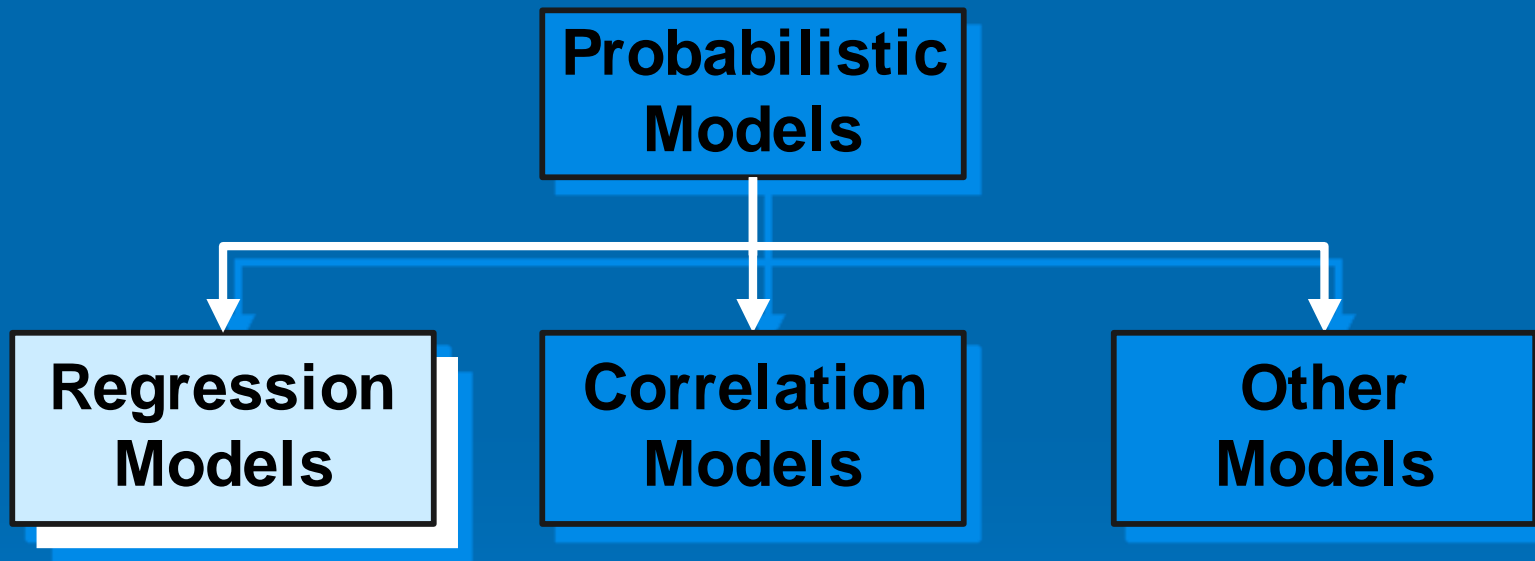


# Learning Objectives...

## 1. Correlation Models

1. Link between a correlation model and a regression model

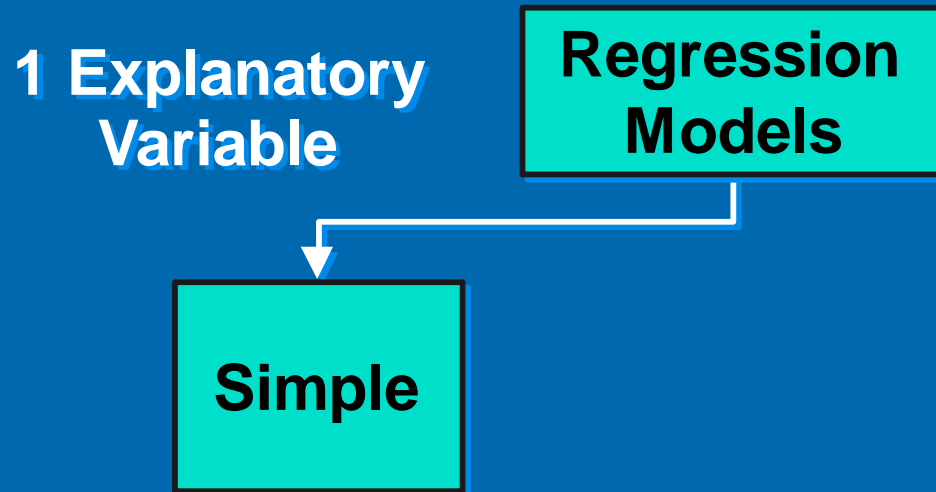
# Types of Probabilistic Models



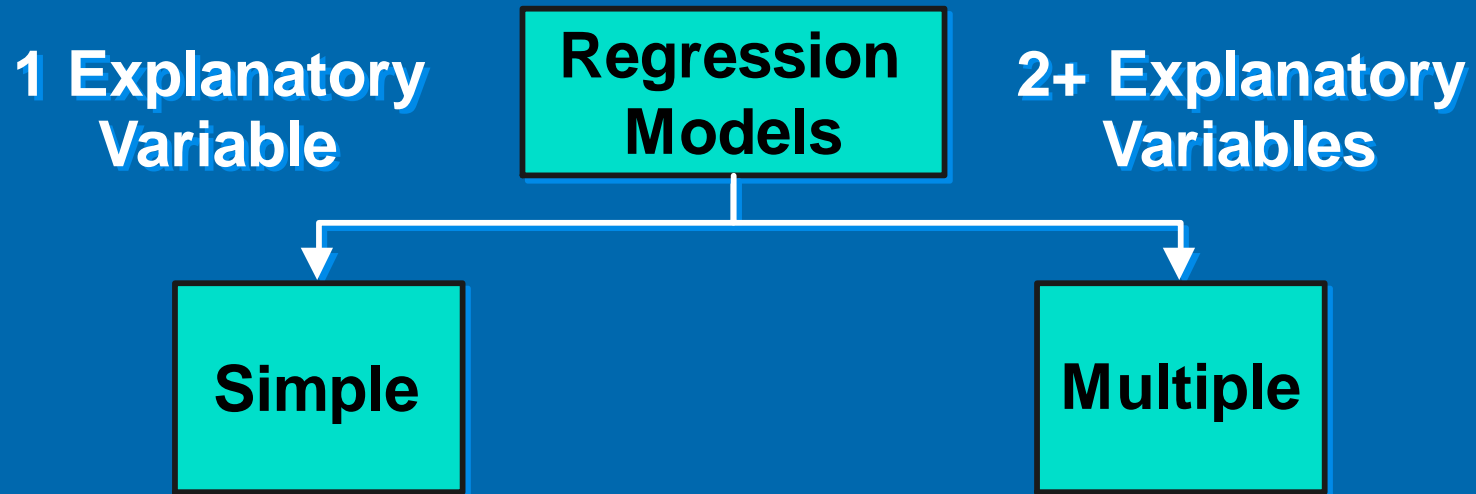
# Types of Regression Models

**Regression  
Models**

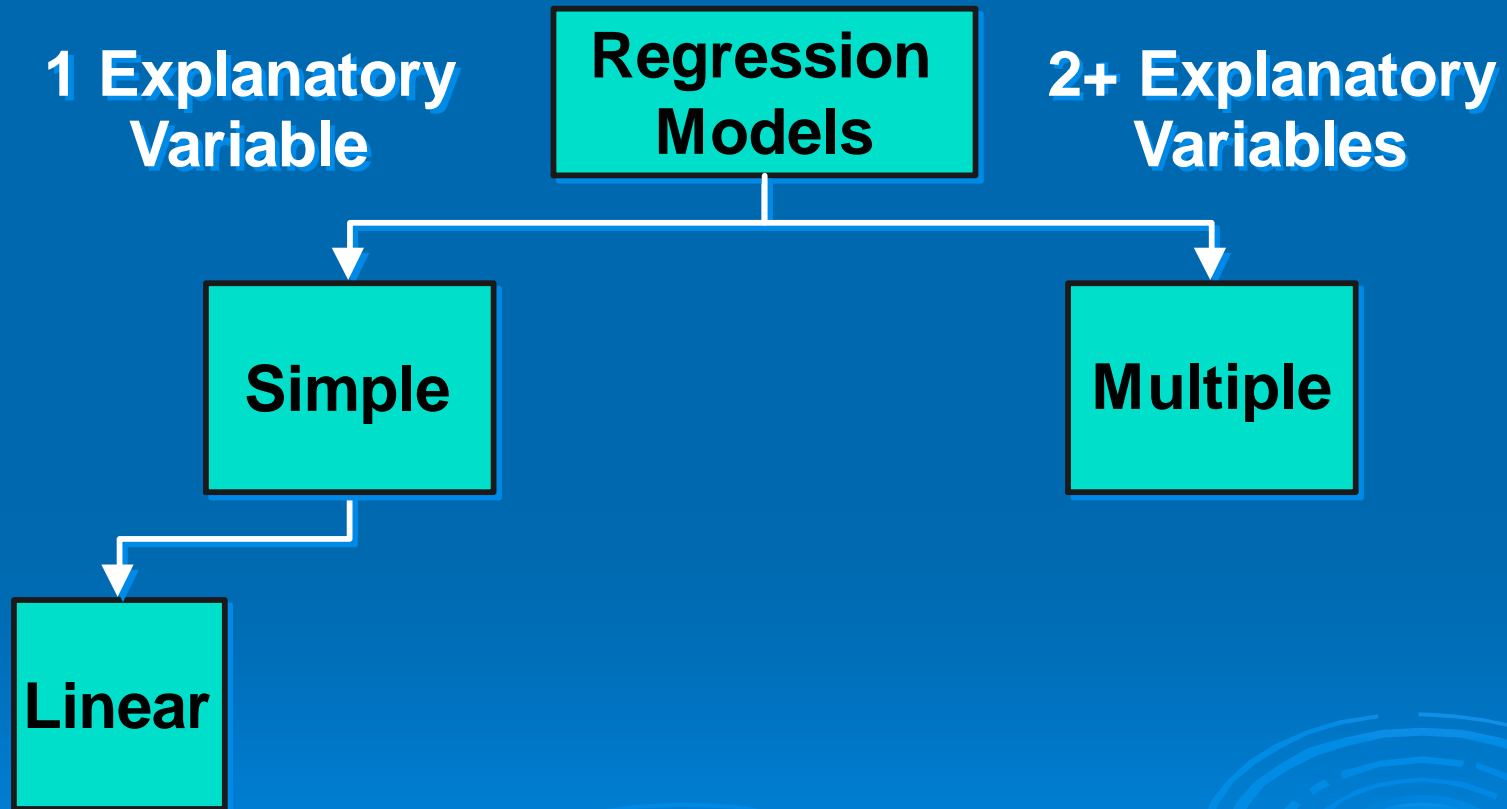
# Types of Regression Models



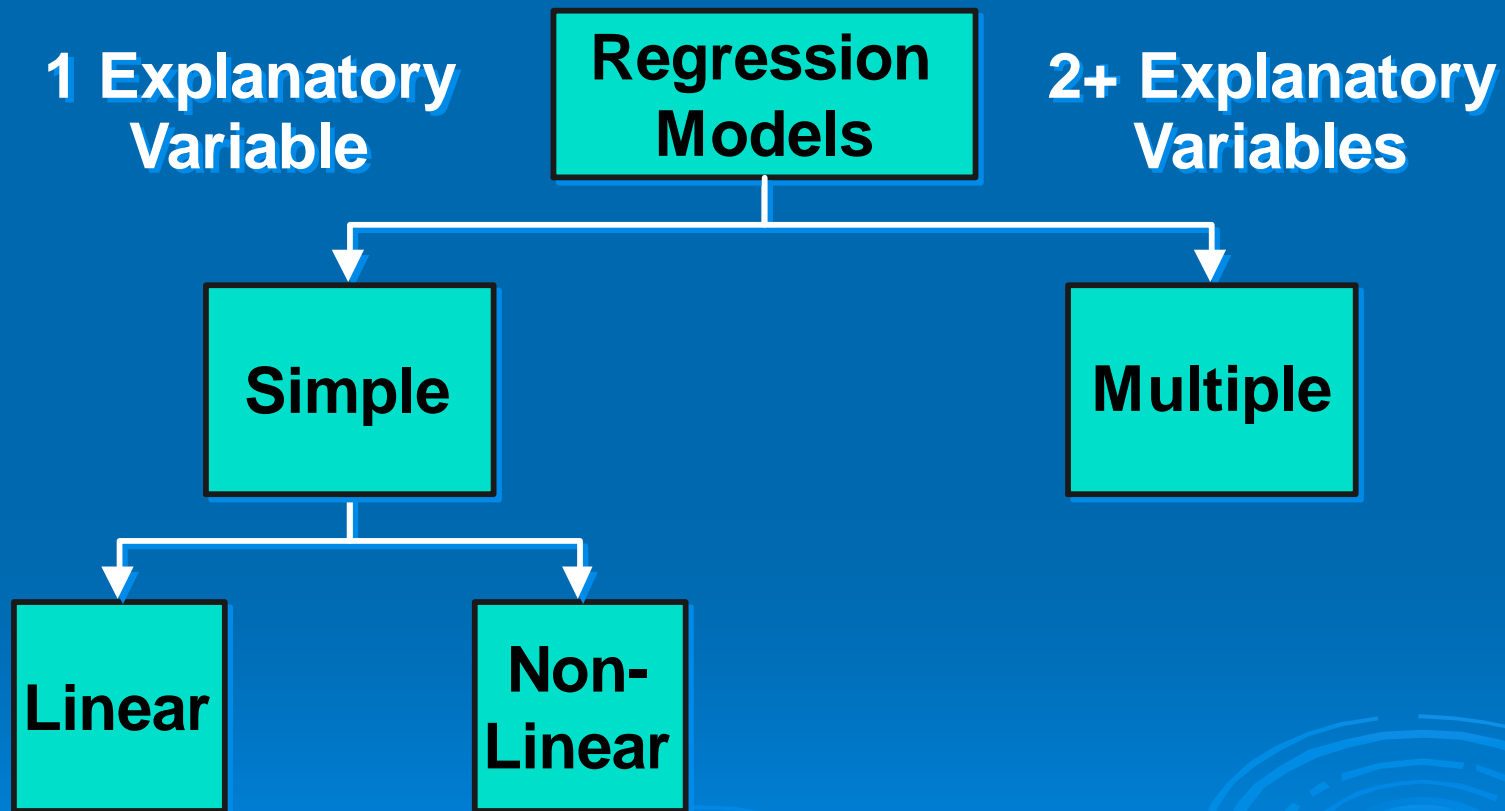
# Types of Regression Models



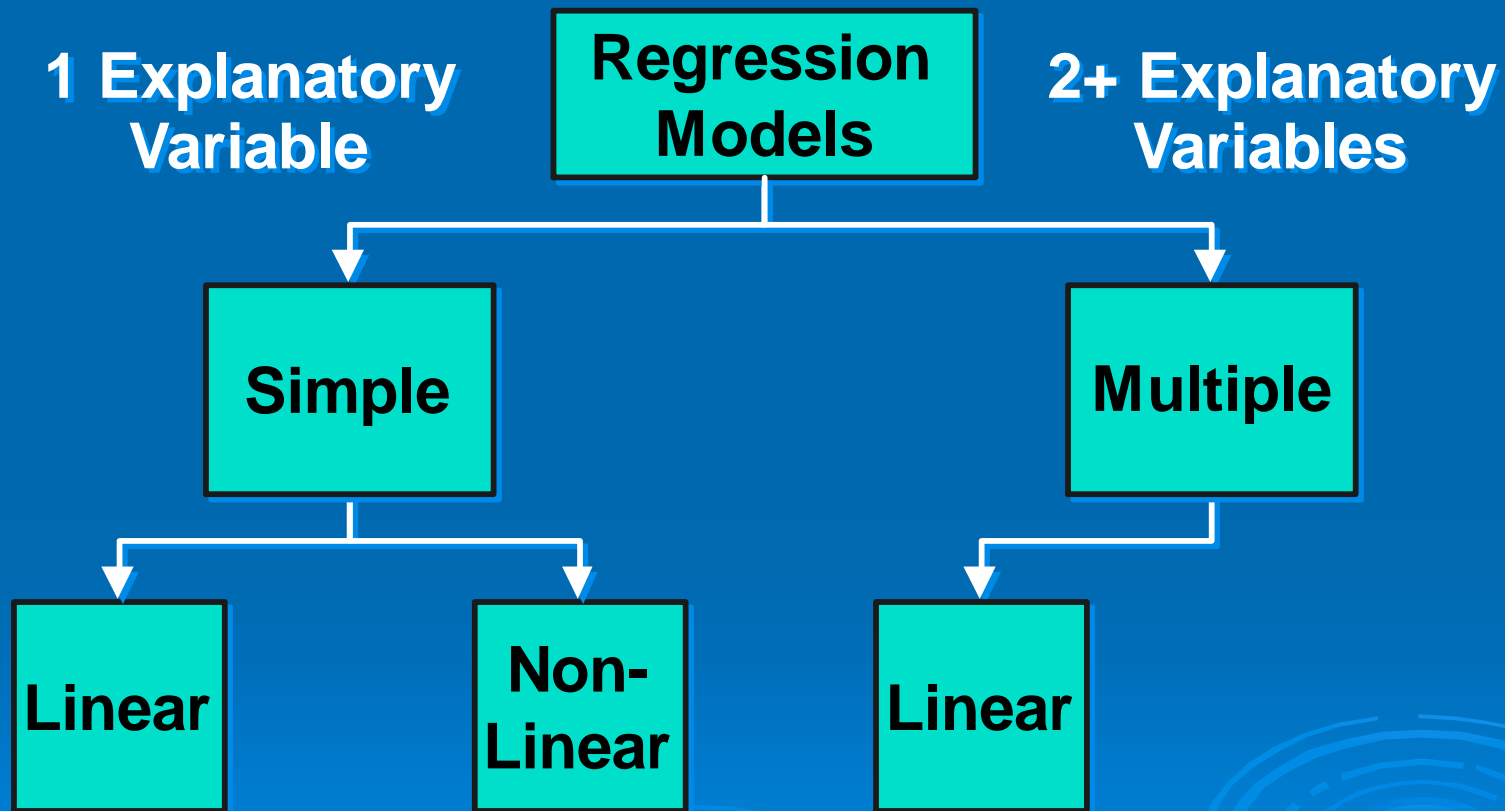
# Types of Regression Models



# Types of Regression Models

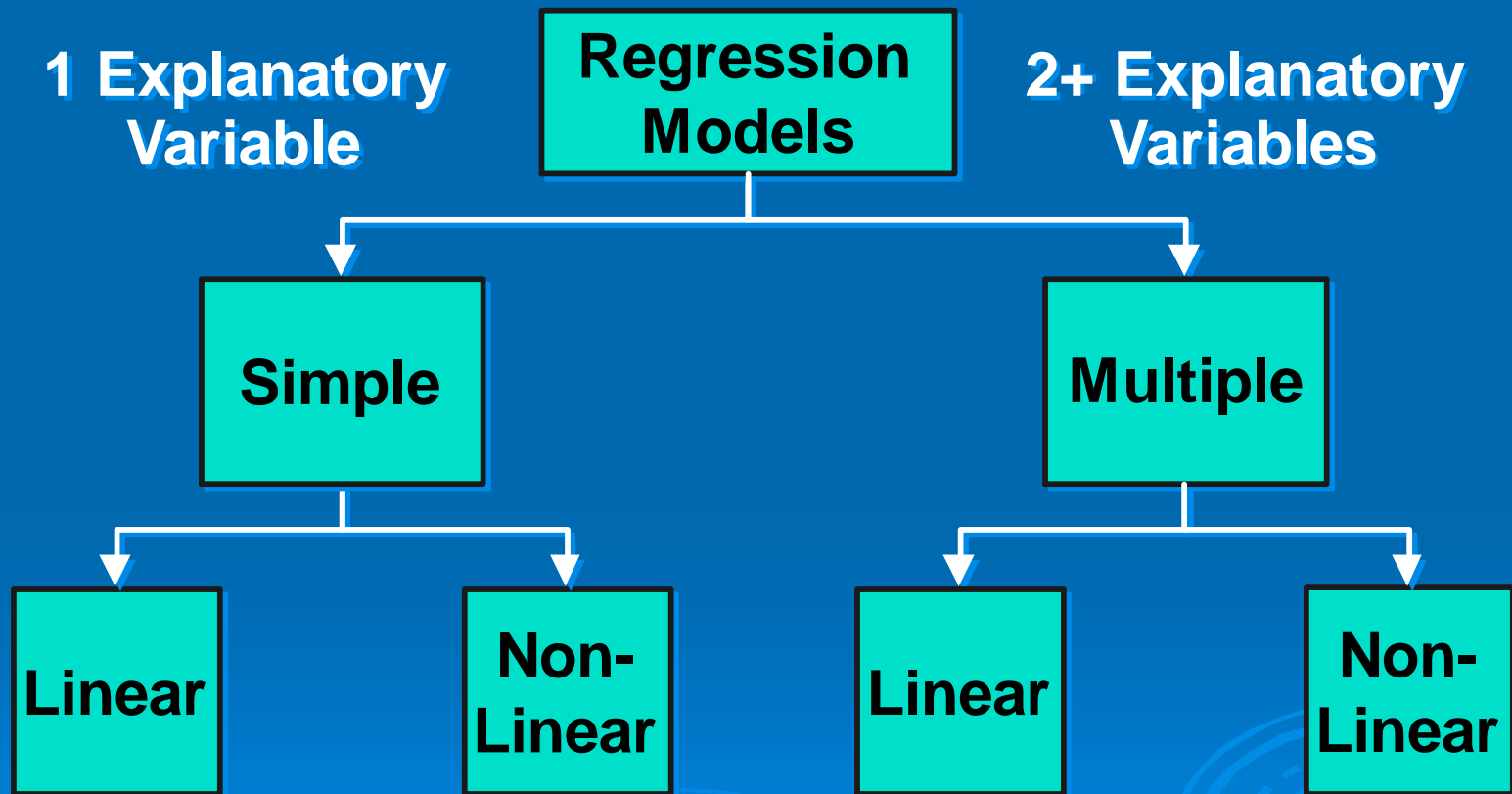


# Types of Regression Models

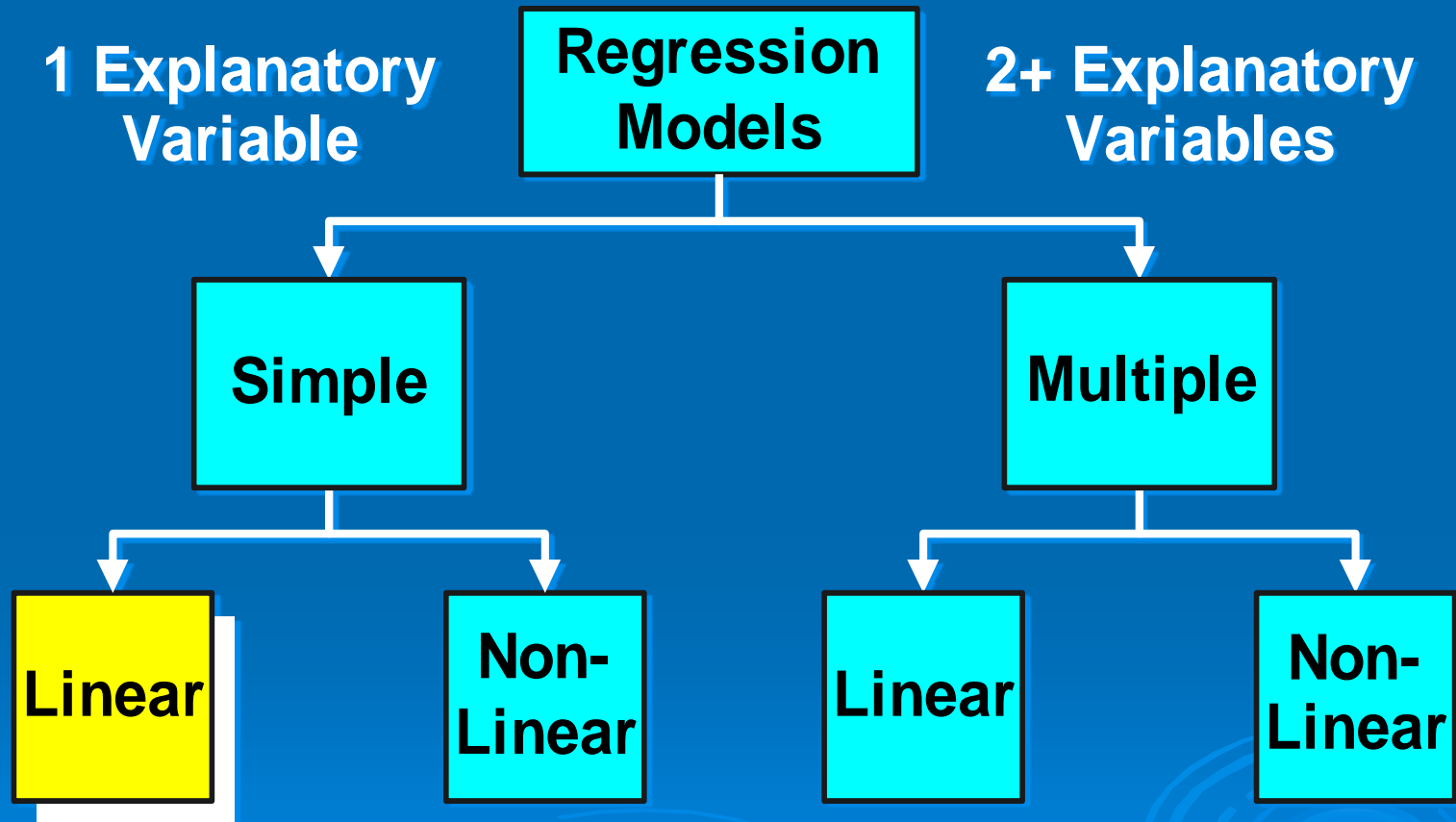




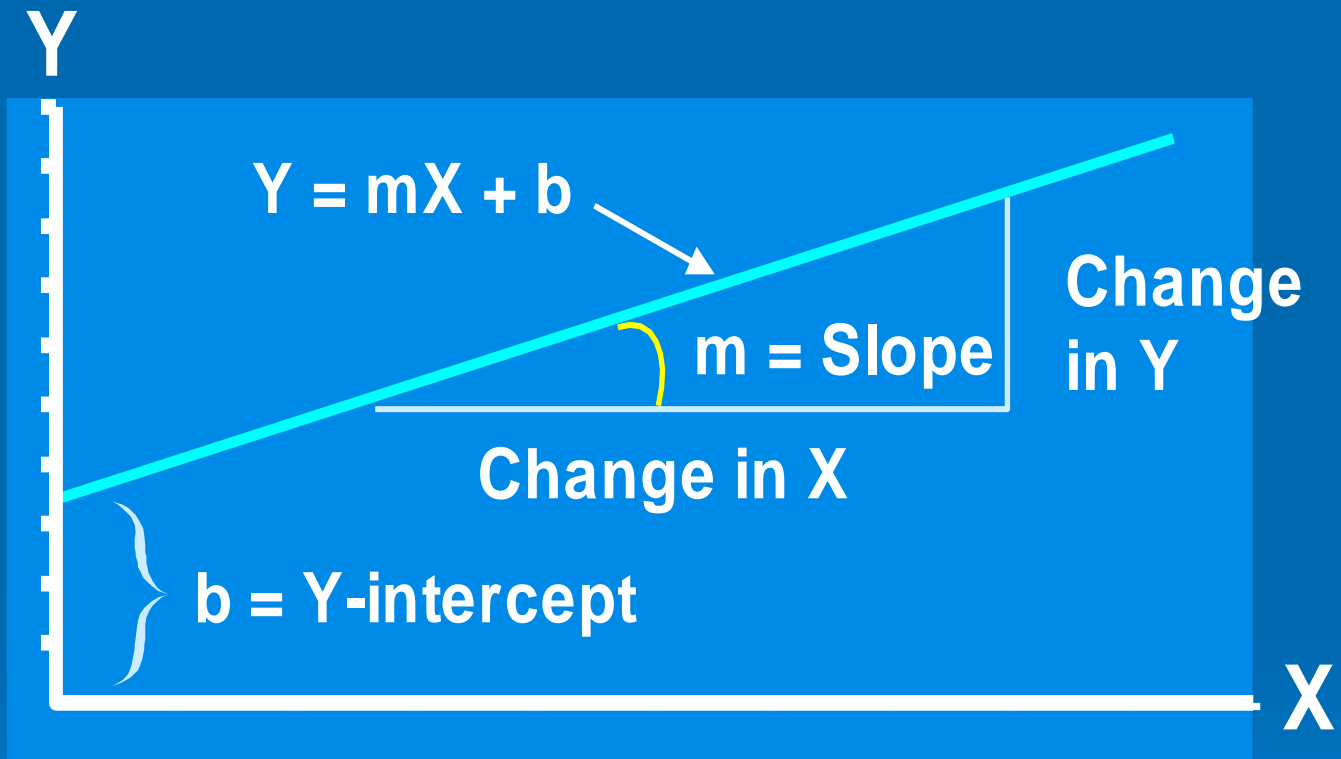
# Types of Regression Models



# Types of Regression Models



# Linear Equations



# Linear Regression Model

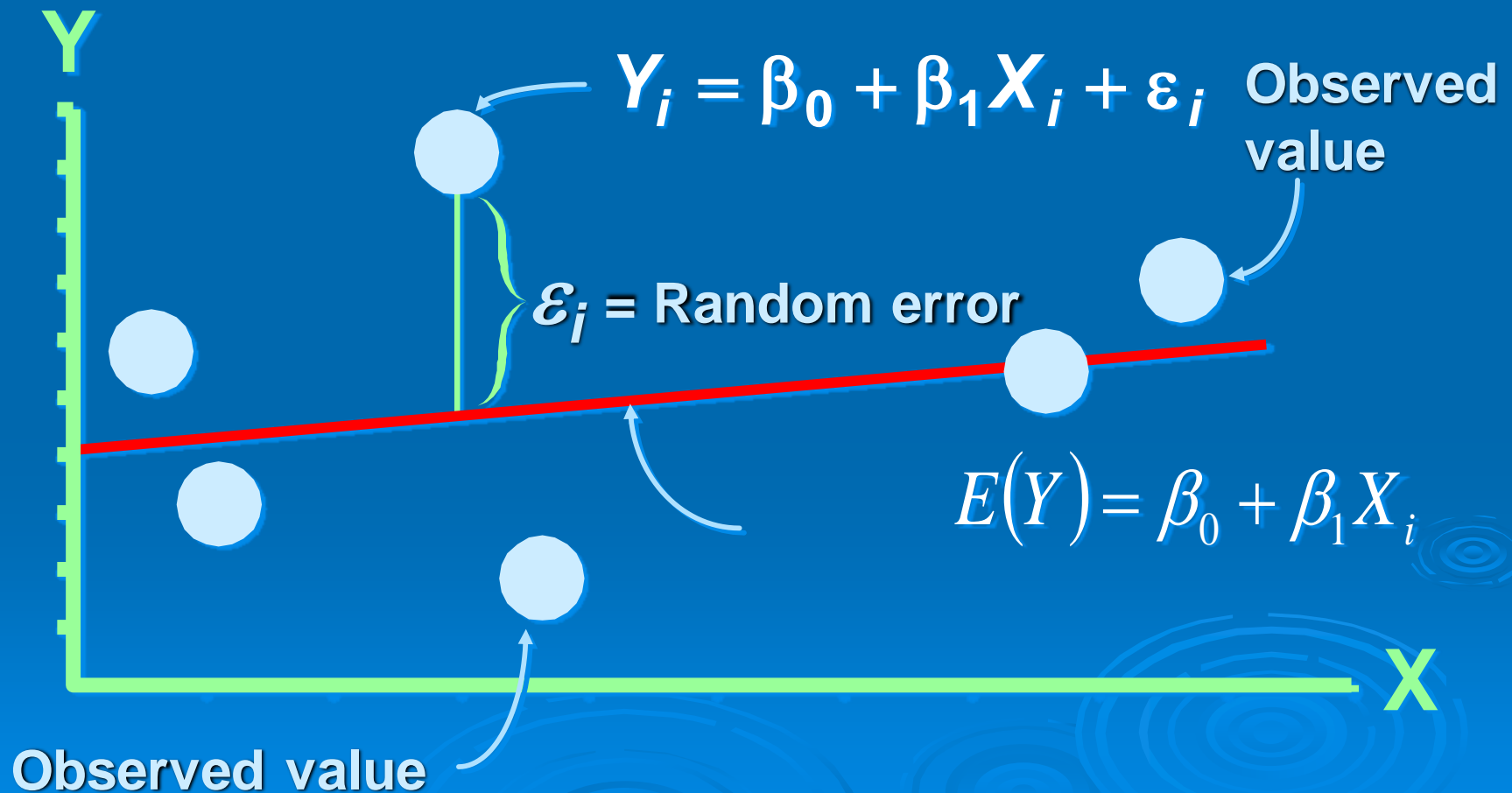
- 1. Relationship Between Variables is a Linear Function

The diagram illustrates the Linear Regression Model equation  $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ . Arrows point from descriptive labels to the corresponding parts of the equation:

- Population Y-Intercept** points to  $\beta_0$ .
- Population Slope** points to  $\beta_1$ .
- Random Error** points to  $\varepsilon_i$ .
- Dependent (Response) Variable (e.g., CD+ c.)** points to  $Y_i$ .
- Independent (Explanatory) Variable (e.g., Years s. serocon.)** points to  $X_i$ .

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

# Population Linear Regression Model



# Example with R

- `library("Hmisc")`
- `mydata <- iris`
- `str(mydata)`
- 'data.frame': 150 obs. of 5 variables:
- `$ Sepal.Length: num 5.1 4.9 4.7 4.6 5 5.4 4.6 5 4.4 4.9 ...`
- `$ Sepal.Width : num 3.5 3 3.2 3.1 3.6 3.9 3.4 3.4 2.9 3.1 ...`
- `$ Petal.Length: num 1.4 1.4 1.3 1.5 1.4 1.7 1.4 1.5 1.4 1.5 ...`
- `$ Petal.Width : num 0.2 0.2 0.2 0.2 0.2 0.4 0.3 0.2 0.2 0.1 ...`
- `$ Species : Factor w/ 3 levels "setosa","versicolor",...: 1 1 1 1 1 1 1 1 1 1 ...`

➤ `cor <- rcorr(as.matrix(mydata[1:4]))`

	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
Sepal.Length	1.00	-0.12	0.87	0.82
Sepal.Width	-0.12	1.00	-0.43	-0.37
Petal.Length	0.87	-0.43	1.00	0.96
Petal.Width	0.82	-0.37	<b>0.96</b>	1.00

➤ `n= 150`

➤ `P`

	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
Sepal.Length		0.1519	0.0000	0.0000
Sepal.Width	0.1519		0.0000	0.0000
Petal.Length	0.0000	0.0000		0.0000
Petal.Width	0.0000	0.0000	<b>0.0000</b>	

➤ **Fit** <- **lm** (**mydata\$Petal.Width** ~ **mydata\$Petal.Length**)

➤ **Summary** (**Fit**)

➤ Call:

➤ **lm**(formula = **mydata\$Petal.Width** ~ **mydata\$Petal.Length**)

➤ Residuals:

➤      Min            1Q            Median            3Q            Max  
➤ -0.56515 -0.12358 -0.01898 0.13288 0.64272

➤ **Coefficients:**

➤                                  Estimate          Std. Error      t value          **Pr(>|t|)**  
➤      (Intercept)              -0.363076      0.039762      -9.131          **4.7e-16 \*\*\***  
➤      mydata\$Petal.Length      0.415755      0.009582      43.387          **< 2e-16 \*\*\***

➤ ---

➤ Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

➤ Residual standard error: 0.2065 on 148 degrees of freedom

➤ Multiple R-squared: 0.9271, **Adjusted R-squared: 0.9266**

➤ F-statistic: 1882 on 1 and 148 DF, **p-value: < 2.2e-16**



# Linear Regression Model

$$\begin{cases} y = \beta_0 + \beta_1 x_1 + e \\ B_0 = -0.363 \quad \beta_1 = 0.415 \end{cases}$$

$$y = -0.363 + 0.415 x_1 + e$$

p-value:  $< 2.2e-16$