Regula-Falsi Method (Method of False Position)

Let $f(x)=0$ be equation and $y=f(x)$ be continuous function between x_0 and x_1 such that $f(x_0)$ and $f(x_1)$ are of opposite signs *i.e.* $f(x_0)f(x_1)<0$. It means that the graph of $y=f(x)$ cuts the x-axis between $x=x_0$ and $x=x_1$. Thus, the root of equation $f(x)=0$ lies between x_0 and x_1. For definiteness, let $f(x_0)$ be $-ve$ and $f(x_1)$ be $+ve$.

Then, the first approximation to the root is

$$x_2 = x_0 - f(x_0)\frac{(x_1 - x_0)}{f(x_1) - f(x_0)}.$$

If $f(x_0)$ and $f(x_2)$ are of opposite sign, the root will lie between x_0 and x_2. Then, the second approximation to the root is

$$x_3 = x_0 - f(x_0)\frac{(x_2 - x_0)}{f(x_2) - f(x_0)}.$$

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$$x_{n+1} = x_{n-1} - f(x_{n-1})\frac{(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}.$$

Working methodology of Regula-Falsi method

Step I: For the given equation $f(x)=0$, define the continuous function $f(x)$.

Step II: Choose two points x_0 and x_1 such that $f(x_0)<0$ and $f(x_1)>0$.

Step III: The first approximation to the root is

$$x_2 = x_0 - f(x_0)\frac{(x_1 - x_0)}{f(x_1) - f(x_0)} \quad \text{Or} \quad x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

Step IV: Evaluate $f(x_2)$. If $f(x_2)<0$, then assign the value of x_2 to x_0

Or If $f(x_2)>0$, then assign the value of x_2 to x_1.

Step V: The second approximation to the root is

$$x_3 = x_0 - f(x_0)\frac{(x_1 - x_0)}{f(x_1) - f(x_0)}$$

Step VI: Repeat this process upto desired accuracy.

Ques: Find the real root of the equation $x^3 - 9x + 1 = 0$ by Regula-Falsi method, correct upto four decimal places.

Solution: The given equation is

$$x^3 - 9x + 1 = 0$$

Let $f(x) = x^3 - 9x + 1$ be the function such that $f(x) = 0$.

Since

$$f(2) = 2^3 - 9(2) + 1 = -9 \quad (-\text{ ve})$$

$$f(3) = 3^3 - 9(3) + 1 = 1 \quad (+\text{ ve})$$

Hence, the root lies between 2 and 3.

So, taking $x_0 = 2$ and $x_1 = 3$, the first approximation to the root is

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} \Rightarrow x_2 = \frac{2(1) - 3(-9)}{1 - (-9)} \Rightarrow x_2 = 2.9$$

Now, $f(2.9) = (2.9)^3 - 9(2.9) + 1 = -0.711$ $(-\text{ ve})$

Thus, the root between 2.9 and 3. Therefore, taking $x_0 = 2.9$ and $x_1 = 3$, so $f(x_0) = -0.711$, $f(x_1) = 1$.

The second approximation to the root is

$$x_3 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} \Rightarrow x_3 = \frac{(2.9)(1) - 3(-0.711)}{1 - (-0.711)} \Rightarrow x_3 = 2.9416$$

Now, $f(2.9416) = (2.9416)^3 - 9(2.9416) + 1 = -0.0207$ $(-\text{ ve})$

Thus, the root between 2.9416 and 3. Therefore, taking $x_0 = 2.9416$ and $x_1 = 3$, so $f(x_0) = -0.0207$, $f(x_1) = 1$.

The third approximation to the root is

$$x_4 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} \quad \Rightarrow \quad x_4 = \frac{(2.9416)(1) - 3(-0.0207)}{1 - (-0.0207)} \quad \Rightarrow$$

$$x_4 = 2.9428$$

Now, $f(2.9428) = (2.9428)^3 - 9(2.9428) + 1 = -0.0003$ (− ve)

Thus, the root between 2.9428 and 3. Therefore, taking $x_0 = 2.9428$ and $x_1 = 3$, so $f(x_0) = -0.0003$, $f(x_1) = 1$.

The fourth approximation to the root is

$$x_5 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} \quad \Rightarrow \quad x_5 = \frac{(2.9428)(1) - 3(-0.0003)}{1 - (-0.0003)} \quad \Rightarrow$$

$$x_5 = 2.942817$$

Hence, the approximate real root correct to four decimal places

is 2.9428