# HW3

March 18, 2018

# 1 HOMEWORK 3

## 1.1 PART 1

```
In [16]: import matplotlib.pyplot as plt
         import numpy as np
         import pandas as pd
         import math
         X_train = np.array(pd.read_csv("./gaussian_process/X_train.csv", header=None))
         X_test = np.array(pd.read_csv("./gaussian_process/X_test.csv", header=None))
         y_train = np.array(pd.read_csv("./gaussian_process/y_train.csv", header=None))
         y_test = np.array(pd.read_csv("./gaussian_process/y_test.csv", header=None))
```

```
In [272]: class GP:
              def __init__(self, alphaS, b):
                  self.alphaS = alphaS
                  self.b = b
              def fit(self, X, y):
                  self.X = X
                  self.y = y
                  self.Kn = np.array([[self.Kern(xi, xj) for xj in X] for xi in X])
                  #print(self.Kn)
                  self.pred_help = np.linalg.inv(self.alphaS*np.identity(self.Kn.shape[0])+self.
                  #print(self.pred_help)

              def predict(self, X):
                  KX = self.KX_calc(X)
                  KDnX = self.KDnX_calc(X)
                  helper_var = np.dot(KDnX,self.pred_help)
                  pred_mean = np.dot(helper_var, self.y).reshape((X.shape[0],))
                  #print(pred_mean)
                  pred_var = np.diag(self.alphaS + KX - np.dot(helper_var,KDnX.T))
                  #print(pred_mean.shape)
                  #print(pred_var.shape)
                  return np.array([pred_mean, pred_var]).T # every row is [mean, var]

              def KDnX_calc(self, X):
                  return np.array([[self.Kern(xi, xj) for xj in self.X] for xi in X])
```

```
    def KX_calc(self, X):
        return np.array([self.Kern(x,x) for x in X])

    def Kern(self,  xi, xj):
        return np.exp(np.sum(np.square(xi-xj))/(-1*self.b))

    def change_pars(self, alphaS, b):
        self.alphaS = alphaS
        self.b = b


def RMSE(y_test, y_pred, leng):
    return np.sqrt(np.sum((y_test-y_pred)**2)/leng)
```

In [282]:
```
def part1():
    bees = [5,7,9,11,13,15]
    alphas = [.1,.2,.3,.4,.5,.6,.7,.8,.9,1]
    gaussian_p = GP(0,0)

    RMSE_arr = []
    for b in bees:
        tmp = []
        for alpha in alphas:
            print("fitting", b, alpha)
            gaussian_p.change_pars(alpha, b)
            gaussian_p.fit(X_train, y_train)
            preds = gaussian_p.predict(X_test)
            means = preds[:,0]
            tmp.append(RMSE(y_test.reshape(y_test.shape[0],), means, y_test.shape[0]))
        RMSE_arr.append(tmp)
    return RMSE_arr

RMSE_arr = part1()
```

```
fitting 5 0.1
fitting 5 0.2
fitting 5 0.3
fitting 5 0.4
fitting 5 0.5
fitting 5 0.6
fitting 5 0.7
fitting 5 0.8
fitting 5 0.9
fitting 5 1
fitting 7 0.1
fitting 7 0.2
```

```
fitting 7 0.3
fitting 7 0.4
fitting 7 0.5
fitting 7 0.6
fitting 7 0.7
fitting 7 0.8
fitting 7 0.9
fitting 7 1
fitting 9 0.1
fitting 9 0.2
fitting 9 0.3
fitting 9 0.4
fitting 9 0.5
fitting 9 0.6
fitting 9 0.7
fitting 9 0.8
fitting 9 0.9
fitting 9 1
fitting 11 0.1
fitting 11 0.2
fitting 11 0.3
fitting 11 0.4
fitting 11 0.5
fitting 11 0.6
fitting 11 0.7
fitting 11 0.8
fitting 11 0.9
fitting 11 1
fitting 13 0.1
fitting 13 0.2
fitting 13 0.3
fitting 13 0.4
fitting 13 0.5
fitting 13 0.6
fitting 13 0.7
fitting 13 0.8
fitting 13 0.9
fitting 13 1
fitting 15 0.1
fitting 15 0.2
fitting 15 0.3
fitting 15 0.4
fitting 15 0.5
fitting 15 0.6
fitting 15 0.7
fitting 15 0.8
fitting 15 0.9
fitting 15 1
```

```
In [281]: pdd = pd.DataFrame(RMSE_arr)
          pdd.columns = [.1,.2,.3,.4,.5,.6,.7,.8,.9,1]
          pdd.reset_index()
          pdd.index = [5,7,9,11,13,15]
          pdd
```

```
Out[281]:         0.1       0.2       0.3       0.4       0.5       0.6       0.7  \
          5   1.966276  1.933135  1.923420  1.922198  1.924769  1.929213  1.934634
          7   1.920163  1.904877  1.908080  1.915902  1.924804  1.933701  1.942254
          9   1.897649  1.902519  1.917648  1.932514  1.945699  1.957235  1.967403
          11  1.890507  1.914981  1.938849  1.957936  1.973216  1.985764  1.996375
          13  1.895849  1.935586  1.964597  1.985502  2.001314  2.013878  2.024310
          15  1.909603  1.959549  1.990804  2.011915  2.027370  2.039465  2.049463

                  0.8       0.9       1.0
          5   1.940583  1.946820  1.953213
          7   1.950380  1.958093  1.965438
          9   1.976492  1.984741  1.992341
          11  2.005603  2.013835  2.021345
          13  2.033307  2.041317  2.048642
          15  2.058105  2.065845  2.072976
```

### 1.1.1 C

The best value of RMSE is 1.890507 which is when b = 11 and alpha_squared = .1

Here, we change the variance and kernel width. We notice that a low variance (alpha_sqrd = 1) works best in general the true underlying Gaussian distribution of our model has a low variance. We also notice that as we change the kernel width (which the larger it is the more we want close values to have a high kernel value) the RMSE converges towards its optimal value (b=11) from both sides.

Now, compared to HW1 in which we used least squares (lambda=0) and got an RMSE value of around 2.6 and also ridge regression with p=2 and lambda=23 we got an RMSE around 2.2

Clearly, gaussian processes with the kernel we used outperform those models however calculating the inverse of variance+Kn is computationally heavier than LS and RR.

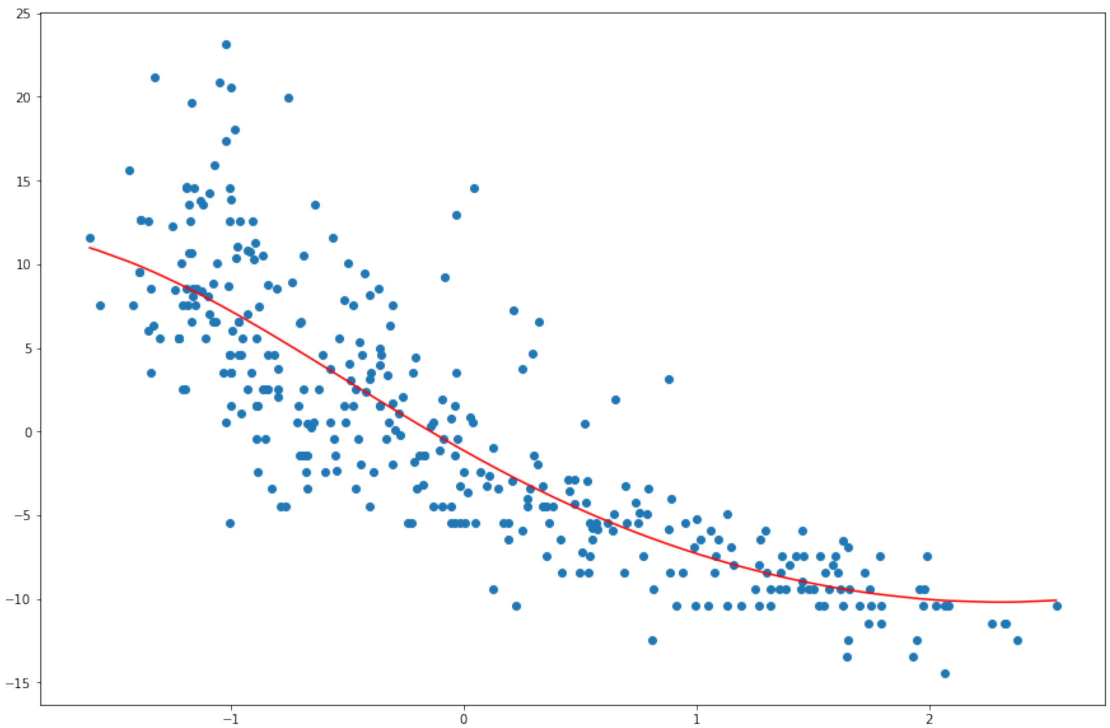
### 1.1.2 D

```
In [295]: gaussianp = GP(2,5)
          gaussian_p.fit(X_train[:,3], y_train)

          train_preds = gaussian_p.predict(X_train[:,3])
          train_means = train_preds[:,0]
          train_acc = RMSE(y_train.reshape(y_train.shape[0],), train_means, y_train.shape[0])

          test_preds = gaussian_p.predict(X_test[:,3])
          test_means = test_preds[:,0]
          test_acc = RMSE(y_test.reshape(y_test.shape[0],), test_means, y_test.shape[0])
```

```
        print("train RMSE: ",train_acc)
        print("test RMSE: ",test_acc)
```

```
train RMSE:  4.253743788
test RMSE:  3.40888298673
```

```
In [317]: plt.figure(figsize=(15, 10))
          plt.scatter(X_train[:,3], y_train)
          order = np.argsort(X_train[:,3])
          xs = np.array(X_train[:,3])[order]
          ys = np.array(train_means)[order]
          plt.plot(xs, ys, color="red")
          plt.show()
```




## 1.2 PART 2

```
In [318]: X_train_B = np.array(pd.read_csv("./boosting/X_train.csv", header=None))
          X_test_B = np.array(pd.read_csv("./boosting/X_test.csv", header=None))
          y_train_B = np.array(pd.read_csv("./boosting/y_train.csv", header=None))
          y_test_B = np.array(pd.read_csv("./boosting/y_test.csv", header=None))
```

```
In [571]: class LS_classifier:
              def fit(self, X, y):
```

```
        self.X = X
        self.y = y
        self.w = np.linalg.inv(np.dot(X.T, X)).dot(X.T).dot(self.y)

    def predict(self, X, use_alpha=False):
        predictions = np.sign(np.dot(X,self.w))
        if use_alpha:
            return self.alpha*predictions
        return predictions

    def set_alpha(self, alpha):
        self.alpha = alpha

    def get_alpha(self):
        return self.alpha

    def flipw(self):
        self.w = (-1)*self.w

class Booster:
    def __init__(self, X, y, its, Xtest, ytest):
        self.X = np.hstack((X, np.ones((X.shape[0], 1))))
        self.y = y.reshape(y.shape[0],)
        self.Xtest = np.hstack((Xtest, np.ones((Xtest.shape[0], 1))))
        self.ytest = ytest.reshape(ytest.shape[0],)
        self.Xb = self.X
        self.yb = self.y
        self.its = its
        self.weights = np.array([1/X.shape[0] for i in range(X.shape[0])])
        self.classifiers = []
        self.error_arr = []
        self.test_er = []
        self.train_er = []
        self.train_uppab = []
        self.boots = [0 for i in range(self.X.shape[0])]
    def boost(self):
        for t in range(self.its):
            if (t+1)%300==0:
                print("iteration: ",t+1)
#             if t<10:
#                 print(self.weights)
            self.bootstrap()
            tmp_classif = LS_classifier()
            tmp_classif.fit(self.Xb, self.yb)
            preds = tmp_classif.predict(self.X)
            #check
            #print(preds.shape==self.y.shape)
            #error = np.dot(self.weights,np.array(preds!=self.y, dtype=int).T)
```

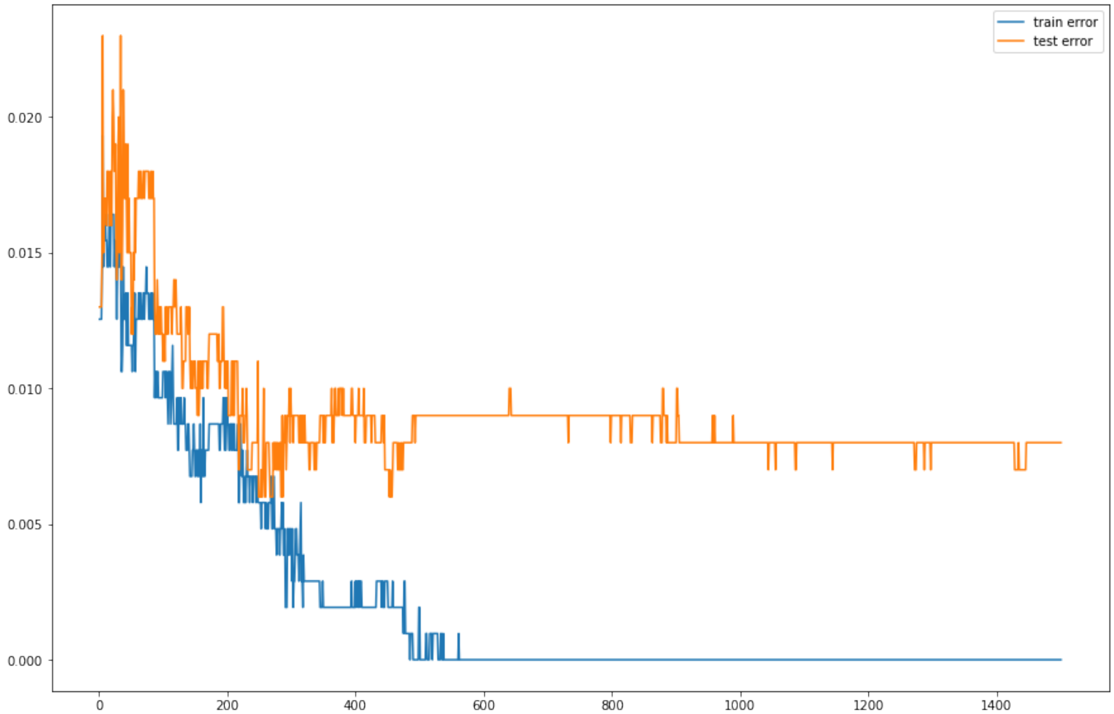
```
            #EPSILON
            error = np.sum(self.weights[(preds!=self.y)])
            if error>0.5:
                tmp_classif.flipw()
                preds = tmp_classif.predict(self.X)
                #error = np.dot(self.weights,np.array(preds!=self.y, dtype=int).T)
                error = np.sum(self.weights[(preds!=self.y)])
            self.error_arr.append(error)

            #print("error ",error)
            alpha = np.log((1-error)/error)*0.5
            #print("alpha ",alpha)
            tmp_classif.set_alpha(alpha)
            #self.weights *= np.exp(-alpha*np.dot(self.y, preds))
            self.weights = normalize(np.multiply(np.exp(-alpha*np.multiply(self.y,pred
            #print("weights ", self.weights)
            #print(np.exp(-alpha*np.dot(self.y, preds)))
            #self.norm_w()
            self.classifiers.append(tmp_classif)


            # TRAIN ERROR
            train_preds = self.predict(self.X)
            train_error = errorcalc(self.y, train_preds, train_preds.shape[0])
            self.train_er.append(train_error)

            # TEST ERROR
            test_preds = self.predict(self.Xtest)
            test_error = errorcalc(self.ytest, test_preds, test_preds.shape[0])
            self.test_er.append(test_error)

            # TRAIN UPPERBOUND
            ZT = np.exp(-alpha)*(1-error) + np.exp(alpha)*error
            if len(self.train_uppab) == 0:
                self.train_uppab.append(ZT)
            else:
                self.train_uppab.append(ZT*self.train_uppab[-1])



    def predict(self, X):
        res = []
        for model in self.classifiers:
            res.append(model.predict(X, True))
        res = np.array(res)
        return np.sign(np.sum(res, axis=0))
```

```python
    def bootstrap(self):
        indix = np.random.choice(np.arange(self.X.shape[0]), size=self.X.shape[0], rep
        #self.Xb = np.take(self.X, indices=indix, axis=0)
        #self.yb = np.take(self.y, indices=indix)
        for i in indix:
            self.boots[i] += 1
        self.Xb = self.X[indix]
        self.yb = self.y[indix]

    def get_error(self):
        return self.error_arr, self.train_er, self.test_er, self.train_uppab, self.boo

    def get_alphas(self):
        res = []
        for model in self.classifiers:
            res.append(model.get_alpha())
        return res


def errorcalc(x, y, l):
    return np.sum(np.array(x==y, dtype=int))/l

def normalize(vec):
    return vec/np.sum(vec)
```

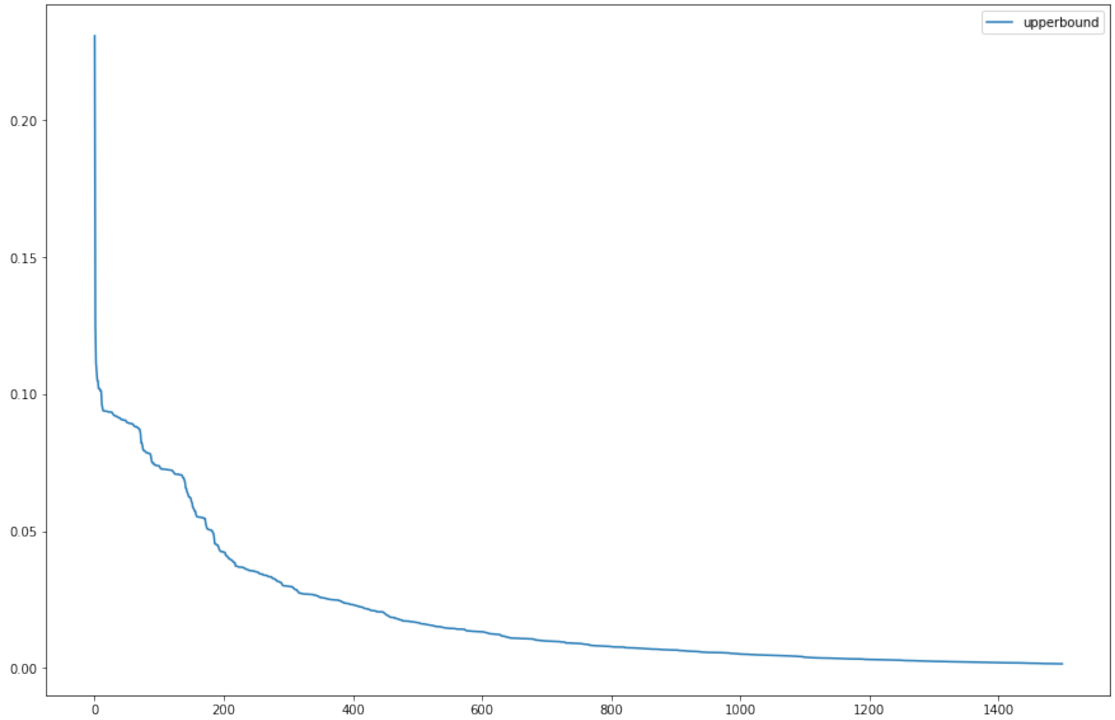
In [578]:
```python
boosty = Booster(X_train_B, y_train_B, 1500, X_test_B, y_test_B.reshape((y_test_B.shap
boosty.boost()
```

```
iteration:  300
iteration:  600
iteration:  900
iteration:  1200
iteration:  1500
```

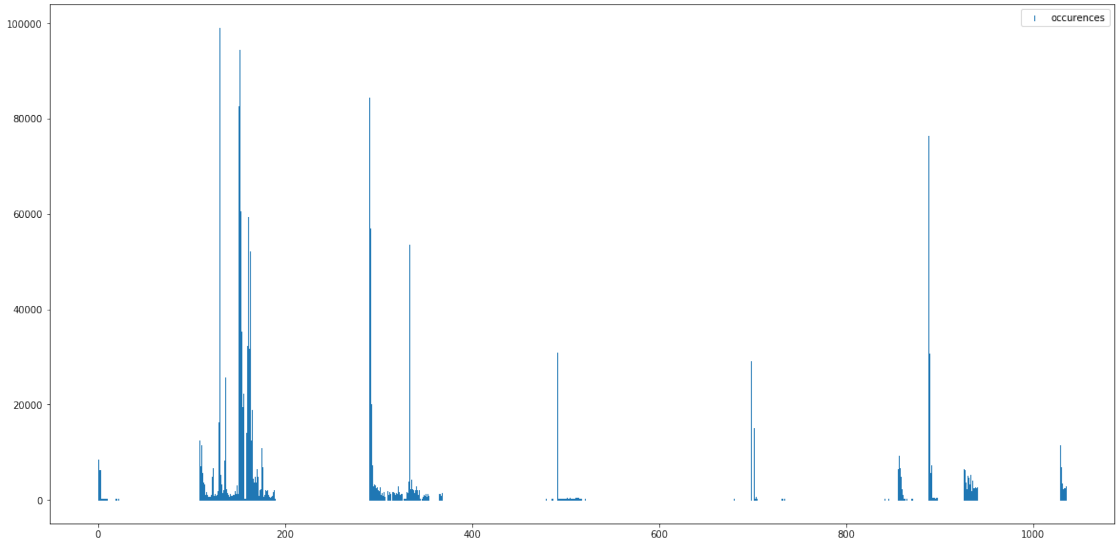
In [579]:
```python
errors = boosty.get_error()

plt.figure(figsize=(15, 10))
plt.plot(range(1,1501), 1-np.array(errors[1]), label= "train error")
plt.plot(range(1,1501), 1-np.array(errors[2]), label= "test error")
plt.legend()
plt.show()
```



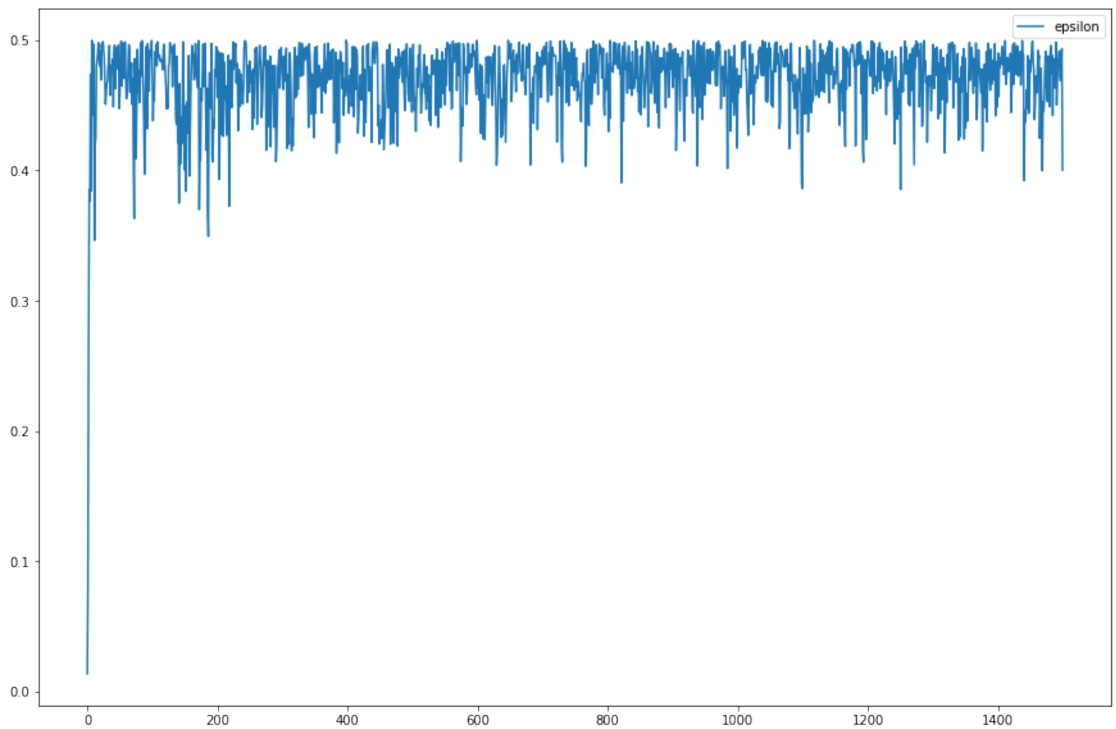

```
In [574]: plt.figure(figsize=(15, 10))
          plt.plot(range(0,1500), np.array(errors[3]), label= "upperbound")
          plt.legend()
          plt.show()
```



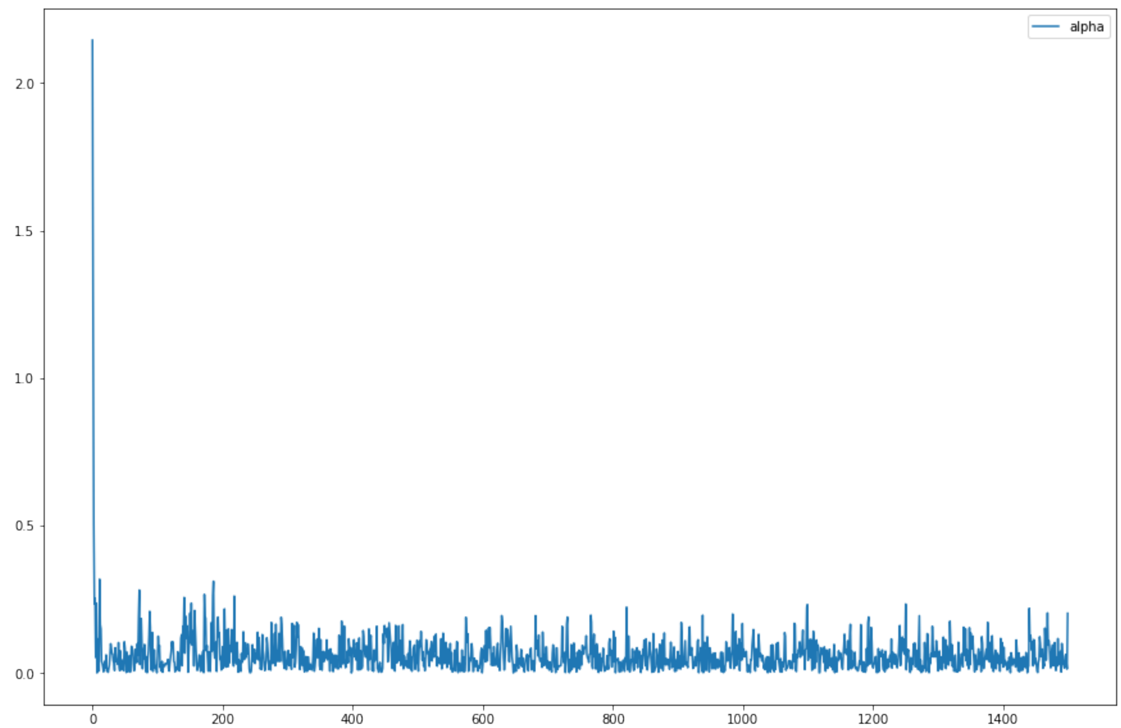

```
In [575]: plt.figure(figsize=(20, 10))
          markerline, stemlines, baseline =plt.stem(range(np.array(errors[4]).shape[0]), np.arra
          plt.setp(markerline, visible=False)
          plt.setp(stemlines, linewidth=1)
          plt.setp(baseline, visible=False)
          plt.legend()
          plt.show()
```



In [576]:
```
plt.figure(figsize=(15, 10))
plt.plot(range(0,1500), np.array(errors[0]), label= "epsilon")
plt.legend()
plt.show()
```




In [577]:
```
plt.figure(figsize=(15, 10))
plt.plot(range(0,1500), boosty.get_alphas(), label= "alpha")
plt.legend()
plt.show()
```


alpha
2.0
1.5
1.0
0.5
0.0
0
200
400
600
800
1000
1200
1400