

ELEN E4903: Machine Learning

Columbia University, Spring 2018

Homework 1: Due February 4, 2018 by 11:59PM

Please read these instructions to ensure you receive full credit on your homework. Submit the written portion of your homework as a *single* PDF file through Courseworks (less than 5MB). In addition to your PDF write-up, submit all code written by you in their original extensions through Courseworks (e.g., .m, .r, .py, .c). Any coding language is acceptable, but do not submit notebooks, do not wrap your files in .rar, .zip, .tar and do not submit your write-up in .doc or other file type. Your grade will be based on the contents of one PDF file and the original source code. Additional files will be ignored. We will not run your code, so everything you are asked to show should be put in the PDF file. Show all work for full credit.

Late submission policy: Late homeworks will have 0.1% deducted from the final grade for each minute late. *Your homework submission time will be based on the time of your last submission to Courseworks. I will not revert to an earlier submission!* Therefore, do not re-submit after midnight on the due date unless you are confident the new submission is significantly better to overcompensate for the points lost. Submission time is non-negotiable and will be based on the time you submitted your last file to Courseworks. The number of points deducted will be rounded to the nearest integer.

Problem 1 (written) – 25 points

Imagine we have a sequence of N observations (x_1, \dots, x_N) , where each $x_i \in \{0, 1\}$. We model this sequence as i.i.d. random variables from a Bernoulli distribution with unknown parameter $\pi \in [0, 1]$ and known parameter, where

$$p(x_i|\pi) = \pi^{x_i}(1 - \pi)^{1-x_i}$$

- (a) What is the joint likelihood of the data (x_1, \dots, x_N) ?
- (b) Derive the maximum likelihood estimate $\hat{\pi}_{\text{ML}}$ for π .

To help learn π , you use a prior distribution. You select the distribution $p(\pi) = \text{beta}(a, b)$.

- (c) Derive the maximum a posteriori (MAP) estimate $\hat{\pi}_{\text{MAP}}$ for π ?
- (d) Use Bayes rule to derive the posterior distribution of π and identify the name of this distribution.
- (e) What is the mean and variance of π under this posterior? Discuss how it relates to $\hat{\pi}_{\text{ML}}$ and $\hat{\pi}_{\text{MAP}}$.

Problem 2 (coding) – 35 points

In this problem you will analyze data using the linear regression techniques we have discussed. The goal of the problem is to predict the miles per gallon a car will get using six quantities (features) about that car. The zip file containing the data can be found on Courseworks.¹ The data is broken into training and testing sets. Each row in both “X” files contain six features for a single car (plus a 1 in the 7th dimension) and the same row in the corresponding “y” file contains the miles per gallon for that car.

Remember to submit all original source code with your homework. Put everything you are asked to show below in the PDF file.

Part 1. Using the training data only, write code to solve the ridge regression problem

$$\mathcal{L} = \lambda \|w\|^2 + \sum_{i=1}^{350} \|y_i - x_i^T w\|^2.$$

- (a) For $\lambda = 0, 1, 2, 3, \dots, 5000$, solve for w_{RR} . (Notice that when $\lambda = 0$, $w_{\text{RR}} = w_{\text{LS}}$.) In one figure, plot the 7 values in w_{RR} as a function of $df(\lambda)$. You will need to call a built in SVD function to do this (all details are in the slides). Be sure to label your 7 curves by their dimension in x .
- (b) The 4th dimension (car weight) and 6th dimension (car year) clearly stand out over the other dimensions. What information can we get from this?
- (c) For $\lambda = 0, \dots, 50$, predict all 42 test cases. Plot the root mean squared error (RMSE)² on the test set as a function of λ —*not* as a function of $df(\lambda)$. What does this figure tell you when choosing λ for this problem (and when choosing between ridge regression and least squares)?

Part 2. Modify your code to learn a p th-order polynomial regression model for $p = 1, 2, 3$. (You’ve already done $p = 1$ above.) For this implementation, do not include the cross terms for this problem, but instead use the method discussed in the slides.

- (d) In one figure, plot the test RMSE as a function of $\lambda = 0, \dots, 500$ for $p = 1, 2, 3$. Based on this plot, which value of p should you choose and why? How does your assessment of the ideal value of λ change for this problem?

¹See <https://archive.ics.uci.edu/ml/datasets/Auto+MPG> for more details on this dataset. Since I have done some preprocessing, you *must* use the data provided with this homework.

² $\text{RMSE} = \sqrt{\frac{1}{42} \sum_{i=1}^{42} (y_i^{\text{test}} - y_i^{\text{pred}})^2}.$