

Homework 1

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1 - list of functions from the lowest growth rate to the highest:

- $2 / N$
- 37
- \sqrt{N}
- N
- $N * \log(\log(N))$
- $N * \log(N) \equiv N * \log(N^2)$
- $N * \log^2(N)$
- $N^{1.5}$
- N^2
- $N^2 * \log(N)$
- N^3
- $2^{N/2}$
- 2^N

2 - a - the sequence in dollars for the fine on day N is $v_N = 2^{N-1}$

b - to reach D dollar we do the inverse which is log base 2 of log base 2 $N = \log_2(\log_2(D))$

3 - a - for this program fraction an addition and an assignment of value will be done for n times, and all that will be done for 23 times. Thus, it will take $2 * N * 23$ TU (Time Units). $O = N$

b - for this program fraction an addition and an assignment of value will be done for -
 $\sum_{i=1}^N i \approx N^2/2$ times. As seen in the book, we consider the worst case for the Big-Oh. $O = N^2$

c - this program fraction uses recurrence. Every time the method is called the number is split into k parts and the program returns 1 when $n \leq k$. The method is called around $\log_k N + \text{constant}$ thus $O = \log(N)$

4 - a. 2.5 ms

b. $2.5 * \log(500) / \log(100) = 3.37$

c. 12.5 ms

d. 62.5 ms

5 - since the A_i are ordered, the array is sorted. We can use binary search to find i (which will be the position of $A_i + 1$). The running time of the algorithm is $O(\log N)$