Questions based on Secant Method

Ques: Find the root of the equation $x^3 - 5x + 1 = 0$ in the interval $(0,1)$ using secant method, correct upto three decimal places.

Solution: The given equation is

$$x^3 - 5x + 1 = 0$$

Let $f(x) = x^3 - 5x + 1$ be the function such that $f(x) = 0$.

Taking the initial approximation as $x_0 = 0$ and $x_1 = 1$, then

$$f(x_0) = (0)^3 - 5(0) + 1 = 1$$

$$f(x_1) = (1)^3 - 5(1) + 1 = -3$$

By secant method, we have

$$x_{n+1} = x_n - f(x_n)\left[\frac{(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}\right] \quad \text{..................(1)}$$

For first approximation, taking $n = 1$, we have

$$x_2 = x_1 - f(x_1)\left[\frac{(x_1 - x_0)}{f(x_1) - f(x_0)}\right]$$

$$\Rightarrow \; x_2 = 1 - (-3)\left[\frac{(1-0)}{(-3-1)}\right] \quad \Rightarrow \quad x_2 = 0.25$$

Now, $f(0.25) = (0.25)^3 - 5(0.25) + 1 = -0.234375$

For second approximation, taking $n = 2$, we have

$$x_3 = x_2 - f(x_2)\left[\frac{(x_2 - x_1)}{f(x_2) - f(x_1)}\right]$$

$$\Rightarrow \; x_3 = 0.25 - (-0.234375)\left[\frac{(0.25-1)}{(-0.234375+3)}\right]$$

$$\Rightarrow \quad x_3 = 0.186441$$

Now, $f(0.186441) = (0.186441)^3 - 5(0.186441) + 1 = 0.074276$

For third approximation, taking $n = 3$, we have

$$x_4 = x_3 - f(x_3)\left[\frac{(x_3 - x_2)}{f(x_3) - f(x_2)}\right]$$

$$\Rightarrow \; x_4 = 0.186441 - (0.074276)\left[\frac{(0.186441-0.25)}{(0.074276+0.234375)}\right]$$

$$\Rightarrow \quad x_4 = 0.201736$$

Now,

$$f(0.201736) = (0.201736)^3 - 5(0.201736) + 1 = -0.000470$$

For fourth approximation, taking $n = 4$, we have

$$x_5 = x_4 - f(x_4)\left[\frac{(x_4 - x_3)}{f(x_4) - f(x_3)}\right]$$

$$\Rightarrow \; x_5 = 0.201736 - (-0.000470)\left[\frac{(0.201736-0.186441)}{(-0.000470-0.074276)}\right]$$

$$\Rightarrow \quad x_5 = 0.201640$$

Hence, the approximate root correct upto four decimal places is 0.201

Ques: Find the root of the equation $x - e^{-x} = 0$ using secant method, correct upto three decimal places.

Solution: The given equation is

$$x - e^{-x} = 0$$

Let $f(x) = x - e^{-x}$ be the function such that $f(x) = 0$.

Taking the initial approximation as $x_0 = 0$ and $x_1 = 1$, then

$$f(x_0) = 0 - e^{0} = -1$$

$$f(x_1) = 1 - e^{-1} = 0.6321$$

By secant method, we have

$$x_{n+1} = x_n - f(x_n)\left[\frac{(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}\right] \qquad \text{..................(1)}$$

For first approximation, taking $n = 1$, we have

$$x_2 = x_1 - f(x_1)\left[\frac{(x_1 - x_0)}{f(x_1) - f(x_0)}\right]$$

$$\Rightarrow \; x_2 = 1 - (0.6321)\left[\frac{(1-0)}{(0.6321+1)}\right] \quad \Rightarrow \quad x_2 = 0.6127$$

Now, $f(0.6127) = 0.6127 - e^{-0.6127} = 0.0708$

For second approximation, taking $n = 2$, we have

$$x_3 = x_2 - f(x_2)\left[\frac{(x_2 - x_1)}{f(x_2) - f(x_1)}\right]$$

$$\Rightarrow \; x_3 = 0.6127 - (0.0708)\left[\frac{(0.6127-1)}{(0.0708-0.6321)}\right]$$

$$\Rightarrow \quad x_3 = 0.5639$$

Now, $f(0.5639) = 0.5639 - e^{-0.5639} = -0.0051$

For third approximation, taking $n = 3$, we have

$$x_4 = x_3 - f(x_3)\left[\frac{(x_3 - x_2)}{f(x_3) - f(x_2)}\right]$$

$$\Rightarrow x_4 = 0.5639 - (-0.0051)\left[\frac{(0.5639 - 0.6127)}{(-0.0051 - 0.0708)}\right]$$

$$\Rightarrow \quad x_4 = 0.5672$$

Now, $f(0.5672) = 0.5672 - e^{-0.5672} = 0.0001$

For fourth approximation, taking $n = 4$, we have

$$x_5 = x_4 - f(x_4)\left[\frac{(x_4 - x_3)}{f(x_4) - f(x_3)}\right]$$

$$\Rightarrow x_5 = 0.5672 - (0.0001)\left[\frac{(0.5672 - 0.5639)}{(0.0001 + 0.0051)}\right]$$

$$\Rightarrow \quad x_5 = 0.5670$$

Hence, the approximate root correct upto four decimal places is 0.567