

Questions based on bisection method

Ques: Find the real root of the equation $x \log_{10} x = 1.2$ by bisection method, correct upto three decimal places.

Solution: The given equation is

$$x \log_{10} x - 1.2 = 0$$

Let $f(x) = x \log_{10} x - 1.2$ be the function such that $y = f(x) = 0$.

Since

$$f(2.74) = 2.74 \log_{10} 2.74 - 1.2 = -0.0005 \quad (- \text{ve})$$

$$f(2.75) = 2.75 \log_{10} 2.75 - 1.2 = 1.1106 \quad (+ \text{ve})$$

Hence, the root lies 2.74 and 2.75.

So, taking $a = 2.74$ and $b = 2.75$, the first approximation to the root is

$$x_1 = \frac{a+b}{2} \Rightarrow x_1 = \frac{2.74+2.75}{2} \Rightarrow x_1 = 2.745$$

$$\text{Now, } f(2.745) = 2.745 \log_{10} 2.745 - 1.2 = 0.0037 \quad (+ \text{ve})$$

Hence, the root lies between 2.74 and 2.745. So, the second approximation to the root is

$$x_2 = \frac{2.74+2.745}{2} \Rightarrow x_2 = 2.7425$$

$$\text{Now, } f(2.7425) = 2.7425 \log_{10} 2.7425 - 1.2 = 0.00161 \quad (+ \text{ve})$$

Hence, the root lies between 2.74 and 2.7425. So, the third approximation to the root is

$$x_3 = \frac{2.74 + 2.7425}{2} \Rightarrow x_3 = 2.74125$$

$$\text{Now, } f(2.74125) = 2.74125 \log_{10} 2.74125 - 1.2 = 0.00052 \quad (+ \text{ve})$$

Hence, the root lies between 2.74 and 2.74125. So, the fourth approximation to the root is

$$x_4 = \frac{2.74 + 2.74125}{2} \Rightarrow x_4 = 2.740625$$

$$\text{Now, } f(2.740625) = 2.740625 \log_{10} 2.740625 - 1.2 = -0.000018 \quad (- \text{ve})$$

Hence, the root lies between 2.740625 and 2.74125. So, the fifth approximation to the root is

$$x_5 = \frac{2.740625 + 2.74125}{2} \Rightarrow x_5 = 2.7409375$$

$$\text{Now, } f(2.7409375) = 2.7409375 \log_{10} 2.7409375 - 1.2 = 0.00025 \quad (+ \text{ve})$$

Hence, the approximate real root correct to three decimal places is 2.740

➤ Bisection method is always convergent.

➤ Bisection method converges linearly.

Or

Bisection method converges linearly.

Or

The order of convergence of bisection method is 1.