

భారతీయ సాంకేతిక విజ్ఞాన సంస్థ హైదరాబాద్
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LEARNING PHASE IN EPOCH

REPORT

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Abstract

Sommario

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CSV Comma Separated Values



Linear Regression

1.1 INTRODUCTION

Machine learning is predictive modelling. We will create models which will predict outcome based on the model's learning from the training data. Sometimes we have to predict the outcome which is a continuous variable. Linear regression is a simple machine learning model which establishes a linear relation between data and outcome. We will discuss the details of establishing the relation based on training data with detailed mathematics and python code.

1.2 MODEL

The model is constructed by simply taking some arbitrary coefficients for creating a linear formula and updating it repeatedly to reach the required state to predict outcome. Everything is explained further.

1.2.1 TERMS USED FREQUENTLY

The following are the terms which we will be frequently using throughout this discussion:

- Features
The input data for the model might have more than one parameter which will decide the outcome. Each parameter is called a feature.

1.2. MODEL

- Instance

The model is mostly trained with a huge set of data. Each individual part of the data with its features and outcome is called an instance.

1.2.2 MATHEMATICAL UNDERSTANDING

Let the dataset have n features. The training dataset contains input X , with m instances. We will consider X as a set of vectors of n dimensions. So $\vec{X}^{(i)}$ represents i^{th} instance of the data.

$$\vec{X}^{(i)} = \sum_{j=1}^n \vec{x}_j^{(i)} \quad (1.1)$$

Here $\vec{x}_j^{(i)}$ is value of the j^{th} feature at i^{th} instance. We consider vectors here for easy mathematics. The outcome of the X is y which is a set of real numbers. While training, we compare the calculated outcome y' and update the model accordingly.

We consider \vec{W} as a vector with dimension n . b is a real number.

$$y' = \vec{W} \cdot X + b \quad (1.2)$$

where

$$\vec{W} \cdot X = \begin{bmatrix} \vec{W} \cdot \vec{X}^{(1)} \\ \vec{W} \cdot \vec{X}^{(2)} \\ \vdots \\ \vec{W} \cdot \vec{X}^{(m)} \end{bmatrix} \quad (1.3)$$

Therefore equation (1.2) can be rewritten as

$$y' = \begin{bmatrix} \vec{W} \cdot \vec{X}^{(1)} + b \\ \vec{W} \cdot \vec{X}^{(2)} + b \\ \vdots \\ \vec{W} \cdot \vec{X}^{(m)} + b \end{bmatrix} \quad (1.4)$$

Our mission here is to find the \vec{W} and b from the given data and use them in

predicting outcomes for any given input data. For that we should have a deeper and clear understanding of the equations.

$$\vec{W} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \quad (1.5)$$

$$\begin{aligned} y'_i = \vec{W} \cdot X^{(i)} + b &= \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \cdot \begin{bmatrix} x_1^{(i)} & x_2^{(i)} & \cdots & x_n^{(i)} \end{bmatrix} + b \\ &= w_1 x_1^{(i)} + w_2 x_2^{(i)} + \cdots + w_n x_n^{(i)} + b \end{aligned} \quad (1.6)$$

Here we can understand w_i as weight of i^{th} feature. The value of each feature is multiplied by it's weight and added to the total. The weight balances the influence of a particular feature on the outcome.

Some features might be generally in higher magnitude, but have a lesser influence on the outcome. Obviously their weight will be small and compensate the high magnitude. Similarly some features might have small magnitudes but higher influence on the outcome. As expected, their weights will be large.

We add b as a constant which will adjust the magnitude of product of weights and features to that of outcome.

1.2.3 FINDING W AND b

Initially we know nothing about W and b . So, we will assume all the weights to be equal to 0. We will compute the y' and compare it y , the original outcome. We define a cost function which represents the error in the calculated outcome.

$$J(W, b) = \frac{1}{m} \sum_{i=1}^m (y_i - y'_i)^2 \quad (1.7)$$

This is called as mean square error. We consider square here to get the absolute value here. We can use other functions like absolute value or 4th power, but square makes the further mathematics simple. Our aim is, as discussed above,

1.2. MODEL

to find the values of W and b where the cost function J is minimum. So, we will update the W and b to achieve minimum using gradient descent. Gradient descent is explained briefly in the next section.

We will find the gradient equation for cost function J .

$$\nabla J(W, b) = \begin{bmatrix} \frac{\partial J}{\partial W} \\ \frac{\partial J}{\partial b} \end{bmatrix} = \begin{bmatrix} \frac{1}{m} \sum_{i=1}^m -2X^{(i)}(y_i - y'_i) \\ \frac{1}{m} \sum_{i=1}^m -2(y_i - y'_i) \end{bmatrix} \quad (1.8)$$

Now we update W and b accordingly.

$$W_{new} = W - \alpha \frac{\partial J}{\partial W} \quad (1.9)$$

$$b_{new} = b - \alpha \frac{\partial J}{\partial b} \quad (1.10)$$

We use the obtained W and b for predicting outcome.

1.2.4 GRADIENT DESCENT

Gradient descent is an iterative algorithm for finding minimum value of a function. The algorithm roughly works in the following steps:

1. Choose a starting point.
2. Calculate the gradient at this point.
3. Make a scaled step against the gradient.
4. Repeat the steps 2 and 3 until minimum is reached, i.e. the gradient reaches almost 0.

GRADIENT

Gradient is slope of the function at a given point in a given direction. Gradient of a n -dimensional function $f(p)$ is given

$$\nabla f(p) = \begin{bmatrix} \frac{\partial f}{\partial x_1}(p) \\ \vdots \\ \frac{\partial f}{\partial x_n}(p) \end{bmatrix} \quad (1.11)$$

ALGORITHM

In the it h iteration,

$$p_{i+1} = p_i - \alpha \nabla f(p_i) \quad (1.12)$$

Where α is the learning rate which controls the step size. The learning rate should be chosen carefully according to number of iterations as

- Small learning rate might make the algorithm stop, i.e reach the maximum iteration, before reaching the minimum.
- Large learning rate might make the algorithm jump too much and thus might skip the minimum.

1.3 QUESTIONS FOR CURIOSITY

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Logistic Regression

2.1 INTRODUCTION

As discussed in the previous chapter, we will build a model which will predict the outcome based on the model's learning from training dataset. Logistic Regression is used for predicting discrete values (only 0 and 1 here) unlike continuous values in Linear Regression. It simply calculates the probability of the value, calculated from Linear Regression, lying on the extremes and output it as 0 and 1.

2.2 MODEL

The model is simply a little additional calculation on Linear Regression. We use sigmoid function to restrict the outcome between 0 and 1. If the outcome is close to 1 then we treat it as 1 and 0 otherwise.

2.2.1 SIGMOID FUNCTION

For restricting the real number value from the outcome of Linear Regression between 0 and 1, we use Sigmoid function. It takes real numbers as input and outputs continuous values between 0 and 1.

$$\text{Sigmoid}(x) = \frac{1}{1 + e^{-x}} \quad (2.1)$$

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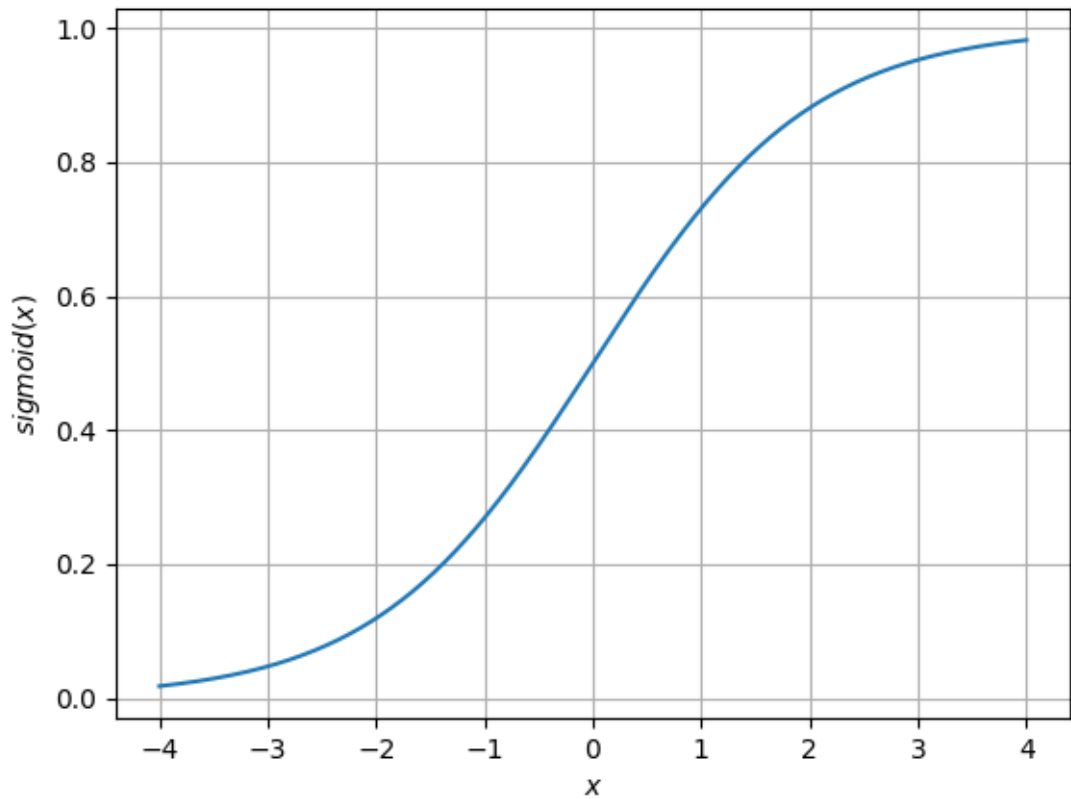


Figure 2.1: Sigmoid function graph

The graph of Sigmoid function is shown in figure 2.1

2.2.2 MATHEMATICAL UNDERSTANDING

As seen in chapter 1 and discussed in section 2.2,

$$y'_i = \frac{1}{1 + e^{-\vec{W} \cdot \vec{X}_{(i)}}} \quad (2.2)$$

2.2.3 FINDING W AND b

We use a little more math here to make the computation easier. We define cost function in a slightly different way

$$h_i = (y'_i)^{y_i} (1 - y'_i)^{1-y_i} \quad (2.3)$$

If you observe keenly and try to understand, the h value gives the absolute difference between the calculated outcome and original outcome. The original outcome is either 0 or 1. We try to spread y' in such a way that the outcome of a particular instance will be close to 0 or 1 depending on the original outcome.

As we are going to use gradient descent, we will be calculating gradient in every round. Calculating gradient for h itself is a little heavy task for computation. We will take $\log(h)$ to make it easier.

So we will be defining the cost function as follows

$$J(W, b) = \frac{1}{m} \sum_{i=1}^m \log h_i \quad (2.4)$$

The gradient for cost function J

$$\nabla J(W, b) = \begin{bmatrix} \frac{\partial J}{\partial W} \\ \frac{\partial J}{\partial b} \end{bmatrix} = \begin{bmatrix} \frac{1}{m} \sum_{i=1}^m X^{(i)}(y_i - y'_i) \\ \frac{1}{m} \sum_{i=1}^m (y_i - y'_i) \end{bmatrix} \quad (2.5)$$

Now we update W and b accordingly

$$W_{new} = W - \alpha \frac{\partial J}{\partial W} \quad (2.6)$$

$$b_{new} = b - \alpha \frac{\partial J}{\partial b} \quad (2.7)$$

We use the obtained W and b for predicting outcome. Notice here that y' contains real number between 0 and 1. We will replace all the values in y' greater than 0.5 with 1 and others with 0.

2.3 QUESTIONS FOR CURIOSITY

Acknowledgments