CHAPTER 6

Sorting and Selection

Algorithm Bubble Sort

This algorithm compares consecutive elements, bubbling the largest elements to the end of the array.

```
bubbleSort(a) {
    n = a.last
    for i = n downto 1 {
        for j = 1 to i-1 {
            if a[j] > a[j+1] {
                temp = a[j]
                a[j] = a[j+1]
                a[j+1] = temp
            }
        }
    }
}
```

NOTE: This is a "traditional" version of BubbleSort. A revised version uses a boolean variable to track whether or not at least one swap has occurred in a pass through the array, thereby eliminating unnecessary passes through the array.

Algorithm Selection Sort

This algorithm sorts the array *a* by first finding the minimum value in the array and swaps it with the first element in the array; then finds the next smallest and swaps it with the second element in the array; and so on.

```
selectionSort(a) {
    n = a.last
    for i = 1 to n-1 {
        minPos = i
        for j = i+1 to n {
            if a[j] < a[minPos]
                minPos = j
        }
        if minPos != i {
            temp = a[minPos]
            a[minPos] = a[i]
            a[i] = temp
        }
    }
}
```

Algorithm 6.1.2 Insertion Sort

This algorithm sorts the array *a* by first inserting *a*[2] into the sorted subarray *a*[1]; next inserting *a*[3] into the sorted subarray *a*[1], *a*[2]; and so on; and finally inserting *a*[*n*] into the sorted subarray *a*[1], ... , *a*[*n* - 1].

```
insertionSort(a) {
    n = a.last
    for i = 2 to n {
        val = a[i]          // save a[i] so it can be inserted
        j = i - 1           // into the correct place
        while (j >= 1 && val < a[j]) {
            a[j + 1] = a[j]    // move larger elements down by
            j = j - 1          // one to make room for val
        }
        a[j + 1] = val        // insert val in the correct place
    }
}
```

Algorithm Merge Sort

This algorithm uses a divide and conquer technique. The array is split in half, the halves are recursively sorted and then merged together.

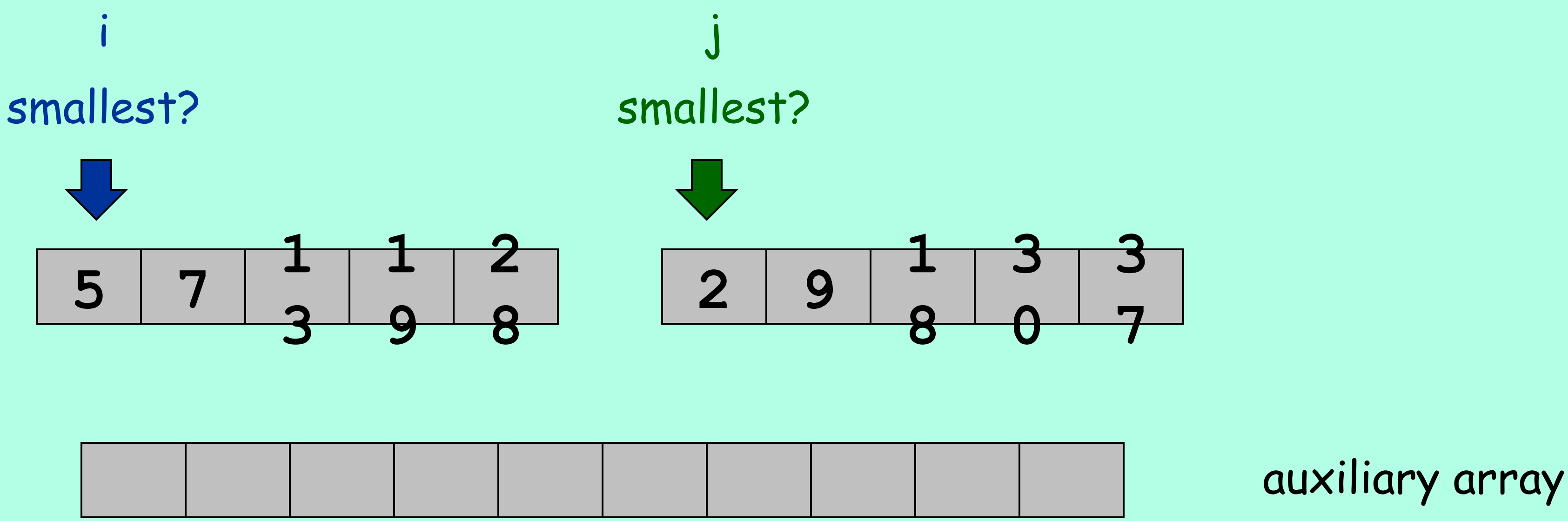
```
mergeSort(a, i, j)
  if (i == j)
    return
  m = (i + j) / 2
  mergeSort(a, i, m)
  mergeSort(a, m+1, j)
  merge(a, i, m, j)

merge(a, i, m, j)
  // merge two sorted subarrays into one
  // see next slides
```

Merging

Merge.

- Each half is in sorted order.
- Use indices i and j to step through the two halves.
- Compare the two elements.
- Insert smallest of two elements into next position of the auxiliary array.
- Repeat until done.



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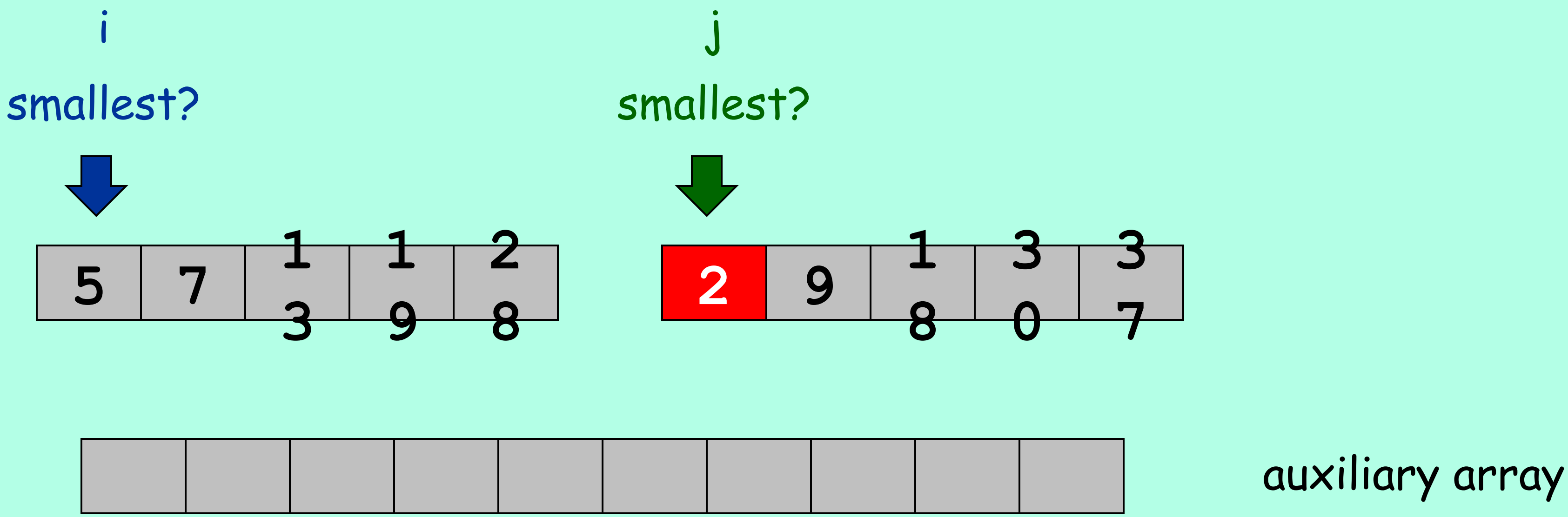

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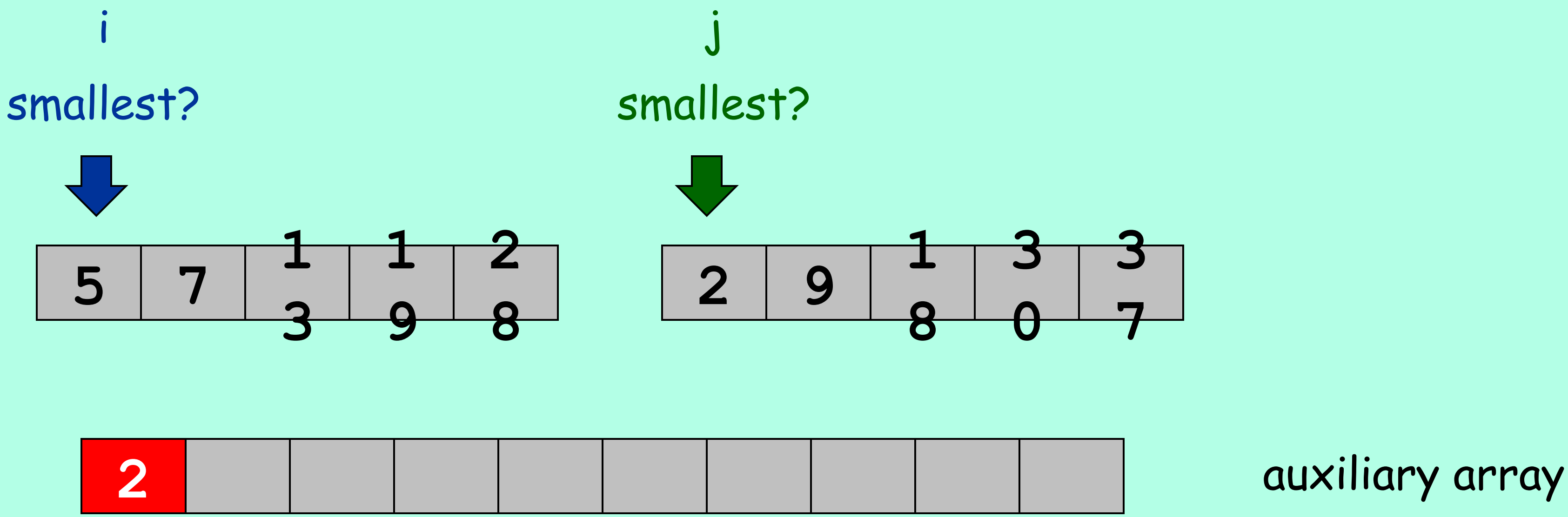

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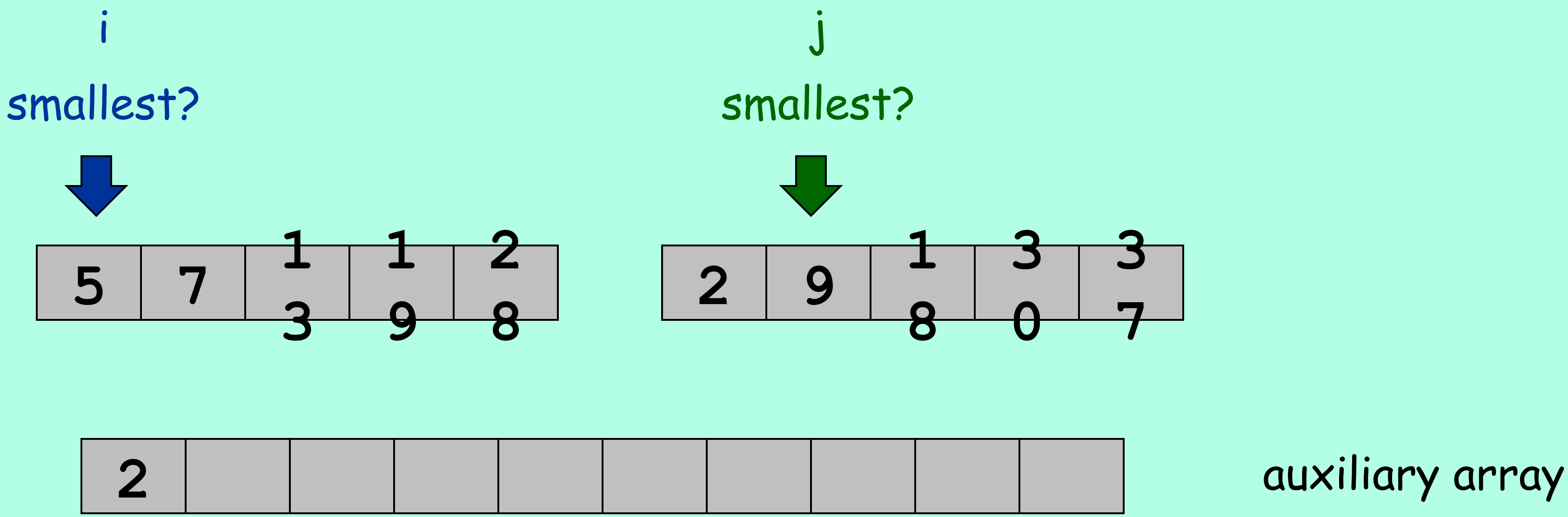

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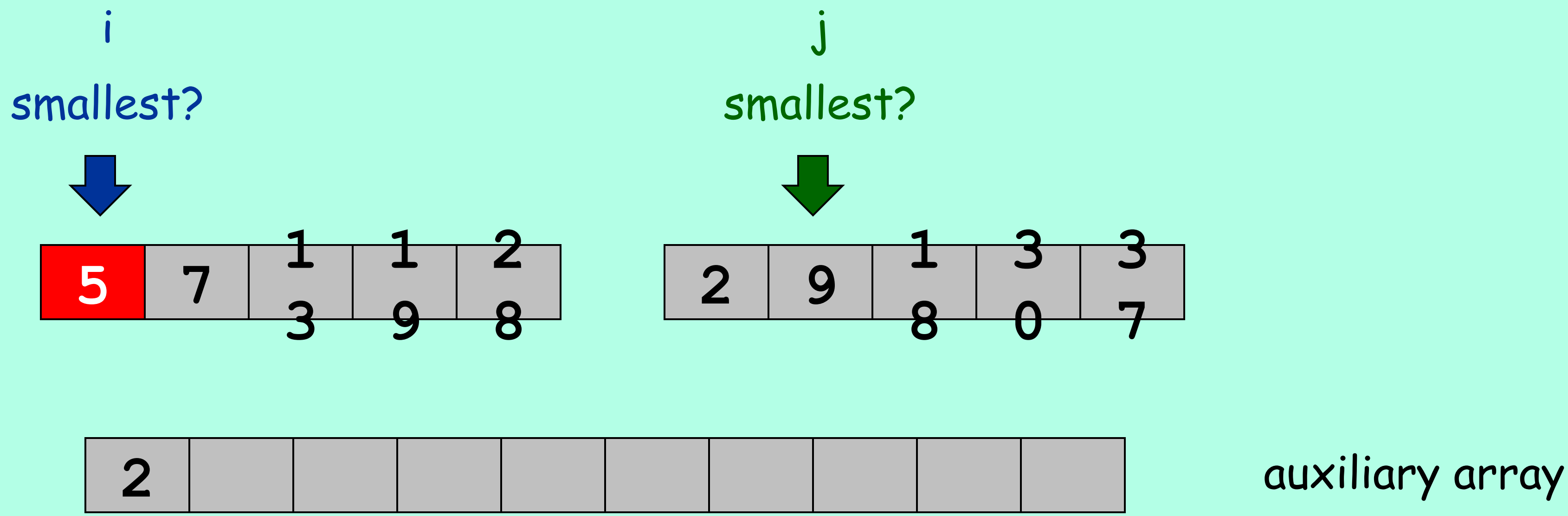

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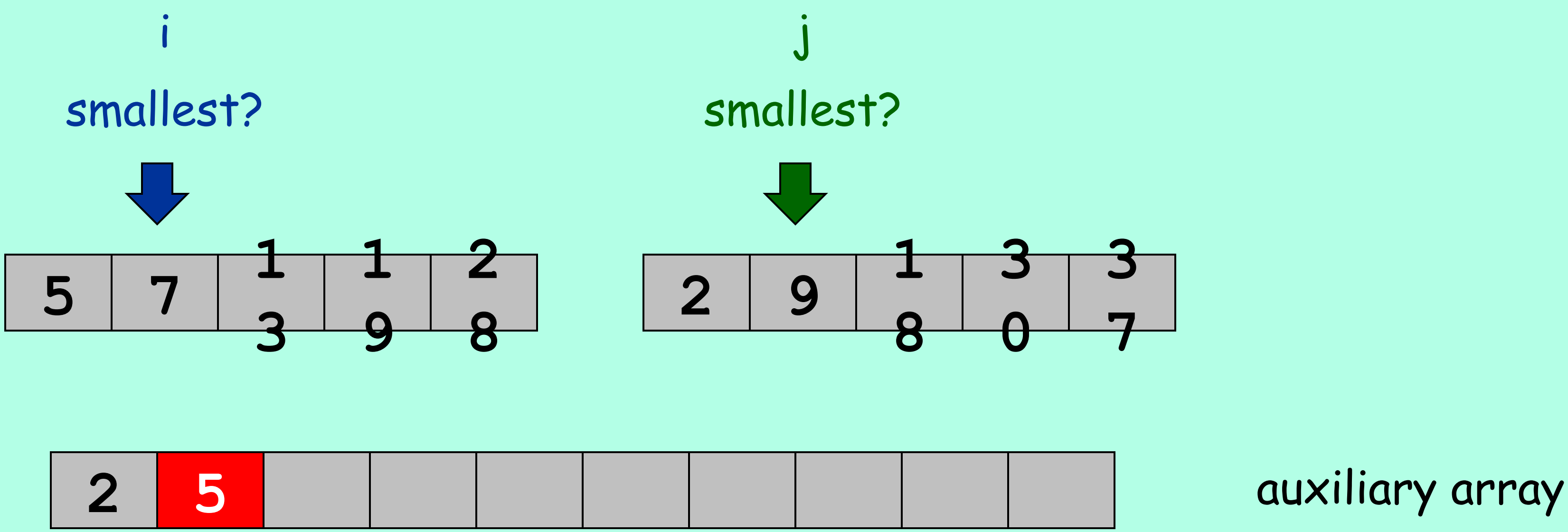

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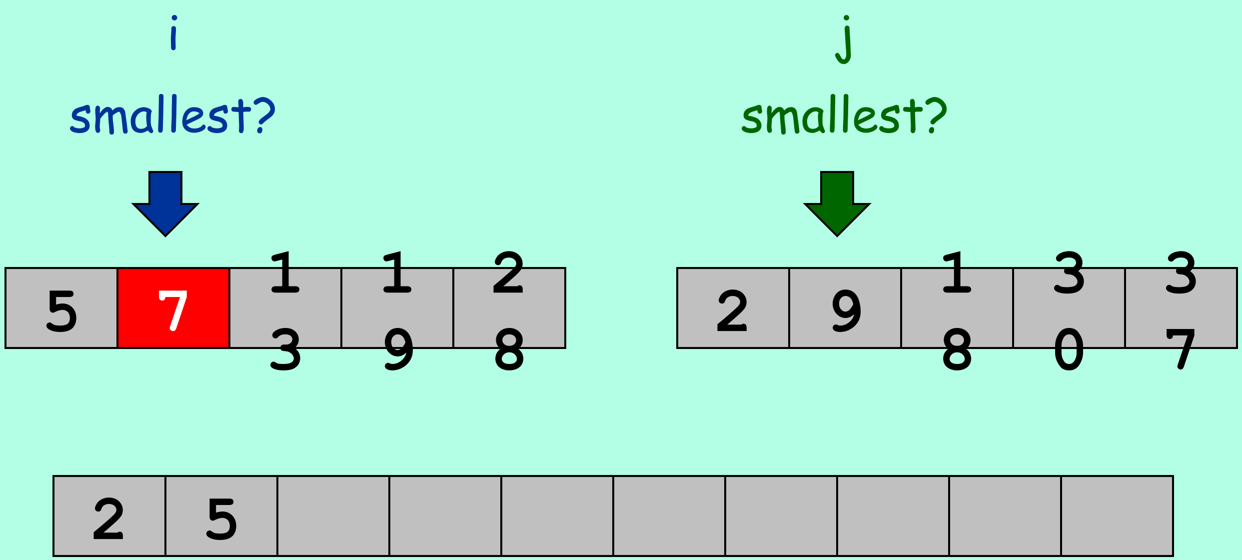

auxiliary array

Merging

Merge.

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- Compare the two elements.
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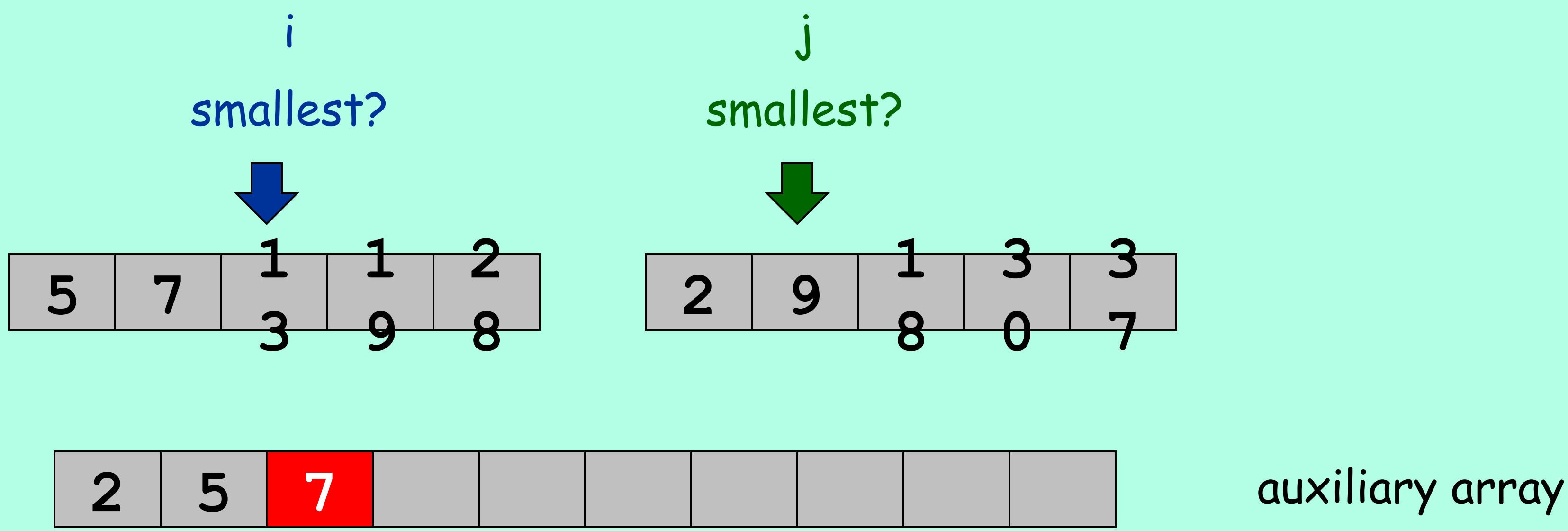
- Repeat until done.



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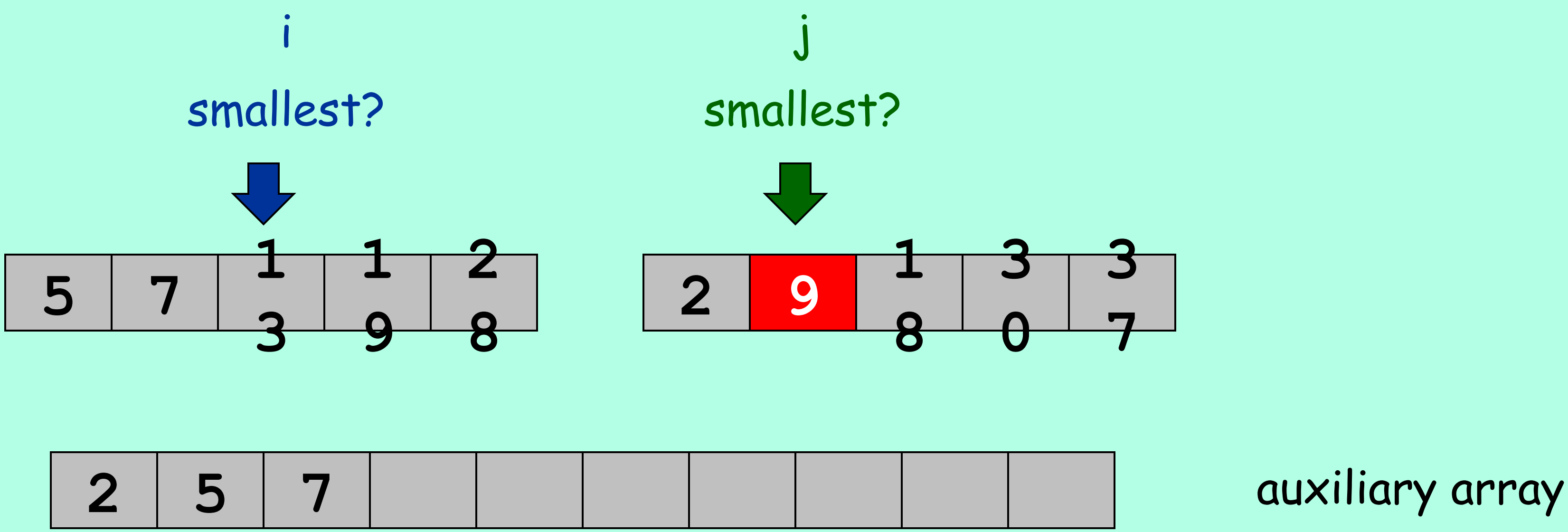

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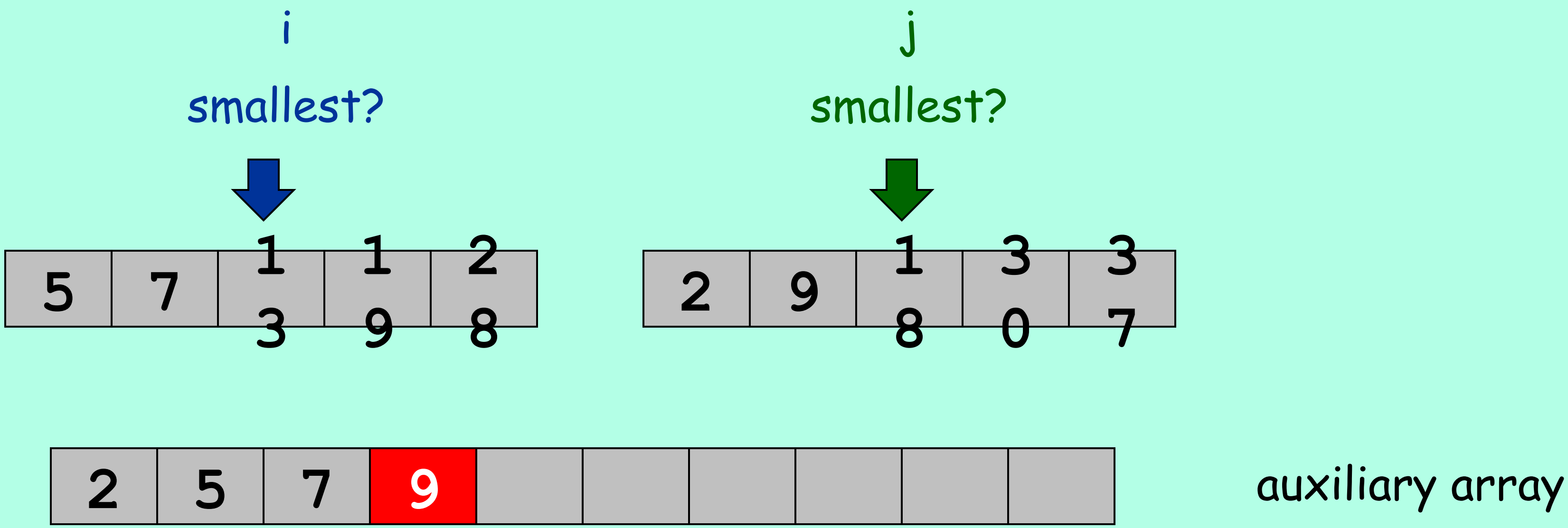

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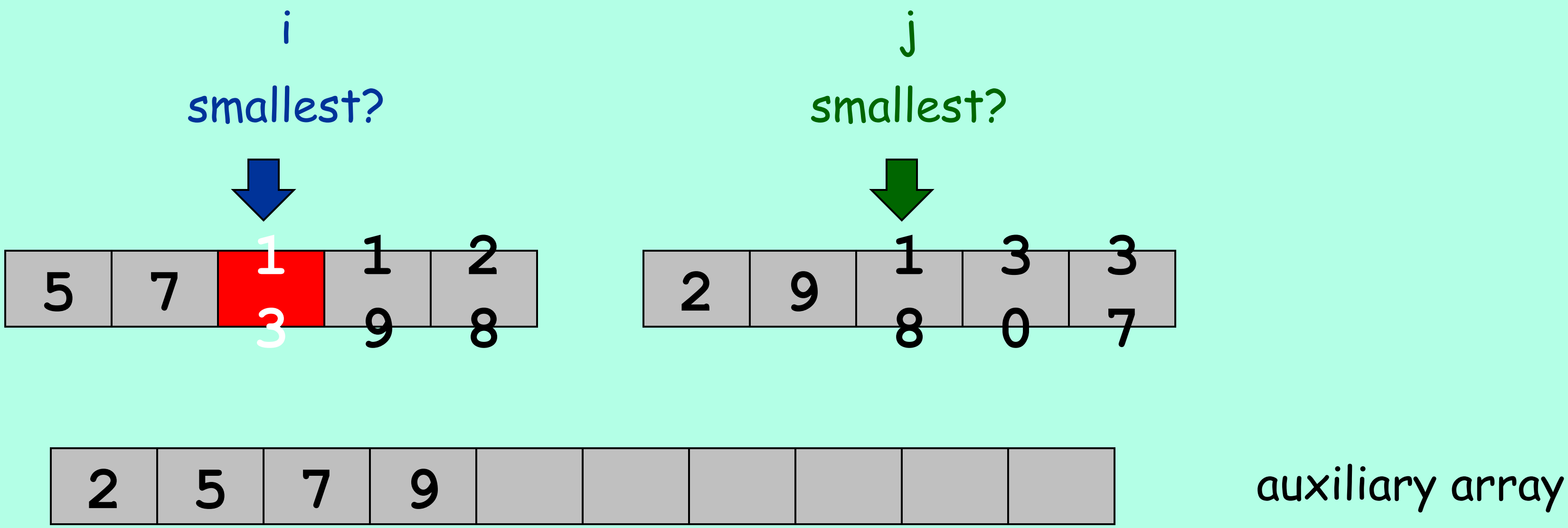

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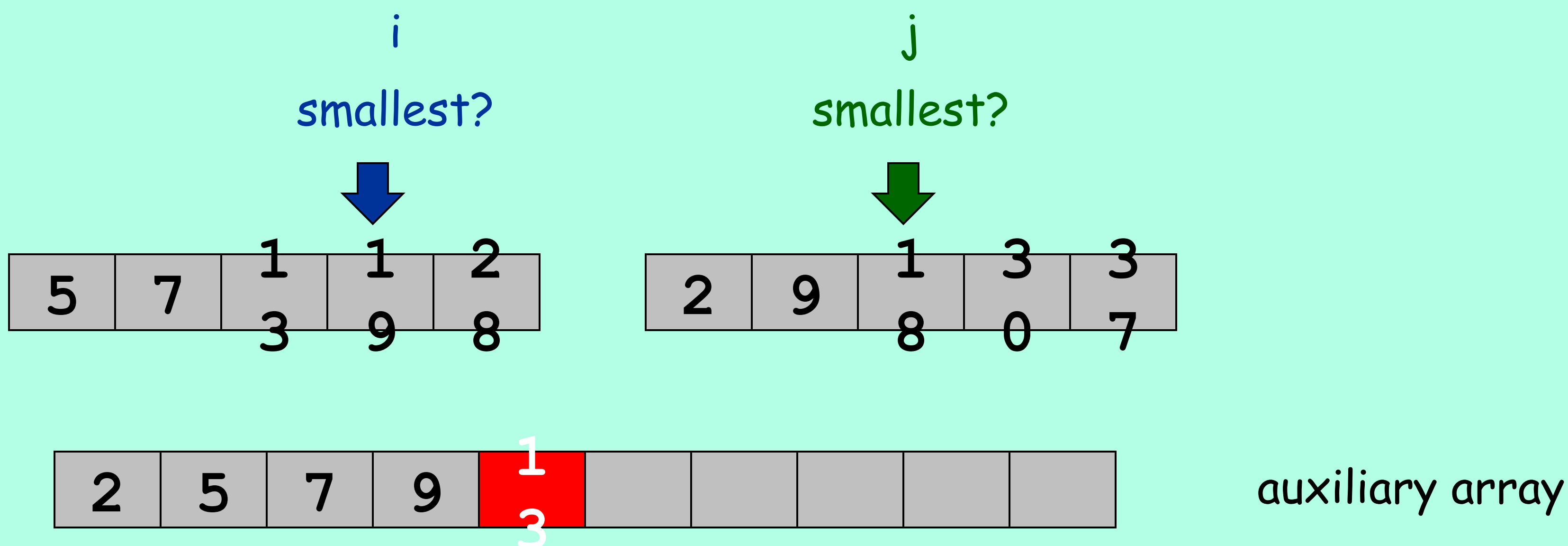

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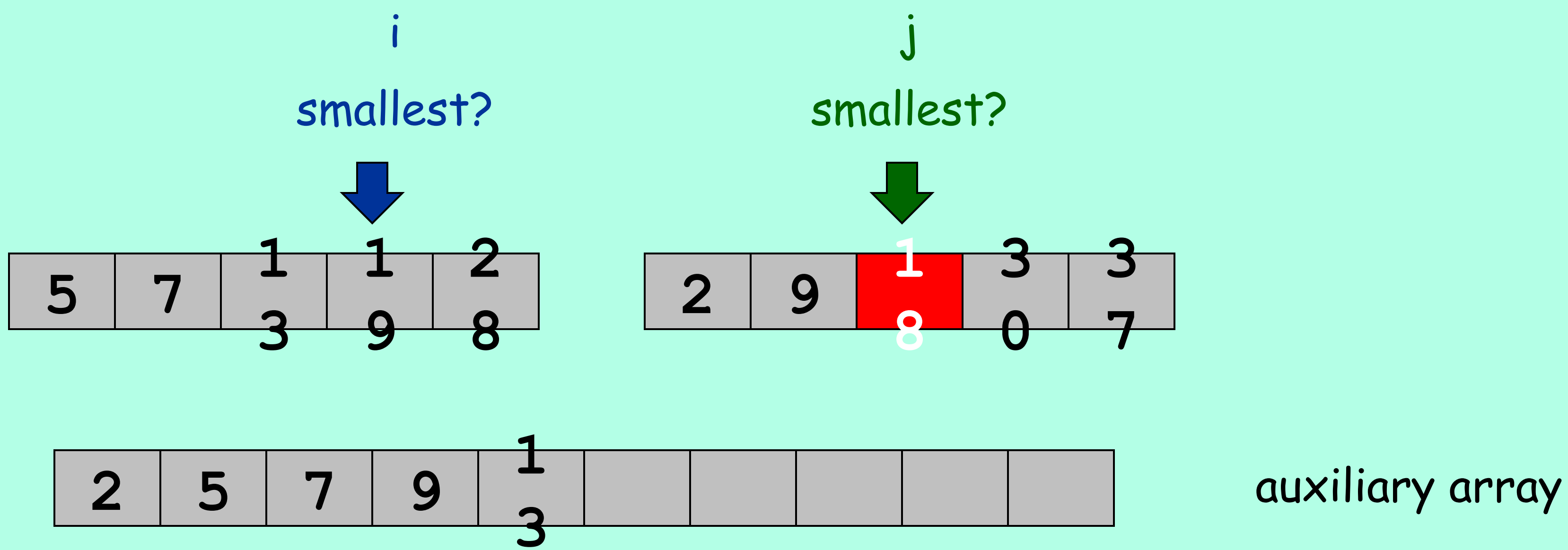

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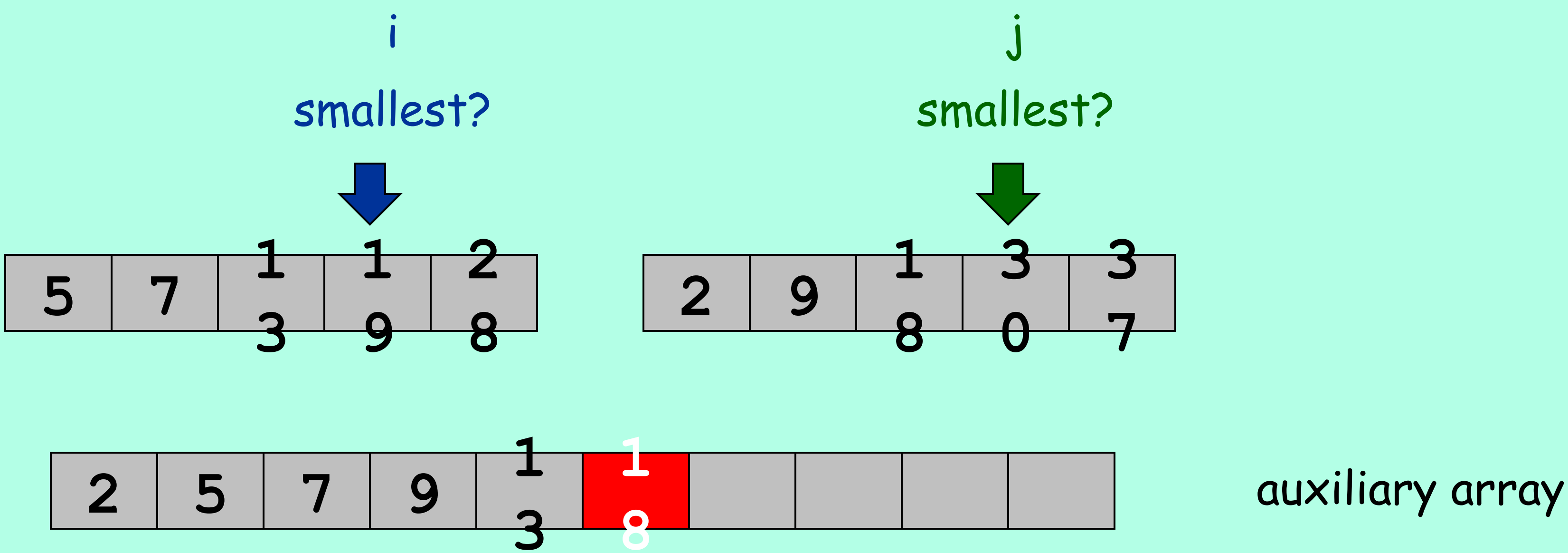

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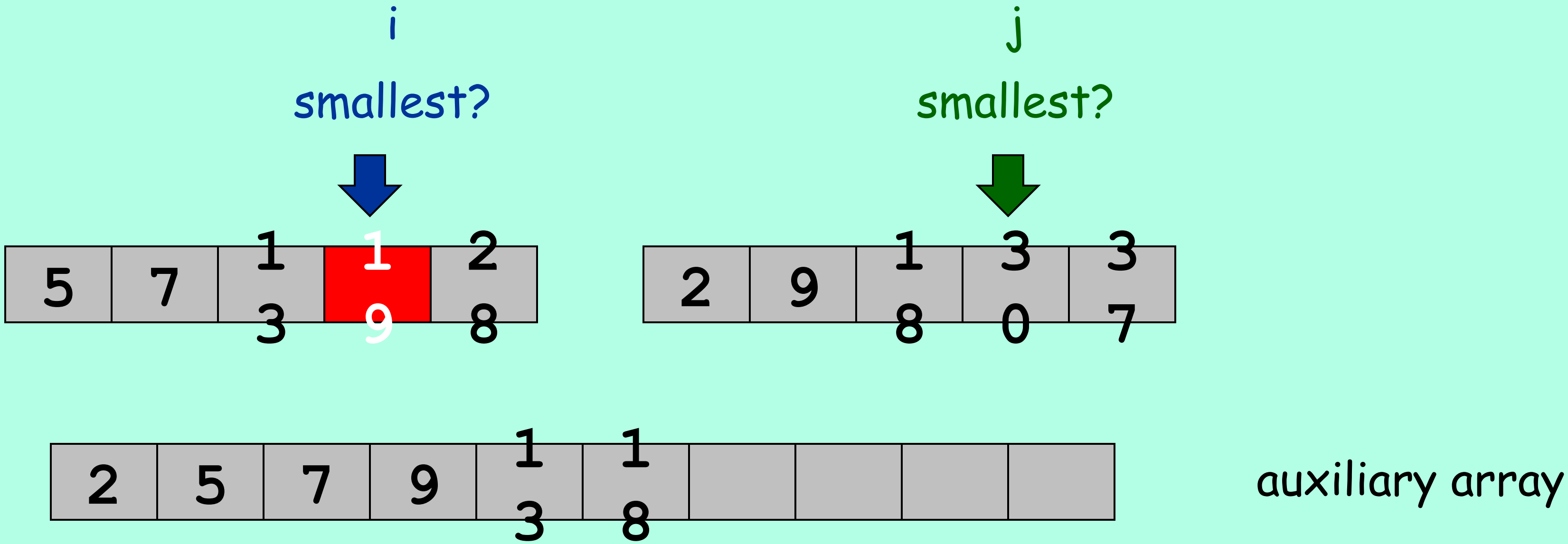

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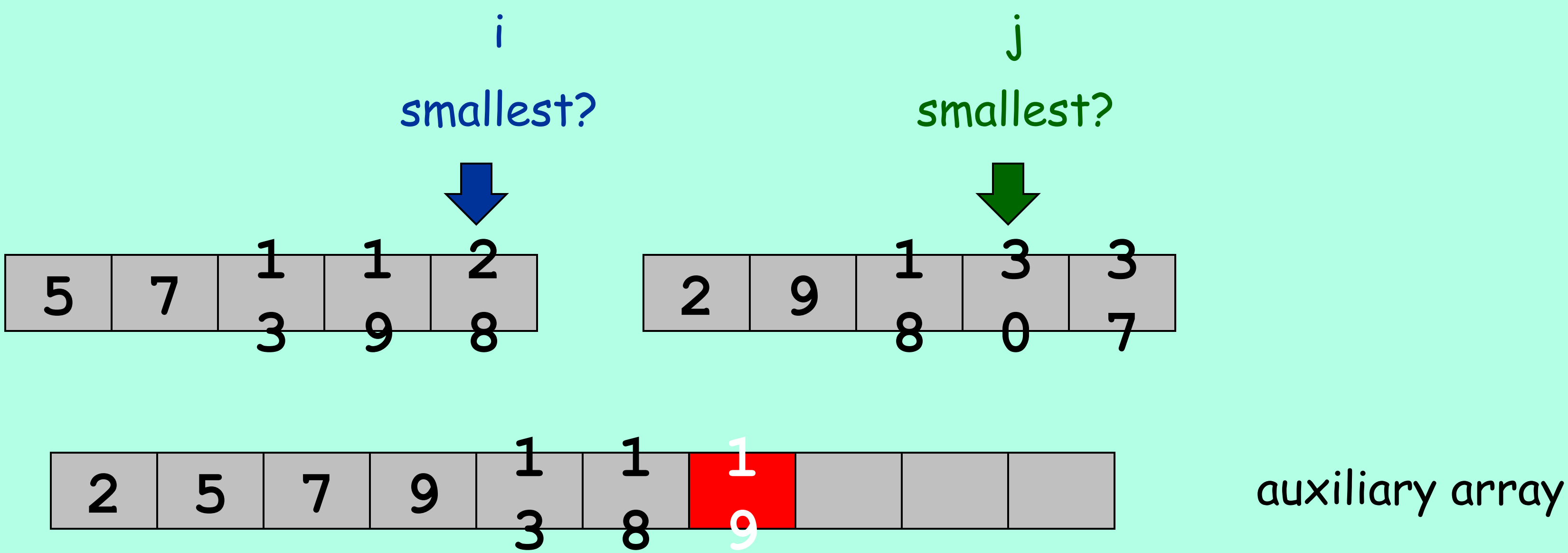

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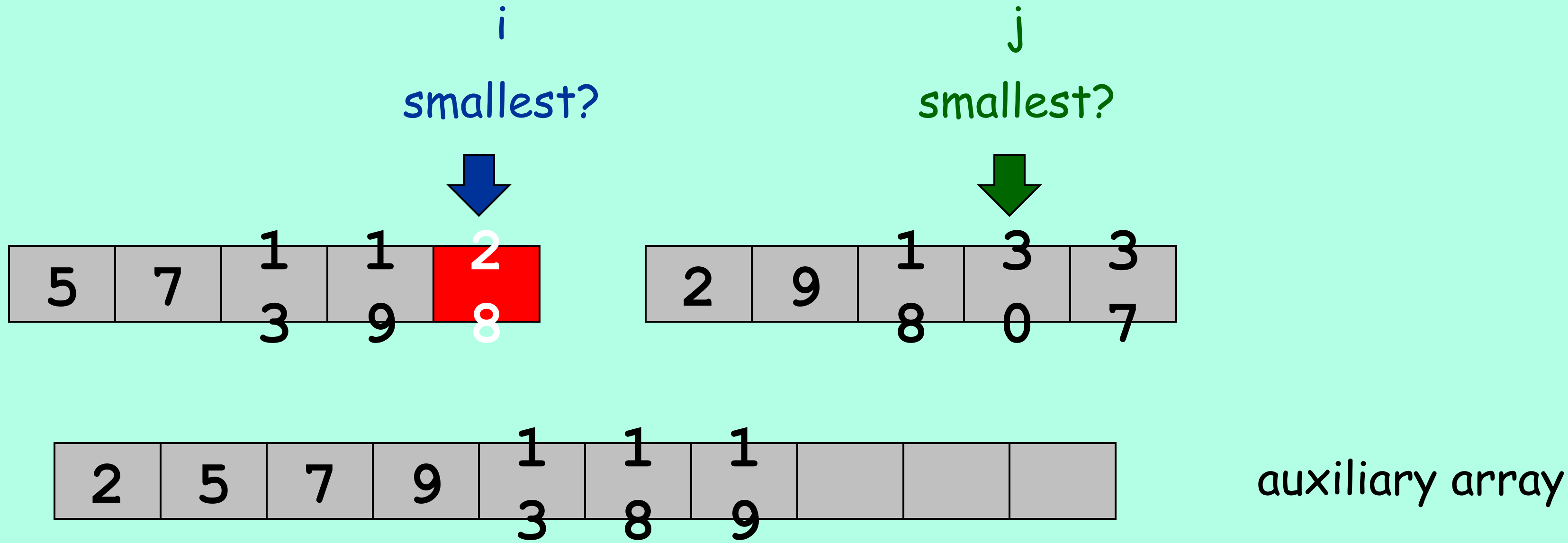

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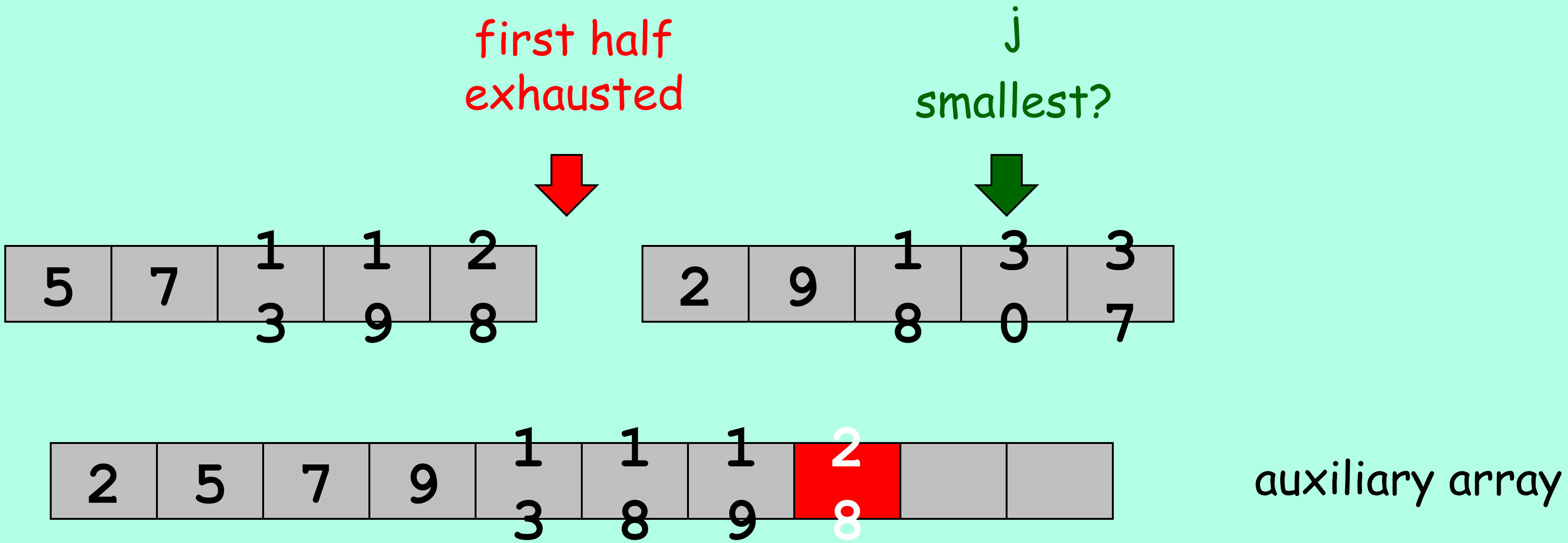

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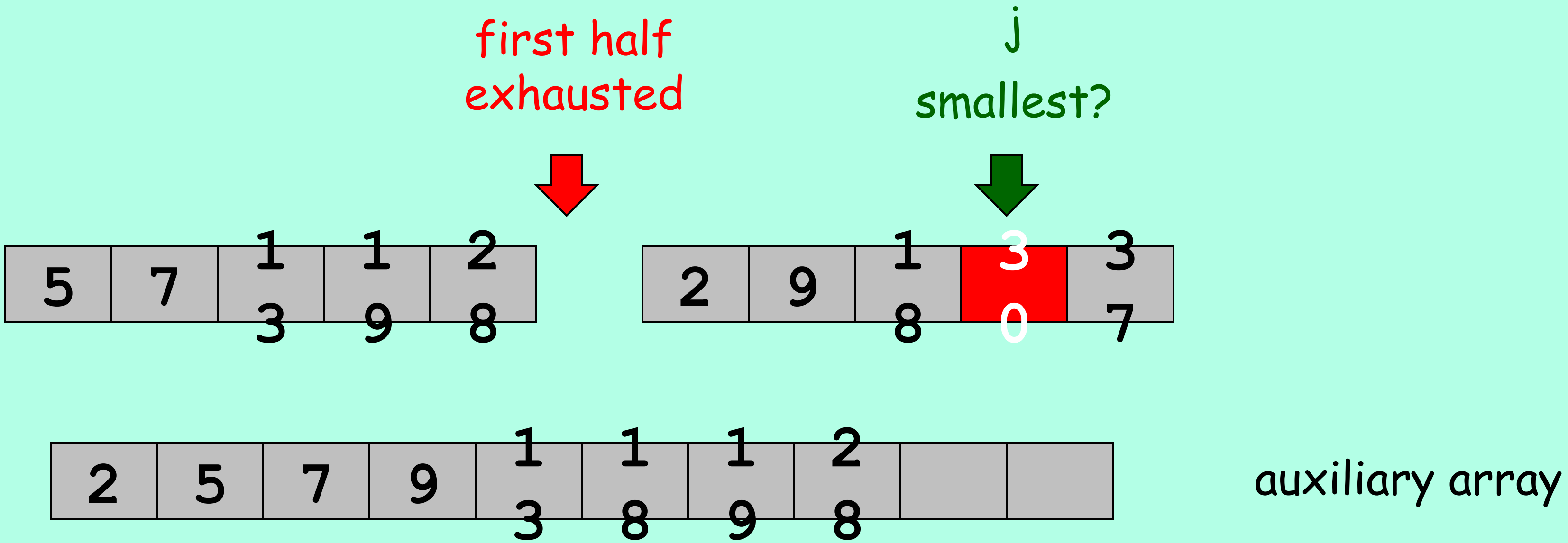

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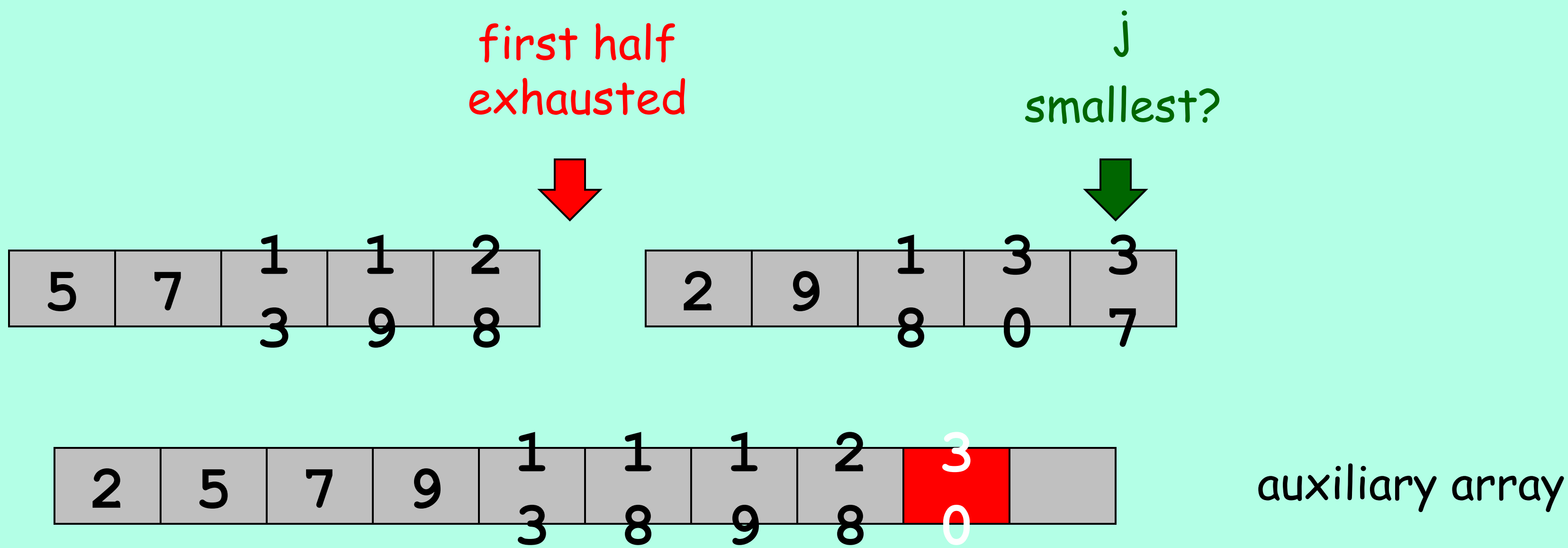

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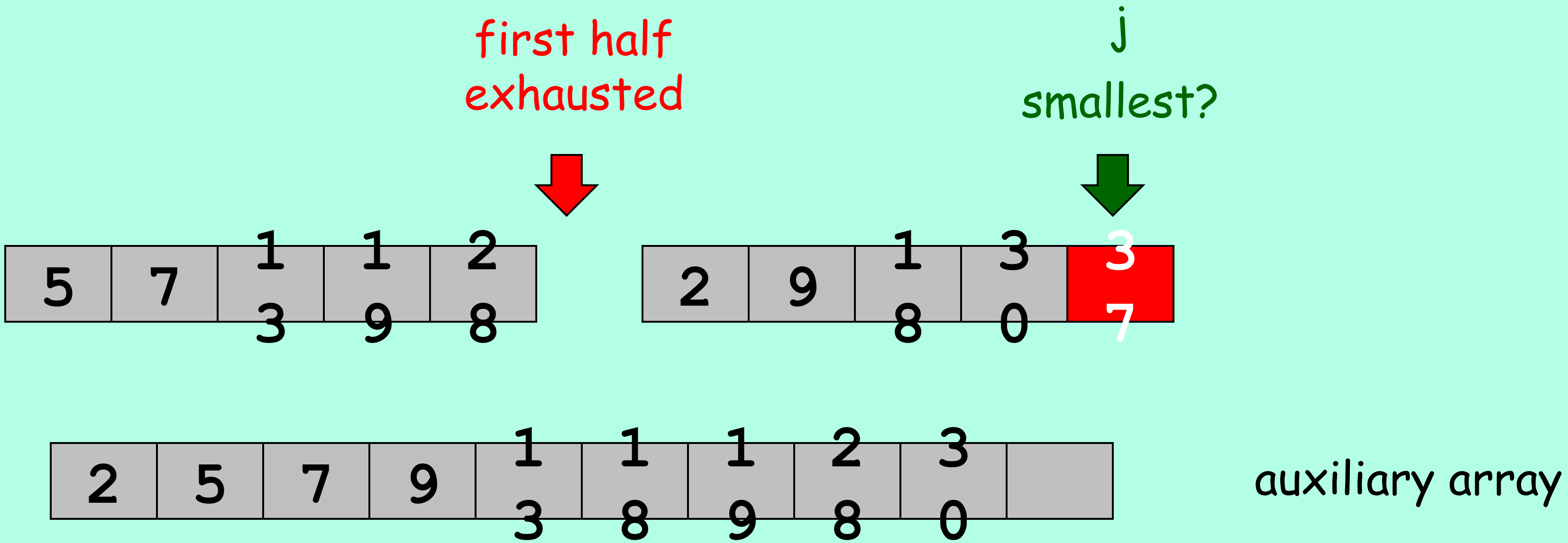

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Merge.

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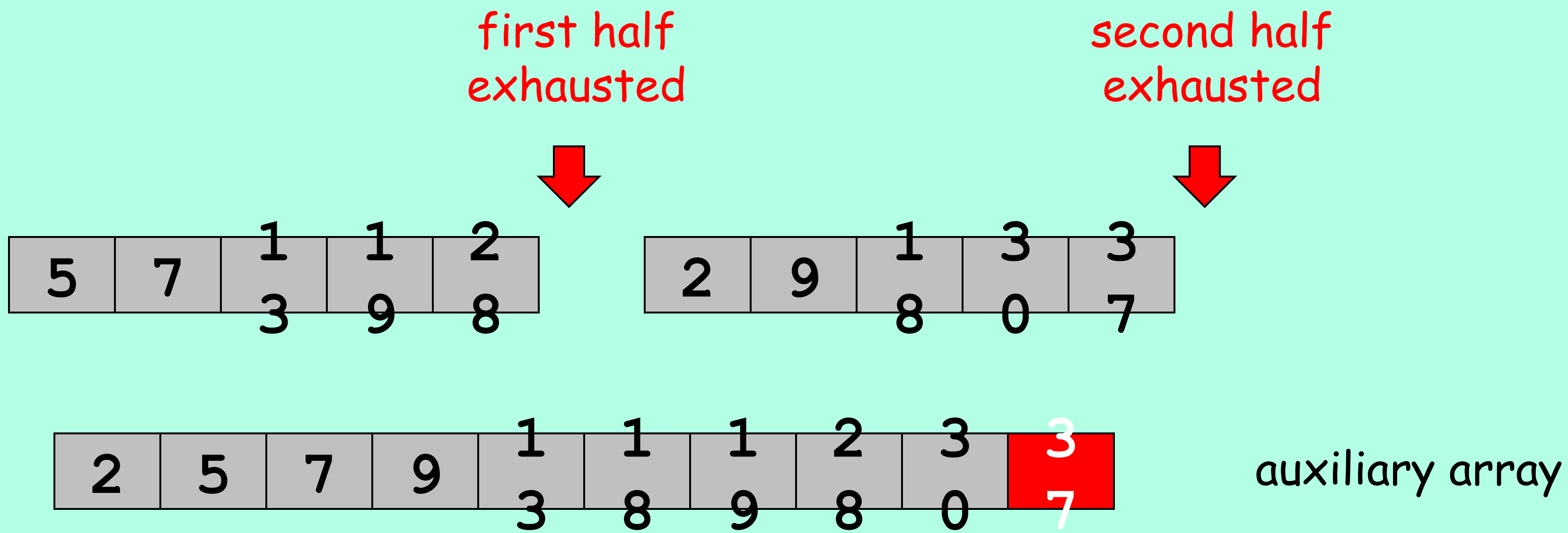
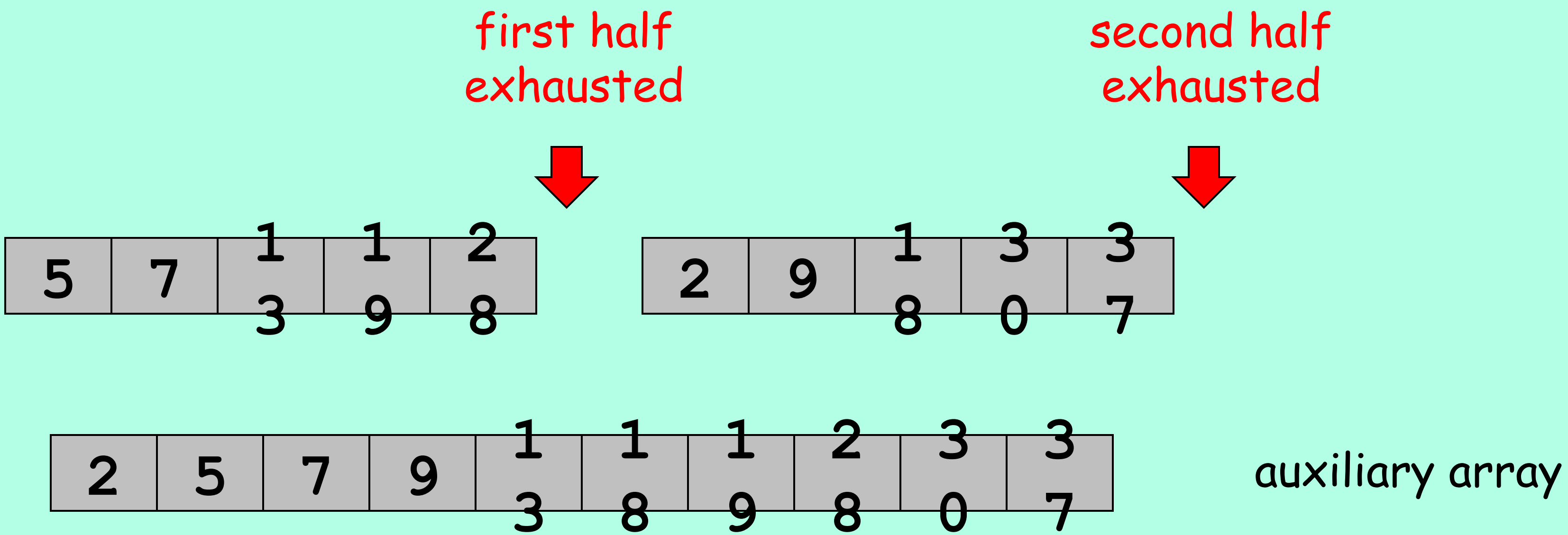


Merging

Merge.

- Each half is in sorted order.
- Use indices i and j to step through the two halves.
- Compare the two elements.
- Insert smallest of two elements into next position of the auxiliary array.
- Repeat until done.



Merging

Determine the running time of the merge operation:

Given two sorted arrays, each with $n/2$ elements, how many comparisons and copies are made?

There is one comparison for each element that is copied to the auxiliary array.

Each of the n elements are copied one at a time to the auxiliary array.

Total running time is $\Theta(n)$.

5	7	13	19	28

2	9	18	30	37

2	5	7	9	13	18	19	28	30	37

auxiliary array

```
merge(a, left, middle, right)
    // create temporary array b of size right - left + 1
    i = left
    j = middle+1
    k = 1
    while (i <= middle && j <= right) {
      if (a[i] < a[j]) {
        b[k] = a[i]
        i = i + 1
      }
      else {
        b[k] = a[j]
        j = j + 1
      }
      k = k + 1
    }
    while (i <= middle) {
      b[k] = a[i]
      k = k + 1
      i = i + 1
    }

    // continued next slide
```

```
// continued from previous slide

    while (j <= right) {
    b[k] = a[j]
      k = k + 1
      j = j + 1
    }
    x = left
    for (t = 1 to right - left + 1) {
      a[x] = b[t]
      x = x + 1
    }
}
```

Algorithm 6.2.4 Quicksort

This algorithm sorts the array $a[i], \ldots, a[j]$ using a divide and conquer approach. The elements to the left of the partition index p are less than a[p] and the elements to the right of p are greater than p.

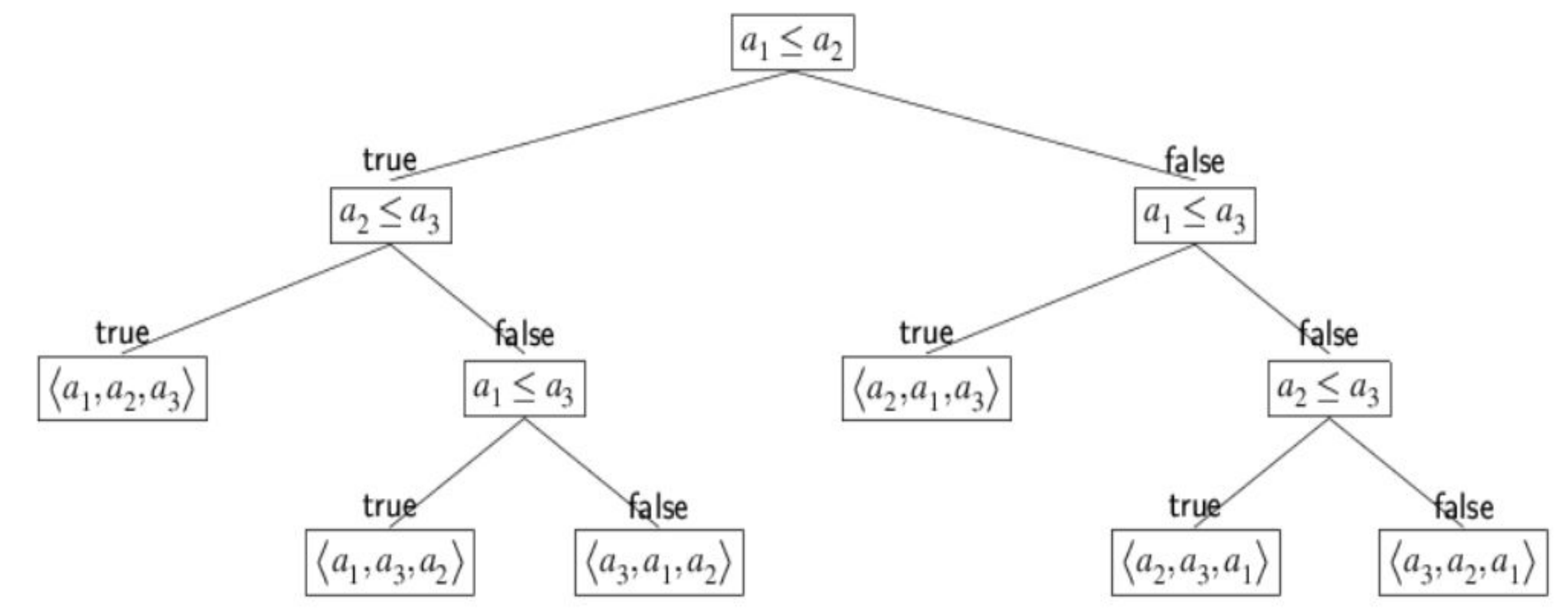
```
Input Parameters: a,i,j
Output Parameters: i

quickSort(a,i,j) {
   if (i < j) {
      p = partition(a,i,j)     // partitions smaller elements
                               // left of a[p], larger to
                               // right of a[p]
      quickSort(a,i,p - 1)
      quickSort(a,p + 1,j)
   }
}
```

A Lower Bound for the Sorting Problem

Theorem Any comparison-based sorting algorithm has worst case time $\Omega(n \lg n)$.

Conclusion: Any comparison-based sorting algorithm must take time **at least** $n \lg n$. There is no hope of finding any faster comparison-based algorithm.



The decision tree for comparing and ordering
an array of n elements has $n!$ leaves.

In the worst case, the number of comparisons $\ge$ height of tree

$$\ge \lg(n!)$$
$$= \Omega(n \lg n)$$

Algorithm 6.4.2 Counting Sort

This algorithm sorts an array $a[1], \ldots, a[n]$ of integers.

Assumption: Each integer is in the range 0 to m, inclusive; usually m is a fairly small value.

It operates by **counting** how many occurrences there are of each integer in the range 0 to m. Next, the array c is used to determine how many values in the array are less than or equal to each integer in the range 0 to m.

```
c[k] = the number of values less than or equal to k
       in the array a
```

```
countingSort(a,m) {
    for k = 0 to m
        c[k] = 0
    n = a.last

    for i = 1 to n
        c[a[i]] = c[a[i]] + 1        // how many of each
value

    for k = 1 to m
        c[k] = c[k] + c[k - 1]       // how many ≤ k

    // sort a with the result in b
    for i = n downto 1 {
        b[c[a[i]]] = a[i]
        c[a[i]] = c[a[i]] - 1
    }

    // copy b back to a
    for i = 1 to n
        a[i] = b[i]
}
```

Counting Sort

	1	2	3	4	5	6	7	8	9	10
a	**4**	**3**	**0**	**3**	**6**	**2**	**0**	**1**	**3**	**4**

Data in the range 0..6

m = 6

	0	1	2	3	4	5	6
c	**0**	**0**	**0**	**0**	**0**	**0**	**0**

```
for i = 1 to n
  c[a[i]] = c[a[i]] + 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	0	1	2	3	4	5	6
c	0	0	0	0	1	0	0

```
for i = 1 to n
  c[a[i]] = c[a[i]] + 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	**4**	**3**	**0**	**3**	**6**	**2**	**0**	**1**	**3**	**4**

	0	1	2	3	4	5	6
c	**0**	**0**	**0**	**1**	**1**	**0**	**0**

```
for i = 1 to n
  c[a[i]] = c[a[i]] + 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	0	1	2	3	4	5	6
c	1	0	0	1	1	0	0

```
for i = 1 to n
  c[a[i]] = c[a[i]] + 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	0	1	2	3	4	5	6
c	2	1	1	3	2	0	1

```
for k = 1 to m
  c[k] = c[k] + c[k - 1]
```

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	0	1 (k)	2	3	4	5	6
c	2	1	1	3	2	0	1

```
for k = 1 to m
    c[k] = c[k] + c[k - 1]
```

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

k

	0	1	2	3	4	5	6
c	2	3	1	3	2	0	1

```
for k = 1 to m
  c[k] = c[k] + c[k - 1]
```

	1	2	3	4	5	6	7	8	9	10
a	**4**	**3**	**0**	**3**	**6**	**2**	**0**	**1**	**3**	**4**

k

	0	1	2	3	4	5	6
c	**2**	**3**	**4**	**3**	**2**	**0**	**1**

```
for k = 1 to m
    c[k] = c[k] + c[k - 1]
```

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

k

	0	1	2	3	4	5	6
c	2	3	4	7	2	0	1

```
for k = 1 to m
  c[k] = c[k] + c[k - 1]
```

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

k

	0	1	2	3	4	5	6
c	2	3	4	7	9	9	10

```
for i = n downto 1
    b[c[a[i]]] = a[i]
    c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	**4**	**3**	**0**	**3**	**6**	**2**	**0**	**1**	**3**	**4**

	1	2	3	4	5	6	7	8	9	10
b										

	0	1	2	3	4	5	6
c	**2**	**3**	**4**	**7**	**9**	**9**	**10**

```
for i = n downto 1
    b[c[a[i]]] = a[i]
    c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	1	2	3	4	5	6	7	8	9	10
b									4	

	0	1	2	3	4	5	6
c	2	3	4	7	8	9	10

```
for i = n downto 1
  b[c[a[i]]] = a[i]
  c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	**4**	**3**	**0**	**3**	**6**	**2**	**0**	**1**	**3**	**4**

	1	2	3	4	5	6	7	8	9	10
b									**4**	

	0	1	2	3	4	5	6
c	**2**	**3**	**4**	**7**	**8**	**9**	**10**

```
for i = n downto 1
  b[c[a[i]]] = a[i]
  c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	1	2	3	4	5	6	7	8	9	10
b							3		4	

	0	1	2	3	4	5	6
c	2	3	4	6	8	9	10

```
for i = n downto 1
  b[c[a[i]]] = a[i]
  c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	1	2	3	4	5	6	7	8	9	10
b							3		4	

	0	1	2	3	4	5	6
c	2	3	4	6	8	9	10

```
for i = n downto 1
    b[c[a[i]]] = a[i]
    c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	1	2	3	4	5	6	7	8	9	10
b			1				3		4	

	0	1	2	3	4	5	6
c	2	2	4	6	8	9	10

```
for i = n downto 1
    b[c[a[i]]] = a[i]
    c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	1	2	3	4	5	6	7	8	9	10
b			1				3		4	

	0	1	2	3	4	5	6
c	2	2	4	6	8	9	10

```
for i = n downto 1
  b[c[a[i]]] = a[i]
  c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	**4**	**3**	**0**	**3**	**6**	**2**	**0**	**1**	**3**	**4**

	1	2	3	4	5	6	7	8	9	10
b		**0**	**1**				**3**		**4**	

	0	1	2	3	4	5	6
c	**1**	**2**	**4**	**6**	**8**	**9**	**10**

```
for i = n downto 1
    b[c[a[i]]] = a[i]
    c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	1	2	3	4	5	6	7	8	9	10
b		0	1				3		4	

	0	1	2	3	4	5	6
c	1	2	4	6	8	9	10

```
for i = n downto 1
    b[c[a[i]]] = a[i]
    c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	**4**	**3**	**0**	**3**	**6**	**2**	**0**	**1**	**3**	**4**

	1	2	3	4	5	6	7	8	9	10
b		**0**	**1**	**2**			**3**		**4**	

	0	1	2	3	4	5	6
c	**1**	**2**	**3**	**6**	**8**	**9**	**10**

```
for i = n downto 1
    b[c[a[i]]] = a[i]
    c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	1	2	3	4	5	6	7	8	9	10
b		0	1	2			3		4	

	0	1	2	3	4	5	6
c	1	2	3	6	8	9	10

```
for i = n downto 1
    b[c[a[i]]] = a[i]
    c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	1	2	3	4	5	6	7	8	9	10
b		0	1	2			3		4	6

	0	1	2	3	4	5	6
c	1	2	3	6	8	9	9

```
for i = n downto 1
    b[c[a[i]]] = a[i]
    c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	1	2	3	4	5	6	7	8	9	10
b		0	1	2			3		4	6

	0	1	2	3	4	5	6
c	1	2	3	6	8	9	9

```
for i = n downto 1
    b[c[a[i]]] = a[i]
    c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	1	2	3	4	5	6	7	8	9	10
b		0	1	2		3	3		4	6

	0	1	2	3	4	5	6
c	1	2	3	5	8	9	9

```
for i = n downto 1
  b[c[a[i]]] = a[i]
  c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	**4**	**3**	**0**	**3**	**6**	**2**	**0**	**1**	**3**	**4**

	1	2	3	4	5	6	7	8	9	10
b		**0**	**1**	**2**		**3**	**3**		**4**	**6**

	0	1	2	3	4	5	6
c	**1**	**2**	**3**	**5**	**8**	**9**	**9**

```
for i = n downto 1
    b[c[a[i]]] = a[i]
    c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	**4**	**3**	**0**	**3**	**6**	**2**	**0**	**1**	**3**	**4**

	1	2	3	4	5	6	7	8	9	10
b	**0**	**0**	**1**	**2**		**3**	**3**		**4**	**6**

	0	1	2	3	4	5	6
c	**0**	**2**	**3**	**5**	**8**	**9**	**9**

```
for i = n downto 1
   b[c[a[i]]] = a[i]
   c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	1	2	3	4	5	6	7	8	9	10
b	0	0	1	2		3	3		4	6

	0	1	2	3	4	5	6
c	0	2	3	5	8	9	9

```
for i = n downto 1
    b[c[a[i]]] = a[i]
    c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	**4**	**3**	**0**	**3**	**6**	**2**	**0**	**1**	**3**	**4**

	1	2	3	4	5	6	7	8	9	10
b	**0**	**0**	**1**	**2**	**3**	**3**	**3**		**4**	**6**

	0	1	2	3	4	5	6
c	**0**	**2**	**3**	**4**	**8**	**9**	**9**

```
for i = n downto 1
  b[c[a[i]]] = a[i]
  c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	1	2	3	4	5	6	7	8	9	10
b	0	0	1	2	3	3	3		4	6

	0	1	2	3	4	5	6
c	0	2	3	4	8	9	9

```
for i = n downto 1
  b[c[a[i]]] = a[i]
  c[a[i]] = c[a[i]] - 1
```

i

	1	2	3	4	5	6	7	8	9	10
a	**4**	**3**	**0**	**3**	**6**	**2**	**0**	**1**	**3**	**4**

	1	2	3	4	5	6	7	8	9	10
b	**0**	**0**	**1**	**2**	**3**	**3**	**3**	**4**	**4**	**6**

	0	1	2	3	4	5	6
c	**0**	**2**	**3**	**4**	**7**	**9**	**9**

```
for i = 1 to n
  a[i] = b[i]
```

i

	1	2	3	4	5	6	7	8	9	10
a	4	3	0	3	6	2	0	1	3	4

	1	2	3	4	5	6	7	8	9	10
b	0	0	1	2	3	3	3	4	4	6

	0	1	2	3	4	5	6
c	0	2	3	4	7	9	9

```
for i = 1 to n
  a[i] = b[i]
```

i

	1	2	3	4	5	6	7	8	9	10
a	**0**	**0**	**1**	**2**	**3**	**3**	**3**	**4**	**4**	**6**

	1	2	3	4	5	6	7	8	9	10
b	**0**	**0**	**1**	**2**	**3**	**3**	**3**	**4**	**4**	**6**

	0	1	2	3	4	5	6
c	**0**	**2**	**3**	**4**	**7**	**9**	**9**

```
countingSort(a,m) {
    for k = 0 to m
        c[k] = 0
    n = a.last

    for i = 1 to n
        c[a[i]] = c[a[i]] + 1        // how many of each
value

    for k = 1 to m
        c[k] = c[k] + c[k - 1]       // how many ≤ k

    // sort a with the result in b
    for i = n downto 1 {
        b[c[a[i]]] = a[i]
        c[a[i]] = c[a[i]] - 1
    }

    // copy b back to a
    for i = 1 to n
        a[i] = b[i]
}
```


$\theta(\mathbf{n+m}) = \theta(\mathbf{n})$ if m is some "small" constant

Running time?

What's going on? Linear time?

This is not a comparison-based sort. It never compares two elements in the array (e.g., `if  a[i] < a[j]`).

Algorithm 6.4.4 Radix Sort

This algorithm sorts the array $a[1], \ldots, a[n]$ of integers.

Assumption: Each integer has k digits.

It sorts the integers by digit, working from the LEAST significant digit to the most significant digit.

```
radixSort(a,k)
{
    // k is the number of digits in each element of  a
   for i = 0 to k-1
      countingSort(a,9) on digit in 10^i place
}
```

```
radixSort(a,k) {
    for i = 0 to k-1
        countingSort(a,9) on digit in 10^i place
}
```

k = 3

329

457

657

839

436

720

355

```
radixSort(a,k) {
    for i = 0 to k-1
        countingSort(a,9) on digit in 10^i place
}
```

k = 3

329

457

657

839

436

720

355

sort by 10^0 = ones digit

```
radixSort(a,k) {
    for i = 0 to k-1
        countingSort(a,9) on digit in 10^i place
}
```

k = 3

329		720
457		355
657		436
839	→	457
436		657
720		329
355		839

```
radixSort(a,k) {
    for i = 0 to k-1
        countingSort(a,9) on digit in 10^i place
}
```

k = 3

329		720
457		355
657		436
839	→	457
436		657
720		329
355		839

sort by 10^1 = tens digit

```
radixSort(a,k) {
    for i = 0 to k-1
        countingSort(a,9) on digit in 10^i place
}
```

k = 3

329	720	720
457	355	329
657	436	436
839	457	839
436	657	355
720	329	457
355	839	657

```
radixSort(a,k) {
    for i = 0 to k-1
        countingSort(a,9) on digit in 10^i place
}
```

k = 3

329	720	720
457	355	329
657	436	436
839	457	839
436	657	355
720	329	457
355	839	657

sort by 10^2 = hundreds digit

```
radixSort(a,k) {
    for i = 0 to k-1
        countingSort(a,9) on digit in 10^i place
}
```

k = 3

329	720	720		329
457	355	329		355
657	436	436		436
839	457	839	→	457
436	657	355		657
720	329	457		720
355	839	657		839

Algorithm 6.4.4 Radix Sort

This algorithm sorts the array $a[1], \ldots, a[n]$ of integers.

Assumption: Each integer has k digits.

It sorts the integers by digit, working from the LEAST significant digit to the most significant digit.

```
radixSort(a,k)
{
    // k is the number of digits in each element of  a
   for i = 0 to k-1
      countingSort(a,9) on digit in 10^i place
}
```

Determine the running time of `radixSort`.

$\theta(\mathbf{kn}) = \theta(\mathbf{n})$ if k is some "small" constant