies:

- Verifying important properties instrinsic to the functions;
- Verifying equivalence of different versions of the same function;
- Verifying that the reverse functions successfully undo the encrypt functions

Many of the basic functions have important properties that can be used to verify them, and many of these properties can be proved efficiently. For example, the product of a number and its multiplicative inverse on a finite field equals to 1. This property is defining characteristic of multiplicative inverse and can be used to verify the function. Similarly we verify commutativity and associativity for finite field multiplication.

Moreover, the AES specification sometimes provide optimization such as look-up table that replaces computation steps. We implement some of these optimizations and ask Cryptol to check equivalence of the optimized and original functions. This is the approach taken for functions such as *subBytes*.

Another way to ensure correctness is make sure that reverse operations used in the decryption function can actually cancel out their corresponding operations in the encryption function. This is approach is used to verify *shiftRows* for example.

In complement with formal verification, we use the automated testing utility provided in Cryptol to test the properties that are defined, especially when the properties could not be proved in a feasible amount of time.

Figure 3.5 summarizes the result of the formal verification process. The result is largely positive: most of the properties can be efficiently proved, while the rest passed the automated tests at least. This gives us a strong confidence that the implementation is correct.

3.4 Review of the Cryptol Language

if-else statement is often the only solution. The advantage of this is that the algorithms specified in imperative style can often be translated with ease. However, whether this is a good language feature is a matter of debate. The language does provide pattern matching, however it is not as powerful as that in Haskell, and is used to only access components of data structures.

Property	Description	Result
polySelfAdd	the sum of two same polynomials on $GF(2^*)$ is 0	proved
mulIden	any number multiplied with the identity element over the field equals the number itself	proved
mulCommutative	commutative property of field multiplication	proved
mulAssociate	associative property of field multiplication	proved
mulInverseCorrect	multiplicative inverse's definition	proved
subByteCorrect	two versions of subBytes are equivalent	proved
shiftRowsCorrect	*shiftRows* applied 4 times results in the original state	proved
rConCorrect	two versions of round constants are equivalent	tested
formByteCorrect	verify *formByte* against its reverse function	proved
toFromStateCorrect	toState is the reverse of fromState	proved
reverseShiftRows	verify *shiftRows* against its reverse function	proved
mixColumnsCorrect	verify *mixColumns* against its reverse function	tested
aesCorrect	decrypt (encrypt (m, k), k) == m	tested

Figure 3.5: Correctness Properties and Results of Verification

Chapter 4

Conclusion and Future Works

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