

# PROBABILITY

**Sample:** set of observations drawn from a population.

**Sample space:** set of all possible outcomes that can happen in a chance situation.

**Event:** a subset of the sample space. A probability is assigned to the event.

**Probability:**  $p = \# \text{ desired outcomes} / \# \text{ possible outcomes}$ .

**Probability Rules:**

1. The probability of an outcome in the sample space is 1.  $P(S)=1$
2. For any event A  $0 \leq P(A) \leq 1$
3. Disjoint events  $P(A \text{ or } B) = P(A)+P(B) = 1, P(A \text{ and } B) = 0$
4. Complement  $P(A) = 1 - P(A^c)$
5. Non disjoint events  $P(A \text{ or } B) = P(A)+P(B) - P(A \text{ and } B)$

**Independent events:** occurrence of one event has no effect on the other.  $P(A \text{ and } B) = P(A) \times P(B)$

**Joint probability:** chance of an outcome of having two events occurring together at the same time.

**Marginal probability:** the probability of observing an outcome with a single variable, regardless of its other variables.

**Conditional probability:** the conditional probability of an event A given that the event B occurs.  $P(A|B) = P(A \text{ and } B)/P(B)$

**Multiplication rule:**  $P(A \text{ and } B) = P(A|B) \times P(B)$ ,  $P(A \text{ and } B) = P(B|A) \times P(A)$

**Bayes Rule:**  $P(A|B) = P(B|A) \times P(A)/P(B)$

**Representation:** Tree/Probability table

## RANDOM VARIABLES

**Random variable:** describes the probability for an uncertain future numerical outcome of a random process.

**Discrete random variable:** the set of possible outcomes is finite.

**Continue random variable:** can take any value within an interval.

**Expected value:** weighted average,  $E(X) = X1 \cdot p(X1) + X2 \cdot p(X2) + \dots + Xn \cdot p(Xn)$

**Variance:** describes the spread is the data from the mean value.  $Var(X) = E[(X-m)^2]$

**Standard deviation:**  $\sqrt{Var(X)}$

**Covariance:** measures variance between two random variables.  $Cov(X,Y) = (\sum (Xi - Xmean)(Yj - Ymean))/n$ . + = same direction

**Correlation:** measures strength of the relationship between variables.  $Corr(X,Y) = Cov(X,Y)/(stdx \cdot stdy)$ . + = variables highly correlated.

**Distance matrix:** squared matrix that contains the distance between the variables of the set.

**I.i.d (Identically independent distributed) random variables:** have same distribution and are mutually independent.

# MATRICES

**$m \times n$**  It is a matrix with  $m$  rows and  $n$  columns.

**Square matrix:** when  $m=n$

**Column vector:** is a matrix with only 1 column

**Row vector:** a matrix with only 1 row

**Transpose matrix:** interchange rows and columns. Notation:  $A' = t(A)$

**Diagonal matrix:** has 0 values except the main diagonal

**Symmetric matrix:** square matrix unchanged when it is transposed.  $A' = A$

**Identity matrix:** diagonal matrix with all elements of the diagonal equal to 1. Notation:  $I$

**Matrix multiplication:**  $A_{m \times n} B_{n \times p} = C_{m \times p}$

**Element-wise multiplication:**  $A_{m \times n} \odot B_{m \times n} = C_{m \times n}$

**Inverse matrix:**  $AA^{-1} = I$

**Trace:** sum of the elements of the diagonal.

**Determinant:** Notation:  $\det(A) = |A|$

**Eigenvalues and eigenvectors**  $Ax = \lambda x$

$\lambda$  is a scalar and is called the eigenvalue of  $A$

$x$  is the eigenvector belonging to  $\lambda$ .

Any nonzero multiple of  $x$  will be an eigenvector.

To find  $\lambda$ :  $|A - \lambda I| = 0$