Questions based on Regula-Falsi method

Ques: Find the real root of the equation $x\log_{10} x = 1.2$ by Regula-Falsi method, correct upto four decimal places.

Solution: The given equation is

$$x\log_{10} x - 1.2 = 0$$

Let $f(x) = x\log_{10} x - 1.2$ be the function such that $f(x) = 0$.

Since

$$f(2.74) = (2.74)\log_{10}(2.74) - 1.2 = -0.0005634 \quad (-\text{ ve})$$

$$f(2.75) = (2.75)\log_{10}(2.75) - 1.2 = 0.00816 \quad (+\text{ ve})$$

Hence, the root lies between 2.74 and 2.75.

So, taking $x_0 = 2.74$ and $x_1 = 2.75$, the first approximation to the root is

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$\Rightarrow \quad x_2 = \frac{(2.74)(0.00816) - (2.75)(-0.0005634)}{(0.00816) - (-0.0005634)}$$

$$\Rightarrow \quad x_2 = 2.7406$$

Now, $f(2.7406) = (2.7406)\log_{10}(2.7406) - 1.2 = -0.0000402 \quad (-\text{ ve})$

Thus, the root between 2.7406 and 2.75.

Therefore, taking $x_0 = 2.7406$ and $x_1 = 2.75$, so

$f(x_0) = -0.0000402$, $f(x_1) = 0.00816$.

The second approximation to the root is

$$x_3 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} \quad \Rightarrow$$

$$x_3 = \frac{(2.7406)(0.00816) - (2.75)(-0.0000402)}{(0.00816) - (-0.0000402)} \Rightarrow \quad x_3 = 2.740646$$

Hence, the approximate real root correct to four decimal places

is 2.7406

Ques: Find the real root of the equation $3x + \sin x - e^x = 0$ by Regula-Falsi method, correct upto four decimal places.

Solution: The given equation is

$$3x + \sin x - e^x = 0$$

Let $f(x) = 3x + \sin x - e^x$ be the function such that $f(x) = 0$.

Since

$$f(0.3) = 3(0.3) + \sin(0.3) - e^{0.3} = -0.154 \quad (-\text{ ve})$$

$$f(0.4) = 3(0.4) + \sin(0.4) - e^{0.4} = 0.0975 \quad (+\text{ ve})$$

Hence, the root lies between 0.3 and 0.4.

So, taking $x_0 = 0.3$ and $x_1 = 0.4$, the first approximation to the root is

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} \Rightarrow x_2 = \frac{(0.3)(0.0975) - (0.4)(-0.154)}{0.0975 - (-0.154)}$$

$$\Rightarrow \quad x_2 = 0.3612$$

Now, $f(0.3612) = 3(0.3612) + \sin(0.3612) - e^{0.3612} = 0.0019$ (+ ve)

Thus, the root lies between 0.3 and 0.3612. Therefore, taking $x_0 = 0.3$ and $x_1 = 0.3612$, so $f(x_0) = -0.154$, $f(x_1) = 0.0019$.

The second approximation to the root is

$$x_3 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} \Rightarrow x_3 = \frac{(0.3)(0.0019) - (0.3612)(-0.154)}{(0.0019) - (-0.154)} \Rightarrow$$

$$x_3 = 0.3604$$

Now, $f(0.3604) = 3(0.3604) + \sin(0.3604) - e^{0.3604} = -0.00005$ $(-\text{ ve})$

Thus, the root lies between 0.3604 and 0.3612. Therefore, taking $x_0 = 0.3604$ and $x_1 = 0.3612$, so $f(x_0) = -0.00005$, $f(x_1) = 0.0019$.

The third approximation to the root is

$$x_4 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$\Rightarrow x_4 = \frac{(0.3604)(0.0019) - (0.3612)(-0.00005)}{(0.0019) - (-0.00005)} \Rightarrow \quad x_4 = 0.36042$$

Hence, the approximate real root correct to four decimal places is 0.3604