5.2 EKF Localization

The Kalman filter is one of the best studied techniques for filtering and prediction of the state of linear systems. Among its virtues, it provides a way to overcome the occasional un-observability problem of the Least Squares approach. Nevertheless, it makes a strong assumption that the two involved process equations (state transition and observation) are linear.

Unfortunately, you should already know that our system of measurements (i.e. the observation function) and motion (i.e. pose composition) are non-linear. Therefore, this notebook focuses from the get-go on the **Extended Kalman Filter (EKF)**, which is adapted to work with non-linear systems. When applied to robot localization, the EKF estimates the robot pose based on sensor measurements and a motion model, taking into account the noise in both the robot's motion and its sensors. Concretely, the robot pose x_t (its state) is modelled as a normal distribution, so $x_t \sim N(\mu_t, \Sigma_t)$,

The EKF algorithm consists of 2 phases: **prediction** and **correction**.

$$
\begin{array}{ll}
\texttt{def ExtendedKalmanFilter}(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t): & \\
\quad \textbf{Prediction.} & \\
\quad \bar{\mu}_t = g(\mu_{t-1}, u_t) = \mu_{t-1} \oplus u_t & \text{(1. Pose predictio} \\
\quad \bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t & \text{(2. Uncertainty of predictio} \\
\quad \textbf{Correction.} & \\
\quad K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1} & \text{(3. Kalman gai} \\
\quad \mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t)) & \text{(4. Pose estimatio} \\
\quad \Sigma_t = (I - K_t H_t) \bar{\Sigma}_t & \text{(5. Uncertainty of estimatio} \\
\quad \texttt{return } \mu_t, \Sigma_t &
\end{array}
$$

Notice that R_t is the covariance of the motion u_t in the coordinate system of the predicted pose $(\bar{x}_t)$, then (Note: J_2 is our popular Jacobian for the motion command, you could also use J_1):

$$R_t = J_2 \Sigma_{u_t} J_2^T \quad \text{with} \quad J_2 = \frac{\partial g(\mu_{t-1}, u_t)}{\partial u_t}$$

Where:

- (u_t, Σ_{u_t}) is the motion command received, and its respective uncertainty.
- (z_t, Q_t) are the observations taken, and their covariance.
- G_t and H_t are the Jacobians of the motion model and the observation model respectively:

$$G_t = \frac{\partial g(\mu_{t-1}, u_t)}{\partial x_{t-1}}, \qquad H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t}$$

At this point the steps in the **prediction** phase are straightforward for us. But, what's the intuition behind the steps in the **correction** one?

3. **Kalman gain** $K_t = \overline{\Sigma}_t H_t^T (H_t \overline{\Sigma}_t H_t^T + Q_t)^{-1}$

   This equation determines how much the prediction should be adjusted based on the new measurement.The Kalman Gain K_t balances the confidence in the prediction with the confidence in the measurement. A high Kalman Gain means the measurement is trusted more than the prediction, while a low Kalman Gain means the prediction is trusted more. If the measurement noise Q_t is high, the gain will be low, meaning the model relies more on its prediction than on the noisy measurement.

4. **Pose estimation** $\mu_t = \overline{\mu}_t + K_t(z_t - h(\overline{\mu}_t))$

   This step updates the current state estiamte (pose) μ_t using the Kalman gain from the previous step and the difference between the observed measurement z_t and the predicted measurement $h(\overline{\mu}_t)$. The term $(z_t - h(\overline{\mu}_t))$ is called the **innovation** or **measurement residual**. It represents the discrepancy between the predicted measurement (based on the current state estimate) and the actual measurement. The innovation provides information on how far off the prediction was. If the innovation is small, the predicted state is close to the actual state; if it's large, there's a significant discrepancy that needs correction. This update moves the estimated pose closer to the observed measurement while accounting for the predicted state and measurement uncertainties.

5. **Uncertainty of the estimation** $\Sigma_t = (I - K_t H_t)\overline{\Sigma}_t$

   This equation updates the covariance matrix Σ_t, representing the uncertainty in the state estimate after incorporating the measurement. It reduces the uncertainty in the state estimate based on the Kalman Gain. If the Kalman Gain is high (i.e., the measurement is reliable), the uncertainty is reduced significantly. If the Kalman Gain is low (i.e., the measurement is less reliable), the reduction in uncertainty is smaller. This adaptability helps maintain a realistic assessment of the confidence in the state estimate.

In this notebook we are going to play with the EKF localization algorithm using a map of landmarks and a sensor providing range and bearing measurements from the robot pose to such landmarks. Concretely, **we are going to**:

1. Implement a class modeling a **range and bearing sensor** able to take measurements to landmarks.
2. Complete a class that implements the robot behavior after completing **motion commands**.
3. Implement the **Jacobian of the observation model**.
4. With the previous building blocks, implement our own **EKF filter** and see it in action.

5. Finally, we are going to consider a more **realistic sensor** with a given Field of View and a maximum operational range.

In [1]:
```
# IMPORTS
import numpy as np
from numpy import random
from numpy import linalg
import matplotlib
matplotlib.use('TkAgg')
from matplotlib import pyplot as plt
from IPython.display import display, clear_output
import time

import sys
sys.path.append("..")
from utils.AngleWrap import AngleWrapList
from utils.PlotEllipse import PlotEllipse
from utils.Drawings import DrawRobot, drawFOV, drawObservations
from utils.Jacobians import J1, J2
from utils.tcomp import tcomp
```

ASSIGNMENT 1: Getting an observation to a random landmark

We are going to implement the `Sensor()` class modelling a range and bearing sensor. Recall that the observation model of this type of sensos is:

$$z_i = \begin{bmatrix} d_i \\ \theta_i \end{bmatrix} = h(m_i, x) = \begin{bmatrix} \sqrt{(x_i - x)^2 + (y_i - y)^2} \\ atan\left(\frac{y_i - y}{x_i - x}\right) - \theta \end{bmatrix} + w_i$$

where $m_i = [x_i, y_i]$ are the landmark coordinates in the world frame, $x = [x, y, \theta]$ is the robot pose, and the noise w_i follows a Gaussian distribution with zero mean and covariance matrix:

$$\Sigma_S = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}$$

For that, complete the following methods:

- `observe()` : which, given a real robot pose (`from_pose`), returns the measurments to the landmarks in the map (`world`). If `noisy=true` , then a random gaussian noise with zero mean and covariance Σ_S (`cov`) is added to each measurement. *Hint you can use* `random.randn()` *for that.*

- `random_observation()` : that, given again the robot pose (`from_pose`), randomly selects a landmark from the map (`world`) and returns an observation from the range-bearing sensor using the `observe()` method previously implemented. The `noisy` argument is just passed to `observe()` . *Hint: to randomly select a landmark, use* `randint()` .

In [4]:
```
class Sensor():
    def __init__(self, cov):
        """
        Args:
            cov: covariance of the sensor.
        """
        self.cov = cov

    def observe(self, from_pose, world, noisy=True, flatten=True):
        """Calculate observation relative to from_pose

        Args:
            from_pose: Position(real) of the robot which takes the observ
            world: List of world coordinates of some landmarks
            noisy: Flag, if true then add noise (Exercise 2)

        Returns:
                Numpy array of polar coordinates of landmarks from the pe
                They are organised in a vertical vector ls = [d_0 , a_0,
                Dims (2*n_landmarks, 1)
        """
        delta = world - from_pose[0:2]

        z = np.empty_like(delta)
        z[0, :] = np.sqrt(delta[0]**2 + delta[1]**2)
        z[1, :] = np.arctan2(delta[1], delta[0]) - from_pose[2]
        z[1, :] = AngleWrapList(z[1, :])

        if noisy:
            z +=  np.sqrt(self.cov) @ random.randn(z.shape[0], z.shape[1]

        if flatten:
            return np.vstack(z.flatten('F'))
        else:
            return z

    def random_observation(self, from_pose, world, noisy=True):
        """ Get an observation from a random landmark

            Args: Same as observe().

            Returns:
                z: Numpy array of obs. in polar coordinates
                landmark: Index of the randomly selected landmark in the
                    Although it is only one index, you should return it a
                    a numpy array.
        """
        n_landmarks = world.shape[1]
        rand_idx = random.randint(n_landmarks)
        world = world[:, [rand_idx]]

        z = self.observe(from_pose, world, noisy)

        return z, np.array([rand_idx])
```

You can use the code cell below **to test your implementation**.

In [5]:
```
# TRY IT!
seed = 0
```

```python
np.random.seed(seed)

# Sensor characterization
SigmaR = 1 # Standard deviation of the range
SigmaB = 0.7 # Standard deviation of the bearing
Q = np.diag([SigmaR**2, SigmaB**2]) # Cov matrix

sensor = Sensor(Q)

# Map
Size = 50.0
NumLandmarks = 3
Map = Size*2*random.rand(2,NumLandmarks)-Size

# Robot true pose
true_pose = np.vstack([-Size+Size/3, -Size+Size/3, np.pi/2])

# Take a random measurement
noisy = False
z = sensor.random_observation(true_pose, Map, noisy)

noisy = True
noisy_z = sensor.random_observation(true_pose, Map, noisy)

# Take observations to every landmark in the map
zs = sensor.observe(true_pose, Map, noisy)

print('Measurement:\n' + str(z))
print('Noisy measurement:\n' + str(noisy_z))
print('Measurements to every landmark in the map:\n' + str(zs))
```

```
Measurement:
(array([[53.76652662],
       [-0.79056712]]), array([0]))
Noisy measurement:
(array([[64.73997127],
       [-0.81342958]]), array([2]))
Measurements to every landmark in the map:
[[ 5.51319938e+01]
 [-1.10770618e+00]
 [ 6.04762304e+01]
 [-1.46219661e+00]
 [ 6.23690518e+01]
 [-5.72010701e-02]]
```

Expected output

```
Measurement:
(array([[53.76652662],
       [-0.79056712]]), array([0]))
Noisy measurement:
(array([[64.73997127],
       [-0.81342958]]), array([2]))
Measurements to every landmark in the map:
[[ 5.51319938e+01]
 [-1.10770618e+00]
 [ 6.04762304e+01]
 [-1.46219661e+00]
```

```
[ 6.23690518e+01]
[-5.72010701e-02]]
```

ASSIGNMENT 2: Simulating the robot motion

In the robot motion chapter we commanded a mobile robot to follow a squared trajectory. We provide here the `Robot` class that implements:

- how the robot pose evolves after executing a motion command (`step()` method), and
- the functionality needed to graphically show its ideal pose (`pose`), true pose (`true_pose`) and estimated pose (`xEst`) in the `draw()` function.

Your mission is to complete the `step()` method by adding random noise to each motion command (`noisy_u`) based on the following covariance matrix, and update the true robot pose (`true_pose`):

$$\Sigma_{u_t} = \begin{bmatrix} \sigma^2_{\Delta x} & 0 & 0 \\ 0 & \sigma^2_{\Delta y} & 0 \\ 0 & 0 & \sigma^2_{\Delta \theta} \end{bmatrix}$$

Hint: Recall again the `random.randn()` function.

In [6]:
```python
class Robot():
    def __init__(self, true_pose, cov_move):
        # Robot description (Starts as perfectly known)
        self.pose = true_pose
        self.true_pose = true_pose
        self.cov_move = cov_move

        # Estimated pose and covariance
        self.xEst = true_pose
        self.PEst = np.zeros((3, 3))

    def step(self, u):
        self.pose = tcomp(self.pose,u) # New pose without noise
        noise = np.sqrt(self.cov_move)@random.randn(3,1) # Generate noise
        noisy_u = u + noise #  Apply noise to the control action
        self.true_pose = tcomp(self.true_pose, noisy_u) #  New noisy pose

    def draw(self, fig, ax):
        DrawRobot(fig, ax, self.pose, color='r')
        DrawRobot(fig, ax, self.true_pose, color='b')
        DrawRobot(fig, ax, self.xEst, color='g')
        PlotEllipse(fig, ax, self.xEst, self.PEst, 4, color='g')
```

It is time **to test** your `step()` function!

In [7]:
```python
# Robot base characterization
SigmaX = 0.8 # Standard deviation in the x axis
SigmaY = 0.8 # Standard deviation in the y axis
SigmaTheta = 0.1 # Bearing standar deviation
R = np.diag([SigmaX**2, SigmaY**2, SigmaTheta**2]) # Cov matrix
```

```python
# Create the Robot object
true_pose = np.vstack([2,3,np.pi/2])
robot = Robot(true_pose, R)

# Perform a motion command
u = np.vstack([1,2,0])
np.random.seed(0)
robot.step(u)

print('robot.true_pose.T:' + str(robot.true_pose.T) + '\'')
```

```
robot.true_pose.T:[[-0.32012577  5.41124188  1.66867013]]'
```

Expected output

```
robot.true_pose.T:[[-0.32012577  5.41124188  1.66867013]]'
```

ASSIGNMENT 3: Jacobians of the observation model

Given that the position of the landmarks in the map is known, we can use this information in a Kalman filter, in our case an EKF. For that we need to implement the **Jacobians of the observation model**, as required by the correction step of the filter.

Implement the function `getObsJac()` that given:

- the predicted pose in the first step of the Kalman filter,
- a number of observed landmarks, and
- the map,

returns such Jacobian. Recall that, for each observation to a landmark:

$$\nabla H = \frac{\partial h}{\partial \{x, y, \theta\}} = \begin{bmatrix} -\frac{x_i - x}{d} & -\frac{y_i - y}{d} & 0 \\ \frac{y_i - y}{d^2} & -\frac{x_i - x}{d^2} & -1 \end{bmatrix}_{2\times 3}$$

Recall that $[x_i, y_i]$ is the position of the i^{th} landmark in the map, $[x, y]$ is the robot predicted pose, and d the distance from such predicted pose to the landmark. This way, the resultant Jacobian dimensions are $(\#observed\_landmarks \times 2, 3)$, that is, the Jacobians are stacked vertically to form the matrix H.

In [34]:
```python
def getObsJac(xPred, lm, Map):
    """ Obtain the Jacobian for all observations.

        Args:
            xPred: Position of our robot at which Jac is evaluated.
            lm: Numpy array of observations to a number of landmarks (ind
            Map: Map containing the actual positions of the observations.

        Returns:
            jH: Jacobian matrix (2*n_landmaks, 3)
    """
    n_land = len(lm)
    jH = np.empty((2*n_land,3))

    for i in range(n_land):
```

```
        # Auxiliary variables
        dx = Map[0, lm[i]] - xPred[0]
        dy = Map[1, lm[i]] - xPred[1]
        d = np.sqrt(dx**2 + dy**2)
        d2 = d**2

        ii = 2*i

        # Build the Jacobian
        jH[ii:ii+2,:] = [
            [-dx/d, -dy/d, 0],
            [dy/d2, -dx/d2, -1]
        ]

    return jH
```

Time **to check** your function!

In [35]:
```
# TRY IT!

observed_landmarks = np.array([0,2])
xPred = np.vstack([-Size+Size/3, -Size+Size/3, np.pi/2]).flatten() # Robo
jH = getObsJac(xPred, observed_landmarks, Map) # Retrieve the evaluated o

print ('Jacobian dimensions: ' + str(jH.shape) )
print ('jH:' + str(jH))
```

```
Jacobian dimensions: (4, 3)
jH:[[-0.70710678 -0.70710678  0.        ]
 [ 0.01415094 -0.01415094 -1.        ]
 [-0.72655713 -0.68710606  0.        ]
 [ 0.01355353 -0.01433173 -1.        ]]
```

Expected output:

```
Jacobian dimensions: (4, 3)
jH:[[-0.71075232 -0.70344235  0.        ]
 [ 0.01308328 -0.01321923 -1.        ]
 [-0.67304061 -0.73960552  0.        ]
 [ 0.01141455 -0.01038723 -1.        ]]
```

ASSIGNMENT 4: Completing the EKF

Congratulations! You now have all the building blocks needed to implement an EKF filter (both prediction and correction steps) for localizating the robot and show the estimated pose and its uncertainty.

For doing that, complete the `EKFLocalization()` function below, which returns:

- the estimated pose (`xEst`), and
- its associated uncertainty (`PEst`),

given:

- the previous estimations (`self.xEst` and `self.PEst` stored in `robot`),
- the features of the sensor (`sensor`),

- the motion command provided to the robot (`u`),
- the observations done (`z`),
- the indices of the observed landmarks (`landmark`), and
- the map of the environment (`Map`).

In [40]:
```
def EKFLocalization(robot, sensor, u, z, landmark, Map):
    """ Implement the EKF algorithm for localization

        Args:
            robot: Robot base (contains the state: xEst and PEst)
            sensor: Sensor of our robot.
            u: Motion command
            z: Observations received
            landmark: Indices of landmarks observed in z
            Map: Array with landmark coordinates in the map

        Returns:
            xEst: New estimated pose
            PEst: Covariance of the estimated pose
    """

    # Prediction
    xPred = tcomp(robot.xEst, u)
    G = J1(xPred, robot.xEst)
    j2 = J2(xPred, u)
    PPred = G @ robot.PEst @ G.T + j2 @ robot.cov_move @ j2.T

    # Correction (You need to compute the gain k and the innovation z-z_p
    if landmark.shape[0] > 0:
        H = getObsJac(xPred.flatten(), landmark, Map) # Observation Jacob
        K = PPred @ H.T @ np.linalg.pinv(H@PPred@H.T + np.diag(np.tile(np
        xEst = xPred + K @ (z-sensor.observe(xPred, Map[:, landmark], Fal
        PEst = (np.eye(3,3)-K@H)@PPred # New estimated Jacobian
    else:
        xEst = xPred
        PEst = PPred

    return xEst, PEst
```

You can **validate your code** with the code cell below.

In [42]:
```
# TRY IT!

np.random.seed(2)

# Create the map
Map=Size*2*random.rand(2,20)-Size

# Create the Robot object
true_pose = np.vstack([2,3,0])
R = np.diag([0.1**2, 0.1**2, 0.01**2]) # Cov matrix
robot = Robot(true_pose, R)

# Perform a motion command
u = np.vstack([10,0,0])
robot.step(u)

# Get an observation to a landmark
```

```
noisy = True
noisy_z, landmark_index = sensor.random_observation(true_pose, Map, noisy

# Estimate the new robot pose using EKF!
robot.xEst, robot.PEst = EKFLocalization(robot, sensor, u, noisy_z, landm

# Show resutls!
print('robot.pose.T:' + str(robot.pose.T) + '\'')
print('robot.true_pose.T:' + str(robot.true_pose.T) + '\'')
print('robot.xEst.T:' + str(robot.xEst.T) + '\'')
print('robot.PEst:' + str(robot.PEst.T))
```

```
robot.pose.T:[[12.  3.  0.]]'
robot.true_pose.T:[[ 1.20000010e+01  3.05423526e+00 -3.13508197e-03]]'
robot.xEst.T:[[ 1.19586407e+01  2.96047951e+00 -1.48514185e-04]]'
robot.PEst:[[ 9.94877200e-03 -4.94253023e-05 -3.18283546e-08]
 [-4.94253023e-05  9.95211532e-03  3.29230513e-08]
 [-3.18283546e-08  3.29230513e-08  9.99795962e-05]]
```

Expected output:

```
robot.pose.T:[[12.  3.  0.]]'
robot.true_pose.T:[[ 1.20000010e+01  3.05423526e+00
-3.13508197e-03]]'
robot.xEst.T:[[ 1.19586407e+01  2.96047951e+00
-1.48514185e-04]]'
robot.PEst:[[ 9.94877200e-03 -4.94253023e-05 -3.18283546e-
08]
 [-4.94253023e-05  9.95211532e-03  3.29230513e-08]
 [-3.18283546e-08  3.29230513e-08  9.99795962e-05]]
```

Playing with EKF

The following code helps you to see the EKF filter in action!. Press any key on the emerging window to send a motion command to the robot and check how the landmark it observes changes, as well as its ideal, true and estimated poses.

Notice that you can change the value of `seed` within the `main()` function to try different executions.

Example

The figure below shown an example of the execution of the EKF localization algorithm with the code implemented until this point.

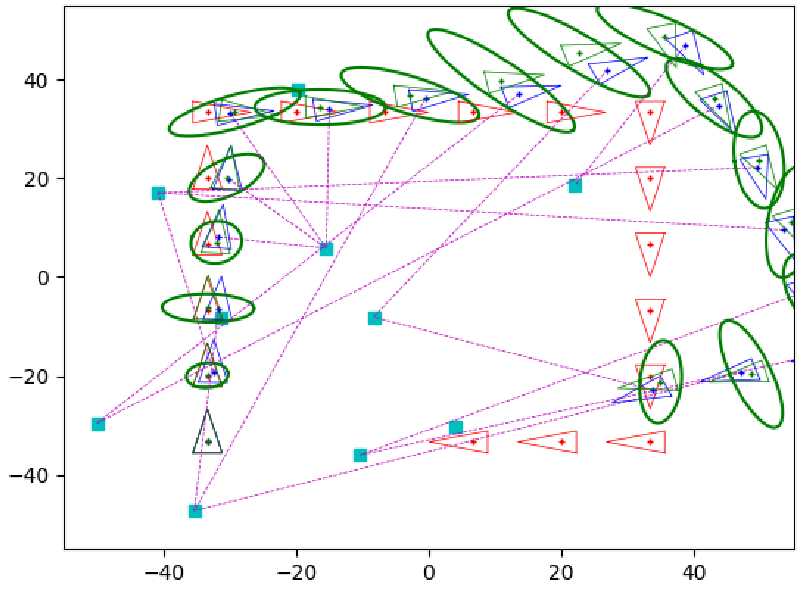


Fig. 1: Execution of the EKF algorithmn for localization,
it shows the true (in blue) and expected (in red) poses,
along the results from localization: pose and ellipse (in green),
the existing landmarks (in cyan),
and each observation made (dotted lines).

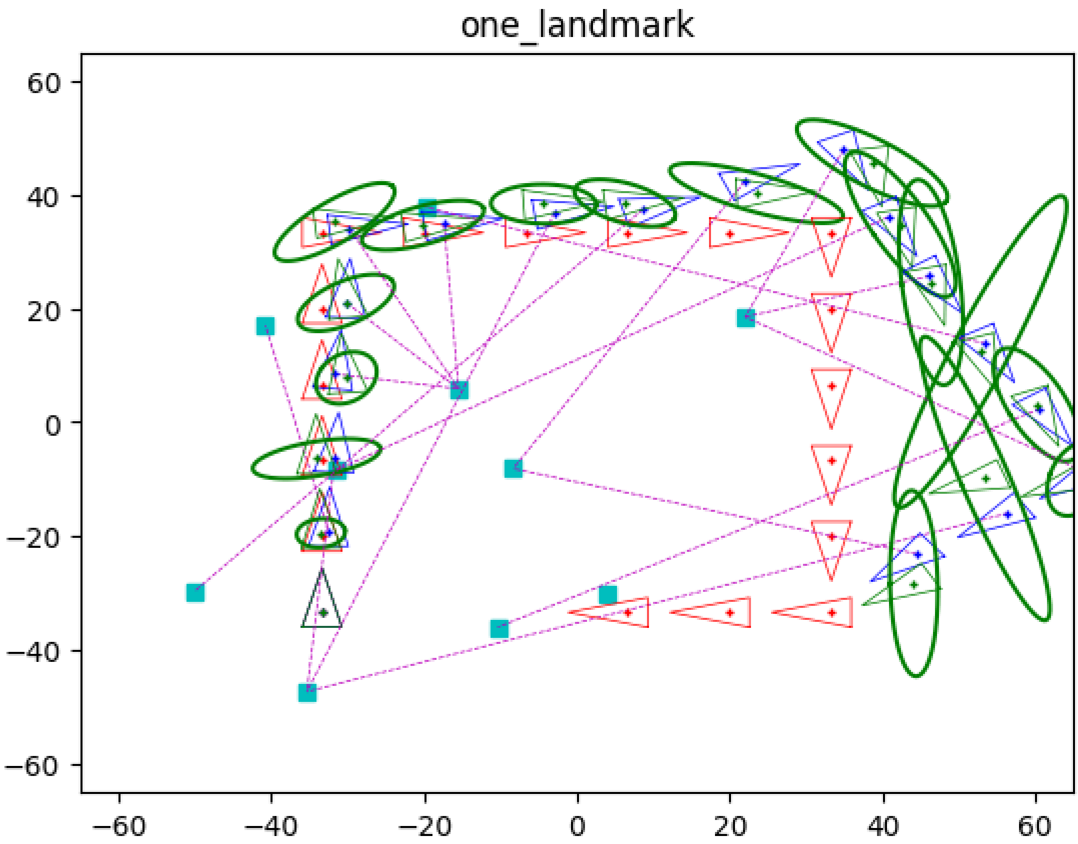
In [43]:
```
def main(robot,
         sensor,
         mode='one_landmark',
         visualization = 'non_stop',
         nSteps=20, # Number of motions
         turning=5, # Number of motions before turning (square path)
         Size=50.0,
         NumLandmarks=10):

    seed = 1
    np.random.seed(seed)

    #Create map
    Map=Size*2*random.rand(2,NumLandmarks)-Size

    # MATPLOTLIB
    if visualization == 'non_stop':
        %matplotlib widget
    elif visualization == 'step_by_step':
        #%matplotlib inline
        matplotlib.use('TkAgg')
        plt.ion()

    fig, ax = plt.subplots()
    plt.plot(Map[0,:],Map[1,:],'sc')
    plt.axis([-Size-15, Size+15, -Size-15, Size+15])
    plt.title(mode)

    robot.draw(fig, ax)
    fig.canvas.draw()
```

```
    # MAIN LOOP

    u = np.vstack([(2*Size-2*Size/3)/turning,0,0]) # control action

    if visualization == 'step_by_step':
        plt.waitforbuttonpress(-1)

    for k in range(0, nSteps-3): # Main loop
        u[2] = 0
        if k % turning == turning-1: # Turn?
            u[2] = -np.pi/2

        robot.step(u)

        # Get sensor observation/s
        if mode == 'one_landmark':
            # DONE (Exercise 4)
            z, landmark = sensor.random_observation(robot.true_pose, Map)
            ax.plot(
                [robot.true_pose[0,0], Map[0,landmark][0]],
                [robot.true_pose[1,0], Map[1,landmark][0]],
                color='m', linestyle="--", linewidth=.5)
        elif mode == 'landmarks_in_fov':
            # DONE (Exercise 5)
            z, landmark = sensor.observe_in_fov(robot.true_pose, Map)
            drawObservations(fig, ax, robot.true_pose, Map[:, landmark])

        robot.xEst, robot.PEst = EKFLocalization(robot, sensor, u, z, lan

        # Drawings
        # Plot the FOV of the robot
        if mode == 'landmarks_in_fov':
            h = sensor.draw(fig, ax, robot.true_pose)
        #end

        robot.draw(fig, ax)
        fig.canvas.draw()

        if visualization == 'non_stop':
            clear_output(wait=True)
            display(fig)
        elif visualization == 'step_by_step':
            plt.waitforbuttonpress(-1)

        if mode == 'landmarks_in_fov':
            h.pop(0).remove()
        fig.canvas.draw()

    if visualization == 'non_stop':
        plt.close()
    elif visualization == 'step_by_step':
        plt.ioff()
```

In [44]:

```
# RUN
mode = 'one_landmark'
# mode = 'landmarks_in_fov'
visualization = 'non_stop'
#visualization = 'step_by_step'

Size=50.0
```

```
# Robot base characterization
SigmaX = 0.8 # Standard deviation in the x axis
SigmaY = 0.8 # Standard deviation in the y axis
SigmaTheta = 0.1 # Bearing standar deviation
R = np.diag([SigmaX**2, SigmaY**2, SigmaTheta**2]) # Cov matrix

true_pose = np.vstack([-Size+Size/3, -Size+Size/3, np.pi/2])
robot = Robot(true_pose, R)

# Sensor characterization
SigmaR = 1 # Standard deviation of the range
SigmaB = 0.7 # Standard deviation of the bearing
Q = np.diag([SigmaR**2, SigmaB**2]) # Cov matrix

sensor = Sensor(Q)

main(robot, sensor, mode=mode, visualization=visualization, Size=Size)
```



ASSIGNMENT 5: Implementing the FoV of a sensor.

Sensors exhibit certain physical limitations regarding their field of view (FoV) and maximum operating distance (max. Range). Besides, these devices often do not deliver measurmenets from just one landmark each time, but from all those landmarks in the FoV.

The `FOVSensor()` class below extends the `Sensor()` one to implement this behaviour. Complete the `observe_in_fov()` method to consider that the sensor can only provide information from the landmkars in a limited range r_l and a limited orientation $\pm\alpha$ with respect to the robot pose. For that:

1. Get the observations to every landmark in the map. Use the `observe()` function previously implemented for that, but with the argument `flatten=False`. With that option the function returns the measurements as:

$$z = \begin{bmatrix} d_1 & \cdots & d_m \\ \theta_1 & \cdots & \theta_m \end{bmatrix}$$

2. Check which observations lay in the sensor FoV and maximum operating distance. *Hint: for that, you can use the* `np.asarray()` *function with the conditions to be fulfilled by the valid measurements inside, and then filter the results with* `np.nonzero()`.
3. Flatten the resultant matrix `z` to be again a vector, so it has the shape $(2 \times \#Observed\_landmarks, 1)$. *Hint: take a look at* `np.ndarray.flatten()` *and choose the proper argument.*

Notice that it could happen that any landmark exists in the field of view of the sensor, so the robot couldn't gather sensory information in that iteration. This, which is a problem using Least Squares Positioning, is not an issue with EKF. ***Hint: you can change the value of*** `seed` ***within the*** `main()` ***function to try different executions.***

In [51]:
```
class FOVSensor(Sensor):
    def __init__(self, cov, fov, max_range):
        super().__init__(cov)
        self.fov = fov
        self.max_range = max_range

    def observe_in_fov(self, from_pose, world, noisy=True):
        """ Get all observations in the fov

        Args:
            from_pose: Position(real) of the robot which takes the observ
            world: List of world coordinates of some landmarks
            noisy: Flag, if true then add noise (Exercise 2)

        Returns:
            Numpy array of polar coordinates of landmarks from the perspe
            They are organised in a vertical vector ls = [d_0 , a_0, d_1,
            Dims (2*n_landmarks, 1)
        """
        # 1. Get observations to every landmark in the map WITHOUT NOISE
        z = self.observe(from_pose, world, False, False)

        # 2. Check which ones lay on the sensor FOV
        angle_limit = self.fov / 2  # auxiliar variable
        distances = z[0, :]  # extract distances from observation matrix
        angles = z[1, :]  # extract angles from observation matrix

        # Filter landmarks within max_range and within the FOV angles
        feats_idx = np.where((distances <= self.max_range) &
                             (np.abs(angles) <= angle_limit))[0]

        if noisy:
            # 1. Get observations to every landmark in the map WITH NOISE
            z = self.observe(from_pose, world, True, False)
```

```
        z = z[:, feats_idx] # extracts the valid observations from z

        # 3. Flatten the  resultant vector of measurements so z=[d_1,thet
        if z.size>0:
            z = np.vstack(z.flatten('F'))

        return z, feats_idx

    def draw(self, fig, ax, from_pose):
        """ Draws the Field of View of the sensor from the robot pose """
        return drawFOV(fig, ax, from_pose, self.fov, self.max_range)
```

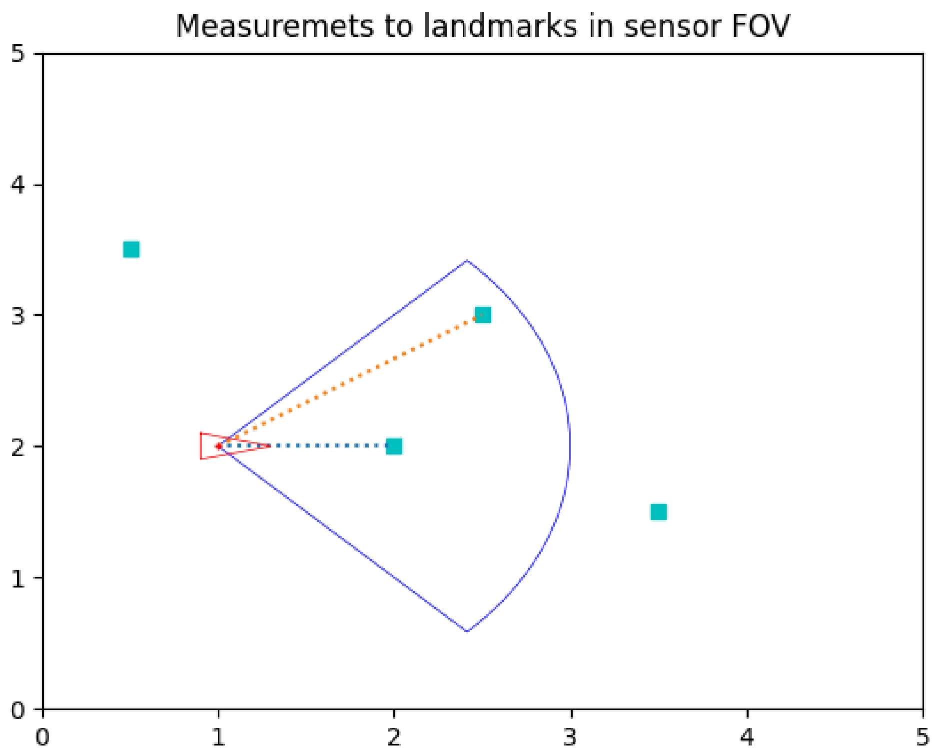
You can now **try** your new and more realistic sensor.

In [52]:
```
# TRY IT!
np.random.seed(0)

# Create the sensor object
cov = np.diag([0.1**2, 0.1**2]) # Cov matrix
fov = np.pi/2
max_range = 2
sensor = FOVSensor(cov, fov, max_range)

# Create a map with three landmarks
Map = np.array([[2., 2.5, 3.5, 0.5],[2., 3., 1.5, 3.5]])

# Take an observation of landmarks in FoV
robot_pose = np.vstack([1.,2.,0.])
z, feats_idx = sensor.observe_in_fov(robot_pose, Map)
print('z:' +str(z))

# Plot results
fig, ax = plt.subplots()
plt.axis([0, 5, 0, 5])
plt.title('Measuremets to landmarks in sensor FOV')
plt.plot(Map[0,:],Map[1,:],'sc')
sensor.draw(fig, ax, robot_pose)
drawObservations(fig, ax, robot_pose, Map[:, feats_idx])
DrawRobot(fig,ax,robot_pose)
```

```
z:[[1.17640523]
 [0.1867558 ]
 [1.84279136]
 [0.49027482]]
```

Out[52]:
```
[<matplotlib.lines.Line2D at 0x754f1015c8c0>]
```

Figure



Expected output:

```
z:[[1.17640523]
 [0.1867558 ]
 [1.84279136]
 [0.49027482]]
```

Playing with EKF and the new sensor

And finally, play with your own FULL implementation of the EKF filter with a more realistic sensor :)

Example

The figure below shows an example of the execution of EKF using information from all the landmarks within the FOV:

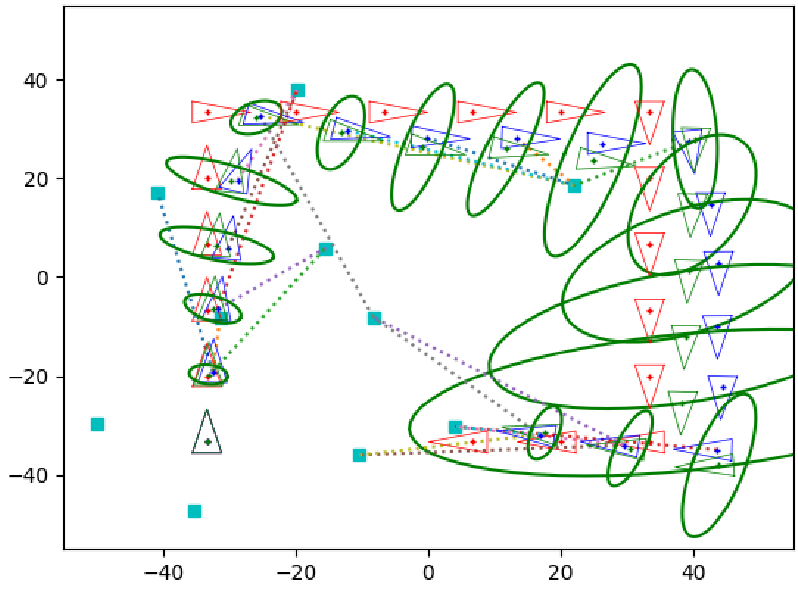
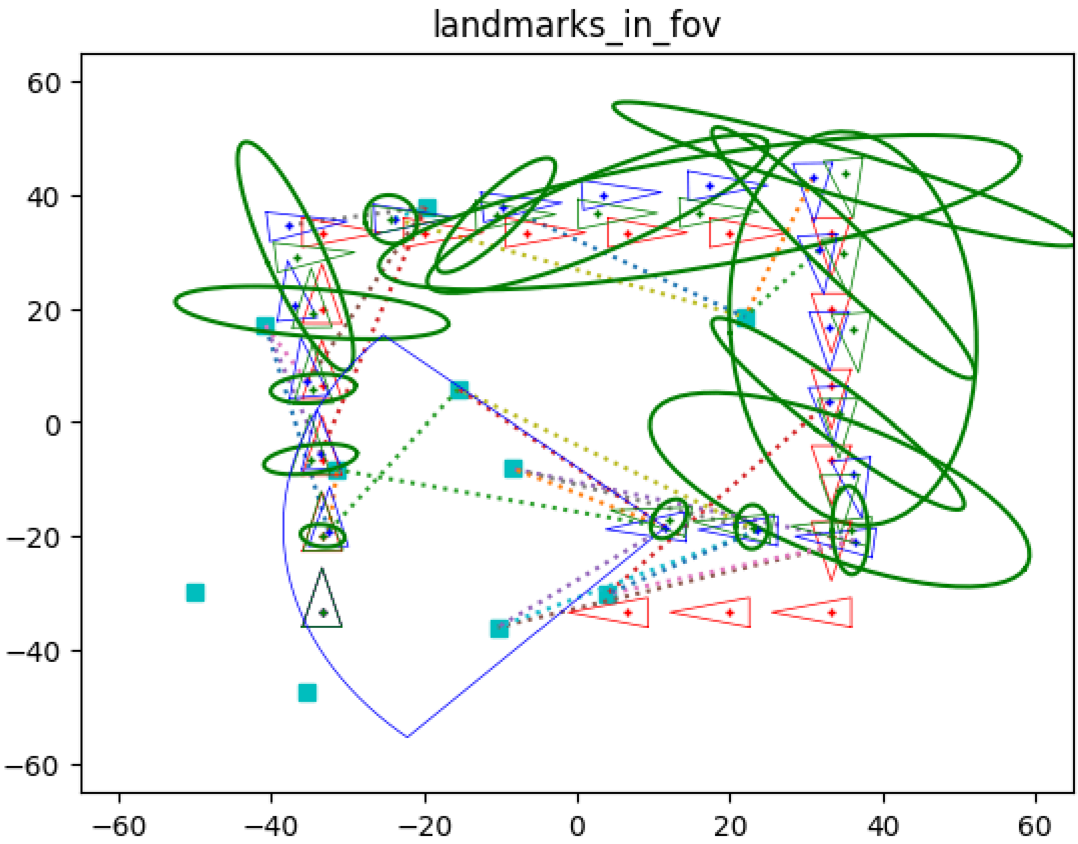


Fig. 2: Execution of the EKF algorithmn for localization. Same as in Fig. 1, except now our robot can observe every lanmark in its f.o.v.

In [53]:
```
# RUN
#mode = 'one_landmark'
mode = 'landmarks_in_fov'
visualization = 'non_stop'
#visualization = 'step_by_step'
Size=50.0

# Robot base characterization
SigmaX = 0.8 # Standard deviation in the x axis
SigmaY = 0.8 # Standard deviation in the y axis
SigmaTheta = 0.1 # Bearing standar deviation
R = np.diag([SigmaX**2, SigmaY**2, SigmaTheta**2]) # Cov matrix

true_pose = np.vstack([-Size+Size/3, -Size+Size/3, np.pi/2])
robot = Robot(true_pose, R)

# Sensor characterization
SigmaR = 1 # Standard deviation of the range
SigmaB = 0.7 # Standard deviation of the bearing
Q = np.diag([SigmaR**2, SigmaB**2]) # Cov matrix
fov = np.pi/2 # field of view = 2*alpha
max_range = Size # maximum sensor measurement range

sensor = FOVSensor(Q, fov, max_range)

main(robot, sensor, mode=mode, visualization=visualization, Size=Size)
```



Thinking about it (1)

Having completed the EKF implementation, you are ready to **answer the following questions**:

- What are the dimensions of the Jacobians of the observation model (matrix H)? Why?

  The Jacobian matrix H of the observation model typically has dimensions of $(\text{observed\_landmarks} \times 2, 3)$. This is because each landmark provides x and y coordinates that need to be differentiated with respect to the robot's x, y, and θ variables. The three columns reflect these derivatives, and the total number of rows depends directly on the count of observed landmarks, as each contributes two entries (for x and y).

- Discuss the evolution of the ideal, true and estimated poses when executing the EKF filter (with the initial sensor).

  In the execution of the Extended Kalman Filter (EKF), the poses evolve in specific ways based on noise and estimation:

```
  - **Ideal Pose**: This represents the theoretical pose
without any noise. When given a motion command, this pose
follows it exactly without deviation.
  - **True Pose**: The true pose is where the robot actually
is, and includes real-world noise. While it follows the same
motion commands, it will deviate due to the inherent
inaccuracies of the system.
  - **Estimated Pose**: The EKF produces an estimated pose
```

```
which attempts to approximate the true pose. Since the true
pose is unknown, the estimated pose relies on sensor
measurements and previous estimations to get as close as
possible to the true location.</i></p>
```

- Discuss the evolution of the ideal, true and estimated poses when executing the EKF filter (with the sensor implementing a FOV). Pay special attention to their associated uncertainties.

*When the EKF is executed with an observation model incorporating a Field of View (FOV), the following dynamics emerge: - **Ideal Pose**: As in the previous case, the ideal pose remains without noise and exactly follows the motion commands:*

```
  - **True Pose**: The true pose still includes real-world
noise and will thus not follow the ideal pose perfectly.
  - **Estimated Pose**: In this scenario, the estimated pose
continues to approximate the true pose but benefits from
additional observations when landmarks fall within the
sensor's FOV. More observations reduce uncertainty in the pose
estimate, as each visible landmark provides additional data to
correct the estimated position. When no landmarks are visible,
uncertainty increases, as there is no information to refine
the prediction. The correction step, crucial for reducing
uncertainty, is skipped in the absence of observations,
resulting in a higher error in the pose estimate.</i></p>
```

- What happens in the EKF filter when the robot performs a motion command, but it is unable to measure distances to any landmark, i.e. they are out of the sensor FOV?

If the robot executes a motion command but cannot detect any landmarks (e.g., they are all outside of the sensor's FOV), it cannot perform a correction on the predicted pose. In this case, the EKF relies solely on its prediction model and provides an estimated position without correction. Over time, this may lead to accumulating errors, as there is no reference information to adjust and refine the estimate.