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# Capital Cost Evaluation for Optimum Process Design of Cryogenic Air Separation

Diploma Thesis  
by  
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# Diploma Thesis for Mr. Robert Pack

## Task Description

Current air separation process optimization is carried out sequentially with various discrete or continuous process parameters, and then iterated with equipment sizing and costing. Typical process parameters include temperature, pressure, flow, pressure drop, product purity, distillation column feed and/or draw locations staging, and etc. Major equipment used in cryogenic air separation include air compressors, turbines, plate and fin heat exchangers, cooler, distillation columns, condensers, reboilers, and etc. Once process optimum condition is identified, individual equipment is sized and cost estimated. Capital cost is then feed back to process to verify the original optimum condition. If the original optimum condition changes, we will have to re-optimize the process parameter and identify the new process optimum condition. New equipment sizing and costing will have to be adjusted accordingly.

The deficiency of the current process is apparent: manual, sequential and iterative. We would like to streamline the optimization process, incorporate the capital equipment cost into the process optimization. The process optimizer we target is gPROMS. The goal is to simultaneously optimize process and equipment to minimize the total process cost with user defined objective functions.

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Tag der Abgabe: xx.xx.2012

# Statutory Declaration

I herewith declare that I have completed the present thesis independently making use only of the specified literature and aids. Sentences or parts of sentences quoted literally are marked as quotations; identification of other references with regard to the statement and scope of the work is quoted. The thesis in this form or in any other form has not been submitted to an examination body and has not been published.

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Date

Signature

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# Nomenclature

## Constants

$g$	gravitational constant	$\left[\frac{m}{s^2}\right]$
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## Greek Letters

$\beta$	aeration factor	$[-]$
$\delta$	film thickness	$[m]$
$\epsilon$	packing void fraction	$[-]$
$\gamma_i$	liquid activity coefficient of component $i$	$[-]$
$\mu_G$	vapour viscosity	$[Pa \cdot s]$
$\mu_L$	liquid viscosity	$[Pa \cdot s]$
$\mu_S$	isentropic expansion coefficient	$\left[\frac{K}{Pa}\right]$
$\mu_{JT}$	<i>Joule-Thompson</i> coefficient	$\left[\frac{K}{Pa}\right]$
$\nu^D$	reflux ratio	$[-]$
$\nu^R$	boilup ratio	$[-]$
$\nu^{vap}$	condenser vapour fraction	$[-]$
$\Phi$	relative froth density	$[-]$
$\Psi_{flood}$	fractional allowed flooding	$[-]$
$\Theta$	corrugation angle	$[rad]$
$\varphi_i^0$	reference vapour fugacity coefficient of component $i$	$[-]$
$\varphi_i$	vapour fugacity coefficient of component $i$	$[-]$
$\zeta_{ij}^F$	splitting variable for feed $i$ on stage $j$	$[-]$



## Nomenclature

$\zeta_j^R$	splitting variable for reflux on stage $j$	$[-]$
$\zeta_{ij}^{SL}$	splitting variable for liquid side draw $i$ on stage $j$	$[-]$
$\zeta_{ij}^{SV}$	splitting variable for vapour side draw $i$ on stage $j$	$[-]$
<b>Variables</b>		
$\Delta p$	pressure drop over entire column	$[Pa]$
$\delta p_{dry}$	dry pressure drop	$[\frac{Pa}{m}]$
$\Delta p_{stage}$	stage-wise pressure drop	$[Pa]$
$\dot{e}_{ij}$	energy flow of to equipment $i$ from energy carrier $j$	$[kW]$
$\dot{m}_i$	mass flow of component $i$	$[\frac{kg}{s}]$
$\ell_{weir}$	length of tray weir	$[m]$
$a$	annuity	$[\$]$
$a_c$	parameter in Peng-Robinson EOS	$[\frac{m^5}{mol^2 s^2}]$
$A_j^{min}$	minimum column area on stage $j$	$[m^2]$
$A_{active}$	active tray area	$[m^2]$
$b$	parameter in Peng-Robinson EOS	$[\frac{m^3}{mol}]$
$C_B^i$	reference cost of equipment $i$	$[\$]$
$C_E^i$	cost of equipment $i$	$[\$]$
$C_{EC}^i$	mass specific cost of energy carrier $i$	$[\frac{\$}{kW}]$
$C_{RM}^i$	mass specific cost of raw material $i$	$[\frac{\$}{kg}]$
$C_0$	initial value of an investment	$[\$]$
$C_1$	Reference equipment cost at time 1	$[\$]$
$C_1$	packing constant	$[\ ]$
$C_2$	Reference equipment cost at time 2	$[\$]$
$C_2$	packing constant	$[\ ]$

## Nomenclature

$C_n$	final value of an investment	[\$]
$C_P$	total process cost	[\$]
$C_P^0$	reference process cost	[\$]
$C_{0e}$	Present value of all expenses	[\$]
$C_{0r}$	Present value of all revenues	[\$]
$C_{\text{column}}$	column cost	[\$]
$C_{\text{internals}}$	cost of column internals	[\$]
$C_{\text{platform}}$	platform cost	[\$]
$C_{EC}$	total cost of energy	[\$]
$C_{FILL}$	cost of raw materials to fill the process	[\$]
$C_{RM}$	total cost of raw materials	[\$]
$C_{\text{tower}}$	tower cost	[\$]
$D$	degression coefficient	[—]
$E$	fractional weld efficiency	[—]
$ent_j$	fractional entrainment for 80% flooding on stage $j$	[—]
$f_C^i$	design complexity correction to equipment cost	[—]
$f_M^i$	material selection correction to equipment cost	[—]
$f_P^i$	pressure correction to equipment cost	[—]
$f_T^i$	temperature correction to equipment cost	[—]
$f_i^L$	liquid fugacity	[—]
$f_i^V$	vapour fugacity	[—]
$F_j$	feed molar flowrate to stage $j$	$[\frac{mol}{s}]$
$F_j$	feed to tray $j$	$[\frac{mol}{s}]$
$F_{Pi}$	compressibility factor of component $i$	[—]

## Nomenclature

$FLV_j$	Sherwood flow parameter on stage $j$	$[-]$
$h_j^L$	molar liquid enthalpy on stage $j$	$[\frac{J}{mol}]$
$h_j^V$	molar vapour enthalpy on stage $j$	$[\frac{J}{mol}]$
$h_j^F$	molar liquid feed enthalpy to stage $j$	$[\frac{J}{mol}]$
$H^R$	reflux enthalpy stream	$[\frac{J}{mol}]$
$h_f$	froth height	$[m]$
$h_L$	dimesionless holdup	$[-]$
$h_w$	weir height	$[m]$
$h_{plate}$	plate spacing	$[m]$
$h_L^{reb}$	reboiler liquid enthalpy	$[\frac{kJ}{kmol}]$
$h_{ow}$	height over weir	$[m]$
$h_V^{reb}$	reboiler vapour enthalpy	$[\frac{kJ}{kmol}]$
$i$	interest rate	$[\%]$
$I_1$	Cost index at time 1	$[-]$
$I_2$	Cost index at time 2	$[-]$
$K_1$	empirical coefficient	$[-]$
$K_{ij}$	equilibrium constant of component $i$ on stage $j$	$[-]$
$L_j$	liquid flow from tray $j$	$[\frac{mol}{s}]$
$L_j$	molar liquid flowrate form stage $j$	$[\frac{mol}{s}]$
$l_{ij}$	liquid molar flowrate of component $i$ from stage $j$	$[\frac{mol}{s}]$
$L_{reb}^{in}$	liquid reboiler influx	$[\frac{kmol}{s}]$
$L_{reb}^{out}$	liquid reboiler outflow	$[\frac{kmol}{s}]$
$m$	parameter in Peng-Robinson EOS	$[-]$
$M^i$	equipment specific factor -	$[$

## Nomenclature

$n_C$	number of components	[—]
$n_F$	number of feeds	[—]
$n_i^{reb}$	molar holdup of component $i$ in reboiler	[ $kmol$ ]
$n_j^L$	liquid molar holdup on stage $j$	[ $mol$ ]
$n_j^V$	vapour molar holdup on stage $j$	[ $mol$ ]
$n_S$	number of column stages	[—]
$n_E$	number of process units	[—]
$n_{ij}$	holdup of component $i$ on stage $j$	[ $mol$ ]
$n_{RM}$	number of different raw materials	[—]
$n_{SL}$	number of liquid side-draws	[—]
$n_{SV}$	number of vapour side-draws	[—]
$NPV$	Net present value	[\$]
$p$	pressure	[ $Pa$ ]
$p$	system pressure	[ $Pa$ ]
$p_i^S$	vapour pressure of component $i$	[ $Pa$ ]
$p_D$	design pressure	[ $psig$ ]
$p_O$	operating pressure	[ $psig$ ]
$POT$	Payout time	[\$]
$POT$	payout time	[ $a$ ]
$q$	interest factor	[—]
$Q^i$	specific quantity for equipment $i$	[ <i>variable</i> ]
$Q_B^i$	equipment specific reference quantity	[ <i>variable</i> ]
$Q_P$	process capacity	[ $\frac{kg}{h}$ ]
$Q_P^0$	reference process capacity	[ $\frac{kg}{h}$ ]

## Nomenclature

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$ROI$	return of investment	$[-]$
$S$	corrugation side	$[m]$
$S$	maximum allowable stress	$[psi]$
$S_j^L$	molar liquid side-draw flowrate form stage $j$	$[\frac{mol}{s}]$
$s_j^L$	dimensionless liquid side-draw from stage $j$	$[-]$
$S_j^V$	molar vapour side-draw flowrate form stage $j$	$[\frac{mol}{s}]$
$s_j^V$	dimensionless vapour side-draw from stage $j$	$[-]$
$S_j^L$	liquid side flow from tray $j$	$[\frac{mol}{s}]$
$S_j^V$	vapour side flow from tray $j$	$[\frac{mol}{s}]$
$T^{bub}$	bubble point temperature	$[K]$
$T^{dew}$	dew point temperature	$[K]$
$T^{sub}$	sub-cooling temperature	$[K]$
$t_p$	shell thickness die to pressure	$[in]$
$t_s$	tower shell thickness	$[in]$
$t_w$	thickness wind and earthquake correction	$[in]$
$t_{op}$	time of process operations	$[s]$
$U_j$	internal energy holdup on stage $j$	$[J]$
$U_{reb}$	reboiler internal energy holdup	$[kJ]$
$v_G$	vapour velocity	$[\frac{m}{s}]$
$V_j$	molar vapour flowrate form stage $j$	$[\frac{mol}{s}]$
$V_j$	vapour flow from tray $j$	$[\frac{mol}{s}]$
$v_j^{flood}$	flooding velocity on stage $j$	$[\frac{m}{s}]$
$v_L$	liquid velocity	$[\frac{m}{s}]$
$v_{ij}$	vapour molar flowrate of component $i$ from stage $j$	$[\frac{mol}{s}]$

## Nomenclature

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$V_{reb}^{out}$	vapour reboiler outflow	$[\frac{kmol}{s}]$
$V_{stage}$	stage volume	$[m^3]$
$W_{tower}$	weight of a tower	$[lbs]$
$x_{ij}$	liquid mole fraction of component $i$ on stage $j$	$[-]$
$x_i^{reb}$	reboiler liquid mole fractions	$[-]$
$y_{ij}$	vapour mole fraction of component $i$ on stage $j$	$[-]$
$y_i^{eq}$	equilibrium vapour concentration of component $i$ on stage $j$	$[\frac{mol}{mol}]$
$y_i^{reb}$	reboiler vapour mole fractions	$[-]$
$z_{ij}^L$	feed mole fraction to stage $j$	$[-]$

## **Abstract**

here comes an abstract

# 1 Introduction

Distillation columns are to this day the most prominent way for separating liquid mixtures. Vast amounts of energy are used during the separation of various product streams. In fact often the majority of the energy required in a given process is consumed in the separation stage. Therefore separation sequences have been under investigation for many years. The synthesis of separation sequences poses great challenges to this day, especially if complex multi-component mixtures are considered.

Several shortcut methods have been developed for screening of potential separation sequences. If several flowsheet alternatives are constructed it becomes necessary to further investigate, which is the most suited to a given task. For this rigorous process models which incorporate tray by tray balances and non-ideal equilibria are employed. One approach, which is currently still employed in industry practice, is to carry out a sequence of process simulations to find the most promising configuration in a manual, trial and error based procedure. However this approach is unlikely to yield near optimal results.

The more reliable alternative to synthesize near optimal flowsheets is to perform mathematical optimization on a given process superstructure. In that context continuous decisions such as operating pressure or temperatures, as well as discrete decisions associated with structural choices on the process equipment have to be made. The resulting problems are called mixed-integer non-linear programs (MINLP), which pose considerable challenges in their solution process. Due to that structural optimization is carried out by experts, with careful consideration when setting up the problem and optimization in specialized programs.

On the other hand process simulation in flowsheeting environments such as Aspen Plus<sup>®</sup> or gPROMS<sup>®</sup> is an integral part of industrial practice. Hence the scope of this thesis is, to implement a flexible, easily employable model for process optimization in the aforementioned modeling environment gPROMS<sup>®</sup> which is capable of automated model initialization as well as structural optimization.

The implemented models will then be applied to the cryogenic air separation process. This process is used to produce highly pure nitrogen, oxygen and noble gases such as argon. While the components themselves do not display any severe non-idealities, the process is highly integrated and coupled in terms of energy and material streams and contains columns with multiple feeds and side draws, and complex column configurations. This makes it especially challenging in terms of process optimization.

The following thesis is structured as follows. First a brief description of the cryogenic air separation process will be given. Subsequently, as economic considerations will always have to be considered, general approaches to estimate the economics associated with process design will be discussed.

In the third chapter a detailed overview of the models implemented in gPROMS<sup>®</sup> will be given. Afterwards, the optimization of chemical processes will be dealt with. Within that section, first some



general theoretical basics of optimization theory are introduced followed by a closer look at solution algorithms for discrete-continuous problems. Finally optimizations will be carried out on the cryogenic air separation process. Using that example, the performance of different solution approaches and optimization objectives in terms of optimality and efficiency will be compared. Finally some possible directions for future research are mentioned.

## 2 Air separation technology

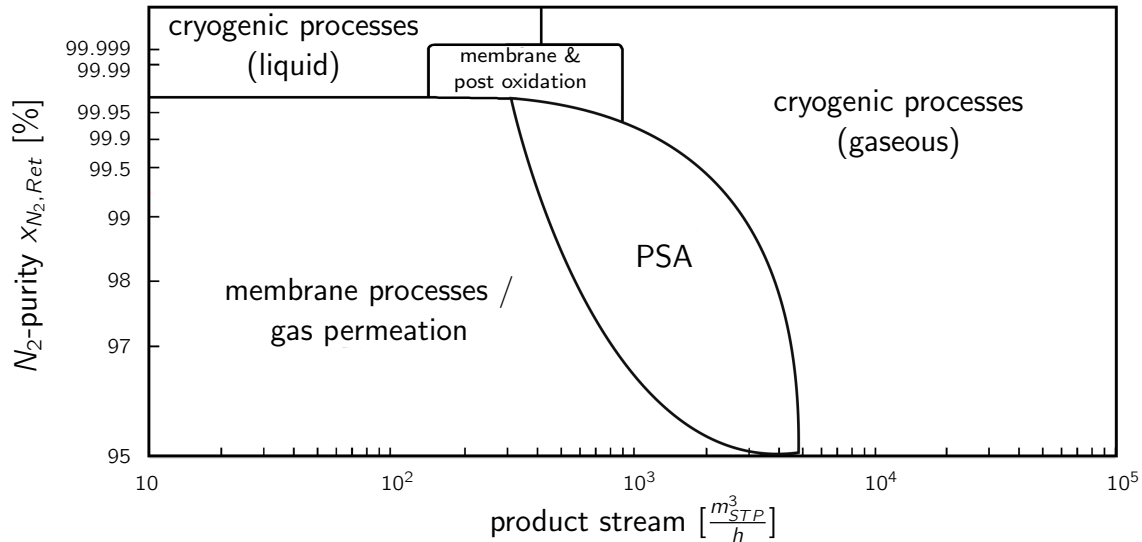


Figure 2.1: Comparison of Air Separation Technologies [32].

The separation of air finds many application in the modern industry. Pure oxygen and nitrogen are essential for many processes. Among the most prominent examples is the production of synthesis gas, which forms the basis for a multitude of very large scale process operations. In addition the air separation processes find application as a utility provider rather than producer of raw material for example in highly efficient integrated combustion cycles [25]. Due to the many different applications, pure nitrogen or oxygen is needed with various demands on purity, flow rate or phase composition. Therefore different technologies are suitable to specific tasks.

In general three main principles of air separation can be identified; adsorption, cryogenic distillation and transport through membranes. Fig. 2.1 illustrates the most economically viable processes depending on product purity and product stream volume. In the remainder of this chapter an overview of the available air separation technologies will be given. Here the main focus lies on introducing several alternative technologies and estimating their applicability to different product demands.

### 2.1 Cryogenic processes

Cryogenic Air Separation finds applications over a great variety of industries among others refining, petrochemicals, medical, food & beverages and environmental [36]. Fig. 2.2 shows the major unit operations associated with process. Initially the air is compressed and later re-expanded to reach the low temperatures required for liquefaction. In an intermediate pre-treatment step most unwanted

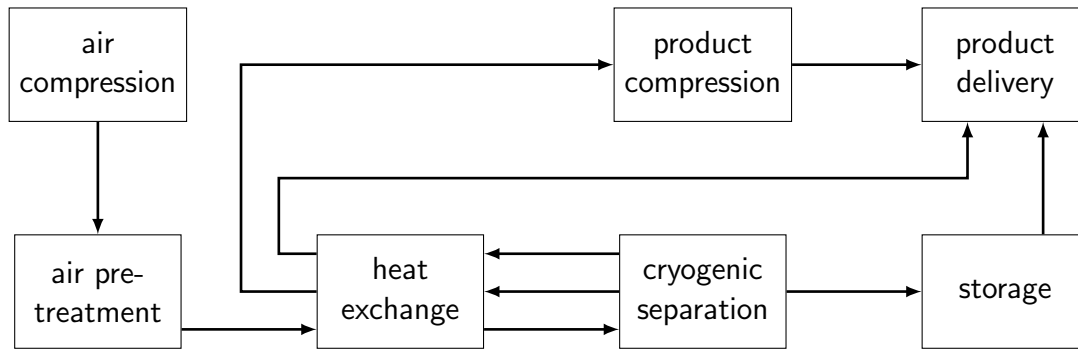


Figure 2.2: Schematic representation of the cryogenic air separation process.

contaminants are removed. It is essential to properly control the amount of carbon dioxide in the gas, as it might freeze out at the low temperatures especially within the heat exchanger. This will result in much decreased performance of the heat exchange unit.

The pre-treated air is then cooled against product streams leaving the separation stage of the process. To which extend the refrigeration is recovered is among the most dominant factors for economic operations. The requirements towards the product streams determine how much can be recovered. Most processes produce gaseous streams at approximately ambient pressure and temperature [37] and hence recover the maximum of energy from the process. However often a subsequent compression step or even liquefaction step is employed to deliver the desired product to the consumer.

In terms of economically viable operations, the compression denotes a sizable contribution to the capital cost and is responsible for the majority of the operating cost. However when recovering liquid product, the compression work needs to increase to compensate for the loss in recoverable refrigeration. Due to the widespread application of the process and several increasingly complex process configurations are being implemented with different strategies, for internal recovery of pressure energy and refrigeration.

The cryogenic process can most economically simultaneously produce large quantities of highly pure oxygen and nitrogen. Furthermore can streams of noble gases, especially argon, be produced. Most alternative technologies will produce one single product stream. Additionally can the economy of scale be largely exploited for this process, which is why current developments are towards large scale single train that can produce an excess of several thousand tonnes of product stream per day [7].

## 2.2 Adsorption processes

Certain materials display a higher affinity to adsorb a specific class of molecules. Nitrogen is more easily adsorbed in zeolites than oxygen. Therefore if air is fed through a zeolite bed, an oxygen rich stream is produced. Since any bed will eventually be saturated and its capability for adsorption be exhausted, the process carries an inherent discontinuity. However for most processes which yield large product streams continuous operations are more cost effective.

To realize a continuous process multiple adsorption vessels are operated simultaneously. Once a bed is saturated, the process is switched to a different vessel and the saturated one is regenerated.

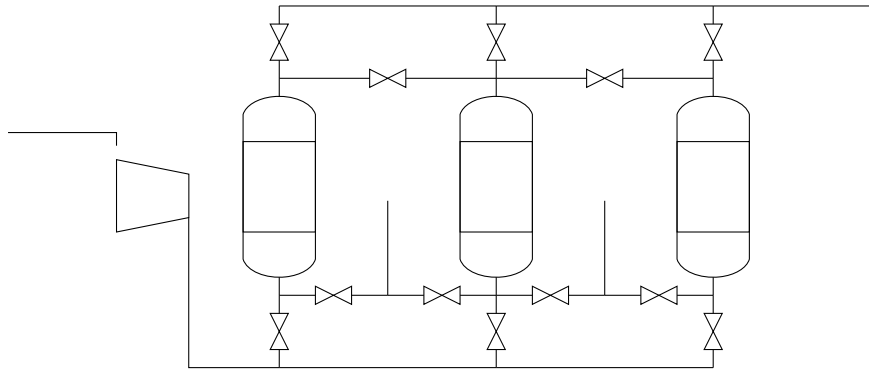


Figure 2.3: Schematic representation of a three bed PSA process.

Regeneration utilizes the change of adsorption capabilities at varying temperature or pressure. By increasing temperature or lowering pressure can the bed therefore be regenerated. Due to the switching during operations these processes are commonly known as temperature swing adsorption (TSA) or pressure swing adsorption (PSA). For most industrial applications PSA is preferred due to simpler operation and faster cycle times [37].

Especially the ability to construct very compact units, have led to implementations of PSA processes at very different scales. Briefcase sized units are constructed for treatment of asthma patients or other medical appliances. Larger scale plants have successfully been utilized, for example in the paper industry during the de-lignation of pulp[29].

The bed size is the determining factor for equipment cost. Since cycle times are limited by the regeneration step, the bed size and therefore the cost increases rapidly with the product flowrate. For very large scale operations the cryogenic process is more economically viable. However the process can be operated much more flexible due to drastically shorter start-up times than the cryogenic process. Furthermore, no by-products can be recovered from the process, without further subsequent separations[9].

For continuous operations two separate bed would suffice, however in order to increase cycle times and process throughput as well as exploit possibilities for pressure energy recovery, process configurations with three or more beds are employed. Variations of the process use sub-atmospheric pressures during the regeneration step. Such processes are commonly referred to as vacuum swing adsorption (VSA).

## 2.3 Membrane processes

In case of the membrane process, the type of membrane used leads to very different process characteristics. For the production of oxygen either polymeric membranes (PEM) or ion-transport membranes (ITM) are installed.

Polymeric membranes operate at near ambient conditions. The driving force for separation is the difference in diffusion rates through the membrane [27]. The product flowrate and selectivity are the determining factors for capital cost. These are mainly dependent on the membrane area. Hence the capital cost for the process is expected to rise somewhat linearly with the product flowrate. The

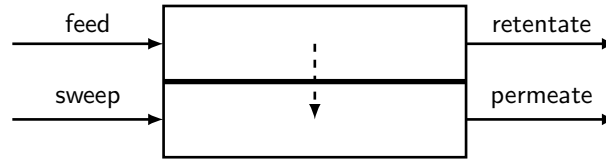


Figure 2.4: Membrane unit for gas permeation.

process can economically generate medium process stream ( $\approx 20 \text{ t/d}$ ) at lower purities. The minimal start-up time and operations near ambient conditions make the process ideal for applications where oxygen enriched process streams suffice and contaminants such as water or carbon dioxide can be tolerated. Membranes which incorporate active species that facilitate oxygen transport through the membrane bear the potential to greatly improve process performance [19]. In general are the alternative technologies more mature than membrane processes.

Processes with ion exchange membranes are drastically different from the ones with PEM's. Foremost they operate at high temperatures ( $\approx 900\text{K}$ ). At the membrane surface oxygen molecules are ionized and transported through the membrane along an externally applied electric potential. They travel through the membrane at very high flow rates. The process is capable of producing virtually pure oxygen. It is especially suited for integration with power generation processes [37].

## 2.4 Hybrid processes

The three different alternatives presented before constitute the major technologies for separations of gases and therefore the production of pure nitrogen and oxygen. All technologies have their specific advantages and disadvantages. As the separation of gases remains a major issue when facing upcoming challenges for our industries substantial efforts are being made to improve current processes in terms of ecological and economic performance. A very promising way to achieve such improvements is to combine these technologies to form hybrid processes. Membranes offer many favourable characteristics but are however rather limited when it comes to large flow-rates and high purities. When lower purities are required or the flow-rates are limited they offer however an efficient and cost effective way for gas separation. Therefore in the past years several alternatives for PSA - membrane [1] or membrane-cryogenic distillation processes [39] have been studied. The membrane can function within the process either as a pretreatment step, where an oxygen enriched stream at medium purities and high flow-rates is produced, or after the main separation, when the main byproducts are already removed, as a last stage to reach very high purities.

## 3 Process economics

Aside from question whether a certain process is capable of producing products according to its specifications, it needs to be investigated if it does so in an economically viable manner. The modeling of process economics is a powerful tool to estimate project profitability. The evaluation of process economics has three major aims in the design phase.

- Compare design options with regard to profitability.
- Economically optimize a given design.
- Estimate project profitability

In any case the total cost of a given project as well as the cash flow structure will have to be analyzed to supply the decision maker with an accurate estimate of the economic conditions. To do so adequate measures to compare and analyze a project in economic terms needs to be employed.

The following sections will first describe how to estimate the total project cost in different stages of the design process. It is evident that, as the more information about a given process becomes available, the more accurately any cost can be estimated. During the design phase of a process roughly three different stages of cost estimation can be formulated.

- An estimate before designing the process yields an order of magnitude estimation for supporting market research efforts. Error  $> 30\%$
- Estimate in the early design phase based on essential process equipment. Error  $\pm 30\%$
- Estimate based on an advanced flowsheet and relevant process parameters. Error  $\pm 20\%$

Once detail engineering commences, even more accurate calculations with errors reducing to  $\pm 5\%$  can be undertaken[31]. At that point a concrete process option will have to be chosen and the investment decision, hence the go-ahead for the project, will already have to be made. All optimization measures within the scope of this thesis will already have concluded at that time.

Within sec. 3.1 first we will take a look at how the total cost of a chemical process might be estimated at different design stages. Subsequently in sec. 3.2 the different ways of evaluating certain investment options and estimating a project's profitability will be discussed.

### 3.1 Project cost

An important factor in every given project is the total cost. During the design of a chemical process many important aspects of the future cost structure are not known a priori, as the final design is still

under development. In general the total cost of implementing and operating a production site can be broken down into several subcategories.

- Battery limit investment
- Utility investment
- Off-Site investment
- Engineering fees
- Working capital

The subsequent sections will present an overview as to how these different costs can be approximated regarding to the level of detail in the available information.

### 3.1.1 Before process design

Before any details about the process to be implemented are known, an estimate can only give an order of magnitude towards cost to be expected. The cost of the new process  $C_P$  can be related to the cost of a reference process  $C_P^0$  by

$$C_P = C_P^0 \cdot \left( \frac{Q_P}{Q_P^0} \right)^D. \quad (3.1)$$

The degression coefficient  $D$  needs to be correlated from historical data.

As the overall price structure will change over time, the reference price will not reflect the current market situation. In order to adjust for that shortcoming several price indices are published all over the world. Some of those tailor to special branches of the industry, others give a picture of the price-development in a economy as a whole. The ratio of prices levels at different times is then assumed to be equal to the ratio of the price indices at the respective times

$$\frac{C_1}{C_2} = \frac{I_1}{I_2}. \quad (3.2)$$

For each index an somewhat arbitrary reference year is chosen. Among the most common indices are the Marshall & Swift index, the Nellson-Farrar-Index or the Chemical Engineering index. Some exemplary values for these indices are given in tab. 3.1. As one can see the development within the process industry very well matches the development in the economy as a whole, which is why for this rough estimate general indices should suffice.

### 3.1.2 During process design

Once the future design has been broken down to fewer potential options and first process flowsheets are available a more elaborate approach, resulting in far improved estimates, becomes possible.

year	Marshall & Swift Installed Equipment Index 1926 $\equiv$ 100		Nelson-Farrar Refinery Construction Index 1946 $\equiv$ 100	Chemical Engineering Plant Cost Index 1957 $\equiv$ 100
	all industries	process industry		
1975	444	452	576	182
1980	560	675	823	261
1985	790	813	1074	325
1990	915	935	1226	358
1995	1027	1037	1392	381
2000	1089	1103	1542	394
2001	1093	1107	1565	396

Table 3.1: Price indices and their development.[10]

### Battery limit investment

The battery limit investment denotes all investments necessary to have all required equipment for process operations installed on-site. This includes structures necessary to house the process as well as delivery and installation of all individual assets. One major part of these cost will be the process equipment. As the exact manufacturers and models of the equipment will not be known in early design stages, an approximation similar to the one in sec. 3.1.1 still needs to be employed. In contrast to before now the cost for individual pieces of process equipment will be considered explicitly. Each piece of equipment will have an specific feature which most heavily affects its cost. For vessels and reactors this might be volume, while for heat exchangers the required heat exchange area is essential. The price for an piece of equipment  $i$   $C_E^i$  can again be approximated by a simple power law

$$C_E^i = C_B^i \left( \frac{Q_E^i}{Q_B^i} \right)^{M^i}. \quad (3.3)$$

A reference price  $C_B^i$  is multiplied by the determining quantity  $Q^i$  normalized to a reference state  $Q_B^i$  and raised to the power  $M^i$  specific to each piece of equipment. Reference prices and quantities for various installations can be obtained from literature (e.g. [35]).

If more detailed information on the process conditions are available, they should also find their way into the cost estimation. Aside from the mere size of the equipment the process conditions will also influence the expected (and actual) cost. The predominant factors to that respect are pressure, temperature, corrosiveness and reactive activity, which will require single units to be manufactured from more resistant materials. Knowledge of a specific category for a piece of equipment to be installed will also lead to refined estimates. For example an plate-fin heat exchanger might be more expensive than a tubular model with the same heat-exchange area. In order to account for all those effect a form factor  $f_F$  can be applied to the equipment cost

$$C_E^i = C_B^i \left( \frac{Q_E^i}{Q_B^i} \right)^{M^i} f_F^i = C_B^i \left( \frac{Q_E^i}{Q_B^i} \right)^{M^i} \underbrace{(1 + f_C^i + f_M^i + f_P^i + f_T^i)}_{=f_F^i}. \quad (3.4)$$

Where  $f_F$  denotes the form factor,  $f_C$  corrects for design complexity,  $f_M$  for material selection,  $f_P$  adjusts for extreme pressures and  $f_T$  for temperature.



As in the previous section all reference prices need to be scaled for the temporal price development. Again the aforementioned indices are used. Furthermore can the prices be corrected for regional differences in price structure. Once more correction factors to prices in an reference region in the world (e.g. USA) are employed.

In addition to the purchased cost, the costs for installing the process equipment have an significant effect on the total needed investment of the process. These installation costs include:

- Installations costs
- Piping ,valves and electrical wiring
- Control system
- Structures and foundations
- Insulation and fire proofing
- Labour fees

To incorporate those additional cost, again correction factors to the main equipment price are used. Depending on the status of the information available they can be expressed as one unified factor or broken down to each specific category. One however needs to bare in mind, that costs for piping and valves – pieces of equipment in direct contact with process media – will be affected by the process conditions in a similar way as the actual equipment, whereas the other categories are more likely to remain unchanged. Thus attention needs to be paid, in with fashion the factors will be applied.

A word should be said about the cost for the control system. Most obtainable data will most likely refer to a decentralized control system, as it has been in use for many years. With ever more powerful computers a centralized approach, namely model predictive control (MPC) is becoming more relevant. As the structure for such a control system may vary significantly from the common designs, the cost factors may as well.

## Services

The utility investments and off-site investments are often referred to as services. Therein included are all measures necessary to supply the process with the media consumed during operation. This includes but is not necessarily limited to generation and distribution of energy, steam and process gases. The utility investments in this context refer to all investments within the greater production site but out of the battery limits of the specific process. Off-site investments contain everything not contained in the site such as roads, power cables, communication systems or waste disposal. All these costs are expressed as fractions of the equipment cost at moderate temperature and pressure. This means, when applying these fractions, the factors  $f_M$ ,  $f_P$  and  $f_T$  should not be considered at this point.

Once again in early design stages one has to resort to factors derived for statistical data, to calculate the cost of raw materials, energy and support media such as lubricants, heat or catalysts. If more detailed information on process streams is available the approach should be refined.

The cost of raw materials  $C_{RM}$  can then be calculated if the material streams of individual raw materials  $m_i$  as well as their specific cost are known.

$$C_{RM} = \left( \sum_i^{N_{RM}} \dot{m}_i \cdot C_{RM}^i \right) \cdot t_{op}. \quad (3.5)$$

Much in the same way the cost for energy can be approximated. This is once more done for each individual process unit, rather than for the whole process as it has been done for the raw materials. Hence with the needed energy for equipment  $i$  with respect to energy carrier  $j$   $e_{ij}$  along with the price for energy carrier  $j$   $C_{EC}^j$  the total energy cost  $C_{EC}$  can be assessed.

$$C_{EC} = \left( \sum_i^{N_E} \sum_j^{N_{EC}} \dot{e}_{ij} \cdot C_{EC}^j \right) \cdot t_{op}. \quad (3.6)$$

### Working capital

The working capital includes all investments necessary for process operations. This means raw materials, payroll, extended credit to customers and so on. In contrast to all other costs the working capital can partially be retrieved when the process ceases operations. How different types of cash flows, extended or owed credit should be handled will be discussed in sec. 3.2. In addition to the cost of raw materials needed during process operations, which generate a product stream, raw materials are also needed to fill all vessels, reactors, columns and piping that make up the process. The cost of these should be considered as investment and not as operation cost [10], since more or less the same amount will remain bound until the process ceases operations and it can (partially) be retrieved.

$$C_{FILL} = \sum_i^{N_{RM}} V_i \cdot C_{RM}^i. \quad (3.7)$$

### Total investment

When all contributions to the total investment are considered an estimate for the total price of the process can be calculated

$$C_P = \sum_i^{N_E} \left[ C_B^i \left( \frac{Q^i}{Q_B^i} \right)^{M^i} \left( f_F \cdot f_{PIPE} + \sum_j f_j \right) \right] + C_{RM} + C_{EC} + C_{FILL} \quad (3.8)$$

It should be emphasized that in early design stages these calculations will at best yield an order of magnitude estimate for the expected cost of implementing a chemical process. Most literature sources give an accuracy of  $\pm 30\%$  [31]. As the project progresses more and more information becomes available a more accurate estimate can be prepared. The most refined ones rely on actual proposals from prospective manufacturers and suppliers.

## 3.2 Investment criteria

The total cost of a project is a very important measure to decide whether to embark on a certain endeavor. However in a complex financial system it cannot be taken as the sole factor to compare investment alternatives. Different other indices are used to measure the attractiveness or feasibility of an investment. One main distinction can be made between different measurements. Those that work with averaged cash flows and consider the project as a single time period (sec. 3.2.1) and models that take into account cash flows made at different points in time – so called multi-period models (sec. 3.2.2).

### 3.2.1 Single period estimation methods

Among the most prominent single period methods are the payout time (POT) and the return on investment (ROI) which will be discussed in this section.

#### Payout time

Payout or amortization time is the time necessary to earn the total investment of the process. It is also often referred to as the break even point. A profitable venture will from that point on begin to make money. A shorter payback time can be considered a measure for a more attractive investment.

$$POT = \frac{\text{capital expenditure}}{\text{incoming cash flow} / \text{period}} \quad (3.9)$$

However many aspects especially in potential profits are not included here.

#### Return on investment

The return on investment is defined as

$$ROI = \frac{\text{average return} / \text{period}}{\text{capital expenditure}} \cdot 100\%. \quad (3.10)$$

It denotes the equivalent to an interest rate if the earned interest is not reinvested such that the same investment is considered in every period. The average return is calculated from the expected returns over certain duration of time. The time considered might either be the total lifecycle of the process or the expected write-off time. As capital expenditure most likely the total investment will be used. However it might give a more precise picture if write-off or the time of individual payments is considered when evaluating the capital expenditure.

### 3.2.2 Multi period estimation methods

As discussed earlier, when using only averaged values and considering only one time period, a distorted picture of the financial situation will evolve. The sources of the money to be invested play a critical

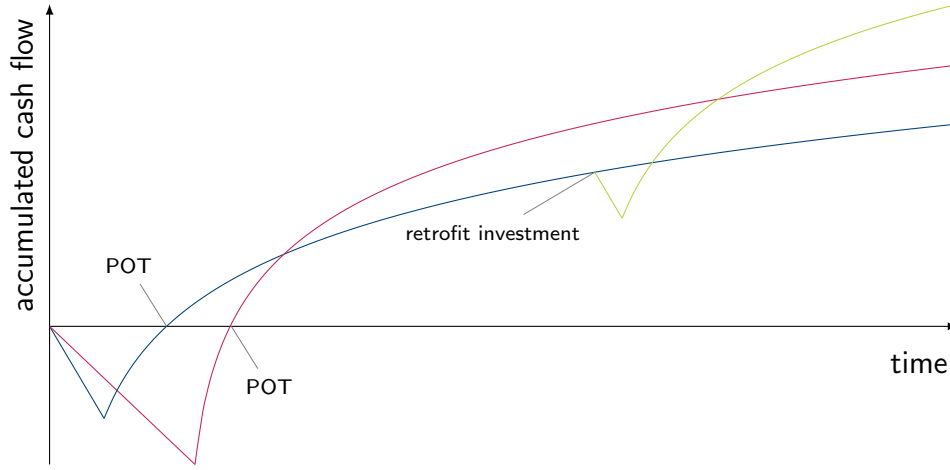


Figure 3.1: Accumulated cash flows over project life cycle.[26]

role in the evaluation process. Cash reserves, loans, preferred stock or other financial instruments are all viable sources for investment capital. Especially for large scale projects often a mixture of many different sources is used to allocate all necessary funds. A generic structure of the cash-flows during the life-cycle of a given project is shown in fig. 3.1. When the project commences no revenue is generated and only investments are made. As process operations begin revenue is generated and the slope of the accumulated cash flow curve switches to an upward directions – assuming the project is profitable. As time progresses the produced product might reduce in value through competitors entering the market or other factors. When the curve intersects the abscissa the project investment is earned and the break point is reached. The two investment alternatives highlight that the amount of initial investment will affect the expected revenue. As more information is gathered during operations, opportunities may arise to optimize the process and leverage so far dormant potentials. These so called retrofit Investments would ideally lead to an improved cash flow structure as indicated by the green line.

Before further investigating multi-period models some basic concepts of financial mathematics and the treatment of interest should be reviewed. When investing the capital  $C_0$  at compounded interest for  $n$  years at a rate of  $i$  %, the compound amount will yield

$$C_n = C_0 \cdot q^n. \quad (3.11)$$

Where  $q$  denotes the interest factor

$$q = 1 + \frac{i}{100}. \quad (3.12)$$

On the other hand the current value of an investment that will yield  $C_n$  in  $n$  years can be calculated by

$$C_0 = C_n \cdot q^{-n} \quad (3.13)$$

The above considerations always assume a single payment at the beginning or end of the entire period. Annuities however are usually in several tranches with regular payments to be made or received at predefined instances. Assuming those payments are of equal size  $a$ , the final value can be computed

by

$$C_n = a \cdot q^\alpha \cdot \frac{q^n - 1}{q - 1}, \quad (3.14)$$

while the current value of an annuity yields

$$C_0 = a \cdot q^{\alpha-n} \cdot \frac{q^n - 1}{q - 1}, \quad (3.15)$$

The factor  $\alpha$  in the previous equations denotes whether payments are made at the beginning of a period ( $\alpha = 1$ ) or at the end ( $\alpha = 0$ )

With that multi-period model can be discussed. Here not only different alternatives can be compared, but also the profitability in comparison with investments in financial products can be assessed.

### Net present value

The net present value (*NPV*) describes the amount of money to be invested if all cash flows – incoming and outgoing – are discounted to the project start ( $t = 0$ ). The present value of all expenses is then

$$C_{0e} = e_0 + e_1 \cdot q^{-1} + \dots + e_n \cdot q^{-n} = \sum_i^n e_i \cdot q^{-i}. \quad (3.16)$$

Here it is given that all expenses are paid at the beginning of each period, as it is the most common case. If revenues  $r_i$  are realized then as well an equivalent formula would be attained. In most cases revenues will come in at the end of a period in which case the present value becomes

$$C_{0r} = r_0 \cdot q^{-1} + r_1 \cdot q^{-2} + \dots + r_n \cdot q^{-(n+1)} = \sum_i^n r_i \cdot q^{-(i+1)}. \quad (3.17)$$

The *NPV* of a given project is then derived from the present values of all expenses and revenues

$$NPV = C_{0r} - C_{0e}. \quad (3.18)$$

For any project to be considered as investment alternative the present value needs to be positive since otherwise the investment would yield losses.

### Discounted cash flow rate of return

The formula for the discounted cash flow rate of return is very similar to the one for a net present value. The difference is, that rather than computing the net present value with given interest rates, the present value is set to zero and the resulting interest rate is then calculated. Other than with the previous methods no analytical solution can be presented but rather an iterative approach to find a solution to

$$0 = \sum_i^n (r_i - e_i) \cdot q^{-i}. \quad (3.19)$$

Here it was assumed that all payments – incoming and outgoing – are made at the beginning of a period. The only variable is the interest rate which is included in the interest factors  $q$ . This method gives the interest rate which would be necessary to earn all time dependent expenses within  $n$  years. In this case a higher  $DCFRR$  indicates a more attractive investment.

#### Annuity method

Within the annuity method two different annuities are calculated and then compared. First the annuity  $a$  of an investment of  $C_0$  at market conditions is of interest.

$$a = C_0 \cdot \frac{q^{n-\alpha}(q-1)}{q^n - 1} \quad (3.20)$$

This annuity is the amount that could be paid out each period if  $C_0$  is invested, compound interest is considered and all funds are used up at the end of  $n$  years.

This is then compared to the annuity of the considered project

$$a_P = \sum_i^n (r_i - e_i) \cdot q^{-i} \cdot \frac{q^{n-\alpha}(q-1)}{q^n - 1}. \quad (3.21)$$

Only if  $a_P > a$  is the project more profitable than simply investing the required capital in a financial product.

#### Interest Rates

With all the presented models it was implicitly assumed that interest rates for debit and credit are equal. The reality however is much different. It will therefore be prudent to use different rates of interest for each case. The basic calculations however remain unchanged. Especially the interest rate that can be earned when investing a certain amount will need to be estimated. As certain products with a known  $ROI$  will not necessarily be the most attractive options on the capital market. Thus in many companies there is an internal value given for this interest rate, which is calculated from historical data.

## 4 Mathematical process model

This chapter will illustrate different process units and their mathematical representation. As depicted in fig. 4.1 three main tasks need to be performed within the cryogenic air separation process. These are pre-purification and compression of raw materials, heat exchange with product streams and separation of streams. While those different stages are highly interdependent they will be discussed separately and mathematical models for the various process units contained in each stage will be presented.

The first section (sec. 4.1) deals with a steady state version of the process model. Especially for steady state models the issue of initializing the variables in such a way that the solver can converge to a solution becomes crucial. With that in mind the strategies developed to initialize in particular the distillation columns will also be elaborated upon.

The second section (sec. 4.2) is devoted to a dynamic version of the models for some process units. In addition to the models themselves several aspects which arise, when considering process dynamics will also be part of that section.

The third section (sec. 4.3) of this chapter takes a closer look at how the economics of the process can be captured within the framework of process simulation and optimization. Understandably, the issue of economic evaluation is closely linked to the minimum sizing of each process unit. As these aspects are for the most part treated uniformly for the steady state case and dynamic case – since only algebraic equations are involved – they will be dealt within a separate section.

All presented models have been implemented in the equation based process simulator gPROMS<sup>®</sup>. Certain aspects arising when implementing the models will be discussed in the last section of this chapter sec. 4.5.

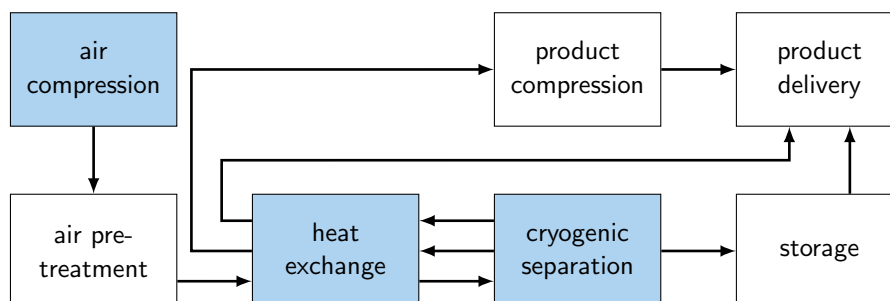


Figure 4.1: simplified cryogenic air separation process.

## 4.1 Steady - state unit models

### 4.1.1 Separation

The separation step within the process occurs within a double effect distillation column. Distillation columns are among the most widely studied pieces of process equipment. While much has been accomplished, the robust simulation poses great challenges to this day. In the context of cryogenic air separation the associated mixture displays only moderate non-idealities and furthermore is a zeotropic one. However further complexity is added due to the strong interdependencies of the different columns arising from thermal and material coupling within the process.

In this section first the mathematical formulation for a general steady-state distillation column model, which has been developed as a multi-purpose, easily applicable model for the process simulator gPROMS<sup>®</sup>, will be presented (sec. 4.1.1).

#### Distillation column model

The core equations decerning the operation of any distillation column, or more general vapour liquid contacting device, are often referred to as the MESH equations . These equations, although rather plain at first glimpse, form a set of highly non-linear, highly coupled equations. The solutions of which poses a non-trivial task for current solution algorithms, whose success is highly dependent on the quality of initial estimates for the involved variables. Therefore a strategy used for the automated generation of such guesses will be described as well.

The model presented here is not only to be used for the simulation of of a given process, but rather for optimization purposes as well. While several of the decision variables associated with the process can be optimized from a generic set of equations, others need further consideration. The optimization decisions which add the greatest complexity for distillation columns are the locations for feeds and side draws, as well the number of trays for a given separation task. Part of the added complexity is the introduction of integer decisions into the model and and the associated need for a super structure which includes all process alternatives to be considered.

The starting point for the model development is a single separation stage within the column with all possible material and energy streams entering and leaving the stage. Fig. 4.2a shows the superstructure to be used for the complete column, while fig. 4.2b shows a super structure for a single separation stage.

As mentioned before the governing equations for a distillation column are known as MESH equations. The acronym MESH stands for material, equilibrium, summation and enthalpy (often represented by the symbol  $H$ ). If not stated otherwise, all equations presented in this section apply to all separation stages. In order to improve readability the indices denoting a stage are sometimes omitted, if not essential for the meaning of a given equation. throughout the description of the models and the thesis the number of stages in a column section will be denoted by  $n_S$ , while  $n_C$ ,  $n_F$ ,  $n_{SV}$  and  $n_{SL}$  represent the total number of components, feeds, vapour and liquid side draws respectively.



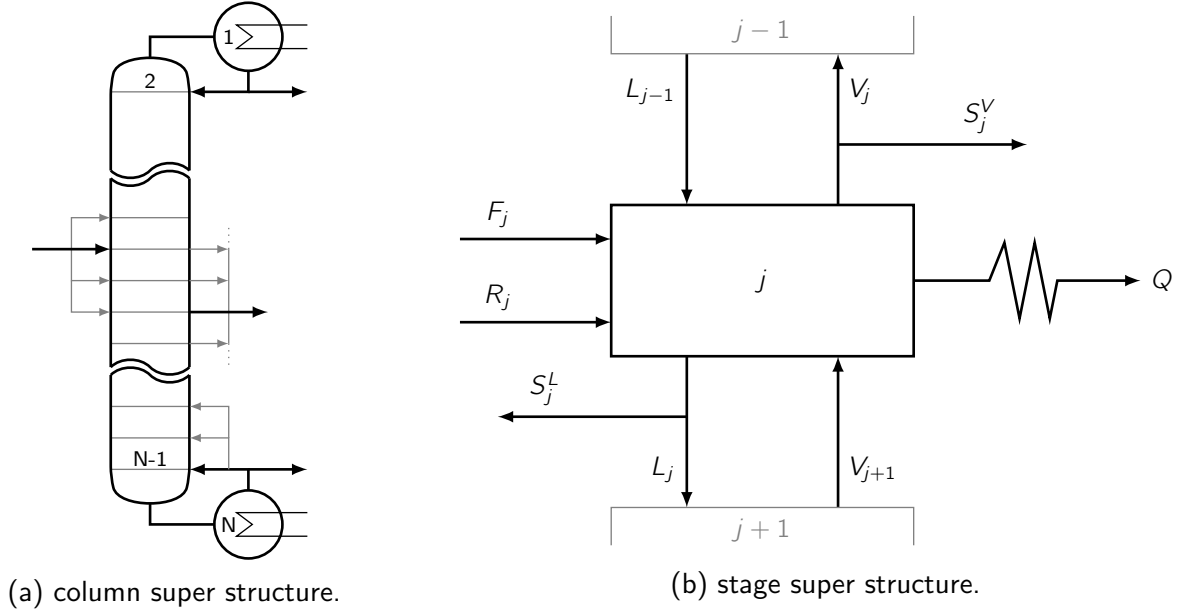


Figure 4.2: superstructures for column and column stages.

**Material balances** in their most general form for an inner column stage can be written as

$$0 = (V_j + S_j^V) \cdot y_{i,j} + (L_j + S_j^L) \cdot x_{i,j} - V_{j+1} \cdot y_{i,j+1} - L_{j-1} \cdot x_{i,j-1} - F_j \cdot z_{i,j},$$

$$i = 1 \dots n_C \quad j = 1 \dots n_S. \quad (4.1)$$

A certain amount of the liquid or vapour leaving a stage can be withdrawn from the column by means of the vapour  $S_j^V$  and liquid  $S_j^L$  side draw streams. During model development it occurred, that when a specification was made on given side-draw, convergence was sometimes hindered. This behaviour could be improved by specifying a dimensionless side-draw,

$$s_j^V = \frac{S_j^V}{V_j}, \quad j = 1 \dots n_S, \quad (4.2)$$

$$s_j^L = \frac{S_j^L}{V_j}, \quad j = 1 \dots n_S, \quad (4.3)$$

rather than an actual stream. Although this technically introduces further non-linearities into the material balances. As it is difficult, to guess the dimensionless side draw for a desired side draw flowrate, the specification can be replaced by a specification on the flowrate, once model initialization has been achieved.

By considering these dimensionless side-draws the material balance can be rewritten as

$$0 = (1 + s_j^V) \cdot V_j \cdot y_{i,j} + (1 + s_j^L) \cdot L_j \cdot x_{i,j} - V_{j+1} \cdot y_{i,j+1} - L_{j-1} \cdot x_{i,j-1} - F_j \cdot z_{i,j},$$

$$i = 1 \dots n_C \quad j = 1 \dots n_S. \quad (4.4)$$

As the model is also to be used for optimization of structural decisions, further extensions are necessary. To accommodate that need, new variables are introduced. Particularly split variables  $\zeta$  as employed by [11] are introduced for feeds, vapour and liquid side draws and the reflux location.

As the feeds are and side draws are to be fed or taken from a single stage, the split variables must assume integer values of either one or zero. Furthermore are those decisions *exclusive or* decisions, hence

$$1 = \sum \zeta. \quad (4.5)$$

In order to optimize the number of stages several superstructures are possible. One can optimize the reboiler reflux location and condenser reflux location or each single one along with the feed and side draw locations. The stage number is then changed as all stages between condenser or reboiler reflux are effectively rendered inactive. The solution of the mass and energy balances for each inactive stage becomes trivial as only one single vapour or liquid stream enters and exits the stage.

While the choice if condenser and or reboiler reflux is optimized is somewhat arbitrary some studies have shown [15] that the strategy of optimizing only feed location and reboiler reflux location possesses some numerical advantages in terms of performance of the solution algorithm. However this observation might be specific to the particular solver used. Furthermore, as will be seen later, there are certain situations where optimizing either reboiler or condenser reflux is beneficial in terms of problem formulation. The developed model therefore offers capabilities to choose, whether the top or bottom reflux is variable. Either reflux will be represented by the generic variable  $R$ .

With the newly introduced split variables for the feed  $\zeta_{ij}^F$  as well as the reboiler reflux  $\zeta_j^R$  and the liquid  $\zeta_{ij}^{SL}$  and vapour  $\zeta_{ij}^{SV}$  side draws, the material balances can be written as

$$\begin{aligned} 0 = & (1 + s_j^V) \cdot V_j \cdot y_{i,j} + (1 + s_j^L) \cdot L_j \cdot x_{i,j} - V_{j+1} \cdot y_{i,j+1} \\ & - L_{j-1} \cdot x_{i,j-1} - \sum_{k=1}^{n_F} \zeta_{kj}^F \cdot F_j \cdot z_{i,j} - \zeta_j^R \cdot R \cdot z_R, \\ & i = 1 \dots C, \quad j = 1 \dots n_S, \quad k = 1 \dots n_F. \end{aligned} \quad (4.6)$$

Furthermore to be able to optimize side draws, the stripping factors have to be reformulated accordingly

$$s_j^V = \frac{\sum_{i=1}^{n_{SV}} \zeta_{ij}^{SV} S_j^V}{V_j}, \quad j = 1 \dots n_S, \quad i = 1 \dots n_{SV}, \quad (4.7)$$

$$s_j^L = \frac{\sum_{i=1}^{n_{SL}} \zeta_{ij}^{SL} S_j^L}{L_j}, \quad j = 1 \dots n_S, \quad i = 1 \dots n_{SL}. \quad (4.8)$$

**Equilibrium** between a vapour and liquid is achieved when the chemical potentials for each component in both phases are equal. This is commonly expressed by an equilibrium  $K_i$  constant. With that the equilibrium equation can be written as

$$y_i^{eq} = K_i \cdot x_i, \quad i = 1 \dots n_C. \quad (4.9)$$

The requirement for equal chemical potentials can be expressed in terms of the vapour  $f_i^V$  and liquid  $f_i^L$  fugacities [2].

$$f_i^V = f_i^L, \quad i = 1 \dots n_C. \quad (4.10)$$

These fugacities are calculated from the liquid activity coefficients  $\gamma_i$ , the pointing factor  $F_{Pi}$ , the reference vapour fugacity coefficient  $\varphi_i$ , the component saturation pressure  $p_i^S$  as well as the system pressure  $p$  along with the vapour and liquid molar fractions

$$\gamma_i F_{Pi} \varphi_i^0 p_i^S x_i = \varphi_i p y_i, \quad i = 1 \dots n_C. \quad (4.11)$$

After simple rearrangement eq. (4.12) an expression for the equilibrium ratios can be derived

$$y_i^{eq} = \underbrace{\frac{\gamma_i F_{Pi} \varphi_i^0 p_i^S}{\varphi_i p}}_{K_i} x_i, \quad i = 1 \dots n_C. \quad (4.12)$$

The equations to determine the quantities used when computing the equilibrium ratios are by themselves functions of temperature, pressure, and vapour as well as liquid molar fractions. They are further discussed in sec. ???. It therefore becomes evident that the equilibrium ratios are a major source for the non-linearities in the presented model.

When simulating a distillation process one might consider only equilibrium stages. This however will in most cases not reflect the true behaviour of a real stage as the time a vapour and liquid phase are in contact might not suffice to achieve equilibrium conditions. This can be accounted for by introduction of Murphee tray efficiencies [17]

$$\eta_{ij}^{eq} = \frac{y_{ij} - y_{ij+1}}{y_{ij}^{eq} - y_{ij+1}}, \quad i = 1 \dots n_C \quad j = 1 \dots n_S. \quad (4.13)$$

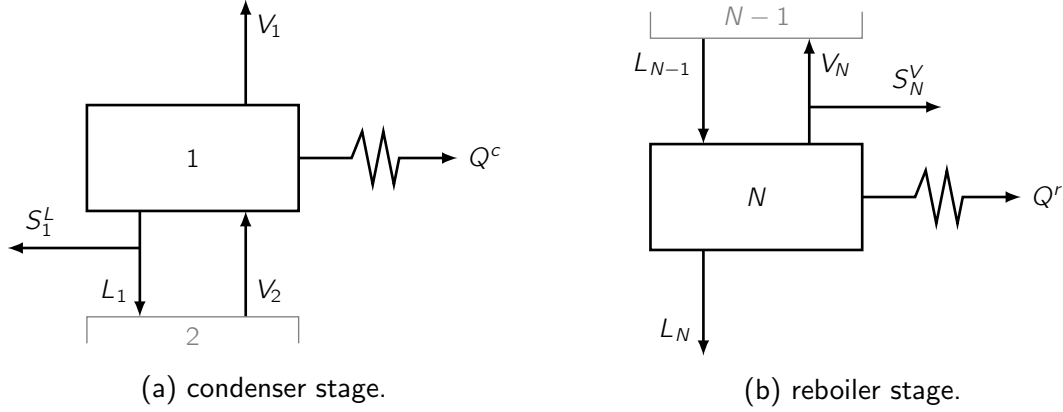
While several models to approximate these efficiencies have been investigated, their predictions are most often rather poor [10] and their evaluation rather complex. Furthermore as indicated in the formula they might differ for different species involved. For the scope of this model however they are not computed separately, but rather supplied and assumed constant for all stages and components.

**Enthalpy balances** are again written considering the previously defined stripping factors and splitting variables

$$\begin{aligned} 0 = & (1 + s_j^V) \cdot V_j \cdot h_j^V + (1 + s_j^L) \cdot L_j \cdot h_j^L - V_{j+1} \cdot h_{j+1}^V \\ & - L_{j-1} \cdot h_{j-1}^L - \sum_{k=1}^{n_F} \zeta_{kj}^F \cdot F_k \cdot h_j^F - \zeta_j^R \cdot H_R - Q_j, \\ & i = 1 \dots n_C, \quad j = 1 \dots n_S, \quad k = 1 \dots n_F. \end{aligned} \quad (4.14)$$

**Condenser and reboiler** are modeled more or less as regular column stages. However they possess certain specialties that are explicitly considered in the column model. For one it is assumed that no feeds enter the condenser stage. Furthermore no vapour side stream is drawn from the condenser as well as no liquid side stream from the reboiler.

Additionally the condenser stage needs to be examined a little further. In terms of operations several assumptions can be made for the condenser. In general one can distinguish a total, partial vapour



and partial vapour liquid condenser. For the total condenser all vapour that enters the respective stage is condensed and only liquid product is drawn. The partial vapour condenser condenses only the vapour the which is to be fed back into the column and all product that is drawn is gaseous. The partial vapour liquid condenser denotes the most general case, where part of the incoming vapour is condensed and product is drawn from the vapour and liquid phase. The most important thing to consider in these different cases is, that while in both partial condensers a vapour liquid equilibrium takes place. Due to the absence of vapour the same does not hold for the total condenser. To accommodate that fact the MESH equations have to be adjusted [28]. While the material and energy balances remain unchanged the equilibrium equations have to be altered. First the vapour and liquid compositions are set equal for all but one component

$$x_{i1} = y_{i1} \quad i = 1 \dots C - 1, \quad (4.15)$$

and the condenser temperature is determined by the bubble point equation

$$1 = \sum_i K_{i1}(T_1, p_1, x_1, y_1) \cdot x_{i1} \quad i = 1 \dots n_C. \quad (4.16)$$

When implementing the model in a process simulator it is sensible to consider, that due to the limited accuracy of computers the omitted component in eq. (4.15) needs to be a non-trace component in the condenser stage. The implemented model therefore has to specify such a component when a total condenser is chosen to avoid numerical difficulties.

In practice it is highly unlikely, that the exact amount of energy required to condensate all liquid will be drawn from the condenser. More likely, if all vapour is condensed, slightly more energy will be withdrawn and sub-cooled liquid will leave the condenser. Therefore the model includes the possibility to specify a degree of sub-cooling  $T^{sub}$  which will be considered when calculating bubble point temperature

$$1 = \sum_i K_{i1}(T_1 + T^{sub}, p_1, x_1, y_1) \cdot x_{i1} \quad i = 1 \dots n_C. \quad (4.17)$$

**Pressure** along the entire column is often specified in the steady-state case. However as it is inconvenient and unpractical to specify a pressure for each stage one might either specify a pressure

at the top and bottom stage and assume a uniform pressure profile along the column, or specify either top  $p_1$  or bottom pressure  $p_N$  along with a total pressure drop  $\Delta p$ , or a stage-wise one  $\Delta p_{stage}$ . However the issue is further complicated if one considers the case of optimization for number of trays. In that case several trays will become inactive. For those trays the mass and energy balances become trivial, as only liquid enters and exits these trays. (For the case employed here, where the reboiler reflux is being optimized). This also means, that from the last active tray down to the reboiler – if present – there should be a uniform profile. It is important to note, that if a uniform pressure profile from the lowest active stage downward is not enforced, the solver will have to compensate for slight changes in the equilibrium due to pressure variations with minimal vapour flow-rates. This is very undesirable, as it might lead to severe problems in the solver, or the calculation of other properties, dependent on these values. To account for this issue, the reboiler reflux split be employed to deactivate the pressure drop

$$p_i = p_{j-1} + \left(1 - \sum_{k=1}^{j-1} \zeta_k^R\right) \cdot \Delta p_{j-1} \quad j = 1 \dots n_S. \quad (4.18)$$

As an alternative to specifying the pressure, one might consider calculating the pressure drop from (semi)-empirical models. There are numerous correlations for different types of column internals. These correlations become particularly important if column dynamics are to be considered, as they form the connection between holdups and flow-rates within the column. Two different pressure drop models have been implemented, one for trayed columns and another one for structured packings. As they are closely tied to dynamics, they will be discussed in more detail in sec. ??.

## Specifications & initialization

The equation systems presented above is comprised of  $n_S n_C$  component balances and equilibrium equations,  $2n_S$  summation equations and  $n_S$  energy balances. This gives a total of  $n_S(2n_C + 3)$  equations. On the other hand there are  $n_S$  temperatures and pressures,  $2n_S$  molar flow rates,  $n_S$  energy streams, and  $n_S n_C$  vapour as well as liquid concentrations. Additionally the feed flow rates compositions and temperatures and the side draw split fractions or flow rates appear as variables. The feeds and side draws would usually be specified, which leaves a total of  $n_S(2n_C + 5)$  variables.

The pressure profile of a distillation column is usually specified. In terms of unit operations this pressure drop is of high significance, as many columns can only be feasibly operated, if the pressure drop does not exceed certain limits. In case of the ASU the production of Argon only became feasible as structured packings, which display a very low pressure drop, became available. This is due to the large number of theoretical stages required to attain the desired Argon purities.

The energies  $Q_i$  denote addition heaters or cooler on the respective stages. For all intermediate stages these values would be specified as well. If all energies are specified, that would – along with the pressure profile – sum up to  $2n_S$  specifications, which leaves  $n_S(2n_C + 3)$  unknowns. As the number of equations and unknowns are the equal, this system is then well specified.

In practice it is often challenging to correctly guess the condenser and reboiler heat loads in advance. This is especially true since they have a tremendous impact on the overall performance of the column. Hence it is often desirable to supply other specifications than the respective heat loads. To allow for

specification	replacement for $H_1$	replacement for $H_N$
reflux or boilup ratio	$0 = L_1 - \nu^D \cdot (V_1 + S_1^L)$	$0 = V_N - \nu^R \cdot L_N$
temperature	$0 = T_1 - T_{spec}$	$0 = T_N - T_{spec}$
product flowrate	$0 = (V_1 + S_1^L) - D$	$0 = L_N - B$
component product flowrate		$0 = L_N \cdot x_{iN} - b_i$
mole fraction	$0 = y_{i1} - y_{i,spec}$	$0 = x_{iN} - x_{i,spec}$

Table 4.1: discrepancy functions for different column specifications.

such specification so called discrepancy functions can be introduced [17], which replace the energy balance for the condenser and / or reboiler stage.

One common specification is the so called reflux ratio  $\nu^D = \frac{L_1}{V_1 + S_1^L}$  for the condenser, or the boil-up ratio  $\nu^R = \frac{L_N}{V_N}$  for the reboiler. They are defined as the ratio of the molar flowrate sent back into the column over the product flowrate which leaves the column. For the reboiler this denotes a liquid stream, while for the condenser the product can be gaseous and liquid. Specifying this leads to

$$0 = L_1 - \nu^D \cdot (V_1 + S_1^L), \quad (4.19)$$

$$0 = V_N - \nu^R \cdot L_N, \quad (4.20)$$

as discrepancy functions. In addition to that further specifications are conceivable. Most commonly distillate ( $D$ ) or bottoms ( $B$ ) flow rates, purities, component flow rates ( $d_i$ ,  $b_i$ ) or temperatures. The corresponding discrepancy functions are summarized in tab. 4.1.

The specifications for the reboiler stage are quite straightforward, in contrast to that, different cases for the condenser have to be considered. In the most general case the top product can be drawn as vapour and liquid. This case is here called a partial vapour liquid condenser. The other cases are a total condenser, where all the vapour entering the condenser stage is condensed, and all product is drawn as a liquid stream, as well as a partial vapour condenser, where only the reflux is condensed and all product is drawn as vapour. As discussed earlier no VLE takes place in the condenser stage, if a total condenser is specified, which needs to be accounted for. Both the total and partial vapour condensed implicitly include an extra specification since in former case the top vapour product flow rate becomes zero and in the latter the top liquid product flowrate. Furthermore a specification of the condenser energy is infeasible as well as implicitly given for the total condenser. In case of the partial vapour liquid condenser no implicit specification is given, which requires an additional specification. In general two top specifications are necessary, whereas only one bottom specification is required. These top specification can include the condenser duty, any top flowrate, the reflux ratio as well as a newly introduced quantity, the top vapour fraction defined as

$$\nu^{vap} = \frac{V_1}{V_1 + S_1^L}. \quad (4.21)$$

As mentioned before the solution of the MESH equations can pose a considerable problem to numerical solvers. It is therefore necessary to supply the solver with feasible estimates for the involved variables that can be used as an initial guess for convergence of the process model. A lot of effort has been spend to formulate robust strategies to initialize distillation column models. One of the most prominent is the so called Inside-Out algorithm first introduced by Boston and Sullivan [6]. Within this algorithm an inner and outer iterative loop are employed. Within the outer loop

approximate parameters for simplified models of phase equilibrium and enthalpy are computed by rigorous thermodynamic models and guesses for stage temperatures and concentrations. Within the inner loop new stage temperatures and concentrations are attained by solving the MESH equations using the simplified thermodynamic models. Once the inner loop converges the simplified model parameters are updated within the outer loop by means of the newly calculated temperatures and concentrations. This algorithm converges in many cases even for very poor initial guesses and has been extended to handle complex columns with side-draws and even reactive distillation [5]. It is still in use in the process simulator Aspen Plus<sup>®</sup>. However as it is used within an modular algorithmic environment it is not applicable to equation based simulators such as gPROMS<sup>®</sup>.

More recently other approaches have been published to attain improved initial guesses. Fletcher and Morton [13] proposed the solution of a column model at infinite reflux and zero feed flow rate. This leads to a much simplified model which can be solved more easily. The computed purities and stage numbers can give valuable insight into the process model. As this approach relies on the solution of a simplified model and has no algorithmic elements, it can be implemented in equation based process simulators.

Another strategy that has been successfully applied to zeotropic and azeotropic mixtures relies on solving the column model for the limiting case of the adiabatic column [4]. The adiabatic column in this case is the column with the minimal entropy production in a real column. To avoid entropy production all streams that come in contact must be in equilibrium. To achieve this the column would have to employ an infinite number of stages and have an infinite number of heat exchangers along its length. The adiabatic column then uses only two heat exchanger in the condenser and reboiler stage and assumes a pinch point at the feed stage.

Furthermore a much simpler approach has proven adequate for many applications [17] which is also employed as a starting point in this work. Therein feed properties are used as initial guesses. First a linear temperature profile from the boiling temperature to dew temperature of the combined feed mixture is used to initialize temperatures, whereas a simple flash at average column pressure and feed temperature yields a vapour and liquid concentration which is used as uniform profile for every column stage. However as the feed might be sub-cooled liquid or super-heated vapour, the TP-Flash is replaced by a specified vapour fraction. As vapour fraction for the flash initial estimates of the vapour and liquid flow rates at the top and bottom of the column are used. The stage-wise molar flow-rates are computed from the constant molal overflow assumption, which postulates a constant heat of vaporization and yields therefore uniform liquid and vapour flow-rates within a column section. A section in this case is denoted by any feed location where the flow-rates change due to the added feed. In the feed stages a super heated or sub-cooled feed is also considered by means of an extended vapour fraction

$$q^F = \frac{h^F - h^L}{h^V - h^L}. \quad (4.22)$$

While this approach leads to model convergence in many cases, it is not entirely robust. While the system considered in this case displays only moderate non-idealities it is highly cupeled. Especially the low pressure column (LPC) has multiple feeds and side draws, which leads to non-convergence if the aforementioned initialization strategy is employed. However the fact that the system is not highly non-ideal can be exploited. Whenever the K-values are not too much dependent on mixture composition an intermediate step can be used to refine concentration guesses. The constant molal

overflow assumption is retained and the equilibrium ratios are computed based on the initial guesses from the first stage. The component balance is then reformulated only in terms of liquid component flow-rates  $l_{ij}$

$$0 = \left( (1 + s_j^V) \cdot K_{ij} \cdot \frac{V_j}{L_j} + (1 + s_j^L) \right) \cdot l_{ij} - \frac{V_{j+1}}{L_{j+1}} \cdot K_{ij+1} \cdot l_{ij+1} - l_{ij-1} - F_j^V \cdot z_{ij}^V - F_j^L \cdot z_{ij}^L, \quad i = 1 \dots n_C \quad j = 1 \dots n_S. \quad (4.23)$$

eq. (4.23) is linear in the liquid component flow rates. Furthermore vapour component flow rates are substituted in the linear component balance and can be computed by

$$v_{ij} = K_{ij} \cdot \frac{V_j}{L_j} \cdot l_{ij} \quad i = 1 \dots n_C \quad j = 1 \dots n_S. \quad (4.24)$$

One of the reasons eq. (4.23) is formulated in terms of component flow rates rather than molar fractions, is that the molar fraction computed in that manner would not be normalized. If the mole fractions are computed from the component flow rates normalization is implicitly given

$$x_{ij} = \frac{l_{ij}}{\sum_k^C l_{kj}} \quad i = 1 \dots n_C \quad j = 1 \dots n_S, \quad (4.25)$$

$$y_{ij} = \frac{v_{ij}}{\sum_k^C v_{kj}} \quad i = 1 \dots n_C \quad j = 1 \dots n_S. \quad (4.26)$$

The total molar flow rates used in eq. (4.23) are computed by solving stage-wise total mass balances under the constant molal overflow assumption. This assumption postulates that the heat of vaporization is independent of system composition. Therefore always the same amount of liquid enters and leaves a given stage

$$0 = L_j + S_j^L - L_{j-1} - (1 - q^F) \cdot F_j \quad j = 1 \dots n_S. \quad (4.27)$$

The vapour total flow rates are then computed from the total mass balances

$$0 = L_j + S_j^L + V_j + S_j^V - L_{j-1} - V_{j+1} - F_j \quad j = 1 \dots n_S. \quad (4.28)$$

As no energy balances are included at this stage, the condenser and reboiler stage are characterized by the reflux ( $\nu^c = \frac{V_1}{L_1}$ ) or boilup ratio ( $\nu^r = \frac{V_N}{L_N}$ ) respectively. This leads to

$$0 = V_1 - \nu^c \cdot L_1, \quad (4.29)$$

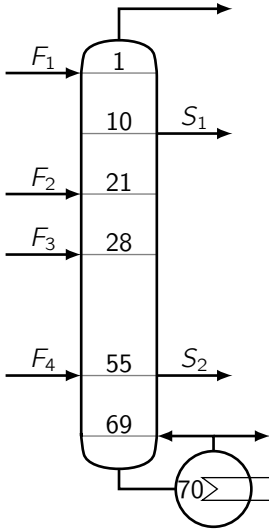
$$0 = L_N - \nu^r \cdot V_N. \quad (4.30)$$

To close the equation system the global mass balance is included

$$0 = V_1 + L_N + \sum_{j=1}^N (S_j^V + S_j^L - F_j), \quad j = 1 \dots n_S. \quad (4.31)$$

For more complex systems more elaborate strategies have been developed, which essentially try to incorporate some of the principals from the inside out algorithm into an equation based environment.





feed specifications						
stream	flow [ $\frac{kmol}{hr}$ ]	$z_{O_2}[-]$	$z_{N_2}[-]$	$z_{Ar}[-]$	$T[K]$	$p[bar]$
$F_1$	2985.77	4.674E-10	0.9999	6.378E-7	79.45	1.3
$F_2$	1836.36	0.2095	0.7812	0.0093	98.91	1.3
$F_3$	7609.06	0.2920	0.6950	0.0130	81.88	1.3
$F_4$	774.94	0.9161	5.393E-12	8.394E-2	92.13	1.8

column specifications					
stages	$S_1$ frac	$S_2$ frac	boilup ratio	$p^{top}$	$p^{bot}$
70	10	0.15	3.5	1.2 bar	1.3 bar

Table 4.2: column specifications.

Figure 4.3: example column.

The process simulator gPROMS<sup>®</sup> allows for definition of standardized initialization procedures as well as different model variants that can be solved consecutively. With that strategies were developed that proved successful for more complex mixtures and even three phase systems. As these strategies are closely linked to the implementation in gPROMS<sup>®</sup> and the programm capabilities, they will be discussed in sec. 4.5

Furthermore it should be noted, that not all specifications are compatible with the initialization strategies described above. While this issue has been addressed, it also will be explained in the implementation section.

**Example** To illustrate how the initialization procedure works an example has been constructed of a rather complex column – or column section – with multiple feeds and side draws (fig. 4.2). It is taken from an example process of cryogenic air separation. The column in question is a column section without an condenser stage and displayed the most difficulties in terms of convergence when constructing the process flowsheet.

In addition to the aforementioned initialization strategy, columns with side draws present are handled in a slightly different manner. Initially the side draws are disregarded. Then the initialization procedure is carried out. Once the column without side draws has converged, a homotopy approach

$$f(\mathbf{x}) = (1 - \alpha) \cdot f_0(\mathbf{x}) + \alpha \cdot f_1(\mathbf{x}) \quad (4.32)$$

is employed, where the parameter  $\alpha$  is initially set to zero and then gradually moved to a value of one. During the initialization homotopies could generally be employed to move from one step to another. While in some cases robustness is improved by such a strategy, it is always computationally far more expensive than simply jumping between different stages.

For clarity reasons the different steps of the initializations procedure are repeated in a tabular manner

- linear temperature profile between dew ( $T^{dew}$ ) and bubble point temperature ( $T^{bub}$ ) of mixed feed.

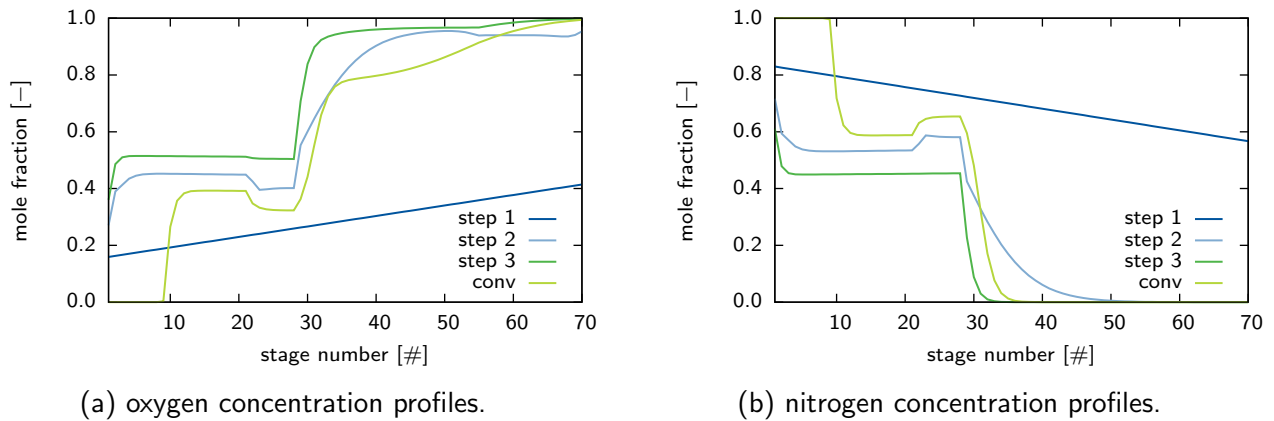


Figure 4.4: initialization example concentration profiles.

- linear profile between feed flash vapour and liquid compositions for liquid stage compositions.
  - constant profile for vapour compositions.
  - molar flow rates from constant molar overflow model.
  - side draw flow rates set to zero.
2.
    - total molar flow rates from constant molar overflow model.
    - simplified equilibrium ratios from initial liquid mole fractions and linear temperature profile.
    - liquid and vapour mole fractions from linearized mass balances.
  3. rigorous solution of MESH equations with side draws still set to zero.
  4. homotopic approach to MESH equations with side draws considered.

The resulting profiles for oxygen and nitrogen concentrations in the example column can be seen in fig. 4.4a and fig. 4.4b.

### 4.1.2 Integrated condenser & reboiler

The integrated condenser reboiler unit is a peculiarity of the ASU process. The fact that condenser and reboiler are linked form the close coupling between both sections of the double effect column. This stems from the fact, that each change in the parameters and operation of this unit will reverberate through the entire process, as they will simultaneously affect boil-up ratio and reflux ratio of the column, both major operational parameters with great effect.

Therefore a separate model for this unit was developed, rather than simply establishing a couple between the condenser and the reboiler in the case for two separately modeled column sections.

#### Reboiler side

The reboiler side is modeled much like an equilibrium stage in the column, considering the entering and leaving streams as depicted in fig. 4.1.2. However in contrast to the equations supplied for a

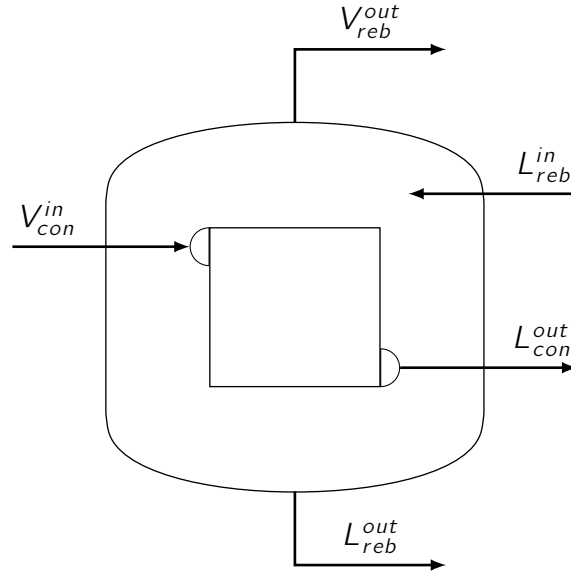


Figure 4.5: Integrated condenser &amp; reboiler unit.

column section, no integer optimization terms will have to be considered. Furthermore no side draws, or separate feeds are recognized. With that the material balances are

$$0 = L_{reb}^{in} x_{i,reb}^{in} - L_{reb}^{out} x_i^{reb} - V_{reb}^{out} y_i^{reb}, \quad i = 1 \dots n_C. \quad (4.33)$$

The energy balance taking into account the transferred energy ( $Q_{CR}$ ) becomes therefore

$$0 = L_{reb}^{in} h_{reb}^{in} - L_{reb}^{out} h_{reb}^L - V_{reb}^{out} h_{reb}^V + Q_{CR}. \quad (4.34)$$

The equilibrium and summation equations are analogous to the ones presented for the column model. A pressure drop again is assigned with relation to the lowest column stage.

### Condenser side

The condenser is modeled as a total condenser. The mass and component balances therefore become trivial

$$x_{i,con} = y_{i,con}^{in}, \quad i = 1 \dots n_C, \quad (4.35)$$

$$V_{con}^{in} = L_{con}^{out}. \quad (4.36)$$

The temperature is set to the bubble temperature of the given mixture at an assigned pressure. The enthalpies of the entering and leaving stream can therefore be calculated from the given EOS. And the energy balance determines the transferred energy

$$0 = V_{con}^{in} h_{con}^{in} - L_{con}^{out} h_{con}^L - Q_{CR}. \quad (4.37)$$

### 4.1.3 Heat exchange

The issue of heat integration is essential to the economic performance of cryogenic air separation. Foremost one must consider the special column configuration used in the process. Since operation of

the condenser in the low pressure section only becomes possible if the reboiler in the high pressure section functions as heat sink, no external utilities are supplied to either unit. Rather they are combined into a single heat exchange unit. Thus the absolute value of the reboiler energy must be matched by the energy recovered from the condenser. Furthermore the material streams entering the process can – and should – exchange heat with the process streams leaving it. The combined condenser / reboiler for LPC / HPC column is assumed as a given heat exchange. This makes sense insofar, as this is a necessity in terms of the actual physical implementation of the process units. Also the usage of the oxygen rich liquid from the HPC as coolant in the Argon condenser is assumed as fixed.

This leaves the process stream leaving the compression stage of the process as well as all product and waste streams leaving the process. All those streams are – for simulation purposes and also in some process implementations – fed into a single multi-stream heat exchange unit. In actual processes all heat exchange and much of the process operations take place in the so called "cold box". As such a heavily insulated area is referred to. This is done to minimize heat exchange with the surroundings. Therefore and for further reasons compact heat exchange units such as plate-fin multi-stream heat exchangers are favoured when dealing with cryogenic processes in general and the cryogenic air separation in particular.

Due to the importance of heat integration to the ASU process some thought should be given as to what modelling approach should be employed. Although the field of heat integration is one of the most intensively studied within process engineering, only a limited amount of approaches is available in open literature [18].

Traditionally heat integration has been carried out in a sequential manner, where it is for the purposes of process optimization assumed, that all heating and cooling is done by external utilities. After an (locally) optimal process configuration is identified, all hot and cold streams within the process are identified, and their temperature intervals fixed. In subsequent steps first the minimum utility requirements, and maximum number of heat exchangers are identified, and a specific heat exchange network (HEN) is designed. While this approach has been successfully applied to a multitude of processes and led to substantial savings, it is questionable if such a sequential approach will yield an optimal or near optimal solution.

Therefore some efforts have been made to develop efficient strategies for simultaneous process optimization and heat integration. Two general approaches can be distinguished. The first one based on the pinch concept. These methods are able to identify the minimum heat requirement as well as stream temperatures during process optimization. The first model along these lines was published by Duran and Grossmann [12] in 1986. They introduced a limited number of quite well behaved constraints into the optimization model to ensure no minimum driving force violations. Recently this model has been extended to handle phase changes and by fixing the utilities to zero been applied to the design of a multi-stream heat exchanger [18]. The major drawback with these methods is that one cannot target area of a given heat exchanger as the approach temperatures are not computed by the model. Therefore the sometimes substantial trade-off between the cost for heat exchange area and utility cost cannot be regarded.

A second approach employs superstructures of a HEN to find optimal matchings of process streams. No pinch point calculations are required, as the actual heat exchange is more or less modeled explicitly. This leads to the benefit, that approach temperatures as well as exchanged heat duties between each

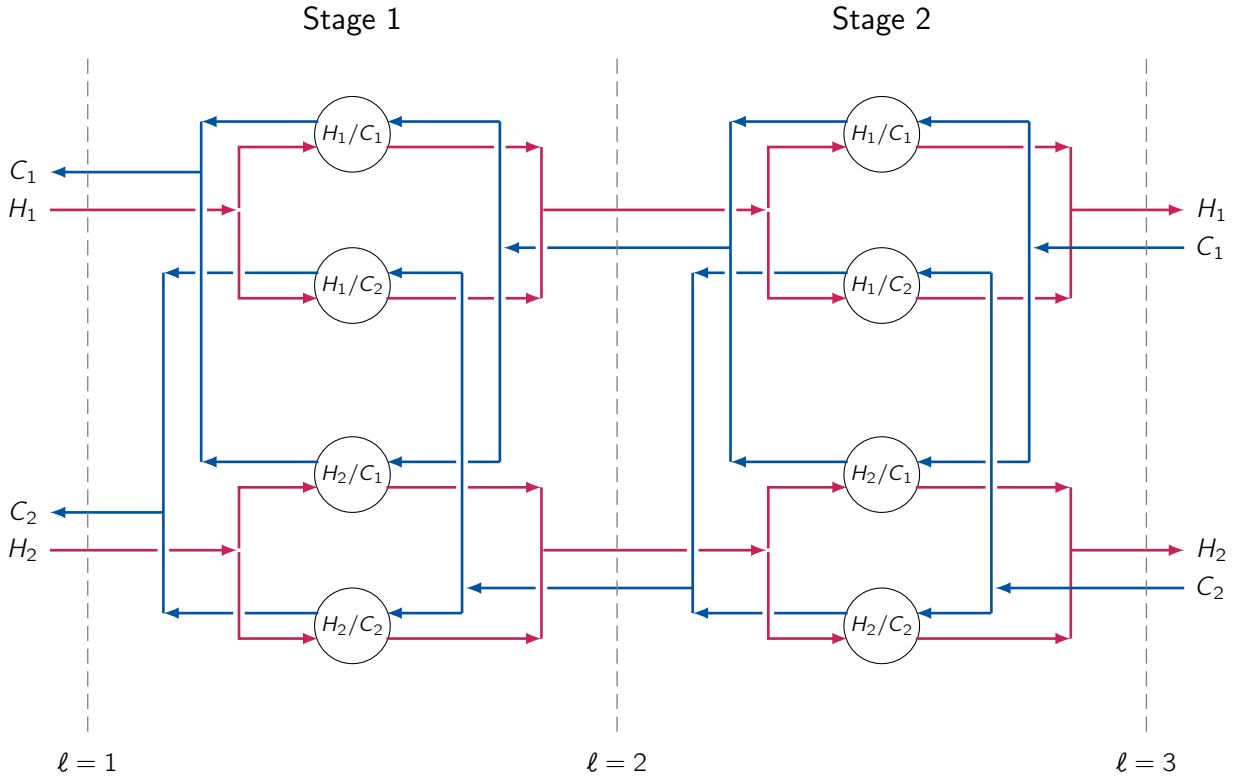


Figure 4.6: Superstructure for multi-stream heat exchanger. [40]

stream coupling are known within the model, and the cost of the designed unit can be considered in an economic objective function.

The approach and respective superstructure adapted in this thesis were first published by Yee and Grossmann [40]. Fig. 4.6 shows the stage wise superstructure for a HEN consisting of two hot and two cold streams.

In this stagewise structure each hot stream can exchange heat with each cold stream within each stage. The following assumptions were made when the model was developed

- Constant heat capacities
- Constant heat transfer coefficients
- Countercurrent heat exchangers
- Isothermal mixing at each stage.

The assumption of constant heat capacities is a common one in the design of HEN's. When no phase boundaries are passed and the temperature range of the involved is not too wide, it is a reasonably good approximation of the real conditions. While constant heat transfer coefficients are assumed the model leaves the flexibility to define different coefficients for each pairing of hot and cold streams. Countercurrent heat exchangers are common in industrial practice. This assumption however does not really pose a limitation, as the model can easily be altered to account for concurrent units.

The last assumption of isothermal mixing is a more major one. It has been introduced as it allows for significant simplifications and leads to a model, where all constraints are linear and all non-linearities

are restricted to the objective function. While that is certainly not true for the entire process model, it should at least allow for some reductions in the model complexity. The assumption states, that regardless of which streams a given stream exchanges heat with, it will leave at the same temperature. Due to that all energy balances around each unit in the superstructure can be eliminated as well as the subsequent mixing of the streams.

**Model equations** First of all a heat balance at each stage is necessary

$$F_i(T_{i,\ell} - T_{i,\ell+1}) = \sum_j q_{ijk} \quad (4.38)$$

$$f_j(T_{i,\ell} - T_{i,\ell+1}) = \sum_i q_{ijk} \quad (4.39)$$

The heat exchange area  $A_{hx}$  can be computed from the exchanged energy, the heat transfer coefficients  $\alpha_{ij}$  and the logarithmic mean temperature difference  $LMTD$ .

$$A_{hx} = \sum_i \sum_j \sum_k \frac{q_{ijk}}{\alpha_{ij} * LMDT_{ijk} + \delta} \quad (4.40)$$

While the small number  $\delta$  is included to avoid problems in the program, when  $LMDT$  becomes zero. In order to avoid further numerical difficulties when the approach temperatures  $\Delta T_{ij}$  at each side of an exchange unit approach zero, it was proposed to use an approximation introduced by Chen [8]

$$LMDT_{ijk} \approx \left[ \Delta T_{ijk} \cdot \Delta T_{ijk+1} \frac{\Delta T_{ijk} + \Delta T_{ijk+1}}{2} \right]^{\frac{1}{3}}. \quad (4.41)$$

While the approach temperatures are defined as

$$\Delta T_{ijk} = \max \{0, T_{ik} - T_{jk}\} \quad (4.42)$$

AS the max function is non-smooth and thus non differentiable at the points  $T_{ik} = T_{jk}$ , gPROMS<sup>®</sup> internally uses a smooth approximation. The exact for of which is unknown to the author.

#### 4.1.4 Compression & expansion

The issue of cooling the ambient air to process temperatures at around 90K is not an easy one. The main hindrance is, that a heat sink at this temperature level is not readily available. Lucky thermodynamics offer a different way to reach such temperatures. In order to do so, the ambient air first needs to be compressed and then expanded again. Cooling then occurs by either exploiting the *Joule-Thompson* effect or isentropic expansion. First a few comments are made about the compression stage, while afterwards the governing principles for cooling by expansion will be described.

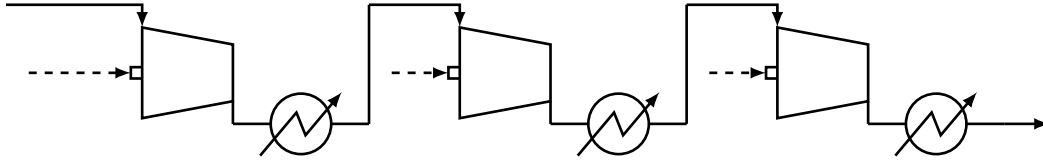


Figure 4.7: Multi-stage compression.

### Multi-stage compression

Compressors and expanders are among the most common process equipment. A multitude of processes utilizes them as primary or auxiliary units. In the context of cryogenic air separation the compression plays a major role, as it enables to reach temperatures needed for liquefying air and gases in general. As the compression of gases is always associated with a significant reduction in volume it requires large amounts of energy. Thus in addition to significantly contributing to the capital cost of the process the compression stage is responsible for the majority of the operating cost encountered in the ASU process.

The rigorous modeling of continuous flow machines in terms of unit operations poses great challenges. For specific units it may be undertaken by means of CFD simulations or employing characteristics diagrams, which require extensive experiments or can be obtained from the manufacturer. For the purposes of process design however a simpler approach with unit efficiencies is appropriate.

As a significant temperature increase goes along with the compression of air and in order to reduce the energy demand of the compression in general, it is beneficial, to use a multi stage compressor with inter-cooling stages as depicted in fig. 4.7. This yields a lower energy consumption as a single stage unit for the same compression ratio.

Subsequently the working equations for the compressor model used in the scope of this thesis are briefly summarized.

A trivial material and component balance around the compressor yields the outlet molar flow-rate  $F^{out}$  as well as the outlet overall composition  $z^{out}$

$$0 = F^{in} - F^{out}, \quad (4.43)$$

$$0 = z_i^{in} - z_i^{out} \quad i = 1 \dots n_C. \quad (4.44)$$

To calculate the mechanical work associated with the desired compression, first the isentropic case

$$S^{in} = S^{out} \quad (4.45)$$

is considered.

$$W_S = F^{in} \cdot (H^{in} - H^{out}) \quad (4.46)$$

$$W \cdot \eta^C = W_S \quad (4.47)$$

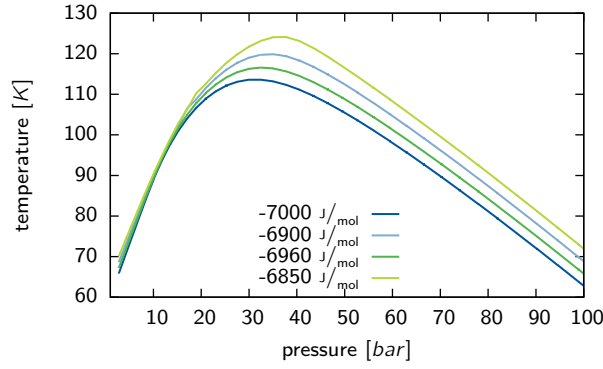


Figure 4.8: Isenthalpes computed by Peng-Robinson EOS.

$$W_S = F^{in} \cdot H^{in}(T^{in}, p^{in}, z_i^{in}) - F^{out} \cdot H^{out}(T^{out}, p^{out}, z_i^{out}) \quad (4.48)$$

$$p^{out} = p^{in} + \Delta p \quad (4.49)$$

### Cooling by expansion

The liquefaction of gases requires temperatures well below ambient conditions. In order to attain such conditions one cannot utilize natural occurring coolants as they are not available at the required temperature level. Rather cooling effects that can occur during the expansion of gases are exploited. First we consider the expansion through an expansion valve or so called *Joule-Thompson* - valve. In an rather accurate approximation, expansion through a valve can approximated as an isenthalpic process ( $h_1 = h_2$ ). To describe the change in temperature during isenthalpic expansion the *Joule-Thompson* coefficient

$$\mu_{JT} = \left( \frac{\partial T}{\partial p} \right)_h, \quad (4.50)$$

which denotes pressure derivative of the temperature at constant enthalpy can be considered. This can be transformed into

$$\mu_{JT} = \frac{1}{c_p} \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right] \quad (4.51)$$

If we employ the Peng-Robinson equation of state, which is appropriate for this mildly non-ideal system [2], we can plot the isenthalpes for ambient air ( $x_{N_2} = 0.7812$ ,  $x_{O_2} = 0.2095$ ,  $x_{Ar} = 0.0093$ ) in a PT-diagram (fig. 4.8). There we can see, that in certain ranges a decrease in pressure at constant enthalpy will result in an increase in temperature, while for other regions temperature decreases. It is interesting to mention that the non-idealities of a given gas give rise to this effect. For an ideal gas the temperature change at isenthalpic expansion would always be zero. Luckily for the cryogenic engineer real gases deviate from ideal behaviour especially at elevated pressures and



low temperatures [3]. It is therefore important to give some consideration to the thermodynamic model used to describe the properties of the system in question, as the non-ideal properties need to be captured appropriately.

A different way of expanding a compressed gas, is by letting it produce work in an fluid kinetic machine. If one assumes an adiabatic devices and disregards irreversible effects, this process can be viewed as isentropic. Analogous to the isenthalpic case an isentropic expansion coefficient can be defined

$$\mu_S = \left( \frac{\partial T}{\partial p} \right)_S = \frac{T}{c_p} \left( \frac{\partial v}{\partial T} \right)_p. \quad (4.52)$$

Here the derivative in the second form corresponds to the volumetric coefficient of thermal expansion  $\beta$ , which is always positive for gases, which in turn means, that an isentropic expansion will always result in an temperature decrease, whereas the isentropic expansion only led to a decrease for specific conditions. Furthermore an isentropic expansion over the same pressure range will always result in lower temperatures than an isenthalpic expansion. The reason that isentropic valves are most commonly used in liquefaction systems, is that those work producing machines cannot handle significant phase changes, which is after all the desired result of liquefaction.

Traditionally only the isenthalpic expansion had been used within the cryogenic air separation process, since – as mentioned before – the air needs to be liquefied in order to be fed into be distilled. However in modern process configurations the isentropic expansion is also considered to recover some of energy form the compression.

## 4.2 Dynamic unit models

### 4.2.1 Distillation column model

The previously described column model is based on the assumption of steady-state operations. This means that all derivatives with respect to time are zero. While a model like that offers valuable insight into the operation of a process many aspects remain unclear. Questions of operability or stability of the process, or if a given steady state can actually be acquired in finite are beyond the scope of steady state models. In order to supply answers to aforementioned questions, dynamic versions of the previously discussed unit models are needed. The aim is to retain the capability for structural optimization for the dynamic models.

First the balance equations have to be rewritten in dynamic form. To do so reservoir terms or holdups are introduced. Namely the component holdups  $n_{ij}$  and internal energy  $U_j$  for each stage. With that

the component balance equations as presented in the previous section can now be rewritten as

$$\begin{aligned} \left(1 - \sum_{k=1}^{j-1} \zeta_k^R\right) \frac{dn_{ij}}{dt} = & (1 + s_j^V) \cdot V_j \cdot y_{ij} + (1 + s_j^L) \cdot L_j \cdot x_{ij} - V_{j+1} \cdot y_{i,j+1} \\ & - L_{j-1} \cdot x_{i,j-1} - \sum_{l=1}^{n_F} \zeta_{lj} \cdot F_j \cdot z_{ij} - \zeta_j^R \cdot V_N \cdot y_{iN}, \\ i = 1 \dots n_C - 1, \quad j = 1 \dots N, \quad k = 1 \dots n_F, \quad l = 1 \dots n_F. \end{aligned} \quad (4.53)$$

While the internal energy balances become

$$\begin{aligned} \left(1 - \sum_{k=1}^{j-1} \zeta_k^R\right) \frac{dU_j}{dt} = & (1 + s_j^V) \cdot V_j \cdot h_j^V + (1 + s_j^L) \cdot L_j \cdot h_j^L - V_{j+1} \cdot h_{j+1}^V \\ & - L_{j-1} \cdot h_{j-1}^L - \sum_{k=1}^{n_F} \zeta_{kj} \cdot F_k \cdot h_j^F - \zeta_j^R \cdot V_N \cdot h_N^V, \\ i = 1 \dots n_C, \quad j = 1 \dots n_S, \quad k = 1 \dots n_F. \end{aligned} \quad (4.54)$$

The summation terms in front of the derivatives are needed if integer optimization is to be carried out with the model. In the steady-state case, no equilibrium took place at inactive stages. However in this case, there are the holdups in connection with the column hydraulics to be considered. While still no equilibrium should take place, even inactive trays will have to have a holdup. By means of the summations, the material balance is effectively transformed into a steady state equation, such that inactive trays will operate at steady-state with constant holdups, and therefore have no effect on the active part of the column.

In addition to the balance equations further constituent equations need to be introduced. From the steady state model we know the equilibrium equations

$$y_{ij} = K_{ij} \cdot x_{ij}, \quad i = 1 \dots n_C \quad j = 1 \dots n_S, \quad (4.55)$$

and the summation equations

$$1 = \sum_i^{n_C} y_{ij} \quad j = 1 \dots n_S, \quad (4.56)$$

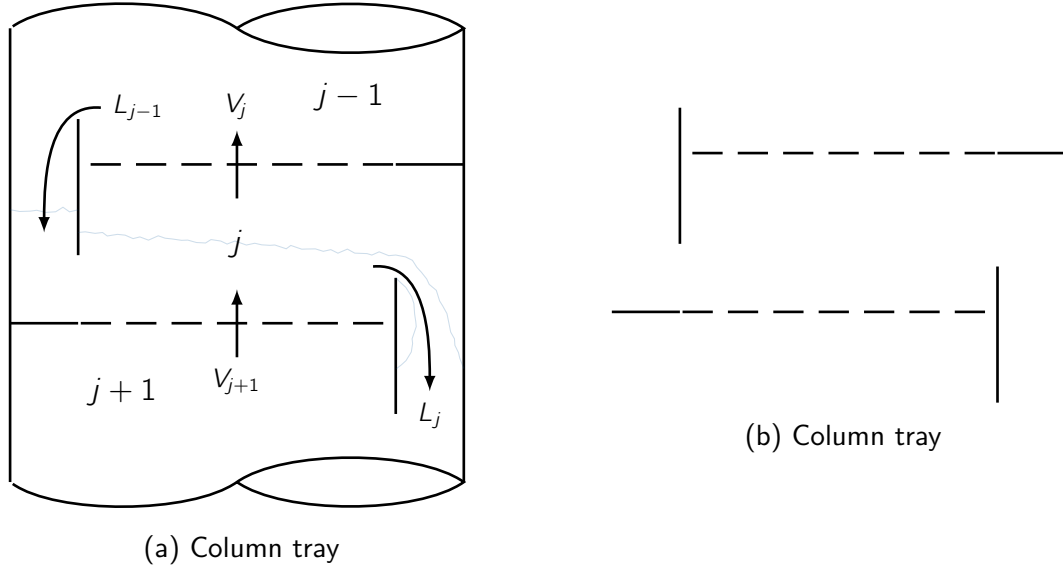
$$1 = \sum_i^{n_C} x_{ij} \quad j = 1 \dots n_S. \quad (4.57)$$

Furthermore defining equations for the accumulation of moles on each stage in vapour  $n_j^V$  and liquid  $n_j^L$  phase are needed

$$n_{ij} = x_{ij} n_j^L + y_{ij} n_j^V \quad i = 1 \dots n_C \quad j = 1 \dots n_S. \quad (4.58)$$

These holdups will fill the volume of a given stage  $V_{stage}$ . Thus a volume constraint can be written as

$$V_{stage} = \frac{n_j^V}{\rho_j^V} + \frac{n_j^L}{\rho_j^L} \quad j = 1 \dots n_S. \quad (4.59)$$



(a) Column tray

(b) Column tray

The internal energy in a stage corresponds to its enthalpy, reduced by pressure term

$$U_j = n_j^L \cdot h_j^L + n_j^V \cdot h_j^V - p_j \cdot V_{stage} \quad (4.60)$$

As we are no longer dealing in steady state hydraulic equations need to be introduced, which determine the liquid and vapour flow rates leaving a separation stage. As the mechanisms driving these flows might be very different depending on the type of internals used, it is not surprising that the corresponding equations are also very different. In the presented model implementation, both trayed columns and structured packings can be handled accordingly.

### Trayed hydraulics

Trayed column hydraulics can be approximated by the following system of equations. All equations presented here were taken from [23].

The liquid flow rates are calculated from the well established francis formula , derived from the law of Bernoulli and taking effects like bubbling into account

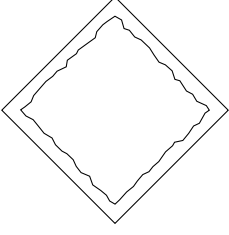
$$L_j = \frac{2}{3} \sqrt{2g} \rho_j^L \ell_{weir} \Phi h_{ow}^{\frac{2}{3}} \quad j = 1 \dots n_S. \quad (4.61)$$

Where  $h_{ow}$  denotes the height of the liquid over the weir, which can be calculated from the froth height  $h_f$  and the weir height  $h_w$

$$h_{ow} = h_f - h_w. \quad (4.62)$$

While the weir height is a tray design parameter the froth height is computed from the clear liquid height and the relative froth density

$$h_f = \frac{n^L MW^L}{A_{active} \rho^L \Phi}. \quad (4.63)$$



Lastly in terms of liquid flow rates, the relative froth density is dependent on the degree of aeration within the liquid, expressed by the aeration factor  $\beta$  from an empirical equation

$$\beta_j = 1 - 0.3593 \left( \frac{V_{j-1} MW_{j-1}^V}{A_{active} \sqrt{\rho_{j-1}}} \right)^{0.177709} \quad j = 1 \dots n_S, \quad (4.64)$$

$$\Phi_j = 2\beta_j - 1. \quad (4.65)$$

The pressure difference between stages is the driving force for the vapour streams. The pressure drop is modeled as having two contributions, the dry and wet pressure drop. While dry pressure drop stems from the vapour flowing through the holes with in tray, the wet pressure drop is caused by the liquid holdup on the stage.

$$p_j - p_{j+1} = h_j^{liq} \rho_j^L g + 0.5 \xi \rho_{j+1}^V \left( \frac{q_{j+1}^V}{A_h} \right)^2 \quad (4.66)$$

### Structured packing hydraulics

Structured packings and their hydraulic behaviour are currently still under investigation. The number of available correlations for calculation of internal flow-rates is much more limited than for trays or even dumped packings. Among the most established models is the one developed by Bravo et al. [33] at the University of Texas. This model has been extended to be valid in the loading region and account for different types of packing material [16]. As main linking factor between vapour and liquid flow-rates as well as the pressure drop, the liquid holdup has been identified by the authors. It is expressed in dimensionless form  $h_t$  in terms of the corrugation side  $S$ , and the film thickness  $\delta$

$$h_L = \frac{4}{S} \delta. \quad (4.67)$$

One very important factor while estimating the hydraulic behaviour is the dry pressure drop per meter packing  $\delta p_{dry}$ . Within in the presented model it is estimated by

$$\delta p_{dry} = \left( \frac{\rho^G}{\rho_{air,1bar}} \right)^{0.4} \left( \frac{C_1 \rho^G v_G^2}{S \epsilon^2 (\sin \Theta)^2} + \frac{C_2 \mu_G v_G}{S^2 \epsilon \sin \Theta} \right) \quad (4.68)$$

Another prerequisite for calculating the holdup is the knowledge of the amount of wetted area of the available surface area within the packing. It seems reasonable to assume that this will be dependent

on the characteristic of the liquid flow through the packing. To characterize the current, well known dimensionless numbers are used. Namely *Weber* ( $We$ ), *Froude* ( $Fr$ ) and *Reynolds* ( $Re$ ) numbers

$$We = \frac{v_L^2 \rho_L S}{\sigma}, \quad (4.69)$$

$$Fr = \frac{v_L^2}{Sg}, \quad (4.70)$$

$$Re = \frac{v_L S \rho_L}{\mu_L} \quad (4.71)$$

With that an approximation for the holdup correction factor  $F_t$  due to partial wetting can be expressed as

$$F_t = \frac{A_1 (We Fr)^{0.15} S^{A_2}}{Re^{0.2} \epsilon^{0.6} (1 - 0.93 \cos \gamma) (\sin \Theta)^{0.3}} \quad (4.72)$$

$$h_L = \left( \frac{4F_t}{S} \right)^{\frac{2}{3}} \left\{ \frac{3\mu_L v_L}{\rho_L \epsilon \sin \Theta g \left[ \left( \frac{\rho_L - \rho_G}{\rho_L} \right) \left( 1 - \frac{\delta p}{\delta p_{flood}} \right) \right]} \right\}^{\frac{1}{3}} \quad (4.73)$$

## 4.2.2 Integrated condenser / reboiler unit

As it has been stated on several occasions the condenser reboiler unit is essential in the cryogenic air separation process. Even more so, when we consider the dynamics behaviour of the process. In order to model this unit, it is assumed that all vapour entering the stage will be condensed, thus the unit is regarded as total condenser. Furthermore the holdups on the condenser side are assumed to negligible. The reboiler side on the other hand is modeled as an equilibrium stage much like the stages within the column.

### Reboiler side

As the reboiler side behaves like an equilibrium stage, the governing equations are very similar to those for a stage inside the columns. And will be given without much further comment.

The material balances

$$\frac{dn_i^{reb}}{dt} = L_{reb}^{in} \cdot x_{i,reb}^{in} - V_{reb}^{out} \cdot y_i^{reb} - L_{reb}^{out} \cdot x_i^{reb}, \quad i = 1 \dots n_C. \quad (4.74)$$

Internal energy balance

$$\frac{dU_{reb}}{dt} = L_{reb}^{in} \cdot h_{in}^{reb} - V_{reb}^{out} \cdot h_V^{reb} - L_{reb}^{out} \cdot h_L^{reb} + Q_{CR}. \quad (4.75)$$

In addition to the balances the equilibrium and summation equations are necessary to complete the equations system.

As for the trays hydraulic equations determine the flowrates leaving the unit. For the liquid outlet Bernoulli's principle was used to derive an equation

$$L_{reb}^{out} = A_{valve} \varrho_L \sqrt{\frac{2 \cdot (\Delta p + \tilde{\varrho}_L g h)}{\tilde{\varrho}_L}}, \quad (4.76)$$

disregarding the height difference an equation for the vapour flowrate can be derived

$$V_{reb}^{out} = A_{valve} \varrho_V \sqrt{\frac{2 \Delta p}{\tilde{\varrho}_V}}. \quad (4.77)$$

### Condenser side

For a total condenser disregarding reservoir terms the mass balances become trivial

$$L_{con}^{out} = V_{con}^{in}. \quad (4.78)$$

To ensure bubble point temperature a summation as well as an equilibrium expression are included

$$1 = \sum_i^{n_C} K_i x_{i,con}, \quad i = 1 \dots n_C, \quad (4.79)$$

$$x_{i,con} = y_{i,con}^{in}, \quad i = 1 \dots n_C - 1. \quad (4.80)$$

An energy balance around the condenser side determines the transferred energy

$$Q_{CR} = L_{con} \cdot (h_{con}^{in} - h_{con}^{out}). \quad (4.81)$$

### Heat exchange

As the transferred energy is determined from the condenser energy balance, the temperature difference between condenser and reboiler can be calculated by means of

$$Q_{CR} = \alpha A_{HX} \cdot (T_{con} - T_{reb}). \quad (4.82)$$

Roffel et al [34] found from process data, that the overall heat transfer coefficient ( $\alpha A$ ) can be approximated by a simple formula in terms of the condenser inlet vapour flowrate

$$\alpha A = c_1 (V_{con}^{in})^{0.8}. \quad (4.83)$$

## 4.3 Sizing & costing models

As discussed earlier economic consideration play a major role in process design. In order to account for the process economics the cost of the process to be implemented needs to be estimated at the design level. However as limited information is available estimation methods have to be employed.

In sec. 3 the general approach for cost estimation of process equipment was introduced, where a specific value such as heat-exchange area or vessel size is used to approximate equipment cost. However for more specific units extended models are available, where statistical data is employed to yield a more realistic fit to cost data. The cost functions and correction factors presented in this chapter are, if not stated otherwise, taken from [35]. Also unless otherwise stated the unit cost is given for the year 2006 ( $CE = 500$ ).

### 4.3.1 Distillation column

The cost of a given distillation column  $C_{\text{column}}$  is comprised of three main parts, the cost for the vessel or tower itself  $C_{\text{tower}}$ , the cost for platforms, ladders, manholes, and nozzles  $C_{\text{platform}}$  and the cost for the internals  $C_{\text{internals}}$

$$C_{\text{column}} = C_{\text{tower}} + C_{\text{platform}} + C_{\text{internals}}. \quad (4.84)$$

The determining factors for the cost of a tower are the construction material and the weight of the vessel. The material is considered by multiplying a base cost by a material factor  $f_M$ . Thus the cost for the tower can be approximated by

$$C_{\text{tower}} = f_M \cdot \exp \left[ 7.2756 + 0.18255 \cdot \ln(W_{\text{tower}}) + 0.02297 \cdot (\ln(W_{\text{tower}}))^2 \right], \quad 9000 \leq W_{\text{tower}} \leq 2.5 \cdot 10^6. \quad (4.85)$$

In this correlation the weight  $W$  is measured in pounds ( $[lbs]$ )

For the support structures the following equation has been presented

$$C_{\text{platform}} = 300.9 \cdot (d_{\text{column}})^{0.63316} \cdot (l_{\text{column}})^{0.80161}, \quad (4.86)$$

where both the diameter  $d_{\text{column}}$  and length  $l_{\text{column}}$  are measured in feet ( $[ft]$ ).

The weight of the column is attained by calculation its volume and multiplying it by the mass density of the construction material  $\tilde{\rho}$

$$W_{\text{column}} = \pi(d_{\text{column}} + t_s)(l_{\text{column}} + 0.8 \cdot d_{\text{column}})t_s \cdot \tilde{\rho}. \quad (4.87)$$

It needs to be noted, that in this formula, all measurements have to be adjusted to the measurement of the density. Newly appearing is the shell thickness  $t_s$  of the tower. In order to compute this, several aspects can be considered.

The ASME pressure vessel code formula is used to compute the minimum thickness due to the design pressure  $p_D$  of a tower

$$t_p = \frac{p_D d_{\text{column}}}{2SE - 1.2p_D}, \quad (4.88)$$

where  $S$  denotes the maximum allowable stress and  $E$  the fractional weld efficiency.

The design pressure is computed from the operating pressure  $p_O$

$$p_D = \max \{ 10, \exp [0.60600 + 0.91615 \cdot \ln(p_O) + 0.0015655 \cdot (\ln(p_O))^2] \} , \quad (4.89)$$

It should be pointed out the the pressures in the previous equation are measured in pound per square inch gauge ( $[psig]$ ).

In addition to the thickness due to pressure, it should be considered – especially for tall vessels – that the vessel might have to withstand external forces such as strong winds or earthquakes. To account for external effects a security addition  $t_w$  to the pressure thickness can be computed

$$t_w = \frac{0.22(d_{\text{column}} + 18)l_{\text{column}}^2}{Sd_{\text{column}}} . \quad (4.90)$$

Therefor the total shell thickness amounts to

$$t_s = t_p + t_w . \quad (4.91)$$

In the previous equations the column diameter and length have frequently been used, however no mention has been made as to how to attain these values. While for a given design these values will be fixed, in the context of an optimization they need to be considered as decision variables. These decisions are closely linked to the choice of column internals. In case of the column height the procedure is very similar for columns with trays and structured packings. In case of the diameter however a clear distinction has to be made. This is due to the fact, that a decision regarding the diameter will have to ensure feasible column operations, which means it will have to be large enough to avoid flooding within the column. Consequently the procedures for determining height and diameter will be dealt with separately for each type of internals.

## Trays

For a trayed column the height can be determined from the number of stages  $n_S$  used multiplied by the plate spacing  $h_{\text{plate}}$ . In addition however it needs to be ensured, that control of the columns is still possible. At the sump of the tower liquid will accumulate during operations. While in an ideal case the liquid level would be constant, that cannot be assumed. Regularly there are three different liquid levels defined in a column. The high level (HLL), normal level (NLL) and the low level (LLL). These levels are defined such that it sufficient time for the liquid to reach these levels ( $t_{\text{min}}^{\text{HLL}}, t_{\text{min}}^{\text{NLL}}, t_{\text{min}}^{\text{LLL}}$ ), if no liquid id withdrawn anymore, or no liquid comes down from the column. Furthermore the times for reaching the reboiler inlet and the lowest plat will ne necessary. What duration to reach those levels will be sufficient needs to be decided by a control engineer.

With these times and heights the height of the tower can be expressed as

$$l_{\text{column}} = n_S \cdot h_{\text{plate}} + \left( \sum_i t_{\text{min}}^i H^i \right) \cdot \frac{L_{n_S}}{A_{\text{column}} \varrho^L} . \quad (4.92)$$



This leaves the diameter to be determined. The most important factor to that regard is the vapour velocity within the tower. It is to be chosen such that no flooding or entrainment will occur. The following equation to compute these operation boundaries are taken from [24].

First the fractional entrainment factor  $ent_j$  for 80% flooding at each stage  $j$  needs to be calculated

$$ent_j = 2.24 \cdot 10^{-3} \cdot 2.377 \exp \left[ -9.394 \cdot FLV_j^{0.314} \right], \quad j = 1 \dots n_S. \quad (4.93)$$

Therein the Sherwood flow parameter  $FLV_j$  is used

$$FLV_j = \frac{\tilde{L}_j}{\tilde{V}_j} \cdot \sqrt{\frac{\tilde{\rho}_j^V}{\tilde{\rho}_j^L}}, \quad j = 1 \dots n_S. \quad (4.94)$$

$$v_j^{flood} = 60 \cdot \left( \frac{\sigma_j^L}{20} \right)^{0.2} \cdot K1_j \cdot \sqrt{\frac{\tilde{\rho}_j^L - \tilde{\rho}_j^V}{\tilde{\rho}_j^V}}, \quad j = 1 \dots n_S. \quad (4.95)$$

$$K1_j = 1.05 \cdot 10^{-2} + 0.1496 \cdot h_{plate}^{0.755} \cdot \exp \left[ -1.463 \cdot FLV_j^{0.842} \right], \quad j = 1 \dots n_S. \quad (4.96)$$

With those values the minimum required column area for each stage  $A_j^{min}$  can be calculated

$$A_j^{min} = \frac{V_j}{\Psi_{flood} \tilde{\rho}_j^V v_j^{flood}}, \quad j = 1 \dots n_S, \quad (4.97)$$

where  $\Psi_{flood}$  is a design parameter which determines the degree of flooding allowed and will usually have a value around 0.8.

## Structured packings

The calculation of the column height is equivalent to the trayed case, with the one difference, that rather than using the plate spacing as the height for each stage a value called height equivalent to theoretical plate (*HETP*) is used. In terms of the column diameter the flooding velocity is determined by the flooding pressure drop as calculated in sec. 4.2.1.

### 4.3.2 Centrifugal pump

Pumps are among the most common units of process equipment. While there are several different kinds of pumps that can be used, the centrifugal pump is one of the most popular choices and denotes a very likely choice for the process conditions considered in this application. Hence other pump types will not be considered at this point.

stages	rpm	case-split	flow ([gpm])	head ([ft])	pow ([Hp])	$f_T$	material	$f_m$
1	3600	VSC	50 - 900	50 - 400	75	1.00	cast iron	1.00
1	1800	VSC	50 - 3500	50 - 200	200	1.50	ductile iron	1.15
1	3600	HSC	100 - 1500	100 - 450	150	1.70	cast steel	1.35
1	1800	HSC	250 - 5000	50 - 500	250	2.00	bronze	1.90
2	3600	HSC	50 - 1100	300 - 1100	250	2.70	stainless teel	2.00
2+	3600	HSC	100 - 1500	650 - 3200	1450	8.90	Hastelloy C	2.95
							monel	3.30
							nickel	3.50
							titanium	9.70

(a) type factors.

(b) material factors.

Table 4.3: Pump cost factors [35].

**Pump** In terms of operations pumps are best described by the volumetric flow transported  $Q$  as well as the pump head  $H$ , the hight that needs to be overcome. Data taken from the company Mosanto was used to correlate the pump cost to a specific value

$$S = Q\sqrt{H}. \quad (4.98)$$

As a reference unit the base price  $C_B$  is estimated for a cast iron single-stage vertically split case at 3600 rpm

$$C_B = \exp \{ 9.7171 - 0.6019 \cdot \ln[S] + 0.0519(\ln[S])^2 \}, \quad 400 \leq S \leq 100000. \quad (4.99)$$

The most influential scaling factors for the pump price are the material, which is accounted for in the material factor  $f_m$ , as well as the rotation, case split orientation (horizontal and vertical), the number of stages, covered flow rate range, pump head range and maximum motor power, which are all agglomerated in the type factor  $f_T$ . Values for these factors are given in tab. 4.3a and tab. 4.3b.

**Electric motor** Separately from the pump itself the motor to drive the compression is considered. While the volumetric flow and the pump head certainly are valid choices to correlate motors for pumps especially, the power consumption is a more general specific value

$$P_C = \frac{P_T}{\eta_P \eta_M} = \frac{P_B}{\eta_M} \quad (4.100)$$

It can be calculated from the theoretic power of the pump  $P_T$  and the efficiencies  $\eta_P$   $\eta_M$ . While an estimate for the expected power consumption might be already available at rather early design stages, the efficiencies will have to be correlated as well if resorting to average values is considered too coarse. Those correlations rely on the volumetric flow in gallons per minute ([gpm]) and the brake horse power  $P_B = \frac{P_T}{\eta_P}$ .

$$\eta_P = -0.316 + 0.24015 \cdot \ln[Q] - 0.01199 \cdot (\ln[Q])^2 \quad 50 \leq Q \leq 5000 \quad (4.101)$$

type motor enclosure	3600 rpm	1800 rpm
open, drip-proof enclosure, 1 to 700 Hp	1.0	0.9
totally enclosed, fan-cooled, 1 to 250 Hp	1.4	1.3
explosion-proof enclosure, 1 to 250 Hp	1.8	1.7

Table 4.4: Type factors for different motor types.

$$\eta_M = 0.80 + 0.0319 \cdot \ln[P_B] - 0.00182 \cdot (\ln[P_B])^2 \quad 1 \leq P_B \leq 1500 \quad (4.102)$$

After having calculated the power which the motor needs to supply its base cost of an open, drip-proof enclosed motor at 3600 rpm can be approximated by

$$C_B = \exp \left\{ 5.8259 + 0.13141 \cdot \ln[P_C] + 0.053255 \cdot (\ln[P_C])^2 + 0.028628 \cdot (\ln[P_C])^3 - 0.0035549 \cdot (\ln[P_C])^4 \right\} \quad 1 \leq P_C \leq 700 \quad (4.103)$$

To adjust the cost for different types of electric motors the type factors from tab. 4.4

## Compressor

The cost of compressors is correlated with their respective power consumption measured in horsepower. Although not the most efficient type of compressor, centrifugal compressors are very popular in the process industry, as they are easily controlled and deliver a very steady flow. However as different types might be employed as well base cost correlations for centrifugal, reciprocating and screw compressors are given.

### Centrifugal compressor

$$C_B = \exp \left\{ 7.5800 + 0.80 \cdot (\ln[P_C]) \right\} \quad 200 \leq P_C \leq 30000 \quad (4.104)$$

### Reciprocating compressor

$$C_B = \exp \left\{ 7.9661 + 0.80 \cdot (\ln[P_C]) \right\} \quad 200 \leq P_C \leq 20000 \quad (4.105)$$

### Screw compressor

$$C_B = \exp \left\{ 8.1238 + 0.7243 \cdot (\ln[P_C]) \right\} \quad 200 \leq P_C \leq 750 \quad (4.106)$$

Again as with most other equipment types correction factors are used to adjust for different realization of this piece of equipment. Here type of motor as well as the construction material have the biggest effects on the unit price and are explicitly considered.

$$C_p = f_D f_M C_B \quad (4.107)$$

The alternatives to the electric motor ( $f_D = 1.0$ ) are a steam turbine ( $f_D = 1.15$ ) or a gas turbine ( $f_D = 1.25$ ). It should however be noted that aside from being the cheapest choice, the electric motor is also the most efficient. Thus the turbines are mostly considered, when process steam or combustion gas is easily available, such that the drawbacks might be eliminated by not having to supply the electric energy for the electric motor. In terms of construction material all base costs are for cast iron or carbon steel. Some appliances may require more resistant and also more expensive materials such as stainless steel ( $f_M = 2.5$ ) or an nickel alloy ( $f_M = 5.0$ ).

### Reboiler / condenser

Reboiler and condenser can be characterized as heat exchangers, and be handled in the same way, as the main difference is whether heat is transferred to or from the process stream. In that sense they must be distinguished when considering the operating cost, as the cost for hot or cold auxiliary streams might differ significantly. As customary for heat exchangers the specific quantity for cost correlations is the necessary heat exchange area  $A$  measured in  $ft^2$ .

Again the construction material as well as the operating conditions have an effect on the final cost

$$C_P = f_P f_M C_B. \quad (4.108)$$

The correction for pressures  $f_P$  takes into account the operating pressure  $P_o$  and is computed by

$$f_P = 0.8510 + 0.1292P_o + 0.0198 * P_o^2. \quad (4.109)$$

The material correction factor  $f_M$

$$f_M = \quad (4.110)$$

### Shell and tube heat exchanger

$$C_B = \exp \{ 11.667 - 0.8709 \cdot (\ln[A]) + 0.09005 \cdot (\ln[A])^2 \} \quad (4.111)$$

### Double pipe

$$C_B = \exp \{ 7.146 + 0.1600 \cdot (\ln[A]) \} \quad (4.112)$$

## 4.4 Thermodynamic models

Aside from the unit operation models, the behaviour of materials in a process needs to be adequately accounted for. This is done by means of so called equations of state (EOS) and excess Gibbs energy models. In terms of thermodynamics there are only a limited amount of variables. Namely the pressure, density and temperature as well as composition. While equations of state can model a

given system in the vapour as well as liquid phase, excess Gibbs energy models only account for the behaviour of a liquid and need to be used in conjunction with other models for the vapour phase. However they have shown considerable better performance for highly non-ideal systems [2]. As mentioned earlier (sec. ??) it is essential to accurately capture the non-idealities of air in order to capture the liquefaction process. In the case of cryogenic air separation, the Peng-Robinson as well as the Benders equation of state have shown satisfactory performance. The Peng-Robinson equation was chosen to be used in the presented model

$$p = \frac{RT}{V - b} - \frac{a_c [1 + m(1 - \sqrt{T_r})]^2}{V^2 + 2bV - b^2} \quad (4.113)$$

$$m = 0.37464 + 1.54226\omega - 0.26992\omega^2 \quad (4.114)$$

$$a_c = 0.45724 \frac{R^2 T_c^2}{p_c} \quad (4.115)$$

$$b = 0.077796 \frac{RT_c}{p_c} \quad (4.116)$$

$$\omega = -1 - \log_{10}(p_r^{sat})_{T_r=0.7} \quad (4.117)$$

However the Peng-Robinson EOS relies on the so called one-fluid theory which models each fluid as pure. To model mixtures the pure component parameters have to be "mixed"

$$a = \sum_{i=1}^C \sum_{j=1}^C y_i y_j a_{ij}, \quad (4.118)$$

$$a_{ij} = \sqrt{a_i a_j} (1 - k_{ij}), \quad (4.119)$$

$$b = \sum_{i=1}^C y_i b_i. \quad (4.120)$$

From that EOS numerous relevant properties such as excess enthalpy, fugacity coefficients or densities can be calculated. For a list of some relevant equations refer to sec. A.1.

## 4.5 Implementation

As mentioned the presented models have been implemented in the process simulator gPROMS<sup>®</sup>. Although application to the cryogenic air separation process served as case where the model would be applied, the aim was to develop – especially in the case of the column models – a flexible model which could be used for a multitude of problems while trying to achieve a reasonable amount of complexity, such that a user mainly used to a pure flowsheeting environment should be able to apply the models with relative ease.

In the following sections first some general aspects of modeling in gPROMS<sup>®</sup> will be pointed out and the structure of the implemented model discussed in more detail. Subsequently several strategies for initializing the column model based on the previously model structure will be presented.

### 4.5.1 Model structure

When discussing the structure of gPROMS<sup>®</sup> models first some aspects of the modeling language have to be discussed. In this context CASE selectors, IF statements and the implementation INITIALIZATION\_PROCEDURE's in gPROMS<sup>®</sup> are most important. Within this section we will first briefly discuss those features and how they are used in gPROMS<sup>®</sup>. Afterwards it will be shown, how these structures are used, to create a model which can quickly be tailored the specific needs of a given user, and how distillation columns can be initialized within that framework.

IF statements and case selectors have many similarities as they can distinguish between different cases within a given model. For many applications they might even be used equivalently. As in almost all contexts the IF statements evaluates a condition and depending on whether it is fulfilled or not and evaluates different formulae accordingly. A CASE statement distinguishes different branches between the model can switch during simulation or initialization. One major difference is, that the number of equations in each CASE branch must match while for IF statements that condition is not enforced. However care must be taken when using these structures – especially during simulation – as they will in many cases introduce discontinuities. Furthermore when using IF statements one must distinguish, whether the evaluated statement is a parameter, or a variable in the model. Given it is a parameter, the statement will remain true (or false) for each instance of simulation. This introduces some flexibility in terms of diverging in the number of equations within model variants.

INITIALIZATION\_PROCEDURE's are used – as the name suggests – to initialize models. The basic idea is to automatically solve a sequence of increasingly complex models, using the solution of a previous model as starting point for converging the current one. Furthermore the switch between models can be done within a either a so called JUMP\_TO or MOVE\_TO statement. Within the former there is a hard cut between the model variants, while in the latter a homotopic approach is employed to get from one model branch to another. While this should be the more robust solution, if the path of the homotopy remains in a feasible region, it necessitates greater care in terms of model robustness and scaling, as the internal integrator is used to move along the homotopic path.

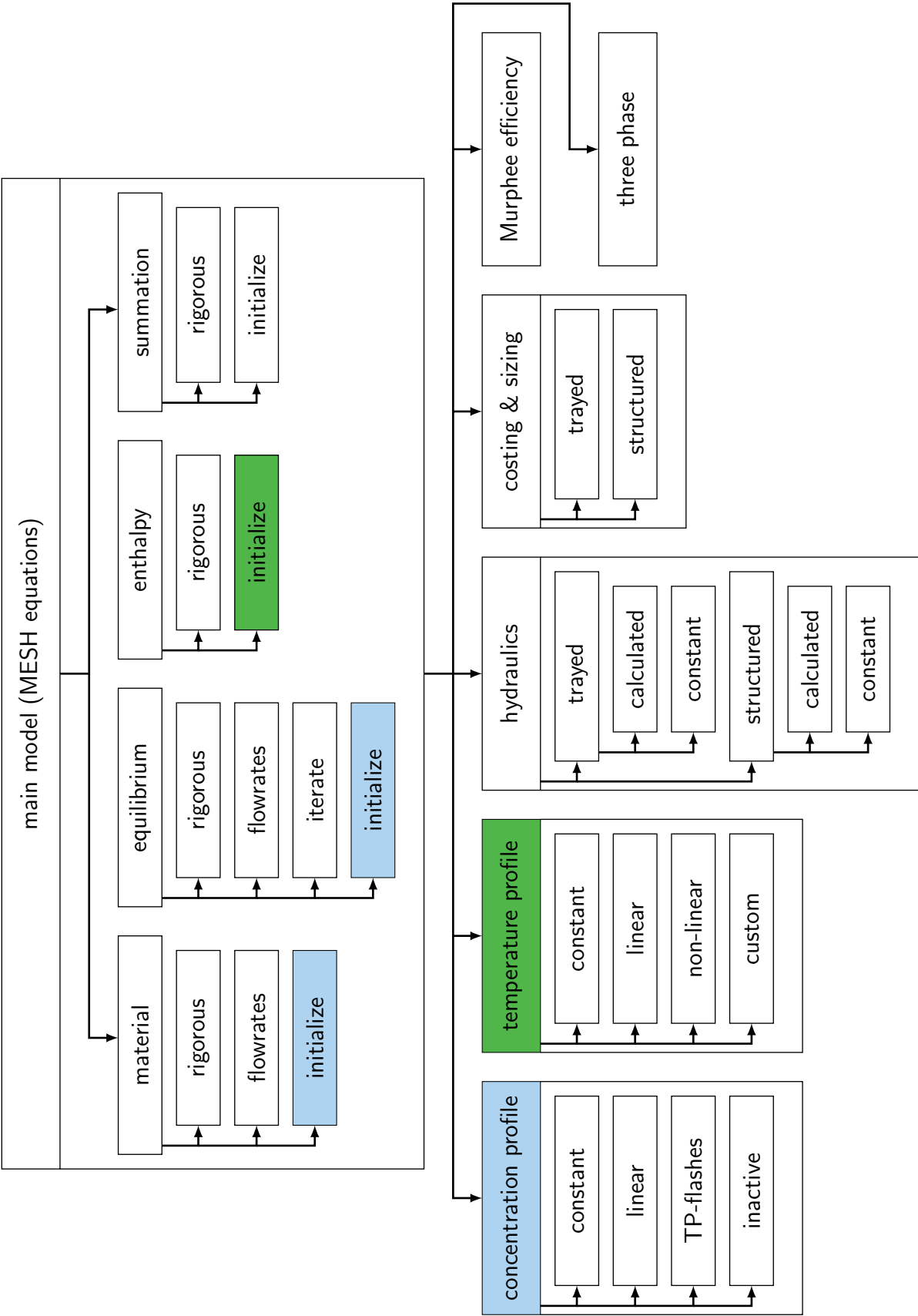


Figure 4.9: Distillation column model structure including sub-models and CASE branches.

The structure of the developed model is depicted in fig. 4.9. The main block denote separate model units that can, with the exception of the main model be activated or deactivated. The connections inside the main blocks denote selectors along with their possible branches.

The selective deactivation of sub-models is done by creating sub flowsheets and then assigning array of a respective model rather than a specific entity. By assigning an array size of zero, the model is effectively deactivated. This is done to reduce the number of variables and computational effort, if certain model capabilities are superfluous. for the rest of this section various model branches will be briefly elaborated upon. Within the main model, all branches labeled "rigorous" correspond to the equations as presented in sec. 4.1.1 and sec. 4.2. For the "flowrates" branches the mass balance has been written in terms of component flowrates rather than total flowrates and mole fractions. The "initilaize" branches of the material, equilibrium and energy selectors rely on the concentration profile and temperature profile sub-models to generate values, which are assigned straight forward to the respective variables. The reason those guesses were not directly implemented in the main model was to retain a level of flexibility of implementing alternative approaches without altering the general model structure. The meaning of the "iterate" branch within the equilibrium selector will be discussed in further detail in the next section.

The profile sub-models give several choices for initial profiles. In the case of the concentration profiles, the edge points will always be determined by a TP or vapour fraction flash of the mixed feed. While for temperature profile model the edge points can be either bubble and dew point temperature of the mixed-feed, or the feed temperatures of the highest (in terms of feed location) and lowest feed.

Having variable pressure along the column, sometimes greatly complicates initial convergence. Therefore can all hydraulics models be turned inactive, in which case they will return a constant previously assigned pressure drop for each stage.

The sub-model "three-phase" somewhat emulates calculations for the case when to liquid phases are present, as it needs to be ensured, that no wrongful information about the phase equilibrium is attained b y the model, which can for instance cause severe errors in the mass balance, while the overall model might still converge. It is simply assumed, that the liquid phases, while separate, are distributed in such a way, that the liquid phase might still be treated as a single phase. Hence in that case, the concentrations and enthalpies of both phases are mixed, while information about both phases is stored in appropriate variables.

### 4.5.2 Initialization procedures

This section deals with different initialization strategies for the distillation columns. Before going deeper into that topic, it needs to be pointed out, that physical property estimation in gPROMS<sup>®</sup> is done by means of so called foreign objects. Those are external objects, to which function calls along with the corresponding parameters can be passed and a return value or vector is returned. These calls range from simple properties like molecular weight of components to flash calculations in many variations.

As an initial remark it should be noted, that for all initialization procedures, if the hydraulic sub-model is activated, first an initial profile is assumed, and after the model has converged, the selector



within the hydraulics sub-model is switched to the calculated pressure drops. No problems with the hydraulics models were encountered, if the specified column geometry allowed for feasible column operations.

### Initial specifications

One issue that comes up when giving specifications for a column is, that not all specifications are compatible with the used initialization procedure. Specifically this is the case for specified duties purities or temperatures as they cannot be considered in the initialize branches.

For those cases it is necessary to supply alternative specifications, namely a reflux or boil-up ratio or any flowrate, which can be fulfilled during initial calculations. Additionally, if "extreme" specifications are given such as very high purities, or extremely low flowrates, convergence might with a given procedure might fail. One strategy, that has proven fruitful in those cases, was to supply a more moderate specification, initialize and converge the model up to the rigorous branches and then switch to the more complex specification. Especially when the switch is made in the rigorous branch, chances are high, that the solution, if a MOVE\_TO switch is employed, will only pass through feasible regions and a final solution can be attained.

These initial specification can be used in conjunction with all initialization procedures discussed in the following sections.

### Flash initialization

The flash initialization, is the simplest, but nevertheless a very effective and efficient way of initializing the model. As initial guess constant profiles are assigned for vapour and liquid concentrations taken from the aforementioned feed flash. For the temperatures a linear profile between dew and bubble point temperature of the feed flash is used. The vapour and liquid flowrates are computed from the constant molar overflow assumption while considering sub-cooled or super-heated feed streams. In terms of the model this strategy translates into starting all selectors in the initialize branches and then simultaneously switching all selectors to the rigorous branches.

### Linearized material balance

As discussed in the example given in sec. 4.1.1 this initialization procedure relies on first initializing the model as for the flash initialization. Afterwards, in an intermediate step, the enthalpy and summation branch are kept in the initialize branch, which means, the initial temperature profile is kept fixed, and the constant molar overflow assumption retained. The material and equilibrium selectors are then switched to the flowrates branches, where a linearized material balance is solved with constant equilibrium ratios and molar flowrates. The constant equilibrium ratios are enforces, by passing the values from the concentration profile sub-model to the physical property calls, rather than the actual computed values for molar concentrations.

The main reason for formulating this mass balance in terms of component flowrates is, that due to the assumptions a solution will yield discrepant results. Especially if the equilibrium ratios in the

converged solution span a large range of values. Due to these discrepancies some molar fractions might even become negative. As the corresponding variable type is bounded to zero or a small negative number, this might cause the solver to fail during convergence of this step.

Furthermore the molar fractions would also have to be re-normalized, as the summation to one is not enforced in this case.

Afterwards all selectors are again switched to the rigorous branch.

```
INITIALISATION_PROCEDURE
START
  material      := initialize ;
  equilibrium   := initialize ;
  energy        := initialize ;
  summation     := initialize ;
  side_scale    := 0 ;
  hydraulics    := constant ;
END
NEXT
  JUMP_TO
    material     := flowrates ;
    equilibrium  := flowrates ;
  END
END
NEXT
  JUMP_TO
    REVERT material ;
    REVERT equilibrium ;
    REVERT energy ;
    REVERT summation ;
  END
END
NEXT
  MOVE_TO
    side_scale := 1 ;
  END
END
NEXT
  JUMP_TO
    REVERT hydraulics ;
  END
END
```

## Iterative approach

Previously it has been said, that the inside-out algorithm has proven very robust when it comes to solving complex distillation problems. However it is based on an modular iterative approach which is incompatible with equation based process simulators.

One of the main aspects is to construct simplified models for enthalpies and the equilibrium, which only give accurate results for the point they have been constructed at, but are much better behaved, than rigorous enthalpy and equilibrium correlations. In each step of the iteration a new model for the physical properties is constructed, at the point, where the previous model evaluation with the previous physical properties model converged to. The iterative process terminates, when no further (or very little) changes occur in the newly constructed parameters for the new physical property models.

In order to mimic such an approach in gPROMS<sup>®</sup>, the branch "iterate" within the equilibrium selector has been created. There the physical properties are called with an new set of concentration variables, which are assigned arbitrary values in all other branches. To do the iterations another structure of gPROMS<sup>®</sup> is used. So called TASKS. With that it becomes possible, to converge a the model, with a given concentration stored in these variables, and after convergence, replace those values with the current values of liquid and vapour mole fractions.

The drawback however, is that the number of iterations to be carried out needs to be specified in advance. This strategy, while rather time consuming due to large overhead in computational time during execution of the TASK, has generated very promising results and with it it was possible to converge columns with very complex multi-component systems.

### SEQUENCE

```

i := 1 ;
WHILE i < number_of_iterations DO
  SEQUENCE
    PARALLEL
      REASSIGN
        column_section.x_iterate(,) := OLD(column_section.x(,));
        column_section.y_iterate(,) := OLD(column_section.y(,));
      END
    SWITCH
      column_section.phase_equilibrium
        := column_section.iterate_to_rigorous;
    END
  END
  i := i + 1 ;
END
END
SWITCH
  column_section.phase_equilibrium := column_section.rigorous;
END
END
```

### **Side draws**

During construction of process flowsheets and application of the developed models it turned out, that columns with side draws add extra complexity in terms of initialization. To accommodate that fact side draw versions of the previously described initialization procedures have been developed. The only difference is, that the side draws are initially set to zero once convergence has been achieved, they are again set to their specified values. Either by an instant change or again in a continuous fashion.

## 5 Process optimization

Process optimization bears great potential for designing ecologically and economically optimal processes. Within this chapter first some basic concepts of constrained optimization are summarized as this proves useful for the remainder of the chapter. The problems arising when optimizing chemical processes involve both continuous and discrete decisions. In order to solve this class of so called mixed-integer non-linear problems (MINLP) designated algorithms have to be employed to accommodate the special nature of the integer variables. The major approaches used today to solve this class of program are summarized in the second section. Lastly these approaches be applied to several problems in the context of optimizing the cryogenic air separation process.

### 5.1 Optimization theory

The purpose of this section is not an exhaustive review of (convex) optimization but rather to mention some elementary concepts and terms that will prove useful in the following sections. Foremost a brief discussion of optimality conditions for constrained problems – the so called Karush-Kuhn-Tucker (KKT) conditions – will be given. Closely linked to those conditions, and in some cases even a prerequisite for their validity, are the constraint qualifications which are subsequently discussed.

#### 5.1.1 Karush-Kuhn-Tucker conditions

This formulation of the KKT conditions is adapted from explanations in [30]. Before the conditions can be stated, some definitions are needed. Considering the most general case of a NLP

$$\begin{aligned} \min_x \quad & f(x) \\ \text{s.t.} \quad & h_i(x) = 0, \quad i \in \mathcal{E} \\ & g_i(x) \leq 0, \quad i \in \mathcal{I} \end{aligned} \tag{P1}$$

where  $f(x) : \mathbb{R}^n \mapsto \mathbb{R}$  denotes the objective function,  $\mathcal{E}$  and  $\mathcal{I}$  are sets of indices corresponding to equality and inequality constraints respectively. Furthermore are the functions  $f$ ,  $g$  and  $h$  differentiable and real-valued.

An active set denotes the union of the set of equality constraints and the inequality constraints where  $g_i(x) = 0$  holds. Thus we can write for the active set at any feasible point  $x$

$$\mathcal{A}(x) = \mathcal{E} \cup \{i \in \mathcal{I} | g_i(x) = 0\}. \tag{5.1}$$

For the constrained problem stated above the Lagrangian is defined as

$$\mathcal{L}(x, \lambda, \mu) := f(x) - \sum_{i \in \mathcal{E}} \lambda_i h_i(x) - \sum_{i \in \mathcal{I}} \mu_i g_i(x) \quad (5.2)$$

With that the optimality conditions for constrained optimization can be stated. First order or necessary KKT conditions must hold at the optimal point  $(x^*, \lambda^*, \mu^*)$

$$\nabla_x \mathcal{L}(x^*, \lambda^*, \mu^*) = 0, \quad (5.3)$$

$$h_i(x^*) = 0, \quad \forall i \in \mathcal{E} \quad (5.4)$$

$$g_i(x^*) \leq 0, \quad \forall i \in \mathcal{I} \quad (5.5)$$

$$\mu_i^* \geq 0, \quad \forall i \in \mathcal{I} \quad (5.6)$$

$$\mu_i^* g_i(x^*) = 0, \quad \forall i \in \mathcal{I} \quad (5.7)$$

$$\lambda_i^* h_i(x^*) = 0, \quad \forall i \in \mathcal{E} \quad (5.8)$$

$$(5.9)$$

The second order or sufficient KKT conditions can be written as

$$w^T \nabla_{xx} \mathcal{L}(x^*, \lambda^*, \mu^*) w > 0, \quad \forall w \neq 0, \quad (5.10)$$

$$\nabla h_i(x^*)^T w = 0, \quad \forall i \in \mathcal{E}, \quad (5.11)$$

$$(5.12)$$

### 5.1.2 Constraint qualifications

Constraint qualifications, can ensure the existence of strictly positive Lagrange multipliers, such that the unconstrained equivalent problem can be constructed. In general one is interested in the weakest possible prerequisite to ensure feasibility of the problem. There are several constraint qualifications proposed in literature, which are differently hard to fulfil and verify. In general a minimum point should satisfy some constraint qualifications or regularity conditions. Several more constraint qualifications can be found in literature, but will not be stated here.

#### Slater's constraint qualification (SCQ)

Among the most widely used constraint qualifications is Slater's constraint qualification

$$\exists \tilde{x}, \forall i \in \mathcal{I} : h_i(\tilde{x}) < 0. \quad (5.13)$$

This qualifications essentially states, that there in fact is a point which will fulfil the constraints.

#### Linear independence constraint qualification (LICQ)

Another widely used is called the linear independence constraint qualification. It states, given the set of feasible solutions of the original problem  $\mathcal{C} = \{x \in \mathcal{X} \mid h_i(x) \leq 0 \forall i \in \mathcal{I}\}$  and the set of active constraints  $\mathcal{A}(\bar{x}) = \{i \in \mathcal{I} \mid h_i(\bar{x}) = 0\}$

$$\{\nabla h_i(\bar{x}) \mid i \in \mathcal{A}(\bar{x})\} \quad \text{is linearly independent.} \quad (5.14)$$

### Mangasarian-Fromovitz constraint qualification (MFCQ)

$$\exists \tilde{u}, \forall i \in \mathcal{A}(\bar{x}) : \nabla h_i(\bar{x})(\tilde{u}) < 0. \quad (5.15)$$

## 5.2 Mixed-integer optimization

In this section some general considerations about mixed-integer non-linear programming (MINLP) will be presented. First the most commonly applied solution techniques will briefly be discussed on the basis of a review on the subject by Grossmann [14]. Then then an alternative approach to solve this class of problems, based on a continuous reformulation of the discrete decision variables, which has recently been proposed [21, 38] will also be introduced. This reformulation is also the basis for a comparison of different approaches to solve this class of problem within the process simulation environment gPROMS<sup>®</sup>.

### 5.2.1 Solution techniques

As previously stated, the following discussion of general solution techniques for MINLP's is largely based in the comprehensive review by I. E. Grossmann [14].

Several approaches to tackle this type of problem have successfully been applied to a multitude of cases for several years now. In general four major types of solution algorithms can be distinguished.

- branch and bound
- outer approximation
- generalized benders
- extended cutting plane
- LP/NLP ...

All these algorithms make use of a limited number of common subproblems, which are then solved in different configurations. Therefore these subproblems can previously be discussed and will be referred to when going elaborating on different algorithms. Furthermore it needs to be emphasised, that the presented solution techniques are designed for convex problems, and only in those cases an optimal solution can be found with a degree of confidence. While they can be applied to the more general non-convex case, only local optimality can be assured.

The general case of a MINLP takes the form

$$\begin{aligned} \min_{x,y} \quad & C = f(x, y) \\ \text{s.t.} \quad & g_j(x, y) \leq 0, \quad j \in \mathcal{J} \\ & x \in \mathcal{X}, \quad y \in \mathcal{Y} \end{aligned} \quad (\text{P2})$$

If the discrete variables in the discrete-continuous program are relaxed, the resulting NLP relaxation can be written as

$$\begin{aligned}
 \min_{x,y} \quad & C_{LB}^k = f(x, y) \\
 \text{s.t.} \quad & g_j(x, y) \leq 0, \quad j \in \mathcal{J} \\
 & x \in \mathcal{X}, \quad y \in \mathcal{Y}_R \\
 & y_i \leq \alpha_i^k, \quad i \in \mathcal{I}_{FL}^k \\
 & y_i \geq \beta_i^k, \quad i \in \mathcal{I}_{FU}^k
 \end{aligned} \tag{P3}$$

Here  $\mathcal{Y}_R$  denotes the relaxed set of the integer set  $\mathcal{Y}$ ,  $\mathcal{I}_{FL}^k$  and  $\mathcal{I}_{FU}^k$  are subsets of the indices denoting the entire set of integer variables. The relaxed integers contained in these sets are bounded to the values  $\alpha_i^k$  and  $\beta_i^k$  respectively. These bounds are lower and upper bound taken from the previous iteration of the algorithm in question. If  $\mathcal{I}_{FL}^k = \emptyset$  and  $\mathcal{I}_{FU}^k = \emptyset$  are empty sets, problem (P3) denotes the fully relaxed problem, initially solved in all algorithms. The optimal solution to this initially solved problem ( $C_{LB}^0$ ) poses an absolute lower bound to (P2) since all variables are fully relaxed.

If all discrete variables are fixed at a given value, the NLP subproblem for fixed  $y^k$  results, as only continuous variables are being considered.

$$\begin{aligned}
 \min_x \quad & C_{LB}^k = f(x, y^k) \\
 \text{s.t.} \quad & g_j(x, y^k) \leq 0, \quad j \in \mathcal{J} \\
 & x \in \mathcal{X}
 \end{aligned} \tag{P4}$$

If (P4) is infeasible the NLP feasibility subproblem for fixed  $y^k$  can be solved.

$$\begin{aligned}
 \min_x \quad & u \\
 \text{s.t.} \quad & g_j(x, y^k) \leq u, \quad j \in \mathcal{J} \\
 & x \in \mathcal{X}, \quad u \in \mathbb{R}
 \end{aligned} \tag{P5}$$

This NLP returns a strictly positive value for  $u$ .

Aside from the presented NLP's a linearized version of (P2) is regularly solved

$$\begin{aligned}
 \min_{x,y} \quad & C_L^k = \alpha \\
 \text{s.t.} \quad & f(x^k, y^k) + \nabla f(x^k, y^k)^T \begin{bmatrix} x - x^k \\ y - y^k \end{bmatrix} \leq \alpha \\
 & g_j(x^k, y^k) + \nabla g_j(x^k, y^k)^T \begin{bmatrix} x - x^k \\ y - y^k \end{bmatrix} \leq 0, \quad j \in \mathcal{J} \\
 & x \in \mathcal{X}, \quad y \in \mathcal{Y}, \quad k = 1 \dots K
 \end{aligned} \tag{P6}$$

The linearized problem can be constructed in several ways from the set of points  $K$  attained in previous iterations. Sometimes only violated or active constraints are linearized. When



objective function and constraints are convex, the objective function is underestimated, while the constraints are overestimated. From the overestimated constraints stems the name outer approximation.

### Branch & bound (B&B)

The branch and bound (B&B) algorithm has originally been proposed for linear problems, but has since been extended to handle non-linear objectives and constraints. As an initial step for the branch and bound the fully relaxed NLP (P3) is solved, which, as mentioned before, provides an absolute lower bound to the program in question. In the rare case that all relaxed integer variables assume integer values the optimal solution has been found and the algorithm can terminate. Otherwise a tree search is performed, exploring the space of the integer variables. In each step an increasing number of integer variables is fixed such that a NLP in the form of (P4) needs to be solved. The solution of these subproblems is a new lower bound for all descendant nodes. Further exploration of branches can be stopped, once a given subproblem returns a value greater than the current upper bound or becomes infeasible.

Due to the large number of NLP subproblems that have to be solved within the tree search, the branch and bound algorithm is most attractive, if the solution is computationally inexpensive, or few nodes have to be explored.

### Outer approximation (OA)

The outer approximation (OA) algorithm relies on consecutively solving (P4) and (P6). Each solution of the NLP with fixed  $y^k$  yields a new point  $(x^k, y^k)$  which is used to construct an updated version of the MILP. The MILP generally includes linearized versions of all constraints and the objective function. As more and more points become available during the iterative process, new constraints are constructed for each available point.

The main theorem for the derivation of the OA algorithm states, that the optimal solution of the problem (P6) constructed from all points  $(x^k, y^k)$ ,  $k \in K^*$ . Where  $K^*$  is made up of all optimal solutions of (P4) where the current  $y^k$  yields a feasible solution, and (P5) where infeasible solutions of the NLP with fixed  $y^k$  are encountered. It should again be emphasized, that this theorem holds only for convex a objective function and constraints.

As the points necessary to construct the aforementioned system are not available when the solution process commences, a smaller systems is constructed and extended as more points become available. The first point again results from solving a fully relaxed system. This again yields an absolute lower bound for the original problem. The solution of each consecutive MILP gives a new lower bound which will always be greater than the bounds from previous iterations. Without any prove this argument is supported by the fact that adding new linear constraints will limit the feasible region of the problem and hence further restrict the possible solutions for the continuous variables.

The optimal points attained from the NLP subproblems with fixed discrete values form upper bounds on the optimal solution. Here no statement can be made about the quality of the bound in each step, but rather is the current upper bound updated, once a lower value is encountered.

The iterative process terminates, once the current upper and lower bound are within a given tolerance. It can be pointed out, that the outer approximation algorithm converges to the optimal solution in a single iteration if objective function and constraints in the original problem are linear, since in that case problems (P2) and (P6) are equivalent.

### Generalized Benders decomposition (GBD)

The generalized Benders decomposition is very similar to the outer approximation algorithm. The main difference lies in way that (P6) is constructed. While for the outer approximation problem all constraints are included, in case of the GBD only active constraints are  $J^k = \{j \mid g_j(x^k, y^k) = 0\}$  considered. Furthermore the set of continuous variables is eliminated by considering the KKT-conditions. The resulting problem

$$\begin{aligned}
 \min_y \quad & C_L^K = \alpha \\
 \text{s.t.} \quad & f(x^k, y^k) + \nabla_y f(x^k, y^k)^T (y - y^k) \\
 & + (\mu^k)^T [g(x^k, y^k) + \nabla_y g(x^k, y^k)^T (y - y^k)] \leq \alpha, \quad k \in \mathcal{K}_{FS} \\
 & (\lambda^k)^T [g(x^k, y^k) + \nabla_y g(x^k, y^k)^T (y - y^k)] \leq 0, \quad k \in \mathcal{K}_{IS}.
 \end{aligned} \tag{P7}$$

Where  $\mathcal{K}_{FS}$  and  $\mathcal{K}_{IS}$  denote the sets of solutions for of feasible and infeasible iterations respectively.

This difference in formulation the MILP in the OA and GBD algorithm is what leads to the major computational differences. In general the MILP in the OA algorithm is expected to return tighter bounds than the respective problem in GBD. However the computational effort to solve the problem might be considerably higher for the OA case. Hence the GBD is expected to need more iterations, before converging, while the cost of a single iteration might be lower.

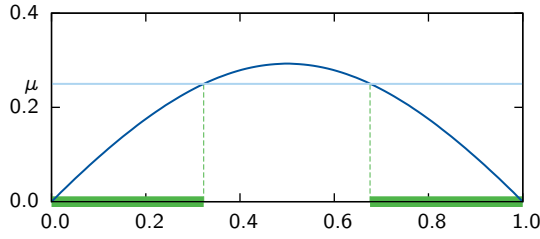
### Extended cutting plane (ECP)

The extended cutting plane method requires no solution any NLP subproblems. Rather a version of the linearized problem is solved during each iteration. After each iteration the problem is extended by an addition cut or linearized constraint. Most commonly a linear version of the most violated constraint is added. A further possibility is to add all violated constraints to the problem. It should be noted, that for the ECP method the objective function has to be linear. If necessary non-linearities in the objective function can be moved to the constraints by introduction of new variables.

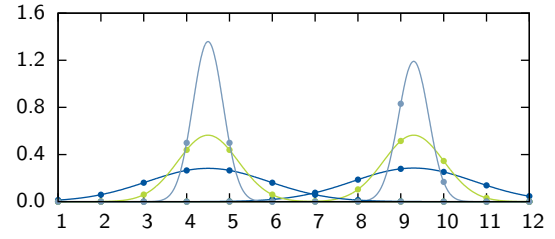
### LP/NLP branch & bound

#### 5.2.2 Continuous reformulation

An alternative approach to solving discrete-continuous problems is to reformulate them as NLP and introduce further constraints which ensure that the discrete decision variables will assume



(a) plot of Fischer-Burmeister function.



(b) plot of DDF function.

integer values. Several different authors have proposed ways of reformulating integer decisions. One common way is to introduce the so called complementary or equilibrium constraints which lead to problems that must be dealt with designated (NLP) solvers. Despite the fact that they possess poor theoretical features they have successfully been applied to problems of different scales. Lately however, alternative approaches have been suggested, which can be implemented in gPROMS<sup>®</sup> with relative ease and then solved with the NLP solvers included in the modeling environment. The first approach presented formulates a so called non-linear complementary problem (NCP) by introducing further constraints. Secondly within the distributed stream method, differentiable distribution functions (DDF) are formulated to model the associated integer decisions. In the following both approaches will be summarized briefly. A more detailed discussion of continuous reformulation techniques can be found in [38].

### Non-linear complementary problem (NCP)

To arrive at a formulation with favourable theoretical properties Kraemer et al. [20] proposed a continuous reformulation based on the Fischer-Burmeister function

$$1 - \sqrt{y_i^2 + (1 - y_i)^2} = 0. \quad (5.16)$$

The zero set of this function contains exactly the values zero and one and hence forces a continuous variable to take either integer value. However it is still very hard to solve complex, large-scale problems by simply enforcing this constraint. In order to facilitate convergence, the FB-function can be relaxed to a fully continuous problem

$$1 - \sqrt{y_i^2 + (1 - y_i)^2} \leq \mu. \quad (5.17)$$

The relaxation factor  $\mu$  is then gradually reduced to zero in a sequence of NLP problems. Fig. 5.1a shows a plot of the FB-function over the domain zero to one. The green lines on the x-axis denote the feasible domain for a given value of  $\mu$ . As one can see, if the relaxation factor is chosen large enough the entire span between zero and one is feasible, while for decreasing values the feasible set becomes disjunct and continuous variables are forced to either side to take on integer values.

### Differentiable distribution function (DDF)

An alternative way of reaching a continuous formulation was brought forth by Lang and Biegler [22] who within their distributed stream method propose the introduction of a differentiable distribution

function (DDF)

$$\psi_j = \frac{\exp \left[ - \left( \frac{j - \sum_i \zeta_{ii}}{\sigma} \right)^2 \right]}{\sum_k \exp \left[ - \left( \frac{k - \sum_i \zeta_{ii}}{\sigma} \right)^2 \right]}. \quad (5.18)$$

This function models a normal distribution around a central tray. As the standard deviation ( $\sigma$ ) is reduced, the spike at a given tray becomes more significant. Again in a series of NLP's  $\sigma$  is driven to small value. At  $\sigma = 0.25$  one can numerically consider the solution as integer. Fig. 5.1b shows a plot of the DDF function for a continuous stage value of 4.5 and 9.3 at  $\sigma = 2, 1, 0.5$ . Please note, that the values for each stage, denoted by dots, must add up to one. Depending on the distribution, a continuous plot can take on values larger than one.

## 5.3 Optimization examples

The simulation of chemical processes supplies valuable insight into the intricacies of a given system. To some extent it is also possible to improve given designs by manual alterations and simulation steps in a form of trial and error procedure. However the results of an approach like that are unlikely to yield even locally optimal results in an acceptable amount of time. Hence optimization approaches are employed to attain improved designs.

The models presented in the previous chapters are capable of projecting various continuous and discrete design decisions. However when synthesizing a process flowsheet, one must take careful consideration as to the optimization objectives. The most obvious objective would be of pure monetary nature, where a process is designed to meet a given set of specifications at minimum cost. An optimization like that can be carried out at steady state. However for a steady state model it is never ensured, that this steady state can actually be reached by the process. Furthermore there is no consideration if the desired steady state can be kept for changing conditions outside the control of the process operator.

In the case that load changes or different operation modes are required, it becomes questionable, if steady state optimization remains feasible, or if a dynamic approach becomes necessary.

In this chapter different optimization studies will be given. For once different solution approaches will be applied to the MINLP arising from a steady state simulation. For once the outer approximation algorithm as implemented in gPROMS<sup>®</sup>, and secondly the continuous reformulation strategy as described in Sec. 5.2.2.

The process under consideration is a generic cryogenic air separation process. As a starting point for the optimization a base case was manually constructed which met certain somewhat arbitrarily chosen, but reasonable constraints.

### 5.3.1 Example process

The developed model were combined to a process flowsheet for the cryogenic air separation process. As no specific process was being evaluated, the process configuration was modeled after an published

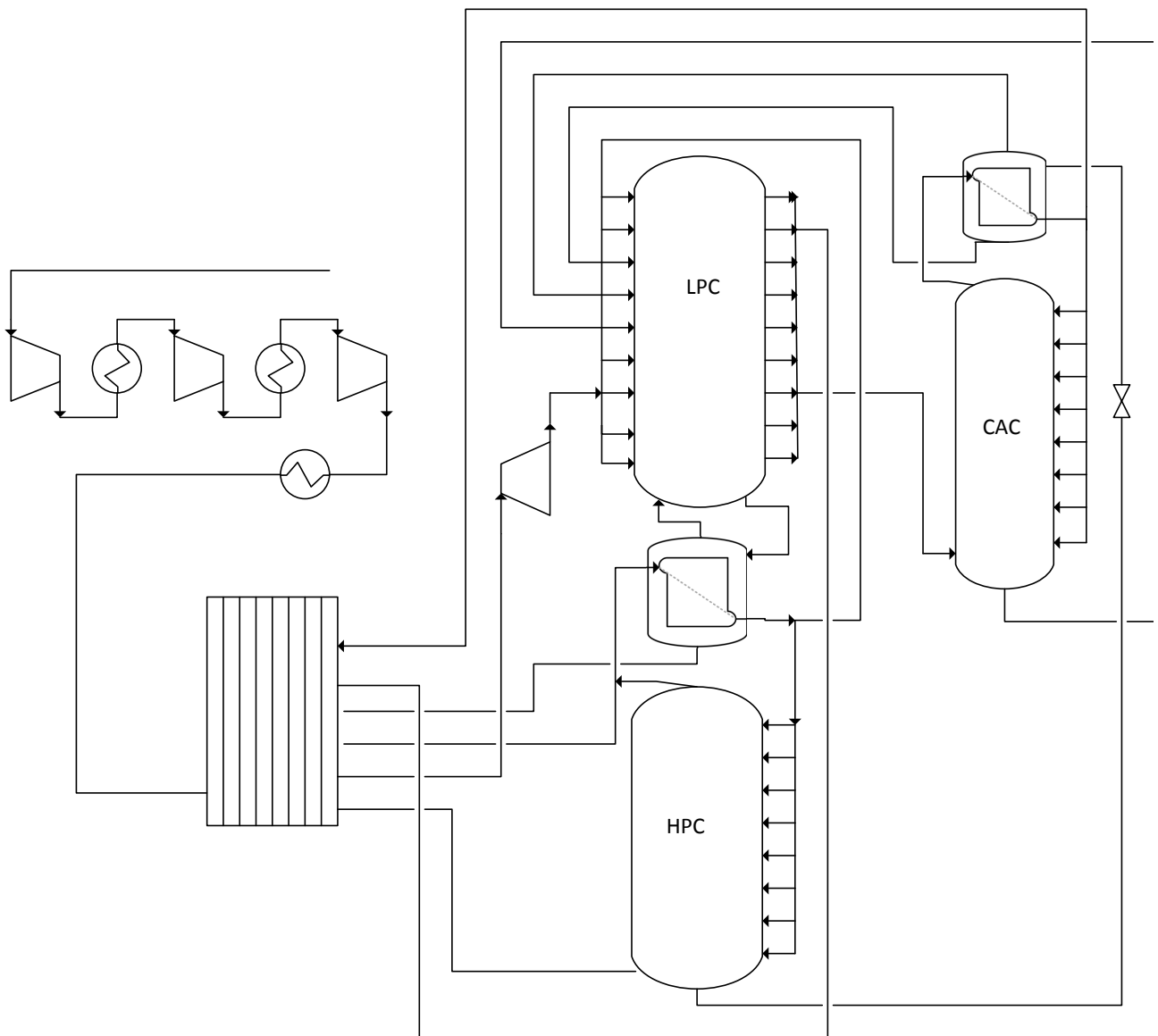


Figure 5.1: Optimization example process.

example process flowsheet. Fig. 5.1 shows the flowsheet considered henceforth. Air is drawn from the surrounding at ambient conditions and compressed with inter-cooling stages. Afterwards the entering process stream exchange heat with the product streams.

The stream is then divided into two separate streams, and compressed in a turbine, and fed to a low pressure column (LPC) while the majority is fed to the high pressure column (HPC). Condenser and reboiler are combined in a single unit. A side column to the LPC generates pure argon. The condensate from the HPC is partially fed into the LPC. The oxygen rich sump product from the HPC is used to condensate the CAC top product, and afterwards fed into the LPC. From the reboiler side liquid oxygen is recovered from the process. From the top of LPC and HPC gaseous nitrogen is drawn.

### 5.3.2 Degrees of freedom

In order to gain more insight into the behaviour of the dynamic models presented above an analysis of the degrees of freedom within the model is at hand. For the degrees of freedom the cost correlations will be disregarded, as they for a closed system of equations given the inputs generated from the column model. Furthermore interdependencies can be disregarded, as the cost model consist only of "forward" computations. In practical terms this statement can be confirmed since the models can be evaluated with and without costing equations. Hence only the stage and hydraulic equations will be considered.

For a given column without condenser or reboiler the model is made up of  $[n_S(5n_C + 24) + n_F]$  differential algebraic equations in  $[n_S(5n_C + 29) + n_F(n_S + n_C + 3) + 1]$  variables. In this isolated case all feed flow rates, their composition and enthalpies would be specified. In this case the feeds include a hypothetical condenser reflux (the upper most feed) as well as reboiler reflux (lowest feed). Along with the feeds and their qualities, the feed splits and reflux split have to be assigned. Lastly the column diameter needs to be known. This yields  $[n_F(n_S + n_C + 2) + n_S + 1]$  specifications. To close the system initial conditions for all states have to be given. There are a total of  $[4n_S]$  dynamic stages in a column section.

The condenser reboiler unit

### 5.3.3 Steady-state optimization

In a first step optimizations were carried out using the steady-state process model. While an effort has been made to create a realistic scenario, the main focus of this section is to demonstrate optimization capabilities within the model environment gPROMS<sup>®</sup> and to compare different solution approaches for complex discrete-continuous problems.

#### Single steady-state operation

In a first case operations for two separate steady-states were optimized. However only structural decisions on the columns were made along with the columns operational parameters. Tab. 5.2 shows

variable	value				
	initial	steady state 1		steady state 2	
		OA	FB(+DDF)	OA	FB(+DDF)
<b>objective</b>	<b>738279.6</b>	<b>336157.3</b>	<b>339387.5</b>	<b>345112.66</b>	<b>345452.5</b>
plant air flowrate	12431.2	12459.9	12433.7	12234.5	12459.9
CAC reflux location	6	16	15	16	16
CAC diameter	3.5	1.172	1.142	1.407	1.383
CRCAC reflux ratio	32	33.847	31.735	49.395	47.112
CRmain reflux ratio	5	4.425	4.038	4 <sup>a</sup>	4 <sup>a</sup>
HPC reflux location	6	15	13	19	20
HPC column diameter	4	2.915	2.900	2.929	3.017
HPC GN2 draw flowrate	2430	1861.67	1961.75	911.67	1092.78
LPC column diameter	5.5	3.942	3.908	4.154	4.242
LPC split air feed location	28	26	25	23	23
LPX oxygeb rich feed location	20	23	22	21	21
CAC reflux feed location	38	38	38	28	28
LPC reflux location	6	15	16	16	17
LPC Watse draw location	8	22	21	20	20
CAC side draw location	38	38	38	36	36
Waste flowrate	7160	7439.883	7413.6724	7581.9365	8257.699
CAC feed flowrate	730	665.17664	624.61566	959.4886	915.698

<sup>a</sup>variable at bound

Table 5.1: steady-state single period optimization results.

a list of all optimization variables along with their initial and optimized values for both steady-states. The optimized values are attained by solving the MINLP by outer approximation or as NCP-problem.

The objective function considers only capital cost for the separation system

$$CAPEX = \left( \sum_c C_c^{\text{cap}} \right) \cdot \left( q^{-a} \frac{q^a - 1}{q - 1} \right) \quad c = \{\text{HPC}, \text{LPC}, \text{CAC}\} \quad (5.19)$$

The difference in the operation modes lies on the requirements on product flow rates and purity. The following constraints were enforced When solving the continuous reformulation of the problem, an extra constraint for each integer variable in form of the relaxed Fischer-Burmeister function was added. Furthermore the closure condition for each location decision was added as an equality constraint.

variable	steady state 1		steady state 2	
	lower bound	upper bound	lower bound	upper bound
nitrogen product flowrate	3300	10000	3000	10000
nitrogen product purity	0.999	1.0	0.995	1.0
oxygen product flowrate	1700	10000	1600	10000
oxygen product purity	0.995	1.0	0.999	1.0
argon product flowrate	20	10000	15	10000
argon product purity	0.995	1.0	0.995	1.0
CAC diameter constraint	0	10	0	10
LPC diameter constraint	0	10	0	10
HPC diameter constraint	0	10	0	10

Table 5.2: steady-state single period optimization results.

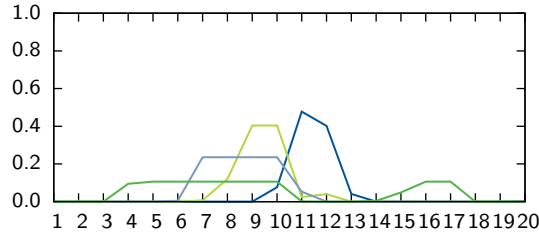


Figure 5.2: convergence failure.

This closure condition needs not to be explicitly considered using the outer approximation approach, since this is handled internally by declaring so called special ordered sets. This allows gPROMS<sup>®</sup> and the OA algorithm to exploit the special structure of the problem due to the *exclusive or* decisions.

In the process of solving the FB-program some difficulties arose. While (almost) integer values were found for most structural decisions, even after the initial iteration, especially the LPC column displayed some convergence problems. As the relaxation factor is reduced, the solutions are pushed towards integer values or in other words, the feasible region between zero and one is divided into two regions which become smaller and smaller. In some cases the discrete solutions remain on one side of the feasible domain, while still fulfilling the closure constraint. As the relaxation factor is reduced, the solver is no longer able to "make the jump" to the other side of the domain. This leads to small values for the integer decisions along most of the column. Fig. 5.2 shows an typical example of this for of convergence. After the initial NLP the feed is evenly distributed between two stages, neither really dominant. As the relaxation is tightened, the values are reduced, and distributed among more stages. As the relaxation factor approaches zero the closure condition can no longer be satisfied, which in turn prevents convergence of the solver.

This issue has been addressed, by [21] who propose a re-initialization strategy based on the magnitude of the Lagrange multipliers associated with the continuous variables. While the results using this adjusted strategy are very promising it could not be implemented within the scope of this thesis. In addition to the adjusted algorithm, the problem formulation can be altered to improve convergence of the solution algorithm. Two strategies can be employed, which exploit the structure of the distillation model, or force the continuous solution to both sides of the feasible domain [20].

The first strategy involves reformulation of purity constraints in terms of the reflux split

$$\sum_j \zeta_j^R x_j \leq x_{\text{spec}}. \quad (5.20)$$

As one will always observe a concentration gradient in a finite distillation column, this formulation favours integer solutions as it competes with a lower capital costs which distributes fractions on lower trays. Hence this strategy is assumed to be most successful for steep concentration gradients at either of the column. It should be considered, that this formulation requires the variable tray location to be on the side at which the specified purity lies.

The second approach relies on the previously introduced differentiable distribution function. Here a sufficient focus on a given tray can be enforced by adjusting the underlying standard deviation. By introducing that function in the first optimization stage, while continuing with the traditional formulation in the subsequent steps, convergence could be achieved.



### 5.3.4 Dual steady-state operation

For a second test case it was assumed, that the plant would have to operate under two distinct operation modes. One requiring a very high nitrogen purity and the other focusing on very pure oxygen. The steady states correspond to the ones from the previous section. To simultaneously consider two steady states, two instances were considered, that were allowed to differ in terms of operational decisions, but fixed for structural decisions. The structural decisions are all stream locations and diameters. The decision variables correspond to the ones from the previous sections.

This problem could only be converged using the reformulation strategy. The outer approximation algorithm did not converge for this particular case. However it needs to be pointed out, that a very similar case for somewhat less complex scenarios was converged by outer approximation. The particular case however could not even be solved using the optimal solution from the continuous reformulation as an initial guess. Otherwise the same initial point as in the previous section was used to initialize the optimization. The results are summarized in tab. ??.

## **6 Conclusion and further research**

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# A Appendix A

## A.1 Peng-Robinson derived properties

The Peng-Robinson equation of state can be rewritten as a cubic polynomial in terms of the compressibility factor  $Z = \frac{pv}{RT}$

$$0 = Z^3 + \alpha Z^2 + \beta Z + \gamma \quad (\text{A.1})$$

$$\alpha = B - 1 \quad (\text{A.2})$$

$$\beta = A - 2B - 3B^3 \quad (\text{A.3})$$

$$\gamma = B^3 + B^3 - AB \quad (\text{A.4})$$

$$A = \frac{ap}{(RT)^2} \quad (\text{A.5})$$

$$B = \frac{bp}{RT} \quad (\text{A.6})$$

The *Joule-Thompson* factor can be expressed as

$$\mu_{JT} = \frac{1}{c_p} \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right] \quad (\text{A.7})$$

and in terms of the compressibility factor

$$\left( \frac{\partial v}{\partial T} \right)_p = \frac{R}{p} \left[ T \left( \frac{\partial Z}{\partial T} \right)_p + Z \right] \quad (\text{A.8})$$

$$\left( \frac{\partial Z}{\partial T} \right)_p = \frac{\left( \frac{\partial A}{\partial T} \right)_p (B - Z) + \left( \frac{\partial B}{\partial T} \right)_p (6BZ + 2Z - 3B^2 - 2B + A - Z^2)}{3Z^2 + 2(B - 1)Z + (A - 2B - 3B^2)} \quad (\text{A.9})$$

$$\left( \frac{\partial A}{\partial T} \right)_p = \frac{p}{(RT)^2} \left( a' - \frac{2a}{T} \right) \quad (\text{A.10})$$

$$\left( \frac{\partial B}{\partial T} \right)_p = \frac{-bp}{RT^2} \quad (\text{A.11})$$

$$a' = \frac{da}{dT} = \frac{1}{2} \sum_{i=1}^c \sum_{j=1}^c w_i w_j (1 - k_{ij}) \left( \sqrt{\frac{a_j}{a_i}} a'_i + \sqrt{\frac{a_i}{a_j}} a'_j \right) \quad (\text{A.12})$$

$$a'_i = \frac{da_i}{dT} = \frac{-m_i a_i}{\left[ 1 + m_i \left( 1 - \sqrt{\frac{T}{T_{ci}}} \right) \right] \sqrt{T T_{ci}}} \quad (\text{A.13})$$

$$\ln \varphi = \frac{1}{RT} \int_0^p \left( V - \frac{RT}{p} \right) dp, \quad (\text{A.14})$$

$$\ln \varphi = (Z - 1) - \ln Z - \int_{\infty}^V \frac{Z - 1}{V} dV \quad (\text{A.15})$$

$$\begin{aligned} \ln \varphi_i = & \frac{b_i}{b} (Z - 1) - \ln \left[ Z \cdot \left( 1 - \frac{b}{V} \right) \right] \\ & + \frac{1}{bRT} \left[ \frac{\sqrt{2}ab_i}{4b} - \sqrt{\frac{aa_i}{2}} \right] \ln \left( 1 + \frac{b}{V} \right) \left( \frac{1 + \frac{b}{V} (1 + \sqrt{2})}{1 + \frac{b}{V} (1 - \sqrt{2})} \right) \end{aligned} \quad (\text{A.16})$$

## **B Appendix B**