M3/4/5 S4 PROBLEM SHEET 4: POISSON PROCESSES

Question 6

Suppose that cars arrive at the petrol station according to a Poisson process, $\{N_t\}_{t\geq 0}$ of rate $\lambda \in \mathbb{R}_+$. In addition, independently, a car is green with probability p; let $\{N_t^g\}_{t\geq 0}$ denote the number of green cars that have appeared. Show that $\{N_t^g\}_{t\geq 0}$ is a Poisson process of rate λp.

Question 7

Let $\{N_t\}$ be a Poisson process of rate $\lambda \in \mathbb{R}_+$ and $X_1, X_2, \ldots$ be a sequence of i.i.d random variables, such that their characteristic function exists. Further $\{N_t\}$ and $\{X_i\}$ are independent. Let

$$Z_t = \sum_{i=1}^{N_t} X_i.$$

Find the Characteristic function of Z_t, $t > 0$.

Question 8

Customers arrive at a bank according to a Poisson process at a mean rate of $\lambda = 10$ per minute. 60% of the customers wish to withdraw money (type A), 30% wish to pay in money (type B), and 10% wish to do something else.

(a) What is the probability that more than 5 customers arrive in 30 seconds?
(b) What is the probability that in 1 minute, 6 type A customers, 3 type B customers, and 1 type C customers arrive?
(c) If 20 customers arrive in 2 minutes, what is the probability that just one wants to carry out a type C transaction?
(d) What is the probability that the first 3 customers arriving require only to make a type A transaction?
(e) How long a time will elapse until there is a probability of 0.9 that at least one customer of type A and one of type B will have arrived? (you will need to solve this numerically).

Question 9

A bank opens at 10.00am and customers arrive according to a non-homogeneous Poisson process at a rate $10(1 + 2t)$, measured in hours, starting from 10.00.

(a) What is the probability that two customers have arrived by 10.05?
(b) What is the probability that 6 customers arrive between 10.45 and 11.00?
(c) What is the probability that more than 50 customers arrive between 11.00 and 12.00?
(d) What is the median time to the first arrival after the bank opens?
(e) By what time is there a probability of 0.95 that the first customer after 11.00 will have arrived?

Question 10

A person makes shopping expeditions according to a Poisson process with rate $\lambda \in \mathbb{R}_+$. The number of purchases he makes is distributed according to a geometric distribution $\mathcal{G}eo(p)$. What are the mean and variance of the number of purchases made in time t?