

## 7.2 Experiments for Hamiltonian Circuit

For the MLP variant that considers Hamiltonian circuits as solutions, the two already introduced sets for this version were tested on MDM-GILS-RVND: a set of 23 instances selected in [1, 2] and a new set of 56 instances selected for this work. All instances of these sets are found in TSPLib.

Tables 7.1, 7.2, and 7.3 show the results on the 23-instance set selected in [1, 2] from TSPLib. For the tables of this chapter, since more than two algorithms are evaluated, the count Better refers to the number of best results that the respective strategy achieved. The obtained results regarding the best solution showed MDM-GILS-RVND found the optimal solutions for all instances along with the other heuristics. Regarding average solution, GILS-RVND, DM-GILS-RVND and MDM-GILS-RVND obtained, respectively, 23, 21 and 20 best results. Considering computational time, the MDM-GILS-RVND and DM-GILS-RVND obtained in average, respectively, 5.48% and 5.06% of less computational time compared to GILS-RVND. Therefore, being MDM-GILS-RVND the fastest heuristic for this group.

Tables 7.4, 7.5 and 7.6 report the results on the 56-instances set. These results demonstrated MDM-GILS-RVND had the best performance among the heuristics in this group, achieving 36 best average solutions against 24 and 22, respectively, for DM-GILS-RVND and GILS-RVND. Regarding best solution, MDM-GILS-RVND also performed better than the other heuristics with 43 best results, while DM-GILS-RVND and GILS-RVND achieved, respectively, 39 and 36. In terms of running time, the MDM heuristic was better, attaining 43 best running times against 12 and 1, respectively, for DM-GILS-RVND and GILS-RVND. The general average of running time obtained by MDM-GILS-RVND was reduced in 23.36% compared to GILS-RVND, against 21.09% obtained by DM-GILS-RVND compared to GILS-RVND. In SST terms, ten instances, being eight instances using Student's t-test and two instances using Wilcoxon test, were statistically significant, where MDM-GILS-RVND was better than GILS-RVND. One instance was statistically significant when GILS-RVND was better than MDM-GILS-RVND in the Student's t-test. Additionally, where MDM-GILS-RVND was better than DM-GILS-RVND, two instances for Student's t-test and one instance for Wilcoxon test were statistically significant.

Instance	OPT/ BKS	GILS-RVND		DM-GILS-RVND		MDM-GILS-RVND	
		Best Solution	Average Solution	Best Solution	Average Solution	Best Solution	Average Solution
dantzig42	12528	<b>12528</b>	<b>12528.0</b>	<b>12528</b>	<b>12528.0</b>	<b>12528</b>	<b>12528.0</b>
swiss42	22327	<b>22327</b>	<b>22327.0</b>	<b>22327</b>	<b>22327.0</b>	<b>22327</b>	<b>22327.0</b>
att48	209320	<b>209320</b>	<b>209320.0</b>	<b>209320</b>	<b>209320.0</b>	<b>209320</b>	<b>209320.0</b>
gr48	102378	<b>102378</b>	<b>102378.0</b>	<b>102378</b>	<b>102378.0</b>	<b>102378</b>	<b>102378.0</b>
hk48	247926	<b>247926</b>	<b>247926.0</b>	<b>247926</b>	<b>247926.0</b>	<b>247926</b>	<b>247926.0</b>
eil51	10178	<b>10178</b>	<b>10178.0</b>	<b>10178</b>	<b>10178.0</b>	<b>10178</b>	10184.3
berlin52	143721	<b>143721</b>	<b>143721.0</b>	<b>143721</b>	<b>143721.0</b>	<b>143721</b>	<b>143721.0</b>
brazil58	512361	<b>512361</b>	<b>512361.0</b>	<b>512361</b>	<b>512361.0</b>	<b>512361</b>	<b>512361.0</b>
st70	20557	<b>20557</b>	<b>20557.0</b>	<b>20557</b>	<b>20557.0</b>	<b>20557</b>	<b>20557.0</b>
eil76	17976	<b>17976</b>	<b>17976.0</b>	<b>17976</b>	<b>17976.0</b>	<b>17976</b>	<b>17976.0</b>
pr76	3445242	<b>3445242</b>	<b>3445242.0</b>	<b>3445242</b>	<b>3445242.0</b>	<b>3445242</b>	<b>3445242.0</b>
pr76r	345427	<b>345427</b>	<b>345427.0</b>	<b>345427</b>	<b>345427.0</b>	<b>345427</b>	<b>345427.0</b>
gr96	2097170	<b>2097170</b>	<b>2097170.0</b>	<b>2097170</b>	2097682.3	<b>2097170</b>	2097682.3
rat99	57986*	<b>57986</b>	<b>57986.0</b>	<b>57986</b>	<b>57986.0</b>	<b>57986</b>	<b>57986.0</b>
kroA100	983128	<b>983128</b>	<b>983128.0</b>	<b>983128</b>	<b>983128.0</b>	<b>983128</b>	<b>983128.0</b>
kroB100	986008	<b>986008</b>	<b>986008.0</b>	<b>986008</b>	<b>986008.0</b>	<b>986008</b>	<b>986008.0</b>
kroC100	961324	<b>961324</b>	<b>961324.0</b>	<b>961324</b>	<b>961324.0</b>	<b>961324</b>	<b>961324.0</b>
kroD100	976965	<b>976965</b>	<b>976965.0</b>	<b>976965</b>	<b>976965.0</b>	<b>976965</b>	<b>976965.0</b>
kroE100	971266	<b>971266</b>	<b>971266.0</b>	<b>971266</b>	<b>971266.0</b>	<b>971266</b>	<b>971266.0</b>
rd100	340047	<b>340047</b>	<b>340047.0</b>	<b>340047</b>	<b>340047.0</b>	<b>340047</b>	<b>340047.0</b>
eil101	27513*	<b>27513</b>	<b>27513.0</b>	<b>27513</b>	27519.8	<b>27513</b>	27519.8
lin105	603910	<b>603910</b>	<b>603910.0</b>	<b>603910</b>	<b>603910.0</b>	<b>603910</b>	<b>603910.0</b>
pr107	2026626	<b>2026626</b>	<b>2026626.0</b>	<b>2026626</b>	<b>2026626.0</b>	<b>2026626</b>	<b>2026626.0</b>
Better	-	23	23	23	21	23	20

\* - Optimality is not proven

Table 7.1: Results on the instances selected from TSPLib in [1, 2]

## 7.3 Experiments for Hamiltonian Path

All 150 instances used in the experiments reported in Section 5.3 were also submitted to MDM-GILS-RVND heuristic, and their results are placed as follows.

Tables 7.7, 7.8 and 7.9 report the results on the 10-instances set selected by the authors of [39]. For this set, as seen in Table 7.9, both MDM-GILS-RVND and DM-GILS-RVND outperformed, in terms of computational time, GILS-RVND, whereas comparing DM and MDM heuristics to each other, they obtained nearly the same number of winnings. Furthermore, MDM-GILS-RVND had the best average computational time in this set requiring 14.47% less computational time than GILS-RVND. In terms of best solution, MDM-GILS-RVND and GILS-RVND found all 10 best results, while DM-GILS-RVND found 9. Considering average solution, MDM-GILS-RVND and GILS-RVND achieved 8 best results, whereas DM-GILS-RVND reached 6 best results.

For sets of 10-customers, 20-customers and 50-customers, their computational results are gathered in Table 7.10, which reports only best solutions, since all obtained average

Instance	GILS-RVND	DM-GILS-RVND		MDM-GILS-RVND	
	Average Time (s)	Average Time (s)	Gap(%) Time	Average Time (s)	Gap(%) Time
dantzig42	<b>0.17</b>	<b>0.17</b>	0.00	<b>0.17</b>	0.00
swiss42	<b>0.16</b>	<b>0.16</b>	0.00	<b>0.16</b>	0.00
att48	0.29	0.29	0.00	<b>0.28</b>	-3.45
gr48	0.31	<b>0.29</b>	-6.45	0.31	0.00
hk48	0.28	0.28	0.00	<b>0.27</b>	-3.57
eil51	0.40	0.40	0.00	<b>0.39</b>	-2.50
berlin52	<b>0.39</b>	<b>0.39</b>	0.00	<b>0.39</b>	0.00
brazil58	0.55	<b>0.52</b>	-5.45	<b>0.52</b>	-5.45
st70	0.99	<b>0.94</b>	-5.05	0.96	-3.03
eil76	1.52	<b>1.44</b>	-5.26	<b>1.44</b>	-5.26
pr76	1.35	<b>1.33</b>	-1.48	<b>1.33</b>	-1.48
pr76r	1.38	<b>1.28</b>	-7.24	1.30	-5.80
gr96	2.84	<b>2.75</b>	-3.17	2.79	-1.76
rat99	5.30	5.02	-5.28	<b>4.87</b>	-8.11
kroA100	4.21	3.58	-14.96	<b>3.51</b>	-16.63
kroB100	4.13	4.01	-2.91	<b>3.77</b>	-8.72
kroC100	3.95	3.62	-8.36	<b>3.61</b>	-8.61
kroD100	4.06	<b>3.47</b>	-14.53	3.57	-12.07
kroE100	4.00	3.69	-7.75	<b>3.51</b>	-12.25
rd100	4.15	3.99	-3.86	<b>3.96</b>	-4.58
eil101	5.79	5.00	-13.64	<b>4.88</b>	-15.72
lin105	3.57	<b>3.44</b>	-3.64	3.51	-1.68
pr107	4.33	<b>4.01</b>	-7.39	4.10	-5.31
Average	-	-	-5.06	-	-5.48
Better	3	13	-	16	-

Table 7.2: Computational time for TSPLib instances selected in [1, 2]

solutions were equal to their respective best solution. The results showed that all optimal values were found by the three heuristics in all executions with different seeds. The three strategies achieved the best solutions an average of 0.01, 0.02 and 0.05 seconds, respectively, for the sets of 10, 20 and 50 customers.

For the 100-customers set, Tables 7.11, 7.12 and 7.13 report its computational results. Regarding best solution, all heuristics found the BKSs of this set. On the other hand, for average solution, GILS-RVND achieved 20 best results, while DM-GILS-RVND and MDM-GILS-RVND achieved 16 best results each. In terms of computational time, DM-GILS-RVND became the best heuristic for this group, achieving 11 best results, while the MDM heuristic reached 9 best results, and, for GILS-RVND, no best result. Also in relation to computational time, DM-GILS-RVND achieved the best average for this set, requiring 10.54% less computational time when compared to GILS-RVND, whereas, for

	DM-GILS-RVND				MDM-GILS-RVND			
	Best	Average	Time	SST	Best	Average	Time	SST
GILS-RVND	0-23-0	2-21-0	0-6-17	0-0	0-23-0	3-20-0	0-4-19	0-0
DM-GILS-RVND	-	-	-	-	0-23-0	1-22-0	7-6-10	0-0

Table 7.3: Summary for Tables 7.1 and 7.2

Instance	GILS-RVND		DM-GILS-RVND		MDM-GILS-RVND	
	Best Solution	Average Solution	Best Solution	Average Solution	Best Solution	Average Solution
gr120	<b>363454</b>	363569.5	<b>363454</b>	363584.8	<b>363454</b>	<b>363454.0</b>
pr124	<b>3154346</b>	<b>3154346.0</b>	<b>3154346</b>	<b>3154346.0</b>	<b>3154346</b>	<b>3154346.0</b>
bier127	<b>4545005</b>	4546378.8	<b>4545005</b>	<b>4545005.0</b>	<b>4545005</b>	4545691.9
ch130	<b>349874</b>	<b>349891.7</b>	<b>349874</b>	349903.5	<b>349874</b>	349903.5
pr136	<b>6199268</b>	<b>6199805.4</b>	<b>6199268</b>	6200032.6	<b>6199268</b>	6200041.6
gr137	<b>4061498</b>	<b>4061498.0</b>	<b>4061498</b>	<b>4061498.0</b>	<b>4061498</b>	<b>4061498.0</b>
pr144	<b>3846137</b>	<b>3846137.0</b>	<b>3846137</b>	<b>3846137.0</b>	<b>3846137</b>	<b>3846137.0</b>
ch150	<b>444424</b>	<b>444424.0</b>	<b>444424</b>	<b>444424.0</b>	<b>444424</b>	<b>444424.0</b>
kroA150	<b>1825769</b>	<b>1825769.0</b>	<b>1825769</b>	<b>1825769.0</b>	<b>1825769</b>	<b>1825769.0</b>
kroB150	<b>1786546</b>	<b>1786546.0</b>	<b>1786546</b>	<b>1786546.0</b>	<b>1786546</b>	<b>1786546.0</b>
pr152	<b>5064566</b>	<b>5064566.0</b>	<b>5064566</b>	<b>5064566.0</b>	<b>5064566</b>	<b>5064566.0</b>
u159	<b>2972030</b>	<b>2972204.2</b>	<b>2972030</b>	2972291.3	<b>2972030</b>	<b>2972204.2</b>
si175	<b>1808532</b>	<b>1808532.0</b>	<b>1808532</b>	<b>1808532.0</b>	<b>1808532</b>	<b>1808532.0</b>
brg180	<b>174750</b>	<b>174750.0</b>	<b>174750</b>	<b>174750.0</b>	<b>174750</b>	<b>174750.0</b>
rat195	<b>218632</b>	218763.2	<b>218632</b>	<b>218760.6</b>	<b>218632</b>	218736.6
d198	<b>1186049</b>	1186098.6	<b>1186049</b>	<b>1186086.2</b>	<b>1186049</b>	1186273.3
kroA200	<b>2672437</b>	2672444.2	<b>2672437</b>	<b>2672437.0</b>	<b>2672437</b>	2672444.2
kroB200	<b>2669515</b>	<b>2674486.0</b>	<b>2669515</b>	2675761.6	<b>2669515</b>	2675993.6
gr202	<b>2909247</b>	2914644.2	<b>2909247</b>	<b>2912564.8</b>	<b>2909247</b>	2913368.4
ts225	<b>13240046</b>	<b>13240046.0</b>	<b>13240046</b>	13240533	<b>13240046</b>	<b>13240046.0</b>
tsp225	<b>402783</b>	403080.2	<b>402783</b>	402970.5	<b>402783</b>	<b>402933.3</b>
pr226	<b>7196869</b>	<b>7196869.0</b>	<b>7196869</b>	<b>7196869.0</b>	<b>7196869</b>	<b>7196869.0</b>
gr229	<b>10725914</b>	<b>10729883.8</b>	<b>10725914</b>	10729943.9	<b>10725914</b>	10731249.9
gil262	285060	285527.1	<b>285043</b>	285343.5	285060	<b>285312.6</b>
pr264	<b>5471615</b>	<b>5471615.0</b>	<b>5471615</b>	<b>5471615.0</b>	<b>5471615</b>	<b>5471615.0</b>
a280	<b>346989</b>	347125.9	<b>346989</b>	<b>347009.6</b>	<b>346989</b>	347106.9
pr299	<b>6556628</b>	<b>6557983.4</b>	<b>6556628</b>	6558164.9	<b>6556628</b>	6559030.8
lin318	<b>5619810</b>	<b>5629995.9</b>	<b>5619810</b>	5630556.9	<b>5619810</b>	5630590.5
rd400	2768830	2776672.7	2767608	<b>2775101.2</b>	<b>2762532</b>	2775707.0
fl417	<b>1874242</b>	1874242.8	<b>1874242</b>	<b>1874242.0</b>	<b>1874242</b>	<b>1874242.0</b>
gr431	21159702	21239150.9	<b>21143311</b>	<b>21210280.6</b>	21180562	21214270.9
pr439	<b>17829541</b>	17887107	<b>17829541</b>	17876876.9	<b>17829541</b>	<b>17868632.7</b>
pcb442	10301705	10323539.7	<b>10290913</b>	10321804.2	10301705	<b>10321465.7</b>
d493	6684190	6691057.1	6680997	6688669.0	<b>6677458</b>	<b>6687268.2</b>
att532	<b>5613010</b>	5632753.5	5622905	5630730.4	5617783	<b>5628346.4</b>
ali535	31870389	31904676.6	31870389	<b>31902870.9</b>	<b>31860679</b>	31910477.9
si535	12247211	12250679.7	<b>12246397</b>	12252151.6	12248066	<b>12251841.0</b>
pa561	<b>658870</b>	<b>661211.6</b>	660249	662216.9	660590	661790.6
u574	9314596	9344178.4	9313459	9350198.1	<b>9308820</b>	<b>9333295.3</b>
rat575	1848869	1859221.1	1847411	1856382.8	<b>1847272</b>	<b>1856335.1</b>
p654	<b>7827273</b>	<b>7827639.2</b>	<b>7827273</b>	7827919.4	<b>7827273</b>	7827867.8
d657	14159477	14220133.3	14125530	<b>14188813.5</b>	<b>14112540</b>	14195797.6
gr666	63571693	63731966.5	63546987	63663647.1	<b>63500984</b>	<b>63612943.5</b>
u724	13506660	13558605.3	<b>13491605</b>	13546178.5	13504408	<b>13537514.7</b>
rat783	3282794	3296069.6	<b>3272226</b>	<b>3290521.7</b>	3275858	3293606.1
dsj1000	7646018508	7685887300.0	<b>7640607124</b>	7671314634.0	7642715113	<b>7664531851.0</b>
dsj1000ceil	7646519008	7683329486.0	<b>7644298506</b>	7680652520.0	7646395679	<b>7676973751.0</b>
pr1002	115550770	116178260.2	115507699	115975798.5	<b>115420846</b>	<b>115874237.0</b>
si1032	<b>46896355</b>	46897662.4	<b>46896355</b>	<b>46896783.6</b>	<b>46896355</b>	<b>46896783.6</b>
u1060	<b>102508056</b>	102759766.0	102558414	102821622.2	102539819	<b>102759493.6</b>
vm1084	94760440	95053081.2	94705227	94982553.5	<b>94670122</b>	<b>94960603.3</b>
pcb1173	30926325	31032128.8	30891188	30972619.2	<b>30890385</b>	<b>30957008.7</b>
d1291	<b>29383346</b>	<b>29477239.4</b>	29392621	29511969.3	29389729	29515210.4
rl1304	144886001	145596878.7	144803181	145558912.3	<b>144592447</b>	<b>145398549.2</b>
rl1323	<b>155697857</b>	156360364.3	155749119	156332300.1	155719283	<b>156273365.5</b>
nrv1379	35360407	35519379.7	35327900	35475906.4	<b>35291795</b>	<b>35456093.0</b>
Better	36	22	39	24	43	36

Table 7.4: Results on the 56-instances set selected from TSPLib

Instance	GILS-RVND	DM-GILS-RVND		MDM-GILS-RVND	
	Average Time (s)	Average Time (s)	Gap(%) Time	Average Time (s)	Gap(%) Time
gr120	9.54	8.24	-13.63	<b>8.10</b>	-15.09
pr124	5.39	<b>5.14</b>	-4.64	5.15	-4.45
bier127	9.25	7.80	-15.68	<b>7.73</b>	-16.43
ch130	9.23	<b>8.46</b>	-8.34	8.88	-3.79
pr136	17.3	<b>14.11</b>	-18.44	14.82	-14.34
gr137	8.11	<b>7.10</b>	-12.45	7.16	-11.71
pr144	9.11	9.06	-0.55	<b>8.80</b>	-3.40
ch150	13.06	10.80	-17.30	<b>10.67</b>	-18.3
kroA150	19.84	<b>15.51</b>	-21.82	15.68	-20.97
kroB150	16.27	14.68	-9.77	<b>14.49</b>	-10.94
pr152	11.23	10.45	-6.95	<b>10.20</b>	-9.17
u159	14.21	<b>12.88</b>	-9.36	12.92	-9.08
si175	19.14	14.92	-22.05	<b>14.85</b>	-22.41
brg180	16.79	<b>16.00</b>	-4.71	16.18	-3.63
rat195	44.69	37.06	-17.07	<b>35.57</b>	-20.41
d198	38.28	<b>31.55</b>	-17.58	31.95	-16.54
kroA200	42.23	<b>33.70</b>	-20.20	33.73	-20.13
kroB200	42.00	36.48	-13.14	<b>36.17</b>	-13.88
gr202	35.95	31.62	-12.04	<b>29.68</b>	-17.44
ts225	<b>26.60</b>	27.28	2.56	27.10	1.88
tsp225	53.89	43.67	-18.96	<b>43.43</b>	-19.41
pr226	34.29	28.86	-15.84	<b>28.85</b>	-15.86
gr229	53.66	43.78	-18.41	<b>41.12</b>	-23.37
gil262	96.12	76.34	-20.58	<b>74.72</b>	-22.26
pr264	47.02	38.74	-17.61	<b>38.68</b>	-17.74
a280	107.18	83.61	-21.99	<b>79.05</b>	-26.25
pr299	104.92	78.64	-25.05	<b>75.93</b>	-27.63
lin318	117.98	100.63	-14.71	<b>90.74</b>	-23.09
rd400	350.84	278.61	-20.59	<b>247.61</b>	-29.42
fl417	382.64	263.61	-31.11	<b>250.61</b>	-34.51
gr431	336.98	264.24	-21.59	<b>245.10</b>	-27.27
pr439	285.56	<b>199.04</b>	-30.30	200.02	-29.96
pcb442	413.41	313.07	-24.27	<b>291.64</b>	-29.46
d493	608.47	410.33	-32.56	<b>390.16</b>	-35.88
att532	988.04	<b>744.64</b>	-24.63	760.49	-23.03
ali535	880.76	587.16	-33.33	<b>570.79</b>	-35.19
si535	498.76	340.83	-31.66	<b>319.01</b>	-36.04
pa561	1155.32	918.00	-20.54	<b>873.20</b>	-24.42
u574	1234.19	893.26	-27.62	<b>854.63</b>	-30.75
rat575	1739.46	1345.04	-22.67	<b>1234.05</b>	-29.06
p654	1755.28	1263.97	-27.99	<b>1239.21</b>	-29.40
d657	2615.66	1868.86	-28.55	<b>1779.16</b>	-31.98
gr666	2296.23	1699.61	-25.98	<b>1609.15</b>	-29.92
u724	4651.76	3505.29	-24.65	<b>3132.70</b>	-32.66
rat783	7044.52	4740.52	-32.71	<b>4475.85</b>	-36.46
dsj1000	18068.70	12612.84	-30.20	<b>12233.54</b>	-32.29
dsj1000ceil	18543.76	12885.48	-30.51	<b>11929.94</b>	-35.67
pr1002	11963.29	8817.55	-26.29	<b>8029.58</b>	-32.88
si1032	2402.72	1975.55	-17.78	<b>1926.99</b>	-19.80
u1060	15680.50	10471.80	-33.22	<b>9910.02</b>	-36.80
vm1084	13894.43	9673.12	-30.38	<b>9468.85</b>	-31.85
pcb1173	20508.89	<b>13903.73</b>	-32.21	14037.76	-31.55
d1291	12171.21	8189.00	-32.72	<b>8072.84</b>	-33.67
rl1304	18617.53	12967.80	-30.35	<b>12407.04</b>	-33.36
rl1323	22758.06	15938.27	-29.97	<b>15115.63</b>	-33.58
nrw1379	49624.72	34547.99	-30.38	<b>32038.65</b>	-35.44
Average	-	-	-21.09	-	-23.36
Better	1	12	-	43	-

Table 7.5: Computational time for the 56-instances set selected from TSPLib

	DM-GILS-RVND				MDM-GILS-RVND			
	Best	Average	Time	SST	Best	Average	Time	SST
GILS-RVND	5-32-19	15-11-30	1-0-55	1-5	7-33-16	12-14-30	1-0-55	1-10
DM-GILS-RVND	-	-	-	-	9-31-16	16-26-14	12-0-44	0-3

Table 7.6: Summary for Tables 7.4 and 7.5