Sorting Algorithms

bubbleSort(a)

```
     n = a.last
     for i = n downto 2
          for j = 1 to i-1
               if a[j] > a[j+1]
                    temp = a[j]
                    a[j] = a[j+1]
                    a[j+1] = temp
```

selectionSort(a)

```
     n = a.last
     for i = 1 to n-1
          minPos = i
          for j = i+1 to n
               if a[j] < a[minPos]
                    minPos = j

          if minPos != i
               temp = a[minPos]
               a[minPos] = a[i]
               a[i] = temp
```

insertionSort(a)

```
{
  n = a.last
  for i = 2 to n
  {
     val = a[i]
     j = i - 1
     while (j >= 1 && val < a[j])
     {
       a[j + 1] = a[j]
       j = j - 1
     }
     a[j + 1] = val
  }
}
```

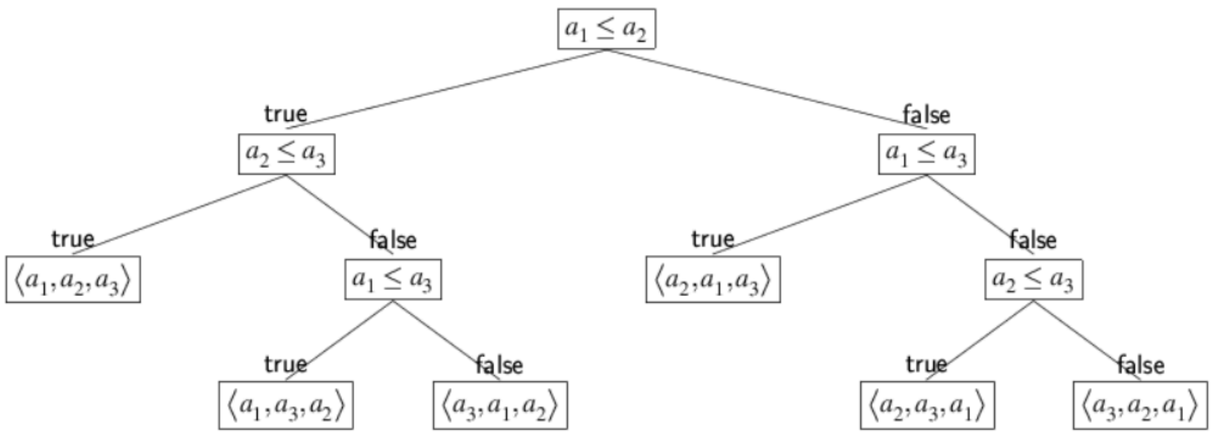
```
mergeSort(a, i, j)
{
  if (i == j)
    return
  m = (i + j) / 2
  mergeSort(a, i, m)       // How do we measure the time
  mergeSort(a, m+1, j)     // for a recursive algorithm?
  merge(a, i, m, j)
}

merge(a, i, m, j)
{
  // Merge two sorted subarrays a[i..m-1] and a[m..j] into one
  // sorted subarray  a[i..j]
  // How much time is required for the merge operation?

}
```

Section 6.3: A Lower Bound for the Sorting Problem

MergeSort and QuickSort are two efficient sorting algorithms, both running in time O(*n* lg n). But can we do any better than this?

Theorem Any <u>comparison-based</u> sorting algorithm has worst case time Ω(n lg n).

Consider the decision tree for an array with 3 elements. There are 6 leaves in this tree, and the tree has height 3.



Generalize: The decision tree for comparing and ordering an array of n elements has n! leaves.

In the worst case, the number of comparisons $\geq$ height of tree

$$\geq \lg(n!)$$
$$= \Omega(n \lg n)$$

Conclusion: Any comparison-based sorting algorithm must take time **<u>at least</u>** n lg n. There is no hope of finding any faster comparison-based algorithm.

Section 6.4 Sorting in Linear Time

CountingSort can only be used on a restricted type of data set. The data must be a set of integers in the range $0 \ldots m$. Generally, the value m should be “small”, say $m = O(n)$.

```
countingSort(a,m) {
     for k = 0 to m
          c[k] = 0
     n = a.last
     for i = 1 to n
          c[a[i]] = c[a[i]] + 1          // how many of each value
     for k = 1 to m
          c[k] = c[k] + c[k - 1]         // how many   k
     // sort a with the result in b
     for i = n downto 1 {
          b[c[a[i]]] = a[i]
          c[a[i]] = c[a[i]] - 1
     }
     // copy b back to a
     for i = 1 to n
          a[i] = b[i]
}
```

Trace through countingSort on the array a below. Note that the range of the data is $0 \ldots 6$. So the call to the algorithm would be `countingSort(a, 6)`. (Show work on separate paper.)

	1	2	3	4	5	6	7	8	9	10
a	**4**	**3**	**0**	**3**	**6**	**2**	**0**	**1**	**3**	**4**

RadixSort can only be used on a restricted type of data set. Every integer in the data set must have a fixed number of digits, say k.

```
radix_sort(a,k) {
     for i = 0 to k - 1
          counting_sort(a,9) on digit in 10^i place
}
```

Determine the running time of each of the algorithms.