Summations

Triangular Numbers: $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$ (proof by induction)

Example 1: Determine the value of the sum $1 + 2 + 3 + \ldots + 50$

Application: Determine the exact number of statements executed in the following algorithm:

```
stockProfit
     n = p.last
     buy = 1
     sell = 2
     profit = p[2] – p[1]
     for i = 1 to n-1
          for j = i+1 to n
               if p[j] – p[i] > profit
                    buy = i
                    sell = j
                    profit = p[j] – p[i]
     println("Buy on day " + buy)
     println("Sell on day " + sell)
     println("Profit: $" + profit)
```

Finite Geometric Sum: $$\sum_{i=0}^{n} ar^i = a \cdot \frac{r^{n+1}-1}{r-1}$$ as long as $r \neq 1$

Example 2: Determine the value of the sum $3 + 6 + 12 + 24 + \ldots + 3072$

Applications of geometric sums and logarithms to binary trees

- A **rooted** binary tree has a root node and every node has at most two children.
- A node with no children is called a **leaf**.
- The **depth** (or level) of a node is the number of edges from the root to the node (i.e., it is the length of a shortest path from the root to the node).
- The **height** of a node is the number of edges on the longest path between that node and a leaf (the leaf must be an ancestor of the node).
- The **height** of a binary tree is the height of the root.
- A **perfect** binary tree is a binary tree in which all interior nodes have two children *and* all leaves have the same *depth* or same *level*.
- A **complete** binary tree is a binary tree in which every level, *except possibly the last*, is completely filled, and all nodes in the last level are as far left as possible.
- NOTE: A heap is a data structure that can be viewed as a complete binary tree with some added constraints on the order of the data in the tree. We will study heaps later.

Example 3: Find the number of nodes in a **perfect** binary tree of height 12.

Example 4: Find the range (min and max) for the number of nodes in a **complete** binary tree of height 6.

Example 5: Find an expression for the number of nodes in a perfect binary tree of height h.

Example 6: Find an expression for the height of a perfect (or complete) binary tree with n nodes (as a function of n).

Example 7: Find the height of a complete binary tree with 2000 nodes.

Arithmetico-geometric series: $\sum_{i=1}^{n} ir^i < \frac{r}{(1-r)^2}$ for $n \geq 1,\ 0 < r < 1$

Example 8: Find an upper bound for the sum $\sum_{i=1}^{n} i\left(\frac{1}{2}\right)^i$