A brief analysis of a Stern-Gerlach apparatus

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In this article, we describe briefly a test of the intrinsic spin of fermionic particles by means of a Stern-Gernlach apparatus. Numerous fields in physics today such as quantum computing, NMR spectroscopy and quantum cryptography benefit from a thorough understanding of intrinsic angular momenta, and consequently, rigorous testing of these principles fundamental to the development of quantum mechanics are necessary. We therefore make use of potassium atoms placed in an oven contained within a vacuum chamber and subsequently apply a magnetic gradient. The field gradient interacts with the magnetic moment of the potassium (mostly accounted for by the electron in the outer shell) and this is detected by means of an electrometer

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INTRODUCTION

One of the most profound discoveries in modern physics has been the concept of an intrinsic spin angular momentum, and therefore production of a magnetic moment capable of interacting with external fields. The theoretical groundwork was constructed by P.A.M Dirac and the first experimental tests were most notably done by O. Stern and W. Gerlach [1] in 1922. The confirmation of an intrinsic magnetic moment not only solved a great many of issues plaguing microscopic physics of the day, but provides a virtually limitless source of experimentation and application. For example, reconstruction of events from collider data, pNMR and MRI and quantum computing and cryptography and atomic clocks all rely on spin to be carried out. Though these experiments tend not to involve a simple splitting of a beam by a magnetic field, the verifiction of concept is important and necessary to further our understanding and widen the types of application in which spin is integrated. We therefore take the effort to repeatedly and systematically verify our most fundamental assumptions. We shall assume the value of μ_B and then find the average lateral splitting distance, s, which is the separation between peaks with the field on and off as will be shown below. This will allow us to finally determine that $\vec{\nabla}B = 68.67 \pm 2.22$ T/m in this apparatus.

SETUP AND CALCULATION

We proceed by creating a vacuum of 5×10^{-6} mm Hg and then placing a slug of potassium $^{41}_{19}K$ atoms into an oven operating at $115 \pm 1°C$. The potassium vapor passes first through .011" collimating slits, then through a manually operated beam gate, then an additional set of collimating slits before entering the magnetic field produced within the pole piece box. Once out of the field gradient, a detector assembly attached to a micrometer is transversely shifted in order to obtain the current produced as a function of position.

Between the collimating slits, a manually operated baffle allows background measurements to be made after each amperage measurement. The test is run first without a magnetic gradient present, and subsequently run again with the gradient on. The field is produced by a DC power supply at 1A. Comparison of results allows a determination of the separation of the current distribution peaks and thus for a calculation of the field gradient. After completion of both runs and subtracting background effects (and multiplying the run with the magnetic field by a factor of 7 for clarity of comparison). The former classical theory would have predicted a beam split into 3 parts, however here we obtain the following plot in Fig. 1.

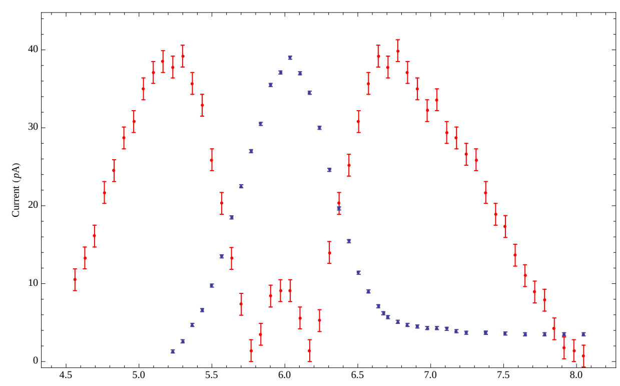

FIG. 1: Distribution of current versus position in the presence of and in the absence of a field gradient clearly showing a split caused by a magnetic moment as predicted in the relativistic quantum theory

validating our fundamental assumption. To begin analysis of the magnetic field gradient from the data, first the necessary constants are collected and shown in Table. 1. Additionally, we may use the following derivation for $\vec{s}$ [2]

$$\vec{a}_z = \frac{\mu_z \vec{\nabla} B}{M}$$

$$\vec{s} = \left(\frac{d}{v}\right)^2 \frac{\vec{a}_z}{2} = \mu_z \vec{\nabla} B \frac{\left[d_1^2 + 2d_1 d_2\right]}{2_{39} m v^2} \tag{1}$$

where $\vec{s}$ is the separation between current distribution peaks (therefore the slight deformation of the expected gaussian shape will not propagate much error), $d^2 = d_1^2 + 2d_1d_2$, the distance d_1 is how far the potassium atoms travel through the pole piece box, d_2 is the distance from the end of the field gradient region to the detector, Fig. 2, and ${}_{39}m$ is the mass of the ${}^{39}_{19}K$ isotope.

TABLE I: Necessary Constants ($c = 1$)

Quantity	*Value*
Most probable beam velocity	$\alpha = 1.32574 \times 10^{-6}$
Bohr Magneton	$\mu_z = 9.27 \times 10^{-24}\ Tm^{-1}$
Distance through field gradient	$d_1 = 10.06 \pm 0.05\ cm$
Distance to detector from pole piece	$d_2 = 31.175 \pm 0.25\ cm$
Average beam deflection	$\vec{s} = 0.803 \pm .005\ mm$
Boltzman Constant	$k_B = 8.617 \times 10^{-5}\ eVK^{-1}$
Temperature	$T = 389.13\ K$
${}^{39}K$ mass	${}_{39}m = 5.8153 \times 10^{-9}\ J$
${}^{41}K$ mass	${}_{41}m = 6.1135 \times 10^{-9}\ J$

By employing the velocity distribution function of the beam intensity $I(v)$

$$I(v)\ dv = 2I_0(v/\alpha)^3 e^{-(v/\alpha)^2}\ d(v/\alpha) \qquad (2)$$

where $\alpha = \sqrt{2k_BT/{}_{41}m}$ is the most probable velocity of atoms in the oven, we find

$$\frac{d}{dv} I(v) = 0 \qquad (3)$$

which leads to the obtained vales of $\alpha = \sqrt{3}v$ with the values shown above. In conjunction with the other values in Table. 1, this allows the calculation of $\vec{\nabla}B$. From this we obtain

$$\vec{\nabla}B = 68.87\ Tm^{-1} \qquad (4)$$

We may calculate the statistical uncertainty in the measurement in the usual way,

$$\frac{\delta(\vec{\nabla}B)}{\vec{\nabla}B} = \pm \sum_{i=1}^{3} \sqrt{\left(\frac{\delta\xi_i}{\vec{\nabla}B}\right)^2 \left(\frac{\partial(\vec{\nabla}B)}{\partial\xi_i}\right)^2} \qquad (5)$$

where $\xi_1 = d_1,\ \xi_2 = d_2$, and $\xi_3 = \vec{s}$. We therefore arrive at

$$\vec{\nabla}B = 68.87 \pm 2.22\ Tm^{-1} \qquad (6)$$

The most probably velocity of the atoms moving in the direction parallel and antiparallel to the field gradient can now be computed which shows

$$v_z = (4.61 \pm .16) \times 10^{-9} \qquad (7)$$

It is worth noting that the velocity v of the atoms in the beam is not the same as the velocity of the atoms in the oven, α. This is due to the fact that the atoms within the slug being heated in the oven are the ${}^{41}K$ isotope which decays in flight to ${}^{39}K$, a 2GeV change in mass.

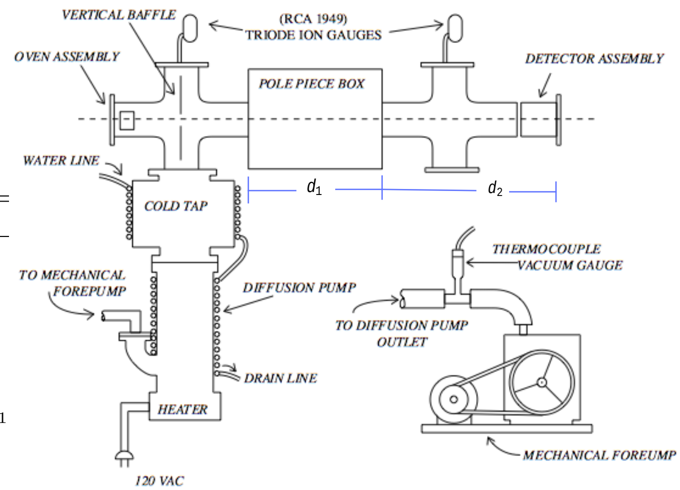

FIG. 2: Apparatus setup. The dashed line is the path of the potassium atoms and the micrometer is attached to the detector assembly on the far right

CONCLUSION

Upon completion of the experiment, it may be reported that nothing unexpected was obtained. That is to say, the conditions and phenomena predicted by the formalism of relativistic quantum mechanics coincide exactly with the results here obtained. The obtained value of $\vec{\nabla}B = 68.87 \pm 2.22$ T/m maintains a margin of statistical error given is on the order or 3% and therefore, *if* there were some type of unknown observable interfering with the spin interaction, the effects would not be able to induce an error to the current theory at an error rate above this limit. Due to the peak splitting into two, as opposed to the classical prediction of three, Dirac's prediction of an intrinsic magnetic moment is verified experimentally here. As there is no known conflict in this domain of theoretical or experimental physics, it seems reasonable to attribute the results here to the resounding success of quantum theory.

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[1] Gerlach, W.; Stern, O. (1922).
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[2] U.W Madison Physics Laboratory 407.
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