Writeup

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Detecting X and Z errors

For X errors:

- If all data qubits in $|\pm x\rangle \equiv |\pm\rangle$ state, ancilla $\rightarrow |0\rangle$
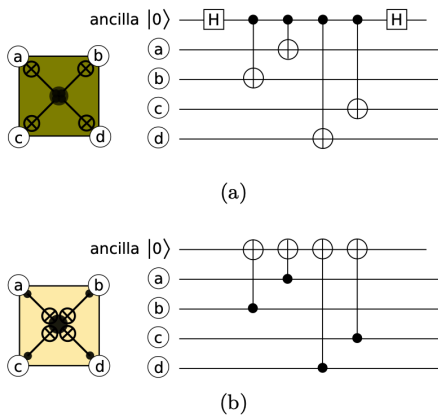- X, Y, Z are the Pauli matrices $\sigma_x, \sigma_y, \sigma_z$

(a)

(b)

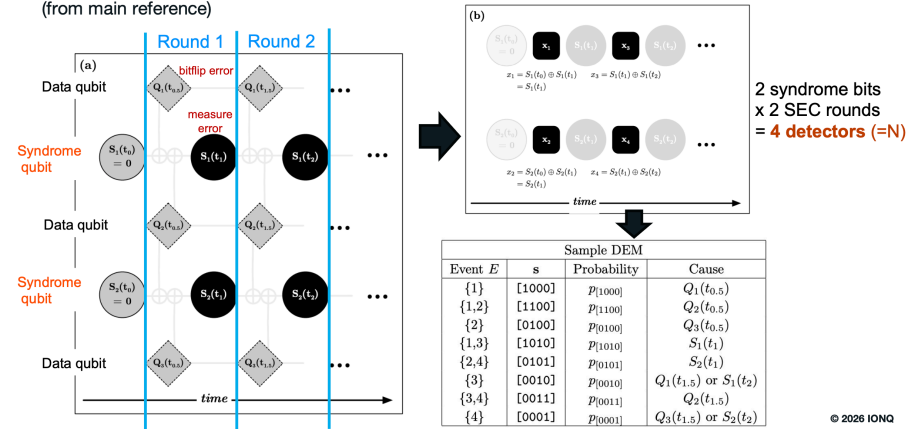FIG. 2. Standard quantum circuits to measure stabilizers (a) $X_a X_b X_c X_d$ and (b) $Z_a Z_b Z_c Z_d$.

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Introduction : Review of DEMs

From Erik's presentation:

Example DEM: 2 rounds of d=3 repetition code

(from main reference)

|	Sample DEM		
Event E	s	Probability	Cause
{1}	[1000]	$p_{[1000]}$	$Q_1(t_{0.5})$
{1,2}	[1100]	$p_{[1100]}$	$Q_2(t_{0.5})$
{2}	[0100]	$p_{[0100]}$	$Q_3(t_{0.5})$
{1,3}	[1010]	$p_{[1010]}$	$S_1(t_1)$
{2,4}	[0101]	$p_{[0101]}$	$S_2(t_1)$
{3}	[0010]	$p_{[0010]}$	$Q_1(t_{1.5})$ or $S_1(t_2)$
{3,4}	[0011]	$p_{[0011]}$	$Q_2(t_{1.5})$
{4}	[0001]	$p_{[0001]}$	$Q_3(t_{1.5})$ or $S_2(t_2)$

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The p_{ij} picture for *Graphlike* Error Models:

- This picture of DEMs came later, and was an attempt to clarify Google's p_{ij} picture of DEMs
- Graphlike means that no error mechanism contributes to more than one pair of detectors(*note these are all single qubit errors*)

- Note that we have *space-like*, *time-like* edges (e.g. p_{12}, p_{13}). This DEM does not have *space-time_like* edges
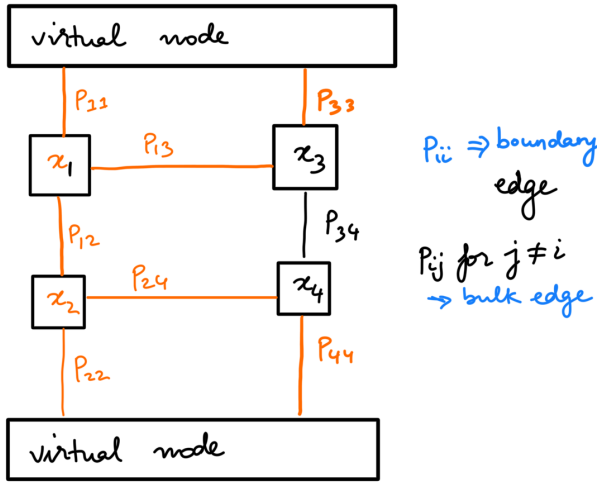
Event	p_{ij}	Cause
{1}	p_{11}	$Q_1(t_{1.5})$
{1,2}	p_{12}	$Q_2(t_{0.5})$
{2}	p_{22}	$Q_3(t_{0.5})$
{1,3}	p_{13}	$S_1(t_1)$
{2,4}	p_{24}	$S_2(t_1)$
{3}	p_{33}	$Q_1(t_{1.5})$ or $S_1(t_2)$
{3,4}	p_{34}	$Q_2(t_{1.5})$
{4}	p_{44}	$Q_3(t_{1.5})$ or $S_2(t_2)$

- Edges define *error mechanisms* that flip one or two of the detectors
- Note that some error mechanisms are indistinguishable e.g. $Q_1(t_{1.5})$ or $S_1(t_2)$ both contribute to p_{33} and cannot be distinguished in this DEM

Inferring DEM from Syndrome data

- Erik showed how $P(s)$ can be inferred from syndrome data in the earlier paper
- Here, we count detector "clicks": $v_i = 1$ if detector on/ error detected and $v_i = 0$ if detector off/ no error
- In the p_{ij} formalism, we have ($\oplus$ denotes XOR)

$$p_{ij} = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\langle v_i v_j \rangle - \langle v_i \rangle \langle v_j \rangle}{1 - 2 \langle v_i \oplus v_j \rangle}} \quad i \neq j$$

- Use the above to infer p_{ii} as

$$p_{ii} = \frac{1}{2} + \frac{\langle v_i \rangle - \frac{1}{2}}{\prod_{i \neq j}(1 - 2p_{ij})}$$

Limitations/Points of Confusion:

- Considered single qubit errors, not *gate errors*
- Gate errors show up as *space time* edges
- Hypergraph/hyper-edges also possible, especially with cycles where an error flips multiple detectors
- This connects to the paper Erik presented on via *reduced DEMs*

What is a Coherent Error

- Consider the example where every qubit has:

$$U = \prod_{j=1}^{n} e^{-i\theta_j Z_j}$$

- Coherent Error Channel

$$\mathcal{E}_{coh}(\rho) = U\rho U^\dagger$$

- If all $\theta_j = \theta$, note this gives a *physical error rate* $p = \sin^2\theta$, which is the dephasing strength you would get from Pauli twirling this coherent error

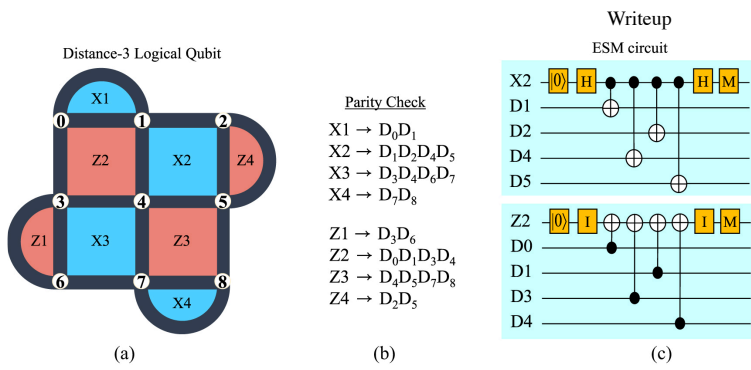$$\mathcal{E}_{twirl}(\rho) = \cos^2\theta\rho + \sin^2\theta Z\rho Z$$

Coherent Errors in the $d = 5$ repetition error

- *also recall X check definition and circuit from Erik*
- We count $\langle v_i \rangle, \langle v_i v_j \rangle$ and reconstruct from $p = \sin^2\theta$ the angles
- Note that in this DEM, only one round of syndrome extraction, 4 detectors, and 5 edges: p_{11}, p_{44} are the boundary, p_{12}, p_{23}, p_{34} are the bulk edges
- Note that each *bulk* edge in DEM maps to noise from two data qubits, each *boundary* edge only relates to one qubit's error
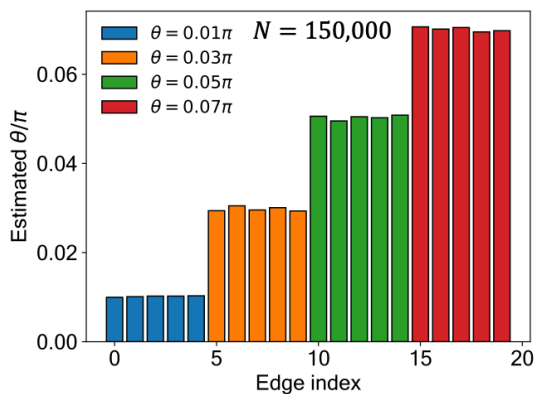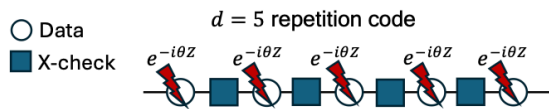
More complicated : X memory rotated surface Code

- *Insert my diagrams about the surface code*

Distance-3 Logical Qubit (a)

Parity Check (b)

$X1 \rightarrow D_0 D_1$
$X2 \rightarrow D_1 D_2 D_4 D_5$
$X3 \rightarrow D_3 D_4 D_6 D_7$
$X4 \rightarrow D_7 D_8$

$Z1 \rightarrow D_3 D_6$
$Z2 \rightarrow D_0 D_1 D_3 D_4$
$Z3 \rightarrow D_4 D_5 D_7 D_8$
$Z4 \rightarrow D_2 D_5$

Writeup

ESM circuit (c)

https://iopscience.iop.org/article/10.1088/2058-9565/accec5?utm_source=researchgate.net&utm_medium=article

- X^L and Z^L are our logical operators

- For the blue and red faces f, we have

$$A_f = \prod_{j \in \partial f} X_j, \; B_f = \prod_{j \in \partial f} Z_j; \quad S_f = A_f \quad \text{if } f \text{ blue}; \qquad S_f = B_f \quad \text{if } f \text{ red}$$

- Note that for the edge cases, ∂f still defines the qubits involved.

- Logical operators:

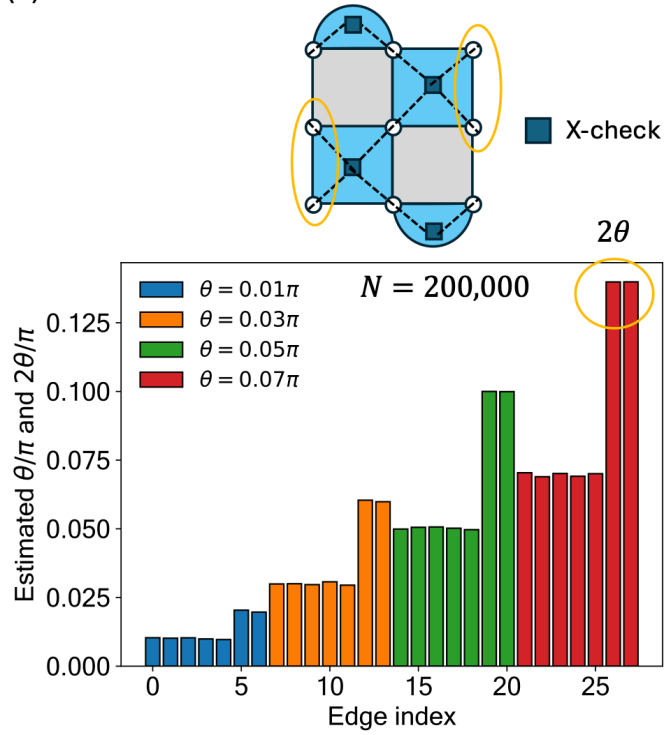$$X^L = \prod_{j \in \text{LEFT}} X_j; \quad Z^L = \prod_{j \in \text{TOP}} Z_j$$

- The DEM looks like this with 7 edges, 4 boundary, 3 bulk

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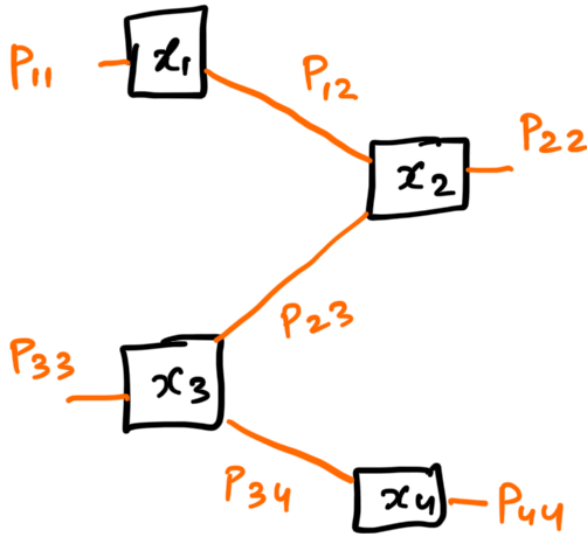
- For bulk edges, $N_d / N_e = 1$, for boundary $N_d / N_e = 2$.

- Coherent enhancement:

(b)

$d = 3$ rotated surface code

- Sample implementation: https://github.com/rootware-ionq/learn_coherent_DEM

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