

$Y = XW + b$ (b) 向量 + 1: 每一项增加

$$L(\hat{y}, y) = \frac{1}{2n} \sum_{i=1}^n (\hat{y}_i - y_i)^2$$

$$\frac{\partial L}{\partial Y} = \nabla Y = \left(\frac{\partial L}{\partial \hat{y}_i} \right)_{n \times 1} \text{ 求 } \frac{\partial L}{\partial \hat{y}_i} = \frac{1}{n} (\hat{y}_i - y_i)$$

$$AB = C \quad \nabla B = A^T \cdot \nabla C$$

$$\therefore \nabla W = X^T \cdot \nabla Y = (w_2)_{n \times 1}$$

$$w := w - \alpha \cdot \nabla W$$

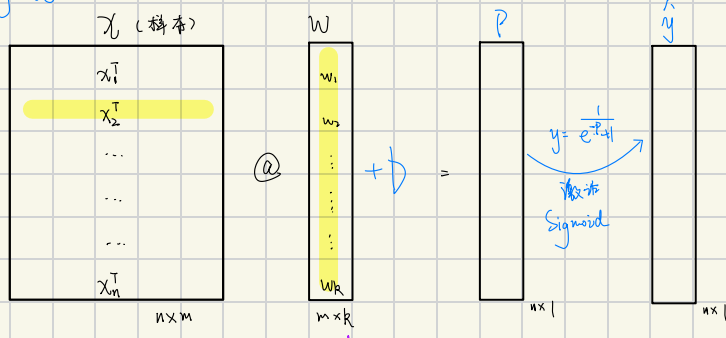
$$\frac{\partial L}{\partial b_i} = \sum_{j=1}^n \frac{\partial L}{\partial \hat{y}_j} \cdot \frac{\partial \hat{y}_j}{\partial b_i} = \frac{\partial L}{\partial y_i}$$

验证 b 为标量

$$b_1 = b_2 = \dots = b_n = \sum \text{ (标量)}$$

$$b := b - \sum \left(\frac{\partial L}{\partial b_i} \right)$$

logistic



$$Y = \text{sig}(XW + b)$$

$$L(\hat{y}, y) = \sum_{i=1}^n \left(y_i \ln \hat{y}_i + (1 - y_i) \ln (1 - \hat{y}_i) \right)$$

$$\frac{\partial L}{\partial \hat{y}_i} = \frac{y_i}{\hat{y}_i} - \frac{1 - y_i}{1 - \hat{y}_i}$$

$$= \frac{y_i - y_i \hat{y}_i - \hat{y}_i + \hat{y}_i^2}{\hat{y}_i (1 - \hat{y}_i)}$$

$$\frac{\partial L}{\partial p_i} = \frac{y_i - \hat{y}_i}{\hat{y}_i (1 - \hat{y}_i)} \cdot \frac{e^{p_i}}{(e^{p_i} + 1)^2} = y_i - \hat{y}_i$$

$$\nabla P = (y_i - \hat{y}_i)_{n \times 1}$$

$$\nabla W = X^T \cdot \nabla P \quad w := w - \alpha \cdot \nabla W$$

$$\frac{\partial L}{\partial b} = \sum (\nabla P) \quad b := b - \alpha \cdot \frac{\partial L}{\partial b}$$