Problem Chosen	2023	Team Control Number
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How to Be A Negotiator on Colorado River

Summary

Located on the Colorado River, Hoover Dam and Glen Canyon Dam hold two of the biggest reservoirs—Lake Powell and Lake Mead, the allocation of the water in which is deeply concerned with the welfare of over 20million people. However, due to the severe drought these years, the design of a better allocation approach is urgently needed.

At first, we developed a set of **fluid mechanics equations** to calculate the flux of Colorado River at certain point. While processing it, we fitted statics using **Adaboost-Decisiontree Regression** to filter the noise. Then, the **water balance equations** and **demand-supply inequations** are constructed to restrict outflow water. The partially sequential relationship of the two dams are also taken into account. Different strategies should be taken corresponding to different water-level in the lake to avoid flooding and damage of the electricity generator engine.

After that, we proposed the **Coordinate Regulation Assessment Model (CRAM)** to give the overall judgment for the whole allocation process of the two dams, synthesizing the benefits of water usage in each state, the benefits of hydroelectricity generation and the water dissipated to Mexico.

For the benefit of water usage, we applied a 2-layer **Analytic Hierarchy Process (AHP)** to determine the weight for factors effecting the benefit of water usage. We considered social, economic and environmental benefit. With the weight of factors calculated, we then determined the total benefit function, and applied the **genetic algorithm** to find the optimization.With suitable parameters, we managed to find a satisfying water allocation for each state and each industry(agriculture, industry and residence).

For the benefit of hydroelectricity generation, we derived the equation of the relationship between flux the electricity generated by the dam generator and calculated the simulation of the total benefit.

For Mexico, we searched for the demand needed and simulate it a **punishing function**. If the water left in Colorado River could not meet the demand of Mexico, the welfare of Mexican will be seriously affected and the overall benefit will suffered a lot, as well.

Standardizing and integrating all three benefits, the CRAM functions as the comprehensive judging criteria to determine how good the allocation is. Again, implement **GA** to find the optimized solution.

The next part of our paper applies different strategies in different conditions, including the drought situation, the change of water and electricity conservation measures, and so on. These also function as the sensitive analysis.At the end , we discussed the strengths and weaknesses of our model.

Keywords: Hydraulic system; GA; Hydroelectricity; Integrated model; Water allocation

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February 10, 2023

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Contents

1 Overview

Background

Lake Mead and Lake Powell are the top two largest artificial reservoirs on the Colorado River, United States. Lake Mead is located in the states of Arizona and Nevada, formed by the Hoover dam. With the maximum capacity of 26.12 million acre-feet, it serves water to nearly 20 million people in Arizona, California, and Nevada, as well as some of Mexico. [1] As the second largest reservoir, Lake Powell, formed by the Glen Canyon Dam, can store 24.3 million acre-feet of water when full. Therefore, it is essential for the Upper Basin states. [2] In addition, these two landscapes attract millions of people by activities including boating, fishing, and water skiing. Considerable amount of hydroelectric power is generated by the turbines on the dam at the same time.

However, climate change has sharply reduced the volume of water feeding the reservoirs. In this case, the water demand of nearby areas becomes more difficult to satisfy. In the late 2019, seven states in Colorado River Basin - Colorado, Wyoming, New Mexico, Utah, California, Nevada and Arizona – claimed that they are ready to restart the negotiations about the future policies to deal with the looming water shortages like what have been done by the 2007 Colorado River Interim Guideline. At the same time, some scholars pointed that due to the fact that "extreme drought conditions blanket more than 75% of the river's Upper Basin, cities, farmers and everyone else in the basin need to use less water." [3] Besides, low water level of the dam also means that the turbines need to stop to avoid damage.

Our Tasks

Since the situation that the water supplement for the reservoir decrease, while the demand from the states can hardly shrink, we need to find a better way to allocate the water in the reservoir. Here are our tasks:

- Build a model to determine how much water should be drawn from each lake to satisfy certain demands, considering the current water level in the lakes, flow of the Colorado River, the amount of water loss due to the evaporation and permeation. Further, predict how long the plan can work and when it expires, how much additional water will be needed.

- Establish a model to estimate the direct benefit and indirect benefit generated by different water allocations.

- Put forward a measure to deal with the situation that there is not enough water for both water usage and hydroelectric power generation, based on the model built.

- Analyze how our model perform when the preconditions change such as the demand for water, hydroelectric power generation increases, and better practice of resource conservation measures.

- Make assessment of our models, find the strengths and weakness of them and make further improvement.

2 Assumptions Notations

Assumptions

- The water drawn from Lake Powell only supplies Colorado, New Mexico and Wyoming, while Lake Mead only supplies California and Arizona.
- The water is drawn averagely in each day, according to the demand of water.
- In each state, water managers are all wise enough to optimize the usage of water to maximize the benefit.

Notations

Abbreviation	***Description***
F	The flux of the Colorado River
$X = \{x_1, x_2, ..., x_i\}$	The set of positions on the river
$T = \{t_1, t_2, ..., t_i\}$	The set of moments
ET	The speed of evapotranspiration
O	The amount of water exchanged between bank storage and the river flux
P	The amount of water taken from the flow for external usage (e.g. residential, industrial, agricultural)
v	The velocity of water flux
A_i	The basal area of the lake i
s	The cross-sectional area of the river
S	The supply of water from the lake i to assigned states
d	The depth of the river at a certain point
w	The width of the river at a certain point
W	Sum of all the water leaving the lake i
D	The demand of water of the states from lake i
TD	The degree centigrade
$\overline{T}$	The average degree centigrade over the time unit
R_a	Extraterrestrial radiation ($mm \cdot day^{-1}$)
e_i	The loss of water from lake i during transportration for demand(e.g. evapotranspiration, osmosis)
e'_i	The loss of water in lake i when storing (e.g. evapotranspiration, osmosis)
ρ	The density of water
X_i	The amount of water assigned for different use
μ	The mechanical efficiency of the hydroelectric generating set of the dam
ρ	The density of the water
E	The demand of electricity
G	The grade of the benefit of the water for specific usage

3 Demand-Supply Model

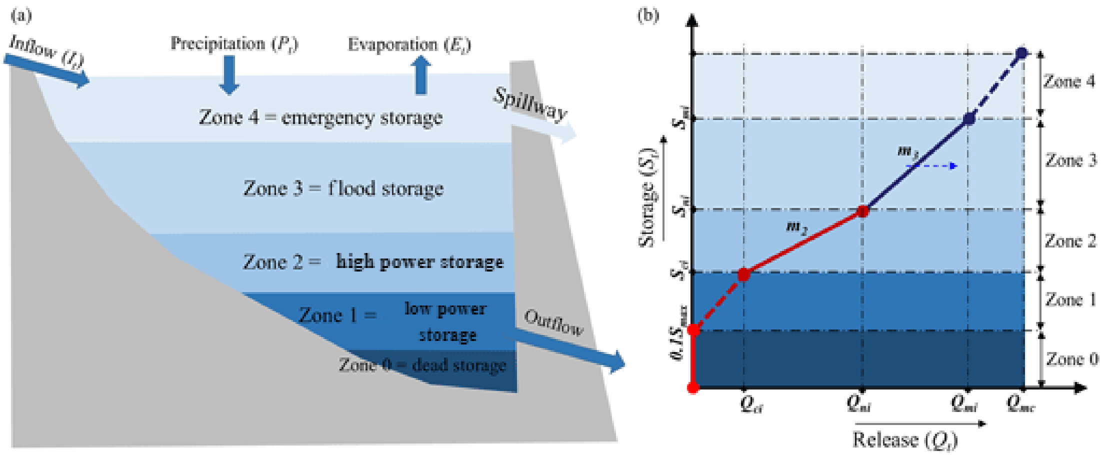

Figure 1: the reservoir operation chart [4]

3.1 The Fluid Mechanics Equations of Colorado River

To acquire the flux running into the Lake Powell and Lake Mead at arbitrary time, we establish a set of continuity equations to simulate the flow of the water in the Colorado river based on flow conservation. And later on, the flux we determined in this fluid mechanics equation will be used to assign the water to different industries of different states.

$$\begin{cases} F_m(x+dx, t+dt) = -ET(x,t) + O(x,t) + F_m(x,t) + F_b \\ F(x,t) = v(x,t) \cdot s(x) \\ s(x_i) \propto D(x_i) \cdot w(x_i) \\ dx = v(x,t)dt \end{cases} \tag{1}$$

solving the above equations gives us

$$\frac{\partial v}{\partial x} d(x) w(x) + v(x,t) \left[\frac{\mathrm{d}d}{\mathrm{d}x} w(x) + \frac{\mathrm{d}w}{\mathrm{d}x} d(x) \right] = -ET(x,t) + O(x,t) + F_b \tag{2}$$

where $ET(x,t)$ is the evapotranspiration at time t (in hours) and at a distance x (in kilometers) away from the upstream. Based on the collected data, we decided to simplify the evapotranspiration and make it an estimated constant which only becomes different when the river enters different states.

$$ET(x,t) = 0.0023TD^{0.5}(\overline{T} + 17.8)R_a = ET_0 \tag{3}$$

[5]

Calculation of the Equation

To solve this fluid mechanic equation, we have to obtain the expression of all the unknown variables. But for a ternary partial differential equations, we can only obtain its numerical solutions which could help little for our model. In this case, we decided to utilize the Adaboost algorithm to give a regression equation based on the data from the past years.

An Adaboost regressor is a meta-estimator that will first fit a regressor on the given data set, then weigh the instance according to the current error rate. If an instance is more likely to be wrongly predicted, it will be given more weight. To improve the regression, it will use subsequent regressors based on the weighed instances. Therefore, those difficult instances will be paid more attention to. Another important characteristic of this algorithm is overfitting can be generally avoided by frequent test.

We first number each day in 202, from 1 to 365. As customary, we shuffled the data and set 70 percent of them as the training data set, 30 percent of them as the test data set. We choose to use cross validation to reduce the probability of overfitting. The figure of predicted values are shown in Figure 2

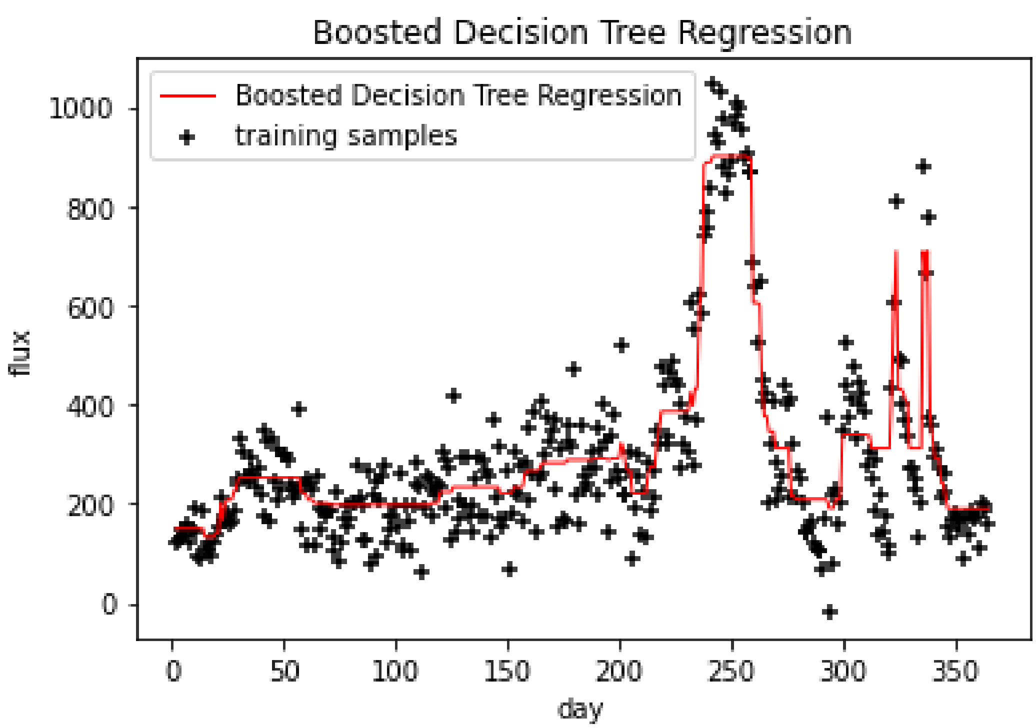

Figure 2: Regression Curve of Inflow for Lake Powell

To show the reliability of the regression, we calculated R^2 using the following formula:

$$R^2 = 1 - \frac{RSS}{TSS}$$

where RSS(Residual Sum of Squares) is the sum of squares of difference between the predicted value and the actual value, TSS(Total Sum of Squares) is the sum of squares of difference between the actual value and the expected value.

In academic researches, a regression is convincing enough if R^2 is larger than 0.7, and ours is 0.731 for the training set, 0.813 for the test set. Thus, this calculation is believable.

3.2 The Water Balance Equations of Colorado River

To determine the amount of water to be drawn from the lakes, we described the relationship between the water flows in and out the two reservoirs by means of linear programming.

In order to meet the electricity demand of the five states, the water used to electronic generation must satisfy:

$$\mu\rho g M F_{ME} t + \mu\rho g P F_{PE} t \geqslant \sum_{i=1}^{5} E_i \tag{4}$$

where:

- M, P are the water levels of Lake Mead and Lake Powell respectively

- F_{ME} is the flux running through the Hoover Dam used for electricity generation

- F_{PE} is the flux running through the Powell Glen Canyon for electricity generation

- E_i is the power demand of the i^{th} state ($i = 1, 2, 3, 4, 5$, representing AZ, CA, WY, NM, and CO respectively)

- t is the time unit

For Lake Powell, its water level must be higher than the dead storage level P_{min} and lower than the maximum level P_{max} to ensure a safe operation of the Glen Canyon Dam:

$$k_P P_{min} \leqslant tF_{PI} + k_P P_0 - tF_{PE} - tF_{PW} - e'_P \leqslant k_P P_{max} \tag{5}$$

where:

- k_p is the volume coefficient, which reflects the numerical relation between the water level P and the reservoir storage V_P, given by the method of linear fitting

- F_{PI} is the flux running into Lake Powell from the upper stream

- P_0 is the initial water level of Lake Powell

Similarly we have the equation for Lake Mead. Additionally, considering that the water outflows from the Glen Canyon Dam supplies part of the inflow of Lake Mead, we can obtain the following equation:

$$k_M M_{min} \leqslant tF_{MI} + k_M M_0 - tF_{ME} - tF_{MW} - e'_M \leqslant k_M M_{max} \tag{6}$$

$$F_{MI} = \lambda F_{PE} + F_{branch} \tag{7}$$

where:

- F_{MI} is the flux running into Lake Mead

- λ is the evapotransporation coefficient

- F_{branch} is the inflow from

Combining (19), (20), and (7) we can now derive the interval of F_{ME} and F_{PE} in terms of the initial water level P_0 and M_0:

$$F_{PI} - F_{PW} + k_p\frac{P_0}{t} - k_P\frac{P_{max}}{t} - e'_P \leqslant F_{PE} \leqslant F_{PI} - F_{PW} + k_p\frac{P_0}{t} - k_P\frac{P_{min}}{t} - e'_P \tag{8}$$

$$\begin{cases} F_{ME} \geqslant \lambda F_{PE} + \left(F_{M,branch} + \dfrac{k_M M_0}{t} - F_{MW} - \dfrac{k_M M_{max}}{t} - e'_M\right) \\ F_{ME} \leqslant \lambda F_{PE} + \left(F_{M,branch} + \dfrac{k_M M_0}{t} - F_{MW} - \dfrac{k_M M_{min}}{t} - e'_M\right) \end{cases} \tag{9}$$

Synthesising (4), (9), and (8), we can produce the graph of the total value range for F_{ME} and F_{PE} represented as the shadowed area:

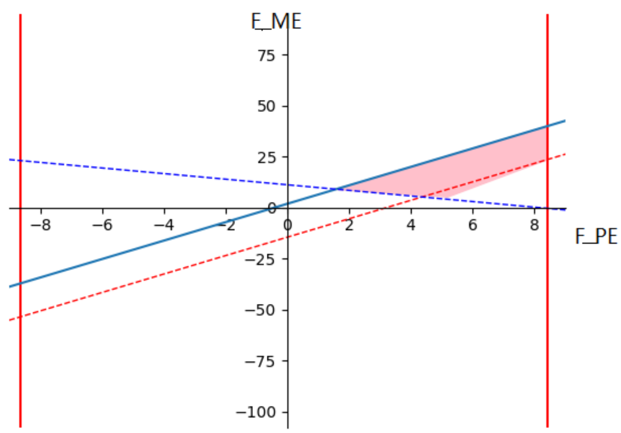

Figure 3: The Flow Control in October

1. When water surface elevation of Lake Mead< 895(feet), which is the deadpool level, F_{ME} and F_{PE} are forced to be 0, since no water leaves the deadpool. Same as for Lake Powell, when its water surface elevation < 3370(feet)

2. When M and N are between the dead storage level and high power level, the arrangement of F_{ME} and F_{PE} should be within the shadowed area, that is:

$$\begin{cases} F_{ME} \leqslant 4.52 \cdot F_{PE} + 1.792 \\ F_{ME} \geqslant 4.52 \cdot F_{PE} - 14.582 \\ F_{ME} \geqslant -0.391043323/0.285 \cdot F_{PE} + 11.09298 \\ 0 \leqslant 4.52 \cdot F_{PE} \geqslant 8.415 \end{cases} \tag{10}$$

3. When M and N are above the flood storage, F_{ME} and F_{PE} should reach the maximum flow in order to avoid possible danger. According to equation (8), we derived that

$$k_P AP + F_{PI}T - F_{PO}T = k_P AP_{min}$$

where F_{PO} is the water out of Lake Powell P_{min} is the low power storage water-level for Lake Powell Since the inflow of Lake Mead has direct relationship with the outflow of Lake Powell, it involves the choose of water-release-strategy stated later. And both of thme are connected in above inequality equations.Therefore, we are going to determine the time when supply cannot meet the demand. From above equation, we calculate that $T_P = 0.295P - 10.8$ when the water runs out. TO prevent this from happening, we need a water input of 901020466 m^3 every year for Mead Powell.

According to the research, the difference of water inflow($m^3/month$) between different seasons may verify as much as $\frac{789432960}{269393997.6} = 2.93$ ($F_{Winter} vs F_{Summer}$) but within a season, the difference of water inflow between months could be estimated to 0.Therefore, we sincerely advised to redefine our model every season with a change of several constant.

4 Trade-off between Usage and Production

Reasonable allocation of the water in the reservoir should consider about the direct benefit, which includes mainly the hydroelectric power generation, and the indirect benefit consists of water usage for agriculture, industry, and residents. Here we build two individual models to assess these two kinds of benefits and then get a joint model for total benefit assessment.

4.1 Assessment of the Indirect Benefit

Here we use the Analytic Hierarchy Process to decide the weight coefficient of all the factors.

4.1.1 Selection of the Factors and Hierarchy Establishment

To allocate water among the five states, we have to figure out how well the water can be utilized in each state. The ability to use water can be evaluated in three different aspects: the benefit produced by unit amount of water in economy, society, and ecology, among which the benefit in economy is considered the most important one. We further dissect the three fields into several specific factors and built a two-layer AHP model as shown in Figure 4.

We use the GDP and efficiency factor to represent each state's ability to use water to benefit economy, where the efficiency factor is given by [6]:

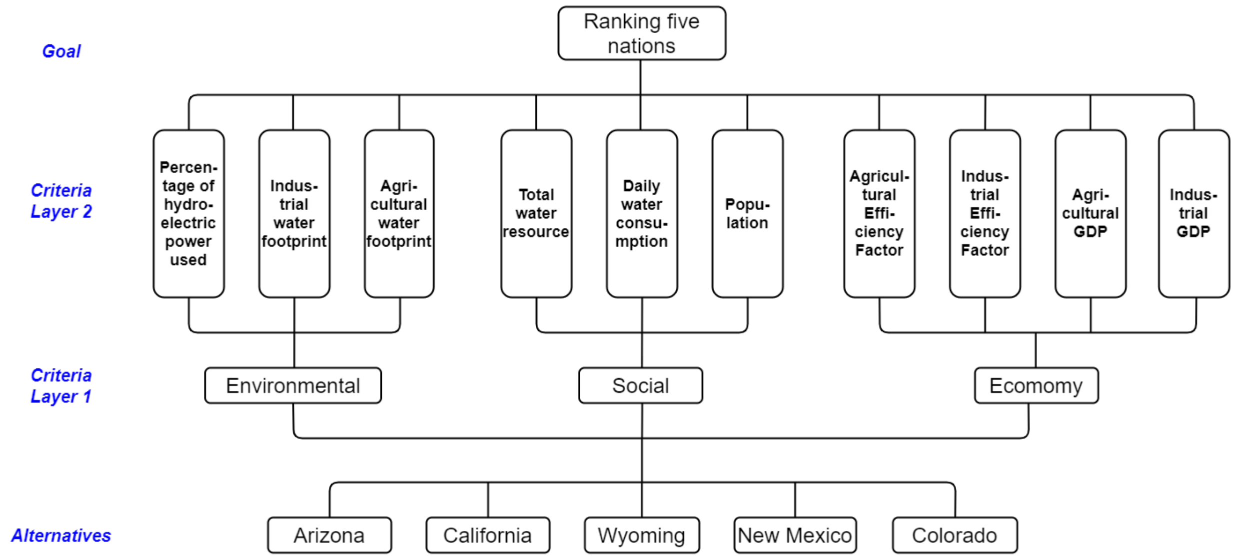

Figure 4: Hierarchy figure

$$k_{eff} = \frac{\text{water for industrial or agricultural usage}}{\text{total amount of water}}$$

For social benefit, since the basic usage of water is to satisfy the demand of residents for household usage. Thus, we assess the water-usage ability in the field of society according to the amount of water resource, daily household water usage and population of each state. The last aspect is environmental benefits. Using hydroelectric power instead of electricity generated from coal or other fossil fuels produces less emission. Transmitting hydroelectric power to a state which is better at hydroelectric power generation means we can save more fuel to reduce the emission of carbon dioxide. "Water footprint" is the total amount of water consumed during production.

4.1.2 Construction of Comparison Matrices

First, by comparing the factors in the Criteria Layer 2 by pairs, we determined a series of weights for these factors with respect to their relative importance.

The comparison matrix for the environmental factors in Layer 2 and the comparison matrix for Criteria Layer 1 are shown below as examples:

Aspects	***Env***	***Social***	***Eco***
Env	1	3	5
Social	1/3	1	3
Eco	1/5	1/3	1

Table 1: Pairwise Comparison Matrix of Criteria Layer 1

PH, ***IF***, ***AF*** stands for the percentage of hydroelectricity power, the industrial water footprint, and the agricultural water footprint, respectively.

Factors	***PH***	***IF***	***AF***
PH	1	1/5	1/3
IF	5	1	3
AF	3	1/3	1

Table 2: Pairwise Comparison Matrix of Environmental Effects in Layer 2

Especially, the larger the amount of available water resource in a state is, the less water we should allocate to it. So when compared with other factors, it is almost the "least important" factor.

To check whether our comparison matrices are reasonable, we calculate the consistency relative indexes (CR) of the comparison matrices, and none of them exceed 0.03. The requirement of CR is that it must be under 0.1 for a consistent enough matrix, so our comparison matrices are sufficiently consistent.

4.1.3 Computation of the Score of Each Alternatives

The grades of the five states derived from the previous analysis are showed in the Table below:

State	***Grade***
AZ	45.387
CA	73.540
WY	45.837
NM	43.771
CO	47.793

Table 3: Grade of Each State

In order to calculate the total benefit of water assignment, we derive a suitable smooth function:

$$y(x, D) = -e^{-x+0.5D-ln(1.5)} + 1.5 \tag{11}$$

That is, with the growth of water supply to each state, its benefit is increasing. However, the maximum can only get close to 1.5 of original benefit, but never exceed. And when the water supply to a state is less than 0.5 of its demand, it will cause it in mess and lose most of its benefit.

Therefore, we could derive the total grade function:

$$G(x) = \sum_{i=1}^{5} y(x_i, D_i) \cdot p_i \tag{12}$$

$p_i$is the grade of the i^{th} state To get the optimization of the benefit function, we apply the heuristic intelligent algorithm, specifically, the genetic algorithm(Second appendix). We produced a group with population 50, and iterates it in 5000 times.The demand Below are the simulation graph of one process:

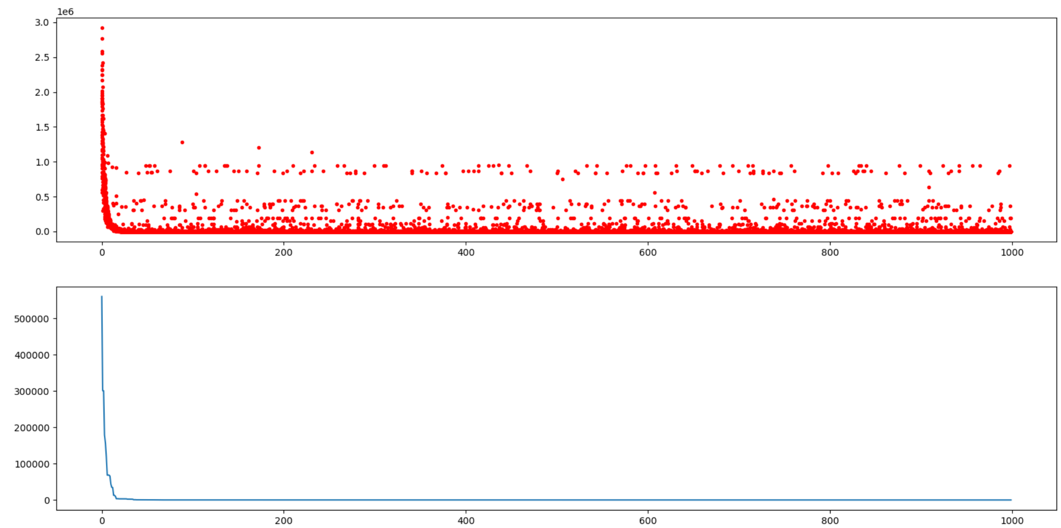

Figure 5: Training Procedure of Genetic Algorithm

4.1.4 Assigning Water to Different Usage

On the basis of AHP developed before, we are able to assign a benefit grade for three different usages in every state, as belowed:

Category	***Grade***	***Category***	***Grade***
CA Residential	29.416	WY Agriculture	16.043
CA Agriculture	25.739	AZ Agriculture	15.886
CO Residential	19.117	NM Agriculture	15.320
CA Industrial	18.385	CO Industrial	11.948
WY Residential	18.335	WY Industrial	11.459
AZ Residential	18.155	AZ Industrial	11.347
NM Residential	17.509	NM Industrial	10.943
CO Agriculture	16.728		

Table 4: Grade of Each State

After that, we use the genetic algorithm to to find the optimal solution assigning a limited or unlimited amount of water to each state. When the supply water is unlimited, the allocation vector is

$$[21.40457.78083.61621.67483.16913.2894.46785.32085.98665.79626.31656.83234.09281.79768.716]$$

Therefore, when the water supply is far beyond the demand, it should be divided separately according to the above chart.

4.2 Computation of the Scores of Hydroelectricity generation

4.2.1 Simulation of the Water Level

According to the water balance equation of the Lake Powell, the change of the volume of water in Lake Powell is calculated by inflow(F_{PI}) - outflow($F_{PW} + F_{PE}$) - water loss(e_P), based on which we define the following equation:

$$k_P A_P (P(X_{PW}, X_{PE}, T) - P_0) = F_{PI}(T) - e_P - F_{PW}(X_{PW}) \cdot T - \int_0^T F_{PE}(X_{PE}, t)dt \quad (13)$$

Where:

- X_{MW} is the total water extracted from Mead Lake for usage in five states,

- X_{ME} is the total water discharged from Mead Lake to generate electricity.

- X_{PW} is the total water extracted from Powell Lake for usage in five states,

- X_{PE} is the total water discharged from Powell Lake to generate electricity.

- F_{PI} is the inflow flux for Lake Powell

- F_{PW} is the outflow flux from Lake Powell to nearby states

- F_{PE} is the outflow flux from Lake Powell to downstream(to generated electricity).

From the above equation, we derive the expression for $P(X1, X2, T)$:

$$P(X_{PW}, X_{PE}, T) = \frac{1}{k_P A_P} \cdot (F_{PI(T)} - F_{PW}(X_{PW}) \cdot T - e_P - \int_0^T F_{PE}(X_{PE}, t)dt) + P_0 \quad (14)$$

The balance equation for Lake Mead is almost the same as the one for Lake Powell ,with the only difference that the inflow of Lake Mead is partially dependent on the outflow of Lake Powell(F_{PE}):

$$\begin{aligned} k_M A_M (M(X_{MW}, X_{ME}, X_{PE}, T) - M_0) = F_{MI}(T) - F_{MW}(X_{MW})T - e_M \\ - \int_0^T F_{ME}(X_{ME}, t)dt + \int_0^T F_{PE}(X_{PE}, t)dt - e_P \end{aligned} \quad (15)$$

Where:

- F_{MI} is the inflow flux for Lake Mead

- F_{MW} is the outflow flux from Lake Mead to nearby states

- F_{ME} is the outflow flux from Lake Mead to downstream(to generated electricity)

And we derived the following equation,similarly:

$$M(X_{MW}, X_{ME}, X_{PE}, T) = \frac{1}{k_M A_M} \cdot (-F_{MW}(X1) \cdot T + F_{MI}(T) - e_M \\ - \int_0^T F_{PE}(X2, t)dt + \int_0^T F_{PE}(X2, t)dt - e_P) + M_0 \tag{16}$$

Since the time period of our assessment model would last for a long time(several years), to simplify the calculation, we take one day as the time unit, and use the discrete sum of daily value to be the representative of the integral over time.

On the basis of this principle, we are able to simplify the water depth function to :

$$M(X_{MW}, X_{ME}, X_{PE}, T) = \frac{1}{k_M A_M}(-F_{MW}(X_{MW}) + F_{MI}(T) - e_M + F_{PE}(X_{PE}, t) - e_T - F_{ME}(X_{ME})) + M_0$$

$$P(X_{PW}, XPE, T) = \frac{1}{k_P A_P}(-F_{PW}(X_{PW}) + F_{PI}(T) - e_P - F_{PE}(X_{PE}, t)) + P_0$$

where;

- $e_M$is the water loss in Lake Mead within a day

- e_P is the water loss in Lake Powell within a day

- e_T is the water loss during transportation from Lake Powell to Lake Mead

4.2.2 Computation of Hydroelectricity

According to the equation of the hydroelectricity, we define the instant amount of power generated by a dam to be of positive correlation with the flux of released water and the water depth. In the case of the Glen Canyon dam, which is:

$$P_{OW}(t) = C \cdot P(t) \cdot F_{PE}(t)$$

$$C = \mu \epsilon \rho g$$

where :

- μ is the mechanical efficiency of turbine

- ϵ is the coefficient to balance the grade of electricity and social benefit

- ρ is the density of water, and g= 9.81 m/s^2

Based on the equation above [7], we are able to get the total amount of electricity generated in the whole year.

$$E(X_{PW}, X_{PE}, X_{MW}, X_{ME}) = \sum_{i=1}^{365} C_P \cdot P(X_{PW}, XPE, i) \cdot F_{PE}(X_{PE}, i) + \sum_{i=1}^{365} C_M \cdot M(M(X_{MW}, X_{ME}, X_{PE}, i) \cdot F_{ME}(X_{PE}, X_{ME}, i) \tag{17}$$

Similar to the calculation of the benefit of water allocation, we derived a function to assess the benefit of generating a certain amount of electricity:

$$E_{benefit}(E, D_e) = -e^{-E+0.7D_e-ln(1.8)} + 1.8 \tag{18}$$

Which means when more hydroelectricity is generated and transmitted to the states, the benefit will increase as well. However, the maximum can only get close to 1.8 times of current benefit, but never exceed. And when the supply to a state is less than 70% of its demand, the benefit will be negative .

4.3 The Coordinate Regulation Assessment Model

In order to mathematically solve the competing interest of water usage and hydroelectricity generation, it is necessary to define a total judging function: $T_{otal(X1,X2)}$–the value of which indicates the total value generated by assigning stated amount of water to different area of usage.

$$T_{otal}(X_{PW}, X_{PE}, X_{MW}, X_{ME}) = max(G_W(X_{PW}, X_{MW})) + G_E(X_{PW}, X_{PE}, X_{MW}, X_{ME}) + G_{MX}$$

- $G_W(X_{PW}, X_{MW})$ is the total benefit of water assigned to all states for usage

- $G_E(X_{PW}, X_{PE}, X_{MW}, X_{ME})$ is the total benefit of electricity generated by releasing water from the dam

- $G_{MX}(X_{ME})$ is the benefit of water delivered to Mexico [8] for usage

where:

$$G_{MX}(X_{ME}) = \frac{X_{ME} - D_{MX} - e_{MX}}{X_{ME}}$$

This is a four-dimension optimization model which could be optimized by genetic algorithm as well.

Constraints

According to equation (5) and (6), we are able to define the inequality relationship between X_{PW}, X_{PE}, X_{MW},and X_{ME}:

$$k_P P_{min} \leqslant tF_{PI} + k_P P_0 - X_{PE} - X_{PW} - e'_P \leqslant k_P P_{max} \tag{19}$$

$$k_M M_{min} \leqslant tF_{MI} + k_M M_0 - X_{ME} - X_{MW} - e'_M \leqslant k_M M_{max} \tag{20}$$

5 Extreme Condition

In the last two sections, we have determined the importance of generating power and supplying enough water to each states. If the water is not enough to satisfy both of the requirement, that is to say, if we draw a certain kind amount of water, the water level in the reservoir will be under the "power pool", which is the lowest level for the turbines to generate hydroelectricity without damaging the machine. In this case, we tested our model with a set of data consists of high water demand and low initial water capacity in the reservoir, several pairs of the inputs [9] and results are shown below:

Elevation (ft)	***Lake Powell***	***Lake Mead***
Dead Pool	3370	895
Power Pool	3490	1050
Max Capacity	3700	1229

Table 5: Data of Lakes

According to the definition before, "Power Pool" stands for the minimum depth for hydroelectricity generation while "Dead Pool" stands for the minimum depth to release water through the dam.For the convinence of computation, we define "depth" as the difference between the elevation the water surface and the dead pool (in meter). In the year of drought, we should assume that the initial value of the depth should be enough to generate electricity, but soon it will be not. So we can choose the depth as shown in the table below to predict the allocation of water. The data [10] is listed in Table 6:

	Lake Powell	***Lake Mead***
Average Depth in Normal Year(m)	68.23	53.34
Water Supply in Normal Year ($10^9 m^3$)	5.92	9.46
Power Release in Normal Year ($10^9 m^3$)	9.5	11.0
Assumed depth in Year ($10^9 m^3$)	40	50
Predicted Water Supply ($10^9 m^3$)	6.96	1.05
Predicted Power Release ($10^9 m^3$)	4.57	4.14

Table 6: Supply and Release against Depth

By comparing the predicted value with the ones in normal year, we can find that our model prefer to reduce the release of both the dams, draw more water from Lake Powell while draw less Water from Lake Mead. This strategy is reasonable because under this situation, releasing too much water may be just releasing water as long as the water level is under "Power Pool". In addition, the reduction of power release of Lake Powell is smaller than the one of Lake Mead. This is because, in our model, the inflow of Lake Mead is positively related to the release of Lake Powell. If there isn't enough water for Lake Mead to supply, the total benefit will be lost. Therefore, transfer more water from Lake Powell to Lake Mead in years of Drought. This preference agrees with the proposition of scholars which says "Fill Mead First"(cite). It proves that our model is practical facing with drought.

6 Possible Changes in the Future and Sensitivity Analysis

6.1 Water Demand Grows or Shrinks

Our model in 4.1.4 for allocating water for different usage, the total benefit of water supply is determined by AHP based on the main features including population, water demand and corresponding GDP in agriculture, industry of each state respectively. When the relative parameters change, the influence of supplied water will change in a way. Specifically, we derived three comprehensive evaluation coefficients to reflect the importance to satisfy residential, agricultural and industrial demands of water. To evaluate the influence of change of population, agriculture and industry, we changed each coefficient about $\pm 5\%$ to see how much water supplied for these usages change by percent in Table 7:

usage	***Resident***	***Agriculture***	***Industry***
Population Grows	0.00600428	-0.00419459	-0.00261282
Population Shrinks	-0.00511894	0.00187593	0.00191996
Agriculture Grows	-0.00035407	0.00108004	-0.00132086
Agriculture Shrinks	0.00722841	-0.00232866	0.00450179
Industry Grows	-0.00128285	-0.00057085	0.00041466
Industry Shrinks	0.00877599	0.00400196	-0.00521726

Table 7: Water Supply and Usage

Here we can find that any growth will bring a more water allocated for this usage while allocating less to other two usages. Besides, the scale of change is relatively small, which proves that our model is robust and error of estimation is acceptable.

6.2 Proportion of Renewable Energy Increases

To obtain the data of the reliability of hydroelectricity for each state, we collected the data about electricity consumption and the amount of hydroelectricity transferred by state, then let the mean value of the ratio to stand for the proportion of renewable energy. To simulate the increase, we should change the ratio and get the new value of the grades, as shown below:

Proportion of Hydroelectricity	***20%***	***23%***	***26%***
Water Supply from Lake Mead ($10^9 m^3$)	4.61	4.53	4.39
Water Supply from Lake Powell ($10^9 m^3$)	8.23	8.19	8.12
Power Release of Lake Mead ($10^9 m^3$)	2.89	3.18	3.24
Power Release of Lake Powell ($10^9 m^3$)	4.57	4.73	4.99

Table 8: Supply,Release and Proportion of Hydroelectricity

We can see that as the dependence of hydroelectricity increases, both of the lakes will increase the power release to optimize the total benefit.

6.3 Additional Water and Electricity Conservation Measures

In current situation, some of the water resource is certainly wasted, which means that the effect of the water supply on development is not maximized. Therefore, if additional water and electricity conservation measures are performed, supplying same amount of water can derive more benefit. In order to simulate this situation, we need to change the coefficients stand for the water utilization ratio and electricity ratio, which are related to the model of social benefit assessment.In the simulation, our model indicates that more water should be drawn to meet the demands.

6.4 Other Possible Changes of Constants that May Count

While computing the benefit of generating hydroelectricity, we chose to fit the relationship of power release and the amount of power generated, which reflects the efficiency of the turbines. To analyze the weights of these two constants, we can change them by $\pm 5\%$. The results are shown below:

Change Rate	-10%	-5%	5%	10%
Water Supply from Lake Mead ($10^9 m^3$)	0.8%	0.2%	-0.5%	-1.1%
Water Supply from Lake Powell ($10^9 m^3$)	1.4%	0.9%	1.2%	2.3%
Power Release of Lake Mead ($10^9 m^3$)	-3.3%	2.2%	0.3%	1.7%
Power Release of Lake Powell ($10^9 m^3$)	3.1%	2.8%	0.9%	1.2%

Table 9: Influence of Turbine Coefficient

The results indicate that when the water is scarce, we should meet the demand of water usage while generating power with less water release. In this way, the total social benefit is maximized according to a series of objective standards. In real world, additional water may be transferred from other basin to make up for the shortage, but this method should not be considered in our model since the main purpose is to deal with the water in the Colorado River.

7 Strengths and Weaknesses

Strengths

- Our model are robust when the constants change, which means even if our estimation of constants is with deviation in a way, the final result won't be affected significantly.

- Our model takes the influence of the release of Lake Powell on the inflow of Lake Mead into account, which is vital for the practice of coordinate regulation. When facing with worse drought, this method will be of more importance.

- The application of Adaboost-Decision Tree Regression can avoid overfitting in a way to reduce the influence of occational data, such as a flood happens once in a century. This method ensures our model can fit into most of conditions without large modification.

- Based on the water balance equation of Colorado, our model can be applied to other basins by simply changing the factors and constants. Iterative algorithm makes it possible to change the number of reservoirs.

- All the essential data used come from official website such as United States Bureau of Reclamation and United States Geological Survey.

Weaknesses

- Our model didn't take the additional water supplement such as rainfall, and water transferred from other basin, which are common in real life. In such a short time, we can only focus on the allocation of the water already been flowing in the Colorado River. In this case, we discuss this in the further discussion.

- The data of the inflow of Lake Mead is hard to find, roughly fitted relationship between the release of Lake Powell and the inflow of Lake Mead may helps but not so accurate. But the sensitivity analysis showed relatively reasonable trend when the parameters change. So our regression functions are enough for rough analysis.

- The basic unit that can be analyzed with enough confidence is month, mean values helps to make up for the deficiency of daily data but may sacrifice the accumulation value of the whole year.

8 Conclusion

At first, we developed a set of fluid mechanics equations to calculate the flux of Colorado River at certain point, and constructed the water balance equations and demand-supply in-equations are to restrict outflow water and sequential relationship of the two dams. Then, the Coordinate Regulation Assessment Model calculated the total benefit of water used in agriculture, residence and industry. To find out the priority, we applied a 2-layer Analytic Hierarchy Process. We synthesized the benefits of water usage in each state, the benefits of hydroelectricity generation and the water dissipated to Mexico in the model.the CRAM functions as the comprehensive judging criteria to determine how good the allocation is. Again, implement genetic algorithm to find the optimized solution. The CRAM functions as the comprehensive judging criteria to determine how good the allocation is. Again, implement genetic algorithm to find the optimized solution for water allocation. After the establishment of the Coordinate Regulation Assessment Model, we tested our model with normal conditions and extreme condition of drought. We let the total benefit, including the direct benefit by generating power and indirect benefit related to society, agriculture, industry and ecology, to be the goal to optimized. The constrains consist of water balance equation and restricted water level of the reservoirs. In addition, to show how our model performs under different situations, we modified certain parameters in the equations, figuring out the way the results change. It is to the sensitivity analysis. For those features we didn't pay much attention to, we tested their importance as well. The result of sensitivity shows that our model reflects well when the critical factors change and keeps relatively robust when those features are less important change. It is proved that our model can be applied under a wide range of situations.

Especially, we discussed how much water can flow into the Gulf of California. It is unfortunate that with the same initial condition, the amount of water released by Hoover Dam is close to the average value of recent years. As we all know, the Colorado River haven't reached the Gulf of California for more than ten years except 2014. It is to say that if the rainfall shortage and hot temperatures persist, our model indicates that no water will left when the Colorado River reaches the gulf.

In the end, we made an objective analysis of strengths and weaknesses of our model and hopes to make it more perfect by discussing the possible way to improve the accuracy.

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Appendices

First Appendix: Ease the Thirst in the Colorado Basin

The Colorado River is one of the most important river in the south-west America, for the water resource it provides in the vast, arid flatlands.

The allocation of the precious water resource in the dry south-west America is always a focus point, which is not only an issue among the states in the Colorado Basin, but also concerns to the right of Mexico, making this problem even more complicated. To reach a consensus on this problem, dozens of documents and contracts has been signed over the decades, including the famous Colorado River Contract signed in 1922.

However, the contract is generally becoming out of date due to global climate change and consistent drought in the Colorado Basin. The Colorado River system is losing at a speed around one million acre-feet per year. Facing the huge pressure under water shortage, changes need to be make to allocate the water in a wiser and more efficient way among the five states and Mexico.

Trying to find a better solution for the allocation of water and hydroelectricity power in the Colorado Basin, our team developed a synthesised model to simulate the water flow of the Colorado River and estimated the benefit the river could bring to the five states with proper arrangement.

Our team uses the water balance equations together with the fluid mechanics equations as constraint equations and then derived the amount of water to be drawn from the two lakes for industrial, agricultural, and residential needs of the five states. The model suggests that the water to be drawn from the Lake Powell and Lake Mead follows a linear relation and are restricted in a small interval. This is the basis and a general guidance for the development of the rest of the model.

After that, we proposed the Coordinate Regulation Assessment Model to give an overall judgment for the whole allocation process of the two dams, synthesizing the benefits of water usage in each state, the benefits of hydroelectricity generation and the water dissipated to Mexico.

After the establishment of the Coordinate Regulation Model, we tested our model with normal conditions and extreme condition of drought. We let the total benefit, including the direct benefit by generating power and indirect benefit related to society, agriculture, industry and ecology, tobe the goal to optimized. The constrains consist of water balance equation and restricted water level of the reservoirs. In addition, to show how our model performs under different situations, we modified certain parameters in the equations, figuring out the way the results change. It is to the sensitivity analysis. For those features we didn't pay much attention to, we tested their importance as well. The result of sensitivity shows that our model reflects well when the critical factors changeand keeps relatively robust when those features are less important change. It is proved that ourmodel can be applied under a wide range of situations.

For the benefit of water usage in each state, we applied a 2-layer Analytic Hierarchy Process(AHP) to determine the weight for each factor effecting the benefit of state using water. Weconsider the effect from three area: social benefit, economic benefit and environmental benefits.Factors of these areas include the GDP of different industries, the utilization factor, the waterfootprint, the population, the ecological water and so on. With the weight of factors calculated, weare able to determine the total benefit function. To find the optimized solution to maximize it, the genetic algorithm is applied. Based on an appropriate rate of heredity and variation, this heuristicmethod managed to find a satisfying water allocation solution for each state and each industry agriculture, industry and residence.

When the water recommend more water to be released from the Lake Powell to the Lake Mead when there is not enough water to meet all the demand for electricity and water. This is reasonable because the water flows out the Lake Powell for generating electricity can be once again used to supply the demand for water and electricity in the lower basin, and thus increase the utilization of the water.

In general, facing the pressure of consistent drought and increase of demand, wish our model can solve part of the crisis of the Colorado Basin.

Second appendix

Here are simulation programmes we used in our model as follow.

Python implementation of the genetic algorithm:

```
# -*- coding: utf-8 -*-
"""
Spyder Editor

This is a temporary script file.
"""
import numpy as np
from sko.GA import GA
D=[1.6860438,7.29233376,0.15295152,0.395453688,1.215717888]
P_O=[45.38743692064355,73.54045343340329,45.837167366214445,43.77136446236359,
     47.79297689572304]
def g_benefit(P):
    x1,x2,x3,x4,x5=P
    y=-(P_O[0]*(-np.exp(-x1+0.5*D[0]-np.log(1.5))+1.5)
    +P_O[1]*(-np.exp(-x2+0.5*D[1]-np.log(1.5))+1.5)
    +P_O[2]*(-np.exp(-x3+0.5*D[2]-np.log(1.5))+1.5)
    +P_O[3]*(-np.exp(-x4+0.5*D[3]-np.log(1.5))+1.5)
    +P_O[4]*(-np.exp(-x5+0.5*D[4]-np.log(1.5))+1.5))
    return y
constraint_ueq = (
    lambda P: P[0] +P[1]-D[0]-D[1]
    , lambda P: -(P[0] +P[1]-D[0]-D[1])
    ,lambda P:  P[2]+P[3]+P[4]-D[2]-D[3]-D[4]
    ,lambda P: -( P[2]+P[3]+P[4]-D[2]-D[3]-D[4])
)

ga = GA(func=g_benefit, n_dim=5, size_pop=50, max_iter=1000, prob_mut=0.001, lb=[0, 0,0,0,
best_x1, best_y = ga.run()
print('best_x1:', best_x1, '\n', 'best_y:', best_y )

import pandas as pd
import matplotlib.pyplot as plt

Y_history = pd.DataFrame(ga.all_history_Y)
fig, ax = plt.subplots(2, 1)
ax[0].plot(Y_history.index, Y_history.values, '.', color='red')
Y_history.min(axis=1).cummin().plot(kind='line')
plt.show()
```

Third appendix

Python implementation of the Adaboost-Decisiontree:

```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.tree import DecisionTreeRegressor
from sklearn.ensemble import AdaBoostRegressor
import pandas as pd

data = pd.read_excel('inflow_day.xlsx')
x_train = data.iloc[0:366,0][:, np.newaxis]
y_train = data.iloc[0:366,1]

PWin = AdaBoostRegressor(DecisionTreeRegressor(max_depth=4),n_estimators=15, random_state=
PWin.fit(x_train,y_train)
y_pred = PWin.predict(x_train)
print(PWin.predict(x_train)[1])

# Plot the results
plt.figure()
plt.scatter(x_train, y_train, color='black', marker='+', label="training samples")
plt.plot(x_train, y_pred, c="r", label="Boosted Decision Tree Regression", linewidth=1)

plt.xlabel("day")
plt.ylabel("flux")
plt.title("Boosted Decision Tree Regression")
plt.legend()
plt.show()
```