Secant Method

Secant method is an improved version of Regula-Falsi method, as it does not require the condition that $f(x_0)f(x_1)<0$.

Taking x_0 and x_1 as initial limits of interval, the first approximation to the root is

$$x_2 = x_1 - f(x_1)\left[\frac{(x_1 - x_0)}{f(x_1) - f(x_0)}\right]$$

The second approximation to the root is

$$x_3 = x_2 - f(x_2)\left[\frac{(x_2 - x_1)}{f(x_2) - f(x_1)}\right]$$

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In general, the successive approximation to the root is

$$x_{n+1} = x_n - f(x_n)\left[\frac{(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}\right]$$

Remarks:

- In this method, $f(x_0)$ and $f(x_1)$ are not necessarily of apposite sign.
- In this method, it is not necessary that the interval (x_0, x_1) must contain the root.
- If at any approximation, $f(x_n) = f(x_{n-1})$, this method fails and shows that it does not converges always.

- If the secant method converges, its rate of convergence is faster than that of the Regula-Falsi method.
- The order of convergence of secant method is 1.62

Ques: Determine the root of the equation $\cos x - xe^x = 0$ using secant method, correct upto four decimal places.

Solution: The given equation is

$$\cos x - xe^x = 0$$

Let $f(x) = \cos x - xe^x$ be the function such that $f(x) = 0$.

Taking the initial approximation as $x_0 = 0$ and $x_1 = 1$, then

$$f(x_0) = \cos 0 - 0 = 1$$

$$f(x_1) = \cos 1 - e^1 = -2.1780$$

By secant method, we have

$$x_{n+1} = x_n - f(x_n)\left[\frac{(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}\right] \qquad \text{..........................(1)}$$

For first approximation, taking $n = 1$, we have

$$x_2 = x_1 - f(x_1)\left[\frac{(x_1 - x_0)}{f(x_1) - f(x_0)}\right]$$

$$\Rightarrow x_2 = 1 - (-2.1780)\left[\frac{(1-0)}{(-2.1780 - 1)}\right] \quad \Rightarrow \quad x_2 = 0.3147$$

Now, $f(0.3147) = \cos(0.3147) - 0.3147e^{0.3147} = 0.5198$

For second approximation, taking $n=2$, we have

$$x_3 = x_2 - f(x_2)\left[\frac{(x_2 - x_1)}{f(x_2) - f(x_1)}\right]$$

$$\Rightarrow \quad x_3 = 0.3147 - (0.5198)\left[\frac{(0.3147-1)}{(0.5198+2.1780)}\right] \quad \Rightarrow \quad x_3 = 0.4467$$

Now, $f(0.4467) = \cos(0.4467) - 0.4467e^{0.4467} = 0.2036$

For third approximation, taking $n=3$, we have

$$x_4 = x_3 - f(x_3)\left[\frac{(x_3 - x_2)}{f(x_3) - f(x_2)}\right]$$

$$\Rightarrow \quad x_4 = 0.4467 - (0.2036)\left[\frac{(0.4467-0.3147)}{(0.2036-0.5198)}\right] \quad \Rightarrow \quad x_4 = 0.5318$$

Now, $f(0.5318) = \cos(0.5318) - 0.5318e^{0.5318} = -0.0432$

For fourth approximation, taking $n=4$, we have

$$x_5 = x_4 - f(x_4)\left[\frac{(x_4 - x_3)}{f(x_4) - f(x_3)}\right]$$

$$\Rightarrow \quad x_5 = 0.5318 - (-0.0432)\left[\frac{(0.5318-0.4467)}{(-0.0432-0.2036)}\right] \quad \Rightarrow$$

$$x_5 = 0.5168$$

Now, $f(0.5168) = \cos(0.5168) - 0.5168e^{0.5168} = 0.0029$

For fifth approximation, taking $n=5$, we have

$$x_6 = x_5 - f(x_5)\left[\frac{(x_5 - x_4)}{f(x_5) - f(x_4)}\right]$$

$$\Rightarrow \; x_6 = 0.5168 - (0.0029)\left[\frac{(0.5168 - 0.5318)}{(0.0029 + 0.0432)}\right] \quad \Rightarrow \quad x_6 = 0.5177$$

Now, $f(0.5177) = \cos(0.5177) - 0.5177e^{0.5177} = 0.0002$

For sixth approximation, taking $n = 6$, we have

$$x_7 = x_6 - f(x_6)\left[\frac{(x_6 - x_5)}{f(x_6) - f(x_5)}\right]$$

$$\Rightarrow \; x_7 = 0.5177 - (0.0002)\left[\frac{(0.5177 - 0.5168)}{(0.0002 - 0.0029)}\right] \quad \Rightarrow \quad x_7 = 0.51776$$

Hence, the approximate root correct upto four decimal places is 0.5177