Optimal Adaptive Neuro-Fuzzy Control of COVID-19 Outbreak Based on Genetic Algorithm

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***Abstract*— In this article, we designed an optimal ANFIS controller in order to control the novel Coronavirus disease 2019 (COVID-19). The nonlinear model of COVID-19 contains eight state variables which are Susceptible, Infected, Exposed, Quarantined, Hospitalized, Recovered, Insusceptible, and Deceased population. We considered the Vaccination rate of Susceptible class, Social distancing, and Treatment rate of the Infected population as the control inputs of the system. After that, we implemented the Genetic algorithm to create the optimal control values. By repeating this process for some different initial conditions, we created some set of necessary data, which include the values of state variables and their corresponding control values. Having created these data, we trained the ANFIS structure to determine the coefficients of Takagi-Sugeno fuzzy Membership functions, and finally we used this trained Network as the system controller. The results show the prodigious effect of the proposed controller for decreasing the population of Infected and Deceased about 85% and 99% at the end of the day 50, respectively. Also the Exposed and the Susceptible classes decreased about 95% and 93% with the presence of the controller, respectively. Thus, the proposed Controller works propitiously for our purposes.**

Keywords*- *COVID-19, ANFIS Controller, Genetic Algorithm, Optimization

1. Introduction

Coronavirus disease 2019 (COVID-19) is a contagious disease caused by severe acute respiratory syndrome Coronavirus 2 (SARS-CoV-2) keeping on spreading in the World [1]. The first case was identified in Wuhan, the capital of China, and then permeated in the whole country [2]. The first pandemic of twenty-first century vehemently spread [3] and the governmental agencies were compelled to apply lockdown and social distancing [1]. Till December 5, 2021 the number of confirmed cases has reached 266,035,254 according to WHO.

In order to study the dynamic behavior of this disease, many mathematical models have been introduced. The initial ones have been developed [4] and were extended to more generalized ones [5-7]. They give a lucrative and an inherent insight to deal with such diseases. Many models are linked with data via statistical methods and they can be described as a system of linear or nonlinear, ordinary or partial differential equations [8,9]. In relation with vaccination strategy, we have taken into account a SEIQHRDP model [10] simultaneously with control of COVID-19 without quarantine.

The main goal of this study is to reduce the number of Infected, Exposed, Susceptible, and more importantly deceased people by using the minimum of three main control inputs which are vaccination, treatment, and social distancing, respectively. So we implemented the optimal ANFIS controller capable of reaching us to our desired purpose. ANFIS is a network-based structure able to combine both Neural Networks and Fuzzy Inference systems to obtain Takagi-Segno optimization parameters [11-13]. The Genetic Algorithm (GA) is an optimization algorithm, which belongs to the larger part of evolutionary algorithms. It is based on Natural Selection Theory which implies finding the best (i.e. Optimal) answer to a particular problem. In this study, we implemented the GA to find the optimal control efforts in relation with an integral objective function.

This article has been classified into following sections. Section 2 explains the nonlinear dynamic of the system based on a system of differential equations. Section 3 explains Genetic Algorithm for finding the values of control inputs. Section 4 has been created to describe the ANFIS controller corresponding to its trained structure. Section 5 scrutinizes the simulation results, and finally section 6 summarizes conclusions and remarks.

2. Mathematical Model

Here in this section, a nonlinear epidemiological model of COVID-19 has been introduced based on the general SEIR model with eight time-varying states containing Susceptible (S(t)), Exposed (E(t)), Infected (I(t)), Quarantined (Q(t)), Hospitalized (H(t)), Recovered (R(t)), Insusceptible (P(t)), and finally Deceased population (D(t)). As stated above, eight state variables have been considered to model this pandemic. The Susceptible class penetrates the Exposed group when the Infected class makes contact with them at a rate of β. Moreover, cancellation of public meetings, and wearing face masks will result in great decrease of the Susceptible population at a rate of α. Also, the Exposed population joins the Infected class with the rate of γ. Additionally, the Infected group penetrates the quarantined class with a rate of δ. In addition, the Hospitalized individuals move to recovered population with a rate of ζ, but the rest enters the deceased population with mortality rate of η. Finally, the Quarantined individuals either recover from the disease with a rate of λ or does lose their lives with a rate of κ, and joins the Deceased class. The conceptual flow diagram of this model has been shown in [10].

As mentioned before, in this article we consider a single control scenario by implying 3 control inputs to the system. The control input $\upsilon(t)$ indicates the vaccination rate of susceptible class during treatment period. Also, the control input $\sigma(t)$ shows the reduction rate in contact between susceptible and infected group. Moreover, the control input $\tau(t)$ denotes the hospitalization and treatment rate of infected individuals. The nonlinear system of differential equations of the system then without any control input signal is as follows [10]:

$$\begin{aligned}
\frac{dS(t)}{dt} &= -\beta S(t)I(t) - \alpha S(t) \\
\frac{dE(t)}{dt} &= \beta S(t)I(t) - \gamma E(t) \\
\frac{dI(t)}{dt} &= \gamma E(t) - \delta I(t) \\
\frac{dQ(t)}{dt} &= \delta I(t) - \lambda Q(t) - \kappa Q(t) \\
\frac{dH(t)}{dt} &= -\zeta H(t) - \eta H(t) \\
\frac{dR(t)}{dt} &= \lambda Q(t) + \zeta H(t) \qquad (1) \\
\frac{dD(t)}{dt} &= \kappa Q(t) + \eta H(t)
\end{aligned}$$

$$\frac{dP(t)}{dt} = \alpha S(t)$$

What we are trying to do achieve is control the system with the control inputs. By penetrating these signals into the system equations we will have [10]:

$$\frac{dS(t)}{dt} = -\beta S(t)I(t)(1 - \sigma(t)) - \alpha S(t) - \upsilon(t)S(t)$$

$$\frac{dE(t)}{dt} = \beta S(t)I(t)(1 - \sigma(t)) - \gamma E(t)$$

$$\frac{dI(t)}{dt} = \gamma E(t) - \delta I(t) - \tau I(t)$$

$$\frac{dQ(t)}{dt} = \delta I(t) - \lambda Q(t) - \kappa Q(t) \qquad (2)$$

$$\frac{dH(t)}{dt} = \tau(t)I(t) - \zeta H(t) - \eta H(t)$$

$$\frac{dR(t)}{dt} = \lambda Q(t) + \upsilon(t)S(t) + \zeta H(t)$$

$$\frac{dD(t)}{dt} = \kappa Q(t) + \eta H(t)$$

$$\frac{dP(t)}{dt} = \alpha S(t)$$

3. Optimal Dataset Generation

From the beginning of the year 1950, efforts have been taken in order to simulate evolutions over computers. After that, in the earliest 1970s, John Holland, an Australian-physicist, Introduced Genetic Algorithm as a public tool for optimization. The Genetic Algorithm (GA) is an optimization and search technique based on the principles of genetics and natural selection. In this study, The GA is applied to minimize the following cost function:

$$J = \int_0^t (\psi(S - S_d)^2 + (E - Ed)\hat{\ }2 + (I - I_d)^2 + (D - D_d)^2 + \pi u^2)dt \qquad (3)$$

Where ψ and π are constant coefficients with which we can make the order of each term the same as each other. The cost function has been considered to minimize deviation of susceptible, exposed, and infected classes from their desired reduction functions which are:

$$\begin{aligned} S_d &= S_0 e^{-at} \\ E_d &= E_0 e^{-at} \\ I_d &= I_0 e^{-at} \\ D_d &= D_0 e^{-at} \end{aligned} \qquad (4)$$

Where S_0, E_0, I_0, and D_0 are the initial values of the optimal states, and a is the exponential coefficient equal to 0.06, respectively.

The GA makes initial population for control inputs randomly, and checks their cost function values based on objective function (3). In the next step of the algorithm, the initial population will be sorted, and at least half of the sorted population or more, with higher values of the cost function will be discarded. After that, the two parents will produce some new offspring, and the GA calculates their cost values. Then, the selected offspring will be replaced by the discarded population. Finally, the new population will be sorted. This process will carry on until the convergence criterion is passed.

As stated above, the GA selects optimum control inputs for each day of simulation. What we are trying to do achieve is find the optimal control values for each day. In order for the GA to do that, we first define a distinct number of initial population—which is an empty structure containing all control inputs and their cost function values for all simulation days. But Instead of creating optimal data step by step, and day by day, we considered a more accurate method. At first, the GA creates a matrix of control input values which is of size of the number of the initial population multiplies by number of required simulation days. Then, the GA begins the optimization process and creates all control values for all required days of simulation. Indeed, for all time steps, control inputs are set for the system and the GA keeps on selecting the optimum values based on minimizing the objective function. As a matter of fact, this method is the same as solving the system incrementally, but this works efficiently rather than solving the system step by step from the very beginning. Consequently, the sorted matrix will be created, and the first row is the optimum solution for the problem. Then these control inputs will be reshaped into a matrix of size of three multiplies by simulation days. Finally, these optimum control inputs will be given to the system to solve the nonlinear dynamic with Fourth Order Runge-Kutta in order to obtain all output states. Therefore, the optimal data set will be created for training the ANFIS structure.

For more accuracy, we need more data for training. By repeating the above process for different initial conditions, more data set will be on hand. As a result, by juxtaposing the created dataset of different initial conditions, the ANFIS structure will be trained to tune Takagi-Sugeno parameters of membership functions.

4. ANFIS Controller

Neuro-Fuzzy systems mostly contain five essential layers discussed in [13]:

- ❖ Fuzzification
- ❖ Implication
- ❖ Normalization
- ❖ Aggregation
- ❖ Defuzzification

As Neural networks can learn from the input data and fuzzy systems can apply the human knowledge for creating fuzzy rules, an Adaptive Neuro-Fuzzy system can learn while approximating nonlinear functions with their desired parameters.

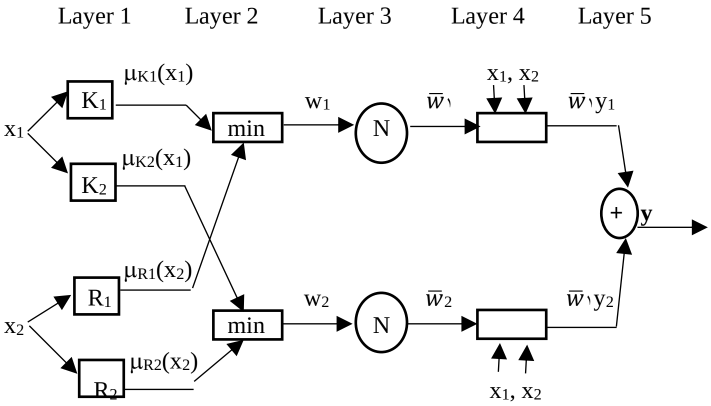


Fig. 2. ANFIS structure

A Takagi-Sugeno structure has been shown in Fig. 2 with two inputs, and one output. Fuzzy Logic "If-then" rules then will be of form:

$$If\ x1\ is\ K1\ and\ x2\ is\ R1\ then\ y1 = a1x1 + b1x2 + c1 \quad (5)$$

$$If\ x1\ is\ K2\ and\ x2\ is\ R2\ then\ y2 = a2x1 + b2x2 + c2 \quad (6)$$

Where a_i, b_i, and c_i are coefficients of Takagi-Sugeno output hyperplane, and $\mu(x_i)$ is the Membership function for the ith input, respectively. The output will then be calculated using Weighted Average defuzzification.

$$y = \bar{w}_1 y1 + \bar{w}_2 y2 \qquad (7)$$

$$\text{where } \bar{w}_1 = \frac{w1}{w1+w2}, \bar{w}_2 = \frac{w2}{w1+w2} \qquad (8)$$

5. Simulation Results

This section indicates the recommended method results simulated in MATLAB R2021. The obtained results show the incredibly significant effectiveness of the proposed controller. According to the nonlinear system without any control strategy, the Infected population will increase in the very first days, but it will decrease through each day passes. Furthermore, the deceased class will dramatically escalate from the very first days. Correspondingly, the exposed group will reach to its maximum value when the Susceptible class is decreasing exponentially. In return, with the presence of the ANFIS controller, the Infected class will dwindle in population exponentially without having any maximum pick value. According to the vaccination of the Susceptible class, the Deceased population will considerably decrease in comparison with the case of no control. On the other hand, the Exposed class will approximately follow its desired exponential function while the Susceptible group has a similar behavior to exposed population.

Fig. 3 depicts the behavior of control inputs for 50 days. Fig. 4 describes the control states discussed above. Also the initial and final condition of all states are shown in Table 1 for 50 days.

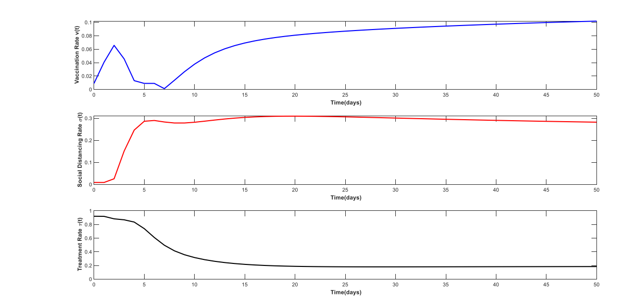
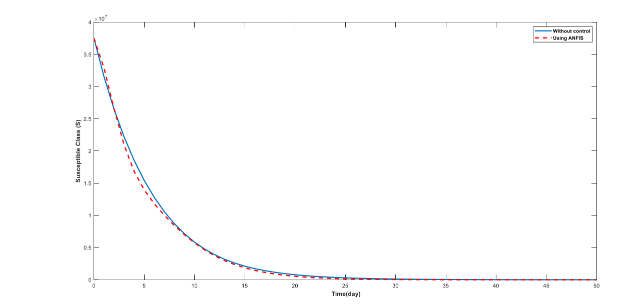
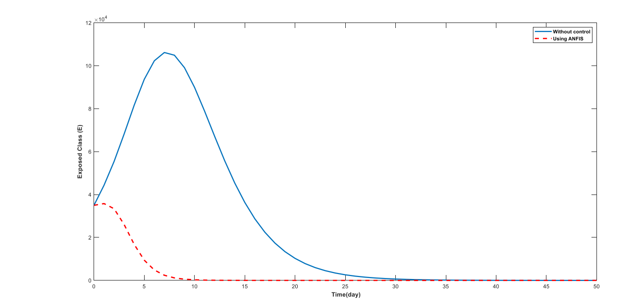


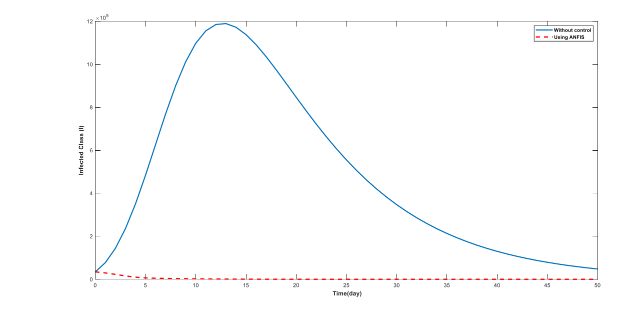
Fig. 3. Control inputs



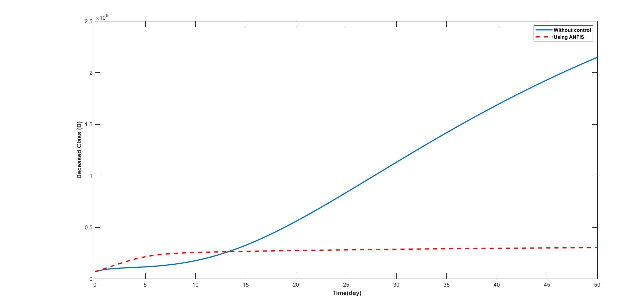
(a)



(b)



(c)



(d)

Fig. 4. (a) Susceptible S, **(b)** Exposed E, **(c)** Infected I, and **(d)** Deceased groups with and without control.

Class Name	Initial conditions of all states	Final values without control	Final values With control
S	37500000	4245	245
E	35000	0	0
I	34500	52970	0
H	40000	1055800	15160
Q	40000	0	0
R	47000	1337610	4221880
D	7000	210810	30210
P	37500	35079550	33473500

Table 1. Initial and final values of all classes in conjunction with the case of no control and control for 50 days

Vaccination of susceptible group plays a dominant role in controlling the epidemic of COVID-19. Actually, the integration of the term $\upsilon(t)S(t)$ over a distinct time period shows the number of susceptible people who get vaccinated. Also the integration of other two terms $\sigma(t)SI$ and $\tau(t)I$ shows the number of individuals who should keep their physical distance in order not to spread the virus, and those who should be hospitalized for the treatment, respectively.

Fig. 5 describes the behavior of all states for 50 days of simulation. In the absence of control, the Quarantined class will increase in population because of severity of the epidemic in the very first days. As the social distancing and vaccination rate penetrates to control the epidemic, the Quarantined class will dwindle day by day considerably. Also the Hospitalized and Insusceptible classes behave similarly for both case of controlled and uncontrolled system. Additionally, the Recovered class decreases for first days but it will increase after about 14 days. The reason is that the recovery period length of COVID-19 is approximately 2 or 3 weeks. After that, by passing days, the more people get vaccinated the better the pandemic can be controlled.

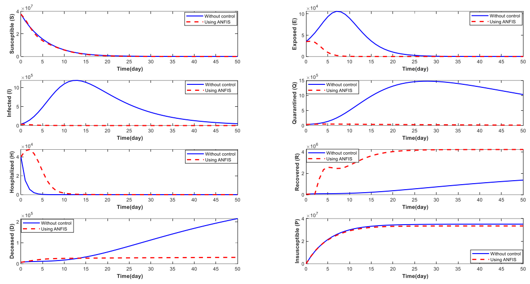


Fig. 5. All output states simulated for 50 days

6. CONCLUSIONS

In this study, we developed an intelligent controller in relation with controlling the widely-spread COVID-19. Although the uncontrolled system could spontaneously manage to control the epidemic, it took at least a hundred days. Nevertheless, the ANFIS controlled the epidemic way better in conjunction with reducing the maximum peak of the Infected, and the deceased population. The reason is that vaccination of the Susceptible class has a highly significant impact on control of COVID-19. Additionally, the obtained results and the proposed controller could better control the epidemic in comparison with [**11**]. because of the modified optimization process we mentioned in section 3. Furthermore, because the upper bound for the vector of initial conditions should not exceed the total number of population, we considered dividing it by *c* which indeed is a real number between one and two. Also we have taken the initial conditions and system parameters from [10]. Also, in this article we controlled the Deceased population in comparison with [**11**] in which it has not been controlled.

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