

# Capital Cost Evaluation for Optimum Process Design of Cryogenic Air Separation

Diploma Thesis  
by  
Robert Pack

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# Nomenclature

$\dot{e}_{ij}$	Energy flow of to equipment $i$ from energy carrier $j$	$[kW]$
$\dot{m}_i$	Mass flow of component $i$	$[\frac{kg}{s}]$
$a$	Annuity	$[\$]$
$C_B^i$	Reference cost of equipment $i$	$[\$]$
$C_E^i$	Cost of equipment $i$	$[\$]$
$C_{EC}^i$	Mass specific cost of energy carrier $i$	$[\frac{\$}{kW}]$
$C_{RM}^i$	Mass specific cost of raw material $i$	$[\frac{\$}{kg}]$
$C_0$	Initial value of an investment	$[\$]$
$C_1$	Reference equipment cost at time 1	$[\$]$
$C_2$	Reference equipment cost at time 2	$[\$]$
$C_n$	Final value of an investment	$[\$]$
$C_P$	Total process cost	$[\$]$
$C_P^0$	Reference process cost	$[\$]$
$C_{0e}$	Present value of all expenses	$[\$]$
$C_{0r}$	Present value of all revenues	$[\$]$
$C_{EC}$	Total cost of energy	$[\$]$
$C_{FILL}$	Cost of raw materials to fill the process	$[\$]$
$C_{RM}$	Total cost of raw materials	$[\$]$
$f_C^i$	Design complexity correction to equipment cost	$[-]$
$f_M^i$	Material selection correction to equipment cost	$[-]$

## Nomenclature

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$f_P^i$	Pressure correction to equipment cost	[—]
$f_T^i$	Temperature correction to equipment cost	[—]
$F_j^L$	Liquid feed to tray $j$	$[\frac{mol}{s}]$
$F_j^V$	Vapour feed to tray $j$	$[\frac{mol}{s}]$
$i$	Interest rate	[%]
$I_1$	Cost index at time 1	[—]
$I_2$	Cost index at time 2	[—]
$L_j$	Liquid flow from tray $j$	$[\frac{mol}{s}]$
$M^i$	Equipment specific factor	[—]
$N_E$	Number of equipment pieces in the process	[—]
$N_{RM}$	Number of raw materials	[—]
$NPV$	Net present value	[\$]
$POT$	Payout time	[\$]
$q$	Interest factor	[—]
$Q^i$	Specific quantity for equipment $i$	[variable]
$Q_B^i$	Equipment specific reference quantity	[variable]
$Q_P$	Process capacity	$[\frac{kg}{h}]$
$Q_P^0$	Reference process capacity	$[\frac{kg}{h}]$
$S_j^L$	Liquid side flow from tray $j$	$[\frac{mol}{s}]$
$S_j^V$	Vapour side flow from tray $j$	$[\frac{mol}{s}]$
$t_{op}$	Time of process operations	[s]
$V_j$	Vapour flow from tray $j$	$[\frac{mol}{s}]$
$x$	Degression coefficient	[—]

# 1 Introduction

Air separation technology, or – more generally speaking – gas separation technology, lies at the heart of the modern process industry. Highly pure oxygen and nitrogen are used in many industrial applications. Modern power generation processes, such as the currently developed OXICOAL process, rely on incineration with pure oxygen to produce flue gases with very high carbon dioxide content for further storage. Nitrogen is essential to many widely used processes such as the production of ammonia in the Haber-Bosch synthesis, as fertilizer or in many organic reactions.

## 2 Air separation technology

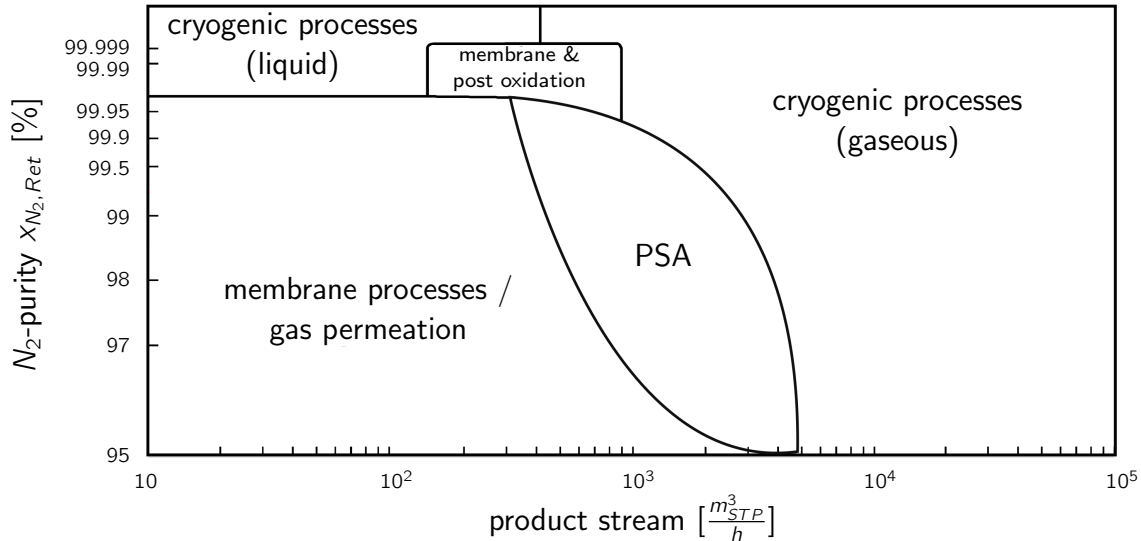


Figure 2.1: Comparison of Air Separation Technologies [4].

There are several ways besides cryogenic air separation that can be employed to separate gas mixtures. In this chapter different competing technologies and their main applications will be discussed. The predominately used technologies are cryogenic distillation, pressure swing adsorption (PSA) as well as gas permeation (GP). In the distillation process the gas is first liquefied. Separation is achieved by the different concentration differences in vapor and liquid phase. PSA relies on the different affinities of gaseous species to adsorb to certain materials in order to extract a component from a mixture. During gas permeation membranes are used. Each species migrates in different quantities through a given membrane depending on process parameters and membrane structure.

fig. 2.1 illustrates the most economically viable processes depending on product purity and product stream volume. It can be seen that alternative air separation processes cannot supply the high quality or quantity of the cryogenic process. Due to that cryogenic air separation is thought to be the main supplier of highly pure gases in industrial quantities for years to come [1]. The alternative processes however offer some very appealing characteristics, which make them the favorable choice when lower quantities of product or more moderate purity is required. The cryogenic process is always connected with a considerable energy consumption for the liquefaction and compression. Due to that smaller implementations of the process are very unlikely to yield economically sound solutions to a separation problem.

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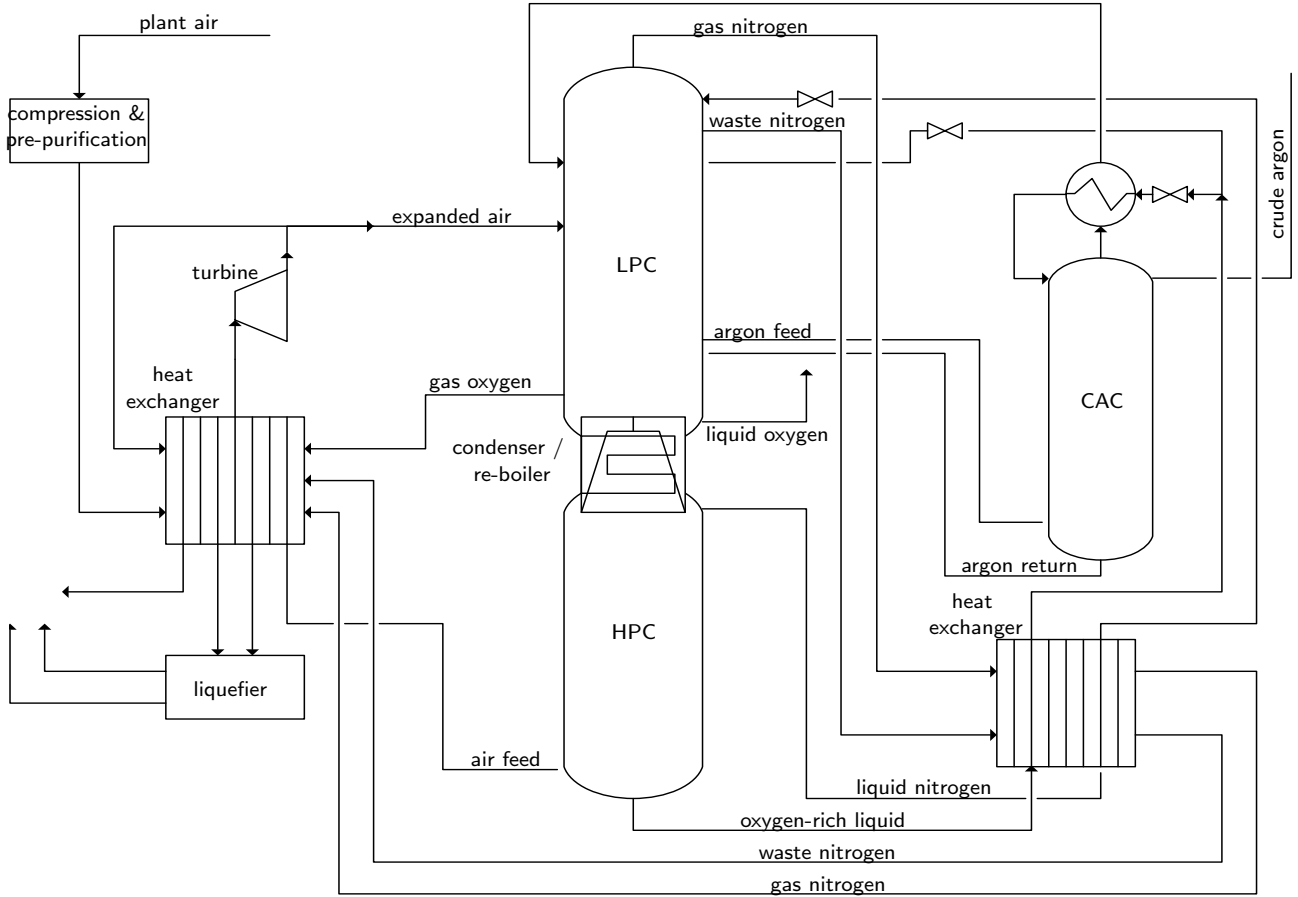


Figure 2.2: Schematic representation of the cryogenic air separation process.

## 2.1 Cryogenic air separation

Cryogenic Air Separation finds applications over a great variety of industries among others refining, petrochemicals, medical, food & beverages and environmental [5]. Furthermore prospective processes for power generation from fossil sources in form of the integrated gaseous combined cycle (IGCC) integrates the air separation process in order to enable more environmentally friendly power generation [2].

As can be seen in fig. 2.2 double effect heat integrated distillation column lies at the heart of the air liquefaction processes. It consists of a high pressure column (HPC) operating at  $0.68 \text{ MPa}$  and temperatures below  $130 \text{ K}$  as well as a low pressure column (LPC) which operates at around  $0.13 \text{ MPa}$  and comparable temperatures. In order to also attain highly pure argon as a product the process may also include a crude argon column (CAC) which works at slightly lower pressures than the LPC.

The plant air entering the process is initially purified, where carbon and nitrogen oxides as well as solid contaminants are removed, and then compressed to process conditions. The compressed air is then cooled against product streams namely liquefied nitrogen, oxygen and argon. The air stream is then divided into several sub-streams. One of those is fed into the HPC bottom, while another is expanded by means of a turbine and further cooled down through the Joule- Thompson effect. Aside from further cooling energy from the initial compression is thus partially recovered. This expanded air stream is then fed into the LPC. At the bottom of the LPC liquid as well as gaseous oxygen are

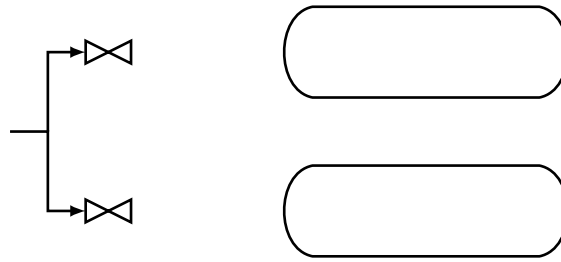


Figure 2.3: Schematic representation of the PSA process.

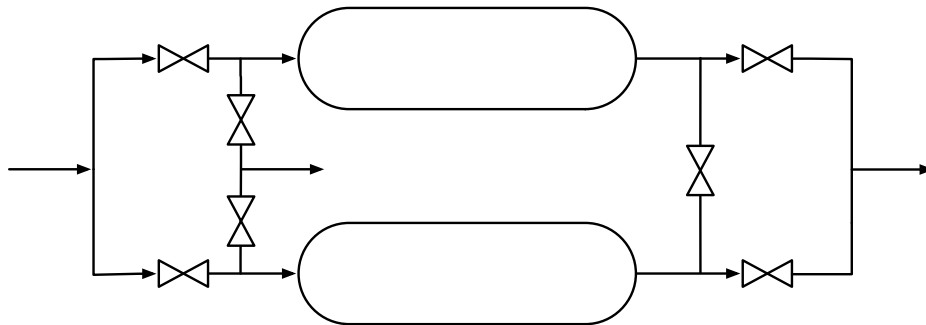


Figure 2.4: Schematic representation of the PSA process.

recovered as desired products. The bottom and top streams from the HPC are made up of an oxygen rich liquid as well as liquid nitrogen. The liquid nitrogen stream is led through a heat exchanger and fed as reflux into the top of the LPC. The bottom stream is, after heat integration, partially fed into the LPC as well as CAC. From the lower part of the LPC a side stream is drawn and led into the bottom of the CAC. At the same point the reflux from the CAC is fed back into the LPC [6].

## 2.2 Pressure swing adsorption

Pressure Swing Adsorption has been employed to separate gaseous mixtures for some time. During the 80's and 90's commercial applications for the production of oxygen or nitrogen have gained more and more attention. Especially the ability to construct very compact units the size of a briefcase, have led to the implementation of PSA processes for treatment of asthma patients or other medical appliances. But also larger scale plants have successfully been utilized, for example in the paper industry during the de-lignation of pulp. It remains true however, that for large scale industrial settings with high product quality demands, cryogenic separation remains the most viable alternative.

Separation is achieved during the PSA process by adsorption of one component in the mixture to a given bed. Once the bed is saturated with the adsorbing species, it has to be regenerated in order to continue production. The ability to adsorb a certain species is dependent on the system pressure. At higher pressures more gas can be adsorbed than at lower pressures. Thus by reducing the pressure in the reaction vessel, the Adsorbent can be regenerated.

In order to avoid non-continuous processes, two or more reaction vessels are employed. Therefore the saturated vessel can be regenerated, while the other one continues production. By alternating

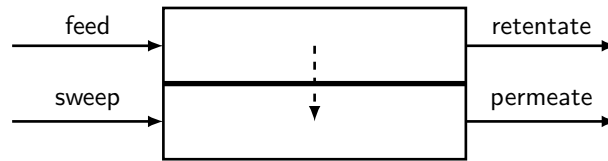


Figure 2.5: Membrane unit for gas permeation.

adsorption and regenerating in the different vessels continuous production can be achieved. A schematic for a simple two bed cycle is shown in fig. 2.4. Ambient air is first led through the first reaction vessel at the elevated pressure. Within the vessel nitrogen is adsorbed until saturation is reached. At that point the ambient air is led through the second vessel. A fraction of the product stream is fed into the first vessel and used as sweep for the regeneration of the adsorbent at lower pressure.

Depending on the size of the process two different pressure level are used. One cycle adsorbs the nitrogen at a pressure of approximately 7.5 bar while regeneration is done at ambient pressure. Within the alternative approach adsorption occurs under ambient condition, while for the regeneration step a vacuum pump reduces the vessel pressure. This process is called Vacuum Pressure Swing Adsorption (VPSA).

An important role when designing the product is the choice of the adsorbent. For almost all current applications of PSA aluminosilicates or zeolites have been designed, tailored to the specific separation task. Their main advantages include a high selectivity towards a specific gas to be adsorbed as well as a very homogenous distribution of diameters in the molecular sieve.

## 2.3 Gas permeation

find paper: DOI: 10.1002/cite.330480804

add reference: ISBN-10: 354034327X

The separation of mixed gases by membrane process is called gas permeation. Its main strength in comparison with alternative processes are the low energy consumption and the possibility to produce flexible mobile units. As mentioned before it is not however capable of producing high quantity highly pure product streams. As fig. 2.1 illustrates the main application for the gas permeation process are small to moderate product streams at intermediate purities.

fig. 2.5 shows the schematic for a single stage membrane unit. Within the feed stream the gaseous mixture is fed into the unit, which can quickly be implemented. Within the unit one or more species migrate favorably through the membrane. In this case mostly dense polymer membranes are employed used. There have been some impressive results with metallic membranes, but due to the very high material costs they have not been adapted by the industry. Furthermore, since gaseous phases often have rather small molecular species, porous membranes cannot achieve desired separation. The driving force the separation process is a difference in partial pressure or species activity across the membrane. According to the molecular structure of each species, the structure of the separating

membrane as well as the process parameters pressure and temperature, they permeate through the membrane in different quantities.

The process of permeation can be subdivided into three separate steps. Sorption at the membrane / feed interface, diffusion through the mostly dense polymer membrane and finally desorption at the permeate side of the membrane.

# 3 Process design under uncertainty

Once a particular reaction or production sequence has been developed in a laboratory it needs to be scaled up to produce the desired product in quantities large enough to meet market demand to a price that allows competitive prices. This is where process design comes into play. At the begin of the design process often only the raw material as well as the desired products, along with the necessary chemical reactions are specified. It is then the task of the process engineer to find a series of production steps which lead to the production of this product. Each step denotes a specific function and the connected functions form a process. At the end each specific function has to be assigned a specific technical realization.

Usually there are numerous options available to perform certain tasks and in order to be realized any endeavor will need to yield acceptable profits. Different options will therefore have to be compared from an engineering and an economic standpoint alike. For complex processes the amount of possible options becomes enormous. Too many to be manually examined and evaluated.

Number of options for separation problem – ref douglas

Process Engineers have to rely on heuristics to limit the number of options and identify the most promising candidates for further studies. These candidates can then be compared in terms of economic performance. In order to do so one has to rely on simulation studies and experiments. As experiments and the construction of pilot plants are very costly the aim is to minimize those. With the rise in available computing power the numerical simulation of processes has become a very powerful tool to support the design process and has the ability to replace numerous experiments. Not only can possible options be evaluated but also can optimal process configurations from a limited pool of options be generated.

To conduct simulation studies first a process model needs to be created. This models in its most general case is a set of partial differential and algebraic equations. When formulating the model great care needs to be applied, as any simulation study can only be as accurate as the underlying model. The process engineer is faces with the task to find an adequate representation of the physical system which delivers meaningful results within an reasonable amount of time. Further enhanced is this problem when the model is to be used in model predictive control, where (approximate) solutions of complex systems might need to be available within seconds or fractions of a second.

## 3.1 Recourse models

## 3.2 Chance constrained optimization

## 4 Process economics

Aside from question whether a certain process is capable of producing products according to its specifications, it needs to be investigated if it does so in an economically viable manner. The modeling of process economics is a powerful tool to estimate project profitability. The evaluation of process economics has three major aims in the design phase.

- Compare design options with regard to profitability.
- Economically optimize a given design.
- Estimate project profitability

In any case the total cost of the project as well as the cash flow structure will have to be analyzed to supply an accurate estimate of the economic conditions. Furthermore an adequate measure to compare and analyze a project in economic terms needs to be employed.

The following sections will first describe how to estimate the total project cost in different stages of the design process. It is evident that, as the more information about a given process becomes available, the accuracy of any cost estimate will increase as well. During the design phase of a process roughly three different stages of cost estimation can be formulated.

- An estimate before designing the process yields an order of magnitude estimation for supporting market research efforts. Error  $> 30\%$
- Estimate in the early design phase based on essential process equipment. Error  $\pm 30\%$
- Estimate based on an advanced flowsheet and relevant process parameters. Error  $\pm 20\%$

Once detail engineering commences, even more accurate calculations with errors reducing to  $\pm 5\%$  can be undertaken[3]. At that point a concrete process option will have to be chosen and the investment decision will already have to be made. All optimization measures within the scope of this thesis will have concluded at that time.

Within sec. 4.1 first we will take a look at how the total cost of a chemical process might be estimated at different design stages. Subsequently in sec. 4.2 the different ways of evaluating a certain investment options and estimating a projects profitability will be discussed.

### 4.1 Project cost

An important factor in every given project is the total cost. During the design of a chemical process many important aspects of the future cost structure are unknown, as the final design is in development.

In general the total cost of implementing and operating a production site can be broken down into several subcategories.

- Battery limit investment
- Utility investment
- Off-Site investment
- Engineering fees
- Working capital

#### 4.1.1 Before process design

Before any details about the process to be implemented are known, an estimate can only give an order of magnitude towards cost to be expected. The cost of the new process  $C_P$  can be related to the cost of a reference process  $C_P^0$  by

$$C_P = C_P^0 \cdot \left( \frac{Q_P}{Q_P^0} \right)^x. \quad (4.1)$$

The degression coefficient  $x$  needs to be correlated from historical data.

As the overall price structure will change over time, the reference price will not reflect the current market situation. In order to adjust for that shortcoming several price indices are published all over the world. Some of those tailor to special branches of the industry, others give a picture of the price-development in a economy as a whole. The price ration of the prices at different times will then be equal to the ration of the price indices at the respective times

$$\frac{C_1}{C_2} = \frac{I_1}{I_2}. \quad (4.2)$$

For each index an somewhat arbitrary reference year is chosen. Among the most common indices are the Marshall & Swift index, the Nelson-Farrar-Index or the Chemical Engineering index. Some exemplary values for these indices are given in tab. 4.1. As one can see the development within the process industry very well matches the development in the economy as a whole, which is why for this rough estimate general indices should suffice.

#### 4.1.2 During process design

Once the future design has been broken down to fewer potential options and first process flowsheets are available a more elaborate approach becomes possible.

year	Marshall & Swift Installed Equipment Index 1926 $\equiv$ 100		Nelson-Farrar Refinery Construction Index 1946 $\equiv$ 100	Chemical Engineering Plant Cost Index 1957 $\equiv$ 100
	all industries	process industry		
1975	444	452	576	182
1980	560	675	823	261
1985	790	813	1074	325
1990	915	935	1226	358
1995	1027	1037	1392	381
2000	1089	1103	1542	394
2001	1093	1107	1565	396

Table 4.1: Price indices and their development.

add citation: PE-VT Script

### Battery limit investment

The battery limit investment denotes all investments necessary to have all required equipment for process operations installed on-site. This includes structures necessary to house the process as well as delivery and installation of all individual assets. One major part of these cost will be the process equipment. As the exact manufacturers and models of the equipment will not be known in early design stages, an approach similar to the one in sec. 4.1.1 still needs to be employed. In contrast to before now the cost for individual pieces of process equipment will be considered separately. Each piece of equipment will have a specific feature that most influences its cost. For vessels and reactors this might be volume, while for heat exchangers the required heat exchange area is essential. The price for a piece of equipment  $i$   $C_E^i$  can again be approximated by a simple power law

$$C_E^i = C_B^i \left( \frac{Q_E^i}{Q_B^i} \right)^{M^i} . \quad (4.3)$$

A reference price  $C_B^i$  is multiplied by the determining quantity  $Q^i$  normalized to a reference state  $Q_B^i$  and raised to the power  $M^i$  specific to each piece of equipment. Reference prices and quantities for various installations can be obtained from literature.

add reference

If information on the process conditions are available, they also can be considered in price calculations. Aside from the mere size of the equipment the process conditions will also have an effect on the expected cost. The predominant factors to that respect are pressure, temperature and the question whether corrosive or reactive media, which will require more resistant materials, will be present. Furthermore if a more detailed choice of process equipment is known this is considered as well. For example a plate-fin heat exchanger might be more expensive than a tubular model with the same heat-exchange area. In order to account for all those effects a form factor  $f_F$  is applied to the equipment cost

$$C_E^i = C_B^i \left( \frac{Q_E^i}{Q_B^i} \right)^{M^i} f_F^i = C_B^i \left( \frac{Q_E^i}{Q_B^i} \right)^{M^i} \underbrace{(1 + f_C^i + f_M^i + f_P^i + f_T^i)}_{=f_F^i} . \quad (4.4)$$



Where  $f_F$  denotes the form factor,  $f_C$  corrects for design complexity,  $f_M$  for material selection,  $f_P$  adjusts for extreme pressures and  $f_T$  for temperature.

As in the previous section all reference prices need to be corrected to compensate for the temporal price development, Here again the already discussed indices are used. Furthermore can regional changes in price structures be considered. Here again have correction factors to prices in an reference region in the world (e.g. USA) been published.

add ref.

In addition to the purchased cost, the costs for installing the process equipment have an significant effect on the total needed investment of the process. These installation costs include:

- Installations costs
- Piping ,valves and electrical wiring
- Control system
- Structures and foundations
- Insulation and fire proofing
- Labour fees

Those again can be expressed as correction factors to the equipment price. Depending on the status of the information available they can be expressed as one unified factor or broken down to each specific category. One however needs to bare in mind, that costs for piping and valves – pieces of equipment in direct contact with process media – will be affected by the process conditions in a similar way as the actual equipment, whereas the other categories are more likely to remain unchanged. Thus attention needs to be paid, in with fashion the factors will be applied.

A word should be said to the cost for the control system. Most obtainable data will most likely refer to a decentralized control system, as it has been in use for many years. With ever more powerful computers a centralized approach, namely model predictive control (MPC) is becoming more relevant. As the structure for such a control system may vary significantly from the common designs, the cost factors may as well.

### Services

The utility investments and off-site investments are often referred to as services. Therein included are all measures necessary to supply the process with the media required for operations. This includes but is not limited to generation and distribution of energy, steam, process gases. The utility investments in this context refer to all investments within the greater production site but out of the battery limits of the process. Off-site investments contain everything not contained in the site such as roads, power cables, communication systems or waste disposal. All these costs are expressed as fractions of the equipment cost at moderate temperature and pressure. This means, when applying these fractions, the factors  $f_M$ ,  $f_P$  and  $f_T$  should not be considered at this point.

Once again in early design stages one has to resort to factors derived for statistical data, to calculate the cost of raw materials, energy and support media such as lubricants, heat or catalysts. If more detailed information on process streams is available the approach should be refined.

The cost of raw materials  $C_{RM}$  can then be calculated if the streams of individual raw materials  $m_i$  as well as their specific cost are known.

$$C_{RM} = \left( \sum_i^{N_{RM}} \dot{m}_i \cdot C_{RM}^i \right) \cdot t_{op}. \quad (4.5)$$

Much in the same way the cost for energy can be calculated. This is once more done for each individual piece of equipment, rather than for the whole process as it has been done for the raw materials. Hence with the needed energy for equipment  $i$  with respect to energy carrier  $j$   $e_{ij}$  along with the price for energy carrier  $j$   $C_{EC}^j$  the total energy cost  $C_{EC}$  can be assessed.

$$C_{EC} = \left( \sum_i^{N_E} \sum_j^{N_{EC}} \dot{e}_{ij} \cdot C_{EC}^j \right) \cdot t_{op}. \quad (4.6)$$

### Working capital

The working capital includes all investments necessary for process operations. This means raw materials, payroll, extended credit to customers and so on. In contrast to all other costs the working capital can partially be retrieved when the process stops operations. How different types of cash flows, extended or owed credit should be handled will be discussed in sec. 4.2. In addition to the cost of raw materials needed during process operations, which generate a product stream, raw materials are also needed to fill all vessels, reactors, columns and piping that make up the process. The cost of these needs to be considered as investment and not as operation cost, since more or less the same amount will remain bound until the process ceases operations and it can (partially) be retrieved.

$$C_{FILL} = \sum_i^{N_{RM}} V_i \cdot C_{RM}^i. \quad (4.7)$$

### Total investment

When all contributions to the total investment are considered an estimate for the total price of the process can be calculated

$$C_P = \sum_i^{N_E} \left[ C_B^i \left( \frac{Q_i}{Q_B^i} \right)^{M^i} \left( f_F \cdot f_{PIPE} + \sum_j f_j \right) \right] + C_{RM} + C_{EC} + C_{FILL} \quad (4.8)$$

It should be emphasized that in early design stages these calculations will at best yield an order of magnitude estimate for the expected cost of implementing a chemical process. Most literature sources give an accuracy of  $\pm 30\%$  [3]. As the project progresses more and more information becomes available an a more accurate estimate can be prepared. Those often rely on actual proposals from prospective manufacturers and suppliers.

## 4.2 Investment criteria

The total cost of a project is a very important measure to decide whether to undergo a certain endeavor. However in a complex financial system it cannot be taken as the sole factor to compare investment alternatives. Different other indices are used to measure the attractiveness of an investment. One main distinction can be made between different measurements. Those that work with averaged cash flows and consider the project as a single time periods (sec. 4.2.1) and models that take into account cash flows made at different points in time – so called multi-period models (sec. 4.2.2).

### 4.2.1 Single period estimation methods

Among the most prominent single period methods are the payout time (POT) and the return of investment (ROI).

#### Payout time

Payout or amortization time is the time necessary to earn the total investment of the process. It is also often referred to as the break even point. A profitable venture will from that point on begin to make money. A shorter payback time is a measure for an more attractive investment.

$$POT = \frac{\text{capital expenditure}}{\text{incoming cash flow} / \text{period}} \quad (4.9)$$

#### Return of investment

The return of investment is defined as

$$ROI = \frac{\text{average return} / \text{period}}{\text{capital expenditure}} \cdot 100\%. \quad (4.10)$$

It denotes the equivalent to an interest rate if the earned interest is not reinvested such that the same investment is considered in every period. The average return is calculated from the expected returns over certain duration of time. The time considered might either be the total lifecycle of the process or the expected write-off time. As capital expenditure most likely the total investment will be used. However it might give a more precise picture if write-off or the time of individual payments is considered when evaluating the capital expenditure.

### 4.2.2 Multi period estimation methods

As discussed earlier, when using only averaged values and considering only one time period, a distorted picture of the financial situation will evolve. The source of the money to be invested plays a critical role in the evaluation process. Cash reserves, loans, preferred stock or other financial instruments are all viable sources of investment capital. Often a mixture of many different sources is used to allocate

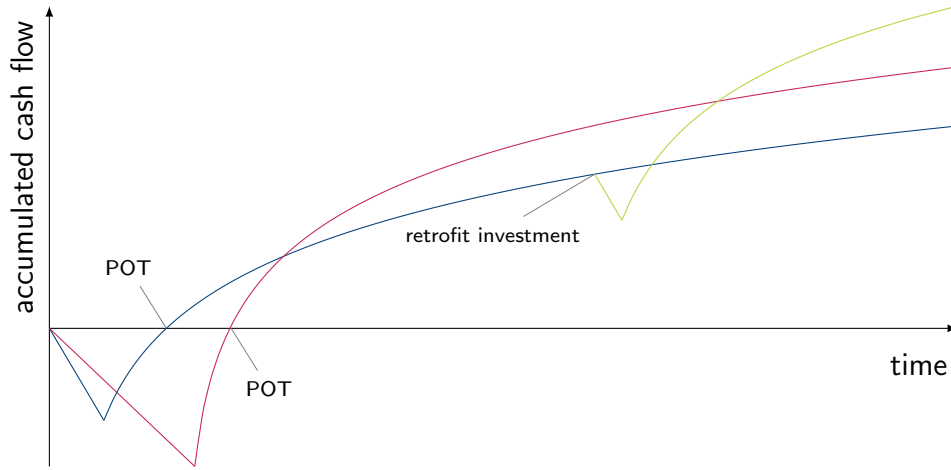


Figure 4.1: Accumulated cash flows over project life cycle.

all necessary funds. A generic structure of the cash-flows during the life-cycle of a given project is shown in fig. 4.1. When the project commences no revenue is generated and only investments are made. As process operations begin revenue is generated and the slope of the accumulated cash flow curve switches to an upward directions – assuming the project is profitable. As time progresses the produced product might reduce in value through competitors entering the market or other factors. When the curve intersects the abscissa the project investment is earned and the break point is reached. The two investment alternatives highlight that the amount of initial investment will affect the expected revenue. As more information is gathered during operations, opportunities may arise to optimize the process and leverage so far dormant potentials. These so called retrofit Investments would ideally lead to an improved cash flow structure as indicated by the green line.

add ref: (fig. 4.1) Script PE-VT

Before further investigating multi-period model some basic ideas about financial mathematics and the treatment of interest should be reviewed. When investing the capital  $C_0$  at compounded interest for  $n$  years at a rate of  $i$  %, the compound amount will yield

$$C_n = C_0 \cdot q^n. \quad (4.11)$$

Where  $q$  denotes the interest factor

$$q = 1 + \frac{i}{100}. \quad (4.12)$$

On the other hand the current value of an investment that will yield  $C_n$  in  $n$  years can be calculated by

$$C_0 = C_n \cdot q^{-n} \quad (4.13)$$

The above considerations always assume a single payment at the beginning or end of the entire period. Annuities however are usually in several trances with regular payments to be made or received at predefined instances. Assuming those payments are of equal size  $a$ , the final value can be computed by

$$C_n = a \cdot q^\alpha \cdot \frac{q^n - 1}{q - 1}, \quad (4.14)$$

while the current value of an annuity yields

$$C_0 = a \cdot q^{\alpha-n} \cdot \frac{q^n - 1}{q - 1}, \quad (4.15)$$

The factor  $\alpha$  in the previous equations denotes whether payments are made at the beginning of a period ( $\alpha = 1$ ) or at the end ( $\alpha = 0$ )

With that multi-period model can be discussed. Here not only different alternatives can be compared, but also the profitability in comparison with investments in financial products can be assessed.

### Net present value

The net present value (*NPV*) describes the amount of money to be invested if all cash flows – incoming and outgoing – are discounted to the project start ( $t = 0$ ). The present value of all expenses is then

$$C_{0e} = e_0 + e_1 \cdot q^{-1} + \dots + e_n \cdot q^{-n} = \sum_i^n e_i \cdot q^{-i}. \quad (4.16)$$

Here it is given that all expenses are paid at the beginning of each period, as it is the most common case. If revenues  $r_i$  are realized then as well an equivalent formula would be attained. In most cases revenues will come in at the end of a period in which case the present value becomes

$$C_{0r} = r_0 \cdot q^{-1} + r_1 \cdot q^{-2} + \dots + r_n \cdot q^{-(n+1)} = \sum_i^n r_i \cdot q^{-(i+1)}. \quad (4.17)$$

The *NPV* of a given project is then derived from the present values of all expenses and revenues

$$NPV = C_{0r} - C_{0e}. \quad (4.18)$$

For any project to be considered as investment alternative the present value needs to be positive since otherwise the investment would yield losses.

### Discounted cash flow rate of return

The formula for the discounted cash flow rate of return is very similar to the one for a net present value. The difference is, that rather than computing the net present value with given interest rates, the present value is set to zero and the resulting interest rate is then calculated. Other than with the previous methods no analytical solution can be presented but rather an iterative approach to find a solution to

$$0 = \sum_i^n (r_i - e_i) \cdot q^{-i}. \quad (4.19)$$

Here it was assumed that all payments – incoming and outgoing – are made at the beginning of a period. The only variable is the interest rate which is included in the interest factors  $q$ . This method gives the interest rate which would be necessary to earn all time dependent expenses within  $n$  years. In this case a higher *DCFRR* indicates a more attractive investment.

### Annuity method

Within the annuity method two different annuities are calculated and then compared. First the annuity  $a$  of an investment of  $C_0$  at market conditions is of interest.

$$a = C_0 \cdot \frac{q^{n-\alpha}(q-1)}{q^n - 1} \quad (4.20)$$

This annuity is the amount that could be paid out each period if  $C_0$  is invested, compound interest is considered and all funds are used up at the end of  $n$  years.

This is then compared to the annuity of the considered project

$$a_P = \sum_i^n (r_i - e_i) \cdot q^{-i} \cdot \frac{q^{n-\alpha}(q-1)}{q^n - 1}. \quad (4.21)$$

Only if  $a_P > a$  is the project more profitable than simply investing the required capital in an financial product.

### Interest Rates

With all the presented models it was implicitly assumed that interest rates for debit and credit are equal. The reality however is much different. It will therefore be prudent to use different rates of interest for each case. The basic calculations however remain unchanged. Especially the interest rate that can be earned when investing a certain amount will need to be estimated. As certain products with a know *ROI* will not necessarily be the most attractive options on the capital market. Thus in many companies there is an internal value given for this interest rate, which is calculated from historical data.

# 5 Mathematical process model

## 5.1 Physical models

### 5.1.1 Distillation columns

$$0 = F_j^V + F_j^L + V_{j+1} + L_{j-1} - V_j - S_j^V - L_j - S_j^L \quad (5.1)$$

$$0 = V_{j+1}y_{i,j+1} + L_{j-1}x_{i,j-1} + F_j^V z_{i,j}^V + F_j^L z_{i,j}^L - (V_j + S_j^V) y_{i,j} - (L_j + S_j^L) x_{i,j} \quad (5.2)$$

$$0 = V_{j+1}H_{j+1}^V + L_{j-1}H_{j-1}^L + F_j^V H_j^{FV} + F_j^L H_j^{FL} - (V_j + S_j^V) H_j^V - (L_j + S_j^L) H_j^L \quad (5.3)$$

$$y_{i,j} = \gamma_{i,j} \cdot K_{i,j} \cdot x_{i,j} \quad (5.4)$$

### 5.1.2 Heat exchanger

### 5.1.3 Compressors

### 5.1.4 Turbine

## 5.2 Economic models

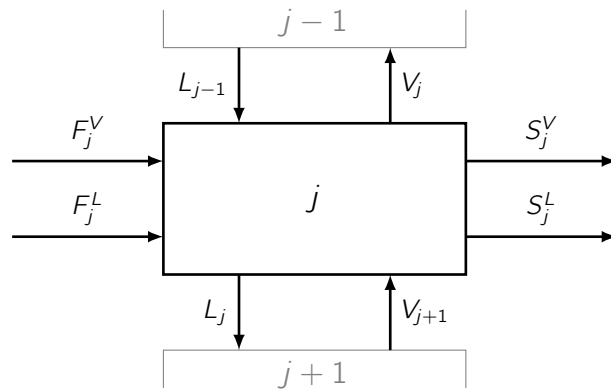


Figure 5.1: Distillation column tray model.

## **6 Conclusion and further research**



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# Todo list

■ add citation . . . . .	2
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■ add citation: PE-VT Script . . . . .	10
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■ add ref. . . . .	11
■ add ref: (fig. 4.1) Script PE-VT . . . . .	14