Big O Notation Solutions

Example 1: Use the definition of O to show that $30n^2 + 12n + 15 = O(n^2)$

Proof: We need to find constants $c > 0$ and n_0 such that $30n^2 + 12n + 15 \le c \cdot n^2$ for all $n \ge n_0$.

For $n \ge 1$, we have:

- $12n \le 12n^2$
- $15 \le 15n^2$

Therefore: $30n^2 + 12n + 15 \le 30n^2 + 12n^2 + 15n^2 = 57n^2$

Choose $c = 57$ and $n_0 = 1$. For all $n \ge 1$: $30n^2 + 12n + 15 \le 57n^2$

Therefore, $30n^2 + 12n + 15 = O(n^2)$. □

Example 2: Use the definition of Ω to show that $30n^2 + 12n + 15 = \Omega(n^2)$

Proof: We need to find constants $c > 0$ and n_0 such that $30n^2 + 12n + 15 \ge c \cdot n^2$ for all $n \ge n_0$.

For $n \ge 1$: $30n^2 + 12n + 15 \ge 30n^2$

Choose $c = 30$ and $n_0 = 1$. For all $n \ge 1$: $30n^2 + 12n + 15 \ge 30n^2$

Therefore, $30n^2 + 12n + 15 = \Omega(n^2)$. □

Example 3: Use the definition of Θ to show that $30n^2 + 12n + 15 = \Theta(n^2)$

Proof: By definition, $f(n) = \Theta(g(n))$ if and only if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.

From Example 1: $30n^2 + 12n + 15 = O(n^2)$ From Example 2: $30n^2 + 12n + 15 = \Omega(n^2)$

Therefore, $30n^2 + 12n + 15 = \Theta(n^2)$. □

Example 4: Use the definition of "big Oh" to show that $2n^2 - 6n = O(n^2)$

Proof: We need to find constants $c > 0$ and n_0 such that $2n^2 - 6n \le c \cdot n^2$ for all $n \ge n_0$.

For $n \ge 3$, we have $n^2 \ge n$, so $6n \le 6n^2$. Therefore: $2n^2 - 6n \le 2n^2 - 0 = 2n^2$

But we need to be more careful. Let's choose $n_0 = 3$. For $n \ge 3$: $6n \le 2n^2$ (since $6n \le 2n^2$ when $n \ge 3$)
Therefore: $2n^2 - 6n \ge 2n^2 - 2n^2 = 0$

Actually, let's approach this differently: $2n^2 - 6n \le 2n^2$ for all $n \ge 1$

Choose $c = 2$ and $n_0 = 1$. For all $n \ge 1$: $2n^2 - 6n \le 2n^2$

Therefore, $2n^2 - 6n = O(n^2)$. □

Example 5: Use the definition of "big Omega" to show that $2n^2 - 6n = \Omega(n_0)$

Proof: We need to find constants $c > 0$ and n_0 such that $2n^2 - 6n \geq c \cdot n_0$ for all $n \geq n_0$.

For large n, the $2n^2$ term dominates, so we need to find when $2n^2 - 6n \geq c \cdot n_0$.

Rearranging: $2n^2 - 6n \geq c \cdot n_0$ $2n^2 - c \cdot n_0 \geq 6n$ $n_0(2 - c) \geq 6n$ $n_0(2 - c) \geq 6$

Choose $c = 1$. Then we need $n_0(2 - 1) \geq 6$, so $n_0 \geq 6$. Choose $n_0 = 3$ (since $3\Omega = 9 \geq 6$).

For $n \geq 3$: $2n^2 - 6n \geq n_0$

Therefore, $2n^2 - 6n = \Omega(n_0)$. □

Example 6: Conclusion?

From Examples 4 and 5:

- $2n^2 - 6n = O(n_0)$
- $2n^2 - 6n = \Omega(n_0)$

Therefore, $2n^2 - 6n = \Omega(n_0)$.

Example 7: Find the mistakes in the following "proofs"

First "proof" mistake: The error is using $C = -4$, which violates the requirement that $c > 0$. Constants in Big O definitions must be positive.

Second "proof" mistake: Again, $C = -4 < 0$, which is invalid. Additionally, the inequality $2n^2 - 6n \geq -4n^3$ doesn't establish that $2n^2 - 6n = \Omega(n_0)$, because we need $2n^2 - 6n \geq c \cdot n_0$ for some positive c.

Example 8, 9 & 10

Beyond the scope of this course. See me in office hours if you'd like to learn more.

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